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School of Economics and Management

# **Estimation of Time-Varying Hedge Ratios for Coffee**

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## **Authors**

Al Azzawi, Lena 19880520-4008  
Esteban Lombana Betancourt, Alejandro 19831105-T752

## **Supervisor**

Anders Vilhelmsson

## **Abstract**

This paper will gain better insights of how to calculate the hedge ratio to reduce the basis risk and protect the market against the price volatility, which is caused by the mismatch between the spot and future prices. This will be done by calculating the time-varying hedge ratio for the Colombian mild Arabica coffee, using two BGARCH models, the diagonal BEKK and diagonal VECH. Four different hedging strategies performance are compared with the minimum variance criterion, during the period of 10 years between January 2003 and March 2013.

We can conclude that the time-varying hedge ratio has the smallest minimum variance out of the four portfolios. Further we can conclude that, the time-varying hedge ratio hasn't a significant difference in the performance, compared to OLS static hedge ratio or the naïve hedge. Therefore the reduction of the basis risk is marginal.

Keywords: Hedge ratio, basis risk, GARCH, BEKK, VECH, futures contracts, coffee.

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We would also like to thank our friends and families, who have supported us during this project.

After this thesis period we can now conclude that through this thesis we have achieved a deeper knowledge about BGARCH models to estimate time-varying hedge ratios. We believe in that this thesis will be clear and interesting for our readers and that it will work as an inspiration source for further research regarding this area.

Lund, 27 May 2013

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Lena Al Azzawi

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Alejandro Esteban Lombana Betancourt

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## 1. Introduction

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*This introducing chapter gives a presentation of the study scale, with a description of the problem background. Further this chapter will introduce the study problem and its purpose. The chapter also includes an outline for the study.*

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### 1.1 Problem Background

Suppose there's a firm in New York with a container full of coffee and they want to hedge the commodity against falling coffee prices. A hedging strategy should be chosen, by using a position in coffee futures market where the agents have the opportunity to reduce the exposure to market risk.

Before taking any decision it is important to answer some questions. Which is the hedge strategy to follow? What type of coffee is in the container? How does the quality of the coffee affect the basis risk in the estimation of the number of contracts to be used in the hedging strategy? What is the best method to calculate the minimum variance hedge ratio?

An agent in the economy faces many risks and the most common one is the market risk, caused by the price fluctuations. There are some techniques to hedge that risk using the derivatives securities markets. One of them is to use the commodities future market where the agents have the opportunity to reduce the exposure to price risk.

A future contract is an agreement to buy or sell an underlying asset at the specified price and at a particular time in the future. (Czekierda & Zhang, 2010)

A hedge strategy can be classified depending of the position assume in the futures contracts. A short position (or short hedge) in the futures market is used when the hedger is long in the spot market. A long position in the futures market is assumed when the hedger is short in the spot market. Following these strategies the loss caused by the movements of the price in one of the markets is offset by the gains in the other market (Hull, 2006).

The hedge ratio determines the quantity of contracts to sell or buy in the futures market to hedge a position in the spot market. In Hull (2006) book, "options, futures and other derivatives" the minimum variance hedge ratio is estimated using an ordinary least square (OLS) regression model. The main problem with the OLS model is the homoscedasticity assumption (Park & Bera, 1987) because it is well known that the variance of the returns

could be high or low for extended periods of time (volatility clustering). Also Myers and Thompson (1989) studied that when news about spot and futures prices are known by the market the commodity prices are well represented by time-varying covariance matrix. There are also some other problems like for example cointegration between spot and future prices according to Brenner and Kroner (1993), it will result in a downward bias on the estimated hedge ratio. The MGARCH models are used to estimate hedge ratio because it allows that the covariance matrix change over time, taking into account the new information given by the market.

## 1.2 Problem Discussion

The futures contract is for the Arabica coffee but in this type of coffee there are three classifications: Colombian Mild Arabica (Colombia, Kenya and Tanzania), other mild Arabica (Bolivia, Burundi, etc...<sup>1</sup>) and other natural Arabica (Brazil, Ethiopia, Paraguay, Timor-leste, Yemen). The principal difference between the three categories is the quality of the coffee that is determined in the grading and classification process. It allows the market to know the different characteristics of the coffee according to some criteria: Altitude and region, botanical variety, preparation (wet or dry process), bean size, number of defects, roast appearance, cup quality and density of the beans (The coffee exporter's guide 2012). These characteristics are represented in the spot price for the different coffee categories (Colombian mild Arabica, other milds and natural Arabica), so a cross-hedge is applicable in this situation.

A perfect hedge must be done using futures contracts for each of the categories, but the futures market is divided in two coffee main species: Robusta and Arabica coffee. There's not a specific market for the different categories in the Arabica coffee so the futures contract only represents an average range of qualities and prices (The coffee exporter's guide 2012). The asset to be hedged, in this case the Colombian mild Arabica is not exactly

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<sup>1</sup> Other mild Arabicas: Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, India, Jamaica, Malawi, Mexico, Nicaragua, Panama, Papua New guinea, Peru, Rwanda, Venezuela, Zambia, Zimbabwe

the same as the asset underlying in the futures contract. This difference will be important in the moment when the contract is closed out increasing the basis risk.

The basis is considered and indicator of market conditions (ICO 107<sup>th</sup> session) for a specific type of coffee. Some factors like the weather, internal conflicts in the regions where the coffee is cultivated, frost and diseases like broca affect the supply and demand of the coffee. Also the delivery location is important to determine the price because there are some transportation and warehouse costs implicit in the spot price of the coffee. These entire factors plus the quality of the coffee is represented in the basis risk for each coffee variety (The coffee exporter's guide 2012).

The purpose of this paper is to calculate and evaluate the performance of the time-varying hedge ratio for the Colombian mild Arabica coffee because it seems to better reflect the market conditions reducing the basis risk when there's new information available in the market. The conditional covariance matrix will be modeled using the bivariate GARCH methodology and the diagonal BEKK parameterization of Engle and Kroner (1995). To compare the performance of the dynamic hedge ratio will be constructed four different hedge portfolios according to different strategies:

1. The unhedged portfolio (no-hedge)
2. The naively hedged portfolio (which is a hedge ratio of one)
3. The OLS hedge portfolio (using the OLS estimation hedge ratio)
4. The dynamic bivariate GARCH model

### **1.3 Research Question**

*How to calculate the hedge ratio for the Colombian mild Arabica coffee and how to find the best hedge ratio, comparing the volatility of each portfolio?*

## 1.4 Delimitations

The aim of this study is to calculate the time-varying hedge ratio for the Colombian mild Arabica coffee. As mentioned in the problem discussion, there are three classifications of the Arabica coffee. In this study we will only focus on the future price and spot prices of the Colombian mild Arabica coffee in ICE.

## 1.5 Outline

The thesis consists of five chapters; an outline of various parts of the research is presented below.

