Wind effects on high rise buildings

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Vindeffekter på höga byggnader

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Lund, 2007

KEYWORDS

First natural frequency of high rise buildings, wind effects, comfort criteria due to wind acceleration, wind induced acceleration, wind velocity pressure, Turning Torso, Smáratorg Tower.

ABSTRACT

The wind effects on high rise buildings were studied by finding and reading books, articles and studying equations. This was done to calculate the first natural frequency of high rise buildings, wind induced acceleration on high rise buildings and how the comfort criteria of acceleration performing on high rise buildings acts on human bodies living in the building. The buildings that were studied are Turning Torso in Malmö, Sweden and briefly Smáratorg Tower in Kópavogur, Iceland. The information on the building Turning Torso is from 17. March 2000 (this is not the final dimension on the building it was later strengthened due to wind load). This information was then used to calculate the first natural frequency, wind induced acceleration and compared to the data that engineers at Turning Torso worked with. The most important results were that there will be excessive movement in the top floors of Turning Torso so that sensitive people may perceive motion and hanging objects may move. For Smáratorg Tower the movement is so excessive that majority of people perceive motion.

The aim was to make a diploma work that can be used in practice, which can be a guide to design high rise buildings due to wind effects in the early states of development.

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PREFACE

I would like to thank Professor Sven Thelandersson for the suggestion of this diploma work, guidance and thoughts on it in the development of the thesis. I would also like to thank the librarians at the institute Väg och Vatten, for support and helping me to find several books and articles on this subject. The architects at Arkís in Iceland is also thanked for all the drawings on the Smáratorg Tower, along with engineers at Ferill in Iceland specially Snæbjörn Kristjánsson for the insight in all the engineering aspect of Smáratorg Tower.
I also would like to thank Reverend Fridgeir Torfi Ásgeirsson, Architect and Computer Engineer, for spiritual guidance and an excessive interest in this thesis.

At last but not least I would like to thank the love of me live Ásta Andrésdóttir and my daughter Elsa Björg Ámundadóttir for keeping me going and filling me with energy every day.

Lund 2007

Ámundi Fannar Sæmundsson

SUMMARY

The tolerance to wind action in- and outside of buildings is an important factor in the structural design of high rise structures. The sway that the structural system may be able to endure still must be reduced to the tolerable limits for humans.

This thesis was done first by gathering literature and information on the matter at hand; this was mainly done on the internet. This is not an easy task, because the literature is sizable and some books are hard to come by, but with the help of excellent librarians, various search engines like Electronic Library Information Navigator (ELIN), Libris, Lovisa and many more the search was made much easier. Some information was received from companies that are involved in building high rise buildings; this took some weeks to obtain. Several articles that were of interest were ordered, but due to the long time it elapsed before the author got them; they were of no use and thus canceled.

It is of interest to find the first natural frequency of vibration for tall slender buildings like Turning Torso and Smáratorg Tower the highest resident buildings in Sweden and Iceland respectively. Two methods were introduced to acquire this, one is a “rule of thumb” that has been used for many years and the other is a more complex method based on the equation of motion.

Because designing codes and standards do not always include mean hourly wind velocity which is needed to find the accelerations in the top floors of high rise buildings, method is included to adjust wind averaged over a certain period of time to wind with mean over one hour.

There are some methods to evaluate the wind velocity pressure from a given wind velocity. These methods differ widely, but they all show that engineers at Turning Torso have underestimated and/or made some approximations when calculating the wind velocity pressure, thus the wind velocity pressure is lower in their calculations.

The along wind acceleration on the top floors is computed with the method engineers at Turning Torso used along with three other methods. These methods differ widely. The methods are used to find the peak acceleration and then the outcome is compared and the comfort criterion is applied for these values.

The comfort criterion due to acceleration in the top floors of a high rise building is introduced and its many parameters discussed and how it will affect human beings. This will involve the kinesiological, psychological response and physiological reactions. When the comfort criterion is established the structural system of a high rise building can be designed so the sway in the top floors is within the limit tolerable for humans.

The values for the accelerations computed were compared to the comfort criterion. The results for Turning Torso are not acceptable for 37.8 m/s wind velocity, the results are all in the zone where sensitive people perceive motion and hanging objects may move. For some calculations the movement is even in the zone where it can produce motion

sickness and desk work is slightly affected. If calculations are done with the wind speed that engineers at Turning Torso used 70 m/s the results become more extreme. They are well over the limit where humans have difficulties walking erect. This means that people can get hurt due to excessive acceleration. This should not be accepted and some measures need to be taken to minimize this unnecessary acceleration. It has to be noted here that the information used in this thesis for Turning Torso is from 17. March 2000, it was later stiffened due to excessive motion in the top floors. This defect in the design cost several hundred million Swedish kronor to repair.

For the Smáratorg Tower, results are over the limit where humans begin to perceive motion and in the zone where the majority of people perceive motion and can affect desk work. To get better results on Smáratorg Tower, more information is required. All the accelerations exceed the limit stated in the comfort criterion, therefore a more detailed computation must be done.

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1 INTRODUCTION

If the height of structures today and the height of structures planned to be built are inspected, it is clear that the structures in the future will be higher and higher. The height of the tallest building changes year by year because skyscrapers are constructed constantly world wide. With this development that buildings are rising, there will be a larger awareness of occupants comfort due to wind induced acceleration in the top floors of a high rise structure.
Still, nowadays high rise buildings are constructed so that they sway so much in wind that occupants complain of movement and even motion sickness.

1.1 BACKGROUND

The structural systems of high rise buildings are usually sensitive to the effects of wind.
With the increasing need to improve the performance of constructed facilities it has placed a growing importance on the problem of wind effects on structures.
The available literature is sizable. It is felt that researchers and designers working in this area will benefit from a review devoted to the fundamentals of wind effect on skyscrapers.

1.2 PURPOSE

The purpose of this diploma work is to compare different methods to find the wind induced acceleration in the top floor of a tall slender structure, and to include the comfort criteria needed for comparison with the wind induced acceleration. The ambition is also to make a good work guideline that can be used in practice within the early state design of a tall slender structure. This will be a guideline to quickly estimate the acceleration in the top floors of a skyscraper.

1.3 LIMITATIONS

The scope of wind is many folded, to limit the many effects of wind in this study only direct positive pressure of wind (along wind) and gust effects are looked at, neglecting drag wind, clean-off-wind, turbulent flow as “eddys and vortexes” and more. The extreme wind conditions like hurricanes and tornadoes are not looked at. Mass dampers in buildings will not be studied or discussed, neither twisting frequencies nor twisting motions, the same is true for across wind motions (see Figure 3-3 for definitions on twisting, across and along wind motion).

The first natural frequency of high-rise buildings has to be found and how it reacts under wind load.

The acceleration in the top floors of a high rise building will be found with different methods and the wind velocity pressure.

A brief look is taken at acceptable accelerations in buildings caused by wind, so that people are not affected of the building swaying in the wind.

The high rise buildings Turning Torso in Malmö, Sweden and Smáratorg Tower in Kópavogur, Iceland are studied.

2 BUILDING DESCRIPTION

The two structures looked at in this thesis are the highest building in Sweden, Turning Torso and the highest in Iceland, Smáratorg Tower.

2.1 TURNING TORSO

Turning Torso is a skyscraper in Malmö, Sweden. It was designed by the Spanish architect Santiago Calatrava and officially opened on 27 August 2005.
Turning Torso is based on a sculpture by Santiago Calatrava called Twisting Torso. The Twisting Torso sculpture is a white marble piece based on the form of a twisting human being (see Figure 2-1 for a sketch that Santiago Calatrava made of this sculpture). This is the first twisting skyscraper of its kind in the world, and has already inspired variations such as Infinity Tower in Dubai and Calatrava's own Chicago Spire in Chicago and Torres de Calatrava in Valencia, **[17].** One reason for the building of Turning Torso was to reinstate a recognizable skyline for Malmö since the removal of the 130 meter high Kockums Crane in 2002, which was located less than a kilometer from Turning Torso. Kockumskranen, which was a large crane that had been used for ship building somewhat symbolized the city.
The tower reaches a height of 190.4 meters with 54 stories (for comparison the pylons in Öresunds bridge are 204 meter high). When completed, it was the tallest building in Scandinavia (it is visible from Copenhagen in Denmark, in the distance across the Öresund), and Europe's second highest apartment building, after the 264 meter high Triumph-Palace in Moscow. The 84 meter high Kronprinsen was the tallest building in Malmö before Turning Torso, **[21].** Two higher buildings are proposed to be built in Sweden in the year 2010, one is Telefonplanskrapan in Stockholm estimated 200 meter and the other one is Malmö Tower, in Malmö estimated 216 meter,**[19].**

The foundation consists of a heavily reinforced and pre-stressed tension cone shell that transfers the actions coming from the core to a compression ring which is equilibrated by 78 bored vertical piles and the circular compression slab constituting the entrance floor. The tension shell extends from the entrance level around 6.5m into the ground; it has a slope of about 23° and a thickness varying from 2.7m to 1.1m. The radius of the compression ring and slab is roughly 40m. An inner ring of 12 piles is located at the tip of the tension cone shell. These piles allow for a partial erection of the elevator and the structural core before completion of the static system constituted by the tension shell, compression ring and slab, **[25].** This compression ring is made into a man made pool that the building is positioned in with two sculptures by Santiago Calatrava, **[17].**
Turning Torso is composed by nine geometrically equal modules. Each module consists of five apartment floors. The modules are separated by combined terrace and technique floors. The floors have a pentagonal plan and each apartment floor is 3.1m high and the terrace and technique floor 4.0m high, that makes the module height of 19.5 m for each module. Each floor is placed in a clockwise rotated position with respect to the one below. The top-most segment is twisted 90° clockwise with respect to the ground floor, which corresponds to a rotation angel of 10° per module.
The two bottom modules are intended as office space, modules three to nine contain 149 luxury apartments but the two top floors hold a conference centre, **[21].**

The main load bearing structural element is a circular concrete core whose centre corresponds exactly to the rotation centre of the floors and approximately to their gravity center. The inner radius of the structural core is 4.3m and constant over the height. The thickness of the structural concrete core equals 2.0m in the basement and in the entrance floor, 1.5m along the first module and it is then reduced by 0.1m per module down to 0.7m at the top of the tower. Inside the structural core the elevator and staircase are placed which represent a secondary structural element. The vertical dead and live loads on the slabs are transferred to the structural core and to the columns situated at the corners of the floor plan. The column loads are introduced back into the core at the bottom of each module through steel corbels consisting of diagonal ties and horizontal struts. These corbels are located either at the bottom floor of each module or in the terrace and technique floor between the modules. In order to prevent cracking due to bending moments induced by wind loading, a post tensioning of the structural core is envisaged. The post tensioning extends over the first 80m of the structural core. The cables are anchored at different heights of the core, leading to a gradual decrease of the prestressing over the height. The floors consist of a grid of radial and edge steel girders that support a composite steel sheet concrete deck floor. The concrete core is strengthened and stiffened by a steel truss which is erected on the outside of the building in front of the triangular tips of the modules. Consequently, the truss exhibits the same torsional clockwise rotation like the floor sequence. The truss odes are connected to the structural core through corbels located in the terrace and technique floors and consisting of a diagonal and a horizontal steel girder, **[25].**

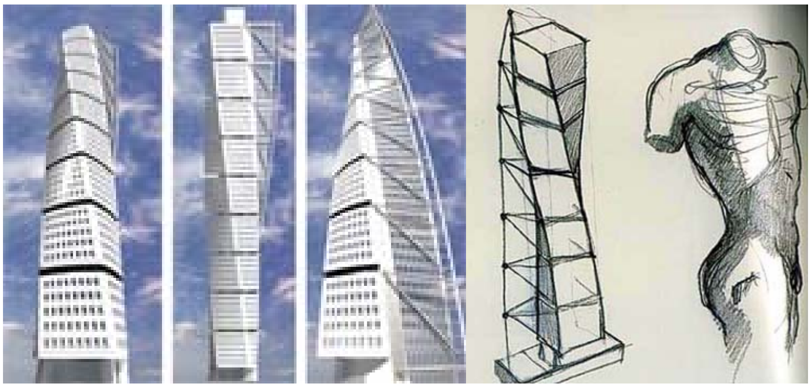

Figure 2-1 Shows a computer image of Turning Torso in Malmö, Sweden and a sketch that Santiago Calatrava made of Turning Torso.

Just for interest, Turning Torso was featured on an episode of the Discovery Channel's Extreme Engineering filmed in February 2004, **[17].**

2.2 SMÁRATORG TOWER

The tower is located in Smárahverfi in Kópavogur, Iceland where the shopping mall Smáralind is located. Smáratorg 3 is the street address to the skyscraper Smáratorg Tower and the highest building in Iceland that reaches a height of 77.9 m, replacing the beautiful church Hallgrímskirkja, that is 74.5 m high and is build to resemble columnar rock (stuðlaberg) that remind you of Icelandic nature. (Journalists in Iceland have always used Hallgrímskirkja as a reference to measure heights on high structures, distances and high things like mountains, and now they have to update that to the new office building Smáratorg Tower). The skyscraper in Smáratorg 3 was designed by the architectural company Arkis in Iceland and all engineering work was done by the engineering companies Ferill and VSÓ in Iceland.

The tower is two folded, that is 20 floors of offices and service floors and a two floor shopping space around it. There is a car parking in the basement and on the 2:nd floor. In the basement and on the first floor of the tower there will be a technical room, elevators and staircases. On the first floor there will also be shops. The main entrance to the building is on the second floor on the south side of the building, where there are also offices, lobby and conference rooms. Floors 3 to 19 contain offices and on the 20th floor is a restaurant and a cafeteria. Technical rooms for the elevators are on the 20th floor as is an indrawn 55 m^2 balcony. Each floor has the modular height of 3.5 m with the exception of the first that is 5.95 m and the twentieth that is 6.1 m.

Figure 2-2 Shows a computer image of the Smáratorg Tower in Kópavogur, Iceland.

The main structural element is concrete. The structural core that consists of restrooms, elevators and staircases, is made out of concrete as are the 16 columns that are evenly spaced around the edge of each floor. The main element on the outside of the building is a combination of aluminum and glass; this will set the face of the building. There is not much unusual with this tower. It is in fact just a simple cube or prism structure that is a light glass block stretching 20 floors up, **[23], [16].**

The building is under construction when this thesis is written.

3 METHODS

The methods can be divided into three main phases:

- The first natural frequency of vibration
- Wind
- Acceleration and comfort criteria

3.1 THE FIRST NATURAL FREQUENCY OF VIBRATION

Only the first natural frequency of vibration is of importance for this thesis, because it can be used in hand calculations. Higher natural frequencies require computer programs to analyse.

3.1.1 THE EQUATION OF MOTION

To find the first natural frequency of vibration for a generalized SDF (single degree of freedom) system, the following equations can be used:

$$\omega = \sqrt{\frac{\tilde{k}}{\tilde{m}}} \qquad \textbf{(Equation 3-1)}$$

$$f = \frac{\omega}{2\pi} \qquad \textbf{(Equation 3-2)}$$

where ω is the angular frequency and f is the corresponding natural frequency. The equation of motion can be used to find the generalized stiffness $\tilde{k}$ and generalized mass $\tilde{m}$ of the structure, with the integrals:

$$\tilde{m} = \int_0^H m(x)[\psi(x)]^2 \, dx \qquad \textbf{(Equation 3-3)}$$

$$\tilde{k} = \int_0^H EI(x)[\psi''(x)]^2 \, dx \qquad \textbf{(Equation 3-4)}$$

Where H is the height of the structure, $m(x)$ is the mass per unit height of the structure at the height x from the ground, E is Youngs modulus for the material, I(x) is the moment of inertia at the height x from the ground and $\psi(x)$ is the shape function. The accuracy of the SDF system formulation depends on the assumed shape function $\psi(x)$ in which the structure is constrained to vibrate. The shape function can be assumed as one of the three following relations:

$$\psi(x) = \frac{3x^2}{2H^2} - \frac{x^3}{2H^3} \qquad \textbf{(Equation 3-5)}$$

$$\psi(x) = 1 - \cos\left(\frac{\pi x}{2H}\right) \qquad \textbf{(Equation 3-6)}$$

$$\psi(x) = \frac{x^2}{H^2}$$ **(Equation 3-7)**

These three shape functions are for a long slender structure that is rigid at the base where x = 0. Thus the shape functions satisfy $\psi(0) = 0$ and $\psi'(0) = 0$. See Figure 3-1 for definition on the shape function. The shape function given in equation 3-7 satisfies the displacement boundary conditions at the base of the structure but violates a force boundary condition at the free end. It implies a constant bending moment over the height of the structure, but a bending moment at the free end of a cantilever is unrealistic unless there is a mass at the free end with a moment of inertia. Thus, a shape function that satisfies only the geometric boundary conditions do not always ensure an accurate result for the first natural frequency, **[3].**

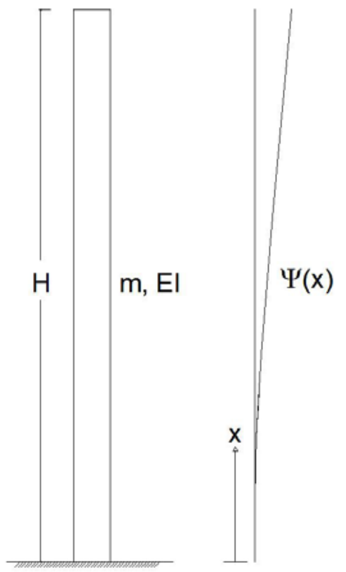

Figure 3-1 Shows the input data for the first natural frequency and how the shape function is defined.

3.1.2 THE "RULE OF THUMB"

It is regularly good to have some quick way to find an approximate solution for a problem, just to get a clue how big the number sought after is.
As a "rule of thumb" the frequency in Hz of a tall building is equal to 100 divided by the building height in feet (1 ft = 0.3048 m), **[6].** This can be transferred to work with the metric system, i.e. the "rule of thumb" becomes:

$$f = \frac{30.48}{H}$$ **(Equation 3-8)**

The frequency in Hz of a tall building is equal to 30.48 divided by the building height in meters.

