Lund University

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First-Year Master thesis, June 2013

The Role of Gold in a Portfolio in Different Market Conditions

Is gold still an attractive investment after the financial crisis in 2008?

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ABSTRACT

This thesis examines whether gold is still attractive to be invested as one of the portfolio component after the financial crisis in 2008. Besides, the study suggests the appropriate weight of gold in the portfolio investment during the normal and crisis periods. This paper uses the U.S. stock, bond, and gold data from 1997 until 2013 to investigate optimal weights by constructing the optimal portfolio obtained by the Variance Minimization and the Sharpe Ratio Maximization under the Markowitz Mean-Variance framework. The result indicates that gold gradually decreases its importance through time, particularly the last study period after the U.S. debt crisis in 2011. The two optimization frameworks show the same outcome of significant drop in the gold's weight in the last study period, resulting from both its bad performance and the highly positive correlation between gold and stock. Regarding to the two different market conditions, investors are suggested to invest more proportion of gold during the crisis period than that during the normal period. However, the level of the optimal weight depends on different investment objectives, which suggests a higher fraction under the Sharpe Ratio Maximization framework.

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1. INTRODUCTION

1.1. Background and Literature Review

1.1.1. Asset Price Bubble and Quantitative Easing

Since the incident of global financial crisis in 2008, three of the world's largest central banks namely the U.S. Federal Reserve, the European Central Bank and the Bank of Japan have implemented Quantitative Easing (QE) and other asset purchase programs in hope to revive severe economic downturn. Nevertheless, this unorthodox way of pumping money into the economy leads to a big surge in money supply. One of the apparent effects of excess liquidity is the devaluation of the U.S. dollar. As the price of gold is often tied to the value of the dollar, what is bad for the dollar is usually good for the gold's price. Precious metal like gold is regarded as a safe haven asset because the price typically increases during times of inflation and devaluation of the dollar. As the market expects that the dollar will be further devalued in the future due to a high possibility of more QE from the three major industrialized countries in order to stimulate their not-fully-recovered economy, the price of gold is also anticipated to remain in an upside trend.

In addition, stock market exhibits similar behavior regarding to the injection of QE. The stock price goes up every time there is an expectation of future money injection. Hence, it seems that this unorthodox way of pumping money into the economy lead to a possibility of a co-movement between gold and stock and a so-called "asset price bubble" phenomenon.

Fundamentally, during high market uncertainty and economic downturn, investors usually include gold in their investment portfolio to reduce potential loss due to the crash in stock market. The reason is because it exhibits safe haven characteristics, i.e., uncorrelated or low positive or negative with equity during bad market condition. Nevertheless, since a heavy injection of QE after financial crisis 2008, it appears to be that this characteristic of gold gradually disappears. Instead, gold becomes a similar asset class to stock, which its price moves in accordance to market condition.

1.1.2. Gold as a Stand-alone Asset

"Gold has a long history as both a store of value and a medium of exchange, due to its unique properties as an efficient hedge against inflation, political instability, and currency risk." Johnson and Soenen (1997).

Many research papers support the idea that ***gold is an inflation-hedging asset*****.** Ghosh, Levin, Macmillan, and Wright (2004) confirm that gold can be considered as a long-run inflation hedge. The more recent study by Worthington and Pahlavani (2007) suggest that gold is the inflation hedge if there is a strong co-integrating relationship between gold and inflation.

Gold's currency-risk hedging property is also supported by some studies. Capie, Mills, and Wood (2005) conclude that gold has been a hedge against the U.S. dollar as gold cannot be reproduced by monetary authorities like currency. The more current studies confirm that gold is a hedge against the U.S. dollar. One of them is Joy (2011) investigates the relationship between the price of gold and the U.S. dollar based on multivariate GARCH model of dynamic conditional correlations. The result shows that the increase in gold price tends to be associated with the decrease in the value of the U.S. dollar.

In terms of ***hedging against negative stock market environment***, Baur and McDermott (2010) investigates whether gold is both a safe haven and hedge asset by using descriptive and econometric analysis under the study period of 1979-2009. "Basically, a strong (weak) hedge is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio on average. Furthermore, a strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets". The study finds that gold is both a strong safe haven and hedge asset for developed countries but not for Australia, Canada, Japan, and large emerging markets such as the BRIC countries.

1.1.3. Gold as an Alternative Investment in a Portfolio

Due to its return normally independent from other assets, gold has played a significant role in the portfolio diversification. There are many papers examine whether gold improves the portfolio's performance. Although the method utilized in their study is different, most of the papers divide the whole sample period into several subsamples for the same purpose.

Chua, Sick and Woodword (1990) investigates whether there is a diversification benefits when adding gold bullion and gold stock into the U.S. common stock portfolio during 1971-1988 using CAPM and Markowitz mean-variance framework. They conclude that investors can rely on gold bullion as an investment for portfolio diversification both in the entire study period and subsample periods.

Johnson and Soenen (1997) investigates whether investing in gold is an attractive asset choice for investors in seven major industrialized countries: Canada, France, Germany, Japan, Switzerland, the U.K., and the U.S. during 1978-1995 by using method developed by Graham and Harvey (1994). Gold proved to improve investment portfolio of Canada, France, Germany, Japan, and Switzerland for only the sub-period of 1978-1983, whereas there is no evidence of positive impact of adding gold to investment portfolio of the U.S. and U.K. for any study period. The average weight of gold for the entire period of study for France, Germany, Japan, Switzerland, and the U.K. is approximately 22 percent. While there is no weight of gold suggested for Canada and the U.S.

According to Demidova-Menzel and Heidorn (2007), their paper examines the role of gold on portfolio investment from the perspective of the U.S. and European investors during 1974-2006 and 1998-2006, respectively. The study periods are divided into three sub-periods for the U.S. portfolio and two sub-periods for Euro portfolio. They show that adding gold improves USD and Euro investment portfolio in the period when gold's return is substantial and its correlation is low with other assets. However, this is not true in the period when correlation between stock and gold is still low but its return is almost zero.

Additionally, Dempster and Artigas (2010) examine how gold has performed relative to the other three traditional inflation hedges: TIPS, S&P GSCI, and BB REITs during three periods of the study: 1974–2009, 1993–2009, and 1997–2009 by using Sharpe ratio. They show that the four potential inflation hedges, gold is proved to be the most effective portfolio diversifier against the other assets held by the typical U.S. investor. The required allocation to gold in the portfolio in order to achieve minimum variance ranges from 4.0 to 6.3 percent, while the required allocation to achieve the maximum Sharpe ratio ranges from 7.0 to 9.9 percent.

The studies mentioned above show that the correlation between the gold's return and the asset's return in the portfolio such as stock price and bond remains low throughout the study period, implying the presence of diversification benefits to the given portfolio. However, the positive impact of gold on investment portfolio's performance depends on how the period of study is divided. During the sub-period of study when gold performs well, adding gold improves portfolio's performance. On the other hand, during the sub-period when gold's performance is bad, gold should not be included as it deteriorates the overall performance of the portfolio.

Until the latest research in 2010 from Baur and Lucey, they study the role of gold in different market conditions (bull or bear market) to examine the property of gold as a hedge or a safe haven by performing a regression model. The U.S., U.K. and German stock and bond's return and gold's returns since 1995 to 2005 are used for the sample of the study. Their empirical finding shows that gold is a safe haven for stocks in extreme negative market condition. However, when they explore only specific length of time, it shows that gold functions as a safe haven around 15 trading days. If investors hold it more than 15 days, they lose money from gold investment.

After reviewing the previous researches, it is possible to conclude that although gold itself has many benefits such as inflation hedge and currency hedge, it turns out that including gold in a portfolio does not always enhance the portfolio's performance. Moreover, currently there is a new factor such as the QE that possibly affects the investment perspectives in gold.

1.2. Motivation

After the QE is implemented, the return of gold and stock tends to have positive relationship, indicating by some possible change in their correlation. According to Sharpe (1966), the correlation between gold's return and asset's return in the portfolio and the performance of gold are two components contributing to the improvement in the portfolio's performance. Therefore, when considering the uncertainty in the performance of gold and the possibility of fading in its diversification benefits after 2008, the issue whether *gold is still an attractive asset component in a portfolio after the financial crisis in 2008* is interesting to be further studied.

In the sense of investment perspective in gold, although this question is possibly considered to be the most important, there are several investment aspects that are unable to be ignored. One of them is providing the appropriate proportion of gold in the portfolio to investors by taking different market conditions into account. This specific study is motivated by two main parts. The first one is that the optimal weight of gold in a portfolio is suggested by some previous studies, but the data is not divided by taking the different market environment into account. Based on the latest paper of Baur and Lucey (2010), their findings implicitly indicate the importance of considering the role of gold in different market conditions, especially the crisis period. This is the second motivation part that leads this study to investigate the appropriate weight of gold when different market environment is taken into the consideration. Besides the study focusing on the period after the financial crisis in 2008, the author is motivated to expand and divide the study period to cover all the crises in stock market for the past twenty years. At least, this thesis is able to contribute more insightful aspect of the suitable weight for different market conditions, which is an important part of the investment perspective in gold.

1.3. Purpose

The purpose of this thesis is to investigate whether gold is still attractive to be invested as one of portfolio components after the financial crisis in 2008. Furthermore, the study aims to fulfill other essential investment perspectives for investors who are considering gold as an alternative asset in their portfolios by suggesting the appropriate weight of gold in the investment portfolio for the different market conditions: normal and crisis periods. In order to see the whole investment picture of gold in each market condition, this paper uses the U.S. stock, bond, and gold data from 1997 until 2013 to investigate optimal weights by constructing the optimal portfolio obtained by the Variance Minimization and the Sharpe Ratio Maximization under the Markowitz Mean-Variance framework.

Research questions

(1) Does gold still remain importance in the portfolio after the financial crisis 2008?
  - Is there any significant change in correlation pattern among stock, bond and gold before and after the crisis?
  - Does the optimal proportion of gold in a portfolio significantly change before and after the crisis?

(2) How much proportion of gold should be invested in the portfolio for different investment objectives and different market conditions?

(3) After including gold in a portfolio, what is a pattern of the optimal weight regarding to the change in market condition over time?

1.4. Thesis Structure

This paper is organized as follow. After this introduction, section 2 provides the theoretical background. Sections 3 describes the data and methodology. Section 4 analyzes the basic statistical result of each individual asset. Section 5 discusses the empirical result. Section 6 presents the concluding discussion.

2. THEORETICAL BACKGROUND

This part of the thesis introduces some key concept and definition that are applied throughout the rest of this paper. Before scoping down details to a portfolio selection, this section begins with the concept of risk measure, which is one of important components for Markowitz Mean-Variance optimization. The remainder of this sections devote for introducing the modern portfolio theory pioneered by Harry Markowitz, which is extensively used in this study for optimizing portfolio. Then, the concept of Sharpe ratio is provided.

2.1 Risk Measure

According to Capital Asset Pricing Model (CAPM), there are two types of risk which are relevant to portfolio management. The two types of risk are the total risk of a portfolio and the market risk or systematic risk (Fama and French, 2004). Portfolio's total risk is measured by variance or standard deviation of the portfolio while market risk is quantified by the portfolio's beta.

Total risk of the portfolio is the function of correlation coefficient and variance of each asset in the portfolio. The key to reduce the total risk is the correlation. This thesis emphasizes on the role of gold in the portfolio. Therefore, the correlation of gold and the other assets is examined. If the correlation coefficient is sufficiently low, gold will reduce the total risk of the portfolio without reducing its expected return. It implies that at the same level of expected return the risk of the portfolio decreases. This phenomenon is called diversification benefits.

Another type of risk that is pertinent to this thesis is the market risk. It plays a significant role in the study period because one of the thesis's purposes is investigating the importance of gold in different market conditions. The systematic risk of the portfolio is the weighted average of the betas of each asset (Chua, Sick and Woodward, 1990). Fundamentally, it is the risk that cannot be eliminated through diversification. According to Basel Committee on banking supervision, there are several main sources of market risk that impacts the movement of market price such as interest rate risk, foreign exchange rate risk, commodity position risk and equity position risk (Basel Committee on Banking Supervision, 1996).

2.2 Portfolio Optimization

In the Mean-Variance framework of Markowitz, return is quantified as expected return or mean and variance is the measure of risk. One of the important assumptions is that investors maximize return and minimize risk. For investors allocating their specified assets in the portfolio, there are a number of approaches to optimize a portfolio relative to their investment objectives. However, only methods relevant to this study are discussed.

2.2.1. Minimum Variance Portfolio (MVP)

For investors that aim to achieve minimum variance, the following optimization problem minimizes the variance of the given portfolio (Kempf and Memmel, 2002):

$$\text{Minimize } w'Vw$$

$$\text{Subject to } w'e = 1$$

V is the variance-covariance matrix of assets in the portfolio. w is the vector of the portfolio's weight and e is the column vector of 1. The result of the optimization leads to Minimum Variance Portfolio. The weights, w_{mv} of the Minimum Variance Portfolio are given by:

$$w_{mv} = \frac{V^{-1}e}{e'V^{-1}e}$$

The expected return and the portfolio's variance of the Minimum Variance Portfolio are written as:

$$\mu_{mv} = \mu' w_{mv} = \frac{\mu' V^{-1}e}{e'V^{-1}e}$$

and

$$\sigma_{mv}^2 = w'_{mv} V w_{mv} = \frac{1}{e'V^{-1}e}$$

Recently, Minimum Variance Portfolio has prompted an interest from investors and researchers due to its advantages over other approaches of portfolio optimization. One of the most important benefits is that this method does not require estimating expected portfolio's return, which is difficult to estimate. Moreover, the inaccurate estimation can lead to suboptimal portfolio selection and poor performance of the portfolio (Jorion, 1991). All stocks are assumed to have equal expected returns with different risk. As a result, the component of the Minimum Variance Portfolio depends only on the covariance matrix of stock returns. As the covariance matrix can be approximated much more accurately than the expected returns, the risk of estimation is expected to be reduced by employing Minimum Variance Portfolio as an approach for the portfolio optimization (Kempf and Memmel, 2002).

