Bachelor Thesis

Variables Important for Bankruptcy Prediction: A Logit Binary Approach

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Abstract

The purpose of this bachelor thesis is to estimate our own bankruptcy prediction model using logit binary data. Our choice of variables is based on Altman's Z-score model 1968. A comparison is then done between results in Altman and our findings. We perform our estimates on 114 listed Nordic companies, where 37 of them went bankrupt during 2002-2012.

We find that our estimated model can categorize defaulting and non-defaulting firms best, two years prior to the event of bankruptcy. This is done with a 76,8 per cent accuracy. Finally, we show that our model can predict bankruptcy of Nordic firms better than Altman's Z-score model.

Keywords: bankruptcy prediction, Z-score model, logit binary, maximum likelihood

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I. Introduction

Throughout this chapter we give background information together with our motivation. We then outline the purpose of the thesis, discuss the thesis delimitations and finally outline previous research on the same topic.

A. *Background and Motivation*

The uncertain economic environment during the recent years have stressed the importance of managing credit risk, i.e. the *"risk that a borrower may default on his obligations; a danger that interest payments and repayment of principal will not occur"*.[1] During the financial crisis of 2008 and 2009 the correlated defaults and the resulting bankruptcy wave evolved to a systemic risk in the financial sector, which in turn had a major negative impact on the entire global economy. A proper prediction of firm bankruptcies might therefore be extremely important and of great interest for a wide range of relevant financial actors.

There are many different approaches to forecast the complex problem of bankruptcy, but the two most influential models that are worth mentioning are due to Edward I. Altman and J. Ohlson (Standard & Poor (2012)). Altman (1968) used a multiple discrete analysis to estimate a model called the Z-score model, which has been broadly used by risk departments globally. Ohlson (1980) estimated another influential model using a logit binary approach based on variables other than those used by Altman. Both studies developed a score in order to measure firms' probability of default. These measures have become fundamental when assessing the credit risk of different firms. (Mester (1997))

In our thesis we have chosen to re-estimate Altman's variables in his Z-score model from 1968, but with modern data from Nordic firms. The estimation is completed using the same method as Ohlson (1980), i.e. the logit binary model. This is interesting since Altman's parameters are calculated with data from American companies before 1968. To our knowledge, parameters derived from Altman's variables have never been estimated on data from listed Nordic firms. Therefore it will be interesting to investigate differences in significances, marginal effects, and overall prediction power of different variables, between our estimation and that of Altman (1968).

For notice, we refer Altman's re-estimated Z-score model on Nordic listed firms as *our model* in the rest of the thesis.

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[1] Reuters Glossary, 1989

B. *Purpose*

The purpose of this thesis is to estimate a bankruptcy prediction model based on the variables in Altman's Z-score model and test it on listed Nordic firms.

C. *Delimitations*

We are aware that our thesis is subject to limitations such as the fact that our data contains companies originating from three different Nordic countries. The financial statements of our data might therefore be exposed to varying laws and regulations. Also, there is a risk of differences in the economical environments that could affect the probability of bankruptcy. However, since we use data from Nordic companies we assume that the markets are relatively comparable to each other. In chapter *III B* we discuss how we normalize the data in order to compare them among firms with different sizes and to eliminate the effect of local currencies.

Our sample contains of bankruptcy and non-bankruptcy firms between the years 2002-2012. Since part of our data runs over a time span where it was affected by the global economic crisis, this could have had an influence on the number of bankruptcies. To improve the model further it would therefore be desirable to remove time fluctuating factors. This would be possible by estimating a regression with data from a very short timespan. Unfortunately, the low frequency of bankruptcies on the Nordic stock exchanges makes the data insufficient with a shorter timespan.

The review of the Z-score model is accomplished by calculating our own parameters in a logit binary regression. We choose the same variables as Altman but instead with data gathered from Nordic companies. Following Altman we estimate the model using data from one up to five years before the bankruptcies occurred.

Finally, due to lack of time we restricted our research on Altman's and Ohlson's models and therefore ignored other possible methods

D. *Outline of the Thesis*

The thesis is outlined as follows: previous research is discussed in chapter II. (the methodology including the models used in this thesis is presented in chapter III), chapter IV introduces the model validation tests, chapter V lays out the analysis of our data and finally, in chapter VI, the conclusion is presented.

II. Previous Research

Studies of bankruptcy predictions vary amongst each other and have diverged procedures. Altman's model is based on a multiple discrete analysis that have been widely used but also criticised. Schumway for instance, argues that hazard models are more appropriate for default prediction. His conclusions arise from the static model's inability to calculate on explanatory variables varying with time, the incapability of accounting for firm's period at risk and the hazard models advantage when calculating with large quantities of data. Schumway resembles the hazard model with a binary choice model with the capability to account for all available years of data for each firm. (Schumway (2001))

Moreover, James Ohlson attempted to estimate a logit binary forecasting model with related variable selection as in Altman's Z-score model. Ohlson added explanatory variables and increased the data to improve the forecasting estimates. His selection of data is, just like Altman's, only listed manufacturing firms that had been traded over the counter (OTC), from 1970-1976. Ohlson seemed to choose firms more restrictive than Altman and succeeded to collect 105 bankruptcy firms and 2058 non-bankruptcy firms, Ohlson for example got rid of the matchmaking part of bankrupt and surviving firms, which he argued was arbitrary and therefore inaccurate for precise estimates. The matchmaking part is necessary in the multiple discriminate analysis (MDA), which Altman uses, but not in the static logit model that Ohlson uses. Ohlsons objective critics are a major reason for our choice of static logit as a model for estimation. (Ohlson (1980))

Nevertheless, Altman's model has since 1968 improved and he has among other enhancements developed a copyrighted model called the ZETA model (Altman, Haldeman, and Narayanan (1977)). The credit score models are despite the critics widely used (Mester (1997)). By calculating more accurate parameters on new data and practicing them on specific markets the result can be of great interest.

III. Methodology

This section describes the substratum for our model, Altman´s Z-score model. It follows with a presentation of the logit binary model. Then we summarize this section by giving an exhibition of the maximum likelihood estimation.

A. *Altman's Z-score Model*

Edward I. Altman presented a paper in 1968, where he identifies a discriminant function (equation 1). In the model information from a company's entire profile is evaluated and an analysis whether a company is close to default or not is possible.

Altman's Z-value is derived through a multiple discrete analysis (MDA). He collected his data from 33 bankrupt firms within the manufacturing area, which he matched with 33 handpicked healthy firms in the same size and operating sector. The variables Altman chose to derive his parameters from were selected to describe five standard ratio categories: liquidity, profitability, leverage, solvency, and activity ratios. Altman first compiled 22 variables describing the upper standard ratio categories. He then reduced his selection to five by the criterion: popularity in literature and potential relevancy to the study.