<i>Chapter 1 Introduction</i>	<ul style="list-style-type: none"><li>• <i>This chapter will consist of a presentation of the the background that is related to the chosen topic. This will be followed by a discussion that results in the purpose and the research question.</i></li></ul>
<i>Chapter 2 Theoretical Framework</i>	<ul style="list-style-type: none"><li>• <i>The chapter is based on selected theories of hedge ratio and the colombian coffee.</i></li></ul>
<i>Chapter 3 Methodology</i>	<ul style="list-style-type: none"><li>• <i>In this chapter the choice of method and approach used in this study will be represented.</i></li></ul>
<i>Chapter 4 Empirical Results/Analysis</i>	<ul style="list-style-type: none"><li>• <i>This chapter represents the data description and the collection of the secondary data. Further this part of the thesis will link the collected data to the theoretical framework.</i></li></ul>
<i>Chapter 5 Conclusion</i>	<ul style="list-style-type: none"><li>• <i>This chapter will, according to the purpose of this study, answer the research question. In the end of this chapter there will be suggestions for further studies.</i></li></ul>

## 2. Theoretical framework

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*In this following section the theoretical framework will be presented. The chapter is based on a review of selected theories. First theory about the basis risk will be introduced and how it affects the hedge ratio.*

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### 2.1 Basis risk

Different hedging methods are used in different situations, some situations desire to use derivatives to hedge a certain risk in the future (Lien & Tse, 2002). Investors use hedging to protect themselves from any unexpected loss that can be caused from the uncertain future shock of the market but hedging is also used for better financial management and planning (Froot A., Scharfstein S., and Stein C., 1993, Mattus, 1993).

Seen from a historical view, hedgers see hedging in terms of the basis risk. The explanation for this is because hedging is seen as an act of arbitrage between cash and futures prices (Castellino, M., G., 2006), which depend on the length of time that the positions must be held before the profit will be realized (Figlewski, 1983).

It is important to define the basis risk because it helps to understand the importance of the hedge ratio. The basis can occur from a number of different sources (Figlewski, 1983), but basically it is the difference between two market prices. The basis, or the differential, is the difference at a given time between the spot price, of the asset to be hedged, and the futures price of contract, ( $b_t = S_t - F_t$ ).

Futures on coffee are used to manage the market risk exposure. If there is a match between the hedged instrument and the contract, then the hedger wants to close his position at the maturity date of the contract. If a hedge has to be closed earlier than maturity of the contract then the hedger is exposed to a so-called basis risk (Czekierda & Zhang, 2010), so there are two main reasons for the existence of the basis risk.

First, the asset to be hedged, in this case the Colombian mild Arabica, is not exactly the same as the asset underlying the futures contract. The second reason depends on the closed out date of the futures contract, if it is near the expiration day the basis risk is going to be

lower. Since the hedger is only interested in reducing the risk, the hedger wants to have a small variance as possible, to make sure that the basis risk is as small as possible.

For the Colombian mild Arabica there's a big difference between the prices that compose the basis in some periods of time. The coffee is not a homogenous product so the different types of Arabica coffee have different prices that reflect the quality of the coffee. This over price pay for the coffee is the differential.

## **2.2 Minimum variance hedge ratio**

The hedge ratio is the number of units of the futures asset that are purchased relative to the number of units of the spot asset. To choose the hedge ratio that minimizes the variance of the returns of the portfolio that contains the spot and futures position; might be the best strategy to find the optimal hedge ratio. The disadvantage of the mean variance hedge ratio is that it ignores the expected return on the hedged portfolio and therefore it is not consistent with mean-variance framework (Sheng-Syan Chen, 2002; Weichen 2009).

The variance minimizations purpose assumes a high degree of risk aversion. Further it can be shown that, when expected returns to holding futures are zero, then the minimum variance hedging rule is also the expected utility-maximizing hedging rule. This makes minimum variance hedging rule more applicable (Baillie & Myers, 1989), and the advantage is the simple calculation and understanding (Sheng-Syan Chen, 2002; Weichen 2009).

The minimum variance hedge ratio is a strategy to optimize the hedge ratio, with the goal to minimize the variance of the hedge portfolio, it is employed to test the hedge effectiveness in coffee future markets (Baillie & Myers, 1989)

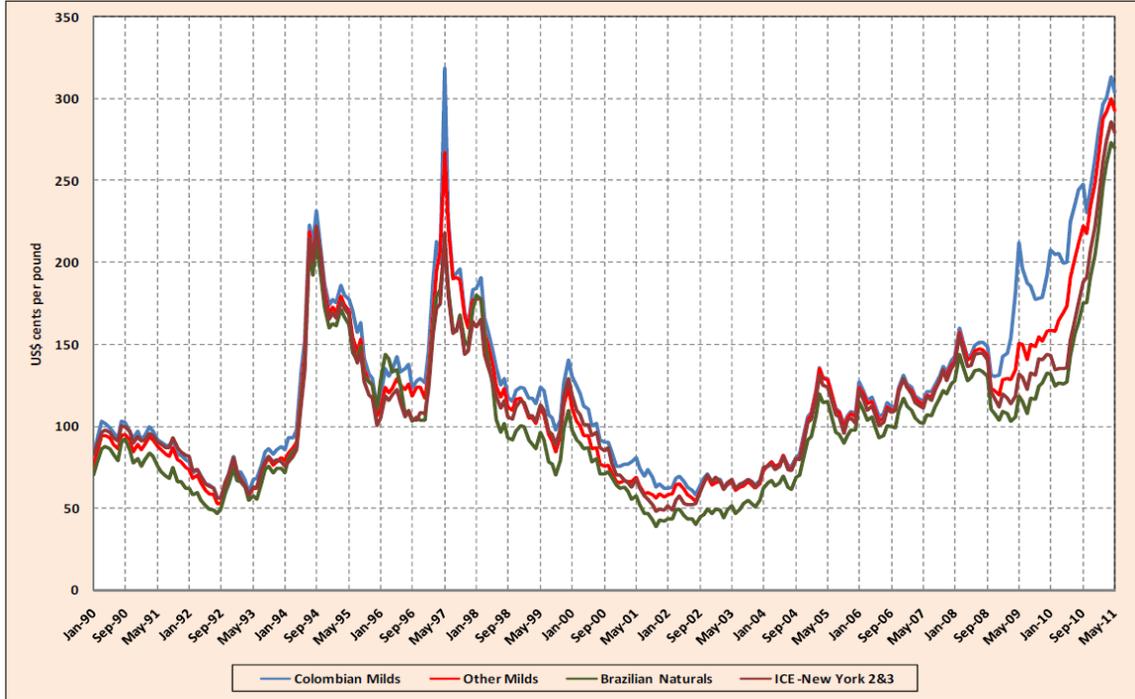


Table 2.2. *Data statistics for the whole period (January 1990 – May 2011)*<sup>2</sup>.

The graph shows the spot prices of the different categories of Arabica Coffee with the futures price. The blue line represents the Colombian Mild Arabica and there are moments in time that the difference respect to the futures price can be large for example in the years 2009 and 2010, so this difference composed by the basis risk and the differential determines the loss or profit of the portfolio.

The profit or loss in the portfolio can be expressed in this way<sup>2</sup>:

$$\pi = S_t N_{t-1}^c - (F_t - F_{t-1}) N_{t-1}^f$$

Where  $N_{t-1}^c$  the number of units of coffee to be hedged and  $N_{t-1}^f$  is the number units of futures contracts. The division  $h = N_{t-1}^f / N_{t-1}^c$  is the hedge ratio. The first equation after evaluating the expected value in t-1 can be written as:

$$E_{t-1}(\pi) = S_{t-1} N_{t-1}^c + (E_{t-1}(\Delta S_t) - h_{t-1} E_{t-1}(\Delta F_t)) N_{t-1}^c$$

<sup>2</sup> It is important to look the similarity of the expression to the effective price that is obtained for the hedge asset. The effective price is represented by the equation:  $S_t + F_{t-1} - F_t$  it is the same equation of the profits or loss of the portfolio, but it is not affected by the number of units. The equation can be organized to get  $F_{t-1} + b_t$  so the basis is implicit in the portfolio profits equation.

