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Alternative Mechanisms for Kidney Exchange

- Could More Swedish Patients Receive a New Kidney?

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Abstract

Patients who are in need of a new kidney and who have a living donor may not be able to receive a kidney from the donor because of blood- or tissue-type incompatibilities. If patient-donor pairs exchange kidneys, more kidney transplants can be performed. In Sweden, no system for kidney exchange exists. This thesis investigates how many more Swedish kidney patients could receive a new kidney if a system for kidney exchange would be organized. Simulations based on a priority mechanism are conducted in order to answer this question. The results suggest that by using a priority mechanism, more Swedish patients could receive a new kidney.

Keywords: Kidney Exchange, Priority Mechanism, Sweden

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Contents

1 Introduction

When treating patients suffering from serious kidney disease, transplantation is the preferred treatment (Roth et al., 2005b, p. 457). There are three ways in how a patient can receive a new kidney, either from a living donor with whom the patient has a relation, or from a donor with whom the patient has no relation, or from a cadaveric donor. In Sweden, the most common donations are either from a living donor with whom the patient has a relation or from a cadaveric donor. In table 1, the development of kidney transplants in Sweden for the period 2002-2011 are shown. In the second and third column, the number of kidney transplants from living donors with whom the patient has a relation and cadaveric donors are shown respectively.

Table 1: Swedish Kidney Transplants

	Living	*Cadaveric*	*Total Transplants*	*Dead*	*Total Patients*
Year					
2002	114	194	308	8	583
2003	130	215	345	5	532
2004	142	230	372	6	504
2005	172	219	391	19	485
2006	131	234	365	18	479
2007	123	256	379	21	505
2008	136	283	419	28	452
2009	164	229	393	19	480
2010	168	202	370	12	593
2011	184	251	435	27	634

It is clear from table 1 that there is a lack of supply of kidneys in Sweden. One of the reasons that patients do not receive a new kidney through transplantation is because every donor cannot donate to every patient. Due to mismatch of blood- and/or tissue-type, a donor might not be able to donate his/her kidney to a patient (Roth et al., 2004, p. 461).

In Sweden today, patients who have a compatible living donor undergo a surgery and receive a new kidney. Patients who have an incompatible or no donor are put on a wait-list for receiving a cadaveric kidney. Patients usually wait between two to four years before receiving a cadaveric kidney (Lennerling, 2012, p. 15-17). The responsibility for kidney transplantations in Sweden, is divided among four hospitals which are responsible for one geographical area respectively. The intention is that kidneys of donors from a certain area should be used in that area. However, exchanges do occur between the hospitals and between the Scandinavian countries (Sahlgrenska Universitetssjukhuset, 2004, chap. 1, p.

2). In order to assign kidneys to the patients on the cadaver wait-list, patients are given a priority. In general, the patient who has spent most time on the wait-list has the highest priority. However, when determining a patient's priority other characteristics are taken into considerations as well such as tissue-type incompatibility and age. Furthermore, patients on the cadaver wait-list are divided into different groups depending on blood-type. Thus, patients on the cadaver wait-list are matched with cadaveric donors of the same blood-type (Sahlgrenska Universitetssjukhuset, 2004, chap. 4, p. 14). If, in Sweden today, a patient has an incompatible donor the donor is sent home and the patient is put on the cadaver wait-list. Two questions arise, is this the only way to organize a kidney exchange system? And, could there be a better way to organize such a system?

In 2004, a centralized kidney exchange clearinghouse was established in New England, U.S.A. The clearinghouse uses a computerized matching algorithm to match patients and donors such that as many patients as possible receive a new kidney (Roth et al., 2005a). In 2012, 2000 patients had received a new kidney from participating in the centralized clearinghouse (Andersson and Lyttkens, 2012, p. 2277).

In comparison to the Swedish system, the clearinghouse in New England offered two additional ways in which patients could improve their situation. In addition to receiving a kidney from a living donor with whom the patient has a relation or from a cadaveric donor, the patients could participate in *paired exchange* and *list exchange*. Paired exchange involves two patient-donor pairs. The basic idea is to exploit opportunities for exchanges of kidneys between patient-donor pairs. Even though the patients are incompatible with their donors, the donors might be compatible with the patient of the other pair. Thus, if the patients exchange donors two kidney transplantations can be carried out. List exchange occurs when a patient with an incompatible donor chooses to donate his/her donor's kidney to a patient on the cadaver wait-list. In return, the patient receives higher priority on the cadaver wait-list. Both type of exchanges are welfare improving for patients since they increase the supply of kidneys, thus allowing for more transplantations to be carried out (Roth et al., 2004, p. 459). The question has been raised by Andersson and Lyttkens (2012), whether a more advanced kidney exchange system should be organized in Sweden where patients, to a larger extent, could participate in these exchanges. However, up until 2012 only one paired exchange has been carried out and list exchange is not an option for Swedish patients (Lennerling, 2012, p. 15).

When organizing a kidney exchange system, the main issue is how to allocate a finite number of kidneys to a finite number of patients such that as many patients as possible receive a new kidney. In order to solve this problem, three matching mechanism have been considered for matching patients with kidneys. The first mechanism is the *Top Trading Cycles mechanism* (TTC) and can be applied when participating in a list exchange is not an option for patients. What the Top Trading Cylces mechanism tries to do is to find ways in which patient-donor pairs can trade kidneys with each other such that the patients receive a new kidney. As a simple example, consider three patient-donor pairs: $(1, a)$, $(2, b)$ and $(3, c)$ where all patients are incompatible with their donor and thus no kidney transplantations can be performed. However, donor a is compatible with patient 2, donor b is compatible with patient 3 and donor c is compatible with patient 1. If the three pairs trade their kidneys in the way indicated by their compatibilities, all patients will receive a new kidney. Moreover, such exchanges could be extended to include any number of patient-donor pairs. The TTC mechanism seeks to find opportunities for such trades among patient-donor pairs in order for as many patients as possible to be given a new kidney.

The second mechanism is the *Top Trading Cycles and Chains mechanism* (TTCC). In addition to participate in trades as explained for the TTC mechanism, a patient can also choose to donate his/her donor's kidney to a patient on the cadaver wait-list. Thus, the patient can participate in a list exchange and is rewarded high priority on the cadaver wait-list. Moreover, more than one patient-donor pair can participate in such a donation. Consider an example of two patient-donor pairs: $(1, a)$ and $(2, b)$ where both patients are incompatible with their respective donors. Furthermore, donor b is compatible with patient 1 while donor a is incompatible with patient 2. Hence, the two pairs cannot trade kidneys such that both patients receive a new kidney. However, patient 2 would like to improve his/her situation by donating his/her donor's kidney to a patient on the cadaver wait-list. The TTCC mechanism then seeks to find a way in which as many patients as possible can benefit from this type of exchange. In our example, donor b would donate his/her kidney to patient 1 while donor a donates his/her kidney to a patient on the cadaver wait-list. Thus, patient 1 and a patient on the cadaver wait-list receive a new kidney. Moreover, patient 2 receives high priority on the cadaver wait-list which increases his/her probability of receiving a new kidney. The TTCC mechanism seeks to match patients with kidneys through these two exchanges.