2.2.2. Sharpe Ratio

Recent literatures about the portfolio optimization in different types of alternative investment assets employ Sharpe Ratio Maximization as an approach to achieve optimal portfolio and asset allocation. This measure, based on Capital Market Line (CML), is defined as the expected return in excess of the risk-free rate over its standard deviation (Sharpe, 1966). It is expressed as (Taylor and Francis Group, LLC, 2007)

$$\frac{E(R_p)-R_{rf}}{\sigma_p}$$

Where $E(R_p)$ is the expected return of a portfolio

R_{rf} is the risk-free rate

σ_p is the standard deviation of a portfolio

Basically, there are two forms of Sharpe ratio classified by its purpose. The Sharpe ratio can be used ex post, meaning after the event, or ex ante, meaning in the future. Ex post version of Sharpe ratio is typically used as a performance assessment for a portfolio over a specific period of time. However, when the Sharpe ratio is in ex ante form, it is used to make a prediction how the portfolio is likely to perform in the future. The utilization of two versions of Sharpe ratio is justified by the supposition that the portfolio's return distribution is constant over time (Hodges, Taylor and Yoder, 1997).

In fact, Sharpe ratio exhibits a number of shortcomings. Firstly, it assumes frictionless financial markets. As a result, it is possible to borrow to invest more than 100 percent in the risky asset. This is not always achievable. Another drawback is that the risk-free rate is constant and identical for lending and borrowing. In its computation, the selection of risk-free rate is significant because it affects rankings of the mutual funds. However, the impact is minimal. Furthermore, its interpretation is difficult when it results in a negative number, which means that if risk increases, the Sharpe ratio also increases (Cogneau and Hubner, 2009).

Despite its drawbacks, it is still commonly employed by financial institutions to evaluate and compare the performance of mutual funds. This is because of its simplicity to compute and interpret.

3. DATA AND METHODOLOGY

3.1. Data

In order to construct optimal portfolios to analyze the whole investment picture of gold in different market conditions, S&P 500 composite total return index for the U.S. from Bloomberg, the 10-year benchmark U.S. government bond return index, London Gold Bullion (U.S. dollar per Troy ounce) and the 3-month U.S. Treasury Bill rate for the risk-free rate from Thompson Financial DataStream are used. The data cover a 17 year from January 1, 1997 until May 1, 2013, leading to a sample size of 4261 observations for daily return.

One of the reasons of selecting only stock, bond and gold in the portfolio is that the study focuses on the role of gold in the traditional portfolio and gold is assumed to be only alternative investment. From the paper of Gurnani, Hentschel and Vogt (2012), the traditional portfolio comprises mainly of bonds and listed equity. Another reason is that including more sophisticated type of financial assets such as futures and option possibly makes the result more difficult to analyze and interpret. For selecting the proxy of the stock market, the U.S market is the most developed and generally used as a benchmark for other stock markets. The S&P500, including 500 leading companies in major industries of the U.S. economy and representing approximately 70 percent of the total market capitalization, reflects the movement of the U.S. market as a whole. Besides the ability of indicating the movement, the S&P 500 index is more diversified than the DJIA. The S&P 500 composite used in this study is the total return index rather than the price index. The total return index takes both the capital gain and dividend payment into account, which superior reflects what investors consider and require from the investment.

The study period from 1997 is able to capture 4 extreme events of the U.S. stock market, which are specified as a crisis period. They are the Dot-Com Bubble, the Dot-Com Bubble Burst, the global financial crisis in 2008 and the U.S. debt crisis in 2011. The rest of these periods are considered as a normal period. Through the rest of this paper, this type of subsampling is called *the market condition approach*. In order to represent the major circumstance of the market, firstly the approximated date of each event is gathered. Then, the periods are chosen by calculating the peaks and troughs within the full sample of interest (See Appendix Exhibit 1). The next section describes the methodological method implemented for this study.

3.2. Methodology

The statistical calculation and portfolio optimization in this thesis are entirely conducted based on the Excel program. The first step is transforming the data into daily return. The 10-year benchmark U.S. government bond return index, London Gold Bullion and S&P 500 composite total return index are converted to the daily return by using the formula$\left(\frac{P1}{P0} - 1\right)$.

For the 3-month US Treasury Bill rate, the formula is$\left(1 + \frac{R}{(100*4)}\right)^{\wedge\left(\frac{1}{90}\right)} - 1$.

In order to examine the risk and return of each individual asset for the whole study period, the basic statistic is obtained by using Descriptive Statistics function in the Data Analysis tools. Next, the data is divided into seven subsample periods based on the market condition which are

(1) Dot-Com Bubble between 1 Jan 1997 and 1 Sep 2000
(2) Dot-Com Bubble Burst between 4 Sep 2000 and 9 Oct 2002
(3) Normal period between 10 Oct 2002 and 9 Oct 2007
(4) Global Financial Crisis 2008 between 10 Oct 2007 and 9 Mar 2009
(5) Normal period between 10 Mar 2009 and 29 Apr 2011
(6) U.S. Debt Crisis between 2 May 2011 and 3 Oct 2011
(7) Normal between 4 Oct 2011 and 1 May 2013

Then, functions and tools in the Excel program are used to calculate four important factors of each individual asset. The four factors related to the study are

(1) Average daily return: The function is AVERAGE.
(2) Standard deviation: The function is STDEV.P.
(3) Correlation is calculated by using Correlation function in the Data Analysis tools.
(4) Shape Ratio of each individual asset

$$\frac{E(R_i) - R_{rf}}{\sigma_i}$$

Where $E(R_i)$ is the average return of an asset obtained from (1)

R_{rf} is the average return of the 3-month U.S. Treasury Bill rate

σ_i is the standard deviation of an asset obtained from (2)

In addition, to be able to answer the research questions of this thesis the data is further divided into 24 subsample periods corresponding to six months (180 days) and 12 subsample periods corresponding to 1 year (360 days) of historical daily return (See Appendix Exhibit 2). Through the rest of this paper, this type of subsampling is called *the fixed estimation window approach.* Investors are assumed to reallocate their portfolio every 180 or 360 days. Although there is some event occurs in the stock market, the investors cannot suddenly adjust their portfolio in the middle of their fixed holding period. Therefore, it is possible that there is a lagged adjustment in some period of time. The reason behind this fixed estimation window approach is that this thesis aims to investigate the change in optimal weight of gold through time. In order to see the trend over time, the analysis requires more observations. Therefore, more subsample periods lead to more observations of the weight, but conversely too short estimation period causes the unstable result (Bengtsson and Holst, 2002). Moreover, in general short-term investors, defined as the 1 year or less investment horizon, usually adjust their portfolio twice or once a year. Therefore, regarding to the 17-year length and the practical investment strategy of investors, the 180 and 360 holding period is selected for this study.

Next step, the optimal portfolio, comprising of the U.S. stock, bond and gold, is constructed based on the Markowitz Mean-Variance framework. The Mean-Variance optimization has different strategies for a portfolio selection. However, there are only two moments taken into account, which are the probability distributions of the asset returns and variance. As a result, a rational investor maximizes expected returns given the acceptable level of risk, or alternatively, minimizes the variance given a certain willingness of expected returns. Nonetheless, in order to get rid of the pre-specific risk and return level, this thesis constructs a portfolio by minimizing variance and maximizing the risk-adjusted return measured by Sharpe ratio subject to same constraints, which are

(a) The portfolio weights sum to one. $\sum_{i=1}^{n} w_i = 1$

(b) Short sales is not allowed, i.e., $w_i \geq 0$.

There are several reasons to impose the short sales constraint. The first reason is that the short selling is very difficult or even impossible to do for an ordinary investor. As a result, no short selling restriction makes the portfolio selection problem more realism. (Bengtsson and Holst, 2002) Another reason is that short selling is more concerned since it impacts the patterns of stock prices and worsens the downturn in the crisis period. This claim is supported by the evidence of the Short Selling Ban announced by the U.S SEC during the financial crisis in 2008. A large group of financial stock is prohibited in short selling (Gruenewald, Wagner, and Weber, 2010). Therefore, in order to make the model more realistic and independent from the regulatory effect, the optimization model limits the short selling.

3.2.1 Variance Minimization Framework

Using this framework, the optimal weights lead to Minimum Variance Portfolio.

Model Framework:

$$\begin{aligned} & Min_w && w'Vw \\ & \text{Subject to} && w'e = 1 \\ & && w_i \geq 0 \end{aligned}$$

Where V is the variance-covariance matrix of assets in the portfolio

w is the vector of portfolio weight

e is the column vector of 1

3.2.2 Sharpe Ratio Maximization Framework

Using this framework, the optimal weights lead to a portfolio giving the highest excess return over a risk-free asset per a unit of total risk.

Model Framework:

$$\begin{aligned} & Max_w && \frac{E(r_p) - r_f}{\sigma_p} \\ & \text{Subject to} && \sum_{i=1}^{n} w_i = 1 \\ & && w_i \geq 0 \end{aligned}$$

Where w_i is the optimal weight of each asset

r_f is the risk-free rate

$E(r_p)$ is the expected return of a portfolio

σ_p is the standard deviation of a portfolio

Then, the total numbers of optimal portfolios are provided in the table 1 below. It clearly presents that the number is different among the three subsampling. The data divided based on the market condition (Normal/Crisis) generates seven optimal portfolios while the 180-day and 360-day estimation windows offer 24 and 12 portfolios, respectively. However, the table presents only a result under one optimization framework. As the thesis constructs the portfolio based on the Variance Minimization and the Sharpe Ratio Maximization, there will be two tables show the optimal weight from each optimization framework.

Lastly, in order to suggest the suitable gold's weight for the normal and crisis period and make the result comparable across the three subsampling, the optimal weights obtained from the fixed estimation window are assigned based on the period of each market condition. The average weight for each period is weighted by the number of days. More details are referred to the Exhibit 3 in the Appendix. Besides, the same calculation is repeated when computing the weighted average weight for the whole study period.

Table 1: Total Numbers of Optimal Portfolios from different subsampling

Subsample Criteria					
(1) Market Condition		**(2) Fixed Estimation Window**			
Normal/Crisis		**180-day**		**360-day**	
Period	*Weight*	*Period*	*Weight*	*Period*	*Weight*
1		1		1	
2		2		2	
3		3		3	
4		4		4	
5		5		5	
6		6		6	
7		7		7	
		8		8	
		9		9	
		10		10	
		11		11	
		12		12	
		13			
		14			
		15			
		16			
		17			
		18			
		19			
		20			
		21			
		22			
		23			
		24			

4. DATA ANALYSIS

All the four assets' time series data are converted into daily return for the descriptive statistics analysis (See Table 2). Over the sample period from 1 January 1997 to May 1, 2012, gold contributes the highest daily return at 0.0380 percent or 14.67 percent per year while S&P500 stock index unsurprisingly performs as the second best at 0.0331 percent or 12.64 percent per year. The risk-free asset generates the lowest return at 0.0071 percent or 2.57 percent per year.

In terms of volatility, S&P 500 has the highest standard deviation on average while the 3-month U.S. Treasury Bill rate has the lowest among the assets. The low volatility of the 3-month U.S. Treasury Bill rate is consistent with the fact that it is the risk-free asset therefore its return has to be less fluctuated. However, one of the reasons that S&P500 is the most volatile is that the study period covers three main crisis situations occurred in the past 17 years. In some certain period, stock is greatly fluctuated and encounters the extreme negative event.

Table 2: Statistical Description of the individual assets

FRTBS3M		USBD10Y index		S&P(TRI)		GOLDBLN	
Mean	0,0071%	Mean	0,0092%	Mean	0,0331%	Mean	0,0380%
Standard Error	8,7801E-07	Standard Error	7,37783E-05	Standard Error	0,000197346	Standard Error	0,000168
Median	5,4313E-05	Median	0	Median	0,000327247	Median	0
Mode	2,77743E-06	Mode	0	Mode	0	Mode	0
Standard Deviation	5,73133E-05	Standard Deviation	0,004815977	Standard Deviation	0,012882015	Standard Deviation	0,010971
Sample Variance	3,28482E-09	Sample Variance	2,31936E-05	Sample Variance	0,000165946	Sample Variance	0,00012
Kurtosis	-1,578644866	Kurtosis	2,814489737	Kurtosis	7,559068329	Kurtosis	6,495201
Skewness	0,137799733	Skewness	-0,017143529	Skewness	-0,026502969	Skewness	-0,19538
Range	0,00017201	Range	0,069688509	Range	0,206070475	Range	0,173244
Minimum	0	Minimum	-0,028326497	Minimum	-0,090259093	Minimum	-0,09663
Maximum	0,00017201	Maximum	0,041362011	Maximum	0,115811383	Maximum	0,076613
Sum	0,300735666	Sum	0,392709448	Sum	1,408946682	Sum	1,620857
Count	4261	Count	4261	Count	4261	Count	4261

When the data is divided into seven sub periods owing to the two market condition, the descriptive statistics is shown in the table 3. This section is outlined by firstly describing the return and risk of each asset in the crisis and normal period. Then, the correlation among three assets in the crisis and normal period and the change in the correlation through time are discussed.

Among the three crisis periods, the daily stock return on average significantly decreases approximately 0.1 to 0.2 percent. The Global Financial Crisis 2008 has the sharpest drop at 0.19 percent. On the other hand, bond always outperforms followed by gold. However, when the risk is taken into consideration, it is consistent with the fact that stock is the most volatile asset during the crisis, indicating with the higher average standard deviation of 0.02 than the average standard deviation of gold and bond of 0.01 and 0.006, respectively.

During the normal periods, it is consistent for every period that the stock return recovers from its previous crisis period. The stock return in the normal period after the Global Financial Crisis 2008 (10 March 2009 and 29 April 2011) gains the most at 0.14 percent. Moreover, it is noticeable to point out that gold also has higher return than its return in the former crisis. One of the reasons is likely that investors tend to allocate their portfolio to the safe haven asset like gold especially after the major uncertain situation just happened. Bond, which always outperforms in the crisis period, turns to be less interesting, reflecting by its negative return. It is possible to be explained that during normal period, investors are more confident in the market situation. Therefore, they reallocate their portfolio to risker assets such as stock and gold rather than holding the lower return asset like bond. In addition, the normal period after the U.S. debt crisis in 2011, gold no longer contributes a positive return. It is the first time that gold has the highest variation over stock but conversely generates unsatisfied average return of -0.03 percent.

When investigating the correlation pattern between each asset, the correlation between gold and stock is slightly negative and almost close to zero in some periods, indicating the diversification benefits to a portfolio. The most negative correlation is found in the U.S. debt crisis period, at -0.27. Surprisingly, in the latest normal period after the U.S. debt crisis in 2011, the correlation between gold and stock appears to be largely positive number of 0.3. This finding considerably contradicts with the latest research of Hood and Malik (2013). Based on daily data from November 1995 to November 2010, they use the regression model to analyze the safe haven and hedge property of precious metals including gold relative to S&P500 index. Their portfolio analysis shows that adding gold in a stock portfolio will contribute a safe haven and hedge especially during market downturns. Also, the negative correlation between gold and stock market supports strong diversification benefits of gold in the portfolio. Nevertheless, it is possible to conclude that after taking the crisis in 2011 into account, there is important change in the relationship between gold and stock.

