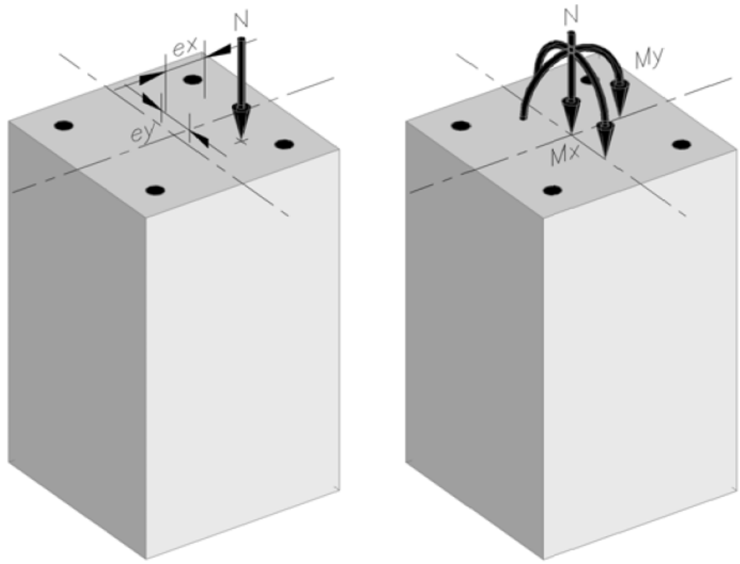
Development of a computer program to design concrete columns for biaxial moments and normal force



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Development of a computer program to design concrete columns for biaxial moments and normal force

Utveckling av ett datorprogram för att bestäma kapacitien i betongpelare som belastas av biaxiel böjning och normalkraft

Valdimar Örn Helgason

2010

Abstract

Design of a concrete column with biaxial moments and normal force is unpractical without the aid of a computer program. It is very time consuming to go through all calculations and to find a suitable layout of a cross-section and positioning of reinforcement for the final design. The goal of this thesis is to create a user friendly computer program for this problem. A critical pier in the Öland Bridge was used as a study case in this thesis.

Keywords: Reinforced concrete; Stress/Strain Relationship; Design of Concrete Columns; Uniaxial Moment; Interaction Diagram; Biaxial Moments; Interaction Surface

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Foreword

This master thesis was written under the administration of the Division of Structural Engineering at the University of Lund during the period October 2009 – June 2010 under the supervision of Dr. Fredrik Carlsson.

I would like to thank my supervisor Dr. Fredrik Carlsson for all the help and time spent during the work of this thesis. I would also like to thank my friends and colleagues Bzav Abdulkarim, Daniel Honfi, Ívar Björnsson and Jóhannes Helgi Jóhannesson for their support and advice during my time at LTH.

I hope this thesis will help readers to gain further knowledge on the design of reinforced concrete columns for biaxial moments and normal force.

This master thesis is dedicated to my dear grandfather, Valdimar Helgason, and my family for all their love, help and patience during my academic years.

Lund, June 2010
Valdimar Örn Helgason

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Symbols

Theory

A_g Gross cross-sectional area of concrete
A_s Area of reinforcement
$A_{s,Tot}$ Total area of reinforcement
A_{Tot} Total area
b Width of a cross-section
f_{cc} Design value for compressive strength of concrete
f_{cck} Characteristic compressive cylinder strength of concrete at 28 days (capacity to resist compressive stress)
f_{ct} Design tensile strength of concrete (capacity to resist tensile stress)
f_d Design value for material strength
f_k Characteristic value for material strength
f_{st} Design value for yield strength of steel
f_{uk} Characteristic ultimate strength of steel
f_{yk} Characteristic yield strength of steel (capacity to resists stress)
d Effective height of reinforcement
e Eccentricity (Chapters 2.9 and 2.10)
e Height from a chosen point of an area to the center of gravity of the corresponding area (Eq. 2.13)
E_{ck} Characteristic modulus of elasticity of concrete
E_d Design value for modulus of elasticity
E_k Characteristic value for modulus of elasticity
E_s Design value for modulus of elasticity of steel
E_{sk} Characteristic modulus of elasticity of steel
F_c Force from concrete
F_s Force from reinforcement
h Height of a cross-section
M_{cr} Cracking moment equal to moment at which section first cracks
M_{ult} Ultimate moment capacity for a specified cross-section
N Axial force
N_{ult} Ultimate capacity for axial force
$N_{ult,C}$ Ultimate compression capacity for a specified cross-section
$N_{ult,T}$ Ultimate tension capacity for a specified cross-section
x Height of the neutral axis
Z Lever arm
γ_n Safety factor, depends on safety class (Chapter 2.6.1)
ε_{c0} Compressive strain in the concrete at the peak stress f_{cc}
ε_{cu} Ultimate compressive strain in the concrete
ε_s Strain in reinforcement
ε_{sy} The strain in reinforcement when it yields
$\eta\gamma_m$ See chapter 2.6.2
ρ Density of concrete

σ_s Stress in reinforcement

Computer program

A_c Area of concrete
A_s Area of reinforcement
b Inner height of a cross-section
B Outer height of a cross-section
f_{cc} Design value for compressive strength of concrete
f_{cck} Characteristic compressive cylinder strength of concrete at 28 days (capacity to resist compressive stress)
f_{st} Design value for yield strength of steel
f_{yk} Characteristic yield strength of steel (capacity to resists stress)
e_c Lever arm for concrete force
E_s Design value for modulus of elasticity of reinforcement
E_{sk} Characteristic value for modulus of elasticity of reinforcement
l Inner width of a cross-section
L Outer width of a cross-section
M_x Moment around x-axis
M_y Moment around y-axis
N Axial force
Ø Diameter of reinforcement
t Thickness of hollow cross-section
x' Transformed x coordinate
y' Transformed y coordinate
γ_n Safety factor, depends on safety class (Chapter 2.6.1)
ε_{cu} Ultimate compressive strain in the concrete
ε_{sy} The strain in reinforcement when it yields
η Heigth of neutral axis for rotated cross-section
φ Rotation of cross-section
φ_{crit} Critical rotation of cross-section

Design of structural members for axial force and biaxial moments in Eurocode

A Area
a Exponent for Eq. 4.9 and 4.10 (Table 4.1)
f_{cc} Design value for compressive strength of concrete
f_{cck} Characteristic compressive cylinder strength of concrete at 28 days (capacity to resist compressive stress)
f_{st} Design value for yield strength of steel
f_{yk} Characteristic yield strength of steel (capacity to resists stress)
e_i Eccentricity along axis i
E_s Design value for modulus of elasticity of reinforcement
E_{sk} Characteristic value for modulus of elasticity of reinforcement
I_i Moment of inertia around respective axis i
i_i Radius of gyration for axis i

M_{di}	Design moment around respective axis i
M_{Rdi}	Moment capacity around respective axis i
N_d	Design axial force
N_{Rd}	Axial force capacity
α	Coefficient which takes account of the long-term effects of the compressive strength and of the unfavorable effects resulting from the way in which the load is applied, $\alpha = 0,85$ if the section is at least as wide at the extreme compressive fibre as it is elsewhere in the compressive zone.
γ_m	Safety factor for material strength

1 Introduction

1.1 Background

Structural members carrying vertical loads in bridges withstand normal force and biaxial moments caused by eccentricity of traffic load or structural system. Analysis and design of a structural concrete member for biaxial moments is difficult because a trial and adjustment procedure is necessary to find the inclination and depth of the neutral axis satisfying the equilibrium equations. The neutral axis is not usually perpendicular to the resultant eccentricity. In design a section and reinforcement pattern could be assumed and the reinforcement area successively corrected until the section capacity approaches the required value. Therefore, the direct use of equations in design is impractical without the aid of a computer program.

1.2 Purpose and goal

The purpose of this master thesis is to understand the theory for design of structural concrete members affected by normal force and biaxial moments. Also a computer program is developed for design is such situations. The results from the computer program will be compared to the results from the approximate method suggested in Eurocode if aid of a computer program is not available.

By using a computer program for the design an exact curvature of interaction lines can be created resulting in precise information about the capacity of a structural member under effects from a biaxial moments caused be eccentricity of the axial load.

The goal is to create a tool for a reliable, quicker and practical design of a structural concrete member that has to resists biaxial moments and axial force.

If there is interest in accessing the computer program I can be contacted at ogwaldo@hotmail.com.

1.3 Limitations

The limitation of the computer program is that only rectangular, solid or hollow, cross-sections can be designed. If a hollow section is designed the thickness of the concrete sections has to be constant.

1.4 Case study

The cross-section of a critical column, pier 28, in the Öland Bridge is used as a case study in this master thesis. Due to alkali reactions in the original columns in the Öland Bridge the columns were strengthened by casting a new layer of reinforced concrete around the original cross-section. Only the new cross-section is considered to be the load bearing structure, so the case study is a rectangular hollow cross-section, see fig. 1-1.

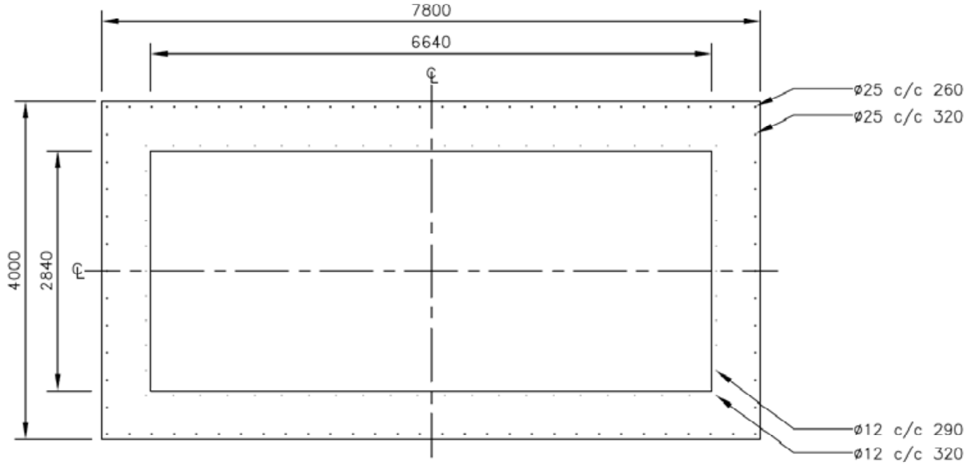


Figure 1-1 Cross-section of pier 28 in the Öland Bridge

2 Theory

2.1 Reinforced concrete

Reinforced concrete is perhaps the most widespread structural material presently used around the world. It is a composite material which consists of concrete and steel.

Concrete has many advantages as a structural material but has one great disadvantage of being weak in tension. The concrete cracks under the smallest tensile force, i.e. when the stresses in the structural concrete member are greater than the tension strength f_{ct}.

Steel has high tensile strength and is placed as reinforcement at tensile zones in the structural concrete member, i.e. once the concrete cracks the tensile force is carried by the reinforcement crossing the cracks. Therefore the advantages of both materials are utilized when the concrete area resists compressive stresses while the reinforcement bars resists tensile stresses.

2.2 Stress/strain relationships

2.2.1 Concrete

The short term stress/strain relationship for compressed concrete is illustrated in Fig. 2-1. The stress/strain relationship is approximately linear for low values of strain, i.e. the slope is reasonably constant for the initial part. The stress/strain relationship becomes substantially non-linear as the stress increases until the point where little extra stress causes a large increase in strain. The peak in stress is known as the compressive strength, f_{cck}, and the corresponding strain is ε_{c0}. Beyond this point the stress must be decreased if failure in the concrete is to be prevented. Finally the concrete will crush when the strain in the extreme concrete fiber in compression reaches its ultimate compressive strain, ε_{cu}.

An average slope, known as the secant modulus, of the line joining the origin to the point corresponding to 40 per cent of the characteristic compressive strength is used to determine the modulus of elasticity, E_{ck}.

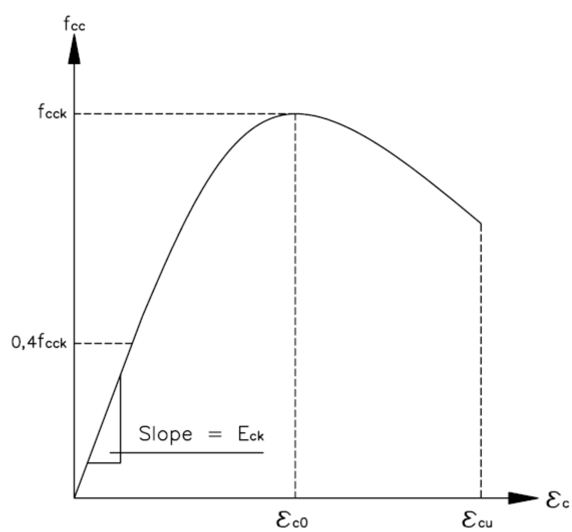


Figure 2-1 Stress/strain relationship for concrete

2.2.2 Reinforcement

The stress/strain relationship for steel, in compression or tension, begins with a straight line from the origin to the proportional limit. That region is called the linear/elastic region. Beyond that point the strain begins to increase more rapidly for each increment in stress, the stress/strain curve has a smaller and smaller slope, until the steel yields (the yield point). Beyond the yield point considerable elongation of the steel occurs with no noticeable increase in the tensile force, known as the yielding phenomenon or perfectly plastic region. The corresponding stress for the yield point is known as the yield stress, f_{yk}, of the steel. The yield strength of the reinforcement is though defined as the stress at which the strain has exceeded the value predicted by the linear relationship by 0,002, known as the offset method. After the steel has undergone large strains that occur during yielding the steel begins to strain harden, that results in increased resistance of the steel to further deformation. At last the load reaches its maximum value and the corresponding stress is called the ultimate stress, f_{uk}. A typical stress/strain diagram for reinforcement in tension or compression is illustrated in Fig. 2-2.

The slope of the linear/elastic region is used to determine the modulus of elasticity for the steel, E_{sk}.

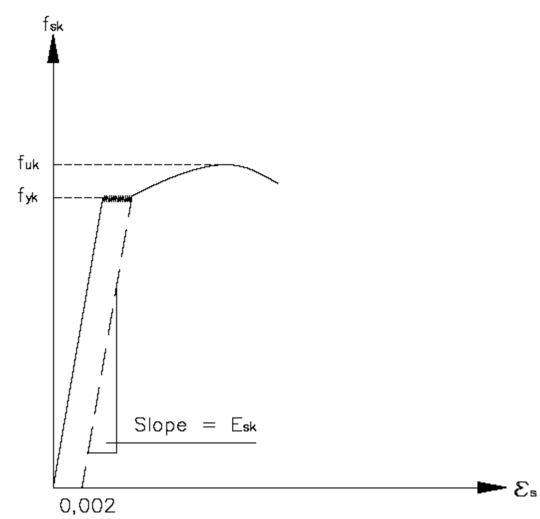


Figure 2-2 Stress/strain relationship for reinforcement

2.3 Simplified stress/strain diagrams

Instead of using the actual stress/strain diagrams that have been explained simplified stress/strain diagrams are used for design in this thesis.

2.3.1 Concrete

A parabolic-rectangular diagram is used for the concrete, Fig. 2-3.

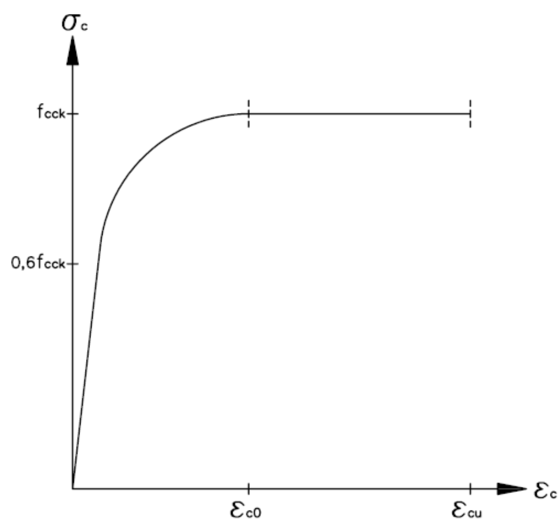


Figure 2-3 Simplified stress/strain diagram for concrete

2.3.2 Reinforcement

A horizontal top branch diagram is used for the reinforcement, Fig. 2-4. In that case no limit on the steel strain is necessary.

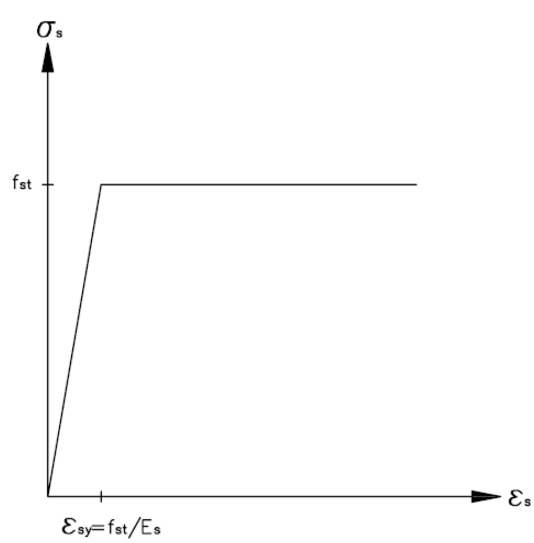


Figure 2-4 Simplified stress/strain diagram for steel

2.4 Modes of failure

There are five modes of failure for reinforced concrete sections which all depend on the amount of reinforcement placed in the section.

2.4.1 Ultimate moment capacity less than cracking moment

As has been mentioned above, concrete cracks easily in tension. When the applied moment in the section has reached the cracking moment, M_{cr}, the ultimate moment capacity, M_{ult}, is greatly affected by the area of reinforcement, A_s. If that area is particularly small the moment capacity after first cracking will be less than M_{cr} which will result in a sudden failure of the member, see fig. 2-5. To prevent such sudden failure sufficient reinforcement must be provided so that $M_{ult} > M_{cr}$.

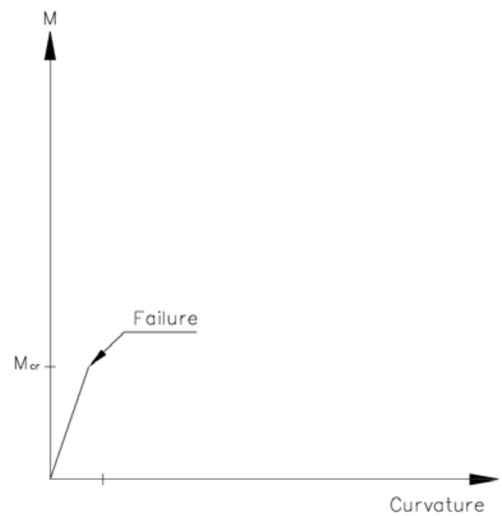


Figure 2-5 Mode of failure 1, $M_{ult} < M_{cr}$

2.4.2 Very small reinforcement

When the moment capacity is increased with more reinforcement, so it will be greater than the cracking moment, but the area of reinforcement is still considered small, failure mode 2 occurs. The reinforcement yields before the maximum compressive strain in the concrete reaches the maximum compressive strain, ε_{cu}, and plastic deformations will occur, see fig. 2-6. That results in considerable deformations before the reinforcement will fail completely.