The third mechanism is the *priority mechanism*. In general, a priority mechanism only consider pairwise exchanges, i.e. trades between two patient-donor pairs. Moreover, participating in list exchange is a possibility for patients and every patient is given a priority. A priority mechanism starts with the patient who has the highest priority and seeks to match him/her with another pair, if trading kidneys is a possibility. The priority mechanism continues in this fashion according to the priority ordering, matching compatible patients, never sacrificing a higher priority patient for a lower priority patient. Whenever a patient cannot be found a matching, the priority mechanism skips that patient. In this way, a priority mechanism matches as many patients as possible.

The purpose of this thesis is to investigate how many more Swedish patients could receive a new kidney if a more sophisticated matching mechanism was used. In order to answer this question, different scenarios will be simulated using a priority mechanism.

2 A Kidney Exchange Model

The model presented in this section is based on Roth et al. (2004), Sönmez and Ünver (2011) and Roth et al. (2005b). In a kidney exchange matching problem, there exist $N = \{n_1, \ldots, n_n\}$ patient-donor pairs where the patient is in need of a new kidney. Let $N = N_I \cup N_C$ be such that N_C is the set of patients who have a compatible living donor and N_I be the set of patients who have an incompatible living donor. Any $n_i \in N$ is referred to as a patient-donor pair. Moreover, let p_i and d_i denote the patient and the donor of pair n_i respectively. In addition, there exist a set $C = \{c_1, \ldots, c_n\}$ of cadaver donors not associated with any patient in particular when entering the kidney exchange matching problem. Let $M = \{m_1, \ldots, m_n\}$ be the set of patients who do no not have a living donor.

A patient is *compatible* with a donor if the donor can medically donate his/her kidney to the patient. Two pairs $n_1, n_2 \in N$ are *mutually compatible* if the donor of pair n_1 is compatible with the patient of pair n_2 and the donor of pair n_2 is compatible with the patient of pair n_1. Furthermore, the option of participating in a list exchange is denoted by w.

Whether a donor can medically donate to a patient or not is determined by two genetic characteristics. The first characteristic is the *ABO blood-type* of the patient and the donor. There exist four blood-types: O, A, B and AB, given no other complications:

- A donor of blood-type O can donate to any patient
- A donor of blood-type A can either donate to a patient of blood-type A or AB
- A donor of blood-type B can either donate to a patient of blood-type B or AB
- A donor of blood-type AB can only donate to a patient of blood-type AB

Thus, given no other complications a patient of blood-type AB can receive any kind of kidney, while a patient of blood-type O can only receive kidneys of blood-type O. The second genetic characteristic that affect compatibility is tissue type, or the HLA type of the patient and the donor. HLA type is determined by six proteins and an increase in mismatch between the patient and the donor decreases the chance of survival of the transplanted organ. Therefore, prior to transplantation a crossmatch test is conducted. If the patient has preformed antibodies against the donor's kidney, donation is not possible and the patient and the donor are incompatible. Incompatibility due to HLA type mismatch is known as *positive crossmatch*.

For each pair $n_i \in N$, let $\succeq_{n_i}$ denote patient of pair n_i's *preference relation* over $N \cup w$. Moreover, let $\succ_{n_i}$ be the strict preference relation and $\sim_{n_i}$ be the indifference relation associated with $\succeq_{n_i}$. The preference relation $\succeq_{n_i}$ is determined by the compatibility between the patient of the pair and all donors and if w is considered acceptable. A patient is assumed to be indifferent between any compatible donor and to strictly prefer any compatible donor to an incompatible. Moreover, a patient who is incompatible with his/her donor strictly prefers his/her donor to any other incompatible donor. In other words, the patient strictly prefers to remain unmatched than to be matched with another incompatible donor. It is assumed that only patients incompatible with their donor, i.e. $n_i \in N_I$ consider the wait-list option, w acceptable. There is a decreased chance of survival for a cadaveric kidney compared to for a kidney from a living donor. Therefore, patients strictly prefer a compatible living donor to a compatible cadaveric donor. For any pair with an incompatible donor, $n_i \in N_I$:

- $n_1 \sim_{n_i} n_2$ for any $n_1, n_2 \in N$ and $n_1 \neq n_2$, where the donors of pair n_1 and n_2 are compatible with the patient of pair n_i
- $n_1 \succ_{n_i} n_i$ for any $n_1 \in N$, where the donor of the pair n_1 is compatible with the patient of pair n_i
- $n_i \succ_{n_i} n_1$ for any $n_1 \in N$, where the donor of the pair n_1 is incompatible with the patient of pair n_i
- $n_1 \succ_{n_i} w$ for any $n_1 \in N$, where the donor of the pair n_1 is compatible with the patient of pair n_i

For any pair with a compatible donor, $n_c \in N_C$:

- $n_1 \sim_{n_c} n_2$ for any $n_1, n_2 \in N$ and $n_1 \neq n_2$, where the donors of pair n_1 and n_2 are compatible with the patient of pair n_c
- $n_1 \sim_{n_c} n_c$ for any $n_1 \in N$, where the donor of pair n_1 is compatible with the patient of pair n_c
- $n_c \succ_{n_c} n_1$ for any $n_1 \in N$, where the donor of pair n_1 is incompatible with the patient of pair n_c

A kidney exchange problem is denoted $(N, \succeq)$, where $\succeq = (\succeq_{n_j})_{n_j \in N}$ is the collection of preferences for all pairs. A *matching* μ is a function such that for any pair $n_j \in N$, either:

- $\mu(n_j) = n_k$ for any $n_k \in N$ or
- $\mu(n_j) = w$ or
- $\mu(n_j) = n_j$

Thus, for any matching μ and pair $n_j \in N$, $\mu(n_j) = n_j$ means that pair n_j is unmatched. If for any two pairs $n_j, n_k \in N$, a matching μ is such that $\mu(n_j) = n_k$ and $\mu(n_k) = n_j$, pair n_j and n_k are mutually compatible and a kidney exchange is carried out. For a matching μ, any kidney belonging to a pair $n_j \in N$ can only be assigned to one patient. However, the wait-list option w can be assigned to any number of patients. A *mechanism* φ is a systematic procedure which finds a matching μ for a kidney exchange problem.

For any kidney exchange problem $(N, \succeq)$, a matching needs to be selected. Due to blood-type incompatibility and positive crossmatch, performing a kidney exchange between a patient and a donor might not be possible. For the same reasons, trying to organize exchanges between patients and donors is not always straightforward. Therefore, a sophisticated mechanism is required which ensures that as many patients as possible are given a new kidney. Such mechanisms will be discussed in the following section.

3 Theoretical Overview

This section will present three mechanisms applicable for selecting a matching to a kidney exchange problem. In section 3.1 the Top Trading Cycles mechanism will be discussed. Section 3.2 will outline the Top Trading Cycles and Chains mechanism. Moreover, priority mechanisms will be discussed in section 3.3. Finally, section 3.4 present a discussion of why these mechanisms are desirable.

The simulations presented in this paper are based on a priority mechanism. However, since the TTC and TTCC mechanisms are related to the priority mechanism and illustrate important concepts of the matching procedure well, these mechanisms will too be presented.