In terms of correlation between bond and stock, it is independent with the market condition, i.e., it maintains the negative correlation for every period. However, the magnitude of the negative correlation is stronger in the U.S. debt crisis 2011 and its next normal period, which is approximately -0.6 compared to the correlation before the crisis of -0.3. Based on the paper of Stivers and Sun (2002), its higher negative correlation between bond and stock is supported by the higher uncertainty in the market.

The change in the correlation of gold to be positive implies that gold does not longer have the diversification benefits, but it turns out to offer higher risk to a portfolio when stock goes down. Conversely, the greater negative correlation between bond and stock possibly results in a bigger role of bond in the portfolio.

Table 3: Basic statistics of individual asset according to different market conditions

period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)			
Risk free rate	**0,01378%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	-0,0265%	0,0086	-0,0469
stock	0,0880%	0,0121	0,0614
bond	0,0012%	0,0040	-0,0318
Correlation	gold	stock	bond
gold	1		
stock	0,0097	1	
bond	-0,0985	0,0358	1

period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)			
Risk free rate	**0,00877%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	0,0288%	0,0078	0,0256
stock	-0,1068%	0,0144	-0,0802
bond	0,0267%	0,0048	0,0372
Correlation	gold	stock	bond
gold	1		
stock	-0,1822	1	
bond	0,1578	-0,3309	1

period 3: Normal period (10 Oct 2002-9 Oct 2007)			
Risk free rate	**0,00782%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	0,0696%	0,0102	0,0603
stock	0,0642%	0,0084	0,0671
bond	-0,0062%	0,0041	-0,0339
Correlation	gold	stock	bond
gold	1		
stock	-0,0428	1	
bond	0,0652	-0,2413	1

period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)			
Risk free rate	**0,00425%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	0,0768%	0,0187	0,0388
stock	-0,1901%	0,0235	-0,0829
bond	0,0442%	0,0070	0,0570
Correlation	gold	stock	bond
gold	1		
stock	-0,0850	1	
bond	0,0286	-0,4202	1

period 5: Normal period (10 Mar 2009-29 Apr 2011)			
Risk free rate	**0,00037%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	0,0977%	0,0107	0,0912
stock	0,1408%	0,0123	0,1144
bond	-0,0030%	0,0058	-0,0058
Correlation	gold	stock	bond
gold	1		
stock	0,0837	1	
bond	-0,0180	-0,3665	1

period 6: US Debt Crisis (2 May 2011-3 Oct 2011)			
Risk free rate	**0,00008%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	0,0752%	0,0148	0,0506
stock	-0,1713%	0,0169	-0,1015
bond	0,1219%	0,0059	0,2050
Correlation	gold	stock	bond
gold	1		
stock	-0,2667	1	
bond	0,3168	-0,6274	1

period 7: Normal (4 Oct 2011-1 May 2013)			
Risk free rate	**0,00021%**		
	mean	**Standard Deviation**	**Sharpe ratio**
gold	-0,0252%	0,0115	-0,0222
stock	0,1015%	0,0095	0,1065
bond	0,0084%	0,0042	0,0195
Correlation	gold	stock	bond
gold	1		
stock	0,2995	1	
bond	-0,1245	-0,6285	1

5. EMPIRICAL RESULT

In this section the result of the Variance Minimization and the Sharpe Ratio Maximization framework under two different subsample criteria are analyzed and compared. Under each sub period criterion, the discussion begins with the overview of gold's weight in the portfolio for the entire period of study as well as during the normal and crisis period. Then, the change in the weight of gold over different market conditions is presented. The last part of the discussion under each framework is the comparison of the two subsample criteria. This section ends with the comparison of the result across the two frameworks.

Table 4: Comparison the optimal weight of gold based on two subsample criteria (Market Condition and Two Fixed Estimation Window) by using two optimization frameworks. (Variance Minimization and Sharpe Ratio Maximization)

Minimizing Variance			
Period	weight of Gold		
	market condition	180 days	360 days
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	18,51%	19,73%	19,94%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	19,64%	19,38%	17,05%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	8,64%	10,06%	9,66%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	8,63%	9,49%	9,84%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	13,09%	8,10%	7,38%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	3,30%	5,04%	5,42%
period 7: Normal (4 Oct 2011-1 May 2013)	1,41%	1,82%	1,94%
Weighted average of total obsevations	**12,02%**	**12,20%**	**11,78%**
Weighted average of Normal Period	**11,40%**	**11,54%**	**11,33%**
Weighted average of Crisis Period	**13,90%**	**14,28%**	**13,21%**
Standard deviation	**6,50%**	**6,28%**	**5,87%**

Maximizing Sharpe Ratio			
	weight of Gold		
Period	market condition	180 days	360 days
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	0,00%	15,33%	17,61%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	26,86%	49,26%	70,54%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	42,72%	35,90%	38,71%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	19,98%	71,65%	53,61%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	31,16%	24,67%	22,52%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	0,00%	17,62%	16,55%
period 7: Normal (4 Oct 2011-1 May 2013)	0,00%	2,63%	4,46%
Weighted average of total obsevations	**22,35%**	**30,92%**	**33,34%**
Weighted average of Normal Period	**22,62%**	**23,62%**	**25,29%**
Weighted average of Crisis Period	**21,49%**	**53,88%**	**58,64%**
Standard deviation	**16,19%**	**21,64%**	**21,69%**

Note: Market condition refers to the sub period based on crisis and normal period.

5.1. Result of Variance Minimization Framework

The optimal portfolio obtained by minimizing variance implies type of investor who aims to minimize risk in their investment decision.

5.1.1. Sub Period based on Crisis and Normal Period

5.1.1.1. Overview

From 1 January 1997 until 1 May 2013 period, the average optimal weight of gold in the portfolio suggested by Minimum Variance Portfolio framework is approximately 12 percent. When considering two different market conditions, the model yields 11.4 percent on average for the normal period and 13.9 percent on average for the crisis period (See Table 4). These results suggest that different market conditions, at least under these periods of study, appears to affect the outcome to some extent. The conclusion is consistent with the finding of Baur and Lucey (2010) that gold is recommended to be included in the portfolio in order to lessen the impact of the crisis regarding to its safe haven property.

5.1.1.2. The Pattern of Change in Weight over Different Market Conditions

Under this subsample criterion, the result shows no pattern of higher gold's weight in the crisis period than that in the normal period. Another interesting finding is that the weight of gold is almost zero (1.4 percent) in the last study period after the U.S. debt crisis from 4 October 2011 to 1 May 2013 (See Table 4). Under the Variance Minimization framework, it is important to examine the correlation and the variance of each asset in the portfolio. Besides an increase in the importance of bond in the portfolio as discussed in the Data Analysis section, gold loses much of its diversification benefits in this period because the correlation between gold and stock turns to be quite highly positive number of 0.3 (See Table 3). Another reason is that the variance of gold in the last period becomes the highest although it always places as the second highest among the three assets in the previous periods (See Table 3).

5.1.2. Sub Period based on Fixed Estimation Window: 180 and 360 days

5.1.2.1. Overview

The optimal weight for gold investment in portfolio slightly varies under two different estimation windows. The suggested average investment proportion of gold for the 180-day estimation period is 12.2 percent and for the 360-day estimation period is 11.8 percent (See Table 4).

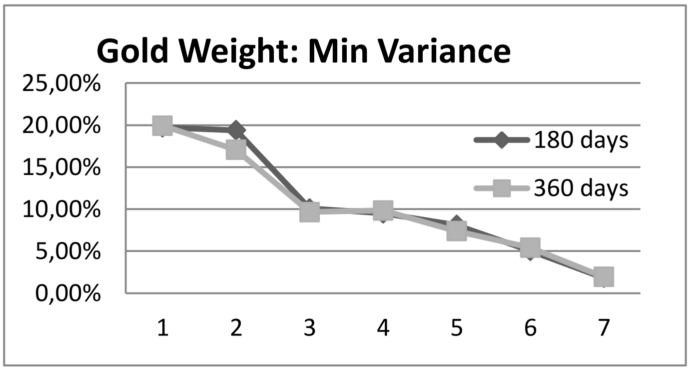
Moreover, the optimal weight of the 180-day estimation period has more variation than that of the 360-day estimation period. It is indicated by the standard deviation of 0.075 and 0.069 respectively (See Appendix Exhibit 2). The lower variation in the 360-day estimation period is consistent with the referred content of Bengtsson (2010). He refers the recommendation of Swedish Financial Supervisory Authority about the twelve-month estimation window, leading less impact from low probability event on the portfolio's weight.

When taking the market condition into account, under the 180-day estimation period the optimal weight in the crisis period is higher than that in the normal period. It is approximately 14.3 and 11.5 percent, respectively. This result is consistent with the 360-day estimation period, which provides the optimal weight of around 13.2 percent during the crisis period and 11.3 percent during the normal period (See Table 4). These results are also in line with the finding of Baur and Lucey (2010).

5.1.2.2. The Pattern of Change in Weight over Different Market Conditions

The results of the 180-day and 360-day estimation windows obviously show a pattern that after every crisis occurs, gold gains less proportion in the portfolio, indicating the less importance of gold after the crisis (See Figure 1).

Figure 1: The optimal weight of gold based on two fixed estimation windows under the Variance Minimization framework



Description
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)
period 3: Normal period (10 Oct 2002-9 Oct 2007)
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)
period 5: Normal period (10 Mar 2009-29 Apr 2011)
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)
period 7: Normal (4 Oct 2011-1 May 2013)

Moreover, the two different holding periods suggest almost no optimal weight of gold in the portfolio in the last period (4 October 2011 to 1 May 2013), only 1.82 percent for 180 days and 1.94 percent for 360 days (See Table 4). The explanation behind this phenomenon is similar to the reason discussed in the section of market condition criterion (5.1.1.2.).

5.1.3. Conclusion: Comparison between Two Different Subsample Criteria (Market Condition and Two Fixed Estimation Windows)

Under the Variance Minimization framework, all of two subsample criteria provide consistent picture about the optimal weight of gold in the portfolio. The appropriate proportion of gold is approximately 12 percent for the entire study period. Additionally, they suggest a higher optimal gold's weight in the crisis than that in the normal period. One possible explanation is that the pattern of variance and correlation among the three assets is the same. Also, there is the minimal change in different market conditions, especially during the crisis period. Thus, the adjustment of the optimal weight in the fixed estimation window does not provide any different outcome comparing to the market condition criterion.

Furthermore, the two different subsample criteria show the similar declining trend of gold's weight in the portfolio over time. It is noticeable that the last period after the U.S debt crisis in 2011, the optimal weight of gold drops to roughly 1 to 2 percent.

However, when considering the pattern of change in weight of gold over different market conditions, the two different subsample criteria display somewhat different view. While the market condition criterion yields no pattern in gold's weight in the portfolio, the 180-day and 360-day estimation windows demonstrate that after every crisis occurs, gold gains lesser proportion in the portfolio.

5.2. Result of Sharpe Ratio Maximization Framework

The optimal portfolio obtained by maximizing portfolio implies type of investor who considers both risk and return in their investment decision. The investors choose a portfolio giving the highest excess return over the risk-free rate under its unit of risk measured by the standard deviation.

5.2.1. Sub Period based on Crisis and Normal Period

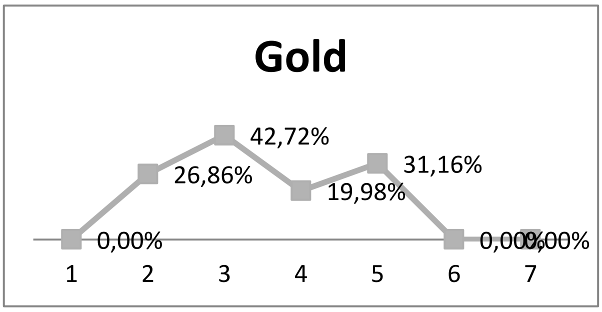
5.2.1.1. Overview

For the whole sub period based on the market condition, gold has an average proportion in a portfolio approximately at 22.3 percent. Nevertheless, when taking the average weight of gold conditioned on the crisis and normal time, the result suggests higher gold's proportion during the normal period at 22.6 percent than that during the crisis period at 21.5 percent (See Table 4). Although the outcome seems contrast with the paper of Baur and Lucey (2010) mentioned in the motivation part above, the optimal weight during the normal period is not significantly higher than the other.

5.2.1.2. The Pattern of Change in Weight over Different Market Conditions

The result obviously shows a pattern that after every crisis happens, gold gains more proportion in the portfolio (See Figure 2). For instance, after the Dot-Com Bubble Burst in 2002 the optimal weight of gold increases by roughly 60 percent in the next normal period. The main reason is that the performance of gold improves by approximately 140 percent from the bubble burst period. However, analyzing the Sharpe Ratio Maximization is essential to consider the other assets' return relative to their risk in the portfolio. The gold is closely well-performed with stock while it significantly outperforms bond. Regarding to the Sharpe ratio of each individual asset, gold and stock contribute higher Sharpe ratio over bond (See Table 3). Consequently, it is consistent with the result of decrease in the bond's proportion.

Figure 2: The optimal weight of gold based on the crisis and normal period under the Sharpe Ratio Maximization Framework



Description
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)
period 3: Normal period (10 Oct 2002-9 Oct 2007)
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)
period 5: Normal period (10 Mar 2009-29 Apr 2011)
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)
period 7: Normal (4 Oct 2011-1 May 2013)

However, since the latest U.S. Debt crisis in 2011, the pattern has changed from the previous periods. The optimal weight of gold in the portfolio drops to zero percent. The important factor is increase in the importance of bond. This situation is possible to be explained by the better bond's performance relative to its risk because investors turn to safer assets during highly uncertain market. Moreover, gold itself turns to be a risker asset but generating lower return, especially in the last study period that the return of gold is negative, but the standard deviation is highest among the three assets. Additionally, it cannot be refused that another reason is the impact of dramatic change in the correlation among these three assets, which is already explained in the Data Analysis above.

5.2.2. Sub Period based on Fixed Estimation Window: 180 and 360 days

5.2.2.1. Overview

For the 180-day holding period, the average of optimal weight in gold is slightly lower than the 360-day holding period, which is 30.92 percent and 33.34 percent, respectively (See Table 4).