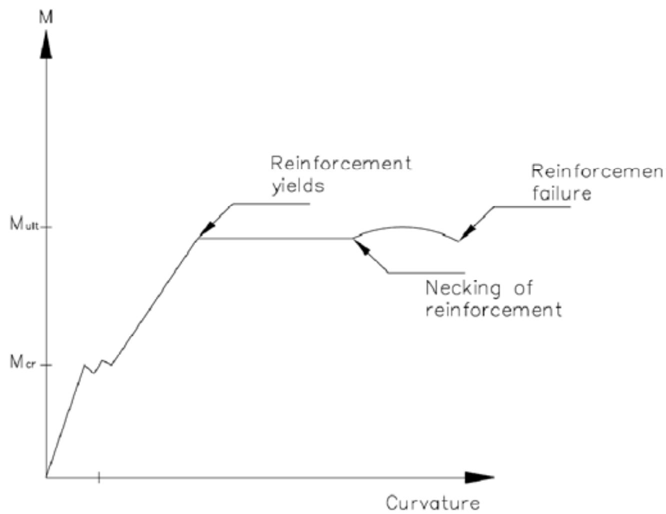


Figure 2-6 Mode of failure 2, very small reinforcement

2.4.3 Under-reinforced section

Larger area of reinforcement is used in the section where the reinforcement yields but the strain in the concrete reaches its maximum before the reinforcement fails. Thus, the failure of the section is due to crushing of the concrete in compression, see fig. 2-7. Because the amount of reinforcement is still considered small the section is termed under-reinforced.

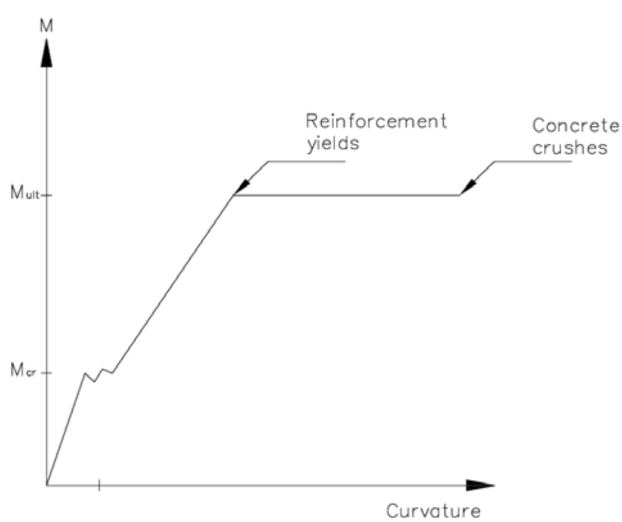


Figure 2-7 Mode of failure 3, under-reinforced

2.4.4 Balanced design

If the area of reinforcement in increased further to a level where the concrete and the reinforcement fail simultaneously, the concrete crushes at the same time as the reinforcement first yields, see fig. 2-8. This is referred to as balanced design.

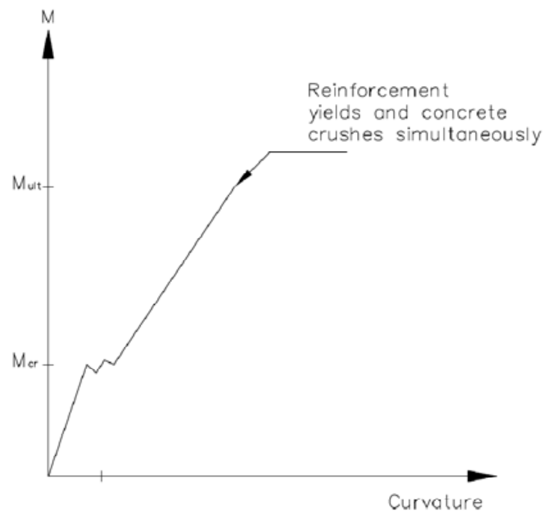


Figure 2-8 Mode of failure 4, balanced design

2.4.5 Over-reinforced section

If the area of reinforcement is increased still further the concrete will crush before any yielding in the reinforcement will occur, see fig. 2-9. In that case there is too much reinforcement in the section and this is termed over-reinforced.

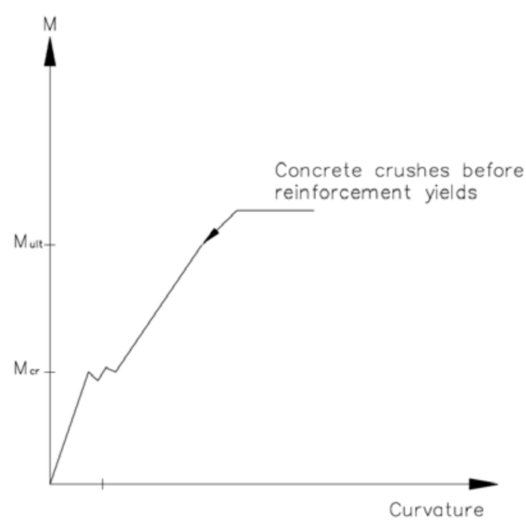


Figure 2-9 Mode of failure 5, over-reinforced

2.5 Design possibilities for reinforced sections

The fourth and fifth modes of failure act as failure of a brittle material, i.e. there is no plastic deformation of the reinforcement prior to collapse. Since there will be no warning of failure and it will happen suddenly these modes are unacceptable forms of failure in design situations. To prevent a brittle failure design codes specify maximum allowable values for the area of reinforcement.

Large plastic deformation will occur prior to failure for modes two and three, there is a ample warning of imminent failure. Those modes are referred to as ductile failures and are highly desirable in design of reinforced concrete structures.

2.6 Safety factors for materials

Material strengths are stochastic variables and estimates of their characteristic values must be made for design.

Characteristic strengths are values below which a minimum number (usually 5 per cent) of specimens are expected to fail. To allow for the possibility that the material resistance is less than its characteristic value BBK 04 states safety factors to estimate the design values for three different safety classes. The safety classes are determined by the following consequences of structural failure and are taken into account in the design value by the parameter γ_n, which is different for the different safety classes.

2.6.1 Safety classes

- Safety class 3: Very serious consequences, significant risk for serious injury. γ_n = 1,2 and probability of failure can be 10^{-6}
- Safety class 2: Severe consequences, some risk for serious injury. γ_n = 1,1 and probability of failure can be 10^{-5}
- Safety class 1: Minor consequences, insignificant risk for serious injury. γ_n = 1,0 and probability of failure can be 10^{-4}

2.6.2 Ultimate limit state

Ultimate limit state is reached when the structure collapses due to loss of equilibrium, stability or to failure by rupture of structural members.

The factor $\eta\gamma_m$ takes into account the long-term effects of the material strength and of the unfavorable effects resulting from the difference from the materials being treated at laboratory and at construction site.

The design value for strength is estimated by:

$$f_d = \frac{f_k}{\eta\gamma_m\gamma_n} \qquad \text{(Eq. 2.1)}$$

Where $\eta\gamma_m$ is:
1,5 for concrete
1,15 for reinforcement

The design value for stiffness is estimated by:

$$E_d = \frac{E_k}{\eta \gamma_m \gamma_n} \quad \text{(Eq. 2.2)}$$

Where $\eta\gamma_m$ is:
1,2 for concrete
1,05 for reinforcement

2.7 Design for axial force

Axial force can both act on the structural member as tension or compression.

2.7.1 Tension

As has been mentioned above concrete cracks under the smallest tensile force and therefore only the reinforcement in tension is used for the design for tensile forces. The tensile strength of a structural member of reinforced concrete is:

$$N_{ult,T} = f_{st} A_{s,Tot} \quad \text{(Eq. 2.3)}$$

Where:
f_{st} = Design value for tensile strength of the reinforcement
$A_{s,Tot}$ = Total area of all the the reinforcement

2.7.2 Compression

Concrete has high compression strength and the reinforcement has the same strength in compression as in tension. Therefore both materials can be used to determine the capacity for compression. The compression strength of a structural member of reinforced concrete is:

$$N_{ult,C} = f_{cc}\left(A_g - A_{s,Tot}\right) + f_{st} A_{s,Tot} \quad \text{(Eq. 2.4)}$$

Where:
f_{cc} = Design value for compressive strength of the concrete
A_g = Gross cross-sectional area of the concrete
$A_{s,Tot}$ = Total area of all the the reinforcement
f_{st} = Design value for tensile strength of the reinforcement

2.8 Ultimate moment capacity

The ultimate moment capacity, M_{ult}, can be calculated for a structural member of reinforced concrete with specified dimensions and areas of reinforcement with combination of the simplified design stress/strain diagrams for concrete and steel. The ultimate compressive strain in the concrete is stated in BBK 04 as:

$$\varepsilon_{cu} = \left(0,4 + \frac{\rho}{2200} 0,6\right) 0,0035 \quad \text{(Eq. 2.5)}$$

With $\rho = 2200$ kg/m^3 for normal or heavy concrete which yields $\varepsilon_{cu} = 0,0035$

Equilibrium of forces from the concrete and the reinforcement can be used to determine the ultimate moment capacity of a section in bending. Moment is taken around the center of gravity for the cross-section with forces from the compressed area of the concrete and compressed/tensile area of the reinforcement. BBK 04 states that 80 percent of the compressed concrete area can be used. The lever arm of the concrete force is taken from the center of gravity of the compressed concrete zone to the center of gravity of the whole section. The appropriated signs for forces in equations for moment capacity are negative for tension and positive for compression.

The concrete force is:

$$F_c = 0,8xbf_{cc} \qquad \text{(Eq. 2.6)}$$

Where:
x = Height of the neutral axis, see fig. 2-10 and 2-12
b = Width of the cross-section
f_{cc} = Design value for compressive strength of the concrete

The force from the reinforcement is determined by the strain in the steel with:

$$\varepsilon_s = \varepsilon_{cu} \frac{x-d}{x} \qquad \text{(Eq. 2.7)}$$

Where:
ε_{cu} = 0 0035
d = Effective height of the reinforcement, see fig. 2-10 and 2-12

If the steel has yielded the strain will be set equal to the strain where the steel yields which is:

$$\varepsilon_{sy} = \frac{f_{st}}{E_s} \qquad \text{(Eq. 2.8)}$$

Where:
f_{st} = Design value for the yield strength of the reinforcement
E_s = Design value for the modulus of elasticity for the reinforcement

The stress in the reinforcement will then be:

- When the reinforcement has yielded: $\varepsilon_s \geq \varepsilon_{sy} \Rightarrow \sigma_s = f_{st}$ (Eq. 2.9)
- When the reinforcement has not yielded: $\varepsilon_s < \varepsilon_{sy} \Rightarrow \sigma_s = \varepsilon_s E_s$ (Eq. 2.10)

The force from each rebar will then be:

$$F_s = A_s \sigma_s \qquad \text{(Eq. 2.11)}$$

Where:
A_s = The area of the reinforcement
σ_s = The stress in the reinforcement

2.8.1 Rectangular solid cross-section

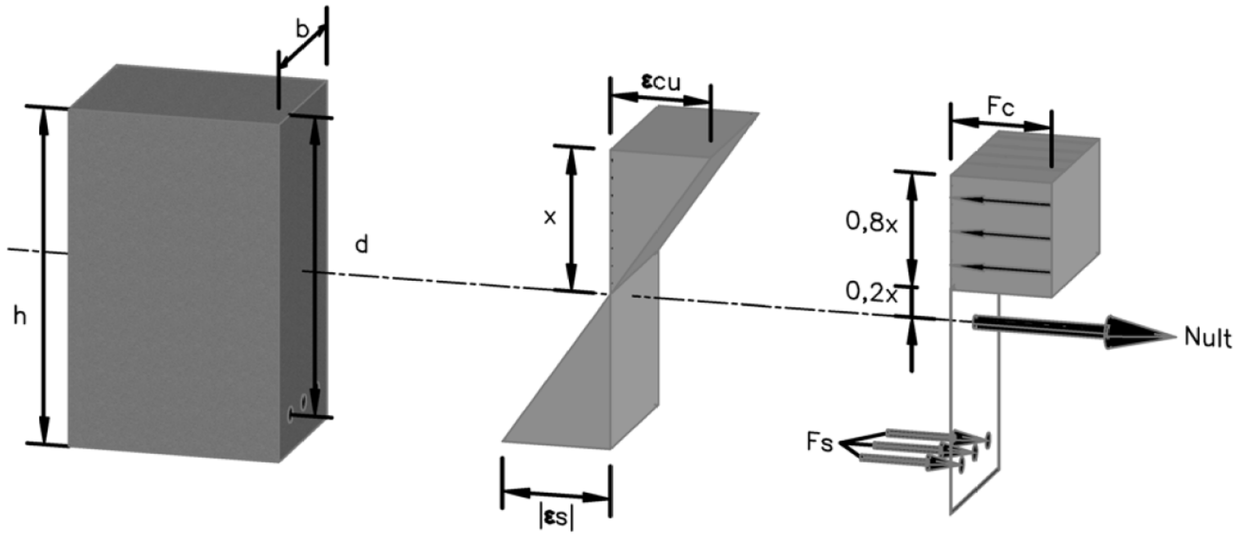


Figure 2-10 Explanation for symbols and strain/stress distribution for a structural concrete member with a rectangular solid cross-section

The equilibrium for the ultimate moment capacity of the beam in fig. 2-10 will then be, see fig. 2-11:

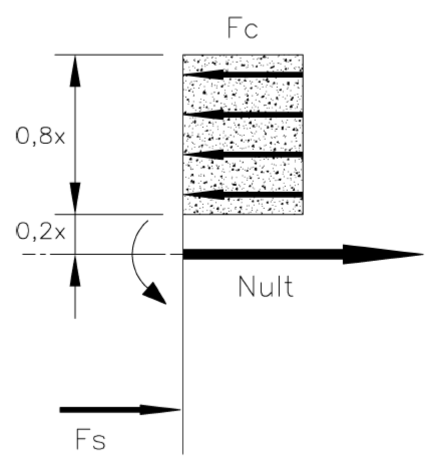


Figure 2-11 Equilibrium for the ultimate moment capacity for a rectangular solid cross-section

$$M_{ult} = F_c\left(\frac{h}{2} - 0,4x\right) + F_s\left(x - d\right) \qquad \text{(Eq. 2.12)}$$

The reinforcement is in tension so the sign of the force will be negative as well as the lever arm since the effective height of the bar is greater than the neutral axis resulting in a positive corresponding moment from the reinforcement. The lever arm for the axial force is zero, i.e. the axial force will not affect the results as long as the moment is taken around the center of gravity for the cross-section.

2.8.2 Rectangular hollow cross-section

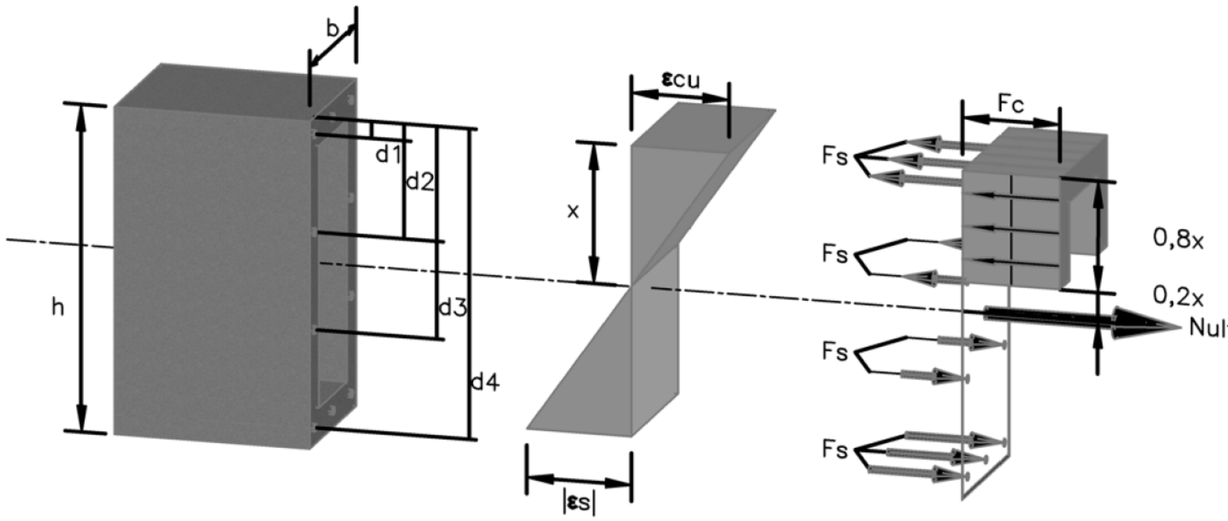


Figure 2-12 Explanation for symbols and strain/stress distribution for a structural concrete member with a rectangular hollow cross-section

Since the center of gravity for the concrete force is not placed at the center of the compressed concrete the first step for rectangular hollow cross-section is to determine the lever arm using the following procedure.

The lever arm, Z, from the neutral axis is determined by dividing the compressed concrete area to a system of sub-areas. The height from the bottom of the compressed area to the center of gravity for the corresponding area, e, is determined with:

$$e = \frac{\sum A_i e_i}{A_{Tot}} \qquad \text{(Eq. 2.13)}$$

Where:
A_i = The area of sub-area i
e_i = The distance from the bottom of the whole area to the center of gravity of sub-area i, Fig. 2-13.
A_{Tot} = Total area of all sub-areas

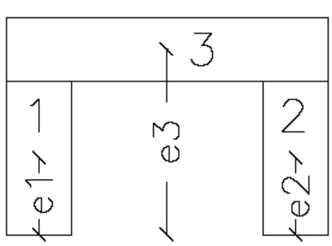


Figure 2-13 Explanation of sub-areas and the corresponding distance from a chosen point to their center of gravity

The lever arm will then be: Z = 0,2x+e (Eq. 2.14)

The equilibrium for the ultimate moment capacity of the beam in fig. 2-12 will then be, see fig. 2-14:

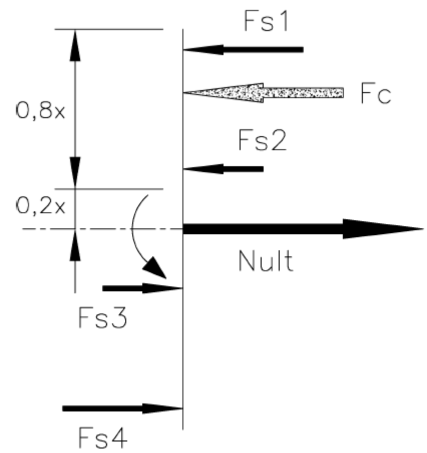


Figure 2-14 Equilibrium for the ultimate moment capacity for a rectangular solid cross-section, the force from the concrete is shown as a arrow placed at the center of gravity for the compressed concrete area

$$M_{ult} = F_c \cdot Z + \sum F_s \left(x - d \right) \qquad \text{(Eq. 2.15)}$$

2.9 Design of structural concrete members for axial force and uniaxial moment

When an applied axial load of a structural member has eccentricity, e, part of the load is carried in bending caused by the eccentricity, see fig. 2-15.