3.1 The Top Trading Cycles Mechanism

Consider a kidney exchange system where list exchange is not an option, such as in Sweden today. What the *Top Trading Cylces mechanism* (TTC) tries to do is to find ways in which patient-donor pairs can trade kidneys with each other in order for as many patients as possible to receive a new kidney. Moreover, these trades can include any number of patient-donor pairs.

Formally, the preference relation is slightly changed such that for each pair $n_i \in N$, let $\succeq_{n_i}$ denote patient of pair n_i's preference relation over N. In a setting where list exchange is not an option, David Gale's Top Trading Cycles mechanism can be used for selecting a matching (Roth et al., 2004, p. 465).

Let a *cycle* be an ordered list of patients and donors, $(d_1, p_1, d_2, p_2, \ldots, d_i, p_i)$ such that donor d_1 points toward patient p_1, patient p_1 points toward donor $d_2, \ldots$, donor d_i points toward patient p_i and patient p_i points toward donor d_1 (Roth et al., 2004, p. 466) (Roth et al., 2005a, p. 377). Gale's Top Trading Cycles algorithm can now be introduced:

Algorithm 1. *The Top Trading Cycles Algorithm*

Step 1: *Each patient starts by pointing toward his/her most preferred donor. At least one cycle is formed. For each cycle formed, the corresponding trades are carried out, i.e., each patient receives the kidney from the donor at whom he/she is pointing. After the trades are conducted, the pairs belonging to the cycles are removed.*

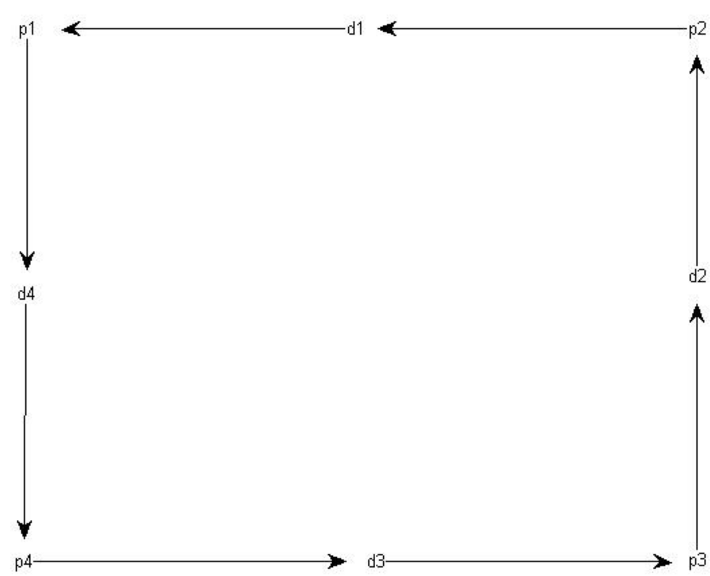
Step k: *In general, every remaining patient points toward his/her most preferred donor of those still available. At least one cycle is formed and the corresponding trades are carried out. All pairs which participated in a trade are removed.*

The algorithm continues in the same fashion until no agents are left and patients receive the kidney of the donor at whom they were pointing to when being removed by the algorithm (Roth et al., 2004, p. 462).

Let us consider a simple example of four patient-donor pairs to clarify how the algorithm works in comparison to the system currently used in Sweden. For simplicity, assume that there exist no positive crossmatch between any patient and any donor. As mentioned earlier, for each pair n_i let p_i and d_i denote the patient and the donor of the pair n_i respectively. Consider the following four patient-donor pairs and their corresponding blood-types:

- n_1: p_1 - O, d_1 - A
- n_2: p_2 - A, d_2 - B
- n_3: p_3 - B, d_3 - AB
- n_4: p_4 - AB, d_4 - O

Figure 1: Example of the TTC Algorithm - One Cycle is Found



Assume that the patients have the following preferences over all pairs:

- n_1: $n_4 \succ_{n_1} n_1 \succ_{n_1} n_2 \sim_{n_1} n_3$
- n_2: $n_1 \succ_{n_2} n_4 \succ_{n_2} n_2 \succ_{n_2} n_3$
- n_3: $n_2 \succ_{n_3} n_4 \succ_{n_3} n_3 \succ_{n_3} n_1$
- n_4: $n_3 \succ_{n_4} n_4 \succ_{n_4} n_1 \succ_{n_4} n_2$

In Sweden today, only patients of compatible pairs would receive a new kidney. Thus, only the patient of the pair n_4 would receive a new kidney. The remaining three patients would be put on the wait-list for cadaveric donors. If the TTC mechanism was to be used instead, in step 1 of the algorithm, the following cycle would be formed: (d_1, p_1, d_4, p_4, d_3, p_3, d_2 p_2,). The cycle is depicted in figure 1. The TTC mechanism selects the following matching:

- $\mu(n_1) = n_4$
- $\mu(n_2) = n_1$
- $\mu(n_3) = n_2$
- $\mu(n_4) = n_3$

Using the TTC mechanism, all four patients receive a new kidney.

3.2 The Top Trading Cycles and Chains Mechanism

In order to incorporate the possibility for patients to participate in list exchange, the *Top Trading Cycles and Chains Mechanism* (TTCC) can be used. In addition to participate in cycles as explained for the TTC mechanism, a patient can also choose to donate his/her donor's kidney to a patient on the cadaver wait-list. Thus, the patient can participate in a list exchange and is rewarded high priority on the cadaver wait-list. Moreover, more than one patient-donor pair can participate in a list exchange. The TTCC mechanism then seeks to find a way in which as many patients as possible can benefit from participating in cycles and list exchanges.

As outlined in section 2, for each pair $n_i \in N$, let $\succeq_{n_i}$ denote patient of pair n_i's preference relation over $N \cup w$. Let a $w - chain$ be an ordered list of pairs: (d_1, p_1, d_2, p_2,. . ., d_{i-1}, p_{i-1}, d_i, p_i) such that donor d_1 points toward patient p_1, patient p_1 points toward donor d_2,. . ., donor d_{i-1} points toward patient p_{i-1}, patient p_{i-1} points toward donor d_i, donor d_i points toward the patient p_i and patient p_i points toward w. As a w-chain is selected, patient p_1 receives a kidney from donor d_2,. . ., patient p_{i-1} receives a kidney from donor d_i, patient p_i is rewarded high priority on the cadaver wait-list and the kidney of donor d_1 is donated to a patient on the cadaver wait-list (Roth et al., 2004, p. 466). At the same step of the mechanism there might exist more than one w-chain. Therefore, a chain selection rule is needed to select which w-chain to carry out[1]. In similarity with the TTC mechanism, some patients will receive a final assignment and be removed by the TTCC mechanism. However, depending on the chain selection rule some pairs might be given an assignment while not being removed. These pairs are thus given a passive role during a segment of the algorithm.

