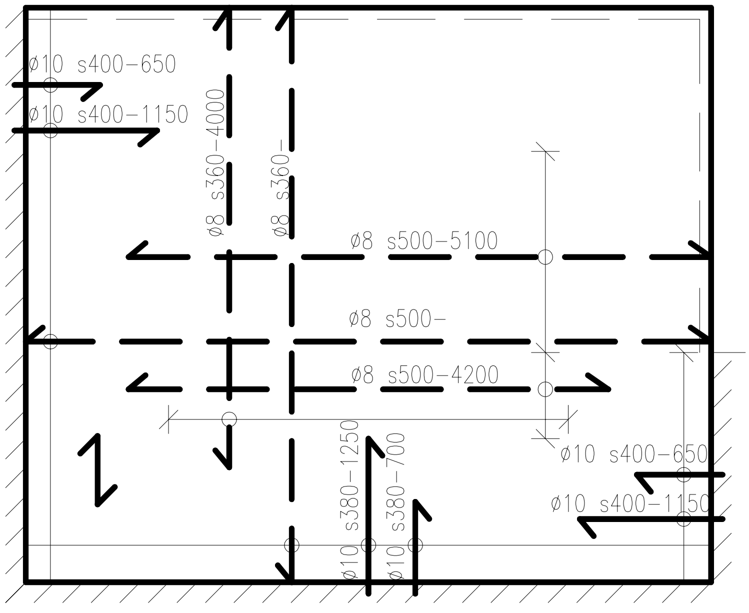
Automatic Reinforcement Generation in FEM-Plate®



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Master's Thesis

Automatic Reinforcement Generation in FEM-Plate®

Examensarbete

Automatisk armeringsgenerering i FEM-Plate®

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Lund 2006

Abstract

In this master's thesis automatic generation of reinforcement distribution in the structural engineering program FEM-Design Plate was investigated. The reinforcement distribution proposed was to be as accurate as possible, both from a theoretical and a practical point of view.

For this purpose, calculation routines considering current national codes and engineering practice have been developed. These routines are based on the results of calculations performed in FEM-Plate. The resulting reinforcement distribution shows that creating a proposal that would fully meet the requirements of an engineer is difficult to accomplish.

Keywords: FEM-Design, FEM-Plate, flow chart, slab, reinforcement, support moment, bay moment, nodes, elements, bar spacing, bar dimension.

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Master's Thesis

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Lund, May 2006

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Summary

Title: Automatic Reinforcement Generation in FEM-Plate

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Problem: FEM-plate requires that the user defines the slab, its elements and the loads acting on it. The program then performs an initial calculation that provides a required reinforcement displayed in each element over the slab, together with the corresponding deflections and crack widths. The program does not currently suggest a reinforcement distribution that considers practical and economical aspects.

Objective: The purpose of this thesis is to analyze and suggest methods by which the program automatically generates a reinforcement distribution, emanating from the calculated moment distribution, adjusted to current requirements given in national codes and production aspects. The goal is to create an interactive program that incorporates the engineer without eliminating him.

Method: The first step is to perform an analysis of simple examples in FEM-Plate and by hand. This way the results provided in FEM-Plate can be derived and therefore better understood. The second step is to convert the results provided in FEM-Plate so that they can be used in the calculation program MATLAB. Thereafter, preliminary routines are created based on primarily the Swedish national code. By interviewing engineers active in both production and design an insight into the procedure used in commercial design was gathered. The routines are then adapted so that the program works in a way more accurate from a commercial viewpoint. Finally, the routines are demonstrated by solving simple cases involving concrete slabs loaded with distributed loads.

Conclusion: By means of the developed routines, it is possible to create a solution that provides an acceptable reinforcement distribution. The solutions must however only be considered as rough estimates. This means that they give an initial idea of how and where the reinforcement needs to be placed. It is difficult to incorporate the experience of an engineer and the choices that he would make. Therefore, a suggestion that provides a fine reinforcement distribution is preferred to a rough as it allows the engineer to decide in which manner the reinforcement should be distributed.

Keywords: FEM-Design, FEM-Plate, flow chart, slab, reinforcement, support moment, bay moment, nodes, elements, bar spacing, bar dimension

Sammanfattning

Titel: Automatisk armeringsgenerering i FEM-Plate

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Problemställning: Betongbjälklaget och de krafter som verkar på detta definieras i FEM-Plate av användaren. Efter utförd beräkning redovisas den erfordrade armeringen i varje element samt nedböjning och sprickbildning. För närvarande redovisar FEM-Plate ingen armeringsfördelning som tar hänsyn till produktionstekniska och ekonomiska aspekter.

Syfte: Syftet med detta examensarbete är att utreda och föreslå metoder för hur programmet automatiskt ska generera olika armeringsförslag. Förslaget ska ta hänsyn till beräknad momentfördelning och eventuell genomstansning. Målet är att skapa ett interaktivt program som underlättar konstruktörens arbete utan att eliminera honom.

Metod: Till en början skapas och utreds ett antal enkla exempel med hjälp av FEM-Plate och för hand. På så sätt kan de ur FEM-Plate erhållna resultaten härledas, vilket ökar förståelsen för beräkningsgången i programmet. Därefter överförs resultaten från FEM-Plate för att kunna användas i beräkningsprogrammet MATLAB. I nästa steg skapas preliminära rutiner baserade på svensk nationell standard. För att kunna skapa ett program som tar hänsyn till inte bara teoretiska regler utan även praktiska, intervjuas ett antal ingenjörer. Detta ger en inblick i den arbetsgång som utnyttjas i den kommersiella sfären. Därefter anpassas rutinerna så att de tar hänsyn till de synpunkter som framkommit vid intervjuerna. Slutligen implementeras rutinerna på ett antal enkla exempel, varefter de kalibreras och kontrolleras med hjälp av FEM-Plate.

Slutsatser: Det är fullt möjligt att skapa ett armeringsförslag som uppfyller alla rent teoretiska krav. Problemet ligger i att detta förslag ofta ger en orimlig armeringsfördelning som inte kan utnyttjas på en byggarbetsplats. Istället ger förslaget en första bild av en armeringsfördelning. Det är svårt att bygga in den erfarenhet som en konstruktör har och de val han hade gjort. Det är därför bättre att sträva efter att redovisa ett preliminärt förslag som ger konstruktören möjligheten att själv bestämma hur den slutliga fördelningen skall göras.

Nyckelord: FEM-Design, FEM-Plate, flödesschema, platta, armering, armeringsfördelning, stödmoment, fältmoment, noder, element, s-avstånd, armeringsdimension

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1 Introduction

1.1 Background

StruSoft is an international company established in 2002. Its core business is to develop computer programs for the building industry. StruSoft provides programs that can handle a wide variety of problems encountered by a structural engineer. One of these is FEM-Design, a suite of programs based on the Finite Element Method. The suite can deal with all types of structural elements and is used for among other the analysis and design of concrete slabs, walls and buildings.

FEM-Plate is one of the programs in the suite. It can be used for analysis and design of complete floor systems. The user establishes a model of the floor structure and specifies the loads and supports. The calculations performed on this model result in among other an illustration of the moment distribution and corresponding reinforcement.

FEM-plate requires that the user defines the slab structure and the loads acting upon it. The program then generates a finite element model of the slab and performs an initial calculation that provides the moments in each element together with the corresponding punching and shear capacities, deflections and crack widths. The moments are thereafter directly translated into a reinforcement quantity in each element. However, this reinforcement quantity is not practically applicable. Instead, the user must define a practical reinforcement distribution based on the resulting moments and perform additional calculations to control if this reinforcement is sufficient. The objective is to have the program suggest a reinforcement distribution that is based not only on theoretical considerations but also practical.

1.2 Purpose

The purpose of this thesis is to analyze and suggest methods by which the program automatically generates a reinforcement distribution, emanating from the calculated moment distribution, adjusted to current requirements given in national codes and production aspects. The goal is to create an interactive program that incorporates the engineer without eliminating him.

1.3 Formulation of the Task

The goal of this thesis is to create flow charts that can be used as templates by StruSoft programmers. These flow charts are to incorporate all the information necessary to create a program for solving the task of finding a satisfactory reinforcement distribution. When finished, the program is to consider not only international codes but also economical and production aspects. The ambition is in other words to consider not only the theoretical demands but also the practical ones. This means that the flow chart must contain information about;

- necessary input parameters
- assumptions made when programming
- conditions dependent on building codes
- subsidiary conditions such as production aspects

1.4 Method

All programming and numerical calculations are carried out in MATLAB. The task has been carried out as follows;

1. Simple examples have been analyzed in FEM-Plate and by hand calculations. In this way the results provided in FEM-Plate can be derived and therefore better understood.
2. The results provided in FEM-Plate have then been converted so that they can be used in the calculation program MATLAB. Thereafter, preliminary routines have been created based on primarily the Swedish national code.
3. Interviews have then been carried out with engineers active in both production and design. The routines have thereafter been adapted so that the program works in a way “more accurate” from a commercial viewpoint.
4. The routines are then used to solve simple cases and the results are checked in FEM-Plate.
5. The routines are calibrated.

As StruSoft is a commercial business they greatly value the opinion of active engineers. Therefore it was important to create not only theoretically correct routines, but such ones that would also consider the opinions and requests of active engineers. Designing the routines according to standards and national codes would give a satisfactory result in theory. The problem is that this result would not necessarily be commercially satisfactory, considering economy and production aspects.

1.5 Limitations

The project is limited to studying the placement of reinforcement around:

1. Columns placed at edges, corners and inside the plate
2. Rectangular holes
3. Points affected by singularities such as the endpoints of edge supports
4. Arbitrary plate geometries

The routines handle only geometries parallel to the x and y axis. The calculations are also limited to dealing only with design in the ultimate limit state and slabs of constant thickness.

1.6 Outline of the Report

Chapter 2 offers a brief description of concrete and reinforced concrete. The theoretical background to the behaviour of transversally loaded slabs and slabs loaded in flexure is thereafter given and finally, design methods for slabs, based on the theory of elasticity and the ultimate limit state, are presented.

Chapter 3 explains the results received from FEM-Plate and how these are used when developing the calculation routines. It also offers a brief description of how the finite element mesh is generated in FEM-Plate and of the program's peak smoothing function.

Chapter 4 offers a summary of the opinions gathered in the interviews with engineers active in both production and design.

Chapter 5 offers an in-depth description of the calculation routines developed using the information gathered from FEM-Plate. The connection between these routines is shown using flow charts. The flow charts are given in Appendix A.

In *chapter 6* four slabs are presented. These slabs are analyzed using the routines described in *chapter 5*. The resulting amount and distribution of reinforcement is then shown and the pros and cons of the results for each slab are discussed. Lastly, there is a discussion of alternative analyzing methods, methods that could refine the calculation routines but that have not been implemented into the current routines.

Chapter 7 offers concluding remarks.

2 Theory

This chapter offers a brief description of concrete and reinforced concrete. The theoretical background to the behaviour of transversally loaded slabs and slabs loaded in flexure is thereafter given. The design methods for slabs, based on the theory of elasticity and the ultimate limit state, are explained as a conclusion of the chapter.

2.1 Concrete and Reinforced Concrete

Concrete is a mixture of aggregate (sand, gravel and crushed rock) held together by cement paste. Because of the wide variety of areas in which concrete is used, different admixtures are commonly added to change certain characteristics of the concrete such as workability, durability, and time of hardening.

Concrete has high compressive strength and low tensile strength. The tensile strength varies from about 8 to 15% of its compressive strength. The low tensile strength is caused by very fine cracks, so called micro cracks. When for example a concrete beam is subjected to compressive forces, the micro cracks close permitting compression transfer meaning that they have little or no effect on the results. When subjected to a tensile force the cracks open, thus significantly lowering the beams load bearing capacity.

Concrete without reinforcement is very brittle as the concrete in itself has low tensile strength. Steel reinforcement bars on the other hand have tensile strengths of more than 100 times that of regular concrete. The reinforcing steel has the ability to compensate for the tensile strength lacking in regular concrete. The combination of concrete and reinforcing steel is made possible by the similar coefficients of thermal expansion. Concrete and steel reinforcement bond well together with little probability of slippage and thus they will act as a unit. This bond can be varied depending on the surface of the bars. [8] 1.6

2.2 Transversally Loaded Slabs

Transversally loaded slabs are classified as being one-way or two-way depending on how they are supported. If they are supported on two opposite edges they are referred to as one-way slabs. The reason for this is that the bending is in one direction only, perpendicular to the supported edges. A slab supported on all four edges is called a two-way slab as bending occurs in two directions. However, a rectangular slab with one edge more than two times as long as the short edge acts as a one way slab with bending primarily in the short direction. [8] 4.7

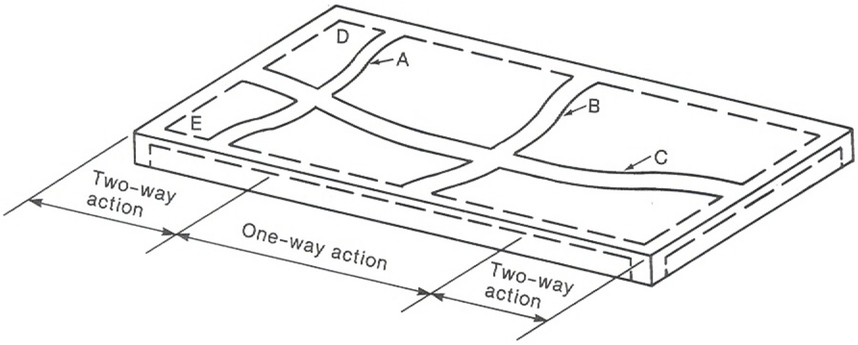


Figure 2.1: *Illustration of a slab with one edge more than two times as long as the short edge. [7] 10.4*

For calculation purposes a one-way slab can be assumed to be a beam with varying width. This assumption simplifies calculations, that can be carried out as for regular beams with bending reinforcement placed perpendicular to the supports. [8] 4.7

2.3 Behaviour of a Slab Loaded in Flexure

Before cracking, a slab can be modelled as elastic which means that for short-term loads the deformations, stresses and strains can be calculated using elastic analysis. [7] 13.4

As the loads increase, cracks will appear in the slab. This means that the slab is no longer isotropic as the crack patterns may differ in two directions, and the slab no longer has constant stiffness. However, tests show that as long as the reinforcement is below its yielding stress, the slab can still be described using the theory of elasticity giving an adequate prediction of the moments. [7] 13.4

With increasing moments the reinforcement eventually yields and the theory of elasticity can no longer predict the behaviour of the slab. A plastic hinge forms and the moments redistribute from yield regions to areas of the slab that are still elastic. [7] 13.4

The load bearing capacity is somewhat underestimated in the current design methods as they do not account for the effects of the arches created under flexure. These arches can jam the hinges, assuming that the concrete is stiff enough to provide reactions for the arches. In reality a slab is rarely supported in a way that allows for the arch phenomena to occur. One part of a slab can on the other hand support another part and in that way counteract the horizontal displacements. [3] 6.5:242

2.4 Design Methods

In the design of continuous slabs it is necessary to consider both the ultimate limit state and the serviceability limit state. For the ultimate limit state it is necessary to study failure by flexure, shear, bond and in some cases torsion. For the serviceability state one must ensure safety against excessive deflections, crack width and vibrations.

There are different ways of treating the design of a slab. These methods are based on either the limit states design theory or theory of elasticity. Methods emanating from the limit states design theory are the yield line analysis and the strip method. The calculation of the moment performed in FEM-Plate is performed using the finite element method, and is based on theory of elasticity.

2.4.1 Theory of Elasticity

The theory of elasticity is the most common way of calculating the section forces in a concrete structure. However, it is often difficult to use for hand calculations. This is mainly due to the fact that the governing slab equation is a forth degree differential equation dependant on deflection, load and flexural rigidity. [9] 7.21

If the reinforcement in a slab is calculated according to the theory of elasticity, the distribution of the reinforcement will often be non-economical and therefore not practically applicable.

2.4.2 Limit States Design Theory

The basic concept of limit design for a reinforced concrete member is that the member can withstand an increasing load after cracking, without failure. However, with increasing loads, the reinforcement will start to yield. The yielding will occur first in the region with the highest moment. When this happens this portion of the reinforcement will act as a plastic hinge, only able to withstand the plasticizing moment while an increase in local angular deformation occurs. If the loads are increased the additional moments will be redistributed to sections of the member that have lower stress, consequently spreading the plastic hinge. The load bearing capacity of the member is reached when there can appear no more plastic hinges in the member or under excessive angular deformations. [8] 14.4

2.5 Yield Line Analysis

The yield line theory is based on the concept that the reinforcement in the slab will yield, thus creating plastic hinges. The hinges in turn divide the slab into a series of smaller, elastic plates. It can be quite difficult to find this ultimate failure pattern for complex slabs. Therefore it is a method best used to check the load bearing capacity and not for actual design. Yield line analysis is a limit state method that will give an upper bound solution.

The choice of yield lines may be done arbitrarily. However, an incorrect choice will result in uncertain results, meaning that the load bearing capacity of the slab is overestimated. This means that a well chosen yield line figure will give a higher amount of reinforcement than a poorly chosen figure. A correct choice of yield lines will result in the true failure load. All other yield lines overestimate the failure load.

Once the yield lines have been chosen the calculation of the moments, or loads, can be done by using either the equilibrium method or the virtual work method. In the equilibrium method, equilibrium equations are written for each plate segment. Special care must be taken to ensure that all the forces acting on each element are accounted for, especially when several yield lines intersect with free edges. Consequently, the virtual work method is more commonly used.

The main idea behind the virtual work method is that the external work done by the loads when displaced, is equal to the internal work done by the rotating yield lines. The total external work is the sum of the work for each plate while the total internal work is the sum of the internal work done on each yield line.

External work = $\sum \int \int w \delta dxdy = \sum W \Delta_c$ *(2.1)*

where
w = load on an element of area
δ = deflection of that element
W = total load on a plate segment
Δ_c = deflection of the centroid of that segment

and internal work = $\sum m_b l \theta$ *(2.2)*

where
m_b = bending moment per unit length of yield line
l = length of the yield line
θ = angel change at that yield line

As the yield lines are assumed to have formed prior to the imposing of the virtual displacement, no elastic deformations occur during the virtual displacement. [7] 15.3

2.6 Strip Method

In the strip method, the slab is considered to work as slices of perpendicular beams. The twisting moment, m_{xy}, in the slab is assumed to be zero which means that the load carried in the x and y direction is larger than the actual load, thus overestimating the moments, m_x and m_y acting on the strips. The strip method is a limit state method that gives a lower bound estimate of the slab capacity. [10] 5.2.2

The choice of strips and the division of the load q into q_x and q_y can be done arbitrarily. The choice will however affect the amount of reinforcement required in the slab. It is therefore clear that the choice affects not only the utilization of the reinforcement but also the economical aspects. Economy therefore states that the division into strips be made so that the design moments are as small as possible.

The advantage of the strip method is that when dividing the slab into strips parallel to the x and y axis, the moment distribution can be calculated in the same way as for a regular beam spanning in one direction. The only requirement is that the divided loads, q_x and q_y, when added result in the original load, q. [10] 5.2.2

If a calculation is performed using both the strip method and yield line analysis, and the resulting load bearing capacities coincide, then the upper and lower limit values coincide. This means that the exact solution according to the theory of plasticity has been found.

2.7 Finite Element Method

The finite element method is a numerical approach by which general differential equations can be solved approximately. When using the finite element method to calculate the moment in a slab, the calculation is done according to the theory of elasticity. This means the capacity of the member is exhausted as soon as the strength of the material has been reached. The calculations are often simplified by adding certain assumptions. This is also the case regarding the calculations in FEM-Plate. One assumption is that the relation between force and deformation is linear in both loading and unloading. This assumption is not fully correct as the stress-strain relation of the concrete is curved. The implication of this simplification is that the calculation of the force and moment distributions is done on uncracked concrete while the design of the cross section is done on cracked concrete.

When designing in the ultimate limit state, the reduction of stiffness may be neglected in the case of a cracked cross section. However, it may not be neglected when moments and forces of second order appear. If the cracking and the consequent reduction of stiffness is considered it will give a more accurate result but significantly complicate the calculations. [3] 3.2:21

3 Features of FEM-Plate

This chapter explains how the results given from FEM-Plate are used when developing the calculation routines. It also offers a brief description of how the generation of the finite element mesh is done in FEM-Plate and of the program's peak smoothing function.

3.1 Analysis Calculations

FEM-Plate displays a large number of results. The following results are taken from the program and used in the development of the calculation routines;

- finite element mesh, including amount of elements that the model consists of, amount of nodes and the size of the moments in each node and element
- slab properties, including geometry, coordinates, thickness, restraints and type of concrete
- column properties, including coordinates, geometry, restraints and reaction forces
- coordinates for the nodes
- coordinates for holes and free edges

3.2 Finite Element Mesh

When generating a finite element mesh, FEM-Plate uses triangular and quadratic elements with six and eight nodes respectively. The mesh generating tool automatically generates the most balanced mesh. This is done by considering the minimum division numbers and the average element size and thereby creating a mesh with a small bandwidth giving better computer efficiency.

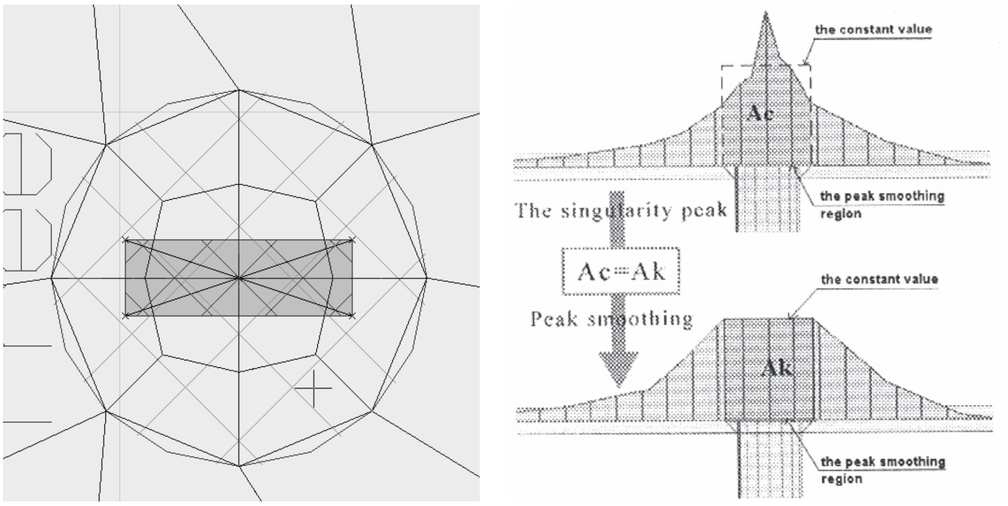
The mesh is first created using only triangular elements. It is thereafter converted into a mixed quadrate-triangle mesh in which the triangles, as far as possible, are merged into quadrates. Again, the mesh with the globally optimal shape is chosen. It is possible to choose and refine the mesh and therefore alter the element size around specific areas of interest, for example free edges and columns. This will however effect the size of the calculated moments in the altered elements.

3.3 Peak Smoothing Function

When using the finite element method singularity problems may occur. This can occur when the calculated results converge to the theoretical solution as an effect of the mesh refinement. The singularity problem means that the inner stresses increase when refining the mesh and at certain places the FEM-theory will give infinite inner stresses. Areas that are especially effected by this problem are for example point supports such as columns and the endpoints or intersections of walls.

The disturbance in the inner stresses caused by singularities effect an area within a short distance of the singularity. For design purposes, this distance is defined in the respective national code. FEM-Plate has a built in function, *peak smoothing,* which is developed by StruSoft and designed to handle the singularity phenomena.

The peak smoothing function creates a region that encloses the inner forces that have undergone a substantial change as a result of the mesh refinement. This region is called the active zone and it is shown in Figure 3.1.



(a) (b)

Figure 3.1: *The active zone around a rectangular column is depicted in (a). (b) is an illustration of the peak smoothing function. [5] 9.6*

The use of the peak smoothing function results in the calculation of a new constant value for the inner stresses above the active zone. When calculating the new value, the principle of energy conservation is taken into account. This means that the new value is determined so that the size of the original and the new stress figure becomes the same. [5] 9

3.4 Design Forces

The moments used for the calculation of the reinforcement are obtained from a file containing *design forces*. This file supplies information about the number of elements in the defined slab geometry. Furthermore it supplies the nodes belonging to each element. The bay and support moment in the x and y direction are given in each node and finally, the file supplies the mean moment calculated in each element.

4 Interviews with Designers and Production Engineers

Four interviews have been carried out with structural and production engineers in order to gather information about the practical design aspects of slabs. The purpose of this chapter is to offer a summary of the opinions gathered in the interviews. This summary includes both general design rules used by the engineers in specific cases, such as holes, and opinions as to what they think the program should consider when calculating the reinforcement distribution in a slab.

Many of the decisions made when designing a slab are based on experience and rule of thumb. This means that much of what is mentioned in this chapter can not be found in national codes.

4.1 Practical Design Aspects

Many of the rules of thumb described were based on practical demands on the construction site. Such demands are for example to use the same bar dimension in the slab, something that applies to both the top and bottom reinforcement, and to not use more than one layer of reinforcement in one direction. If possible, use the same bar dimension in the top and bottom of the slab as this simplifies the placing of the reinforcement on the construction site. This demand does not apply to pre-cast slabs as the bottom reinforcement is handled at a factory.

The reinforcement due to bay moment is placed in the whole slab. This means that it is placed all the way to the supports even when it is unnecessary. The main reason for this is to minimize the effects of an accident. By placing the reinforcement in the whole bottom area a continuous collapse of the building can be avoided.

The reinforcement due to column punching is, for practicality reasons, rarely bended and is instead placed as a net above the column in the same layer as the bending reinforcement.

Holes are divided into small and large holes. A hole can be considered large if one of the sides is more than two times the plate thickness. However, this can not be used as a general rule as the location of the hole in the slab is important.