3.2 WIND

"Some inhabitants in existing buildings have experienced motion sickness caused by building sway; people feel the movement and sense the twisting of the building. At times minor damage to furniture and equipment has occurred, strange creaking sounds from shaking elevator shafts and air leakage around windows were noticed and unpleasant whistling of wind around the sides of the building itself was heard. Some building occupants find it impossible to use balconies except on totally calm days because of constantly turbulent winds on the building face. The list of examples can go on and on. What is important however is the need to recognize that a concern for human tolerance and the activities to be performed in and around the building must be a major factor in the design of today's high-rise building,." **[8].**

"Tornados are most powerful of all winds. However the probabilities of one striking at any one location are exceptionally low compared to extreme winds. It has therefore been generally considered that the cost of designing structures to withstand tornado

effects is significantly higher than the expected loss associated with the risk of a tornado strike. The only buildings where tornados have to be considered in design are structures that if failure would happen, have exceptionally critical consequence (like nuclear power plants)," **[9].**

Some structures, mainly those that are tall and/or slender, will respond dynamically to the effects of wind. The best known structural collapse due to wind was the Tacoma Narrows Bridge which occurred in 1940 at a wind speed of only about 19 m/s. It failed after it had developed a coupled torsional and flexural mode of oscillation with the period of 5 seconds; this lasted about an hour before the bridge collapsed, **[13],** This bridge was exceptionally long and narrow, the center span was 853.44 m and the width was 11.89 m. The bridge underestimated life time was 4 months, no one was injured or killed in this affair except a dog that was to afraid to leave the car it was in when the bridge finally gave in and collapsed. Of course this is a bridge but interesting nevertheless.

An important problem associated with wind induced motion of buildings is concerned with human response to vibration and perception of motion. At this point it will be sufficient to note that humans are astonishingly sensitive to vibration to the extent that motions may feel uncomfortable even if they correspond to relatively low levels of stress and strain. Therefore serviceability considerations will rule the design for most tall buildings and not strength issues.

The along wind response is due to the mean and the fluctuating drag force, which in turn are due to positive pressures on the buildings windward face and negative pressures (suctions) on the buildings leeward face (see Figure 3-3 for definition on wind- and leeward faces). It is not meaningful to distinguish between along- and across-wind responses if wind direction is not normal to one of the buildings faces. No practical methods are available for calculating responses in high rise buildings except for along-wind response normal to a rectangular building surface, **[10].** The across-wind response of a building is usually larger than the response in along-wind direction especially in tall slender buildings, one of the reasons can be that the air acts only with its ordinary speed and mass in the along-wind direction, but in the reflected wind (eddys and vortexes) the speed is slightly lower but the mass is considerably increased by the compression that the air gets when it hits the obstacle, and the momentum of any motion is composed of multiplying speed and mass. For good solution on across wind forces a wind tunnel test is required, **[6].**
In addition to the along wind and cross wind vibrations, a building can vibrate torsionally due to the random effects of wind.

3.2.1 DESIGN CRITERIA

In terms of designing a structure for wind loads the following basic design criteria need to be satisfied.

- Stability against falling down, uplift and/or sliding of the structure as a whole.
- Strength of the structural components of the structure is required to be enough to withstand imposed loading without failure during the lifetime of the structure.

The ULS (Ultimate Limit State)[1] wind speed satisfies stability and strength limit state requirements, in most international codes.

- Serviceability where an overall deflection is expected to remain within acceptable limits. An additional criterion that requires careful consideration in wind sensitive structures such as tall buildings is the control of sway acceleration when subjected to wind loads under serviceability conditions. This criterion is based on human tolerance to vibration discomfort in the upper levels of buildings.

Wind response is relatively sensitive to both mass and stiffness and response accelerations can be reduced by increasing either or both of these parameters. However this is in conflict with earthquake design optimization where loads are minimized in buildings by reducing both mass and stiffness. Increasing the damping results in a reduction in both, wind and earthquake responses, **[13].**

3.2.2 GUST EFFECTS

The wind has two components, one static and one dynamic, the dynamic character of wind is a generally constant mean wind velocity and a varying gust velocity, **[8].**
A gust is essentially a pocket of higher velocity wind within the general moving fluid air mass. The resulting effect of a gust is that of a brief increase, or gush, in the wind velocity, usually of not more than 15% of the sustained velocity and for only a fraction of a second in duration. Because of both its higher velocity and its smashing effect, the gust actually represents the most critical effect of the wind, **[1].**
The pressure due to the fluctuating component of wind, or gust, is difficult to calculate. It depends not only on the local terrain and nature of wind, but also on size, shape and dynamic properties of the building itself. As far as the vibration of tall buildings is concerned, the fluctuating component of wind pressure is the important component to consider.
Due to the mean component of wind load, tall buildings will deflect along the wind direction and vibrate about this deflected shape caused mainly by the fluctuating wind pressure component. In relation to occupant comfort, only the horizontal vibration needs to be considered.

[1] A structure satisfies the ultimate limit state criteria if all factored stresses are below the factored resistance calculated for the section under consideration

3.2.3 HOW TO FIND DESIGN WIND SPEED AT ANY LOCATION

This information can be retrieved from a close by airport, a local weather station, a helicopter landing site or if there have been similar or high constructions in the area that have been affected by the wind. However to acquire the design wind velocity the data collected needs some processing.
Information can also be found in books and standards for any land with wind records for any specified location. These winds are usually design wind velocity with the return period of 50 years and the peak wind velocity for seconds, minutes or 1 hour and the reference height of 10 m.
It should be noted that for the comfort criteria acceleration in the top floors of a structure, the design mean 20 min wind velocity with the return period of 6 years is recommended.**[9][6]**

3.2.4 DEPENDENCE OF WIND SPEED ON AVERAGING TIME

The value for the mean wind speeds depends upon the averaging time. As the length of the averaging interval decreases, the maximum mean speed corresponding to that length increases. The relation between the wind speed averaged over t seconds, $v_t(z)$, and the hourly speed, $v_{3600}(z)$, may be written as:

$$v_t(z) = v_{3600}(z)\left(1 + \frac{\xi^{1/2} \cdot c(t)}{2.5 \cdot \ln(z / z_0)}\right) \qquad \textbf{(Equation 3-9)}$$

where the coefficient $c(t)$ is determined on the basis of statistical studies of wind speed records. Values for $c(t)$ are listed in Table 3-2, which correspond to open terrain conditions (z_0 = 0.05 m) and an elevation z = 10 m. It is suggested that the values of the coefficient $c(t)$ listed in Table 3-2 are acceptable for roughness lengths z_0 up to 2.50 m. Values for ξ are listed in Table 3-1 corresponding to various roughness lengths z_0. Surface roughness lengths z_0 values are listed in Table 3-3, **[9].**

Type of terrain	Costal	Open	Sparsely built-up suburbs	Densely built-up suburbs	Centers of Large cities
z_0	0.005	0.07	0.30	1.00	2.50
ξ	6.50	6.00	5.25	4.85	4.00

Table 3-1 Approximate ratio for various surface roughness categories.

t [s]	1	10	20	30	50	100	200	300	600	1000	3600
c(t)	3.00	2.32	2.00	1.73	1.35	1.02	0.70	0.54	0.36	0.16	0.00

Table 3-2 Coefficient c(t) for various times. Corresponds to open terrain conditions (z_0=0.05m) and an elevation z=10m. Is assumed to works for terrain conditions up to z_0=2.50m.

Table 3-2 and equation 3-9 are combined to make Figure 3-2 and there the ratio of probable wind over period t and over one hour can be witnessed.

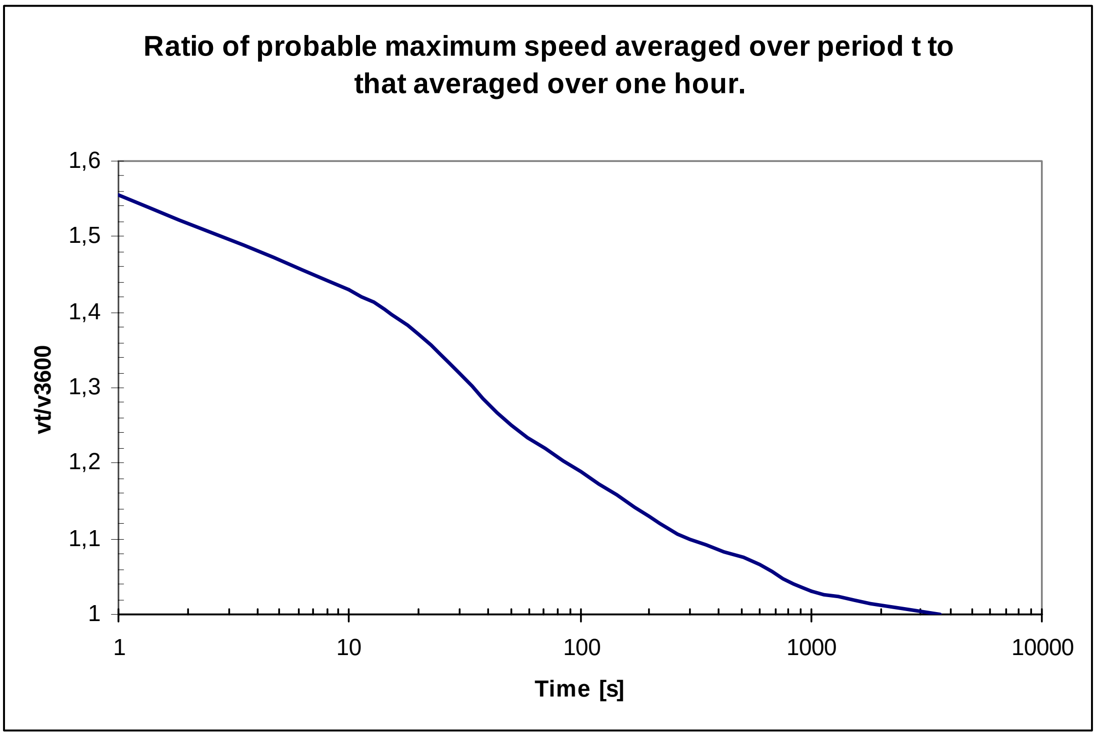

Figure 3-2 Ratio of probable maximum wind speed averaged over period t to that averaged over one hour. This corresponds to open terrain conditions (z_0 = 0.05 m) and an elevation z = 10 m. Is assumed to works for terrain conditions up to z_0=2.50m.

Figure 3-2 shows how the wind for shorter periods of time is a great deal higher than the wind for one hour. An example if an average wind is given for 10 seconds it can be seen on Figure 3-2 that it is 43 % higher than the average one hour wind and for 600 seconds the difference is 6.7 % (note that this is for the reference height z = 10 m).

3.2.5 WIND VARIATION WITH HEIGHT

Terrain type	Terrain category	Roughness length, z_0 [m]	Terrain parameter β [-]	Length, z_{min} [m]
I.	Exposed open terrain with few or no obstructions and water surface at serviceability wind speeds.	0.01	0.17	2
II.	Water surfaces, open terrain, grassland with few, well scattered obstructions having heights generally from 1.5 to 10 m.	0.05	0.19	4
III.	Terrain with numerous closely spaced obstructions 3 to 5 m high such as areas of suburban housing.	0.30	0.22	8
IV.	Terrain with numerous large, high (10 to 30m high) and closely spaced obstructions such as large city centers and well developed industrial complexes. (At least 15% of the area is built.)	1.00	0.24	16

Table 3-3 surface roughness length, Terrain parameter and the height which the exposure factor is constant for different terrain type. If the wind blows from e.g. terrain type I to III then the roughness length and terrain parameter do not change until 10 km into terrain type III
[2]

The wind velocity changes with height and is increased with height from the ground. The wind variation with height expresses that characteristic mean wind velocity, can be found with the following formula:

$$v_{mk}(z) = v_{ref} \cdot \sqrt{C_{\exp}(z)} \qquad \textbf{(Equation 3-10)}$$

where v_{ref} can be found in Swedish standards and the books **[2]** and **[5]** as the mean wind speed for 10 min, the reference height 10 m and the return period of 50 years. The exposure factor $C_{\exp}(z)$ is defined with

$$C_{\exp}(z) = \left(\beta \cdot \ln\left(\frac{z}{z_0}\right)\right)^2 \qquad z \geq z_{\min} \qquad \textbf{(Equation 3-11)}$$

where z is the height from the ground, z_0 is the roughness length, β is the terrain parameter and the value for z_{min} can be found in Table 3-3. If $z < z_{min}$ then the exposure factor is: **[2]**

$$C_{\exp}(z) = C_{\exp}(z_{\min})$$

3.2.6 REFERENCE- AND CHARACTERISTIC VELOCITY PRESSURE

The reference velocity pressure for any given location is defined as:

$$q_{ref} = \frac{1}{2} \cdot \rho_{Air} \cdot v_{ref}^2 \qquad \textbf{(Equation 3-12)}$$

where v_{ref} can be found in Swedish standards and the books **[2]** and **[5]** as the mean wind speed for 10 min, the reference height 10 m and the return period of 50 years for different locations. The density of the air is ρ_{Air} (typically equal to $1.25\,kg/m^3$).
The characteristic velocity pressure for any given location is defined as:

$$q_k = C_{dyn} \cdot C_{\exp} \cdot q_{ref} \qquad \textbf{(Equation 3-13)}$$

where the exposure factor is defined in equation 3-11 and for a static structure the wind gust factor C_{dyn} is defined as:

$$C_{dyn} = 1 + \frac{6}{\ln(H/z_0)} \qquad H \geq z_{\min} \qquad \textbf{(Equation 3-14)}$$

where H is the structure height, the roughness length $z_{0,}$ the terrain parameter β and the value for z_{min} can be found in Table 3-3. If H < z_{min} than the wind gust factor is $C_{dyn}(H) = C_{dyn}(z_{\min})$, the wind gust factor only depends on the structures height and the roughness length, **[2].**

Another way to find the wind load is given by **[6]**. The mean wind load in the direction of the wind for a unit area of the building is given by

$$q = C_D \cdot 0.047289 \cdot V^2 \qquad \textbf{(Equation 3-15)}$$

Where q is the pressure in $\left[\mathrm{N/m^2}\right]$, C_D is the drag coefficient [-] and V is the velocity in $\left[\mathrm{km/hour}\right]$. The drag coefficient for common shapes of tall buildings in a constant velocity field is approximately 1.3, **[6].**

3.2.7 ALONG-WIND RESPONSE

To compute the along-wind response of a structure it has to be assumed that the wind direction is perpendicular to one surface of the structure.

3.2.7.1 CLOSED-FORM EXPRESSIONS FOR THE ALONG-WIND RESPONSE

This chapter provides closed-form expressions for the along-wind response based on the logarithmic description of the wind profile, and the use of mean hourly wind speeds near the top of the structure. The fundamental mode of vibration is assumed to be approximately linear. The contribution of the second and higher vibration modes to the response in negligible. These equations are in principle only acceptable if $n_1 \cdot H / \overline{V}(H) \geq 0.1$ which is the case for most high rise structures.

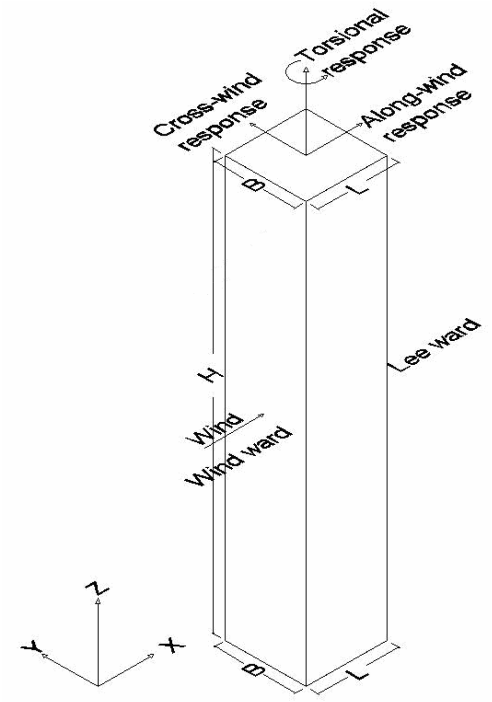

Figure 3-3 Shows where the wind hits the building and how the depth and width are determined along with the responses.