The Top Trading Cycles and Chains algorithm can now be introduced:

Algorithm 2. *The Top Trading Cycles and Chains Algorithm*

[1]For a discussion of different chain selection rules and their implications see Roth et al. (2004)

Step 1: *At the start, all patients are active and all kidneys are available. At each stage, every patient points toward his/her most preferred donor of those still available, or w. Every passive patient points toward his/her given assignment and each donor points toward the patient of his/her pair.*

Step 2: *There exists either a cycle, a w-chain, or both. If a cycle does not exist, go to step 3. Otherwise, locate the cycle, perform the corresponding trades and remove all pairs forming the cycle. Continue in the same fashion, each remaining patient points toward his/her most preferred donor of those still remaining and carry out every cycle found. Repeat this procedure until no more cycles are formed.*

Step 3: *If there are no patients left, the algorithm is terminated. Otherwise, there exists at least one w-chain. In accordance with the chain selection rule, a w-chain is selected and the assignments for the patients in the w-chain is final. The chain selection rule also determines whether the w-chain is removed or not.*

Step 4: *After the selection of a w-chain, new cycles might be able to be formed. Repeat step 2 and 3 with the remaining active patients and donors until no patient remains.*

Once the algorithm is done, every patient of each pair is either assigned a kidney or the list exchange option w. However, every patient might not receive a new kidney through transplantation since a patient might prefer to be matched with his/her incompatible donor rather than to be matched with a donor of a different pair (Roth et al., 2004, p. 468-469).

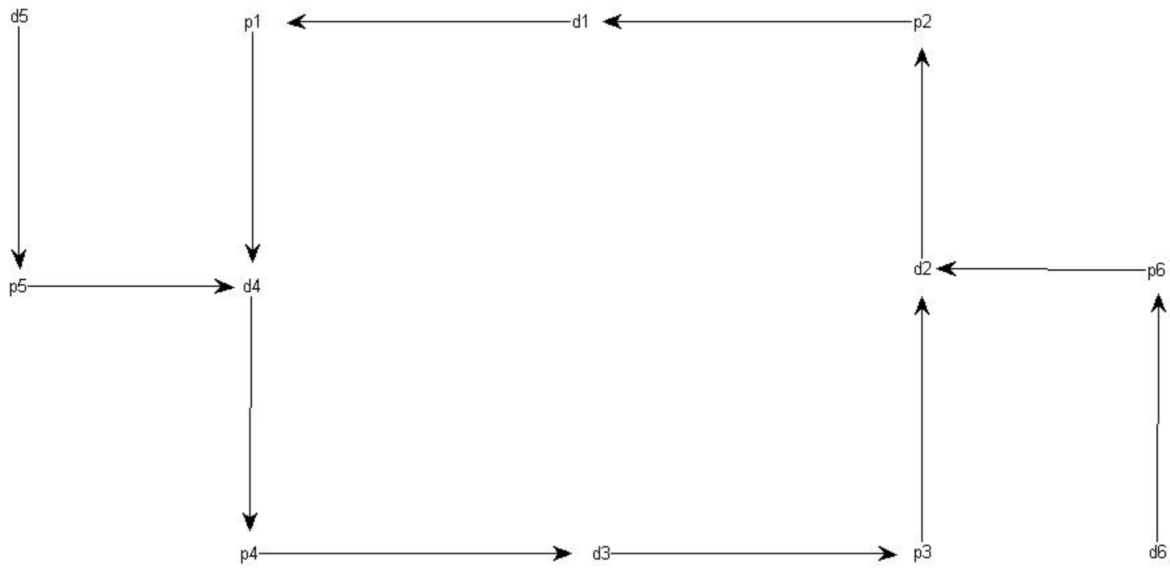
A simple example can be considered to illustrate the TTCC mechanism. This example is constructed in such a way that no considerations of chain selection rules have to be made. Assume that our kidney exchange population is extended from the example in section 3.1 to include the following two patient-donor pairs and their corresponding blood-types:

- n_5: p_5 - O, d_5 - B
- n_6: p_6 - B, d_6 - AB

Assume that preferences for the six pairs are extended to be the following:

- n_1: $n_4 \succ_{n_1} n_1 \succ_{n_1} n_2 \sim_{n_1} n_3 \sim_{n_1} n_5 \sim_{n_1} n_6 \succ_{n_1} w$
- n_2: $n_1 \succ_{n_2} n_4 \succ_{n_2} n_2 \succ_{n_2} n_3 \sim_{n_2} n_5 \sim_{n_2} n_6 \succ_{n_2} w$
- n_3: $n_2 \succ_{n_3} n_5 \succ_{n_3} n_4 \succ_{n_3} n_3 \succ_{n_3} n_1 \sim_{n_3} n_6 \succ_{n_3} w$
- n_4: $n_3 \succ_{n_4} n_6 \succ_{n_4} n_4 \succ_{n_4} n_2 \succ_{n_4} n_5 \succ_{n_4} n_1 \succ_{n_4} w$
- n_5: $n_4 \succ_{n_5} w \succ_{n_5} n_5 \succ_{n_5} n_2 \sim_{n_5} n_3 \sim_{n_5} n_6 \sim_{n_5} n_1$
- n_6: $n_2 \succ_{n_6} n_5 \succ_{n_6} n_4 \succ_{n_6} w \succ_{n_6} n_6 \succ_{n_6} n_3 \sim_{n_6} n_1$

Figure 2: Example of the TTCC Algorithm, Round 1 - One Cycle is Found

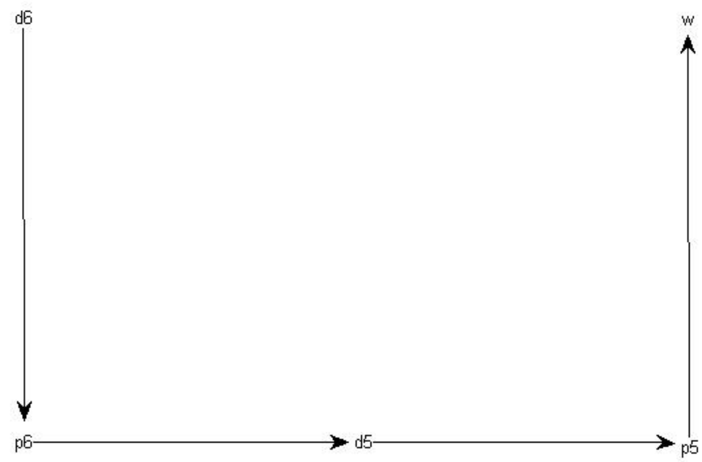


With the current Swedish system, the patient belonging to pair n_4 would be the only one receiving a new kidney. The remaining patients would be put on the cadaver wait-list. However, applying the TTCC algorithm to this kidney exchange problem would yield a completely different outcome. In the first step of the first round of the algorithm, every patient points toward his/her most preferred donor. In step 2, the following cycle is formed: (d_1, p_1, d_4, p_4, d_3, p_3, d_2 p_2,). The pairs forming the cycle are removed and their assignment is final. Next, a second round of step 2 is conducted and the following w-chain is formed: (d_6, p_6, d_5, p_5). In step 3 of the algorithm the w-chain is carried out. Thus, patient p_5 receives high priority on the cadaver wait-list, patient p_6 receives a new kidney from donor d_5 and a patient on the cadaver wait-list receives a new kidney from donor d_6. In figure 2 and figure 3 the first and second round of the TTCC algorithm are depicted. The TTCC algorithm selects the following matching:

- $\mu(n_1) = n_4$
- $\mu(n_2) = n_1$
- $\mu(n_3) = n_2$
- $\mu(n_4) = n_3$
- $\mu(n_5) = w$
- $\mu(n_6) = n_5$

Using the TTCC mechanism for selecting a matching to the kidney exchange problem, five patients with a living donor are given a new kidney. In addition, one patient on the cadaver wait-list is given a new kidney from a living donor. Thus, in total six patients are given a new kidney and one patient has increased his/her chance of receiving a cadaveric kidney.

