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Abstract

In this thesis, the impact of unknown parameter estimation in the case of the volatility parameter in the Black-Scholes model on hedging is investigated.

The objective is to develop a systematic and robust model of hedging to challenge the conventional delta and improved mean-variance hedge, and hopefully improve upon risk assessment values. The proposed algorithms were tested in situations of uncertainty to prove their accuracy and robustness. Call option prices were drawn from the Black-Scholes model and hedged accordingly with the various routines. Risk measures were then calculated in the form of the Value at Risk and the Expected Shortfall to further assess their overall performance. In reality most hedging is done with no knowledge of some crucial parameters, so a scenario is created to somehow replicate such a condition to further assess the performance of the routines. A simulated vector of possible values of the volatility parameter is drawn to further test the algorithms and aid in our assessment .

Chapter 1

1.1 Introduction

Modern day finance is riddled with the burden of parameter estimation and the risk involved with this task. Parameter estimation in financial models has become a very important and crucial practice, due to the need for the best or optimal forecast for price movements. An optimal model gives an advantage to determine a profit or loss outcome, and the emphasis on adequate estimates is very crucial. Financial analysts build various models in hope of simulating as realistically as possible, the trends of the various markets be it commodities, stocks, derivatives and so on, to gain an advantage in their daily business activities. Risk managers and hedge managers are examples of such professions who use their models to give out estimates of risk and formulate appropriate strategies to curb them.

Due to the 2008 recession, the overall tolerance for risk has hit a low point with investors wary and cautious in our markets today. The need for adequate estimates and the appropriate strategies cannot be ignored. The slightest of advantages noticed in models and their parameters, in estimation and forecasting is welcomed and garnered towards consumer satisfaction and profitability.

The focus of this work is on the misrepresentation of the volatility parameter and its use in hedging. The choice of the volatility parameter in hedging strategies is somewhat inconclusive and the focus here is to investigate it. Recent studies developed look into the pricing and hedging techniques which has been very beneficial and has allowed for more insight . In $[Carr 2005]$ an expression was developed for profit from using volatilities. $[Henrard\ 2003]$ derived these results and constructed simulations to look into the properties of the path-dependent profit. $[Carr\ and\ Verma(2005)]$is another study which investigates the problem of hedging using stochastic implied volatility. The choice of our parameters in pricing, or in hedging has been of concern for many analysts and experts, and revised methods are sought after for better pricing and hedging. In $[Lindstrom 2010]$ the effect of the parameter uncertainty is also analyzed on option pricing with an in depth look also leading to new discoveries

The choice of implied vs historical volatility is of high concern in pricing and hedging strategies. Recent developments looking into the efficiency and importance of the implied volatility suggest the implied volatility as a very useful estimators and perform much better than most estimators. $[Poon\ and\ Granger]$ It is even assumed of the existence of a relationship between these two volatilities which can be used in forecasting strategies.That is another topic all in itself, but our main interest for this thesis is to investigate the effect of volatility misrepresentation on the hedge weights and in effect the hedge error as well. In $[Willmot, Ahmad]$ the effect of the different volatilities on profit making was investigated and was shown to be quite interesting

the final conclusion said the overall profit was not affected by the choice of parameter.This gave rise in the topic for which further comparison and investigation was of interest. the effect of using varying volatility parameters in calculating the hedge weight is the focus of this thesis. Similarly, all work is kept in the Black-Scholes world.

For this thesis we choose to concentrate on the estimation or better yet the mis-specification of the volatility parameter in models. We further analyze the impact the volatility parameter being mispecified will have on hedging the option. We take into consideration the effects of the estimates on the hedge weight. And later, the overall performance on the portfolio and the hedging error are analyzed. This work is done in the Black-Scholes world, which assumes constant volatility throughout the life of the claim amongst other things.

- Chapter 2 gives an overview of the financial system or way of life, introducing the markets, some models and literature to help in our understanding

- Chapter 3 highlights some of the mathematics behind derivatives

- Chapter 4 focuses on the pricing of derivative instruments

- In Chapter 5, the topic of hedging is viewed

- In Chapter 6, the analysis and methodology of the investigation is carried out

- In Chapter 7 conclusions are drawn and recommendations for further studies are given

Chapter 2

2.1   Financial Markets

They are markets for the sale and purchase of financial instruments such as commodities, derivatives, currencies etc. Financial markets are of various types and sizes. A typical example is the capital market consisting of the stock markets and the bond markets. Capital markets are of large importance in our business world today. They are used in raising long term funds for businesses and even governments all over the world and have become crucial in day to day operations.

Other notable markets are the money markets; which facilitate the trade of financial instruments with high liquidity and short maturity times; the commodity market involving the trading of its namesake, commodities; futures markets, insurance markets, foreign exchange markets etc.

2.2   Derivatives

Another type of market, which we are very interested in, is the derivatives market which deals with the trade of derivative instrument. The growth of this financial instrument, ie derivatives, cannot be ignored. Over the last decade the derivative market has been so large it has been estimated as of 1996 to be $ $1200 trillion$. Derivatives have essentially become the wild child of modern finance. They are feared and equally loved by most traders today due to their risky nature, and more oft than not, are a misunderstood aspect of investment.

Such is the situation of these instruments but none can deny the fact of its huge impact on our markets today. According to one of the world's leading quantitative finance experts, Paul Wilmot, $1.2 quadrillion is the so-called notional value of the worldwide derivatives market. To put that in perspective, the world's annual gross domestic product is between $50 $trillion$ and $60 $trillion$ as at early 2012.We, here of, give insight to this fascinating financial instrument.

2.2.1   History Of Derivatives

Options are one of the most common forms of derivatives used on the markets in present day, albeit their misrepresentation, and the lack of due diligence on the part of their critics. The history of derivatives is highly debatable, but in .[Thales in Politics is in BookI, Chapter 11, sections 5-10] they are said to have been a tool present in human transactions dating as far back as the Greek civilization. Aristotle in his Politics provides a reference to the use of options involving a successful speculation by the philosopher Thales. Another reference

to the history of an option like contract can also be referenced to the Bible in Genesis 29 when Rachel, daughter of Laban is offered to Jacob in return for seven years of dedicated labour to Laban.

Examples of options are the European call and put options, the American option, Asian option, Binary options, Russian options.

A derivative is known as a contract between two consenting parties who lay out conditions under which the payments are to be made between them. It can also be seen as a security which derives its value from a well specified group of assets called underlying assets, the underlying being stocks, commodities etc[*Aberg*] They are essential in modern day finance even with their associated risk and uncertainties.

Derivatives have an ability to transfer risk and can be used as insurance which make it one of the more invaluable tools used by traders, risk managers, hedgers etc.

Assume a company 'Q' is making a big purchase from another company 'Y' in another country. In a month or tw,o they estimate to close the deal and Q knows that, it needs a lot of money, ie foreign currency at that time. The financial risk faced, is that the said currency may rise which will be expensive for company Q. The company essentially can write up a contract to lock the exchange rate in the currency it so desires for the trade. A derivative eases the complexity of this situation by doing just that. The company Q can buy forwards locking in the purchase or may go in for an option giving it the option to buy the currency at a proposed rate. This is one of the many uses of derivatives, and below are brief descriptions about them.

2.2.2 Types Of Derivatives

There are two types of derivatives namely the commitments and contingent claims which are all traded on two types of markets; exchange traded and over the counter trades. Derivatives that are exchange traded contracts can be termed as standardized contracts that are traded via derivative trading platforms such as the Eurex, the Korea Exchange etc. Over the counter derivatives are basically any forms of contracts or transactions created privately or anywhere other than derivative exchanges between two consenting parties. We delve further into our explanation of the derivatives by defining a forward contract

2.2.2.1 Forward Contracts

A forward contract, or forward for short, is an agreement between two parties in which the buyer of the contract must buy the underlying asset at a predefined time from the other party, that is, the seller of the contract, at a price agreed upon at the start. This form of commitment is private and is of a largely unregulated market. This is highly due to the fact that they are different, and tailored or customized, to suit the needs of the parties involved. The futures contract is a variation of the forward being the fact that it essentially is also a forward with an additional feature of being a public and standardized contract that takes place in a derivative exchange

The payoff function is that of;

$$\text{payoff} = S - K$$

from the payers position. Where S is the stock price at maturity and K is the agreed price at the start of the contract termed the strike

2.3 Options

They are financial instruments representing a mutual agreement between two parties with one party offering the right to sell or buy an agreed upon financial asset at a later date at an agreed price, and the other party having the choice to exercise this claim.The second party is not obliged in any way to exercise the claim.

By far the most popular types of options are the equity options. These are options written or traded, based on individual stocks. another popular type of options is the Index options, which is written on market indices such as the $S\&P500, NASDAQ, FTSE100$ etc other examples are bond options,interest options, currency options and options on futures

2.3.0.2 European Call

The European call option is a financial contract between two parties being the seller of this contract and the buyer. The holder/buyer has the 'option' not the obligation, to buy the underlying asset from the seller of the option at a predefined price(the strike K) at a certain time. The seller of the contract is obligated to sell the underlying should the buyer decide to exercise this right.
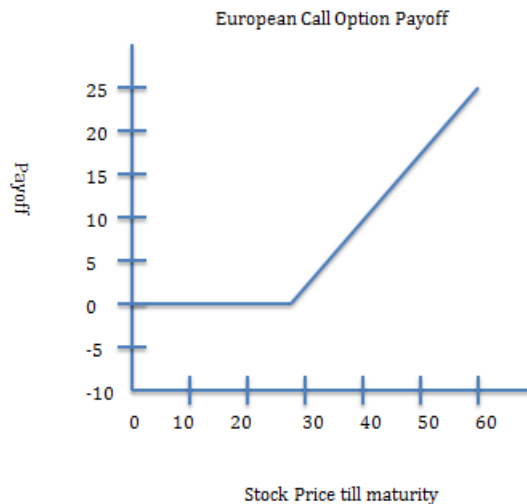
Figure 2.1: payoff diagram of call option

The payoff function is ;

$$max(S - K, 0)$$

2.3.0.3 European Put

[ht] The European put option is an option where the buyer of this contract has the right to sell the underlying to the seller at a strike price at a certain time.