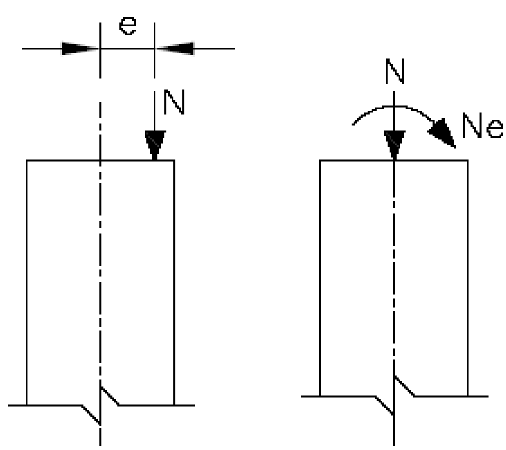


Figure 2-15 Uniaxial moment caused by eccentricity of an axial force

Ultimate moment and axial force capacity can be determined for a specified section by the location of the neutral axis. By moving the neutral axis from pure compression to pure tension a unique combination of moment and axial force capacity can be plotted as an interaction diagram.

2.9.1 Interaction diagram

Interaction diagram is a graph illustrating the capacity of a structural concrete member to resist a range of combinations of moment and axial force. By changing the location of the neutral axis, giving different size of compressive and tension zones, each case will lead to a different capacity calculated from the strain distribution. First the section is in pure compressions, then it will be over-reinforced until it reaches the point where it is balanced designed. After the point of balanced design the section will reach pure bending, then be under-reinforced and finally be in pure tension, see fig. 2-16.

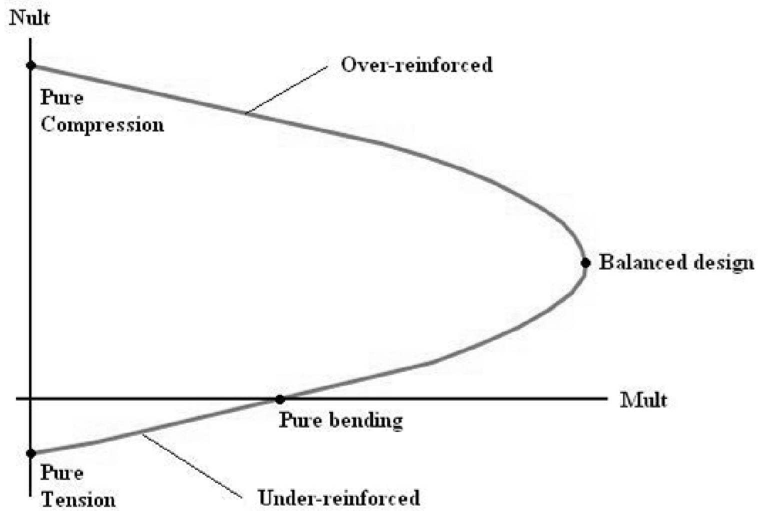


Figure 2-16 Interaction diagram

2.10 Design of structural concrete members for axial force and biaxial moments

If a structural member is simultaneously subjected to bending about two, usually perpendicular, axes it is referred to as biaxial moments. Like uniaxial moment biaxial moments is caused by eccentric loading of the structural member, see fig. 2-17.

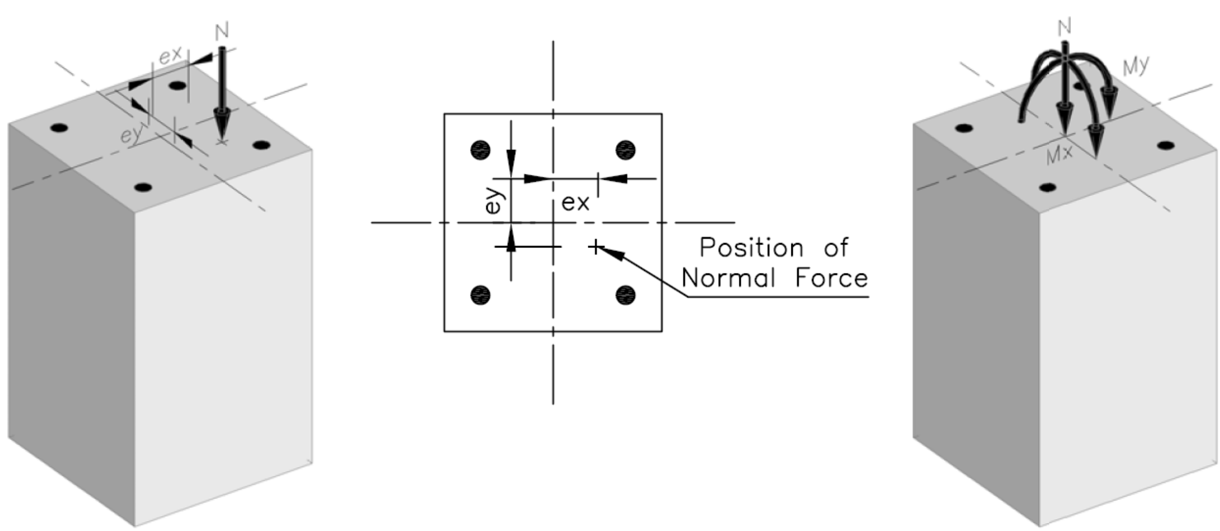


Figure 2-17 Biaxial moments caused by eccentricity of axial force

2.10.1 Stress/strain distribution

The strain increases linearly with distance from the neutral axis, in an elevation perpendicular to the plane of the neutral axis. So the distribution of the strain is triangular and the corresponding stresses in the reinforcement, now all at different distances from the neutral axis, are determined from the stress/strain relationship, see fig. 2-18 and 2-19.

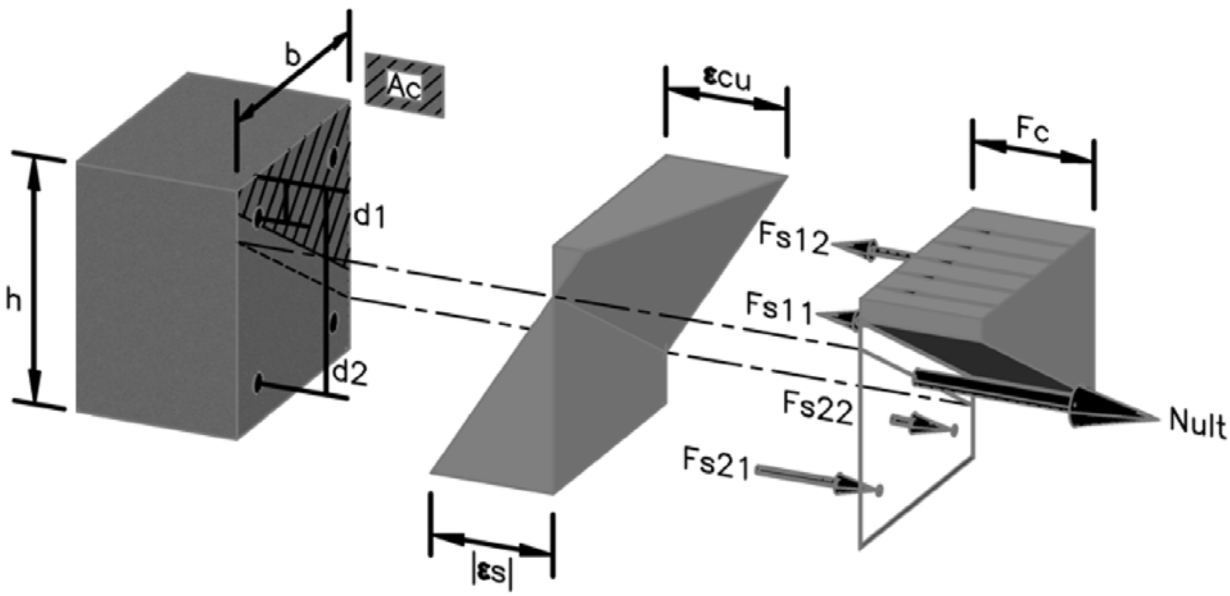


Figure 2-18 Explanation of strain/stress distribution for a structural concrete member with a biaxial moments in three dimensions

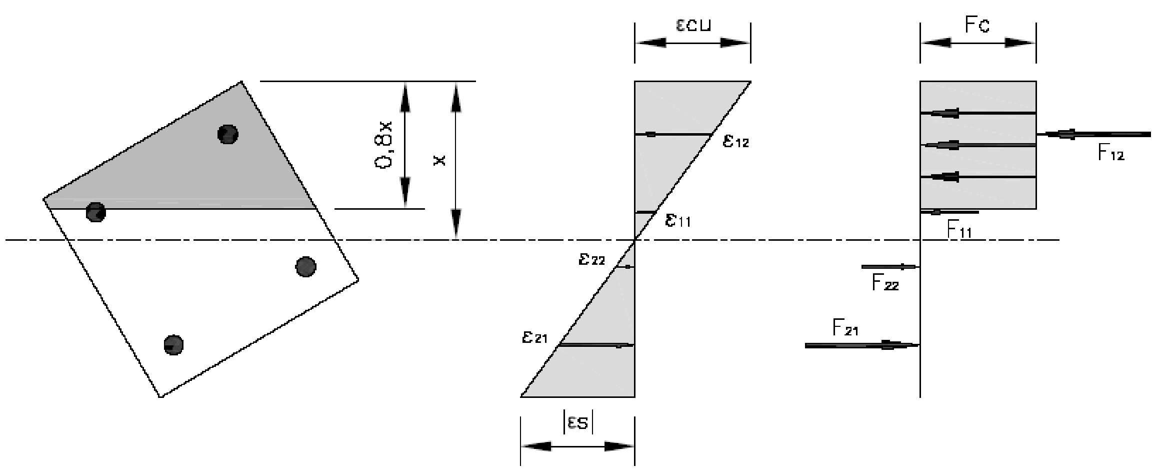


Figure 2-19 Explanation of the strain/stress distribution for a structural concrete member with a biaxial moments in two dimensions

The shape of the compression zone of the concrete depends on the inclination of the neutral axis, see fig. 2-20.

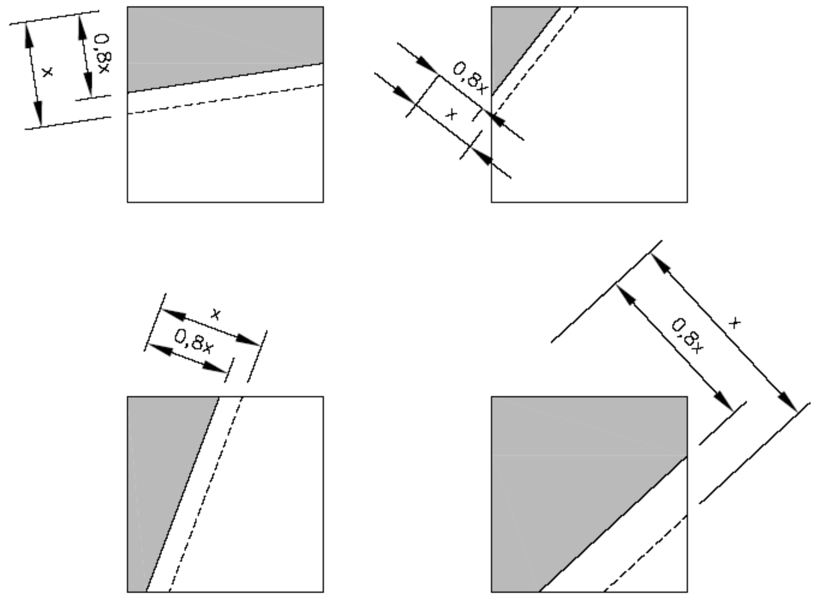


Figure 2-20 Four different shapes of the concrete zone for a rectangular cross-section

2.10.2 Interaction surface

For structural concrete members affected by biaxial moments the same principles are used as for a structural member affected by uniaxial moment, by varying the inclination of the neutral axis and creating an interaction diagram. By creating interaction diagrams at various angles a series of diagrams is created to form the interaction surface, see fig. 2-21. Each point on this surface represents one particular set of axial load and bending about the major axes, x and y, see fig. 2-21.

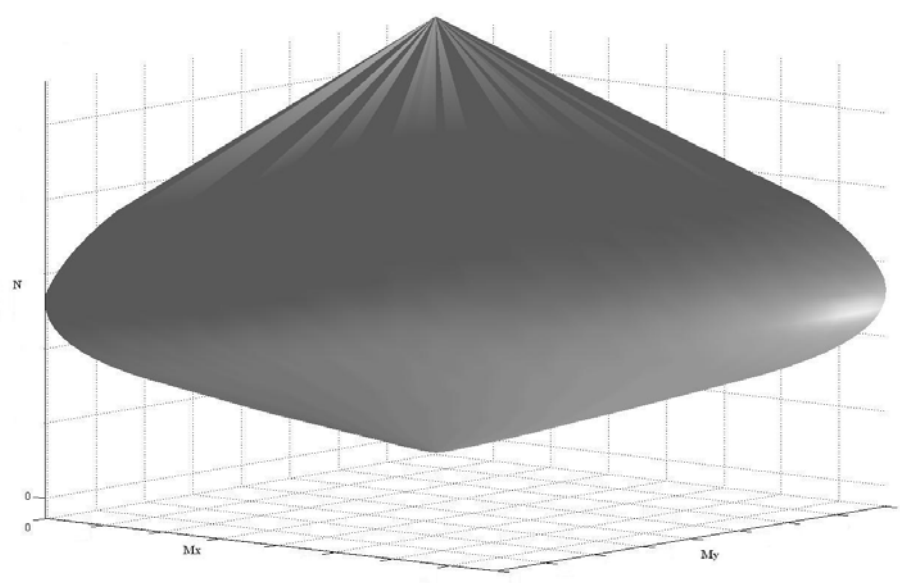


Figure 2-21 Example of interaction surface

If a horizontal section is taken through the interaction surface, for given axial load, an interaction line is created which shows the possible combination of moments about the major axes, see fig. 2-22. The shape of the interaction line varies with the section geometry and the level of the axial load.

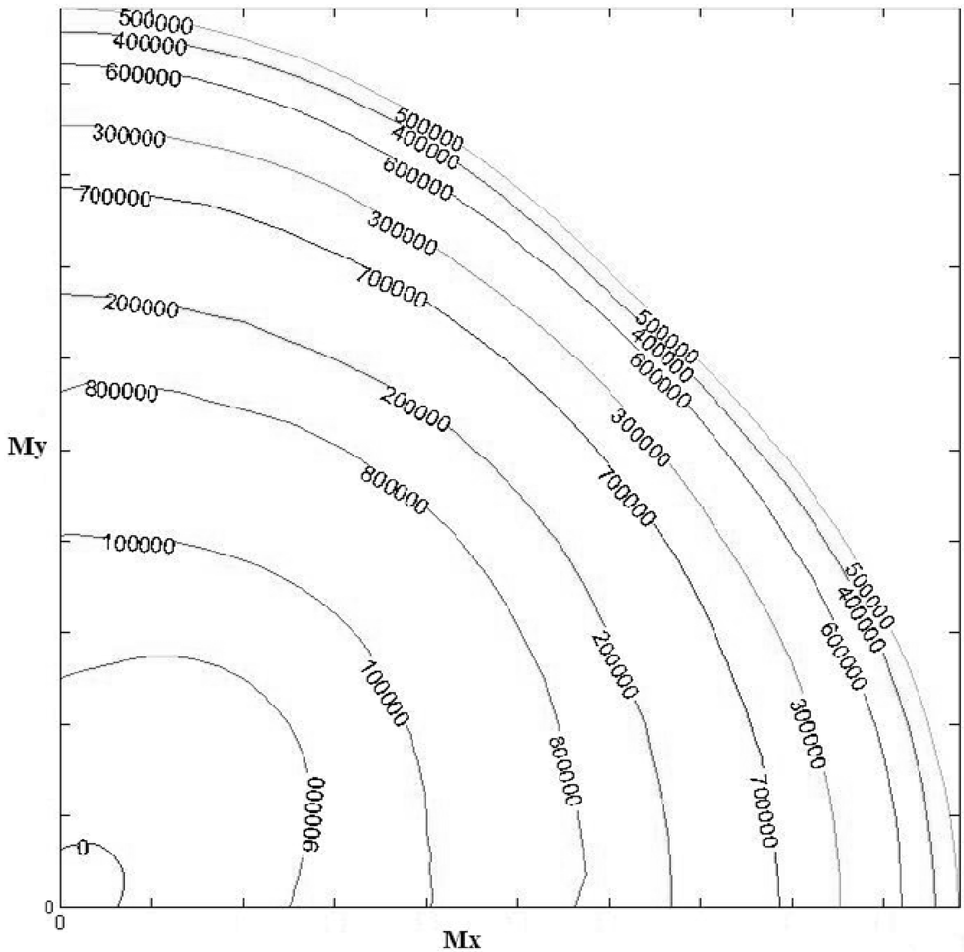


Figure 2-22 Series of interaction lines for different axial forces

3 Computer program

MATLAB was used to create a computer program to design concrete columns for biaxial moments and normal force. All necessary input values for the geometry, amount of reinforcement, material properties and required capacity are defined in an Excel sheet and from those informations a coordinate system for the cross-section and the reinforcement is created. Point (0,0) in the coordinate system is always located at the center of gravity for the cross-section. MATLAB calculates the capacity of the selected cross-section and creates the corresponding figures for the design.