The term  $S_{t-1}N_{t-1}^c$  is already known at time  $t-1$ , so to reduce the variance of  $\pi$  is necessary to minimize the variance of  $E_{t-1}(\Delta S_t) - h_{t-1}E_{t-1}(\Delta F_t)$  that is expressed by the equation:

$$\sigma_{p,t}^2 = \sigma_{s,t}^2 + h_{t-1}^2 \sigma_{f,t}^2 - 2h_{t-1} \sigma_{sf,t}$$

To find the hedge ratio which minimizes the risk we calculate the derivative of  $\sigma_{p,t}^2$  respect to  $h_{t-1}^2$  and equating to zero the hedge ratio is:

$$h_{t-1} = \frac{\sigma_{sf,t}}{\sigma_{f,t}^2}$$

Finally, the hedge ratio is the covariance of spot and futures prices divided by the variance of futures prices. Here the hedge ratio is time dependent, but it is also valid for the OLS estimator when the portfolio is time separable (Hull, 2006).

## 2.4 Previous empirical work

Since the creation of the futures markets works as an instrument to hedge the market risk, many research papers have been studying the estimation of the hedge ratio to improve the hedging performance. Anderson, Danthine (1981) developed a theory of the utility maximization function to find the optimal futures position. According to the optimal equation there are two important components the speculative part and the pure hedge part. The hedgers are not interested in the speculative part so they reformulated the equation where the speculative part is given by the basis risk. They found some difficulties for the empirical application of their theory like for example assuming that the agent knows the relevant moments of the probability distribution prices.

Stephen Figlewski (1983) examines the different sources of basis risk on the Standard and Poor's 500 index contract. According to Figlewski (1983) the cross-hedge is one of the main reasons for having basis risk because it is very difficult to replicate a portfolio with all the 500 stocks of the index. Although it seems that the commodities don't have this problem because the underlying asset is the same as the futures contract, the basis cross-hedge problem can be found in the coffee because it is not a homogenous product having different range of prices according to its quality. Further Figlewski (1983) uses an OLS regression method to estimate the hedge ratios and compares its performance solving in some way the calculation of the relevant moments of the probability distribution prices.

Cecchetti, Cumby, Figlewski (1988) estimates optimal hedge ratios basically taking into account two factors, the maximization of the expected utility function and allowing a time-varying distribution using and ARCH model, but to ensure the positive definiteness they constraint the correlation of the returns to be constant. In other papers like for example Briys. Crouhy, Schlesinger (1993) it is interesting to analyze the hedging behavior of an agent when there's the existence of a background risk like the basis. Their conclusion is that basis risk noise makes hedging less effective, but there exist an effect that partially controls the basis risk by using less futures contracts. Based on that the agent could use fewer futures contracts' reducing the basis risk, therefore the importance lies on how the hedge ratio is estimated

Baillie, Myers (1991) uses a BGARCH model for the estimation of the optimal commodities futures hedge and they compares the variance reduction between a constant and time-varying hedge ratio. It improves the way to hedge the underlying asset because now the variance-covariance matrix changes over time reflecting the market conditions when there's new information available in the market. They didn't use an utility function representing the risk aversion of the agent because the expected returns to holding futures are zero. It is like using a high degree of risk aversion where the minimum variance hedging rule is also the expected utility maximizing hedging rule. The proof of this result is available in Baillie and Myers (1991).

Basically, the minimum variance hedging rule is the risk criterion applied in this paper because the firm is looking for different objectives instead of only maximizes the returns of the portfolio. For this reason it is not used any utility function to represent the risk aversion of the agent because the main objective is to reduce the variance of the coffee price where the expected returns of the portfolio are zero. One of the objectives that a firm wants to achieve when hedging is to mitigate the underinvestment problem Bessembinder (1991). It happens because the hedge decreases the sensitivity of the senior claim value when there's an increment in the invested capital, it contribute to the equity holders to capture a larger portion of the benefits from the investment and improves the contract terms the firm negotiate with the different economical agents (customers, creditors and managers). This cannot be achieve or duplicate by individual hedging of the shareholders and it is independent of the agent's risk preferences. Stultz (1996) and Leland (1998) provide

evidence that hedging with derivatives to reduce the firm's cash flow volatility can improve the firm's debt capacity. According to Froot, Scharfstein and Stein (1993) hedging certain risks reduces the firm probability to borrow costly external debt and also it increases the probability to execute profitable investments. Campbell and Kracaw (1987) study that when the borrower has the incentive and opportunity to increase the firm's risk; it generates some deadweight costs that can be reduced via hedging. There are some arguments about tax incentives to hedge. Smith and Stultz (1985) and Graham and Smith (1999) argue that firms with a convex tax rate structure have an incentive to hedge reducing the firm's expected tax liabilities.

In the literature there are different approaches to deal with the estimation of an efficient hedge ratio to decrease the basis risk. As it was mentioned before the basis explains the coffee's market conditions. For example the weather can affect the coffee's supply increasing the basis risk. Manfredo and Richards (2009) study how to hedge with weather derivatives to reduce the basis risk. A coffee grower for example can buy a CDD futures or CDD call options to hedge against rising temperatures, but the spatial risk arises more in the tropical countries where the coffee grows. You can find a country with different weathers for a particular region, so it is very complicated to structure a financial instrument. Golden, Yang and Zou (2010) measure the effectiveness of using a basis hedging strategy to reduce the financial consequences of weather. They found is more effective the basis hedge strategy in winter than in summer and also to use some regional indices (RMS)<sup>3</sup> than the CME<sup>4</sup>. It seems a good way to hedge the basis risk, but the problem is the no-existence of these indices in the different regions where coffee grows. Another way to deal with the problem of basis risk is estimating the parameters with an OLS model, but it seems restrictive to assume that the optimal hedge ratio is constant over time (Abdulnasser & Youssef, 2012). In this regard, to solve this restriction the time-varying hedge ratio can be estimated with GARCH models like the BEKK and diagonal VECH, which will be used in this paper. There are other types of GARCH models, the nonparametric copula-based GARCH model was used by Power, Vedenov, Anderson and Klose (2013) to estimate the hedge ratio when the joint dependence structure is allowed to be possible no elliptical. They

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<sup>3</sup> The risk management solutions, Inc. provides products and services for the risk management of natural hazard risks like for example the weather.

<sup>4</sup> The Chicago mercantile Exchange has some financial products to manage the weather risk.

didn't find any improvement in the variance reduction compared to other models like OLS or BEKK models, but depending of the risk criterion, that kind of estimation may be preferable for different approaches. For example they found that the copula-based GARCH model performs better when the risk criterion is measured by the expected short fall.

The remainder of this paper proceeds to describe the data and model utilized, followed by an analysis of the empirical results, before the conclusion is presented.

### 3. Methodology

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*The following chapter will describe the research methods used in this thesis. A presentation of the data collection and general method will be given.*

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#### 3.1. Purpose of the research

Generally speaking, the purpose of a research is often to find the answer for a specific question by applying scientific procedures (Kothari, 2008). This paper will gain better insights of how to calculate the hedge ratio to reduce the basis risk to protect the market against the price volatility, because of the mismatch between the spot and future prices, in this case, for the Colombian mild Arabica coffee. This is the main goal of this study. The research will investigate how the estimation of a dynamic hedge ratio can improve the variance reduction of the portfolio decreasing the basis risk. So the agents could protect it selves against the negative impact of volatility.

Further Jacobsen (2002) is writing about two alternative approaches that can be employed: deductive and inductive approach. This research have chosen the deductive approach since the approach bases the research on theoretical framework and further collects empirical data, with the aim to find a result and conclusion.

#### 3.2 General Method of Work

Two models are going to be estimated. The first econometric model is a simple linear regression model using the ordinary least square (OLS) technique to estimate the coefficient. It is represented by the equation:

$$\Delta S_t = a - b\Delta F_t + u_t$$

In this case by definition  $b = \frac{\sigma_{fs}}{\sigma_f^2}$  and it is the hedge ratio. It doesn't depend on time.

