Unlimited Prices: An Extreme Value Distribution Approach to Estimating Art Prices

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Abstract

I set out to construct a valuation model for paintings using a newly created sample consisting of paintings sold at Impressionist and Modern art auctions at Sotheby's between the latter half of 2003 until the end of 2006. I create a valuation model using the standard hedonic regression methods used by other researchers in the art market and describe a new way of viewing the dynamics of the art market, leading to an extreme value distribution approach to estimating the hedonic regression. The resulting models, using both the standard method and the extreme value method, are then compared to the performance of the Sotheby's own pre-sale estimates.

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Chapter 1

Introduction

I had the pleasure of attending the autumn evening sale of Post-War and Contemporary art at Christie's in New York. In the course of 73 works, a new world high for the category was set at 412 million USD. This beat the previous world high of 375 million USD, set 24 hours earlier at the Contemporary Art evening sale at Sotheby's. The world of art auctions at this level is quite a strange place, at one point a bidder tried to raise the bid with 50000 USD when the bid was at several million, less than the standard bid increment, only to be met with some muffled chuckles by the audience and a snide remark by the auctioneer before accepting the bid. During the morning auction at Christie's the following day, I saw one of Josef Albers' *Hommage to the Square* paintings sell for 1,900,000 USD only to see a similar one sell for 600,000 USD about an hour later, despite the latter having received a higher pre-sale estimate.

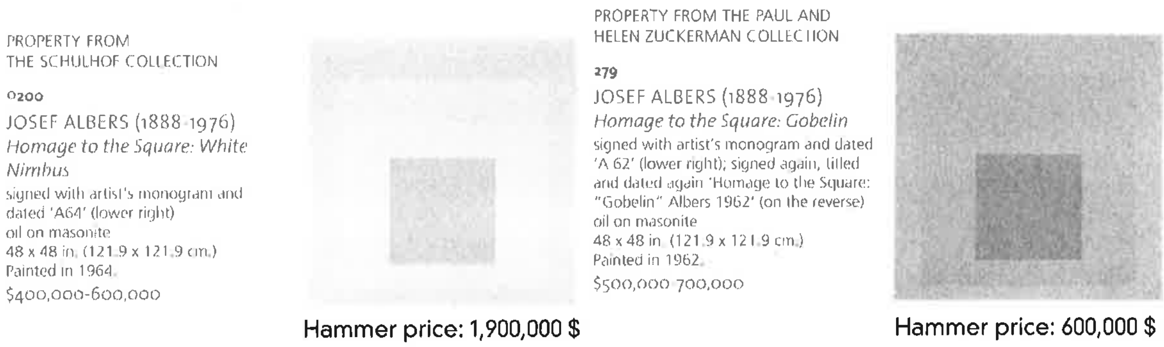

Figure 1.1: Which one do you prefer?

The world of art auctions is only one part of the art market, commonly called the *secondary market*, along with art galleries and art dealers directly representing artists, commonly called the *primary market*. These in turn are part of the larger art world where a lot, if not most, of what is being created by artists is without a quantifiable value nor even up for sale.

My goal is to investigate the possibility and viability of using a mathematical model to provide a second source of valuation of paintings to human experts. While a mathematical model will struggle to capture the subtleties of the art market, it will provide consistent valuations not affected by current mood and attention of a human expert, and can be used as a reality check.

My work is focused on the *secondary market* of auction houses, quite simply because that is the only place where somewhat reliable, publically available, data can be found. I will begin by briefly describing auctions and the art market and then give an overview of current research before moving on to my main purpose: creating a valuation model for paintings.

Chapter 2

Dynamics of the art market

2.1 About auctions

There are many types of auctions, the most famous being the *English auction*[1]. The *English auction* is an ascending price auction where the auctioneer starts the price low and raise it until there only is one bidder left. Other types of auctions include the *Dutch auction*, where the auctioneer starts at a very high price and lowers it until a participant stops the auction and claims the item at that price; the *second-price sealed-bid auction*, where participants place sealed bids and the highest bidder pays the price given by the second highest bidder; and the *first-price sealed-bid auction*, where participants place sealed bids and the highest bidder pays the price they themself has given. Milgrom and Weber (1982) provide a theoretical foundation for the different types of auctions. As the auctions at the major auction houses are *English auctions*, the focus will be on those.

You have seen versions of the *English auction* in a lot of places, from ebay to movies, it is the most action packed of the standard auction types with an auctioneer rapidly stating bid levels and gesturing to show who currently has the highest bid. The bid is increased using set *bid increments*, usually about 10 % higher than the previous bid, but in the end it is up to the auctioneer to determine which increments are acceptable. When the bidding stops, the item, usually called the *lot*, is said to be *knocked down* or *hammered down*, a point driven home by the auctioneer hitting the podium with a special hammer. The final bid at which the item is *knocked down* or *hammered down* is called the *hammer price*. This process is repeated for each lot in the sale.

Every lot that has been *hammered down* has not neccessarily been sold. If the highest bid has failed to reach the *reserve price*, a minimum price required to sell the work set by the seller, the lot will go unsold. The *reserve price* is set by the seller together with the auction house. It is never set above the lower pre-sale estimate given by the auction houses. Auction houses operating in New York City are legally required to state whether a lot has sold or not. This has increasingly become standard practice and many auction houses now state whether a lot has sold or been passed regardless of location, but there are still many auction houses who don't. The passed lots are said to have been *bought in*. *Buy in* rates vary between different kinds of auctions. Ashenfelter (1988) and Abowd and Ashenfelter (1988), in an unreleased article, found typical *buy in* rates for wine auctions to range between five and ten percent and *buy in* rates up to one third for Impressionist paintings with rates for most other auctions falling between these. Worth noting is that the *buy in* rate in my sample of Impressionist and Modern art is around 25 %.

In order to get the bidding started, the auctioneer is allowed to place *chandelier bids*, where the auctioneer may place bids on behalf of the seller up to the *reserve price*. It is customary at the large auction houses to start the bidding quite close to the *reserve price* but to leave a few *bid increments* margin. If a particular lot has received significant interest before the auction starts, usually in the form of *absentee bids*, the auctioneer may choose to start the bidding at a higher level.

The seller receives the *hammer price*, however this is not the price paid by the buyer: instead the buyer pays the *premium price*: the *hammer price* and a *buyer's premium* charged by the auction house for

[1]As Ashenfelter (1988) points out: the English auction is really Roman. The word auction comes from Latin's "auctio" meaning *to ascend*.

facilitating the sale. The *buyer's premium* is usually a percentage of the *hammer price* and, currently, tend to start at around 25% of the *hammer price* and decrease as the *hammer price* goes higher. The *premium price* is usually the one stated in the auction results. The auction houses also charge a *seller's commision* which is also a percentage of the *hammer price* paid by the seller to the auction house. The transaction fees make up a significant part of the total price paid.

Wilson (1977) showed, for sealed-bid auctions, that as the number of participants in the auction increase, the price becomes a better estimator of the true value of the lot. At the extreme end of auctions, there are only a few potential buyers, as such the realised prices show more noise and are a worse estimator of the true price. Since all bids are placed in the open at English auctions, the participants gain information about each other's valuation during the bidding process. Milgrom and Weber (1982) showed that this led to a higher expected price at auction as bidders adjust their own valuations upward during bidding. Personally, I believe there is a fair amount of competition between bidders, where two rivals will keep raising the bid, neither backing down, even when the bid exceed their own valuation, simply to not let the other bidder "win".

2.2 Dynamics of the Art Market[2]

Moving on to specifics of the art market. First of all: the art market, tradeable art, is a subset of the art world in which much art is of a nature that can't be sold in regular markets. The art market can then be divided into two parts: the *primary market* and the *secondary market*. We begin with the *primary market*, this is where most aspiring artists begin their career, where individual artists sell their work through local galleries, show it at local exhibitions and sell it directly to potential customers. Sadly, this is also where many careers end: as with most artistic endeavours in any creative field, the supply of would-be artists is larger than the demand for their work. Only when an artist's work has been bought in the *primary market* can they be considered an official artist.

Next we have the *secondary market* where an artist's works are resold to new owners and more prestigious galleries and museums. Trade in the *secondary market* is carried out mainly through art dealers and auction houses. Only very few artists successfully transition from the *primary market* to the *secondary market*. In the *secondary market*, works from deceased artists are traded along works from living artists meaning that an aspiring artist not only need to beat living rivals, but also deceased ones.

These divisions aren't absolute: it is possible for an artist to sell works on the *primary market* while at the same time being traded on the *secondary market*. The characteristics of *primary* and *secondary markets* repeat at many hierarchical levels: the ground floor being the local markets that, in most cases, cater to a single smaller town. Moving up, the markets cover a larger and larger area. At the very top we have the international galleries representing world famous artists, prominent museums and the top auction houses located in world leading cities like New York, London and Shanghai.

The entire art market is characterised by extremely low fungibility: it is hard to substitute art from one artist with art from another. In most cases it is even hard to substitute one artwork from a given artist with another artwork from the same artist. Owning one artwork can be considered having a monopoly of a unique resource. Tastes in art also vary over time, both when it comes to particular artists and when it comes to medium, topic and colours used. Buelens and Ginsburgh (1993) find that the prices attained by specific artists vary over time. Even though some works of a particular artist are enjoying good fortunes doesn't mean that the artists entire œuvre is performing as well: as artist's expression change over time, so does the reception of the works.

Almost all research into the art market has looked at the *secondary market*, and the auction houses in particular, as that is the only data that is available somewhat publically and consistently. Most art galleries, even at the international level, are privately held and reluctant in releasing sales figures.

In his influential article, Baumol (1986) investigated the nature of art prices. He makes the case that there is no equilibrium price for art, as opposed to regular markets, arguing that common frameworks, applied to stock prices, will fail to produce satisfactory results. The lack of an equilibrium and intrinsic value mean that the price of any particular artwork float more or less freely.

[2] This section relies in part on Gérard-Varet (1995)

2.3 Art as an investment

As traditional markets have been quite sluggish in the wake of the financial crisis there is an increased interest in alternative investments. This include purchases of art for other reasons than the pure aesthetical appeal, as stated by Czujack (1997): a painting provides consumption services, in the form of aesthetic-prestige services and decorative services, as well as financial services to its owner. Most of the research into the financial side of art has been devoted to the development of art indices and investigations into the characteristics of these indices.

Ashenfelter and Graddy (2003) have compiled a good review of investigations into the financial performance of art up to that point. Their article contains the results from price indices created by Anderson (1974), Stein (1977), Baumol (1986), Frey and Pommerehne (1989), Buelens and Ginsburgh (1993), Pesando (1993), Goetzmann (1993, 1996), Barre, Docclo and Ginsburgh (1996), Pesando and Shum (1996), Czujack (1997), and Mei and Moses (2001). The estimated real rates of return vary from 8,3% for Czujack (1997) looking at Picasso paintings to 0,55% for Baumol (1986) looking at paintings in general. Most of the indices are created using either repeat sales regressions (RSR) or hedonic regressions (HR) which will both be described in section 3.1. Pesando and Shum (2007) had cautious advice: investigating the real return of prints during the period from 1979 to 2003, they found it to be marginally more than 1%, less than the real return of U.S. Treasury bills. Renneboog and Spaenjers (2010, 2011) investigated the real rate of return of the global and Russian art market and found them both to be 3,97%, the global index during the period from 1957 to 2007 and the russian index from 1967 to 2007. For the period 1997 to 2007 they found the global index to have a real rate of return 6,30% compared to 12,37% for the Russian index. While there is quite a large difference in returns based on the period being used, there is less disagreement in the fact that art returns tend to exhibit large variance.

There is no consensus among articles about the correlation between art returns and the stock and bond markets. Goetzmann (1993) found significant correlation between art returns and returns of the London Stock Exchange. Pesando and Shum (2007) found a portfolio of prints to have a positive correlation with stock returns. Renneboog and Spaenjers (2011) found a large correlation between lagged returns on a global stock portfolio and art returns. Goetzmann, Renneboog and Spaenjers (2010) found a positive correlation between the log-returns of historical British stock prices and the rate of return of art. In the same article they also found a rather strong correlation between income inequality and the rate of return of art. Campbell (2007) used available indices to investigate art returns compared to traditional investments and found low correlation to other asset classes.

One thing to remember when considering art as a purely financial investment: the indices aggregate a very large number of transactions, some in the millions, of unique artworks. As a lone investor, you will struggle to attain a balanced art portfolio that can be used to capture the general art market. There are art funds that attempt to remedy this, but the basic problem persist: each artwork is unique and indivisible, you can't buy shares of the Mona Lisa. Kraeussl and Wiehenkamp (2010) attempts to remedy this by creating a call option on an underlying art index, while it is hard to get liquidity for financial products like this, it shows promise as a way to hedge against movements in the art price.