3.1 Input values

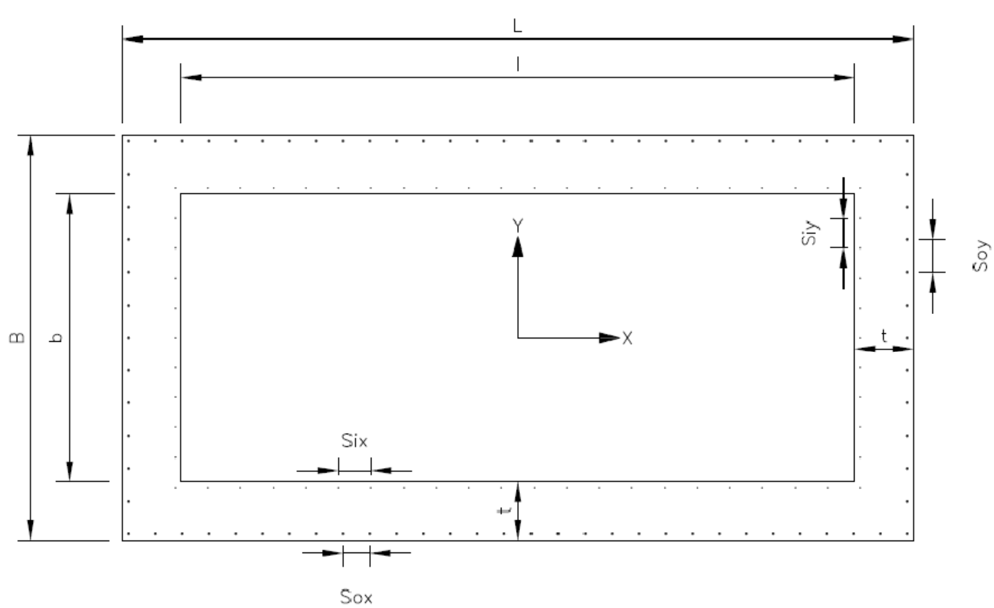
3.1.1 Screen shot

Each input value is referred to the number which is in the designated cell in the screen shot. If the cell is marked with – than it means that the value in that cell is calculated in Excel and should not be changed.

Geometry

	mm
B	**1.1**
L	**1.2**
t	**1.3**
b	**1.4**
l	**1.5**

A_c - mm^2



Concrete Cover **1.6** mm

Reinforcement

Outside Bars				Inside Bars			
x (Pcs.)	Sox (mm)	y (Pcs.)	Soy (mm)	x (Pcs.)	Sox (mm)	y (Pcs.)	Soy (mm)
2.1	-	**2.2**	-	**2.3**	-	**2.4**	-

	∅	A_s
	mm	mm^2
Outside Bars	**2.5**	-
Inside Bars	**2.6**	-

Materials

Safety Class, γ_n **3.1**

Concrete

f_{cck}	f_{cc}	ε_{cu}
MPa	MPa	-
3.2	-	**3.3**

Reinforcement

f_{yk}	f_{st}	E_{sk}	E_s	ε_{sy}
MPa	MPa	MPa	MPa	-
3.4	-	**3.5**	-	-

Numbers of checks

Rotation 0 - 90°	**4.1**	Spacing between calculations in degrees (1° - 90°)
Neutral axis	**4.2**	Number of calculations for neutral axis

Loads Combinations

	N	M_x	M_y
	kN	kNm	kNm
LC 1	**5.1.1**	**5.1.2**	**5.1.3**
LC 2	**5.2.1**	**5.2.2**	**5.2.3**
LC 3	**5.3.1**	**5.3.2**	**5.3.3**

3.1.2 Geometry

As stated in the introduction only rectangular cross-sections can be designed with the computer program. Six values are needed to create the cross-section, explained below.

B: The outer height of the cross-section, cell 1.1

L: The outer width of the cross-section, cell 1.2

t: Thickness of the concrete section. Only used for calculation of a hollow section and must be the same for the whole section, put as 0 for a solid section, cell 1.3

b: The inner height of the cross-section. Only used for calculation of a hollow section, put as 0 for a solid section, cell 1.4

l: The inner width of the cross-section. Only used for calculation of a hollow section, put as 0 for a solid section, cell 1.5

Concrete cover: The distance from the outer surface of the cross-section to the outer limit of the reinforcement, cell 1.6

3.1.3 Reinforcement

Input values are defined for the number of the outside (cells 2.1 and 2.2) and inside (cells 2.3 and 2.4) reinforcement, see fig. 3-1. The program calculates the spacing of the bars with respect to the concrete cover. If the cross-section is solid input values for the inside reinforcement should be put as 0.

It needs to be stated that the first and last bar in each layer of reinforcement, red bars in fig. 3-1 are included in the number of reinforcement for the x direction.

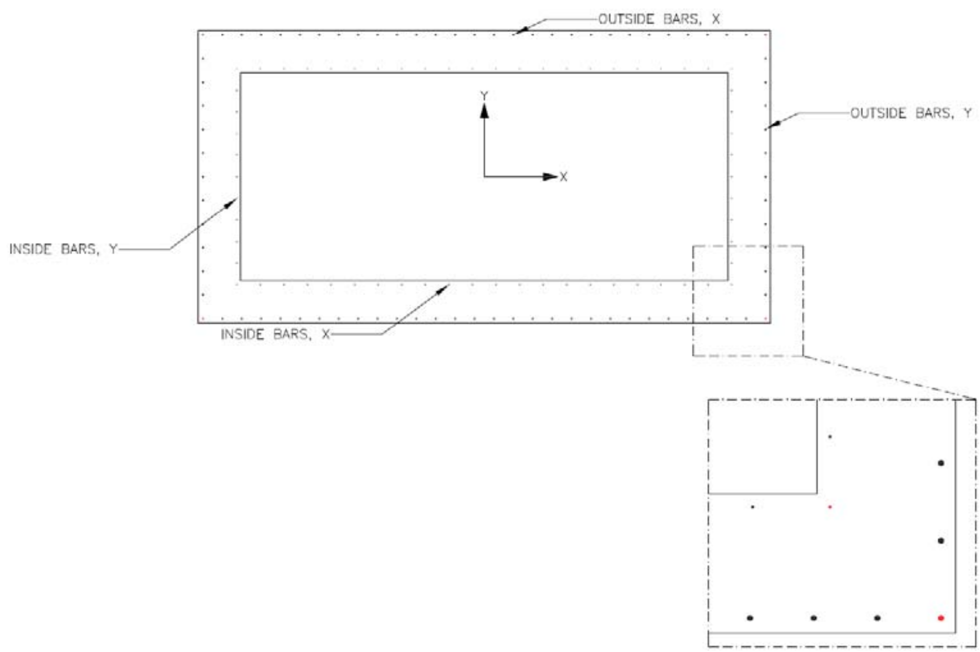


Figure 3-1 Explanation of input values for the reinforcement bars

The diameter of the reinforcement is then defined, both outside (cell 2.5) and inside (cell 2.6). If there is no inside reinforcement that should already have been defined as zero amount of bars so it does not matter if the value of the diameter of inside bars is not zero.

3.1.4 Materials

The first input value for the materials is the factor γ_n (cell 3.1) which depends on the safety class for the design, see the chapter 2.6 (Safety factor for materials).

3.1.4.1 Concrete

The input values for the concrete are:

The characteristic compressive strength, f_{cck}, cell 3.2

and

The ultimate compressive strain, ε_{cu}, cell 3.3

3.1.4.2 Reinforcement

The input values for the reinforcement are:

The characteristic yield strength, f_{yk}, cell 3.4

and

The characteristic modulus of elasticity, E_{sk}, cell 3.5

3.1.5 Number of checks

Used to define the accuracy of the calculations. Less spacing between rotation of the cross-section and the neutral axis leads to more accuracy of the result.

3.1.5.1 Rotation

The rotation of the cross-section is defined (cell 4.1) as spacing between calculations from 0° to 90°, i.e. if the value is chosen as 30° calculations will be made for 0°, 30°, 60° and 90°.

3.1.5.2 Neutral axis

Number of calculated neutral axes is defined here (cell 4.2), pure compression and pure tension is always included in the calculations.

3.1.6 Loads

Used to define the combinations of axial force and bending about the major axes, x and y, that should be checked for the design (cells 5.1.1 to 5.3.3). These combinations are plotted on interaction lines for the defined axial force, i.e. showing if the capacity of the section fulfills the requirements.

3.2 MATLAB functions

Five MATLAB functions were created for the calculations of the capacity of the cross-section. Each function will be explained on the next pages.

3.2.1 Input

Function file used to collect all input values and the coordinate system for the cross-section and all reinforcement.

3.2.2 Transform

Function file used to transform global node position vectors (x,y) to local position vectors (x',y') for a given rotation (φ) of the coordinate axis. The corresponding formulas are used:

$$x' = x\cos\varphi - y\sin\varphi \quad \text{(Eq. 3.1)}$$

$$y' = x\sin\varphi + y\cos\varphi \quad \text{(Eq. 3.2)}$$

3.2.3 Rforce

Function used to calculate the force vector for a given position matrix and corresponding area vector which represents the location and area properties of the reinforcement bars. Input includes position matrix of the concrete cross-section, reinforcement bars and depth of the neutral axis. The strain and the corresponding stress in the reinforcement is calculated and then the force from each bar.

3.2.4 ConcF

Function used to find the compressed area of concrete and corresponding center of gravity for a given number of neutral axes which are to be checked.

Further explanations will be described below in similar steps as the function works.

3.2.4.1 Critical rotation

The concrete node numbering is changed after certain rotation of the cross-section, φ_{crit}, since node 2 needs to be below node 4 in the calculations, see fig. 3-2 and 3-3, and known lengths for the height and width of the cross-section before it is rotated, see Appendix A.

The critical rotation is: $\varphi_{crit} = \tan^{-1}\left(\frac{B}{L}\right)$ (Eq. 3.3)

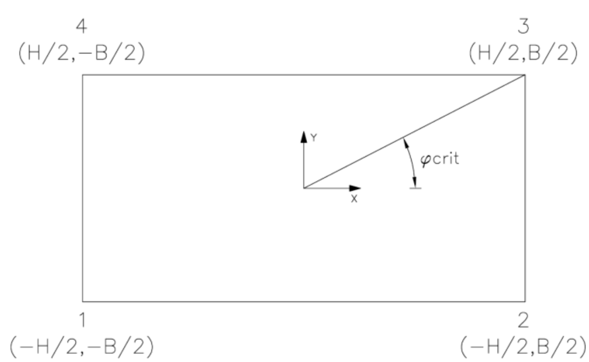


Figure 3-2 Explanation of the critical rotation, φ_{crit}

The same principle is used for the inner box if the cross-section is hollow by substituting B and L with b and l.

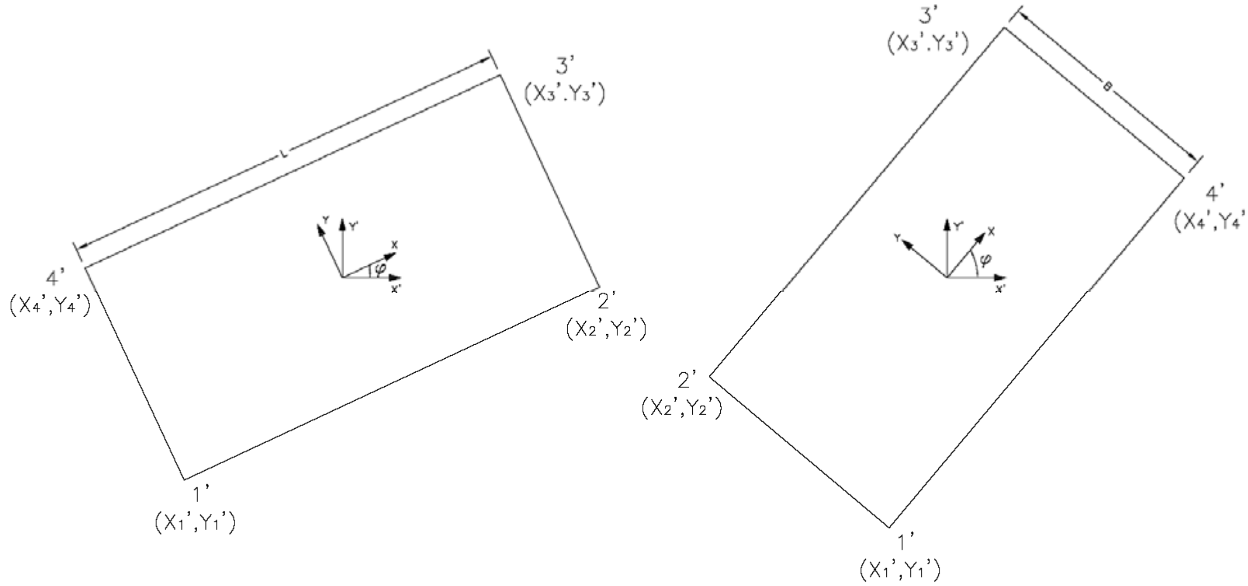


Figure 3-3 Example of how nodes 2 and 4 changes for rotation below and above critical rotation

3.2.4.2 Cases for different positioning of the neutral axis

The neutral axis is always in vertical position and is defined as all area above it is in compression.

If the cross-section has not been rotated or been rotated by 90° it is always a rectangular zone and the area and the corresponding center of gravity is calculated as in chapters 2.8.1 and 2.8.2.

When the cross-section is rotated than it is divided into three zones where the first one is a triangle, the second one is a parallelogram and the third one is a trapezoid until it reaches pure compression and becomes a triangle. Five different cases are considered, see fig. 3-4 and 3-5. The function always calculates the height of the rotated cross section for calculations of the corresponding lever arm.

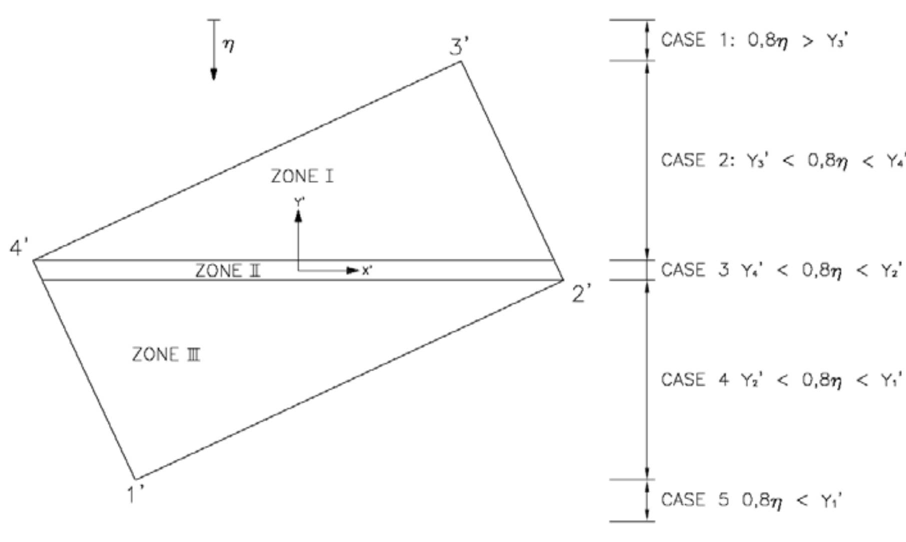


Figure 3-4 Definition of case one to five for $\varphi < \varphi_{crit}$

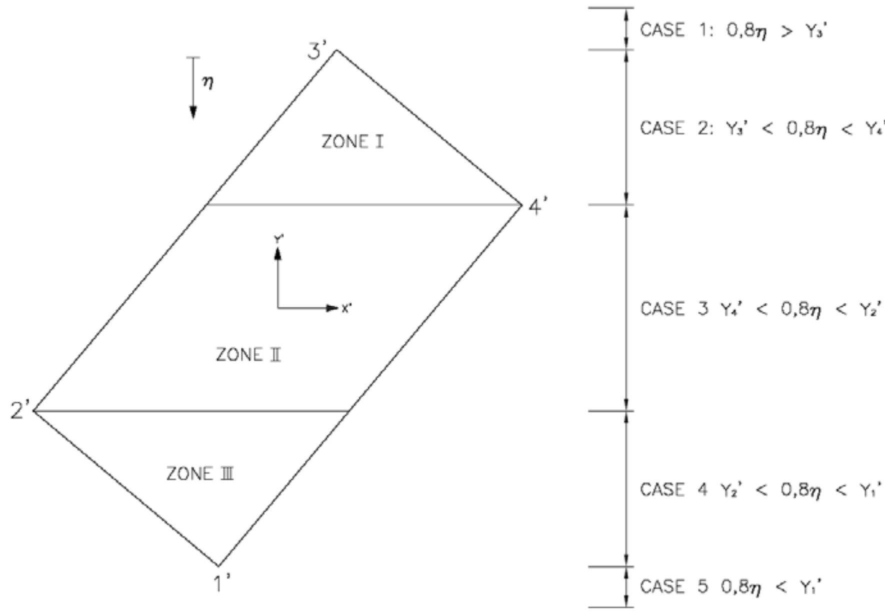


Figure 3-5 Definition of case one to five for $\varphi > \varphi_{crit}$

If the cross-section is hollow the inner rectangle is divided into the same zones and all calculations are the same but with a different critical rotation. When the area and the lever arm for the section are calculated all properties of the inner box are subtracted from the outer box to create the properties for the rectangular hollow section.

The area is calculated by: $A_c = \sum A_{i,\text{Outer Box}} - \sum A_{i,\text{Inner Box}}$ (Eq. 3.4)

and the lever arm by: $e_c = \dfrac{\sum A_{i,\text{Outer Box}} \cdot e_{i,\text{Outer Box}} - \sum A_{i,\text{Inner Box}} \cdot e_{i,\text{Inner Box}}}{A_c}$ (Eq. 3.5)

3.2.4.2.1 Case one

The cross-section is in pure tension so the compressed concrete area and the center of gravity will be zero.

3.2.4.2.2 Case two

The compressed concrete area is in zone I and is a triangle, see fig. 3-6.