Because of the arrangement in Altman's Z-score model (equation (1)) the variables X_1-X_4 must be calculated as absolute percentages and only variable X_5 is shown as a percentage value. (Altman, Haldeman, and Narayanan (1977))

He then evaluated statistical significance and correlation among the variables and chose, despite the fact that variable X_5 was not statistically significant, the model as:

$$Z = 0{,}012X_1 + 0{,}014X_2 + 0{,}033X_3 + 0{,}006X_4 + 0{,}999X_5 \tag{1}$$

where,

X_1 = *Working capital/Total assets*

X_2 = *Retained earnings/Total assets*

X_3 = *Earnings before interest and taxes/Total assets*

X_4 = *Market value equity/Book value of total liabilities*

X_5 = *Sales/Total assets, and*

Z = *Overall index.*

B. *Variables in Altman's Z-score Model*

Below is an explanation of the five variables that Altman use in the Z-score model. By using quotients that provides percentages, Altman normalized his variables and they became comparable even though the data was collected from firms with different sizes.

X_1*: Working capital/Total assets*. This is the financial ratio that Altman founded to be the most valuable variable to predict bankruptcies with. The ratio describes the relation between *working capital* and *total assets*. *Working capital* itself is a measurement that describes the assets in a company that is meant to be put into practice often within less than three years and is calculated as *current assets* minus *current liabilities*. According to Altman, a typical reaction for a company going through an economically difficult time with constant operating losses is decreasing the *current assets*. (Al-Rawi, Kiani, and Vedd (2008))

X_2*: Retained earnings/Total assets. Retained earnings* is the accumulation of all profits retained since the company was founded (Businessdictionary (2012)). The measure is meant to give a description of a firm's age, where a young firm is supposed to have a low quotient and vice versa. This gives an estimate of how companies face risk of default in reality since older firms more rarely are declared bankrupt than younger firms.

X_3*: Earnings before interest and tax (EBIT)/Total assets.* Earnings minus expenses from income tax and interests states how lucrative a firm is. Earning power constitutes a fundamental incentive to operate a firm and the ratio is therefore interesting to study as an explainable variable to bankruptcy.

X_4*: Market value of equity/Book value of total debt.* The *market value* of a firm's equity is *the total market value* of all equity and *the book value of total debt* that includes all accounted debt. If the market value of equity is below the total debt the firm becomes insolvent and eventually bankrupt. Moreover this financial ratio contributes with an important market valuation aspect to the prediction model. (Lennox (1999))

X_5*: Sales/Total assets.* The sales of a firm manifest the manufacturing capability of companies' assets. In Altman's model this financial ratio did not deliver any statistical significance but he still found it to be useful to default prediction because of the relationship to other variables in the model. (Altman (1968))

With the estimated parameters from the variables above, Altman examined his model by testing whether the model succeeded in its forecasting. He then divided the Z-values in three categories based on the range of Z-values that fails to forecast the actual performance of the business.

The most probable Z-values of making false predictions is in Altman's model those between 1.81 and 2.99. Hence, the Z-values between 1.81 and 2.99 are interpreted as a zone of ignorance and are not completely trustworthy. The values larger than 2.99 avoid bankruptcy and the values lower than 1.81 becomes bankrupt without any misclassification. (Altman (1968))

C. *The Logit Binary Model*

The Y-variable that we try to explain is binary, also called dummy, meaning that the dummy variable can only appear as 1 or as 0. A business default may be explained in econometrics through a binary choice between bankruptcy and not bankruptcy (equation (2)). (Verbeek (2012))

$$Y_i = \begin{cases} 1 \; if \; bankruptcy \\ 0 \; if \; not \end{cases} \tag{2}$$

In our thesis we use the binary model where Y_i describes whether the given company have gone bankrupt, 1, or not, 0. In a binary model it is not possible to preform a normal regression analysis as the Y-value only have two different choices. Therefore, equation (3) is impossible to use since the continuous variable Z_i, which can take any value, the coefficients β_1, β_2 and the residual U_i must equal exactly Y_i (1 or 0). It is somewhat unrealistic to imagine the residual to predict the values of $\beta_1, \beta_2, and\ Z_i$ in order for the equation to match Y_i.

$$Y_i = \beta_1 + \beta_2 Z_i + U_i \tag{3}$$

Instead, we assume Z_i affect the probability that Y_i equals 1 or 0, higher values of Z_i generate higher probability of default and the reverse for none default. Therefore it is necessary to transform the Z-value into a probability. There are numerous ways of doing so, in this thesis we will use the logit model (equation (4)) that always give values $0 < (\Pr(Y = 1)) < 1$ (Dougherty (2011)). This transformation creates a distribution drawn as a S-shaped curve (Figure (1)). According to Davidson and MacKinnon (1982) when applying the logit model it is necessary to include at least 50 observations since data containing less rarely have enough explanatory variables to estimate a correct regression. (Davidsson and MacKinnon (1982))

$$Pr(Y=1) = \frac{1}{(1+e^{-z})}, Pr(Y=0) = \frac{e^{-z}}{(1+e^{-z})} \quad (4)$$

Z_i shapes as a linear regression (equation (5)) with help of dependent explanatory variables X_i, which we actually observe. A cumulative Z_i always increase the probability of default, if X_i affects liquidation positively when it increases then $\beta > 0$, but if default is less likely when X_i increases then $\beta < 0$.

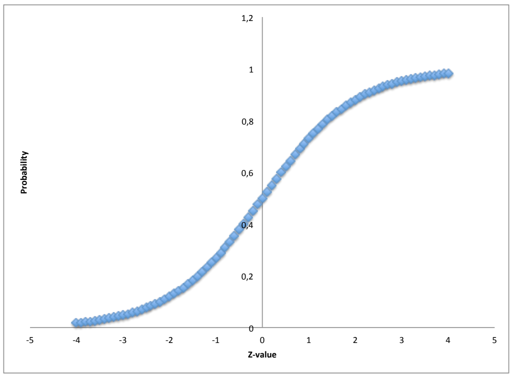

Figure 1. Transformed Z-values into logit probabilities

$$Z_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_K X_{Ki} \quad (5)$$

In equation (5) there are no residuals, instead the unsystematic part of the equation appears when the probability calculation (equation (4)) is drawn against a randomized value between 1 and 0. The randomized value mirrors the estimated probability of default. With this procedure the per cent ratio of the efficiency in our prediction will be shown. We are of course aware of the possibility that a company with high probability of default still can manage to survive in the model. As the random term changes from time to time the interpretation is only to see the effectiveness of the model.