However, the model must meet certain assumptions: The variance of the errors is constant and finite (homoscedasticity) and the errors are linearly independent of one another (no serial autocorrelation). The white test and the Breusch-Godfrey test are two ways to proof if there's problems of heteroscedasticity and serial autocorrelation respectively.

According to some studies like for example Baillie and Myers (1991) argue that commodity prices are better represented with a time-varying covariance matrix, so the OLS assumption of homoscedasticity is not accomplished. To deal with this problem the bivariate GARCH model allows a time-varying covariance matrix.

The bivariate generalized autoregressive conditionally heteroscedastic model (BGARCH) is used to estimate the hedge ratio  $h_{t-1} = \frac{\sigma_{sf,t}}{\sigma_{f,t}^2}$ . The importance of this formula is given by the fact that the variance of futures price and the covariance between movements in the spot and futures prices are changing over time. There are different GARCH models proposed in the econometrics literature like for example the VECH, the diagonal VECH and the diagonal BEKK models.

The VECH model requires the estimation of 21 parameters it is given by the equations:

$$h_{11t} = c_{11} + a_{11}u_{1t-1}^2 + a_{12}u_{2t-1}^2 + a_{13}u_{1t-1}u_{2t-1} + b_{11}h_{11t-1} + b_{12}h_{22t-1} + b_{13}h_{12t-1}$$

$$h_{22t} = c_{21} + a_{21}u_{1t-1}^2 + a_{22}u_{2t-1}^2 + a_{23}u_{1t-1}u_{2t-1} + b_{21}h_{11t-1} + b_{22}h_{22t-1} + b_{23}h_{12t-1}$$

$$h_{12t} = c_{31} + a_{31}u_{1t-1}^2 + a_{32}u_{2t-1}^2 + a_{33}u_{1t-1}u_{2t-1} + b_{31}h_{11t-1} + b_{32}h_{22t-1} + b_{33}h_{12t-1}$$

$h_{iit}$  represent the conditional variance at time t of the returns of coffee spot prices and futures prices and the last equation represents the conditional covariance between the spot and futures price returns. The estimation of this kind of models takes too much time and resources so the model is restricted to the diagonal VECH model:

$$vech(H_t) = w + A_1 vech(\epsilon_{t-1}\epsilon'_{t-1}) + B_1 vech(H_{t-1})$$

Where  $A_1$  and  $B_1$  is assumed to be diagonal reducing the number of parameters to be estimated to only nine. The problem with the diagonal VECH model is that the covariance matrix could be no positive semi-definite. In that case some restrictions can be applied to,  $w$ ,  $A_1$  and  $B_1$  to solve the problem for example using the rank N Cholesky factorized matrix of the coefficient matrix. It estimates the same number of parameters, but ensure that the conditional covariance matrix is positive definite.

Another way to estimate the parameters is the diagonal BEKK model where the variance-covariance matrix is always positive definite and we only have to estimate seven parameters. In this model the conditional covariance matrix is represented by:

$$H_t = W'W + A'H_{t-1}A + B'U_{t-1}U'_{t-1}B$$

For the BGARCH models it is important to correctly specify the mean equation to avoid the autocorrelation of the residuals. The correct specification of the mean equation depends if  $\Delta S_t$  and  $\Delta F_t$  are cointegrated in such case must be use a VEC (vector error correction) model specifies like:

$$\Delta Y_t = \mu + \sum_{i=1}^l \Gamma_i \Delta Y_{t-i} + \pi v_{t-i} + \epsilon_t$$

$$Y_t = \begin{bmatrix} F_t \\ S_t \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_f \\ \mu_s \end{bmatrix} \quad \Gamma_i = \begin{bmatrix} \Gamma_{i,F}^F & \Gamma_{i,S}^F \\ \Gamma_{i,F}^S & \Gamma_{i,S}^S \end{bmatrix} \quad \pi = \begin{bmatrix} \pi_F \\ \pi_S \end{bmatrix} \quad \epsilon_t = \begin{bmatrix} \epsilon_{F,t} \\ \epsilon_{S,t} \end{bmatrix}$$

In this model the returns of the spot and futures price are regressed against the lags of the prices and the error correction term  $v_{t-i}$  that captures the long relationship between the variables.

Finally, there are different hedge ratios to know which have the best performance the returns of each portfolio must be calculated according to the formula:

$$R_p = \Delta S_t - h_t \Delta F_t$$

The variance of the returns of the portfolio must be estimated and compare it with the variance of the unhedged portfolio.

## 4. Empirical Results/Analysis

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*This chapter represents data description and the collection of the secondary data. Further this part of the thesis will link the collected data to the theoretical framework.*

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### 4.1 Data Description

According to Jacobsen (2002) there are two types of data employed in a research: the primary and the secondary data. This study will use the secondary data. The collection of data will be gained from the database; *datastream*, during the period between January 2003 and March 2013 consisting of 2568 daily observations. The futures price of Arabica coffee is obtained from the ICE (intercontinental exchange) and the New York spot prices of Colombian mild Arabica coffee. The choice of the period for the data is motivated with the reason that the diagonal BEKK and the diagonal VECH works better with a bigger amount of observations, therefore the chosen period of 10 years.

The collected data will further be sorted by time, to gain the overview of the data. This will be done in EViews 6.0.

Future contracts expire five times per year - March, May, July, September, and December (delivery months). By using the closest-to-maturity contract, unless the next closest has greater open interest, in which case we switch to this contract, getting a constant time series.

The daily observations that are corresponding to U.S. public holidays are deleted from the observed period, with the aim to avoid the integration of false zero returns (Brooks, Ólan & Persaud, 2002).

The 7.6 in the Appendix is representing the Dickey Fuller unit root test. From the table it is seen that it is not possible to reject the null-hypothesis of non-stationary for the futures and spot price series. The non-stationarity of the price series is consistent with the weak-form efficiency of the futures markets and cash.

The return series are calculated with natural logarithm. The equation for the future price is:

$$LN(\text{future Price}_t / \text{future Price}_{t-1})$$

and for the spot price:

$$LN(\text{spot Price}_t / \text{spot Price}_{t-1})$$

The Jarque-Bera test will give the knowledge of, if the series is normally distributed or not. In the table 7.7 in the Appendix, it is seen that both price series are not normally distributed and therefore the null-hypotesis is rejected.

In the table 7.8, in the Appendix, it presents the Engle Granger results, demonstrating that the null hypothesis of non-stationarity in the residuals of the cointegrating regression is weakly rejected. In the table 7.9 in the Appendix is showing the Johansen test statistics, which is rejecting the null of no-cointegrating vector. There is at most one cointegrating vector. This is given for both the max forms and trace.

According to Baillie and Myers (1991) there isn't an exact one-to-one association in a commodity futures hedge because of the carry costs. However, this does not avoid the existence of some other cointegrating relationship with a [1, 1] cointegrating vector, the data appear to be cointegrated.

## 4.2 OLS model results

The next table shows the OLS estimation results:

OLS Estimation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
C	7.70E-05	0.000128	0.601382	0.5476
D(LNFPRICE)	0.855386	0.006491	131.7754	0.0000

Table 4.2. Data statistics for the whole period (December 1999 – March 2012).