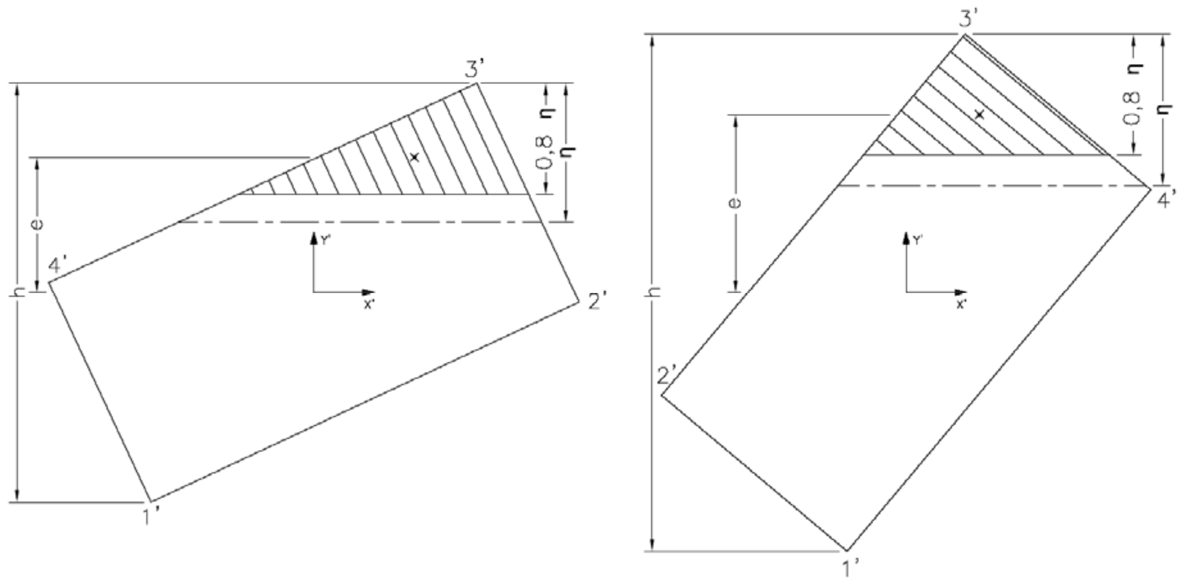


Figure 3-6 Example of case two, $\varphi < \varphi_{crit}$ and $\varphi > \varphi_{crit}$

3.2.4.2.3 Case three

Zone I is all in compression and part of zone II, i.e. triangle and parallelogram, see fig. 3-7.

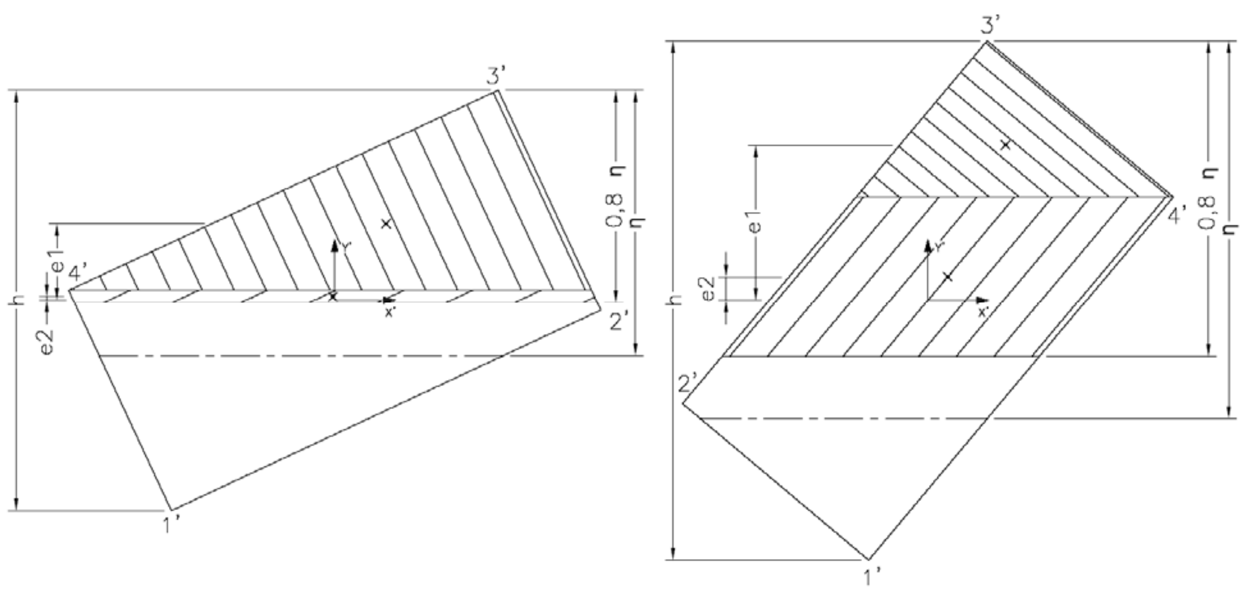


Figure 3-7 Example of case three, $\varphi < \varphi_{crit}$ and $\varphi > \varphi_{crit}$

3.2.4.2.4 Case four

Zone I and II are both in compression and part of zone III, i.e. triangle, parallelogram and trapezoid, see fig. 3-8.

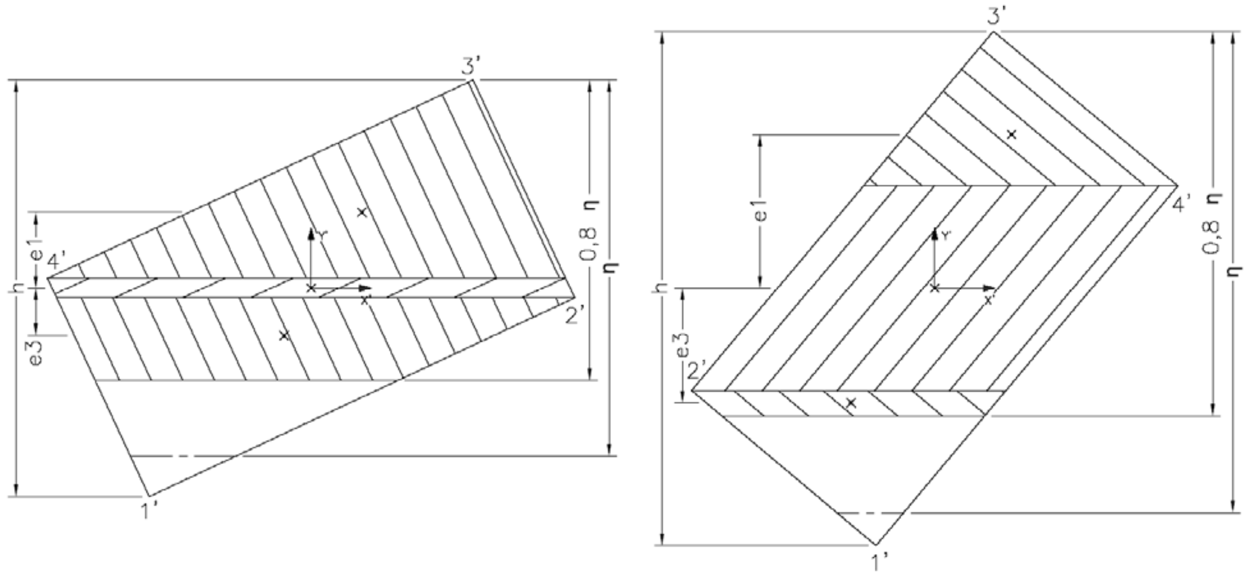


Figure 3-8 Example of case four, $\varphi < \varphi_{crit}$ and $\varphi > \varphi_{crit}$

3.2.4.2.5 Case five

The cross-section is in pure compression so the compressed concrete area will be zone I, II and III. The center of gravity will be at the same point as the moment is taken around so the moment capacity will be zero.

3.2.5 MNCalc

This function returns the results for the design. It sums up the forces from the reinforcement and the concrete for all neutral axes and rotations that have been chosen to be checked. Then it calculates the capacity of the section for axial force and bending. Finally it plots figures of the cross-section, the interaction surface (with and without contour lines), interaction lines for chosen load combinations and development if the interaction lines by showing different lines for decreasing axial force capacity.

3.3 Example of use with the case study

As was mentioned the case study is a rectangular hollow cross-section, see fig. 3-9. Input values are marked as bold.

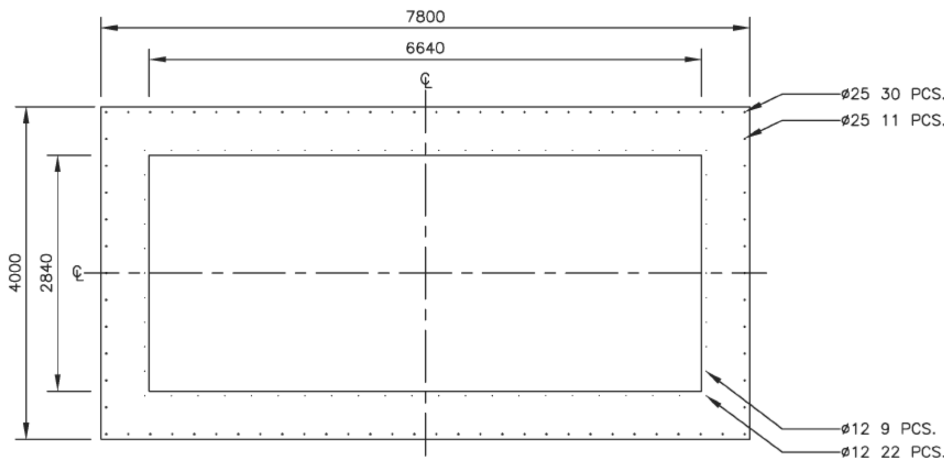


Figure 3-9 Cross-section of the case study

3.3.1 Geometry

Input values for the geometry of the cross-section are:

	mm
B	**4000**
L	**7800**
t	**580**
b	**2840**
l	**6640**

Concrete Cover **50** mm

3.3.2 Reinforcement

Input values for the reinforcement are:

Outside Bars				Inside Bars			
x (Pcs.)	Sox (mm)	y (Pcs.)	Soy (mm)	x (Pcs.)	Sox (mm)	y (Pcs.)	Soy (mm)
30	264,7	**11**	322,9	**22**	321,5	**9**	295,2

	∅	A
	mm	mm^2
Outside Bars	**25**	490,87
Inside Bars	**12**	113,10

3.3.3 Materials

The input values for the materials are:

Safety Class, γ_n	**1,2**

Concrete

f_{cck}	f_{cc}	ε_{cu}
MPa	MPa	-
28,5	15,83	**0,0035**

Reinforcement

f_{yk}	f_{st}	E_{sk}	E_s	ε_{sy}
MPa	MPa	MPa	MPa	-
390	282,61	**200.000**	158.730	0,00178

3.3.4 Number of checks

The input values for the numbers of checks are:

Rotation 0 - 90°	**1**	Spacing between calculations in degrees (1° - 90°)
Neutral axis	**100**	Number of calculations for neutral axis

3.3.5 Load combinations

The load combinations are:

	N	M_x	M_y
	kN	kNm	kNm
LC 1	**41.012**	**109.556**	**30.443**
LC 2	**40.730**	**102.452**	**29.539**
LC 3	**39.450**	**47.127**	**74.349**

3.3.6 Results

Run MATLAB file (MNCalc.m) and the results and corresponding figures can be seen below:

3.3.6.1 Cross-Section

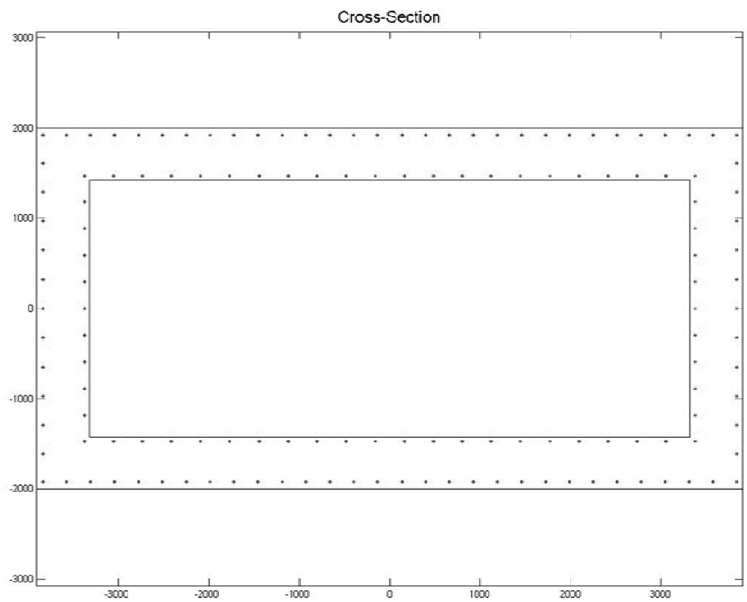


Figure 3-10 The defined cross-section

3.3.6.2 Interaction surface

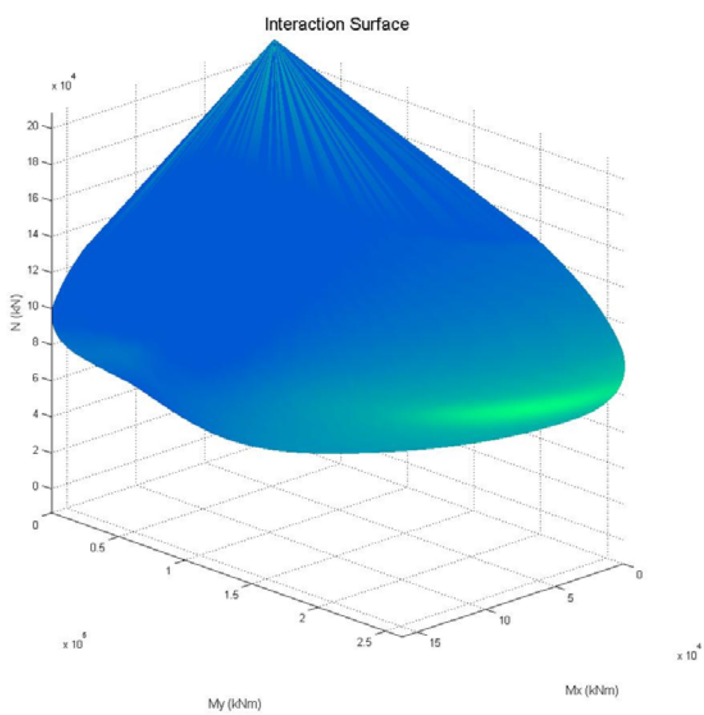


Figure 3-11 The interaction surface for the defined cross-section

3.3.6.3 Interaction lines

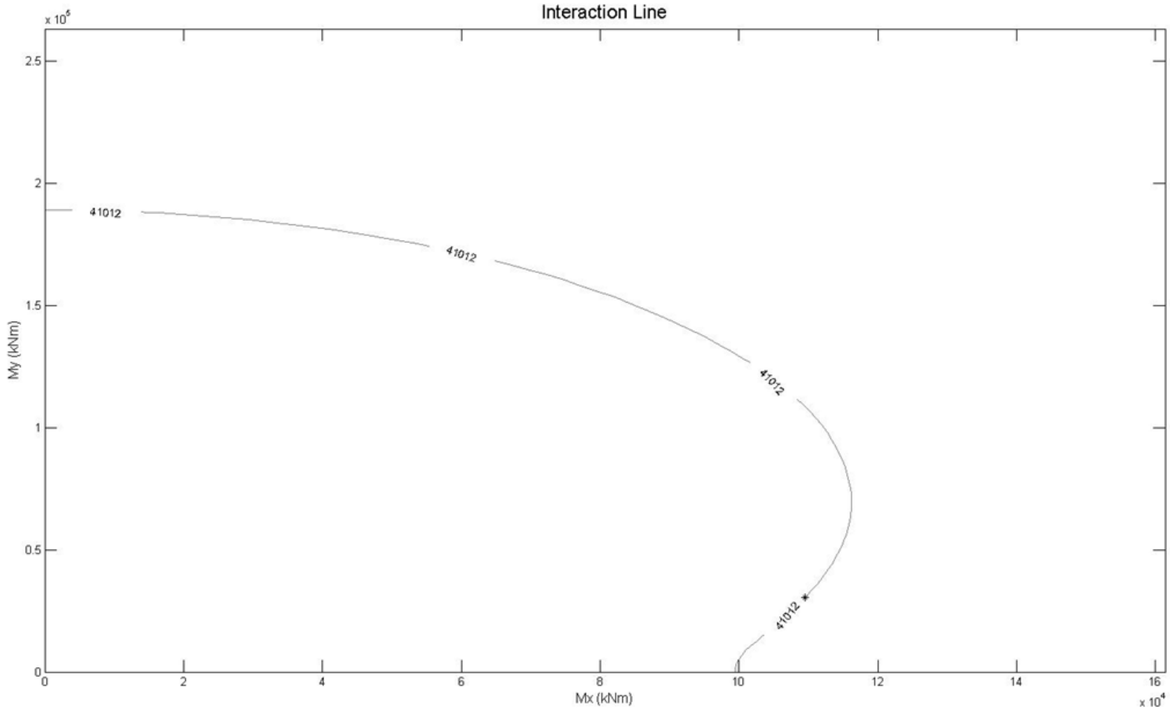


Figure 3-12 Load combination 1, marked with a star, which is just outside the surface of safe design, i.e. the capacity of the cross-section does not fulfill the requirements.

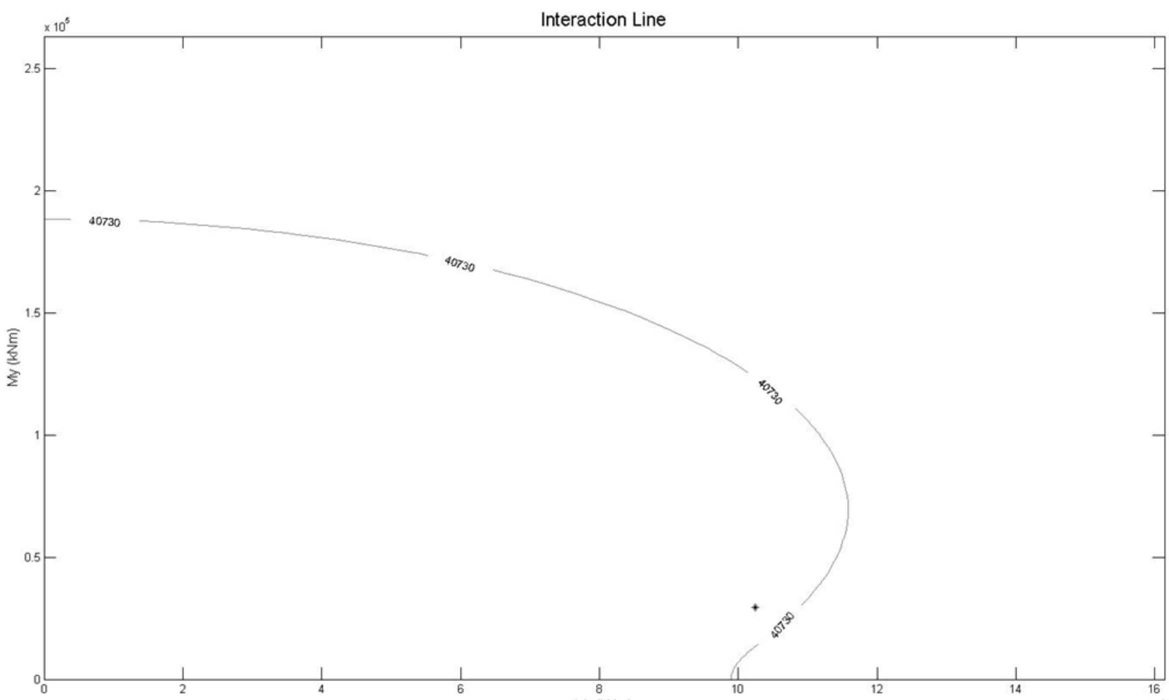


Figure 3-13 Load combination 2, marked with a star, which is inside the surface of safe design, i.e. the capacity of the cross-section fulfills the requirements.