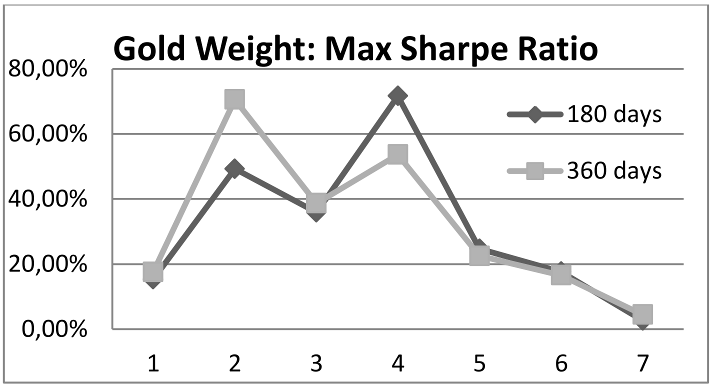
In terms of the variation of gold's weight obtained from the two different holding periods, the result shows that the shorter holding period has higher variation. The standard deviation of optimal weight for 180-day period is at 0.31 while the volatility of optimal weight for 360-day period is at 0.28 (See Appendix Exhibit 2).

When the optimal weight is assigned by taking the market condition into account, the result is noticeably different between the crisis and normal period. It indicates that during the crisis period investors is suggested to hold higher gold's proportion than that in the normal period. For the crisis period the range is between 53 percent and 59 percent while the range in the normal period is between 23 percent and 26 percent (See Table 4).

5.2.2.2. The Pattern of Change in Weight over Different Market Conditions

Both the 180-day and 360-day estimation windows show a pattern that the gold's weight sharply rises in the crisis period and then declines in the normal period (See Figure 3). The result is consistent with the previous claim on the importance of gold in extreme negative event by Baur and Lucey (2010).

Figure 3: The optimal weight of gold based on two fixed estimation windows under Sharpe Ratio Maximization framework



Description
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)
period 3: Normal period (10 Oct 2002-9 Oct 2007)
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)
period 5: Normal period (10 Mar 2009-29 Apr 2011)
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)
period 7: Normal (4 Oct 2011-1 May 2013)

However, the most important finding is that the extreme drop in the gold's weight in the last study period after the US debt crisis happens. It decreases by 85 percent for 180-day period and by 73 percent for 360-day period (See Table 4).

5.2.3. Conclusion: Comparison between Two Different Subsample Criteria (Market Condition and Two Fixed Estimation Windows)

When comparing the optimal weight of gold obtained from two subsample criteria, the result is not consistent. The optimal portfolio obtained based on the market condition suggests slightly more proportion of gold in the normal period than that in the crisis period. Conversely, the optimal weight of gold based on the two fixed estimation windows suggests more gold in the crisis period than the normal period. One of the possible reasons behind the difference is that there is the portfolio adjustment in the middle of the crisis in the fixed estimation window criterion. Therefore, it is possible that in some certain period gold significantly has superior return relative to risk than that of stock and bond, resulting in the higher proportion in the optimal portfolio. For example, in the 180-day estimation window the portfolio reallocated between 31 Oct 2001 and 9 July 2002, which is in the middle of the Dot-Com Bubble Burst, invests 100 percent in gold. Also, the portfolio adjusted on 16 Jan 2008, which is in the middle of Global Financial Crisis 2008, invests all money in gold. The solely gold investment is supported by the good performance of gold in the middle of the two crisis periods.

Based on the market condition and the fixed estimation window criterion, it is possible to conclude that the optimal weight of holding gold during the normal period is roughly between 22 percent and 25 percent. However, for the crisis period the result has a wide range between two subsample criteria. Therefore, it is inappropriate to suggest the suitable weight of gold in the crisis period.

Nevertheless, when analyzing the optimal weight over time, the two different criteria show the same pattern. That is the trend of optimal proportion of gold in the portfolio gradually decreases over time. Therefore, if investors highly weigh the perspective of gold investment on the latest crisis in 2011, they are recommended to consider only the two periods of the U.S. debt crisis in 2011 and the normal period after that. It is obviously seen that the suggested optimal weight decreases to less than 5 percent. It is possible to conclude that the role of gold in the portfolio is less after the latest financial crisis.

5.3. Comparison between Two Frameworks

This section the results between the Variance Minimization and the Sharpe Ratio Maximization frameworks are compared in terms of difference and similarity with mainly focusing on the optimal weight of gold, which is the purpose of this thesis. However, interesting findings of other assets in the portfolio are also discussed.

5.3.1. Difference

5.3.1.1. Optimal Weight of Gold

Overall, the weighted average of optimal weight in gold is obviously higher when using Sharpe ratio in the portfolio optimization. The main reason is the different algorithm of constructing the portfolio. Sharpe ratio maximization takes asset's return into consideration therefore the optimal portfolio is definitely constructed by weighting more fractions into higher asset's return relative to risk like gold. On the other hand, return is not an essential key in the Variance Minimization framework, as a result the weight of gold is less but instead the greater weight of bond takes place with approximately 70 percent of total portfolio on average (See Table 5). Implication of this finding relates to the investment objectives and acceptable risk level of investors. The more aggressive investors, indicated by the Sharpe Ratio Maximization framework, are able to have more weights in the gold, conversely the more risk-averse investors, indicated by the Variance Minimization framework, invests more in the less risky asset like bond.

In terms of the other assets' proportion in the optimal portfolio, it is along with general expectation that in the Variance Minimization framework, bond gains the highest weight in the total portfolio following by stock and gold, respectively. This outcome is possibly explained with the lowest risk of bond that is one of the main factors of the model, which attempts to construct the lowest risk portfolio. However, the role of bond in the optimal portfolio does not maintain its importance in the Sharpe Ratio Maximization framework. On average, the proportion of bond and stock is not too deviated from each other, which is 28 percent for bond and 22 percent for gold. Consequently, the remaining proportion turns toward stock with average weight of almost 50 percent (See Table 5).

Table 5: Comparison the optimal weight of all assets based on the market condition by using two optimization frameworks. (Variance Minimization and Sharpe Ratio Maximization)

Optimal Weight (Subsample by the market condition)						
	Minimizing Variance			Maximizing Sharpe Ratio		
Period	Bond	Stock	Gold	Bond	Stock	Gold
1	74,83%	6,66%	18,51%	0,00%	100,00%	0,00%
2	65,05%	15,30%	19,64%	73,14%	0,00%	26,86%
3	68,49%	22,88%	8,64%	0,00%	57,28%	42,72%
4	76,26%	15,12%	8,63%	80,02%	0,00%	19,98%
5	65,52%	21,40%	13,09%	28,29%	40,56%	31,16%
6	74,71%	21,98%	3,30%	93,67%	6,33%	0,00%
7	72,17%	26,42%	1,41%	62,09%	37,91%	0,00%
Weighted Average of Optimal Weight	**70,27%**	**17,71%**	**12,02%**	**28,49%**	**49,16%**	**22,35%**
Normal period: Weighted Average of	**70,32%**	**18,27%**	**11,41%**	**12,80%**	**64,58%**	**22,62%**
Crisis period: Weighted Average of Optimal	**70,12%**	**15,96%**	**13,92%**	**77,83%**	**0,68%**	**21,49%**
Standard deviation	**0,0430**	**0,0614**	**0,0650**	**0,3573**	**0,3387**	**0,1619**

Note: See Appendix Table D for more details in how to calculate the weighted average number

5.3.1.2. Variation of the Weight

In terms of the variation in the adjusted optimal weight through time, there is the evidence of higher volatility of weight changing when the Sharpe Ratio Maximization is utilized. This claim is indicated by higher standard deviation of all assets' weight for both subsample criteria: the market condition and fixed estimation window (See Table 5). It implies that the pattern of weight adjustment in the Variance Minimization framework is smoother than that in the Sharpe Ratio Maximization framework. There are several causes explaining this outcome.

One of the possible reasons is that, for the weight of gold in the Sharpe Ratio Maximization framework, there is the extreme surge in the gold's proportion during the crisis because its return considerably outperforms the other risky assets like stock, which contributes negative return. Moreover, when examining the weight of every asset, it is noticeable that the weight obtained from the Sharpe Ratio Maximization framework is fluctuated. There is some period that the optimal portfolio invests only stock or only bond and gold. Therefore, the weight ranges from 0 percent to 100 percent. In contrast, the weight of gold in the Variance Minimization framework is evenly distributed to all assets (See Table 5). One possible reason is that when variance and correlation are taken into consideration in the Variance Minimization framework, there is a slight change between these two factors over time. Therefore, the portfolio constructed by the Variance Minimization framework does not encounter with the intensive change in the weight like the Sharpe Ratio Maximization framework.

5.3.2. Similarity

5.3.2.1. The Weight of Gold for Normal and Crisis Period

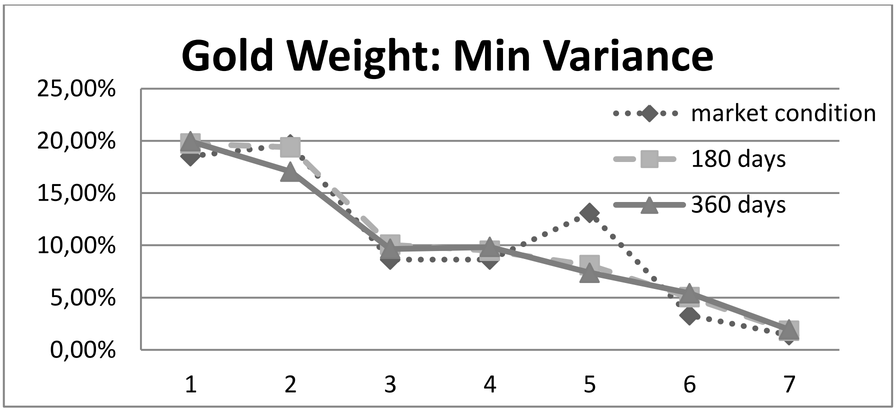
Overall, the Variance Minimization and the Sharpe Ratio Maximization frameworks suggest the higher weight of gold during the crisis than that during the normal period. However, there is a slightly different outcome for the Sharpe Ratio Maximization framework when the data is divided based on the market condition criterion. It suggests somewhat the same weight of gold for the crisis and normal period. One explanation is that there is a small change in the Sharpe ratio of gold relative to stock and bond from one specific market condition to another condition. However, the result is similar to the outcome of the Variance Minimization framework for the two subsample criteria because there is a small discrepancy of gold's weight between the normal and crisis periods. The reason of the minimal difference is due to the slightly unchanged pattern of variance and correlation among the three assets (more details are discussed in the section 5.1.2.3.)

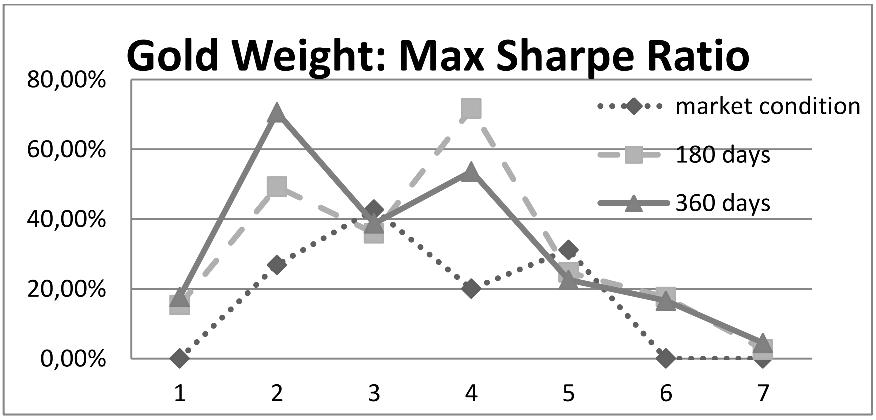
When the pattern over time is compared between the Variance Minimization and the Sharpe Ratio Maximization frameworks, the proportion of gold decreases after every crisis period under the fixed estimation window criterion. However, when investigating the outcome from the market condition criterion, there is no apparent pattern that enables to draw any substantial conclusion.

5.3.2.2. Trend of Gold's Weight over Time

Under the Variance Minimization and the Sharpe Ratio Maximization frameworks, they show that the optimal weight of gold investment in the portfolio declines over time. However, the declining trend for the Variance Minimization framework is steadier than for the Sharpe Ratio Maximization framework (See Figure 4). The reason behind this phenomenon is that the Sharpe Ratio Maximization framework takes both risk and return of the assets into account. Some study subsample period shows the extreme increase in the gold's performance, therefore the weight of gold is greatly adjusted to have higher proportion than that in the previous period. On the other hand, the weight adjustment under the Variance Minimization framework is not highly fluctuated from period to period. It is supported by the slight change in the standard deviation of gold. Therefore, the gold's weight is gradually adjusted, resulting in the smoother movement over time.

Figure 4: Trend of Gold Weight over Time





5.3.2.3. Significant Decline in the Weight of Gold since the US Debt Crisis 2011

In the last two periods of the study, the optimal weight of gold suggested by the two frameworks significantly drop comparing to the previous five periods. Particularly during the last study period, the two optimization frameworks suggest a minimal weight of gold in the portfolio, indicating that gold losses much of its role in the investment portfolio. The reason behind the consistent outcome is explained into two parts. Firstly, the higher standard deviation of gold in the last period reflects the lower proportion in the Variance Minimization framework. Secondly, the bad performance of gold results in the smaller weight in the Sharpe Ratio Maximization framework. Nevertheless, these results are quite different from the previous 6 periods because the risk and return of gold mostly places in the second ranking.

Moreover, the change in correlation among each asset in the portfolio is another factor, which impacts the gold's weight in the two optimization frameworks. However, the explanation of the correlation refers back to the Data Analysis discussed above.

6. DISCUSSION AND CONCLUSION

This thesis investigates the role of gold in the portfolio by using Variance Minimization and Sharpe Ratio Maximization under the Markowitz optimization model. The study period captures the important market events from 1997 to 2011. For analyzing, the data is divided by two different criteria: the market condition (crisis and normal period) and the fixed holding period (180 and 360 days).

In order to conclude the above result, this section refers back to the three research questions in the introduction part. The first question is whether gold still remains importance in the portfolio after the financial crisis 2008. The answer to this question is that gold has reduced its importance, which is consistent for both Variance Minimization and Sharpe Ratio Maximization approach. The most interesting finding is that the optimal weight of gold extremely drops in the last study period, which is the period after the US debt crisis in 2011. The Variance Minimization framework suggests the average optimal weight of 1.7 percent while the Sharpe Ratio Maximization framework offers 2.4 percent. There are two possible reasons to explain this phenomenon. The first reason is that gold's price decreases, resulting in the negative return. The other reason is that during this period, the U.S. central bank embarks in the third-round of QE, which causes gold to lose much of its diversification benefits. This is indicated by the significant change in correlation from low negative number on average in the previous study periods to high positive number of 0.3. In sum, both the bad performance and the positive correlation between gold and stock lead to less attractiveness in gold investment. Moreover, the other portfolio component like bond turns to be more interesting asset, which contributes lower risk and also generates higher return relative to gold during the period. Besides, its correlation with stock is highly negative, implying the diversification benefits. As a result, it is unsurprising that gold decreases in its importance while bond plays the bigger role in the portfolio.