Since the actual beta values are unknown in equation (5), estimation is necessary (equation (6)). The fitted model (equation (6)) is estimated with the maximum likelihood estimator given the explanatory variables X_i.

$$\hat{Z}_i = b_1 + b_2 X_{2i} + b_3 X_{3i} + \cdots + b_K X_{Ki} \quad (6)$$

The probability for Y_i to equal 1 may be written as (equation (7)) which shows exactly how the X_i variables affect the probability for a company to go insolvent. The marginal effect of the explanatory variables is a very practical value, which tell us what will happen with the probability when X_i changes marginally. Derivation of our parameters in the Z-line is interpreted as the marginal effect on the probability of a business default (equation (8)). (Verbeek (2012)) Important to remember is that the estimated betas cannot be interpreted except if they are derived. But, the sign of beta equals the sign of the derived value. When using the logit model it is possible to estimate the marginal effect using *b* and the equation for Z_i, (equation (6)).

$$Pr(Y=1) = \frac{1}{1+e^{-\beta_1+\beta_2 X_{2i}+\beta_3 X_{3i}+\cdots+\beta_K X_{Ki}}} \tag{7}$$

$$\frac{\partial(\Pr Y=1)}{\partial X_i} = \left(\frac{\partial \Pr Y=1}{\partial Z}\right)\left(\frac{\partial Z}{\partial Xi}\right) = \left(\frac{e^{-z}}{(1+e^{-z})^2}\right) b_i \tag{8}$$

When estimating the logit binary model with the maximum likelihood estimator the beta values are not as precise as when doing it on a normal regression. The maximum likelihood estimator demands more quantity of data in order to perform precise estimates.

D. *Maximum Likelihood Model*

This is a general example of how the maximum likelihood estimator estimates (equation (9)).

$$X_i \sim \mathrm{IIDN}(\mu, \sigma) \tag{9}$$

X_i is an independent normally distributed variable with unfamiliar μ and standard deviation σ, whose probability distribution looks like (equation (10)), which provides the classic bell-curve. Because of X_i is an independent variable the product of all the observed X_i creates a probability function (equation (11)) that is dependent of μ. (Dougherty (2011))

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{X-\mu}{\sigma^2})^2} \tag{10}$$

$$f(X_1 + \cdots + X_n) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{X_i-\mu}{\sigma^2})^2} \tag{11}$$

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{X_i-\mu}{\sigma^2})^2} \tag{12}$$

The Maximum Likelihood principle estimates μ with the value that will maximize L (equation (12)), where L is a function of μ instead of X_i in equation (equation (11)). There is of interest to find the specific μ that makes the probability to observe what we actually observe as high as possible, which is an optimization problem. Note, to make the derivation easier we use log-likelihood (equation (13)). Through derivation of (equation (13)) we can prove that the maximum likelihood estimator and μ equals the true mean of X_i.

$$\ell = log\ L = \sum_{i=1}^{n} log \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma^2})^2} \tag{13}$$

E. Maximum Likelihood in the Logit Model

As we know, the binary model is composed with the Y_i variables that equal 1 or 0 and the parameter Z_i, which reflects the probability of what Y_i will result in. The probability is derived through a transformation of the linear regression Z_i (equation (6)). The probability distribution of Y_i is therefore given by the multiplication of $Pr(Y = 1)$ times $Pr(Y = 0)$ (14).

$$\Pr(Y = y_i) = \left(\frac{1}{1+e^{-Z}}\right)^{Yi} \left(\frac{e^{-z}}{1+e^{-Z}}\right)^{(1-Yi)} \tag{14}$$

The probability distribution in our likelihood model of Y_i is therefore as equation (15).

$$L(\beta_i) = \left(\frac{1}{1+e^{-z}}\right)^{\sum Y_i} \left(\frac{e^{-z}}{1+e^{-z}}\right)^{n-\sum Y_i} \tag{15}$$

Z_i is an artificial linear regression of the explanatory variables X_i and as explained above derived by the parameters β_i. By choosing the combination of β_i that maximizes the likelihood function will give us the estimates (Dougherty (2011)). Theoretically, in order to maximize this function a derivation is needed. Since the likelihood function in practise is very complicated to derive the log-likelihood function (equation (16)) is used instead. (Pampel (2000))

$$\ln\big(L(\beta)\big) = \sum_{i=1}^{n} \left\{\left[Y_i \ln\left(\frac{1}{1+e^{-z}}\right)\right] + \left[(1 - Y_i)\ln\left(\frac{e^{-z}}{1+e^{-z}}\right)\right]\right\} \tag{16}$$

When applying the binary model, as we do in our thesis, EViews uses (equation (15)) and numerically test which β_i that maximizes the function. The β_i that maximizes the likelihood are the estimates. This estimation technique demands that the regression fulfils the assumptions that the distribution in (equation (15)) is symmetric around its μ. (Verbeek (2012))

IV. Model Validation Tests

In this section we explain the criterion for our variable selection. It follows of a presentation of the model validation tests we use, such as type I and type II error, chi-squared test, and hypothesis test.

A. *Criterion for Variable Selection*

When choosing our variables we have gone through a procedure that is mainly based upon James Olsson's report from 1980. Olsson handpicked and included companies that where classified as industrials, he excluded utilities, transportation-, and financial service companies such as banks, insurance firms, brokerages, etc. (Ohlson (1980)). The reason for excluding such companies is because they are structurally different and have to face eventual bankruptcies with other conditions**.** Altman, in difference to Olsson and us, made a careful selection of non-bankrupt firms where he identified two similar businesses where one was bankrupt and the other one not. (Altman 1968))

The data is collected mainly from Datastream and complemented with data from Retriever. Our total sample of data consist of 395 firms, 56 of them are bankrupt and 339 of them are healthy firms. The bankrupt group consist of firms that filed for bankruptcy in Norway, Sweden, and Denmark between 31/12/2002 and 05/09/2012. We located which firms that went bankrupt by doing back-up research on all companies that had been delisted from the largest stock exchanges in the chosen countries. The information in Sweden was available from Skatteverket. In Norway and Denmark the information was acquired from historical information letters issued by the Nasdaq OMX and Oslo Bors. The firms we were unable to find or with insufficient data was excluded.

B. *Type I and Type II Error*

There are numerous ways to test a model's accuracy. Testing the model for type I and type II errors is one of them. A model can be inaccurate through two different ways, these mistakes are known as type I or type II errors. In our thesis a type I error denotes when the model incorrectly predicts a bankrupt company to survive, whereas a type II error represents when the model predicts a surviving company to go bankrupt. (Verbeek (2012))

		Model	
		Bankrupt	Non-Bankrupt
	Bankrupt	Correct prediction	Type I error
Actual	Non-Bankrupt	Type II error	Correct prediction

Table 1. Type I and type II error

Either of the two different errors are connected with a mistake in the model's accuracy. When decreasing the probability of a model's misspecification, it leads to a low chance of a type I error but instead higher probability for a type II error. That is the reason why it is of great importance to be aware such errors and balance the error probability.