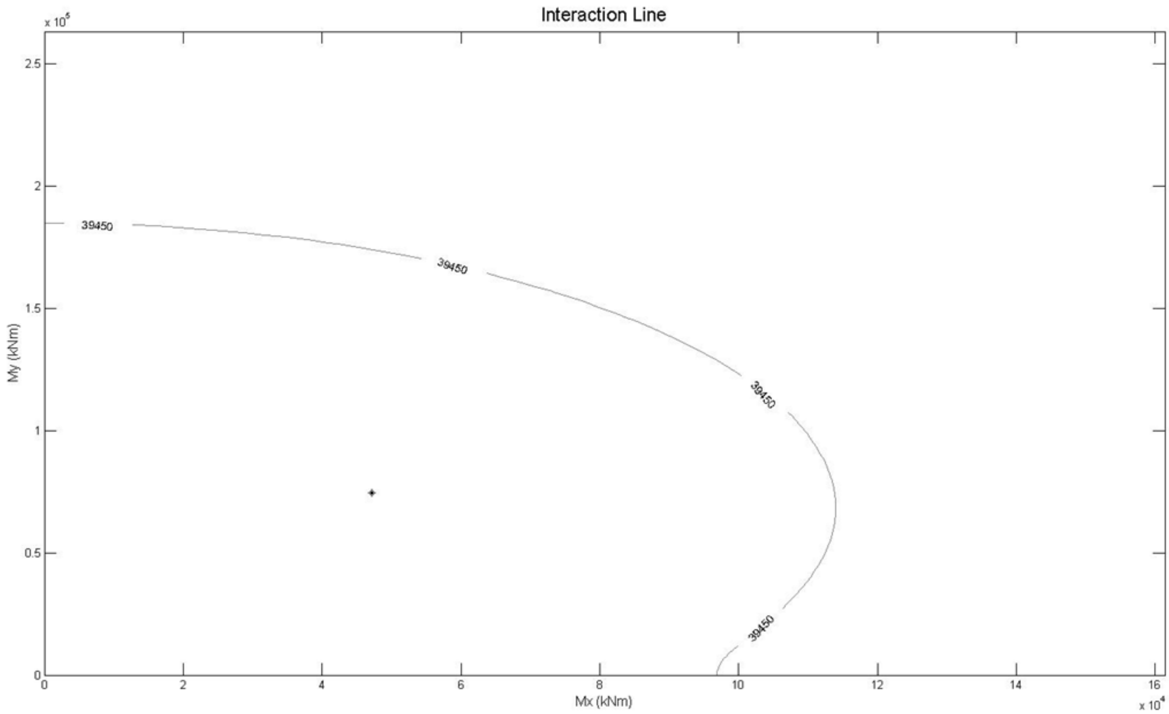


Figure 3-14 Load combination 3, marked with a star, which is inside the surface of safe design, i.e. the capacity of the cross-section fulfills the requirements.

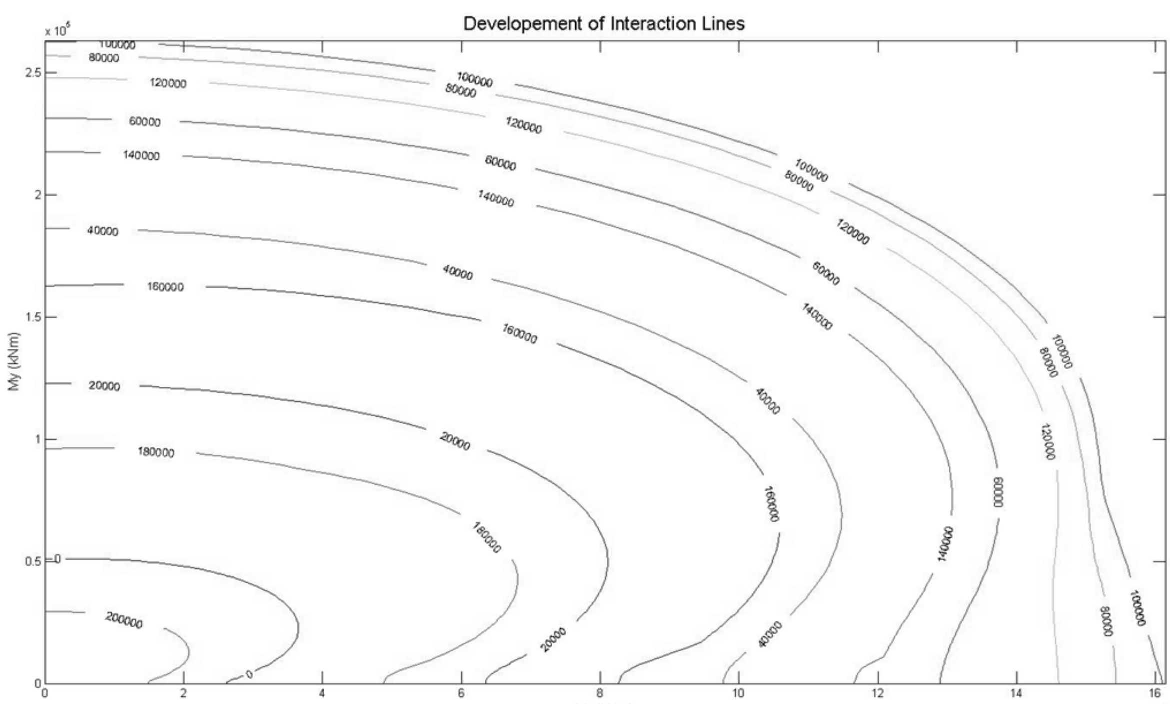


Figure 3-15 Development of the interaction lines for the defined cross-section

4 Design of structural concrete members for axial force and biaxial moments in Eurocode

Chapter 5.8.9 in EN 1992-1-1-2004 includes a simplified design of structural concrete members in biaxial moments if a computer program is not used for the design.

First it should be stated that the safety factors for materials are not the same in Eurocode and BBK 04. The strength of the materials according to Eurocode is:

Concrete:

$$f_{cc} = \frac{\alpha f_{cck}}{\gamma_m} \quad \text{(Eq. 4.1)}$$

Where:
α = coefficient which takes account of the long-term effects of the compressive strength and of the unfavorable effects resulting from the way in which the load is applied, α = 0,85 if the section is at least as wide at the extreme compressive fibre as it is elsewhere in the compressive zone.
γ_m = 1,5

Reinforcement:

$$f_{st} = \frac{f_{yk}}{\gamma_m} \quad \text{(Eq. 4.2)}$$

$$E_s = \frac{E_{sk}}{\gamma_m} \quad \text{(Eq. 4.3)}$$

Where:
γ_m = 1,15

The structural concrete member can be designed with separate uniaxial bending about both major axes, i.e. only bending about one axis is considered in turn, if the relative eccentricities satisfy one of the following conditions:

$$\frac{\frac{e_y}{B_{eq}}}{\frac{e_x}{L_{eq}}} \leq 0,2 \quad \text{or} \quad \frac{\frac{e_x}{L_{eq}}}{\frac{e_y}{B_{eq}}} \leq 0,2 \quad \text{(Eq. 4.4.a and 4.4.b)}$$

Where:

$$i_i = \sqrt{\frac{I_i}{A}} \quad \text{(Eq. 4.5)}$$

$$B_{eq} = i_x \cdot \sqrt{12} \quad \text{(Eq. 4.6)}$$

$$L_{eq} = i_y \cdot \sqrt{12} \tag{Eq. 4.7}$$

$$e_i = \frac{M_{di}}{N_d} \tag{Eq. 4.8}$$

If the relative eccentricities do not satisfy one of the following conditions Eurocode suggests that the load contour method can be used if there is no aid of a computer program.

The load contour method is based on representing simplified interaction line for the axial force acting on the structural concrete member. The curve of the interaction line varies for different values of the axial force, see fig. 4-1, and can be approximated by a non-dimensional interaction equation:

$$\left(\frac{M_{dx}}{M_{Rdx}}\right)^a + \left(\frac{M_{dy}}{M_{Rdy}}\right)^a = 1 \tag{Eq. 4.9}$$

Where the exponent a, for rectangular cross-sections, can be found with linear interpolation from table 4.1:

Table 4.1

N_d/N_{Rd}	0,1	0,7	1,0
a	1,0	1,5	2,0

Where N_{Rd} is the ultimate compression capacity of the cross-section, equation 2.4.

As can be noticed in fig. 4-1 as the design axial force decreases more safety is included in the design, a approaches 1 and more of the actual curvature is neglected, than for design of axial force close to the ultimate compression capacity of the structural concrete member, a≈2.

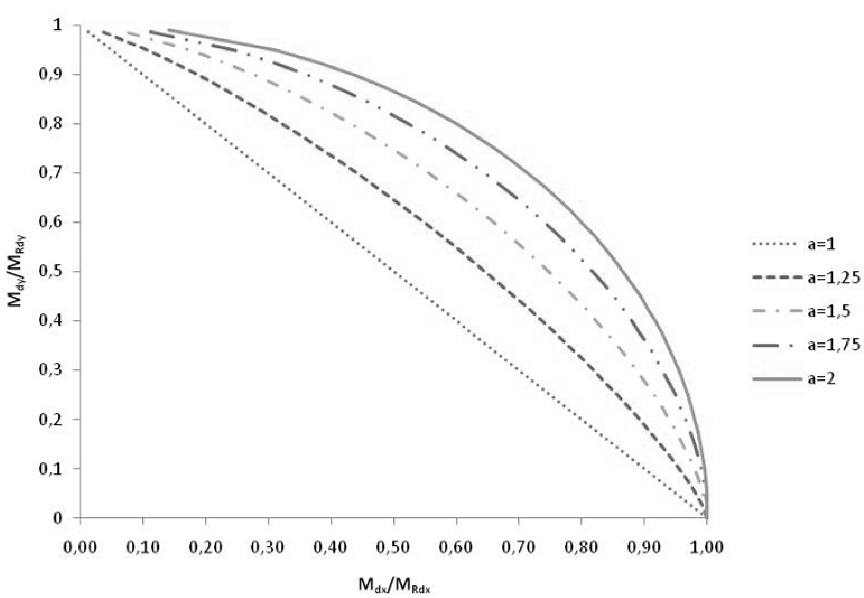


Figure 4-1 Example of how the curve varies for a different values of a

So Eurocode suggests that the following criteria must be satisfied in the absence of an accurate cross-section design for biaxial moments.

$$\left(\frac{M_{dx}}{M_{Rdx}}\right)^{a}+\left(\frac{M_{dy}}{M_{Rdy}}\right)^{a}\leq 1 \qquad \text{(Eq. 4.10)}$$

4.1 Results for design of the case study according to Eurocode

Detailed calculations can be seen in Annex B.

The load combinations are:

N	Mx	My
kN	kNm	kNm
41.012	109.556	30.443
40.730	102.452	29.539
39.450	47.127	74.349

The relative eccentricities for all the load combinations do not satisfy the conditions, equations 4.4.a and 4.4.b, so the load contour method is used with a as 1,07. a is approximately the same for all load combinations.

The axial load for all combinations is around 40.500 kN so the moment resistance about both major axes with that axial load is approximately:

$M_{Rdx} \approx 103.000$ kNm, see fig. 4-2.

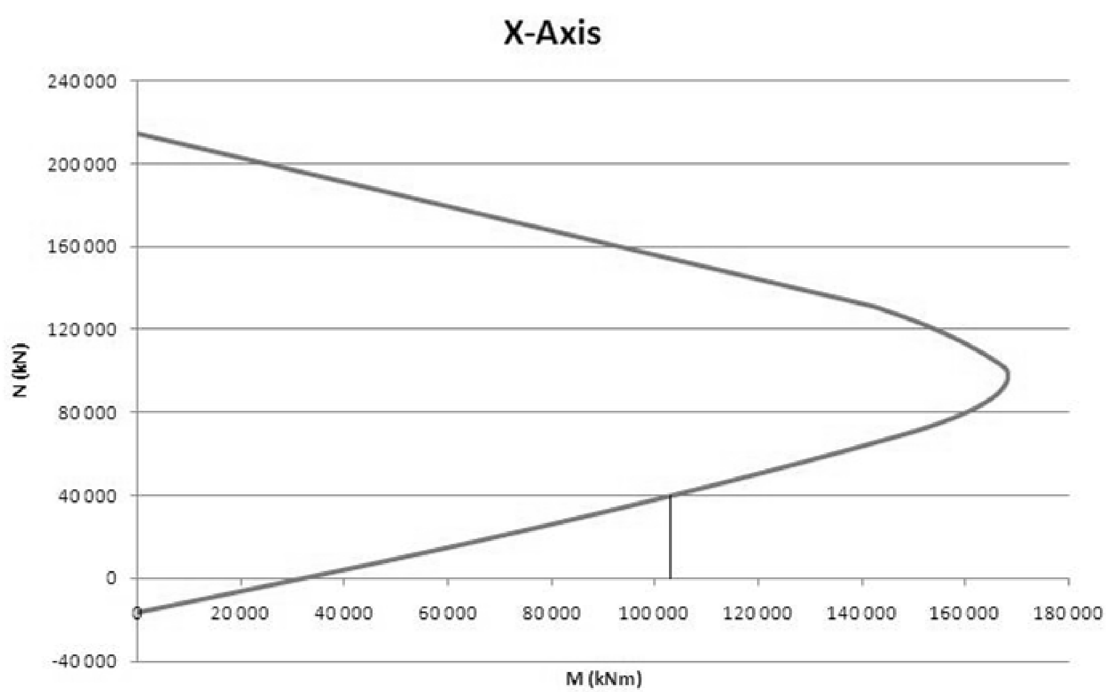


Figure 4-2 Interaction diagram for bending around x-axis for the case study

$M_{Rdy} \approx 195.000$ kNm, see fig. 4-3.

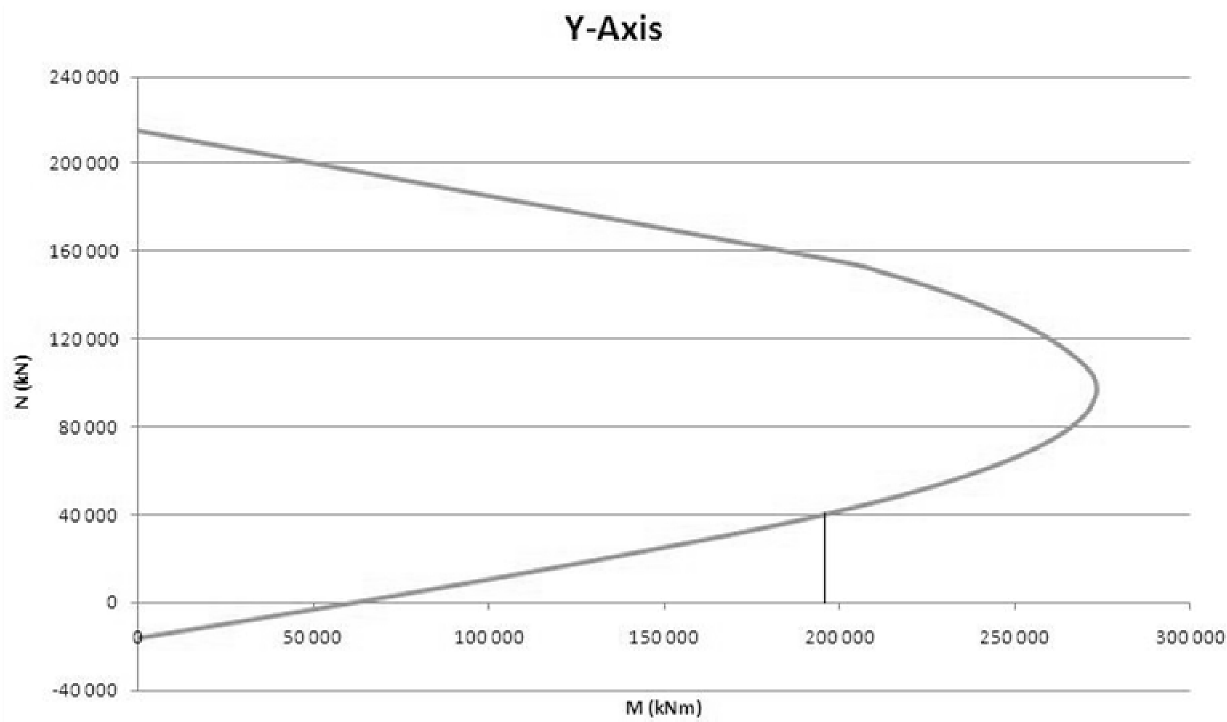


Figure 4-3 Interaction diagram for bending around y-axis for the case study

The results from equation 4.10 will then be:

LC 1	1,20
LC 2	1,12
LC 3	0,79

or can be expressed graphically, see fig 4-4:

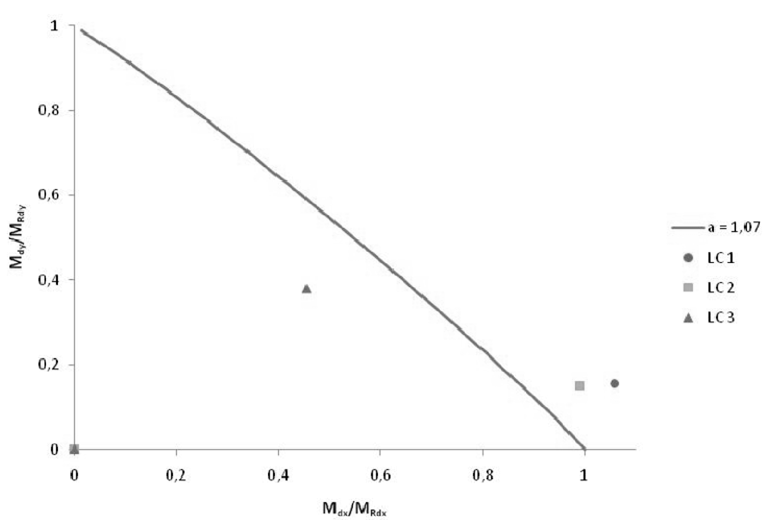


Figure 4-4 Simplified interaction line with the load contour method

5 Conclusion

The advance of using a powerful design tool such as a computer program for design of a reinforced concrete column can easily been seen when comparing the results for load combination 2 of the study case.