The second question is how much proportion of gold is appropriate to be invested in the portfolio for different investment objectives in different market conditions. With regards to different investment goals, investors aiming to minimize the total risk of their portfolio construct the optimal portfolio by using the Variance Minimization framework. However, investors considering both risk and return construct their portfolio by using the Sharpe Ratio Maximization framework. Under the Variance Minimization framework, the suggested optimal weight of gold in the portfolio for the normal period is 11.4 percent and for the crisis period is 13.8 percent. On the other hand, the Sharpe Ratio Maximization framework provides a higher weight, which is 23.8 percent for the normal period and 44.7 percent for the crisis period. However, in order to suggest the suitable weight for each specific market condition, it is necessary to discuss its robustness over different subsampling. It means that this suggested weight is still valid although the data is divided differently. Under the Variance Minimization framework, the optimal weight of gold for both normal and crisis period does not significantly deviate among three subsample approaches (market condition, 180-day and 360 day). It is possible to conclude that the suggested weight is robust although the holding period changes. In contrast, this conclusion is not able to apply with the optimal weight obtained from the Sharpe Ratio Maximization. There is the significant difference in the optimal weight between the market condition and the two fixed estimation windows criterion. The suggested weight is unable to use as a benchmark for the future investment because the change in holding period and the specific market conditions during the investment period impact the optimal weight. However, one possible implication from the Sharpe Ratio framework is that investors are recommended to take the holding period and the performance of gold during their investment into consideration when making a decision on the appropriate weight for gold investment.

The last question is that after including gold in the portfolio, what the pattern of its optimal weight is regarding to the change in market conditions over time. In order to answer this question, it is necessary to consider the trend of gold's weight in the portfolio relative to the other assets' proportion through time. Both the Variance Minimization and the Sharpe Ratio Maximization frameworks generate the decreasing pattern of the gold's weight relative to the others' asset in the portfolio. Also, this pattern is consistent across all the different ways of subsampling. The pattern implies the importance of gold has declined over time. Furthermore, the volatility of the weight in the pattern is interesting to discuss. The pattern under the Sharpe Ratio Maximization framework is apparently more fluctuated. During some crisis period the weight of gold sharply rises, while the Variance Minimization framework does not show the extreme change in the weight. It is possible to conclude that investors aiming to achieve the maximum reward/risk portfolio are recommended to accept the sudden and great adjustment of their portfolio allocation when the market condition changes.

In conclusion, the most interest finding of this thesis is that gold does not maintain its importance when the time passes, especially the last study period after the U.S. Debt crisis in 2011. This is the period that the third round of QE is implemented. The finding in the smaller proportion of gold in the portfolio provides another up-to-date picture for gold investment, which investors are able to apply with their investment decision. Besides, policy makers and regulators gain more insightful understanding in the effect of their policy. The evidence indicates that the policy significantly affects the correlation among the assets in the market, particularly between gold and stock. Hence, it possibly distorts the general belief in the benefit of gold as a safe haven and hedge asset, which are generally used as an investment criterion to include gold in the portfolio.

However, this thesis is limited by using only the data of the U.S stock market. Therefore, the conclusion is possibly different if other countries' stock markets take into account in the analysis. Besides, future research is likely to expand the current study by including more asset classes in the benchmark portfolio.

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8. APPENDIX

Exhibit 1: The figure shows how the data is divided based on the stock market condition.

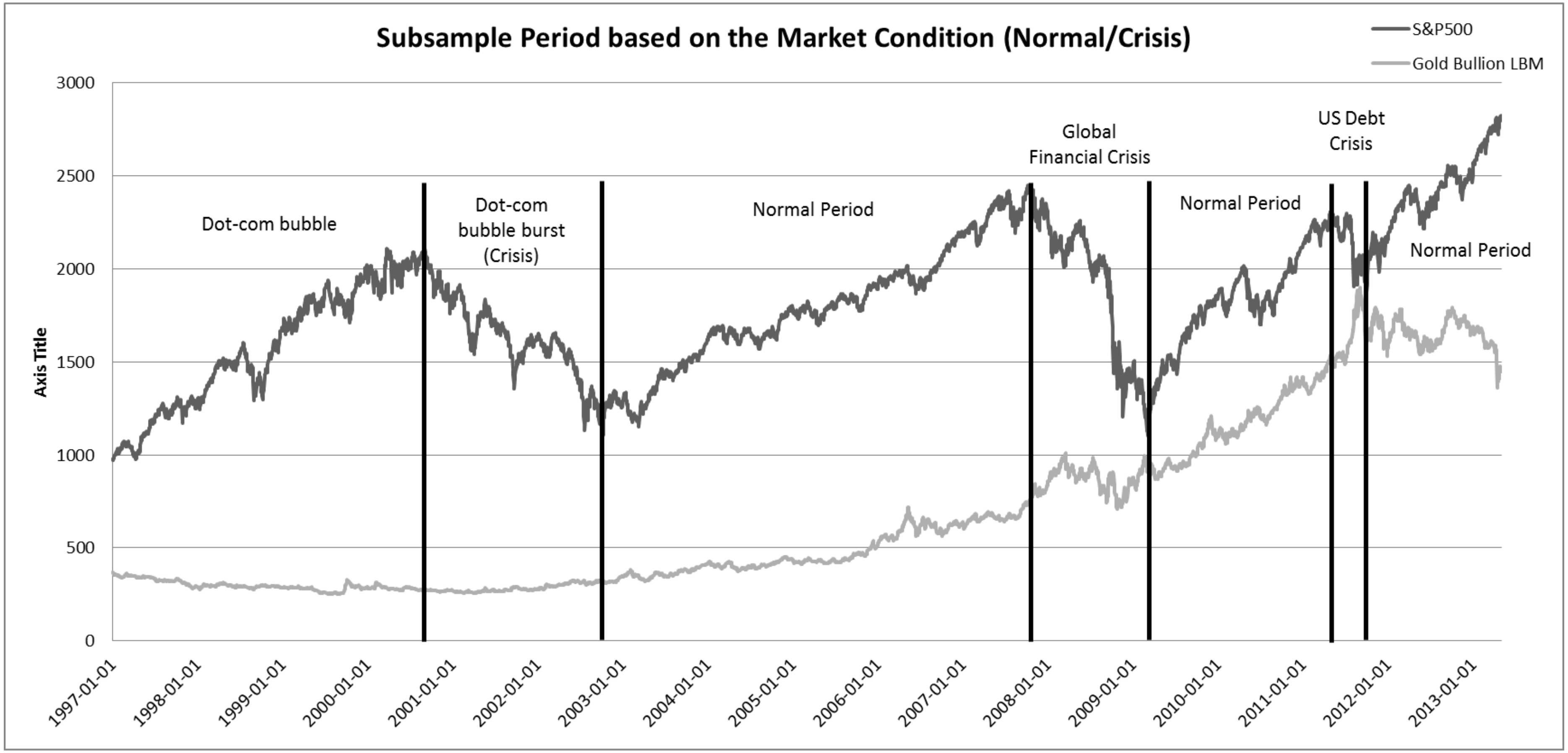


Exhibit 2: How to calculate the weighted average of gold's weight when the data is divided based on the two fixed estimation windows (180 and 360 days) under two optimization frameworks (Variance Minimization and Sharpe Ratio Maximization)

Under 180-day estimation window, the entire period of study is divided into 24 subsample periods, in which there are 180 days for the first 23 periods and 121 days for the last period. Thus, to compute the average number of gold's weight for the entire study period, it is necessary to weigh each optimal weight with its number of day to total observation days.

For example, in the Exhibit 2.1, there are 4261 observations. The period 1 has 180 days so the weight of this period is 4.2 percent. Then, 4.2 percent is multiplied with the optimal weight of gold in this period. The process continues until 24 subsample periods is obtained. The average of optimal gold's weight is achieved by summing up all weighted 24 subsample of gold's weight.

The process for calculating the average of optimal gold's weight for 360-day estimation window is the same as for 180-day estimation window (see Exhibit 2.2).

Exhibit 2.1: Overall weighted average of optimal weight under 180-day estimation window

	Period	Minimizing Variance			Maximizing Sharpe Ratio					Weighted average of weight in gold	
	(180-day subsample)	Bond	Stock	Gold	Bond	Stock	Gold	Days	Weight	Min variance	Max Sharpe ratio
1	1 Jan 1997 - 9 Sep 1997	75,45%	0,00%	24,55%	0,00%	100,00%	0,00%	180	0,042	1,04%	0,00%
2	10 Sep 1997 - 19 May 1998	73,87%	5,54%	20,60%	52,20%	47,80%	0,00%	180	0,042	0,87%	0,00%
3	20 May 1998 - 26 Jan 1999	66,89%	12,65%	20,46%	77,85%	22,15%	0,00%	180	0,042	0,86%	0,00%
4	27 Jan 1999 - 5 Oct 1999	79,36%	0,40%	20,24%	0,00%	18,44%	81,56%	180	0,042	0,85%	3,45%
5	6 Oct 1999 - 13 Jun 2000	81,72%	6,78%	11,50%	0,00%	100,00%	0,00%	180	0,042	0,49%	0,00%
6	14 Jun 2000 - 20 Feb 2001	64,63%	11,52%	23,85%	100,00%	0,00%	0,00%	180	0,042	1,01%	0,00%
7	21 Feb 2001 - 30 Oct 2001	74,23%	12,84%	12,93%	58,37%	0,00%	41,63%	180	0,042	0,55%	1,76%
8	31 Oct 2001 - 9 Jul 2002	55,55%	20,83%	23,62%	0,00%	0,00%	100,00%	180	0,042	1,00%	4,22%
9	10 Jul 2002 - 18 Mar 2003	64,58%	18,32%	17,10%	65,06%	12,22%	22,72%	180	0,042	0,72%	0,96%
10	19 Mar 2003 - 25 Nov 2003	54,06%	26,57%	19,37%	8,07%	51,56%	40,37%	180	0,042	0,82%	1,71%
11	26 Nov 2003 - 3 Aug 2004	63,52%	30,93%	5,56%	0,00%	100,00%	0,00%	180	0,042	0,23%	0,00%
12	4 Aug 2004 - 12 Apr 2005	63,57%	22,26%	14,17%	0,00%	57,63%	42,37%	180	0,042	0,60%	1,79%
13	13 Apr 2005 - 20 Dec 2005	68,21%	22,23%	9,56%	0,00%	37,89%	62,11%	180	0,042	0,40%	2,62%
14	21 Dec 2005 - 29 Aug 2006	86,74%	10,49%	2,77%	0,00%	47,42%	52,58%	180	0,042	0,12%	2,22%
15	30 Aug 2006 - 8 May 2007	74,44%	20,40%	5,17%	0,00%	84,40%	15,60%	180	0,042	0,22%	0,66%
16	9 May 2007 - 15 Jan 2008	70,42%	20,65%	8,93%	46,55%	0,00%	53,45%	180	0,042	0,38%	2,26%
17	16 Jan 2008 - 23 Sep 2008	67,92%	25,06%	7,02%	0,00%	0,00%	100,00%	180	0,042	0,30%	4,22%
18	24 Sep 2008 -2 Jun 2009	75,56%	10,89%	13,55%	60,53%	0,00%	39,47%	180	0,042	0,57%	1,67%
19	3 Jun 2009 - 9 Feb 2010	66,77%	23,82%	9,41%	39,84%	42,72%	17,44%	180	0,042	0,40%	0,74%
20	10 Feb 2010- 19 Oct 2010	70,66%	24,35%	4,98%	63,02%	20,89%	16,09%	180	0,042	0,21%	0,68%
21	20 Oct 2010 - 28 Jun 2011	58,54%	33,41%	8,04%	0,00%	61,24%	38,76%	180	0,042	0,34%	1,64%
22	29 Jun 2011 - 6 Mar 2012	74,09%	22,70%	3,21%	73,72%	21,52%	4,76%	180	0,042	0,14%	0,20%
23	7 Mar 2012 - 13 Nov 2012	69,31%	28,51%	2,18%	69,44%	28,72%	1,84%	180	0,042	0,09%	0,08%
24	14 Nov 2012 -1 May 2013	75,78%	24,22%	0,00%	69,44%	28,72%	1,84%	121	0,028	0,00%	0,05%
	Standard deviation			**0,0747**			**0,3118**	**4261**	**100,00%**	**12,20%**	**30,92%**

Exhibit 2.2: Overall weighted average of optimal weight under 360-day estimation wind

	Period	Minimizing Variance			Maximizing Sharpe Ratio					Weighted average of weight in gold	
	(360-day subsample)	Bond	Stock	Gold	Bond	Stock	Gold			Min variance	Max Sharpe ratio
1	1 Jan 1997 - 19 May 1998	75,13%	1,77%	23,10%	0,00%	100,00%	0,00%	360	0,084	1,95%	0,00%
2	20 May 1998- 5 Oct 1999	69,81%	9,10%	21,09%	0,00%	53,13%	46,87%	360	0,084	1,78%	3,96%
3	6 Oct 1999 - 20 Feb 2001	77,28%	9,29%	13,43%	0,00%	100,00%	0,00%	360	0,084	1,13%	0,00%
4	21 Feb 2001 - 9 Jul 2002	65,57%	16,52%	17,91%	0,00%	0,00%	100,00%	360	0,084	1,51%	8,45%
5	10 Jul 2002 - 25 Nov 2003	60,74%	20,21%	19,05%	37,31%	22,42%	40,26%	360	0,084	1,61%	3,40%
6	26 Nov 2003 - 12 Apr 2005	63,77%	26,86%	9,37%	0,00%	75,97%	24,03%	360	0,084	0,79%	2,03%
7	13 Apr 2005 - 29 Aug 2006	77,91%	17,54%	4,55%	0,00%	48,52%	51,48%	360	0,084	0,38%	4,35%
8	30 Aug 2006 - 15 Jan 2008	72,27%	20,90%	6,82%	46,47%	14,03%	39,50%	360	0,084	0,58%	3,34%
9	16 Jan 2008 - 2 Jun 2009	75,07%	14,39%	10,54%	43,08%	0,00%	56,92%	360	0,084	0,89%	4,81%
10	3 Jun 2009 - 19 Oct 2010	68,38%	24,02%	7,60%	56,45%	24,58%	18,98%	360	0,084	0,64%	1,60%
11	20 Oct 2010 - 6 Mar 2012	69,62%	24,96%	5,42%	55,44%	28,00%	16,55%	360	0,084	0,46%	1,40%
12	7 Mar 2012- 1 May 2013	71,75%	27,60%	0,65%	63,20%	36,80%	0,00%	301	0,071	0,05%	0,00%
	Standard deviation			**0,0691**			**0,2814**	**4261**	**100,00%**	**11,78%**	**33,34%**

Exhibit 3: How to assign the optimal gold's weight obtained from the two fixed estimation windows (180 and 360 days) to each different market condition

Under 180-day estimation window, the entire period of study is divided into 30 subsample periods, which the length of each estimation window is different. Thus, to compute the average number of gold's weight for each 7 subsample periods based on the market condition, it is necessary to weigh each optimal weight with its number of day to total observation days in the market condition that it belongs to.

