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Master Thesis

The CP violating NMSSM in light of the newly discovered Higgs boson

Morgan Svensson Seth

Theoretical High Energy Physics
Department of Astronomy and Theoretical Physics
Lund University
Sölvegatan 14 A
SE-223 62 Lund

Supervised by: Johan Rathsman

Abstract

The Next to Minimal Supersymmetric Standard Model (NMSSM) allows a CP violating phase at tree level in the Higgs sector. This CP violation will introduce new mixing between the neutral Higgs states and give rise to changes in the mass spectrum and couplings. We investigate these effects in light of the newly discovered Higgs boson, as its measured properties provides new limits on the Higgs sector. We try to impose these new limits and, in particular, investigate the apparent excess in the $\gamma\gamma$ channel. We also have a small discussion about naturalness in the choice of parameters.

1 Introduction

With the recent discovery of a Higgs boson [1] [2] by the ATLAS and the CMS collaboration the final missing piece of the Standard Model of particle physics might soon be in place. The Standard Model (SM) on the other hand is not the final answer, even if the inclusion of gravity is ignored, as there are both experimental results like dark matter which are not included and theoretical problems within the theory. Still the discovery is of great interest as the properties of the found Higgs will put more constraints on beyond the standard model physics. In addition the measured Higgs shows an interesting excess in the decay Higgs going to two photons, although the errors are too large to confirm anything.

One way of extending the standard model is by making it supersymmetric. Supersymmetry [3], [4], [5] relates fermions and bosons and thus one needs to introduce new bosons corresponding to the known fermions and new fermions corresponding to the bosons. As of now no such particle has been discovered but their presence might still be observable as they provide additional intermediate states in SM processes which can alter for example decay processes and cross sections. More directly one needs to introduce new Higgs bosons to provide mass to all particles so the measured Higgs boson can be a mixture of these states and thus have different properties from the standard model Higgs.

Supersymmmetry is introduced for a number of reasons. The main one is the hierarchy problem where, in the SM, the loop corrections to the Higgs mass gives quadratic terms in the cutoff scale typically at the GUT or Planck scale. Supersymmetry solves this issue as boson and fermion loops come with different sign and the contributions cancel each other. Amongst other benefits, there is a cold dark matter candidate if the theory preserves R-parity, the additional particles can provide unification of the different gauge couplings at the GUT scale and supersymmetry is also a necessary ingredient in several other extensions such as string theory.

The minimal supersymmetric extension of the standard model (MSSM) is made by extending all SM fields into superfields, which include both fermionic and bosonic degrees of freedom, thereby representing both the ordinary particles and their new supersymmetric partners. One also has to include an additional Higgs doublet. This is necessary as in a supersymmetric theory the same Higgs doublet cannot provide mass to, for example, both the up and down quarks and therefore provides the smallest extension which includes all of the SM. Even if we ignore the breaking mechanism of Supersymmetry, which is necessary as no supersymmmetric partner have yet been discovered, the MSSM still has problems. Among them is the so called μ-problem which comes from the μ-parameter which is not determined by supersymmetry breaking and that couples the two Higgs doublets in the superpotential. This parameter has the dimension of mass and can in principle be of the order of the Planck mass which would require large cancellations with soft supersymmetry breaking parameters. Furthermore the Higgs sector is very restricted and at tree level the mass of the lightest Higgs is less than the Z boson mass. So to reach a mass of around 125 GeV, significant loop corrections are required. Although this is not entirely impossible, large amounts of fine tuning of the available parameters is necessary.

The Next to Minimal SuperSymmetric Model (NMSSM) on the other hand can solve the μ-problem. By introducing a new scalar singlet that couples to the two Higgs doublets and giving this field a Vacuum expectation Value (VeV) during the supersymmetry breaking one can arrive at an effective μ parameter of the right order. It also has several other advantages. One example is that the Higgs sector is less restricted and a higher Higgs mass is more easily achieved. In addition, of interest to us is that it allows for CP violation at tree level compared to the MSSM where it is only possible at 1-loop level.

In the CP conserving case the addition of this new singlet gives rise to one additional CP even and one CP odd state compared to the MSSM. The masses of these will depend on new parameters of the NMSSM which are not determined by the vacuum conditions of electroweak symmetry breaking and thus introduces a new mass scale to the Higgs sector. This leads to a change in the mass relations from the MSSM as the Higgs states mix. The CP violation will provide an additional mixing of the Higgs states so no clear CP states exist, and give rise to additional changes in the Higgs masses, couplings to other particles etc.

For analyzing the NMSSM we first use the Sarah [9] [10] [11] version 3.2.0 program for an implementation of the CP violating NMSSM. Using this model we then used SPheno [12] [13] version 3.2.2 to get the phenomenology for different choices of input parameters. Finally we also used HiggsBounds [14] [15] [16] version 4.0.0 to get constraints on the Higgs sector.

This thesis is structured as follows. In section 2 we make an introduction to supersymmetry. In section 3 we introduce the NMSSM, give some relevant properties of the Higgs sector and how to achieve CP violation at tree level. In section 4 we discuss the scanned parameter space and discuss the constraints we impose from experiments and from naturalness. In section 5 we present the results from the different scans we performed. The summary and conclusions are in section 6.

2 The basics of supersymmetry

Supersymmetry provides an interesting theory to study and can be approached from two different viewpoints (for a full review of supersymmetry, including more references to the original literature, see for example [6]). First it provides a solution to some of the remaining issues of the standard model as already mentioned in the introduction, on the other hand it can be seen as an extension of the spacetime symmetries of special relativity. Here, as we focus on the Higgs, we will begin with the quadratically divergent loop contributions to the Higgs mass, the so called hierarchy problem in the standard model.

2.1 SM and the Hierarchy problem

Before we move on to this we begin with a short reminder of the SM and what must be included in a supersymmetric theory consistent with experiments. The SM is a gauge theory, that is we obtain interactions of the fields by demanding that the theory is invariant under a local gauge transformation. As the transformation is local a derivative acting on a field will not be invariant under this gauge transformation and the derivatives are therefore replaced with a covariant derivative of the form

$$\nabla_\mu = \partial_\mu + gA_\mu \tag{1}$$

Here the new A_μ is needed to make the derivative invariant under the gauge transformation. From a physical standpoint this new term acting on a field represents an interaction with a gauge field A_μ with a coupling strength g. The SM is invariant under the combination of three gauge groups on the form

$$U(1)_Y \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C \tag{2}$$

and the fields carry different charges under these groups depending on how they interact with them. Here both quarks and leptons carry weak hypercharge (Y), while colour charge (C) is only carried by quarks. The weak interaction (L) on the other hand is a bit different as it only interacts with the left handed components so we must split the left and right handed components as

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \quad d_R, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R \tag{3}$$

and the same for the second and third generation of quarks and leptons. We did not include any ν_R as, for simplicity, we will treat the neutrinos as massless.

