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The Black-Litterman Model Applied on OMXS30

NEKN01 – Master Essay I

by

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Abstract

This thesis applies the Black-Litterman (BL) model on the stocks that makes up the Swedish stock index OMXS30 during the year of 2012. Public information in the form of stock recommendations from financial institutions has been used as views in the BL-framework. A method of estimating confidence in these views has been analyzed and further clarified. The testing consists of two BL-portfolios along with two Mean Variance-portfolios, allowing me to compare the performance difference that the BL-model has. The two portfolios differ in terms of short selling restrictions. Although the results show a difference in total returns in favor of the restricted BL-portfolio, no significant difference were found compared to the other portfolios or the stock market index, both in terms of returns and in terms of Sharpe ratios.

Acknowledgement
I would like to take this opportunity to thank my supervisor Lu Liu for her help, patience and guidance during this process.
Lastly, I owe a great deal of my results to Carl Johan Arestad and Johan Rahmqvist for their excellent work that inspired me to write this thesis.

Sincerely,

Fredrik Gertzell
20th of May 2013

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1 Introduction

The Mean Variance framework put forward by Harry Markowitz in his *Portfolio Selection* (1952) laid out the foundation of modern portfolio theory. His work inspired William Sharpe (1963) and John Lintner (1965) to further develop it, into what will later become what we now know as the Capital Asset Pricing Model (CAPM) (Levy, 2010).

During my studies, I have come across the portfolio optimization problem in the Mean Variance framework on more than one occasion, and I have always looked at it with a pair of skeptical eyes. Yes, the intuition behind it is easy to understand. Clearly, all investors seek to maximize their return whilst at the same time minimize portfolio variance. However, when testing the model in practice, I have found that it does not have a very realistic implementation in the real world, for real investors.

In the real world, investors have (hopefully) opinions about the markets in which they seek to invest. Trusting blindly on expected returns, purely based on historical data, is something most investors know that one should not do. In fact, all financial products (such as different types of funds) I have come across, always warns the investor that historical returns are not a guarantee for future returns.

What investors do (again, hopefully) is instead to analyze the investment thoroughly to find out if a particular investment has a fair probability of giving the investor a positive return. This should be even truer in our day and age. With all the turmoil that is happening in the world of economics today, fiscal crises, currency crises, debt crises and energy crises, the financial markets are extremely sensitive. This puts a lot of pressure on today's investors. One has to be ready to change opinions about different investments constantly as new information arises. Long gone are the days when you could throw a dart at a financial paper to decide which stocks to buy. This requires a model that allows the investor to change the input variables in the portfolio optimizer. This is what the Black Litterman-model allows us to do, which is the reason I took an interest in it and decided to write this thesis.

The model was introduced by Robert Black and Bob Litterman (1992). They presented a method, based on the MV-framework, where an investor could alter the input variables by introducing views about the assets into the model. Given that no one can be certain about the future, they also presented how this uncertainty of the views should be incorporated in the model. This is what made me take an interest in this model, since it, on paper at least, provides a platform that is more suited for the real world.

1.1 Problem formulation

The MV-framework shows us how we in the most efficient way can select financial assets, such as stocks, to form a portfolio of assets. However, the model does not provide a different way to calculate expected returns and variances, on which the portfolio selection is based, other than calculating it from historical data. Thus, the model, in a way, expects history to repeat itself. Furthermore, it leaves little or no room for an investor to use his own views on the market when choosing portfolio weights. This means that there is no reason for an investor to analyze the fundamentals of a company before buying its stocks.

The Black Litterman model suggests a solution to this problem by allowing investors to implement their own subjective views into the model. However, there are a few different ways that can be implemented. Since the BL-model was presented, there have been a lot of different approaches on how exactly one should implement these views, and further, how to deal with the uncertainty of these views. So, is there a good way of implementing the BL-model on the Swedish stock market, and does it improve portfolio performance? Is the use of public recommendations as views, as suggested by Arestad & Rahmqvists (2012), a good approach for estimating views in the BL-model? Too try and answer these questions, I have formulated the purpose of this thesis below.

1.2 Purpose

The main purpose of this thesis is to give the reader, and myself, and insight to how an investor can apply the Black Litterman model in the real world. The aim is to use public information (in form of stock recommendations) to create portfolios based on the stocks in the Swedish stock index OMXS30 during the year 2012. These portfolios shall then be compared to each other and the stock index to see if they differ in performance. I also aim to further develop the way to calculate confidence in views based on this public information.

1.3 Disposition

This thesis contains the following: Chapter two will cover the theoretical framework that is used in this thesis. More specifically, the Mean Variance framework will be explained, together with an in depth description of the Black Litterman model. It also contains a brief explanation of how to significantly test the Sharpe ratios of portfolios. The chapter will also cover some of the research that has been done on the subject. This chapter should give the reader an understanding of the MV-framework and the Black Litterman model together with its components. The third chapter thoroughly explains the method used in the thesis to give the reader a clear insight about the work process that led to the results. In this chapter I will also present the way I further develop Arestad & Rahmqvists (2012) way of estimating the omega-matrix, which is a matrix containing the uncertainty of the views. The chapter also covers what data has been used in the analysis. The fourth chapter will present the empirical result from the analysis, which will be analyzed and commented as it is presented. The estimated portfolios will be analyzed in how they have performed in respect to each other and in respect to the market portfolio. The fifth chapter will conclude the thesis with a summary of the results and some concluding remarks. I will also present suggestions for further research in this chapter.

2 Theory & Previous research

In this chapter I will cover the theoretical framework used in this thesis. I will also present some of the previous research that has been done on the subject.

2.1 Mean Variance

The Mean Variance (MV)-framework is one of the more important works than has been done in finance. The fact that I'm writing this thesis, some 60 years after Markowitz published his article, should verify this. The MV-framework, as put forward by Harry Markowitz in his *"Portfolio Selection"* (1952), establishes the ground for witch the Black Litterman model stands on. The BL-model is, in essence, the MV-model, but with different method to obtain the input variables. Thus, it is quite important to understand the original model before I explain the BL-model.

Markowitz (1952) points out that an investor seeks to maximize the expected portfolio return and also minimize the portfolio variance. He further argued that the investment rule of thumb at the time, which was to diversify among assets with the highest expected return, could not be correct. The idea was that if you invested in a large number of assets, the law of large numbers would minimize the portfolio variance, using diversification alone. He concluded that a (diversified) portfolio with the highest expected return did not result in lowest possible variance. Instead, he proposed the *E-V rule*. Consider a portfolio *P*, with *N* assets. We have a Nx1 vector of expected returns, $\boldsymbol{\mu}$ and a Nx1 vector of portfolio weights $\boldsymbol{w}$. Lastly, we have a covariance matrix $\sum$ based on the return from the assets. The portfolio expected return and variance, in matrix notation, is given by (Markowitz, 1994):

$$E[R_P] = \boldsymbol{\mu}'\boldsymbol{w} \tag{2.1}$$

$$Var_p = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \tag{2.2}$$

The investor then seeks to maximize the expected return and minimize the variance, given the constraint of a fully invested portfolio (Markowitz 1952):

$$\sum_{i=1}^{N} w_i = 1 \tag{2.3}$$

We can also impose restrictions against short selling assets, that is, we are only allowed to buy assets:

$$w_i \geq 0 \tag{2.4}$$

Markowitz (1994) argues that a portfolio which maximizes the return and minimizes the variance is an *efficient* portfolio. A portfolio is not consider efficient if there exists combinations that render a higher return with unchanged (or lower) variance, or a combination that lowers the variance whilst keeping the expected return unchanged (or higher).

This is something that is easy to understand and also to implement in the real world. All an investor need to find this optimal portfolio is price data for the assets and an application, such as Excel, that can process this data. From that point of view, the MV-framework is very appealing. However, it is not without fault. Over the years, the model has been tested and received various types of criticism.

One common criticism is that the model is very sensitive to changes in the input variables, such as the returns of the individual assets. Best & Grauer (1991) showed that an equally weighted MV-portfolio with 100 assets would lose half of its assets, if the mean of *any one* asset increased with 11,6 %. One would think that reducing the portfolio with half of its assets would affect it drastically but the portfolio mean and variance only changed by 2 %.

2.2 Sharpe ratio

The Sharpe ratio is a well-known measurement to compare asset performance. It is described numerous publications, so I will only briefly describe it. William F. Sharpe (1963) used the work of Markowitz and developed it further, to make it easier to find the sets of efficient portfolios. He suggested a model which describes the return of any asset as a linear relationship with an underlying factor that can explain the return of assets. A few years later, he followed this up with introducing what we now know as the Sharpe ratio. We assume that investors can invest in, and borrow capital, to a certain risk-free rate. Further, we assume that the expected return of an asset is a linear function of its risk in terms of standard deviation (Sharpe, 1966):

$$E[R_i] = r_f + b\sigma_i \tag{2.5}$$

Here, b is what we now call the Sharpe ratio and is defined as (Elton at al., 2011):

$$b = SR_i = \frac{E[R_i] - r_f}{\sigma_i} \tag{2.6}$$

Sharpe then points out that the most efficient portfolio is the one which has the highest value of *b*, or in today's terms, the highest Sharpe ratio.

2.2.1 Testing Sharpe ratios

While most readers are familiar with Sharpe ratios, the statistical properties of the same might be more unknown. The Sharpe ratio gives us a very easy to use measurement to compare the risk-adjusted performance between portfolios. Often investors simply just look at the Sharpe ratios of some portfolios and conclude that one is better than the other. But is it really? While it is a very good tool for evaluating actual performance, we still cannot be sure that this performance was due to competent investment strategy, or simply blind luck. Consequently, the ability to test and compare Sharpe ratios statistically is useful when drawing conclusions from them.