The reinforcement around holes is placed in both the top and bottom of the slab. If the calculated amount of required reinforcement is below 2 θ 12, then 2 θ 12 is placed on each edge. If the required amount exceeds 2 θ 12, the calculated value is chosen. For small holes, the extension of the reinforcement bars is the same as the size of the hole. If the hole is for example 400 mm wide, the reinforcement is extended 400 mm to the left and right of the hole. This will result in a sufficient anchorage length. However, this rule of thumb should not be used for large holes. These must instead be checked according to national building codes.

4.2 Desired Features

An important request was to have the option to choose the length of the reinforcement bars. A second request was to have the program present the calculated lengths of the reinforcement bars as lengths evenly divided from the largest bar length. This means that if the length of the reinforcement bar is 12 meters, then the presented lengths should be 2, 3, 4, 6 meters and so on.

Another common request was to have the option of defining the load bearing direction. This is especially important when dealing with pre-cast slabs as their load bearing capacity is limited to one direction.

When dealing with pre-cast slabs specifically, the user should have the ability to limit not only the largest but also the smallest distance between the reinforcement bars in the bottom of the slab. In some cases, the reinforcement due to bay moment is placed in the slab which is cast on top of the pre-cast slab on the construction site. However, the pre-cast slab must still be reinforced so that it can carry the additional concrete before it has cured. Therefore the pre-cast slab must be given a minimum reinforcement, typically θ 8 with 200 or 300 mm between the reinforcement bars.

The user should also be able to define the areas in which to place the top reinforcement and also the load bearing direction for the top reinforcement.

The top reinforcement should be placed in a zigzag pattern with varying length to create a more economically efficient reinforcement distribution, see Figure 4.1. This placement is also favourable when considering crack patterns as the zigzag placement creates a smoother transition between reinforced and un-reinforced areas.

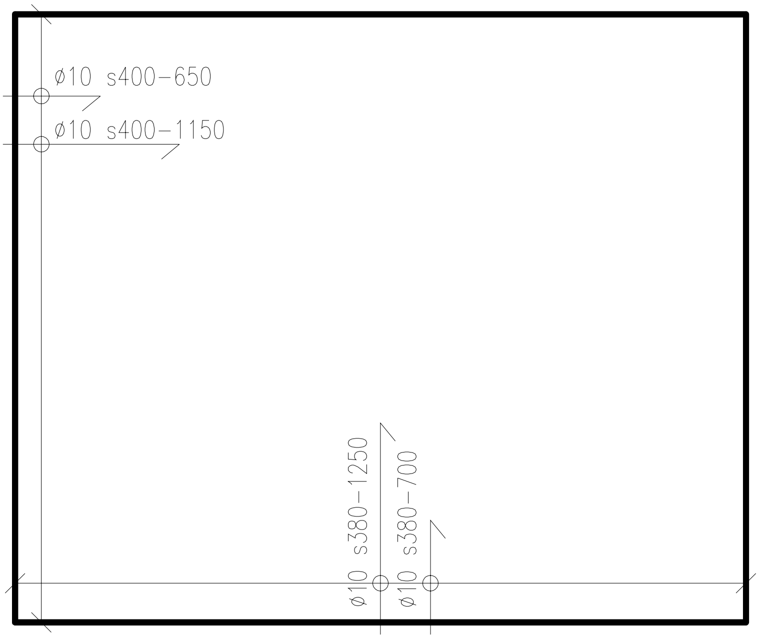


Figure 4.1: *Zigzag pattern for support reinforcement.*

5 Routine Development

In this chapter the routines used to calculate the necessary amount and distribution of reinforcement are explained starting with the routines for a *general slab* and followed by *columns*, *holes* and *free edges*. The routines are written with regard to the Swedish national code and deal with the ultimate limit state.

The connection between the routines thoroughly described below is shown in the flow charts in Appendix A. It is therefore recommended that the flow charts are studied while reading the descriptions.

The criteria taken into consideration are either from national code (NC) or information gathered through interviews (IW) as mentioned in chapter 4 – *Interviews with Designers and Production Engineers*. The requests and rules of thumb have as far as possible been considered when creating the calculation routines.

5.1 General Slab

The general slab has a rectangular shape without holes, free edges or columns. It can have free or fixed supports and an arbitrary number of spans.

5.1.1 *Routine*: MomentRegions

By comparing the nodes of elements, the neighbours of one specific element can be determined. This is done for all elements, hence establishing a register containing every element and its neighbours.

To determine where the various moments appear, the element moments obtained from FEM-Plate are used. These moments are presented in four groups; negative moments in the x and y direction and positive moments in the x and y direction. For each of these, the moment in each element is evaluated. An arbitrary element is controlled for one moment type at a time. If that specific element has a moment that differs from zero, it is collected into a group and its neighbours are controlled.

This procedure is repeated as long as new elements are added to the group. When the procedure stops, a group containing elements with the same type of moment has been defined.

If there are several separated moment areas, a group is created for each one of these areas. First, all elements not belonging to a group are considered. A new group is formed, and the sorting process continues. New groups are created until all areas are represented by a group. This means that every element with a moment different from zero is sorted into a group. The next step is to sort the element numbers, element moments, node numbers and node moments by group and save them in separate cells. This procedure is repeated for all four moment types.

The moments in some elements may be very small. These elements would be sorted into groups even though they in reality can be neglected. This means that the sorting process would create fewer groups with larger content and effect the final result in a negative way. In order to prevent this phenomenon every moment smaller than a certain limit is neglected. Through interviews it was determined that this limit could be chosen as the moment corresponding to half the tensile strength of the concrete. The use of half the tensile strength, as opposed to the whole tensile strength, is to provide additional safety only and was given as a recommendation in the interviews. IW [11]

5.1.2 *Routine*: MinimumReinforcement

The purpose of the minimum reinforcement is to account for the tensile stresses that appear in the bottom of the slab due to the bay moment. If the user does not define a minimum reinforcement, the reinforcement is calculated with regard to present national code.

The calculation is performed for three predefined bar dimensions and the amount of reinforcement is evenly distributed in the x and y direction. The concrete is assumed to be cracked. NC [1] 4.5.6

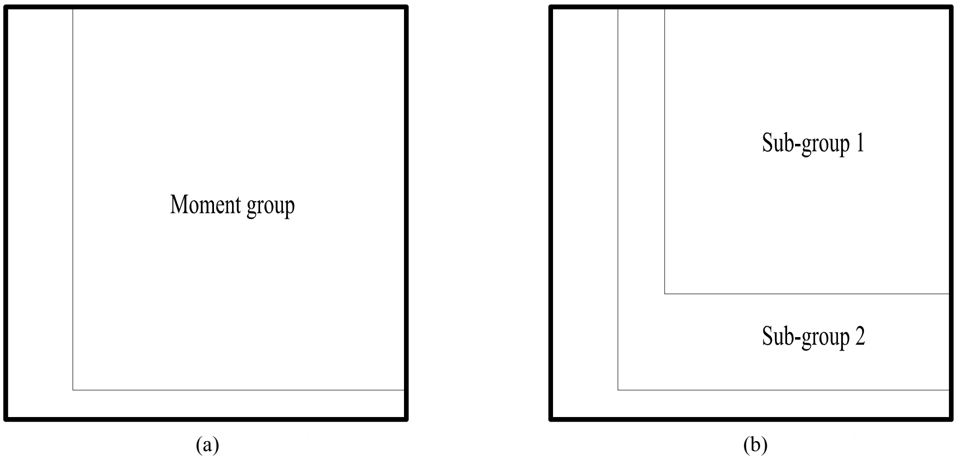
The maximum bar spacing is given the value of two times the slab thickness. NC [2] 3.9.6

The minimum bar spacing is set to 100 mm. NC [3] 3.9:5

5.1.3 *Routine*: ReinforcementAreas

For each of the four moment types, the geometrical distribution of moments is determined. All nodes belonging to a group are divided into two sub-groups. The shape of the resulting reinforcement regions are defined as rectangles. The coordinates of these rectangles are chosen as the smallest and largest x and y coordinates of the group.

In this routine, two sub-groups are created, Figure 5.1. The first sub-group contains all nodes with moments larger than, or equal to, half of the largest node moment in the group. The rest of the nodes are sorted into the second sub-group. This is done as the inner sub-group should contain twice the amount of reinforcement as the outer group for later calculations.



(a) (b)

Figure 5.1: *A moment group is shown in (a) while (b) shows the resulting sub-groups created by the routine* ReinforcementAreas**.**

The reason for dividing each group into two sub-groups is to render an inner region that contains the largest moments and therefore requires a larger amount of reinforcement. The second sub-group is an outer region requiring less reinforcement. This renders a more economical reinforcement suggestion but it also means that the bar dimensions can vary between the regions.

5.1.4 *Routine*: Coordinate

The x and y coordinate for each node in the input vector is controlled. The smallest and largest x and y coordinates are thereafter stored as coordinates for the entire area.

5.1.5 *Routine*: InsideTheRegions

InsideTheRegions determines which elements lie within given coordinates. For all four moment types, each sub-group is analyzed separately.

Based on the coordinates of the sub-group, which were defined in *ReinforcementAreas*, each side of the rectangle is represented as a vector. The four vectors are defined in Figure 5.2.

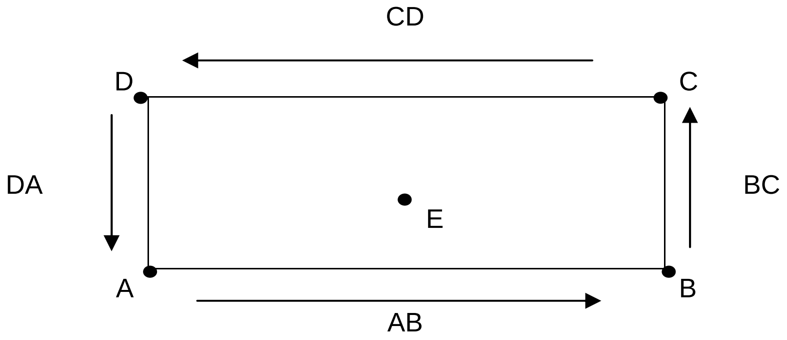


Figure 5.2: *The orientation of the vectors defining a reinforcement region.*

To determine on which side of a line the node E is placed, the determinant between the vector and the node is calculated [Appendix B]. For every element in the entire mesh, all nodes are controlled against the vectors forming the rectangle. If one or more nodes lie inside the rectangle, the element is connected to that specific sub-group. Since there are two sub-groups, where one surrounds the other, some elements in the larger sub-group also appear in the smaller. These elements are deleted from the larger sub-group and in this way they only belong to the smaller.

5.1.6 *Routine*: BiggestMoment

The largest moments in each group are determined by comparing the average moments for the elements in that group.

5.1.7 *Routine*: CheckMmax

As the coordinates for two sub-groups are based on the nodal moments, some errors may occur. The node with the largest moment does not necessarily belong to the element with the largest mean moment. If the maximum moment for the inner sub-group is smaller than the maximum moment for the outer sub-group, a correction must be made. First, the inner sub-group is given the outer sub-groups maximum moment as this moment is the largest in the group. Next, the nodal coordinates for the element with the largest moment are compared to the coordinates for the inner sub-group. If the coordinates for any of the nodes are larger / smaller than the coordinates for the current sub-group, the boundaries are enlarged to enclose the node. When all nodes of the current element have been compared to the boundaries, a check according to *InsideTheRegions* is performed. With the new coordinates as boundaries, the scanning of the elements is once again performed in search of the largest moment.

5.1.8 *Routine*: AdditionalMoment

AdditionalMoment controls which areas require additional reinforcement, taking into consideration the minimum reinforcement applied in the bottom of the slab and all necessary reinforcement in the top of the slab.

Every group of bay moments is compared to the moment corresponding to the minimum reinforcement. The first sub-group, containing the largest moment of the two, is checked first. If the moment is smaller than the moment from the minimum reinforcement, the additional moment is set too zero. If the first sub-group has been set to zero, as will the second. If not, the second sub-group is given a value of the moment corresponding to half the element moment of the first sub-group. The consequence of this is that the first sub-group will have twice the amount of reinforcement as the second group.

The procedure is analogue for the support moments. As the minimum reinforcement is placed only in the bottom of the slab, the comparison between the moments in a group and the moment corresponding to the minimum reinforcement is omitted.

The moment corresponding to the minimum reinforcement, is calculated based on $A_{s\,,min}$.

$$M_{\min re\inf} = A_{s,\min} f_{st} 0.9d \qquad (5.1)$$

where $M_{minreinf}$ = the moment corresponding to the minimum reinforcement

$A_{s,\,min}$ = the minimum reinforcement

f_{st} = tensile strength of steel

d = effective height

All moments are calculated under the assumption that the reinforcement in the y direction has the largest effective height. This means that if the x direction is load bearing, the calculated amount of necessary reinforcement will be slightly over estimated.

5.1.9 *Routine*: AdditionalReinforcement

AdditionalReinforcement calculates the required amount of additional reinforcement for all groups. The groups containing a support moment are subjected to an additional control: if necessary, the reinforcement in the second sub-group is corrected to achieve twice the amount of reinforcement as in the inner sub-group. The criteria to be fulfilled by the correction are that the same bar dimension is used in both sub-groups and, if possible, twice the bar spacing in the second group compared to the first. If these criteria are met, the amount of reinforcement in the first sub-group is twice the amount of that in the second and the distribution of reinforcement is simplified.

The reinforcement quantity is calculated using the equations below:

$$\bar{m} = \frac{M}{bd^2 f_{cc}} = \omega(1-0.5\omega)$$ 5.2 NC [3] 3.6:432

$$A_s = \frac{M}{f_{st} d(1-\frac{\omega}{2})}$$ 5.3 NC [3] 3.6:432

The distance between the bars is then calculated by dividing the cross section area of the reinforcement bar with the reinforcement quantity previously calculated.

The maximum bar spacing is given the value of two times the slab thickness. NC [2] 3.9.6

The minimum bar spacing is set to 100 mm. NC [3] 3.9:5

5.1.10 *Routine*: AreaCheck

This routine controls if any regions are overlapping each other. If that is the case a warning appears and is given as output.

The reason why regions may overlap is that they are formed by the elements contained in the groups. An element is linked to a group if the element has the same type of moment, M_{xs} M_{ys} M_{xb} M_{yb}, as the surrounding elements. In some cases one group can be shaped as area 2 in Figure 5.3.

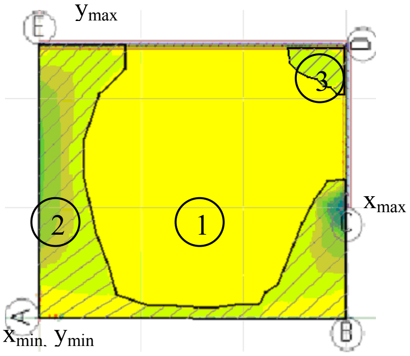


Figure 5.3: *Illustration of overlapping regions.*

The coordinates of the regions are defined by the smallest and largest x and y nodal coordinates. In the case of area 2 in Figure 5.3, the largest x coordinate would be at C and the largest y coordinate at edge D-E. The smallest x and y coordinates would be at A. The coordinates for area 2 would in other words include the entire slab. This means that a part of the region covers area 1 in which none of the groups elements lay. This area requires no reinforcement, but as it is included in the region it leads to an over-reinforcement of the area. The region also covers a second group; area 3. This means that area 3 in the top right corner will receive a reinforcement contribution from both the group covering the entire slab and the correct group.

5.1.11 *Routine*: Anchorage

The anchorage length is calculated for each group and added to the coordinates for each region.

The calculations are performed according to NC [3] 3.9:12

5.2 Column

In the Column routines, the slab's punching capacity is controlled. If necessary, the punching reinforcement is then calculated.

The control of the punching capacity is already performed in FEM-Plate. However, before the punching capacity can be controlled, the user must define the bending reinforcement above the columns. FEM-Plate can then perform a second calculation, checking if the capacity is sufficient. The developed routines incorporate this second step, thereby eliminating the need for a second calculation in FEM-Plate.

The sections of the columns can be square, circular or rectangular and placed arbitrarily.

5.2.1 *Routine*: ColumnRead

Column type, dimensions and placement is collected from FEM-Plate.

5.2.2 *Routine*: NodeColumnMoment

For the centre node of each column the moments are saved and linked to the corresponding column in separate cells according to column geometry.

The coordinates for every column refer to the centre of the column. When a column is present in a slab, the finite element mesh adapts to the column rendering a node placed in the centre of the column.

5.2.3 *Routine*: ColumnPlacement

ColumnPlacement determines if the columns are placed on an edge, in a corner or inside the slab. With that in consideration, parameters regarding punching and shear are calculated.

The effective height, d, is calculated as a mean value of the effective heights in the x and y direction NC [1] 3.12.3. These, in turn, are calculated from the maximum bar dimension in the respective layer.

The calculations in this routine are simplified. The effective height, d, is calculated based on the maximum bar dimension, Φ_{max}, used in the slab. If the dimension of a bar placed above a column is less then Φ_{max}, the effective height based on Φ_{max} is still used. The slab must be of rectangular shape as all controls of coordinates are performed under the assumption that the slab has four sides.

The placement of a column is determined by comparing the boundaries of the columns with the boundaries of the slab. This determines if a column should be categorized as an inside, edge or corner column. A column meets the demands of being on an edge if the distance between the column and the edge of the slab is equal to or less than the thickness of the slab. If this criterion is fulfilled for two edges the column is placed in a corner. [4] 2.2.2.3.2

Depending on column shape and position, the length of the area effected by punching failure and the length of the area effected by shear failure is calculated. NC [3] 6.5:342

The calculation of eccentricity depends on the column placement NC [1] 3.12.3. All calculated values are linked to the corresponding column.

5.2.4 *Routine*: ColumnReinforcement

The coordinates of the columns are compared to the coordinates of the regions for the support reinforcement. After determining in which region a column is placed, the reinforcement of that region is linked and saved to the current column. This supplies information about the content of reinforcement over the columns.

The routine assumes that there is reinforcement in both x and y direction over a column.

5.2.5 *Routine*: PunchingCalculations

The capacity in the slab is calculated according to applicable code. This calculation is performed three times due to the three different bar dimensions in the input. For each alternative a different amount of reinforcement is defined, which means that varying input values are used in the calculations. If the capacity of the slab is insufficient, the value of the reaction force is saved and linked to the current column. The routine also calculates a length, the c-distance, from which the coordinates of the area to be reinforced are determined later on. Finally, the new reinforcement content over each column is determined.

In this routine the effective height, d, is calculated as a mean value of the effective heights in the x and y direction NC [1] 3.12.3. These, in turn, are calculated from the maximum bar dimension in the respective layer.

The c-distance depends on the placement of the column and is calculated according to NC [3] 6.5:342.

The reinforcement content in the top of the slab is determined and controlled according to NC [1] 3.12.3.

5.2.6 *Routine*: PunchingReinforcement

If necessary, additional reinforcement due to punching and shear is added in the top of the slab. The reinforcement content for the additional reinforcement is added to the original content and the spacing between the reinforcement bars is calculated. The calculations are performed separately for the x and y direction.

The amount of reinforcement required, is determined in the same way as for shear reinforcement NC [3] 3.7:42. Equal distribution of the reinforcement between the x and y direction is assumed.

In the calculation of shear reinforcement, the reinforcement angle is set to 90°. NC [3] 3.7:42

If the content of reinforcement is larger than 0.01 it is assumed that the slab thickness or bar dimension is insufficient and no further calculations are performed. NC [1] 3.12.3

5.2.7 *Routine*: PunchingCoordinates

For every column that has a c-distance connected to it, the coordinates for the reinforcement, emanating from the c-distance, are calculated.

In this routine the bar spacing distance is controlled for each column. If the spacing is less than 100 mm the reinforcement dimension is increased according to the input parameters consequently resulting in an increase of spacing. NC [3] 3.9:5

5.3 Hole

The routines are based on rules of thumb and are written for small rectangular holes. They are not adapted for large holes. The routines can easily be adapted for circular holes but this has not been done.

5.3.1 *Routine*: HoleReinforcement

To determine the design moment governing the hole reinforcement, all nodes placed on the edges of the hole are located. The design moment is collected for the corresponding elements. The calculation of reinforcement and number of bars is thereafter done according to equations 5.2 and 5.3. Finally, the coordinates for the reinforcement are calculated.

As long as the necessary reinforcement calculated is less than two 12 mm bars, two 12 mm bars are placed in both bottom and top around the hole. This minimum amount of reinforcement is commonly applied by Swedish engineers. IW [11]

As mentioned in *chapter 4 – Interviews with Designers and Production Engineers*, the extension of the reinforcement around the hole depends on the size of the hole. If the hole is 400 mm wide, the reinforcement is extended 400 mm to the left and right of the hole. The same principle applies to the extension in the y direction. The placement of reinforcement in both the top and bottom of the slab around holes and free edges may seem redundant. Again, it is based on the practice of engineers. IW [11]

When the coordinates of the reinforcement are calculated, the spacing between the bars is set to a minimum distance of 100 mm. NC [3] 3.9:5

According to the interviews, an acceptable width of the reinforcement region around the hole is between four to six times the slab thickness. If the areas calculated exceed this width, the reinforcement must be arranged into two or more layers. This calculation is not performed in this routine but a notification is presented if this would be the case. IW [11]

5.4 Free Edge

The free edge can be of arbitrary length and placement.

5.4.1 *Routine*: FreeSide

The largest bay moment along the free edge is determined by searching all elements attached to the edge. The necessary reinforcement is then calculated according to equation 5.2 and 5.3. Finally, the coordinates for the reinforcement are calculated.

When the coordinates of the reinforcement are calculated, the spacing between the bars is set to a minimum distance of 100 mm. NC [3] 3.9:5.

According to the interviews, an acceptable width of the reinforcement region next to the free edge is between four to six times the slab thickness. If the area calculated exceeds this width, the reinforcement must be arranged into two or more layers. This calculation is not performed in this routine but a notification is presented if this would be the case. IW [11]

5.5 Comments

AdditionalReinforcement

A calculation error has been found in routine 5.1.9 - *AdditionalReinforcement.* When calculating the relative moment $\overline{m}$, according to equation 5.2, the unit of the design moment is in kN instead of N. The consequence of this is that ω is 1000 times smaller than it is supposed to be. The effect on the required reinforcement is negligible as ω is small even when the calculations are performed correctly and the difference of the parenthesis $\left(1-\frac{\omega}{2}\right)$ in equation 5.3 is very close to one in both cases. Even though the error in this routine could easily be fixed, the consequence on the following routines is more difficult to overview.

As described in chapter 5.2 – *Columns*, the calculations of the slabs punching capacity performed in the routines are not necessary as this value is calculated in FEM-Plate. When using the “incorrect” value of the design moment, the slabs capacity, calculated by the written routines, almost corresponds to the capacity calculated by FEM-Plate. This is why the “incorrect” value of the design moment is used. The correct value leads to an overestimation of the slabs load bearing capacity. This overestimation results in no need for additional reinforcement due to column punching, even though FEM-Plate shows that it is necessary. A comparison was performed, forcing the routines to use the slab capacity calculated in FEM-Plate. This resulted in a need for punching reinforcement even when the correct value of the design moment was used. This leads us to believe that the initial error has been compensated for and therefore it has not been corrected. Due to the limited time frame the reason for this additional error has not been investigated.

Hole

The reason that the routines in chapter 5.3 – *Hole* are valid for small holes only is that an additional demand for large holes was discovered at a late stage. This demand deals with the effect of the hole on the concrete surrounding the hole. According to NC [1] (6.1), the effect of a large hole is negligible on a distance of 1.5 times the width (height) of the hole. This requirement has not been taken into account which results in an overestimation of the concrete capacity nearest to the hole. However, this effect is negligible for small holes.

6 Case Studies

In *chapter 6*, four different slabs are analyzed using the routines developed in *chapter 5* and deal with the problems that were mentioned in chapter 1.5 – *Limitations*. With these examples, the resulting amount and distribution of reinforcement is illustrated, and the pros and cons of the routines are discussed. Lastly, there is a discussion of alternative analyzing methods, methods that could refine the calculation routines but that have not been implemented into the current routines.

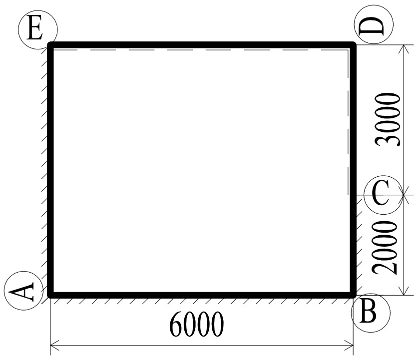
All the examples are taken from [10] Appendix 4. The specifics for each slab are given in the respective table. All four slabs have a concrete cover of 20 mm in both top and bottom and a bar distance round off of 50 mm. The dead weight of the concrete is not included in the loads presented, but is considered in the calculations.

Note that in the figures showing required reinforcement, the green areas represent over-reinforced regions and red areas represent under-reinforced regions.

All dimensions are given in millimetres.

6.1 Slab 1 – Simple and Fixed Supports

Slab 1 is simply supported along C-D and D-E. The slab is fully fixed along A-B, B-C and E-A, see Figure 6.1.