The observed $W^\pm$ and Z bosons both have mass which is not directly possible in a gauge theory as mass terms would violate the gauge symmetry. This electroweak symmetry breaking is done via the Higgs mechanism where a new field is introduced which has weak charge and hypercharge. With a potential with a non zero minimum means this field will have a natural Vacuum expectation Value (VeV). As the vacuum is no longer symmetric under the $U(1)_Y \otimes \mathrm{SU}(2)_L$ transformations the symmetry is said to be spontaneously broken. By the Goldstone theorem any broken continuous symmetry will lead to massless particles known as Goldstone bosons, a total of 3 for $U(1)_Y \otimes \mathrm{SU}(2)_L$ breaking. As the broken symmetries are gauge symmetries the gauge fields obtain mass terms in terms of the VeV. The gauge fields mix with the Goldstone bosons which becomes the longitudinal polarisations of the now massive $W^\pm$ and Z bosons. In addition one combination of the gauge fields, namely the photon, is still massless.

The Higgs mechanism can also generate masses for the fermions which cannot have a mass term without violating the $\mathrm{SU}(2)_L$ symmetry. Excitations from this VeV will constitute the final particle of the SM, the Higgs boson. As the VeV part of the field will be used to generate mass it will couple to particles proportional to their mass. Although the SM explains almost all that we know in particle physics it still has a few remaining problems and, as mentioned earlier, the hierarchy problem is a relevant one for supersymmetry.

The hierarchy problem is why does the Higgs boson obtain a mass so much lower than the Planck mass. For a fermion Yukawa term of the form $-\lambda_f H\bar{f}f$ we get a loop contribution to the Higgs mass as

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \log - \text{term} \tag{4}$$

where Λ_{UV} is the momentum cutoff scale used to regulate the loop integral. The value of this cutoff can be of order of the Planck mass which generates a correction 30 orders of magnitude larger than the observed mass. The cutoff can be lower but then one needs to introduce new physics at this scale to account for the cutoff. For other heavy particles, if there existed a heavy scalar S, introduced perhaps to solve some of the remaining problems, it whould have a coupling to the Higgs field of the form $-\lambda_S|H|^2|S|^2$ which would generate a mass correction on the form

$$\Delta m_H^2 = \frac{|\lambda_s|^2}{16\pi^2}\left(\Lambda_{UV}^2 - 2m_S^2 \ln\left(\Lambda_{UV}/m_s\right) + \ldots\right) \tag{5}$$

which also have the same diverging terms.

To get the low mass for the observed Higgs boson all the large contributions must cancel almost perfectly. There are also Higgs theories where the Higgs is not a fundamental particle and therefore can avoid this issue, but otherwise we have to cancel the contributions somehow. Such a cancellation seems very unnatural for parameters that do not need to be related in any way so a convenient way around this is by invoking a symmetry. In particular we want a symmetry that demands the terms to cancel exactly. The different signs between the fermion and boson contributions suggests that a symmetry relating bosons and fermions might be the answer.

Such a symmetry is called supersymmetry and it does not only provide a reasonable way to cancel the quadratic Λ_{UV} terms, in fact they must always cancel exactly, even for higher order contributions. A symmetry that relates bosons to fermions will, as mentioned previously, have to be related to the spacetime symmetry as they transform different under Lorentz transformations. We will return to this later but first we begin by regarding an operator that transforms fermions to bosons.

2.2 A supersymmetric theory

In writing down supersymmetric theories we will use the two-component Weyl spinor notation, as summarized in the Appendix.

If an operator Q generates the transformation between fermionic and bosonic states the operator must itself carry spin, in the simplest case 1/2. Then the operator will have left handed components Q_α, and the hermitian conjugate will be the right handed component. Now these objects must satisfy anticommutation relations as follows

$$\begin{aligned} \{Q_\alpha, Q^\dagger_{\dot\beta}\} &= \sigma^\mu_{\alpha\dot\beta} P_\mu \\ \{Q_\alpha, Q_\beta\} &= \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \end{aligned} \tag{6}$$

Where the P_μ is a conserved 4-vector quantity. By the Coleman-Mandula theorem [7] the choice for P_μ is greatly restricted and in realistic cases it has to be the ordinary 4-momentum. These operators on the other hand commute with the squared mass operator $-P_\mu P^\mu$ and the generators of the gauge transformations. Therefore the new particles must have the same mass and the same electric, weak isospin and colour charges. This means that supersymmetry must be broken and we will come back to that a bit later, first we see how to form a supersymmetric field theory.

The simplest case is a free field theory containing a two component Weyl fermion ψ. By supersymmetry we must also include a complex scalar field as its superpartner. For free fields the respective contributions to the Lagrangian density are given by

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi \quad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi \tag{7}$$

Now a supersymmetry transformation should turn the scalar field into a fermion field and the simplest possibility is

$$\delta\phi = \epsilon\psi \quad \delta\phi^* = \epsilon^\dagger \psi^\dagger \tag{8}$$

where the ϵ^α is an infinitesimal anticommutating two component Weyl fermion, with dimension $[mass]^{-1/2}$, that parametrizes the supersymmetry transformation. This leads to a change in the scalar part of the Lagrangian as

$$\delta\mathcal{L}_{\text{scalar}} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi \tag{9}$$

If we want this to cancel with the change in the fermion part we must have

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi \quad \delta\psi^\dagger_{\dot{\alpha}} = -i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^* \tag{10}$$

This leads to a change

$$\delta\mathcal{L}_{\text{fermion}} = -\epsilon\sigma^\mu\bar{\sigma}^\nu\partial_\nu\psi\partial_\mu\phi^* + \psi^\dagger\bar{\sigma}^\nu\sigma^\mu\epsilon^\dagger\partial_\nu\partial_\mu\phi^* \tag{11}$$

which can be rewritten using identities for the Pauli matrices as

$$\delta\mathcal{L}_{\text{fermion}} = \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi - \partial_\mu\big(\epsilon\sigma^\nu\bar{\sigma}^\mu\psi\partial_\nu\phi^* + \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi\big) \tag{12}$$

Now the first terms cancel and the second term is a total derivative so the action remains unchanged by this transformation. In addition we get the commutator for two different supersymmetry transformations parametrized by ϵ_1 and ϵ_2 to be

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\phi = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\phi \tag{13}$$

So the commutator gives back the derivative of the field. As the operator $-i\partial_\mu$ corresponds to the generator of spacetime translations P_μ we have the same structure as the commutation relation in eq. 6 and the correct form for the commutator can be derived from this relation. But first there is a small problem. For the fermion field we get

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\psi_\alpha = -i(\sigma^\mu\epsilon_1^\dagger)_\alpha\epsilon_2\partial_\mu\psi + i(\sigma^\mu\epsilon_2^\dagger)_\alpha\epsilon_1\partial_\mu\psi \tag{14}$$