To elucidate, we will test how accurately our model can predict business defaults and non-defaults. This is done by comparing the logit transformed Z-value (equation (7)), i.e. the probability of default, with a randomized value (P*), which has a normal distribution with restrictions to zero and one ($0 \leq X \leq 1$) (equation (17)). If the model's output coincides with the reality it counts as a correct prediction, otherwise it is considered as a type I or type II error. The comparison to the randomized value is necessary since there is an absence of a strict rule when bankruptcies appear.

$$Predict\ Y_i = 1\ if\ Z_i > P^* \tag{17}$$

If the diagonals for "correct predication" is summarized and divided by the total number of observations included in the analysis, the output will, in per cent, tell how successful the model categorize the companies. (Altman (1968))

C. *Chi-Squared Test*

In order to test whether the model is significant we create a similar table as the type I type II error where we generate expected values, E_{ij}. These are then compared to the model and the reality, O_{ij}, in a chi-squared test (equation (18)).

$$X^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}} \tag{18}$$

The output from the chi-squared test, the p-value, is compared to the benchmark 0,05, if the number is less than 0,05 the model is significant and we have enough evidence to say that our model can accurately predict bankruptcy and non-bankruptcy. (Chernoff and Lehmann (1954))

D. *Multicollinearity*

Marno Verbeek (2012) described multicollinearity as a problem when two or more variables in a multivariate regression suffer from high correlation among each other with the consequence of untrustworthy estimates.

When a model is estimated consisting of more than one explanatory variable there is always a risk of a systematic dependency among the variables, resulting in multicollinearity. It frequently occurs among time series data that covers observations over a certain period of time.

A regression that suffers multicollinearity follows the problem of not knowing what parameter that affected the change in the regression. By analysing the dependent parameters correlation it is possible to detect multicollinearity. (Farrar and Glauber (1967))

E. *Heteroskedasticity*

If the disturbance term has the same variance for all observations, it suffers homoskedasticity. If not, we say the regression suffers heteroskedasticity, which means differing dispersion. In a linear OLS regression heteroskedasticity may be seen as a tendency in the assumed stochastic residual variable u_i to spread as the value of X_i increases. This results in an incorrect estimation of the standard errors of the regression's coefficient that makes inference unfeasible. (Dougherty (2011))

In a logistic model the assumptions in OLS of normally distributed residuals with a constant variance is incorrect. The residuals are instead dependant on the probability for an outcome and are neither normally distributed nor homoscedastic. With heteroskedasticity in the residuals as a required circumstance, the analysis of the residuals becomes complex. In a logistic model heteroskedasticity is referred to the deviance residuals, which measures the contribution of deviance of a particular outcome. That gives a measure of the overall lack of fit in the model. These deviance residuals are used to preform hypothesis tests that correspond to the ones in OLS regressions (Cohen, West, and Aiken (2003)). Testing for heteroskedasticity in logistic models appears to be an inquiry that is performed when there is reason to suspect heteroskedasticity and otherwise the test seems to be disregarded. (Hole (2006))

The literature that describes test for heteroskedasticity in binary logistic models is therefore limited and the available information is on a higher econometric level than we are able to master (Davidsson and MacKinnon (1982)). As we do not have any reason to believe that our variables suffers from heteroskedasticity. We can therefore assume that the data is homoskedastic and tests regarding heteroskedasticity are not made.

F. *Hypothesis Testing*

The point of preforming a hypothesis test is to examine if a certain claim concerning the parameters β_i is correct or not. The null hypothesis meaning a parameter equal to zero is said to be true until the test data shows otherwise. The alternative hypothesis claims the null hypothesis not to be true with regards to the small probability for the null hypothesis to be true. (Westerlund (2005)) There are no specific rules to apply when deciding to reject or stay with the given argument, which sometimes leads to mistakes. These mistakes are known as type I or type II errors (table (1)). A type I error denotes when the null hypothesis is true but being rejected by the test, whereas a type II error represents when accepting the null hypothesis even though the alternative hypothesis is true. (Verbeek (2012))

V. Analysis

This section analyses and presents our data. It follows of a discussion how accurate our model can predict bankruptcy. We begin by exanimating the data through varying hypothesis testing. Then follows a presentation of our Z-score model together with an analysis of the estimated parameters. Finally, we analyse and compare the marginal effects from the estimated variables and a comparison to Altman's Z-score model is made.

A. *Data Analysis*

In order to regress our data a selection process for each regression was made. Altman estimated data only from manufacturing firms within a certain asset size in his model. This was done because of structural discrepancy between sectors and also firms with different sizes. Our data selection process is less restrictive then Altman's as an outcome of our different estimation techniques. Altman had to match his bankrupt companies with a corresponding healthy firm, which is not needed in a logit binary estimation. Instead we have studied how our distribution of each variable is shaped and erased the extreme values called black swans. This resulted in five diverse samples for each year prior to bankruptcy.

Our final sample contains of 37 unique default firms on Nordic stock exchanges that filed for bankruptcy between 31/12/2002 and 05/09/2012. In total, 114 unique firms with data in each company's home currency were used in the analysis. Since we normalize the values, as mentioned in chapter *III B*, there will be no difference or skewed interpretation from the values coming from different counties. Each bankrupt firm has one or more corresponding healthy firms with similar economic condition as the bankrupt firm, which help the model to identify factors that lead to default.

i. *Time Fluctuating Factors in our Sample*

The data we have gathered is exposed to structural differences that change over time. Obviously macroeconomic factors fluctuate in a timespan of ten years and different variables could be more or less affected by those circumstances. For example, in financial distress as we experienced during the financial crisis of 2008 and 2009, it is more likely that firms' pay higher interest rates than previous years. It is also more likely that firm's in need of credits become disallowed during similar financial periods. The influence of these macroeconomic factors is therefore important to have knowledge about when assembling a representative sample with the purpose of forecasting bankruptcies. Despite this, we have not put any weight in assembling our sample from a minimized period of time. Instead, we established our data by maximizing the quantities and erasing extremely deviating variables from the sample.

The data used in all our estimations is put together below to demonstrate our samples allocation in time.

Year for data observations	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Total
Number of failing companies	0	0	0	1	0	0	4	10	8	5	9	37
Percentage of total number of failing companies				2,70%			10,81%	27,03%	21,62%	13,51%	24,32%	100,00%

Table 2. Bankruptcy frequency

As table (2) shows our sample is, despite our criteria selection, not distributed widespread over the years. This is mostly a consequence of our problem of finding data for bankruptcies early in our timespan. But, this previously considered problem makes our exposure to fluctuations in macroeconomic slightly less comprehensive. We therefore constrain our model to disregard this factor.

ii. Analysis of the Country Selection

Each year the World Bank put together a index based on empirical data measuring the *"ease of doing business"* in different countries. The index is meant to describe how healthy the business environment is when operating and starting a firm. This data is unfortunately based on larger companies than the ones used in our model. Still, the data presents a description of how equivalent the business climate is in different countries. As we gathered data from Sweden, Norway, and Denmark the ranking between these countries are interesting to analyse. In 2012, Sweden was ranked thirteenth, Denmark fifth and Norway sixth on the assembled rating of all analysed countries. This is pleasant for us, as it strengthens our primary assumption that these three countries are plausible in the explanatory power of a bankruptcy in a regression.

Differences among the countries are nevertheless inevitable. In the index one of the variables represent the ability of resolving insolvency where Norway is ranked as number two and Sweden ranked eighteenth. Yet, due to limitations in our essay, we assume that the parameters have an equally important contribution to the forecasting of bankruptcy regardless of country in our model.