A calculated Sharpe ratio (SR) is only an estimate of its true value. The asymptotic distribution (assuming *IID* returns) of this estimator is given by (Lo, 2002):

$$\sqrt{T}\left(\widehat{SR}_i - SR_i\right) \sim N(0, V_{IID}) \tag{2.7}$$

The asymptotic variance can be written as:

$$V_{IID} = 1 + \frac{1}{2}SR_i^2 \tag{2.8}$$

Now that we know the variance, we can easily calculate the standard error of the estimator:

$$SE\left(\widehat{SR}_i\right) = \sqrt{\frac{(1 + \frac{1}{2}SR_i^2)}{T}} \tag{2.9}$$

Finally, using the standard error, calculating a 95 % confidence interval is very straightforward:

$$\widehat{SR}_i \pm 1{,}96 * SE\left(\widehat{SR}_i\right) \tag{2.10}$$

Now we have the tools to analyze a Sharpe ratio more thoroughly. Using the confidence interval, we can easily see where the "true" Sharpe ratio lies. Naturally, if the confidence interval contains the value zero (e.g. from -0,1 to 0,1), we cannot reject the null hypothesis that the true Sharpe ratio is equal to zero.

Two things need mentioning here that the observant reader might notice. First; the proportion of the standard error to the Sharpe ratio will increase as the Sharpe ratio gets closer to zero. Second; there is a finite limit of this proportion as the Sharpe ratio increases. The second problem will not be of an issue in this thesis, but the first one might. I will explain this further in chapter 4.

Now, suppose that we have two Sharpe ratios that we want to compare, and to be able to say something along the line of "Sharpe ratio A is significantly different from Sharpe ratio B". We begin by defining the null hypothesis, comparing the Sharpe ratios *i* and *n.* (Jobson & Korkie 1981):

$$H_0: SR_i - SR_n = 0 \tag{2.11}$$

We can use the estimated ratios to define a transformed difference between the two, using the sample standard deviation and average excess returns:

$$\widehat{SR}_i - \widehat{SR}_n \equiv \widehat{SR}_{in} = s_n\overline{r_i} - s_i\overline{r_n} \tag{2.12}$$

The asymptotic variance from this transformed difference is given by:

$$\theta = \frac{1}{T}\left[2\sigma_i^2\sigma_n^2 - 2\sigma_i\sigma_n\sigma_{in} + \frac{1}{2}\mu_i^2\sigma_n^2 + \frac{1}{2}\mu_n^2\sigma_i^2 - \frac{\mu_i\mu_n}{2\sigma_i\sigma_n}(\sigma_{in}^2 + \sigma_i^2\sigma_n^2)\right] \tag{2.13}$$

Here, the population mean μ_i, variance σ_i^2 and covariance σ_{in} can be replaced with the sample equivalent. Now we can use the estimated Sharpe ratio and the variance to calculate a z-statistic:

$$z(\widehat{SR}_{in}) = \frac{\widehat{SR}_{in}}{\sqrt{\theta}} \sim N(0,1) \tag{2.14}$$

Now we have the tools to fully analyze the different Sharpe ratios that will be presented in chapter 4.

2.3 Black-Litterman model

Fischer Black and Robert Litterman (1992) presented their model in the paper *"Global Portfolio Optimization"* where they concluded that the portfolio optimization methods (e.g. the MV-framework) that existed at the time were rarely used in the real world. The main reason for this is because of extreme solutions suggested by the MV-model optimization.

These solutions consist of extreme short sale positions, positions that no rational investor would even consider, when optimizing portfolios without short sale constraints. Drobetz (2001) discuss this fact and provides some examples of portfolio weights based on real data that would not exist in a professional fund portfolio. On the other end, when optimizing portfolios with no short selling allowed the model often suggests corner solutions with zero weight in many assets and a high weights in just a few. Green & Hollifield (1992, p 1786) explained this lack of diversification: *"[…] due to the dominance of a single factor in the covariance structure of returns […]"*.

Black & Litterman (1992) pointed to two reasons for these problems. The first is due to the fact that using historical returns as expected returns is not a good way of estimating future expected returns.[1] The second reason is that the MV-framework is very sensitive to changes in input variables, as discussed above. What the BL-model does is; presenting a new method of estimating expected return and then combining these with an investors views about these returns.

In this section I will present the theoretical framework for the BL-model, mainly following the approach suggested by He & Litterman (1999) and Meucci (2010).

2.3.1 Returns

To estimate expected returns we begin by realizing that they are in fact random variables that we cannot observe. The BL-model starts with the CAPM equilibrium distribution, a distribution that we then can change as we add views as new information. First, assume that asset returns are normally distributed with a mean and covariance matrix (He & Litterman, 1999):

$$r \sim N(\mu, \Sigma) \tag{2.15}$$

The mean returns are in turn assumed to be normally distributed around its equilibrium risk premium:

$$\mu = \pi + \epsilon \tag{2.16}$$

This equilibrium is then what is our starting point. The equilibrium returns vector (π) is calculated as:

[1] For further reading on this subject, see Merton (1980).

$$\boldsymbol{\pi} = 2\lambda(\boldsymbol{w}_{m,i}\boldsymbol{\Sigma}) \tag{2.17}$$

Thus, the equilibrium returns depend on the covariance matrix and the market weights. These market weights can be calculated using the market value (*mw)* of the assets which we choose to use (Black & Litterman, 1992):

$$w_{m,i} = \frac{mv_i}{\sum_i mv_i} \tag{2.18}$$

Meucci (2009) points out that these weights need not be calculated this way. What matters is that we have a reference portfolio that we want to impose our views on. Furthermore, λ is a risk aversion parameter that can be estimated as (Idzorek, 2005):

$$\lambda = \frac{(E[r_{mt}] - r_f)}{\sigma_m^2} \tag{2.19}$$

Meucci (2009) means that this is not a sensitive parameter, however, I say that it in fact can be. It all depends on what type of return one uses in the calculation. If we use historical data to calculate the expected return, we could end up with a negative value of λ, which would imply risk seeking rather than risk aversion. Giacometti et al. (2007) point out that λ is a scaling factor that greatly can affect the equilibrium returns. Black & Litterman suggested setting λ exogenously to 1,2 (Meucci 2010).

2.3.2 Views

The views are presented into the model via two components. First, we have the P-matrix, which tells the model which asset that is affected by a specific view. The P-matrix is a *KxN* matrix, with *K* number of views and *N* number of assets. The matrix consists of values 1, 0 or -1. 1 means that we have an asset will be affected positively by a view, thus, -1 imply that an asset will be affected negatively. A zero simply means that the asset is not affected by the view. If a row in the P-matrix sums to 1, then we have an *absolute view* and if it sums to zero, we have a *relative* view. The views are represented in a v-vector with *K* number of views, containing the views in form of returns (Idzorek, 2005, p. 11). Consider the following example, with 6 assets and three views.

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 0.10 \\ 0.05 \\ 0.06 \end{bmatrix} \tag{2.20}$$

Here, asset four is believed to have a return of 10 % (an absolute view as the row sums to 1), the spread between asset two and three is believed to increase with 5 % (a relative view) and asset six is believed to have a return of 6 %.

2.3.3 Confidence in the views

When it comes to specify the confidence that the investor has in her views, there are some different ways one can approach this. The confidence determines the variance of the views; this variance is presented in the form of the $\mathbf{\Omega}$-matrix. This matrix is a very crucial part of the BL-model, and, unfortunately, very complicated in its nature (Idzorek, 2005). Black & Litterman (1992) does not provide a detailed explanation of how the $\mathbf{\Omega}$-matrix is estimated, but they present it as a diagonal matrix, assuming no covariance between views:

$$\mathbf{\Omega} = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix} \tag{2.21}$$

Meucci (2010) suggests using the covariance matrix as a starting point and then using an overall confidence level, c, for all views:

$$\mathbf{\Omega} = \frac{1}{c}\boldsymbol{P}\mathbf{\Sigma}\boldsymbol{P}' \tag{2.22}$$

The problem with this is that we cannot assign different confidence for different views. Another problem is that $c \in (0, \infty)$, which means that the investor has to be able to answer the question: “On a scale from zero to infinity, how confident are you in your overall analysis?”, which is asking quite a lot.

Walters (2011) summarizes four different approaches to the Omega matrix. First, using the variance in returns, either like Meucci (2010) above (ignoring a diagonal matrix), or:

$$\mathbf{\Omega} = \mathrm{diag}(\boldsymbol{P}(\tau\mathbf{\Sigma})\boldsymbol{P}') \tag{2.23}$$

The second way (also presented by Mankert, 2006) is to specify a 68 % confidence interval around the expected return from the view. Since a 68 % interval approximately corresponds to 1 standard deviation in the normal distribution, one can use this as the standard deviation in the view. The squared value is of course the variance, which will then be a diagonal element in the Omega-matrix. The third way suggested is to calculate views from some kind of factor model that describes the return of an asset. Given this regression, we can use the variance in the residuals as variance in views (Walters 2011).

The fourth way suggested by Walters is the one put forward by Idzorek (2005). Idzorek lets the investor assign a confidence for each view between 0 – 100 %. Using this confidence, we calculate portfolio weights. Then, we calculate portfolio weights as if confidence would have been 100 %. These two different weights are then compared to how they differ from the market weights. The ratio between these differences gives the implied confidence:

$$confidence = \frac{(\widehat{w} - w_{mkt})}{(w_{100\%} - w_{mkt})} \tag{2.24}$$

2.3.4 Weights

Assuming that we now have the different variables discussed above, we can start calculating the mean returns within the BL-framework (Black & Litterman, 1992) where we combine the equilibrium returns and the views:

$$\boldsymbol{\mu}_{BL} = ((\tau\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}'\boldsymbol{\Omega}^{-1}\boldsymbol{P})^{-1}((\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\pi} + \boldsymbol{P}'\boldsymbol{\Omega}^{-1}\boldsymbol{v}) \tag{2.25}$$

This is then, I believe, the most important formula in the BL-model. Now we have the returns which will be used to calculate optimal weights, in the same way that we would in a normal MW-portfolio according to this calculation:

$$\boldsymbol{w}_{msr} = \left((E[\boldsymbol{R}] - r_f)\boldsymbol{\Sigma}^{-1}\boldsymbol{I}'\right)^{-1}(E[\boldsymbol{R}] - r_f)\boldsymbol{\Sigma}^{-1} \tag{2.26}$$

Substituting expected return and covariance matrix with the BL-equivalent renders:

$$\boldsymbol{w}_{msr(BL)} = \left((\boldsymbol{\mu}_{BL} - r_f)\boldsymbol{\Sigma}_{BL}^{-1}\boldsymbol{I}'\right)^{-1}(\boldsymbol{\mu}_{BL} - r_f)\boldsymbol{\Sigma}_{BL}^{-1} \tag{2.27}$$

Note that this assumes that we have total returns in our views. If we have *excess returns*, one does not need to subtract the risk-free rate from the returns. The BL-covariance matrix is given by (Meucci 2010):

$$\boldsymbol{\Sigma}_{BL} = \boldsymbol{\Sigma} + \boldsymbol{\Sigma}_{BL}^{\mu} \tag{2.28}$$

$$\boldsymbol{\Sigma}_{BL}^{\mu} = ((\tau\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}'\boldsymbol{\Omega}^{-1}\boldsymbol{P})^{-1} \tag{2.29}$$

2.3.5 The scalar *tau*

In many of the equations covered up to this point there has been a scalar, denoted τ (tau). There are no clear rules for how this scalar should be calculated. Black & Litterman (1992) gave no further information apart from tau being a constant. However, He & Litterman (1999) used a value of tau=0,05. Idzorek (2005) concluded that the scalar is very hard to specify, just as the uncertainty in the views. Blamont & Firoozye (2003, p.102) concluded that it need to be between 0 and 1, meaning *"…that the variance in expected return is smaller than the variance of the actual returns"*. Walters (2010) examined how different values of tau affect the distribution of the estimate of the mean return. Using a mean return of 8 % and standard deviation of 15 %, he showed that a tau of 0,2 would lead to a standard error of 20 % in a 99 % confidence interval, which obviously means that the estimate is not very good. In chapter 3 I will discuss the value of tau used in this thesis.

2.3.6 Using analyst recommendations as views

As mentioned, this thesis uses almost the same method as described by Arestad & Rahmqvist (2012). They constructed BL-portfolios using the 36 stocks that the mutual fund *Nordea Sverigefond* consists of. As views, they collected public information in the form of analyst recommendations. They used the number of compiled views as an estimator of the confidence in views. (This will be discussed further in chapter 3.) They were not able to find any significant difference in performance between their portfolios and the mutual fund.

He, Grant & Fabre (2013) studied performance of analyst recommendations on stocks from the Australian stock index S&P/ASX 50. They were able to find that a portfolio containing stocks with positive consensus recommendations outperformed a portfolio containing stocks with negative recommendations. Further, they used these recommendations as views in a BL-portfolio. When the portfolio was balanced on a daily basis it significantly outperformed the index, however, after transaction costs it did not. Monthly rebalancing reduced these transaction costs, but also lowered the performance.

3 Method

In this chapter I will provide an in-depth description of how the underling theories have been used in this thesis to construct different portfolios based on the BL-model.

3.1 Selection of market

When choosing to test the BL- model, two major questions arise. First, on what market should I test the model? If an investor aims at obtaining a portfolio that is as diversified as it is possible, naturally one would need to apply the model on the entire world market. This would first require that we identify which types of assets that would be included in such a portfolio. The assets would range from common stocks and bonds to commodities and currencies. After identifying the asset classes, one needs to gather data from all over the world, covering all asset classes. This would, by far, be the best diversified portfolio that is attainable to an investor. However, the BL-model needs the investor to have views on at least some of these markets; otherwise we would just perform a normal portfolio optimization within the MV-framework.

This raises the second question; what should I use as views in the model? Since the goal is to back-test the model during one year, it would mean that I would have had to collect a vast amount of data. It would mean collecting data from all available asset classes, from all different countries during one year. One can easily see that this would be a daunting task to say the least. Furthermore, my goal was to do the test from the eyes of a Swedish investor. Investors tends to be home biased, which means that they prefer to invest in their home country (Tesar & Werner, 1995). From an diversification point of view, this is naturally not a good approach, but it is what most investors does, since they believe that they have a better knowledge and understanding about their home market.

So, given this reasoning, what would be an appropriate market to analyze? Some criteria for the market in question are necessary to narrow down the possibilities. My main goal was that I wanted to study the Swedish stock market. Swedish market, since I am home biased just as most investors, and the stock market since it is a financial market that is easy to understand for most investors.

The other criteria for the market in question are that I need to be able to collect views for the stocks that it includes. I need not only be able to collect views, since a BL-model without views is somewhat pointless, but I need to be able to collect views on a somewhat regular basis to get a big enough sample to analyze. This means that if I want to be able to collect views on a regular basis, I need to stick with stocks that have a high trading volume. The reason for this is simply that the most traded stocks are those that attract most attention from the investor collective, and consequently, from the financial firms that publishes the recommendations.

Following this reasoning, I can conclude that what I need is Swedish stocks with a high trading volume (and thus, a lot of views), preferably collected in an index. The index OMXS30 fits this description very nicely indeed. It contains the 30 most traded stocks on the Swedish stock market. It is also often used as a benchmark index for index funds, funds that many common investors choose to invest in. So this gives us a benchmark index to which we can compare the BL-portfolio against.

3.2 Views

Following in the footsteps of Arestad & Rahmqvist (2012) I have chosen to use the online feature called *Stockwatch*, from the newspaper *Dagens Industri*, to collect my views. On Stockwatch there is a feature that lets the user get access to an archive of analyses on different stocks made by financial companies. These analyses nearly always end up in an investing recommendation for the specific stock. The recommendations sometimes use different terminology. Some comes in the form of "performance" notations, where they for instance says that particular stock will "outperform", which implies that it is a buy recommendation. Other financial institute just simply recommends that you should buy the stock. Whatever the terminology used, all recommendations can be broken down into three different categories: Buy, Sell or Neutral. Buy and sell are self-explanatory while a neutral view means either that the stock is correctly valued by the market, or that it simply is expected to perform along with the market. Either way, the neutral recommendation does imply that an investor should have a neutral attitude towards that stock.

A recommendations generally comes with a *target price*. This target price is what is essential when converting the views into data that I can use in the BL model. What the target price gives is consequentially an expected return. Given the target price and the spot price of the stock, at the time of the recommendation, we can calculate the expected return which then is used in the BL-model.

When collecting the views I have categorized them as: 1 (buy) -1 (sell) and 0 (neutral). This is only to easy identify them in the data sheet. Naturally, one can choose to label the recommendations to their own liking. The target price and the date is then matched with the corresponding date and spot price from the daily price data from the stock in question, to create the expected return. This is then done repeatedly for every view available, for each of the 30 stocks, during the whole year of 2012. During this period, with 30 stocks, this adds up to a total of 1474 individual views. They were distributed with 40 % buy, 40 % neutral and 20 % sell recommendations.

Lastly, I use the expected returns, from each month in 2012 to create a total expected return for every month, calculated as the arithmetic mean from the expected returns from that month. This gives us 12 summarized expected returns (one for each month) for all 30 stocks. In some cases there were no views available for certain months. What I chose to do here was to assume that no new information has arrived, thus the previous views should still be considered to hold. The only thing I have to do is to reduce the expected return with 1/12 for every month that has no views. This simply because one month has passed and we need to adjust the 12 month expected return accordingly.

There are, of course, different ways one can approach the expected returns. In reality, recommendations tend to sometimes come in a cluster, often on a single day. This is often after a company has released new public information, such as an annual report. This can result in many views in one specific month, but allocated to only one day. One way of summarize these views would then be to do it after the company has released the new information. However, this would greatly increase the work load as far as this thesis goes, but for a real life situation it should be a good way to approach it. The problem that arises when doing this though, is how to treat views that do not come in conjunction with corporate events. One would then need further assumptions in how to treat these views. For this thesis, a monthly summarize of views works well enough since it can mimic an investor that rebalances her portfolio on a monthly basis.

In terms of expected return horizon, I have treated all target prices as a 12-month price estimate. This means that all expected returns in my modeling are annual returns. The main reason for this is that when reading through all 1474 recommendations, I found that it is not very common that the analysis speaks about what horizon their target price is related to. But the once that does in fact tells us about a horizon for the target price has always used a 12 month horizon. This lead me to believe that it might be industry standard to use yearly returns when estimating target prices. This assumption is not something that is empirically proved in any way, but it is an assumption that needs to be made to fit the purpose of this thesis.

3.3 Confidence in views

When it comes to calculate the confidence in the views, and thereby the Omega matrix, there are a few ways one can approach this, as I have covered earlier. I have chosen to use a similar way that Arestad & Rahmqvist (2012) did in their paper. They used a new way of estimating the diagonal omega matrix. Like many other methods of estimating the omega matrix, they start by using the variance in the stock return as a starting point for the variance in views. This variance was then divided by the compiled number of views, according to the formula below (with stock *k* at time *t*). This gives us a diagonal omega matrix, since we consider the variance in views to be non-correlated.

$$\omega_{k,t}^2 = \frac{\sigma_{k,t}^2}{m_{k,t}} \tag{3.1}$$

Here, *m* is the compiled views. However, they do not specify in detail how they calculate *m*. They do provide one example where they say (Arestad & Rahmqvist, 2012, p. 23):

> *"[…] if three analysts suggested 'buy' for H&M in Dec. 2009 then our variance for this particular view is divided buy 3 during the subsequent year."*

The problem here is that they do not specify what they do if a stock, for instance, had 3 buy, 2 neutral and 1 sell recommendation.

I suggest one way of dealing with this issue. First, one identifies the type of recommendation that is the *dominant* type of view during one month. The dominant view is simply the one with the most recommendations. Not necessarily in absolute majority, but simply the type of view that has most recommendations. If we take the example above (3 buy, 2 neutral, 1 sell) then *buy* is the dominant view. Here, I suggest a way to calculate *m* based on this dominant view in the following way:

$$m_{k,t} = 1 + \left(\sum V_D - \sum V_{ND}\right) \tag{3.2}$$

Here, V_D is the dominant view and V_{ND} are the non-dominant views, i.e. the rest of the views that are not the dominant one. Following the example above (3 buy, 2 neutral, 1 sell) it becomes:

$$m_{k,t} = 1 + (3 - 3) = 1 \tag{3.3}$$

This would then capture the collective ambivalence among the financial institutions, where they clearly do not have the same view on how the stock will perform. This leads to us *not* reducing the variance in the views, but rather let it be the same variance as the empirical yearly return variance. To show the difference when the collective recommendations are more concurrent, consider the following example: 6 buy, 1 neutral, 0 sell. Clearly, during this month, the collective of financial institutions have a much more clear view on how the stock will perform. In this case, *m* will be:

$$m_{k,t} = 1 + (6 - 1) = 6 \tag{3.4}$$

To capture the fact that the collective of analysts seem more "sure" of their opinions, the variance in views will be 6 times smaller than the variance in returns for this particular stock for this month.

Now, we still need to impose some assumptions when calculating *m*. The observant reader will probably quickly realize that there will be times when *m* can be equal to zero, or even be negative. Naturally, this will not work since we cannot divide by zero and we cannot have negative variance. To overcome this, I simply use the following restriction:

$$m_{k,t} \geq 1 \tag{3.5}$$

If we simply say that *m* cannot be lower than 1, we deal with both the problem of negative values and *m* being equal to zero. Seeing as $m_{k,t} = 1$, by definition, does not lower the variance in views, it make sense to have this restriction since we will only reduce the variance in views when we have clear dominant view from the collective of analysts.

In some rare cases, a problem can occur even if we have a clearly dominant view. This is if the dominant view has no target prices. To illustrate this problem, consider the following example from the data. In September 2012 the Ericsson B stock had 4 recommendations; 3 neutral and 1 buy. Clearly, the dominant view here is neutral. However, none of the neutral recommendations had a target price. The buy recommendation however, had a target price which, when put in relation to the spot price, would give an expected return of 21,2 %. Naturally, we cannot use this expected return in our BL-estimate without skewing the results. Therefore, we need a rule that states that if $m_{k,t} > 1$, we need to make sure that the expected return reflects the type of dominant view that we have.

One could of course come up with different ways of dealing with this, for example reducing the expected return in some way, to reflect that neutral is the dominant view. The problem is that it is hard to set up strict rules for exactly how to do this. In the real world, one could use more subjective methods by setting the expected return in relation to the dominant view and adjust the expected return accordingly. But, since this is such a rarely occurring problem, the easiest solution is simply to disregard from the views that month and use the expected return calculated from previous month's views. This is by no means perfect, but given the rarity of the problem, it fits in the purpose of this thesis.

3.4 Calculating portfolio weights

Now that we have a vector of expected returns from the views, an omega matrix with the variance in the views and a P-matrix we can start calculating the portfolio weights for each month. Note that we have views (with expected returns) for all stocks and all months, except for January 2012, where there were no views for Securitas B. This means that for that month, the views vector is a 1x29 vector and that the P-matrix is a 29x30 matrix. For all other months, we have views for all stocks, making the views vector a 1x30 and the P-matrix a 30x30 matrix.

Since we are using views from previous month to calculate portfolio weights (i.e. views from January to calculate portfolio weights at February 1st), our portfolio will begin in February 1st and ends at January 31st 2013. At the first (trading) day in each month, we rebalance the portfolio based on the views on the month before.

Four portfolios will be constructed; two based on the MV-framework and two that are based on the BL-model. The two different types of portfolios are: *unrestricted*, where we allow short selling and *restricted*, were no short selling is allowed. However, both portfolios will have the restriction of being fully invested. The portfolio weights for the unrestricted portfolio can be calculated using matrix algebra according to the formula for portfolio weights in the BL model, discussed in the theory section (see equation 2.26-2.27).

However, before we can do this, we need to calculate the mean return given from the BL framework (μ_{BL}). This is also done by matrix algebra using formula (2.25) discussed in the theory section. Two things need to be said here. First; the value of tau (τ). As covered in the theory section, there are a few different ways to estimate this. When first coming in contact with the BL-model, we used the method that Walters (2011) suggested:

$$\tau = 1/N \tag{3.6}$$

Given that a value close to zero is a common recommendation and that I use daily observations, both suggestions of estimating tau are met. Since there are different numbers of trading days in one year, tau is a bit different from month to month, but the range in which is varies is very small. In the 12 different weight calculations, tau ranges from 0,00361 to 0,00398.

The next thing that needs to be mentioned is the vector of π. If we recall from the theory section we can calculate the market weights using the market value of the stocks. These equilibrium market weights are calculated for each new month, using the spot price at the corresponding date and the number of shares that are outstanding for each stock. This gives us the market value for each stock to use in equation (2.18).

λ can, as we saw earlier, either be set exogenously to 1.2, or estimated by using expected return and variance in the market (in this case OMXS30). The downside with setting it to 1.2 exogenously is that we do not take into account the current market climate, and thus not using the correct level of risk aversion that actually exists in the market right now. Granted, calculating the risk aversion based on the last 12 months might also not capture the current risk aversion in a correct way, but it should be better than assuming that the level of risk aversion does not changes. The upside of using lambda equal 1.2 is of course that we have risk aversion in the model. If we instead estimate lambda, we will get negative values, i.e. risk seeking, if the mean return from the latest 12 months is less than the risk free rate.

In a downward market, we could argue the fact that investors are looking to buy stocks cheaply and thus investing in the more volatile stock market even if the market is in a downward trend. However, given the nature of the data I have been using, lambda will be negative 11 out of 12 months. This is simply not very realistic, since it would imply that the investors during this period were not risk avert, but rather risk seeking. Seeing as this is a period with great uncertainty in the markets, with the Euro crises and other turbulence, a lack of risk aversion is highly unlikely. Therefore, I have chosen to set lambda to 1,2. I did try using the estimated lambdas, but the results from that indicated that the BL framework is most likely built on the assumption of a positive lambda.

To calculate the restricted portfolio weights we cannot use the formula presented in chapter 2. Instead this must be done by finding the optimal solution given the restrictions. This is done using the tool Solver in Excel. What we do is the standard optimization in the MV-framework, that is, we maximize the portfolio Sharpe ratio (in matrix form) subject to the restrictions in the following way:

$$\max \frac{\boldsymbol{w}'_{BL}(\boldsymbol{\mu_{BL}} - r_f)}{(\boldsymbol{w}'_{BL}\boldsymbol{\Sigma}_{BL}\boldsymbol{w}_{BL})^{1/2}} \tag{3.7}$$

$$s.t \sum_{i=1}^{N} w_{i,BL} = 1 \quad and \;\; w_{i,BL} \geq 0 \tag{3.8}$$

Finally, when constructing the MV-portfolios, the same formulas will be used. The only difference is that I will be using the vector $\boldsymbol{\pi}$ as expected return, instead of $\boldsymbol{\mu}_{BL.}$ The main reason for this is, as mentioned in chapter 2; mean returns are not good estimates of expected returns. The last thing to change is to use the covariance matrix based on the returns, instead of the BL counterpart, when calculating the weights for the MV-portfolios. What this means is that I will be using the equilibrium returns as presented in the BL-framework (instead of historical mean returns), but without implementing any views. This will better show the difference that the views has in the portfolios.

3.5 Testing

Now that we have our sets of portfolio weights over 12 months, we can construct the portfolios and look at their performance during a 12 month period. Using the daily returns of the stocks, we can create the portfolio daily returns by multiplying the returns and the weights for every day in the month that apply to the weights of that month. These returns will then be compared with the returns of OMXS30, to see if there is any significant difference in the returns. Furthermore, the portfolios Sharpe ratios will be statistically tested and compared. Two types of hypothesizes will be tested regarding the Sharpe ratios; first, if they are significantly different from zero, second, if they are different from the Sharpe ratio of OMXS30. The mathematics about statistics testing of Sharpe ratio is covered section 2.2.

3.6 Data

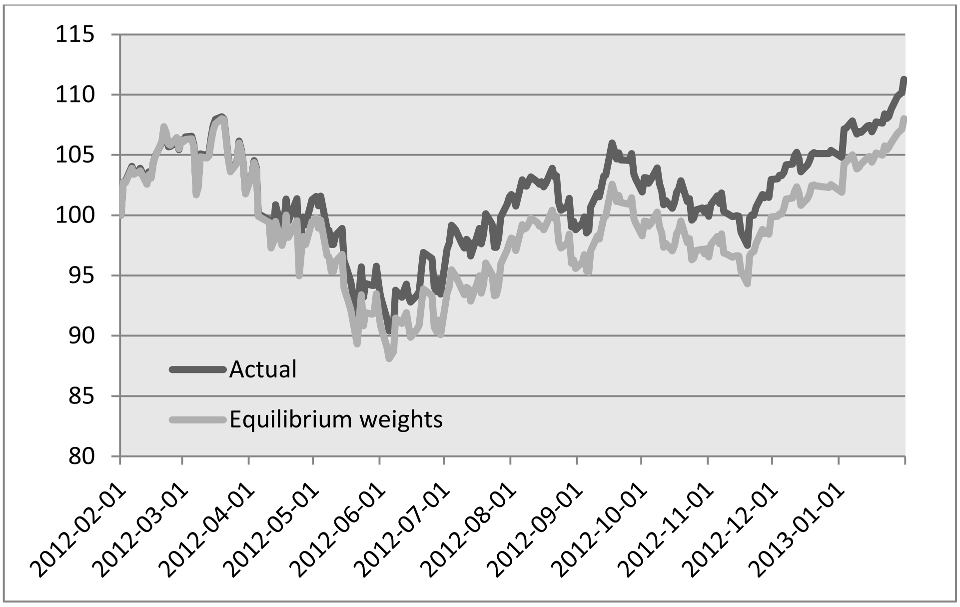
The collection of data regarding the views has already been covered, but some things should be mentioned regarding the collection and processing of the stock price data. Daily stock prices have been collected from Nasdaq OMX Nordic. The prices are closing prices which is the last quoted price on a given trading day. Using these closing prices I have created a *moving* 12 month average return. This is done by subtracting the log price of a stock at January 1st 2011 from the log stock price at January 1st 2012. This is then the 12 month return at January 1st 2012. This is then repeated for every trading day in 2012 and what we end up with is a time series that shows us how the 12 month return has progressed during 2012. These returns are the ones that are being used in the calculation for the covariance matrix, which in turn of course are the base of a lot of other calculations. Daily log returns for the stocks has also been calculated, which will be the basis for which the portfolio returns are calculated.

The risk free rate that is being used is the Swedish 1 month Treasury bill (SSVX 1M) and is collected from the Swedish central bank (Riksbanken). There are of course many different rates one can choose to use as the risk free rate, such as treasury bills with longer maturity or government bonds with even longer maturity. Naturally, most investors would consider any T-bill or bond issued from the Swedish government to be risk-free, but I feel that to have minimal risk, one need to have as low time to maturity as possible. Hence, I have chosen to use the 1 month T-bill. The effects that using this risk free rate will have on the results should not be considered substantial. Naturally, one could test a whole range of portfolios that uses different risk free rates, but that goes beyond the scope of this thesis.

4 Results

In this chapter I will present the results from the study. I will begin by discuss the impact that the estimated market equilibrium weights have on the results. Then I will present how the portfolios have performed during the period. They will be shown in relation to how OMXS30 have performed during the same period. Further, I will look at the daily returns from the portfolios and OMXS30 to see if we can find any significant difference in the daily returns between the portfolios and OMXS30. I will also test if we can see any significant difference between the portfolios Sharpe ratios, since they are more interesting than just returns. Finally, I will discuss the difference in weights between the portfolios to show the difference it makes when we impose short selling restrictions and how that effect portfolio optimization in the MV-framework. The results presented in this chapter will be analyzed and discussed as they are presented.

4.1 Market index



Graph 1: OMXS30 with actual weights and calculated equilibrium weights

Given the fact that we need to estimate the market weights every month, it is clear that the performance of a portfolio using these weights will not be identical to that of the market index. In the chart above, I have plotted the real OMXS30 and how it would have performed if it were weighted using the equilibrium weights every month. As we can see, the performance does differ, but it is clear that the two series are very closely correlated. The correlation between the two is in fact 0,962. Even though, it will have an effect on the BL portfolios, but given the high correlation the effects should not be of a great importance.

The problem occurs because of the fact that I am performing monthly calculation of the weights. Given how Nasdaq OMX calculate the index[2], the true market weights of the index should be very close, if not identical, to the ones I calculate each month. But as trading goes on during the month, the true market weights will change each day, but my weights will only change once every month. This is most likely what causes the difference in performance between the two, rather than the way the market weights are calculated. Since the method for this thesis is based on monthly rebalancing of portfolios, this is a fact that we simply have to accept. Also, Nasdaq OMX does not supply data over how the index weights have changed over time, meaning that the estimated weights have to be used. It should be pointed out that all comparisons to OMXS30 will be made with the true performance of the index.

4.2 Overall portfolio performance

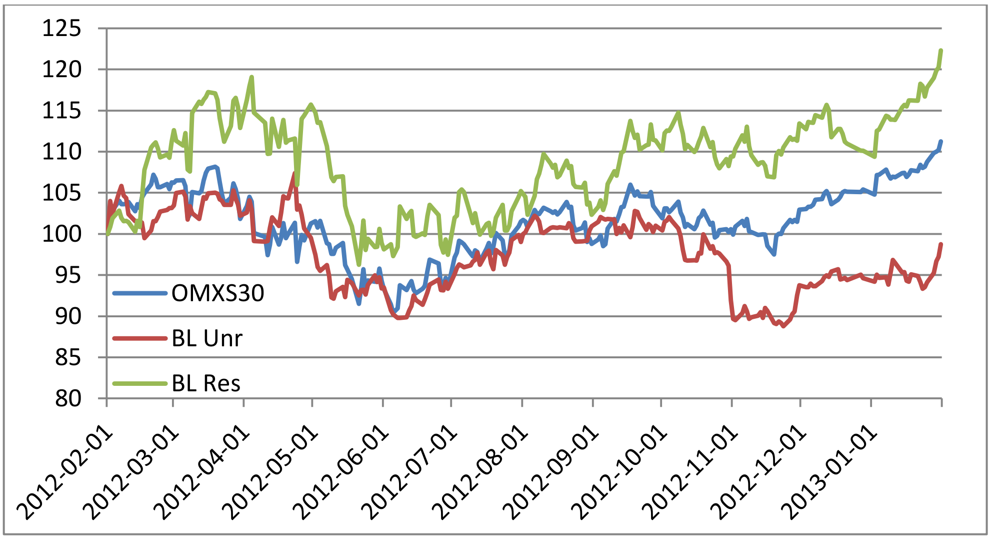
I begin with showing the overall return from the four portfolios and the market index. In terms of this total return, the restricted BL-portfolio is the clear winner. The two unrestricted portfolios got a similar return, far worse than the restricted portfolios and the index.

	OMXS30	BL Unr	BL Res	MV Unr	MV Res
Total return	10,86%	-3,06%	20,87%	-2,62%	12,58%

Table 1: Total return of market index and portfolios

Now, let's take a graphic look at how the four different portfolios have performed during the analyzed period.

4.2.1 BL-Portfolios



Graph 2: BL-portfolios performance

[2] For a detailed explanation, see "Rules for the construction and maintenance of the OMX Stockholm 30 Index"

Let us first look at how the two BL portfolios have performed during the 12 month long time period. As we can see in the chart above, the unrestricted portfolio follow the index rather well for the first few months. This is somewhat surprising given that this period feature a downward movement for the index. Since the unrestricted portfolio has short selling at its disposal, I expected it to perform better during a down period like this. Perhaps we could not expect it to have positive return during this period, since it at the same time also takes positive position, but I expected the short positions to have better hedging effect against negative movements in the market index. The restricted portfolio however, performs better in this period. The key to this is in the very beginning, were it chose the right stocks to invest in. After these first few months, it mirrors the index rather well, but given its "early lead", it stays above the index for the rest of the period.

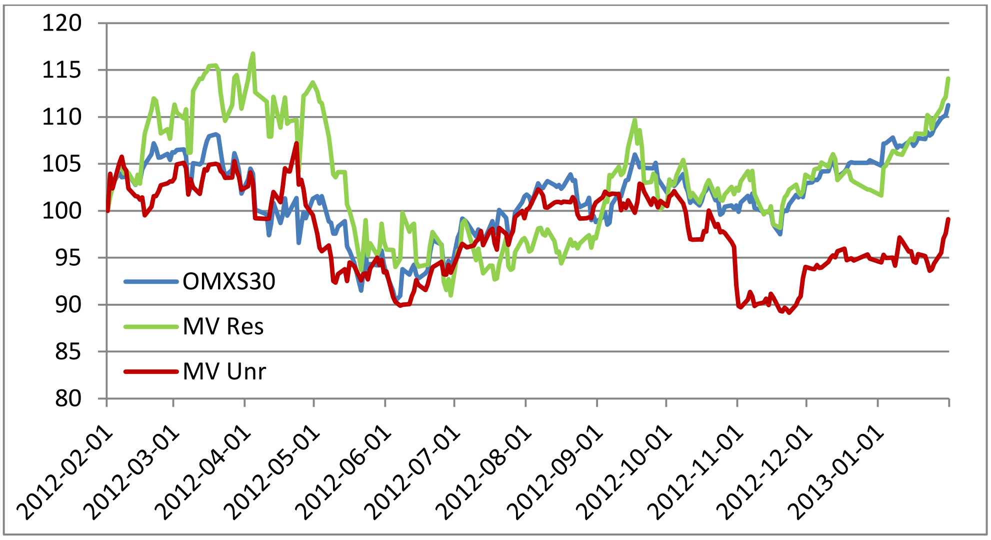
The market hits its bottom in late May / beginning of June, which the two portfolios also do. After this trend change, both portfolios follow the movement of the index. It appears that the restricted portfolio is the most volatile of the three, which might not be so strange given that it is not as diversified as the other two. Remember that we have 30 stocks to choose from, which mean that if the restricted portfolio is using only a few of these stocks (which it often does), it will be less diversified, meaning (theoretically) higher variance. During this period, the combination of short and long positions seems to reduce the volatility of the unrestricted portfolio.

In the late fall and winter, the unrestricted portfolio starts performing way worse than the other two. Again, this is the risk with using short positions. However, in the period between mid-September and mid-November, the market is again in a downward trend, dropping around 9 points. Again, it is a bit peculiar that the unrestricted portfolio yet again fail to utilize short positions to reduce losses in a downward market. But unlike the downward period in the first half of the year, the unrestricted portfolio not only follows the market down, it performed far worse. What is also interesting is that the restricted portfolio does not drop as much as the index during this period and then proceed to perform well for the rest of the period.

From the bottom in late November, the unrestricted portfolio fails to come back to the same level as the index during the last part of the period. Here, my choice of time period for the study limits us to see if it ever would have recovered to the index level.

4.2.2 MV-portfolios

Now, let’s move on to the MV-portfolios. These are interesting too look at since we want to see if the BL framework has done any difference against the normal MV-framework. That is, has the input of views in the model affected how the MV-portfolios perform?



Graph 3: MV-portfolios performance

As we can see, the unrestricted MV-portfolio seems to behave very much like the unrestricted BL-portfolio, which it in fact does to a big extent (see correlations below). Given this fact, the performance of the unrestricted MV-portfolio is not something we need discuss further.

What is interesting is the restricted MV-portfolio. Just as the BL equivalent, it takes an “early lead” over the index. But remember that the BL-portfolio from that point kept outperforming the index; this is not case for the restricted MV-portfolio. After the early lead, it rather quickly falls to the levels of the index, and from that point follows it rather good, expect for a short period during the late summer, where it performs worse than the index.

4.2.3 Price correlation

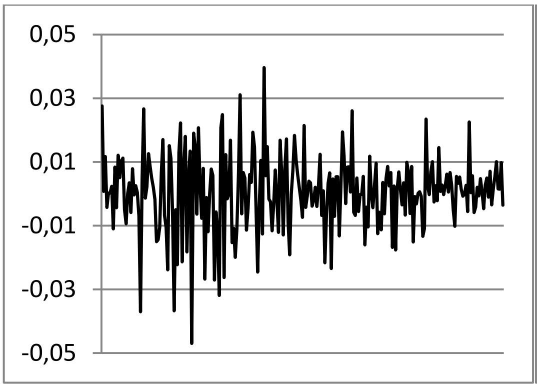
Finally, before analyzing the actual daily returns, I present the correlation between the prices of the portfolios and the index. As one easily can see by just examining the charts above, the correlation between the portfolio prices are high. However, the restricted portfolios looks more correlated with the index than the unrestricted BL-portfolio. Given the fact that we have no short selling in the unrestricted models, this is something I expected. The results in the table below confirm this suspicion. As we can see, the correlation between the restricted portfolios and the index is indeed higher than the correlation between the index and the unrestricted portfolios.

Remember that I pointed out that the two unrestricted portfolios performed rather similar, this is confirmed by the correlation table, with a correlation coefficient of 0,9998. Another interesting thing is that correlation between the restricted portfolios and the unrestricted ones are quite low.

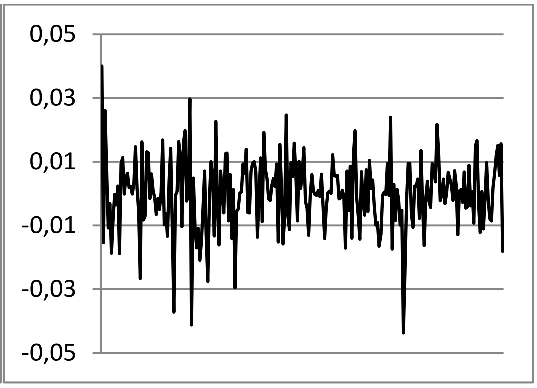
	Omx	BL Unr	BL Res	MV Unr	MV Res
Omx	1				
BL Unr	0,421462	1			
BL Res	0,794050	0,199042	1		
MV Unr	0,435597	0,999802	0,210717	1	
MV Res	0,683142	0,433132	0,815405	0,437595	1

Table 2: Price correlation between portfolios

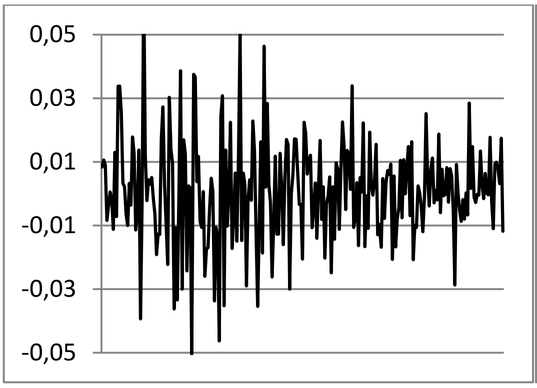
4.3 Daily returns



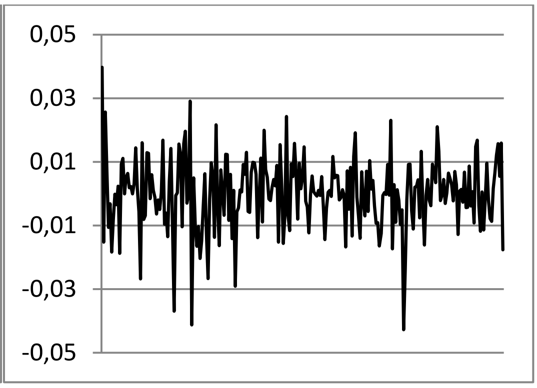
Graph 4: OMXS30 daily returns



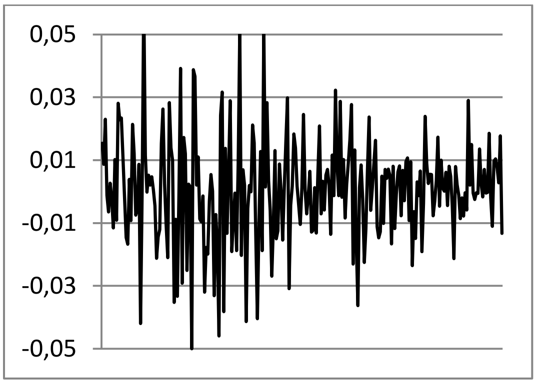
Graph 5: BL Unr daily returns



Graph 6: BL Res daily returns



Graph 7: MV Unr daily returns



Graph 8: MV Res daily returns

	ADF	P-value
Omx	-17,079	0,00000
BL Unr	-16,4785	0,00000
BL Res	-16,1378	0,00000
MV Unr	-16,0017	0,00000
MV Res	-10,2709	0,00000

Table 3: Augmented Dickey Fuller test

The charts above shows the daily returns for the four portfolios and the index. Just by looking at these time series, they all appear to be stationary. To be sure, the augmented Dickey-Fuller test is performed. The ADF null hypothesis of a unit root is rejected in all five series, thus we conclude that they are stationary, which is good news for when we analyze these daily returns further.

The daily returns consist of 251 observations. The tables below show some statistical properties of the daily returns:

	Mean	Variance	Std. Dev	Skewness	Excess Kurtosis	Jarque Berra	J-B P-value
OMXS30	0,00048	0,00014	0,01178	-0,40298	1,73053	38,11359	0,000000
BL Unr	-0,00006	0,00013	0,01127	-0,35762	1,80894	39,57243	0,000000
BL Res	0,00088	0,00026	0,01606	0,16927	1,58548	27,48815	0,000001
MV Unr	-0,00004	0,00012	0,01111	-0,36224	1,82865	40,46149	0,000000
MV Res	0,00061	0,00027	0,01657	0,07556	1,38211	20,21659	0,000020

Table 4: Properties of daily returns

	Omx	BL Unr	BL Res	MV Unr	MV Res
Omx	1				
BL Unr	0,502591	1			
BL Res	0,872920	0,291805	1		
MV Unr	0,520307	0,999602	0,309481	1	
MV Res	0,835636	0,340000	0,930154	0,355899	1

Table 5: Correlation between daily returns

First, some comments about the correlation matrix between the daily returns. All of the pairs, except from BL Unr / MV Res, show a higher correlation coefficient in the daily return then they did in prices. As we can see, the returns between the restricted portfolios and the index are very high (0,8729 and 0,8356), which is a first indication that they probably are not significantly different from the index, despite their outperforming of the index. Further, the correlation between the two restricted portfolios is also very high (0,9302), which would indicate that the BL-portfolios does not differ from the MV-portfolios. This is also confirmed by the (very) high correlation between the two unrestricted portfolios (0,9996)

As we saw when examining the portfolio performance above, the two unrestricted portfolios performed the weakest. This is (naturally) also true for the daily returns, were the average daily return is negative for these two portfolios. However, they are in turn the portfolios with the lowest variance. The restricted BL-portfolio performed best with an average daily return of 0,088 %. However, the variance for the restricted BL-portfolio was almost twice as big as the variance for OMXS30 (this is also true for the restricted MW-portfolio), which will affects its risk adjusted return, as I will discuss below.

All five series shows a high excess kurtosis, e.g. they have fat tails. Remember that a normal distribution has an excess kurtosis equal to 0 (Verbeek 2012). We can also see that we have negative skewness in all three series. Thus, everything points to the fact that they are not normally distributed. The Jarque-Berra test, with H_0*: Normal distribution*, confirms this with rejection of the null with very low p-values.

The fact that we have non-normal distribution in returns is not a surprise. In fact, Cont (2000) points out that this is one of the common stylized facts about many financial time series. The kurtosis is generally larger the shorter the time frame. As an example, Cont says that the kurtosis of the 5-minute return of the S&P500 future is roughly 16. To compare the portfolios, I will be using a t-test, which has fatter tails than the normal distribution, however, as the degrees of freedom (*N-K*) increases it gets closer to the normal distribution (Verbeek, 2012), as it is in my case with 251 observations. Given the fact that this thesis only aims to compare the properties of portfolios, the non-normality should not matter when comparing portfolio returns. Furthermore, when it comes to compare portfolio performance, I feel that the Sharpe ratio is a more relevant measurement.

	1-tail	2-tail
BL Unr	0,300266161	0,759362
BL Res	0,374371145	0,7823536
MV Unr	0,304489012	0,5316325
MV Res	0,460152065	0,9816935
BL against the MV equivalent		
Unr	0,493598811	0,9871976
Res	0,425307034	0,8506141

On the left is a selection of t-tests performed on the daily returns. The first section tests the difference between the portfolio returns and the return of OMXS30. The second section compares the BL-portfolios against the equivalent MV-portfolio (i.e. restricted vs. restricted & unrestricted vs. unrestricted).

Table 6: Significance tests, P-values

As we can see, none of the portfolios daily return differs from the daily return of OMXS30. Thus, when it comes to daily returns, we cannot say that the BL- or the MV-portfolios is any different from the index. In this case, buying an index fund is preferable to active portfolio management using the MV-framework (since it does not require any effort from the investor). Furthermore, we cannot say that implementing the views from the BL-framework has made any significant difference from the normal MV-framework. Thus, the views collected could not improve the MV-portfolios in terms of daily returns.

4.4 Sharpe ratios

Even though I could not find any significant difference in returns, remember that the MV-framework is about return *and* variance. Using the daily returns, Sharpe ratios have been calculated for the portfolios and the index. Remember, since they are calculated on daily data (return, standard deviation and risk-free rate), they are smaller than what the reader familiar with Sharpe ratios are perhaps used to. In the table below, we can see the Sharpe ratios, its variance under H_0: $SR_i=0$ and the confidence intervals for this null hypothesis. Another hypothesis that is being tested is weather the Sharpe ratio of a portfolio is equal to the Sharpe ratio of OMXS30; H_0: $SR_i=SR_{OMXS30}$. This is particularly interesting, since it tells us if there is a significant different between the *risk-adjusted* return between a portfolio and the risk-adjusted return we get by investing in the index.

			Confidence interval		*H_0 SR_i=SR_{OMX}*	
	S.R.	S.E. under H_0	Lower	Upper	Z-test	P-value
OMXS30	*0,03754*	*0,06314*	*-0,08621*	*0,16130*		
BL Unr	*-0,00869*	*0,06312*	*-0,13241*	*0,11502*	*-0,73411*	*0,23144*
BL Res	*0,05268*	*0,06316*	*-0,07112*	*0,17648*	*0,47543*	*0,31724*
MV Unr	*-0,00737*	*0,06312*	*-0,13109*	*0,11634*	*-0,38049*	*0,35179*
MV Res	*0,03447*	*0,06314*	*-0,08929*	*0,15822*	*-0,01887*	*0,49247*

Table 7: Sharpe ratios based on daily data

As we can see, the confidence interval for all portfolios (and OMXS30) contains the value zero. Thus, we cannot reject (on a 95 % significance level) the null that the Sharpe ratio is equal to zero for any portfolio (or the index). We can also see that none of the portfolios has a Sharpe ratio that is significantly different from the Sharpe ratio of the index. Of course, this is expected since none of the Sharpe ratios could be said to be different from zero.

However, there might be a problem with these results. The reason for this is because of how we calculate the significance tests. Lo (2002) shows that a Sharpe ratio of 0,5, with 250 observations, have a standard error of 0,067. As we can see, this is not much higher than the standard errors that my Sharpe ratios have. Another example from Lo; a Sharpe ratio of 1,5 with the same number of observations has a standard error of 0,092, which is a much smaller proportion to the Sharpe ratio than the proportion I have, and the one with a SR=0,5. Given the fact that we have 250 observations and the way we calculate the standard error, the standard error is unlikely to be proportionally correct to the Sharpe ratio when we are using daily data. In fact, the standard error of the Sharpe ratio for the restricted BL-portfolio is as we see 0,0754. In contrast, the standard error of a Sharpe ratio of -0,00869 (BL Unr, based on daily data) is 0,06312. The different is likely to be negligible.

Naturally, the ideal situation would be to use longer period returns, such as monthly or annual. The problem is of course that we only have data from one year, meaning that if we were to use annual data, we would only have one observation, which is no good for making inference. What I can do is to use weekly and monthly data, but then the problem is that the number of observations reduces, giving higher standard errors. However, to see if there is any difference, let’s look at the Sharpe ratios using monthly data (12 observations) and weekly data (50 observations). First, the ones based on monthly data.

			Confidence interval		*H_0 SR_i=SR_{OMX}*	
	Sharpe	**S.E. under H_0**	**Lower**	**Upper**	**Z-test**	**P-value**
OMXS30	0,1235	0,2898	-0,4444	0,6915		
BL Unr	-0,1392	0,2901	-0,7078	0,4293	-1,0309	0,1513
BL Res	0,2369	0,2927	-0,3368	0,8106	0,2689	0,3940
MV Unr	-0,1320	0,2899	-0,7003	0,4363	-1,0093	0,1564
MV Res	0,1310	0,2899	-0,4373	0,6992	0,0279	0,4889

Table 8: Sharpe ratios based on monthly data

As we can see, the Sharpe ratios are now bigger, but so are the standard errors. Consequently, there is no difference in the inference we make. None of the Sharpe ratios are significantly different from zero and therefore, none of the portfolios Sharpe ratio are different from the index Sharpe ratio.

Now, let's see if there is any difference when we use weekly data.

			Confidence interval		H_0 SR_i=SR_{OMX}	
	Sharpe	S.E. under H_0	Lower	Upper	Z-test	P-value
OMXS30	0,0844	0,1417	-0,1933	0,3621		
BL Unr	-0,0208	0,1414	-0,2981	0,2564	-0,8700	0,1922
BL Res	0,1156	0,1419	-0,1625	0,3937	0,0709	0,4717
MV Unr	-0,0180	0,1414	-0,2952	0,2592	-0,8666	0,1931
MV Res	0,0707	0,1416	-0,2068	0,3482	-0,4437	0,3286

Table 9: Sharpe ratios on weekly data

As we can see, there is still no difference in the results of the tests. Even with 50 observations, and fairly more "normal" looking Sharpe ratios, we still cannot say that there is any difference between them, since none of the ratios are significantly different from zero. Therefore, we can conclude that none of the portfolios differ to the index, neither in returns nor in Sharpe ratios.

4.5 Weights

One of the goals with the BL-model is to reduce the extreme corner solutions that are common with portfolio optimizing in the MV-framework. Another one is to reduce the extreme short sale positions often found in the unrestricted portfolios. When it comes to the unrestricted portfolios, there is close to no difference between the most extreme weights. The most extreme short sale weight in the BL-portfolio was -0,721, and -0,719 in the MV-portfolio. On the other end, the most extreme long position in the BL-portfolio was 0,757 and in the MV-portfolio it was 0,742. In both cases, the BL-portfolio was the more extreme. However, the difference is so small it is negligible.

When it comes to the restricted portfolios the interesting thing would be to see if the number of stocks in the portfolio is higher in the BL-portfolio than in the MV-portfolio and thereby reducing the "corner" solutions. The average number of stocks used in the restricted MV-portfolio was 2,66. Two times was only one stock used, seven times was *three or more* stocks used and two times was *four or more* stocks used. The maximum number of stocks used was 5. The MV-portfolio used four stocks that the BL-portfolio never used.

In the restricted BL-portfolio the average number of stocks was 3. One time was only one stock used, *three or more* stocks were used on nine occasions, *four or more* was used four times. The maximum number of stocks was the same as in the MV-portfolio, 5. Just like the MV-portfolio, the BL-portfolio used four stocks that the MV counterpart never used.

Based on this, yes the restricted BL used more stocks than MW on most occasions, but one have to ask if it managed to diversify better than MW in a satisfactory manner. I would say no, especially since we had 30 stocks to choose from. The conclusion has to be that the BL-model has not been able diversify much better than the MV-model. One explanation for this might lie in the way I estimated the confidence in views. The average confidence in views was only 2,16. It is quite possible that this did not reduce the variance enough to have a large enough effect on the expected returns in the BL-framework. Since the confidence (theoretically) could be set to infinity, reducing the variance in views to zero, I should perhaps have considered a method that could reduce the variance in views to a greater extent.