This can be rewritten using the Fierz identity and the result is

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\psi_\alpha = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\psi_\alpha + i\epsilon_{1\alpha}\epsilon_2^\dagger\bar{\sigma}^\mu\partial_\mu\psi + i\epsilon_{2\alpha}\epsilon_1^\dagger\bar{\sigma}^\mu\partial_\mu\psi \tag{15}$$

The first term is the desired one but we also get two additional terms. These vanish on shell, when the classical equation of motion $\bar{\sigma}^\mu\partial_\mu\psi = 0$ are satisfied. However in a quantum theory, where one can have off-shell solutions, the equations of motion are not satisfied. In order to solve this an auxiliary field F with no kinetic term is introduced. It has dimension of $[mass]^2$ and a potential term

$$\mathcal{L}_{\text{auxiliary}} = F^*F \tag{16}$$

This gives the equations of motion $F = F^* = 0$ but by introducing supersymmetry transformations as

$$\delta F = -i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi \quad \delta F^* = -i\partial_\mu\psi^\dagger\bar{\sigma}^\mu\epsilon \tag{17}$$

and in addition adding an extra term to the transformation of ψ and $\psi^\dagger$

$$\delta\psi_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F \quad \delta\psi_{\dot{\alpha}}^\dagger = -i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^*\epsilon_{\dot{\alpha}}F \tag{18}$$

we arrive at what we want, that the contributions cancel up to a total derivative term. Finally the equation

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})X = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu X \tag{19}$$

holds for all $X \in \{\phi, \phi^*, \psi, \psi^\dagger, F, F^*\}$ from which it can be shown that the algebra for the supersymmmetry generators is, up to a multiplicative factor, as in eq. 6.

2.3 Chiral interactions

For a proper supersymmetric theory one also needs to introduce interactions, both gauge interactions and non gauge couplings. We begin with the non gauge interactions and try to introduce a potential term to a model similar to the one above. For interactions to be interesting we introduce a number i of different supersymmmetric pairs. So for each fermion field ψ_i we have a corresponding scalar field ϕ_i with an auxiliary field F_i as in the above section. The most general renormalizable potential terms we can write are

$$\mathcal{L}_{int} = \Big(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i + x^{ij}F_iF_j + cc\Big) - U \tag{20}$$

where W^{ij}, W^i, x^{ij} and U are polynomials in the scalar fields ϕ, ϕ^* of degree 1, 2, 0 and 4 respectively. Since the free terms are invariant under supersymmetry transformations by themselves the interaction terms must also be invariant. This eliminates the U term and the x^{ij} as it turns out that nothing can cancel the change in the Lagrangian from these terms under a supersymmetry transformation. The remaining terms, proportional to W^{ij} and W^i, will be related and therefore similar names have been introduced. For the moment we will assume no relation between these and instead derive the necessary relation.

We divide the variations into two parts and demand that they cancel separately. First we consider the case where the variation acts on the W_{ij} and we get

$$\delta\mathcal{L}_{int}|_{on-W_{ij}} = \Big(-\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k}(\epsilon\psi_k)(\psi_i\psi_j) - \frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k^*}(\epsilon^\dagger\psi_k^\dagger)(\psi_i\psi_j) + c.c.\Big) \tag{21}$$

The contribution $(\epsilon\psi_k)(\psi_i\psi_j)$ does not occur anywhere else and the only way to have them cancel is via the Fierz identity

$$(\epsilon\psi_k)(\psi_i\psi_j) + (\epsilon\psi_i)(\psi_j\psi_k) + (\epsilon\psi_j)(\psi_k\psi_i) = 0 \tag{22}$$

Thus the $\frac{\delta W^{ij}}{\delta\phi_k}$ must be entirely symmetric under interchange of i, j, k. In addition, for the term $(\epsilon^\dagger\psi_k^\dagger)(\psi_i\psi_j)$ there is no such relation and the W^{ij} must thus not contain ϕ^*. In other words, the W^{ij} must be a holomorphic function of ϕ. So now we have

$$W^{ij} = M^{ij} + y^{ijk}\phi_k \tag{23}$$

In fact it is convenient to write this term as

$$W^{ij} = \frac{\delta^2}{\delta\phi_i\delta\phi_j}W \tag{24}$$

where we have introduced the so called superpotential W

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k \tag{25}$$

which is not an ordinary potential, it is a holomorphic function in the scalar fields ϕ. From the variational part that contains a spacetime derivative we have

$$\delta\mathcal{L}_{int}|_\partial = \big(iW^{ij}\partial_\mu\phi_j\psi_i\sigma^\mu\epsilon^\dagger + iW^i\partial_\mu\psi_i\sigma^\mu\epsilon^\dagger + c.c.\big) \tag{26}$$

this will turn out to be a total spacetime derivative if

$$W^i = \frac{\delta W}{\delta\phi_i} \tag{27}$$

and so the relation between the W^i and W^{ij} is established in terms of the superpotential. In fact the superpotential does more than that. The new terms F_iW^i and $F_i^*W^{i*}$ gives the equations of motion for the F field as

$$F_i = W_i^*, \quad F_i^* = W_i, \tag{28}$$

so the F terms in the ordinary potential can be entirely rewritten in terms of the superpotential, in what is called an F-term.

2.4 Gauge interactions

We also need to incorporate gauge interactions in our picture. As the approach will be fairly similar to the previous one we will mostly state the results with little or no derivations. For a gauge field A^a_μ where a runs over the adjoint representation of the gauge group (a=1,. . . ,8 for SU(3) 1, 2, 3 for SU(2) and 1 for U(1)). We also need to introduce the corresponding two component Weyl fermions λ^a. These will now transform under the gauge transformations as

$$A^a_\mu \to A^a_\mu + \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c \tag{29}$$

$$\lambda^a \to \lambda^a + g f^{abc} \lambda^b \Lambda^c \tag{30}$$

where Λ parametrizes an infinitesimal gauge transformation, g is the coupling strength and f^{abc} are the structure constants of the gauge group. As before we need an auxiliary field D^a with the same gauge transformation as the λ^a, dimension $[mass]^2$ and an equation of motion $D^a = D^{a*} = 0$. The Lagrangian is given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a \tag{31}$$

with the usual field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{32}$$

and covariant derivative of the λ^a as

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c \tag{33}$$

The supersymmetric transformations are given by

$$\begin{aligned} \delta A^a_\mu &= -\frac{1}{\sqrt{2}} \big(\epsilon^\dagger \bar{\sigma}^\mu \lambda^a + \lambda^{\dagger a} \bar{\sigma}_\mu \epsilon \big) \\ \delta \lambda^a_\alpha &= \frac{i}{2\sqrt{2}} (\sigma^\mu \bar{\sigma}^\nu \epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon_\alpha D^a \\ \delta D^a &= \frac{i}{\sqrt{2}} \big(- \epsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \lambda^a + \nabla_\mu \lambda^{\dagger a} \bar{\sigma}^\mu \epsilon \big) \end{aligned} \tag{34}$$

and the supersymmetric transformations also satisfy

$$(\delta_{\epsilon_1} \delta_{\epsilon_2} - \delta_{\epsilon_2} \delta_{\epsilon_1}) X = i(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger) \nabla_\mu X \tag{35}$$

for $X \in \{F^a_{\mu\nu}, \lambda^a, \lambda^{\dagger a}, D^a, D^{a*}\}$ similar to the previous result, but with the covariant derivative instead of the ordinary derivative.