It would have been interesting for us to analyse how the index corresponds to the mean Z-score of each country in our model. However, since our sample from both Norway and Denmark is significant smaller than Sweden analysis is not possible. (World Bank and IFC (2012))

iii. *Logit Transformation*

As we transform all Z-values from each company into probability parameters using the logit model, our data through one to five years ahead of failure shapes as follows in figure (2). In theory this curve is supposed to look like figure (1), which gives the best interpretation and shows how the dataset cover both bankruptcies and non-bankruptcies. The transformed Z-values also tell us how well the model explains failures and survivors. A misshaped S-curve indicates a defective model.

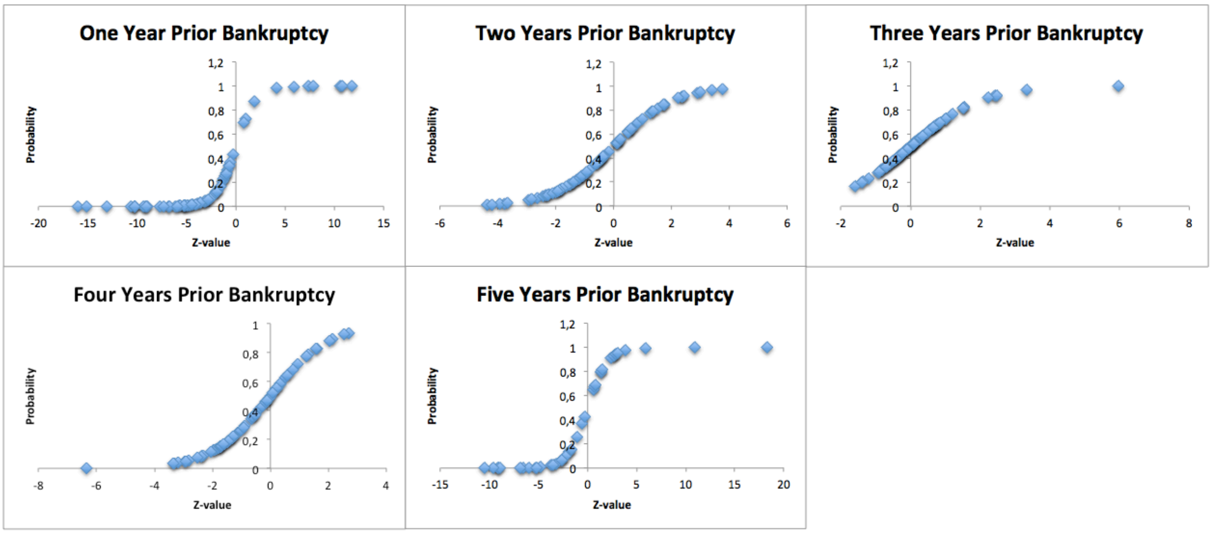

Figure 2. Our transformed Z-values into logit probabilities

It is well defined in the graphs in figure (2) that the probabilities lay within the restriction, $0 < (\Pr(Y = 1)) < 1$. The most important understanding of the S-curve is that the larger value of the risk variable Z_i the higher is the probability of default, which is exactly how the plotted data in figure (2) works. The data testing values two years prior to bankruptcy seems to have an S-curve that looks like the optimal one (figure (1)).

The intuitional interpretation of the diagram is as follows: the probability parameter is represented on the Y-axis and the Z-value on the X-axis. When a company have a Z_i value that represents 0,5 on the Y-axis, the probability of default is fifty-fifty and the larger the Z_i value gets the higher the probability of default will be and vice verse. Again, the S-curve with data two years prior to the event of bankruptcy have the best representation of both default and non-default companies. We therefore suspect that our model can predict and categorize bankruptcies and surviving companies best two years prior to the actual failure. But further tests on the data are necessary.

B. *Test for Misspecification*

This sub-section tests the properties of our model.

i. Test of the Model's Accuracy

First of all, we tested if our model could properly explain business defaults through a chi-squared test. In order to decide how many years back the model could explain and predict company defaults we ran the data through *five tests*, where each test represented one to five years prior failure.

The first test, one year prior failure, is represented by 66 companies where 14 of them went bankrupt and 52 survived. Since firms that are on the edge of bankruptcy normally show signs of failure, a high score of accuracy is expected.

		Model		
		Bankrupt	Non-Bankrupt	TOT
	Bankrupt	10	4	14
Actual	Non-Bankrupt	5	47	52
	TOT	15	51	66

	Number Correct	Per cent Correct	Per cent Error	n
Type I	10	0,714	0,286	14
Type II	47	0,904	0,096	52
Total	57	0,864	0,136	66

Chi-squared, p 9.643E-07

Table 3. One year prior default – type I, type II errors and chi-squared

As shown in the matrix above (table (3)) the model is accurate when classifying the sample one year prior failure. A type II error is shown only as 9,6 per cent and a type I error as 28,6 per cent. The chi-squared test on the sample presents a p-value, 0,0000009643, which confirms the model with significance. The output from the results is promising, but not enough to be convinced that our model can classify failures and surviving firms**.** Also, the sample in this test consists of 52 surviving firms and only 14 bankruptcies. We must therefore do further research.

The observations two years prior bankruptcy is our largest sample containing 82 firms where we have 30 bankruptcies. The result is not as accurate as when modelling data one year prior, but 70 per cent correct observations for bankruptcies profits the model and is acknowledging evidence. The model does not only with significance classify failures correctly but also surviving firms with 80,8 per cent accuracy.

		Model		
		Bankrupt	Non-Bankrupt	TOT
	Bankrupt	21	9	30
Actual	Non-Bankrupt	10	42	52
	TOT	31	51	82

	Number Correct	Per cent Correct	Per cent Error	n
Type I	21	0,700	0,300	30
Type II	42	0,808	0,192	52
Total	63	0,768	0,232	82

Chi-squared, p 4.954E-06

Table 4. Two years prior default – type I, type II errors and chi-squared

As seen in table (4) the chi-squared test of the model gives an output p-value of, 0,000004954, meaning it is significant on a 99 per cent level. Hence, the test is accurate two years prior to the event of failure. Data tested two years after the event of failure gives a 30 per cent chance of a type I error and 19,2 per cent for a type II error, which is impressively small and confirms our model. Even though the model is accurate two years prior bankruptcy it is necessary to apply additional tests.

Our third test includes a total of 80 companies where 27 are failures. Since the model have limitations we predict it not to be as accurate as it is two years prior. As shown in the matrix in table (5) a type II error is present 43,4 per cent and a type I error 29,6 per cent. Even though the chi-squared test's p-value is higher than before, the test is still significant with a level of 0,022**.**

		Model		
		Bankrupt	Non-Bankrupt	**TOT**
	Bankrupt	19	8	27
Actual	Non-Bankrupt	23	30	53
	TOT	42	38	80

	Number Correct	**Per cent Correct**	**Per cent Error**	**n**		
Type I	19	0,704	0,296	27		
Type II	30	0,566	0,434	53		
Total	49	0,613	0,388	80	**Chi-squared, p**	0,022

Table 5. Three years prior default – type I, type II errors and chi-squared

In conclusion with table (5) the model cannot accurately categorize the groups, and the interpretation becomes less clear.