C 20/25	f_{cck} f_{ctk}	19 MPa 1.45 MPa
Ps 500	f_{stk}	500 MPa
Slab thickness	t	0.160 m
Load		12.5 kN/m^2

Figure 6.1: *Slab 1 – geometry and support conditions.*

6.1.1 Results

Bottom X - As seen in Figure 6.2 the reinforcement is divided into three regions:

- region 1 - θ 8 s150
- region 2 - θ 8 s320
- region 3 - θ 8 s200

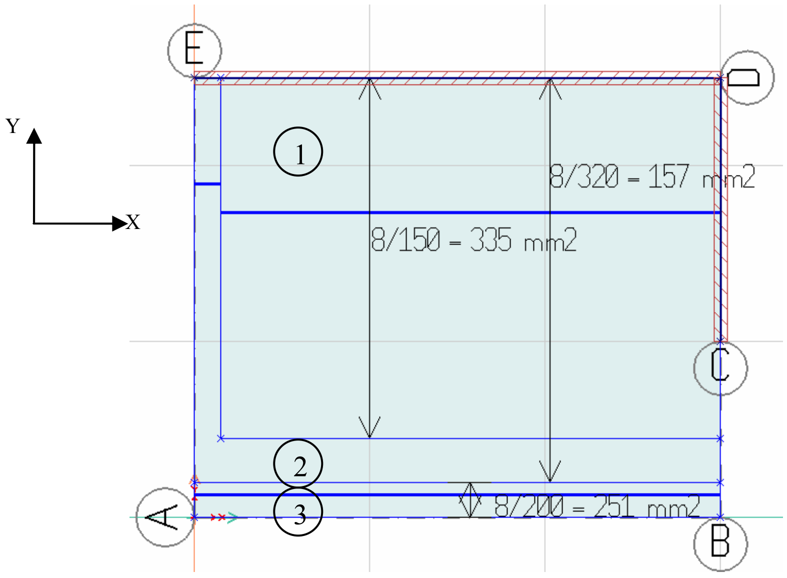


Figure 6.2: *Distribution of the reinforcement, bottom X.*

Reinforcement regions 1 and 2 are the result of creating sub-groups based on moments as was explained in chapter 5.1.3 – *ReinforcementAreas*. The capacity of the concrete in region 3 is larger than the bay moment. This means that there is no need for reinforcement in this region. However, as was explained in chapter 5.1.2 - *MinimumReinforcement*, all areas unaffected by the bay moment are given the minimum reinforcement as a default value.

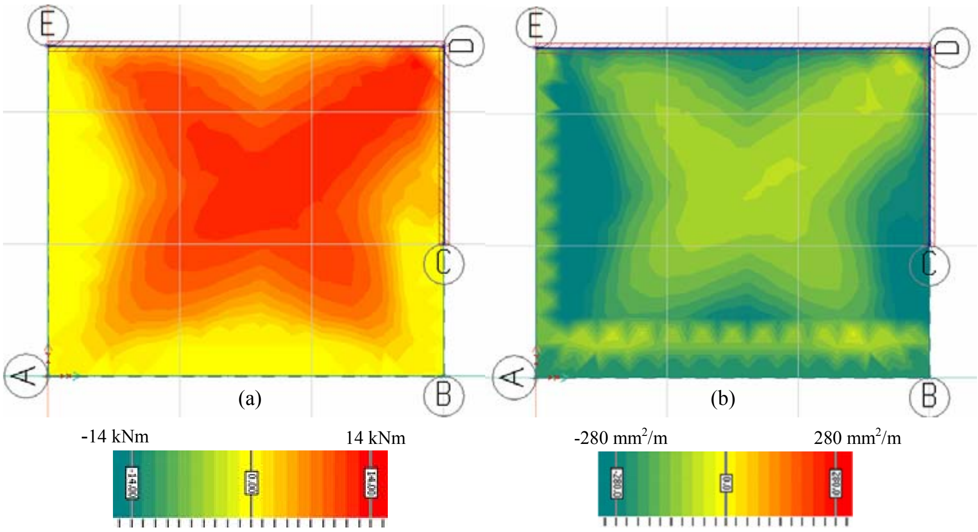


Figure 6.3: *Distribution of moment (a) and missing reinforcement (b) – bottom X.*

Bottom Y – As shown in Figure 6.4 the reinforcement is divided into three regions:

- region 1 - θ 8 s150
- region 2 - θ 8 s300
- region 3 - θ 8 s200

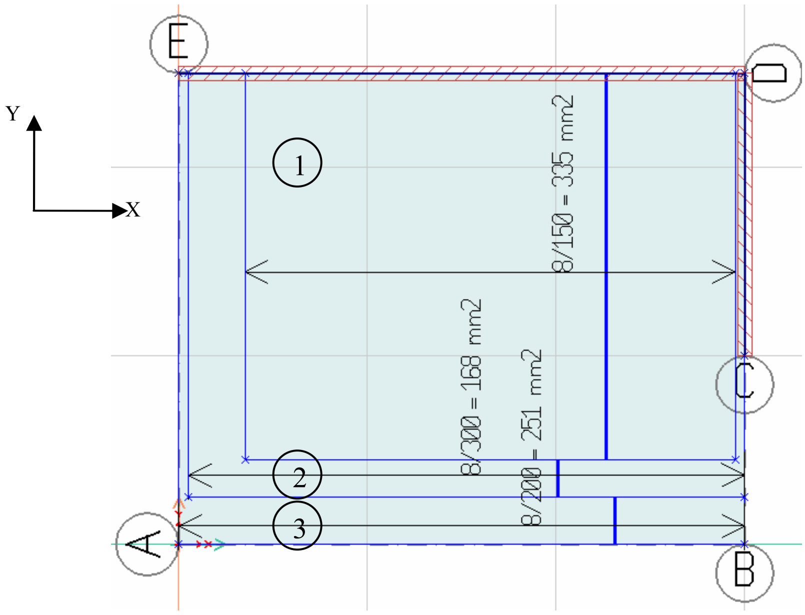


Figure 6.4: *Distribution of reinforcement, bottom Y.*

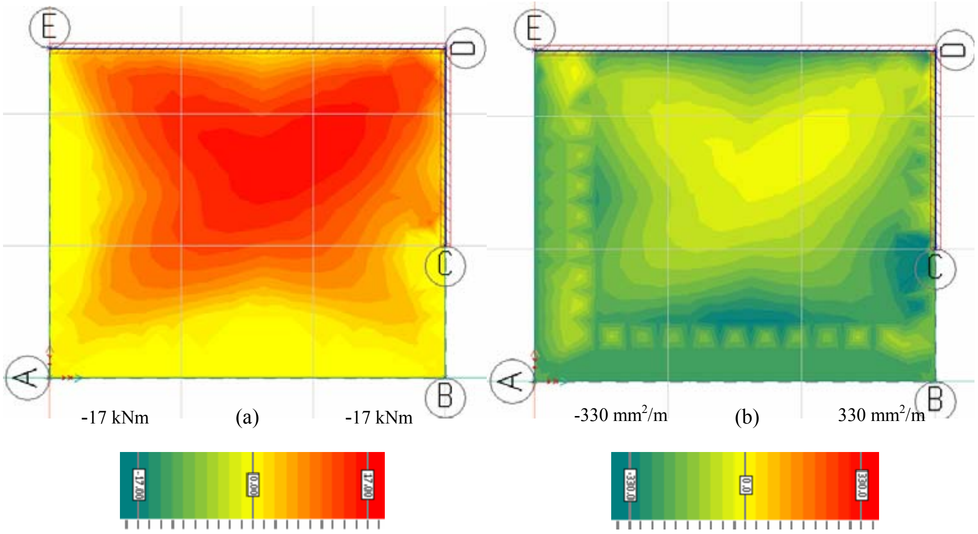


Figure 6.5: *Distribution of moment (a) and missing reinforcement (b) – bottom Y.*

Top X – As seen in Figure 6.6 the reinforcement is divided into four regions:

- region 1 - θ 16 s100
- region 2 - θ 16 s200
- region 3 - θ 16 s200
- region 4 - θ 10 s320

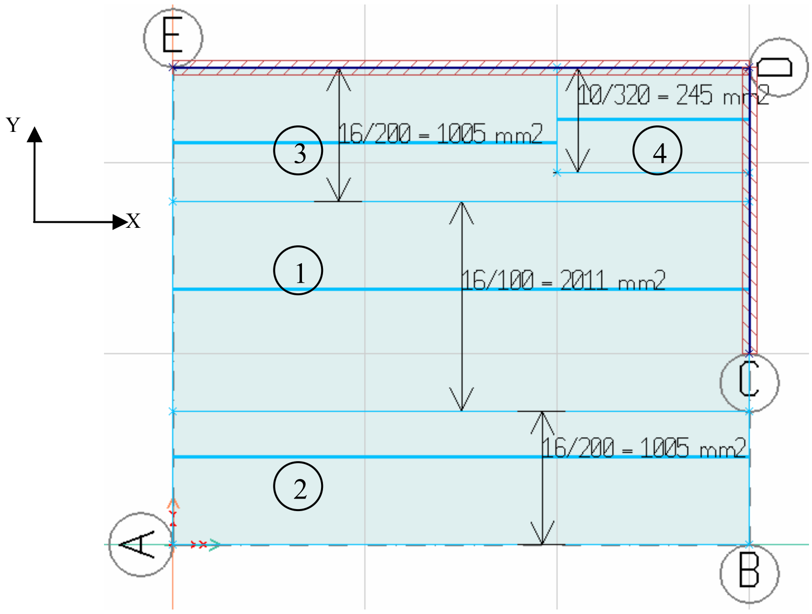


Figure 6.6: *Distribution of reinforcement, top X.*

Region 1 extends across the slab which is clearly unnecessary as there is no support moment in the middle of the slab. This results in an excess of reinforcement of approximately 2011 mm^2/m. This is due to the fact that there is a high support moment along E-A and at point C. These moments are united into one region due to the small moments scattered along A-B. The reinforcement in region 4 is due to the small moments in the intersection of walls C-D and D-E.

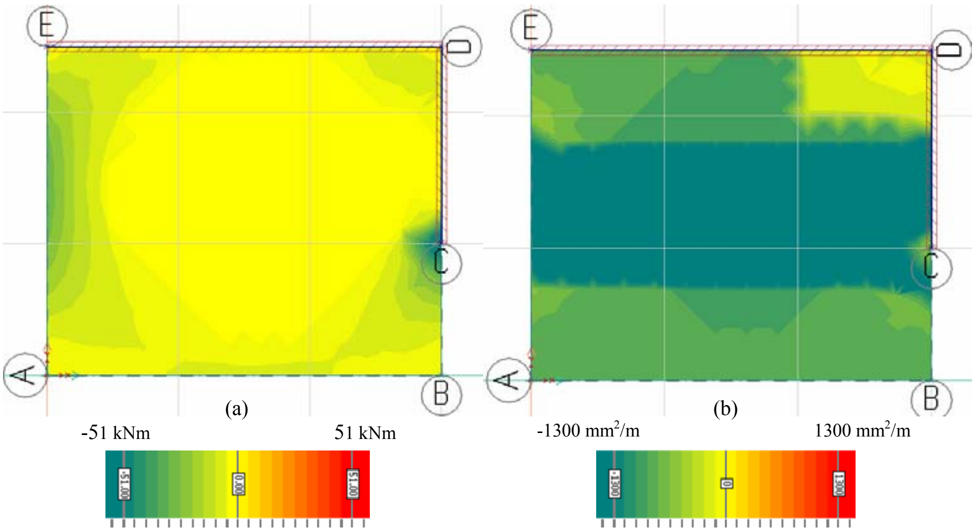


Figure 6.7: *Distribution of moment (a) and missing reinforcement (b) – top X.*

Top Y – As seen in Figure 6.8 the reinforcement is divided into three regions:

- region 1 - θ 10 s150
- region 2 - θ 10 s300
- region 3 - θ 10 s320

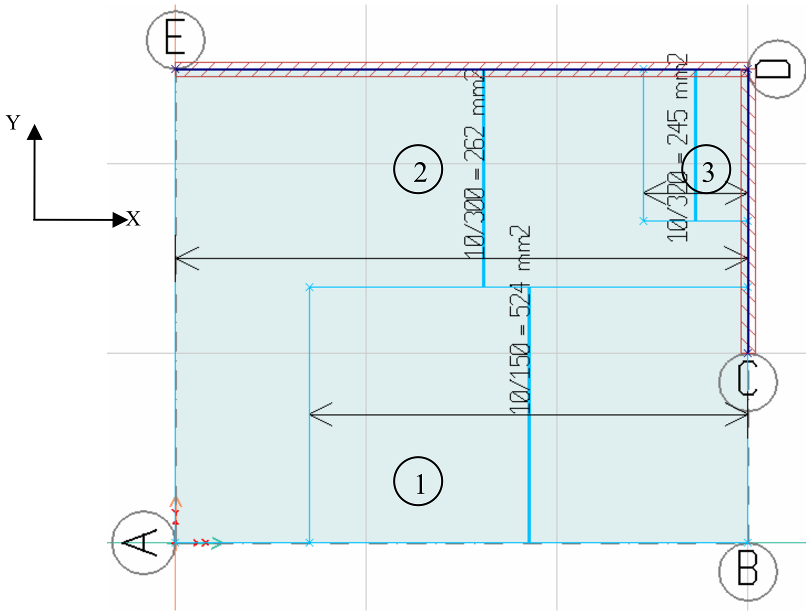


Figure 6.8: *Distribution of reinforcement, top Y.*

As in *top X* a large part of the slab is over-reinforced. In this case the excess reinforcement amounts to approximately 524 mm^2/m, located in the top left of region 1. As in the previous case this is due to the division of the slab into regions performed by the routines. The top right coordinate of region 1 is on C-D. The bottom left coordinate will be left of the large support moment along A-B. This results in a large region that includes an area with a moment equal to or close to zero.

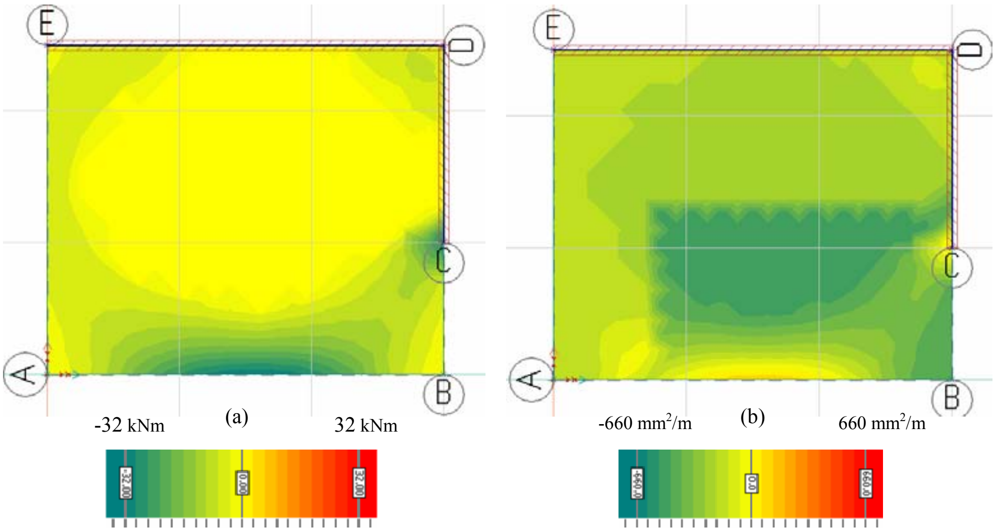
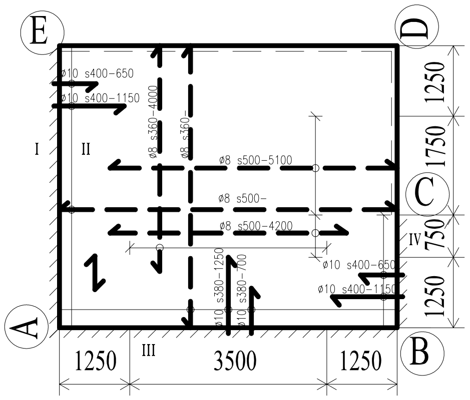


Figure 6.9: *Distribution of moment (a) and missing reinforcement (b) – top Y.*

6.1.2 Comparison with Hand Calculation

Figure 6.10 shows the reinforcement distribution resulting from a simple hand calculation based on the standard method. The calculation is presented in Appendix C.

- Support I-II: Φ10 s200
- Support III-II: Φ10 s190
- Support IV-II: Φ10 s200
- Bay area, x-direction: Φ8 s250
- Bay area, y-direction: Φ8 s180



C 20/25	f_{cck}	19 MPa
	f_{ctk}	1.45 MPa
Ps 500	f_{stk}	500 MPa
Slab thickness	t	0.160 m
Load	q	16.3 kN/m^2

Figure 6.10: *Resulting reinforcement distribution based on a hand calculation.*

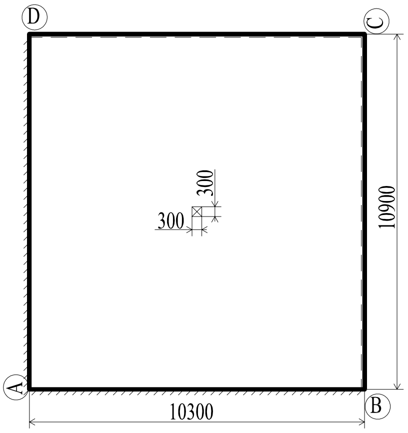
6.1.3 Comments

As can be seen in Figure 6.7, a large part of region 1 is over-reinforced. This phenomenon can be attributed to the current way of creating regions. The smallest moment, along edge A-B, is larger than the capacity set for the concrete. This means that one continuous region starting form edge E-A, via A-B, to the highest moment at point C is created. The coordinates of the region are thereafter extracted. These are the largest and smallest x and y values (chapter 5.1.10 – *AreaCheck)*. The largest x value is on edge B-D, the largest y value on edge E-D. This means that the whole slab will be one region and the centre of the slab is therefore over-reinforced. In Figure 6.6, regions 2 and 3 cover half the maximum moment and therefore have wider spacing. Region 4 is a separate region originally inside region 1. It therefore has separate reinforcement.

The reinforcement distribution in the bottom of the slab calculated by hand, Figure 6.10, corresponds well with the distribution provided by the calculation routines, Figure 6.2 and Figure 6.4. However, when comparing the reinforcement in the top of the slab, Figure 6.10 with Figure 6.6 and Figure 6.8, the difference is substantial. This is due to the singularity problem which gives a high moment at C, Figure 6.7 and Figure 6.9. This high moment requires a larger amount of reinforcement. This, combined with the division into moment regions which in this case is clearly faulty, gives a highly over-reinforced reinforcement suggestion, Figure 6.7 and Figure 6.9. The reinforcement in the intersection between edge C-D and D-E is also unnecessary. Due to the small value of this support moment, it is usually neglected.

6.2 Slab 2 – Simple and Fixed Supports with Centre Column

Slab 2 is simply supported along B-C and C-D. The slab is fully fixed along edge A-B and D-A. It is also supported by a column, see Figure 6.11.



C 25/30	f_{cck} f_{ctk}	24 MPa 1.70 MPa
Ps 500	f_{stk}	500 MPa
Slab thickness	t	0.250 m
Load		8 kN/m^2

Figure 6.11: *Slab 2 – geometry and support conditions.*

6.2.1 Results

Bottom X – As seen in Figure 6.12 the reinforcement is divided into two regions:

- region 1 - θ 8 s100
- region 2 - θ 8 s200

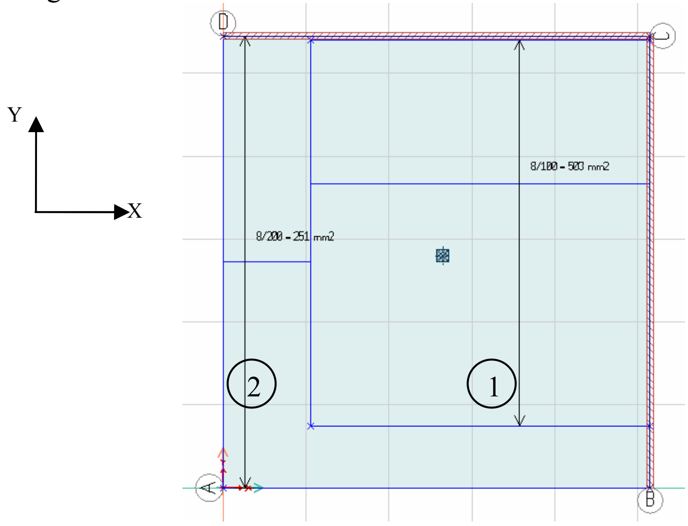


Figure 6.12: *Distribution of reinforcement, bottom X.*

The area around the column has no bay moment which means that there is a patch surrounding the column that is over-reinforced with an excess of reinforcement of approximately 500 mm^2/m. The amount of reinforcement is governed by the largest bay moment which is to the right of the column. As the moment areas around the column are attached the whole area surrounding the column is given the same amount of reinforcement.

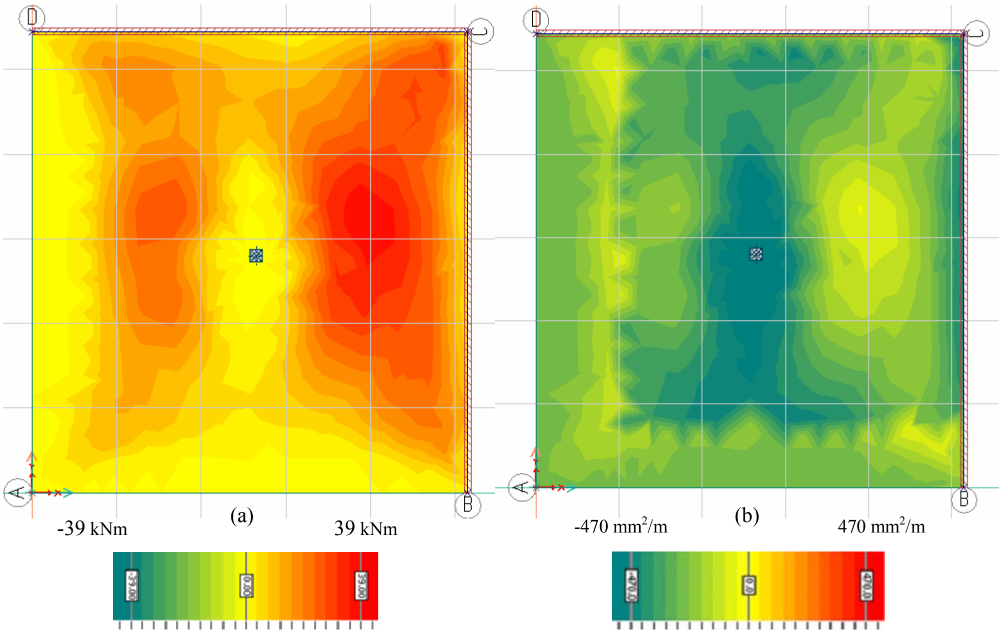


Figure 6.13: *Distribution of moment (a) and missing reinforcement (b) – bottom X.*

Bottom Y –As seen in Figure 6.14 the reinforcement is divided into five regions:

- region 1 - θ 8 s100
- region 2 - θ 8 s250
- region 3 - θ 8 s150
- region 4 - θ 8 s350
- region 5 - θ 8 s200

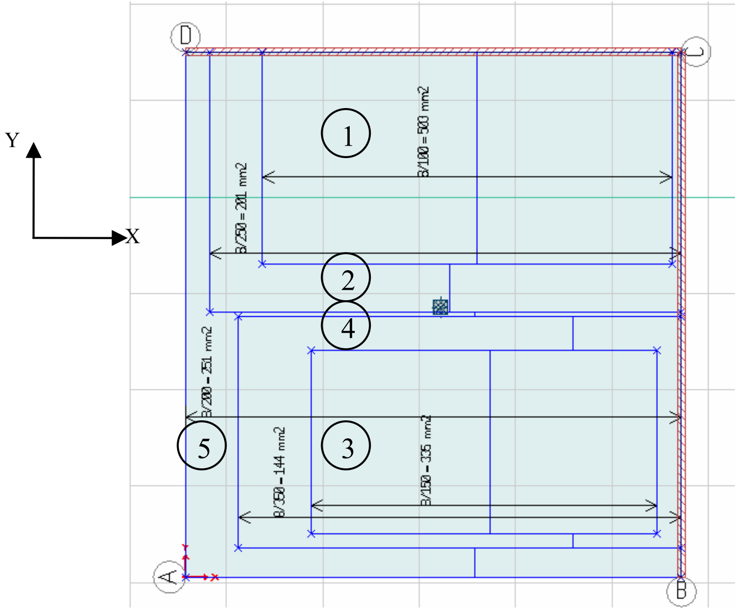


Figure 6.14: *Distribution of reinforcement, bottom Y.*

Region 1 covers the largest bay moment. Since the moment is smaller in the corresponding region 3, the reinforcement in this region can be placed with wider spacing. Around these tightly reinforced regions there are two smaller regions, regions 2 and 4, with wider spacing and lastly the minimum reinforcement, region 5.