We must finally introduce the gauge coupling together with the ϕ and ψ fields previously discussed. To do this we must change all the ordinary spacetime derivatives to covariant ones

$$\nabla_\mu X_i = \partial_\mu X_i, -igA^a_\mu T^{aj}_i X_j \tag{36}$$

for $X_i \in \{\psi_i, \phi_i, F_i\}$ where we let the fields transform under the gauge group in a representation with the hermitian matrices $(T^a)^i_j$ satisfying $[T^a, T^b] = if^{abc}T^c$.

This couples the vector bosons to the fermions and scalars. The λ^a and D^a must also have the corresponding couplings in order for the theory to be supersymmetric and the terms turn out to be

$$\mathcal{L}_{extra} = -\sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a \tag{37}$$

Finally the supersymmetric transformations for the ϕ, ψ, and F must also have covariant derivatives and an additional term in the δF as

$$\begin{aligned}
&\delta\phi = \epsilon\psi \\
&\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \nabla_\mu \phi + \epsilon_\alpha F \\
&\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \psi_i + \sqrt{2}g(T^a\phi)\epsilon^\dagger \lambda^{\dagger a}
\end{aligned} \tag{38}$$

and similar for the hermitian conjugates. As a final point the equations of motion for the D field will now be

$$D^a = -g(\phi^* T^a \phi) \tag{39}$$

So similarly to the F field, which could be expressed in terms of the superpotential by the so called F-term the D^a can be expressed in the scalar fields which is called the D-term.

2.5 Superfields

Now we have a fully functioning supersymmetric model but there is another approach which is more close to special relativity. One can express a supersymmetric theory by introducing new anticommutating coordinates $\theta^\alpha, \theta^\dagger_{\dot{\alpha}}$ in addition to the x^μ spacetime coordinates. Therefore we need to know a bit about anticommutating Grassman variables η.

For a function in the η variable, using that $\eta^2 = 0$ we get that a Taylor expansion in η terminates

$$f(\eta) = f_0 + f_1\eta \tag{40}$$

from this expansion derivatives of a function is simply

$$\frac{d}{d\eta} f(\eta) = f_1 \tag{41}$$

but more interesting, if we want to define integration in a way with similar properties as the normal integral, such as integration by parts, one defines it as

$$\int f(\eta)d\eta = f_1 \tag{42}$$

So integration and derivation are the same. For our case we need the new variables θ^α and $\theta^\dagger_{\dot{\alpha}}$ to be complex anticommutating two component spinors with dimension $[mass]^{-1/2}$. As they combine with the totally antisymmetric $\epsilon_{\alpha\beta}$

$$\theta\theta = \theta^\alpha \epsilon_{\alpha\beta} \theta^\beta \tag{43}$$

the squared terms are not zero but the higher order terms are. Thus a field $S(x^\mu, \theta, \theta^\dagger)$ over superspace can be expanded in the θ and $\theta^\dagger$ with coefficients only depending on x^μ

$$S(x^\mu, \theta, \theta^\dagger) = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c + \theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d \tag{44}$$

In superspace we can define supersymmetry transformations by defining the operators

$$\hat{Q}_\alpha = i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu \quad \hat{Q}^\dagger_{\dot{\alpha}} = -i\frac{\partial}{\partial\theta^{\dot{\alpha}\dagger}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu \tag{45}$$

From this we can see that they fulfill the anticommutation relations required for a supersymmetry transformation

$$\begin{aligned} \{\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot{\beta}}\} &= 2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu \\ \{\hat{Q}_\alpha, \hat{Q}_\beta\} &= \{\hat{Q}^\dagger_{\dot{\alpha}}, \hat{Q}^\dagger_{\dot{\beta}}\} = 0 \end{aligned} \tag{46}$$

The supersymmetry transformation δ_ϵ of a superfield S is defined in terms of the $\hat{Q}$ as

$$\delta_\epsilon S = -\frac{i}{\sqrt{2}}(\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger)S \tag{47}$$

For the physical fields to be invariant under supersymmetry transformations some constraints must be put upon them. First are the chiral superfields Φ which can be described by the so called chiral covariant derivatives given by

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu \quad D^\dagger_{\dot{\alpha}} = -\frac{\partial}{\partial\theta^{\dot{\alpha}\dagger}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu \tag{48}$$

Some algebra shows that these commute with the supersymmetry transformation δ_ϵ so the derivatives are covariant with respect to supersymmetry.

We can now define a chiral superfield Φ as satisfying

$$D^\dagger_{\dot{\alpha}}\Phi = 0 \tag{49}$$

This constraint can be solved and the solution can be written as

$$\Phi = \phi + i\theta^\dagger\bar{\sigma}^\mu\theta\partial_\mu\phi + \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\partial_\mu\partial^\mu\phi + \sqrt{2}\theta\psi - \frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\bar{\sigma}^\mu\partial_\mu\psi + \theta\theta F \tag{50}$$

where our naming of the variables ϕ, ψ and F comes as, if one does the algebra, they have the same properties and supersymmetry transformations as the corresponding fields in the previous sections.

We also need to introduce the vector superfield V which comes from imposing that the field is real, that is $V = V^*$. This will lead to the following constraints on a general superfield

$$a = a^*, \quad \xi^\dagger = \chi^\dagger, \quad c = b^*, \quad v_\mu = v_\mu^*, \quad \eta^\dagger = \zeta^\dagger, \quad d = d^* \tag{51}$$

By redefining the fields

$$\eta_\alpha = \lambda_\alpha - \frac{i}{2}(\sigma^\mu\partial_\mu\xi^\dagger)_\alpha, \quad v_\mu = A_\mu, \quad d = \frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a \tag{52}$$

we can also get the gauge fields with the same properties as in the previous section. The vector field will be

$$\begin{aligned} V(x^\mu, \theta, \theta^\dagger) =& a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* + \theta^\dagger\bar{\sigma}^\mu\theta A_\mu + \theta^\dagger\theta^\dagger\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger) + \\ &+ \theta\theta\theta^\dagger(\lambda^\dagger - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\xi) + \theta\theta\theta^\dagger\theta^\dagger\big(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\big) \end{aligned} \tag{53}$$

Now there are some additional fields, the a, ξ and b fields. Fortunately one can do a supergauge transformation to eliminate these new fields in the so called Wess-Zumino gauge.