When testing data four years prior bankruptcy, the test is not significant according to the chi-squared p-value. The sample contains 68 firms where 23 have gone bankrupt.

		Model		
		Bankrupt	Non-Bankrupt	**TOT**
	Bankrupt	8	15	23
Actual	Non-Bankrupt	15	30	45
	TOT	23	45	68

	Number Correct	**Per cent Correct**	**Per cent Error**	**n**		
Type I	8	0,348	0,652	23		
Type II	30	0,667	0,333	45		
Total	38	0,559	0,441	68	**Chi-squared, p**	0,905

Table 6. Four years prior default – type I, type II errors and chi-squared

The model has a type I error 65,2 per cent, which is more like a guessing game rather than a bankruptcy prediction model.

Since our sample five years prior to the event of bankruptcy only contains of 39 observations it does, according to Davidson and MacKinnon, not include enough information in order to draw conclusions from the regression. Interestingly, our model estimates with 71,4 per cent accuracy the bankrupt group but as mentioned, it is impossible to make any interpretation.

We generalize three, four, and five years prior bankruptcy as long-range predictive data, as Altman (1968) does in his article of the Z-score model. This is done since the model's effectiveness decreases as the years increase from the event of bankruptcy. As a result of tables (3-6) and the discussion above it is accurate and significant to forecast business failure two years prior the bankruptcy event. The following years substantially decrease the chance for our model to predict and correctly categorize failing or non-failing companies.

The type I error from table (6) gives a clear message that it is no longer possible to predict bankruptcies four years prior business failure. The total per cent of correct ratios in the type I type II error matrix is presented in table (7).

Five Year Predictitve Accuracy of total sample

Year Prior to Bankruptcy	n	Hits	Misses	Per cent Correct
1st	66	57	9	86,36%
2nd	82	63	19	76,83%
3rd	80	49	31	61,25%
4th	68	38	30	55,88%
5th	39	27	12	69,23%

Table 7. Five years predictive accuracy of total sample

Our table (7) shows that, the model works best one year prior failure with a total of 86,36 per cent predictive accuracy, which confirms the results of the chi-squared test. But as mentioned, companies normally have an abnormal change in their variables close to bankruptcy. It is therefore encouraging that our model classifies with 76,8 per cent probability two years prior bankruptcy, before most of the abnormal changes in the data occurs.

ii. Test of the Parameters

In order to test the significance in each of our coefficients we performed a t-test. We are testing the null hypothesis that the coefficient equals zero. We calculate a Z-statistic, which we reject under a significance level if the value of the statistic exceeds the table value. By rejecting the null hypothesis the tested coefficient have significance in our model. The test is executed on all our regressions from five years prior to bankruptcy until one year before with different confidence intervals. By investigating this on our regressions we can draw conclusions under which confidence interval our coefficients are significant. We can also see how the significance in each coefficient fluctuates compared to the year prior to bankruptcy and tell if there are any interesting differences.

Below you find our five hypothesis tests on the significance of the five coefficients included in our model each year prior to bankruptcy. Important to notice is that by looking at the p-values in table (8) it is possible to tell whether we can reject the null hypothesis or not. It is possible to use different approaches regarding the level of accuracy. If the p-value exceeds for example 0,05, it is more than a 5 per cent probability that the null hypothesis is true (Dougherty (2011)). The test is generated using EViews.

	Test of Parameters									
	1 Year Prior		**2 Year Prior**		**3 Year Prior**		**4 Year Prior**		**5 Year Prior**	
Variable	**Coefficient**	**Prob.**	**Coefficient**	**Prob.**	**Coefficient**	**Prob.**	**Coefficient**	**Prob.**	**Coefficient**	**Prob.**
X1	0,036	0,249	0,026	0,020	0,013	0,240	0,000	0,996	0,018	0,589
X2	0,043	0,111	0,022	0,165	-0,002	0,919	0,008	0,704	0,079	0,190
X3	-0,246	0,011	-0,119	0,000	-0,043	0,055	-0,072	0,083	-0,588	0,048
X4	-0,039	0,024	-0,011	0,008	-0,006	0,176	-0,011	0,003	-0,024	0,119
X5	-1,397	0,222	-0,350	0,461	0,394	0,235	0,336	0,437	5,759	0,078
Obs with Dep = 0	52		52		53		45		32	
Obs with Dep = 1	14		30		27		23		7	

Table 8. One to five years prior bankruptcy – test of parameters

With regards to the differences in the number of observations among the tests, the significances should be interpreted with caution. For example, the last five years prior to bankruptcy prediction do not have enough observations to draw any subjective conclusions regarding the significances.

In line with the conclusions from the chi-squared test, the strongest predicting power for bankruptcy is given one and two years prior the event of bankruptcy. These two models are thus the most interesting to evaluate. Altman chose to focus his analyse on the two years prior to bankruptcy as his one year prior to bankruptcy did not give unbiased estimates. Therefore we have also chosen to analyse our forecasting model two years prior to bankruptcy.

C. *Our Bankruptcy Prediction Model*

Henceforth in this thesis, we refer to the estimations based on the data from two years prior to bankruptcy, when we analyse the results. When we discuss Altman's model we also refer to his two years prior to bankruptcy model. Our two years prior to bankruptcy model is given by (formula (21)).

$$Z = 0{,}026424X_1 + 0{,}022123X_2 - 0{,}119066X_3 - 0{,}010699X_4 - 0{,}349868X_5 \quad (21)$$

where our model's, as well as Alman's, variables are as follows,

X_1 = *Working capital/Total assets*

X_2 = *Retained earnings/Total assets*

X_3 = *Earnings before interest and taxes/Total assets*

X_4 = *Market value equity/Book value of total liabilities*

X_5 = *Sales/Total assets, and*

Z = *Overall index.*

i. *Significance of* β_2

In our two years prior to bankruptcy model the parameter estimated on variable X_2 lack significance under the null hypothesis on 95 per cent confidence interval unlike parameters X_1, X_3 and X_4. Instead we can reject the null hypothesis under 80 per cent significance level.

The variable X_2, *Retained earnings/Total assets,* is described as a measurement of a firm's age where the ratio is supposed to increase as the firm matures. However, it is not certain that the variables in our sample actually fulfil its purpose. This is due to our data that primarily is derived from stock exchanges mainly containing firms with focus on growth and expansion. Table (9) tells which stock exchanges our data is derived from.

	Bankruptcies	Non-Bankrutpcies	Total
Sweden - First North, Aktietorget, NGM-Equity	13	11	24
Sweden - Small Cap	6	7	13
Norway	6	32	38
Denmark	5	2	7
Total	30	52	82

Table 9. Countries and trading markets for our data

On other Nordic stock exchanges that contain more mature firms, the bankruptcies occurred with a low frequency. This made it difficult to establish a quantitative sample. Therefore, Altman's data on X_2 seems to be more reliable as a company with focus on growth generally is younger and might differ structurally compared to well-established firms.