The payoff function is ;

$$max(K - S, 0)$$

The European type options are very important in financial markets and are often considered as the building blocks to more complex options. They are often referred to as vanilla options and the markets that trade these options are referred to as vanilla markets

European Put Option Payoff
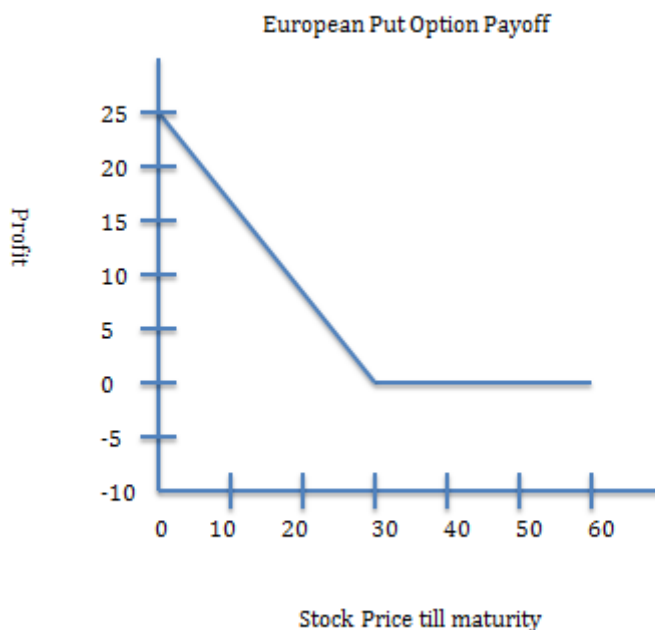
Figure 2.2: payoff diagram of put option

2.4 Moneyness

2.4.0.4 In-the-money

The European call option is said to be in the money (ITM) if the strike K is below the price of the underlying, ie,$K < S_t e^{rT}$ since the payoff is positive if the stock evolves with the short rate.

In the case of the European put option, it is in-the-money when the strike K is above the market price of the underlying,$K > S_t e^{rT}$

2.4.0.5 At-the-money

Put and call options are said to be at-the-money (ATM) if the strike is equal to the maturity price, ie $K = S_t e^{rT}$

2.4.0.6 Out-of the -money

European call options are said to be out of the money (OTM) if the strike is higher than the price of the underlying asset, that is , $K > S_t e^{rT}$. For the European put option to be out of the money, the strike must be less than the price of the underlying asset, that is, $K < S_t e^{rT}$.

Note that when a European call option is in-the-money then the European put with the same strike on the same asset is out-of the –money and vice versa.

2.5 Other Types Of Options

There are other notable options to be noted and brief descriptions of these options are given below. Following Aberg(2010) we have;

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2.5.1 The Binary Option

This is an option whose payout is fixed after the underlying asset price exceeds a certain level or strike. It is also known as the digital option or the 'all or nothing' option If the stock reaches a certain level (K) before a fixed time point (maturity time T), the binary option gives a unit amount of the money or otherwise nothing. The payoff function is as follows;

$$\phi(S_T) = 1_{\{S_T \geq K\}}(S_T) = \begin{cases} 1 & S_T \geq K, \\ 0 & S_T \leq K. \end{cases}$$

2.5.2 The American Option

This type of option is somewhat similar to the European option, but it is subject to being exercised anytime during the life of the contract. This option allows option holders to exercise the option at any time prior to and including its maturity time, T, thus increasing the value of the option to the holder relative to the European option. Its payoff structure is as follows;

For the American call option

$$\phi(S) = max(S_\tau - K, 0)$$

where τ is any stopping time smaller than or equal to T

For the American put option, the payoff is ;

$$\phi(S) = max(K - S_\tau, 0)$$

In this case, the holder has the option to sell the underlying asset at the optional time $\tau \leq T$.

2.5.3 The Asian Option

It is a type of option which has payoff depending on the mean price of the underlying asset over the life of the contract, as opposed to the price at maturity, as in the case of the European option.

The payoff function of the Asian call is ;

$$\phi(S) = max(M_S[0, T] - K, 0)$$

where $M_S[0, T]$ is the mean of the underlying asset S during a time interval $[0, T]$

2.5.4 The Barrier Option

This is basically an option whose payoff depends on the underlying asset price reaching or exceeding a predetermined price.
Let $H_d < S_o < H_u$ be the barriers in the down and up case respectively.
The payoff functions of the four different cases are then defined as;

$$\phi_{do}(S) = (S_T - K)^+ * 1_{\{m_S(T) \geq H_d\}},$$

$$\phi_{di}(S) = (S_T - K)^+ * 1_{\{m_S(T) < H_d\}},$$

$$\phi_{uo}(S) = (S_T - K)^+ * 1_{\{M_S(T) \leq H_u\}},$$

$$\phi_{ui}(S) = (S_T - K)^+ * 1_{\{M_S(T) > H_u\}}.$$

where $m_s(t)$ and $M_s(t)$ are the running minimum and maximum at time t, ie;

$$m_s(T) = \inf_{0 < u < T} S_u \quad and \quad M_s(T) = \sup_{0 < u < T} S_u$$

Chapter 3

Mathematical Theory

In pricing of assets on financial markets in continuous time, asset prices are modeled as continuous time stochastic process.

3.1 Stochastic Processes

A collection of random variables on a probability space (Ω, F, P) is defined as a *stochastic process*

Furthermore, a stochastic process is a diffusion process if its local dynamics can be approximated by a stochastic difference equation such as;

$$X(t + \Delta t) - X(t) = \mu(t, X(t))\Delta t + \sigma(t, X(t))Z(t).$$

where Z is a normally distributed noise term which is independent of everything prior to time t and μ and σ are given deterministic functions.

The function μ is called the local drift term of the process and σ, the diffusion term

3.2 Brownian Motion

A stochastic process is called a Wiener process if the following conditions hold;

- W(0) = 0.

- The process W has independent increments, ie if r < s ≤ t < u, then W(u) - W(t) and W(s) - W(r) are independent stochastic variables.

- For s < t the stochastic variable W(t) - W(s) has the Gaussian distribution $N[0, \sqrt{t - s}]$.

- W has continuous trajectories.

Consider X as a stochastic process and also that there exists a real number x_o and two adapted processes μ and σ such that the relation below stands for all $t \leq o$;

$$X(T) = a + \int_0^t \mu(s)ds + \int_0^t \sigma(s)dW(s),$$

where a is given real number.

3.3 Ito Formula

consider a process X that has stochastic differential as;

$$dX(t) = \mu(t)dt + \sigma(t)dW(t),$$

where μ and σ are adapted processes.

Define Z as Z(t)=f(t,X(t)).Then Z has a stochastic differential given by

$$df(t, X(t)) = \{\frac{df}{dt} + \mu\frac{df}{dx} + \frac{1}{2}\sigma^2\frac{d^2f}{dx^2}\}dt + \sigma\frac{df}{dx}dW(t).$$

Chapter 4

PRICING OF DERIVATIVES

4.1 Risk Neutral Valuation

The arbitrage free price of the claim $\phi(S(t))$ is given by the formula

$$\pi(t; \phi) = F(t; S(t))$$

where F is given by the formula;

$$F(t, s) = e^{-r(T-t)} E_{t,s}^{Q}[\phi(S(t))]$$

It is seen that the price of the derivative, given today's time t and today's stock price s, is computed by taking the expectation of the final payment $E_{t,s}^{Q}[\phi(S(t))]$ and then discounting this expected value to present value using the discount factor $e^{-r(T-t)}$.

It is also noted that the expectation taken over the payment is not the P-measure but rather the Q-measure, which is sometimes called the **risk adjusted measure**.

4.2 Black Scholes and Merton Formula

In 1973 a publication on the pricing of options was released by two economists, Fischer Black and Myron Scholes which redefined the world of derivative pricing. This discovery was birthed when Fischer Black started out working to discover a pricing model for stock warrants.

Soon after this discovery, Myron Scholes joined Black and their efforts together resulted in the discovery of the renowned Black-Scholes valuation model for derivatives. Their model is in fact a follow up on research by a PhD candidate at the University of Chicago, James Boness. Their improvements on the Boness model gave the analytical model, which we now know as the Black-Scholes model, a closed form solution to price vanilla options.

The overall framework of the model is that it assumes are;

- Stock returns are normally distributed

- No dividends are paid out

- Interest rates do not change during the option life

- Commissions don't exist

- Volatility is held constant

- Options can only be exercised at expiration

- Markets can't be predicted.

It is noted that some of the assumptions held by the model are unrealistic and invalid, giving results which are not always accurate. The financial industry utilizes more sophisticated models in pricing these days which give more accurate results. The Black-Scholes model is mostly used as a benchmark for further developments.

The Black-Scholes model

$$dB(t) = rB(t)dt$$

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t)$$

where α and σ are constants.

The arbitrage free price of a simple claim $\phi(S(t))$ is given as;

$$F(t, s) = e^{-r(T-t)}E_{t,s}^{Q}[\phi(S(t))]$$

Where the Q-dynamics are

$$dS(u) = rS(u)dt + \sigma S(u)dW(u)$$

$$S(t) = s$$

where S is given by;

$$S(T) = S(t) * exp\{(r - \frac{1}{2}\sigma^2)(T - t) + \sigma(W(T) - W(t))\}$$

This gives rise to the following integral;

$$F(t, s) = e^{-r(T-t)} \int_{-\infty}^{\infty} \phi(se^z)f(z) \ dz$$

where f is the density of a random variable Z with the distribution

$$N[(r - \frac{1}{2}\sigma^2)(T - t), \ \sigma\sqrt{T - t}]$$

The integral above, for the general choice of contract function ϕ, must be calculated numerically. There are however, a few particular cases where the function can be evaluated more or less analytically, and the best known of these is in the valuation of the European call option. This claim is of the form $\phi(x) = max(x - K, 0)$.

In this case we obtain

$$E_{t,s}^Q[\max(se^z - K, 0)] = 0 * Q(se^z \leq K) \quad + \quad \int_{ln\frac{K}{S}}^{\infty} (se^z - K)f(z)dz$$

And through calculations we arrive at the price of the claim.