As has been stated in the thesis the computer program gives an exact curve of the interaction surface and interaction lines when the method from Eurocode gives a simplified solution. In fig. 5-1 the results for design interaction line from the computer program and the simplified method are compared. It can clearly be seen that the simplified method neglects a lot of design surface, i.e. the surface on the right side from the blue line. This results in a save design using the computer program for load combination 2 while the design is not save with the load contour method from Eurocode, see fig. 5-2 and 5-3.

These results prove the value of using a computer program though the contour method from Eurocode is a good design tool when use of computer program is not available.

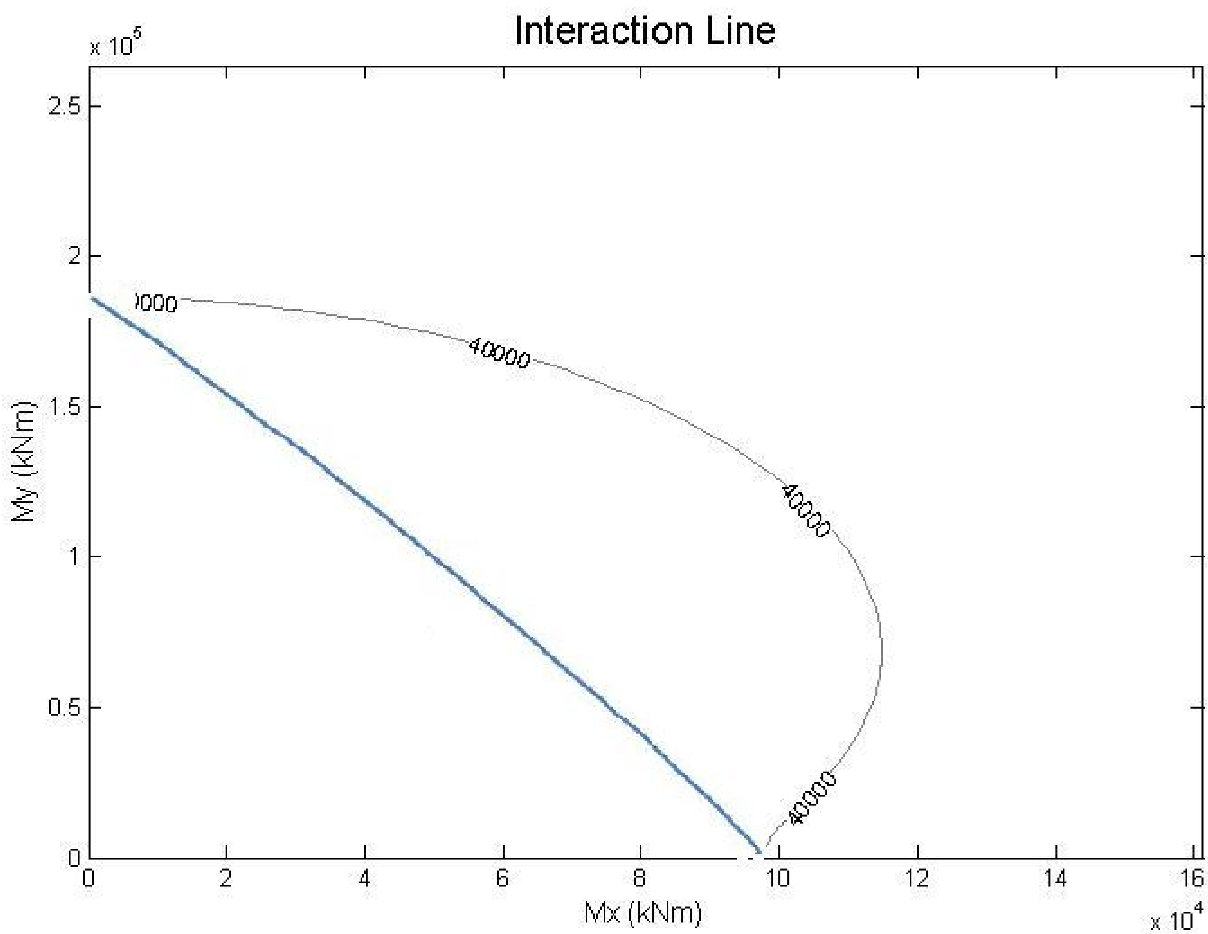


Figure 5-1 Interaction line for chosen axial load from the computer program compared to interaction line, blue line, from the simplified method Eurocode suggests

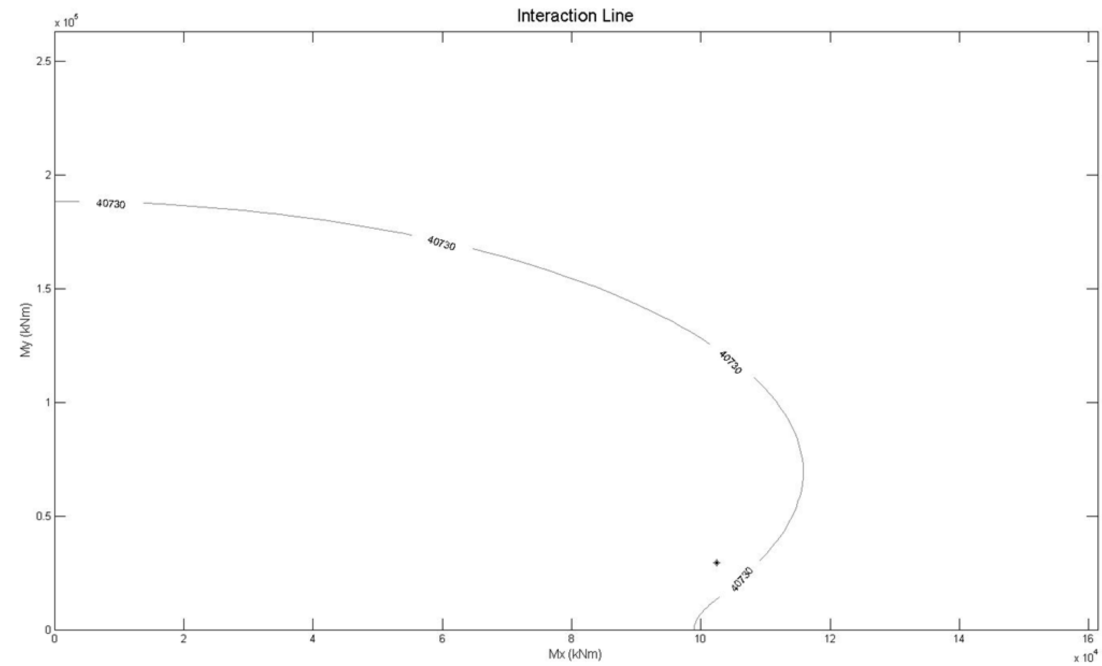


Figure 5-2 Interaction line from the computer program, requirements fulfilled – safe design

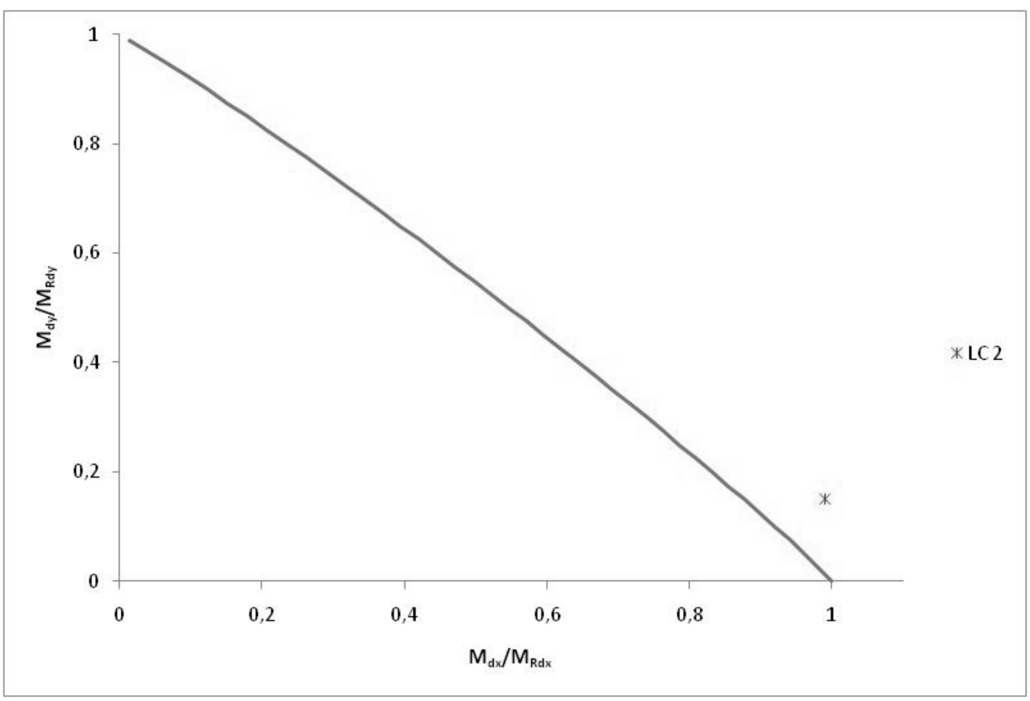


Figure 5-3 Interaction line from contour method in Eurocode, requirements not fulfilled – not safe design

6 References

6.1 Literature

BBK (2004). *Boverkets handbok om betongkostruktioner.* Boverket, Karskrona, Sweden.

EC2 (2004), *Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings,* European Prestandard EN 1992-1-1-2004:E, European Committee for Standardisation TC 250, Brussels, Belgium.

Engström, Björn. (2004), *Beräkning av betongkonstruktioner.* Chalmer Tekniska Högskola, Gothenburg, Sweden.

Eugene J. O´Brien and Andrew S. Dixon. (1999), *Reinforced and Prestressed Concrete Design: The Complete Process.* Harlow: Longman Scientific & Technical, London, United Kingdom.

Gere, James M. (2004), *Mechanics of materials, sixth edition.* Thomson Learning, London, United Kingdom.

Isaksson, Tord and Mårtensson, Annike. (2008), *Byggkonstruktion, Regel- och formelsamling.* Studentlitteratur, Malmö, Sweden.

Park, Robert and Pauley, Thomas. (1975), *Reinforced concrete structures.* John Wiley & Sons, Inc., USA.

6.2 Computer programs

AutoCAD 2009, Autodesk.

MathType (Version 6.6); Design Science.

Matlab R2008b (Version 7.7.0.471), The MathWorks.

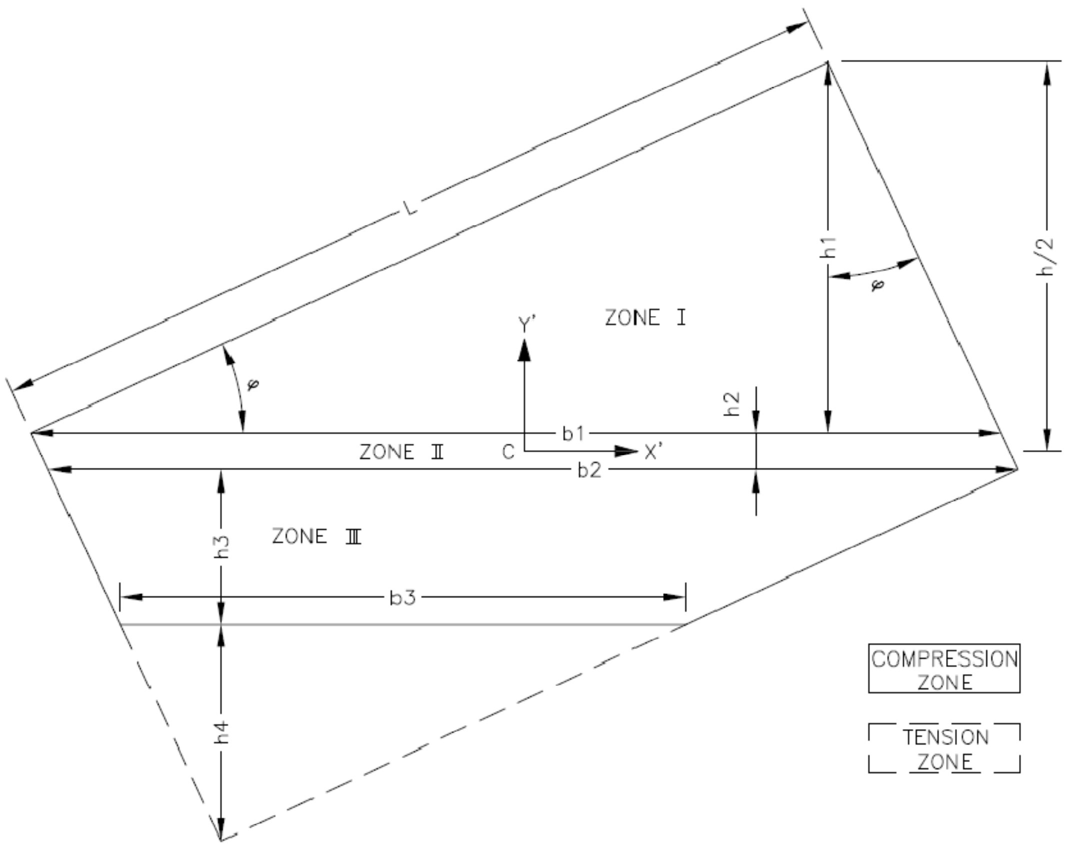
Microsoft Office Excel 2007, Microsoft.

Microsoft Office Word 2007, Microsoft.

Microsoft Paint, Microsoft.

Annex A. Properties of plane areas

$\varphi < \varphi_{crit}$



Zone 1:

$$b_1 = h_1\left(\tan\varphi + \frac{1}{\tan\varphi}\right)$$

$$A_1 = \frac{h_1 b_1}{2}$$

$$e_1 = \frac{h}{2} - \frac{2h_1}{3}$$

Zone 2:

$$b_2 = \frac{L}{\cos\varphi}$$

$$A_2 = b_2 h_2$$

$$e_2 = \frac{h}{2} - \left(h_1 + \frac{h_2}{2}\right)$$

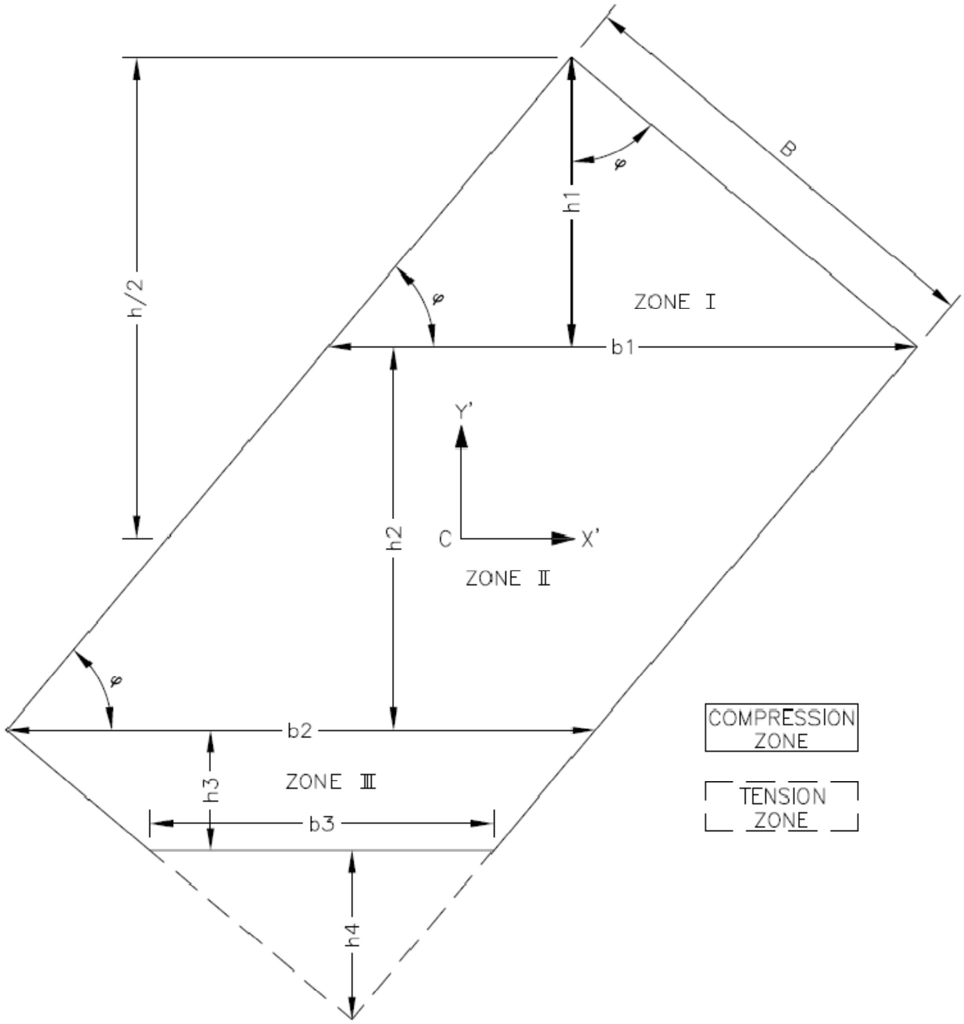
Zone 3:

$$b_3 = h_3\left(\tan\varphi + \frac{1}{\tan\varphi}\right)$$

$$A_3 = \frac{h_3\left(b_2 + b_3\right)}{2}$$

$$e_3 = \frac{h}{2} - \left(h_1 + h_2 + \frac{h_3\left(b_2 + 2b_3\right)}{3\left(b_2 + b_3\right)}\right)$$

$\varphi > \varphi_{crit}$



Zone 1:

$$b_1 = h_1\left(\tan\varphi + \frac{1}{\tan\varphi}\right)$$

$$A_1 = \frac{h_1 b_1}{2}$$

$$e_1 = \frac{h}{2} - \frac{2h_1}{3}$$

Zone 2:

$$b_2 = \frac{B}{\sin\varphi}$$

$$A_2 = b_2 h_2$$

$$e_2 = \frac{h}{2} - \left(h_1 + \frac{h_2}{2}\right)$$

Zone 3:

$$b_3 = h_3\left(\tan\varphi + \frac{1}{\tan\varphi}\right)$$

$$A_3 = \frac{h_3\left(b_2 + b_3\right)}{2}$$

$$e_3 = \frac{h}{2} - \left(h_1 + h_2 + \frac{h_3\left(b_2 + 2b_3\right)}{3\left(b_2 + b_3\right)}\right)$$

Annex B. Detailed calculations for biaxial moments according to Eurocode

Material properties.