Just as we integrate a Lagrangian density over the entire spacetime we can construct a superspace Lagrangian by also integrating over the superspace coordinates. To ensure that the theory is supersymmetric one needs to form terms that are invariant under the supersymmetry transformation.

For a vector field it turns out that the so called D-term

$$[V]_D = \int d^2\theta d^2\theta^\dagger\, V(x^\mu, \theta, \theta^\dagger) \tag{54}$$

is invariant under supersymmetry transformations and thus is a valid term in a Lagrangian. Another way to form an invariant term is for a chiral field in a so called F-term

$$[\Phi]_F = \int d^2\theta\, \Phi = \int d^2\theta d^2\theta^\dagger\, \delta^2(\theta^\dagger)\Phi \tag{55}$$

As a product of chiral superfields is a chiral superfield, any holomorphic function of superfields must also be chiral. Thus we once again introduce the superpotential W but now as a function of the chiral superfields Φ. In addition if we form a vector superfield from a chiral one as $\Phi\Phi^*$ we can form an ordinary Lagrangian density as

$$\mathcal{L} = [\Phi^i \Phi_i^*]_D + [W(\Phi)]_F \tag{56}$$

If we put in the same superpotential as in eq. 25 but with a superfield Φ instead of the scalar field ϕ we obtain the same Lagrangian density.

We do not introduce the gauge interactions here, it requires a bit of extra work but one obtains the same result as in the previous section. Instead the important idea is that we can formulate the same theory in terms of superfields and a superpotential. Thus to specify a supersymmetric theory we as usual need the field content, although this time in terms of superfields and their charges under gauge groups. In addition the rest of the physically allowed interactions can be expressed in terms of the superpotential.

2.6 Supersymmetry breaking and the MSSM

We know that supersymmetry cannot be an exact symmetry since the superpartners must have the same mass and charges as the ordinary particles and no supersymmetric partner has been found. To explain this some sort of breaking mechanism is needed to make the supersymmetric partners too massive for detection. Usually supersymmetry is broken by spontaneous symmetry breaking, just like the electroweak breaking, that is the fundamental theory is supersymmetric but we get a vacuum state that is not. There are many ways that supersymmetry can be broken, through so called F and D-term breaking, through gauge and gravity mediated breaking. In most practical cases however we make no assumptions how the supersymmetry is broken and simply parametrize the breaking by the so called soft breaking terms

$$\mathcal{L}_{soft} = -\Big(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i + c.c\Big) - (m^2)^i_j\phi^{j*}\phi_i \tag{57}$$

The terms consist of masses M_a for the gauge fermions λ, scalar squared masses b^{ij} and $(m^2)^i_j$, scalar couplings a^{ijk} and finally so called tadpole couplings t^i. These terms might seem a bit arbitrary but a supersymmetric theory with these soft breaking terms will not have quadratic divergences. In addition the different ways of generating supersymmetry breaking will in most cases give terms of this form.

To see how a realistic supersymmetric theory looks like, we can use the Minimal Supersymmetric Standard Model (MSSM) as an example. In this model we need to specify superfields so that all the SM fields are included. First we have the gauge superfields in Table 1. and the chiral superfields in Table 2. In these tables we have introduced the naming convention for the supersymmetric particles (sparticles). The supersymmetric partner field is usually indicated by a $\sim$ on top. The scalar superpartners are named by adding an ”s” before the ordinary particles name and for the fermionic superpartners we end their name with ”-ino”.

In addition, just as the B and the three W bosons mix to the γ, Z and $W^{\pm}$, the Higgsinos, winos and binos are mixed and the physical states are called charginos and neutralinos.

Name	SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$
bino, B boson	$\hat{B}$	$\lambda_{\tilde{B}}$	B	$U(1)$
winos, W bosons	$\hat{W}$	$\lambda_{\tilde{W}}$	W	SU(2)
gluino, gluon	$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)

Table 1: Gauge superfields (SF) in the MSSM and the corresponding ordinary and supersymmetric particles.

Name	SF	Spin 0	Spin $\frac{1}{2}$	$(U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
squarks, quarks	$\hat{Q}$	$(\tilde{u}_L,\tilde{d}_L)$	(u_L,d_L)	$(\frac{1}{6},\mathbf{2},\mathbf{3})$
(× 3 families)	$\hat{u}$	$\tilde{u}^*_R$	$u^\dagger_R$	$(-\frac{2}{3},\mathbf{1},\mathbf{3})$
	$\hat{d}$	$\tilde{d}^*_R$	$d^\dagger_R$	$(\frac{1}{3},\mathbf{1},\mathbf{3})$
sleptons, leptons	$\hat{L}$	$(\tilde{\nu},\tilde{e}_L,)$	(ν,e_L)	$(-\frac{1}{2},\mathbf{2},\mathbf{1})$
(×3 families)	$\hat{e}$	$\tilde{e}^*_R$	$e^\dagger_R$	$(1,\mathbf{1},\mathbf{1})$
Higgs, Higgsinos	$\hat{H}_u$	(H^+_u,H^0_u)	$(\tilde{H}^+_u,\tilde{H}^0_u)$	$(\frac{1}{2},\mathbf{2},\mathbf{1})$
	$\hat{H}_d$	(H^0_d,H^-_d)	$(\tilde{H}^0_d,\tilde{H}^-_d)$	$(-\frac{1}{2},\mathbf{2},\mathbf{1})$

Table 2: The Chiral superfields in the MSSM, their charges under the gauge groups and the corresponding ordinary and supersymmetric particles.

Apart from the supersymmetric partners the only extra bit compared to the standard model is an additional Higgs doublet. It is necessary, as the superpotential must be a holomorphic function in the superfields which prevents the use of both the H and $H^\dagger$ to generate the up and down quarks respectively as is done in the SM. Instead we need two different Higgs fields to make up the mass terms without using complex conjugates.

Before we come to the superpotential we want to consider the so called R parity. In the SM one has the lepton and baryon number for which no experimental violation have been seen. In supersymmetric theories this is introduced in terms of matter parity

$$P_M = (-1)^{3(B-L)} \tag{58}$$

which forbids terms that violate conservation of baryon and lepton number in the superpotential. This gives the quarks and leptons a matter parity of 1 and the Higgs fields matter parity of -1. For a supersymmetric theory we rewrite this in terms of R-parity as

$$P_R = (-1)^{3(B-L)+2s} \tag{59}$$

which is obtained by adding a term proportional to the spin "s". This is the same as matter parity as the product of $(-1)^{2s}$ for all particles in a vertex must be +1 by conservation of angular momentum. Using R-parity instead of matter parity shows the difference between the ordinary and supersymmetric partners. Sparticles have an R-parity of −1 while ordinary particles all get +1. Thus R parity means that there can be no mixing between the ordinary particles and the sparticles. In addition any interaction vertex must involve an even number of sparticles. This has important phenomenological consequences. First this means that we always produce an even number of sparticles in collider experiments. In addition a single sparticle cannot decay into a state of only ordinary particles which means any free sparticle that is not the lightest one must decay into a final state with at least one particle being the lightest sparticle. In addition this lightest sparticle (referred to as the LSP) must be stable and such provides a candidate for dark matter. As the name Minimal suggests, R-patrity is always assumed to hold in the MSSM.