For example, in the Exhibit 3.1, in order to find the weighted average of gold's weight for the Dot-Com Bubble period, there are 958 observations in the period and 6 subsample periods. The period 1 (1 January 1997-9 September 1997) has 180 days so the weight of this period is 18.79 percent (180/958). Then, 18.79 percent is multiplied with the optimal weight of gold in the period 1. The process continues until the first 6 subsample periods is obtained. The weighted average of gold's weight for the Dot-Com Bubble period is achieved by summing up all first weighted 6 subsample of gold's weight.

Furthermore, in order to compute the average optimal gold's weight for the normal and crisis periods, the 7 subsample periods are divided into two groups: 4 periods for the normal condition and 3 periods for the crisis condition. The total observations of the normal and crisis are gathered (3233 days for the normal and 1028 days for the crisis period).

For instance, the 958 observations of Dot-Com Bubble are divided by the total 3233 observations of the normal period. These weights are multiplied with the weighted average of gold's weight for the Dot-Com Bubble period. The process is repeated for the rest of the 3 subsamples in the normal period. Lastly, the weighted average of gold's weight in the normal period is achieved by summing up all weighted 4 subsample of gold's weight. The number of the crisis period is obtained by the same process.

The process for calculating the average optimal gold's weight for 360-day estimation window is the same as for 180-day estimation window (see Exhibit 3.2).

Table 3.1: Weighted average of optimal weight for normal and crisis period under 180-day estimation window

		Gold optimal weight				Weighted average of gold proportion		Normal period			Crisis period		
Sub period based on market condition	**Sub period (180 day)**	**Min Variance**	**Max Sharpe ratio**	**Days**	**Weight**	**Min Variance**	**Max Sharpe ratio**	**Weight**	**Min Variance**	**Max Sharpe ratio**	**Weight**	**Min Variance**	**Max Sharpe ratio**
1 Jan 1997-1 Sep 2000	1 Jan 1997 - 9 Sep 1997	24,55%	0,00%	180	18,79%	4,61%	0,00%						
(Dot-com Bubble)	10 Sep 1997 - 19 May 1998	20,60%	0,00%	180	18,79%	3,87%	0,00%						
	20 May 1998 - 26 Jan 1999	20,46%	0,00%	180	18,79%	3,84%	0,00%						
	27 Jan 1999 - 5 Oct 1999	20,24%	81,56%	180	18,79%	3,80%	15,33%						
	6 Oct 1999 - 13 Jun 2000	11,50%	0,00%	180	18,79%	2,16%	0,00%						
	14 June 2000-1 Sep 2000	23,85%	0,00%	58	6,05%	1,44%	0,00%						
				958	**1**	**19,73%**	**15,33%**	**29,63%**	**5,85%**	**4,54%**			
4 Sep 2000-9 Oct 2002	4 Sep 2000-20 Feb 2001	23,85%	0,00%	122	22,26%	5,31%	0,00%						
(Dot-com Bubble Burst)	21 Feb 2001 - 30 Oct 2001	12,93%	41,63%	180	32,85%	4,25%	13,67%						
	31 Oct 2001 - 9 Jul 2002	23,62%	100,00%	180	32,85%	7,76%	32,85%						
	10 Jul 2002 - 9 Oct 2002	17,10%	22,72%	66	12,04%	2,06%	2,74%						
				548	**1**	**19,38%**	**49,26%**				**53,31%**	**10,33%**	**26,26%**
10 Oct 2002-9 Oct 2007	10 Oct 2002 - 18 Mar 2003	17,10%	22,72%	114	8,74%	1,49%	1,99%						
(Normal Period)	19 Mar 2003 - 25 Nov 2003	19,37%	40,37%	180	13,80%	2,67%	5,57%						
	26 Nov 2003 - 3 Aug 2004	5,56%	0,00%	180	13,80%	0,77%	0,00%						
	4 Aug 2004 - 12 Apr 2005	14,17%	42,37%	180	13,80%	1,96%	5,85%						
	13 Apr 2005 - 20 Dec 2005	9,56%	62,11%	180	13,80%	1,32%	8,57%						
	21 Dec 2005 - 29 Aug 2006	2,77%	52,58%	180	13,80%	0,38%	7,26%						
	30 Aug 2006 - 8 May 2007	5,17%	15,60%	180	13,80%	0,71%	2,15%						
	9 May 2007 - 9 Oct 2007	8,93%	53,45%	110	8,44%	0,75%	4,51%						
				1304	**1**	**10,06%**	**35,90%**	**40,33%**	**4,06%**	**14,48%**			
10 Oct 2007-9 Mar 2009	10 Oct 2007 - 15 Jan 2008	8,93%	53,45%	70	18,97%	1,69%	10,14%						
(Global Financial Crisis 2008)	16 Jan 2008 - 23 Sep 2008	7,02%	100,00%	180	48,78%	3,42%	48,78%						
	24 Sep 2008 -9 Mar 2009	13,55%	39,47%	119	32,25%	4,37%	12,73%						
				369	**1**	**9,49%**	**71,65%**				**35,89%**	**3,41%**	**25,72%**
10 Mar 2009-29 Apr 2011	10 Mar 2009 - 2 June 2009	13,55%	39,47%	61	10,91%	1,48%	4,31%						
(Normal Period)	3 Jun 2009 - 9 Feb 2010	9,41%	17,44%	180	32,20%	3,03%	5,62%						
	10 Feb 2010- 19 Oct 2010	4,98%	16,09%	180	32,20%	1,60%	5,18%						
	20 Oct 2010 - 29 Apr 2011	8,04%	38,76%	138	24,69%	1,99%	9,57%						
				559	**1**	**8,10%**	**24,67%**	**17,29%**	**1,40%**	**4,27%**			
2 May 2011-3 Oct 2011	2 May 2011 - 28 June 2011	8,04%	38,76%	42	37,84%	3,04%	14,66%						
(US Debt Crisis)	29 Jun 2011 - 3 Oct 2011	3,21%	4,76%	69	62,16%	1,99%	2,96%						
				111	**1**	**5,04%**	**17,62%**				**10,80%**	**0,54%**	**1,90%**
4 Oct 2011-1 May 2013	4 Oct 2011 - 6 Mar 2012	3,21%	4,76%	111	26,94%	0,86%	1,28%						
(Normal Period)	7 Mar 2012 - 13 Nov 2012	2,18%	1,84%	180	43,69%	0,95%	0,80%						
	14 Nov 2012 -1 May 2013	0	1,84%	121	29,37%	0,00%	0,54%						
				412	**1**	**1,82%**	**2,63%**	**12,74%**	**0,23%**	**0,33%**			
			Normal Period	3233				**100,00%**	**11,54%**	**23,62%**	**100,00%**	**14,28%**	**53,88%**
			Crisis Period	1028									
				4261									

Table 3.2: Weighted average of optimal weight for normal and crisis period under 360-day estimation window

		Gold optimal weight				Weighted average of gold proportion		Normal period			Crisis period		
Sub period based on market condition	**Sub period (360-day)**	**Min Variance**	**Max Sharpe ratio**	**Days**	**Weight**	**Min Variance**	**Max Sharpe ratio**	**Weight**	**Min Variance**	**Max Sharpe ratio**	**Weight**	**Min Variance**	**Max Sharpe ratio**
1 Jan 1997-1 Sep 2000	1 Jan 1997 - 19 May 1998	23,10%	0,00%	360	37,58%	8,68%	0,00%						
(Dot-com Bubble)	20 May 1998- 5 Oct 1999	21,09%	46,87%	360	37,58%	7,93%	17,61%						
	6 Oct 1999 - 1 Sep 2000	13,43%	0,00%	238	24,84%	3,34%	0,00%						
				958	**1**	**19,94%**	**17,61%**	**29,63%**	**5,91%**	**5,22%**			
4 Sep 2000-9 Oct 2002	4 Sep 2000 - 20 Feb 2001	13,43%	0,00%	122	22,26%	2,99%	0,00%						
(Dot-com Bubble Burst)	21 Feb 2001 - 9 Jul 2002	17,91%	100,00%	360	65,69%	11,76%	65,69%						
	10 Jul 2002 - 9 Oct 2002	19,05%	40,26%	66	12,04%	2,29%	4,85%						
				548	**1**	**17,05%**	**70,54%**				**53,31%**	**9,09%**	**37,60%**
10 Oct 2002-9 Oct 2007	10 Oct 2002 - 25 Nov 2003	19,05%	40,26%	294	22,55%	4,30%	9,08%						
(Normal Period)	26 Nov 2003 - 12 Apr 2005	9,37%	24,03%	360	27,61%	2,59%	6,63%						
	13 Apr 2005 - 29 Aug 2006	4,55%	51,48%	360	27,61%	1,26%	14,21%						
	30 Aug 2006 - 9 Oct 2007	6,82%	39,50%	290	22,24%	1,52%	8,78%						
				1304	**1**	**9,66%**	**38,71%**	**40,33%**	**3,90%**	**15,61%**			
10 Oct 2007-9 Mar 2009	10 Oct 2007 -15 Jan 2008	6,82%	39,50%	70	18,97%	1,29%	7,49%						
(Global Financial Crisis 2008)	16 Jan 2008 - 9 Mar 2009	10,54%	56,92%	299	81,03%	8,54%	46,12%						
				369	**1**	**9,84%**	**53,61%**				**35,89%**	**3,53%**	**19,24%**
10 Mar 2009-29 Apr 2011	10 Mar 2009 - 2 Jun 2009	10,54%	56,92%	61	10,91%	1,15%	6,21%						
(Normal Period)	3 Jun 2009 - 19 Oct 2010	7,60%	18,98%	360	64,40%	4,89%	12,22%						
	20 Oct 2010 -29 Apr 2011	5,42%	16,55%	138	24,69%	1,34%	4,09%						
				559	**1**	**7,38%**	**22,52%**	**17,29%**	**1,28%**	**3,89%**			
2 May 2011-3 Oct 2011	2 May 2011 - 3 Oct 2011	5,42%	16,55%	111	100,00%	5,42%	16,55%						
(US Debt Crisis)						**5,42%**	**16,55%**				**10,80%**	**0,59%**	**1,79%**
4 Oct 2011-1 May 2013	4 Oct 2011 - 6 Mar 2012	5,42%	16,55%	111	26,94%	1,46%	4,46%						
(Normal Period)	7 Mar 2012- 1 May 2013	0,65%	0,00%	301	73,06%	0,48%	0,00%						
				412	**1**	**1,94%**	**4,46%**	**12,74%**	**0,25%**	**0,57%**			
			Normal Period	3233				**100,00%**	**11,33%**	**25,29%**	**100,00%**	**13,21%**	**58,64%**
			Crisis Period	1028									
				4261									

Exhibit 4: How to calculate the weighted average number of optimal weight (Overall, Normal, Crisis period) based on the market condition (crisis and normal) under two optimization frameworks (Variance Minimization and Sharpe Ratio Maximization)

The study period is divided into 7 periods based on the crisis and normal periods, which the length of estimation window is different. Therefore, in order to calculate the overall average number, it is necessary to weigh each optimal weight with its number of day to total observation days. For example, in the Exhibit 4.1 there are 4261 observations. The period 1 has 958 days so the weight of this period is 22.48 percent. Then, 22.48 percent is multiplied with the optimal weights of all assets in this period. For instance, gold has 18.51 percent but has only 4.16 percent as a component of weighted average weight of gold for the whole period. The process is repeated for the rest of the 6 subsamples period. Lastly, the overall weighted average of gold's weight is achieved by summing up all weighted 7 subsample of gold's weight.

However, for the normal and crisis period, the calculation is the same except the number of total observations. There are 3233 observations in the normal and 1028 observations in the crisis period.