To find the peak along-wind deflection x_{pk} at elevation z [m] the following equation can be used:

$$x_{pk}(z) \approx \frac{0.5 \cdot \rho_{Air} \cdot u_*^2 \cdot C_D \cdot B \cdot z}{M_1 \cdot (2\pi \cdot n_1)^2} \left(\mathrm{J} + 3.75(\mathcal{B} + \mathrm{R})^{1/2} \right) \qquad \textbf{(Equation 3-16)}$$

Where the density of the air is ρ_{Air} (usually equal to $1.25\, kg/m^3$). The "rule of thumb" that geologists use to find the pressure on the ground is to say that for each plan the structure has the weight 10 kN/m^2 per plan. This can be used to estimate the total weight of the structure per meter. M_1 in [kg] is the fundamental modal mass of the building found with:

$$M(z) = B \cdot L \cdot \rho_b(z) \qquad \textbf{(Equation 3-17)}$$

$$M_1 = \frac{1}{H^2} \int_0^H M(z) \cdot z^2 \cdot dz \qquad \textbf{(Equation 3-18)}$$

where M(z) is the mass of structure per unit height, $\rho_{\mathrm{b}}(z)$ is the bulk mass of structure per unit volume, width is B (normal to the wind direction), and the depth is L (parallel to the wind direction). The friction velocity u_* in [m/s] is:

$$u_* = \frac{\overline{V}(H)}{2.5 \cdot \ln(H / z_0)} \qquad \textbf{(Equation 3-19)}$$

where $\overline{V}(H)$ is the mean hourly wind speed [m/s] at elevation H and z_0 is the surface roughness length. In Table 3-3 are examples of surface roughness lengths.

$$\mathrm{J} = 0.78 \cdot Q^2 \qquad \textbf{(Equation 3-20)}$$

$$Q = 2 \cdot \ln\left(\frac{H}{z_0}\right) - 1 \qquad \textbf{(Equation 3-21)}$$

$$\mathcal{B} = \frac{6.71 \cdot Q^2}{1 + 0.26 \cdot (B/H)} \qquad \textbf{(Equation 3-22)}$$

$$\mathrm{R} = \frac{0.59 \cdot Q^2 \cdot N_1^{-2/3}}{\zeta_1} \cdot \frac{C_{Df}^2}{C_D^2} \cdot \frac{\mathrm{C}(\eta_1)}{1 + 3.95 \cdot N_1 \cdot (B/H)} \qquad \textbf{(Equation 3-23)}$$

where ζ_1 is the damping ratio (it is usually set equal to 1% for steel and 2% for reinforced concrete frames, respectively). The quantities J, $\mathcal{B}$ and R are measures of the mean, quasi-static and resonant response, respectively. The logarithmic variable Q is needed in other equations, as is the variable N_1 given by:

$$N_1 = \frac{n_1 \cdot H}{u_* \cdot Q} \qquad \textbf{(Equation 3-24)}$$

where n_1 is the natural frequency of vibration in fundamental mode of vibration. The drag coefficient C_D and the reduced drag coefficient C_{Df}^2 are given by:

$$C_{Df}^2 = C_w^2 + C_l^2 + 2 \cdot C_w \cdot C_l \cdot \mathrm{C}(\eta_2) \qquad \textbf{(Equation 3-25)}$$

$$C_D = C_w + C_l \qquad \textbf{(Equation 3-26)}$$

where C_w and C_l are the average pressure coefficients on the windward face and on the leeward face respectively. The mean pressure and suction coefficients are functions of the shape of the structure. In the case of a tall building with a rectangular shape in plan, it may be assumed that $C_w = 0.8$ and $C_l = 0.5$. The drag coefficient C_D depends on the form of the building and the roof slope. For buildings hexagonal or octagonal in plan, values may be reduced by 20 %. For buildings that are round or elliptical in plan, values may be reduced by 40 % and 27% respectively, **[8].** The variables $\mathrm{C}(\eta_1)$ and $\mathrm{C}(\eta_2)$ are found with the following equations:

$$\mathrm{C}(\eta_i) = \frac{1}{\eta_i} - \frac{1 - e^{-2\eta_i}}{2 \cdot \eta_i^2} \qquad \textbf{(Equation 3-27)}$$

$$\eta_1 = 3.55 \cdot N_1 \qquad \textbf{(Equation 3-28)}$$

$$\eta_2 = \frac{12.32 \cdot N_1 \cdot \Delta}{H} \qquad \textbf{(Equation 3-29)}$$

where Δ is the smallest of the dimensions *H, B* and *L*. The building's height is *H*, width is *B* (normal to the wind direction), and the depth is *L* (parallel to the wind direction). The indicator *i* is the number 1 or 2 needed to calculate $\mathrm{C}(\eta_1)$ and $\mathrm{C}(\eta_2)$. The peak acceleration at elevation z can be found with the following approximate formula:

$$\ddot{x}_{pk}(z) \approx 4.0 \cdot \frac{0.5 \cdot \rho_{Air} \cdot u_*^2 \cdot C_D \cdot B \cdot z}{M_1} \cdot \mathrm{R}^{1/2} \qquad \textbf{(Equation 3-30)}$$

If the fundamental modal shape deviates much from a straight line the ratio *p* between the along-wind response which is calculated by taking the dynamic amplification into account on the one hand and by neglecting it on the other can be written in the form:

$$p \approx \frac{J + 3.75 \cdot (\mathcal{B} + \mathrm{R})^{1/2}}{J + 3.75 \cdot \mathcal{B}^{1/2}} \qquad \textbf{(Equation 3-31)}$$

If the fundamental modal shape is not a straight line, as was assumed in the development of equation 3-16 and equation 3-30, corrections may be applied to the calculated deflections and accelerations to account for the nonlinearity of the modal shape, **[10].**

3.2.7.2 ALONG-WIND RESPONSE OF STRUCTURES WITH AN APPROXIMATELY LINEAR FUNDAMENTAL MODAL SHAPE

This is in praxis the same method as in chapter 3.2.7.1 with the difference that the root mean squared (RMS) deflection and acceleration are included here along with the calculation of the peak factors for the deflection and acceleration. The peak factor is assumed to be 4 for the acceleration in chapter 3.2.7.1.

z_d is a zero plane displacement and for practical calculations it may be assumed that it is equal to zero. Hence:

$$Q = 2 \cdot \left(1 - \frac{z_d^2}{H^2}\right) \ln\left(\frac{H - z_d}{z_0}\right) - 1, \quad z_d = 0 \Rightarrow Q = 2 \cdot \ln\left(\frac{H}{z_0}\right) - 1 \qquad \textbf{(Equation 3-32)}$$

where z_0 is the surface roughness length and *H* is the height of the structure. The logarithmic variable Q is needed in equations 3-20, 3-22, 3-23 and 3-24. The wind friction pressure q_* in [N/m^2] is found with:

$$q_* = \frac{1}{2} \rho_{Air} \cdot u_*^2 \qquad \textbf{(Equation 3-33)}$$

where the density of the air is ρ_{Air} (usually equal to $1.25\,kg/m^3$), q_* is on the along-wind side of the structure and the friction velocity u_* can be found with equation 3-19. $\bar{x}$ is the mean displacement at the top of the structure given by:

$$\bar{x} = \frac{q_* \cdot C_D \cdot B \cdot H}{M_1 \cdot (2\pi \cdot n_1)^2} \cdot J$$ **(Equation 3-34)**

Where C_D is the drag coefficient given with equation 3-26, B is the width (normal to the wind direction), H is the height, n_1 is the first natural frequency of vibration and J is the mean response given with equation 3-18. M_1 is the fundamental modal mass given with equation 3-18. The σ_x is the root mean squared value of the fluctuating deflection given by:

$$\sigma_x = \frac{q_* \cdot C_D \cdot B \cdot H}{M_1 \cdot (2\pi \cdot n_1)^2} \cdot \left(\frac{\xi \cdot \mathscr{B}}{6} + R \right)^{1/2}$$ **(Equation 3-35)**

where ξ is the approximate ratio for various surface roughness categories. Values for ξ can be found in Table 3-1. The quasi-static response $\mathscr{B}$ can be found with equation 3-22 and the resonant response R can be found with equation 3-23. K_x is the peak factor (usually around 3 to 4) which can be found with the following equation:

$$v_x = n_1 \left(\frac{R}{R + (\xi \cdot \mathscr{B}/6)} \right)^{1/2}$$ **(Equation 3-36)**

$$K_x = \left(1.175 + 2 \cdot \ln(v_x \cdot T)\right)^{1/2}$$ **(Equation 3-37)**

where T is the duration of the storm and indicates that the expected peak values of the fluctuations will be higher if the duration of the storm increases. The assumed storm duration is implicit in the use of design mean speed (usually between 600 and 3600 s).

$$G = 1 + K_x \frac{\sigma_x}{\bar{x}}$$ **(Equation 3-38)**

where G is the gust response factor and

$$X_{pk} = G \cdot \bar{x}$$ **(Equation 3-39)**

where X_{pk} is the peak displacement at the top of structure

$$\sigma_{\ddot{x}} = \frac{q_* \cdot C_D \cdot B \cdot H}{M_1} \cdot R^{1/2}$$ **(Equation 3-40)**

where $\sigma_{\ddot{x}}$ is the root mean squared value of the acceleration at the top of the structure.

$$K_{\ddot{x}} = \left(1.175 + 2 \cdot \ln(n_1 \cdot T)\right)^{1/2}$$ **(Equation 3-41)**

where $K_{\ddot{x}}$ is the peak factor (usually about 4)

$$\ddot{X}_{pk} = K_{\ddot{x}} \sigma_{\ddot{x}} \qquad \textbf{(Equation 3-42)}$$

where $\ddot{X}_{pk}$ is the peak acceleration at the top of the structure, which is needed for comparison with the comfort criterion, **[9].**

3.2.7.3 WIND INDUCED VIBRATIONS

This is the method used by the Turning Torso engineers, there is not given an explanation or reference to this method in their papers. This method could not be studied as required but is included for comparison.
In terms of a model approach, a wind action of a given intensity can be subdivided into a stationary portion and a turbulent portion:

$$q = q_s + (1 + \beta) \cdot q_d \qquad \textbf{(Equation 3-43)}$$

Where q is the total wind pressure, q_s is the stationary wind pressure, q_d is the peak value of the distribution of wind pressures due to turbulences (gusts),

$$q_{d,\beta} = (1 + \beta) \cdot q_d \qquad \textbf{(Equation 3-44)}$$

This is the peak value of the distribution of wind pressures due to gusts corresponding to a wind action and β is the amplification coefficient considering the dynamic nature of wind pressures due to gusts. The dynamic amplification coefficient β is:

$$\beta = \frac{\alpha}{1 - \alpha^2} \sqrt{1 + \alpha^2 - 2 \cdot \alpha \cdot \sin\left(\frac{\pi}{2 \cdot \alpha}\right)} \qquad \textbf{(Equation 3-45)}$$

where α is:

$$\alpha = \frac{f_{q_{wd}}}{f_B} \qquad \textbf{(Equation 3-46)}$$

where $f_{q_{wd}}$ is the frequency of the harmonic distribution of the wind pressures due to gusts and f_B is the natural frequency of structure subjected to wind action.
The stationary portion results in static wind pressures and constant deformations of the structure the wind is acting on. While the turbulent portion of wind action gives rise to vibration which are superposed to the constant deformation pattern. That means that the vibration takes place with respect to the deformed position due to the stationary portion of wind action.
Consequently, only the gust portion of wind action is of relevance for problems of human perception of vibrations and the related comfort conditions in buildings.
Wind pressures due to turbulences are of uncontrollable nature. For analysis purposes, the gusts are usually assumed to have a harmonic distribution over time.

It has to be pointed out that the increase with the height above ground of the gust wind pressures is significantly less pronounced than for the stationary wind pressure, **[25].** To find the maximum acceleration for harmonic vibration the following equation can be used:

$$a_{\max} = A \cdot \omega^2 = A \cdot (2\pi \cdot f)^2 \qquad \textbf{(Equation 3-47)}$$

Where A is the amplitude of vibration and f is the frequency of vibration. Amplitude A should not be confused with overall lateral deflection δ of the building. The maximum deflection is given by $\delta = \Delta + A$ in which Δ is the mean static deflection about which the building will then oscillate with an amplitude A.
[4] The amplitude A for a rigid[2] structure can be found with:

$$A = \frac{q_{d,\beta} \cdot H^4}{8 \cdot E \cdot I} \qquad \textbf{(Equation 3-48)}$$

Where $q_{d,\beta}$ is in N/m, H is the height of the structure, E is the corresponding Youngs modulus and I is the moment of inertia, **[7].**

3.2.7.4 GUST LOADING FACTORS METHOD (ALAN G. DAVENPORT)

The superimposed effect of wind is the foundation of gust factor approach, which is the separation of wind loading into mean static and fluctuating components. The mean load factor is evaluated from the mean wind speed using pressure and load coefficients. The fluctuating loads are determined separately by a method which makes an allowance for the intensity of turbulence at the site, size reduction effects and dynamic amplification, **[13].**

3.2.7.4.1 The peak factor method

The average largest response during a period T in seconds (T is the time period the reference wind is taken over usually between 10 min and 1 hour) is given by:

$$Y_{\max} = \overline{Y} + g \cdot \sigma_Y \qquad \textbf{(Equation 3-49)}$$

Where $\overline{Y}$ is the mean response to the mean wind load and g is the peak factor given by:

$$g = \sqrt{2 \cdot \ln(\nu \cdot T)} + \frac{0.57}{\sqrt{2 \cdot \ln(\nu \cdot T)}} \qquad \textbf{(Equation 3-50)}$$

where ν is the number of times the mean value is crossed per unit time. For a lightly damped system this can be approximated to the first natural frequency of the system that is $\nu \approx n_0$. The second part of equation 3-49 is the only part of interest, because it

[2] The concept of rigid motion is that of a physically inflexible solid, which must be moved as a single entity so that its movement is completely determined by the displacement of a single "point" and the orientation of the solid body about that point.

has to do with the fluctuating nature of wind forces associated principally with gusts. This part is corresponding to the amplitude A from chapter 3.2.7.3 and equation 3-48 thus:

$$A = g \cdot \sigma_Y \qquad \textbf{(Equation 3-51)}$$

There is not given a good explanation to find the RMS (root mean squared) deflection σ_Y by Davenport so the values can be found with equations in chapter 3.2.7.2 and then the maximum acceleration for harmonic vibration can be found with equation 3-47.

3.2.7.4.2 Gust pressure factor method

It is assumed that the mean wind load can be found with:

$$q(z) = \frac{1}{2} \cdot \rho \cdot V^2 \cdot C_D \qquad \textbf{(Equation 3-52)}$$

Where V is the velocity near the top of the structure, ρ is the air density and C_D is the drag coefficient.
The gust pressure factor method is intended to take account of the superimposed dynamic effect of gusts. The gust factor is used in combination with the mean load so that the total wind loading is:

$$q(z)_{\max} = G \cdot q(z) \qquad \textbf{(Equation 3-53)}$$

Where G is the “gust factor” and $p(z)$ is the mean wind load. The “gust factor” is given by:

$$G = 1 + g \cdot r \cdot \sqrt{\hat{\mathbf{B}} + \hat{\mathbf{R}}} \qquad \textbf{(Equation 3-54)}$$

Where g is the peak factor found with equation 3-50, r is the roughness factor, $\hat{\mathbf{B}}$ is the excitation by background turbulence and $\hat{\mathbf{R}}$ is the excitation by turbulence resonant with structure found by:

$$\hat{\mathbf{R}} = \frac{s \cdot F}{\zeta_1} \qquad \textbf{(Equation 3-55)}$$

Where s is the size reduction factor, F is the gust energy ratio and ζ_1 is the damping ratio of the structure (it is usually set equal to 1% for steel and 2% for reinforced concrete frames, respectively). The size reduction factor is found by:

$$s = \frac{\pi}{3}\left(\frac{1}{1+\frac{8}{3 \cdot \xi_0}} \cdot \frac{1}{1+10 \cdot \frac{B}{H \cdot \xi_0}}\right) \qquad \textbf{(Equation 3-56)}$$

Where B and H are the width and height of the structure respectively and ξ_0 is the dimensionless frequency denoted as:

$$\xi_0 = \frac{H \cdot n_0}{V} \quad \textbf{(Equation 3-57)}$$

The gust energy ratio F is found by:

$$F = \frac{\left(\frac{1219.2 \cdot n_0}{V}\right)^2}{\left(1 + \left(\frac{1219.2 \cdot n_0}{V}\right)^2\right)^{4/3}} \quad \textbf{(Equation 3-58)}$$

The roughness factor r is given by:

$$r = 4 \cdot \sqrt{z_0} \cdot \left(\frac{9.14}{H}\right)^{2 \cdot \alpha} \quad \textbf{(Equation 3-59)}$$

where α is equal to 0.40 for city conditions and 0.16 for open country conditions and z_0 is the surface roughness length. The excitation by background turbulence B is given by:

$$\hat{\mathbf{B}} = 2 \cdot \left(1 - \frac{1}{\left(1 + \left(\frac{457.2}{H}\right)^2\right)^{1/3}}\right) \quad \textbf{(Equation 3-60)}$$

Note that in equation 3-58, 3-59 and 3-60 the constants needed to be altered to fit the metric system. Also note that equation 3-60 was altered from a constant given in **[11]** which was clearly a mistake, to the equation 3-60. This was done be studying Fig.6. in **[11]** that shows the values for excitation by background turbulence B compared with the height of structure above ground, and finding the equation that fits the curve on that figure, the result is equation 3-60. (If equations 3-58, 3-59 and 3-60 would be used with ft instead of meters the constants ought to be changed from-to 1219.2 m = 4000 ft, 9.14 m = 30 ft and 457.2 m = 1 500 ft).

3.3 ACCELERATION AND COMFORT CRITERIA

It should be understood that a building is not something that will move about like a vehicle or an airplane. These structures are supposed to move, but to the layman a building is not. To use data from tests that were not planned for tall buildings is uncertain. To establish perception threshold it could be done by using artificial tests, but it is tricky to accept that tolerance thresholds can be properly determined from test environments. The psychological factors involved in perceiving tall building motion are unique and difficult to duplicate artificially.
It must be understood that perception of motion is not the same as tolerance. It would be expensive to design and construct a building that would not sway in the worst storm or during large earthquakes. Some movement in the top floors must be allowed, but the purpose is to establish levels of motion and corresponding occurrence rates which are acceptable to both building occupants and the building owner. The swaying in the top floors of a building need to have some threshold, so buildings can be created without being an embarrassment to the design engineers and the users, **[6].**

The perception and tolerance of motion for human is kinesiological, psychological response and physiological reactions.
The psychological perception and tolerance threshold varies widely depending on the motion and following human factors:
Individual difference: All humans are different and have different sensitivity for motion and vibration.
Gender: The responses of men and women are essentially the same, although women are slightly more sensitive than men.
Age: The sensitivity of humans to motion is an inverse function of age, with children being the most sensitive.
Body type: Sensitivity to motion is not a function of body type, whether “tall or short” or “fat or thin”.
Body posture: The degree of motion sensitivity is proportional to the distance of a subjects head from the floor. The higher the head is from the floor, the greater the sensitivity.
Body orientation: Humans are more sensitive to fore-and-aft (back and front) motion than to side-to-side motion. This is because the head can move more freely in the fore-and-aft direction.
Expectancy and experience: People that know the building is going to move will have less tolerance for the acceleration than the people not expecting movement. The acceleration threshold is approximately twice for people not knowing movement is going to occur compared to people expecting movement.
Body movement: Perception thresholds for walking subjects are generally higher than for standing subjects. A walking person can endure more acceleration, vibration than a person that is standing still.
Visual clues: Visual clues play an important part in confirming perception of motion. The eyes can perceive the motion of objects in a building, and can observe the rotation and swaying of the building relative to the world outside.
Acoustic clues: Acoustic clues also affect human perception. Buildings can make sounds when swaying and sound of wind whistling outside the building.