The superpotential for the MSSM (the neutrinos are assumed massless for simplicity) is given by

$$W_{MSSM} = (y_u^{ij}\hat{u}((\hat{Q}^T)^i\epsilon\hat{H}_u^j) + y_d^{ij}\hat{d}^i((\hat{Q}^T)^j\epsilon\hat{H}_d) + y_e^{ij}\hat{e}^i((\hat{L}^T)^j\epsilon\hat{H}_d) + \left[\mu(\hat{H}_u^T\epsilon\hat{H}_d)\right] \quad (60)$$

where the index i and j runs over the 3 generations. The y^{ij} matrices are dimensionless couplings, with the diagonal part giving the fermion mass terms and the off diagonal elements giving rise to the CKM mixing for quarks. In addition there is a term with the two Higgs fields corresponding to the Higgs mass term of the SM. Finally supersymmetry breaking is needed but unless a specific breaking scenario is considered it is introduced via soft breaking terms as in eq. 57. We get a number of new interactions and the general form of these new interactions can be seen in fig. 1. Of course the interacting particles must carry correct charge for gaugino interactions and all of the usual quantum numbers must be conserved.

In addition to the scalar interactions the extra Higgs doublet gives rise to several new Higgs states. In total there are now 8 real degrees of freedom for the Higgs fields. As in the SM three off these will end up as longitudinal components of the $W^{\pm}$ and Z bosons but it now leaves a total of five Higgs fields remaining. Of these, two will be charged and for the three uncharged, two will be scalar and one will be a pseudoscalar. The Higgs sector will be the focus of the remaining paper but in our case it will be the NMSSM which mainly differ from the MSSM by its two additional Higgs states.

3 CPV in the NMSSM

For the NMSSM (see [8] for a general introduction to the NMSSM) one wants to generate the μ-term in the Higgs potential dynamically. For that we will introduce a new Higgs superfield $\hat{S}$ with no electric charge and a singlet under $SU(2)$ and $SU(3)$. This will give rise to two additional Higgs states and as they mix with the MSSM Higgs states they will alter their properties.

Here we use the Z_3-symmetric version and extend the MSSM superpotential, with the μ-term set to 0, as

$$W_{NMSSM} = W_{MSSM} + \lambda\hat{S}(\hat{H}_u^T\epsilon\hat{H}_d) + \frac{\kappa}{3}\hat{S}^3 \quad (61)$$

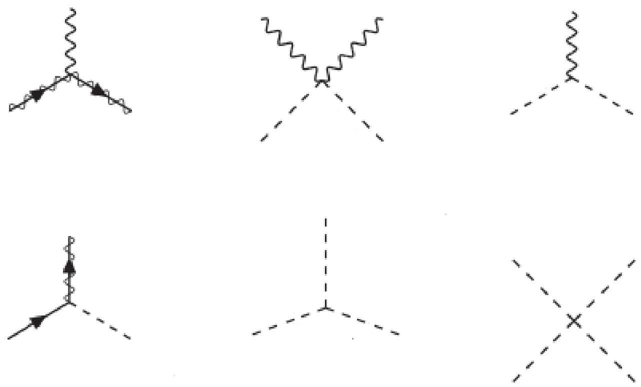

Figure 1: New interactions from the supersymmetric theory. For the Feynman diagrams a solid line with a wavy line on top corresponds to the gauginos and the dotted line to scalars, both Higgs fields and sfermions. Reading from top left we have: the gauginos can interact with the ordinary gauge fields, the scalars can also interact with the gauge fields and on the second row the gaugino can also interact similar to a gauge boson but it turns a fermion to a scalar and finally the sfermions and Higgs can have 3 and 4 scalar interactions.

In addition supersymmetry must be broken and new supersymmetry breaking terms must be introduced for the new singlet field S in the ordinary potential as

$$V_{NMSSM} = V_{MSSM} + m_S^2|S|^2 + (\lambda A_\lambda S(H_u^T \epsilon H_d) + \frac{1}{3}\kappa A_\kappa S^3 + h.c.) \tag{62}$$

The new λ parameter will give rise to an effective μ term as λv_s where v_s is the vacuum expectation value for the new S field. The κ term is necessary since without it the superpotential would have a global $U(1)$ Peccei-Quinn symmetry. This symmetry will be broken when the S-field acquires a VeV and by the Goldstone theorem there whould be a massless Perci-Quinn mode which has not been observed. Thus these two parameters are the essential to the NMSSM. For the other supersymmetry breaking parameters we use generic soft breaking terms.

This will give us the tree level Higgs potential as

$$V_{\text{Higgs,NMSSM}} = V_D + V_F + V_{soft} \tag{63}$$

where

$$\begin{aligned}
V_D &= \frac{g_2^2}{8}(|H_u^\dagger H_d|^2) + \frac{g_1^2}{8}(|H_u^\dagger H_u|^2 - |H_d^\dagger H_d|^2) \\
V_F &= |\lambda|^2|S|^2(H_u^\dagger H_u + H_d^\dagger H_d) + |\lambda|^2|H_u^T \epsilon H_d|^2 + |\kappa|^2||S|^4 - (\lambda\kappa^* H_d^T \epsilon H_u S^{*2} + h.c.) \\
V_{soft} &= m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_S^2|S|^2 + (\lambda A_\lambda S(H_u^T \epsilon H_d) + \frac{1}{3}\kappa A_\kappa S^3 + h.c.)
\end{aligned} \tag{64}$$

In the Higgs sector of the MSSM, any possible complex phase in the Higgs sectors parameters can be rotated away by redefining the fields and thus the MSSM must be CP conserving at tree level. In the NMSSM this is not possible for the following reason. The phase of $H_d^T \epsilon H_u$ and S can be used to rotate away any complex phase in κA_κ and λA_λ but there are still complex phases in λ and κ. These phases only enter as physical parameters in the term $\lambda\kappa^*$ so a phase $\phi = arg(\lambda\kappa^*)$ must be introduced.

There is also the possibility of having spontaneous CP violation in the Higgs sector by having relative phases for the complex VeV of the Higgs fields. One of the phases, taken to be v_d, can be rotated away and the v_u and v_s have phases ϕ_u and ϕ_s respectively. In the end, taking into account the so called tadpole equations (eq. 65 below), the only independent CP violating parameter will enter as a linear combination of these as $I = \text{Im}(\lambda\kappa^* e^{\phi_u - 2\phi_s})$, so one can still treat $\phi = \text{Arg}[\lambda\kappa^*]$ as the CP violating phase. In addition there are several possibilities to introduce CP violation at 1-loop level, just as in the MSSM, but the focus of this paper is on the tree-level CP-violation so we will not look at what effects these forms of CP violation have.