The price of a European call option with strike price K and maturity time T is given by the formula

$$\pi(t) = F(t, S(t))$$

where

$$F(t, S(t)) = sN[d_1(t, s)] - e^{-r(T-t)}KN[d_2(t, s)]$$

with N as the cumulative distribution function for the N[0,1] distribution and

$$d_1(t, s) = \frac{ln\frac{K}{S} + (r + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2(t, s) = d_1(t, s) - \sigma\sqrt{T - t}$$

4.3   Other Pricing Models

Other notable derivative pricing models are the

4.3.1   Binomial model

The binomial model of option pricing uses the principle of the probability of upwards and downwards movement of the stock price. It assigns probabilities to upward movement of the stock and downward movement during the maturity time of the option. Consider a single period case where the times of interest are the time now $t = 0$ and the time at maturity $T = 0$. If S_o is the current price today at maturity, it is safe to assume the price to be

$$S_T = \begin{cases} S_o u & P_u \\ S_o d & 1 - P_u \end{cases}$$

where $u = $ *the probability of the stock price moving up* and $d = $ *the probability of the stock price going down*

Now take a derivative with its associated payoff as;

$$\phi(S_T) = \begin{cases} \phi(uS_o) \\ \phi(dS_o) \end{cases}$$

In finding the price of the derivative at $t = 0$, the hedge is needed, which is the portfolio $h = [h_s h_b]$ having the same payoff as the derivative. This gives us

4.3.2 Heston

This model was proposed by Steve Heston in $[Heston93]$. It rids the pricing of the option from the Black-Scholes assumption of constant volatility, and as such, is termed as one of the stochastic volatility models.

It is of the form:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^1,$$

$$dV_t = k(\theta - V_t)dt + \sigma \sqrt{V_t} dW_t^2,$$

$$dW_t^1 dW_t^2 = \rho dt.$$

where S_t and V_t , are the stock price and instantaneous variance respectively, and are the Wiener processed with correlation . is described as a CIR process. $[CIR\ 85]$. Other notable models are the

4.3.3 Merton

In $[Merton\ 1976]$ Merton derived a model which introduced jumps in addition to the classical Brownian motion. He proposed a formula which was derived for a more general case when the underlying stock returns a generated by a mixture of both continuous and jump processes.

Where

$$\pi_{JD}(S, K, \sigma, T, r, v, \lambda, m) = \sum_{k=0}^{\infty} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} \pi_{BS}(S, K, \sigma_k, r_k, T)$$

where;

$$\sigma_k = \sqrt{\sigma^2 + \frac{kv^2}{T}} and r_k = r - \lambda(m-1) + \frac{k.log(m)}{T}$$

Other renowned models in use are the Bates model $[Bates\ 1996]$, Variance Gamma model $[Madan, Seneta\ 1990]$

4.3.4 Fourier Pricing

Another method of pricing which has developed in recent times, and is used in analysis, is the pricing of options using Fourier transforms. Applied Fourier transforms has been used to value options in recent developments such as $[Bakshi\ and\ Chen]$, $[Scott]$, $[Bates\ 1996]$, $[Heston]$, and $[Chen\ and\ Scott]$.

The approach is the to find the delta numerically and the risk neutral probabilities of finishing in the money. These in addition with the stock price and the strike price is used to derive the option price. $[Carr\ and\ Madan]$ later developed a method which took advantage of the computational prowess of the Fast Fourier Transform(FFT) which was lacking in previous studies. Their approach was to assume knowledge of the characteristic function of the risk neutral density. Taking this characteristic function, they developed a simple analytical expression for the Fourier transform of the option value. the FFT is then used to numerically derive the option value.

4.4 Volatility

It is the standard deviation of the log price process . It is the statistical measure of the dispersion of returns for a given security or market index. It can also be defined as a variable in option-pricing models which shows the extent to which the return of the underlying asset will fluctuate between now and the option's expiration. [investopedia] The fluctuations in the value of the volatility are a result of changes in the emotions of the investors or in other terms the risk appetite of the individual. There are two main types of volatility that are used in the option markets namely; the historical volatility and the implied volatility.

4.4.1 Historical Volatility

It is often referred to as the actual volatility or realized volatility. It is the measure of the stock's price movement based on historical prices and it is sometimes used to measure the activity of the stock price over time. Suppose we want to value a European call option with six months left to maturity. An obvious idea is to use historical stock price data in order to estimate σ . In reality, however, the volatility is not constant over time. It is standard to use the historical data for a period of the same length as the time to maturity, which in this case will be data for the last six months. In order to obtain the estimate of σ , assume the standard Black-Scholes geometric Brownian motion model, ie

$$dBt = rB(t)dtdS(t) = \alpha S(t)dt + \sigma S(t)dW(t),$$

under the measure P. Observe the stock price process S at n + 1 discrete equidistant points t0,t1,...,tn, where Δt denotes the length of the sampling interval, ie , $\Delta t = t_i$ - t_{i-1} . Thus observe $S = S_{t0}, S_{t1}, ..., S_{tn}$, and in order to estimate σ assume S has $\log -normal$ distribution.

Define ϵ_1,..., ϵ_n by;

$$\epsilon_i = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right)$$

Note that are independent, normally distributed random variables with

$$E[\epsilon_i] = \left(\alpha - \frac{1}{2}\sigma^2\right)\Delta t$$

$$Var[\epsilon_i] = \sigma^2$$

Using elementary statistics an estimate of σ is obtained, ie ,

$$\sigma^* = \frac{S_\epsilon}{\sqrt{\Delta t}}$$

where the sample variance S_ϵ^2 is given by;

$$S_\epsilon^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\epsilon_i - \bar{\epsilon})^2,$$

where

$$\bar{\epsilon} = \frac{1}{n}\sum_{i=1}^{n}\epsilon_i$$

The standard deviation, D, of the estimate is approximately given by;

$$D(\sigma^*) \approx \frac{\sigma^*}{\sqrt{2n}}$$

4.4.2 Implied Volatility

Originally the volatility in the Black and Scholes model was estimated as the standard deviation of the log-returns, ie the historical volatility. As the vanilla market matured the pricing of the vanilla options became more independent of the Black and Scholes formula. It became more interesting to take the quoted price in the market, and compute the volatility. Thus the term , implied volatility. In a sense the volatility was first set using the historical stock prices but is nowadays set using the vanilla options.

It is the estimated volatility of a security's price. In addition to known parameters such as stock price(S_t),interest rate(r), time to maturity and strike price(K), implied volatility is used in the calculation of option prices. Implied volatility can be derived from the Black-Scholes model. To define the implied volatility as in [$Aberg$]first consider the Black-Scholes formula

$$\Pi_{BS}(\sigma; T, M) = S_t(N[d(T, M, \sigma)]) - e^{-M}N[d(T, M, \sigma) - \sigma\sqrt{\tau}]$$

Where

$$d(T, M, \sigma) = \frac{M}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2}, \quad M = \ln\left[\frac{S_t}{Ke^{-r\tau}}\right] \text{ and } \tau = T - t$$

The Black-Scholes formula here is viewed as a function of the volatility ,T and M are parameters. Consider now a market quote of a call option Π_{EC} , the implied volatility $\sigma_{iv}(T, M)$ at this expiry/ strike price is defined as the volatility in the Black-Scholes formula that implies the price , ie, the inverse of the Black-Scholes at Π_{EC}.

$$\Pi_{EC} = \Pi_{BS}(\sigma_{iv}(T, M); T, M) \Rightarrow \sigma_{iv}(T, M) = \Pi_{BS}^{-1}(\Pi_{EC}; T, M)$$

where the inverse in the Black-Scholes formula is taken on the volatility. This formula however cannot be computed in closed form expression. this is due to the presence of the normal distribution functions that are also not represented in closed from expressions.. The popularity of the implied volatility comes from the fact that it is much easier to evaluate from the different vanilla prices on the market. It is noted that if the quotes given by the Black-Scholes model, the implied volatility would be a flat over the strikes due to the constant volatility in the Black-Scholes model for all strikes

4.5 Volatility Smile or Skewness

In using the Black-Scholes pricing model, the volatility can be calculated by using the market prices of relevant options. The issue of constant volatility being unrealistic resurfaces here in the sense that, when the calculated implied volatilities of a group of options with the same expiration date is plotted against different or ascending strike prices, there is a disparity present. This disparity is in the form of a curve or skewness which is due to the fact that the implied doesn't stay constant over the variations of strike prices.

Volatility increases as the option becomes
increasingly in-the-money or out-of-the-money

Implied Volatility

In-the-money puts

In-the-money call

Out-of-the-money call
options

Out-of-the-money puts

Strike price
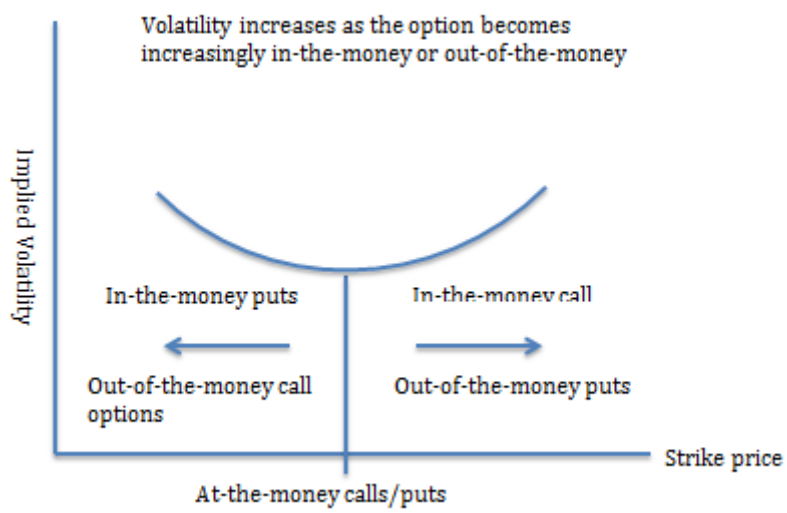
At-the-money calls/puts

Figure 4.1: smile ref:optionsguide.com

Chapter 5

HEDGING

The act of protecting your portfolio is just as important as the process of improving the value of your portfolio. Having priced and sold a security, the seller needs to hedge this position until it matures, which is a more serious commitment.