The effects of tall building motions on the kinesiological and physiological reactions of the human body are important. The following factors are important to physiological reactions:
Swaying state of the body: With regard to body orientation motion in the for-and-aft direction caused greater sway than that in side-to-side direction. The balance states of the females are more sensitive than for males. This is important because the swaying state threshold is similar to the perception and tolerance thresholds.
Acceleration induced on the body: A standing person is mechanically like an inverted pendulum and the motion of the floor will cause a person to pivot about his ankles imparting angular acceleration to the body and head. The accelerations of the body and head are larger than the acceleration of the floor.
Motion effects on human task performance: Human task performance is directly related to floor acceleration.
Physiological reactions and motion sickness: Even though the level of motion is low, people become sick when exposed to motion for a long time.
Sense organ: The skin can sense vibration when parts of the body are touching a source of vibration. The deep senses perceive vibration by the stimulation of muscle spindle, tendon organ and pacinian corpuscle (responsible for sensitivity to deep pressure touch and high frequency vibration). The deep senses are important to control body motion. The vestibular organ sense directly perceives motion and maintains equilibrium and the vision and auditory sense confirm the suspicion that motion is occurring. It is conceivable that the vision and hearing senses may be the most important for sensing large amplitude motion. Motions only sensed with the eyes can produce motion sickness.
Difficulty of walking: When subjects walk in the same direction as the floor motion, they experience difficulty in maintaining balance primarily in the fore and aft (back and front) direction.
When walking at right angel to the motion, subjects experience difficulty in keeping balance in both fore and aft (back and front) and side to side direction.
Movement of furniture and fixtures:
Hanging objects begin to move when the acceleration reaches $0.05\, m/s^2$ (0.005 g)[3] and begin to make intermittent sound at $0.2\, m/s^2$ (0.02 g) and make a sound at $0.4\, m/s^2$ (0.04 g). Furnitures begin to fall over when acceleration reaches $0.85\, m/s^2$ (0.085 g).
The threshold accelerations for high rise buildings are:[4]

- The perception threshold is less than $0.05\, m/s^2$ (0.005 g)
- The limit of psychological and task performance is about $0.4\, m/s^2$ (0.04 g)
- The limit of walking is $0.5\, m/s^2$ (0.05 g) to $0.7\, m/s^2$ (0.07 g)
- For safety considerations the maximum limit of building motion should not be allowed to exceed $0.85\, m/s^2$ (0.085 g)

or more detailed:

- For accelerations less than $0.05\, m/s^2$ (0.005 g) a human cannot distinguish motion, and task performance and walking are not affected at all.

[3] "g" is the earths gravity.
[4] It has to be noted that the acceleration looked at is the peak acceleration, this is not clear from the literature, **[6]**, but it is indicated.

- Between $0.05\,m/s^2$ and $0.10\,m/s^2$ (0.005 g and 0.01 g) some people can identify motion, and some furniture and fixtures such as pendant lights and water begin to move slightly, but these movements are generally not observable except to a person who looks directly at them.
- Between $0.1\,m/s^2$ and $0.25\,m/s^2$ (0.01 g and 0.025 g) most people are able to perceive motion and this level of motion slightly affects desk work. If the motion continues for several hours there are some people who complain of motion sickness. People can walk without hindrance.
- Between $0.25\,m/s^2$ and $0.4\,m/s^2$ (0.025 g and 0.04 g) desk work becomes difficult and at times almost impossible. Most people can walk and go up and down stairs without too much difficulty. Furniture and fixtures start to make sounds.
- Above $0.4\,m/s^2$ (0.04 g) the effect of the period of motion is important and people strongly perceive motion. Standing people lose their balance and find it hard to walk naturally
- Over $0.5\,m/s^2$ (0.05 g) the effect of the period of motion is greater. Most people cannot tolerate the motion and are unable to walk.
- About $0.6\,m/s^2$ (0.06 g) people very strongly perceive motion and cannot walk. This is considered to be the limit of walking ability.
- Acceleration over $0.85\,m/s^2$ (0.085 g) a few objects begin to fall and it is expected that some people may be injured even if they remain still. Accelerations greater than $0.85\,m/s^2$ are undesirable for building motion, **[6].**

There is no generally accepted international standard for comfort criteria in tall building design. However a sizeable amount of research has been carried out into the important physiological and psychological parameters that affect human perception to motion and vibration in the low frequency range of 0-1 Hz that is common in tall buildings. Table 3-4 gives some guidelines to general human perception levels. Table 3-4 includes the various parameters stated prior. The acceleration looked at is the peak acceleration.[5]

[5] Again it is not clear from the literature, **[13],** if the acceleration looked at is the RMS or the peak, but by studying this it is most likely to be the peak acceleration.

Peak Acceleration	Perceptibility
0.000g <a< 0.005g	People cannot perceive motion.
0.005g <a< 0.010g	Sensitive people can perceive motion and hanging objects may move slightly.
0.010g <a< 0.025g	Majority of people will perceive motion, level of motion may affect desk work and long-term exposure may produce motion sickness.
0.025g <a< 0.040g	Desk work becomes difficult or almost impossible but ambulation is still possible.
0.040g <a< 0.050g	People strongly perceive motion, have difficulties to walk naturally and people standing may lose their balance.
0.050g <a< 0.060g	Most people cannot tolerate motion and are unable to walk naturally.
0.060g <a< 0.070g	People cannot walk or tolerate motion.
a> 0.085g	Objects begin to fall and people may be injured.

Table 3-4 Human perceptibility threshold for harmonic vibration (Valid for frequency range 0 to 1 Hz), [13].

Based upon interviews of people in two buildings after severe storms the return periods for storms causing a root mean squared horizontal acceleration at the building top which exceeds tentative values $\overline{\sigma}^* = 0.05\,m/s^2$ (0.005 g) shall not be less than R^* = 6 years. To investigate the occupants comfort a recurrence of wind between 6 and 10 years may be sufficient, depending on the nature of the building. The root mean squared shall represent an average over the 20 min period of highest storm intensity and be spatially averaged over the building floor, **[9],[6].**

"*In North America codes a proposed ranges of acceptable peak accelerations with 10-year return period at the top floors are 0.015 g to 0.020 g for offices and 0.010 g to 0.015 g for residential buildings. However, it has been determined that acceptable accelerations levels decrease as the oscillation frequency increases, so it has been suggested that these limits be reduced for higher frequencies of vibration, from the values just stated, which are assumed to be valid for frequencies of 0.1Hz, to about half of those values for frequencies of 1Hz,*" **[10].**

Peak acceleration	Perceptibility
0.000g <a< 0.005g	imperceptible
0.005g <a< 0.015g	perceptible
0.015g <a< 0.050g	disturbing/unpleasant
0.050g <a< 0.100g	annoying
0.100g <a< 0.150g	very annoying
0.150g <a>	intolerable

Table 3-5 Human perceptibility threshold for harmonic vibration (Valid for frequency range 1 to 10 Hz), [25].

Table 3-5 is the perceptibility limit engineers at Turning Torso used. This table has higher values for the thresholds of perceptibility.

3.4 DRIFT RATIO CRITERION

It has been expressed by several engineers, working in the design of high rise buildings, that an occupants comfort criterion should be expressed in terms of a drift ratio, which is a dimensionless number obtained by dividing the displacement at the top of the building by the building height. This ratio is often expressed in the form 1/N, in which N is a number like 500.
No number has formally been suggested for limiting human discomfort caused by building motion. Since a drift ratio is equivalent to a stiffness constraint, it may not be a good parameter to use for controlling human discomfort. This is because human perception and discomfort appear to be closely related to building acceleration, which is only weakly affected by a change in stiffness. The drift ratio criterion can be misleading as an indicator of the expected performance of a motion with regard to human response to tall building wind induced motions, **[6].**

3.5 WIND TUNNEL TESTS

Wind tunnel testing is a powerful tool that allows engineers to determine the nature and intensity of wind forces acting on complex structures. Wind tunnel testing is particularly useful when the complexity of the structure and the surrounding terrain, resulting in complex wind flows, does not allow the determination of wind forces using simplified code provisions. Wind tunnel testing involves blowing air on the building model under consideration and its surroundings at various angles relative to the building orientation representing the wind directions. This is typically achieved by placing the complete model on a rotating platform within the wind tunnel. Once testing is completed for a selected direction, the platform is simply rotated by a chosen increment to represent a new wind direction, **[13].**

Errors and uncertainties with wind tunnel simulations can be large. These errors are due to the violation of the Reynolds number in the laboratory, flow simulation problems, possible blockage problems and possible measurement problems. Comparison on measurements from 6 wind tunnels around the world showed that the results varied widely among wind tunnels, the coefficients of variation being as high as 30%, **[10].**

3.6 WIND FORCES IN THE FUTURE

In the future the temperature of the earth will be climbing due to greenhouse effects. Greenhouse effects have started to set its impact on the weather today and one of its effects is more weather extremes. These increases in extremes are important from the wind engineering viewpoint and can have its pros and cons. The pros are that wind can be used to provide power for mills and electricity from windmills for example. The cons are that wind related disasters are the most costly in terms of property damage and fatalities. Wind engineers should be significantly worried with wind relations with human activities, especially if winds are getting more violent as is expected in the future.

4 RESULTS

In this chapter the dimensions of Turning Torso and Smáratorg Tower are included, the first natural frequency is to be found, the wind induced vibrations, the acceleration due to gust effects of wind and these results compared to the acceleration and comfort criteria.

4.1 DIMENSIONS OF TURNING TORSO

The height of Turning Torso is 190.4 m but the height of the core over the ground that can be assumed to be participating in the bending is 175.5 m.
The width of Turning Torso is B = 26.5 m and the depth is L = 16.5 m.

Because of the 90° twist of the Turning Torso, engineers for Turning Torso have estimated a reduction form factor for the shape as $\alpha = 0.8$.This assumption for the reduction form factor will be used in further calculation, a discussion on this reduction will materialize later.

The first natural frequency engineers at Turning Torso got with FEM modeling is 0.198 Hz

The core of Turning Torso is constructed with C55/65 concrete thus the Youngs modulus is E_c=36000 N/mm^2.

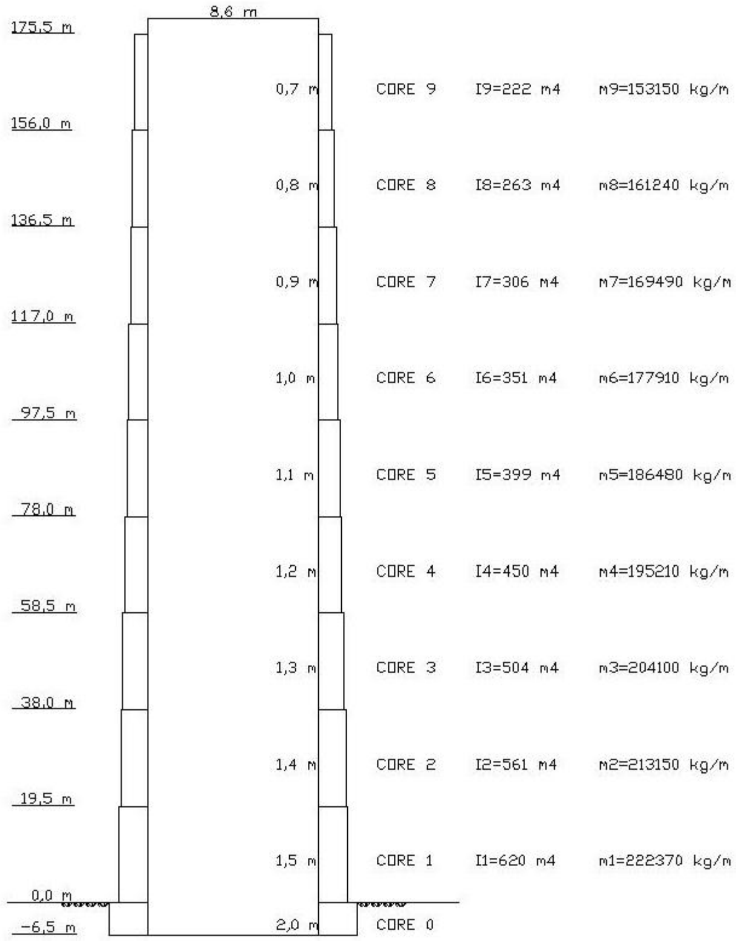

Figure 4-1 Shows a schematic drawing that holds the modular height for the structural core for Turning Torso with the information for the moment of inertia and the mass per meter for each core.

The dead load slab construction (includes steel grid, composite floor and pavement) is approximately equal to 2500 kN/floor.
The dead load façade (includes entire edge construction and facade) is approximately equal to 480 kN/floor.
The live load on floors is approximately equal to 300 kN/floor.
This corresponds to the total load of 3280 kN/floor plus the load from the structural core.

The wind velocities that engineers at Turning Torso used for calculations are:
V_q = 50 m/s at ground level, corresponding to q = 1.5 kN/m^2.

V_q = 70 m/s at top of tower, corresponding to q = 3.0 kN/m^2.

Engineers at Turning Torso used the gust wind pressure equal to $q_d = 0.7\,kN/m^2$ (corresponding to a wind action with a recurrence period of 50 years), **[25].** The selection of this gust is not given any explanation for what wind velocity it should correspond to.

4.2 THE FIRST NATURAL FREQUENCY OF VIBRATION

The largest dynamic response generally occurs in the lowest mode or modes of vibration of the structure with higher frequencies relatively inactive. There are reasons for this principally that the greatest energy in the wind almost invariably exists at lower frequencies. Thus, estimation of the amplitudes in the lowest modes alone is adequate for many design purposes, **[11].**

4.2.1 THE EQUATION OF MOTION

To compute the first natural frequency of Turning Torso the equation of motion is used. The shape function selected is shown in equation 3-5 because it has shown to be the most accurate of these three shape functions. The height H is 175.5 m, Youngs modulus is for the structural core that has C55/65 concrete is E=36000 N/mm^2 and for the moment of inertia is shown in Figure 4-1 how it changes with height. The mass $m(x)$ is for the corresponding core component and changes with height similar to the moment of inertia, but there is a constant mass for each floor that is combined from the buildings self weight, the weight from furniture and occupants (it is a “rule of thumb” that the weight of a building is 10 kN/m^2 for each floor of the building plus all unordinary structural components. This “rule of thumb” is known in the geotechnical community and used to determine the pressure that acts on the ground from a structure **[22]**). To get the mass for the core it is assumed that the reinforced concrete has the self weight of 2550 kg/m^3. It is known that the total load without the structural core is 3280 kN/floor and there are 6 floors (with the combined terrace and technique floors) and the height of each module is 19.5 m. Then the mass $m(x)$ has the weight per meter:

$$m(x) = \frac{3280kN/floor \cdot 6\,floor}{19.5m} = 1010kN/m = 101000kg/m$$

Plus the weight from the corresponding structural core per meter, that is needed in the calculation, see Figure 4-1 for the total mass per meter of each building part.
With this information the generalized mass $\tilde{m}$ and generalized stiffness $\tilde{k}$ can be found with equation 3-3 and 3-4 respectively.

$$\tilde{m} = \int_{0m}^{19.5m} 222370\,kg/m \cdot \left[\frac{3x^2}{2H^2} - \frac{x^3}{2H^3}\right]^2 dx + \int_{19.5m}^{38.0m} 213150\,kg/m \cdot \left[\frac{3x^2}{2H^2} - \frac{x^3}{2H^3}\right]^2 dx + \ldots$$

$$\ldots + \int_{156.0m}^{175.5m} 153150\,kg/m \cdot \left[\frac{3x^2}{2H^2} - \frac{x^3}{2H^3}\right]^2 dx = 6747300\,kg$$

$$\tilde{k} = \int_{0m}^{19.5m} 36\frac{GN}{m^2}\cdot 620m^4 \cdot \left[\frac{6}{2H^2} - \frac{6x}{2H^3}\right]^2 dx + \int_{19.5m}^{38.0m} 36\frac{GN}{m^2}\cdot 561m^4 \cdot \left[\frac{6}{2H^2} - \frac{6x}{2H^3}\right]^2 dx + \ldots$$

$$\ldots + \int_{156.0m}^{175.5m} 36\frac{GN}{m^2}\cdot 222m^4 \cdot \left[\frac{6}{2H^2} - \frac{6x}{2H^3}\right]^2 dx = 10402000\,N/m$$

Then the first natural frequency can be found with equation 3-1 and 3-2

$$\omega = \sqrt{\frac{10402000N/m}{6747300kg}} = 1.24163s^{-1}$$

$$f = \frac{1.24163}{2\pi} = 0.1976\,Hz$$

This first natural frequency is actually the same natural frequency as the engineers for Turning Torso got from their FEM model, it only differs with 0.2 %.

Let us see what value the other shape functions give for the first natural frequency. The shape function obtained with equation 3-6 gives, with the same calculations:

$$f = 0.1988\,Hz$$

This is again a reasonable outcome for the first natural frequency or only differs with 0.4 % from the first natural frequency from the FEM model.

The shape function given in equation 3-7 gives, with the same analogous calculations:

$$f = 0.2203\,Hz$$

$$\psi(x) = \frac{3x^2}{2H^2} - \frac{x^3}{2H^3} \qquad \textbf{(Equation 3-5)}$$

$$\psi(x) = 1 - \cos\left(\frac{\pi x}{2H}\right) \qquad \textbf{(Equation 3-6)}$$

$$\psi(x) = \frac{x^2}{H^2} \qquad \textbf{(Equation 3-7)}$$

Figure 4-2 Shows the shape functions, this is just recapitulated here for convenience to the reader.

This is not a reasonable outcome for the first natural frequency since it differs with 11.3 % from the first natural frequency from the FEM model. This shows that the shape functions given in equation 3-5 and 3-6 are ideal for the calculations of the first natural frequency, but the shape function given in equation 3-7 gives a poor approximation of the first natural frequency.

If the approximation is made that the “rule of thumb” known in the geotechnical community will apply then the mass per meter is:

$$m(x) = \frac{10kN/(m^2\cdot floor)\cdot(16.5m\cdot 16.5m + 16.5m\cdot 10.3m/2)\cdot 6\,floor}{19.5m} = 1100kN/m = 110000kg/m$$

Then the frequency obtained with this mass and from shape functions shown in equation 3-5, 3-6 and 3-7 are 0.1924Hz, 0.1935Hz and 0.2145Hz respectively. These frequencies differ with 2.8 %, 2.3 % and 8.3 % respectively, compared with the first

natural frequency from the FEM analysis. This difference is acceptable and should be a good approximation for the first natural frequency.
See appendix for matlab application program used to compute the first natural frequency.

4.2.2 THE "RULE OF THUMB"

If the result for the first natural frequency of vibration of a tall slender structure is to be found rapidly the "rule of thumb" can be used. The frequency from the "rule of thumb" for Turning Torso is:

$$f = \frac{30.48 Hz \cdot m}{175.5 m} = 0.174 Hz$$

This frequency is less than the first natural frequency of Turning Torso (0.198Hz) the difference is 12%.