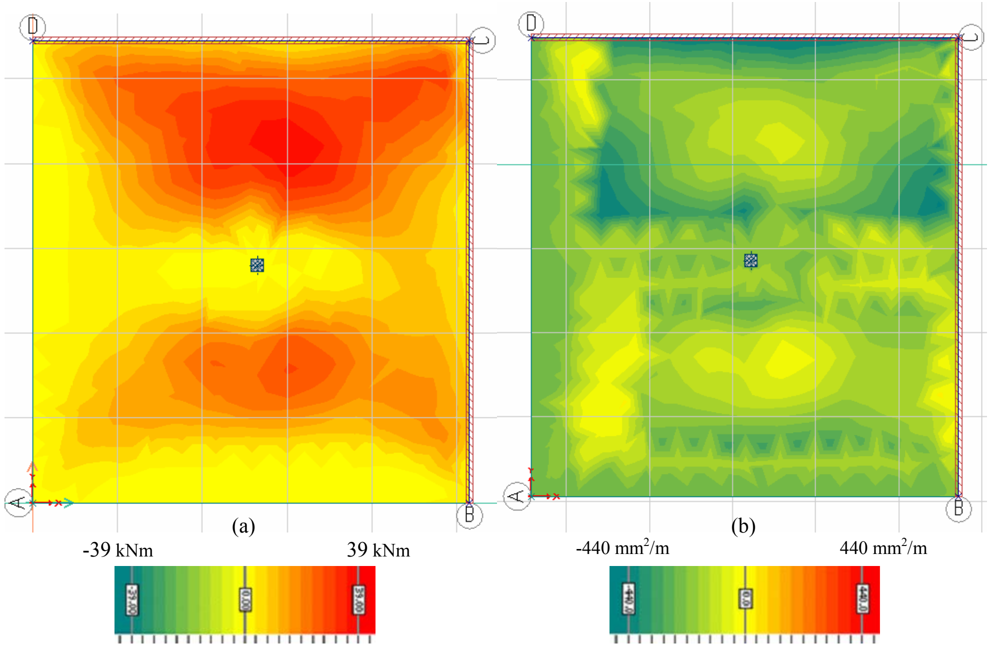


Figure 6.15: *Distribution of moment (a) and missing reinforcement (b) – bottom Y.*

Top X – As seen in Figure 6.16 the reinforcement is divided into five regions:

- region 1 - θ 16 s160
- region 2 - θ 12 s200
- region 3 - θ 10 s150
- region 4 - θ 10 s300
- region 5 - θ 10 s500

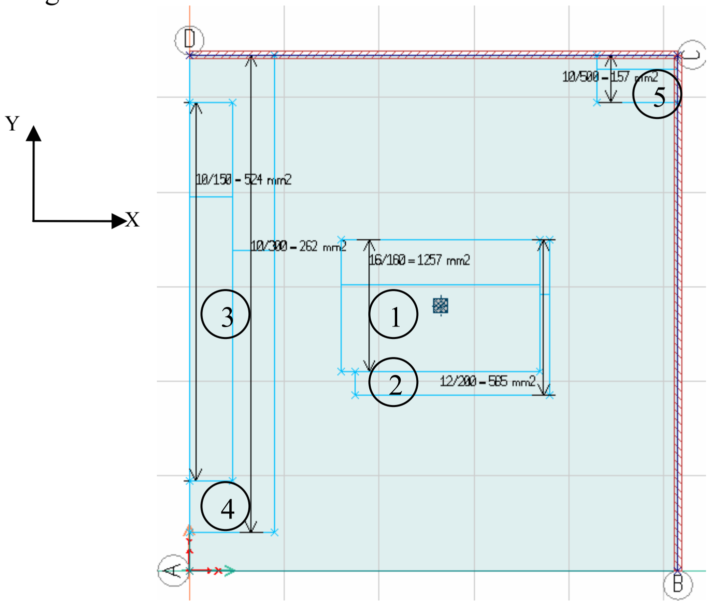


Figure 6.16: *Distribution of reinforcement, top X.*

The reinforcement above the column is divided into two types of reinforcement. One deals with the bending moment and the other with the reaction forces that can cause column punching if the capacity in the slab is insufficient. Region 1 defines the reinforcement due to column punching and includes the bending reinforcement. The surrounding region, region 2, is only due to bending. Regions 3 and 4 handle the support moment along edge D-A. This shows the overlapping built into the routines so as to reduce the amount of reinforcement. It is obvious that the area above the column is over-reinforced. There is an excess of reinforcement of about 1257 mm^2/m in the far edges of region 1. This reinforcement is necessary for punching reasons.

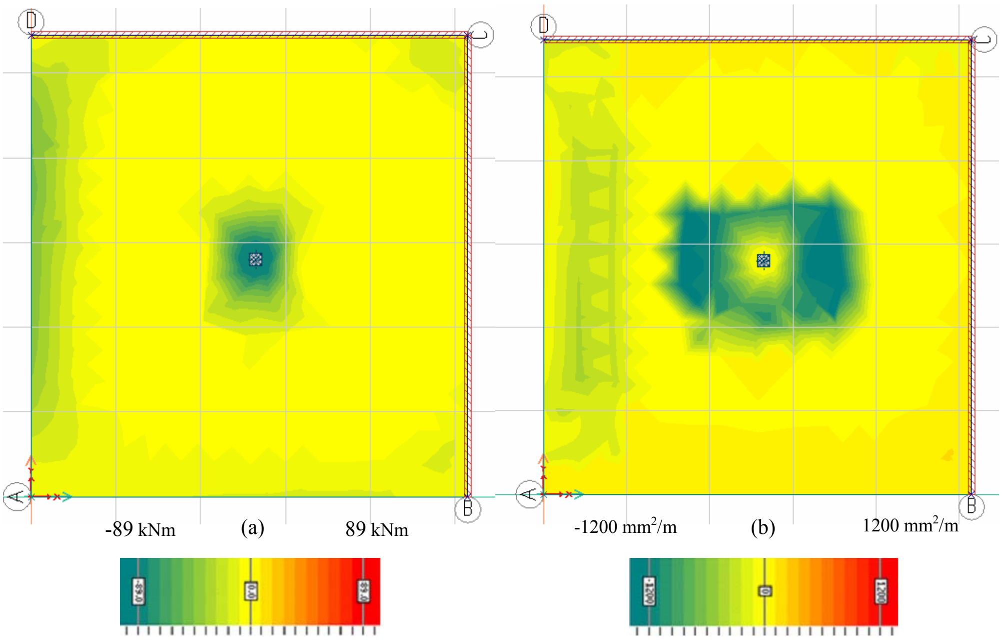


Figure 6.17: *Distribution of moment (a) and missing reinforcement (b) – top X.*

Top Y – As seen in Figure 6.18 the reinforcement is divided into five regions:

- region 1 - θ 16 s170
- region 2 - θ 12 s200
- region 3 - θ 12 s200
- region 4 - θ 10 s200
- region 5 - θ 10 s400
- region 6 - θ 10 s500

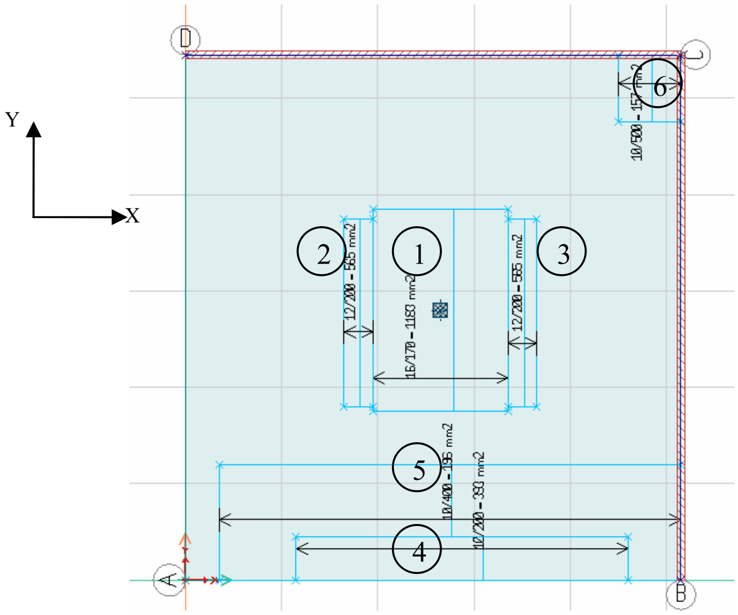


Figure 6.18: *Distribution of reinforcement, top Y.*

As in the previous case, region 1 handles both bending moment and punching forces. The region is surrounded by two smaller regions which handle the remainder of the bending moment. There is an excess of reinforcement of about 1183 mm^2/m in the far edges of region 1 which again is necessary for punching reasons.

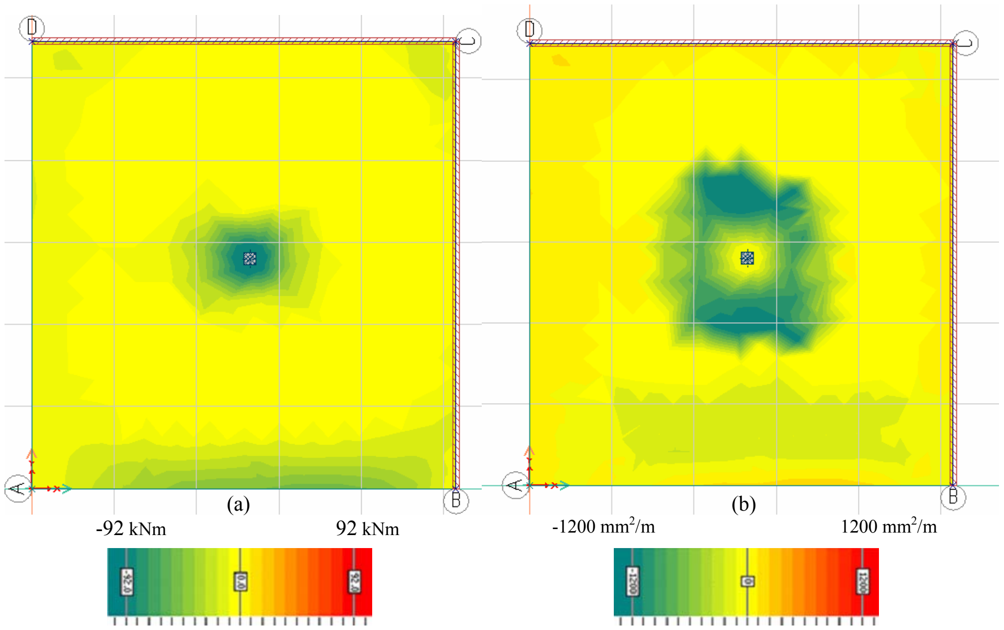


Figure 6.19: *Distribution of moment (a) and missing reinforcement (b) – top Y.*

6.2.2 Comments

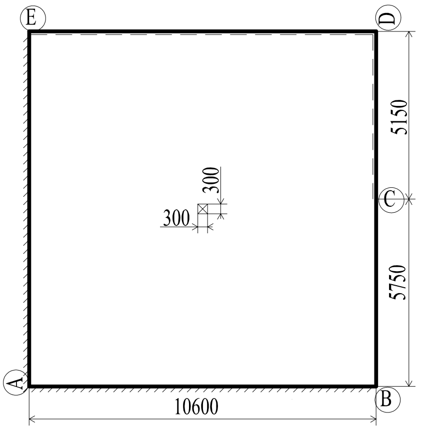
The difference in reinforcement distribution between *bottom X*, Figure 6.12, and *bottom Y*, Figure 6.14, may seem somewhat illogical as the slab is more or less identical in the x and y direction. This difference can be attributed to the process of creating moment groups. When studying Figure 6.13 (a) and Figure 6.15 (a), both figures show two distinct areas with large bay moments. When creating moment regions the moments smaller than the concrete capacity are neglected, thus creating groups. In this case, the moments connecting the two areas with large bay moments in *bottom X*, Figure 6.13 (a), are larger than the concrete capacity. This results in one large moment group that covers the column. In the case of *bottom Y*, Figure 6.15 (a), the moments connecting the two areas with larger bay moments are smaller than the concrete capacity, and two separate moment groups are created on each side of the column.

In *slab 2, bottom Y,* Figure 6.14, there is a strip of minimum reinforcement only 0.1 meters wide, dividing region 2 and 4. It is obvious that special reinforcement would not be placed within a strip that thin. The same can be said for the strip between region 1 and the right edge of the slab. This strip is only 0.2 meters wide and the reinforcement would most likely be extended to the edge of the slab.

Region 5 in *top X,* Figure 6.16, and region 6 in *top Y,* Figure 6.18, are due to a small support moment at the intersection of edges B-C and C-D. In reality, this reinforcement area is unnecessary. However, it is shown in the figures as it is given as a result from the calculations.

6.3 Slab 3 – Simple and Fixed Supports with Free Edge and Centre Column

Slab 3 is simply supported along edges C-D and D-E and fully fixed along edges A-B and E-A. Line B-C is a free edge. It is also supported by a column, see Figure 6.20.



C 28/35	f_{cck} f_{ctk}	27 MPa 1.80 MPa
Ps 500	f_{stk}	500 MPa
Slab thickness	t	0.250 m
Load		8 kN/m^2

Figure 6.20: *Slab 3 – geometry and support conditions.*

6.3.1 Results

Bottom X - As seen in Figure 6.21 the reinforcement is divided into five regions:

- region 1 - θ 8 s150
- region 2 - θ 8 s300
- region 3 - θ 8 s100
- region 4 - θ 8 s200
- region 5 - θ 8 s200

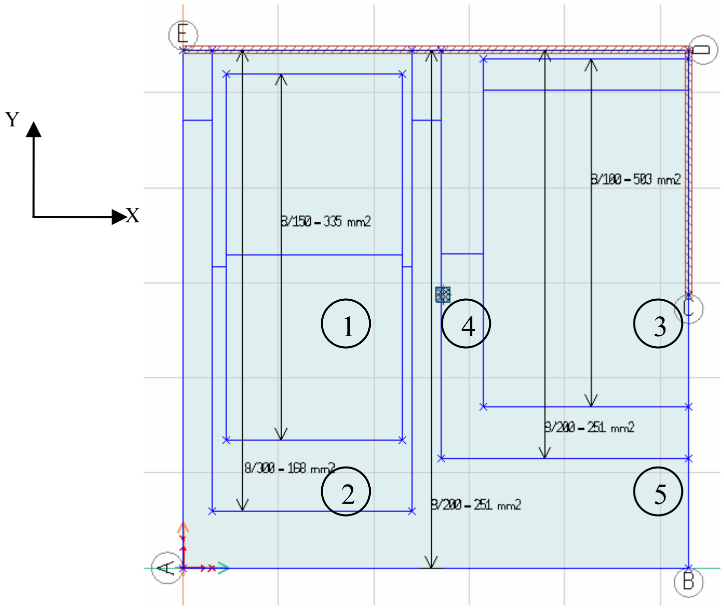


Figure 6.21: *Distribution of reinforcement, bottom X.*

The distance between the reinforcement bars is smaller in region 3 and 4 than in region 1 and 2 which is logical considering the moment distribution. It can also be seen that the free edge has virtually no effect on the required reinforcement in the x direction.

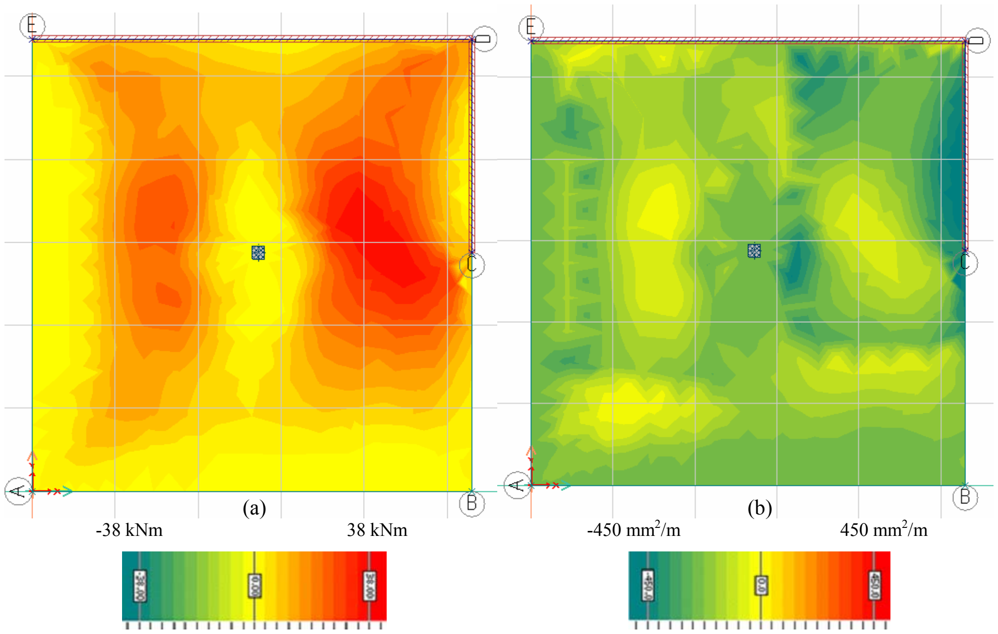


Figure 6.22: *Distribution of moment (a) and missing reinforcement (b) – bottom X.*

Bottom Y – As seen in Figure 6.23 the reinforcement is divided into six regions:

- region 1 - θ 8 s100
- region 2 - θ 8 s250
- region 3 - θ 8 s100
- region 4 - θ 8 s200
- region 5 - θ 8 s200
- region 6 - θ 12 s100

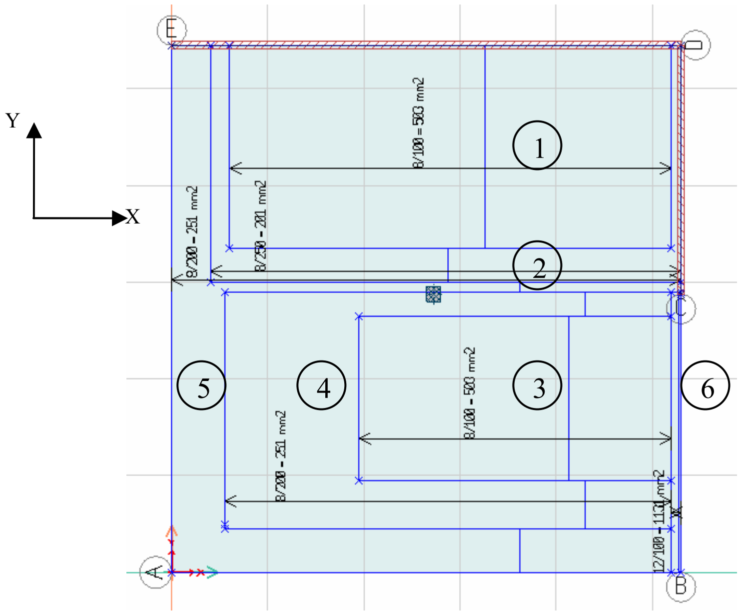


Figure 6.23: *Distribution of reinforcement, bottom Y.*

As in *bottom X* there are two reinforcement regions on each side of the column. And again there is reinforcement placed with smaller spacing in region 3 and 4 corresponding to the largest moment. There is a significant difference in the bay moment of slab 3, Figure 6.22 (a), compared to that of slab 2, Figure 6.15 (a). This is due to the free edge, region 6. The edge has therefore been additionally reinforced according to chapter 5.4 – *Free Edge*.

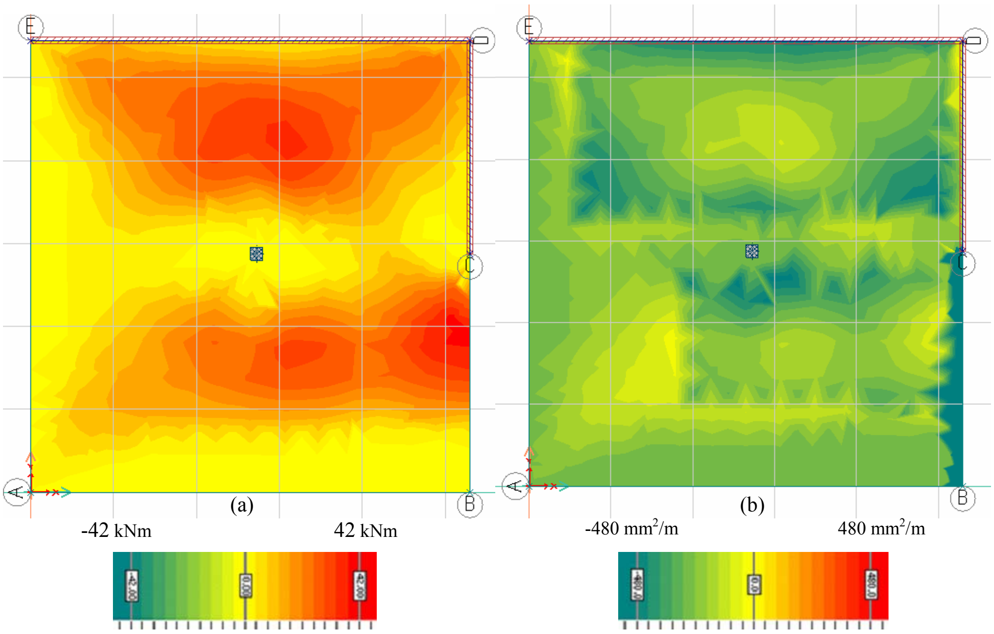


Figure 6.24: *Distribution of moment (a) and missing reinforcement (b) – bottom Y.*

Top X – As seen in Figure 6.25 the reinforcement is divided into nine regions:

- region 1 - θ 16 s170
- region 2 - θ 12 s200
- region 3 - θ 12 s200
- region 4 - θ 10 s150
- region 5 - θ 10 s300
- region 6 - θ 10 s150
- region 7 - θ 10 s300
- region 8 - θ 10 s350
- region 9 - θ 10 s500

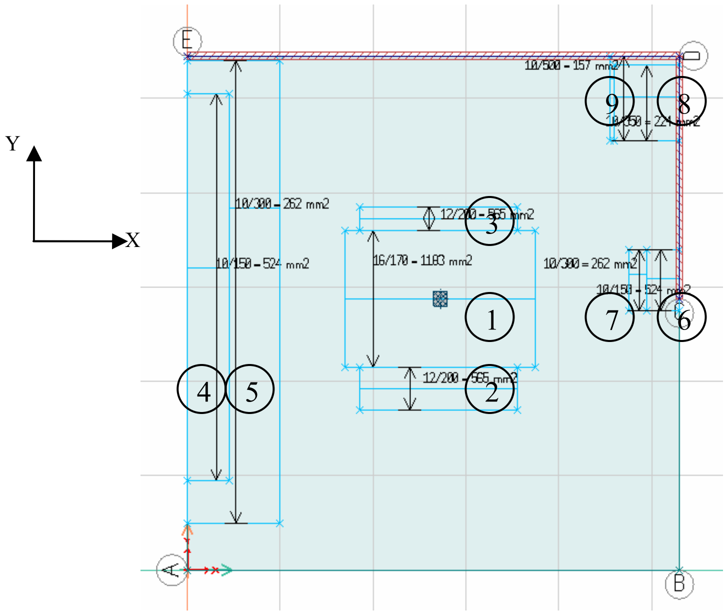


Figure 6.25: *Distribution of reinforcement, top X.*

The distribution of the reinforcement is largely the same as for *slab 2*, Figure 6.16. The largest difference is the addition of regions 6 and 7. The origin of these regions is the singularity problem discussed in chapter 3.3 – *Peak Smoothing*, which occurs in point C. Again, there are two small reinforcement regions, regions 8 and 9, in the top right corner due to the intersection of the walls.

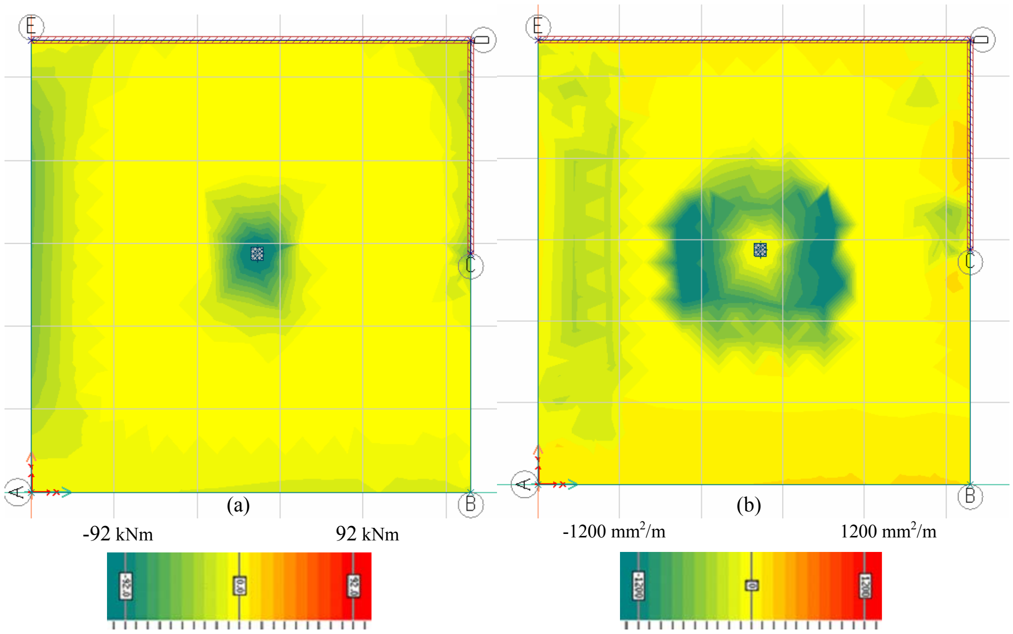


Figure 6.26: *Distribution of moment (a) and missing reinforcement (b) – top X.*

Top Y – As seen in Figure 6.27 the reinforcement is divided into nine regions:

- region 1 - θ 12 s200
- region 2 - θ 16 s180
- region 3 - θ 12 s200
- region 4 - θ 10 s100
- region 5 - θ 10 s200
- region 6 - θ 12 s100
- region 7 - θ 12 s100
- region 8 - θ 12 s200
- region 9 - θ 12 s100

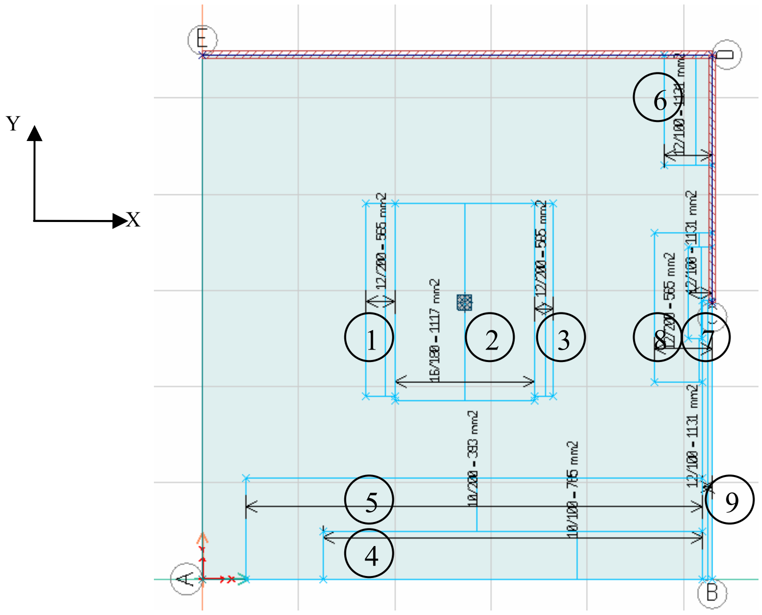


Figure 6.27: *Distribution of reinforcement, top Y.*

There is a clear difference in the amount of reinforcement in *top Y* compared to *top X*. The free edge gives the addition of region 9, a region that can not be motivated out of a moment perspective but that is placed there none the less IW [11]. Regions 7 and 8 are needed as a consequence of the singularity problem discussed earlier. The reinforcement due to column punching divides the bending reinforcement into two regions, regions 1 and 3.