Figure 3: Example of the TTCC Algorithm, Round 2 - One w-chain is Found



3.3 Priority Mechanisms

When the theory regarding kidney exchange was developed, patients were assumed to have strict preferences and unbound sizes of cycles/w-chains were considered possible. However, based on medical expertise Roth et al. (2005b) developed the theory to be compatible with binary preferences and cycles/w-chains limited to two patient-donor pairs. It seemed reasonable to assume that any patient would consider any compatible living donor equally preferable. Moreover, any exchange of kidneys is preferred to be conducted simultaneously for incentive reasons. A paired exchange involves four simultaneous surgical teams, a three-way exchange six simultaneous surgical teams etc. Thus, for logistical reasons large cycles/w-chains of pairs would be difficult to perform. Therefore, limiting the cycles/w-chains to only include two pairs was considered a first step for a practical implementation of a kidney exchange mechanism in New England (Roth et al., 2005b, p. 153).

Given the constraints of limiting the cycles/w-chains to include only two pairs and for patients to have binary preferences, the TTC and TTCC mechanisms could no longer be used. Both mechanisms require patients to have strict preferences over all donors. Therefore, *Priority Mechanisms* were considered for solving the, slightly different, kidney exchange problem. In order to apply a priority mechanism, a priority ordering has to be given to the patients. In accordance with the priority ordering, a priority mechanism tries to match mutually compatible patients. The mechanism starts by trying to find a mutually compatible patient for the patient who has the highest priority continuing with the patient who has the second highest priority etc. If no match can be found for a patient, the mechanism skips that patient. Moreover, a patient with higher priority is never sacrificed for a patient with lower priority. In this way, the priority mechanism matches as many patients as possible (Roth et al., 2005b, p. 158).

Formally, let an *individual rational matching* be such that for any $n_k \in N$ if $\mu(n_k) \neq n_k$ then $\mu(n_k) \succ_{n_k} n_k$. In other words, a matching μ will only be considered if two pairs are mutually compatible. Let $\mathcal{M}$ be the set of individual rational matchings. Since only individual rational matchings are considered, focus can be put on a reduced kidney exchange problem induced by a *mutual compatibility matrix.* For any patients of pair $n_i, n_j \in N$, let $R = [r_{n_i,n_j}]_{n_i \in N, n_j \in N}$ be an $\mid N \mid \times \mid N \mid$ mutual compatibility matrix, with element:

$$r_{n_i,n_j} = \begin{cases} 1 & \text{if } n_j \succ_{n_i} n_i \text{ and } n_i \succ_{n_j} n_j \\ 0 & \text{otherwise} \end{cases}$$

(N, R) is considered a *reduced problem* of $(N, \succeq_{n_i})$ (Roth et al., 2005b, p. 156).

As mentioned in section 1, allocating cadaveric kidneys to patients is done in accordance with the priority of each patient. A priority mechanism extends this thinking to patients with living donors as well. Let a *priority ordering* be such that the ith patient of pair n_i has priority i. Let the natural ordering (1, 2, 3,. . ., i) be the priority ordering of all patients such that patient belonging to pair n_i is the patient with priority i for each $n_i \in N$. A *priority function* is a non-negative function $\pi : N \longrightarrow \mathbb{R}_+$, if it is increasing in priority, i.e. if $\pi(n_i) \geq \pi(n_{i+1})$.

A hospital H wants to maximize the number of transplants possible given its preferences $\succ_H$ over all matchings. Let $M_\mu = \{n_i \in N : \mu(n_i) \neq n_i\}$, i.e. the number of patients matched at matching μ. If $\succ_H$ is *responsive* to the priority ordering it is a *priority preference.* Then, $\mu \succ_H \mu'$ if $M_{\mu'} \subset M_\mu$, or if for any two patients who belong to pair $n_i, n_j \in N$, $i < j$, $M_\mu \backslash M_{\mu'} = \{n_i\}$ and $M_{\mu'} \backslash M_\mu = \{n_j\}$. Hence, a matching μ is preferred to some other matching μ' if more patients are matched in μ or if patients of higher priority are matched in μ.

Let every matching in $\mathcal{E}^i$ be a *priority matching.* Given a priority ordering (1, 2, 3,. . . , i) and a reduced problem (N, R), a *priority mechanism* selects a matching in the following way (Roth et al., 2005b, p. 158-159):

- let $\mathcal{E}^0 = \mathcal{M}$, be the set of all matchings
- In general for $j \leq i$, let $\mathcal{E}^j \subseteq \mathcal{E}^{j-1}$ be such that:

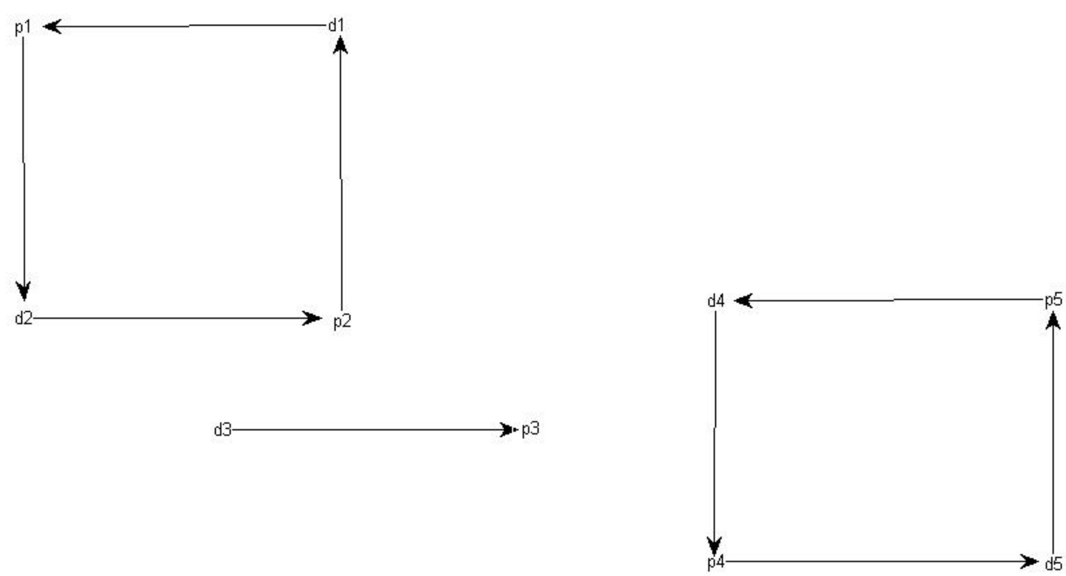
$$\mathcal{E}^j = \begin{cases} \{\mu \in \mathcal{E}^{j-1} : \mu(n_j) \neq n_j\} & \text{if } \exists\ \mu \in \mathcal{E}^{j-1} : \mu(n_j) \neq n_j, \\ \mathcal{E}^{j-1} & \text{otherwise} \end{cases}$$

Respecting the priority ordering of patients, a priority mechanism selects a priority matching such that as many patients as possible are matched. Only mutually compatible pairs are matched and a patient for whom a matching cannot be found is skipped.