Exhibit 4.1: Overall weighted average number of optimal weight

							Overall: Weighted Average of Optimal Weight							
	Minimizing Variance			Maximizing Sharpe Ratio					Minimizing Variance			Maximizing Sharpe Ratio		
Period	Bond	Stock	Gold	Bond	Stock	Gold	Days	weight	Bond	Stock	Gold	Bond	Stock	Gold
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	74,83%	6,66%	18,51%	0,00%	100,00%	0,00%	958	22,48%	16,82%	1,50%	4,16%	0,00%	22,48%	0,00%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	65,05%	15,30%	19,64%	73,14%	0,00%	26,86%	548	12,86%	8,37%	1,97%	2,53%	9,41%	0,00%	3,45%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,49%	22,88%	8,64%	0,00%	57,28%	42,72%	1304	30,60%	20,96%	7,00%	2,64%	0,00%	17,53%	13,07%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	76,26%	15,12%	8,63%	80,02%	0,00%	19,98%	369	8,66%	6,60%	1,31%	0,75%	6,93%	0,00%	1,73%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	65,52%	21,40%	13,09%	28,29%	40,56%	31,16%	559	13,12%	8,60%	2,81%	1,72%	3,71%	5,32%	4,09%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	74,71%	21,98%	3,30%	93,67%	6,33%	0,00%	111	2,61%	1,95%	0,57%	0,09%	2,44%	0,16%	0,00%
period 7: Normal (4 Oct 2011-1 May 2013)	72,17%	26,42%	1,41%	62,09%	37,91%	0,00%	412	9,67%	6,98%	2,55%	0,14%	6,00%	3,67%	0,00%
Standard deviation	**0,0430**	**0,0614**	**0,0650**	**0,3573**	**0,3387**	**0,1619**	**4261**	**100%**	**70,27%**	**17,71%**	**12,02%**	**28,49%**	**49,16%**	**22,35%**

Exhibit 4.2: Weighted average number of optimal weight for the normal period

Period	Minimizing Variance			Maximizing Sharpe Ratio			Normal period: Weighted Average of Optimal Weight							
	Bond	Stock	Gold	Bond	Stock	Gold			Minimizing Variance			Maximizing Sharpe Ratio		
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	74,83%	6,66%	18,51%	0,00%	100,00%	0,00%	Day	Weight	Bond	Stock	Gold	Bond	Stock	Gold
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	65,05%	15,30%	19,64%	73,14%	0,00%	26,86%	958	29,63%	22,17%	1,97%	5,49%	0,00%	29,63%	0,00%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,49%	22,88%	8,64%	0,00%	57,28%	42,72%								
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	76,26%	15,12%	8,63%	80,02%	0,00%	19,98%	1304	40,33%	27,62%	9,23%	3,48%	0,00%	23,10%	17,23%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	65,52%	21,40%	13,09%	28,29%	40,56%	31,16%								
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	74,71%	21,98%	3,30%	93,67%	6,33%	0,00%	559	17,29%	11,33%	3,70%	2,26%	4,89%	7,01%	5,39%
period 7: Normal (4 Oct 2011-1 May 2013)	72,17%	26,42%	1,41%	62,09%	37,91%	0,00%								
Standard deviation	**0,0430**	**0,0614**	**0,0650**	**0,3573**	**0,3387**	**0,1619**	412	12,74%	9,20%	3,37%	0,18%	7,91%	4,83%	0,00%
							3233	**100%**	**70,32%**	**18,27%**	**11,41%**	**12,80%**	**64,58%**	**22,62%**

Exhibit 4.3: Weighted average number of optimal weight for the crisis period

Period	Minimizing Variance			Maximizing Sharpe Ratio			Crisis period: Weighted Average of Optimal Weight							
	Bond	Stock	Gold	Bond	Stock	Gold			Minimizing Variance			Maximizing Sharpe Ratio		
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	74,83%	6,66%	18,51%	0,00%	100,00%	0,00%	Day	Weight	Bond	Stock	Gold	Bond	Stock	Gold
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	65,05%	15,30%	19,64%	73,14%	0,00%	26,86%								
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,49%	22,88%	8,64%	0,00%	57,28%	42,72%	548	53,31%	34,68%	8,16%	10,47%	38,99%	0,00%	14,32%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	76,26%	15,12%	8,63%	80,02%	0,00%	19,98%								
period 5: Normal period (10 Mar 2009-29 Apr 2011)	65,52%	21,40%	13,09%	28,29%	40,56%	31,16%	369	35,89%	27,37%	5,43%	3,10%	28,72%	0,00%	7,17%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	74,71%	21,98%	3,30%	93,67%	6,33%	0,00%								
period 7: Normal (4 Oct 2011-1 May 2013)	72,17%	26,42%	1,41%	62,09%	37,91%	0,00%	111	10,80%	8,07%	2,37%	0,36%	10,11%	0,68%	0,00%
Standard deviation	**0,0430**	**0,0614**	**0,0650**	**0,3573**	**0,3387**	**0,1619**								
							1028	**100%**	**70,12%**	**15,96%**	**13,92%**	**77,83%**	**0,68%**	**21,49%**

Exhibit 5: The average daily return of individual assets for the two estimation windows (180 and 360 days)

The two tables below present the daily return on average of each individual asset for each subsample period. The return is calculated by fixing the estimation window as 180 days in the Exhibit 5.1 and 360 days in the Exhibit 5.2.

Exhibit 5.1

	(180-day subsample)	3-month treasury bill rate	US 10-year bond yield	S&P500	GOLD
1	1 Jan 1997 - 9 Sep 1997	0,0140%	0,0013%	0,1403%	-0,0750%
2	10 Sep 1997 - 19 May 1998	0,0139%	0,0242%	0,1084%	-0,0362%
3	20 May 1998 - 26 Jan 1999	0,0127%	0,0381%	0,0830%	-0,0245%
4	27 Jan 1999 - 5 Oct 1999	0,0125%	-0,0606%	0,0328%	0,0846%
5	6 Oct 1999 - 13 Jun 2000	0,0150%	-0,0072%	0,0822%	-0,0646%
6	14 Jun 2000 - 20 Feb 2001	0,0160%	0,0297%	-0,0656%	-0,0655%
7	21 Feb 2001 - 30 Oct 2001	0,0095%	0,0252%	-0,0897%	0,0515%
8	31 Oct 2001 - 9 Jul 2002	0,0048%	-0,0150%	-0,0466%	0,0662%
9	10 Jul 2002 - 18 Mar 2003	0,0038%	0,0390%	-0,0293%	0,0471%
10	19 Mar 2003 - 25 Nov 2003	0,0027%	-0,0073%	0,1201%	0,0851%
11	26 Nov 2003 - 3 Aug 2004	0,0029%	-0,0079%	0,0322%	0,0034%
12	4 Aug 2004 - 12 Apr 2005	0,0059%	0,0023%	0,0520%	0,0515%
13	13 Apr 2005 - 20 Dec 2005	0,0093%	-0,0066%	0,0419%	0,0932%
14	21 Dec 2005 - 29 Aug 2006	0,0127%	-0,0169%	0,0289%	0,1200%
15	30 Aug 2006 - 8 May 2007	0,0136%	0,0057%	0,0893%	0,0715%
16	9 May 2007 - 15 Jan 2008	0,0110%	0,0407%	-0,0348%	0,1617%
17	16 Jan 2008 - 23 Sep 2008	0,0046%	-0,0038%	-0,0639%	0,0111%
18	24 Sep 2008 -2 Jun 2009	0,0007%	0,0181%	-0,0657%	0,0686%
19	3 Jun 2009 - 9 Feb 2010	0,0003%	0,0013%	0,0832%	0,0561%
20	10 Feb 2010- 19 Oct 2010	0,0004%	0,0571%	0,0625%	0,1271%
21	20 Oct 2010 - 28 Jun 2011	0,0003%	-0,0211%	0,0697%	0,0682%
22	29 Jun 2011 - 6 Mar 2012	0,0001%	0,0584%	0,0415%	0,0707%
23	7 Mar 2012 - 13 Nov 2012	0,0003%	0,0245%	0,0247%	0,0235%
24	14 Nov 2012 -1 May 2013	0,0002%	-0,0013%	0,1274%	-0,1411%

Exhibit 5.2

	Period	Average Daily Return			
	(360-day subsample)	**3-month treasury bill rate**	**US 10-year bond yield**	**S&P500**	**GOLD**
1	1 Jan 1997 - 19 May 1998	0,0139%	0,0127%	0,1244%	-0,0556%
2	20 May 1998- 5 Oct 1999	0,0126%	-0,0112%	0,0579%	0,0300%
3	6 Oct 1999 - 20 Feb 2001	0,0155%	0,0113%	0,0083%	-0,0650%
4	21 Feb 2001 - 9 Jul 2002	0,0072%	0,0051%	-0,0682%	0,0588%
5	10 Jul 2002 - 25 Nov 2003	0,0033%	0,0158%	0,0454%	0,0661%
6	26 Nov 2003 - 12 Apr 2005	0,0044%	-0,0028%	0,0421%	0,0274%
7	13 Apr 2005 - 29 Aug 2006	0,0110%	-0,0117%	0,0354%	0,1066%
8	30 Aug 2006 - 15 Jan 2008	0,0123%	0,0232%	0,0272%	0,1166%
9	16 Jan 2008 - 2 Jun 2009	0,0027%	0,0071%	-0,0648%	0,0399%
10	3 Jun 2009 - 19 Oct 2010	0,0004%	0,0292%	0,0728%	0,0916%
11	20 Oct 2010 - 6 Mar 2012	0,0002%	0,0186%	0,0556%	0,0695%
12	7 Mar 2012- 1 May 2013	0,0002%	0,0141%	0,0660%	-0,0427%

Exhibit 6: The risk and return of each optimal portfolio obtained from two subsample criteria: Market Condition (Crisis and Normal Period) and two fixed estimation windows (180 and 360 days) under two optimization frameworks (Variance Minimization and Sharpe Ratio Maximization)

The three tables below show the important information of each optimal portfolio obtained by the Variance Minimization and the Sharpe Ratio Maximization framework. It is noticeable that Sharpe ratio obtained from the two frameworks is too low or too high in some periods. However, it is possible because some study periods take the extreme event in the stock market into account and the estimation window may be too short. However, when considering longer estimation windows such as during the Dot-Com Bubble Burst (4 Sep 2000-9 Oct 2002) and the Normal period (10 Oct 2002-9 Oct 2007) in the Exhibit 6.1. The Sharpe ratio during these two periods indicates more logical number. Nevertheless, the value of portfolios' Sharpe ratio is not used in this thesis's analysis.

Exhibit 6.1

Subsample based on Market Condition		**Minimizing Variance**			**Maximizing Sharpe Ratio**		
Period	**Date**	**Expected Return of a portfolio**	**Standard deviation of a portfolio**	**Sharpe Ratio**	**Expected Return of a portfolio**	**Standard deviation of a portfolio**	**Sharpe Ratio**
1	Dot-com bubble (1 Jan 1997-1 Sep 2000)	0,0019%	0,003346	-3,56%	0,0880%	0,012088	6,14%
2	Dot-com bubble burst (4 Sep 2000-9 Oct 2002)	0,0067%	0,003576	-0,58%	0,0273%	0,004384	4,22%
3	Normal period (10 Oct 2002-9 Oct 2007)	0,0164%	0,003179	2,71%	0,0665%	0,006367	9,22%
4	Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	0,0116%	0,005225	1,40%	0,0507%	0,006821	6,81%
5	Normal period (10 Mar 2009-29 Apr 2011)	0,0409%	0,004061	9,99%	0,0867%	0,005924	14,57%
6	US Debt Crisis (2 May 2011-3 Oct 2011)	0,0559%	0,003669	15,21%	0,1033%	0,004966	20,79%
7	Normal (4 Oct 2011-1 May 2013)	0,0326%	0,002477	13,06%	0,0437%	0,002828	15,39%

Exhibit 6.2

Subsample based on fixed estimation window (180 days)		Minimizing Variance			Maximizing Sharpe Ratio		
Period	**Date**	**Expected Return of a portfolio**	**Standard deviation of a portfolio**	**Sharpe Ratio**	**Expected Return of a portfolio**	**Standard deviation of a portfolio**	**Sharpe Ratio**
1	1 Jan 1997 - 9 Sep 1997	-0,0175%	0,002952	-10,65%	0,1403%	0,009713	13,01%
2	10 Sep 1997 - 19 May 1998	0,0164%	0,002911	0,86%	0,0644%	0,005645	8,95%
3	20 May 1998 - 26 Jan 1999	0,0310%	0,003121	5,85%	0,0480%	0,003654	9,67%
4	27 Jan 1999 - 5 Oct 1999	-0,0308%	0,003850	-11,25%	0,0750%	0,008280	7,55%
5	6 Oct 1999 - 13 Jun 2000	-0,0077%	0,003406	-6,66%	0,0822%	0,014205	4,73%
6	14 Jun 2000 - 20 Feb 2001	-0,0040%	0,002856	-6,99%	0,0297%	0,003675	3,73%
7	21 Feb 2001 - 30 Oct 2001	0,0138%	0,003753	1,15%	0,0362%	0,004927	5,40%
8	31 Oct 2001 - 9 Jul 2002	-0,0024%	0,003619	-1,99%	0,0662%	0,007676	7,99%
9	10 Jul 2002 - 18 Mar 2003	0,0279%	0,003144	7,64%	0,0325%	0,003433	8,35%
10	19 Mar 2003 - 25 Nov 2003	0,0444%	0,003816	10,93%	0,0957%	0,005697	16,32%
11	26 Nov 2003 - 3 Aug 2004	0,0051%	0,003884	0,58%	0,0322%	0,006785	4,32%
12	4 Aug 2004 - 12 Apr 2005	0,0203%	0,002944	4,91%	0,0518%	0,004946	9,28%
13	13 Apr 2005 - 20 Dec 2005	0,0137%	0,002829	1,58%	0,0738%	0,005660	11,39%
14	21 Dec 2005 - 29 Aug 2006	-0,0083%	0,002702	-7,80%	0,0768%	0,008674	7,39%
15	30 Aug 2006 - 8 May 2007	0,0262%	0,002292	5,50%	0,0865%	0,005250	13,89%
16	9 May 2007 - 15 Jan 2008	0,0359%	0,003064	8,13%	0,1053%	0,005655	16,68%
17	16 Jan 2008 - 23 Sep 2008	-0,0178%	0,003879	-5,79%	0,0111%	0,018086	0,36%
18	24 Sep 2008 -2 Jun 2009	0,0158%	0,006757	2,24%	0,0380%	0,009332	4,00%
19	3 Jun 2009 - 9 Feb 2010	0,0260%	0,003730	6,87%	0,0458%	0,004969	9,16%
20	10 Feb 2010- 19 Oct 2010	0,0619%	0,002909	21,15%	0,0695%	0,003083	22,41%
21	20 Oct 2010 - 28 Jun 2011	0,0164%	0,003398	4,74%	0,0691%	0,006253	11,01%
22	29 Jun 2011 - 6 Mar 2012	0,0549%	0,003512	15,62%	0,0553%	0,003524	15,68%
23	7 Mar 2012 - 13 Nov 2012	0,0245%	0,002487	9,76%	0,0245%	0,002487	9,76%
24	14 Nov 2012 -1 May 2013	0,0299%	0,001975	15,00%	0,0331%	0,002058	15,96%

Exhibit 6.3

Subsample based on fixed estimation window (360 days)		Minimizing Variance			Maximizing Sharpe Ratio		
Period	Date	Expected Return of a portfolio	Standard deviation of a portfolio	Sharpe Ratio	Expected Return of a portfolio	Standard deviation of a portfolio	Sharpe Ratio
1	1 Jan 1997 - 19 May 1998	-0,0011%	0,002964	-5,07%	0,1244%	0,010478	10,54%
2	20 May 1998- 5 Oct 1999	0,0038%	0,003596	-2,46%	0,0448%	0,007923	4,07%
3	6 Oct 1999 - 20 Feb 2001	0,0007%	0,003204	-4,60%	0,0083%	0,013210	-0,54%
4	21 Feb 2001 - 9 Jul 2002	0,0026%	0,003776	-1,20%	0,0588%	0,008438	6,12%
5	10 Jul 2002 - 25 Nov 2003	0,0314%	0,003557	7,90%	0,0427%	0,004213	9,36%
6	26 Nov 2003 - 12 Apr 2005	0,0121%	0,003476	2,22%	0,0386%	0,005610	6,09%
7	13 Apr 2005 - 29 Aug 2006	0,0019%	0,002828	-3,22%	0,0721%	0,007098	8,60%
8	30 Aug 2006 - 15 Jan 2008	0,0304%	0,002714	6,68%	0,0607%	0,004432	10,91%
9	16 Jan 2008 - 2 Jun 2009	0,0002%	0,005788	-0,42%	0,0258%	0,011490	2,01%
10	3 Jun 2009 - 19 Oct 2010	0,0444%	0,003358	13,12%	0,0518%	0,003627	14,17%
11	20 Oct 2010 - 6 Mar 2012	0,0306%	0,003574	8,51%	0,0374%	0,003952	9,41%
12	7 Mar 2012- 1 May 2013	0,0281%	0,002314	12,02%	0,0332%	0,002496	13,21%