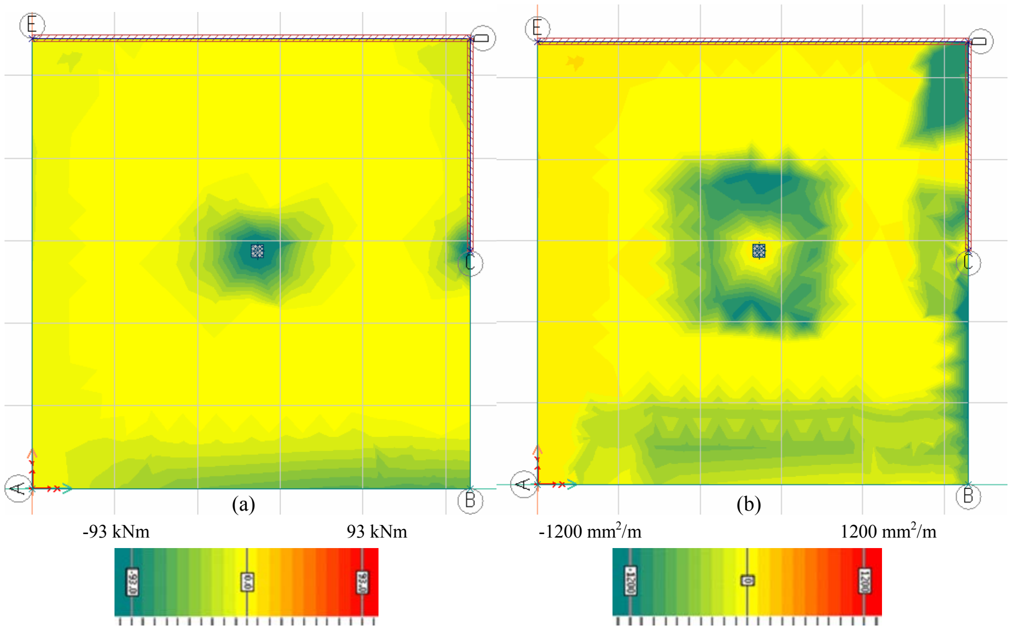


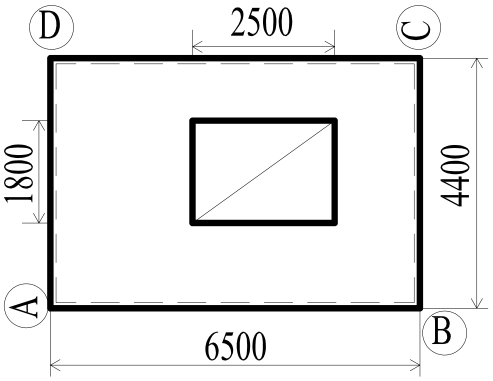
Figure 6.28: *Distribution of moment (a) and missing reinforcement (b) – top Y.*

6.3.2 Comments

Although *slab 2*, Figure 6.11, and *slab 3*, Figure 6.20, are similar in appearance, the variation in reinforcement distribution is very large. This can be seen when comparing the reinforcement for *bottom X*, Figure 6.12 and Figure 6.21. The variation is due to the difference in the quality, and therefore the tensile capacity, of the concrete. The quality of the concrete is higher in *slab 3* which means that the capacity is higher. In *slab 2*, Figure 6.12, one reinforcement area covers the whole column whereas in *slab 3*, Figure 6.21, there is a division into two areas, one on the left side of the column and one on the right side. When comparing the moments in *bottom X* for slabs 2 and 3, Figure 6.13 (a) and Figure 6.22 (a), two distinct areas with high bay moments can be seen in both figures. These moment areas are in both cases connected by smaller moments in reality creating one large moment group surrounding the column. However, as the concrete capacity is higher than the moment connecting these two moment groups in *slab*, *3* Figure 6.22 (a), two separate moment areas are created resulting in a more complicated reinforcement distribution. In the case of *slab 2*, Figure 6.13 (a), the smallest moments surrounding the column and connecting the moment areas, exceed the tensile capacity of the concrete, and therefore only one reinforcement region is created.

6.4 Slab 4- Simple Supports and Hole

Slab 4 is a slab with a hole simply supported along A-B, B-C, C-D and D-A, Figure 6.29.



C 25/30	f_{cck} f_{ctk}	24 MPa 1.70 MPa
Ps 500	f_{stk}	500 MPa
Slab thickness	t	0.200 m
Load		9 kN/m^2

Figure 6.29: *Slab 4 – geometry and support conditions.*

6.4.1 Results

Bottom X – As seen in Figure 6.30 the reinforcement is divided into eight regions:

- region 1 - θ 8 s200
- region 2 - θ 12 s100
- region 3 - θ 8 s200
- region 4 - θ 8 s100
- region 5 - θ 8 s100
- region 6 - θ 8 s200
- region 7 - θ 12 s100
- region 8 - θ 8 s200

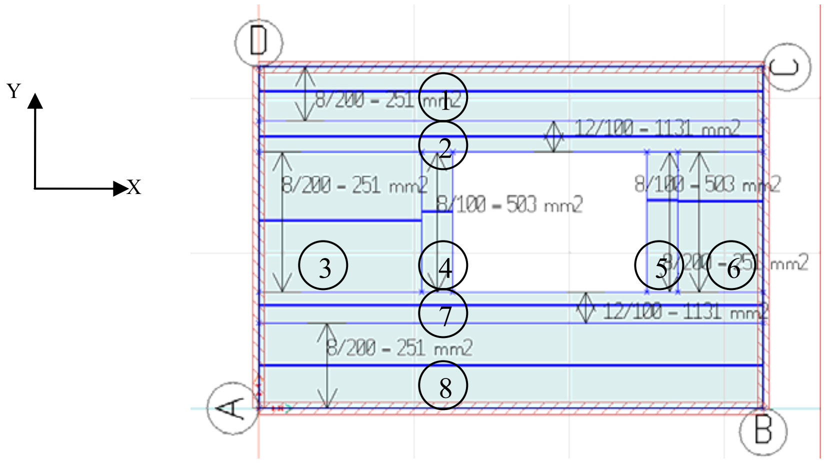


Figure 6.30: *Distribution of reinforcement, bottom X.*

Since there is a hole in the slab, there is special reinforcement placed in strips in both the x and y direction. As was explained in chapter 5.3.1 - *HoleReinforcement*, the reinforcement bars used are always 2 θ 12 s100 provided that the necessity is lower than the capacity of this reinforcement. In this case this reinforcement is represented by regions 2 and 7.

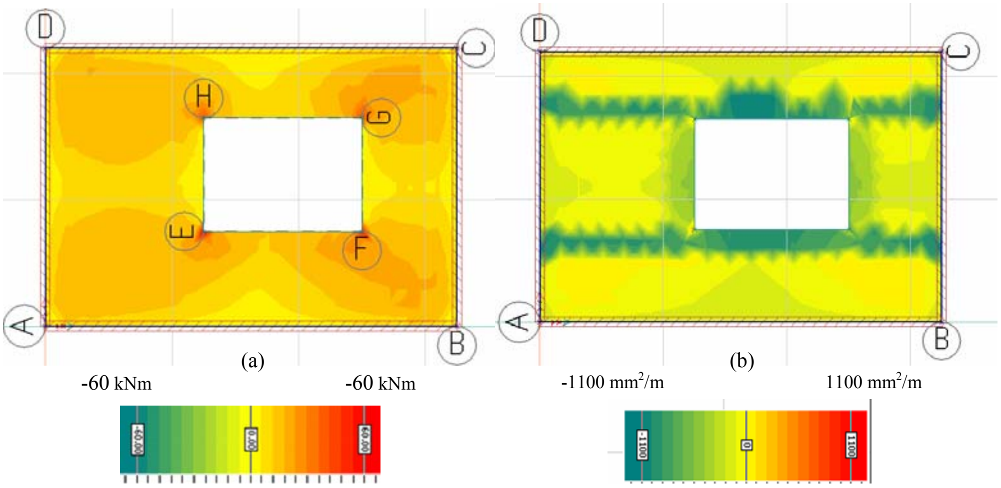


Figure 6.31: *Distribution of moment (a) and missing reinforcement (b) – bottom X.*

Bottom Y – As seen in Figure 6.32 the reinforcement is divided into eight regions:

- region 1 - θ 8 s150
- region 2 - θ 12 s100
- region 3 - θ 8 s150
- region 4 - θ 10 s100
- region 5 - θ 10 s100
- region 6 - θ 8 s150
- region 7 - θ 12 s100
- region 8 - θ 8 s150

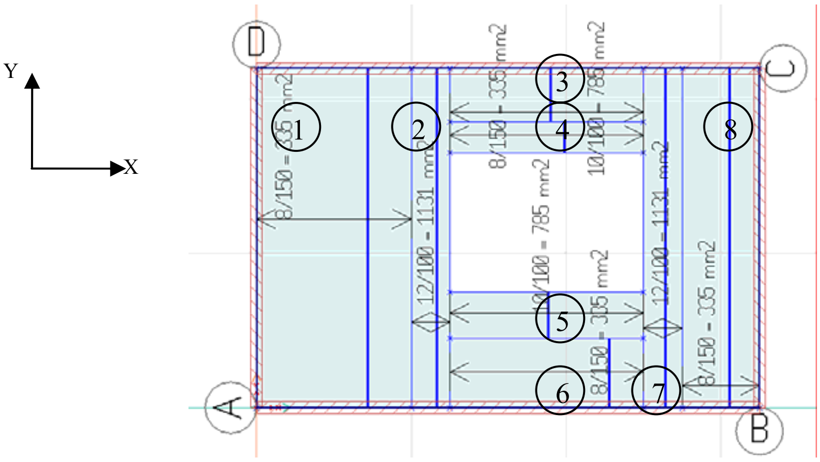


Figure 6.32: *Distribution of reinforcement, bottom Y.*

Regions 2 and 7 consist of 2 θ 12 s100 each due to the hole.

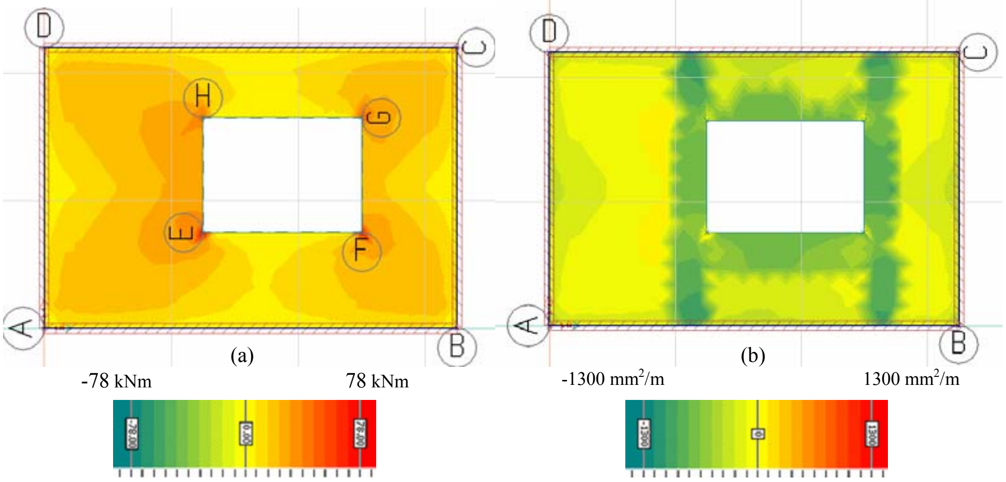

(a) (b)

Figure 6.33: *Distribution of moment (a) and missing reinforcement (b) – bottom Y.*

Top X – As seen in Figure 6.34 the reinforcement is divided into six regions:

- region 1 - θ 10 s400
- region 2 - θ 10 s350
- region 3 - θ 12 s100
- region 4 - θ 12 s100
- region 5 - θ 10 s400
- region 6 - θ 10 s400

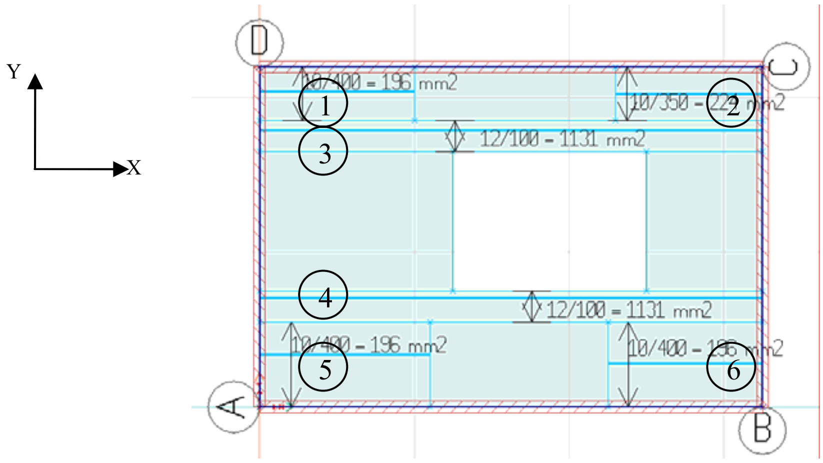


Figure 6.34: *Distribution of reinforcement, top X.*

Regions 3 and 4 consist of 2 θ 12 s100 each due to the hole. The illustration of the missing reinforcement in Figure 6.35, is somewhat misleading. The orange area, symbolizing the missing reinforcement, amounts to approximately 60 mm^2/m which originates from a small moment. The capacity of the concrete is sufficient to withstand this moment.

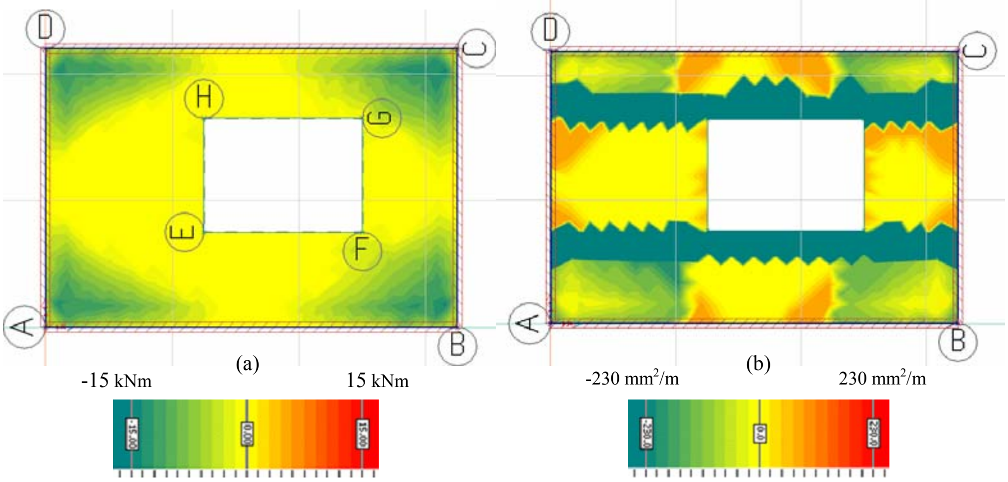


Figure 6.35: *Distribution of moment (a) and missing reinforcement (b) – top X.*

Top Y – As seen in Figure 6.36 the reinforcement is divided into seven regions:

- region 1 - θ 10 s400
- region 2 - θ 10 s400
- region 3 - θ 12 s100
- region 4 - θ 10 s400
- region 5 - θ 12 s100
- region 6 - θ 10 s400
- region 7 - θ 10 s400

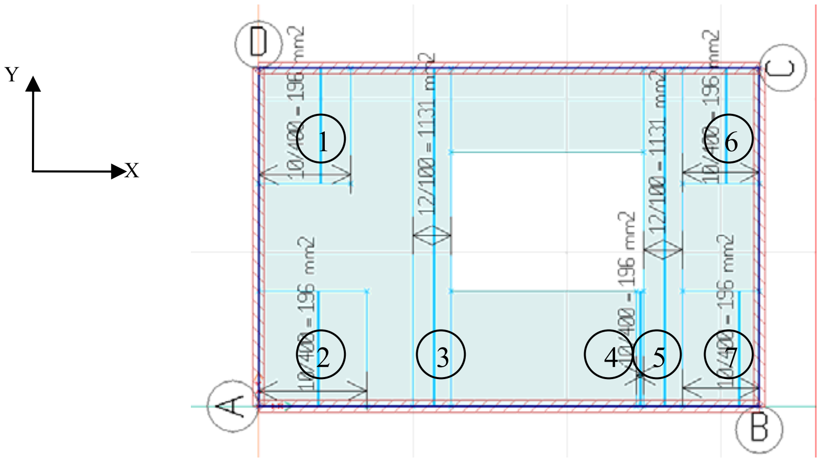


Figure 6.36: *Distribution of reinforcement, top Y.*

The distribution of the reinforcement is basically the same as in the *top X* direction, Figure 6.34.

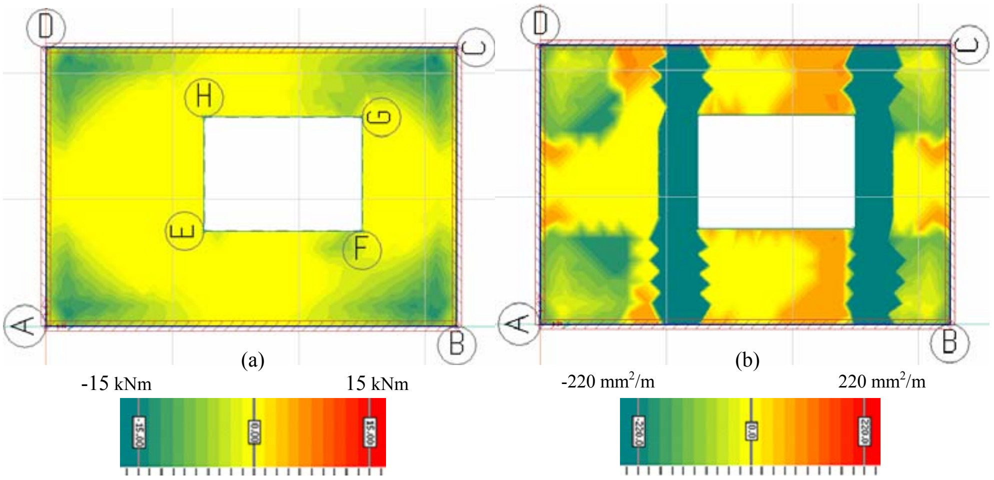


Figure 6.37: *Distribution of moment (a) and missing reinforcement (b) – top Y.*

6.4.2 Comments

The hole presented in this case is clearly a large hole. Still, the routines developed in chapter 5.3 – *Hole* were used to analyze it. According to NC [1] 6.1, the effect of a large hole is negligible on a distance of 1.5 times the width (height) of the hole. This requirement has not been taken into account which results in an overestimation of the concretes capacity nearest to the hole. However, this effect is negligible for small holes. The result presented is therefore not entirely correct but is still presented to show how the routine will work when dealing with a slab with a hole.

6.5 Discussion

Minimum Reinforcement

In the design practice, there is always a minimum reinforcement placed in the bottom of the slab even when it is not required according to the calculations. The reason for the use of minimum reinforcement is to increase the ductility of the concrete. The amount of reinforcement used in the bottom of the slab varies depending on the engineer responsible for the design and the specific building. Therefore the user should be able to define the minimum reinforcement as bar dimension and spacing distance, for x and y direction separately. In the routines used, the minimum reinforcement is assumed to be the same in both directions.

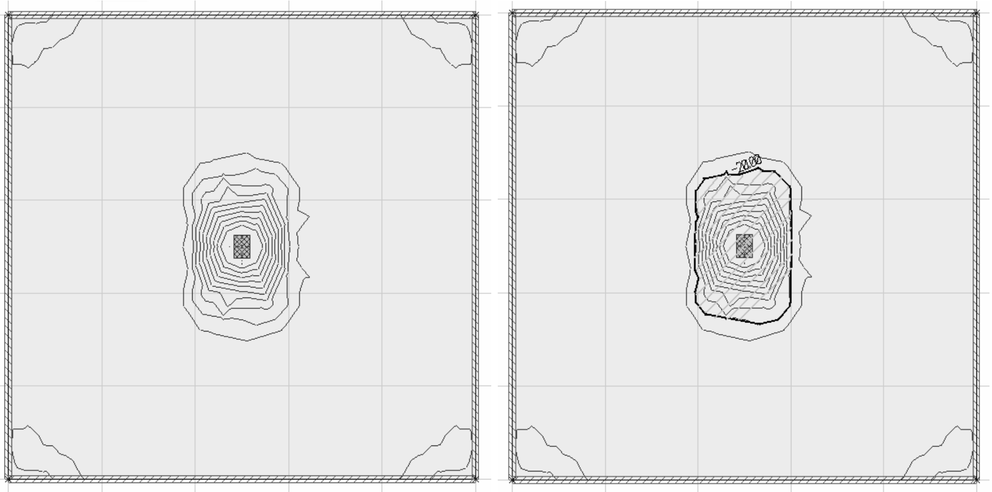
The defined / default minimum reinforcement is currently not compared to the calculated minimum reinforcement required due to the bay moment. The calculated minimum reinforcement should be compared to the defined / default minimum reinforcement, and the largest of these should be chosen as reinforcement in the slab.
Note: This comparison has been added in the flow chart.

Creating Moment Groups

The current way of creating moment groups can in some cases form unnecessarily large groups. An example of this is a slab supported by many columns. The current way of creating groups, based on the capacity of the concrete, can lead to a connection between the support moments of the different columns, forming one large moment group covering the entire slab. This means that the amount of reinforcement above all columns would be the same. If there are large differences in the support moments above the columns, some of these would be over-reinforced. This problem can be avoided if each column was represented by its own group. A way of doing this would be to consider the largest moment appearing over the columns. All neighbours to the element containing the largest moment are controlled. The elements containing a smaller moment are stored in the same group. This procedure continues until the moment in the neighbouring elements starts to increase. A group has now been formed, linked only to the current column. The procedure starts over again, ignoring the elements already sorted into a group. Using this sorting procedure, every column would be given its own moment group and the necessary reinforcement would be more adapted to the requirements of each column.

Creating Sub-groups

When defining the sub-groups, chapter 5.1.3 - *ReinforcementAreas*, their boundaries are determined using node moments. This may seem strange as the design of the reinforcement is done using the mean moment. The reason for this is that the node moment provides a more accurate value when defining a boundary. Figure 6.38 shows a slab supported by a column. The contour lines illustrate the support node moments. In this case, the 20 kNm line represents half the maximum moment. This means that the boundary for sub-group 1 will follow the 20 kNm contour line. The use of the node moment gives a well defined boundary. However, the largest node moment of an element is not a representative value of the moment for the entire element. The mean moment on the other hand is a weighted value of the node moments of an element. Therefore, the mean moment is used to calculate the necessary reinforcement, but not to define the sub-groups as this would result in a far to rough sub-group boundary.



(a) (b)

Figure 6.38: *In (a), the contour lines for the support moment in the x direction are shown. (b) shows the 20 kNm boundary.*

Creating Reinforcement Regions

When creating reinforcement regions, these are based on the moment distribution. A more efficient alternative would be to create regions dependant on a pre-defined difference in the spacing of the bars. The user could for example define that the switch between regions is made every 100 mm. This would mean that if there is a region that demands bars spaced with 150 mm the next region would be created when the spacing required was 250 mm. This would give a result more adapted to the practical and economical demands on a construction site. However, this does not necessarily solve the problem of small distances between the edges of two regions. A way of handling this would be to define the smallest distance acceptable between two regions.

Reinforcement over Supports

In order to decrease the amount of reinforcement used over supports and along fixed edges, every second bar length is made shorter. The longer bars cover the entire moment area while the shorter bars cover a smaller region where the amount of reinforcement is based on a larger moment. The criterion governing this is that the inner area should contain twice the amount of reinforcement as the outer, as discussed in chapter 5.1.3 - *ReinforcementAreas*. The result of this is that bars with the same dimensions, but different lengths, are placed next to each other. The handling is simplified as only two bar lengths are used and the margin of error on the construction site is reduced as only one bar dimension is used. A second reason for the use of two lengths is to avoid crack initiation. The ending of all reinforcement in the same place leaves a defined boundary which is avoided when using two lengths. The same applies to reinforcement over large regions.

Bar Length / Dimension / Spacing

To minimize the waste when cutting the bars into the calculated lengths, the lengths could be adapted to the length of the uncut reinforcement bars. If the original reinforcement bars have a length of 12 meters, acceptable lengths for the reinforcement regions would be 6, 4, 3 meters and so on. This has not been done in the current routines. All resulting lengths of the reinforcement bars are considered acceptable in the routine.

To simplify the reinforcement process and minimize the risk of errors on the construction site, variation in bar dimension and bar spacing should be kept to a minimum. This is something the routine does not consider as can be seen in the cases above. These parameters can currently vary freely which results in as many as three different bar dimensions in the same direction. If only one bar dimension was used, as was requested in the interviews, it would result in a very rough reinforcement distribution. Therefore, the design engineer should have the option of using only one bar dimension, but it should not be a default value.

Anchorage

All coordinates are presented with the anchorage length included. This results in an overestimation of the structural reinforcement in FEM-Plate. This overestimation is accepted as it simplifies the direct transfer of the reinforcement layout to a reinforcement drawing. To achieve satisfactory anchorage length along the edges the reinforcement bars must be bent down. If the reinforcement length necessary would cross the edge of the slab, the reinforcement would have to be bent down at the edge and extended at the bottom of the slab. This would give the reinforcement the necessary anchorage. An example of the anchorage problem is shown in *slab 4 – Simple Supports and Hole*. For anchorage purposes the reinforcement would have to be given an additional extension. For the bottom reinforcement, FEM-Plate would give an addition to the capacity in the top and vice versa. Therefore, this addition has been omitted when drawing the reinforcement in FEM-Plate. However, this information is important for anchorage reasons and should therefore be stored and presented. The calculations are performed for anchorage consisting of straight bars only. No consideration has been taken to anchorage by the edges.