Let us consider a simple example where a priority mechanism, at each step, intends to match the patient in consideration with the patient who has the highest priority of the remaining patients. Consider the following five patient donor-pairs with corresponding blood-types and the priority ordering (1, 2, 3, 4, 5):

- n_1: p_1 - O, d_1 - A
- n_2: p_2 - A, d_2 - O
- n_3: p_3 - O, d_3 - AB
- n_4: p_4 - B, d_4 - A
- n_5: p_5 - A, d_5 - B

Figure 4: Example of the Priority Mechanism - Two Cycles are Found



All cycles found by the priority mechanism are depicted in figure 4. Using the current Swedish system, only the patient of the pair n_2 would receive a new kidney. The priority mechanism selects the following matching:

- $\mu(n_1) = n_2$
- $\mu(n_2) = n_1$
- $\mu(n_3) = n_3$
- $\mu(n_4) = n_5$
- $\mu(n_5) = n_4$

Using the priority mechanism, four patients receive a new kidney. The only patient not receiving a kidney is the patient belonging to pair n_3.

3.4 The Axioms

The reason for considering the mechanisms discussed in section 3.1 - 3.3 is that they satisfy desirable axioms. In this context, two important axioms are *Pareto efficiency* and *strategy-proofness*. A matching μ is Pareto efficient if there exists no other matching ν where all patients are weakly better off and some patients are strictly better off. Hence, in a Pareto efficient matching no patient's situation can be improved without making the situation of another patient worse. Moreover, a Pareto efficient mechanism always chooses a Pareto efficient matching (Roth et al., 2004, p. 472). A mechanism is strategy-proof if no patient can gain from lying. There are two ways in which patients can lie, either by misrepresenting their preferences or, if the patient has more than one donor, by not reporting all of his/her donors.

Efficiency is important since it is essential that patients are as well off as possible. A situation not desirable is when there exist two patient-donor pairs who do not receive a new kidney even though they are mutually compatible. An efficient mechanism assures that this is not the case. Strategy-proofness implies that by truthfully reporting their preferences and available donors, patients maximize their chance of receiving a new kidney (Roth et al., 2005b, p. 159). A strategy-proof mechanism makes it easier and safer for patients to participate. Since no advanced strategies for reporting preferences and donors have to be considered, the system is fair and no time has to be spent learning the system in order to develop advanced strategies.

Formally, a matching μ is Pareto efficient if there exists no other matching ν such that: $\nu \succeq_{n_i} \mu \; \forall \; n_i \in N$ and $\nu \succ_{n_i} \mu$ for some $n_i \in N$ (Roth et al., 2005b, p. 156). Let $\mathcal{R}_{n_i}$ be the strategy space of the patient belonging to pair n_i and $\succeq'_{n_i}$ be a misrepresentation of preferences. If $\forall \; n_i \in N$, $\forall \succeq_{n_i}, \succeq'_{n_i} \in \mathcal{R}_{n_i}$ and $\forall \succeq_{-n_i} \in \mathcal{R}_{-n_i}$ a mechanism φ makes truthful preference revelation a dominant strategy, then: $\varphi[\succeq_{-n_i}, \succeq_{n_i}](n_i) \; \succeq_{n_i} \; \varphi[\succeq_{-n_i}, \succeq'_{n_i}](n_i)$ (Abdulkadiroğlu and Sönmez, 2010, p.7).

For a kidney exchange problem and strict preferences the TTC mechanism is efficient (Ma, 1994) and makes truthful preference revelation a dominant strategy for patients (Roth, 1982).

Theorem 1. *The Top Trading Cycles mechanism is efficient and strategy-proof.*

Extending the kidney exchange problem to include list exchange, Roth et al. (2004) show that for any chain selection rule which do not remove a selected w-chain at any non-terminal stage, the TTCC mechanism is efficient. For a wide range of chain selection rules, the TTCC mechanism makes truthful preference revelation a dominant strategy.

Theorem 2. *For specific choices of chain selection rules*[2]*, the Top Trading Cycles and Chains mechanism is efficient and strategy-proof.*

Restricting the attention to paired exchange and binary preferences, Roth et al. (2005b) show that a priority mechanism is efficient and that truthful preference revelation as well as reporting the full set of available donors are dominant strategies for patients.

Theorem 3. *A Priority mechanism is efficient and strategy-proof.*

4 Simulations

The object of the simulations is to determine how many Swedish patients would have received a new kidney if a priority mechanism had been used for finding a matching to the 2011 matching problem. The priority mechanism will start with the patient who has the highest priority, and at each step, try to match the patient with the patient who has the highest priority of the remaining patients. Only pairwise exchanges will be considered and for a match between two patient-donor pairs to occur the pairs have to be mutually compatible. When the cycles/w-chains are limited to two pairs, finding the maximal number of exchanges possible is best done by using Jack Edmond's[3] algorithm (Roth et al., 2005a, p. 378). However, for computational simplicity the simulations in this paper will not be based on Jack Edmond's algorithm. Thus, the simulated results can be improved.

Patients will have binary preferences which are generated in accordance with section 2. Following Roth et al. (2005a, p. 378), two different scenarios for list exchanges are considered. Either list exchange is unavailable, or 40 percent of the patients with an incompatible donor consider participating in a w-chain an acceptable option. Furthermore, no dynamic considerations are made. All patients and donors are assumed to be available at the same time which is not very realistic. In this sense, the results from the simulations can be seen as an upper bound. Moreover, the simulations will consider so called *altruistically unbalanced exchange*. Consider two pairs $n_1, n_2 \in N$ where the donor of pair n_1 is compatible with both patients, while the donor of pair n_2 is only compatible with the patient of pair n_1. Since pair n_1 and n_2 are mutually compatible, a kidney exchange can be performed. However, since the donor of pair n_1 is compatible with the patient of his/her pair, such an exchange would be unlikely to occur in real life (Roth et al., 2005a, p. 377). Even so, considerations of altruistically unbalanced exchange make it possible to define an upper bound for what the mechanism could achieve.

[2]for more details see (Roth et al., 2004).

[3]For more details regarding the algorithm see Roth et al. (2005b)

In 2011, the number of patients receiving a kidney from a living donor with whom the patient has a relation was 184. However, there is no data on how many of the remaining 423 patients who actually have a living donor with whom the patient is incompatible. Therefore, when conducting the simulations, out of the 423 patients, the share of patients who have an incompatible donor will be varied between 0 and 100 %. Hence, due to the uncertainty regarding patients with incompatible donors different scenarios will be simulated as a sensitivity analysis. In real life, whether a patient has a donor or not probably depends on the size of the patient's social network, i.e. the size of the patient's family and the number of his/her friends.

According to Roth et al. (2005a, p. 377), the performance of a kidney exchange matching is largely dependent on the following four factors:

1. The number of patients and donors registered in the database.
2. Whether list exchange is an option or not.
3. The maximum number of patients who can participate in a cycle/w-chain.
4. If altruistically unbalanced exchange is possible.