We investigated our sample and its component in order to analyse whether the variables fulfils their purpose in the model. We started to examine the values of *retained earnings* in order to see how they were correlated with bankruptcies.

The sample distribution of X_2 stretches from values between -58 and +51, with an equally large sample group around the mean of approximately +1. It is therefore of interest to investigate whether our bankruptcy data is allocated among low values of X_2 as it originally was meant. In the numerator of X_2, which is *retained earnings*, we can see a clear pattern where the sample representing healthy companies has 70 per cent of its population with a positive value. As *retained earnings* represents accumulated earnings over the years and is suppose to be higher if the company has lived long this seems to be correct. Focusing on the negative *retained earnings* values we found that 70 per cent of the bankrupt sample was represented here. This appears to bring good explanatory power.

When organizing the X_2 by value we find a small group from our bankrupt sample among the higher total values. This may explain a part of our lowered explanatory power compared to Altman. Even though, the total data seams to bring good explanatory power. The characteristics of *retained earnings* divided by *total assets* may differ a bit in our model in comparison to Altman's. The evidence is not strong enough to draw a conclusion that our sample, consisting mainly of growth companies, has some structural differences that lower the significance level in our coefficient compared to Altman. To investigate this further we examine the correlation matrix of our variables.

By studying the correlation matrix in (table (10)) we observe a correlation of approximately 0,62 per cent between X_2 and X_3, which indicates that there could exist multicollinearity between the parameters. Both of the variables contain components measuring *earnings* divided by *total assets* and can thereby explain the multicollinearity. The variable X_2 can be seen as a measure of the firm's total profitability over the years and X_3 as the profitability during a certain year. By studying formula (22) we observe that X_3's numerator *Earnings before interest and taxes* in fact is a component of X_2's numerator as well.

Two Years Prior Default	X1	X2	X3	X4	X5
X1	1				
X2	0,184	1			
X3	0,346	0,619	1		
X4	0,014	0,157	-0,110	1	
X5	0,111	0,206	0,048	0,118	1

Table 10. Correlation matrix

The correlation between the variables appears to be stronger for younger firms. Formula (22) is an outcome of how *retained earnings* is put together.

$$X_2 = \frac{Retained\ earnings}{Total\ assets} = \frac{\sum_{t=1}^{n} Earnings_t + \cdots + Earnings_n - \sum_{t=1}^{n}(Dividends_t)}{Total\ assets} \quad (22)$$

In young businesses it is more likely that the earnings from the latest year have a substantial weight in the total quotient. In those firms the correlation between X_2 and X_3 therefore strong. This may be an explanation to the lack of significance in β_2, as our sample mostly is based on younger firms. In our multivariate regression, the X_3 variable explains the profitability part for X_2. Even if we believe that we notice an explanatory pattern in X_2's data, with a strong correlation against X_3 the significance is hard to prove. In order to prove X_2's significance the sample must increase.

To investigate our models overall accuracy without $\beta_2$we performed a chi-square test on a new estimated regression without any estimated parameter on X_2. With this test it was possible to see how much X_2 influenced our Z-value. The answer was that our original regression predicted approximately five per cent better on the two years before bankruptcy forecast. This alone does not give much information since increasing the amount of variables adds explanatory power per definition. However, in the unlikely scenario of no correlation between the explanatory variable and the dependant variable there will be no explanatory effect of the added variable. The motive to keep X_2 in our model is because an explanatory trend in the variable is observed. Also, a likely reason for the low significance could due to of the correlation between X_2 and X_3.

ii. *Significance of* β_5

In contrast to the other estimated parameters in our model, $\beta_5's$ significance is much less extensive under the null hypothesis. This may appear as a failure, but we keep in mind that not even Altman succeeded to get significance on this parameter. Hence, the purpose of regressing X_5 in the model was not based on the variables significance. He explains the insignificance as an effect of the multivariate regression where the other variables erase the importance of X_5.

Altman (1968) explained this as *"Because of its unique relation to other variables in the model, the Sales/Total assets ratio ranks second in its contribution to the overall discriminating ability of the model"*.[2] Altman refers to an outcome of the high negative correlation he found between X_3 and X_5 in the bankrupt group.

When we compare our correlations with Altman's we did not find such a relationship. Altman got a (-0,78) correlation between X_3 and X_5 while we received a correlation of approximately (+0,05) (table (13)). Because of the low correlation this doesn't appear to be a preferable explanation to the lack of significance in X_5 that we've received in our model.

To figure out what could cause the insignificance we examined our sample of X_5. By sorting the different observations according to size we examine whether we can observe a pattern in the distribution. In line with our lack of significance in β_5, a pattern is hard to distinguish. This tells us that the *sale* rate in our data does not have any explanatory power. The explanation may be consequences of some troubled companies practicing massive sales in order to solve liquidity problems. These firms do therefore not break-even, examining only their sales we disregard that the products may be sold even though making losses. The reason to some of the high *sales* ratios among the bankrupt group might be found here. The uncorrelated pattern between the dependant variable and our X_5 variable could be a result of how some firm's solve their liquidity problems through sales and others may not have that as an option.

iii. *Marginal Effects*

The marginal effect is calculated through the partial derivative of $X_{i.}$ Because variables X_1 to X_4 in our model are written as whole percentages, in comparison to X_5, the marginal effects among X_1-X_4 and X_5 will differ. In order to make the marginal effect comparable we transform them into the same unit. A unit increase for variable X_1- X_4 is 100 times larger than an equally large unit change for variable X_5. Therefore, we multiply the margin effect on the X_1- X_4 variables with 100. Our marginal effects are as follows in table (11).

[2] Altman, 1968

Average Marginal Effects				
B1	**B2**	**B3**	**B4**	**B5**
0,689	0,577	-3,104	-0,279	-0,091

Table 11. Two years prior bankruptcy – average marginal effects

As we can see the largest effect on the bankruptcy prediction is given by X_3. The effect is negative and when the ratio of X_3, *Earnings before interest and taxes/Total asset*, increases the total risk of bankruptcy decreases. The multicollinearity that our model suffers from could affect the marginal effect of X_3, and should be interpreted with precaution.

The second highest effect is given by X_1, *Working capital/Total assets*, which has a positive marginal effect and gives the percentage of liquid assets in a firm. If the percentage of liquid assets in a company is high it implies that a company face a higher risk of bankruptcy. For example, a firm with only *current assets* face a higher risk for bankruptcy than a firm with low percentage of *current assets*.

Variable X_2, *Retained earnings/Total assets*, is rated as number three and is supposed to mirror the age of a firm. An old firm is supposed to have a high value and is presumed more unlikely to face default. The marginal effect in our model is positive which means that a higher value of the variable brings greater probability of bankruptcy. This is the opposite marginal effect than what could be expected as a higher value is supposed to lower the probability for bankruptcy. However, the interpretation of X_2 is not obvious neither a fact to the lack in significance nor because of the high correlation we observed between X_3 and X_2. Due to the lack of significance, the true parameter value can be found on a wide range around the estimated value. Therefore it is possible that the marginal effect also is incorrect.