The best way to define hedging is to view it as insurance. When investors hedge they are insulating themselves as much as possible from a negative event. This is not to say the negative event won't happen but the exposure to this mishap will be minimal.

A hedge can be constructed from many types of financial instruments, including stocks, exchange-traded funds, insurance, forward contracts, swaps, options, many types of over-the-counter and derivative products, and futures contracts.

Let X be a contingent claim with maturity T. if there exists a self-financing portfolio h such that;

$$P(V^h(T) = X) = 1 \quad a.s.$$

then h is a hedge or a replicating portfolio of the claim.

To hedge, the price of the derivative must be unique otherwise there is an arbitrage opportunity. An arbitrage opportunity is unrealistic because there exists, possibility of earning money with no risk involved.

If all derivatives in the market can be hedged, the market is said to be complete. A good market is one seen as complete and free from arbitrage. In such a market, all the derivatives can be priced, and the simple task of finding their corresponding hedges is needed. Thus if there is a derivative which has two hedges with different prices, it admits arbitrage, and if there is a derivative without a hedge the market is not complete.

5.1 The Greeks

The Greeks are the sensitivities of the option price to changes in other relevant parameters. The name *Greeks* is a result of the sensitivities being represented with Greek letters. Below is an introduction to them;

5.1.1 Delta

This is the rate of change of the option to the change in the stock or underlying price. it is sometimes referred to as the hedge ratio.

$$\Delta = \frac{\partial \pi}{\partial S}$$

where Π is the option price and S is the stock price

It always lies between 0 and 1 for call options and between 0 and -1 for put options. If the delta is calculated for a call option and for example it is .60, it is interpreted as; for every unit increase 1, in the stock price, there is an increase of .60 in the call price. It is used sometimes as a proxy for moneyness as well. If say a call's delta has been calculated to be close to 1, it is often seen to be deep in-the-money. Similarly if the calculated delta tends to draw closer to 0, it is seen to deep out-of the-money. At-the-money calls have deltas to be around 0.5.

5.1.2 Gamma

This is the rate of change of delta with respect to change in the stock price

$$\Gamma = \frac{\partial^2 \pi}{\partial S^2}$$

5.1.3 Vega

This is the sensitivity measure of the change in the price of the derivative with respect to a change in the volatility.

$$\nu = \frac{\partial \pi}{\partial v}$$

5.1.4 Rho

This is the sensitivity measure of the change in the price of the derivative with the respect to a change in the interest rate.

$$\rho = \frac{\partial \pi}{\partial r}$$

5.1.5 Theta

This is the sensitivity measure of the change in the price of the derivative with the respect to a change in the time

$$\Theta = \frac{\partial \pi}{\partial t}$$

5.2 Delta Hedging

This is one of, if not, the most fundamental form of hedging. It aims to reduce the risk associated with movements in stock prices by offsetting long and call positions in options. Its explanation is given in the form of an example below; Take an option of price, Π, with an underlying asset price of S. The delta is consequently calculated to give a value of Δ . This just means if the price of the underlying asset goes up by a unit say 1, the price of the option will also go up by Δ. Say a broker takes a short position on our option in question. Let

us also assume it to be a call. If he/she shorts this call, the amount if shares to be held so that a small change in the underlying will offset by the change in the option is thus a Δ number if shares.

5.3 Portfolio

The portfolio is a combination of assets, be it the stocks and bank account, and at times the derivative movements are also added.

Following $[Bjork\ Chapter 6]$

Let the $N-dimensional$ price process $S(t); t \geq 0$ be given. A portfolio strategy, simply called a portfolio, is any F_t^S-adapted N-dimensional process $h(t); t \geq 0$.

The portfolio h is said to be Markovian if it is of the form

$$h(t) = h(t, S(t)),$$

for some function

$$h : \mathbb{R}_+ \times \mathbb{R}^\mathbb{N} \to \mathbb{R}^\mathbb{N}.$$

The value process of the portfolio is thus;

$$V^h(t) = \sum_{i=1}^{n} h_i(t) S_i(t)$$

where $h_i(t)$ is the number of asset $S_i(t)$

A consumption process is any F_t^S-adapted one-dimensional process $c(t); t \geq 0$.

A portfolio–consumption pair portfolio–consumption pair (h,c) is called self-financing if the value process V^h satisfies the condition;

$$dV^h(t) = \sum_{i=1}^{N} h_i(t) dS_i(t) - c(t) dt$$

i.e. if

$$dV^h(t) = h(t) dS(t) - c(t) dt$$

For a given portfolio h the corresponding relative portfolio u is given by

$$u_i(t) = \frac{h_i(t) S_i(t)}{V^h(t)}, i = 1$$

where we have

$$\sum_{i=1}^{N} u_i(t) = 1.$$

The self-financing condition can now easily be given in terms of the relative portfolio.

5.4 Risk Measures

Value At Risk

For investors the loss of money is not desired and knowledge of potential exposure to some drastic losses are highly favored. VaR is such a measure which deals with questions such as *What is my worst case scenario*? .It is used as a measure for the risk of loss on a specific portfolio of financial assets. It can be written as;

$$P(L > VaR_\alpha) = \alpha$$

where L is the loss

Expected Shortfall

A measure viewed as an improvement in the capabilities of the VaR estimate is the Expected Shortfall.It is regarded as a more better estimate considering it gives a better scope of critical situations which can occur. It produces an estimate of the average loss when the extreme loss actually occurs. It is sometimes considered as the Conditional VaR estimate. It answers the question *'If the worst happens what is my expected loss?'*ie

$$E[L|L > VaR_\alpha]$$

Chapter 6

Analysis

Now we analyze the hedge errors for different hedging settings under the Black Scholes model. It is assumed that a more holistic approach of taking into account the uncertainty of hedging with an unknown volatility should be addressed. The unknown volatility being that which may be different from the actual volatility used in finally hedging on the market.

It is assumed that the volatility is unknown. If an initial estimate of the volatility is used to generate reasonable and similar values for the volatility, through an algorithm which is described below, better hedge weights should be derived to give even better risk measures.

A number of simulation scenarios are also considered, with performance based on the risk measures calculated. The focus is to find the most appropriate hedge type out of the three proposed and test their robustness and accuracy. They were further analyzed in different hedge periods to give more results.

In evaluating the performance of the hedge weight in the portfolio the proposed scenarios were considered;

- Hedge with *delta*

- Hedge with *hedgeweight*

- Hedge with *hedgeweightsim*

- Hedge with simulated version of delta, simulated version of hedgeweight and a simulated version of hedgeweightsim

The above terms are self explanatory for some and the others will be explained along the course of analysis. The *delta* above signifies the classic delta algorithm , the second ,ie the *hedgeweight*, is the mean variance hedge weight, and the third routine, ie, *hedgeweightsim* a reformulated version of the mean variance hedge weight method.

In the first scenario where we hedge with delta, Black-Scholes prices generated are hedged accordingly using the classic delta hedge and later portfolio values are constructed to aid in the calculation of the hedge errors. This method is straight forward and is used somewhat as a benchmark.

The second scenario stems from the mean-variance hedge and is used to calculate the hedge weight labeled *hedgeweight*. It challenges the classic delta hedge for a better hedging error and a robust algorithm and was

developed to aid in the assessment.It is calculated as such in the algorithm;

$$a = \frac{\pi_E^C(S_t e^{\Delta \sigma^2}, K) - \pi_E^C(S_t, K)}{S_t e^{\Delta \sigma^2}}$$

where $\pi_E^C(S_t e^{\Delta \sigma^2}, K)$ represents a modified version of the Black-Scholes call price and $\pi_E^C(S_t, K)$ represents the classic Black-Scholes call price. Δ here is also defined as the step parameter.

The third scenario termed the *hedgeweightsim* algorithm, is a robust algorithm constructed to take in a generating value for the volatility and thereby generating subsequent volatility values from the chi-square distribution. These values together in a vector, are considered a single unit and hedge weights are calculated. This method characterizes a multi-version of the above *hedgeweight* algorithm. This is assumed to cater to the uncertainty in the estimation of unknown values of the volatility in the hedging process. The formula below represents the hedgeweight used in the portfolio;

$$a = \frac{E[\pi_E^C(S_t e^{\Delta \sigma^2}, K) - \pi_E^C(S_t, K)]}{S_t E[e^{\Delta \sigma^2}]}$$

With the fourth scenario, a vector of 100 volatilities is generated and stored in a vector called *simvol*22. Each member of *simvol*22 is used as a single generating value for the *hedgeweightsim* algorithm above. Results were derived from the sum of all hedge errors pooled together from all 100 simulations and recorded. The same technique was implemented using each volatility value to subsequently derive hedges based on the *delta* and single *hedgeweight* algorithms. Also values were recorded and risk measures calculated from them.

6.1   Numerical Implementation

All calculations and simulations were done in Matlab. The work has been implemented in the classic Black Scholes world with all simulations and calculations conforming to the noted Black-Scholes assumptions. Monte Carlo simulations was used to generate stock and subsequent derivations were conducted.

The data used in the analysis was as follows;
A number of 10,000 trajectories of the stock price path were drawn with an initial value of 100, a discounting rate r, of 0.05 and a volatility of 0.65. Next, was to price the call options based on the stock prices and a strike price K of 85 according to the Black-Scholes model. Subsequently prices were generated along the paths for all trajectories.
A 100 values was set for the,*simvol*22, vector for the simulated version of events. It should be stated also that the degrees of freedom f was set to 60.
With respect to the time steps of events, the number of daily steps were set to 365, the weekly set to 52 and the monthly set to 12.

In our implementation key risk measures were calculated namely the Value at Risk(VaR) and Expected Shortfall. After the different forms of hedge weights were derived according to the different techniques, they were combined with the stock prices, the bank and the bank weight to form the portfolio values of the assets. Hedge errors were then calculated from there by subtracting the portfolio values of the assets from the Black-Scholes prices ie;

$$HedgeError_i = V_i - \Pi_i$$

where V is the portfolio value and Π is the Black-Scholes price

The mean hedge errors and the mean squared hedge errors were consequently calculated from these values to aid in the analysis.