Heteroskedasticity Test: White			
F-statistic	70.20656	Prob. F(2,2565)	0.0000
Obs*R-squared	133.2813	Prob. Chi-Square(2)	0.0000
Scaled explained SS	1157.053	Prob. Chi-Square(2)	0.0000

### Breusch-Godfrey Serial Correlation LM Test

F-statistic	11.88697	Prob. F(5,2561)	0.0000
Obs*R-squared	58.24557	Prob. Chi-Square(5)	0.0000

### Heteroskedasticity Test ARCH

F-statistic	7.208620	Prob. F(5,2557)	0.0000
Obs*R-squared	35.62550	Prob. Chi-Square(5)	0.0000

The coefficient of the model or the hedge ratio is significant according to the t-statistic, but the white test shows problems of heteroskedasticity because the null hypothesis of no heteroskedasticity is rejected at the 1% level of significance. Also the Breusch-Godfrey test rejects the null hypothesis of no autocorrelation at the 1% level of significance. The main problem when both assumptions of the OLS model are violated is that the standard errors could be wrong. Although the coefficient is consistent and unbiased, it may not have the minimum variance among the class of unbiased estimators. It means that doing any inference about the coefficient could be incorrect.

### 4.3 Diagonal VECM and BEKK model results

As it was analyzed in the data description section there's not seem to be a strong cointegration between the variables. However, the model used in this paper for the mean equation is a VECM model. The main objective of this model instead of doing a VAR model is to avoid the loss of long run information (Brooks, 2008).

The mean equation is the same for each model. A vector error correction model (VECM) with two lags for each variable and the basis as an error correction term. The number of lags was chosen according to the information criteria such as Akaike information criteria (AIC), Schwarz information criteria (SC) and Hannan-Quin information criteria (HQ). The

detailed results are presented in table 7.3 in the appendix. Only the AIC criteria recommend using three lags in the mean equation, but the HQ and SC criteria recommend using two lags and following the idea of a parsimonious model it was decided to use only two lags.

The next table shows the estimation of the BGARCH model. It is used two ways to estimate the coefficients: the diagonal VECH using the rank N Cholesky factorized matrix to guarantee the positive definiteness of matrix  $H_t$  and a diagonal BEKK model.

**Covariance specification: Diagonal BEKK**

$$\text{GARCH} = M + A1 * \text{RESID}(-1) * \text{RESID}(-1)' * A1 + B1 * \text{GARCH}(-1) * B1$$

**Transformed Variance Coefficients**

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	8.03E-05	5.24E-06	15.31959	0.0000
M(1,2)	6.46E-05	4.00E-06	16.16894	0.0000
M(2,2)	5.81E-05	3.67E-06	15.83885	0.0000
A1(1,1)	0.308604	0.010741	28.73014	0.0000
A1(2,2)	0.293622	0.011389	25.78070	0.0000
B1(1,1)	0.845432	0.008742	96.70737	0.0000
B1(2,2)	0.865678	0.007693	112.5353	0.0000

**Covariance specification: Diagonal VECH**

$$\text{GARCH} = M + A1 * \text{RESID}(-1) * \text{RESID}(-1)' + B1 * \text{GARCH}(-1)$$

**Transformed Variance Coefficients**

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	6.77E-06	1.04E-06	6.500477	0.0000
M(1,2)	5.53E-06	7.57E-07	7.310923	0.0000
M(2,2)	4.52E-06	6.55E-07	6.906176	0.0000
A1(1,1)	0.017925	0.001714	10.45931	0.0000

A1(1,2)	0.018514	0.001483	12.48779	0.0000
A1(2,2)	0.019126	0.001897	10.08286	0.0000
B1(1,1)	0.964249	0.003461	278.6064	0.0000
B1(1,2)	0.963665	0.002862	336.6547	0.0000
B1(2,2)	0.966476	0.002949	327.6761	0.0000

It was estimated in two different ways the coefficients of the conditional variance-covariance matrix because each model imposes different restrictions in the estimation process of the regressors and could give different values for the variance and covariance estimates (Kroner & Ng 1998). The hedge ratio is given by the formula  $h_{t-1} = \frac{\sigma_{sf,t}}{\sigma_{f,t}^2}$ , so it is important to compare both models and be sure the values don't differ too much from one model to another, preserving a similar hedge ratio and returns volatility of the portfolio.

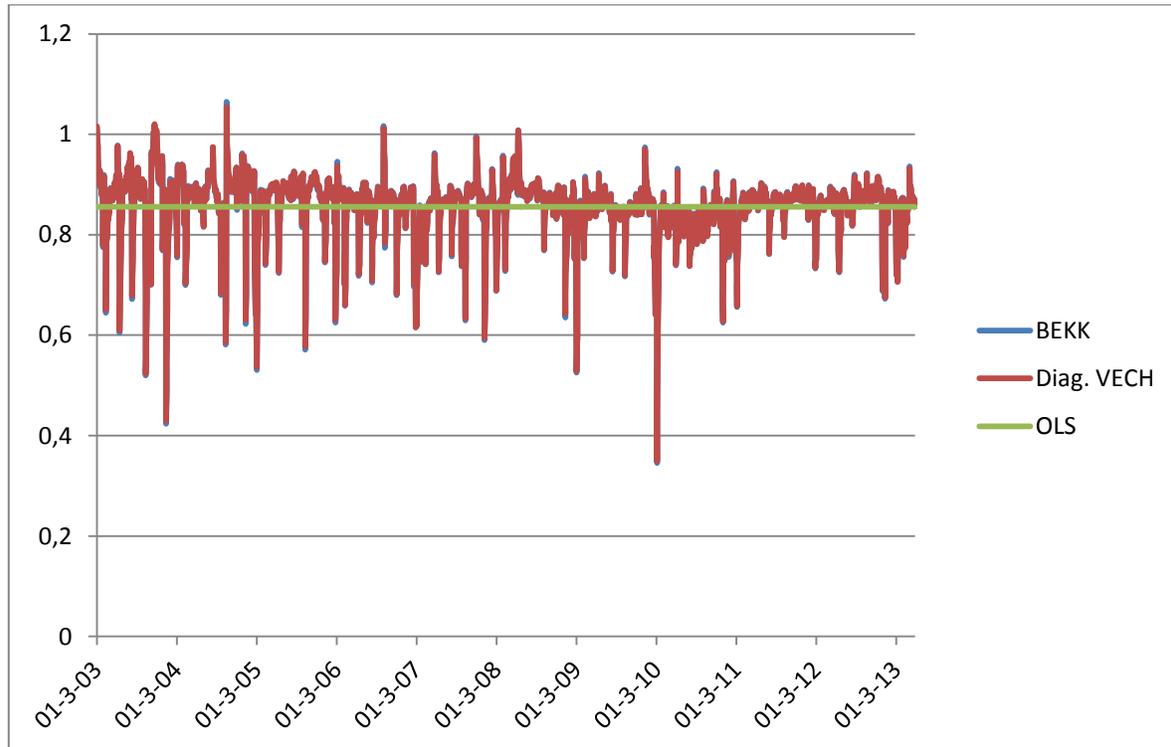
The parameters of both models were estimated using the Marquardt optimization algorithm for maximizing the likelihood function. The estimators for the diagonal VECH and BEKK model are positive, the sum of the coefficients in the conditional variance equations is less than one and the null hypothesis of the coefficients equal to zero is rejected at the 1% level of significance. It means that the BGARCH models are stationary in variance and there's a strong relationship between cash and futures markets.

The assumption of no serial autocorrelation in the residuals is tested using the portmanteau test Ljung-box Q-statistic. In the table 7.4 the null hypothesis of no serial autocorrelation cannot be rejected in any of the twelve lags for both models. Also the  $Q^2$  statistic cannot be rejected so there's no evidence of heteroskedasticity in the models. The only problem is to assume the residuals are normally distributed because table 7.5 shows the rejection of the null hypothesis using the Jarque-bera normality test. According to the test the main problem is the excess of kurtosis in the residuals.