A second aspect that has not been considered is the possibility of collisions between two reinforcement regions. This is a possible outcome if the distance between the two reinforcement regions is small. If the reinforcement was placed without the anchorage length included, there would be no problem. But when including the anchorage length, it may cover the strip dividing two regions, thus creating a reinforcement collision. Since the reinforcement is placed on the same depth in the slab this would not be practically possible, but on the other hand such a small strip between two areas would not be left without reinforcement on a construction site.

7 Concluding Remarks

Out of the four points defined in chapter 1.5 – *Limitations*, all have been dealt with apart from point 4, arbitrary geometries. This was mainly due to lack of time. However, the difference between an arbitrary geometry and a rectangular slab is, as we see it, negligible. The information about plate geometry, including slab edges, is already available in FEM-Plate in its existing state. Therefore if a calculation demands a comparison between a slab edge and coordinates of some sort it can easily be performed using the plate information. The placement of the reinforcement is more complicated as the routines developed require the slab geometry to be rectangular. If used on a slab with varying geometry, the reinforcement will inevitably be placed in regions outside the defined geometry. This does in reality not pose a problem in FEM-Plate, as the program automatically neglects reinforcement placed outside the slab. The current routines have been implemented on examples with arbitrary geometry with satisfactory results. However, no further study of this case has been performed and therefore the examples documented only include rectangular slabs.

The examples show a large variation in both reinforcement dimensions and spacing. It was earlier mentioned that production aspects often demand that this variation is minimized so that errors on the construction site can be avoided. The current division into reinforcement regions is in many cases unreasonable. If an engineer was to use the proposed reinforcement distribution, without any modification, the solution would be expensive, time consuming and clumsy as it would demand that the workers on the construction site be very precise. An alternative would be to propose a rough distribution which might render a result easier to overview. However, the objective of this thesis was never to eliminate the engineer, but instead give him a tool to help simplify the reinforcing process.

When studying the routines, some of the calculations and steps in these may seem clumsy and illogical. The limited time frame and the fact that the development of the routines was a secondary objective led to the use of, in a few cases, inefficient routines. This is mainly due to the reformulation of the prerequisites. During the course of the thesis, discussions and interviews led to a revision of the calculation process. But instead of rewriting the routines, which is time consuming, the existing routines were adapted accordingly. An example of such a routine is *InsideTheRegions*, where the node moment is used as a source of definition for moment areas. The thought behind this was to start from a graphic image of a logical reinforcement distribution, and to translate this into the computer. This way of sorting the moment has gradually changed. Time permitting; the routines would have been adapted. But as the time frame was limited, they have been left unchanged.

The engineers using pre-cast slabs in their structures expressed wishes for the ability to define a main direction of the reinforcement. As pre-cast slabs are heavily reinforced in one direction it is important for this reinforcement to have the largest effective height possible. For this procedure to work satisfactory the main direction would have to be defined by the user. This could be done in an advanced input mode. It could contain not only the main direction but also minimum and maximum bar distance (if these are to differ from the national code), if the results are to be presented as only one bar dimension or with varying dimensions and if the coordinates of the regions should be adapted to lengths evenly dividable from the maximum bar length.

During the work within this thesis it was seen that an active choice of reinforcement distribution was often required to render a reasonable result. The decisions and choices that an engineer makes are often dependant on the current case studied. With growing experience, it is easier to define an effective distribution. This experience is difficult to incorporate into routines and therefore, a more detailed proposal is preferred as it gives the engineer the freedom of choice. In this way, the program will be able to help the engineer in his work, allowing him to make active choices from a reasonable suggestion.

8 Bibliography

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9 Appendix

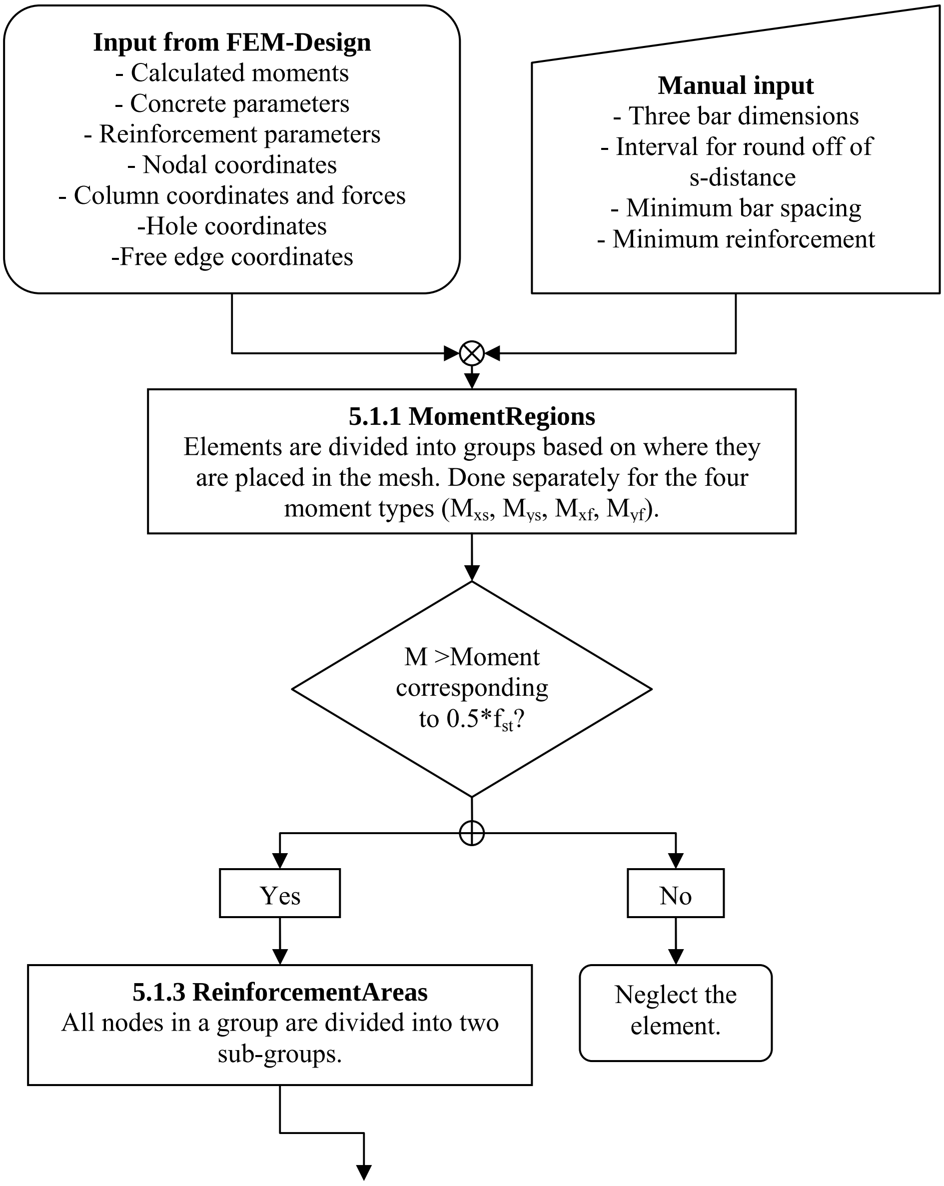
Appendix A - Flow Chart

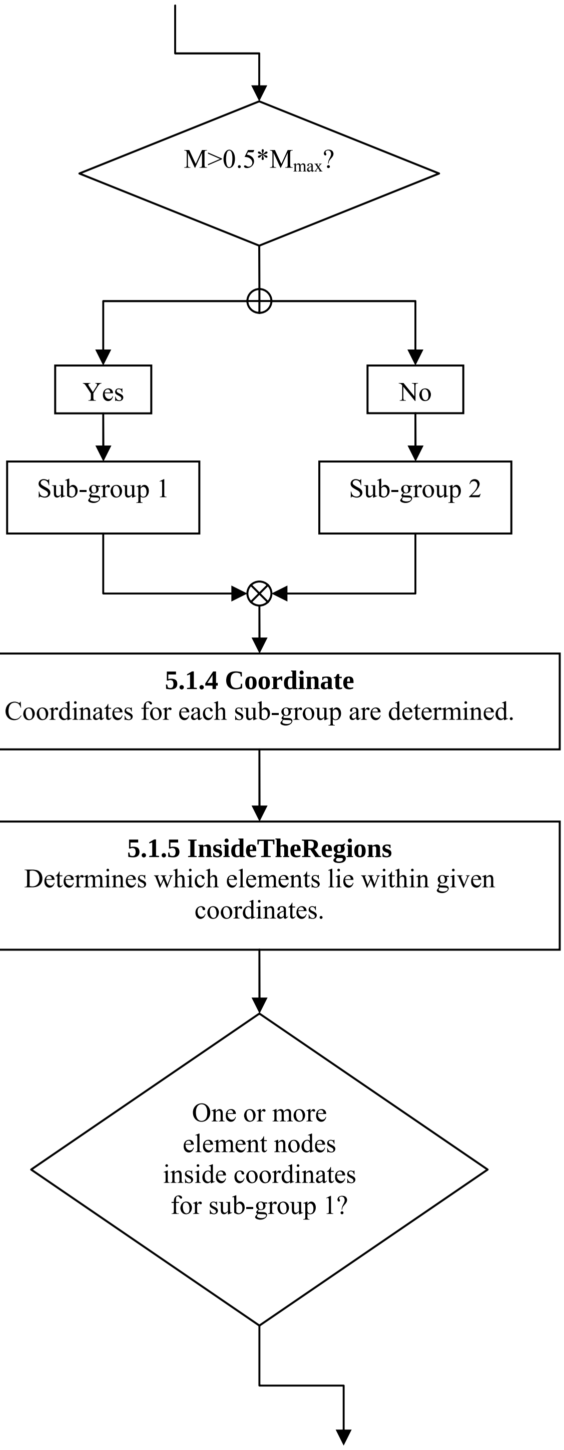
Appendix B – Determination of Node Placement

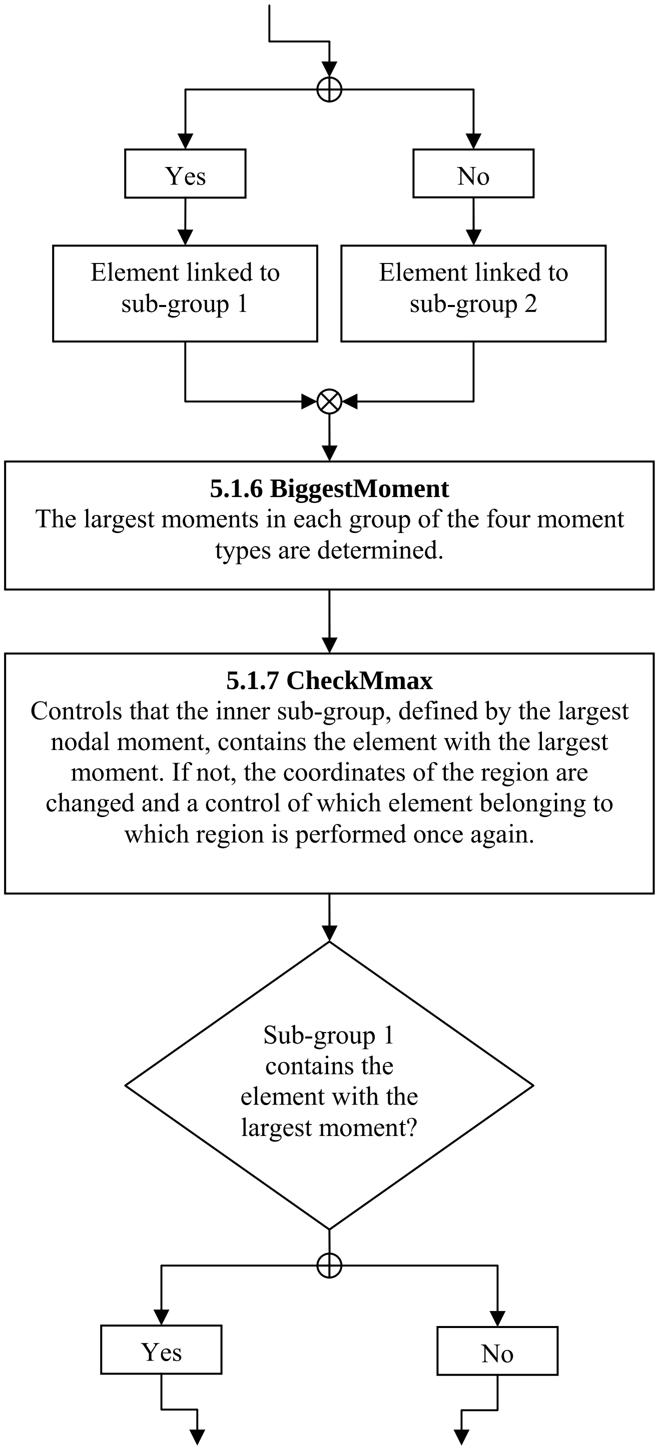
Appendix C – Presentation of Hand Calculations

9.1 Appendix A – Flow Chart

9.1.1 General Slab

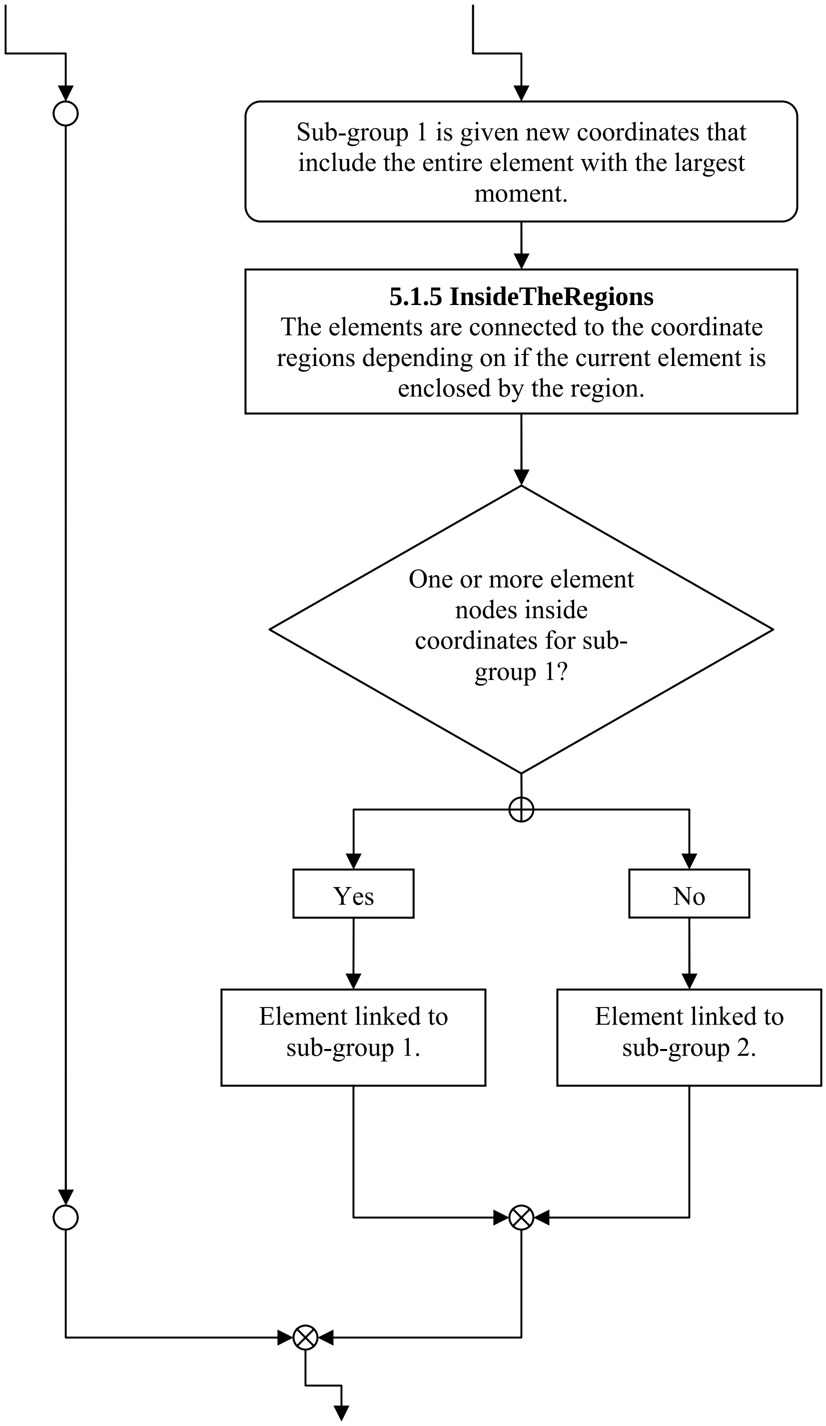


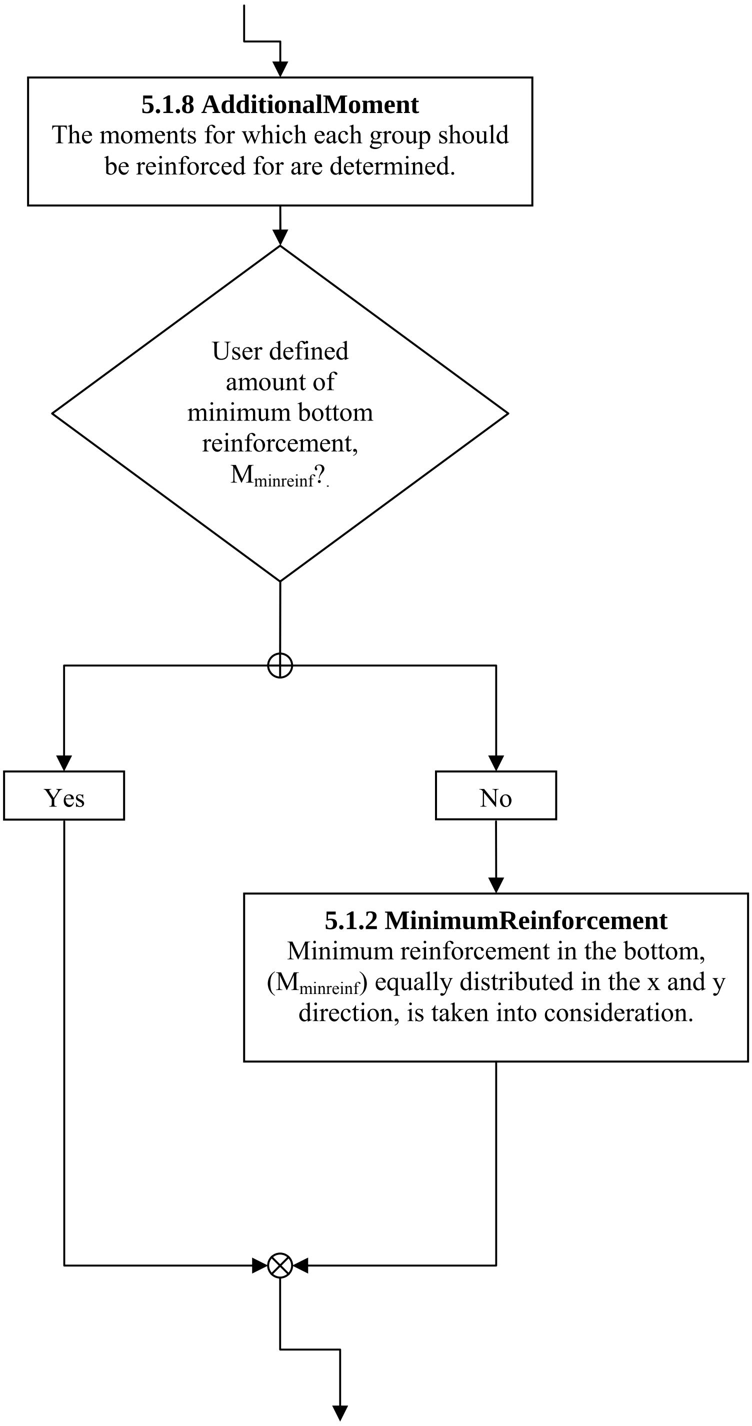

M>0.5*Mmax?
Yes
No
Sub-group 1
Sub-group 2
5.1.4 Coordinate
Coordinates for each sub-group are determined.
5.1.5 InsideTheRegions
Determines which elements lie within given coordinates.
One or more element nodes inside coordinates for sub-group 1?

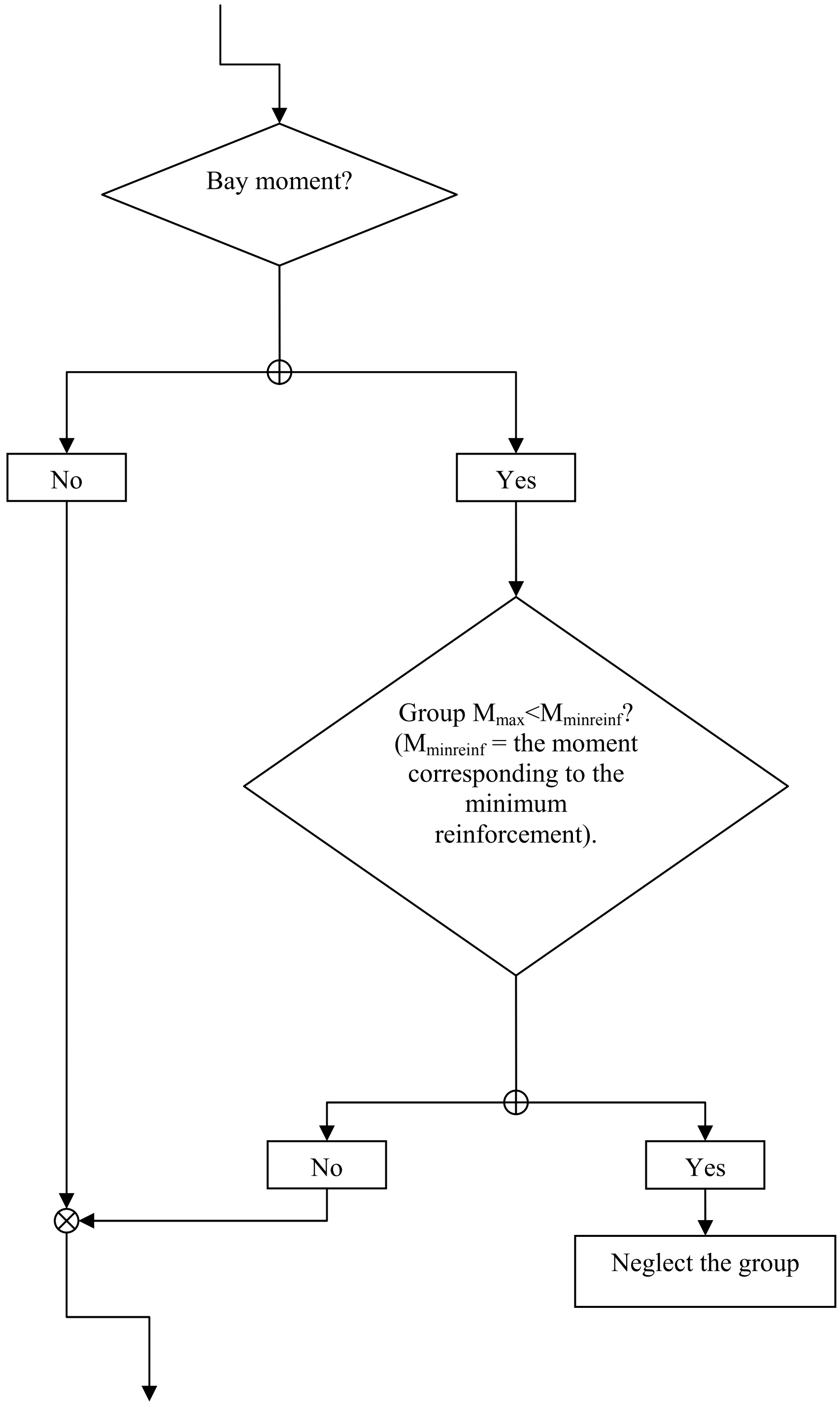


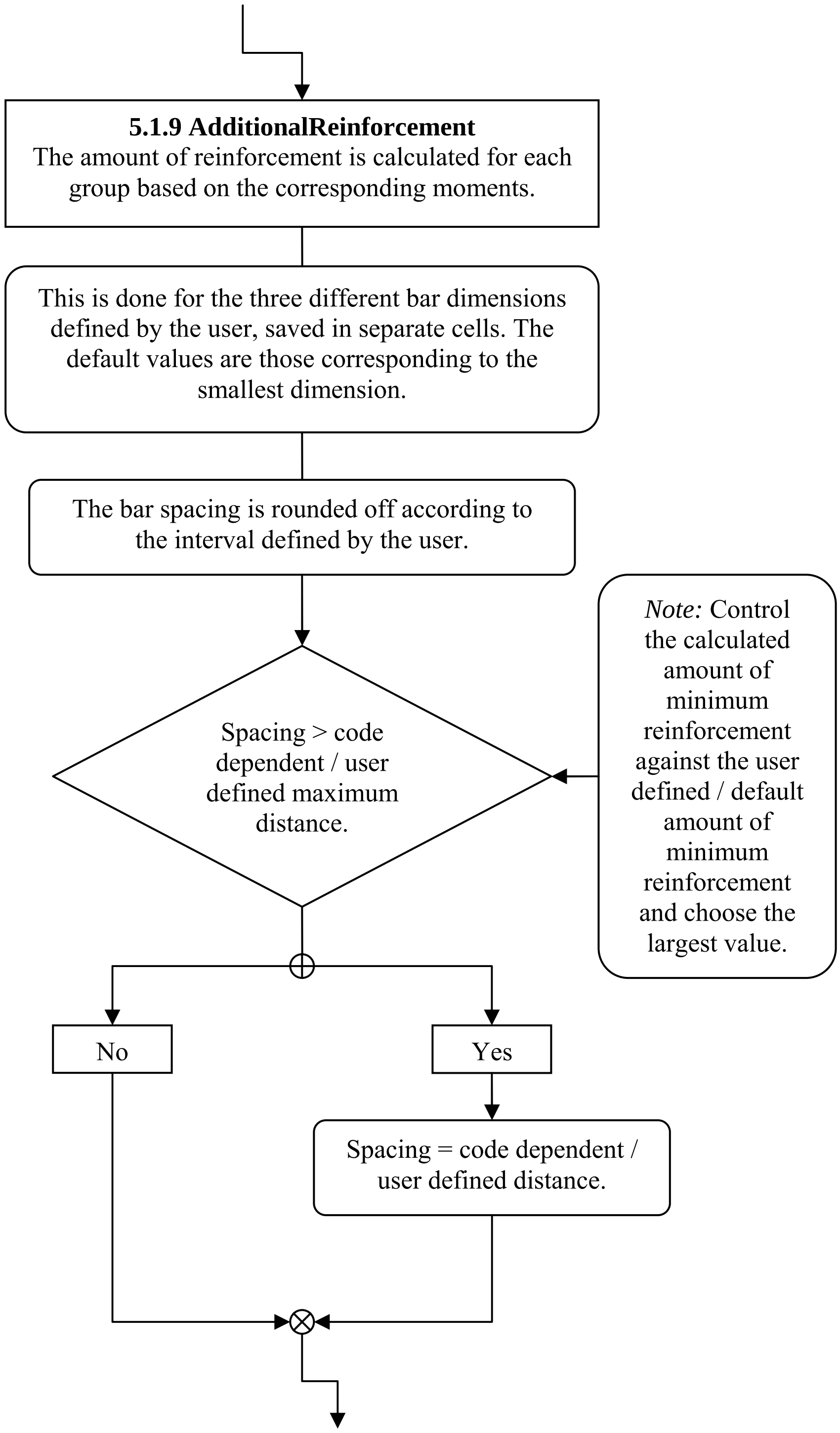
5.1.6 BiggestMoment
The largest moments in each group of the four moment types are determined.