The Value at Risk was also calculated at VaR_1 and VaR_{99} respectively to depict the best case and worst case scenarios at a 99% confidence interval. In this case, the object of interest will be the worst case form of loss attainable, which will be the VaR_{99}

The next notable measure of interest calculated, was the Expected Shortfall. It produces an estimate of the average loss possible when the extreme loss limit is reached. In this work, due to the equation of the hedge error, the VaR_{99} is regarded as the extreme loss estimate.
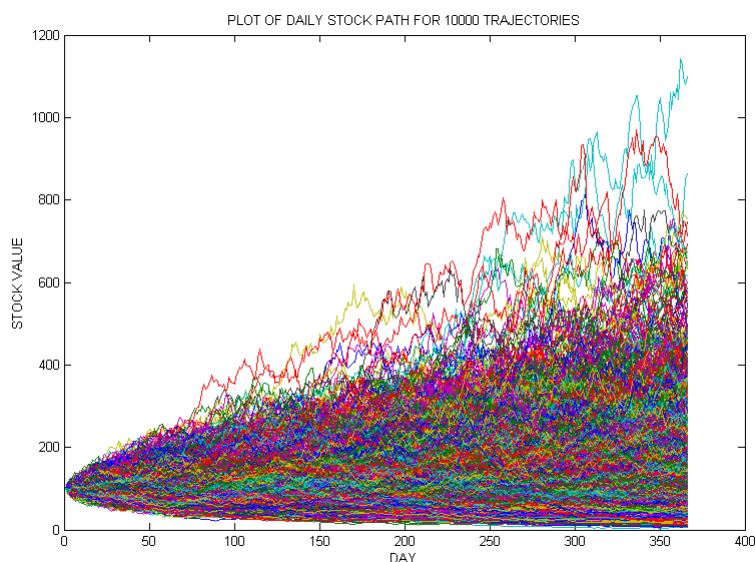

Figure 6.1: 10000 stock price trajectories

Figure 6.1 represents the stock price paths generated from the Black-Scholes model consisting of 10000 trajectories. It was generated with an initial value of 100 for all 10000 trajectories and follows a geometric Brownian motion with a strike k of 85.

6.1.1 Delta

To get the hedge was quite straight forward in the delta algorithm setting. Using the Black-Scholes model of pricing, automatically produces our required delta hedge which is *normcdf*($d1$) in the pricing algorithm.

6.1.2 Hedgeweight

The mean variance hedge weight implemented in the *hedgeweight* algorithm was calculated as follows;

Calculation of the Hedgeweight

Consider a portfolio V to be;

$$V = a * S_i + b * B_i$$

The hedgeweight a was calculated to be

$$a = \frac{E[\pi_E^C(S_t e^{\Delta\sigma^2}, K) - \pi_E^C(S_t, K)]}{S_t E[e^{\Delta\sigma^2}]}$$

We also acknowledge that the variance σ has distribution

$$\sigma^2 \in \chi^2(f)\frac{\sigma_o^2}{f}$$

And has expectation to be;

$$E[\sigma^2] = \sigma_o^2$$

We consider also the density as

$$f_x\{x\} = \frac{x^{\frac{f}{2}-1}.2^{\frac{-f}{2}}}{\Gamma(\frac{f}{2})}.e^{\frac{-x}{2}} \quad x > 0$$

Thus the expectation of the denominator will be;

$$\int_0^\infty (e^{\frac{\Delta\sigma_o^2 x}{f}} - 1) \ f_x(x) \ dx = \int_0^\infty (e^{-x[\frac{1}{2} - \frac{\Delta\sigma_o^2}{f}]}x^{\frac{f}{2}} - 2^{\frac{-f}{2}}) \ dx - 1$$

$$= 2^{\frac{-f}{2}}\left[\frac{1}{2} - \frac{\Delta\sigma_o^2}{f}]^{\frac{-f}{2}}\right] - 1$$

$$= \left[1 - \frac{2\Delta\sigma_o^2}{f}\right]^{\frac{-f}{2}} - 1$$

$$\overset{=}{f \to \infty} \quad e^{\Delta\sigma_o^2} - 1$$

In the calculation of the numerator for the value of a that is;

$$E[\pi_C^E(S_t e^{\Delta\sigma^2}, K) - \pi_E^C(S_t, K)]$$

Together with our representation of the Black-Scholes prices in Fourier form we calculate as such;

$$\frac{1}{2\pi i} \int_{-i\infty+z}^{i\infty+z} \frac{e^{-r(T-t)}.e^{-lnk.z+(z+1)lnS}}{z(z+1)} \quad . \quad e^{(T-t)k(z+1)} \ dz$$

where

$$k(z) = \left(r - \frac{\sigma^2}{2}\right)z + \frac{z^2\sigma^2}{2}$$

Now assume that

$$E^Q[S_T^z|F_t] = e^{(T-t)k(z)}$$

Thus

$$E^Q[e^{-r(T-t)}S_T|F_t] \begin{cases} = & e^{-r(T-t)}e^{-r(T-t)k(1)}S_t \\ \\ = & e^{(T-t)[k(1)-r]}S_t \\ \\ = & S_t \end{cases}$$

since $k(1) = r$

Back to the calculation of the numerator gives us;

$$\frac{1}{2\pi i}e^{-r(T-t)} \int_{-i\infty+z}^{i\infty+z} \frac{e^{-r(T-t)}.e^{-lnk.z+(z+1)lnS}}{z(z+1)} \quad . \quad e^{(T-t)k(z+1)} \quad . \quad E[e^{(z+1)\Delta\sigma^2} - 1]dz$$

$$\frac{1}{2\pi i}e^{-r(T-t)} \int_{-i\infty+z}^{i\infty+z} \frac{e^{-r(T-t)}.e^{-lnk.z+(z+1)lnS}}{z(z+1)} \quad . \quad e^{(T-t)k(z+1)} \quad . \quad [h_1(z+1) - h_2(z+1)] \ dz$$

where

$$h_1(z) \quad = \quad E[e^{\sigma^2\left(\frac{(z)^2-(z)}{2}\right)(T-t)+(z+1)\Delta}]$$

$$= \frac{1}{\left(1 - \sigma_o^2\left[\frac{z^2-z}{f}(T-t) + \Delta(z)\right]\frac{2}{f}\right)^{\frac{f}{2}}}$$

$$\xrightarrow{f=\infty} e^{(T_t)\left(\frac{z^2-z}{2}\right)\sigma_o^2+\Delta(z)\sigma_o^2} \tag{6.1}$$

And

$$h_2(z) \quad = \quad E[e^{\sigma^2\left(\frac{(z+1)^2+(z+1)}{2}\right)(T-t)+(z+1)\Delta}]$$

$$h_1(z) \quad = \quad \frac{1}{\left(1 - \sigma_o^2\left[\frac{z^2-z}{f}(T-t)\right]\right)^{\frac{f}{2}}}$$

$$\xrightarrow{f=\infty} e^{(T_t)\left(\frac{z^2-z}{2}\right)\sigma_o^2} \tag{6.2}$$

As previously calculated, we know the values of the hedge error etc, according to the delta hedge . Now the values according to the mean variance hedge given in the calculation above as a which is;

$$a = \frac{E[\pi_E^C(S_t e^{\Delta\sigma^2}, K) - \pi_E^C(S_t, K)]}{S_t E[e^{\Delta\sigma^2}]}$$

This mean variance hedge-weight, a is calculated as an expectation of the difference in a Black-Scholes of the modified stock price, which was calculated using the product of the original stock price and an exponential value, and the original Black-Scholes value and dividing the difference by the expectation of the modified stock price minus the original stock price.

6.1.3   Hedgeweightsim

The algorithm termed *hedgeweightsim* is all that is left now to be described. This routine took the previous *hedgeweight* routine for the singular case and made it a more vector based one of different values. As previously explained, the mean was taken over difference in the modified price and the original price of the option and divided by the stock

It is assumed taking the mean over these possible outcomes will create a reduction in overall hedge errors formulating a better mean hedgeerror and mean square hedge error.

6.2   Simulated Version

The last algorithm used in the comparison was implemented as follows;

- Generate random values from the volatility

- Store vector as *simvol*22

- Use each of the values in *simvol*22 and run through all three algorithms.

- Pool together all hedge errors from the respective algorithms and perform necessary investigations on them

To give a prelimpse of how the comparison procedure will be implemented, a test run of different volatility values are used. Consider a situation where two different volatilities are proposed. These volatility values differ from the actual volatility. As the actual volatility was set to 0.65,it was preferable to choose implied volatilities as 0.85 and 0.45 to see if choice of implied volatility also affected events.
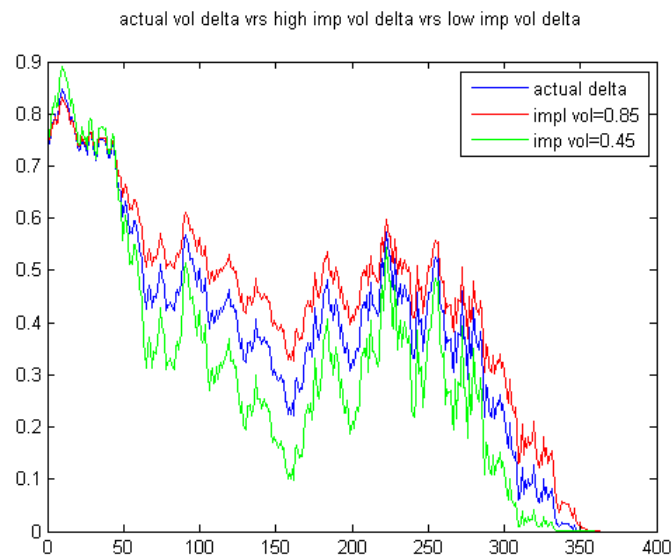

Figure 6.2: comparison of deltas

Figure 6.2 is a sample from the delta trajectories to illustrate the difference in the deltas based on the different volatilities.