The mean equation for the GARCH model was calculated using the change in the logarithm prices, the table 7.10 in the Appendix shows the constant term is not significant at 1% level. It implies that the minimum variance hedging rule is consistent with the expected utility maximization depending only in the conditional covariance matrix of futures and spot prices without any form of utility function or measure of the risk aversion degree.

## 4.4 Hedge ratios performance

The next graph shows the dynamic hedge ratio estimated with the BGARCH models and also includes the OLS hedge ratio.



The different restrictions impose in the variance-covariance matrix by each of the models used to calculate the time-varying hedge ratio don't have any effects in the final results, so it is the same to estimate the hedge ratio with a diagonal VECH or BEKK model at least in this case using the coffee prices returns. Looking at the graph the minimum time-varying hedge ratio is 0.346 and the maximum is above one 1.065, but is interesting to see how fast it goes back again to the average (0.861) and fluctuates between 0.8 and 1 values. It is an important fact in the moment for planning the strategy to hedge the coffee price. For example if the hedge ratio is below 0.8 it's better to hedge the coffee with an average ratio because there are some costs in rebalancing the portfolio. In the graph it seems the OLS hedge ratio is an average of the time-varying hedge ratio, so the variance of the portfolio returns must be similar.

The next table reports the annually volatility and returns of the different portfolios strategies:

				Dynamic Hedge Ratio	
	Unhedged	Naive	OLS	BEKK	Diag. VECH
Mean	0.00034943	3.02326E-05	7.63933E-05	0.000111509	0.000111206
Variance	0.00032637	0.000050148	0.000042019	0.000030643	0.000030883
Standard deviation	0.01806578	0.00708149	0.006482235	0.005535634	0.00555728
Annual Volatility	28.68%	11.24%	10.29%	8.79%	8.82%
Annual return	8.81%	0.76%	1.93%	2.81%	2.80%
Reduction		-60.80%	-64.12%	-69.36%	-69.24%

The variance reduction between the BEKK (-69.36%) model is almost the same of the diagonal VECH (-69.24%) because the annual volatility is similar in both models indicating the same performance for the time-varying hedge ratios. Now, if it is compared the four strategies is always better to hedge the coffee price because there´s a volatility reduction of at least 60.80% with the naive hedge. Also the dynamic hedge has better performance against the other strategies, but it only reduces the volatility in 1.5% and 2.4% approximately compare with the OLS hedge and the naive hedge respectively. The dynamic portfolio strategy is more efficient than using a naive hedge because the hedger can improve the returns in 2% and get the minimum variance of the four strategies.

Although it is complicated to use a dynamic hedge because you must rebalance the portfolio daily and it has some transaction costs like for example the broker´s commission fees and operative cost. Exist the alternative to rebalance the portfolio in the moment of the roll-over where it can be calculated the hedge ratio to be use during that period until the next maturity date of the futures contract. The other alternative is to use the OLS hedge ratio, the volatility is only 1.5% higher compared to the dynamic strategy and the return 0.9% lower, so the difference is marginal.

## 5. Conclusion

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*This chapter will, according to the purpose of this study, answer the research question. In the end of this chapter there will be suggestions for further studies.*

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The objective of this paper is to reduce the basis risk using a dynamic hedging model for the Colombian milds Arabica because it is the coffee variety with a higher basis risk. The minimum variance hedging rule is the risk criterion chosen to measure the effectiveness of the dynamic hedge compare to static hedge strategies. The approach to estimate the dynamic hedge ratio was the diagonal VECM and BEKK models because in other, previous studies these models perform better to reduce the variance of the portfolio. The results are summarized as follows.

The time-varying hedge ratio strategy improved the performance in reducing the variance of the portfolio and the basis risk compare with the OLS and naive portfolio, but it is a marginal reduction of the variance. Although there are some moments in time the hedge ratio deviate from the average, it returns very quickly again to fluctuate between a range of 0.8 and 1, so there are no real incentives to use a hedge ratio out of this range. There's not a big improvement in the portfolio returns that justifies the implementation of a dynamic strategy because it involves a substantial increase in numerical complexity to estimate the conditional variances and covariances using a large number of parameters. Also the different GARCH models could lead to different hedge ratios estimation so it takes time and resources to know which model fits better the minimum-variance risk criterion.

From a theoretical point of view it is very interesting to analyze that the OLS hedge ratio which has a number of major methodological drawbacks like the no possibility of cointegration between the spot and futures prices and the homoscedasticity assumption has a similar performance than the bivariate GARCH models, so the reduction of the basis risk is similar for both models.

## **5.1 Further studies**

We feel that, concluded from above, a further extension of this thesis subject could be an examination of the usage of another GARCH model. There could also be a matter of interest to compare the hedge ratio of different types of coffee. Or another angle of the thesis could be, using a different criterion to measure the risk.

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## 7. Appendix

### Breusch-Godfrey Serial Correlation LM Test:

F-statistic	11.88697	Prob. F(5,2561)	0.0000
Obs*R-squared	58.24557	Prob. Chi-Square(5)	0.0000

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 05/06/13 Time: 14:06

Sample: 1/02/2003 3/28/2013

Included observations: 2568

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.31E-06	0.000127	-0.010347	0.9917
D(LNFPRICE)	0.002316	0.006435	0.359889	0.7190
RESID(-1)	-0.128942	0.019753	-6.527636	0.0000
RESID(-2)	-0.054944	0.019859	-2.766650	0.0057
RESID(-3)	-0.035780	0.019877	-1.800088	0.0720
RESID(-4)	-0.055451	0.019860	-2.792088	0.0053
RESID(-5)	-0.061283	0.019727	-3.106612	0.0019
R-squared	0.022681	Mean dependent var	3.14E-20	
Adjusted R-squared	0.020392	S.D. dependent var	0.006482	
S.E. of regression	0.006416	Akaike info criterion	-7.257364	
Sum squared resid	0.105419	Schwarz criterion	-7.241415	
Log likelihood	9325.455	Hannan-Quinn criter.	-7.251582	
F-statistic	9.905810	Durbin-Watson stat	1.998404	
Prob(F-statistic)	0.000000			

**Table 7.1** Breusch-Godfrey Serial Correlation LM Test (January 2003 – Mars 2013).

**Heteroskedasticity Test: White**

F-statistic	70.20656	Prob. F(2,2565)	0.0000
Obs*R-squared	133.2813	Prob. Chi-Square(2)	0.0000
Scaled explained SS	1157.053	Prob. Chi-Square(2)	0.0000

Test Equation:  
 Dependent Variable: RESID^2  
 Method: Least Squares  
 Date: 05/06/13 Time: 14:07  
 Sample: 1/02/2003 3/28/2013  
 Included observations: 2568

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.48E-05	3.78E-06	6.549546	0.0000
D(LNFPRICE)	0.001085	0.000171	6.348844	0.0000
(D(LNFPRICE))^2	0.043445	0.004435	9.796754	0.0000
R-squared	0.051901	Mean dependent var	4.20E-05	
Adjusted R-squared	0.051162	S.D. dependent var	0.000175	
S.E. of regression	0.000171	Akaike info criterion	-14.51271	
Sum squared resid	7.47E-05	Schwarz criterion	-14.50587	
Log likelihood	18637.32	Hannan-Quinn criter.	-14.51023	
F-statistic	70.20656	Durbin-Watson stat	1.810188	
Prob(F-statistic)	0.000000			