4.3 WIND

Wind is due to the movement of air and is mainly horizontal. Wind is a product of the uneven heating of the Earth's surface. The two major influences on the atmospheric circulation are the differential heating between the equator and the poles, and the rotation of the planet (Coriolis effect).

4.3.1 ALTERING WIND SPEED WITH AN AVERAGING TIME TO AVERAGING HOUR.

To find the wind for Turning Torso the reference wind v_{ref} can be found in Swedish standards and the books **[2]** and **[5]** as the mean wind velocity for 10 min, the reference height 10 m and the return period of 50 years. This wind for Malmö, Sweden is $v_{ref} = 26 m/s$ for terrain type II and roughness length z_0=0.05m.

Because the formulas presented in chapter 3.2.7 expect mean hourly wind velocity and not mean 10 minutes wind velocity it has to be changed so the results can be used for calculations of along-wind response. This can be done with the use of Table 3-1, Table 3-2, Table 3-3 and equation 3-9. Let us change the mean reference 10 minutes wind velocity $v_{ref} = 26 m/s$ (t = 600 s) to mean hourly wind velocity through equation 3-9 at the elevation z = 10 m, for this the approximate ratio is selected $\xi = 6.0$ and the coefficient $c(t) = 0.36$ for 600 seconds:

$$v_{ref,3600}(z) = \frac{v_t(z)}{\left(1 + \frac{\xi^{1/2} \cdot c(t)}{2.5 \cdot \ln(z/z_0)}\right)} = \frac{26.0 m/s}{\left(1 + \frac{6.0^{1/2} \cdot 0.36}{2.5 \cdot \ln(10m/0.05m)}\right)} = 24.4 m/s$$

Now the new mean hourly wind velocity can be used in further calculations.

4.3.2 WIND VARIATION WITH HEIGHT

For design purpose it is the wind speed near the top of the structure that is needed to compute, this height corresponds to $z = 175.5m$ for Turning Torso. With this information and the terrain parameter β from Table 3-3 the exposure factor is:

$$C_{\exp}(z = 175.5m) = \left(0.19 \cdot \ln\left(\frac{175.5m}{0.05m}\right)\right)^2 = 2.406$$

And the characteristic mean wind velocity at the height 175.5 m is:

$$v_{mk}(z) = 24.4\,m/s \cdot \sqrt{2.406} = 37.8\,m/s$$

This wind velocity $37.8\,m/s$ and the wind velocity $70\,m/s$ that engineers at Turning Torso used at the top of the structure are the one used in further calculations.
But if the same method is used to find the wind velocity near the top of the structure for the wind $50\,m/s$ as engineers at Turning Torso used at ground level, it becomes:

$$v_{mk}(z) = 50\,m/s \cdot \sqrt{2.406} = 77.6\,m/s$$

at the top. This difference is possibly because of some rough estimation on the behalf of engineers at Turning Torso, or perhaps because of different computation methods.

4.3.3 REFERENCE- AND CHARACTERISTIC VELOCITY PRESSURE

To find the reference velocity pressure, the reference velocity is the same as before $v_{ref} = 24.4\,m/s$ and the density of the air is $\rho_{Air} = 1.25\,kg/m^3$ as recommended, then the pressure is:

$$q_{ref} = \frac{1}{2} \cdot 1.25\,kg/m^3 \cdot (24.4m/s)^2 = 372.1\,N/m^2$$

The exposure factor is the same as in chapter 4.3.2 $C_{\exp}(z = 175.5m) = 2.406$, with the height H=175.5m and the roughness length $z_0 = 0.05$ m, the wind shock/gust factor C_{dyn} is:

$$C_{dyn} = 1 + \frac{6}{\ln(175.5m/0.05m)} = 1.735$$

Now the characteristic velocity pressure near the top of the structure can be found as:

$$q_k = 1.735 \cdot 2.406 \cdot 372.1\,N/m^2 = 1553\,N/m^2 = 1.553\,kN/m^2$$

This pressure for the wind velocity that engineers at Turning Torso used at ground level $q_{ref}(50m/s) = 1.563\,kN/m^2$ is:

$q_k = 1.735 \cdot 2.406 \cdot 1.563 kN/m^2 = 6525 N/m^2 = 6.525 kN/m^2$

If the mean velocity pressure near the top of the structure is found with the method given in equation 3-15 there the wind velocity is in kilometers pre hour, $37.8\, m/s \cdot 3.6(km \cdot s/m \cdot hour) = 136\, km/hour$

$q = (1.3 \cdot 0.8) \cdot 0.047289 \cdot (136)^2 = 1.307 kN/m^2$

This pressure for the wind velocity that engineers at Turning Torso used at the top of the tower $70\, m/s \cdot 3.6(km \cdot s/m \cdot hour) = 252\, km/hour$ is:

$q = (1.3 \cdot 0.8) \cdot 0.047289 \cdot (252)^2 = 3.123 kN/m^2$

The difference from these calculations is substantial or 16 % for the reference wind and 52 % for the wind velocity engineers at Turning Torso used. These wind pressures are found with widely different parameters where one depends on the reference velocity, exposure factor and a gust factor, while the other depends on the wind near the top and the drag coefficient. The author does not recommend these methods to find the wind pressure. The method used in chapter 3.2.7.4.2 to find the maximum wind pressure is rather recommended, there the gust factor and drag coefficient are included for the paramount solution.

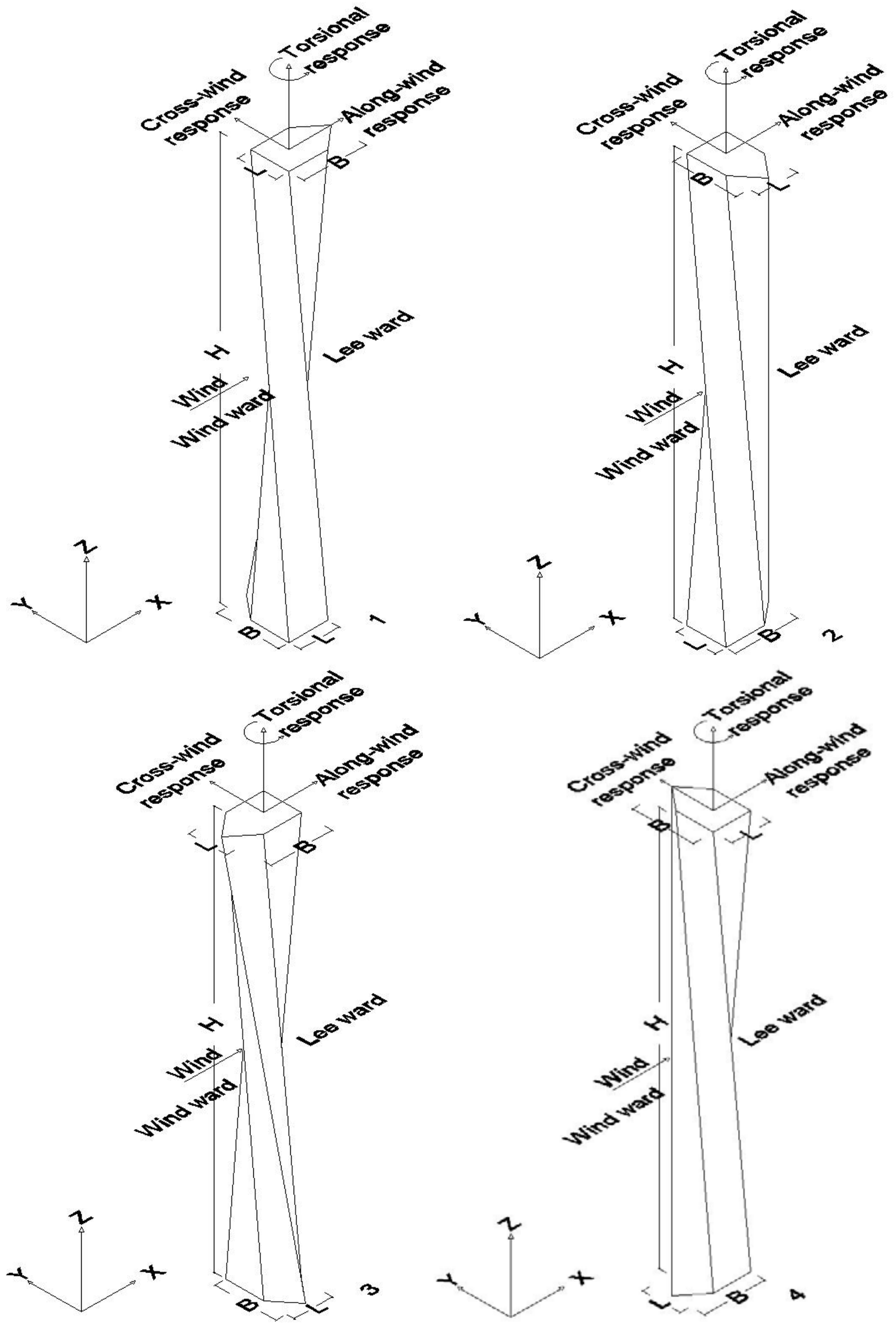

Figure 4-3 Shows where the wind hits Turning Torso and how the responses are determined. It also shows how the depth and width changes/shifts with height.

4.3.4 ALONG-WIND RESPONSE

First we have to ensure that $n_1 \cdot H / \overline{V}(H) \geq 0.1$ holds:
$0.198Hz \cdot 175.5m / 70m/s = 0.50 > 0.1 \Rightarrow OK!$
This shows that the equations are operative. Along wind causes a fluctuating load on a structure. This implies that the structure starts to vibrate. If these vibrations are significant, the dynamic response must be calculated and the acceleration due to gust. Finally it is compared to the acceleration and comfort criterion.

4.3.4.1 CLOSED-FORM EXPRESSIONS FOR THE ALONG-WIND RESPONSE

If Figure 4-3 is examined it demonstrates how the width and depth changes from top to bottom. Because of this unique form Turning Torso has, the wind will never work with its full force on the whole width as assumed in these calculations, thus the assumption that Turning Torso has the reduction form factor $\alpha = 0.8$ is acceptable. This reduction form factor corresponds to a building that is hexagonal or octagonal in plan, **[8]**. Then the average pressure coefficient C_w on the windward face and C_l on the leeward face is reduced from the rectangular building values ($C_w = 0.8$ and $C_l = 0.5$) with the reduction form factor. Then the values $C_w = 0.64$ and $C_l = 0.4$ are obtained and used in further calculations.

For these calculations the wind is assumed to hit Turning Torso like the orientation in structures marked 2 or 4, in Figure 4-3. Specifically the wind will hit Turning Torso at the top where the width is at its maximum and then the width will decrease as the height descends.

The surface roughness length is assumed to be in terrain type II in Table 3-3. Then the value $z_0 = 0.05$ is established.

The density of the air is assumed to be $\rho_{air} = 1.25\, kg/m^3$ as recommended; the damping ratio is selected to $\zeta_1 = 0.02$ as suggested for concrete structures

The fundamental modal shape is assumed to be linear so equations in chapter 3.2.7 are applicable.

4.3.4.1.1 Calculations for 37.8 m/s wind

The design wind velocity obtained from calculation in chapter 4.3.1 is assigned through equations to Turning Torso to find the response the structure gets. Lets start with the first natural frequency n_1= 0.198 Hz and apply the equations from chapter 3.2.7.1 to this information.
The friction velocity u_* is:

$$u_* = \frac{37.8\, m/s}{2.5 \cdot \ln(175.5m / 0.05m)} = 1.85\, m/s$$

The dimensionless variable Q is:

$$Q = 2 \cdot \ln\left(\frac{175.5m}{0.05m}\right) - 1 = 15.33$$

Then the measures of the mean and quasi-static response J and $\mathcal{B}$ can be found:

$$\mathrm{J} = 0.78 \cdot 15.33^2 = 183.23$$

$$\mathcal{B} = \frac{6.71 \cdot 15.33^2}{1 + 0.26 \cdot (26.5m / 175.5m)} = 1516.70$$

The dimensionless variable N_1 is:

$$N_1 = \frac{0.198Hz \cdot 175.5m}{1.85m/s \cdot 15.33} = 1.22$$

Now the dimensionless variables η_1 and η_2 can be established with Δ = L = 16.5 m:

$$\eta_1 = 3.55 \cdot 1.22 = 4.35$$

$$\eta_2 = \frac{12.32 \cdot 1.22 \cdot 16.5m}{175.5m} = 1.42$$

Now in turn the $\mathrm{C}(\eta_1)$ and $\mathrm{C}(\eta_2)$ can be calculated:

$$\mathrm{C}(\eta_1) = \frac{1}{4.35} - \frac{1 - e^{-2 \cdot 4.35}}{2 \cdot 4.35^2} = 0.20$$

$$\mathrm{C}(\eta_2) = \frac{1}{1.42} - \frac{1 - e^{-2 \cdot 1.42}}{2 \cdot 1.42^2} = 0.47$$

Now the dimensionless drag coefficient C_D and C_{Df}^2 can be calculated:

$$C_D = 0.64 + 0.40 = 1.04$$

$$C_{Df}^2 = 0.64^2 + 0.40^2 + 2 \cdot 0.64 \cdot 0.40 \cdot 0.47 = 0.81$$

With this information the dimensionless measures of the resonant response R can be calculated:

$$\mathrm{R} = \frac{0.59 \cdot 15.33^2 \cdot 1.22^{-2/3}}{0.02} \cdot \frac{0.81}{1.04} \cdot \frac{0.20}{1 + 3.95 \cdot 1.22 \cdot (26.5m / 175.5m)} = 534.40$$

In Figure 4-1 the mass of the building per unit height is listed as m1 to m9 this is the information needed to find the modal mass M_1 for Turning Torso:

$$M_1 = \int_{0m}^{19.5m} 222.370\,kg/m \cdot \frac{z^2}{H^2}dz + \int_{19.5m}^{38.0m} 213.150\,kg/m \cdot \frac{z^2}{H^2}dz + \ldots$$

$$\ldots + \int_{156.0m}^{175.5m} 153.150\,kg/m \cdot \frac{z^2}{H^2}dz = 9.821.400\,kg$$

See appendix for matlab application program used to compute the fundamental modal mass.
Now all the information needed to find the peak along-wind deflection at elevation z has been gathered, thus:

$$x_{pk}(z) \approx \frac{0.5 \cdot 1.25kg/m^3 \cdot (1.85m/s)^2 \cdot 1.04 \cdot 26.5m \cdot 175.5m}{9.821.400kg \cdot (2\pi \cdot 0.198Hz)^2} \cdot$$

$$\cdot\left(183.23 + 3.75(1516.70 + 534.40)^{1/2}\right) = \underline{0.241m}$$

The peak acceleration at elevation z can also be found with this information:

$$\ddot{x}_{pk}(z) \approx 4.0 \cdot \frac{0.5 \cdot 1.25kg/m^3 \cdot (1.85m/s)^2 \cdot 1.04 \cdot 26.5m \cdot 175.5m}{9.821.400kg} \cdot 534.40^{1/2} = \underline{0.098m/s^2}$$

This acceleration is corresponding to 0.010g needed for comparison.

To verify how accurate this result is if the fundamental modal shape is not a straight line, as was assumed, the ratio p it can be found:

$$p \approx \frac{183.23 + 3.75 \cdot (1516.70 + 534.40)^{1/2}}{183.23 + 3.75 \cdot 1516.70^{1/2}} = 1.072 = 107.2\%$$

This result shows that if the building will be designed as rigid[6], the response will be underestimated by 7.2 %.

The same equations were used to estimate the deflection and acceleration for other frequencies as well, the frequencies found by means of "rule of thumb" and the three found with different shape functions in chapter 4.2.1. The frequency used here to show the handling of equations in chapter 3.2.7.1 and engineers at Turning Torso used in their calculations, is also computed like a squared prism structure with the average pressure coefficient $C_w = 0.8$ on the windward face and $C_l = 0.5$ on the leeward thus $\alpha = 1.0$. This computation is marked with the darkest gray in Table 4-1 and is only for comparison with a squared prism structure.

[6] For design purposes buildings are classified as rigid if their wind induced resonant effects are negligible.

Review for wind of 37.8 m/s					
Frequency [Hz]	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Deflection from gust [m]**	**Height / Deflection [1/L]**	**Underesti-mation [%]**
0.1980	0.122	0.0124	0.301	583	7.2
0.1740	0.112	0.0114	0.318	552	9.3
0.1924	0.096	0.0097	0.243	722	7.6
0.1935	0.095	0.0097	0.240	731	7.6
0.1976	0.098	0.0100	0.242	725	7.3
0.1980	0.098	0.0100	0.241	729	7.2
0.1988	0.097	0.0099	0.239	735	7.2
0.2145	0.085	0.0087	0.193	910	6.2
0.2203	0.087	0.0089	0.192	914	5.8

Table 4-1 Shows the acceleration and deflection from gust for a given frequency for 37.8 m/s wind for chapter 4.3.4.1. The underestimation if the building is designed as rigid is specified in the last column.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000 kg$ found with the “rule of thumb” used in the geotechnical community.

4.3.4.1.2 Calculations for 70 m/s wind

The same calculations were completed on behalf of the wind that engineers at Turning Torso used. The results for these calculations can be found in Table 4-2. As before the frequency marked with the darkest gray is for a squared prism structure.

Review for wind of 70 m/s					
Frequency [Hz]	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Deflection from gust [m]**	**Height / Deflection [1/L]**	**Underesti-mation [%]**
0.1980	0.773	0.0788	1.170	150	21.5
0.1740	0.695	0.0708	1.257	140	26.0
0.1924	0.603	0.0614	0.948	185	22.4
0.1935	0.600	0.0611	0.936	188	22.2
0.1976	0.620	0.0632	0.940	187	21.5
0.1980	0.619	0.0631	0.936	188	21.5
0.1988	0.616	0.0628	0.927	189	21.3
0.2145	0.545	0.0556	0.741	237	18.9
0.2203	0.560	0.0571	0.735	239	18.1

Table 4-2 Shows the acceleration and deflection from gust for a given frequency for 70 m/s wind for chapter 4.3.4.1. The underestimation if the building is designed as rigid is specified in the last column.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000 kg$ found with the “rule of thumb” used in the geotechnical community.