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	Exp Shortfall
Delta	-0.0188	28.8594	[-12.3274 15.1315]	19.5775
Delta2	-0.0294	33.1675	[-13.7778 12.9901]	16.1458
Delta3	0.0263	37.8520	[-10.5621 20.0033]	25.7262

Table 6.1:

In this simple setting of comparison,volatility values were set as 0.65 for Delta, 0.85 for Delta2 and, 0.45 for Delta3. All other values remained constant and the hedge errors were derived accordingly. The table gives out the VaR values and teh Expected Shortfall which will be used in the comparison. Values produced here are to be regarded as trivial

6.3 Daily Hedging

The hedge errors calculated by the three different algorithms are illustrated below in the table. They are calculated over the span of a year with hedging done daily;

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	EXP SHORTFALL
Delta	-0.0025	1.0516	[-2.6834 2.7579]	3.4224
HedgeWeightMV	-0.0024	1.0508	[-2.6817 2.7251]	3.4180
HedgeWeightMVSim	-0.0017	1.0594	[-2.7666 2.6237]	3.3110

Table 6.2: Hedge Error measures for the Daily Hedged delta,hedgeweight and hedgeweightsim algorithms.

From the values shown in Table 6.2, it is apparently clear the hedgeweight values for the mean squared hedge errors always seemed to have an upper hand, even though it did seem minute. The proposed *hedgeweightsim* algorithm displayed quite good results as well. It had the best out of the three with regards to the mean hedge error and the expected shortfall.
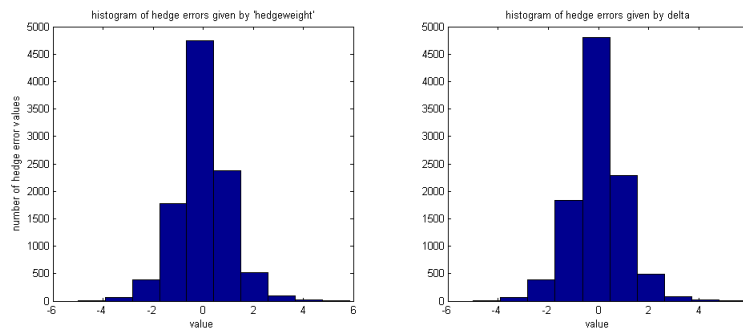

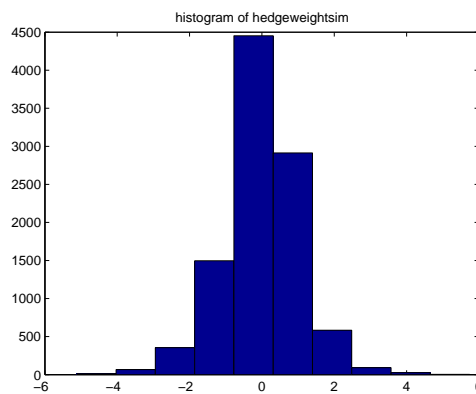Figure 6.3: delta histogram vrs hedgeweight histogram

Figure 6.4: hedgeweightsim histogram

Fig 6.3 above shows the histograms of the hedge errors given by the 'hedgeweight ' and the delta respectively. Fig 6.4 following also represents the histogram of the hedge errors given by the hedgeweightsim algorithm. This is as expected with both graphs having a somewhat similar pattern

6.4   Simulated Version of the Daily hedge steps

To truly assess the situation of uncertainty, a more realistic scenario of not knowing what the true value of the volatility was considered. It was decided the actual volatility should be generating element. The sample volatilities were in turn generated from the chi-square distribution . The choice of the chi-square distribution is purely subjective and any suitable choice of distribution can be used. The chi-square was chosen here in view of its association with it being the distribution in most cases with the volatility parameter in stochastic models.

From this generated vector of volatilities, the subsequent hedge errors and statistics are constructed to compare with our previous values

Below is a brief setup of the procedure implemented;

- Generate random values for the volatility

- for each of the volatilities, treat as the actual volatility to derive Black-Scholes prices, deltas and hedge-weights.

- With these values derive the hedge errors based on each volatility

- Pool together all values of the hedge errors and derive necessary statistics and measures.
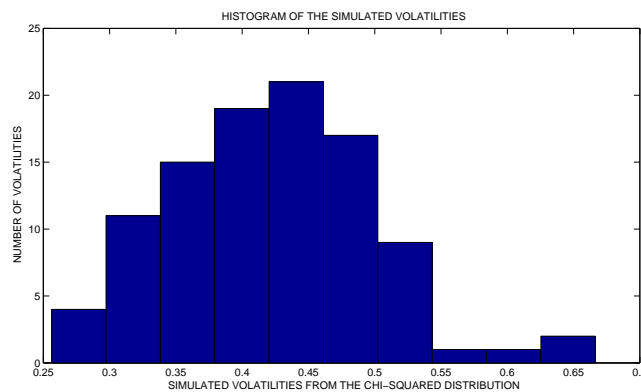

Figure 6.5: simvol22 histogram

Fig 6.5 is a diagram showing the histogram of the 100 simulated volatilities.

It is representing the 100 other values which can be used for our volatility in pricing and hedging. We should however note that in this generation, the actual volatility was used. This gives an overview of results to be obtained even if the estimate is somewhat close to the actual volatility. It is stored in a vector called simvol22
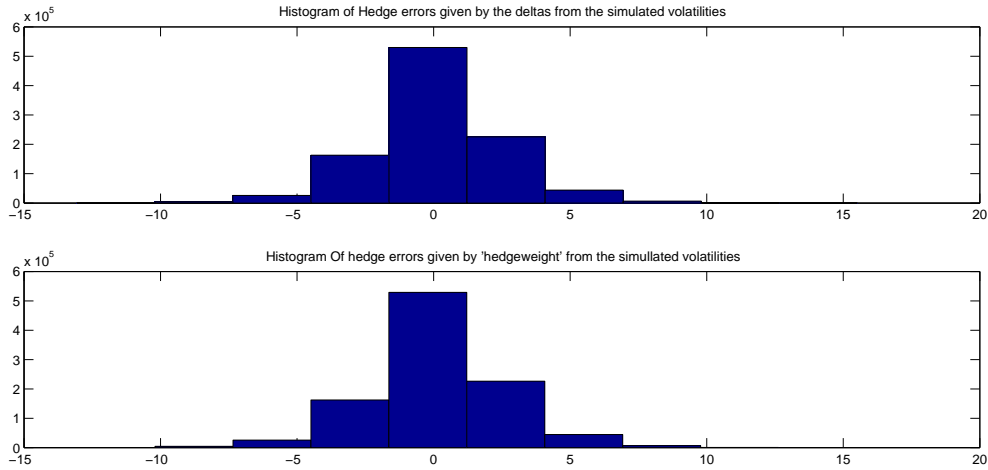
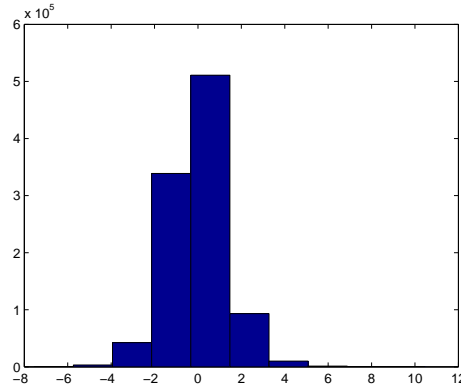Figure 6.6: simulated histogram for delta and hedgeweight

Figure 6.7: histogram for simulated hedgeweightsim

Fig 6.6 and 6.7 illustrate the histograms of the pooled hedge errors given by the simulated volatilities' deltas and hedge-weights respectively. As can be seen these histograms are similarly clustered around the mean but in the case of the simulated volatilities have longer tails. It can also be noted that there is a change in the skewness of the histogram when comparing the *hedgeweightsim* algorithm's histogram to the rest.

Data was retrieved as follows in Table 6.3;

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	SHORTFALL
DeltaSim	0.1060	5.7085	[-6.2877 6.5164]	7.7597
HedgeWeightMV	0.1061	5.7080	[-6.2839 6.5177]	7.7597
HedgeWeightSim	-0.0015	1.6400	[-3.2152 3.3959]	4.2743

Table 6.3: Hedge Error measures for the daily simulated delta,hedgeweight and hedgeweightsim algorithms

From each of the simulated volatilities, the hedge errors were calculated individually just as in the case of the single *hedgeweight* algorithm in the previous section. To compare the statistics and measures against the conventional delta and hedge-weight, all the calculated hedge errors were pooled together as one array and statistics were calculated accordingly to compare the measures.

6.5   Weekly Comparison

In this section, unlike the previous one where hedging was done daily over a year, hedging is done weekly in this case and subsequent data and graphs are produced

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	EXP SHORTFALL
Delta	0.0119	7.0493	[-6.5562 7.2449]	9.1527
HedgeWeightMV	0.0111	7.0139	[-6.5394 7.1915]	9.1153
HedgeWeightSim	0.0108	7.0138	[-6.5634 7.1847]	9.0523

Table 6.4: reults for the weekly hedged algorithms. These are the values from the case in which hedging is done in weeks within the year. Note the similarity in values for all algorithms

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	EXP SHORTFALL
Simulated Delta	0.0554	11.8080	[-8.8240 9.2897]	11.6762
Simulated HedgeWeightMV	0.0546	11.7760	[-8.8149 9.2740]	11.6482
Simulated HedgeWeightSim	0.0112	7.6104	[-6.7232 7.5133]	9.5314