Since the simulations only consider two-way exchanges, the matching could be improved by increasing the number of participants in the cycles/w-chains. However, Roth et al. (2007) simulate different sizes of cycles/w-chains. They conclude that two-way exchanges account for a large part of the potential gains when using a matching mechanism. Increasing to three-way exchanges, almost all gains are exploited.

4.1 The Patient-Donor Population

In 2011, the number of patients in need of a kidney was 634. As a first step in the simulations and in accordance with section 3.3, patients are randomly assigned a priority. However, in the same year 27 patients on the cadaver wait-list died. Since these patients are probably too weak to survive an operation, the simulations will not consider these 27 patients. As a second step, 27 patients are removed at random from the total list of patients leaving the list at 607 patients in need of a new kidney. Next, the 607 patients have to be given a blood-type. The distribution of blood-types for the Swedish population is shown in table 2 (Blodcentralen, 2012). Since no data is available of the actual distribution of blood-types among the Swedish kidney patient-donor population, the information in table 2 will be used to assign blood-types to all patients.

In proceeding, 184 patients are randomly selected to form the set of patients with compatible donors, i.e. N_C, leaving the cadaver wait-list at 423 patients. The next step is to assign a blood-type to each donor of each compatible pair. Since the patients and the donors of each pair are compatible, no considerations have to be made regarding positive crossmatch. Conditional on the blood type of the patient, a blood-type has to be assigned to each donor of each compatible pair. Given the patient's blood-type and based

Table 2: Distribution of Blood-types For The Swedish Population.

Blood-type	Frequency
O	38 %.
A	44 %.
B	12 %.
AB	6 %.

on the information in table 2, the following distribution of blood-types is used for assigning the compatible donors a blood-type:

Table 3: Distribution of Blood-types For Donors of Compatible Pairs.

		Frequency of Donor Blood-type			
		O	A	B	AB
Patient Blood-type	O	100 %	0 %	0 %	0 %
	A	46 %	54 %	0 %	0 %
	B	76 %	0 %	24 %	0 %
	AB	38 %	44 %	12 %	6 %

As the set N_C is fully defined, the next step is to randomly select the patients who belong to the set N_I, i.e. the patients who have an incompatible donor. As discussed in section 4, different shares of patients belonging to this set will be considered. After the share is determined, patients are selected at random from the cadaver wait-list to form part of the set N_I. These patients are removed from the cadaver wait-list while maintaining their priority.

Assigning blood-types to the donors of the pairs belonging to N_I is not as simple as for the pairs forming the set N_C. Since, as discussed in section 2, incompatibility between the patient and the donor can stem from either blood-type mismatch or positive crossmatch. There is no data available on the risk for positive crossmatch between Swedish patient-donor pairs. Therefore, simulations will be based on data from Roth et al. (2005a, p. 378). The risk for positive crossmatch between a patient and a random donor will be considered to be 11%. Furthermore, for a female patient whose donor is her spouse the risk is increased to 33% due to that antibodies can develop during child birth. There is no Swedish data available on how many of the patients are females nor how many donors are husbands of the patients. Therefore, data from Roth et al. (2007, p. 839) will be used. Data which is based on the American patient-donor population. The data is presented in table 4. Based on the information in table 4 and the risks for positive crossmatch, 11% and 33%, the risk for positive crossmatch between an incompatible patient-donor pair is 15%.

When generating incompatible donor blood-types, two scenarios are considered:

Table 4: Patient Gender and Unrelated Living Donors

Gender	Frequency
Female	40.90 %.
Male	59.10 %.
Unrelated Living Donor	**Frequency**
Spouse	48.97 %.
Other	51.03 %.

Either, the patient and the donor are blood-type compatible but there is a positive crossmatch between the two generating the incompatibility. In these cases, donor blood-types follow the distribution presented in table 3. Otherwise, patients are blood-type incompatible and, conditional on the blood-type of the patient, donor blood-types are simulated using the distribution presented in table 5:

Table 5: Distribution of Blood-types For Donors of Blood-Type Incompatible Pairs.

		Frequency of Donor Blood-type			
		O	A	B	AB
Patient Blood-type	O	0 %	71 %	19 %	10 %
	A	0 %	0 %	67 %	33 %
	B	0 %	88 %	0 %	12 %
	AB	0 %	0 %	0 %	0 %

A patient having blood-type AB cannot be blood-type incompatible with a donor. Therefore, whenever a patient has blood-type AB and is incompatible with the donor the pair has a positive crossmatch. Thus, the blood-type of the donor is generated based on the distribution in table 3.

So far, all patients and living donors have been given a blood-type. The only set of agents not assigned a blood-type is the set of cadaveric donors, i.e. C. It is assumed that the distribution of blood-types of the cadaveric donors follow the distribution of the Swedish population. Therefore, 251 cadaveric donors are randomly assigned a blood-type based on table 2.

4.2 The Matching

In order to conduct the matching, compatibilities and thus preferences have to be determined between all patient-donor pairs. For any pair $n_i \in N$, the compatibility of the pair has been determined. Since all patients and donors have been assigned a blood-type and the risk for positive crossmatch between a random donor and a patient is 11 %, compatibilities between all patient-donor pairs can be decided.

Regardless if list exchange is an option or not, the first step of the matching is, in accordance with the patient priority ordering, to match the compatible and incompatible patient-donor pairs with each other. As a second step, compatible patient-donor pairs, not matched at the first step, are matched with each other. If list exchange is available, the following third step is conducted: Patient-donor pairs, not matched in the previous steps, who prefer to participate in a list exchange do so by forming a w-chain with another pair. The patient who initiates the w-chain receives top priority on the cadaver wait-list. Next, patients with a living donor who were not matched in the previous steps are added to the cadaver wait-list. As a fourth step, patients on the cadaver wait-list are matched with cadaveric donors of the same blood-type and in accordance with the priority ordering. If list exchange is not a possibility, the third step vanishes and we go directly from step 2 to step 4. Conducting the simulations, 100 Monte-Carlo trials are performed to obtain an average result for the different matching mechanisms.

5 Results

Simulations of the priority mechanism, with and without list exchange, suggest that an adoption of either mechanism is favorable to the current system. Results are presented in tables 6, 7, and in figure 5. In table 6, results from using the priority mechanism not allowing for list exchange are presented. As mentioned in section 4, the number of patients who have an incompatible donor is not known. There are 423 patients who either have an incompatible donor or no living donor. In the first column of table 6, the share of the 423 patients having an incompatible donor is shown. Thus, if share is 50% on average 211.5 patients have an incompatible donor and 211.5 patients do not have a living donor. The second column reports the number of paired kidney exchanges which have been conducted. The third column reports the number of compatible pairs matched. The fourth column is the number of patients who have been matched with a cadaveric donor. Column five reports the total number of patients who have received a new kidney from using the priority mechanism. Finally, the share of total patients receiving a new kidney is reported in column 6. In table 7, results from using the priority mechanism with list exchange are presented. Column 1-3 report the same as in table 6. However, column 4 reports the number of kidney exchanges occurring from patients participating in list exchanges. Column 5 reports the number of patients matched with a cadaveric donor. In column 6, the total number of patients receiving a new kidney from using the priority mechanism is reported. In column 7, the share of total patients receiving a new kidney is reported.