**Table 7.2** *Heteroskedasticity Test: White (January 2003 – Mars 2013).*

**VAR Lag Order Selection Criteria**

Endogenous variables: LNFPRICE LNNSPCOLMILDS  
 Exogenous variables: C  
 Date: 05/07/13 Time: 05:26  
 Sample: 1/02/2003 3/28/2013  
 Included observations: 2568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	2971.114	NA	0.000339	-2.312394	-2.307837	-2.310742
1	15748.03	25523.98	1.62e-08	-12.26015	-12.24648	-12.25519
2	15772.18	48.19989	1.60e-08	-12.27584	-12.25306*	-12.26758*
3	15776.43	8.472799	1.60e-08*	-12.27603*	-12.24414	-12.26447
4	15777.20	1.539784	1.60e-08	-12.27352	-12.23251	-12.25865
5	15780.29	6.154233	1.60e-08	-12.27281	-12.22269	-12.25464
6	15786.28	11.92652	1.60e-08	-12.27436	-12.21513	-12.25289
7	15791.09	9.563287*	1.60e-08	-12.27499	-12.20664	-12.25021
8	15791.38	0.570817	1.60e-08	-12.27210	-12.19464	-12.24402

\* indicates lag order selected by the criterion  
 LR: sequential modified LR test statistic (each test at 5% level)  
 FPE: Final prediction error  
 AIC: Akaike information criterion  
 SC: Schwarz information criterion  
 HQ: Hannan-Quinn information criterion

**Table 7.3** *VAR Lag Order Selection Criteria (January 2003 – Mars 2013).*

### BEKK

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: no residual autocorrelations up to lag h

Date: 05/09/13 Time: 13:38

Sample: 1/03/2003 3/28/2013

Included observations: 2567

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.813517	0.9366	0.813834	0.9366	4
2	1.372437	0.9946	1.373190	0.9946	8
3	4.727741	0.9665	4.732420	0.9663	12
4	12.31200	0.7222	12.32851	0.7211	16
5	21.22244	0.3841	21.25635	0.3822	20
6	29.82287	0.1907	29.87693	0.1889	24
7	30.19580	0.3539	30.25088	0.3513	28
8	34.11457	0.3663	34.18190	0.3633	32
9	34.53659	0.5382	34.60540	0.5349	36
10	38.27319	0.5482	38.35661	0.5444	40
11	40.49563	0.6226	40.58862	0.6186	44
12	41.64574	0.7293	41.74413	0.7256	48

\*The test is valid only for lags larger than the System lag order.  
df is degrees of freedom for (approximate) chi-square distribution

**Table 7.4.1** BEKK (January 2003 – Mars 2013).

### Diagonal VECH

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: no residual autocorrelations up to lag h

Date: 05/09/13 Time: 13:47

Sample: 1/03/2003 3/28/2013

Included observations: 2567

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.780757	0.9410	0.781061	0.9410	4
2	1.361870	0.9948	1.362628	0.9948	8
3	4.782209	0.9649	4.786968	0.9647	12
4	12.40873	0.7154	12.42539	0.7142	16
5	21.36021	0.3762	21.39434	0.3743	20
6	29.99446	0.1849	30.04882	0.1831	24
7	30.36559	0.3460	30.42097	0.3434	28
8	34.30745	0.3577	34.37515	0.3546	32
9	34.72469	0.5292	34.79386	0.5259	36
10	38.46964	0.5392	38.55346	0.5354	40
11	40.69734	0.6140	40.79074	0.6100	44
12	41.85652	0.7214	41.95536	0.7177	48

\*The test is valid only for lags larger than the System lag order.  
df is degrees of freedom for (approximate) chi-square distribution

**Table 7.4.2** Diagonal VECH (January 2003 – Mars 2013).

### Diagonal VECH

System Residual Normality Tests  
Orthogonalization: Cholesky (Lutkepohl)  
Null Hypothesis: residuals are multivariate normal  
Date: 05/09/13 Time: 13:53  
Sample: 1/03/2003 3/28/2013  
Included observations: 2567

Component	Skewness	Chi-sq	df	Prob.
1	0.053276	1.214339	1	0.2705
2	-0.273293	31.95447	1	0.0000
Joint		33.16881	2	0.0000

Component	Kurtosis	Chi-sq	df	Prob.
1	4.343741	193.1284	1	0.0000
2	19.65469	29667.98	1	0.0000
Joint		29861.11	2	0.0000

Component	Jarque-Bera	df	Prob.
1	194.3427	2	0.0000
2	29699.93	2	0.0000
Joint	29894.27	4	0.0000

**Table 7.5.1** *Diagonal VECH (January 2003 – Mars 2013).*

### BEKK

System Residual Normality Tests  
Orthogonalization: Cholesky (Lutkepohl)  
Null Hypothesis: residuals are multivariate normal  
Date: 05/09/13 Time: 13:55  
Sample: 1/03/2003 3/28/2013  
Included observations: 2567

Component	Skewness	Chi-sq	df	Prob.
1	0.051961	1.155139	1	0.2825
2	-0.315514	42.59028	1	0.0000
Joint		43.74542	2	0.0000

Component	Kurtosis	Chi-sq	df	Prob.
1	4.291034	178.2749	1	0.0000
2	19.87915	30473.04	1	0.0000
Joint		30651.31	2	0.0000

Component	Jarque-Bera	df	Prob.
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1	179.4300	2	0.0000
2	30515.63	2	0.0000
Joint	30695.06	4	0.0000

**Table 7.5.2 BEKK (January 2003 – Mars 2013).**

**Group unit root test: Summary**

Series: LNFPRICE, LNSPCOLMILDS  
Date: 05/13/13 Time: 09:35  
Sample: 1/02/2003 3/28/2013  
Exogenous variables: Individual effects  
Automatic selection of maximum lags  
Automatic lag length selection based on SIC: 0  
Newey-West automatic bandwidth selection and Bartlett kernel  
Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-1.42497	0.0771	2	5136
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-0.61559	0.2691	2	5136
ADF - Fisher Chi-square	4.43629	0.3502	2	5136
PP - Fisher Chi-square	4.41661	0.3526	2	5136

\*\* Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.

**Table 7.6 Group unit root test: Summary (January 2003 – Mars 2013).**

**Descriptive statistics**

	LNFPRICE	LNSPCOLMILDS
Mean	4.818883	4.883852
Median	4.807499	4.877104
Maximum	5.724075	5.780558
Minimum	4.045679	4.072269
Std. Dev.	0.388072	0.409597
Skewness	0.071996	-0.007952
Kurtosis	2.654415	2.519538
Jarque-Bera Probability	14.99739	24.72732
	0.000554	0.000004
Sum	12374.89	12541.73
Sum Sq. Dev.	386.5902	430.6647
Observations	2568	2568

**Table 7.7 Group unit root test: Summary (January 2003 – Mars 2013).**

### Cointegration

Date: 05/13/13 Time: 09:40  
 Series: LNFPRIE LNNSPCOLMILDS  
 Sample: 1/02/2003 3/28/2013  
 Included observations: 2568  
 Null hypothesis: Series are not cointegrated  
 Cointegrating equation deterministic: C  
 Automatic lags specification based on Schwarz criterion (maxlag=27)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
LNFPRIE	-3.562032	0.0275	-25.31982	0.0185
LNNSPCOLMILDS	-3.535859	0.0296	-24.78066	0.0208

\*MacKinnon (1996) p-values.