2.4 Sources of bias

It is important to remember that the indices are created using subsamples of realised sales, selection bias in the data need to be considered. Samples based on sales in the *secondary market* are likely to be truncated both at the high and low end. Works that have become worthless since their first appearance on the *secondary market* will not reappear at auction, removing the worst performing works from the sample. Works that have increased in renown since being bought at auction often end up being sold through dealers directly to potentail customers, or get donated to museums, truncating the very high end. As Goetzmann (1993) points out: this means auction transactions may not reflect the stylistic risk, the risk that their investment will become worthless as the artist falls from fashion, to investors.

However, what I consider one of the largest sources of bias is the sale rate: the works that fail to reach their *reserve price* and are *bought in* are universally excluded. Ashenfelter and Graddy (2011) find that the price shocks, difference of between the bid and the estimate from the auction house, for unsold items is consistently negative. They also found the reserve price to be about 70% of the low estimate from the auction houses. Beggs and Graddy (2008) find that paintings that have come to auction and failed to sell return significantly less when they are sold in the future. Beggs and Graddy (2009) also looked at anchoring[3] at auctions and found that the current price is influenced by the previous price, meaning that buyers anchor their bids on either the previous price or on the pre-sale estimate.

This brings us to the questions: are the auction houses unbiased in their estimates? Mei and Moses (2005) found an upward bias in the price estimates for expensive paintings over a long period. They also investigate the effect price estimates from the auction houses have on returns by looking at sales before and after 1973, the year price estimates first appear in their sample, and find that the entrance of pre-sale estimates affected art prices. However, their data does not contain *bought-in* paintings. McAndrew, Smith and Thompson (2011) attempt to remedy this by estimating the distribution of reserve prices and incorporating this information into their data. They find that the price estimates from the auction houses, with consideration to the *bought in* works, are unbiased.

Looking at specific auctions, there are internal structures. Auction houses have a tendency to cluster similar artist and similar artworks during auctions. Beggs and Graddy (1997) found that the presale estimate declines with order in both Contemporary and Impressionst and Modern auctions, meaning that auction houses place their highest expected value lots toward the start of the auctions. They also found that the valuation decline through the auction might induce a decline in the price relative the presale estimate. Pesando and Shum (2007-2) find evidence of "irrational exuberance" in auctions specifically for Picasso where the special sale of a large private collection realised extraordinary prices, not seen before or after the auction, consistent with the general idea among art professionals that art from renowned private collections attain higher prices.

The auction houses themselves have quite a strict selection regimen where they won't accept works they believe will be too difficult to sell. As Barber and Odean (2008) show, buying behaviour is affected by attention. Works being sold at auctions generate attention for the artists making them more available for buyers. The question remains a chicken and egg one: do artists sell because they get accepted to auctions or do they get accepted to auctions because they sell? If you don't get accepted to an auction you won't be able to sell and thus won't get accepted to auctions.

It is also important to remember that the samples used are quite restrictive, often only Christie's and Sotheby's. The estimated rates of return to art investments are representative of investment into art being sold at the very top of the art market, not neccessarily the art market at large.

[3]For an interesting example, see Kahnemann and Tversky (1982)

Chapter 3

Methodology and data set

3.1 Methodology[1]

The two dominant methods used to create art indices are the *repeat sales regression* (RSR) and the *hedonic regression* (HR). Both are a way of dealing with the varying quality of unique underlying products. Both methods have found use mainly for computing real estate indices, a market that share many similarities with the art market. Triplett (2006) compiled a handbook to using hedonic methods for the computation of indices which include both methods with examples of application to information technology. Chanel, Gérard-Varet and Ginsburgh (1996) performed comparisons between the results of RSR and HR and argued that for the case of heterogeneous commodities with infrequent trading, art being the example they chose, it was preferable to base an index on a HR using all sales, not just resales. Ginsburgh, Mei and Moses (2005) showed that RSR may be seen as a special case of HR, none the less, I will give a quick overview and present some benefits and drawbacks of both methods

3.1.1 Repeat sales regression

RSR is based on the sale and resale of specific objects. The main assumption is that, since it is the same object, quality is constant. That way, any difference in price may be attributed to the evolution of the market. By combining multiple resales, an aggregate index is created. Whether or not the assumption of constant quality hold is up for debate, as mentioned earlier: tastes change over time, affecting the percieved quality of the artwork.

The basic RSR requires at least $2N$ sales of N objects over a time period $t = 0, 1, \ldots, T$. $\mathrm{P}_{i,t}$ is the price for painting i at time t. The most common form assumes that the returns can be represented by a term μ_t, considered the average return in period t, plus an error term:

$$r_{i,t} = \mu_t + \varepsilon_{i,t} \tag{3.1}$$

Then the logarithm of the relative return for object i held between its purchase date, b_i, and its sale date, s_i, is:

$$r_i = \ln(\mathrm{P}_{i,s}) - \ln(\mathrm{P}_{i,b}) = \sum_{t=b_i+1}^{s_i} r_{i,t} = \sum_{t=b_i+1}^{s_i} \mu_t + \sum_{t=b_i+1}^{s_i} \varepsilon_{i,t} \tag{3.2}$$

Let $\mathbf{r}$ represent the vector of the logarithm of N repeated sales observations. C is a $N \times T$ matrix of dummy variables to indicate the holding period of each work, taking the value -1 at the purchase date, 1 at the sales date and 0 all other dates. The average return each period can then be estimated using OLS as:

$$\hat{\mu} = (\mathbf{C}'\mathbf{C})^{-1}\,\mathbf{Cr} \tag{3.3}$$

[1]This section relies in part on Ginsburgh, Mei and Moses (2005)

Improvements on this method were proposed by Goetzmann (1993) where he showed that a WLS regression on the form:

$$\hat{\mu} = \left(\mathbf{C}'\boldsymbol{\Omega}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}\boldsymbol{\Omega}^{-1}\mathbf{r} \tag{3.4}$$

Provide the maximum likelihood estimate of $\hat{\mu}$. Under the assumption that $\varepsilon_{i,t}$ are independent in time, the weights in $\boldsymbol{\Omega}$ are the times between sales.

The main drawback of RSR lies in the fact that only a few of the sales are repeat sales, meaning most data will be discarded. Identifying repeat sales is itself quite difficult, several paintings by the same artist may share title and characteristics, in order to be sure you need to look at the actual paintings. It also requires the mix of objects sold each period to have the same quality.

Since RSR lacks the explicit pricing of characteristics, it can't be used directly to determine the price of works that haven't been sold at auction. A way around this problem has been developed by *artnet*, outlined in their white paper, artnet (2012), where they describe a method of using comparables: identifying similar artworks based on their characteristics and using their indices to price them.

3.1.2 Hedonic regression

The theory behind HR is built on the proposition that the value of an object to a buyer is based on the characteristics of the object. This approach requires a specification of the relevant characteristics, something easier said than done. As Triplett (2006) points out: the most important thing when doing a HR is knowing your market: a misspecified model will not perform satisfactory. If relevant characteristics are omitted, the residuals may reflect the specification errors, another concern is that one or more of the included characteristics will adapt to account for the omitted characteristic, leading to incorrect estimation. One often discussed problem with HR is multicollinearity in the data, Triplett dedicates the entire chapter six in his book to discussing the issue. Extreme cases of mulitcollinearity may result in negative values for one or more coefficients that were expected to have a positive sign. As Triplett points out, even if individual parameters may be poorly estimated, the price may be estimated with low variance if the overall fit of the regression is good.

Assume that the price of an object can be considered as a function of characteristics, some time-invariant and some time-varying. $\mathrm{P}_{i,t}$ is the price for painting i at time t, $\mathbf{p}$ is the vector of all the log prices. With $\mathrm{v}_{i,k}$, $k = 1, 2, \ldots, m$, time-invariant characteristics, $\mathrm{w}_{i,j,\tau}$, $\tau = 0, 1, \ldots, T$, $j = 1, 2, \ldots, n$, time-varying characteristics.

$$p_{i,t} = \sum_{k=1}^{m} \alpha_k \mathrm{v}_{i,k} + \sum_{\tau=0}^{T}\sum_{j=1}^{n} \beta_{j,\tau}\mathrm{w}_{i,j,\tau} + \varepsilon_{i,t} \tag{3.5}$$

The standard approach is to estimate the coefficients, α and β, using OLS is by combining the time-invariant and time-varying characteristics, $\mathrm{v}_{i,k}$ and $\mathrm{w}_{i,j,\tau}$, into the matrix $\mathbf{C}$ and combining α and β into the parameter vector θ and estimating:

$$\hat{\theta} = \left(\mathbf{C}'\mathbf{C}\right)^{-1}\mathbf{C}\mathbf{r} \tag{3.6}$$

3.2 Data set

3.2.1 Description

The data set used consist of all painting sales of Impressionist and Modern art at Sotheby's from the latter part of 2003 until the end of 2006, the start date was chosen due to the required data on the artworks being unavailable before this. Only auctions that are exclusively Impressionist and Modern art have been included, this means that sales at Sotheby's locations other than London and New York have been left out as these, during the sampled period, include both Modern and Contemporary art. The data has been collected from the publically available auction results at Sotheby's website. The reason Sotheby's only has been used for data collection is threefold. First of all: they show both sold and *bought-in* artworks in their online auction results, this means that the characteristics of the *bought-in* works can be recorded and compared to the sold works. Second, where as some investigations have found a difference in the prices attainted at Sotheby's and Christie's in the same location[2], other investigations have found them to not differ significantly[3]. Thirdly: I have hand collected the data, gathering all sales from both auction houses for a similar period would unfortunately have required too much time.

Most of the sales during a year takes place during spring and autumn auctions, May and November for New York, February and June for London. This leads me to aggregate the sales taking place into semi-annual periods.

The following information is recorded: Name of the auction and information on the auction including date and location, the name of the artist, the name of the painting, the *premium price* (this is set to 0 if the painting was *bought in*), the pre-sale estimates, if the painting is signed, monogrammed or stamped with the artists mark, whether the painting has its title written somewhere on itself, if the painting is dedicated to a third party, whether the painting has been dated, information on the medium and support, the height and width of the painting measured in meters, the number of separate provenance entries, the number of exhibitions where the painting have been shown, the amount of literature that mention the painting, whether it has been included in one or more *Catalogues Raissonés* or *Catalogues Critiques*, whether a trusted third party has authenticated the painting and whether or not the provenance lists the artists, the artist's estate or a family member of the artist.

The following artworks are excluded from the data: sculptures, double-sided paintings and paintings missing information on one of the aspects which are always present: height, width, artist, support and medium.

The result is a sample of 3776 sold paintings to be used for modelling and a sample of 1273 *bought-in* paintings to be used for comparison.

A separate verification sample has also been collected, comprising every fifth work brought to auction in the first half of 2007, resulting in 224 sold paintings. The unsold artworks have also been gathered but not used for verification.

As described in the previous chapter, the results attained using this data set will only be representative for works with quality similar to those in the sample.

3.2.2 Data adaption

In order to use the collected data in the hedonic regression, it first has to be adapted. The height and width of the artworks turned out, not unexpectedly, to be highly correlated: most paintings are close to a square shape. Instead, the area of the painting squared, the length of the diagonal and the difference from 1 to 1 in aspect ratio (diffasp) are used. These parameters, though they might seem a bit odd, are used to capture three expected features of the art market. The diagonal is used to have a linear measure of painting size, my assumption is that a larger painting is prefered to a smaller one. However I do expect there to be an upper limit to how large a painting can become while still fitting the homes of the collectors. Therefore I use the area squared to, expectedly, act as a punishment on very large paintings. Diffasp is based on the assumption that reasonably square paintings are more sought after.

As the goal of my work is to create a valuation model, not an index, I eschewed using time dummies to capture the price evolution over time and instead used an external index to capture the evolution over time. I used MSCI's WORLD Standard (Large+Mid Cap) index. I used the closing values from the final trading day each month to create a mean value for each semi-annual period. The latter half of 2003 was then set to one. The same methodology was used with S&P 500 and the gold price. In the end, the MSCI index was chosen on the basis of earlier articles[4].

Using the medium and support information: dummy variables were created. When it came to the medium, I considered the first medium listed to be dominant. A painting listed with multiple mediums thus get placed

[2]Pesando (1993)

[3]Pesando and Shum (2007-2), Baumol (1986)

[4]See section 2.3

under the first medium's dummy variable. As each painting only has one support, this was used to create the dummy variables.

By looking for specific keywords in the title of each painting, dummy variables were created to capture the topic of the painting. As with the medium information, certain categories of keywords were considered dominant over others in cases where multiple categories were identified. See Appendix A.5 for the complete list of keywords.

One of the main assumptions used is that each artist is internally consistent in quality compared to others: there are differences between works that will, hopefully, be captured by other variables but a Picasso is always a Picasso regardless of the magnitude of the particular painting. As mentioned earlier, different parts of an artist's works may be considered as differing in quality, however: for the subsample paintings appearing at auction, I consider the assumption of constant quality valid. Since a lot of the artists in the sample appear very few times, most only appearing once, they are aggregated into groups: one containing the seldom traded artists defined as those that appear five times or less in the sample, and one containing the medium traded artists defined as those that appear between six and fourteen times in the sample. Artists that appear fifteen or more times in the sample are given their own dummy variable.