From table 6, it is clear that maintaining the current system (not allowing for list exchange) but selecting a matching with the priority mechanism is preferable to the current situation. Looking at table 7 and thus extending the current system to include list exchange further improves the kidney match. Figure 5 compares the performance of the priority mechanism with and without list exchange for different numbers of incompatible donors. From figure 5, it is possible to conclude that including list exchange

Table 6: Number of Patients Matched - Without List Exchange

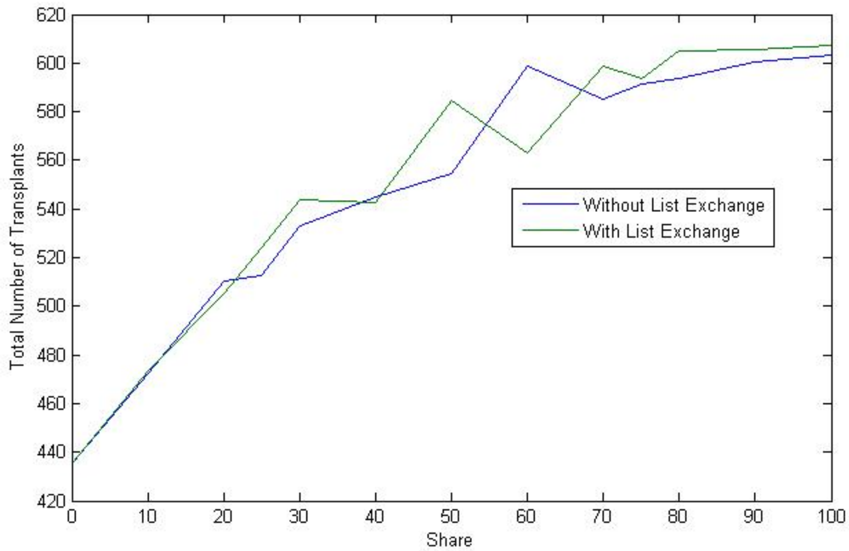
	Paired	*Compatible*	*Cadaveric*	*Total*	*Percentage*
Share					
0%	182.78	1.22	251	435	71.66 %
25%	261.78	1.62	249.15	512.22	84.44 %
50%	312.38	2.05	240.24	554.67	91.38 %
75%	374.18	1.93	215.07	591.18	97.39 %
100%	417.24	1.84	184.17	603.25	99.38 %

Table 7: Number of Patients Matched - With List Exchange

	Paired	*Compatible*	*List Exchange*	*Cadaveric*	*Total*	*Percentage*
Share						
0%	182.46	1.54	0	251	435	71.66 %
25%	264.64	1.89	2.1	250.88	519.51	85.59 %
50%	335.94	1.66	19.4	223.83	580.83	95.69 %
75%	379.86	2.01	10.02	213.45	605.34	99.73 %
100%	416.88	2.06	0	187	605.94	99.83 %

is preferable. As the share of patients who have an incompatible donor increases, the number of patients who receive a new kidney increases at a diminishing rate. In the simulated environment, if at least 75% of the patients have a donor nearly every patient receives a new kidney when using the priority mechanism with list exchange. Given that every patient has a donor and that the priority mechanism without list exchange is used, only four patients are left without a new kidney. For not very unrealistic shares of patients with incompatible donors such as 50% and 75%, both mechanisms match at least 91% of the patients with a new kidney. Since only 71.66% of all patients were given a new kidney in 2011, adopting either version of the priority mechanism in Sweden would greatly improve the chances for patients to receive a new kidney. Looking at column four in table 7, it might seem odd that the number of patients matched in a w-chain decreases after share reaches 50% and is zero for a share of 100%. However, given the way the simulations are conducted this is to be expected. When two patient-donor pairs participates in a w-chain, one pair donates a kidney to a patient on the cadaver wait-list. As share increases, the number of patients on the cadaver wait-list, i.e. patients who have no donor, decreases. As share reaches 100%, there are no such patients and no w-chains are possible.

Figure 5: Performance of The Priority Mechanism With and Without List Exchange



6 Concluding Remarks

Are the results from these simulations realistic? As mentioned in section 1, since 2004 a system has been used in New England to match kidney patients with kidneys. As the simulations conducted in this paper, the system is to a large extent based on the theory outlined in section 3. As mentioned in section 1, the system in New England has generated positive results as did the simulations in this paper. From these experiences, it seems reasonable to assume that adopting a new system in Sweden, based on the mechanisms discussed in section 3, would give more patients a new kidney. However, the simulations conducted in this paper are to a certain extent not based on data from the Swedish kidney patient-donor population. Regarding patient-donor blood types it is reasonable to assume that, in accordance with the law of large numbers, as the number of patients and donors increases, blood-type distributions follow those of table 2, table 3 and table 5. However, the risk for positive crossmatch is solely based on data from the American kidney patient-donor population introducing uncertainty to the simulations. If the risk for positive crossmatch does not differ too much between the Swedish and American kidney transplant populations the results are still valid.

Regardless of how similar the simulated patient-donor population is to the real population, it is apparent from theoretical, empirical, and simulated evidence that adopting a mechanism based on the discussion in section 3 would increase the number of kidney transplants in Sweden. Thus, importance is not to be put on the exact simulated numbers, emphasis should rather be put on that the numbers suggest that an implementation of a matching mechanism would improve the situation for Swedish kidney patients. If decision makers would be interested in changing the current system, more exact simulations could be conducted given the existence of the required data. In such simulations, different scenarios could be considered regarding for example the size of the cycles/w-chains, dynamic settings and the inclusion of altruistically unbalanced exchange. When only paired exchange is a possibility, simulations could be based on Jack Edmond's algorithm further increasing the number of paired matches. As mentioned in section 4, and suggested by table 6 and table 7, gains from kidney exchange is, amongst other things, dependent on how many patients who have a donor. Thus, in order to simulate the effects of a new matching mechanism, having access to data regarding the number of patients who have an incompatible donor is crucial.

Results suggest that extending the current system to include the possibility for patients to participate in list exchange would further increase the number of patients who receive a new kidney. This makes intuitive sense as well, since patient-donor pairs are given incentives to donate a kidney. Moreover, the costs of setting up a new system based on the priority mechanism ought to be marginal in comparison to the number of patients who would benefit from such a system. However, coordination between hospitals is required and as many hospitals as possible should form part of the system. Since the performance of the matching algorithms improves as the patient-donor population grows, a Scandinavian or even European kidney exchange system could be organized. Such a system would be even more beneficial for the patients.

If a system was to be set up in Sweden, a starting point could be to only conduct paired exchanges and to not allow for altruistically unbalanced exchange. As patients and doctors learn and hopefully trust the system, the system could be extended to include three-way exchanges and altruistically unbalanced exchange. Such an extension would further improve the performance of the matching mechanism.

In conclusion, theoretical, empirical and simulated evidence presented in this paper suggest that decision makers could, rather easily, organize a system which would improve the situation for many of the Swedish patients in need of a new kidney.

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