Analysis of a model for traffic networks

Gustav Nilsson

Traffic networks are today an essential part of our community. They are used for transportation of both people and goods. A good theoretic model for traffic networks can therefore be useful both for understanding the networks behavior and for being able to control the network. With good control strategies congestion can then be avoided.

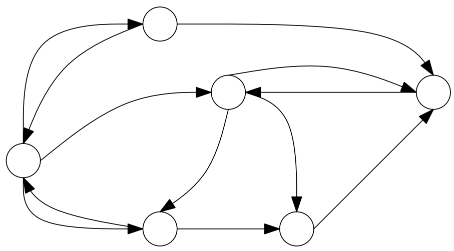

Figure 1: *A simple graph that can exemplifies the structure of the network. The nodes (circles) can be seen as junctions and the links (arrows) can be seen as roads.*

Traffic networks today

Investigations show that traffic networks are already overloaded today in several places, and the demands will increase in the near future. According to the European Union report [European Union, 2011], goods transportation will increase by about 80 % by 2050 and transportation of people will increase by 50 % by the same time. The report also claims that congestion costs about one percent of the European Unions total gross domestic product. Developing good control strategies that avoid congestion is therefore of paramount importance both for economic and environmental reasons.

Our model

In our model for traffic networks, the structure of the network is described by a *graph*. A graph is built up by two components, namely nodes and links which connect nodes. In a traffic network, links can be seen as roads and nodes as junctions, see Figure 1.

We assume that the flow on a link can only go in one direction, so to model a common road two links are needed. In a traffic network different drivers want to reach different destination, but they share common roads. This kind of problem is commonly called a *multi commodity problem*. In our model, for simplicity, we assume that for each destination there is a static inflow of vehicles at some junction.

Some assumptions about the behavior of the traffic and the drivers are also made. For each road, we assume that the velocity of the vehicles lowers when the density increases, and vice versa. It is also assumed that all vehicles in one direction on one road move with the same velocity. When a driver enters a junction, we assume that the driver has a preferred choice about which of the outgoing roads she wants to take. The only variable affecting the driver's choice is the traffic density on the outgoing roads. If the density is high on the preferred road, the driver might chose another one.

Analysis

In the thesis [Nilsson, 2013], it has been shown that if the network is acyclic, i.e., no vehicle can come back to a junction it already visited, and the network follows the aforementioned assumptions, the traffic density on every road reaches asymptotically a value which is independent of the initial conditions. So it does not matter how many vehicles there are on the road when we start observing the network.

Investigations have also been made on how sensitive the traffic network model is to perturbations. Perturbations in a traffic network can for instance occur when a car accident happens, or a road work limits the velocity. When something like that happens, it should nonetheless affect the network in a limited way, so that it does not cause big problems somewhere else.

Control approach

Since the driver in our model only takes the densities on the roads that are leaving the junction into account when making her decisions, it is impossible for her to take the traffic situation further ahead into account. But, by introducing traffic lights that stop cars from entering a congested road, a density increment will occur on the roads before the real congestion. This can then affect the driver's route selection and avoid huge congestions in some cases.

Since a traffic light in the model only takes into account the densities on the roads stemming from a junction when it decides if it should be green or red, the control is *distributed.* A control strategy is distributed when it does not require information about the whole system to be able to control it. The benefits of distributed control are that there is no need for communication links between the traffic lights, and it makes it easier to scale the network, i.e., build new roads and junctions with traffic lights.

Other applications

Even if the model is proposed having traffic networks in mind, there exist other applications where the same model can be used, to mention a few:

Data network Communications between computers, e.g., browsing the web or chatting with somebody, take place by exchange of *data packets* among computers. The data packages are sent through the network along common network media, just like vehicles share roads.

Production chain In a factory, where different products are made, some of the parts in the assembly chain might use the same machine. Thus machines can be seen as links and places where the products are split to further processing as nodes.

Delivery chain When goods and letters are delivered to their recipients, common resources like flights, trains and trucks are used. Here, the transports can be seen as links and the terminals where the goods are reloaded as nodes.

References

[European Union, 2011] European Union (2011). *Transport 2050: The major challenges, the key measures.* `http://europa.eu/rapid/press-release_MEMO-11-197_en.htm`.

[Nilsson, 2013] G. Nilsson, *A multi-commodity dynamical model for traffic networks*, Master's Thesis, Masters Thesis, ISRN LUTFD2/TFRT–5925–SE, Department of Automatic Control, Lund University, Sweden, 2013.