Table 6.5: simulated version of weekly hedged algorithms. This table gives results for the simulated versioin of weekly hedging done wthin the year. Note the difference in the values for the *hedgeweightsim* algorithm from the other algorithms. Also note the similarity with the previous non-simulated weekly values
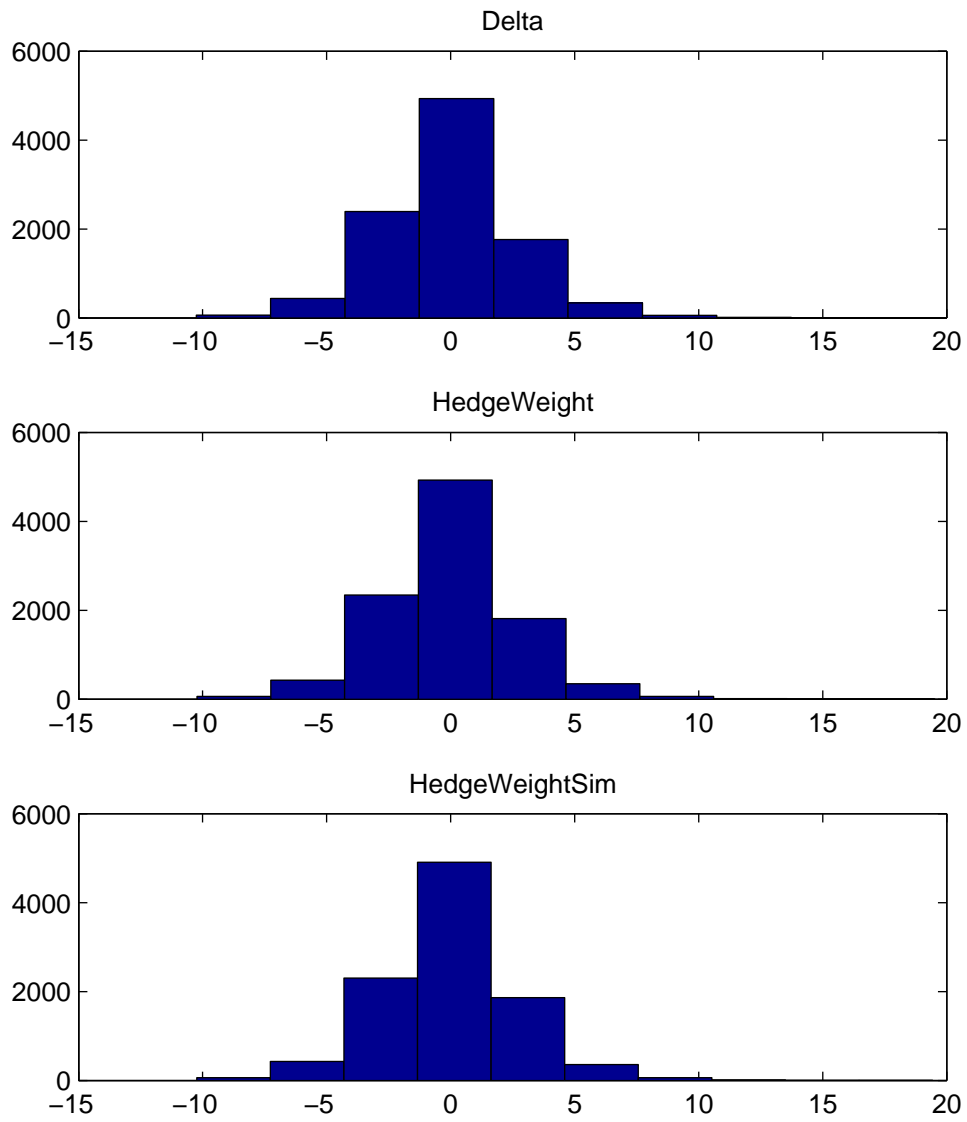
Figure 6.8: histogram of hedge errors for weekly delta,hedgeweight and hedgeweightsim.This is a figure of histograms generated from the weekly hedge errors constructed from the delta, hedgeweight and hedgeweightsim algorithms
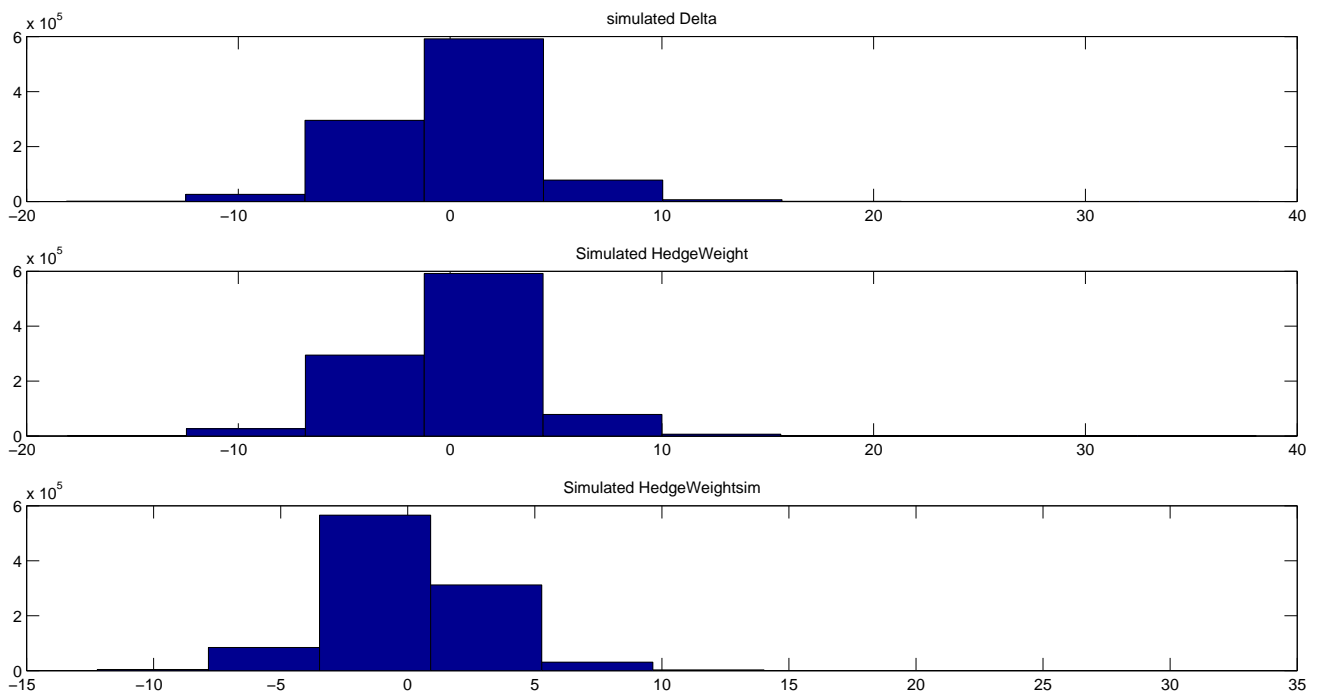
Figure 6.9: histogram of hedge errors for weekly simulated delta,hedgeweight and hedgdeweightsim. The figure above gives a histogram based comparison of the simulated weekly hedged hedge errors according to the delta, hedgeweight and hedgeweightsim.

6.6 Monthly Comparison

For this section hedging is done monthly to also give a view of the hedge weights in this scenario.

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	EXP SHORTFALL
Delta	0.0022	28.5563	[-12.2156 15.2900]	18.6748
HedgeWeightMV	-0.0147	27.9263	[-12.1739 14.6556]	18.0901
HedgeWeightSim	-0.0164	27.9283	[-12.2102 14.5229]	17.9464

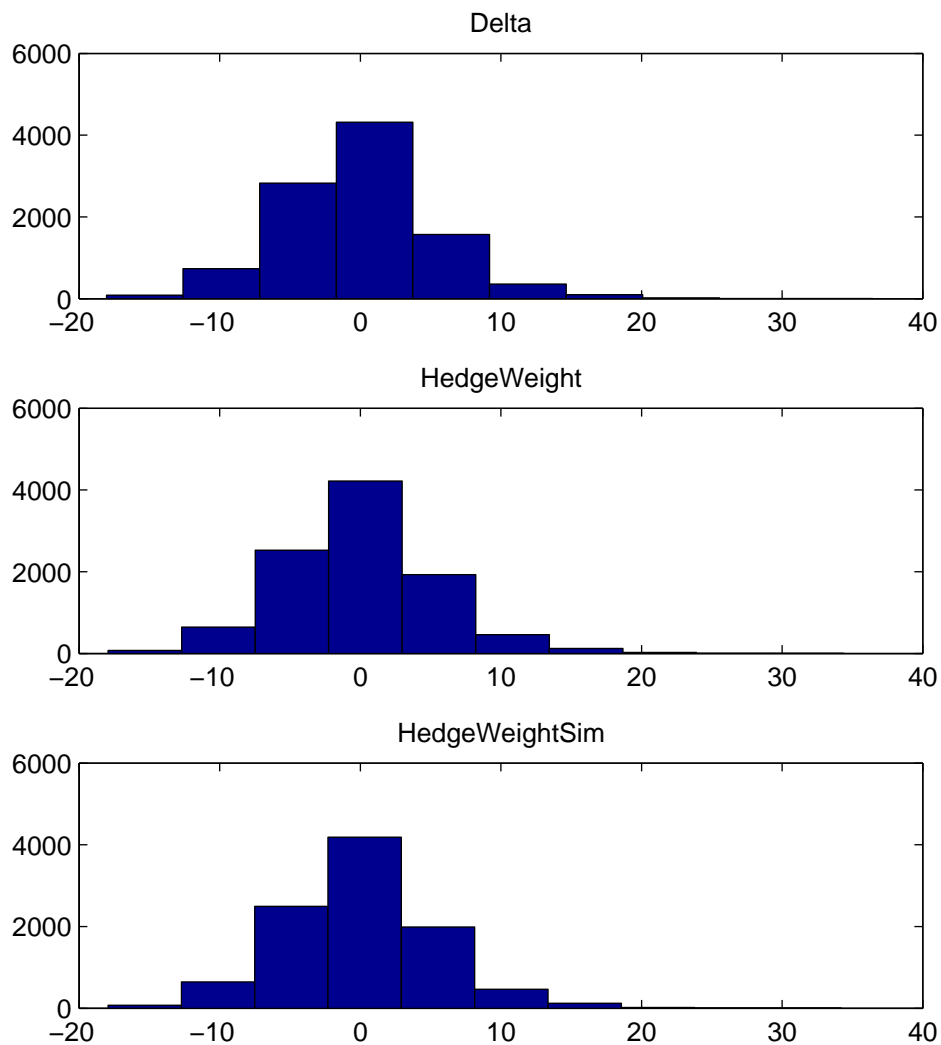Table 6.6: Hedge Error measures for the monthly hedged delta,hedgeweight and hedgeweightsim algorithms

Figure 6.10: histogram of hedge errors for monthly delta,hedgeweight and hedgdeweightsim. The figure given is a comparison of the histograms of the monthly hedged hedge errors given by delta, hedgeweight and hedgeweightsim