4.3.4.2 ALONG-WIND RESPONSE OF STRUCTURES WITH AN APPROXIMATELY LINEAR FUNDAMENTAL MODAL SHAPE

The same constants are used here as in chapter 4.3.4.1.

4.3.4.2.1 Calculations for 37.8 m/s wind

The design wind velocity obtained from calculation in chapter 4.3.1 is used to find the response the structure gets. The first natural frequency n_1 = 0.198 Hz will be used to define equations in chapter 3.2.7.2.

The zero plane displacement z_d is assumed to be equal to zero, then the dimensionless variable Q is:

$$Q = 2 \cdot \ln\left(\frac{175.5m}{0.05m}\right) - 1 = 15.33$$

Because Q is the same here as in chapter 4.3.4.1 recapturing values for the friction velocity u_*, mean mean-response J, quasi-static response $\mathcal{B}$, resonant response R and the drag coefficient C_D will be done along with the fundamental modal mass M_1:

$$u_* = 1.85 m/s$$

$$\mathrm{J} = 183.23$$

$$\mathcal{B} = 1516.70$$

$$C_D = 1.04$$

$$\mathrm{R} = 534.40$$

$$M_1 = 9.821.400 kg$$

The friction pressure that the wind has on the along-wind side of the building is:

$$q_* = \frac{1}{2} \cdot 1.25 kg/m^3 \cdot (1.85 m/s)^2 = 2.14 N/m^2$$

The mean displacement, $\bar{x}$, at the top of Turning Torso is:

$$\bar{x} = \frac{2.14 N/m^2 \cdot 1.04 \cdot 26.5m \cdot 175.5m}{9.821.400 kg \cdot (2\pi \cdot 0.198 Hz)^2} \cdot 183.23 = 0.125m$$

The root mean squared value of the fluctuating deflection, σ_x, can be found with the approximate ratio $\xi = 6.00$ obtained from Table 3-1:

$$\sigma_x = \frac{2.14N/m^2 \cdot 1.04 \cdot 26.5m \cdot 175.5m}{9.821.400 \cdot (2\pi \cdot 0.198Hz)^2} \cdot \left(\frac{6.00 \cdot 1516.70}{6} + 534.40 \right)^{1/2} = 0.031m$$

The variance, v_x, is now of interest:

$$v_x = 0.198Hz \left(\frac{534.40}{534.40 + (6.00 \cdot 1516.70 / 6)} \right)^{1/2} = 0.10Hz$$

The dimensionless peak factor, K_x, can now be found with the duration of the storm 3600 seconds:

$$K_x = (1.175 + 2 \cdot \ln(0.10Hz \cdot 3600s))^{1/2} = 3.60$$

The dimensionless gust response factor, G, can now be calculated:

$$G = 1 + 3.60 \frac{0.01m}{0.037m} = 1.89$$

The peak displacement, X_{pk}, at the top of Turning Torso is:

$$X_{pk} = 1.89 \cdot 0.037m = 0.24m$$

The root mean squared value, $\sigma_{\ddot{x}}$, of the acceleration at the top of Turning Torso needs to be found:

$$\sigma_{\ddot{x}} = \frac{2.14N/m^2 \cdot 1.04 \cdot 26.5m \cdot 175.5m}{9.821.400kg} \cdot 534.40^{1/2} = 0.0244m/s^2$$

The peak factor, $K_{\ddot{x}}$, can be calculated now:

$$K_{\ddot{x}} = (1.175 + 2 \cdot \ln(0.198Hz \cdot 3600s))^{1/2} = 3.78$$

Now the peak acceleration, $\ddot{X}_{pk}$, at top of Turning Torso can be calculated:

$$\ddot{X}_{pk} = 3.78 \cdot 0.0073m/s^2 = 0.092m/s^2$$

Now the peak acceleration is computed and can be used for comparison. This acceleration is corresponding to 0.0094 g.

To verify how accurate this result is if the fundamental modal shape is not a straight line, as was assumed, the ratio p can be found:

$$p \approx \frac{183.23 + 3.75 \cdot (1516.70 + 534.40)^{1/2}}{183.23 + 3.75 \cdot 1516.70^{1/2}} = 1.072 = 107.2\%$$

This result shows that if the building will be designed as rigid, the response will be underestimated by 7.2 %.

This formulation was used to find the responses for additional frequencies found with “rules of thumb”, different shape functions and different modal mass M_1 (found with the “rule of thumb” known in the geotechnical community). There is also a calculation for Turning Torso as a squared prism structure, this calculation is marked with the darkest gray in Table 4-3 and is only for comparison with a squared prism structure.

Review for wind of 37.8 m/s							
Frequency [Hz]	RMS Acceleration [m/s^2]	RMS acceleration / Gravity [g]	Peak Acceleration [m/s^2]	Peak acceleration / Gravity [g]	Deflection from gust [m]	Height / Deflection [1/L]	Underesti-mation [%]
0.1980	0.0305	0.0031	0.115	0.0118	0.295	594	7.2
0.1740	0.0280	0.0029	0.105	0.0107	0.311	564	9.3
0.1924	0.0239	0.0024	0.090	0.0092	0.238	736	7.6
0.1935	0.0237	0.0024	0.090	0.0091	0.235	745	7.6
0.1976	0.0245	0.0025	0.093	0.0094	0.237	740	7.3
0.1980	0.0244	0.0025	0.092	0.0094	0.236	743	7.2
0.1988	0.0243	0.0025	0.092	0.0094	0.234	749	7.2
0.2145	0.0213	0.0022	0.081	0.0082	0.189	927	6.2
0.2203	0.0218	0.0022	0.083	0.0085	0.189	931	5.8

Table 4-3 Shows the acceleration and deflection from gust for a given frequency for 37.8 m/s wind for chapter 4.3.4.2. The underestimation if the building is designed as rigid is specified in the last column.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000 kg$ found with the “rule of thumb” used in the geotechnical community.

4.3.4.2.2 Calculations for 70 m/s wind

The same calculations were completed on behalf of the wind that engineers at Turning Torso used. The results for these calculations can be found in Table 4-4. As before the frequency marked with the darkest gray is for a squared prism structure.

Review for wind of 70 m/s							
Frequency [Hz]	**RMS acceleration [m/s²]**	**RMS acceleration / Gravity [g]**	**Peak acceleration [m/s²]**	**Peak acceleration / Gravity [g]**	**Deflection from gust [m]**	**Height / Deflection [1/L]**	**Underesti-mation [%]**
0.1980	0.193	0.0197	0.731	0.0746	1.162	151	21.5
0.1740	0.174	0.0177	0.651	0.0664	1.244	141	26.0
0.1924	0.151	0.0154	0.569	0.0580	0.941	187	22.4
0.1935	0.150	0.0153	0.566	0.0577	0.929	189	22.2
0.1976	0.155	0.0158	0.586	0.0597	0.934	188	21.5
0.1980	0.155	0.0158	0.585	0.0596	0.929	189	21.5
0.1988	0.154	0.0157	0.583	0.0594	0.921	191	21.3
0.2145	0.136	0.0139	0.518	0.0528	0.737	238	18.9
0.2203	0.140	0.0143	0.534	0.0544	0.732	240	18.1

Table 4-4 Shows the acceleration and deflection from gust for a given frequency for 70 m/s wind for chapter 4.3.4.2. The underestimation if the building is designed as rigid is specified in the last column.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000kg$ found with the "rule of thumb" used in the geotechnical community.

4.3.4.3 WIND INDUCED VIBRATIONS

This is the method engineers at Turning Torso used. This method is extremely sensitive to the frequency selected for the peak value of the turbulent wind pressure. Engineers at Turning Torso only looked at the responses obtained from gusts forming at 4 seconds interval and used that to judge against the comfort criteria. They do not have an explanation, assumptions or any general thoughts how this interval is practical. The author thinks this is a poor selection of frequency interval for the estimation on occupant comfort and safety, for the most response of the building there should have been investigated the gust with approximately the same frequency as the first natural frequency of the building.
This method can not be as accurate as the ones illustrated in chapter 4.3.4.1 and 4.3.4.2 because of the neglecting of drag coefficient, surface roughness length and more.

4.3.4.3.1 Four second gust

The wind pressure due to gusts is assumed to be constant over the height above ground.
Engineers at Turning Torso used the peak value of turbulent wind pressure equal to $q_d = 0.7\, kN/m^2$, this will be used in this calculation blindly. The natural frequency of Turning Torso is $f_B = 0.198$ Hz.
Assuming $f_{q_{wd}} = 0.0625\,\text{Hz}$ (corresponding to a period $T_{q_{wd}} = 16$ s and hence, to a time of development of a single gust of 4 s), one gets:

$$\alpha = \frac{0.0625Hz}{0.198Hz} = 0.3125$$

$$\beta = \frac{0.3125}{1-0.3125^2}\sqrt{1+0.3125^2 - 2\cdot 0.3125\cdot \sin\left(\frac{\pi}{2\cdot 0.3125}\right)} = 0.45$$

Then the turbulent portion of wind action is:

$$(1+0.45)0.7kN/m^2 = 1.02\,kN/m^2$$

The horizontal displacement of Turning Torso due to this gust wind pressure equals:

$$A = \frac{q\cdot H^4}{8\cdot E\cdot I} = \frac{1.02\,kN/m^2\cdot 26.5m\cdot(175.5m)^4}{8\cdot 36GPa\cdot 410m^4} = 22\,cm$$

Considering a harmonic vibration of Turning Torso with amplitude of 22 cm and the frequency of the gust excitation $0.0625\,\mathrm{Hz}$, one gets a maximum acceleration of:

$$a_{\max} = 0.22m\cdot(2\pi\cdot 0.0625Hz)^2 = 0.034m/s^2$$

This acceleration is equal to 0.0035g

4.3.4.3.2 Three second gust

If the assumption is $f_{q_{wd}} = 0.083\,\mathrm{Hz}$ (corresponding to a period $T_{q_{wd}} = 12$ s and hence, to a time of development of a single gust of 3 s) this is the frequency that is recommended by most standards and is considered most likely to occur**, [10].** One gets $\alpha = 0.42$ and $\beta = 0.66$ then the turbulent portion of wind action is:

$$(1+0.66)\cdot 0.7\,kN/m^2 = 1.16\,kN/m^2$$

The horizontal displacement of Turning Torso due to this gust wind pressure equals $A = 25\,cm$ and then the maximum acceleration is $a_{\max} = 0.068m/s^2$ this acceleration is equal to 0.0069g

4.3.4.3.3 Gust with approximately the same frequency as the first natural frequency of the building

If the assumption made is $f_{q_{wd}} \approx 0.198\mathrm{Hz}$ (corresponding to a period $T_{q_{wd}} = 5.05$ s and hence, to a time of development of a single gust of 1.26 s) this frequency is the same as the first natural frequency of Turning Torso. One gets $\alpha = 1.0$ and $\beta = 0.0$ then the turbulent portion of wind action is:

$$(1+0.0)\cdot 0.7\,kN/m^2 = 0.7\,kN/m^2$$

The horizontal displacement of Turning Torso due to this gust wind pressure equals $A = 14.9\,cm$ and then the maximum acceleration is $a_{\max} = 0.231 m/s^2$ this acceleration is equal to 0.0236 g.

4.3.4.4 GUST LOADING FACTORS METHOD (ALAN G. DAVENPORT)

Because of its simplicity, the gust loading factor method has been developed, applied and received a widespread acceptance around the world and is employed in wind loading codes and standards in almost all major countries (e.g., Australian, Canada, USA, Japan, Europe), **[12], [14].**

4.3.4.4.1 The peak factor method

4.3.4.4.1.1 Calculations for 37.8 m/s wind

Lets find the acceleration through the peak factor suggested by Davenport, 1967 for the first natural frequency 0.198 Hz. The duration of the storm is T = 3600 seconds and it is understood that the system is lightly damped, thus making $\nu \approx n_0$ first natural frequency of the system. Now all the information needed to find the peak factor is established:

$$g = \sqrt{2 \cdot \ln(0.198 Hz \cdot 3600 s)} + \frac{0.57}{\sqrt{2 \cdot \ln(0.198 Hz \cdot 3600 s)}} = 3.782$$

The RMS deflection obtained in chapter 4.3.4.2 is remembered and repeated here for convenience $\sigma_x = 0.0309 m$. Note that this is for drag coefficient $C_D = 1.04$.
The amplitude A due to gust can now be found:

$$A = 3.782 \cdot 0.0309 m = 0.1169 m$$

Then the maximum acceleration for harmonic vibration can be found with equation 3-47:

$$a_{\max} = 0.1169 m \cdot (2\pi \cdot 0.198 Hz)^2 = 0.181 m/s^2$$

This peak acceleration corresponds to 0.0184 g needed for comparison in the comfort criteria.
This formulation was used to find the responses for additional frequencies found with “rules of thumb”, different shape functions and different modal mass M_1 (found with the “rule of thumb” known in the geotechnical community). There is also a calculation for Turning Torso as a squared prism structure, this calculation is marked with the darkest gray in Table 4-5 and is only for comparison with a squared prism structure.

Review for wind of 37.8 m/s						
Frequency [Hz]	**Peak factor**	**RMS Deflection from gust [m]**	**Peak deflection from gust [m]**	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Height / Deflection [1/L]**
0.1980	3.782	0.0386	0.1461	0.226	0.0230	1201
0.1740	3.748	0.0416	0.1559	0.186	0.0190	1125
0.1924	3.774	0.0313	0.1182	0.173	0.0176	1485
0.1935	3.776	0.0309	0.1167	0.172	0.0176	1504
0.1976	3.781	0.0310	0.1174	0.181	0.0184	1495
0.1980	3.782	0.0309	0.1169	0.181	0.0184	1502
0.1988	3.783	0.0306	0.1158	0.181	0.0184	1515
0.2145	3.803	0.0245	0.0930	0.169	0.0172	1886
0.2203	3.810	0.0243	0.0925	0.177	0.0181	1897

Table 4-5 Shows the peak acceleration and deflection obtained with different first natural frequencies for the structure. The RMS deflection obtained from chapter 4.3.4.2 is included.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000kg$ found with the "rule of thumb" used in the geotechnical community.

4.3.4.4.1.2 Calculations for 70 m/s wind

The same calculations were completed on behalf of the wind that engineers at Turning Torso used. The results for these calculations can be found in Table 4-6. As before the frequency marked with the darkest gray is for a squared prism structure.

Review for wind of 70 m/s						
Frequency [Hz]	**Peak factor**	**RMS Deflection from gust [m]**	**Peak deflection from gust [m]**	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Height / Deflection [1/L]**
0.1980	3.782	0.1690	0.6393	0.989	0.1009	275
0.1740	3.748	0.1872	0.7017	0.839	0.0855	250
0.1924	3.774	0.1379	0.5206	0.761	0.0775	337
0.1935	3.776	0.1360	0.5134	0.759	0.0774	342
0.1976	3.781	0.1359	0.5140	0.792	0.0808	341
0.1980	3.782	0.1352	0.5114	0.792	0.0807	343
0.1988	3.783	0.1339	0.5065	0.790	0.0806	347
0.2145	3.803	0.1052	0.3999	0.726	0.0740	439
0.2203	3.810	0.1037	0.3952	0.757	0.0772	444

Table 4-6 Shows the peak acceleration and deflection obtained with different first natural frequencies for the structure. The RMS deflection obtained from chapter 4.3.4.2 is included.

Note that for the frequencies 0.1924 Hz, 0.1935 Hz and 0.2145 Hz the modal mass is $M_1 = 10.348.000kg$ found with the "rule of thumb" used in the geotechnical community.

4.3.4.4.2 Gust pressure factor method

4.3.4.4.2.1 Calculations for 37.8 m/s wind

Let us find the max wind velocity pressure through the gust factor method, with the wind velocity 37.8 m/s and the drag coefficient C_D = 1.04, first the mean wind pressure is found through equation 3-52:

$$q(175.5m) = \frac{1}{2} \cdot 1.25kg/m^3 \cdot (37.8m/s)^2 \cdot 1.04 = 923N/m^2$$

Now the gust factor needs to be evaluated and for that the dimensionless frequency ξ_0 can be found with equation 3-57:

$$\xi_0 = \frac{175.5m \cdot 0.198Hz}{37.8m/s} = 0.92$$

The peak factor is the same as in chapter 4.3.4.4.1.1 or $g = 3.782$ and the dimensionless roughness factor r found with equation 3-59, with $\alpha = 0.16$ and the surface roughness length $z_0 = 0.05m$ is:

$$r = 4 \cdot \sqrt{0.05m} \cdot \left(\frac{9.14}{175.5m}\right)^{2 \cdot 0.16} = 0.35$$

Now the dimensionless excitation by background turbulence $\hat{\mathbf{B}}$ is found with equation 3-60:

$$\hat{\mathbf{B}} = 2 \cdot \left(1 - \frac{1}{\left(1 + \left(\frac{457.2}{175.5m}\right)^2\right)^{1/3}}\right) = 0.99$$

The dimensionless size reduction factor s is established through equation 3-56:

$$s = \frac{\pi}{3}\left(\frac{1}{1 + \frac{8}{3 \cdot 0.92}} \cdot \frac{1}{1 + 10 \cdot \frac{26.5m}{175.5m \cdot 0.92}}\right) = 0.10$$

The dimensionless gust energy ratio F is created trough equation 3-58:

$$F = \frac{\left(\frac{1219.2 \cdot 0.198 Hz}{37.8 m/s}\right)^2}{\left(1 + \left(\frac{1219.2 \cdot 0.198 Hz}{37.8 m/s}\right)^2\right)^{4/3}} = 0.28$$

Now all the information needed together with the damping ratio $\zeta_1 = 0.02$ needed to find the dimensionless excitation by turbulence resonant with the structure $\hat{\mathbf{R}}$ is found with equation 3-55:

$$\hat{\mathbf{R}} = \frac{0.10 \cdot 0.28}{0.02} = 1.43$$

Now all the information needed to evaluate the "gust factor" G has been gathered:

$$G = 1 + 3.78 \cdot 0.35 \cdot \sqrt{0.99 + 1.43} = 3.04$$

Now the maximum wind velocity pressure due to gust can be found as:

$$q(z)_{\max} = 3.04 \cdot 929 N/m^2 = 2827 N/m^2$$

In Table 4-7 is a review for the wind pressure for 37.8 m/s wind and for different frequencies found with different methods. There is also a calculation for Turning Torso as a squared prism structure, this calculation is marked with the darkest gray in Table 4-7 and is only for comparison with a squared prism structure.