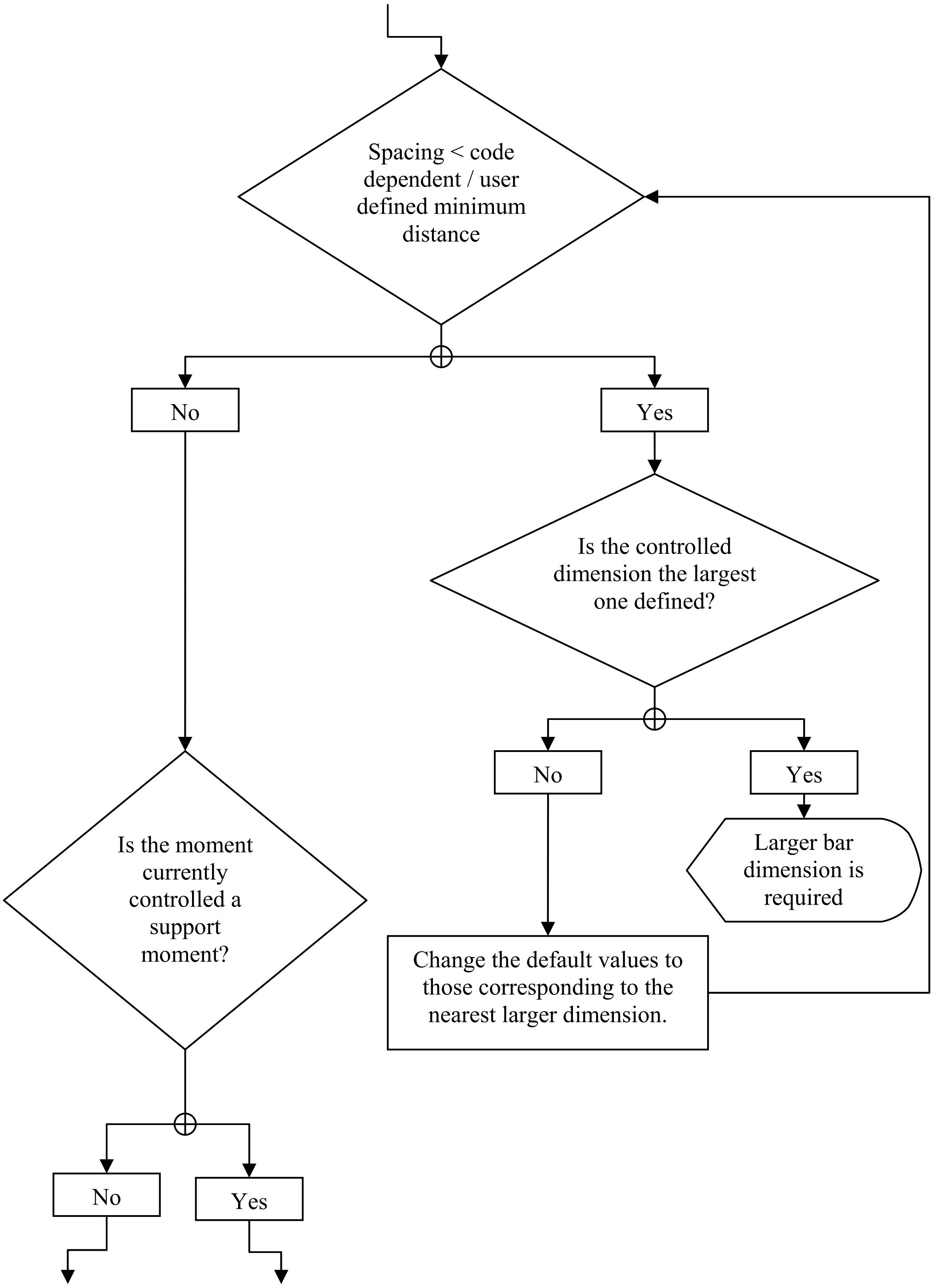
5.1.7 CheckMmax
Controls that the inner sub-group, defined by the largest nodal moment, contains the element with the largest moment. If not, the coordinates of the region are changed and a control of which element belonging to which region is performed once again.

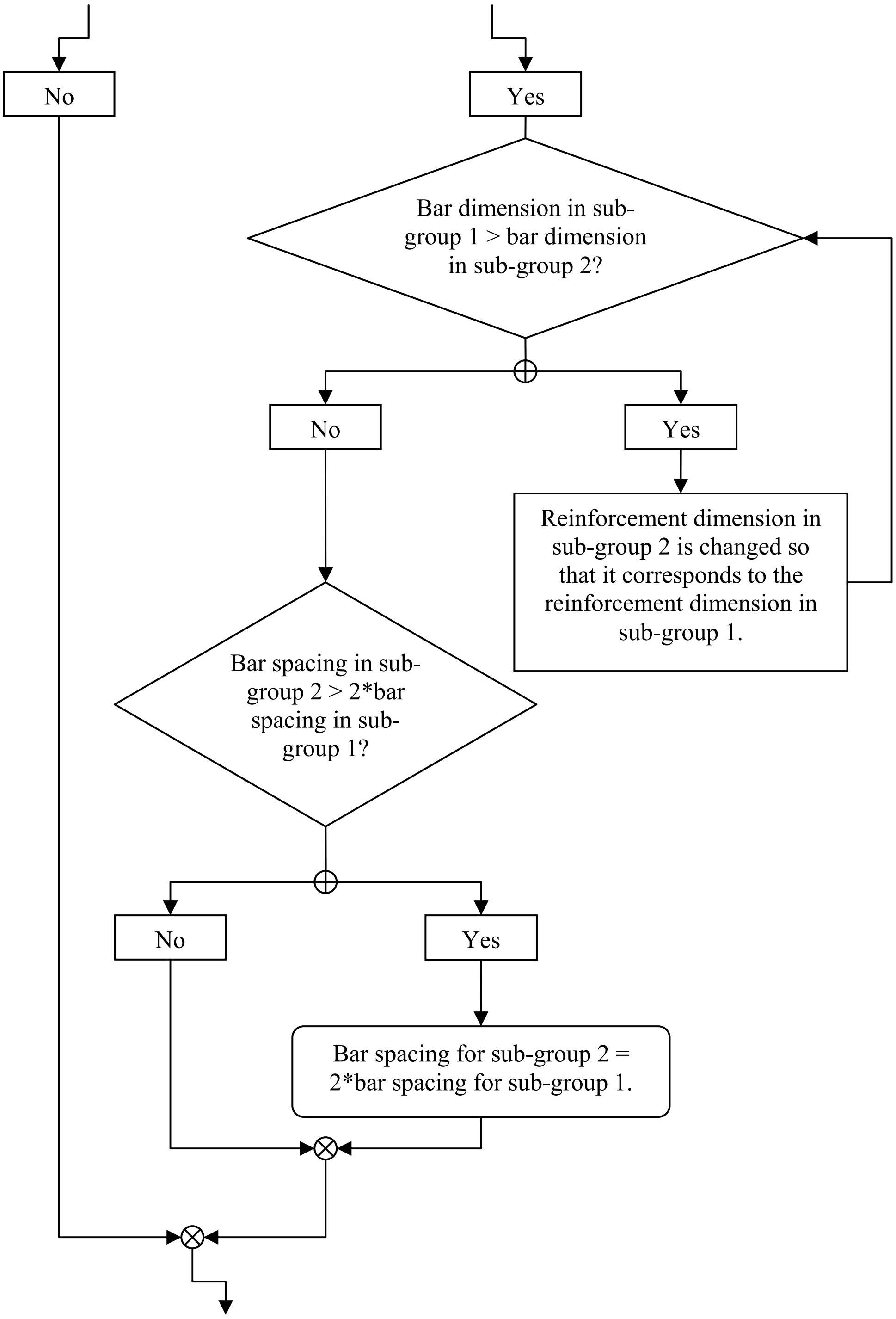

Sub-group 1 is given new coordinates that include the entire element with the largest moment.
5.1.5 InsideTheRegions
The elements are connected to the coordinate regions depending on if the current element is enclosed by the region.
One or more element nodes inside coordinates for sub-group 1?
Yes
No
Element linked to sub-group 1.
Element linked to sub-group 2.

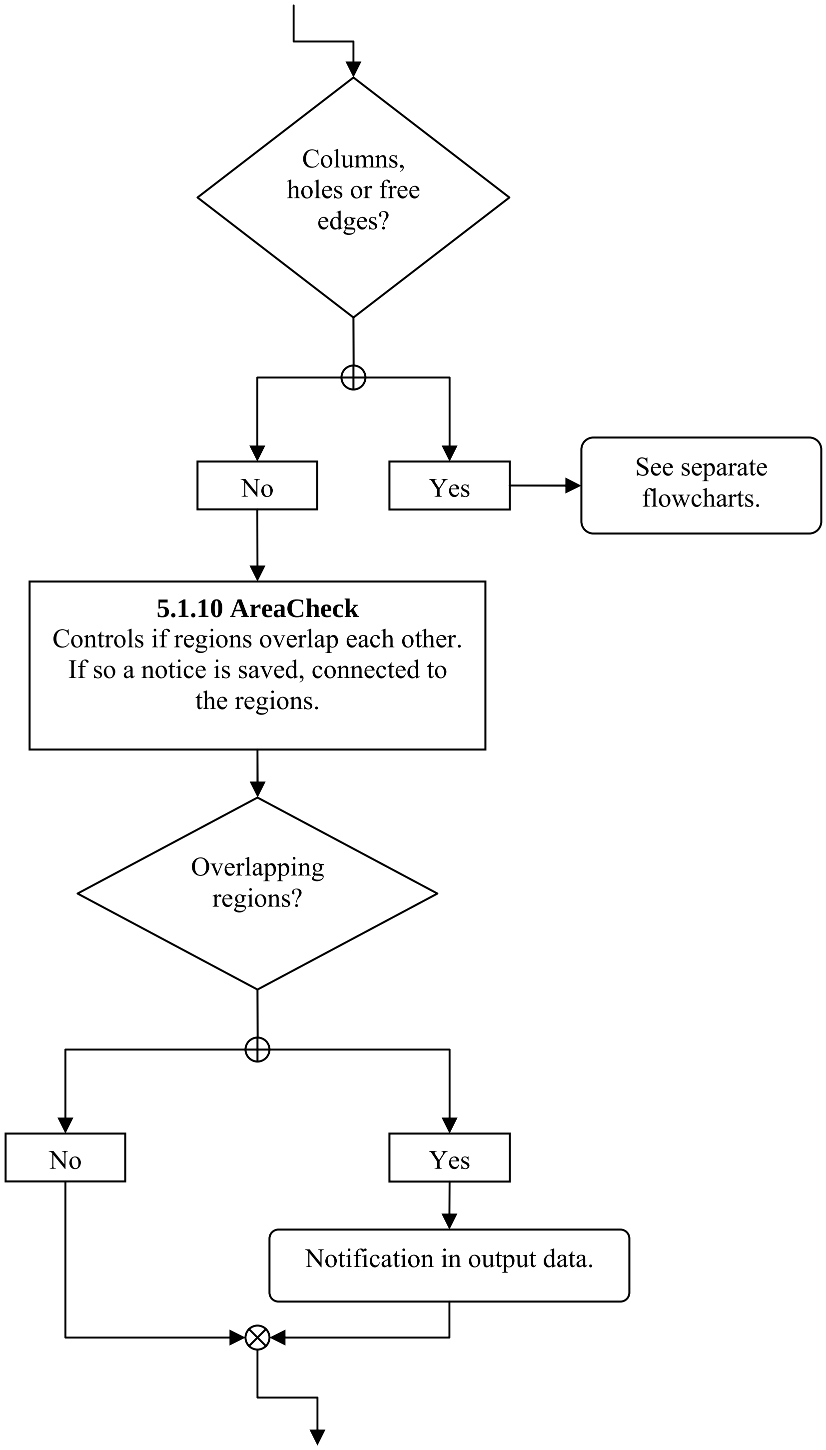

5.1.8 AdditionalMoment
The moments for which each group should be reinforced for are determined.
User defined amount of minimum bottom reinforcement, $M_{minreinf}$?.
Yes
No
5.1.2 MinimumReinforcement
Minimum reinforcement in the bottom, ($M_{minreinf}$) equally distributed in the x and y direction, is taken into consideration.

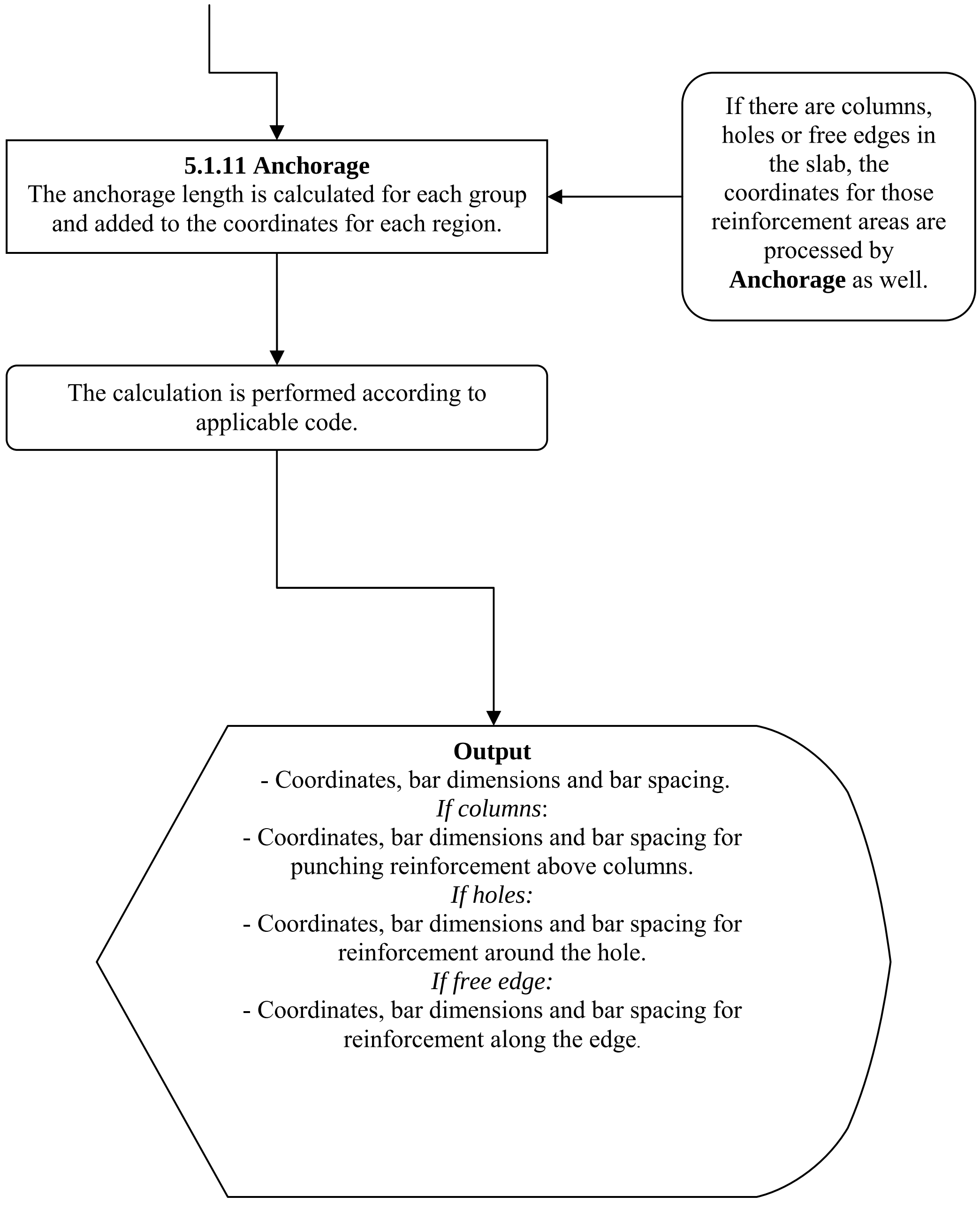

Bay moment?
No
Yes
Group M_{max}<$M_{minreinf}$?
($M_{minreinf}$ = the moment corresponding to the minimum reinforcement).
No
Yes
Neglect the group


5.1.9 AdditionalReinforcement
The amount of reinforcement is calculated for each group based on the corresponding moments.
This is done for the three different bar dimensions defined by the user, saved in separate cells. The default values are those corresponding to the smallest dimension.
The bar spacing is rounded off according to the interval defined by the user.
Spacing > code dependent / user defined maximum distance.
Note: Control the calculated amount of minimum reinforcement against the user defined / default amount of minimum reinforcement and choose the largest value.
No
Yes
Spacing = code dependent / user defined distance.

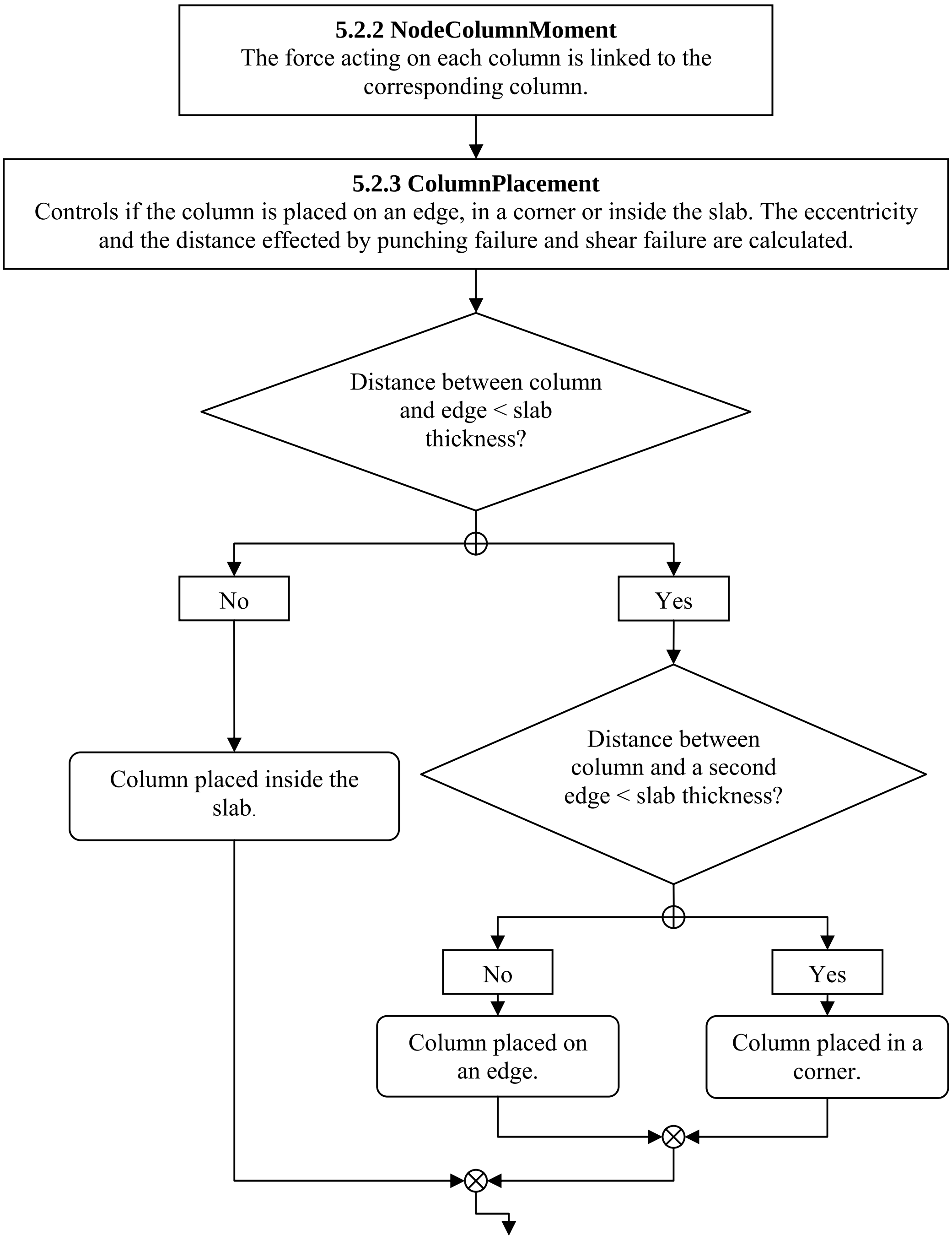

Spacing < code dependent / user defined minimum distance
No
Yes
Is the controlled dimension the largest one defined?
No
Yes
Larger bar dimension is required
Change the default values to those corresponding to the nearest larger dimension.
Is the moment currently controlled a support moment?
No
Yes

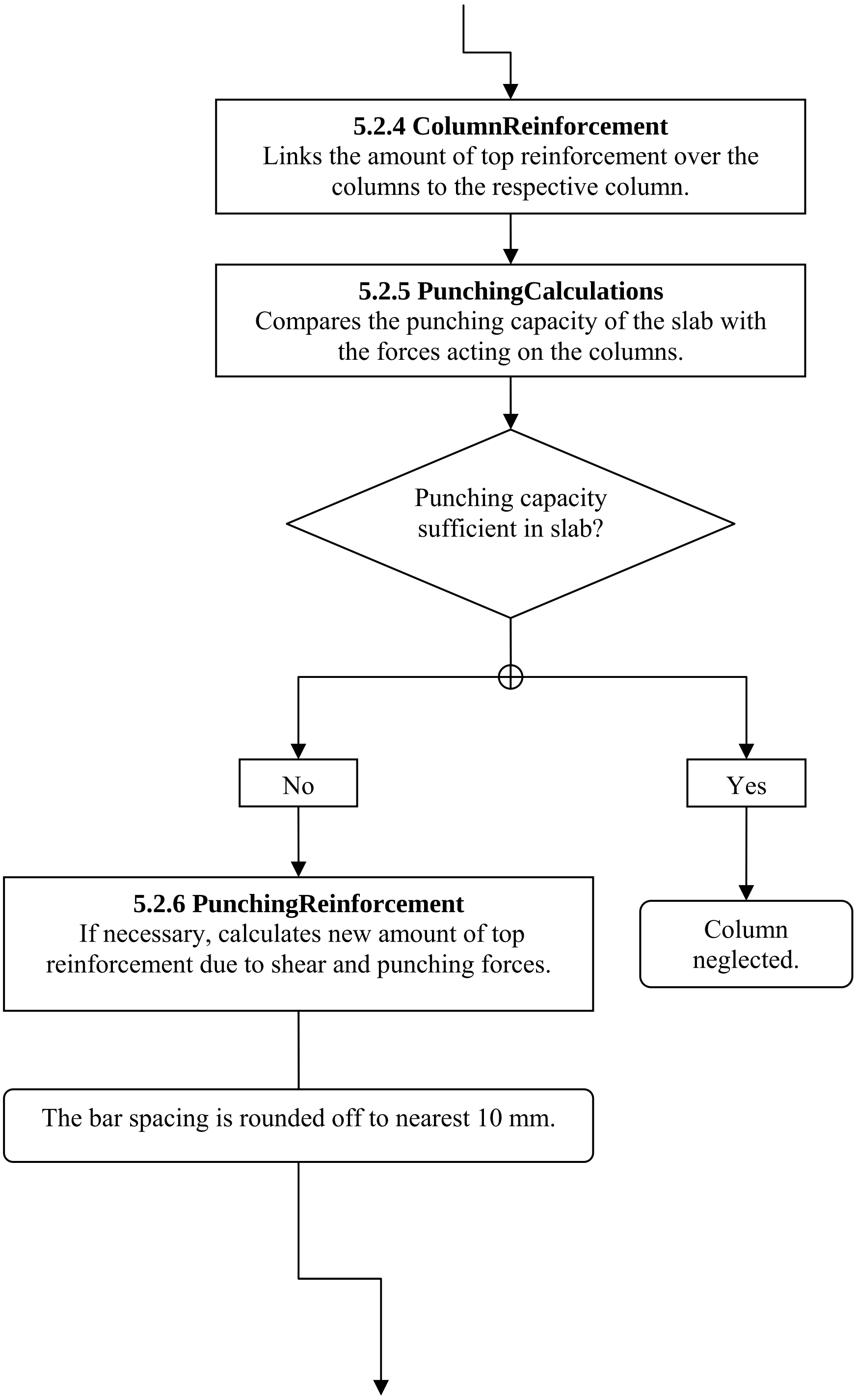

No
Yes
Bar dimension in sub-group 1 > bar dimension in sub-group 2?
No
Yes
Reinforcement dimension in sub-group 2 is changed so that it corresponds to the reinforcement dimension in sub-group 1.
Bar spacing in sub-group 2 > 2*bar spacing in sub-group 1?
No
Yes
Bar spacing for sub-group 2 = 2*bar spacing for sub-group 1.