Exhibit 7: The optimal weight of all assets in the portfolio obtained from two subsample criteria: Market Condition (Crisis and Normal Period) and two fixed estimation windows (180 and 360 days) under the Variance Minimization and the Sharpe Ratio Maximization frameworks

Optimal Weight (Subsample by the market condition)						
	Minimizing Variance			Maximizing Sharpe Ratio		
Period	Bond	Stock	Gold	Bond	Stock	Gold
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	74,83%	6,66%	18,51%	0,00%	100,00%	0,00%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	65,05%	15,30%	19,64%	73,14%	0,00%	26,86%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,49%	22,88%	8,64%	0,00%	57,28%	42,72%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	76,26%	15,12%	8,63%	80,02%	0,00%	19,98%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	65,52%	21,40%	13,09%	28,29%	40,56%	31,16%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	74,71%	21,98%	3,30%	93,67%	6,33%	0,00%
period 7: Normal (4 Oct 2011-1 May 2013)	72,17%	26,42%	1,41%	62,09%	37,91%	0,00%

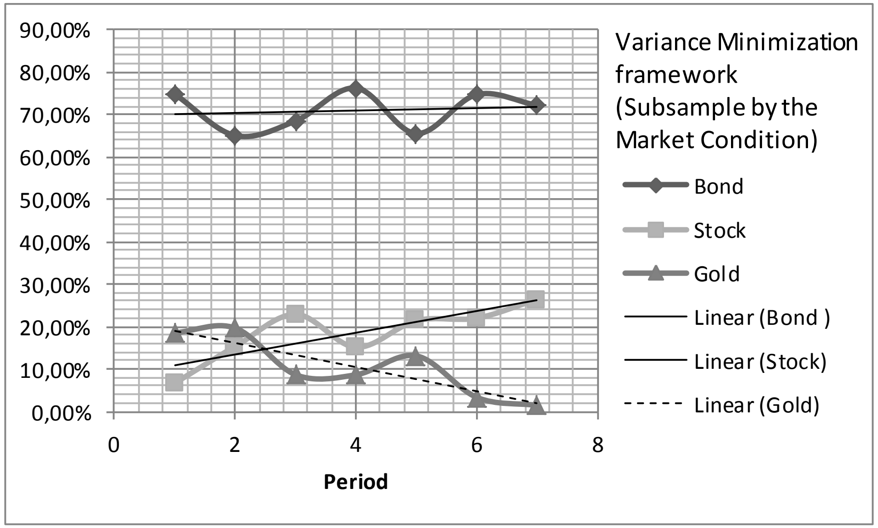
Optimal Weight (Subsample by 180-day estimation window)						
	Minimizing Variance			Maximizing Sharpe Ratio		
Period	Bond	Stock	Gold	Bond	Stock	Gold
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	74,80%	5,46%	19,73%	30,49%	54,19%	15,33%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	64,80%	15,83%	19,38%	49,27%	1,47%	49,26%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,26%	21,68%	10,06%	10,73%	53,37%	35,90%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	70,86%	19,65%	9,49%	28,35%	0,00%	71,65%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	66,95%	24,95%	8,10%	39,73%	35,60%	24,67%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	68,21%	26,76%	5,04%	45,82%	36,55%	17,62%
period 7: Normal (4 Oct 2011-1 May 2013)	72,50%	25,68%	1,82%	70,59%	26,78%	2,63%

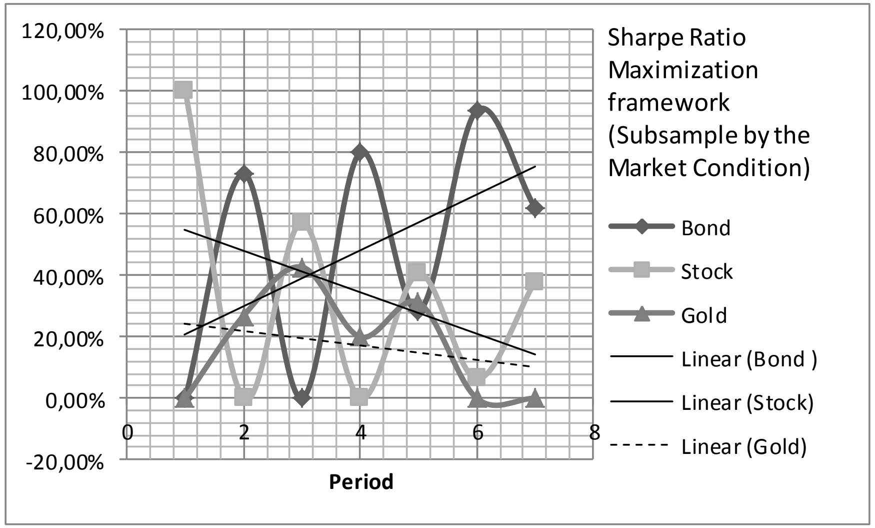
Optimal Weight (Subsample by 360-day estimation window)						
	Minimizing Variance			Maximizing Sharpe Ratio		
Period	Bond	Stock	Gold	Bond	Stock	Gold
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)	73,66%	6,39%	19,94%	0,00%	82,39%	17,61%
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)	67,60%	15,35%	17,05%	4,49%	24,96%	70,54%
period 3: Normal period (10 Oct 2002-9 Oct 2007)	68,88%	21,46%	9,66%	18,75%	42,54%	38,71%
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)	74,54%	15,63%	9,84%	43,72%	2,66%	53,61%
period 5: Normal period (10 Mar 2009-29 Apr 2011)	69,42%	23,20%	7,38%	54,74%	22,74%	22,52%
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)	69,62%	24,96%	5,42%	55,44%	28,00%	16,55%
period 7: Normal (4 Oct 2011-1 May 2013)	71,18%	26,89%	1,94%	61,11%	34,43%	4,46%

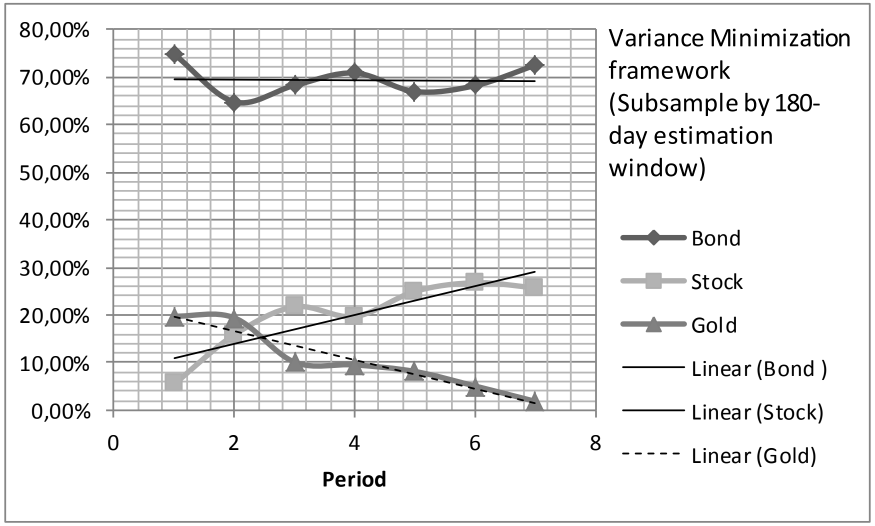
Exhibit 8: The graph comparing the optimal weight of all assets in the portfolio obtained from two subsample criteria: Market Condition (Crisis and Normal Period) and two fixed estimation windows (180 and 360 days) under the Variance Minimization and the Sharpe Ratio Maximization frameworks

The figures present the optimal weight of all assets in the 7 subsample periods based on the two market conditions. For the two fixed estimation windows, how to obtain the weighted average of each asset's weight is explained in the Exhibit 3. Both of two optimization frameworks indicate the declining trend over time for gold. However, the graph of the Sharpe Ratio Maximization shows more fluctuation of the change in the weight.

Period
period 1: dot-com bubble (1 Jan 1997-1 Sep 2000)
period 2: dot-com bubble burst (4 Sep 2000-9 Oct 2002)
period 3: Normal period (10 Oct 2002-9 Oct 2007)
period 4: Global Financial Crisis 2008 (10 Oct 2007-9 Mar 2009)
period 5: Normal period (10 Mar 2009-29 Apr 2011)
period 6: US Debt Crisis (2 May 2011-3 Oct 2011)
period 7: Normal (4 Oct 2011-1 May 2013)







80,00%
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2
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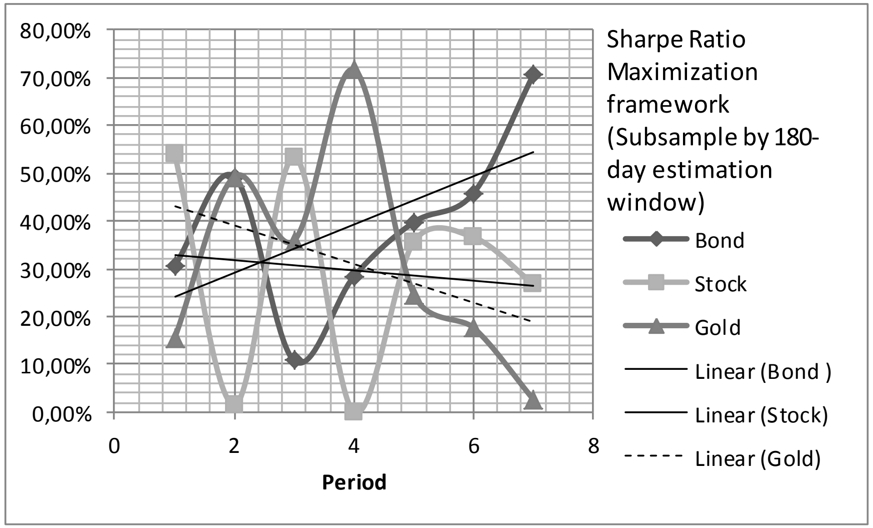
8
Period
Variance Minimization framework (Subsample by 180-day estimation window)
Bond
Stock
Gold
Linear (Bond)
Linear (Stock)
Linear (Gold)



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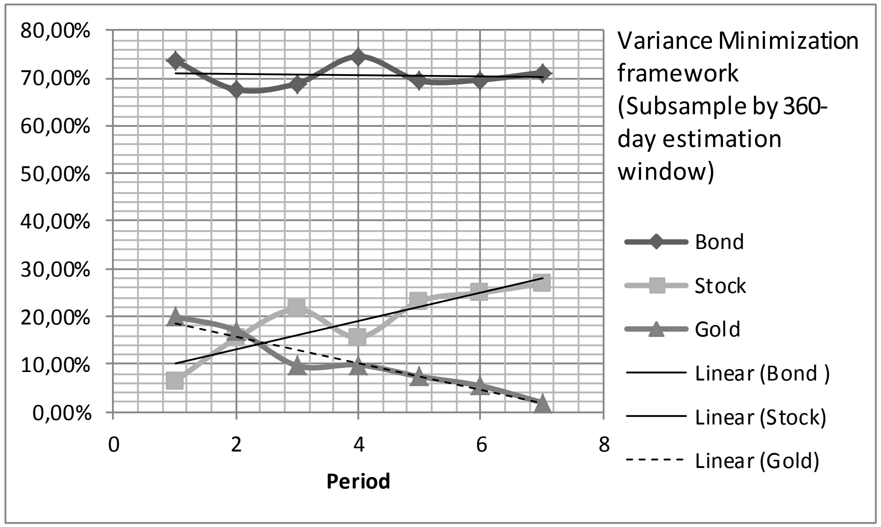
Period
Sharpe Ratio Maximization framework (Subsample by 180-day estimation window)
Bond
Stock
Gold
Linear (Bond)
Linear (Stock)
Linear (Gold)



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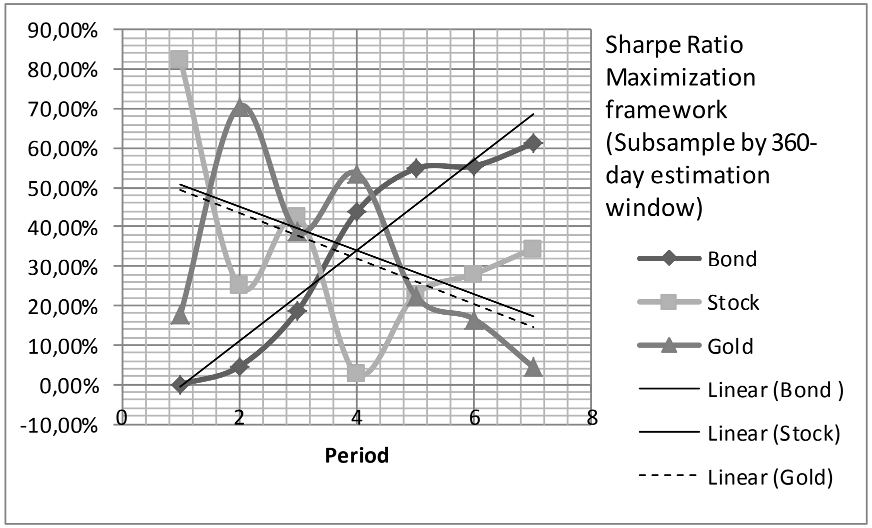
Period
Variance Minimization framework (Subsample by 360-day estimation window)
Bond
Stock
Gold
Linear (Bond)
Linear (Stock)
Linear (Gold)



90,00%
80,00%
70,00%
60,00%
50,00%
40,00%
30,00%
20,00%
10,00%
0,00%
-10,00%
0
2
4
6
8
Period
Sharpe Ratio Maximization framework (Subsample by 360-day estimation window)
Bond
Stock
Gold
Linear (Bond)
Linear (Stock)
Linear (Gold)