Review for wind of 37.8 m/s			
Frequency [Hz]	**Mean wind pressure [N/m^2]**	**Gust factor**	**MAX wind pressure due to gust [N/m^2]**
0.1980	1161	3.04	3534
0.1740	929	2.97	2755
0.1924	929	3.03	2811
0.1935	929	3.03	2814
0.1976	929	3.04	2826
0.1980	929	3.04	2827
0.1988	929	3.05	2829
0.2145	929	3.09	2872
0.2203	929	3.11	2887

Table 4-7 Shows mean and max wind pressures and the gust factor for different frequencies and 37.8 m/s wind.

4.3.4.4.2.2 Calculations for 70 m/s wind

The same calculations were completed on behalf of the wind that engineers at Turning Torso used. The results for these calculations can be found in Table 4-8. As before the frequency marked with the darkest gray is for a squared prism structure.

Review for wind of 70 m/s			
Frequency [Hz]	**Mean wind pressure [N/m^2]**	**Gust factor**	**MAX wind pressure due to gust [N/m^2]**
0.1980	3981	2.76	10982
0.1740	3185	2.69	8556
0.1924	3185	2.74	8734
0.1935	3185	2.75	8744
0.1976	3185	2.76	8782
0.1980	3185	2.76	8785
0.1988	3185	2.76	8793
0.2145	3185	2.80	8932
0.2203	3185	2.82	8982

Table 4-8 Shows mean and max wind pressures and the gust factor for different frequencies and 70 m/s wind.

In Table 4-8 it can be seen that the estimation that the engineers at Turning Torso made for the wind pressure corresponding to the wind 70 m/s is too low or 3000 N/m^2 they estimated compared to 3185 N/m^2 (this is around 6 % difference).

These calculations are mainly performed to see how accurate the estimations engineers at Turning Torso made and because their wind induced acceleration depended on the assumed wind pressure.

4.4 SMÁRATORG TOWER

Lets apply the formulation learned in this report to the highest building in Iceland. The assumed height of the building is H = 77.9 m which corresponds of 20 floors, the width is B = 38.4 m and the depth is L = 20.4 m.
The structural core of the building is made out of columns at the edge and walls in the middle where the staircase and elevators are. The columns are made out of C40/50 concrete with E_{cm} = 31.5 GPa and the walls are made out of C30/37 concrete with E_{cm} = 28.8 GPa. The moment of inertia was found by studying the drawings obtained from Arkis, an architectural firm in Iceland, then the structural core was redrawn in AutoCad and the moment of inertia was calculated for each elevation (see the appendix for a method to find the moment of inertia with AutoCad and a schematic figure of one of the floors used to find the moment of inertia). The information was made into a schematic drawing see Figure 4-4 where the moment of inertia is shown for each core but the mean moment of inertia is $I_{1,mean} = 197.4m^4$ with this method. When this moment of inertia is found the columns are neglected and thought to have no contribution to the inertia.
From information obtained from engineers in Iceland the mean moment of inertia is $I_{2,mean} = 282m^4$.

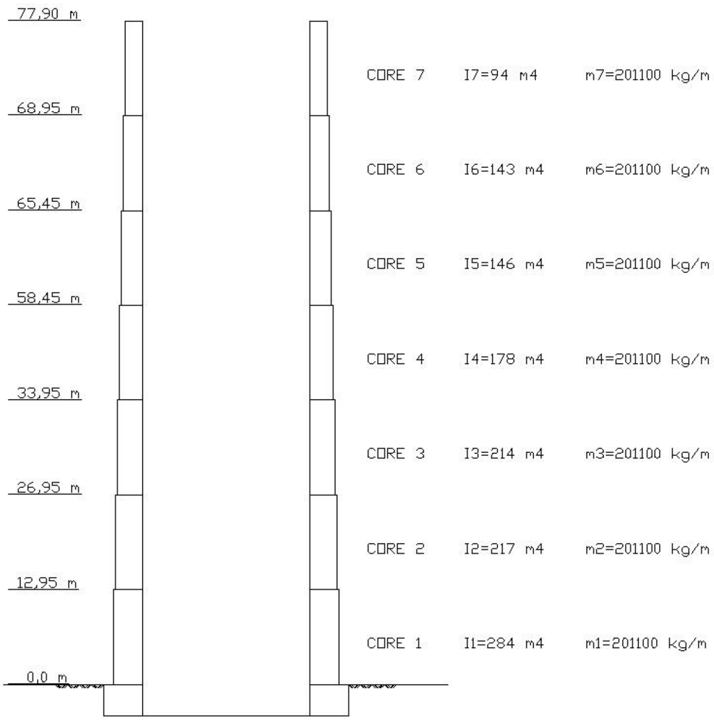

Figure 4-4 Shows the modular height for the structural core for Smáratorg Tower with the information for the moment of inertia and the mass per meter fore each core.

The mass is not known for the building and then the known “rule of thumb” can be used to estimate the mass:

$$M(z) = \frac{10kN/(m^2 \cdot floor)\cdot(38.4m \cdot 20.4m)\cdot 20 floor}{77.9m} = 2011kN/m = 201100kg/m$$

This information is shown in Figure 4-4 for each core. With this information the generalized modal mass can be found:

$$M_1 = \int_{0m}^{77.9m} 201.100 kg/m \cdot \frac{z^2}{H^2} dz = \frac{201.100 kg/m \cdot 77.9m}{3} = 5.221.900 kg$$

If the first natural frequency is found through chapter 3.1.1 and the shape function given in equation 3-5 for the two mean moments of inertia found, for $I_{1,mean} = 197.4m^4$ and $I_{2,mean} = 282m^4$ respectively:

$f_{1,mean} = 0.545Hz$ and $f_{2,mean} = 0.595Hz$

The first natural frequency engineers in Ferill Iceland got was 0.5 Hz (This was found through computation with the earthquake program ETABS), **[24].** If the first natural frequency is found with the "rule of thumb" it is:

$$f = \frac{30.48Hz \cdot m}{77.9m} = 0.391Hz$$

The wind at the reference height 10 m and the surface roughness length z_0 = 0.05 m is $v_{ref} = 35.5m/s$ for 10 min wind, **[24].** The average hour reference wind is $v_{ref,3600} = 33.3m/s$, thus the wind near the top of the structure is $v_{mk} = 46.5m/s$ needed to compute the acceleration in the top floors. With this information the acceleration in the top floors can be found with equation in chapter 3.2.7.1 and 3.2.7.2 and the peak factor method in chapter 3.2.7.4.1 the results are listed in Table 4-9, Table 4-10 and Table 4-11 respectively.

Values in Table 4-9 are from the chapter 3.2.7.1 Closed-form expressions for the along-wind response

Review for wind of 46.5 m/s					
Frequency [Hz]	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Deflection from gust [m]**	**Height / Deflection [1/L]**	**Underesti-mation [%]**
0.391	0.238	0.0243	0.144	540	7.6
0.500	0.179	0.0182	0.086	909	4.4
0.545	0.162	0.0165	0.072	1088	3.6
0.595	0.145	0.0148	0.060	1306	3.0

Table 4-9 Shows the acceleration and deflection from gust for a given frequency for 46.5 m/s wind. The underestimation if the building is designed as rigid is specified in the last column.

Values in Table 4-10 are from the chapter 3.2.7.2 Along-wind response of structures with an approximately linear fundamental modal shape

Review for wind of 46.5 m/s							
Frequency [Hz]	**RMS acceleration [m/s^2]**	**RMS acceleration / Gravity [g]**	**Peak acceleration [m/s^2]**	**Peak acceleration / Gravity [g]**	**Deflection from gust [m]**	**Height / Deflection [1/L]**	**Underesti-mation [%]**
0.391	0.059	0.0061	0.236	0.0240	0.145	537	7.6
0.500	0.045	0.0046	0.180	0.0183	0.086	904	4.4
0.545	0.040	0.0041	0.163	0.0166	0.072	1082	3.6
0.595	0.036	0.0037	0.148	0.0151	0.060	1299	3.0

Table 4-10 Shows the acceleration and deflection from gust for a given frequency for 46.5 m/s wind. The underestimation if the building is designed as rigid is specified in the last column.

Values in Table 4-11 are from the chapter 3.2.7.4.1 The peak factor method

Review for wind of 46.5 m/s						
Frequency [Hz]	**Peak factor**	**RMS Deflection from gust [m]**	**Peak Deflection from gust [m]**	**Peak Acceleration [m/s^2]**	**Acceleration / Gravity [g]**	**Height / Deflection [1/L]**
0.391	3.957	0.0192	0.0761	0.459	0.0468	1024
0.500	4.019	0.0111	0.0445	0.439	0.0447	1752
0.545	4.040	0.0092	0.0370	0.434	0.0443	2104
0.595	4.062	0.0076	0.0308	0.431	0.0439	2528

Table 4-11 Shows the peak acceleration and deflection obtained with different first natural frequencies for the structure. The RMS deflection obtained from equations in chapter 3.2.7.2 is included.

4.5 ACCELERATION AND COMFORT DISCUSSION

Turning Torso and Smáratorg Tower will be discussed separately.

4.5.1 TURNING TORSO

The comfort criteria for Turning Torso will be divided into two discussions, first for 37.8 m/s wind and second for 70 m/s wind that engineers at Turning Torso used.

4.5.1.1 CONSIDERATIONS FOR 37.8 M/S WIND

The name of the chapters used to find the acceleration for this wind is used to distinguish, and make it easier to compare.

4.5.1.1.1 Closed-form expressions for the along-wind response

The peak acceleration obtained in chapter 4.3.4.1 varies with different frequencies from 0.0087 g to 0.0114 g, if Turning Torso was a squared prism structure the acceleration would exceed 0.0124 g. These limits are all over the perceptibility of 0.005 g which is the limit set for humans to perceive motion. This means that sensitive people will perceive motion for most frequencies. At the frequency found with the “rule of thumb” majority of people will perceive motion, the motion will affect desk work and long term exposure may produce motion sickness.

4.5.1.1.2 Along-wind response of structures with an approximately linear fundamental modal shape

The peak acceleration obtained with computation in chapter 4.3.4.2 varies with different frequencies from 0.0082 g to 0.0107 g and for the squared prism shape the acceleration is 0.0118 g. These accelerations are also over the perceptibility value that humans can distinguish motion. Occupants in Turning Torso would feel some inconvenience due to this wind acceleration, the movement can slightly affect desk work and hanging objects may move.

4.5.1.1.3 Gust loading factors method (Alan G. Davenport)

The peak acceleration obtained with computation in chapter 4.3.4.4 varies with different frequencies from 0.0172 g to 0.0190 g and if Turning Torso was a squared prism structure the acceleration would be 0.0230 g. These limits are all in the zone where majority of people will perceive motion, level of motion may affect desk work and long term exposure may produce motion sickness.

4.5.1.1.4 Recapitulate

The results for all the methods used to find the peak acceleration are poor for Turning Torso occupants and will be of inconvenience to them. Further calculations have to be done, for example to find wind induced acceleration with the mean 20 min wind with recurrence time of 6 years as is recommended.

4.5.1.2 CONSIDERATIONS FOR 70 M/S WIND

The author supposes that this wind should be looked at just to confirm that people will not have to witness the circumstances where they can not walk upright and be hurt by falling objects like furniture. That acceleration shall not exceed 0.04 g, the zone where people strongly perceive motion and find it hard to walk naturally.

4.5.1.2.1 Closed-form expressions for the along-wind response

The peak acceleration obtained in chapter 4.3.4.1 varies with different frequencies from 0.0556 g to 0.0708 g and if Turning Torso was a squared prism structure the acceleration would exceed 0.0788 g. These limits are in the zone where most people cannot walk or tolerate motion up to the limit where people strongly perceive motion and cannot walk or tolerate motion. This means that there is hindrance for people to walk naturally and that they can be injured due to acceleration induced by this wind velocity.

4.5.1.2.2 Along-wind response of structures with an approximately linear fundamental modal shape

The peak acceleration obtained with computation in chapter 4.3.4.2 varies with different frequencies from 0.0528 g to 0.0664 g and for the squared prism shape the acceleration is 0.0746 g. These accelerations are in the zone where people cannot tolerate the motion and are unable to walk. This means that people can be injured and walking ability is hindered due to the acceleration by this wind velocity. These values can all be considered to be on the edge where human cannot walk. This is without a doubt a really poor result.

4.5.1.2.3 Wind induced vibrations

The accelerations obtained with this method vary widely depending on the selected frequency of the harmonic distribution of the wind pressure due to gust. The accelerations computed in chapter 4.3.4.3 are 0.003 g for 4 second gust, 0.007 g for 3 second gust and 0.024 g for gust with the same frequency as the first natural frequency of Turning Torso. For the 4 second gust the acceleration is in the zone where people cannot perceive motion. For the 3 second gust the acceleration is in the zone where sensitive people can feel motion and hanging objects may move. For the last one the acceleration is in the zone where majority of people perceived motion and desk work may be affected.

4.5.1.2.4 Gust loading factors method (Alan G. Davenport)

The peak acceleration obtained with computation in chapter 4.3.4.4 varies with different frequencies from 0.0740 g to 0.0855 g and if Turning Torso was a squared prism structure the acceleration would be 0.1009 g. These accelerations are all in the zone where people very strongly perceive motion and cannot walk. For the acceleration that exceeds 0.085 g furniture and lose objects can fall and some people may be injured even if they remain still. Wind induced accelerations over 0.085 g should never be witnessed in buildings.

4.5.1.2.5 Recapitulate

For all methods used to find the wind induced acceleration the level of motion is so large that people cannot walk naturally or not at all, this means that people can be injured due to this motion.

For the method engineers at Turning Torso used the acceleration is much lower and strongly depends on the frequency of the harmonic distribution selected, this method is not recommended by the author and should not be used to find the acceleration in the top floors of high rise buildings.

To use this high wind velocity to find the designing acceleration in the top floors in a structure is a poor work method, it should only be included to ensure that people will not be injured.

4.5.2 SMÁRATORG TOWER

The comfort criteria for Smáratorg Tower will only be for 46.5 m/s wind

4.5.2.1 CONSIDERATIONS FOR 46.5 M/S WIND

The name of the chapters used to find the acceleration for this wind is used to distinguish, and make it easier to compare.

4.5.2.1.1 Closed-form expressions for the along-wind response

The peak acceleration obtained in chapter 4.4 varies with different frequencies from 0.0148 g to 0.0243 g. It is safe to say that the acceleration for all frequencies is in the zone where majority of people perceive motion and desk work may be affected. For the frequency 0.391 Hz, the acceleration is between the zone where desk work may be affected and the zone where desk work becomes difficult.

4.5.2.1.2 Along-wind response of structures with an approximately linear fundamental modal shape

The peak acceleration obtained in chapter 4.4 varies with different frequencies from 0.0151 g to 0.0240 g. It is safe to say that the acceleration for all frequencies is in the zone where majority of people perceive motion and hanging objects may move. For the frequency 0.391 Hz, the acceleration is between the zone where desk work may be affected and the zone where desk work becomes difficult.

4.5.2.1.3 Gust loading factors method (Alan G. Davenport)

The peak acceleration obtained in chapter 4.4 varies with different frequencies from 0.0439 g to 0.0468 g. These accelerations are all in the zone where people strongly perceive motion, have difficulties to walk naturally and people standing may lose their balance.

4.5.2.1.4 Recapitulate

For all methods the peak acceleration is over the zone where majority of people perceive motion and even up to the zone where people strongly perceive motion. This is not acceptable for an office building where desk work may be affected in this excessive motion.

Further calculations have to be done, for example to find wind induced acceleration with the mean 20 min wind with recurrence time of 6 years as is recommended.

5 DISCUSSION AND CONCLUSION

The designer of a high rise flexible structure ought to consider the potential problems of excessive building motions. The designer must attend to this problem, first by predicting the probable motion and second by judging what is tolerable. If it is found that the predicted motion is not acceptable, the design should be modified to decrease the potential motions.

In this thesis there has been gathered information and methods to evaluate from a early state design the first natural frequency of a structure, how to find wind speeds and change them to be eligible for equations to compute the RMS and peak deflection and acceleration. The RMS and peak acceleration is the movement at the top of the structures needed to compare with the comfort criteria for humans. These equations for the acceleration are very limited and only work for the first natural frequency of the structure. There are also included different ways to estimate the wind velocity pressure on the side of the structure for any given wind velocity. The comfort criterions are looked at for tall slender buildings with low values for the first natural frequency. The studies that have been made in the area were used to establish an acceptable acceleration comfort criteria for humans.

Standards will have to include more wind velocities with the recurrence of different intervals like 6, 10 and 20 years or methods to evaluate these wind velocities. This will be necessary because structures are being built higher and higher and then there follows a larger request to for occupants comfort and thus the need to evaluate the effects of acceleration in the top floors.

The method to compute the first natural frequency included in this study has shown to be accurate and is recommended by the author. This method is easy to use with the help of matlab or similar programs and can straightforwardly be calculated by hand.

If the wind velocity pressure computed from the 70 m/s wind is compared to the pressure engineers at Turning Torso used, it is clear that it is underestimated. This can be because of different methods used and/or assumptions made by the engineers at Turning Torso.

The most important results are that for 37.8 m/s wind, the wind induced accelerations in the top floors of Turning Torso are severe enough to make occupants perceive motion and even become motion sick. The wind induced acceleration due to 70 m/s wind is furthermore unacceptable, this is because the wind induced acceleration is over the limit where people cannot walk properly and up to the level where people cannot walk, furniture and loose objects can begin to fall. This acceleration is so excessive that people can easily be injured. The results obtained indicate that Turning Torso should be stiffened or some methods taken to minimize the acceleration and thus the injuries that occupants can witness in this wind induced motion. It has to be noted that the information worked with for Turning Torso are from 17. March 2000, this is in the early state design. Turning Torso was later stiffened due to this problem, this flaw in the design cost several hundred millions Swedish kronor to repair.
The author has suggested that accelerations from high winds should be within 0.04 g, so occupants do not injure themselves due to excessive accelerations.

The most important results for Smáratorg Tower are that the peak acceleration is severe enough to make majority of people perceive motion and even strongly perceive motion. This means that desk work may be affected and even up to the point where people may loose their balance. This is not acceptable in an office building. Further calculations have to be complete with more information on Smáratorg Tower, to get more accurate results. It has to be noted that for each floor in Smáratorg Tower there is space reserved for installing of dampers, to minimize this acceleration, **[24]**.