All collected prices are *premium prices*: the *hammer price* and *the buyers premium* added together. The *hammer price* is calculated by subtracting the *buyers premium* charged at the sale location. The *hammer prices* for sales in London are converted from GBP to USD using the exchange rate at the sale date. I do not translate the *hammer prices* in USD from their nominal values. The art market is increasingly global, with buyers and sellers at auction from many countries[5], using a national CPI to translate the nominal prices would not capture the real prices experienced by buyers and sellers. As Aït-Sahalia Y, Parker J A and Yogo M (2004) point out: the consumption of basic goods, used to calculate CPI's, is distinguished from the consumption of luxury goods. Buying exclusive art at auction is a luxury good to the utmost degree.

The information on whether a painting is signed, monogrammed or stamped is translated into three categories: signed, marked (containing the paintings that are monogrammed and stamped) and unmarked.

The dating of the paintings is used to place them in groups based on which decade they were created. There is one group for paintings created prior to 1890, one for paintings created after 1970 and then one for each decade between. In some cases the dating of a painting is given as an estimated interval, if this interval is a decade or less, the painting is given the middle year, rounded up, otherwise it is marked as undated and put in the same group as all the undated paintings. Each group is then given a dummy variable.

Finally we have the provenance data. In order to avoid dependence, if a painting is indicated as being from the artist, the artist's estate or the artist's family members, one provenance entry is subtracted from the number of provenance entries. Likewise, if a painting is indicated as appearing in a *Catalogue Raissoné* or *Catalogue Critique*, one literature entry is subtracted. The square root of the number of provenance entries along with the natural logarithm of the number of exhibition and literature entries plus one is then used for the estimation. It is worth noting that I count exhibitions that have travelled to different locations as one exhibition since the selection of the painting has only been made once. Czujack (1997) ranks the provenance entries for Picasso paintings by influence of the owner, I have chosen to not rank individual provenance entries, exhibitions or works of literature by importance, quite simply because I do not feel my personal knowledge enable me to do this in a satisfactory way.

3.2.3 Comparison between sold and *bought-in* paintings

Is there a difference, quality wise, between the works that sell and those that are *bought-in*? To answer that question I compiled descriptive statistics for both the sold and *bought-in* paintings, it can be found in appendix A.1 for the sold paintings and in appendix A.2 for the *bought-in* paintings.

To determine whether there is a difference, I investigate whether the 95% confidence intervals for the corresponding variables overlap. Most variables overlap but there are some differences. Starting with the location parameter: the number of works brought to auction in London is significantly greater among the unsold works. This doesn't say anything about the quality of the paintings but it means the buy-in rate in London has been higher than that of New York in my sample.

[5]Bill Ruprecht, Chairman, President and Chief Executive of Sotheby's, stated in their press release "Sotheby's Announces Fourth Quarter and Full Year 2012 Results", that Sotheby's had received bids from over 120 countries during 2012.

Moving on, I find that significantly more works are included in *Catalogues Raissonés* or *Catalogues Critiques* among the *bought-in* paintings. As this is seen as raising the quality, it would imply a higher quality among the *bought-in* paintings compared to the sold paintings. At the same time, significantly fewer paintings are signed and more are stamped, monogrammed or unmarked in the unsold sample. As a signature is considered the better mark of authenticity, the unsold sample is considered as having lower quality.

Significantly fewer paintings using gouache and more paintings using charcoal as medium are present in the *bought-in* sample. As gouache is generally considered as attaining a higher price than charcoal, the *bought-in* sample is considered as having lower quality.

The *bought-in* sample contains significantly fewer paintings created after 1970. The *bought-in* sample also contain significantly more works brought to auction during the second half of 2003 and significantly fewer works brought to auction during the second half of 2005. None of these factors can be considered affecting the quality of the paintings in the sample.

The artists present in the different samples were also investigated. The only differences, all significantly fewer in the *bought-in* sample, were David Burliuk, Emilio Grau Sala, Jean Dufy, Jean-Pierre Cassigneul, Louis Valtat and Reuven Rubin. The differences were generally slight and all, except Jean Dufy and Louis Valtat, appear few times in the sample, just over the threshold to be included as a separate dummy variable. Given the number of artists included, some are bound to show spurious differences.

In conclusion: there are signs of quality difference between the sold and *bought-in* paintings, mainly to the advantage of the sold paintings, however it is only found in some variables and can not be considered large.

3.2.4 Expected signs

Going by read articles and my own knowledge of the art market, I expect that different characteristics will receive different parameter values. In this section I will give an overview of my expectations before performing the estimation. In several cases when it comes to dummy variables, the expected sign depend on their relation to each other.

Characteristic	Expected sign	Motivation
AREA SQUARED	Negative	There is an upper limit to how large a painting can become and still fit in the home of a collector, I expect that this will put negative pressure on the largest paintings.
DIAGONAL	Positive	A larger painting should be prefered to a smaller one. Using the diagonal to keep track of size means that the characteristic grows linearly.
DIFFASP	Negative	Most paintings are relatively square, paintings that are a lot larger in one dimension compared to the other should be less desireable.
INDEX	Positive	By the results of earlier articles, an increasing MSCI World index should be associated with higher prices for paintings.
PROVENANCE	Positive	The more of a paintings previous history is known, the more assured it is that it isn't a forgery. The value of a painting should therefore increase with the number of provenance references.
EXHIBITION	Positive	The more exhibitions a painting have been displayed at, the more likely it is to be a work of importance that has stood up to scrutiny.
LITERATURE	Positive	Same as with EXHIBITION: the more literature mention a painting, the more likely it is to be a work of importance that has stood up to scrutiny.
LOCATION	Unsure	I expect that location should matter less and less as time goes on as the art market becomes more global. Since my sample is slightly older, it may have a significant impact though I am unsure whether positive or negative.

DEDICATED	Unsure	A dedication might increase the likelihood of a painting being a genuine work and therefore have a positive sign. It might also have a negative sign as artists rarely give away their best work for free. It might also make it harder for a would-be buyer to view the painting as "theirs".
TITLED	Unsure	Whether or not a painting is titled somewhere on the work shouldn't matter. It might make the painting easier to attribute but I suspect that the effect is marginal at best.
INC CR/CC	Positive	The inclusion in a *Catalogue Raissoné* or *Catalogue Critique* should increase the value of a painting as it means it is considered an official work of the artist.
FROM ARTIST	Positive	If the provenance lists the artist, the artist's family or the artist's estate as a previous owner it should be seen as decreasing the risk of the painting being a forgery and therefore increase the value. It is possible that the opposite will happen: if the artist's estate is comprised of works of lesser quality that have been unable to sell while the artist was alive, it might decrease the value.
AUTHENTICATED	Positive	As with INC CR/CC, the value should increase as the authenticity of the painting has been assured by a trusted third party.
SIGNED	Set to 0	Most paintings in the sample are signed, as such: signed paintings will be set to zero.
MARKED	Negative	Relative to a signed painting, one which is stamped or monogrammed should be worth less.
UNMARKED	Negative	A painting without any marking from the artist should be worth less relative to a signed painting.
CANVAS	Set to 0	Most paintings in the sample are created using canvas as the support. It is also generally viewed as fetching the highest prices.
OTHER SUPPORT	Negative	As canvas generally is viewed as fetching the highest prices, I expect all other support dummies to have a negative sign.
OIL	Set to 0	Most paintings in the sample are created using oil as the medium. It is also generally viewed as fetching the highest prices.
OTHER MEDIA	Negative	As with canvas relative other supports, the rest of the media are expected to fetch lower prices.
NO TOPIC FOUND	Set to 0	I expect that the paintings where I am unable to identify a topic, are of similar quality to the sample at large.
STILL LIFE	Positive	Still lives were a popular topic, I expect them to have a positive sign.
PEOPLE	Positive	Various scenes with people became more popular as well.
WOMAN/GIRL	Positive	Artists, mostly men, have always liked painting the female form, I expect these to be popular.
NUDE	Positive	Same as above, most nudes are of females.
PORTRAIT	Positive	Quite often commisioned works with a backstory.
HEAD	Positive	Also a popular motif, a lot of experimentation during this period used the head as the subject.
STUDY	Negative	Most often lesser works done in preparation for larger ones.
OTHER TOPICS	Unsure	I am unsure of what sign to expect from the other topics.
NO DATE	Set to 0	This group is the largest, as such it is set to 0.
OTHER DATES	Positive	I expect paintings without a date to generally be lesser works, as such paintings that have been dated should fetch a higher value than the undated ones.

S. T ARTISTS	Negative	The market for artists in this group is probably not as established, that might make it harder to authenticate the works and decrease the number of potential buyers.
M. T. ARTISTS	Set to 0	I expect the quality of paintings from this group to be similar to the general sample.
H. T. ARTISTS	Positive	I expect the highly traded artists to be better established with a larger pool of potential buyers. I expect them to, generally, receive positive signs though without a doubt there will be some that receive negative signs. Certain more famous artists, such as Cézanne, Monet and Picasso will most probably receive positive signs.

Chapter 4

Estimation

4.1 Standard method

As the goal of my work is developing a way to estimate the value of paintings, I came to the conclusion that HR would be my best option. While *artnet's comparables* method is interesting, I lack the expertise and database to properly implement it. RSR would not fit my goal of being able to provide valuations of paintings that haven't been sold previously.

As my semi-annual sample cover a period from the latter half of 2003 until the second half of 2006, I feel the time period is too short to show significant changes in style preferences by buyers. Therefore I assume all my characteristics to be time-invariant. The method used to create the model is outlined in section 3.1.2.

Looking at the data, it is apparent that there are large differences in prices attained during the evening auctions compared to those attained during day auctions. Even internally during the different types of auctions, there are large differences. To account for these differences I created a model with a threshold based on the number of provenance, exhibition and literature entries found for a particular painting. If the total number is 8 or higher, I group the painting in the "high part" of the model, otherwise it is placed in the "low part". Both the 'high' and the 'low' groups share intercept and artist dummy variables. As such, the form of my threshold model is:

$$p_i = \alpha_0 + \mathrm{I}_{i,low}\left\{\sum_{k=1}^{n}\alpha_k \mathrm{v}_{i,k}\right\} + \mathrm{I}_{i,high}\left\{\sum_{j=1}^{m}\alpha_j \mathrm{v}_{i,j}\right\} + \sum_{l=1}^{o}\beta_l \mathrm{v}_{i,l} + \varepsilon_i \tag{4.1}$$

Where p_i is the natural logarithm of the *hammer price*, α_0 is the intercept, the coefficients α_k and α_j descripe the same variables but belong to the "high" and "low" parts of the model and β_l are the coefficients for the shared artist dummy variables.

Combining all coefficients into a vector, θ, $(m + n + o + 1) \times 1$, the characteristics of all paintings into a matrix, $\mathbf{X}$, $N \times (m+n+o+1)$, and all logarithms of the *hammer price* into a vector, $\mathbf{p}$, $N \times 1$: the coefficients were estimated using OLS as:

$$\hat{\theta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}\mathbf{p} \tag{4.2}$$

The initial model contained all adapted parameters described in section 3.2.2. Using likelihood ratio (LR) tests, and the 95% confidence interval for the parameters, the next model was created by removing the parameter closest to zero and investigating the LR to double-check the potential significance of the parameter. This was repeated until only significant characteristics were left in the final model. A summary of the resulting model, including adjusted R^2, can be seen in appendix A.3.

Log-*hammer prices* are estimated as:

$$\widehat{\ln(\mathbf{HP})} = \mathbf{X}\hat{\theta} \tag{4.3}$$

The residual, e_{OLS}, is calculated as:

$$e_{OLS} = \ln(\mathbf{HP}) - \widehat{\ln(\mathbf{HP})} \tag{4.4}$$

While my final model scored well using adjusted R^2 compared with other author's models[1], the residuals were unsatisfactory: they weren't normally distributed[2].

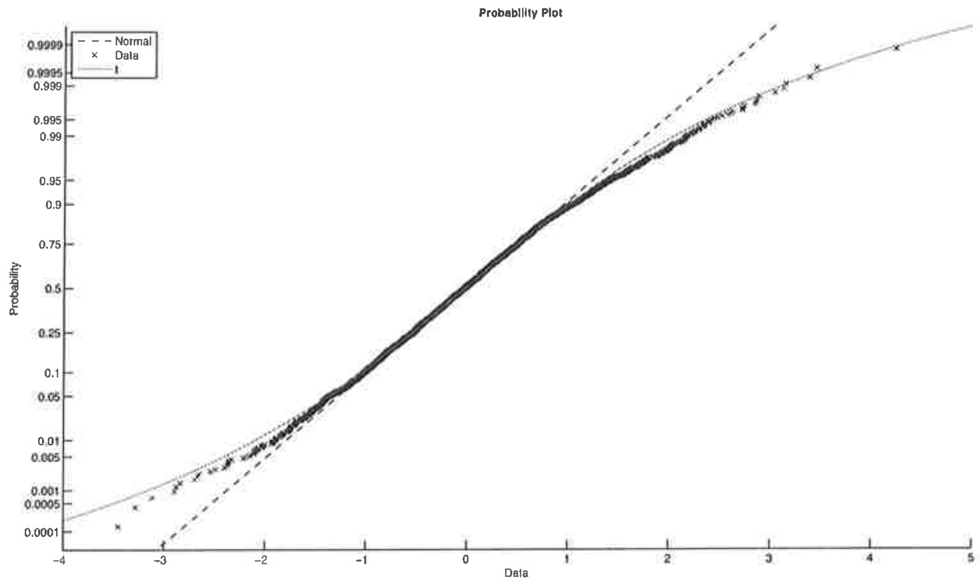

Figure 4.1: Probability plot of the OLS residuals, showing the normal and t-distribution.

One of the implicit assumptions in most[3] articles I've read is that the *hammer prices* are log-normally distributed, seen by the use of OLS with the log-prices as the dependent variable, this should have led to acceptably normal residuals. The log-normal distribution has the pleasant characteristic that it is bounded downwards at 0 and that it is easy to translate into the regular normal distribution, making it easy and quick to work with.

Investigating the distribution of the log-prices, there was too little weight in the lower tail and too much in the upper tail to be considered normal. However, the double-log of the prices proved a closer match to the normal distribution. This led me to view the art market in a new way.

4.2 Extreme value approach

As I described in section 2.2: there are many levels of the art market, each divided into *primary* and *secondary* markets. From each level, only a few artists make it to the next with prices for these artists increasing accordingly. Only very few artists make it to the international art market.

A mathematical way of viewing this dynamic is to consider the artists in each level as the block maxima of artists in the level below. This would suggest that one of the constituent distributions of the GEV[4] distribution could be used to describe the distribution of hammer prices.

[1]My model scored an adjusted R^2 of 0,7118. Chanel, Gérard-Varet and Ginsburgh (1996) reported between 0,630 to 0,747 for three different models, Renneboog and Spaenjers (2011) reported 0,7048, Czujack (1997) using a sample of only Picasso reported 0,81, Goetzmann (1993) reported 0,70.

[2]One of the assumptions when using OLS is that the residuals are normally distributed

[3]McAndrew, Smith and Thompson (2010) investigate the bias of auction house estimates explicitly assuming log-normality among hammer ratios.

[4]Generalized Extreme Value distribution, one of the constituent distributions which led me down this path is the *Gumbel* distribution, which is a double-exponential.

Looking at the realities of the art market: the lowest price possible for an artwork is zero and the majority of art created is of low value. At the same time there is no theoretical upper limit for the price[5]. As we also know from the earlier sections, auction houses are restricive in the artworks they accept: they won't accept works of too low value and those they think will have trouble selling.

These characteristics fit the Fréchet distribution, which has support from any real number to infinity, a heavy tail and a PDF that starts at zero for the lowest support and then quickly develops hump before slowly decreasing. To test my intuition, I fit a GEV distribution to the exponential of the residuals from the OLS-estimation to determine how well it corresponded: the distribution fit well and the sign of the shape parameter indicated that the data was Fréchet distributed. The fitted PDF of the GEV to the exponential of the OLS residuals can be seen in figure 4.2.

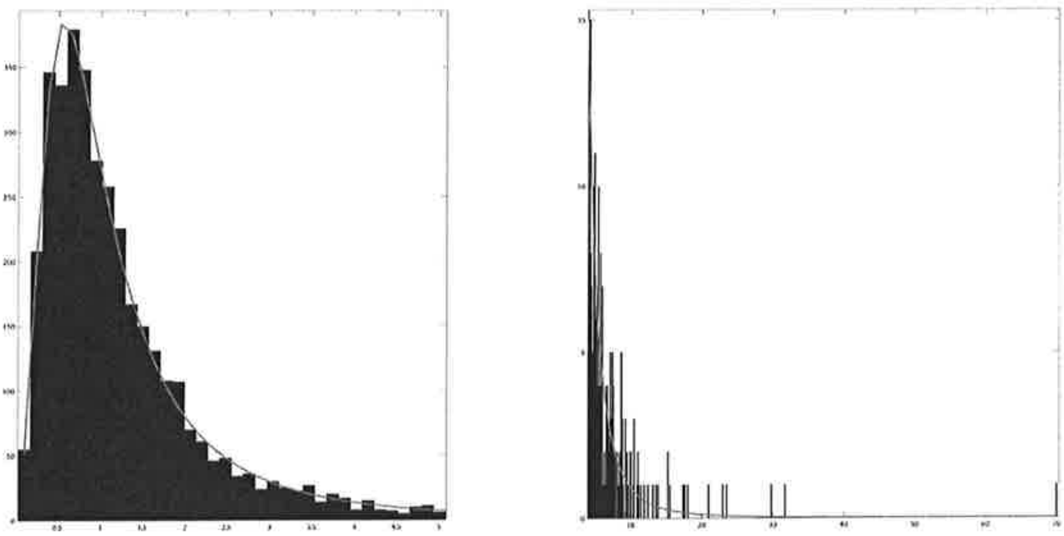

Figure 4.2: The PDF of the GEV distribution fit to the OLS residuals, shown on the histogram of the OLS residuals. The figure has been split in two to increase the visibility of the tail.

The PDF and CDF of the Fréchet distribution are as follow.

$$\begin{aligned}
Parameters: &\quad \alpha \in (0,\infty)\ \text{shape} \qquad \sigma \in (0,\infty)\ \text{scale} \qquad \mu \in (-\infty,\infty)\ \text{location} \\
Support: &\quad x > \mu \\
CDF: &\quad \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right\} && (4.5) \\
PDF: &\quad \frac{\alpha}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{-1-\alpha}\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right\} && (4.6) \\
Mean: &\quad \mu+\sigma\Gamma\left(1-\frac{1}{\alpha}\right)\ for\ \alpha>1,\ \infty\ \text{otherwise} && (4.7) \\
Median: &\quad \mu+\frac{\sigma}{\sqrt[\alpha]{ln(2)}} && (4.8)
\end{aligned}$$

For the purpose of modeling *hammer prices*, the support needs to be from zero and incorporate the characteristics of the paintings. I therefore set the location parameter $\mu = 0$ and incorporate the characteristics

[5] Remember Baumol (1986): prices float freely. The current record for most expensive painting sold is Paul Cézanne's "The Card Players" that was bought by the Royal Family of Qatar for somewhere between 250 and 300 million USD in April 2011.

of the paintings into the scale parameter $\sigma = e^{\mathbf{X}\theta}$, resulting in the following PDF and CDF:

$$CDF: \quad \exp\left\{-\left(\mathbf{HP}e^{-\mathbf{X}\theta}\right)^{-\alpha}\right\} \tag{4.9}$$

$$PDF: \quad \alpha e^{-\mathbf{X}\theta}\left(\mathbf{HP}e^{-\mathbf{X}\theta}\right)^{-1-\alpha}\exp\left\{-\left(\mathbf{HP}e^{-\mathbf{X}\theta}\right)^{-\alpha}\right\} \tag{4.10}$$

Where **HP** is the vector of *hammer prices.* Using the same structure of characteristics as the OLS model, $\theta, (m+n+o+1) \times 1$, $\mathbf{X}, N \times (m+n+o+1)$, yield the following model structure:

$$\mathbf{HP} = e^{\mathbf{X}\theta}\varepsilon_{\text{Fréchet}} \tag{4.11}$$

Where $\varepsilon_{\text{Fréchet}}$ is multiplicative, Fréchet distributed, noise.

To estimate the unknown parameters θ and the shape parameter α, a numerical maximum likelihood estimation is performed using the log-likelihood function derived from equation (4.10):

$$\ln \mathrm{L} = -\mathbf{X}\theta + \ln(\alpha) - (1-\alpha)\left(\ln(\mathbf{HP}) - \mathbf{X}\theta\right) - \left(\mathbf{HP}e^{-\mathbf{X}\theta}\right)^{-\alpha} \tag{4.12}$$

As with the OLS model, the initial model contained all gathered characteristics of the paintings. Confidence intervals were calculated using profile likelihood. The parameters closest to zero were then removed, stepwise, until the final model only contained parameters statistically significant at the 95% level. Some cases, where a parameter was expected to have an effect from market knowledge, received the expected sign and were only insignificant by a very small margin, have been left in the model if it was significant at the 90% level. These are reported with a 90% confidence interval and are marked with an asterisk. The complete list of included parameters can be found in appendix A.4.

The *hammer prices* are estimated as:

$$\widehat{\mathbf{HP}} = \frac{e^{\mathbf{X}\hat{\theta}}}{\sqrt[\alpha]{\ln(2)}} \tag{4.13}$$

The reason the median is used as opposed to the mean is down to the nature of the Fréchet distribution: the heavy tail means that a few very large observations increase the mean a lot. Thus the median better represent the value of the noise.

As the noise is multiplicative, the residual, $e_{\text{Fréchet}}$, is calculated as:

$$e_{\text{Fréchet}} = \frac{\mathbf{HP}}{\widehat{\mathbf{HP}}} \tag{4.14}$$

With multiplicative noise, in this case, we want the median value of the residual to equal 1 and the residual to fit the pertinent distribution. The median of the residual is 1.0422, acceptably close to 1. The residual also fit the Fréchet distribution. The fitted PDF of the GEV distribution, specified as a Fréchet distribution, to a histogram of the residuals can be seen in figure 4.3.

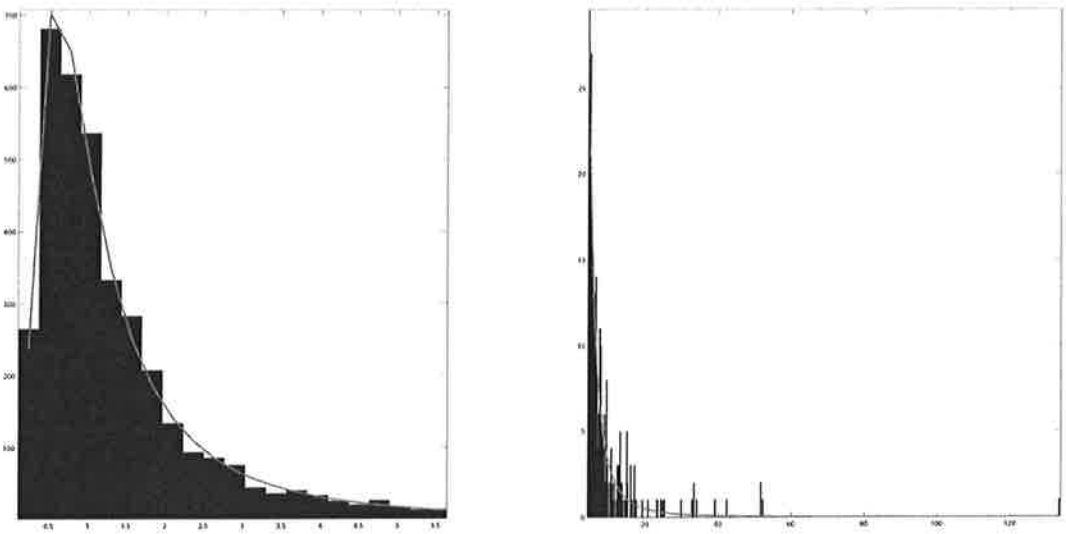

Figure 4.3: The PDF of the GEV distribution fit to the residuals shown on the histogram of the residuals for the EV model. The figure has been split in two to increase the visibility of the tail.

Chapter 5

Results

5.1 Final models

Both in the case of the OLS model and the EV model, not all parameters in the data turn out to have a significant impact. The OLS model end up with 106 parameters, the EV model with 118 parameters plus the shape parameter. In both models, most of the parameter signs turn out as expected, but some differ. These differences will be given some attention. The final parameters, their values and a short discussion on how they comply with expectations are presented in appendix A.3 for the OLS model and in appendix A.4 for the EV model.

Common among all models is that TITLED, expectedly, lacks significance. In all but the "high part" of the EV model, FROM ARTIST is insignificant, and there it receives a negative sign. I would have expected it to receive a positive sign as it would increase the likelihood of the work being genuine. I elaborated on in section 3.2.4. DEDICATED received negative signs in both the OLS and EV model but lacked significance in all but the "high part" of the OLS model. I would have expected UNMARKED to become negative in all models, instead it turned out insignificant in some parts and even recieved a positive sign for the "high part" of both the OLS and EV models. Most topic dummies proved to have a smaller effect than I had expected, some of them having the opposite sign to what I expected, though my personal expectations are open for critique from art scholars.

The fact that the LOCATION parameter received a positive sign in the "low part" of both the OLS and the EV model while being insignificant in the "high part" of both is quite interesting. It could mean that the buyers at the very top of the market are more mobile, removing differences by sale location by actively purchasing in several places, while the lower parts attract a larger proportion of local buyers with differing wealth by location.

The *shape parameter* estimated in the EV model control the characteristics of the Fréchet distribution. The final value of the shape parameter, $\alpha = 1,3097$, has some implications. The variance of the Fréchet distribution is:

$$Variance: \quad \sigma^2\left(\Gamma\left(1-\frac{2}{\alpha}\right)-\left(\Gamma\left(1-\frac{1}{\alpha}\right)\right)^2\right) \text{for } \alpha > 2, \qquad \infty \text{ otherwise} \tag{5.1}$$

As the shape parameter is less than 2, the variance is infinite. In reality that is an impossibility: while the maximum price paid for a painting is ever increasing, you can never sell a painting for more than all the wealth of the planet, and realistically not much more than a few billion USD. I view the infinite variance as a mathematical artefact.

5.2 Accuracy

The most important test of the models lie in how well they manage to predict the *hammer prices*, both in absolute and relative terms. In this section I will compare the accuracy of the created models in terms of estimation errors and in comparison to the pre-sale estimates from the auction houses. I will report the results both using *in sample* data and using verification data. For the OLS model, I have transformed the model back to *hammer prices.*

The confidence intervals for the OLS model are the standard, normal confidence intervals (C.I.). For the EV model, the C.I. are constructed by running a monte carlo using GEV noise created from the parameter estimates of the EV model residual. The confidence intervals are the relevant percentiles of the values attained. The hit rates for the 95% confidence intervals can be seen in Table 5.1.

In sample	OLS model	EV model	Auction house
Paintings inside 95% C.I.	94,41%	94,94%	
Paintings below 95% C.I.	2,22%	0,85%	
Paintings above 95% C.I.	3,36%	4,21%	
Verification			
Paintings inside 95% C.I.	60,71%	88,84%	
Paintings below 95% C.I.	9,82%	2,23%	
Paintings above 95% C.I.	29,46%	8,93%	

Table 5.1: Hit rates for 95% confidence intervals.

By looking at the absolute error, I get a measure of the distance from the realised *hammer price* to my estimates and the point estimate from the auction house. I have used the arithmetic mean of the high and low pre-sale estimate from the auction house as the point estimate[1]. The results are presented, for both the general models and for the submodels containing paintings with fewer than eight combined provenance entries, designated "low part", in Table 5.2. The percentages given in the parentheses are the differences from the auction house result.

In sample	OLS model	EV model	Auction house
Mean absolute error	289290 (91,56%)	297130 (96,75%)	151020
Mean absolute error, low part	309290 (93,86%)	317990 (99,32%)	159540
Median absolute error	28346 (44,69%)	26012 (32,77%)	19591
Median absolute error, low part	28574 (42,87%)	26273 (31,36%)	20000
Verification			
Mean absolute error	607070 (120,15%)	443080 (60,68%)	275760
Mean absolute error, low part	696860 (117,89%)	481790 (54,23%)	312380
Median absolute error	110170 (217,15%)	43439 (25,06%)	34736
Median absolute error, low part	120700 (178,17%)	52096 (20,06%)	43392

Table 5.2: Mean and median absolute error.

The pre-sale interval given by the auction house contained about 24% of the *hammer prices* in the sample[2]. In order to compare the characteristics of the C.I. of my models to that of the pre-sale interval, I assumed that the auction house estimates are created to cover about 20% in general and created 20% C.I. for both models resulting in the hit rates in Table 5.3. The assumption that the auction house's pre-sale intervals are created to contain 20% of hammer prices might be quite weak as the pre-sale intervals from the auction house in the validation sample has a lot lower hit rate compared to the *in sample* intervals.

The size of the 20% confidence intervals is displayed in Table 5.4. As with the absolute error, both the mean and the median have been investigated for both the general models and the "low part" of the models.

[1] I investigated the buy-in rate in my sample and found it to be 25,21%, I then calculated the number of bids inside, above and below the high and low pre-sale estimates. By placing all the unsold paintings below the low pre-sale estimate, I found that 45,40% of total lots exceeded the high pre-sale estimate, 36,56% were lower than the low pre-sale estimate, and 18,05% were inside. As the number of shortfalls and exceedances are quite evenly matched, I assume that the auction houses aim for the price to be in the middle of their intervals.

[2] I call this the "hit rate".

In sample	OLS model	EV model	Auction house
Paintings inside 20% C.I.	22,33%	20,34%	24,13%
Paintings below 20% C.I.	39,99%	26,30%	15,17%
Paintings above 20% C.I.	37,67%	53,36%	60,70%
Verification			
Paintings inside 20% C.I.	8,93%	17,41%	14,73%
Paintings below 20% C.I.	31,25%	20,09%	6,70%
Paintings above 20% C.I.	59,82%	62,50%	78,57%

Table 5.3: Hit rates for 20% confidence intervals.

The percentages given in the parentheses are the differences from the auction house results.

In sample	OLS model	EV model	Auction house
Mean interval	123430 (17,00%)	71980 (-31,77%)	105500
Mean interval, low part	39281 (9,47%)	26328 (-26,63%)	35884
Median interval	271150 (171,15%)	18018 (80,18%)	10000
Median interval, low part	21729 (117,29%)	14766 (47,66%)	10000
Verification			
Mean interval	77269 (-33,64%)	77039 (-33,84%)	116440
Mean interval, low part	45201 (-60,55%)	81703 (-28,70%)	114580
Median interval	28590 (42,95%)	27296 (36,48%)	20000
Median interval, low part	20794 (3,97%)	31528 (57,64%)	20000

Table 5.4: Size of 20% confidence intervals.

5.3 Discussion

Comparing the OLS model to the EV model and beginning from the top, *in sample*: both of them capture about the same amount of the prices in their 95% C.I., the main difference being that the missed prices are spread equally above and below for the OLS model and mainly above for the EV model. Moving on to the absolute errors: both the OLS and the EV models have about the same mean absolute error while the EV model is a slight improvement when it comes to the median absolute error.

Looking at the 20% C.I., both the OLS and EV models capture about 20% with the OLS model having the shortfalls and exceedances spread evenly above and below while the EV model has more about double the number of exceedances compared to shortfalls. The size of both the mean and median confidence intervals are quite a bit smaller using the EV model compared to the OLS model.

Moving on to the verification sample: the 95% C.I. of the EV model captures 88,84% of the prices with about 2% below and 9% above. The 95% C.I. of the OLS model capture 60,71% of the prices with about 10% falling below and 29% above. The mean absolute error is about 50% higher for the OLS model compared to the EV model while the median error is a bit over twice as large.

Looking at the 20% C.I., the EV model capture 17,41% of the prices with about 20% falling below and 63% above. The OLS model capture 8,93% of the prices with about 31% falling below and 60% above. The mean interval size for the entire model is almost the same with both the OLS and the EV model, while the mean interval for the "low part" of the OLS model being about half the size of the EV model. The median interval size for the entire model is about the same for both the OLS and EV model with the median interval size for the "low part" of the OLS model being about a third smaller than the EV model.

Looking at the results, I feel that the EV model does a better job than the OLS model at estimating the prices, both *in sample* and especially using the verification sample. The characteristics of the exceedances and shortfalls also better match those given by the auction houses. I think one of the main reasons for the EV model achieving better results come from the form of the loss function: using OLS, you're minimising using a quadratic loss function, the square part of least squares. As such, the few, very large values in the sample will receive a large weight. Using WLS instead of OLS, it is possible to decrease the weight of these large observations. Though the model will fit the large prices worse as a result, it will improve the fit to more numerous, smaller prices. In the EV model, the loss function better accounts for the heavy tails.

How do the EV model stack up compared to the pre-sale valuation from the auction house? Again, beginning *in sample*: as the auction houses don't give any 95% C.I., I'll start with the absolute errors. The mean absolute errors for both the entire model and the "low part" are about twice as large as for the auction house estimate. When looking at the median error, this shrinks to being about 30% larger for the EV model compared to the auction house estimate. The 20% C.I. capture about the same amount with the auction house beating the EV model with 24,13% inside the interval compared to 20,34%. The auction house pre-sale interval has about 15% of prices falling below and 61% above compared to 26% below and 53% above for the EV model. The mean interval size for the EV model is about a third smaller compared to the auction house's. The median interval size for the entire EV model is about 80% larger than that of the auction house with the interval size for the "low part" of the EV model being about 50% larger.

Moving on to the verification data: the EV model's mean absolute error end up around 60% larger than the auction house estimate error. The median error being around 25% larger. The 20% C.I. of the EV model capture more of the prices, with a hit rate of 17,41% compared to 14,73%, again with fewer prices exceeding the interval and more falling short. The mean interval size is about 30% smaller for the EV model compared to the auction house pre-sale interval. The median interval size being about 35% larger for the entire EV model compared to the auction house pre-sale interval with the "low part" being about 60% larger compared to the auction house pre-sale interval.

While being an improvement compared to the OLS model, particularly using the verification data, the EV model isn't as precise as the auction house's estimates. This is not at all unexpected, whereas the auction houses are staffed with knowledgeable experts and have direct access to the paintings to examine them, that is not possible using the model.

It is worth noting the threshold used to determine whether a painting belonged in the "low part" or the "high part" of either model. I have used the total number of provenance entries[3] to classify the paintings, this is not a perfect system. Particularly when it comes to famous paintings: a human expert can identify these directly, my model can not. When one of these painting have been in the ownership of a private collector for a long time, it might not have shown up at exhibitions and the number of owners in the provenance are few, that would place it in the "low part" and lead to a larger estimation error under the assumption that higher quality work have more provenance entries.

Another advantage the auction houses have is the *reserve price*, as it is set at or below the lower pre-sale estimate, usually at around 70% of the lower pre-sale estimate, there's a limit to the number of works falling short of the lower pre-sale estimate of the auction house. The EV model doesn't have this benefit. As was shown by Beggs and Graddy (2009), there are anchoring effects at auctions, further benefiting the accuracy of the auction house estimates as bidders may use their estimates as reference points.

There are still questions about how to take *bought in* paintings into account. Most of the times when unsold paintings have been taken into account, the *reserve price* has been used as a replacement for the *hammer price*. This is not entirely consistent with how English auctions work, where the valuation of the second highest bidder decide the final price. Using the *reserve price* would mean using the highest valuation, that of the seller, as the price of the work. A more consistent approach would be using the highest bid by an auction participant below the *reserve price*. However, some paintings don't receive any bids except *chandelier bids* on behalf of the seller. I don't know how to account for these paintings without using the *reserve price*. This information on the high bid below the *reserve price* is, to my knowledge, not recorded. Even though the *reserve prices* are recorded by the auction houses, they are kept secret by the auction houses...

[3]Provenance, Exhibitions and Literature mentions.

Chapter 6

Conclusion

I have created valuation models for paintings, based on a newly created sample of painting sales at Impressionist and Modern art auctions, using both the standard estimation techniques of the field to creating hedonic regression models and a newly developed method using extreme value distributions, more precisely the Fréchet distribution, to estimate the hedonic regression. The extreme value approach shows promise, attaining smaller absolute valuation errors compared to the standard approach and achieving confidence intervals closer in characteristic and size to those of the auction houses.

The extreme value approach is the result of a new way of viewing the art market, described in section 4.2, where the artists and prices in each level of the *primary* and *secondary market* is seen as block maxima of the level below, motivating the use of a GEV family distribution. The characteristics of art prices, that they are strictly positive and, mostly, without an upper limit correspond to the characteristics of the Fréchet distribution.

In my thesis, I have concentrated on creating a valuation model whereas most articles I have read have concentrated on creating price indices. It would be interesting to increase the time covered in the sample to create a price index based on the extreme value approach and compare it to existing indices.

My own experience with the sale of the two *Josef Albers*, I mentioned in the introduction, show the nature of the art market: given the unique nature of each painting and the small number of transactions, you might be approximately right in your valuation, but you will be absolutely wrong.

Appendix A

Appendix

A.1 Sold paintings

Appendix A.1 presents the descriptive statistics for the hedonic variables for the sold paintings in the sample that have been used as the basis for the hedonic regression. ASPECT is the height divided by the width of the artwork. Unlike other industries where aspect ratio is given as width divided by height, the opposite is used in the art world. PROVENANCE, EXHIBITION and LITERATURE are the number of each type of provenance listed for the artwork. LOCATION is a dummy variable that equals one the artwork has been sold at Sotheby's in London, the variable equals zero if the artwork has been sold at Sotheby's in New York. DEDICATED is a dummy variable that equals one if the artwork is dedicated to someone. TITLED is a dummy variable that equals one if the title of the artwork is written somewhere on the artwork. INC(CR/CC) is a dummy variable that equals one if the artwork is included in one or more *Catalogues Raissonés* or *Catalogues Critiques.* AUTHENTICATED is a dummy variable that equals one if a trusted third party has confirmed the artwork's authenticity. FROM ARTIST is a dummy variable that equals one when the artist, the artist's estate or a family member of the artist is listed in the provenance notes. SIGNED, MARKED and UNMARKED are take the value one depending on whether the artwork is signed, stamped or monogrammed, or not marked by the artist. The support dummies indicate whether the artworks supporting material. The medium dummies indicate which medium is listed first for the artwork: it is assumed to be the dominant medium used to create the artwork. The topic dummies indicate the main subject of the artwork, the keywords listed in appendix A.5 are used to determine the topic. The lowest number rank category is then assumed to dominate other potential categories for the artwork and is set to one. The period dummies indicate what decade the artwork was created. If a particular artwork isn't dated, it appears in the NO DATE category. The time dummies indicate when the artwork was sold. The mean and standard deviation (**S.D.**) are given for each variable. The number of ones and zeros for dummy variables are also included. Descriptive statistics for artist dummies have been omitted from this appendix.

	Mean	**S.D.**	**1**	**0**
Total number of observations: 3776				
Variables				
Size variables				
HEIGHT	0,5118	0,2618		
WIDTH	0,5320	0,3168		
Provenance variables				
PROVENANCE	2,5344	2,1501		
EXHIBITION	1,0215	2,4203		
LITERATURE	1,1528	2,1605		

Dummies				
Location				
LOCATION	0,4033	0,0080	1523	2253
Authenticity dummies				
DEDICATED	0,0286	0,0027	108	3668
TITLED	0,1033	0,0050	390	3386
INC(CR/CC)	0,3647	0,0078	1377	2399
AUTHENTICATED	0,2206	0,0067	833	2943
FROM ARTIST	0,1925	0,0064	727	3049
SIGNED	0,8363	0,0060	3158	618
MARKED	0,0755	0,0043	285	3491
UNMARKED	0,0882	0,0046	333	3443
Support dummies				
CANVAS	0,5469	0,0081	2065	1711
PAPER	0,3154	0,0076	1191	2585
BOARD	0,0493	0,0035	186	3590
PANEL	0,236	0,0025	89	3687
CARD	0,0196	0,0023	74	3702
OTHER	0,0453	0,0034	171	3605
Medium dummies				
OIL	0,6322	0,0078	2387	1389
CRAYON	0,0291	0,0027	110	3666
CHARCOAL	0,0199	0,0023	75	3701
PENCIL	0,0519	0,0036	196	3580
GOUACHE	0,0646	0,0040	244	3532
PASTEL	0,0196	0,0023	74	3702
PEN	0,0487	0,0035	184	3592
WATERCOLOUR	0,0895	0,0046	338	3438
OTHER	0,0445	0,0034	168	3608
Topic dummies				
UNTITLED	0,0114	0,0017	43	3733
URBAN	0,0697	0,0041	263	3513
LANDSCAPE	0,0503	0,0036	190	3586
BOAT	0,0132	0,0019	50	3726
BUILDING	0,0381	0,0031	144	3632
NATURE	0,0379	0,0031	143	3633
ANIMAL	0,0180	0,0022	68	3708
STILL LIFE	0,0781	0,0044	295	3481
PEOPLE	0,0381	0,0031	144	3632
WOMAN/GIRL	0,1221	0,0053	461	3315
NUDE	0,0363	0,0030	137	3639
PORTRAIT	0,0281	0,0027	106	3670
HEAD	0,0249	0,0025	94	3682
ABSTRACT	0,0159	0,0020	60	3716
STUDY	0,0204	0,0023	77	3699
NO TOPIC FOUND	0,3975	0,0080	1501	2275

Period dummies				
NO DATE	0,2293	0,0068	866	2910
PRE-1890	0,0588	0,0038	222	3554
1890–1899	0,0448	0,0034	169	3607
1900–1909	0,1046	0,0050	395	3381
1910–1919	0,1133	0,0052	428	3348
1920–1929	0,1184	0,0053	447	3329
1930–1939	0,1009	0,0049	381	3395
1940–1949	0,0683	0,0041	258	3518
1950–1959	0,0651	0,0040	246	3530
1960–1969	0,0516	0,0036	195	3581
POST-1970	0,0448	0,0034	169	3607
Time dummies				
2003–2	0,0712	0,0042	269	3507
2004–1	0,1803	0,0063	681	3095
2004–2	0,0591	0,0038	223	3553
2005–1	0,2222	0,0068	839	2937
2005–2	0,1062	0,0050	401	3375
2006–1	0,2376	0,0069	897	2879
2006–2	0,1234	0,0054	466	3310

A.2 Bought-in paintings

Appendix A.2 presents descriptive statistics for the hedonic variables for the *bought-in* paintings in the sample. The hedonic variables are the same as in appendix A.1. The hedonic variables in the *bought in* sample whose 95 % confidence interval don't overlap with the 95% confidence interval of the hedonic variables of sold paintings are marked with an asterisk. Descriptive statistics for artist dummies have been omitted from this appendix.

	Mean	S.D.	1	0
Total number of observations: 1273				
Variables				
Size variables				
HEIGHT	0,5073	0,2494		
WIDTH	0,5270	0,2668		
Provenance variables				
PROVENANCE	2,6803	2,1418		
EXHIBITION	0,8979	1,8894		
LITERATURE	1,0660	1,5633		
Dummies				
Location				
LOCATION*	0,5043	0,0140	642	631
Authenticity dummies				
DEDICATED	0,0165	0,0036	21	1252
TITLED	0,1084	0,0087	138	1135
INC(CR/CC)*	0,4171	0,0138	531	742
AUTHENTICATED	0,2404	0,0120	306	967
FROM ARTIST	0,2058	0,0113	262	1011
SIGNED*	0,7730	0,0117	984	289
MARKED*	0,1060	0,0086	135	1138
UNMARKED*	0,1210	0,154	333	1119
Support dummies				
CANVAS	0,5059	0,0140	644	629
PAPER	0,3456	0,0133	440	833
BOARD	0,0479	0,0060	61	1212
PANEL	0,0369	0,0053	47	1227
CARD	0,0173	0,0037	22	1251
OTHER	0,0463	0,0053	47	1226
Medium dummies				
OIL	0,6064	0,0137	772	501
CRAYON	0,0228	0,0042	29	1244
CHARCOAL*	0,0361	0,0052	46	1227
PENCIL	0,0715	0,0072	91	1182
GOUACHE*	0,0393	0,0054	50	1223
PASTEL	0,0275	0,0046	35	1238
PEN	0,0566	0,0065	72	1201
WATERCOLOUR	0,0935	0,0082	119	1154
OTHER	0,0463	0,0059	59	1214

Topic dummies				
UNTITLED	0,0110	0,0029	14	1259
URBAN	0,0558	0,0064	71	1202
LANDSCAPE	0,0495	0,0061	63	1210
BOAT	0,0110	0,0029	14	1259
BUILDING	0,0503	0,0061	64	1209
NATURE	0,0416	0,0056	53	1220
ANIMAL	0,0204	0,0040	26	1247
STILL LIFE	0,0660	0,0070	84	1189
PEOPLE	0,0353	0,0052	45	1228
WOMAN/GIRL	0,1453	0,0099	185	1088
NUDE	0,0393	0,0054	50	1223
PORTRAIT	0,0424	0,0056	54	1219
HEAD	0,0157	0,0035	20	1253
ABSTRACT	0,0110	0,0029	14	1259
STUDY	0,0291	0,0047	37	1236
NO TOPIC FOUND	0,3763	0,0136	479	794
Period dummies				
NO DATE	0,2357	0,0119	300	973
PRE-1890	0,0801	0,0076	102	1171
1890-1899	0,0487	0,0060	62	1211
1900-1909	0,1037	0,0085	132	1141
1910-1919	0,1155	0,0090	147	1126
1920-1929	0,1257	0,0093	160	1113
1930-1939	0,1021	0,0085	130	1143
1940-1949	0,0746	0,0074	95	1178
1950-1959	0,0511	0,0062	65	1208
1960-1969	0,0393	0,0054	50	1223
POST-1970*	0,0236	0,0043	30	1243
Time dummies				
2003–2*	0,1108	0,0088	141	1132
2004–1	0,1987	0,0112	253	1020
2004–2	0,0456	0,0058	58	1215
2005–1	0,2302	0,0118	293	980
2005–2*	0,0778	0,0075	99	1174
2006–1	0,2082	0,0114	265	1008
2006–2	0,1288	0,0094	164	1109

A.3 OLS hedonic regression results

Appendix A.3 presents the results of the hedonic regression using the OLS approach outlined in section 4.1 with the logarithm of the *hammer price* as the dependent variable. Descriptive statistics for the data sample used for the hedonic regression can be found in appendix A.1. The final model is composed of a *Low* and a *High* part that depend on the number of provenance entries for each artwork. Both of these parts share artist dummies and intercept. MSCI's WORLD Standard (Large+Mid Cap) index is used as an external signal. The artist dummies are divided into three groups: *Seldom traded artists*, *Mid traded artists* and *Highly traded artists*. Artists in the two lower groups are aggregated while the artists in the highly traded group are given their own dummy variables. A complete list of highly traded artists included is presented at the bottom of the table, see appendix A.4.

Variables that aren't statistically significant at the 95 % level are removed from the final model.The exception from this are a few of the variables in the *High* and *Low* parts that are expected to have an impact on the price, that are significant at the 90 % level using. These are marked with *. For each variable, I present the coefficient (θ_i), the 95 % or 90 % confidence interval, the price impact for the coefficient ($exp(\theta_i) - 1$) and the number of each dummy variable in the final model (N). Dummy variables with fewer than 15 values have been aggregated.

A.3.1 Discussion

Beginning with the "low part" of the model, the LOCATION parameter receives a positive sign, meaning that paintings sold in London tend to attract a higher price. Unexpectedly, UNMARKED turn out as insignificant while MARKED receive a negative sign. When it comes to the support dummies, only PAPER turn out as significant with a negative sign. For the medium dummies, GOUACHE and PASTEL aren't significant and WATERCOLOUR is only significant at the 90% level with a negative sign[1]. Almost all topic dummies lack significance, the exceptions being PEOPLE and STUDY, both receiving their expected signs. Not all period dummies are significant but they do receive their expected signs. As such, the period dummy for 1950–1959 is significant at the 90% level and kept in the model.

The "high part" carries a few more surprises, not unexpectedly as it is created using a smaller part of the sample compared to the "low part". DIFFASP receive the expected sign but lacks significance. Unlike in the "low part", LOCATION is not significant. Even more unexpectedly: UNMARKED receives a positive sign while MARKED receives a negative one. AUTHENTICATED turns out insignificant while DEDICATED is receives a negative sign. The medium dummies act the same as in the "low part" but here, WATERCOLOUR is insignificant too. The only significant topic dummy is NUDE, which unexpectedly receives a negative sign. Unlike the "low part", the largest number of paintings in the "high part" are dated as being prior to 1890, as such it is set to 0, other periods that are significant receiving negative signs.

A.3.2 Parameters

	θ_i	Confidence interval: 95%	$exp(\theta_i) - 1$	N
INTERCEPT	8,6388	[8,4002 | 8,8774]		
Adjusted R^2	0,7118			
Low model				
Variables				
DIAGONAL	1,3361	[1,2075 | 1,4648]	280,43 %	
$AREA^2$	-0,1377	[-0,1754 | -0,1000]	-12,86 %	
DIFFASP	-0,1811	[-0,2930 | -0,0691]	-16,56 %	
MSCI World Index	0,5194	[0,3206 | 0,7182]	68,10 %	
Provenance variables				
PROVENANCE	0,2847	[0,2370 | 0,3323]	32,93 %	
EXHIBITION	0,3697	[0,2817 | 0,4576]	44,73 %	
LITERATURE	0,5350	[0,4294 | 0,6407]	70,75 %	

[1] As mentioned earlier, parameters that receive their expected sign and are significant at the 90% level are kept in the model.

Dummies				
Location				
NEW YORK	[set to 0]			1873
LONDON	0,1369	[0,0712 | 0,2026]	14,67 %	1241
Authenticity dummies				
INC(CR/CC)	0,3111	[0,2241 | 0,3982]	36,49 %	1022
AUTHENTICATED	0,1730	[0,0853 | 0,2606]	18,88 %	789
FROM ARTIST	0,1357	[0,0476 | 0,2239]	14,54 %	498
SIGNED	[set to 0]			2641
MARKED	-0,1349	[-0,2597 | -0,0100]	-12,62 %	228
Support dummies				
CANVAS	[set to 0]			2065
PAPER	-0,3586	[-0,4653 | -0,2520]	-30,14 %	1019
Medium dummies				
OIL	[set to 0]			1934
CRAYON	-0,6107	[-0,8170 | -0,4044]	-45,70 %	94
CHARCOAL	-0,6364	[-0,8864 | -0,3865]	-47,08 %	57
PENCIL	-0,6775	[-0,8471 | -0,5079]	-49,21 %	169
PEN	-0,7812	[-0,9596 | -0,6028]	-54,21 %	152
WATERCOLOUR	-0,1366	[-0,2542 | -0,0190]*	-12,77 %	295
OTHER	-0,5130	[-0,6734 | -0,3526]	-40,13 %	147
Topic dummies				
No topic found	[set to 0]			1192
PEOPLE	0,2619	[0,0986 | 0,4252]	29,94 %	115
STUDY	-0,2751	[-0,5004 | -0,0498]	-24,05 %	60
Period dummies				
NO DATE	[set to 0]			846
PRE-1890	0,3971	[0,2158 | 0,5784]	48,75 %	106
1890-1899	0,4674	[0,3003 | 0,6345]	59,99 %	122
1900-1909	0,3921	[0,2796 | 0,5046]	48,01 %	300
1910-1919	0,4061	[0,2938 | 0,5183]	50,09 %	290
1930-1939	0,2051	[0,0973 | 0,3128]	22,76 %	313
1950-1959	0,1228	[0,0190 | 0,2266]*	13,46 %	225

	θ_i	Confidence interval: 95%	$exp(\theta_i) - 1$	N
High model				
Variables				
DIAGONAL	1,5902	[1,3721 | 1,8083]	390,48 %	
$AREA^2$	-0,0628	[-0,0919 | -0,0337]	-6,08 %	
MSCI World Index	0,7650	[0,4458 | 1,0842]	114,89 %	
Provenance variables				
PROVENANCE	0,3678	[0,2482 | 0,4875]	44,46 %	
EXHIBITION	0,2006	[0,0989 | 0,3023]	22,22 %	
LITERATURE	0,7122	[0,6127 | 0,8118]	103,85 %	
Dummies				
Authenticity dummies				
INC(CR/CC)	0,2494	[0,0977 | 0,4012]	28,33 %	355
DEDICATED	-0,0877	[-0,1724 | -0,0060]*	-8,39 %	229
SIGNED	[set to 0]			517
MARKED	-0,5836	[-0,8362 | -0,3310]	-44,21 %	57
UNMARKED	0,3064	[0,0843 | 0,5286]	35,86 %	88
Support				
CANVAS	[set to 0]			
PAPER	-0,3628	[-0,5763 | -0,1493]	-30,43 %	172

Medium dummies				
OIL	[set to 0]			453
CRAYON	-1,1842	[-1,6518 | -0,7167]	-69,40 %	16
CHARCOAL	-0,6954	[-1,1359 | -0,2550]	-50,11 %	18
PENCIL	-0,9349	[-1,3150 | -0,5548]	-60,74 %	27
PEN	-1,6191	[-1,9913 | -1,2469]	-80,19 %	32
OTHER	-0,4921	[-0,8982 | -0,0861]	-38,87 %	21
Topic dummies				
NO TOPIC FOUND	[set to 0]			309
NUDE	-0,5891	[-0,9604 | -0,2177]	-44,52 %	23
Period dummies				
NO DATE	-0,9533	[-1,3566 | -0,5501]	-61,45 %	20
PRE-1890	[set to 0]			116
1900-1909	-0.2437	[-0,4524 | -0,0350]	-21,63 %	95
1920-1929	-0,4165	[-0,6312 | -0,2019]	-34,07 %	88
1930-1939	-0,3487	[-0,5888 | -0,1086]	-29,44 %	68
1940-1949	-0,5269	[-0,8240 | -0,2298]	-40,96 %	40
1960-1969	-0,9517	[-1,3382 | -0,5653]	-61,39 %	24

	θ	Confidence interval: 95%	$exp(\theta_i) - 1$	N
Artist dummies				
Mid Traded Artist	[set to 0]			421
Seldom Traded Artist	-0,6579	[-0,7617 | -0,5540]	-48,21 %	421
ALBERT ANDRÉ	-1,1793	[-1,5528 | -0,8058]	-69,25%	21
ALBERT GLEIZES	-0,4458	[-0,8854 | -0,0062]	-35,97%	15
ALBERT LEBOURG	-1,4487	[-1,8897 | -1,0077]	-76,51%	15
ALEXEJ VON JAWLENSKY	1,0105	[0,6280 | 1,3931]	174,71%	20
ALFRED SISLEY	0,6377	[0,2488 | 1,0267]	89,22%	20
AMEDEO MODIGLIANI	1,7152	[1,3341 | 2,0964]	455,79%	21
ANDRÉ DERAIN	-0,6083	[-0,9439 | -0,2726]	-45,57%	26
ANDRÉ LHOTE	-0,6977	[-1,0190 | -0,3763]	-50,22%	29
AUGUSTE HERBIN	-0,3391	[-0,6585 | -0,0197]	-28,76%	29
BERNARD BUFFET	-0,5655	[-0,8399 | -0,2910]	-43,19%	42
CAMILLE PISSARRO	0,5118	[0,2564 | 0,7671]	66,82%	49
CARLOS NADAL	-0,6246	[-1,0545 | -0,1947]	-46,45%	16
CHARLES CAMOIN	-0,7360	[-1,1181 | -0,3538]	-52,10%	20
CLAUDE MONET	0,6386	[0,3028 | 0,9745]	89,39%	29
DAVID BURLIUK	-0,8689	[-1,2107 | -0,5272]	-58,06%	26
EDGAR DEGAS	1,3102	[0,9887 | 1,6317]	270,69%	35
EGON SCHIELE	1,5154	[1,2454 | 1,7854]	355,12%	50
EMIL NOLDE	0,6563	[0,3255 | 0,9870]	92,76%	28
ÉMILE-OTHON FRIESZ	-0,9895	[-1,3903 | -0,5887]	-62,82%	18
EMILIO GRAU SALA	-0,4430	[-0,7306 | -0,1554]	-35,79%	37
FERNAND LÉGER	0,7602	[0,5268 | 0,9935]	113,87%	60
FRANCOIS GALL	-1,3435	[-1,7846 | -0,9023]	-73,91%	15
GEORGES BRAQUE	0,7062	[0,3784 | 1,0341]	102,63%	28
GEORGES D'ESPAGNAT	-1,0525	[-1,3947 | -0,7103]	-65,09%	25
GUSTAVE LOISEAU	-0,3008	[-0,5474 | -0,0543]	-25,98%	51
HENRI LE SIDANER	-0,5230	[-0,8000 | -0,2461]	-40,73%	41
HENRI LEBASQUE	0,2862	[0,0388 | 0,5336]	33,13%	51
HENRI MANGUIN	-0,8361	[-1,2613 | -0,4110]	-56,66%	16
HENRI MARTIN	0,4232	[0,1599 | 0,6865]	52,68%	45
HENRI MATISSE	1,9120	[1,6857 | 2,1383]	576,68%	67
HENRY MORET	-0,4953	[-0,8389 | -0,1518]	-39,06%	26

JEAN DUFY	-0,2997	[-0,5128 | -0,0867]	-25,90%	75
JEAN-BAPTISTE-ARMAND GUILLAUMIN	-0,8619	[-1,1264 | -0,5975]	-57,77%	46
JEAN-PIERRE CASSIGNEUL	-0,4160	[-0,8206 | -0,0114]	-34,03%	18
JOAN MIRÓ	1,1353	[0,8990 | 1,3716]	211,21%	57
KEES VAN DONGEN	0,9612	[0,7067 | 1,2157]	161,48%	47
LE PHO	-1,0688	[-1,4308 | -0,7068]	-65,66%	23
LOUIS VALTAT	-0,5310	[-0,7106 | -0,3514]	-41,20%	107
MAN RAY	-0,5380	[-0,9641 | -0,1119]	-41,61%	17
MARC CHAGALL	1,6203	[1,4464 | 1,7943]	405,49%	118
MAURICE UTRILLO	0,2313	[0,0063 | 0,4564]	26,03%	63
MAX ERNST	0,7654	[0,4069 | 1,1240]	114,99%	23
MAXIMILIEN LUCE	-0,8012	[-1,0029 | -0,5994]	-55,12%	81
MOÏSE KISLING	-0,3740	[-0,7113 | -0,0367]	-31,20%	26
PABLO PICASSO	1,6716	[1,5344 | 1,8088]	432,08%	258
PAUL KLEE	0,6640	[0,3332 | 0,9949]	94,26%	32
PAUL SIGNAC	0,2926	[0,0050 | 0,5803]	34,00%	39
PIERRE BONNARD	0,8610	[0,5404 | 1,1815]	136,54%	29
PIERRE-AUGUSTE RENOIR	1,0847	[0,8991 | 1,2702]	195,85%	99
PIERRE-EUGÈNE MONTÉZIN	-0,7405	[-0,9947 | -0,4863]	-52,31%	49
RAOUL DUFY	0,3779	[0,1878 | 0,5680]	45,92%	92
RENÉ MAGRITTE	1,5131	[1,2250 | 1,8012]	354,09%	40
SALVADOR DALÍ	0,7211	[0,4302 | 1,0121]	105,67%	36
TSUGUHARU FOUJITA	0,7165	[0,4510 | 0,9820]	104,73%	44
WASSILY KANDINSKY	1,1764	[0,7945 | 1,5583]	224,27%	23

Highly traded artists without significant parameter values:
ALBERT MARQUET, ANDRÉ MASSON, CHAIM SOUTINE, ÉDOUARD VUILLARD, EUGÈNE BOUDIN, FRANCIS PICABIA, GEORGE GROSZ, GEORGES ROUAULT, GUSTAV KLIMT, HENRY MOORE, O.M., C.H., JEAN METZINGER, LYONEL FEININGER, MARIE LAURENCIN, MARINO MARINI, MAURICE DE VLAMINCK, PAUL DELVAUX, REUVEN RUBIN.

A.4 EV hedonic regression results

Appendix A.4 presents the results of the hedonic regression using the extreme value approach outlined in section 4.2 with the *hammer price* as the dependent variable. Descriptive statistics for the data sample used for the hedonic regression can be found in appendix A.1. The final model is composed of a *Low* and a *High* part that depend on the number of provenance entries for each artwork. Both of these parts share artist dummies and intercept. MSCI's WORLD Standard (Large+Mid Cap) index is used as an external signal. The artist dummies are divided into three groups: *Seldom traded artists*, *Mid traded artists* and *Highly traded artists*. Artists in the two lower groups are aggregated while the artists in the highly traded group are given their own dummy variables. A complete list of the artists included in the different groups is presented at the bottom of the table. Variables that aren't statistically significant at the 95 % level using profile likelihood are removed from the final model.The exception from this are a few of the variables in the *High* and *Low* parts that are expected to have an impact on the price that are significant at the 90 % level using profile likelihood. These are marked with *. For each variable, I present the coefficient (θ_i), the 95 % or 90 % confidence interval, the price impact for the coefficient ($exp(\theta_i) - 1$) and the number of each dummy variable in the final model (N). Dummy variables with fewer than 15 values have been aggregated.

A.4.1 Discussion

Begining with the "low part" of the model, the LOCATION parameter receive a positive sign, meaning that paintings sold in London tend to attract a higher price. UNMARKED receive the expected negative sign and a point estimate lower than MARKED. Several support dummies receive significant values, all with negative sign, only CARD and OTHER being insignificant compared to CANVAS. All medium dummies are significant and receive their expected, negative, signs. The URBAN and ABSTRACT topic dummies receive a positive signs. PEOPLE, WOMAN/GIRL, HEAD receive their expected positive signs, but WOMAN/GIRL is only significant at the 90% level. STUDY receives its expected negative sign. Most period dummies receive their expected, positive signs, but POST–1970 unexpectedly receives a negative sign.

As with the OLS model, the "high part" offers a few surprises. AUTHENTICATED lacks significance, FROM ARTIST receives a negative sign where a positive sign was expected and UNMARKED receives a positive sign where a negative sign was expected. When it comes to medium dummies, GOUACHE, PASTEL and WATERCOLOUR aren't significant. Among the topic dummies, URBAN receives a positive sign and so do HEAD. NUDE and PORTRAIT unexpectedly receive negative signs. Among the period dummies, works created prior to 1890 is again set to 0. Compared to the OLS model, fewer periods are significant and receive their expected negative signs. The period from 1950–1959 receives a positive sign.

A.4.2 Parameters

	θ_i	Confidence interval: 95%	$exp(\theta_i) - 1$	N
INTERCEPT	8,3135	[8,2968 | 8,3384]		
Shape parameter	1,3097	[1,2809 | 1,3398]		
Low model				
Variables				
DIAGONAL	1,1919	[1,1597 | 1,2253]	229,12 %	
DIFFASP	-0,2315	[-0,2894 | -0,1776]	-20,67 %	
AREA^2	-0,1109	[-0,1435 | -0,0891]	-10,49 %	
MSCI World Index	0,4078	[0,3866 | 0,4290]	50,35 %	
Provenance variables				
PROVENANCE	0,2633	[0,2427 | 0,2838]	30,11 %	
EXHIBITION	0,3286	[0,2616 | 0,3940]	38,91 %	
LITERATURE	0,4614	[0,3612 | 0,5301]	58,62 %	
Dummies				
Location				
NEW YORK	[set to 0]			1873
LONDON	0,2133	[0,1707 | 0,2556]	23,78 %	1241

Authenticity dummies				
INC(CR/CC)	0,3688	[0,3209 | 0,4144]	44,49 %	1022
AUTHENTICATED	0,1376	[0,0838 | 0,1904]	14,76 %	789
SIGNED	[set to 0]			2641
MARKED	-0,1291	[-0,2305 | -0,0322]	-12,91 %	228
UNMARKED	-0,1491	[-0,2468 | -0,0555]	-13,85 %	245
Support dummies				
CANVAS	[set to 0]			2065
PAPER	-0,1486	[-0,1961 | -0,1023]	-13,81 %	1019
BOARD	-0,1917	[-0,3165 | -0,0736]	-17,45 %	152
PANEL	-0,1788	[-0,3584 | -0,0123]	-16,37 %	75
Medium dummies				
OIL	[set to 0]			1934
CRAYON	-0,9391	[-1,0997 | -0,7908]	-60,90 %	94
CHARCOAL	-0,7643	[-0,9722 | -0,5748]	-53,44 %	57
PENCIL	-0,9344	[-1,0530 | -0,8222]	-60,72 %	169
GOUACHE	-0,2505	[-0,3569 | -0,1488]	-22,16 %	207
PASTEL	-0,4158	[-0,6195 | -0,2291]	-34,02 %	59
PEN	-1,1414	[-1,2670 | -1,0238]	-68,06 %	152
WATERCOLOUR	-0,3173	[-0,4062 | -0,2320]	-27,19 %	295
OTHER	-0,6437	[-0,7711 | -0,5240]	-47,46 %	147
Topic dummies				
No topic found	[set to 0]			1192
URBAN	0,1028	[0,0025 | 0,1991]	10,82 %	232
PEOPLE	0,2284	[0,0845 | 0,3639]	25,66 %	115
WOMAN/GIRL	0,0714	[0,0061 | 0,1350]*	7,41 %	380
HEAD	0,2433	[0,0686 | 0,4058]	27,54 %	79
ABSTRACT	0,3061	[0,0931 | 0,5011]	35,81 %	54
STUDY	-0,4451	[-0,6472 | -0,2599]	-35,92 %	60
Period dummies				
NO DATE	[set to 0]			846
PRE-1890	0,4002	[0,2502 | 0,5411]	49,22 %	106
1890-1899	0,4162	[0,2768 | 0,5482]	51,63 %	122
1900-1909	0,2616	[0,1737 | 0,3466]	29,90 %	300
1910-1919	0,3517	[0,2624 | 0,4383]	42,15 %	290
1930-1939	0,2351	[0,1491 | 0,3184]	26,51 %	313
1940-1949	0,1197	[0,0162 | 0,2189]	12,71 %	218
1960-1969	0,1682	[0,0510 | 0,2798]	18,31 %	171
POST-1970	-0,1299	[-0,2499 | -0,0160]	-12,18 %	164

	θ_i	Confidence interval: 95%	$exp(\theta_i) - 1$	N
High model				
Variables				
DIAGONAL	1,3871	[1,3260 | 1,4467]	300,31 %	
DIFFASP	-0,2286	[-0,3679 | -0,0960]	-20,44 %	
AREA^2	-0,0545	[-0,0915 | -0,0268]	-5,30 %	
MSCI World Index	0,3873	[0,3412 | 0,4326]	47,30 %	
Provenance variables				
PROVENANCE	0,4018	[0,3761 | 0,4271]	49,45 %	
EXHIBITION	0,2258	[0,1890 | 0,2617]	25,33 %	
LITERATURE	0,6550	[0,6104 | 0,6988]	92,51 %	

Dummies				
Authenticity dummies				
INC(CR/CC)	0,3012	[0,2205 | 0,3795]	35,14 %	355
FROM ARTIST	-0,0987	[-0,1998 | -0,0020]	-9,40 %	229
SIGNED	[set to 0]			517
MARKED	-0,2198	[-0,4271 | -0,0299]	-19,73 %	57
UNMARKED	0,3390	[0,1739 | 0,4933]	40,36 %	88
Support				
CANVAS	[set to 0]			
BOARD	-0,5003	[-0,7725 | -0,2577]	-39,37 %	34
PAPER	-0,3285	[-0,4457 | -0,2174]	-28,00 %	172
Medium dummies				
OIL	[set to 0]			453
CRAYON	-1,3854	[-1,7940 | -1,0404]	-74,98 %	16
CHARCOAL	-0,4271	[-0,8094 | -0,1000]	-34,76 %	18
PENCIL	-1,1738	[-1,4813 | -0,9038]	-69,08 %	27
PEN	-1,2235	[-1,5049 | -0,9739]	-70,58 %	32
OTHER	-0,6502	[-1,0019 | -0,3459]	-47,80 %	21
Topic dummies				
NO TOPIC FOUND	[set to 0]			309
URBAN	0,3094	[0,0241 | 0,5634]	36,26 %	31
NUDE	-0,5870	[-0,9221 | -0,2952]	-44,40 %	23
PORTRAIT	-0,2895	[-0,5909 | -0,0235]	-25,14 %	28
HEAD	0,5498	[0,1281 | 0,9066]	73,29 %	15
Period dummies				
NO DATE	-0,7110	[-1,0721 | -0,3996]	-50,88 %	20
PRE-1890	[set to 0]			116
1900-1909	-0.2563	[-0,4152 | -0,1079]	-22,61 %	95
1920-1929	-0,1547	[-0,2763 | -0,0083]*	-12,92 %	88
1950-1959	0,4229	[0,0715 | 0,7278]	52,63 %	21

	θ	Confidence interval: 95%	$exp(\theta_i) - 1$	N
Artist dummies				
Mid Traded Artist	[set to 0]			421
Seldom Traded Artist	-0,5137	[-0,5882 | -0,4423]	-40,17%	421
ALBERT ANDRÉ	-0,7511	[-1,1025 | -0,4469]	-52,81%	21
ALBERT LEBOURG	-1,0485	[-1,4710 | -0,6930]	-64,95%	15
ALEXEJ VON JAWLENSKY	1,1617	[0,8016 | 1,4742]	219,53%	20
ALFRED SISLEY	1,2258	[0,8654 | 1,5383]	240,67%	20
AMEDEO MODIGLIANI	2,2767	[1,9261 | 2,5818]	874,42%	21
ANDRÉ DERAIN	-0,6232	[-0,9367 | -0,3478]	-46,38%	26
ANDRÉ LHOTE	-0,3359	[-0,6319 | -0,0742]	-28,53%	29
CAMILLE PISSARRO	0,9618	[0,7377 | 1,1667]	161,64%	49
CHAIM SOUTINE	0,4750	[0,1045 | 0,7947]	60,80%	19
CHARLES CAMOIN	-0,3768	[-0,7377 | -0,0652]	-31,39%	20
CLAUDE MONET	1,2005	[0,9051 | 1,4634]	232,17%	29
DAVID BURLIUK	-0,3675	[-0,6813 | -0,0919]	-30,75%	26
EDGAR DEGAS	1,3640	[1,0966 | 1,6040]	291,17%	35
ÉDOUARD VUILLARD	0,7828	[0,4978 | 1,0372]	118,75%	31
EGON SCHIELE	1,9691	[1,7486 | 2,1720]	616,45%	50
EMIL NOLDE	0,9530	[0,6519 | 1,2198]	159,35%	28
ÉMILE-OTHON FRIESZ	-0,8256	[-1,2079 | -0,4987]	-56,20%	18
EUGÈNE BOUDIN	0,2469	[0,0361 | 0,4403]	28,01%	55
FERNAND LÉGER	0,6903	[0,4887 | 0,8759]	99,42%	60
FRANCOIS GALL	-0,8333	[-1,2559 | -0,4775]	-56,54%	15

GEORGES BRAQUE	1,0917	[0,7915 | 1,3581]	197,95%	28
GEORGES D'ESPAGNAT	-0,6504	[-0,9710 | -0,3701]	-47,82%	25
GEORGES ROUAULT	0,5080	[0,2276 | 0,7585]	66,21%	32
GUSTAV KLIMT	0,7006	[0,4154 | 0,9549]	101,49%	31
HENRI LEBASQUE	0,5964	[0,3769 | 0,7968]	81,56%	51
HENRI MANGUIN	-0,5535	[-0,9609 | -0,2081]	-42,51%	16
HENRI MARTIN	0,8530	[0,6192 | 1,0662]	134,66%	45
HENRI MATISSE	2,0514	[1,8627 | 2,2279]	677,91%	67
HENRY MOORE, O.M., C.H.	0,3835	[0,0322 | 0,6883]	46,74%	21
JEAN-BAPTISTE-ARMAND GUILLAUMIN	-0,4219	[-0,6539 | -0,2118]	-34,42%	46
JOAN MIRÓ	1,3614	[1,1545 | 1,5520]	290,18%	57
KEES VAN DONGEN	1,1059	[0,8770 | 1,3149]	202,19%	47
LE PHO	-0,4517	[-0,7864 | -0,1599]	-36,35%	23
LYONEL FEININGER	0,3827	[0,1336 | 0,6078]	46,63%	40
MARC CHAGALL	2,005	[1,8645 | 2,1392]	642,52%	118
MARIE LAURENCIN	0,3916	[0,0905 | 0,6579]	47,93%	28
MAURICE DE VLAMINCK	0,2918	[0,1161 | 0,4552]	33,88%	78
MAURICE UTRILLO	0,7717	[0,5757 | 0,9531]	116,35%	63
MAX ERNST	1,1088	[0,7750 | 1,4015]	203,07%	23
MAXIMILIEN LUCE	-0,4362	[-0,6089 | -0,2761]	-35,35%	81
PABLO PICASSO	1,9765	[1,8816 | 2,0694]	621,74%	258
PAUL DELVAUX	0,6625	[0,2557 | 1,0083]	93,95%	16
PAUL KLEE	0,8839	[0,6037 | 1,1341]	142,04%	32
PAUL SIGNAC	0,6999	[0,4473 | 0,9281]	101,36%	39
PIERRE BONNARD	1,3265	[1,0320 | 1,5892]	276,80%	29
PIERRE-AUGUSTE RENOIR	1,3281	[1,1727 | 1,4742]	277,38%	99
PIERRE-EUGÈNE MONTÉZIN	-0,3873	[-0,6116 | -0,1832]	-32,11%	49
RAOUL DUFY	0,5634	[0,4023 | 0,7144]	75,67%	92
RENÉ MAGRITTE	1,4778	[1,2295 | 1,7039]	338,32%	40
REUVEN RUBIN	0,5006	[0,0936 | 0,8465]	64,97%	16
SALVADOR DALÍ	0,9598	[0,6968 | 1,1968]	161,11%	36
TSUGUHARU FOUJITA	1,0342	[0,7973 | 1,2493]	181,29%	44
WASSILY KANDINSKY	1,1817	[0,8473 | 1,4747]	225,98%	23

Highly traded artists without significant parameter values:
ALBERT GLEIZES, ALBERT MARQUET, ANDRÉ MASSON, AUGUSTE HERBIN, BERNARD BUFFET, CARLOS NADAL, EMILIO GRAU SALA, EUGÈNE BOUDIN, FRANCIS PICABIA, GEORGE GROSZ, GUSTAVE LOISEAU, HENRI LE SIDANER, HENRY MORET, JEAN DUFY, JEAN METZINGER, JEAN-PIERRE CASSIGNEUL, LOUIS VALTAT, MAN RAY, MARINO MARINI, MOÏSE KISLING.

Mid traded artists:
ACHILLE LAUGÉ, ALBERTO GIACOMETTI, ALFRED KUBIN, ANDRÉ DUNOYER DE SEGONZAC, ARISTIDE MAILLOL, BALTHUS, BEN NICHOLSON, O.M., BLANCHE HOSCHEDÉ-MONET, CAMILLE BOMBOIS, CLAUDE-ÉMILE SCHUFFENECKER, CONSTANTIN TERECHKOVITCH, EDVARD MUNCH, ERNST LUDWIG KIRCHNER, FERDINAND DU PUIGAUDEAU, FRANCISCO BORÈS, HENRI DE TOULOUSE-LAUTREC, HENRI FANTIN-LATOUR, HENRI-EDMOND CROSS, HERMANN MAX PECHSTEIN, HONORÉ DAUMIER, ISMAËL DE LA SERNA, JACQUES MARTIN FERRIÈRES, JACQUES VILLON, JAMES ENSOR, JUAN GRIS, JULES PASCIN, KURT SCHWITTERS, LE CORBUSIER, LÉON DE SMET, LOVIS CORINTH, MAURICE DENIS, MAX BECKMANN, MAX LIEBERMANN, ODILON REDON, OSSIP ZADKINE, OTTO DIX, PAUL CÉZANNE, PAUL GAUGUIN, PAUL SÉRUSIER, PIET MONDRIAN, TAKANORI OGUISS, TAMARA DE LEMPICKA, THÉO VAN RYSSELBERGHE, VINCENT VAN GOGH, WLADYSLAW SLEWINSKI.