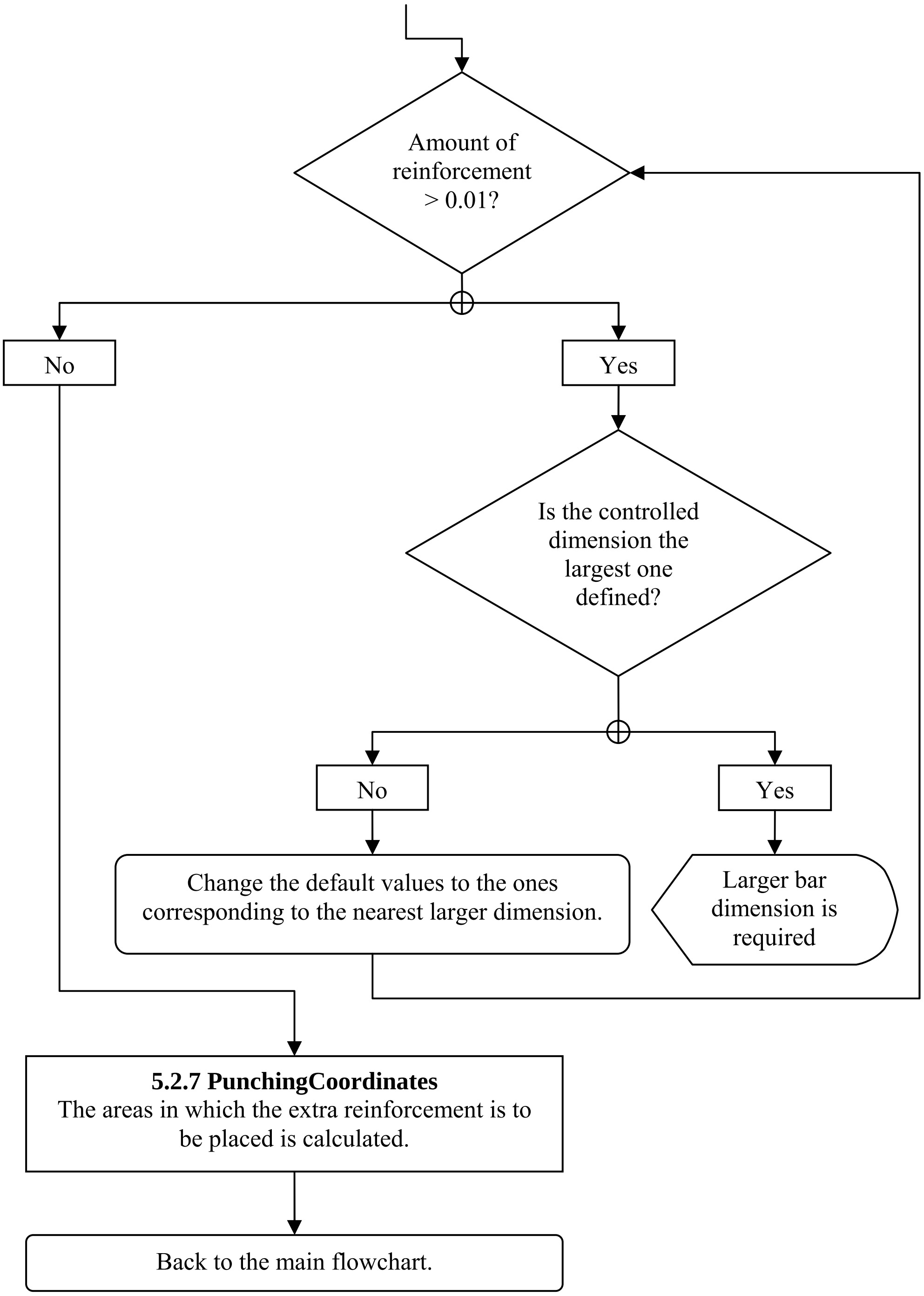

Columns, holes or free edges?
No
Yes
See separate flowcharts.
5.1.10 AreaCheck
Controls if regions overlap each other. If so a notice is saved, connected to the regions.
Overlapping regions?
No
Yes
Notification in output data.


5.1.11 Anchorage
The anchorage length is calculated for each group and added to the coordinates for each region.
If there are columns, holes or free edges in the slab, the coordinates for those reinforcement areas are processed by **Anchorage** as well.
The calculation is performed according to applicable code.
Output
- Coordinates, bar dimensions and bar spacing.
If columns:
- Coordinates, bar dimensions and bar spacing for punching reinforcement above columns.
If holes:
- Coordinates, bar dimensions and bar spacing for reinforcement around the hole.
If free edge:
- Coordinates, bar dimensions and bar spacing for reinforcement along the edge.

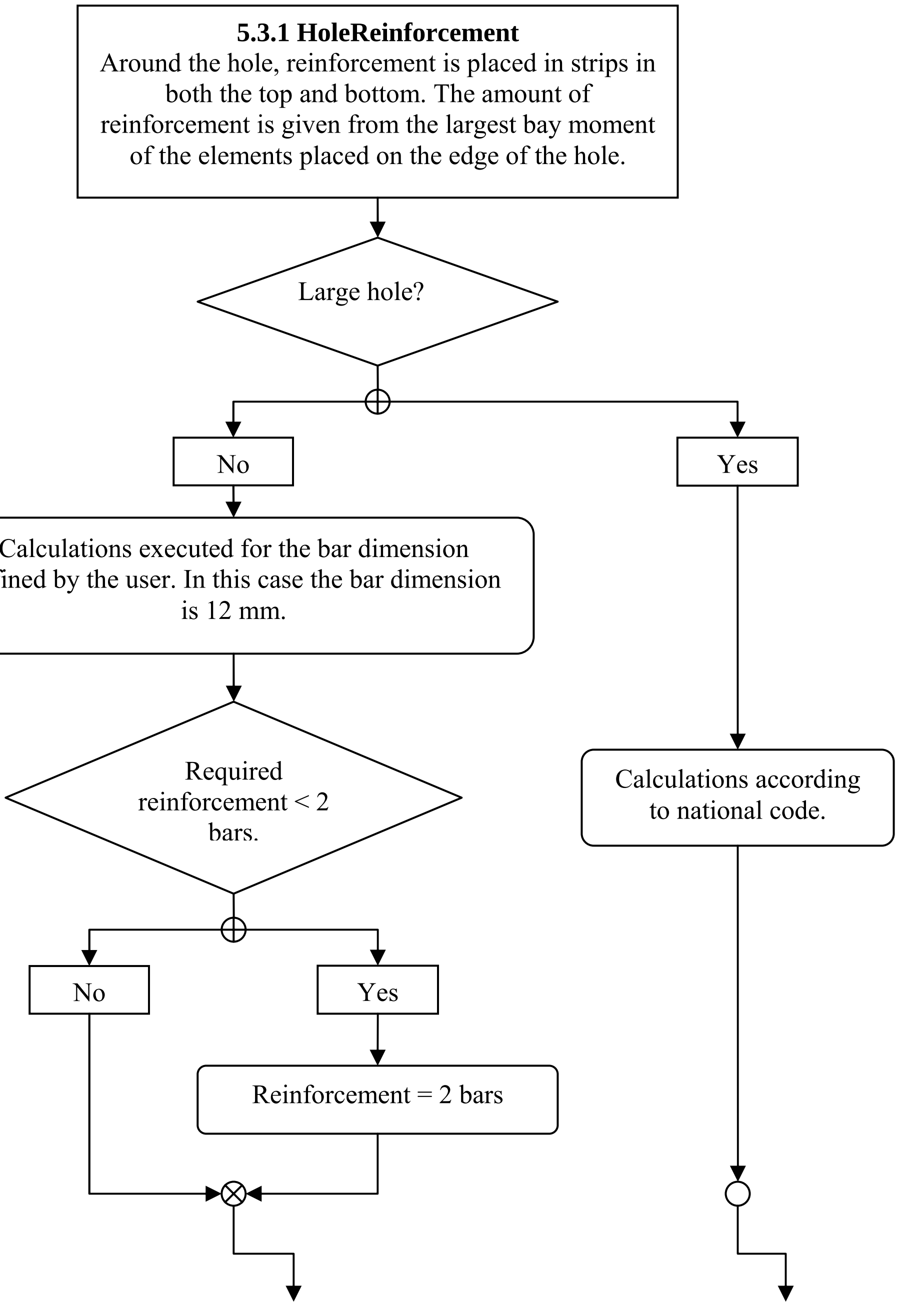
9.1.2 Columns

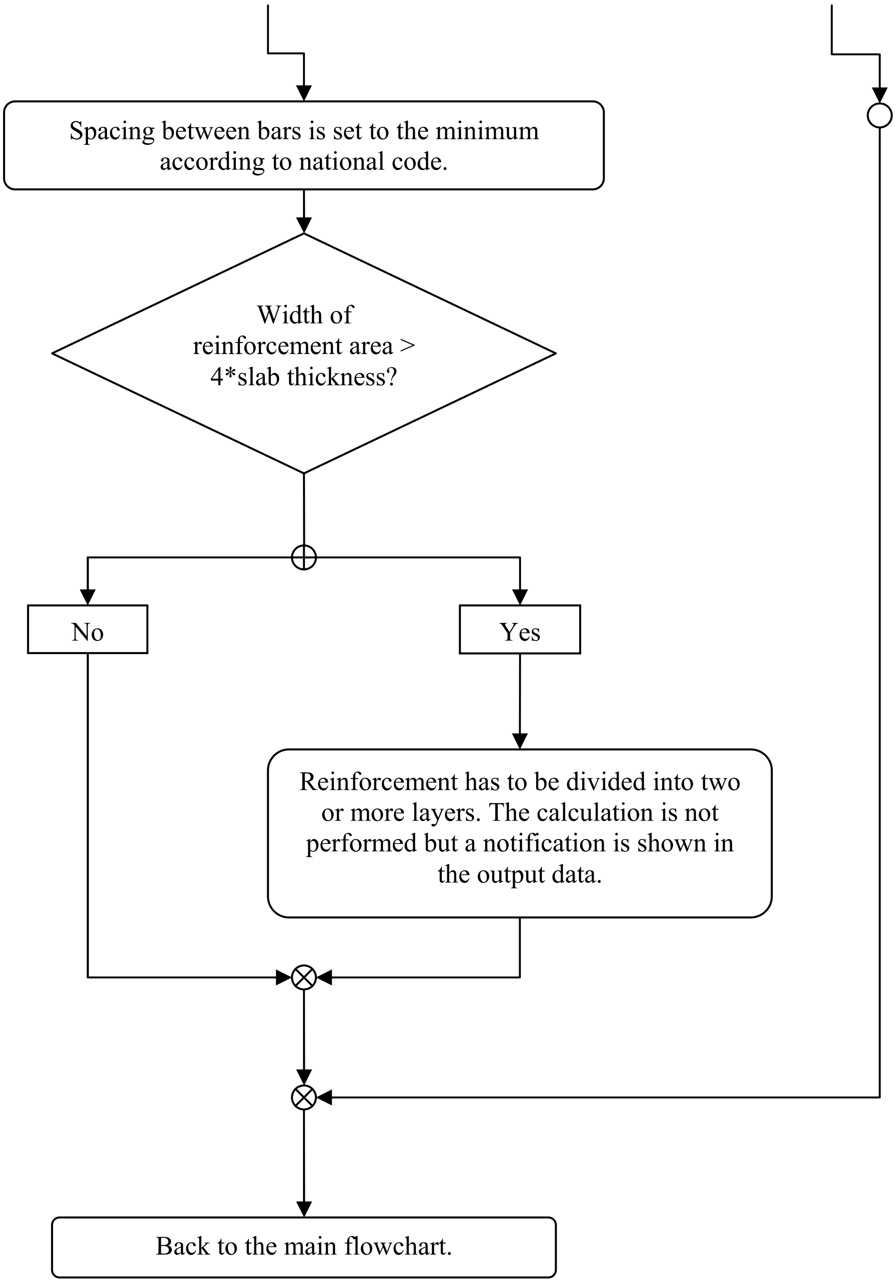

5.2.2 NodeColumnMoment
The force acting on each column is linked to the corresponding column.
5.2.3 ColumnPlacement
Controls if the column is placed on an edge, in a corner or inside the slab. The eccentricity and the distance effected by punching failure and shear failure are calculated.
Distance between column and edge < slab thickness?
No
Yes
Column placed inside the slab.
Distance between column and a second edge < slab thickness?
No
Yes
Column placed on an edge.
Column placed in a corner.


5.2.4 ColumnReinforcement
Links the amount of top reinforcement over the columns to the respective column.
5.2.5 PunchingCalculations
Compares the punching capacity of the slab with the forces acting on the columns.
Punching capacity sufficient in slab?
No
Yes
5.2.6 PunchingReinforcement
If necessary, calculates new amount of top reinforcement due to shear and punching forces.
Column neglected.
The bar spacing is rounded off to nearest 10 mm.

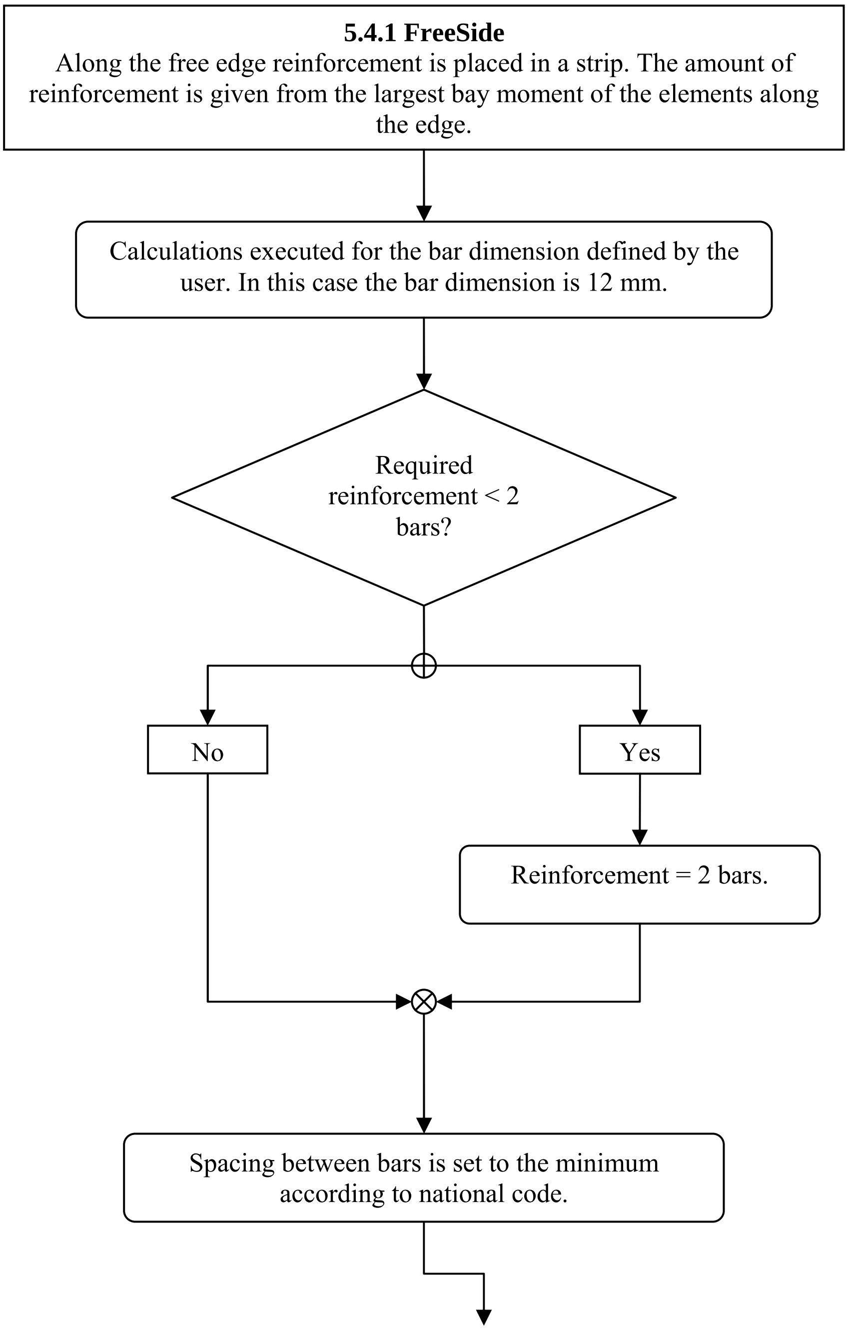

Amount of reinforcement > 0.01?
No
Yes
Is the controlled dimension the largest one defined?
No
Yes
Change the default values to the ones corresponding to the nearest larger dimension.
Larger bar dimension is required
5.2.7 PunchingCoordinates
The areas in which the extra reinforcement is to be placed is calculated.
Back to the main flowchart.

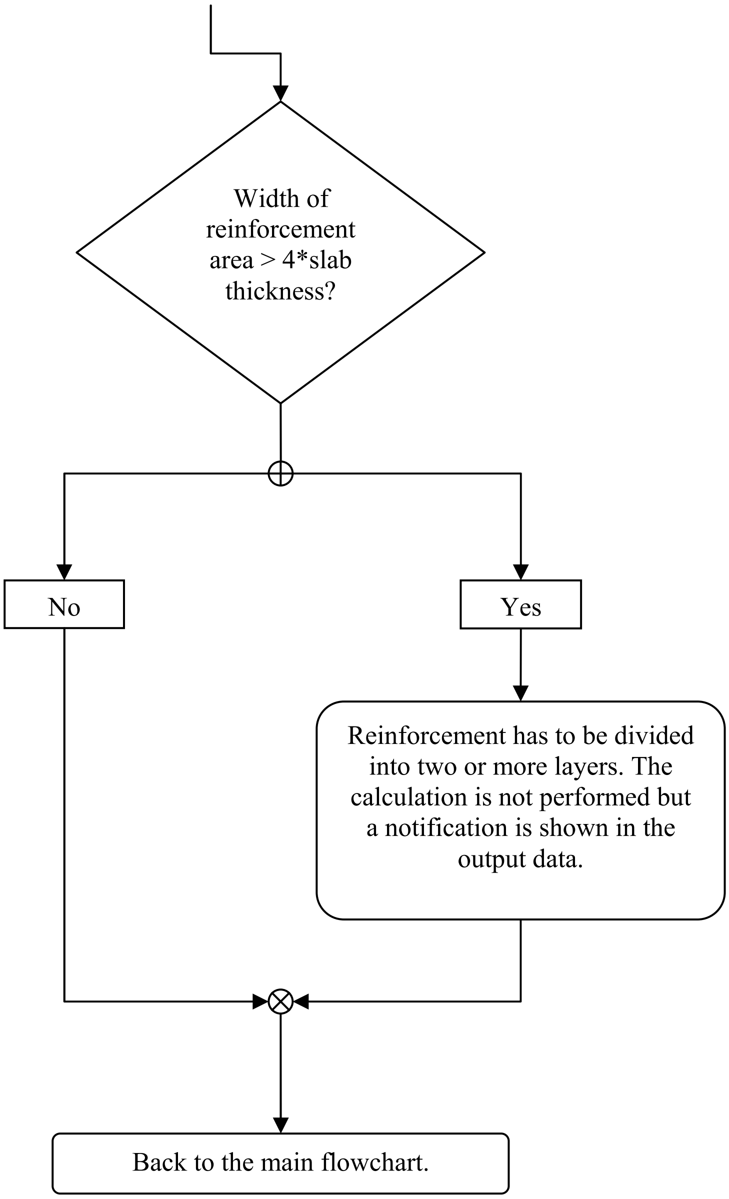
9.1.3 Hole


5.3.1 HoleReinforcement
Around the hole, reinforcement is placed in strips in both the top and bottom. The amount of reinforcement is given from the largest bay moment of the elements placed on the edge of the hole.
Large hole?
No
Yes
Calculations executed for the bar dimension defined by the user. In this case the bar dimension is 12 mm.
Required reinforcement < 2 bars.
Calculations according to national code.
No
Yes
Reinforcement = 2 bars


Spacing between bars is set to the minimum according to national code.
Width of reinforcement area > 4*slab thickness?
No
Yes
Reinforcement has to be divided into two or more layers. The calculation is not performed but a notification is shown in the output data.
Back to the main flowchart.

9.1.4 Free Edge

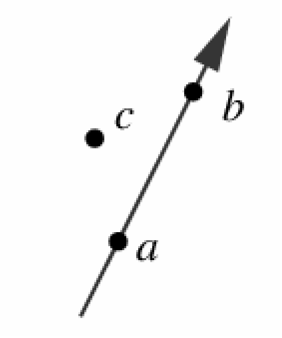



Width of reinforcement area > 4*slab thickness?
No
Yes
Reinforcement has to be divided into two or more layers. The calculation is not performed but a notification is shown in the output data.
Back to the main flowchart.

9.2 Appendix B – Determination of Node Placement

To determine on which side of a line a node is placed, the determinant between the vector $\overrightarrow{ab}$ and the node is calculated.

a – starting point of the vector $\overrightarrow{ab}$
b - end point of the vector $\overrightarrow{ab}$
c – node of interest



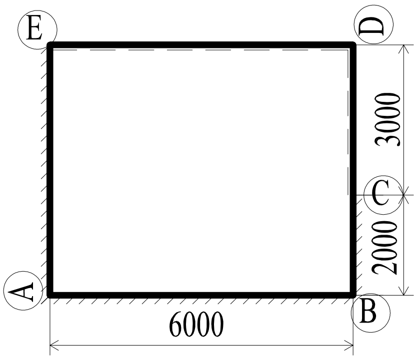
$$M = \begin{bmatrix} a_x & a_y & 1 \\ b_x & b_y & 1 \\ c_x & c_y & 1 \end{bmatrix}$$

If the determinant of M is positive, the node is located to the left of the vector $\overrightarrow{ab}$ and therefore inside the region. If the value is zero, it is located on the vector $\overrightarrow{ab}$ and a negative determinant means that the node is located to the right of $\overrightarrow{ab}$.[12]

APPENDIX B – DETERMINATION OF NODE PLACEMENT

9.3 Appendix C – Presentation of Hand Calculation for Slab 1

The example calculated is presented in [10] Appendix 3 as example 2:5. To correspond with slab 1 studied in chapter 6 – *Case Studies* the load is, instead of q=12.5 kN/m^2, set to q=16.3 kN/m^2 (12.5+dead weight).



C 20/25	f_{cck} f_{ctk}	19 MPa 1.45 MPa
Ps 500	f_{stk}	500 MPa
Slab	d t	6 x 5 m 0.160 m
Load	q	16.3 kN/m^2

The calculations are performed according to the standard method presented in [10] 4.

As part of the edge B-D is a fixed support, a factor φ must be calculated according to equation [10] (4.4)

$$\varphi = \frac{L_s}{L} \longrightarrow \varphi = \frac{2}{5}.$$

The moments are then calculated using equation [10] (4.5)

$$m = \varphi \alpha_{fixed} q b^2 + (1-\varphi)\alpha_{free} q b^2 .$$

The factors α are collected from [10] Appendix 2, where α_{free} corresponds to elementary case 4 and α_{rigid} corresponds to elementary case 8. The quotient a/b = 1.2 as a=6m and b=5m.

$$m_{xs} = \frac{2}{5}\cdot 0.0432 \cdot 16.3 \cdot 5^2 + (1-\frac{2}{5})\cdot 0.0467 \cdot 16.3 \cdot 5^2 = 18.51 \text{ kNm/m}$$

$$m_{ys} = \frac{2}{5}\cdot 0.0450 \cdot 16.3 \cdot 5^2 + (1-\frac{2}{5})\cdot 0.0575 \cdot 16.3 \cdot 5^2 = 21.45 \text{ kNm/m}$$

$$m_{xf} = \frac{2}{5}\cdot 0.0210 \cdot 16.3 \cdot 5^2 + (1-\frac{2}{5})\cdot 0.0270 \cdot 16.3 \cdot 5^2 = 10.05 \text{ kNm/m}$$

$$m_{yf} = \frac{2}{5}\cdot 0.0289 \cdot 16.3 \cdot 5^2 + (1-\frac{2}{5})\cdot 0.0400 \cdot 16.3 \cdot 5^2 = 14.53 \text{ kNm/m}$$

The following equations are used to determine the amount of reinforcement

$$\overline{m} = \frac{M}{bd^2 f_{cc}} = \omega(1 - 0.5\omega)$$ NC [3] 3.6:432

$$A_s = \frac{M}{f_{st} d(1 - \frac{\omega}{2})}$$ NC [3] 3.6:432

This results in the following reinforcement:

- Bay reinforcement
  - x-direction: Φ8 s250
  - y-direction: Φ8 s180
- Support reinforcement
  - x-direction: Φ10 s200
  - y-direction: Φ10 s190

The reinforcement distribution is determined according to [10] 4.6. The distribution is based on following parameters

- b/4=1.25 m
- d=0.13 m
- e_{X1}=1.02 m (${e_{X2}}$=0.57 m)
- e_{Y1}=1.12 m (${e_{Y2}}$=0.68 m)

and results in the following reinforcement drawing: