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The Risk-Free Rate's Impact on Stock Returns with Representative Fund Managers

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Abstract

In this thesis, the risk-free rate's impact on stock market excess returns was examined. Firstly, theoretical arguments were made for that a low risk-free rate might lower the excess return on the stock market, since this increases the incentive for fund managers to increase variance of returns. Under the assumption that fund managers affect the preferences of the representative investor, propositions regarding stock returns and the risk-free rate were made. Using the time series of stochastic volatility risk premium estimates created by Bollerslev, Gibson and Zhou's (2011), it was tested if investor risk aversion is lower when the risk-free rate is low. The risk-free rate's impact on the cross-section of stock returns was tested through the same methodology used by Black, Jensen, Scholes (1972) with independent variables linked to the risk-free rate added. Support for lower risk aversion during periods of a low risk-free rate was found. In opposite to the proposition regarding the cross-section of stock returns, the tests suggest that excess returns for all portfolios are higher when the risk-free rate is low.

Keywords:

Asset Pricing, Agency Problem, Fund Managers, Risk-Free Rate, Risk Aversion,
Stock Returns

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1. Introduction

The fund management industry has grown sharply since the 1970s and this development has increased their fraction of the stock market activity dramatically. This has resulted in an increased possibility for diversification and access to new markets for households, but not without introducing other problems. Rajan (2005) points out that discretionary investment management creates an agency problem since the fund has an incentive to maximize its profits rather than the risk-adjusted returns. This may result in excessive risk taking, the funds can increase their expected compensation through increasing the variance of their returns. Rajan also points out that the fund industry's increased fraction of the market should be reflected in asset prices. Furthermore, he claims that the incentive to increase variance is greater when the risk-free rate is low. However, there is no thorough theoretical discussion or empirical evidence for the latter statement in his article.

In the aftermath of the financial crisis, the weak state of most developed economies combined with unprecedented expansionary monetary policy has resulted in a long period with low levels of the risk-free interest rate (risk-free rate). Therefore, the problem described is highly relevant for the current state of the financial market. If the agency problem between fund investors and fund managers is aggravated when the risk-free rate is low, the investors are obviously affected. Moreover, monetary policy makers ought to be aware of the effect that monetary stimulus has on the financial market. Finally, corporations should be interested in the risk-free rate's impact on their cost of capital in order to plan their financing activities.

Consequently, it should be of great interest to test Rajan's reasoning empirically and that is the aim of this thesis. More specifically, we will examine how the risk-free rate affects stock market returns and focus on the impact of a low risk-free rate. In section 2, we will review asset pricing and agency theory to establish a foundation for propositions regarding how the risk-free rate affects the behavior of stock market returns, assuming that fund managers' preferences affect stock returns. The propositions are deduced in section 3 and regards investor risk aversion, systematic risk and idiosyncratic risk. In section 4, the proposition regarding risk aversion will be tested by using a time series of risk aversion estimates created by Bollerslev, Gibson and Zhou's (2011) (henceforth BGZ) and examining the explanatory power of variables linked to the risk-free rate. In section 5 we test the proposition regarding systematic risk by following Black, Jensen and Scholes (1972) (henceforth BJS) and adding independent variables linked to the risk-free rate. The proposition regarding idiosyncratic risk will be left untested. In section 6 we will conclude our findings and discuss how they relate to the participants mentioned above.

2. Theory and Empirical Evidence

2.1 Background on the Capital Asset Pricing Model

Markowitz (1952) is the antecessor of modern portfolio theory. Markowitz states that an investor will gain expected return by taking on more variance and consequently a reduction of variance lower the expected return of the portfolio. He rejects the law of large numbers, unrealistically stating that diversification can be achieved without giving up any expected return. The reason why the law of large numbers cannot be applied to stock returns is that they are too intercorrelated. Hence, the rule that an investor should maximize discounted expected return must also be dismissed. Markowitz presents the expected return–variance of returns (E-V) rule. The rule simply remarks that expected return is desirable and variance is not. Markowitz points out that when following the (E-V) rule, investors must diversify among different industries to avoid covariance between stocks in the same industry. Furthermore, the investor must also be aware of the covariance between securities from different industries (Markowitz, 1952).

In the genesis of the Capital Asset Pricing Model (CAPM), Sharpe (1964) and Lintner (1965) added two main assumptions to Markowitz portfolio model, based on the mean-variance choice. The first of the new assumptions is that there is complete agreement among investors concerning the joint distribution of asset returns. The second assumption is that all investors can borrow and lend money at the risk-free rate (Fama & French, 2004). The first assumption implies that the market portfolio must be the mean-variance efficient portfolio, since all investors will hold the same portfolio. The second assumption determines the return on a zero-beta security, which investors will combine with the market portfolio to adjust the risk of the total portfolio. However, risk-free borrowing and lending is an assumption that is more or less unrealistic empirically (Fama & French, 2004). Therefore, Black (1972) developed a CAPM model similar to the predecessor, except the assumption of risk-free borrowing and lending. He showed that variance-efficiency could instead be obtained by allowing unrestricted short sales of risky assets. Fama and French (2004) emphasize that the CAPM is based on unrealistic assumptions both in the elder and newer versions. However, the model still provides insight into asset pricing and its imperfections gives academics a reason to search for improvements.

2.2 Modern Asset Pricing Theory

In this section we present asset pricing theory based on Cochrane (2000). Cochrane focus on consumption-based asset pricing. He points out that although the CAPM was formulated before the consumption-based model, the former model is a specialization of the latter. The fundament behind consumption-based asset pricing is that the expected discounted payoff should determine the price of an asset. The discount factor should correspond to the level where the marginal utility loss of consuming less today equals the marginal utility gain of the expected proceeds in the future. This can be expressed as:

(1) $$p_t(x_{t+1} \, \mathrm{I} \, \Omega) = \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right)$$

Where $(x_{t+1} \mathrm{I} \, \Omega)_t$ represents the expected payoff *x* at time *t+1* given the information set Ω at time *t*. Throughout the rest of this thesis we will denote this as $E(x_{t+1})_t$. β is an impatience coefficient and $u'(c)$ is the marginal utility of consumption. The discount factor is represented by *m*:

(2) $$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

The risk correction component of the discount factor should depend on the asset's covariance with marginal utility of consumption. Since the utility of consumption is a decreasing function, marginal utility is higher when an individual's level of consumption is low and vice versa. Consequently, an individual will prefer an asset with a high payoff in states where overall consumption of the individual is low. Thus, the required excess return of an asset should equal:

(3) $$E_t(R^i) - R^f = \frac{cov(u'(c_{t+1}), R^i_{t+1})}{E(u'(c_{t+1}))}$$

Where R^i represents 1 plus the return of stock i and $E(R^i m) = 1$. Moreover, R^f represent 1 plus the risk-free rate. Throughout this thesis $r^i = R^i - 1$. As stated in equation 3, the only variance that should contribute to the risk premium is the covariance with marginal utility of consumption and this is referred to as systematic risk. Variance uncorrelated with the marginal utility of consumption is referred to as idiosyncratic risk and should not receive any risk compensation, because only covariance adds variance to an investor's consumption for marginal purchases of an asset. This is shown below where ξ is the purchased amount of an asset with payoff x. For small changes in ξ, the changes in ξ^2 become very small and the last component of the equation becomes negligible.

(4) $$\sigma^2(c + \xi x) = \sigma^2(c) + 2\xi cov(c,x) + \xi^2\sigma^2(x)$$

If the utility of consumption is assumed to follow a power utility function of $u'(c) = c^{-\gamma}$ then equation 2 can be written as:

(5) $$m = \beta(\frac{c_{t+1}}{c_t})^{-\gamma}$$

Where γ is risk aversion. Then the expected return can be expressed as a function of its regression coefficient for changes in consumption ($\beta_{i,\Delta c}$):

(6) $$E(R^{i)} = R^f + \beta_{i,\Delta c}\lambda_{\Delta c}$$

The factor risk premium $\lambda_{\Delta c}$ can be expressed as:

(7) $$\lambda_{\Delta c} = \gamma var(\Delta c)$$

Where γ is the risk aversion and $var(\Delta c)$ is the variance of consumption growth. However, the consumption-based model has not performed well in explaining empirical returns. One possible explanation for this is flawed consumption data. Therefore, various pricing models have been developed that use a factor to represent marginal utility growth of consumption. The most famous factor-pricing model is the CAPM. The CAPM uses total wealth as a proxy for consumption. Hence, expected return of an asset is determined by the risk-free rate, the return on total wealth and the asset's regression coefficient for total wealth.

The latter is known as the the beta value. Moreover, the market portfolio is used as a proxy for total wealth. As a result:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx R^f + (R^m - R^f)\beta^i \quad (8)$$

Equation 8 will be used to test the risk-free rate's impact on the cross-section of stock returns. Note that β^i is the regression coefficient from equation 6 and β is the impatience coefficient from equation 1. Using returns on the market portfolio as a proxy for changes in consumption, equation 8 can be expressed as a function of risk aversion and variance of the market portfolio returns:

$$R^m - R^f \approx \gamma var(R^m) \quad (9)$$

2.3 The Representative Investor

Cochrane (2000) shows that the law of one price and absence of arbitrage is sufficient to create an equilibrium for stock market returns. However, it is common in the asset pricing research to assume that there is a representative investor. This idea was introduced by Rubinstein (1974) who addressed the question of finding equilibrium returns when investors' resources, beliefs, and preferences differ. Rubinstein shows that under certain assumptions, a composite of the investors' different resources, beliefs, and preferences can be created and that the composite of those variables can be linked to exogenous parameters of the economy. Therefore, equilibrium security returns can be determined as if the market did consist of individuals with identical resources, beliefs, and preferences. We will not describe the assumptions required to prove the algebraic relation. For this thesis, the notion of a representative is sufficient to discuss how the difference between fund managers' and individual investors' preferences might affect stock returns. Rubinstein attributes the following characteristics to the representative investor (Rubinstein, 1974, page 234):

1. "His initial wealth, optimal present consumption, and optimal future wealth for every state are arithmetic averages of their corresponding aggregate values"
2. "Any homogeneous economic characteristic shared by all actual individuals is also an economic characteristic of the composite individual"

3. "His beliefs for each state are a function of the beliefs of all actual individuals for the corresponding state and at most also depend on the tastes of all actual individuals"
4. "His rate of patience and taste parameters are, respectively, a function of the rates of patience and taste parameters of all actual individuals and at most also depend on the beliefs of all actual individuals"
5. "Equilibrium rates of return are determined as if there exist only composite individuals"

Of these characteristics, number 4 will be useful in the rest of this thesis.

2.4 Stochastic Volatility, a Tool to Estimate Risk Aversion

From equation 6 and equation 7 it is evident that the factor risk premium for asset returns depends on risk aversion and the variance of the factor. However, the variance of the factor is not necessarily constant. For example, under the CAPM framework one should also be concerned with the volatility of the market portfolio's variance (see equation 8 and equation 9).

Since the variance of returns is stochastic, this adds another dimension of risk, in effect, uncertainty regarding the variance of returns (Carr & Wu, 2009). Various derivatives can be used to trade the risk associated with stochastic variance. Consequently, an investor can take a position in variance and empirical studies have concluded that the stochastic risk premium is negative. This implies that investors are willing to pay a premium to hedge the exposure to variance risk in the stock market (Carr & Wu, 2009).

The stochastic risk premium can be used to derive an estimation of risk aversion. Assuming a linear volatility risk premium and an affine stochastic volatility model result in the following relationship (BGZ):

(10)
$$-\pi var_t = cov_t\left(\frac{du'(w_t)}{u'(w_t)}, dvar_t\right)$$

Where π represent the stochastic volatility risk premium, $u'(w_t)$ represent marginal utility of wealth and therefore $du'(w_t)$ represent changes in marginal utility of wealth. Assume that the representative investor has the following power utility function of wealth (BGZ):

(11) $$U_t = e^{-\beta t}\frac{W_t^{1-\gamma}}{1-\gamma}$$

Where β represents the impatience coefficient. Moreover, if the representative investor holds the market portfolio in equilibrium, then marginal utility of wealth is (BGZ):

(12) $$u'(w_t) = e^{-\beta t}W_t^{-\gamma}$$

Moreover, BGZ states that it follows from Itô's formula that:

(13) $$cov_t\left(\frac{du'(w_t)}{u'(w_t)}, dvar_t\right) = -\gamma\rho\sigma var_t$$

Where σ is the volatility of the variance and ρ is the covariance between the Brownian motion processes for stock prices and the volatility of stock returns. Combining equation 10 and equation 13 yields:

(14) $$-\pi var_t = -\gamma\rho\sigma var_t$$

Hence:

(15) $$\gamma = \frac{\pi}{\rho\sigma}$$

Moreover, BGZ's analysis of S&P500 data finds that $\rho = -0.8$ and $\sigma = 1.2$. In effect:

(16) $$\gamma \approx -\pi.$$

BGZ use this relationship to create a time series of risk aversion estimations through GMM estimation. They use model-free volatility measures to calculate risk-neutral implied volatility from options on NYSE firms. Moreover, five-minute returns on the S&P500 index are used to calculate realized volatility on the stock market. With these inputs, BGZ created a time series of the risk premium for stochastic volatility in S&P500 options through GMM estimation. This is used as a time series for risk aversion estimates.

2.5 Excess Return of the Market Portfolio and the Risk-Free Rate

In order to test the risk-free rate's affect on the cross-section of stock returns, we will use the CAPM. Hence, we want to state how the risk-free rate is expected to affect excess return. In equation 8, the excess return is formulated as $(R^m - R^f)$. The simplest way to calculate expected excess return on the market portfolio at a given point in time would be to use the historical average of R^m and subtract the current level of the risk-free rate (Merton, 1980), Then the risk-free rate's impact on a given stock's returns would be:

$$\frac{\Delta Excess\ return}{\Delta R^f} = -\beta \tag{17}$$

This follows from:

$$Excess\ return = (R^m - R^f)\beta \tag{18}$$

This assumes that R^m is independent of the risk-free rate. However, Merton (1980) points out two flaws in this approach. Firstly, if this was the case, the factor risk premium could become negative during high levels of the risk-free rate. Under this scenario investors would have to pay a premium to take on risk and this does not make sense unless investors are risk seeking. Secondly, the required return of the market portfolio ought to depend on the level of inflation. Since the risk-free rate takes inflation into account, so does the return on the market portfolio if it depends on the level of the risk-free rate. Hence, the required return on the market portfolio ought to depend on the risk-free rate and therefore the excess return ought to be independent of the risk-free rate.

2.6 Empirical Tests of the CAPM

Fama and French (2004) criticize the CAPM's importance because of its poor empirical record. The CAPM's empirical failure may be a result of the simplifying assumptions mentioned above, which causes difficulties when being implemented in reality. Earlier research of the CAPM test three aspects; (1) If beta is the only variable with explanatory power and if stock returns are linearly related to their betas. (2) If the expected return of the market portfolio exceeds the expected return of zero beta assets. (3) If zero-beta assets' expected returns are equal to the risk-free rate and if the beta premium equals the expected market return minus the risk-free rate.

Fama and French (2004) remark that the relationship between beta and the average returns is too flat in the earlier tests of the CAPM. In effect, the return on a zero-beta security is not the risk-free rate and the beta risk premium is lower than the excess return of the market portfolio. The earlier tests supported the claim that the market portfolio is mean-variance efficient and that the beta premium is positive.

However, recent tests of the CAPM and the Black version of CAPM provides evidence that the beta is not the only factor that explain stock returns (Fama & French, 2004). Using a cross-sectional regression, Fama and French (1992) finds evidence for that the following variables contribute to stock returns: firm size, the earnings-to-price ratio, the debt-to-equity ratio and the book-to-market ratio.

Some researchers go as far as saying that the CAPM has never been tested. Roll (1977) argues that the market portfolio is empirically and theoretically inadequate, because it is not clear which assets that can be excluded. As a result the CAPM use proxies and not the true market portfolio.

2.7 The Security Market Line and Betting Against Beta

One of the earlier tests of the CAPM was BJS's article. They carry out empirical tests on all securities listed on the New York Stock exchange between 1926 and 1966 to examine whether or not the returns are consistent with the CAPM. BJS divide stocks into 10 different portfolios based on their beta values. They then regress the excess returns of the portfolios with the excess return on the market portfolio as the independent variable. Following from equation 8, the regression used is:

$$r_t^j - r_t^f = \alpha^j + \beta^j(r_t^m - r_t^f) + e_t^j \qquad (19)$$

Where $(r_t^m - r_t^f)$ is the excess return on the market portfolio, e^j is assumed to be an independent and normally distributed random variable and the intercept α^j is expected to be zero if the CAPM holds. Consistent with the CAPM, they find a linear relationship between excess returns and beta values. However, for the high-beta portfolios the intercept was negative and for the low-beta portfolios the intercept was positive. Moreover, the intercept and slope of the portfolios varies over time. These findings are inconsistent with the CAPM and the BJS's explanation is the unrealistic assumption mentioned above that investors can borrow and lend at the risk-free rate. Since this assumption does not hold in practice, investors use more risky assets to substitute leverage and therefore stocks with high (low) beta values have lower (higher) required return than predicted by the CAPM.

Frazzini and Pedersen (2010) point out that "many mutual fund families offer balanced funds where the "normal" fund may invest 40% in long-term bonds and 60% in stocks, whereas as the "aggressive" fund invests 10% in bonds and 90% in stocks" (page. 2). Hence, if these funds want to increase their risk, one way is to reallocate their investments to assets with high beta values. In line with BJS, they argue that restricted borrowing flattens the security market line. Moreover, they augment BJS's findings further by constructing a portfolio with a long position in low beta assets and a short position in high beta assets. Furthermore, they use a risk-free asset to adjust the portfolio's overall beta to zero. They find that this betting-against-beta portfolio generates Fama-French abnormal returns of 0.69 percent per month with a t-statistic of 6.55 for U.S. stocks.

2.8 Limits of arbitrage

In the financial theory, arbitrage is generally described as an event where an investor makes money without taking on risk and without any initial capital. Simply explained, the arbitrageur makes an almost simultaneous purchase and sale of the same or essentially the same security on two different markets with a surplus as result.

Shleifer and Wishny (1997) points out that this view is not confirmed by professional investors, which states that an arbitrageur is both risky and requires capital. This may also be the reason why the number of professional arbitrageurs is very limited, and why they usually use other people's money to make their investments. Numerous academics argue that the existence of arbitrageurs in financial markets is favorable because they minimize possible mispricing of listed securities.

However, Shleifer and Vishny argue that arbitrageurs' ability to bring efficiency to security markets is limited. They describe a situation when the arbitrageurs are constrained because the investors are withdrawing their money from the fund, due to recent failures in capturing expected returns for arbitrage. These withdrawals tend to occur when the arbitrageur is in the most crucial need of capital to regain control of their position and to not lose more money (Shleifer & Vishny, 1997).

The constrained flexibility to recover their positions is therefore limiting their role as creators of market efficiency. This failure is particularly prominent in extreme circumstances, when prices are far out of line and arbitrageurs are fully invested. They are therefore unable to keep their position and have to exit the market. However, because of this kind of fire sales security prices may diverge even more from their fundamental prices (Shleifer & Vishny, 1997).

2.9 Jensen's Alpha and the Sharpe-Ratio

2.9.1 Jensen's Alpha

Until Jensen (1967) presented his alpha constant, later known as Jensen's alpha, financial theories lacked tools to measure the performance of risky investments. Jensen expressed two major dimensions of portfolio performance. First, he considered fund managers' ability to outperform the market by successfully predicting future security prices. Secondly, Jensen considered the fund managers' ability to minimize risk, through diversified allocation of investments. Hence, Jensen was interested in the manager's individual skill relative to the market. To clarify, the word performance in Jensen's findings is used to embrace a fund manager's ability to forecast future movements in the market and not the mean-variance efficiency of the fund's portfolio. Jensen uses the same regression as BJS (equation 19) but applied it to fund returns instead of stock returns (however, Jensen's alpha was developed before BJS's article).

$$r_t^j - r_t^f = \alpha^j + \beta^j (r_t^m - r_t^f) + e_t^j \tag{20}$$

The α_j is supposed to capture any effect of skill. If a fund manager consistently beats the market, the excess returns will be higher than the risk premium for systematic risk and the intercept will therefore be positive. The error term e_t^j is expected to be zero and independent.

2.9.2 The Sharpe ratio

Sharpe (1966) tested the relationship between volatility and return on 34 mutual funds. He developed a ratio, later known as the Sharpe ratio, which measures the excess return relative to the risk exposure for a given portfolio. This ratio can be used to compare the performance of different funds. The original Sharpe ratio was formulated as:

$$Sharpe\ ratio = \frac{E(r^i - r^f)}{\sigma(r^i)} \tag{21}$$

Where r_f is the risk-free rate and r_i the return of the portfolio in consideration.

2.10 Fund manager's Historical Performance

Fama and French (2010) finds evidence that the fund industry on average performed in line with the market portfolio before expenses, for the period 1984 to 2006. Hence, the funds underperformed the market portfolio by approximately the managing expense. Fama and French continue to discuss if differences in fund performance is attributed to luck or skill. They find that skill is most likely not the prominent factor and concludes that it is often the result of other factors. They also mention that fund managers delivered better results when the number of funds, and consequently the competition, was low. Another explanation given by Fama and French is that a successful fund manager often enrolls other positions with greater benefits. However, Cuthbertson et al. (2008) find strong evidence that previous poor performers will continue to underperform. Blake and Timmermann (1998) points out that the research has a survivorship bias because some funds "die", which is often believed to be the bad performing funds. However, they find that 89 percent of the UK funds graded as "dead" were merged with other funds and superior performance is as likely to be the reason for the merger. In effect, only 11 percent of the "dead" funds were closed down.

French (2008) studies how much investors pay in fees for the prospect of beating the market, by investing in actively managed funds instead of investing in the market portfolio. The market portfolio is according to the CAPM the most efficient investment strategy when assuming market efficiency to be justifiable (Sharpe, 1964). However, French (2008) makes it simpler and probably more realistic by looking at passively managed mutual funds instead of the market portfolio. Further, French shows that the average difference between an active and a passive investment strategy was 0.67 percent annually, from 1980 to 2006. This is significantly lower then the extra fee an investor pays for actively managed funds.

2.11 Agency Problems and Asset Pricing

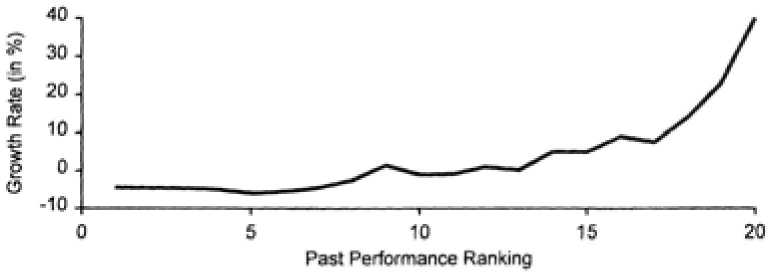
Jensen and Meckling (1976) define an agency relationship as a contract under which a principal engage an agent to perform some service on their behalf, which implies decision-making authority. The agency theory puts considerations to the issues that occur when an agent's actions is not fully aligned with the principal's interest. The principal can mitigate the divergences of interest by suitable incentive programs to the agent. The cost of the agency problem is defined as the sum of; monitoring expenditure, bonding expense by the agent and the residual loss (Jensen & Meckling, 1976).

Chevalier and Ellison (1997) points out that there is an agency problem between investors in mutual funds (principals) and the mutual fund managers (agents). The firm that operates the mutual fund wants to maximize its profits and the fund managers act on behalf of the firm. The investor in the mutual fund wants to maximize their risk-adjusted return. When these two objectives do not coincide there is an agency problem since investors cannot directly observe the information that the fund managers possess and how it is used. Hence, it is difficult for the investors to monitor whether or not the fund managers are striving to maximize risk-adjusted return or if they pursue their own interests. Chevalier and Ellison only discuss the agency problem for mutual funds, but we argue that the same underlying problem exists in all sorts of investment management.

Chevalier and Ellison also point out that mutual fund companies are compensated based on a percentage of assets under management and that this gives them an incentive to maximize the size of the fund. Moreover, they find a somewhat convex relationship between inflow to funds and year-end portfolio returns subtracted by the market return. As a result, fund managers have an incentive to increase the variance of returns since this will maximize their expected compensation, provided that these benefits are greater than the expected cost of bad reputation by producing lower risk-adjusted returns. Chevalier and Ellison argue that this incentive is the greatest for funds that towards the end of the year is trailing behind its peers in terms of return. In an empirical study they find that total standard deviation and the unsystematic risk of funds with this incentive structure increase between September and December. However, they do not find statistical significant results for an increase in systematic risk.

Sirri and Tufano (1998) find a more distinct convex relationship between fund inflows and past performance. However, they use a ranking procedure, where the funds' return are compared to the other funds in the sample, as the proxy for fund performance. Figure 1, provides the relationship between fund inflow and past performance in their sample. Sirri and Tufano also perform a piecewise regression and find support for the convex relationship. When performing the same piecewise regression with past raw returns they find similar results, even when including Jensen's alpha as an explanatory variable.

Figure 1 – Fund Inflows and Past Performance



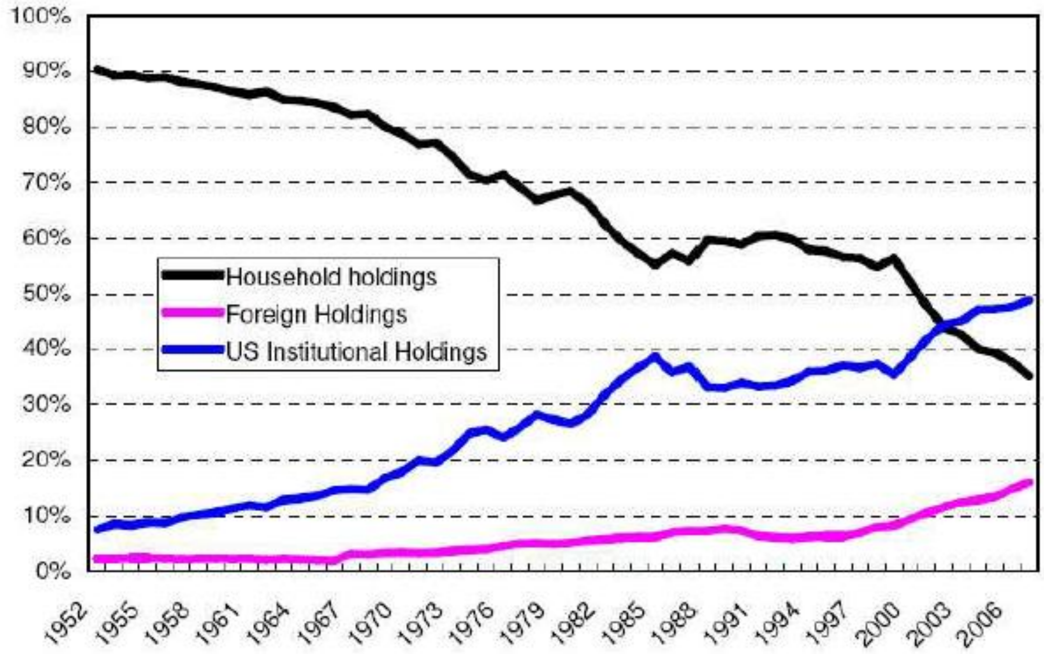
Source: Sirri & Tufano (1998), with permission from the authors.

Moreover, Agarwal, Daniel and Naik (2004) use a method similar to Sirri and Tufano's (1998) and find that the relationship between performance and inflows are convex for hedge funds as well.

From section 2.10, one can conclude that the results ought to be similar when using fund returns compared to market returns and fund returns compared to the fund industry since the fund industry's return before cost on average follows the market return. Chevalier and Ellison (1997) argue that fund investors focus on past realized returns instead of the Sharpe-ratio when choosing which funds to invest in and that this explains the convexity. Sirri and Tufano (1998) point out that investors evaluate the performance compared to the other funds, therefore the return of a fund relative to its competitors ought to determine fund inflows. To summarize, there is empirical support for that fund managers face a convex shaped relationship between their performance and fund inflows. This leads to an incentive for fund managers to increase the variance of their returns in order to maximize their expected compensation.

Brennan and Li (2008) links the agency problems of fund manager to asset pricing and points out that the assumption of utility maximizing individuals (as expressed in equation 1) may not be valid anymore. The fraction of stocks held by households decreased from approximately 85 percent in the mid-1960's to approximately 35 percent in 2008. Meanwhile, the fraction of stocks held by funds has increased dramatically. The fraction of domestic institutions did account for 49 percent in 2008 (Brennan and Li, 2008). This development is illustrated in figure 2.

Figure 2 – US Common Stock Holdings



Source: Brennan & Li (2008), with permission from the authors.

As discussed above, fund managers' preferences for securities may differ from individual investors due to agency problems. Therefore, Brennan and Li (2008) argue that the demand for securities ought to diverge from traditional asset pricing models such as the CAPM when fund managers become more dominant in the market. Provided that individual investors cannot exploit this fully, as in section 2.8, the changes in demand will be reflected in security prices.

3. Propositions for Asset Pricing

So far, we have presented research that indicates a convex relationship between recent fund returns and fund inflows. Hence, the benefit for funds that outperform their peers to a certain degree is greater than the cost for those funds that trail behind by the same degree. This leads to an incentive for fund managers to increase the variance of their returns. In effect, they ought to take on more risk than individual investors prefer since it is hard to monitor the information the fund possess and how it is used. We have also presented arguments for that fund managers' preferences should be reflected in stock prices because of their increasingly large share of the market.

In this section we will develop theoretical arguments that builds on the theory presented above to show that the incentive to increase variance is greater when the risk-free rate is low. We will also suggest various ways in which fund managers can increase variance. Lastly, from these two discussions, three propositions will be deduced regarding the return on the stock market.

In order to develop the propositions, a few assumptions are necessary; (1) A fund's compensation is proportional to its assets under management. (2) There is a convex relationship between fund inflows and recent returns relative to other funds. (3) Fund managers have to decrease the Sharpe-ratio of their portfolio in order to increase variance. (4) To some point, the increased expected compensation from increasing variance exceeds the expected reputation costs the fund may incur from a low Sharpe-ratio. (5) Fund managers' preferences do impact the preferences of the representative investor.

The purpose of assumption 1 and 2 is to establish the convex relationship between fund compensation and its returns, which creates an incentive to increase variance as discussed above. From assumption 3 and 4, it follows that fund managers should act on the incentive to increase variance, deviating from their investors' preferences. From assumption 5, it follows that fund managers' preferences impact the equilibrium returns.

Assumption 1 is borrowed from Chevalier and Ellison (1997), and since many firms have performance based compensation systems it is not true. However, due to its option like nature, performance based compensation is likely to increase this convexity and since all funds do not have performance based compensation, the effect is excluded from the analysis. Assumption 2 derives from the empirical findings presented in section 2.11.

Assumption 3 follows from Frazzini and Pedersen's (2010) observation that most fund managers are restricted from using leverage. Assumption 4 derives from the arguments presented in section 2.11 that investors primarily focus on return instead of the Sharpe-ratio. Assumption 5 derives from Brennan and Lis' (2008) discussion. Under assumption 1-5, the market does not consist of individuals that maximize their utility of future consumption. Instead, the utility maximizing individuals are mixed with profit maximizing fund managers with an incentive to increase variance.

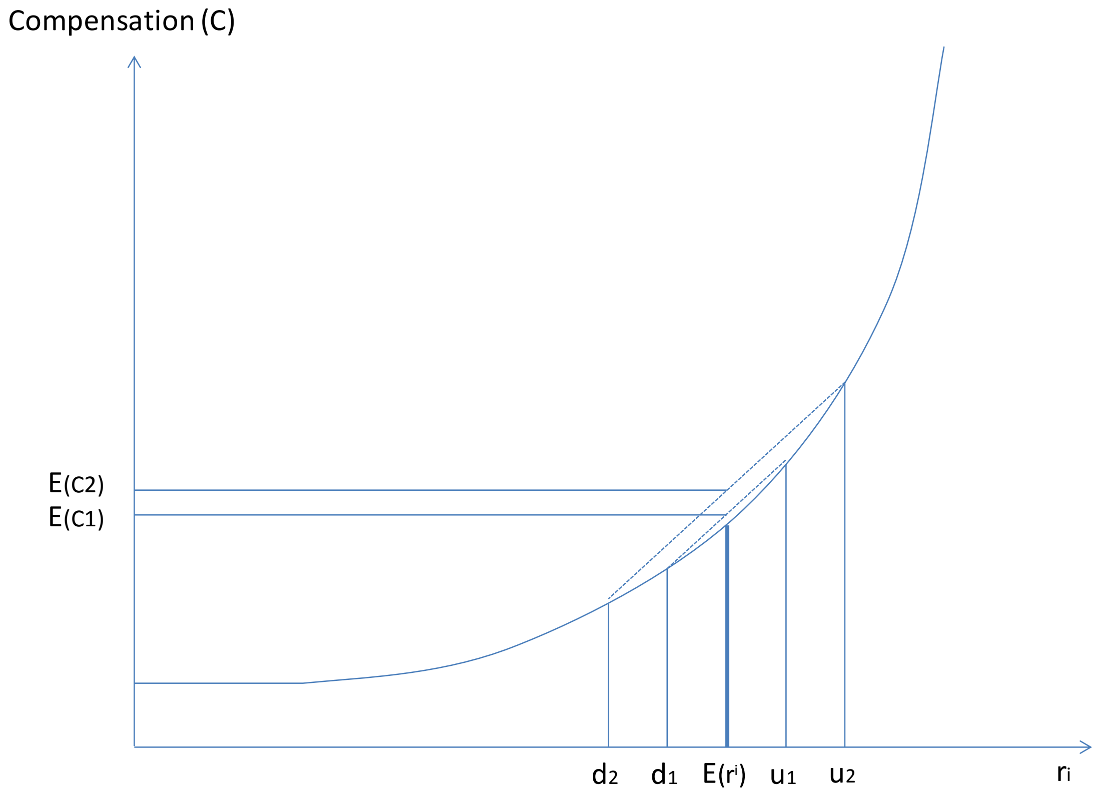
3.1 Fund Manager Incentive and the Risk-Free Rate

Next, we examine the incentive to increase variance further and how a low risk-free rate affects this incentive. This is demonstrated by a simplified example:

- Consider a fund manager that can choose between two portfolios in time *t*.
- The portfolio returns are divided into a risk-free component (r_{t+1}^{f}) and a risk premium (r_{t+1}^{r}). In effect, the portfolio returns are represented by:
  $r_{t+1}^{i} = r_{t+1}^{f} + r_{t+1}^{r}$
- In the short-run, r^{f} is assumed to be constant. In effect, $r_{t}^{f} = r_{t+1}^{f}$
- In *t+1*, r_{t+1}^{r} only have two possible outcomes for each portfolio, one *up* state (r_{t+1}^{ru}) and one *down* state (r_{t+1}^{rd}), the two states are equally probable. We will now drop the time-script notation and let the subscript represent the portfolio.
- For Portfolio 1 (P_1), $r_1^{ru} = 8\%$ and $r_1^{rd} = 4\%$. This result in expected risk premium and variance of: $E(r_1^{r}) = 6\%$, $\sigma^2(r_1^{r}) = 4\%$
- For Portfolio 2 (P_2), $r_2^{ru} = 10\%$ and $r_2^{rd} = 2\%$. This result in expected risk premium and variance of: $E(r_2^{r}) = 6\%$, $\sigma^2(r_2^{r}) = 16\%$
- The relationship between the fund's compensation and r^{i} is convex as established by assumption 1 and 2 and illustrated in figure 3.
- Consider two states of risk-free rate: $r^{f} = 2\%$ and $r^{f} = 10\%$

The mean-variance structure of the portfolios is established by assumption 3, in order to increase variance, the fund manager has to decrease the return per unit of variance. However, our example is somewhat extreme where the fund manager cannot increase return at all by increasing variance, the purpose of this is to simplify and make the example more distinct. The fund's expected compensation for the two different portfolios is illustrated in figure 3.

Figure 3 – Fund Compensation and Portfolio Return



Due to the convexity of the slope, expected compensation is greater for portfolio 2 since it has a greater dispersion around its mean. In effect, portfolio 2 is the most attractive choice for the fund manager. For the risk-averse investor, portfolio 1 is obviously more appealing than portfolio 2 since the portfolios have the same expected return and portfolio 1 has lower variance. The return of the two portfolios during the two states of the risk-free rate is calculated in table 1:

Table 1.

Example Illustrating Fund Manager Incentives

	Portfolio	
	$\boldsymbol{P_1}$	$\boldsymbol{P_2}$
$Rf = 2\%$	$2\% + \begin{Bmatrix} 8\% = 10\%\ if\ up \\ 4\% = 6\%\ if\ down \end{Bmatrix}$	$2\% + \begin{Bmatrix} 10\% = 12\%\ if\ up \\ 2\% = 4\%\ if\ down \end{Bmatrix}$
$Rf = 10\%$	$10\% + \begin{Bmatrix} 8\% = 18\%\ if\ up \\ 4\% = 14\%\ if\ down \end{Bmatrix}$	$10\% + \begin{Bmatrix} 10\% = 20\%\ if\ up \\ 2\% = 12\%\ if\ down \end{Bmatrix}$

The distance between the expected return and the *up*/*down* state are presented in table 2 as the bolded number within the bracket:

Table 2.

Example Illustrating Fund Manager Incentives

	Portfolio	
	$\boldsymbol{P_1}$	$\boldsymbol{P_2}$
$Rf = 2\%$	$\mu(r_1^i) = 8\%, r_1^i = 8\% * (1 \pm \mathbf{0.25})$	$\mu(r_2^i) = 8\%, r_2^i = 8\% * (1 \pm \mathbf{0.50})$
$Rf = 10\%$	$\mu(r_1^i) = 16\%, r_1^i = 16\% * (1 \pm \mathbf{0.125})$	$\mu(r_2^i) = 16\%, r_2^i = 16\% * (1 \pm \mathbf{0.25})$

From table 2, one can conclude that the dispersion around the expected return increase, and that the effect from choosing portfolio 2 is greater when the risk-free rate is low. This effect is also illustrated in figure 4 and figure 5.

Figure 4 – Fund Compensation and Portfolio Return, Low Risk-Free Rate

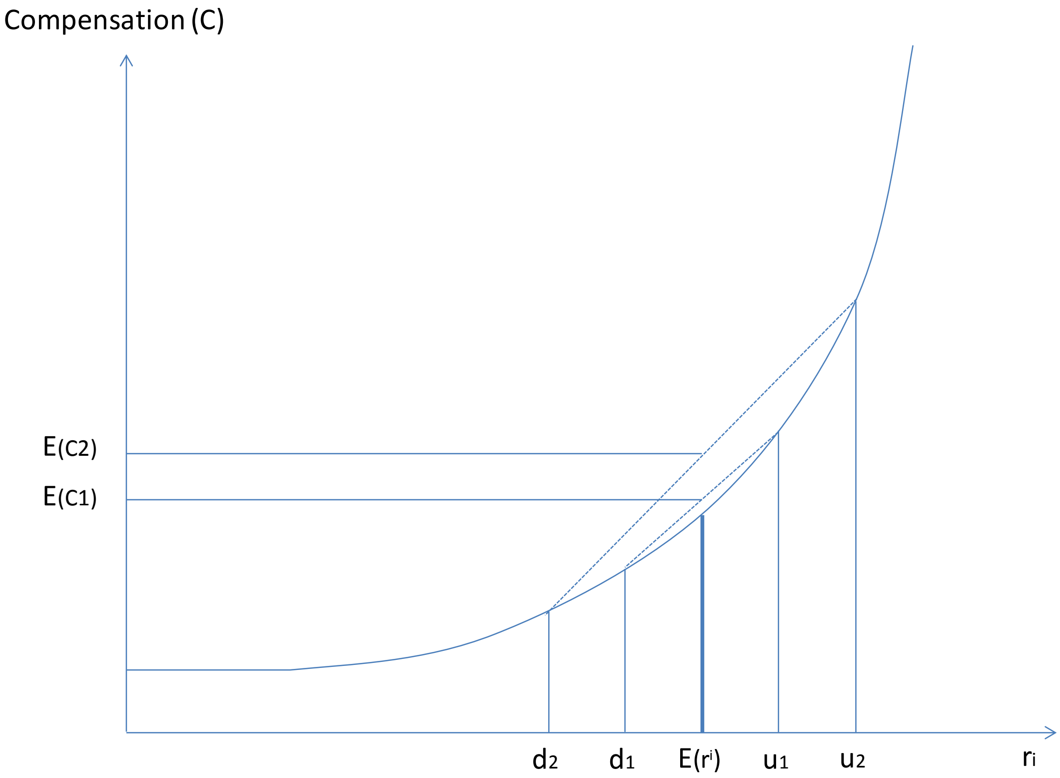
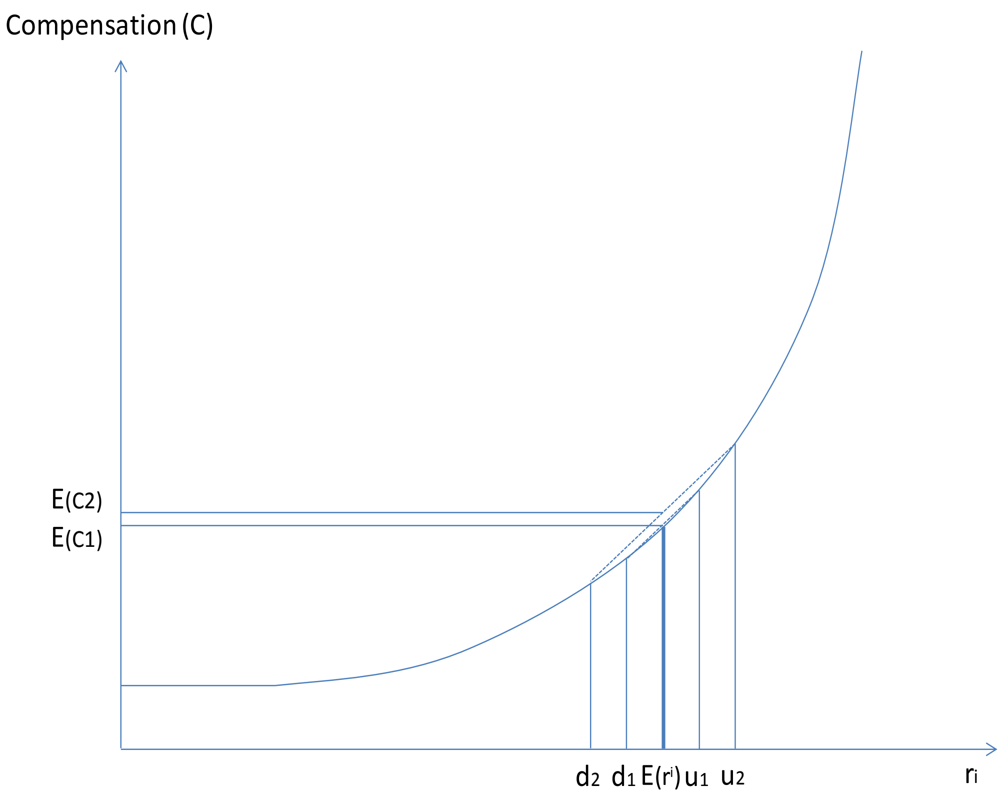


Figure 5 – Funds Compensation and Portfolio Return, High Risk-Free Rate



When studying figure 4 and figure 5 one might argue that when the risk-free rate is low, $E(r^i)$ should move to the left on the x-axis. That is true, but remember that the relationship between fund inflows and returns depend on the performance relative to the rest of the fund industry, as argued by Sirri and Tufano (1998). Therefore, the graphed relationship should also shift to the left. In order to make the results more comparable, the scale of the x-axis is adjusted to keep $E(r^i)$ in the same position.

The difference in expected compensation $E(C)$ between the two portfolios is clearly greater when the risk-free rate is low. Hence, we conclude that the benefit from increasing variance is greater for the fund when the risk-free rate is low. The Sharpe-ratio from section 2.9.2 is presented in table 3 for both portfolios in both states.

Table 3.

Example Illustrating Fund Manager Incentives

	Portfolio	
	$\boldsymbol{P_1}$	$\boldsymbol{P_2}$
$Rf = 2\%$	$\frac{8\% - 2\%}{\sqrt{4\%}} = 30\%$	$\frac{8\% - 2\%}{\sqrt{16\%}} = 15\%$
$Rf = 10\%$	$\frac{16\% - 10\%}{\sqrt{4\%}} = 30\%$	$\frac{16\% - 10\%}{\sqrt{16\%}} = 15\%$

The Sharpe-ratio is the same for both portfolios regardless of the risk-free rate. Therefore, the adverse effect for the investor (or residual cost as in section 2.11) is the same for both states of the risk-free rate when the fund manager choose portfolio 2. As established by assumption 4, the fund manager can decrease the Sharpe-ratio to some extent before the expected reputation cost from a lower Sharpe-ratio exceeds the benefit from increased expected compensation. The benefit will exceed the costs to a greater extent when the risk-free rate is low, since a marginal increase in variance will contribute more to expected compensation. Therefore, the incentive for fund managers is greater when the risk-free rate is low.

One insight from the numerical example is that given assumption 1-5, expected compensation increase with the dispersion of returns relative to the expected return. This can be represented by the coefficient of variation ($\sqrt{\sigma^2}/E(r^i)$), which we will use in order to generalize our theoretical argument. For the generalized theoretical argument we keep assumption 1-5 and add the following assumptions:

- The return of a portfolio is still characterized by: $r^i = r^f + r^r$
- Changes in the risk premium is independent of the risk-free rate: $\frac{\partial r^r}{\partial r^f} = 0$
- To increase variance, the fund manager has to decrease the Sharpe-ratio
- Expected compensation increase when the return's coefficient of variation increase: $\frac{\partial E(compensation)}{\partial(\sqrt{\sigma^2(r^i)}/E(r^i))} > 0$
- Expected reputation cost increase when the Sharpe-ratio decreases. To make the comparison with expected compensation more intuitive, we invert the Sharpe-ratio: $\frac{\partial E(reputation\ cost)}{\partial(\sqrt{\sigma^2(r^r)}/E(r^r))} > 0$
- In the short-run, r^f is assumed to be constant. Moreover, r^f can be either *high* or *low*

Since the risk-free rate is constant in the short-run:

(22) $$\sigma^2(r^i) = \sigma^2(r^r)$$

Portfolio returns are higher when the risk-free rate is high. Hence, for a given level of variance, the increase of the coefficient of variance for a marginal increase of variance must be greater when the risk-free rate is low:

(23) $$\left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^i))}{\partial\sigma^2(r^r)}\right]_{low\ r^f} > \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^i))}{\partial\sigma^2(r^r)}\right]_{high\ r^f}$$

Since the risk premium is independent of the risk-free rate, the inverted Sharpe-ratio must be independent of the risk-free rate:

(24) $$\left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^r))}{\partial\sigma^2(r^r)}\right]_{low\ r^f} = \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^r))}{\partial\sigma^2(r^r)}\right]_{high\ r^f}$$

Therefore, the following relationship must hold:

$$(25) \qquad \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^i))}{\partial\sigma^2(r^r)}\right]_{low\ r^f} - \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^r))}{\partial\sigma^2(r^r)}\right]_{low\ r^f} > \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^i))}{\partial\sigma^2(r^r)}\right]_{high\ r^f} - \left[\frac{\partial(\sqrt{\sigma^2(r^r)}/E(r^r))}{\partial\sigma^2(r^r)}\right]_{high\ r^f}$$

Hence, the difference between the increase in expected compensation and the increase in expected reputation cost, for a marginal increase of variance, must be greater when the risk-free rate is low:

$$(26) \qquad \left[\frac{\partial E(comp)}{\partial\sigma^2(r^r)}\right]_{low\ r^f} - \left[\frac{\partial E(rep.cost)}{\partial\sigma^2(r^r)}\right]_{low\ r^f} > \left[\frac{\partial E(comp)}{\partial\sigma^2(r^r)}\right]_{high\ r^f} - \left[\frac{\partial E(rep.cost)}{\partial\sigma^2(r^r)}\right]_{high\ r^f}$$

If fund managers increase variance until:

$$(27) \qquad \left[\frac{\partial E(comp)}{\partial\sigma^2(r^r)}\right] = \left[\frac{\partial E(rep.cost)}{\partial\sigma^2(r^r)}\right]$$

Then the equilibrium level of variance will be greater for low levels of r^f.

3.2 Ways for Fund Managers to Increase Variance

To see how the incentive structure presented above might affect stock returns, the various ways in which fund managers can increase variance is examined. In this section we use equation 19 to describe the return of a fund's portfolio but move r^f to the right hand side of the equation and denote excess return on the market portfolio as λ^m:

$$(28) \qquad r_t^j = r_t^f + \alpha^j + \beta^j \lambda^m + e_t^j$$

In line with the discussion in section 2.5 we accept Merton's (1980) claim that the risk premium *ought* to be independent of the risk-free rate, but we would like to explicitly state that this assumes a utility maximizing individual as the representative investor. Moreover, we assume that fund managers cannot outperform the market, (in line with section 2.10) resulting in a α^j of 0 and we continue to assume that e^j is independent and normally distributed. Hence, the variance of returns can be expressed as:

$$\sigma^2(r^j) = \sigma^2(r^f) + {\beta^j}^2 \sigma^2(\lambda^m) + \sigma^2(e^j) \tag{29}$$

This expression is derived in Appendix 1.

To identify how fund managers can increase the variance of their portfolios and how it relates to the stock market, we divide the funds' investments into two portfolios; investments in the stock market and investments in other markets. Using equation 28 and assuming that α^j is zero, the total return of the two portfolios is:

$$r^{fund} = w^{sm}[r^f + \lambda^m \beta^{sm} + e^{sm}] + w^{om}[r^f + \lambda^m \beta^{om} + e^{om}] \tag{30}$$

Where the superscript *sm* refers to the *stock market* and the superscript *om* refers to *other markets*. w^{sm} refers to the fraction of the fund's total investments allocated to the stock market portfolio and w^{om} to the fraction invested in the other markets portfolio. β^{sm} is the beta value for the stock market portfolio and β^{om} is the beta value for the other markets portfolio. Markowitz (1952) shows that the variance of two securities combined (which also applies for two portfolios) is:

$$\sigma^2(r^j) = {w^x}^2 {\sigma^x}^2 + {w^y}^2 {\sigma^y}^2 + 2 w^x w^y \sigma^{x,y} \tag{31}$$

Where $\sigma^{x,y}$ is the covariance between the securities *x* and *y*. Substituting equation 29 into equation 31 the variance of a fund's portfolio is:

$$\sigma^2(r^{fund}) = {w^{sm}}^2 \left[{\beta^{sm}}^2 \sigma^2(\lambda^m) + \sigma^2(e^{sm}) \right] + {w^{om}}^2 \left[{\beta^{om}}^2 \sigma^2(\lambda^m) + \sigma^2(e^{om}) \right] + 2 w^{sm} w^{om} \sigma^{sm,om} \tag{32}$$

Where β^{sm} refers to the systematic risk of the stock market portfolio and β^{om} refers to the systematic risk of the other markets portfolio. $\sigma^2(e^{sm})$ refers to the idiosyncratic risk of the stock market portfolio and $\sigma^2(e^{om})$ refers to idiosyncratic risk of the other markets portfolio. $\sigma^{sm,om}$ refers to the covariance between stock market portfolio and the other markets portfolio. We impose the following restrictions on equation 32:

$$\sigma^2(e^{sm}) > \sigma^2(e^{om}), E(e^{sm}) = 0, E(e^{om}) = 0$$

$$\beta^{sm} > \beta^{om}$$

Given equation 32 and its restrictions, the four most effective methods for fund managers to increase variance is to; (1) allocate a greater fraction of investments to the stock market, (2) increase the beta value of the stock market portfolio, (3) increase expected volatility of e^{sm} and (4) increase the covariance between the two portfolios. We will focus on the three first methods.

3.3 Impact on Stock Returns

To summarize our theoretical arguments, a large fraction of the stock market participants are fund managers and we argue that they have an incentive to increase variance that is greater when the risk-free rate is low. The three main ways of doing this is to allocate a larger fraction of their investments to the stock market, to increase the beta value of their investments in the stock market and to increase investments in stocks with idiosyncratic risk. Lastly, we consider how this may affect the equilibrium of stock market returns. We assume that there is a representative investor as in section 2.3. Moreover, characteristic 4 in section 2.3 establish that the preferences of the representative investor is a function of the preferences of each actual investor. Since a large fraction of the market consists of funds, the fund managers' preferences ought to have a great impact on the preferences of the representative investor, this is also in line with Brennan and Lis' (2008) discussion. From the discussion regarding fund managers' incentives, the various ways to pursue these incentive and their impact on the equilibrium of stock returns we derive the following propositions:

3.4 Proposition 1 – Risk Aversion and the Risk-Free Rate

The demand for stocks will be higher when the risk-free rate is low, since fund managers will allocate a larger fraction of their investments to the stock market in order to increase the variance of return. The variance increases both through a higher degree of systematic risk and a higher degree of idiosyncratic risk, as established by the restrictions on equation 32. This reflects a lower risk aversion in the stock market.

3.5 Proposition 2 – Systematic Risk and the Risk-Free Rate

The return of high beta stocks will be lower when the risk-free rate is low, since fund managers will allocate a greater fraction of their stock market investments to high-beta stocks in order to increase the variance of return. The variance is increased through increasing exposure to systematic risk, as established by the restriction on equation 32.

3.6 Proposition 3 – Idiosyncratic Risk and the Risk-Free Rate

The return of stocks with high idiosyncratic risk will be lower when the risk-free rate is low, since fund managers will allocate a greater fraction of their stock market investments to stocks with high idiosyncratic risk, in order to increase the variance of the return. Due to time constraints, we will leave this proposition to be tested in future research.

4. Proposition 1 – Risk Aversion and the Risk-Free Rate

4.1 Methodology and Results

In order to test proposition 1, the BGZ time series of risk aversion estimates is used. The series contain 172 estimates for the period 1990, January to 2004, April. As discussed in section 2.4, the time series is an estimation of stochastic volatility risk premium in the option market and we argue that this has two weaknesses. Firstly, the leap from stochastic volatility risk premium to risk aversion requires the assumptions described in section 2.4. Moreover, the stochastic volatility risk premium is derived through GMM estimations. Hence, our statistical analysis will be performed on a sample that is derived from statistical analysis. In effect, the data representing risk aversion are estimates, not precise observations.

Secondly, the time series reflect risk aversion in the option market and the participants in this market may not have the same incentive as the ones in the stock market. For example, many fund managers are constrained from investing in the option market. Our proposition builds on the assumption that fund manager’s incentives differ from mean-variance maximizing investors and the BGZ estimations may not capture this effect if there are limits of arbitrage (as discussed in section 2.8) between these markets. Nevertheless, the BGZ observations are the best estimates of risk aversion in the stock market that we possess.

We start by performing a time series regression with the risk-free rate as an explanatory variable for risk aversion, assuming that the risk-free rate is exogenous. Throughout the rest of this thesis, the risk-free rate will be defined as the return on the three-month Treasury Bill. When performing an Augmented Dickey-Fuller test the p-value is 0.67 and the null hypothesis that the risk-free rate has a unit root cannot be rejected, in effect, the risk-free rate appears to be non-stationary. For the first difference of the risk-free rate (Δr_t^f) the p-value becomes zero for the Augmented Dickey-Fuller test and therefore Δr_t^f is used in our regressions. For the BGZ time series, the null hypothesis has a p-value of 0.0065 in the Augmented Dickey-Fuller test so no adjustment is necessary.

In line with De Boef and Keele (2008), we want to start without any restrictions on the regression that cannot be supported by theory. Since it is costly to restructure a fund's portfolio, changes in risk aversion cannot be assumed to occur instantly when the incentive to increase variance changes, therefore four lags of Δr_t^f is included in our regression. Additionally, the relationship ought to be non-linear since the incentive to increase variance is greater when the risk-free rate is low. Therefore, Δr_t^f-squared is included in the regression. Moreover, we include an ARMA (2,2) model in the regression to capture the effect of previous values of risk aversion and previous residuals.

$$(33)\quad ra = \alpha + \beta_1 \Delta r_t^f + \beta_2 \Delta r_{t-1}^f + \beta_3 \Delta r_{t-2}^f + \beta_4 \Delta r_{t-3}^f + \beta_5 \Delta r_{t-4}^f + \beta_6 ra_{t-1} + \beta_7 ra_{t-2} + \beta_8 e_{t-1} + \beta_9 e_{t-2} + e_t$$

We start out with equation 33 and then remove the variables with the highest p-value step-by-step until we are left with only significant estimations. The results of the first regression are presented in table 4.

Table 4
Risk Aversion, ARMA (2,2) and Δr_f
Sample Size: 172
Period: 1990, January to 2004, April

Variable	Coefficient	t-Statistics	Probability
$\hat{\alpha}$	1.7379	9.2276	0.0000
$\hat{\beta}_1$(Δr*f*)	0.0361	0.3941	0.6940
$\hat{\beta}_2$(Δr*f-1*)	-0.0138	-0.1136	0.9097
$\hat{\beta}_3$(Δr*f-2*)	-0.0065	-0.0481	0.9617
$\hat{\beta}_4$(Δr*f-3*)	0.0286	0.2344	0.8150
$\hat{\beta}_5$(Δr*f-4*)	0.0044	0.5331	0.5948
$\hat{\beta}_6$(Δrf^2)	0.1744	1.5620	0.1203
$\hat{\beta}_7$(AR,1)	1.5009	6.1168	0.0000
$\hat{\beta}_8$(AR,2)	-0.5766	-2.6224	0.0096
$\hat{\beta}_9$(MA,1)	-0.1723	-0.6645	0.5073
$\hat{\beta}_{10}$(MA,2)	-0.0221	-0.1524	0.8791
Adj. R-squared	0.9042		

None of the Δr_t^f parameters are close to significant but the Δr_t^f-squared has a fairly low p-value. However, when gradually removing the least significant estimates, one is left with an AR(2) process.

Due to the suspected non-linearity, we want to test the relationship between risk aversion and the risk-free rate during different levels of the risk-free rate. To do this, we divide the observations of risk aversion into three categories based on the levels of the risk-free rate at the time. We arbitrarily choose the thresholds for the different categories but try to achieve a somewhat equal number of observations in each group.

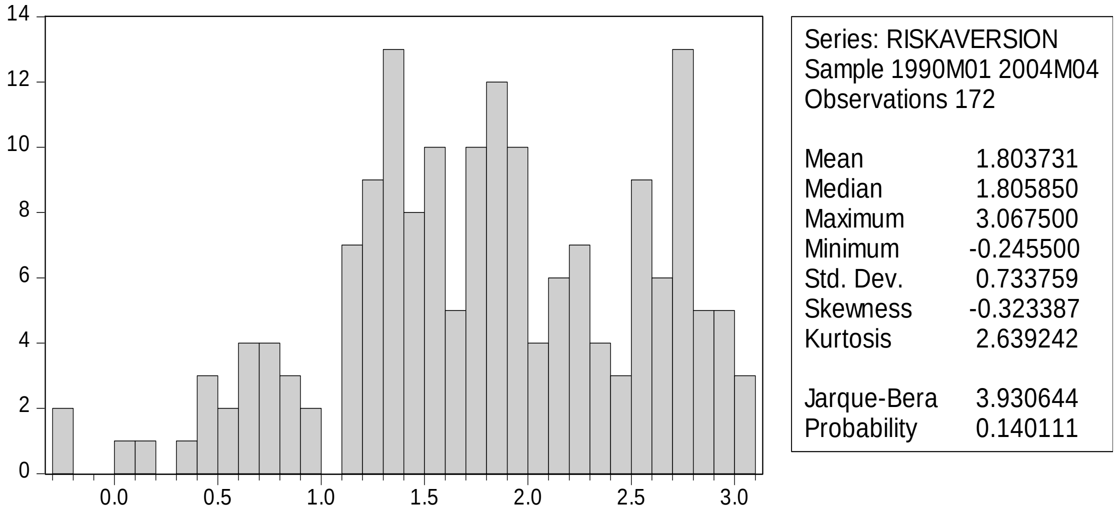
When the risk-free rate is below 3.5 percent the observation is sorted into the category *low,* which include 53 observations. This differs from how a *low* risk-free rate is categorized in proposition 2 below where the threshold is 3 percent. We would like to use the threshold 3 percent as in proposition 2 but this would only yield 38 observations for proposition 1, which is undesirable. When the risk-free rate is between 3.5 and 5 percent the observation is sorted into the category *medium*, which include 51 observations. When the risk-free rate is above 5 percent the observation is sorted into the category *high*, which include 68 observations. The small sample for the different categories is not very desirable, but until more estimates are provided this sample will have to do. Summary statistics for the three samples are presented in table 5.

Table 5
Risk Aversion During
Different States of r_f
Sample Size: 172
Period: 1990, January to 2004, April

	Risk Aversion		
Interest Level	***High r_f***	***Low r_f***	***Medium r_f***
Mean	1.9389	1.5120	1.9267
Median	2.0138	1.4140	1.8957
Maximum	2.9481	2.7777	3.0675
Minimum	0.1670	-0.2455	0.6373
Std. Dev.	0.7862	0.6327	0.6828
Skewness	-0.6331	-0.3843	-0.1542
Kurtosis	2.3939	4.3587	2.1352
Jarque-Bera	5.5830	5.3812	1.7911
Probability	0.0613	0.0678	0.4084
Sum	131.8438	80.1341	98.2639
Sum Sq. Dev.	41.4138	20.8154	23.3120
Observations	68	53	51

Interestingly, the mean risk aversion is considerably lower for the group with a low risk-free rate and this is in line with our proposition. The median is also lower for this group, which indicates that the difference in mean is not explained by outliers. However, we need to test whether or not the difference is statistically significant. Since the samples do not have equal variance we use Welch's t-test (Welch, 1951). To perform this test we have to assume that risk aversion is normally distributed. This may be a bold assumption considering the histogram for the total sample in figure 6.

Figure 6 – Histogram of Risk Aversion Estimates



Nevertheless one cannot reject that the sample is normally distributed at a 10 percent significance level using a Jarque-Bera test. We test if the mean risk aversion in the *low* category is lower than the medium and high category by performing two separate tests. The results are presented in table 6.

Table 6
Total Sample Size: 172
Period: 1990, January to 2004, April
Risk Aversion, Welch's Equality of mean

Group	***p-value***
High and Low Group	0.0012
Medium and Low Group	0.0018

The null hypothesis of equal means can be rejected with 1 percent significance in both tests. Hence, we conclude that risk aversion is lower when the risk-free rate is low.

Furthermore, we want to examine *how* the risk-free rate affects risk aversion when the risk-free rate is low. When testing if changes in the risk-free rate impact risk aversion with independent variables that was assumed to be linear in their parameters, we could not find a significant relationship. However, when studying figure 7, 8 and 9, there appear to be a somewhat linear relationship between risk aversion and the risk-free rate for the *low* group, the relationship then disappear for the other groups.

Figure 7 – Risk Aversion Estimates and the Risk-Free Rate

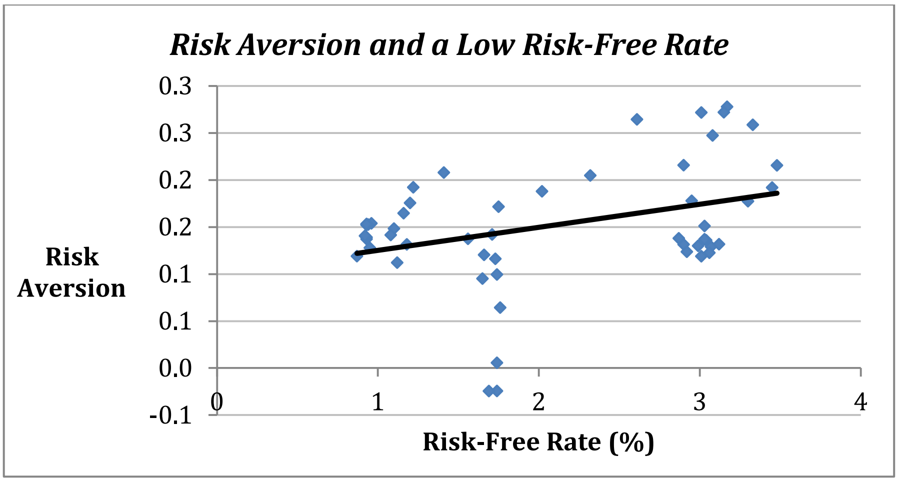


Figure 8 – Risk Aversion Estimates and the Risk-Free Rate

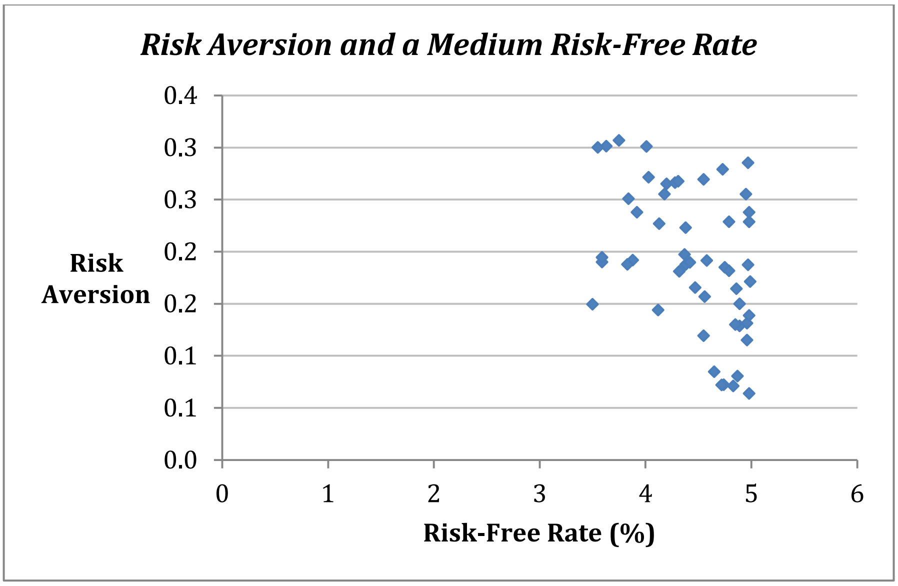
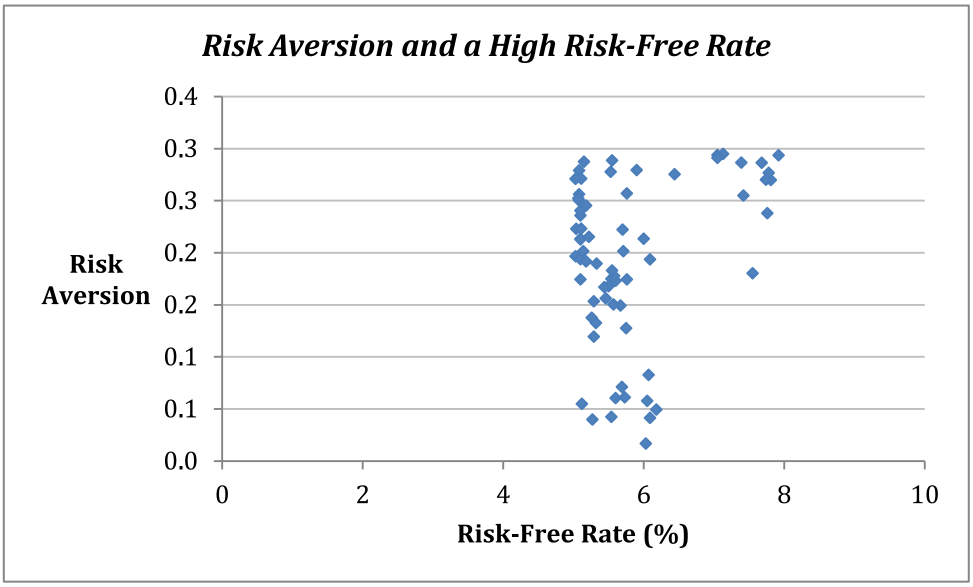


Figure 9 –Risk Aversion Estimates and the Risk-Free Rate



Therefore, we construct a regression with an interaction term for a dummy variable (*low*) and Δr_t^f. The dummy variable equals 1 when the risk-free rate is below 3.5 percent and equals 0 otherwise. From table 4 it is evident that an AR(2) process describe a large part of the movements in risk aversion. Consequently the AR(2) process is also included in the regression, which result in:

(34) $$ra = \alpha + \beta_1 low\Delta r_t^f + \beta_2 ra_{t-1} + \beta_3 ra_{t-2} + e_t$$

Hence, the first difference of the risk-free rate is modeled to affect risk aversion in the following way:

(35) $$\frac{\Delta ra}{\Delta(\Delta r_t^f)} = \begin{Bmatrix} \beta_3 \; if \; R_t^f < 3.5\% \\ 0 \; if \; R_t^f > 3.5\% \end{Bmatrix}$$

From proposition 1, β_1 is expected to be positive since risk aversion should increase as the risk-free rate increase. However, the interaction term is far from statistically significant (table 7).

Table 7
Risk Aversion, AR(2) and Interaction Dummy for r_f
Sample Size: 172
Period: 1990, January to 2004, April

Variable	Coefficient	t-Statistics	Probability
$\hat{\alpha}$	1.7531	8.6861	0.0000
$\hat{\beta}_1$(low*Δrf)	-0.0554	-0.5736	0.5670
$\hat{\beta}_2$(AR,1)	1.3556	19.8193	0.0000
$\hat{\beta}_3$(AR,2)	-0.4399	-6.4216	0.0000
Adj. R-squared	0.9098		

We perform one last test by removing the interaction term but keeping the dummy variable. One might suggest that we are getting close to data mining at this point, but we think that the tests are in line with the argumentation presented in section 3:

(36) $$ra_t = \alpha + \beta_1 low_t + \beta_2 ra_{t-1} + \beta_3 ra_{t-2} + e_t$$

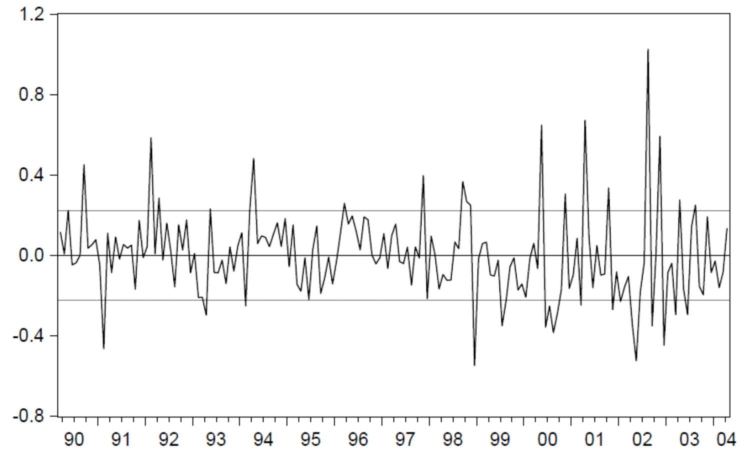
The dummy variable should be interpreted as the difference in the intercept when the risk-free rate is low compared to when it is not (Wooldridge, 2003, page 220). Hence, the dummy variable should take on a negative value to support proposition 1. The results are presented in table 8.

Table 8
Risk Aversion, AR(2) and Dummy Variable for Low r_f
Sample Size: 172
Period: 1990, January to 2004, April

Variable	Coefficient	t-Statistics	Probability
$\hat{\alpha}$	1.7292	8.3497	0.0000
$\hat{\beta}_1$(low)	0.0704	0.8689	0.3861
$\hat{\beta}_2$(AR,1)	1.3629	19.9913	0.0000
$\hat{\beta}_3$(AR,2)	-0.4458	-6.5286	0.0000
Adj. R-squared	0.9100		

The dummy variable is far from significant. When studying the graphed residuals in figure 10, one significant outlier can be identified in August, 2002.

Figure 10 – Residuals for Regression 36



In order to test if excluding the outlier would improve the results, we create a dummy variable that equals 1 for the data point (AUG02DUM). In effect, the observation is excluded from the sample (Brooks, 2008, page 166). Excluding the outlier does not improve the results considerably (table 9) and therefore no indication on *how* the risk-free rate affects risk aversion is found.

Table 9
Risk Aversion, AR(2) and Dummy Variable for Low r_f, Excl.Outlier
Sample Size: 172
Period: 1990, January to 2004, April

Variable	Coefficient	t-Statistics	Probability
$\hat{\alpha}$	1.7255	8.5440	0.0000
$\hat{\beta}_1$(low)	0.0751	0.9954	0.3210
$\hat{\beta}_2$(AUG02DUM)	0.4955	4.1995	0.0000
$\hat{\beta}_3$(AR,1)	1.4111	21.139	0.0000
$\hat{\beta}_4$(AR,2)	-0.4921	-7.3624	0.0000
Adj. R-squared	0.9181		

4.2 Discussion of Results

The findings that the risk aversion is lower when the risk-free rate is low does support proposition 1. However, one has to consider the initial discussion in section 4.1, the sample is not *observations* of risk aversion from the *stock market*. Instead, the sample consists of *estimates* of risk aversion derived from the *option market*. On the other hand, we argue that the imperfection in our proxy for risk aversion does motivate the crude method of comparing the means of the subgroups. The estimation errors in each subgroup probably cancel out each other to some extent when calculating the mean. Moreover, the imperfection of the estimates might explain why we cannot find an OLS regression that describes how the risk-free rate affects risk aversion. If there is a relationship between the true risk aversion and the risk-free rate, estimation errors in our proxy might distort this relationship and ruin the possibility to find how the risk-free rate affects risk aversion.

Another likely explanation is that the functional form for the relationship differs from the ones in our regressions. The mean risk aversion is significantly lower when the risk-free rate is low, but the difference between the high and the medium group is fairly low. This indicates a non-linear relationship, and although the non-linear relationships included in our regressions did not manage to produce significant results, there are many other possible non-linear relationships that have not been tested. For example, it would be useful to include the natural logarithm of the risk-free rate as a parameter in the OLS regression. Moreover, if a relationship that is linear in the parameter cannot be found, a non-linear regression model may be required to identify the relationship.

5. Proposition 2 – Systematic Risk and the Risk-Free Rate

5.1 Methodology and Results

To examine the risk-free rate's effect on the cross-section of stock returns we follow BJS's procedure with some modifications. Daily and monthly stock prices between 1980 and 2012 for the NYSE have been gathered from Thomson Reuters Datastream. Initially the database stated that the number of listed companies was 2,712, but we could only retrieve 1,749 stocks for an unknown reason. Hence, it is hard to say if the omitted data will contribute to any bias. Then daily and monthly arithmetic returns where calculated from the stock prices.

One bias introduced by the data is that the returns for the stocks are not adjusted for dividends. This results in understated returns for companies with high dividends relative to companies with low dividends. Another bias is that we only have historical data for the stocks that were listed as of 2013-04-04. In effect, stocks that have been delisted during the period are not included. This results in a survivorship bias. Similar to Blake and Timmerman's (1998) discussion of survivorship bias in the fund industry, there are two main reasons for that stocks are delisted; bankruptcy or that it has been acquired by another company. Therefore, both well and bad performing firms might be excluded from the sample. The market portfolio's return is defined as the NYSE Composite Price Index, which is the value weighted return on the NYSE without adjustments for dividends. The reason why we started in 1980 is that the expansion of the fund industry intensified around that year (as illustrated in figure 2 above).

Table 10 - Number of Stocks in Portfolios by Year

Year	*Number of Stocks**	*Year*	*Number of Stocks**
1980	476	1997	919
1981	491	1998	966
1982	507	1999	1 001
1983	517	2000	1 046
1984	534	2001	1 075
1985	548	2002	1 119
1986	568	2003	1 143
1987	598	2004	1 175
1988	620	2005	1 236
1989	639	2006	1 295
1990	649	2007	1 344
1991	665	2008	1 423
1992	691	2009	1 454
1993	729	2010	1 486
1994	783	2011	1 561
1995	828	2012	1 633
1996	876		

*Total number of stocks, divided equally into 10 portfolios. The stocks are divided as follows, e.g. year 1980: Portfolio 1-9 contains of 47 stocks each and portfolio 10 contains of 47+6=53 stocks. Consequently portfolio 10 contains some more observations for every year.

In line with BJS we divide the stocks into 10 portfolios based on the ranking of their beta, in order to reduce the problem of cross-sectionally dependent residuals in the time series test. Also in line with BJS, stocks are sorted into the portfolios based on beta values estimated from a preceding period, in order to reduce the regression to mean phenomenon. Portfolio 1 is the portfolio with the lowest beta and portfolio 10 is the one with the highest beta. BJS use monthly returns for the five preceding years to calculate the beta values, we choose daily returns in the preceding year instead since the higher sample frequency provides more precise estimates of variance (Merton, 1980). Therefore, more precise estimates of covariance is achieved with higher sample frequency.

To calculate excess return for the market portfolio and for individual stocks, we subtract $(1+3MTB)^{(1/365)}$ from the daily return, where $3MTB$ is the annualized interest rate for three-month Treasury Bills. The monthly excess return of each portfolio is defined as the equally-weighted average return of the stocks, subtracted by $(1+3MTB)^{(1/12)}$. The choice of equally weighted average return is in line with BJS and Fama and French (1996). The portfolios are updated every year and result in a time series of monthly excess returns for the 10 portfolios between 1981 and 2012. Many of the observations in portfolio 1 are stocks that have not been traded during the year, which results in a beta value of 0. Some stocks with extremely negative beta values are also included in portfolio 1 and for some years, stocks with extremely high beta values are included in portfolio 10. Therefore, one should be cautious when interpreting the results of these portfolios. After this procedure, we replicate BJS's time series regression for each portfolio:

(37) $$r_t^j - r_t^f = \alpha^j + \beta^j (r_t^m - r_t^f) + e_t^j$$

If the CAPM accurately describe stock returns, then the intercept is expected to be zero for all portfolios. The results for the regressions are presented in table 11.

Table 11
Portfolio Excess Return and Market Excess Return
Sample Size: 384
Period: 1981, January to 2012, December
Portfolio Number

Item	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
$\hat{\alpha} \bullet 10^2$	5.6313	0.4323	0.3881	0.4285	0.3606	0.2900	0.5033	0.3977	0.2394	0.6011
t($\hat{\alpha}$)	1.3792	2.6833	2.9811	2.9843	2.5808	1.7424	3.3091	2.5817	1.4759	2.5294
p($\hat{\alpha}$)	0.1686	0.0076	0.0031	0.0030	0.0102	0.0822	0.0010	0.0102	0.1408	0.0118
$\hat{\beta}$	-1.2462	0.6035	0.7315	0.7947	0.9202	0.9746	1.1033	1.2014	1.2540	1.5166
t($\hat{\beta}$)	-0.8044	11.0663	15.2617	17.9623	16.3131	19.6164	18.4101	16.3395	18.8313	15.3136
p($\hat{\beta}$)	0.4217	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Adj. R-squared	0.0032	0.4779	0.6688	0.6693	0.7063	0.6764	0.7419	0.7336	0.7443	0.6654

With Newey-West fixed std. error

When performing White's test for hetroscedasticity, the assumption of homoscedasticity is rejected with a significance level of 0.05 for the f-test for most portfolios. Moreover, when performing a Breusch-Godfrey test for autocorrelation with 10 lags, the assumption of uncorrelated residuals is also rejected with a 0.05 significance level for most of the portfolios. To prevent incorrect inference regarding the significance of the estimated parameters, we use the Newey-West procedure to modify the estimations of the standard errors (Brooks, 2008, 152). This is also in line with Merton (1980), who argues that when estimating parameters for realized returns, the results should be adjusted for hetroscedasticity since the variance of market returns are not constant.

An Augmented Dickey-Fuller test allows us to reject the null hypothesis that the NYSE returns are non-stationary with a significance level of 0.01. The Bera-Jarque test is used to test if the residuals are normally distributed for the portfolios. The test rejects the null hypothesis that the residuals are normally distributed at a 0.01 significance level for all portfolios. The non-normality of the residuals will be discussed in section 5.2. The estimated parameters for portfolio 1 are nonsensical, the beta value is estimated to -1.24 without statistical significance and the adjusted R-squared for the regression is only 0.0058, as mentioned earlier the quality of the data in this portfolio is weak so we do not put any effort into interpreting its results throughout the rest of this thesis.

To examine whether or not the risk-free rate affect stocks with different betas in different ways, the first difference of the risk-free rate is added as an independent variable to the BJS regression. We continue to assume that excess returns on the market portfolio are independent of the risk-free rate. If our proposition holds, including Δr_t^f will pull the intercept towards 0 and increase the adjusted R-squared for the high-beta portfolios and the effects diminish for lower-beta portfolios.

If the excess return on the market portfolio is not independent of the risk-free rate (as discussed in section 2.5), multicollinearity will exist and the OLS regression cannot be used to test our proposition. However, the correlation between excess return on the NYSE and the first difference of the risk-free rate is only -0.08, in effect, their movements appear to be independent of each other. As discussed earlier, fund managers should not be able to instantly reallocate their portfolios when the risk-free rate decrease and therefore we include four lags of Δr_t^f. Moreover, since the incentive is greater during periods of low risk-free rate and decrease as the risk-free rate increase, the relationship may be non-linear and therefore ${\Delta r_t^f}^2$ is included:

$$r_t^j - r_t^f = \alpha^j + \beta_1^j\left(r_t^m - r_t^f\right) + \beta_2 \Delta r_{t-1}^f + \beta_3 \Delta r_{t-2}^f + \beta_4 \Delta r_{t-3}^f + \beta_5 \Delta r_{t-4}^f + \beta_6 {\Delta r_t^f}^2 + e_t^j \quad (38)$$

For every portfolio we run the regression with four lags and ${\Delta r_t^f}^2$, then the estimates with highest p-value are gradually removed. We continue to use White's test for hetroscedasticity and the Breusch-Godfrey test for autocorrelation with 10 lags. The regressions for the portfolios appear to have both hetroscedastic and autocorrelated residuals and therefore the Newey-West procedure are used to modify the estimated standard errors as in the previous regression. At a 0.05 significance level, this result in statistical significance for the third lag in portfolio 2, portfolio 3, portfolio 4 and portfolio 5, these results are presented in table 12.

Table 12.
Portfolio Excess Return, Market Excess Return and r_f
Sample Size: 384
Period: 1981, January to 2012, December

	Portfolio Number			
Item	**P2**	**P3**	**P4**	**P5**
$\hat{\alpha} \bullet 10^2$	0.3836	0.3397	0.3843	0.3014
t($\hat{\alpha}$)	2.4067	2.7468	2.7741	2.2773
p($\hat{\alpha}$)	0.0166	0.0063	0.0058	0.0233
$\hat{\beta}_1$	0.6092	0.7348	0.7985	0.9243
t($\hat{\beta}_1$)	11.2852	15.3389	17.9853	16.2330
p($\hat{\beta}_1$)	0.0000	0.0000	0.0000	0.0000
$\hat{\beta}_4 \bullet 10^2$($\Delta rf$-3)	-0.6739	-0.6354	-0.6579	-0.7491
t($\hat{\beta}_4$)	-2.8306	-2.2107	-2.5024	-2.6660
p($\hat{\beta}_4$)	0.0049	0.0277	0.0128	0.0080
Adj. R-squared	0.4880	0.6773	0.6750	0.7138

With Newey-West fixed std. error

Furthermore, the Jarque-Bera test rejects that the residuals are normally distributed for all portfolios at a significance level of 0.01. As mentioned above, this problem will be discussed in section 5.2. In order to focus on the effect of a *low* risk-free rate, we use the BJS regression and add a dummy variable for when the risk-free rate is below 3 percent (*low)*, which corresponds to 113 of our 384 observations:

(39) $$r_t^j - r_t^f = \alpha^j + \beta_1^j\left(r_t^m - r_t^f\right) + \beta_2^j low + e_t^j$$

The results are presented in table 13.

Table 13
Portfolio Excess Return, Market Excess Return and Dummy Variable for Low r_f (3%)
Sample size: 384
Period: 1981, January to 2012, December

	Portfolio Number									
Item	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
$\hat{\alpha} \bullet 10^2$	2.1062	0.2125	0.1702	0.1925	0.1614	0.0847	0.3001	0.1432	-0.0444	0.1904
t($\hat{\alpha}$)	2.1518	1.0878	1.1139	1.1267	0.9740	0.4079	1.7604	0.9567	-0.2782	0.8048
p($\hat{\alpha}$)	0.0320	0.2774	0.2660	0.2606	0.3307	0.6836	0.0791	0.3394	0.7810	0.4214
$\hat{\beta}_1$	-1.1907	0.6070	0.7349	0.7984	0.9233	0.9779	1.1065	1.2054	1.2584	1.5230
t($\hat{\beta}_1$)	-0.8015	10.9610	14.9068	17.4078	15.8719	19.0646	17.9777	16.0671	18.4720	15.1263
p($\hat{\beta}_1$)	0.4234	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{\beta}_2$(*low*)	0.1192	0.0074	0.0074	0.0080	0.0067	0.0069	0.0069	0.0086	0.0096	0.0139
t($\hat{\beta}_2$)	1.0560	2.3679	2.7463	2.6992	2.1284	2.0069	1.7393	1.8591	2.0761	2.0392
p($\hat{\beta}_2$)	0.2917	0.0184	0.0063	0.0073	0.0339	0.0455	0.0828	0.0638	0.0386	0.0421
Adj. R-squared	0.0064	0.4844	0.6753	0.6757	0.7096	0.6792	0.7444	0.7370	0.7484	0.6706

With Newey-West fixed std. error

As for previous regressions White's test for hetroscedasticity and Breusch-Godfrey for autocorrelation with 10 lags is used. Most portfolios appear to have hetroscedastic and autocorrelated residuals, therefore the Newey-West procedure is used as before. After this correction, the dummy variable is significant at a 0.05 level for all portfolios except portfolio 8 that has a p-value of 0.06 and portfolio 7 that has a p-value of 0.08. Moreover, it can be rejected that the residuals are normally distributed with a significance level of 0.01 using the Jarque-Bera test. As mentioned above, this will be discussed in section 5.2 below. Since the threshold for *low* is defined differently in proposition 1 and proposition 2, one could accuse us of data mining. Therefore, we have provided the results for the same regression but with 3.5 percent as the threshold for a low risk-free rate in appendix 2.

5.2 Discussion of Non-Normality in Residuals

In section 5.1 we could conclude that none of our tests of proposition 2 had normally distributed residuals. In table 14,15 and 16, the skewness and the kurtosis for the residuals are presented. In contrast to the normal distribution that is symmetric (Brooks, 2008, page 161), the skewness is positive for almost all of the portfolios and increase with the beta value. In effect, positive values are more likely than stated in the normal distribution. In our findings, we are most interested in the parameter for a low risk-free rate (the dummy variable). Since this parameter is positive, the skewness of the residuals implies that the p-values from our OLS regressions are lower than they would be using the true distribution. In effect, the certainty regarding our results is weaker than stated by the p-value.

Moreover, the distributions of the residuals have excess kurtosis, which appears to be greater for the portfolios with a high beta value. The normal distribution has a kurtosis of 3 and the excess kurtosis is therefore the kurtosis in excess of 3. This implies that the distribution is more peaked and has fatter tails than the normal distribution (Brooks, 2008, page 162). The fatter tails lead to a higher probability of extreme values. Therefore, the p-values from our OLS regressions are too low since they assume normally distributed residuals. To summarize, the p-values of our parameters are too low since the residuals are positively skewed and has excess kurtosis. This effect is greater for the portfolios with high beta values.

Table 14
Residuals for Regression 37, Skewness and Kurtosis

	Portfolio Number									
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
Skewness	18.8570	-0.4175	0.5219	0.8816	1.0373	0.1964	2.1896	2.6815	2.1434	3.2617
Kurtosis	364.4677	6.6679	8.3280	6.3531	8.9114	14.7471	15.8081	19.4551	16.4067	24.2851

Table 15
Residuals for Regression 38, Skewness and Kurtosis

	Portfolio Number			
	P2	**P3**	**P4**	**P5**
Skewness	-0.4663	0.4620	0.8342	0.9715
Kurtosis	6.6836	8.0073	6.1660	8.7423

Table 16
Residuals for Regression 39, Skewness and Kurtosis

	Portfolio Number									
	P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
Skewness	18.6949	-0.3954	0.3980	0.7484	0.9116	0.1094	2.0400	2.4340	1.8639	3.1871
Kurtosis	360.3002	6.6422	8.0210	5.9155	8.3052	14.1888	14.8160	18.0419	15.2347	24.8656

5.3 Discussion of Results

When examining the results for regression 37 one can conclude that our findings are not in line with BJS. The intercepts are positive for all portfolios, in contrast to BJS that found a negative intercept for the high-beta portfolios. It can be rejected that the intercept is zero at 0.05 significance for most of the portfolios. The positive intercept indicate that there is some other risk factor that is priced by the market that is not strongly related to the systematic risk of stocks. The intercepts also suggest that this additional risk factor has great economic significance. Fama and French (1996) manages to decrease the intercepts with their three-factor model. They make 25 regressions in their study with an average R-squared of 0.93 and with small intercepts just distinguishable from zero. Following from the adjusted R-squared, systematic risk appears to explain a fairly large part of the portfolio returns but also leaves much of it unexplained. The adjusted R-squared does also seem to increase with the portfolio beta but the relationship is far from monotonic.

In regression 38, the third lag of the risk-free rate decrease the intercepts for portfolio 2-5, but it is still significantly different from 0 at a 0.05 level. The adjusted R-squared also increase modestly. Including the variable does not appear to alter the systematic risk's impact on the portfolio returns considerably and no monotonic pattern for the intercept is evident. The parameter for Δr_{t-3}^{f} does not seem to depend on the beta value.

Table 17
Difference in Parameters Between Regression 37 and 38

	Portfolio Number			
Item	**P2**	**P3**	**P4**	**P5**
$\hat{\alpha} \bullet 10^2$	-0.0487	-0.0484	-0.0442	-0.0592
$\hat{\beta}_1$	0.0056	0.0034	0.0038	0.0041
Adj. R-squared	0.0100	0.0084	0.0057	0.0076

We find it difficult to interpret these findings and it is likely that they are part of some greater pattern that is not captured in the regression.

In regression 39, the dummy variable decreases the intercept for all portfolios. When the dummy is included, the intercept seem to decrease to approximately zero for the high-beta portfolios and the intercept is only significantly different from 0 at a 0.05 level for portfolio 1. However, the R-squared does not increase dramatically when the dummy variable is included.

Table 18
Difference in Parameters Between Regression 37 and 39

	Portfolio Number									
Item	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
$\hat{\alpha} \bullet 10^2$	-3.5251	-0.2198	-0.2179	-0.2360	-0.1992	-0.2053	-0.2032	-0.2545	-0.2838	-0.4107
$\hat{\beta}_1$	0.0555	0.0035	0.0034	0.0037	0.0031	0.0032	0.0032	0.0040	0.0045	0.0065
Adj. R-squared	0.0031	0.0065	0.0065	0.0065	0.0033	0.0029	0.0024	0.0034	0.0041	0.0052

The dummy parameter is positive for all portfolios and the annualized difference in return for low levels of the risk-free rate is presented in table 19.

Table 19
Annualized Dummy Parameter from Regression 19

Item	Portfolio Number									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
$(1+\hat{\boldsymbol{\beta}}_2)^{12}$-1	2.8627	0.0925	0.0925	0.1003	0.0834	0.0860	0.0860	0.1082	0.1215	0.1802
$\hat{\boldsymbol{\beta}}_2(low)$	0.1192	0.0074	0.0074	0.0080	0.0067	0.0069	0.0069	0.0086	0.0096	0.0139

These findings are consistent with those for regression 38, the positive dummy variables suggest that the returns are higher when the risk-free rate is low and the parameter in regression 38 suggest that excess returns decrease as the third lag of the risk-free rate increase, at least for low-beta portfolios. When taking the economic significance into consideration, our findings from regression 39 may be a bit unrealistic. According to our findings, the annualized excess return for portfolio 9 is 12 percentage points higher during periods of a risk-free rate below 3 percent. The result for portfolio 10 is even higher, but as mentioned above, the data for portfolio 10 might have some flaws. We cannot find a normative justification for that the excess return should be higher when the risk-free rate is low. Linking back to section 2.5, it is possible that excess return should be dependent of the current level of the risk free rate, in effect:

$$\frac{\Delta Excess\ return}{\Delta R^f} = -\beta$$

This implies that excess return would be higher when the risk-free rate is low. However, this would also imply that the dummy variable should increase monotonically for the portfolios, which is not the case. Moreover, it was concluded above that the correlation between excess return on the NYSE index and the risk-free rate is very low in our sample. Lastly, this explanation would yield significant results for the risk-free rate in regression 38, In effect, the risk-free rate would not only impact excess returns during low levels.

If one wishes to create a story to motivate these findings it ought to include a representative investor who's demand for stocks decrease when the risk-free rate is low.

However, this is the opposite of our narrative created in section 3. Instead, we suggest that there is another causal relationship. For example, low levels of the risk-free rate might be most prevalent after economic shocks. However, to examine this relationship is beyond the scope of this thesis. If this suggestion is correct, the relationship found between excess return and the risk-free rate might instead reflect the effect of economic shocks on excess returns. After an economic shock, uncertainty regarding the future is likely to be greater. Hence, volatility should be greater and following from equation 9, this result in greater excess returns. Moreover, a low level of the risk-free rate might be caused by monetary stimulus. The monetary stimulus might increase the expected cash flows for the firms, which result in high realized returns. However, this requires that the market participants did not anticipate the monetary stimulus. To study this relationship is also beyond the scope of this thesis.

The results may also be exaggerated by the bias created by not adjusting for dividends. Managers might be cautious to pay dividends after economic shocks since the future is uncertain. The access to capital markets might be limited, which makes a cash buffer desirable. This would exaggerate the difference in returns for our data since it does not account for dividends, which would be greater when the risk-free rate is not low.

6. Conclusion

To summarize, we have found support for that risk aversion is lower when the risk-free rate is below 3.5 percent. However, no significant parameter estimate for the risk-free rate was found in the OLS regressions performed. When replicating BJS time series regression with more recent data, we find that all intercepts are positive. Moreover, we find that the return of all portfolios is considerably higher when the risk-free rate is below 3 percent. In effect, stock returns appear to be higher when the risk-free rate is low, regardless of the beta value.

At first glance, one might conclude that combining the findings for the two propositions result in an incoherent picture of investor behavior. The risk aversion is lower when the risk-free rate is low, but the stock returns increase. However, before dismissing our findings one ought to study equation 9 again. The increase in the stock returns can be explained by increased volatility, which is the causal relationship we suggested in section 5.3. This noise makes it hard to disentangle the demand for high-beta stocks in relation to the demand for low-beta stocks. In this scenario, we have not tested proposition 2 properly. However, this reasoning is not supported by data, research in the relationship between stock market volatility and the risk-free rate would be useful to help explain the high excess return for when the risk-free rate is low. Moreover, proposition 1 has the benefit of being independent of the CAPM. As discussed in section 2.2, the CAPM is a factor pricing model in which a stock's regression coefficient with the market portfolio determines the cross-section of returns. In table 11 it is evident that the CAPM does leave a substantial fraction of stock returns unexplained. Hence, the deduction of proposition 2 may rely on a misspecified theoretical framework.

Based on the findings that stock returns are higher during low risk-free rates, it seems to be less attractive for corporations to fund their operations from the equity market during these periods. If this is due to greater volatility, it ought to be reflected in their cost of debt as well and the corporations would have to accept higher funding costs during these periods. However, if this is due to a anomaly in the stock market, it would be wise for corporations to avoid funding their operations through equity issues during periods of a low risk-free rate.

It is important to keep in mind that even though the findings are in line with proposition 1, this does not *prove* that fund managers deviate from their investors' interests and consequentially increase the risk of their portfolios in order to maximize their own compensation. The findings do indeed *support* our theoretical arguments, which were developed before the tests were conducted. As far as we know, our proposition is currently the best explanation for the phenomena observed in the data. If one is willing to conceive that risk aversion decrease when the risk-free rate is low, due to an increased incentive for fund managers to take on excessive risk, we point out two practical implications.

Firstly, fund investors should be aware of the increased agency problem in order to monitor the fund managers more carefully during periods of a low risk-free rate. Although French's (2010) research show that it would be wise for investors to avoid actively managed mutual funds altogether, the reason to avoid them appears to be stronger during periods of a low risk-free rate. Secondly, monetary policymakers ought to take the findings into consideration when forecasting the consequences of their monetary policy decisions. As mentioned above, the proxy for risk aversion is derived from option prices and under a no-arbitrage assumption this should be a good proxy for risk aversion in the entire financial market. Hence, the expansionary monetary policies currently conducted in the developed countries should have consequences for the risk appetite in the financial market.

As mentioned above, further research in the relationship between the risk-free rate and stock market volatility would be useful in order to provide understanding for why the excess returns are greater when the risk-free rate is low. Moreover, it would be interesting to test if the effect of the risk-free rate remains in the Fama and French three-factor model. In addition, further econometric work is necessary to find a more precise description for how the risk-free rate affects risk aversion. Finally, it would be interesting to see empirical tests of proposition 3, perhaps by using the framework of Ang et al. (2006) and adding the dummy variable for a low risk-free rate used in proposition 2.

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APPENDICES

APPENDIX 1

Derivation of variance of returns

$r^j = r^f + \alpha^j + \lambda^m \beta^j + e^j$

Assume that α^j is zero. Assume e^j is independently normally distributed, therefore its covariance with r^f and λ^m is zero. Lastly assume that the risk premium is uncorrelated with the risk-free rate. In effect:

$\sigma^2(\alpha^j) = 0, \sigma_{\lambda^m e^j} = 0, \sigma_{r^f e^j} = 0, \sigma_{r^f \lambda^m} = 0$

Therefore, the variance of portfolio returns is:
$\sigma^2(r^j) = \sigma^2(r^f) + \sigma^2(\lambda^m \beta^j) + \sigma^2(e^j)$

For a given level of β^j the variance of the portfolio becomes:

$\sigma^2(r^j) = \sigma^2(r^f) + {\beta^j}^2 \sigma^2(\lambda^m) + \sigma^2(e^j)$

APPENDIX 2

Table 20
Portfolio Excess Return, Market Excess Return and Dummy Variable for Low r_f(3.5%)
Sample size: 384
Period: 1981, January to 2012, December

	Portfolio Number									
Item	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
$\hat{\alpha} \bullet 10^2$	1.5543	0.1743	0.1742	0.1865	0.1247	0.0372	0.2736	0.1091	-0.1593	0.1216
$t(\hat{\alpha})$	1.8029	0.8384	1.0788	1.0463	0.7140	0.1685	1.5382	0.6865	-1.0074	0.4834
$p(\hat{\alpha})$	0.0722	0.4023	0.2814	0.2961	0.4757	0.8663	0.1248	0.4928	0.3144	0.6291
$\hat{\beta}_1$	-1.1895	0.6071	0.7345	0.7980	0.9235	0.9782	1.1065	1.2054	1.2595	1.5232
$t(\hat{\beta}_1)$	-0.7968	11.0002	15.0079	17.5111	15.9177	19.1336	18.0518	16.1104	18.5540	15.1559
$p(\hat{\beta}_1)$	0.4261	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{\beta}_2$(*low*)	0.1181	0.0075	0.0062	0.0070	0.0068	0.0073	0.0067	0.0084	0.0115	0.0139
$t(\hat{\beta}_2)$	1.2255	2.5164	2.4389	2.4902	2.3059	2.2588	1.8516	2.0323	2.8703	2.2846
$p(\hat{\beta}_2)$	0.2211	0.0123	0.0152	0.0132	0.0217	0.0245	0.0649	0.0428	0.0043	0.0229
Adj. R-squared	0.0067	0.4852	0.6736	0.6745	0.7101	0.6800	0.7444	0.7371	0.7511	0.6711

With Newey-West fixed std. error

Looking at the historical data for the risk-free rate during the period of our sample we found a threshold of 3 percent to be appropriate for a low risk-free rate since this constitute approximately one-third of the sample. But due to lack of observations qualified to this group we decided to use a threshold of 3.5 percent when studying the smaller sample in proposition 1. One might accuse us for data mining and therefore we made the regression for proposition 2 including the higher threshold as used in proposition 1. The results were even slightly more significant using the lower level of the risk-free rate. A comparison between the two regressions shows that the p-value for the 3 percent “dummy” is significant at a 0.05 percent level for all portfolios except 1 and 7. This is better than for the 3.5 percent level, in which portfolio 1, 7 and 8 were insignificant at the 0.05 level. There is no substantial difference when comparing the intercepts, the coefficient estimates or the pattern across the different beta portfolios. The adjusted R-square’s does also appear to be almost unaffected by this modification.

APPENDIX 3 – ARTICLE WRITTEN FOR A SWEDISH BUSINESS PAPER

Fondförvaltarna riskerar investerarnas besparingar för egen vinning

”Lägre räntor ökar risktagandet på aktiemarknaden”

En allt större del av dagens sparande placeras i aktiefonder, vilket gjort fondförvaltarna till den enskilt största aktören på aktiemarknaden. Spararna placerar sina pengar i fonder av flera anledningar. Diversifiering och en tro på att fondförvaltarnas skicklighet ska ge dem en bra avkastning är förmodligen två av de mest centrala motiven. Detta trots att girighet och höga bonusar har skamfilat finansbranschen mer än en gång tidigare. Författarna har studerat närmare hur förändringar i den riskfria räntan påverkar riskaptiten på aktiemarknaden.

1 706 bolag på New York Stock Exchange och deras utveckling sedan 1980 ingår i studien. Foto: Daniel Gardtman

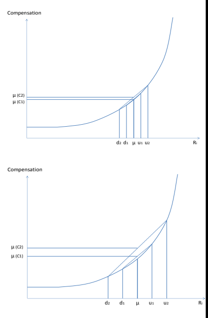
Inspirationen till studien är hämtad från Raghuram Rajan som menar att risken på de finansiella marknaderna ökat kraftigt under de senaste decennierna och pekar på ett samband med den allt större närvaron från fondförvaltare. Det finns en stark enhällighet bland forskare om att fondförvaltarnas kompensationsprogram leder till ett Agent-Principalproblem gentemot dess investerare. Möjligheten till hög kompensation för förvaltarna gör att de inte alltid tar beslut som bäst matchar investerarnas preferenser. De riskerar alltså investerarnas besparingar för egen vinning.

Våra resultat visar på att i tider med låg riskfri ränta ökar risktagandet på aktiemarknaden. Vi illustrerar detta med en figur som visar på sambandet mellan fondförvaltarnas kompensation och olika nivåer av fondavkastning.

Tidigare studier visar på att desto mindre insyn investerarna har, desto högre är incitamentet för fondförvaltarna att öka risken i fonden. Spararna bör därför öka kraven på transparens vad gäller fondernas risktagande.

Våra slutsatser angående den riskfria räntans påverkan på aktiemarknaden bör tas på högsta allvar.

Grafiken till höger visar det konvexa sambandet mellan en fonds avkastning och dess kompensation. Anmärkningsvärt är att förväntad kompensation är högre för de förvaltare som tar högre risk. Detta samband förstärks ytterligare under perioder med låga riskfria räntor.



Beaktning bör tas av beslutsfattare så som, räntedelegationen på riskbanken och när nya regleringar införs inom fondmarknaden.

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