Concrete:

$$f_{cc} = \frac{\alpha f_{cck}}{\gamma_m} = \frac{0,85 \cdot 28,5MPa}{1,5} = 16,2MPa$$

Reinforcement:

$$f_{st} = \frac{f_{yk}}{\gamma_m} = \frac{390MPa}{1,15} = 339,1MPa$$

$$E_d = \frac{E_{sk}}{\gamma_m} = \frac{200.000MPa}{1,15} = 173.913MPa$$

Cross-section properties.

$$B = 4000mm$$
$$b = 2840mm$$
$$L = 7800mm$$
$$l = 6640mm$$
$$A = BL - bl = 4000mm \cdot 7800mm - 2840mm \cdot 6640mm = 12.342.400mm^2$$

$$I_x = \frac{LB^3 - lb^3}{12} = \frac{7800mm \cdot (4000mm)^3 - 6640mm \cdot (2840mm)^3}{12} = 2,89 \cdot 10^{13} mm^4$$

$$i_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2,89 \cdot 10^{13} mm^4}{12.342.400mm^2}} = 1530,9mm$$

$$I_y = \frac{BL^3 - bl^3}{12} = \frac{4000mm \cdot (7800mm)^3 - 2840mm \cdot (6640mm)^3}{12} = 8,89 \cdot 10^{13} mm^4$$

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{8,89 \cdot 10^{13} mm^4}{12.342.400mm^2}} = 2683,8mm$$

$$B_{eq} = i_y \cdot \sqrt{12} = 1530,9mm \cdot \sqrt{12} = 5303,1mm$$

$$L_{eq} = i_x \cdot \sqrt{12} = 2683,8mm \cdot \sqrt{12} = 9296,9mm$$

Capacity of the cross-section.

Bending around x-axis:

Neutral Axis	F'_s	M'_s	F_s	M_s	$\sum F_s$	$\sum M_s$	F_c	M_c	Z_c	Z_N	N	M
mm	kN	kNm	kN	kNm	kN	kNm	kN	kNm	m	m	kN	kNm
Pure Compression					16.029	0	198.566	0			**214.595**	**0**
4000	9.460	30.412	0	0	9.460	30.412	122.146	374.628	3,067	2,000	**131.605**	**141.829**
3900	9.210	29.379	-86	-3	9.124	29.383	120.647	361.304	2,995	1,900	**129.771**	**144.122**
3700	8.964	27.368	-575	-137	8.389	27.505	117.650	335.196	2,849	1,700	**126.038**	**148.436**
3500	8.700	25.423	-1.140	-492	7.560	25.915	114.652	309.808	2,702	1,500	**122.212**	**152.405**
3300	8.501	23.551	-1.869	-1.136	6.631	24.687	111.655	285.139	2,554	1,300	**118.286**	**156.055**
3100	8.326	21.733	-2.745	-2.156	5.581	23.889	108.657	261.189	2,404	1,100	**114.238**	**159.416**
2900	8.132	19.944	-3.763	-3.642	4.369	23.586	105.660	237.959	2,252	0,900	**110.028**	**162.520**
2700	7.943	18.206	-4.971	-5.714	2.973	23.920	102.662	215.448	2,099	0,700	**105.635**	**165.424**
2500	7.765	16.524	-6.252	-8.295	1.513	24.819	99.665	193.657	1,943	0,500	**101.178**	**167.887**
2333,8	7.606	15.163	-6.649	-9.703	957	24.866	97.173	176.092	1,812	0,334	**98.131**	**168.205**
2200	7.485	14.077	-6.965	-10.867	520	24.944	95.169	162.319	1,706	0,200	**95.688**	**168.125**
2000	7.281	12.500	-7.281	-12.500	0	25.000	92.171	142.326	1,544	0,000	**92.171**	**167.326**
1800	7.112	10.977	-7.625	-14.179	-512	25.157	89.174	123.052	1,380	-0,200	**88.662**	**165.942**
1600	6.933	9.507	-7.951	-15.904	-1.018	25.411	86.176	104.498	1,213	-0,400	**85.159**	**163.972**
1400	6.728	8.061	-8.274	-17.677	-1.546	25.738	83.179	86.663	1,042	-0,600	**81.633**	**161.382**
1200	6.553	6.674	-8.615	-19.495	-2.062	26.170	80.182	69.548	0,867	-0,800	**78.119**	**158.213**
1000	6.247	5.285	-8.931	-21.360	-2.684	26.645	77.184	53.152	0,689	-1,000	**74.500**	**154.297**
900	6.099	4.617	-9.127	-22.317	-3.028	26.934	75.685	45.224	0,598	-1,100	**72.658**	**152.081**
800	5.895	3.962	-9.269	-23.273	-3.374	27.235	74.187	37.475	0,505	-1,200	**70.813**	**149.686**
700	5.643	3.335	-9.428	-24.248	-3.785	27.584	70.543	29.628	0,420	-1,300	**66.758**	**143.997**
650	5.531	3.035	-9.538	-24.744	-4.007	27.779	65.504	25.547	0,390	-1,350	**61.498**	**136.348**
600	5.400	2.745	-9.606	-25.230	-4.207	27.974	60.466	21.768	0,360	-1,400	**56.259**	**128.505**
550	5.245	2.466	-9.687	-25.725	-4.443	28.191	55.427	18.291	0,330	-1,450	**50.984**	**120.408**
500	5.131	2.201	-9.846	-26.230	-4.715	28.431	50.388	15.116	0,300	-1,500	**45.673**	**112.056**
450	5.080	1.941	-10.107	-26.757	-5.027	28.698	45.349	12.244	0,270	-1,550	**40.322**	**103.442**
400	5.016	1.686	-10.327	-27.290	-5.311	28.975	40.310	9.674	0,240	-1,600	**34.999**	**94.649**
375	4.994	1.561	-10.476	-27.568	-5.482	29.128	37.791	8.503	0,225	-1,625	**32.309**	**90.134**
350	4.994	1.436	-10.671	-27.857	-5.677	29.293	35.272	7.407	0,210	-1,650	**29.595**	**85.531**
325	4.994	1.311	-10.813	-28.145	-5.818	29.456	32.752	6.387	0,195	-1,675	**26.934**	**80.957**
300	4.994	1.186	-10.872	-28.421	-5.878	29.607	30.233	5.442	0,180	-1,700	**24.355**	**76.453**
275	4.994	1.061	-10.941	-28.700	-5.947	29.761	27.713	4.573	0,165	-1,725	**21.766**	**71.881**
250	4.994	936	-11.025	-28.985	-6.031	29.922	25.194	3.779	0,150	-1,750	**19.163**	**67.236**
225	4.994	812	-11.034	-29.262	-6.040	30.074	22.675	3.061	0,135	-1,775	**16.634**	**62.661**
200	4.994	687	-11.034	-29.538	-6.040	30.225	20.155	2.419	0,120	-1,800	**14.115**	**58.050**
175	4.994	562	-11.034	-29.814	-6.040	30.376	17.636	1.852	0,105	-1,825	**11.595**	**53.389**
150	4.994	437	-11.034	-30.090	-6.040	30.527	15.116	1.360	0,090	-1,850	**9.076**	**48.678**
125	4.482	280	-11.034	-30.366	-6.553	30.646	12.597	945	0,075	-1,875	**6.044**	**42.924**
100	3.361	126	-11.034	-30.642	-7.673	30.768	10.078	605	0,060	-1,900	**2.405**	**35.941**
75	1.494	19	-11.034	-30.917	-9.540	30.936	7.558	340	0,045	-1,925	**-1.982**	**27.460**
50	0	0	-13.275	-31.221	-13.275	31.221	5.039	151	0,030	-1,950	**-8.237**	**15.311**
25	0	0	-16.029	-31.656	-16.029	31.656	2.519	38	0,015	-1,975	**-13.509**	**5.014**
Pure Tension					-16.029	0	0	0			**-16.029**	**0**

Bending around y-axis:

Neutral Axis	F's	M's	F_s	M_s	$\sum F_s$	$\sum M_s$	F_c	M_c	Z_c	Z_N	N	M
mm	kN	kNm	kN	kNm	kN	kNm	kN	kNm	m	m	kN	kNm
Pure Compression					16.029	0	198.566	0			**214.595**	**0**
7800	10.664	57.544	0	0	10.664	57.544	143.502	746.876	5,205	3,900	**154.167**	**203.170**
7575	10.380	54.713	-83	-14	10.297	54.699	140.130	709.783	5,065	3,675	**150.427**	**211.689**
7350	10.121	51.970	-215	-81	9.907	51.890	136.758	673.600	4,925	3,450	**146.665**	**219.657**
7125	9.871	49.289	-386	-218	9.485	49.072	133.386	638.327	4,786	3,225	**142.871**	**227.076**
6900	9.627	46.683	-594	-447	9.033	46.236	130.014	603.965	4,645	3,000	**139.047**	**233.956**
6675	9.383	44.149	-836	-788	8.548	43.360	126.642	570.514	4,505	2,775	**135.190**	**240.300**
6450	9.140	41.670	-1.119	-1.262	8.021	40.408	123.270	537.973	4,364	2,550	**131.291**	**246.113**
6225	8.900	39.273	-1.444	-1.893	7.456	37.381	119.898	506.343	4,223	2,325	**127.354**	**251.410**
6000	8.656	36.935	-1.814	-2.708	6.842	34.227	116.525	475.623	4,082	2,100	**123.367**	**256.194**
5775	8.413	34.664	-2.238	-3.739	6.175	30.925	113.153	445.813	3,940	1,875	**119.328**	**260.476**
5550	8.173	32.476	-2.719	-5.025	5.453	27.451	109.781	416.914	3,798	1,650	**115.235**	**264.278**
5325	7.925	30.334	-3.265	-6.609	4.659	23.725	106.409	388.925	3,655	1,425	**111.068**	**267.597**
5100	7.681	28.271	-3.888	-8.543	3.793	19.728	103.037	361.847	3,512	1,200	**106.830**	**270.465**
4875	7.443	26.283	-4.482	-10.552	2.961	15.731	99.665	335.680	3,368	0,975	**102.626**	**272.455**
4650	7.197	24.346	-4.955	-12.332	2.242	12.014	96.293	310.422	3,224	0,750	**98.535**	**273.200**
4425	6.956	22.491	-5.385	-14.076	1.571	8.415	92.921	286.076	3,079	0,525	**94.492**	**273.034**
4075,0	6.575	19.731	-6.052	-16.928	523	2.803	87.675	250.011	2,852	0,175	**88.198**	**271.236**
3900	6.390	18.413	-6.390	-18.413	0	0	85.052	232.807	2,737	0,000	**85.052**	**269.634**
3700	6.175	16.965	-6.769	-20.161	-594	-3.197	82.055	213.819	2,606	-0,200	**81.460**	**267.237**
3500	5.952	15.555	-7.149	-21.966	-1.197	-6.411	79.057	195.549	2,474	-0,400	**77.860**	**264.215**
3250	5.684	13.883	-7.632	-24.304	-1.947	-10.422	75.311	173.724	2,307	-0,650	**73.363**	**259.597**
3000	5.414	12.288	-8.107	-26.713	-2.693	-14.425	71.564	153.023	2,138	-0,900	**68.871**	**254.007**
2750	5.143	10.778	-8.590	-29.218	-3.447	-18.440	67.817	133.446	1,968	-1,150	**64.370**	**247.468**
2500	4.880	9.360	-9.063	-31.791	-4.183	-22.431	64.070	114.993	1,795	-1,400	**59.887**	**239.986**
2250	4.605	8.010	-9.553	-34.469	-4.948	-26.459	60.323	97.664	1,619	-1,650	**55.376**	**231.512**
2000	4.343	6.761	-10.025	-37.210	-5.682	-30.448	56.577	81.459	1,440	-1,900	**50.895**	**222.130**
1750	4.068	5.581	-10.511	-40.051	-6.443	-34.469	52.830	66.378	1,256	-2,150	**46.387**	**211.742**
1500	3.796	4.492	-10.978	-42.959	-7.182	-38.467	49.083	52.421	1,068	-2,400	**41.901**	**200.437**
1250	3.537	3.491	-11.467	-45.966	-7.930	-42.475	45.336	39.589	0,873	-2,650	**37.406**	**188.172**
1000	3.208	2.540	-11.941	-49.047	-8.732	-46.508	41.589	27.880	0,670	-2,900	**32.857**	**174.752**
750	2.851	1.700	-12.432	-52.221	-9.581	-50.521	37.843	17.295	0,457	-3,150	**28.262**	**160.241**
600	2.540	1.245	-12.679	-54.158	-10.139	-52.913	31.008	11.163	0,360	-3,300	**20.869**	**135.434**
500	2.371	983	-12.923	-55.472	-10.552	-54.489	25.840	7.752	0,300	-3,400	**15.288**	**116.185**
400	2.273	738	-13.298	-56.822	-11.025	-56.084	20.672	4.961	0,240	-3,500	**9.647**	**96.286**
300	2.164	514	-13.586	-58.209	-11.421	-57.695	15.504	2.791	0,180	-3,600	**4.083**	**76.211**
200	2.164	298	-13.864	-59.603	-11.700	-59.306	10.336	1.240	0,120	-3,700	**-1.364**	**56.093**
150	2.164	189	-13.864	-60.296	-11.700	-60.107	7.752	698	0,090	-3,750	**-3.948**	**46.377**
100	1.457	55	-13.864	-60.990	-12.408	-60.935	5.168	310	0,060	-3,800	**-7.240**	**33.843**
75	647	8	-13.864	-61.336	-13.217	-61.328	3.876	174	0,045	-3,825	**-9.341**	**25.789**
50	0	0	-14.836	-61.695	-14.836	-61.695	2.584	78	0,030	-3,850	**-12.252**	**14.604**
25	0	0	-16.029	-62.111	-16.029	-62.111	1.292	19	0,015	-3,875	**-14.737**	**5.026**
Pure Tension					-16.029	0	0	0			**-16.029**	**0**

Load combination 1.

$$N_d = 41.012kN$$
$$M_{dx} = 109.556kNm$$
$$M_{dy} = 30.443kNm$$

$$e_x = \frac{M_{dx}}{N_d} = \frac{109.556 \cdot 10^6 \, Nmm}{41.012 \cdot 10^3 \, N} = 2665,5mm$$

$$e_y = \frac{M_{dy}}{N_d} = \frac{30.443 \cdot 10^6 \, Nmm}{41.012 \cdot 10^3 \, N} = 740,7mm$$

$$\frac{\frac{e_y}{B_{eq}}}{\frac{e_x}{L_{eq}}} = \frac{\frac{740,7mm}{5303,1mm}}{\frac{2665,5mm}{9296,9mm}} = 0,49 > 0,2$$

$$\frac{\frac{e_x}{L_{eq}}}{\frac{e_y}{B_{eq}}} = \frac{\frac{2665,5mm}{9296,9mm}}{\frac{740,7mm}{5303,1mm}} = 2,1 > 0,2$$

$$\frac{N_D}{N_{Rd}} = \frac{41.012kN}{214.595kN} = 0,19$$

$$a = 1,0 + \frac{1,5 - 1,0}{0,7 - 0,1} \cdot \left(0,19 - 0,1\right) = 1,076$$

$$\left(\frac{M_{dx}}{M_{Rdx}}\right)^a + \left(\frac{M_{dy}}{M_{Rdy}}\right)^a = \left(\frac{109.556kNm}{103.000kNm}\right)^{1,076} + \left(\frac{30.443kNm}{195.000kNm}\right)^{1,076} = 1,20 > 1 \Rightarrow \text{ NOT OK!}$$

Load combination 2.

$N_d = 40.730kN$

$M_{dx} = 102.452kNm$

$M_{dy} = 29.539kNm$

$$e_x = \frac{M_{dx}}{N_d} = \frac{102.452 \cdot 10^6 \, Nmm}{40.730 \cdot 10^3 \, N} = 2515,4mm$$

$$e_y = \frac{M_{dy}}{N_d} = \frac{29.539 \cdot 10^6 \, Nmm}{40.730 \cdot 10^3 \, N} = 725,2mm$$

$$\frac{\frac{e_y}{B_{eq}}}{\frac{e_x}{L_{eq}}} = \frac{\frac{725,2mm}{5303,1mm}}{\frac{2515,4mm}{9296,9mm}} = 0,51 > 0,2$$

$$\frac{\frac{e_x}{L_{eq}}}{\frac{e_y}{B_{eq}}} = \frac{\frac{2515,4mm}{9296,9mm}}{\frac{725,2mm}{5303,1mm}} = 2,0 > 0,2$$

$$\frac{N_D}{N_{Rd}} = \frac{40.730kN}{214.595kN} = 0,19$$

$$a = 1,0 + \frac{1,5-1,0}{0,7-0,1} \cdot (0,19 - 0,1) = 1,075$$

$$\left(\frac{M_{dx}}{M_{Rdx}}\right)^a + \left(\frac{M_{dy}}{M_{Rdy}}\right)^a = \left(\frac{102.452kNm}{103.000kNm}\right)^{1,075} + \left(\frac{29.539kNm}{195.000kNm}\right)^{1,075} = 1,12 > 1 \Rightarrow \text{ NOT OK!}$$

Load combination 3.

$$N_d = 39.450kN$$
$$M_{dx} = 47.127kNm$$
$$M_{dy} = 74.349kNm$$

$$e_x = \frac{M_{dx}}{N_d} = \frac{47.127 \cdot 10^6 \, Nmm}{39.450 \cdot 10^3 \, N} = 1194,6mm$$

$$e_y = \frac{M_{dy}}{N_d} = \frac{74.349 \cdot 10^6 \, Nmm}{39.450 \cdot 10^3 \, N} = 1884,6mm$$

$$\frac{\frac{e_y}{B_{eq}}}{\frac{e_x}{L_{eq}}} = \frac{\frac{1884,6mm}{5303,1mm}}{\frac{1194,6mm}{9296,9mm}} = 2,77 > 0,2$$

$$\frac{\frac{e_x}{L_{eq}}}{\frac{e_y}{B_{eq}}} = \frac{\frac{2515,4mm}{9296,9mm}}{\frac{725,2mm}{5303,1mm}} = 0,36 > 0,2$$

$$\frac{N_D}{N_{Rd}} = \frac{39.450kN}{214.595kN} = 0,18$$

$$a = 1,0 + \frac{1,5 - 1,0}{0,7 - 0,1} \cdot (0,18 - 0,1) = 1,070$$

$$\left(\frac{M_{dx}}{M_{Rdx}}\right)^a + \left(\frac{M_{dy}}{M_{Rdy}}\right)^a = \left(\frac{47.127kNm}{103.000kNm}\right)^{1,070} + \left(\frac{74.349kNm}{195.000kNm}\right)^{1,070} = 0,79 < 1 \Rightarrow \text{ OK}$$