Number four in the rating is X_4, *Market value of equity/Book value of total debt*, which has a negative marginal effect. The ratio shows *the market value of equity* in percentage of *the total debts* and it describes how much a company can drop in value before insolvency. The more space available for the assets to decline the better, which our marginal effect indicates.

The least contributory marginal effect is given by X_5, *Sales/ Total assets*. Even though the lack of significance in this parameter the marginal effect is pleasing. It is desirable for a company to have high sales and if the *sales* divided by *total assets ratio* is high, the probability for bankruptcy should decrease. As we observed in our sample, this was not the actual case and we were unable to find any pattern giving explanatory power to the regression. It is therefore pleasing that this variable has the lowest marginal effect in our model. Also, it is satisfying that the small margin effect of X_5 in our model is negative.

D. Comparison to Altman's marginal effects

In the table (12) you find the rating of the marginal effect in our model and Altman's.

Ration on the Relative Contribution of the Variables

Rating	Our Model	Altmans Model
1	X3	X3
2	X1	X5
3	X2	X4
4	X4	X2
5	X5	X1

Table 12. Two years prior failure – ratio of the relative contribution of the variables

In Altman's model the variables contribution to the bankruptcy prediction differs from ours. The highest marginal effect in his model is given by X_3, *Earnings before interests and taxes/Total assets*, which is the same as in our model.

The second highest marginal effect in Altman's model is given by X_5, *Sales/ Total assets.* Altman explains this as an effect of the negative correlation among X_5 and the other variables in the model. In our model, X_5 does not show high correlation among the other variables. Again, X_5 it is insignificant, but it is a satisfactory result that it has the lowest marginal effect in our model. X_1, *Working capital/Total assets* was the ratio that Altman referred to as the best indicator of default, which has the second largest effect on the estimates in our model.

The third highest impact on the prediction of bankruptcy in Altman's model is given by X_4 and in our model it is X_2. Since a parameter with no logical interpretation in the model has a large impact, it is a problem for us. The fourth highest marginal effect in Altman's model is X_2 and in our model it is X_4. The least marginal contribution according to Altman is X_1 and in our model X_5.

The two models are estimated from the same variables but have diverse marginal effects. This could indicate structural differences between the countries and the market that we have analysed. The highest contributor to bankruptcy in both models is X_3, which is a measure of how lucrative a firm is. Since earning money is a great source to why we start businesses and therefore a fundamental ratio despite country or market.

The X_1 variable has the second highest contribution in our model compared to Altman who ranks X_1 as the variable with the smallest impact. This could be interpreted, as it may be more necessary to keep your relative current liabilities low in order to avoid default when you are a Nordic firm than when you are an American. It can also be a consequence of how the bankruptcy rules have changed during the years as Altman's firms went bankrupt during the 1960s. Further differences are small among the rating of marginal effects, which make an analysis hard to perform.

i. Chi-Squared Test of Altman's Z-score Model

Finally, we tested Altman's original Z-score model (formula (1)) against ours (formula (21)) in order to be sure that our estimated beta values can categorize bankruptcies and surviving Nordic firms better than Altmans'. Since our earlier tests have showed that our model is best two years prior the event of bankruptcy, the input for X_i was therefore the same as when we made the test for our model. But, instead of using our estimated beta values Altman's beta values were used. The test was preformed through a chi-squared test, where we found out how accurate Altmans' classification is compared to ours.

		Bankrupt	Non-Bankrupt	TOT
	Bankrupt	17	13	30
Actual	Non-Bankrupt	38	14	52
	TOT	55	27	82

	Number Correct	**Per cent Correct**	**Per cent Error**	**n**
Type I	17	0,567	0,433	30
Type II	14	0,269	0,731	52
Total	31	0,378	0,622	82

Chi-squared, p	0,128

Table 13. Two years prior default with Altman's beta – type I, type II errors and chi-squared

When having the same input, X_i, two years prior default but with Altman's beta values the chi-squared p value in table (13) indicates that that the model is not significant and cannot be used. This is strengthen by the type II error percentage error, 73,1 per cent. In other words, our model predicts bankruptcies and categorizes non-bankruptcies on listed Nordic firms better than Altman's Z-score model does two years prior the event of default.

VI. Conclusion

With regards to the uncertain economical environment and the following effects such as bankruptcies and high credit risks, it has gained importance for a precise default measurement. We estimated our own bankruptcy prediction model based upon the variables in Altman's Z-score model and tested it on listed Nordic firms. We used a logit binary model that gave us a Z-score, which measured the probability of default and non-default.

Our sample contains of 114 unique firms where 37 of them went bankrupt between 31/12/2002 and 05/09/2012. We have used companies from Sweden, Norway, and Denmark with normalized data to clean the differences among local currencies. We studied how accurate our model could categorize these companies as either bankrupt or surviving.

We found that our model works best two years prior the event of bankruptcy and can categorize defaults and non-defaults, on NGM-equity, Aktietorget, First North, and Small Cap, with 76,8 per cent accuracy. When comparing our model to Altman's Z-score model we found that ours can categorize defaults and non-defaults better. With the favourable results of our model, it could be implemented in the loan application processes in order to make more confident and fair decisions regarding to accept loans or not.

Foremost, we have two key questions regarding our research data.

1) Did the amount of data delimitate our thesis and could our model become more accurate with an extension of the collected data?
2) Where the parameters we used the best in order to minimize the mistakes in our estimated bankruptcy prediction model?

We could probably have improved our estimation with a larger sample of bankrupt firms. The fact that our data was derived from mainly growth firms was probably the reason for lack of significance in β_2 as younger firms suffered from multicolinearity between X_2 and X_3. Financial ratios earlier than 2006 was inadequate and therefore our bankrupt sample are clustered from 2007-2012. An archive with more comprehensive data from older companies may therefore increase the sample and improve our model. On the other hand, a more widespread timespan of data may expose our model to structural changes caused by fluctuations in the macroeconomic environment that may harm our estimation. Since we perceived our model to work best two years prior to the event of default, an extension of the timespan would therefore not have gained any positive effect on our results. Also, as the years increase after two years prior bankruptcy, the number of correct hits in the model decreases. This is associated with our models increased incapability to differentiate between bankruptcies and surviving firms over longer time spans.

Furthermore, increasing our sample to Finnish companies may have improved the model without breaking our initial purpose of creating a bankruptcy prediction model for Nordic firms. Due to language and time limitations the data from Finnish firms was very hard to collect.

Another delimitation was our choice of variables. There are many interesting financial ratios that would have explanatory power to a bankruptcy. We chose Altman's variables as delimitation despite the fact that he developed the model 1968. We believe the general factors describing bankruptcy are gathered in the model and therefore we chose to delimitate our estimation to those variables.

A broader and larger dataset and an increased amount of default companies would probably improve our model and, as a result of that, improve the models accuracy. An extension of the number of variables used would also give a clearer picture of a company's economical situation and improve our model. We suggest this to be subject for further research.

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