Intermediate Results:

	LNFPRIE	LNNSPCOLMILDS
Rho - 1	-0.011142	-0.010920
Rho S.E.	0.003128	0.003088
Residual variance	5.04E-05	5.48E-05
Long-run residual variance	3.96E-05	4.28E-05
Number of lags	1	1
Number of observations	2566	2566
Number of stochastic trends**	2	2

\*\*Number of stochastic trends in asymptotic distribution

**Table 7.8** Cointegration (January 2003 – Mars 2013).

### Johansen test

Date: 05/13/13 Time: 10:05  
 Sample: 1/02/2003 3/28/2013  
 Included observations: 2568  
 Trend assumption: Linear deterministic trend  
 Series: LNFPRIE LNNSPCOLMILDS

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.005685	17.91390	15.49471	0.0212
At most 1	0.001274	3.273771	3.841466	0.0704

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

### Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized	Max-Eigen	0.05
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No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.005685	14.64013	14.26460	0.0436
At most 1	0.001274	3.273771	3.841466	0.0704

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b\*S11\*b=I):

LNFPRICE	LNSPCOLMILDS
-21.35823	19.25863
-7.644570	9.488120

Unrestricted Adjustment Coefficients (alpha):

D(LNFPRICE)	0.001478	-4.80E-05
D(LNSPCOLMILDS)	0.001237	-0.000269

1 Cointegrating Equation(s):                      Log likelihood                      15789.46

Normalized cointegrating coefficients (standard error in parentheses)

LNFPRICE	LNSPCOLMILDS
1.000000	-0.901696 (0.02994)

Adjustment coefficients (standard error in parentheses)

D(LNFPRICE)	-0.031577 (0.00828)
D(LNSPCOLMILDS)	-0.026416 (0.00761)

Selected  
(0.05 level\*)  
Number of  
Cointegrating  
Relations by  
Model

Data Trend:	None	None	Linear	Linear	Quadratic
Test Type	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend	No Trend	No Trend	Trend	Trend
Trace	0	0	1	0	2
Max-Eig	0	1	1	0	0

\*Critical values based on MacKinnon-Haug-Michelis (1999)

**Table 7.9** Johansen test (January 2003 – Mars 2013).

Estimation Method: ARCH Maximum Likelihood (Marquardt)  
 Covariance specification: Diagonal BEKK  
 Sample: 1/03/2003 3/28/2013  
 Included observations: 2567

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-0.114681	0.053986	-2.124282	0.0336
C(2)	0.035491	0.042048	0.844058	0.3986
C(3)	0.119325	0.059077	2.019823	0.0434
C(4)	-0.062645	0.046183	-1.356457	0.1750
C(5)	0.023140	0.008926	2.592495	0.0095
C(6)	0.000413	0.000389	1.061197	0.2886
C(7)	0.025551	0.046944	0.544298	0.5862
C(8)	0.085919	0.034507	2.489919	0.0128
C(9)	-0.029428	0.052443	-0.561153	0.5747
C(10)	-0.115865	0.039842	-2.908116	0.0036
C(11)	0.014665	0.008379	1.750199	0.0801
C(12)	0.000324	0.000351	0.924114	0.3554

Variance Equation Coefficients				
C(13)	7.98E-05	5.25E-06	15.21433	0.0000
C(14)	6.44E-05	4.00E-06	16.07549	0.0000
C(15)	5.80E-05	3.67E-06	15.79412	0.0000
C(16)	0.308626	0.010780	28.62886	0.0000
C(17)	0.293807	0.011449	25.66176	0.0000
C(18)	0.845879	0.008799	96.13393	0.0000
C(19)	0.865785	0.007729	112.0183	0.0000
Log likelihood	15909.89	Schwarz criterion		-12.33760
Avg. log likelihood	3.098928	Hannan-Quinn criter.		-12.36521
Akaike info criterion	-12.38091			

Equation:  $D(\text{LNFPRI}) = C(1)*D(\text{LNFPRI}(-1)) + C(2)*D(\text{LNFPRI}(-2)) + C(3)*D(\text{LNPCOLMILDS}(-1)) + C(4)*D(\text{LNPCOLMILDS}(-2)) + C(5)*\text{RESIDB}(-1) + C(6)$

R-squared	0.008611	Mean dependent var	0.000311
Adjusted R-squared	0.006676	S.D. dependent var	0.019712
S.E. of regression	0.019646	Sum squared resid	0.988495
Durbin-Watson stat	2.018724		

Equation:  $D(\text{LNPCOLMILDS}) = C(7)*D(\text{LNFPRI}(-1)) + C(8)*D(\text{LNFPRI}(-2)) + C(9)*D(\text{LNPCOLMILDS}(-1)) + C(10)*D(\text{LNPCOLMILDS}(-2)) + C(11)*\text{RESIDB}(-1) + C(12)$

R-squared	0.003461	Mean dependent var	0.000355
Adjusted R-squared	0.001516	S.D. dependent var	0.018069
S.E. of regression	0.018055	Sum squared resid	0.834826
Durbin-Watson stat	2.002454		

**Table 7.10.1** Mean and variance-covariance equations (January 2003 – Mars 2013)

Estimation Method: ARCH Maximum Likelihood (Marquardt)  
 Covariance specification: Diagonal VECH  
 Date: 05/09/13 Time: 13:44  
 Sample: 1/03/2003 3/28/2013  
 Included observations: 2567

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-0.116211	0.053954	-2.153880	0.0312
C(2)	0.034331	0.041840	0.820528	0.4119
C(3)	0.120558	0.059049	2.041644	0.0412
C(4)	-0.061452	0.046005	-1.335769	0.1816
C(5)	0.023159	0.008834	2.621634	0.0088
C(6)	0.000406	0.000387	1.048197	0.2945
C(7)	0.024554	0.046951	0.522975	0.6010
C(8)	0.086005	0.034269	2.509702	0.0121
C(9)	-0.028751	0.052400	-0.548694	0.5832
C(10)	-0.115797	0.039606	-2.923722	0.0035
C(11)	0.014755	0.008271	1.783891	0.0744
C(12)	0.000315	0.000348	0.906553	0.3646
Variance Equation Coefficients				
C(13)	7.40E-05	5.26E-06	14.06065	0.0000
C(14)	5.91E-05	3.88E-06	15.24592	0.0000
C(15)	5.28E-05	3.57E-06	14.78867	0.0000
C(16)	0.302875	0.010487	28.88065	0.0000
C(17)	0.287675	0.011321	25.41179	0.0000
C(18)	2.52E-05	3.582179	7.05E-06	1.0000
C(19)	0.855357	0.008825	96.92026	0.0000
C(20)	0.875614	0.007748	113.0129	0.0000
C(21)	1.79E-92	4.87E+88	3.7E-181	1.0000
Log likelihood	15909.66	Schwarz criterion		-12.33130
Avg. log likelihood	3.098881	Hannan-Quinn criter.		-12.36181
Akaike info criterion	-12.37916			

Equation:  $D(LNFPRICE) = C(1)*D(LNFPRICE(-1)) + C(2)*D(LNFPRICE(-2)) + C(3)*D(LNSPCOLMILDS(-1)) + C(4)*D(LNSPCOLMILDS(-2)) + C(5)*RESIDB(-1) + C(6)$

R-squared	0.008636	Mean dependent var	0.000311
Adjusted R-squared	0.006700	S.D. dependent var	0.019712
S.E. of regression	0.019646	Sum squared resid	0.988471
Durbin-Watson stat	2.017804		

Equation:  $D(LNSPCOLMILDS) = C(7)*D(LNFPRICE(-1)) + C(8)*D(LNFPRICE(-2)) + C(9)*D(LNSPCOLMILDS(-1)) + C(10)*D(LNSPCOLMILDS(-2)) + C(11)*RESIDB(-1) + C(12)$

R-squared	0.003477	Mean dependent var	0.000355
Adjusted R-squared	0.001531	S.D. dependent var	0.018069
S.E. of regression	0.018055	Sum squared resid	0.834813
Durbin-Watson stat	2.001777		

**Table 7.10.2** Mean and variance-covariance equations (January 2003 – Mars 2013).