The “rule of thumb” to find the first natural frequency has shown to give lower values than the “real value” for the first natural frequency. This “rule of thumb” should be used with care.
The “rule of thumb” to estimate the mass of the building has shown to be a reasonable assumption but gives somewhat higher value than the “real value”. This can be used to help in calculations but not to be taken as a fact.

Further studies
This study has shown the potential of computing the acceleration in the top floors of structures so that the structure can be designed with the aim that occupants will not feel discomfort.
For further studies one might explore how to find the across wind acceleration in the top floors of high rise buildings. This can be prepared by including the methods used in designing codes and standards around the world.
The list of studies for the various effects of wind is everlasting and better studies and understanding is desired in all fields.

Ámundi Fannar Sæmundsson

REFERENCES

BOOKS

In the case of many authors just the first will be mentioned.

[1]
Ambrose, James and Vergun, Dimitry. 1980 "*Simplified Building design for wind and earthquake forces*". John Wiley & sons, Inc.

[2]
Boverket, byggavdelningen. 1997 "*Boverkets handbook om snö- och vindlast*". Utgåva 2, Boverket.

[3]
Chopra, Anil K. 2001 "*Dynamics of structures: Theory and applications to earthquake engineering*" 2nd edition. Prentice-Hall, Inc.

[4]
Higgins, Theodore R. 1979 "*Structural design of tall steel buildings: Council on tall buildings and urban habitat*" Vol. SB. American society of civil engineers.

[5]
Isaksson, Tord and Mårtensson, Annika. 2006 "*Byggkkonstruktion: Tabell- och formelsamling*". Författarna och Studentlitteratur.

[6]
Kavanagh, Tomas C. 1981 "*Planning and environmental criteria for tall buildings: Council on tall buildings and urban habitat*" Vol. PC. American society of civil engineers.

[7]
Mohr, Gunnar. 2003 "*Teknisk Ståbi*" 18th edition. Ingeniören böger, Ingeniören A/S.

[8]
Schueller, Wolfgang. 1977 "*High-rise building structures*" John Wiley & sons, Inc.

[9]
Simiu, Emil and Scanlan, Robert H. 1996 "*Wind effects on structures: fundamentals and applications to design*" 3rd edition. John Wiley & sons, Inc.

[10]
Simiu, Emil and Miyata, Toshio. 2006 "*Design of buildings and bridges for wind: A practical guide for ASCE-7 Standard users and designers of special structures*" 1st edition. John Wiley & sons, Inc.

ARTICLES

[11]
Davenport, Alan G. 1967 "*Gust Loading Factors*." Journal of the structural division. Vol.93, No.ST3, June, 1967.

[12]
Kareem, Ahsan and Zhou, Yin. 2003 "*Gust loading factor – past, present and future*." Jurnal of wind engineering and industrial aerodynamics. Elsevier Ltd. Vol.91, pages 1301-1328

[13]
Mendis, P.. 2007 "*Wind loading on tall buildings*." EJSE Special issue: Loading on structures. Pages 41-54

[14]
Zhou, Yin. 1999 "*Alongwind static equivalent wind loads and responses of tall buildings. Part I: Unfavorable distributions of static equivalent wind loads*." Jurnal of wind engineering and industrial aerodynamics. Elsevier science Ltd. Vol.79, pages 135-150

WEB

[15]
Official homepage of architectss – Architecture Sceintific Soceity
http://architectss.blogfa.com
Downloaded: 13. November, 2007.

[16]
Official homepage of Arkis – Architectural company in Iceland
www.ark.is.
Visited: 30. October, 2007.

[17]
Official homepage of emporis - The Building Industry Platform
www.emporis.com.
Visited: 29. May, 2007.

[18]
Official homepage of postershope
http://www.postershop.co.uk/Anonymous/Anonymous-Lunchtime-atop-Skyscraper-5000174.html
Downloaded: 13. November, 2007.

[19]
Official homepage of skyscraperpage
www.skyscraperpage.com.
Visited: 29. May, 2007.

[20]
Official homepage of VSÓ – Engineering company in Iceland
www.vso.is.
Downloaded: 1. November, 2007.

[21]
Official homepage of wikipedia - The Free Encyclopedia
www.wikipedia.com.
Visited: 29. May, 2007.

ORAL AND E-MAIL REFERENCES

[22]
Dalblom, Ola. 2007, "*Lecture in Grundläggningsteknik, VGT021 2006/2007*"
Professor in Structural mechanics LTH.

[23]
Gunnarsson, Hilmir Þór. 2007, "*Constructions / architectural drawings on Smáratorg Tower*" Constructing architect BFI, Arkís ehf.

[24]
Kristjánsson, Snæbjörn. 2007, "*Engineering information on Smáratorg Tower*"
Structural engineers, Ferill.

[25]
Thelandersson, Sven. 2007, "*Informations and drawings on Turning Torso from June 2000*" Professor in structural engineering, LTH.

IMAGES

All figures are by the author of this thesis except for:
The one on the front page it is from **[18]**
The one of Smáratorg Tower it is from **[20]**
The one of Turning Torso and the sketch made by Santiago Calatrava it is from **[15]**

APPENDIX

HOW TO FIND MOMENT OF INERTIA WITH AUTOCAD.

First the object has to be drawn in AutoCad.

Then all the lines have to be converted to polyline (that are not already) with the command “Edit Polyline” from the “Modify II” toolbox, than select a line, than the text appears on the command line “Object selected is not a polyline. Do you want to turn it into one? <Y>” select Y for yes, then the option “Enter an option [Close/Join/Width/Edit vertex/Fit/Spline/Decurve/Ltype gen/Undo]:” appears select J for Join and join/group the rest of the lines that is needed to join.

Now this has to be changed into region by typing “REGION” in the command line and select the objects.

If the joined objects have an opening in it like an elevator shaft, than the shaft can be subtracted with the command “Subtract” from the “Solids Editing” toolbar.

Now the zero point has to be moved in the mass middle of the object, this is done with the command “Origin UCS” from the “UCS” toolbar.

Now all the information needed can be received with the command “Region/Mass Properties” from the “Inquiry” toolbar.

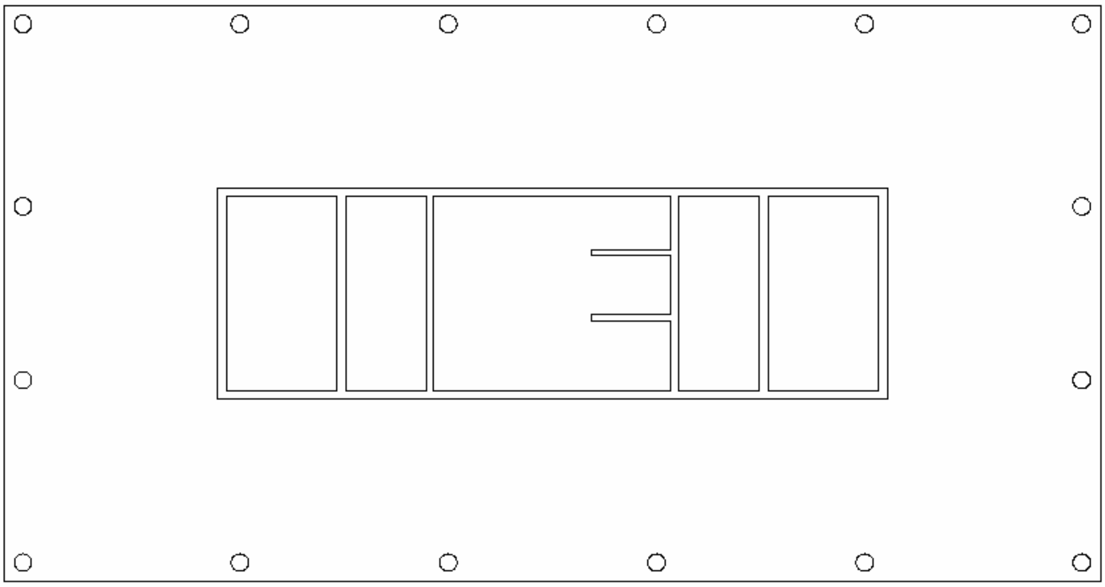

Figure 5-1 Shows one of many schematic figure of a floor in Smáratorg Tower, used to find the moment of inertia. This particular floor rises between 12.95 m and 26.95 m. Note that the pillars were neglected when the moment of inertia was established.

MATLAB CODE TO FIND THE FIRST NATURAL FREQUENCY OF VIBRATION FOR TURNING TORSO.

```
% function Turning Torso Frequency calc
clc
clear all
close all

syms x;
L=175.5;%m
Alfa_concrete=2550;%kg/m3 Weight of reinforced concrete.
core_m=110000;%kg/m 101000
Econcrete=36e9;%Youngs modulus N/m2 the structural core is made out
of C55/65 concrete
Radius=4.3;%m
Diameter=8.6;%m

%The length of each core element of the Turning torso
x2=L-19.5*8;%m
x3=L-19.5*7;%m
x4=L-19.5*6;%m
x5=L-19.5*5;%m
x6=L-19.5*4;%m
x7=L-19.5*3;%m
x8=L-19.5*2;%m
x9=L-19.5;%m
x10=L;%m

%Thickness of each core element
t2=1.5;%m
t3=1.4;%m
t4=1.3;%m
t5=1.2;%m
t6=1.1;%m
t7=1.0;%m
t8=0.9;%m
t9=0.8;%m
t10=0.7;%m

%The total weight of each part of the core per m
m2=Alfa_concrete*((Radius+t2)^2*pi-Radius^2*pi)+core_m;%kg/m
m3=Alfa_concrete*((Radius+t3)^2*pi-Radius^2*pi)+core_m;%kg/m
m4=Alfa_concrete*((Radius+t4)^2*pi-Radius^2*pi)+core_m;%kg/m
m5=Alfa_concrete*((Radius+t5)^2*pi-Radius^2*pi)+core_m;%kg/m
m6=Alfa_concrete*((Radius+t6)^2*pi-Radius^2*pi)+core_m;%kg/m
m7=Alfa_concrete*((Radius+t7)^2*pi-Radius^2*pi)+core_m;%kg/m
m8=Alfa_concrete*((Radius+t8)^2*pi-Radius^2*pi)+core_m;%kg/m
m9=Alfa_concrete*((Radius+t9)^2*pi-Radius^2*pi)+core_m;%kg/m
m10=Alfa_concrete*((Radius+t10)^2*pi-Radius^2*pi)+core_m;%kg/m

I2=((Diameter+2*t2)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I3=((Diameter+2*t3)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I4=((Diameter+2*t4)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I5=((Diameter+2*t5)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I6=((Diameter+2*t6)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I7=((Diameter+2*t7)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I8=((Diameter+2*t8)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
I9=((Diameter+2*t9)^4-Diameter^4)*pi/64; %moment of inertia [m^4]
```

```
I10=((Diameter+2*t10)^4-Diameter^4)*pi/64; %moment of inertia [m^4]

%The shape functions
% Gafla1=x^2/L^2;
% Gafla2=2/L^2;
% Gafla1=1-cos(pi*x/(2*L));
% Gafla2=(cos(pi*x/(2*L))*(pi^2/(4*L^2)));
Gafla1=3*x^2/(2*L^2)-x^3/(2*L^3);
Gafla2=6/(2*L^2)-6*x/(2*L^3);

%Integrate to get k and m
k=int(Econcrete*I2*(Gafla2)^2,x,0,x2)+
int(Econcrete*I3*(Gafla2)^2,x,x2,x3)+
int(Econcrete*I4*(Gafla2)^2,x,x3,x4)+
int(Econcrete*I5*(Gafla2)^2,x,x4,x5)+
int(Econcrete*I6*(Gafla2)^2,x,x5,x6)+
int(Econcrete*I7*(Gafla2)^2,x,x6,x7)+
int(Econcrete*I8*(Gafla2)^2,x,x7,x8)+
int(Econcrete*I9*(Gafla2)^2,x,x8,x9)+
int(Econcrete*I10*(Gafla2)^2,x,x9,x10);

m=m2*int((Gafla1)^2,x,0,x2)+   m3*int((Gafla1)^2,x,x2,x3)+
m4*int((Gafla1)^2,x,x3,x4)+    m5*int((Gafla1)^2,x,x4,x5)+
m6*int((Gafla1)^2,x,x5,x6)+    m7*int((Gafla1)^2,x,x6,x7)+
m8*int((Gafla1)^2,x,x7,x8)+    m9*int((Gafla1)^2,x,x8,x9)+
m10*int((Gafla1)^2,x,x9,x10);

w=sqrt(k/m);
f=w/(2*pi)
```

MATLAB CODE TO FIND THE FIRST NATURAL FREQUENCY OF VIBRATION FOR SMÁRATORG TOWER.

```
% function Highest building in Islands ca 80m Frequency calc
clc
clear all
close all

syms x;
L=77.9;%m
Alfa_concrete=2550;%kg/m3
core_m=201100;%kg/m
Econcrete=28.8e9;%Youngs modulus N/m2 concrete C40/50 and C30/37

%The length of each part of the highest building in Iceland
x2=5.95;%m
x3=12.95;%m
x4=23.45;%m
x5=26.95;%m
x6=33.95;%m
x7=58.45;%m
x8=65.45;%m
x9=68.95;%m
x10=L;%m

%The weight of each part of the core per m
m2=core_m;%kg/m
m3=core_m;%kg/m
m4=core_m;%kg/m
m5=core_m;%kg/m
m6=core_m;%kg/m
m7=core_m;%kg/m
m8=core_m;%kg/m
m9=core_m;%kg/m
m10=core_m;%kg/m

I2=282;%824; %moment of inertia [m^4]
I3=282;%662; %moment of inertia [m^4]
I4=282;%595; %moment of inertia [m^4]
I5=282;%521; %moment of inertia [m^4]
I6=282;%518; %moment of inertia [m^4]
I7=282;%482; %moment of inertia [m^4]
I8=282;%450; %moment of inertia [m^4]
I9=282;%447; %moment of inertia [m^4]
I10=282;%320; %moment of inertia [m^4]

%The shape functions
% Gafla1=x^2/L^2;
% Gafla2=2/L^2;
% Gafla1=1-cos(pi*x/(2*L));
% Gafla2=(cos(pi*x/(2*L))*(pi^2/(4*L^2)));
Gafla1=3*x^2/(2*L^2)-x^3/(2*L^3);
Gafla2=6/(2*L^2)-6*x/(2*L^3);
%Integrate to get k and m
k=int(Econcrete*I2*(Gafla2)^2,x,0,x2)+
int(Econcrete*I3*(Gafla2)^2,x,x2,x3)+
int(Econcrete*I4*(Gafla2)^2,x,x3,x4)+
int(Econcrete*I5*(Gafla2)^2,x,x4,x5)+
```

```
int(Econcrete*I6*(Gafla2)^2,x,x5,x6)+
int(Econcrete*I7*(Gafla2)^2,x,x6,x7)+
int(Econcrete*I8*(Gafla2)^2,x,x7,x8)+
int(Econcrete*I9*(Gafla2)^2,x,x8,x9)+
int(Econcrete*I10*(Gafla2)^2,x,x9,x10);

m=m2*int((Gafla1)^2,x,0,x2)+   m3*int((Gafla1)^2,x,x2,x3)+
m4*int((Gafla1)^2,x,x3,x4)+    m5*int((Gafla1)^2,x,x4,x5)+
m6*int((Gafla1)^2,x,x5,x6)+    m7*int((Gafla1)^2,x,x6,x7)+
m8*int((Gafla1)^2,x,x7,x8)+    m9*int((Gafla1)^2,x,x8,x9)+
m10*int((Gafla1)^2,x,x9,x10);

w=sqrt(k/m);
f=w/(2*pi)
```

MATLAB CODE TO FIND THE FUNDAMENTAL MODAL MASS FOR TURNING TORSO.

```
% function Turning Torso Mass calc
clc
clear all
close all

syms x;
L=175.5;%m
Alfa_concrete=2550;%kg/m3 Weight of reinforced concrete.
core_m=110000;%kg/m 101000
Radius=4.3;%m
Diameter=8.6;%m

%The length of each core element of Turning Torso
x2=L-19.5*8;%m
x3=L-19.5*7;%m
x4=L-19.5*6;%m
x5=L-19.5*5;%m
x6=L-19.5*4;%m
x7=L-19.5*3;%m
x8=L-19.5*2;%m
x9=L-19.5;%m
x10=L;%m

%Thickness of each core element
t2=1.5;%m
t3=1.4;%m
t4=1.3;%m
t5=1.2;%m
t6=1.1;%m
t7=1.0;%m
t8=0.9;%m
t9=0.8;%m
t10=0.7;%m

%The total weight of each part of the core per m
m2=Alfa_concrete*((Radius+t2)^2*pi-Radius^2*pi)+core_m;%kg/m
m3=Alfa_concrete*((Radius+t3)^2*pi-Radius^2*pi)+core_m;%kg/m
m4=Alfa_concrete*((Radius+t4)^2*pi-Radius^2*pi)+core_m;%kg/m
m5=Alfa_concrete*((Radius+t5)^2*pi-Radius^2*pi)+core_m;%kg/m
m6=Alfa_concrete*((Radius+t6)^2*pi-Radius^2*pi)+core_m;%kg/m
m7=Alfa_concrete*((Radius+t7)^2*pi-Radius^2*pi)+core_m;%kg/m
m8=Alfa_concrete*((Radius+t8)^2*pi-Radius^2*pi)+core_m;%kg/m
m9=Alfa_concrete*((Radius+t9)^2*pi-Radius^2*pi)+core_m;%kg/m
m10=Alfa_concrete*((Radius+t10)^2*pi-Radius^2*pi)+core_m;%kg/m

Gafla1=x^2/L^2;

%Integrate to get the fundamental modal mass M1
m=     m2*int((Gafla1),x,0,x2)+    m3*int((Gafla1),x,x2,x3)+
m4*int((Gafla1),x,x3,x4)+    m5*int((Gafla1),x,x4,x5)+
m6*int((Gafla1),x,x5,x6)+    m7*int((Gafla1),x,x6,x7)+
m8*int((Gafla1),x,x7,x8)+    m9*int((Gafla1),x,x8,x9)+
m10*int((Gafla1),x,x9,x10);
```