As already mentioned the parameters of the Higgs potential cannot be chosen independently of each other as the vacuum must have a stationary point at the VeV. These conditions are called the tadpole equations and are given by

$$
\begin{aligned}
&v_d m_{H_d}^2 - R_\lambda v_u v_s \frac{g_1^2+g_2^2}{8} v_d(v_d^2 - v_u^2) + \frac{|\lambda|^2}{2} v_d(v_u^2+v_s^2) - \frac{R}{2} v_u v_s^2 = 0 \\
&v_u m_{H_u}^2 - R_\lambda v_d v_s \frac{g_1^2+g_2^2}{8} v_u(v_d^2 - v_u^2) + \frac{|\lambda|^2}{2} v_u(v_d^2+v_s^2) - \frac{R}{2} v_d v_s^2 = 0 \\
&v_s m_S^2 - R_\lambda v_d v_u + R_\kappa v_s^2 + \frac{|\lambda|^2}{2} v_D(v_d^2+v_s^2) + |\kappa|^2 v_s^3 - R v_d v_u v_s = 0 \\
&I_\lambda v_u v_s + \frac{I}{2} v_u v_s^2 = 0 \\
&I_\lambda v_d v_s + \frac{I}{2} v_d v_s^2 = 0 \\
&I_\lambda v_d v_u - I_\kappa v_s^2 + I v_d v_u v_s = 0
\end{aligned}
\tag{65}
$$

where we have the vacuum expectation values $v^2 \equiv v_d^2 + v_u^2$, v_s for the respective Higgs fields and $\tan\beta = v_u/v_d$. We also use the following shorthand for the real and imaginary parts

$$
\begin{aligned}
R &= Re(\lambda\kappa^* e^{i(\phi_u - 2\phi_S)}) \\
R_\lambda &= Re(\lambda A_\lambda e^{i(\phi_u + \phi_S)}) \\
R_\kappa &= Re(\kappa A_\kappa e^{i3\phi_S})
\end{aligned}
\tag{66}
$$

$$
\begin{aligned}
I &= Im(\lambda\kappa^* e^{i(\phi_u - 2\phi_S)}) \\
I_\lambda &= Im(\lambda A_\lambda e^{i(\phi_u + \phi_S)}) \\
I_\kappa &= Im(\kappa A_\kappa e^{i3\phi_S})
\end{aligned}
\tag{67}
$$

The last three tadpole conditions makes it possible, as previously mentioned, to eliminate two phases and express all the imaginary parts in terms of I. The other three tadpole conditions can be used to eliminate three other parameters. In our study we will solve for the R_κ, R_λ (thus indirectly determine the A_κ and A_λ) and $m_{H_d}^2$. The remaining parameters cannot be removed and will not directly depend on others. Thus we can solve for the physical properties such as the masses of the Higgs bosons in terms of these parameters.

For the NMSSM Higgs sector, the additional S field gives an additional scalar and pseudoscalar Higgs for a total of 3 scalars and 2 pseudoscalars in the CP conserving case. In the CP violating case, the two types will be mixed and there are no clearly defined CP states. The mass matrix for the neutral Higgs bosons will thus be

$$
\begin{pmatrix} M_S & M_{CPV} \\ (M_{CPV})^T & M_{PS} \end{pmatrix}
\tag{68}
$$

Where M_S is the 3×3 mass matrix for the scalars and M_{PS} the 2×2 mass matrix for the pseudoscalars. In the CP conserving case these are the only terms present so they can be diagonalized separately. The CP violation introduces the M_{CPV} term, which depends on the previously mentioned $I = Im(\lambda\kappa^* e^{\phi_u - 2\phi_s})$ and is given by

$$M_{CPV} = \frac{Iv}{2}\begin{pmatrix} 0 & -3v_s s_\beta \\ 0 & -3v_s c_\beta \\ v_s & 4v s_\beta c_\beta \end{pmatrix} \tag{69}$$

where $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$. The other matrices are given by

$$M_S = \begin{pmatrix} m_z^2 c_\beta + (R_\lambda + \frac{Rv_s}{2})\frac{v_s v_u}{v_d} & |\lambda|^2 v_d v_u - m_Z^2 s_\beta c_\beta - R_\lambda v_s - \frac{Rv_s^2}{2} & |\lambda|^2 v_d v_s - R_\lambda v_u - Rv_u v_s^2 \\ |\lambda|^2 v_d v_u - m_Z^2 s_\beta c_\beta - R_\lambda v_s - \frac{Rv_s^2}{2} & m_z^2 s_\beta + (R_\lambda + \frac{Rv_s}{2})\frac{v_s v_d}{v_u} & |\lambda|^2 v_u v_s - R_\lambda v_d - Rv_d v_s^2 \\ |\lambda|^2 v_d v_s - R_\lambda v_u - Rv_u v_s^2 & |\lambda|^2 v_u v_s - R_\lambda v_d - Rv_d v_s^2 & 2|\kappa|^2 v_s^2 + \frac{R_\lambda v_d v_u}{v_s} + R_\kappa v_s \end{pmatrix} \tag{70}$$

$$M_{PS} = \begin{pmatrix} (2R_\lambda + Rv_s)/\sin(2\beta) & (R_\lambda - Rv_s)v \\ (R_\lambda - Rv_s)v & R_\lambda v^2 \sin(2\beta)/(2v_s) - 3R_\kappa v_s + Rv^2\sin(2\beta) \end{pmatrix} \tag{71}$$

Diagonalizing the CP-violating mass matrix gives 5 Higgs states with no definite CP properties denoted $h_1, \ldots, h_5$ with corresponding eigenvalues $m_{h_1}^2, \ldots m_{h_5}^2$. These are as usual arranged in increasing order so h_1 is the lightest Higgs. If the lightest Higgs boson is mostly pseudoscalar then it is possible that the discovered boson corresponds to the next to lightest Higgs but otherwise the discovered boson should be the lightest one. In the CP conserving case the tree level limit on the mass of the lightest CP even boson will be increased to

$$m_{H_1}^{NMSSM} < m_Z^2\Big(\cos^2(2\beta) + \frac{2\lambda^2\sin^2(2\beta)}{g_1^2 + g_2^2}\Big) \tag{72}$$

compared to just the first term $m_Z^2\cos^2(2\beta)$ for the MSSM. For the CP violating case this limit will instead limit the lightest Higgs, without any constraints on its CP properties. This comes about as the CP violation terms in the mass matrix are off diagonal and symmetric, and such terms will decrease the lowest eigenvalue (and increase the highest). For the remaining Higgs particles the change will vary from case to case. There are still some limits to the modification as the diagonalization leaves the trace unchanged so the sum of the masses squared will still remain unchanged by a CP violating term.