Another possibility is that the expected returns given in the recommendations might be calculated in the same manner as the π-vector in the BL-model. It is not impossible that analysts use a model which calculates returns based on market weights and a certain level of risk aversion. Should this be the case, which is not unlikely, the views would not alter the expected returns very much.

5 Conclusion & final remarks

The purpose of this thesis was to give myself, and the reader, an insight in how the BL-model could be implemented in the real world, more specifically, on the Swedish stock market. Using public information in the form of stock recommendations, I created views that consisted of expected returns and the level of confidence in these views. These views consisted of 1474 individual recommendations on the 30 stocks in the OMXS30 index, collected during the year of 2012. The views were then summarized for each month, giving us an "average view" for every 12 months. This is by no means a perfect way of summarizing the public information, but as a method in this thesis it worked rather well. There is a big risk that, by waiting until the end of the month to rebalance the portfolios, a lot of the new information will already be priced by the market. This would in turn mean that the expected returns, calculated from these views, are incorrect when we rebalance the portfolios. This is a great concern when it comes to interpret the result from my analysis. In the real world, an investor need not to be restricted by neither specific dates nor collecting all recommendations available. To fully take advantage of these views, one should of course try to implement them in the model as quickly as possible. But as a demonstration for how it can be done, I believe it has served its purpose.

In this thesis, I further developed the method used by Arestad & Rahmqvists (2012) to collect views from public analyst recommendations, and estimating the confidence in these views. I suggested a more specific method to calculate the confidence in views by determine which type of view (buy, sell or neutral) that was the dominant one for a specific month. Depending on how concurrent the recommendations were, the variance in the views got adjusted accordingly. I believe that this method has worked rather well when it comes to estimate the omega matrix, with the downside that it might not have reduced the variance enough. The upside of this method is that it is strictly mathematical and only based on how concurrent the analysts are. This removes the some of the subjective nature that is a part of the BL-model. In an actual investing situation on the other hand, there are of course different ways to approach the confidence in views. Regardless of the method, I believe that the important thing is to be able to assign each view a seperate level of confidence, rather than an overall confidence for all views. My suggested method, is one possible way of accomplish this.

This thesis presents the results from four portfolios and their performance during the 12 months between February 1st 2012 and January 31st 2013. Two BL-portfolios were constructed, one with short selling constraint and one without these constraints. Two similar

MV-portfolios were also created. In comparison to the index (OMXS30), both of the unrestricted portfolios performed worse than the index. The two restricted portfolios performed better than the index, were the restricted BL-portfolio performed the best. However, none of the portfolios daily returns were significantly different from the daily returns of the index. Further, there was no significant difference between the restricted BL- and MV-portfolios when compared to its counterpart. I would also like to point out that I have not considered the effect of transaction costs in this thesis. To fully be able to (correctly) compare an active strategy (the portfolios presented in this thesis) against a passive strategy (only investing in the index) we need to present the performance after transaction costs.

When comparing Sharpe ratios, I could not find any significant difference between the portfolio Sharpe ratios and the ratio of the index. I used daily, weekly and monthly data to see if there was any difference in the results, however, that did not change the results. Here, once again, the problem is lack of data. Naturally, a longer timeframe might have shown different results. The problem is that I collected 1474 views during 2012, which was very time consuming. If I were to use a longer timeframe, for instance 5 years, and assuming the same amount of views per year, it would add up to 7370 views, which goes beyond the scopes of this thesis.

5.1 Further research

As I have pointed out, one of the downsides with my results are the fact that I only have portfolio data from one year. The issue is the big number of views collected. It would be interesting to see what the results had been during a timeframe of 5 years. This would be a job for someone with more time on their hands, or possibly one could write an application that downloads views for you in an efficient manner. Further, it would be interesting to see a different approach of how to analyze stock recommendations. For instance, one could categorize recommendations depending on what financial institution that made them and then construct a portfolio for each institution. Another interesting topic would be to test different ways of estimating the confidence in views. The method I chose can be done in different ways, e.g. give even higher confidence to concurrent views allowing us to reduce the variance in views to a greater extent, or even calculate the confidence in a different way entirely. Generally, there are a lot of exiting topics one can choose from when examining the BL-model.

References

Arestad, C.J. & Rahmqvist, J. (2012) "Applying the Black-Litterman Model on the Swedish Stock Market", *Master II Thesis,* School of Economics and Management, Lund Univeristy

Best, M.J. & Grauer, R.R. (1991) "Oh the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results." *The Review of Financial Studies*, January, pp. 315-342

Black, F. & Litterman, R. (1992) "Global Portfolio Optimization", *Financial Analysts Journal,* September-October 1992,Vol. 48 Issue 5, pp. 28-43

Blamont, D. & Firoozye, N. (2003) "Asset Allocation Model" *Global Markets Research: Fixed Income Research*, Deutsche Bank, July

Cont, R. (2001) "Empirical properties of asset returns: stylized facts and statistical issues." *Quantitative Finance*, Vol 1, Issue 2, pp. 223-236

Drobetz, W. (2001)."How to avoid pitfalls in portfolio optimization? Putting the Black-Litterman approach at work", *Financial Markets and Portfolio Management*, Vol. 15, Issue 1, pp. 59-75

Elton , E.J., Gruber, M.J., Brown, S.J. & Goetzmann, W.N. (2011) "Modern Portfolio Theory and Investment Analysis" 8th Edition, Hoboken NJ, Wiley & Sons Inc.

Giacometti, R., Bertocchi, M., Rachev, S.T., & Fabozzi, F.J. (2005) "Stable distributions in the Black-Litterman approach to the asset allocation", *PSTAT Recent Technical Report,* University of California, Santa Barbara

Green, R.C. & Hollifield, B. (1992) "When Will Mean-Variance Efficient Portfolios Be Well Diversified?", *The Journal of Finance*, Vol. 47, Issue 5, pp. 1785-1810

He, G. & Litterman, R. (1999) "The Intuition Behind Black-Litterman Model Portfolios", *Fixed Income Research*, December, Goldman Sachs

He, P.W., Grant, A. & Fabre, J. (2013) "Economic value of analyst recommendations in Australia: an application of the Black-Litterman asset allocation model", *Accounting & Finance,* Vol. 53, Issue 2, pp. 441-470

Idzorek, T. M. (2005) "A Step-By-Step Guide to The Black-Litterman Model", *Working paper.*

Jobson, J.D. & Korkie, B.M. (1981) "Performance Hypothesis Testing With the Sharpe and Treynor Measures", *The Journal of Finance*, Vol. 36, No. 4, pp. 889-908

Levy, H. (2010) "The CAPM is Alive and Well: A Review and Synthesis", *European Financail Management,* Vol. 16, Issue 1, pp. 43-71

Lintner, J. (1965) "Security prices, risk, and maximal gains from diversification" *The Journal of Finance*, Vol. 20, No. 4, pp. 587-615

Lo, A.W. (2002) “The Statistics of Sharpe Ratios” *Financial Analysts Journal*, Vol. 58 No. 4, pp. 36-452

Mankert, C. (2006) “The Black-Litterman Model – mathematical and behavioural finance apporaches towards its use in practice”, Licentiate Thesis, *Royal Institute of Technology,* Stockholm

Markowitz, H. (1952) “Portfolio Selection”, *The Journal of Finance*, Vol. 7, No. 1, pp 77-91

Markowitz, H. (1994) “The General Mean-Variance Portfolio Selection Problem [and Discussion]”, *Philosophical Transactions: Physical Science and Engineering*, Vol. 347, No. 1684, Mathematical Models in Finance, pp. 543-549

Merton, R.C. (1980) “On Estimating The Expected Return On the Market: An Exploratory Investigation”, *Journal of Financial Economics*, Vol. 8, Issue 4, pp. 323-361

Meucci, A. (2010) “The Black-Litterman Approach: Original Model and Extensions”, *Working paper*.

Meucci, A. (2009) “Enhancing the Black-Litterman and related approaches: Views and stress-test on risk factors”, *Journal of Asset Management*, Vol. 10, Issue 2, pp.89-96

Sharpe, W.F. (1963) “A Simplified Model for Portfolio Analysis”, *Management Science*, Vol. 9, No. 2, pp. 277-293

Sharpe, W.F. (1966) “Mutual Fund Performance”, *The Journal of Business*, Vol. 39, No. 1, pp. 119-138

Tesar, L.L & Werner I.M. (1995) “Home bias and high turnover”, *Journal of International Money and Finance*, Vol. 14, No. 4, pp. 467-492

Verbeek, M. (2012) “A Guide to Modern Econometrics” 4th Edition, West Sussex, Wiley & Sons Ltd.

Walters (2010) “The Factor Tau in the Black-Litterman Model”, *Department of Computer Science*, Boston University

Walters, J. (2011) “The Black-Litterman Model in Detail”, *Department of Computer Science*, Boston University

Dagens Industri - Stockwatch
http://www.di.se/stockwatch/ (Accessed 2013-05-20)

Nasdaq OMX Nordic
Historical Prices
http://www.nasdaqomxnordic.com/aktier/historiskakurser/ (Accessed 2013-05-20)
Index Methodology
https://indexes.nasdaqomx.com/docs/Methodology_OMXS30.pdf (Accessed 2013-05-20)

Riksbanken (The Swedish Centralbank)
http://www.riksbank.se/sv/Rantor-och-valutakurser/Sok-rantor-och-valutakurser/
(Accesed 2013-05-20)