6.7 Simulated Version of The Monthly Hedged steps

	MEAN HEDGE ERROR	MSE HEDGE ERROR	VaR	EXP SHORTFALL
Simulated Delta	0.1907	32.9716	[-13.4072 16.3165]	20.1133
Simulated HedgeWeightMV	0.1740	32.3648	[-13.2989 15.9023]	19.6154
Simulated HedgeWeightSim	-0.0131	26.3987	[-11.9334 14.4775]	18.0088

Table 6.7: Hedge Error measures for the simulated monthly hedged delta,hedgeweight and hedgeweightsim algorithms
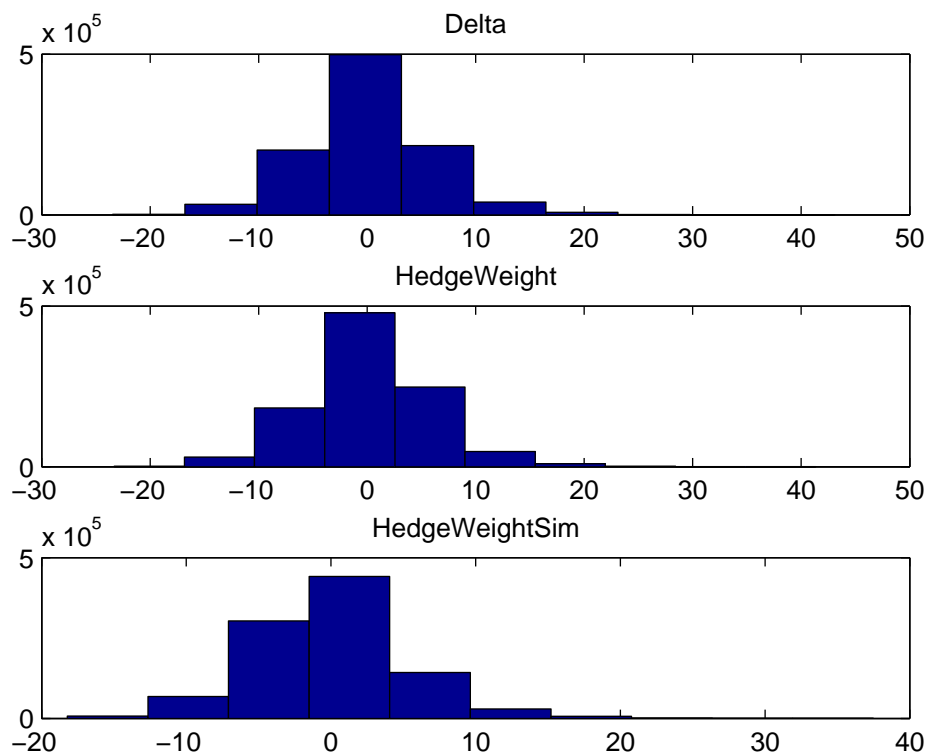

Figure 6.11: histogram of hedge errors for simulated monthly delta,hedgeweight and hedgdeweightsim. This figure compares the histogram of the *simulated* monthly hedged hedge errors given by delta, hedgeweight and hedgeweightsim respectively

Chapter 7

Conclusions

Taking a look at the values in Table 6.2, where the measures on the hedge errors are recorded. This is the case where the hedging is done on a daily basis. Any significant differences between the measures produced for the mean hedge error for *delta*, *hedgeweight* and *hedgeweightsim* algorithms are highlighted and discussed.

Looking at the mean square hedge error, VaR and Shortfall, the values seem not too different from each other, with respect to the different hedge weights algorithms. The histograms of the hedge errors in Fig 6.3 and Fig 6.4, which represent the three algorithms , also shows a similar distribution in the values and similar tail features. It can be said that they are not significantly different from each other.

The issue of uncertainty is then addressed next in the form of simulating a vector of possible volatilities and running them individually through a simulation of each of the three proposed algorithms. Table 6.3 relays the results of events, and what happened when this procedure was implemented. The mean hedge errors and the mse hedge errors for the *delta* and *hedgeweight* algorithms are seen to be clearly affected by the simulation process, producing significantly differing values from before. This can be assumed to be unfavourable as it shows it is highly affected by the uncertainty portrayed somewhat by the introduction of the random volatility values from *simvol*22.

The proposed *hedgeweightsim* algorithm however does not show that much variation in results produced from the previous results from Table 6.2, ie, the non-simulated version of the algorithms. It is seen to have handled the introduction of the random volatility values, and produced somewhat equivalent and favorable results.

Observations drawn from the comparison in the histogram plots in Fig 6.7 show a quite significant variation in the plots for the hedge errors of the *delta* and *hedgeweight* algorithms, and the histogram plot of the hedge errors of *hedgeweightsim* algorithms. Inspection shows the first two have much longer tails and their skew shape somewhat differs from the latter.

In observing the values of the Expected Shortfall also, it should be noted that there exists a noticeable trait which runs across the three hedging periods, ie, daily, weekly and monthly. The expected shortfall values for the simulated *hedgeweigthsim* algorithm differs much more when compared with the shortfall values derived from the other simulated algorithms. It is however, much more similar to the normal hedged algorithm values. It can be said to give an overall, better risk assessment as compared to the other algorithms.

Simulations were done also in different hedging periods such as; monthly and weekly to produce similar results just as in the daily hedging scenario. The simulated results for the *hedgeweightsim* algorithm always strayed towards the initial values for the mean hedge error, mean squared hedge error , the value at risk and the expected shortfall values of the initial scenarios for the *delta*, *hegeweight* and *hedgeweightsim* algorithms. On the issue of the expected shortfall, it showed a significant improvement as compared to the *delta* and *hedgeweight* with respect to the *hedgeweightsim* in the simulated setting.

Overall performance can be concluded as the proposed *hedgeweightsim* algorithm to be a more robust and better routine in hedging infused with uncertainty and depicts a better quality of estimation of hedge weights. This routine/technique wasn't devoid of its faults. It was very costly when it came to the processing time which may not be convenient for some traders. Considering the amount of time the *hedgeweightsim* algorithm took in producing results for the least costly and the fastest simulation setting, which will be work within the monthly number of hedging steps, every 10 volatility values from the 100 generated simvol22 vector took at least 45 minutes to generate the hedge errors. This makes a total of 450 minutes for just the production of hedge errors. Taking into consideration the fact that the monthly hedged scenario will be the fastest of the lot,this makes the algorithm more or less unappealing. An intel i5 duo core 2.4 GHz processor with an 8g RAM was used in running these simulations.

Further Research

Research further into the issue of volatility uncertainty with regards to option pricing and hedging strategies has been covered considerably over the years. This proposed routine and idea admittedly is but a thin layer of events covering uncertainty in hedging. Further development and research can concentrate on probable conditions for dealing with the duration of implementation and more exact results. The algorithm also tackles a more general if not basic routine in pricing and hedging and other models can be incorporated to give a wider scope of implementation, especially with regards to the stochastic volatility models to give a more realist setting .

References

1. Black, F and Scholes, M The pricing of Options and Corporate Liabilities Journal of Political Economy 81, 1973

2. Bjork,T 'Arbitrage Theory in Continuous Time' 2004

3. Aberg Sebastian, Derivative Pricing(Lecture Notes) 2010

4. Carr, P FAQs in option pricing theory. J.Derivatives 2005

5. Henrard, M Parameter Risk in the Black Scholes model. Risk and Insurance 0310002, Economics Working Paper Archive at WUSTL

6. Carr, P and Verma A , Potential Problems in Market Models of implied Volatility.(working paper)

7. Heston Steven ,A Closed-Form Solution for Options withStochastic Volatility with Applications to Bonds and Currency Options,1993

8. D. Goldfarb and G. Iyengar (2003), "Robust portfolio selection problems", Mathematics of Operations Research

9. Wilmott Paul and Ahmad Riaz, Which Free Lunch Would You Like Today, Sir?: DeltaHedging,Volatility Arbitrage and Optimal Portfolios

10. Merton, Robert C., . "Option pricing when underlying stock returns are discontinuous," Journal of Financial Economics, Elsevier, vol. 3(1-2), pages 125-144.1976

11. Bates, D 'Jumps and Stochastic Volatility: the exchage rate process implicit in the Deutchemark options', Review of Financial Studies, vol 9,pp 69-107, 1996

12. Madan D.D and Seneta E, 'The Variance Gamma model for share market returns,' The Journal of Business,vol.63 pg 511-524, 1990

13. Lindstrom Eric, Implications of Parameter Uncertainty on Option Prices Advances in Decision Sciences,Volume 2010 (2010), Article ID 598103, 15 pages.

14. http://www.globalresearch.ca/financial-implosion-global-derivatives-market-at-1-200-trillion-dollars-20-times-the-world-economy/30944

15. Poon,S.H and Granger, C. Forecasting Volatility in Financial Markets, Journal of Economic Literature. 2003

16. Chen, Ren-Raw and Louis Scott , 'Pricing Interest Rate Options in a Two-Factor Cox-Ingersoll-Ross Model of the Term Structure,' Review of Financial Studies, 5, 613-636.1992

17. Carr, Peter and Madan,Dilip, Option Pricing and the Fast Fourier Transform.Journal of Computational Finance, Volume 2 Number 4, pp. 61-73.1999

18. Cox, J.C., J.E. Ingersoll and S.A. Ross . "A Theory of the Term Structure of Interest Rates". Econometrica 53: 385–407.1985

19. Bakshi, Gurdip and Zhiwu Chen , 'An Alternative Valuation Model for Contingent Claims," Journal of Financial Economics, 44, 1, 123-165. 1997

20. John Hull Options, Futures and other Derivatives. Prentice-Hall, third edition, 1997

21. http://www.investopedia.com/terms/v/volatility.aspaxzz2EdYzfOtq12/10/2012

22. Scott, Louis , 'Pricing Stock Options in a Jump-Diffusion Model with Stochastic Volatility and Interest Rates: Application of Fourier Inversion Methods,' Mathematical Finance, 7, 413-426. 1997

23. Geoffrey Poitras The Early History of Option Contracts