However, things might not be as convenient as first imagined. If we solve for R_κ and R_λ using the tadpole equations one gets,

$$R_\lambda = \frac{1}{v_d v_s}\Big(-\frac{1}{2}Rv_d v_s^2 + m_{H_u}^2 v_u + \frac{g_1^2 + g_2^2}{8}v_u(v_d^2 - v_u^2) + |\lambda|^2 v_u(v_d^2 + v_s^2)\Big) \tag{73}$$

$$R_\kappa = \frac{1}{v_s^3}\Big(\frac{1}{2}Rv_d v_u v_s^2 - m_S^2 v_s^2 + m_{H_u}^2 v_u^2 + \frac{g_1^2 + g_2^2}{8}v_u^2(v_d^2 - v_u^2) + |\lambda|^2(v_d^2 v_u^2 + \frac{v_s^2}{2}(v_u^2 - v_d^2)) - v_s^4|\kappa|^2\Big) \tag{74}$$

where we note that these parameters depend on the real part of R which changes with the CP violating phase. This means that a change of phase cannot be done without altering some other parameters which are not the explicit CP violating ones. For our choice of free parameters, the change in the diagonal terms in the mass matrices will cancel so the CP phase will still not change the masses directly at tree level.

The mass of the charged Higgs boson can also be solved for in these parameters and one gets

$$m_{H\pm}^2 = m_W^2 - |\lambda|^2 v^2/2 + (2R_\lambda + Rv_s)v_s/\sin(2\beta) \tag{75}$$

where once again the CP violating contributions cancel and the charged Higgs mass is unaffected by a change in CP phase at tree level.

The implementation of the model is done through the SARAH and SPheno programs. As a supersymmetric model only requires the particle content, gauge groups, superpotential and the supersymmetry breaking terms, a supersymmetric model can be defined rather easily. From this much work is required to derive the ordinary potential, calculate the Feynman rules, derive and diagonalize mass matrices and rotate to mass eigenstates. This is what Sarah does and a bit more. The program also calculates the masses to next to leading order and the renormalization group equations at two loops. In addition, among the many versions of output, it can output the analytical expressions derived in terms of source code for SPheno. With this source code, if values for the parameters are provided, either at a grand unified level or, as in this thesis, at a lower energy scale, Spheno calculates the physical properties. The program solves the two loop renormalization group equations numerically and all the masses of the supersymmetric particles are calculated, including mixing, flavor structure and CP violation, up to one loop level. In addition other measurable properties are then derived such as decay widths and branching ratios. SPheno also calculates changes in several low energy observables compared to the SM such as $b \to s\gamma$ decays and the anomalous magnetic moments.

4 Parameter space and constraints

For the Higgs sector in the CP conserving NMSSM there are six independent parameters usually taken to be κ, λ, A_λ and A_κ from the Higgs potential and tanβ and v_s the VeVs. As the S field was introduced to solve the μ-problem it is convenient to use $\mu_{eff} = \lambda v_s$ instead of v_s.

In addition, as mentioned before, we will also replace A_κ and A_λ, which is necessary in the CP violating case, with $m_{H_u}^2$ and m_S^2 from the Higgs potential. This is not usually done, as the more commonly used approach, even in the CP conserving case, is to replace them with a Higgs mass term such as m_{h^+}. Instead we solve for A_κ and A_λ using $m_{H_u}^2$ and m_S^2 as input in the tadpole conditions. As mentioned in the previous section this is a convenient choice as it does not lead to a direct change of the Higgs masses but only alters the mixing between them.

For the parameters that do not appear directly in the Higgs part of the potential, we consider a universal M_{SUSY} scale for the sfermion masses at the supersymmetry breaking scale. So the corresponding mass matrices M_Q, M_L etc. are diagonal and have all diagonal elements equal to $M_{\rm SUSY}$ which we take to be 1 TeV. The gaugino masses are taken to be related as in the constrained MSSM with unification at $M_{\rm GUT}$, thus we use $M_1 = 100$ GeV, $M_2 = 200$ GeV and $M_3 = 800$ GeV.

For the parameters that are varied, we input their values at the low energy scale Q=120 GeV and the ranges are chosen as follows below. Note that λ and κ are given real ranges and the CP violating phase is varied separately as we will also investigate the CP conserving case for comparison.

$$\begin{aligned}
&\tan\beta \in [1, 60] \\
&\lambda \in [0, 0.7] \\
&\kappa \in [-0.7, 0.7] \\
&\mu \in [100, 1000] \text{ GeV} \\
&A_t = A_b = A_\tau \in [-5000, 5000] \text{ GeV} \\
&m_{H_u}^2 \in [-0.5, 10] \text{ TeV}^2 \\
&m_S^2 \in [-10, 10] \text{ TeV}^2 \\
&\phi = arg(\lambda\kappa^*) \in [0, 2\pi]
\end{aligned} \tag{76}$$

The reasoning behind the choices are as follows. For λ, κ and $\tan\beta$ we demand perturbativity up to the GUT scale and then the parameters have to be constrained within these ranges [17]. In addition it is only the sign of the product $\lambda\kappa$ which is important in the CP conserving case and in that case the sign is assigned to κ. For the μ parameter we have a lower bound from experimental limits on the Higgsino mass. The upper bound is not a fixed one, but since one reason we introduced the NMSSM was to allow a natural low μ parameter we do not allow it to get too large as one needs more fine tuning of the model with a larger μ. For the different A parameters we want to in part consider the m_h^{max} [18] scenario. The A_t parameter describes the mixing between the right and left handed top squarks which contributes to the Higgs mass at loop level. The contribution to the Higgs mass has a maximum for $A_t = \sqrt{6}M_{SUSY}$, and similarly for the other A's. Furthermore, as we will see, the solutions will be unphysical at a bit higher A so we include the full range of allowed A's. The $m_{H_u}^2$ and m_S^2 have no direct limits on them. On the other hand, from a naturalness point of view, they are supersymmetry breaking parameters which we assumed previously to be of the mass scale of order TeV, so the masses should be at most a few TeV. The harder limit on negative $m_{H_u}^2$ arises as other physical mass terms get a negative mass squared, and thus we have not gotten any allowed point outside this range from SPheno (see the left plot in fig. 2).

As mentioned above $m_{H_u}^2$ and m_S^2 are not constrained by other considerations so first we let them vary randomly whithin natural bounds of $[-10, 10]$ TeV^2 for these parameters. The rest of the parameters was varied as above until we got 2000 accepted points in SPheno. Looking at the actual available parameter space as shown in fig. 2 we see to the left that we do not get any accepted points for larger negative values of $m_{H_u}^2$. In addition $m_{H_d}^2$, which in our case is solved for in the tadpole equation, is also

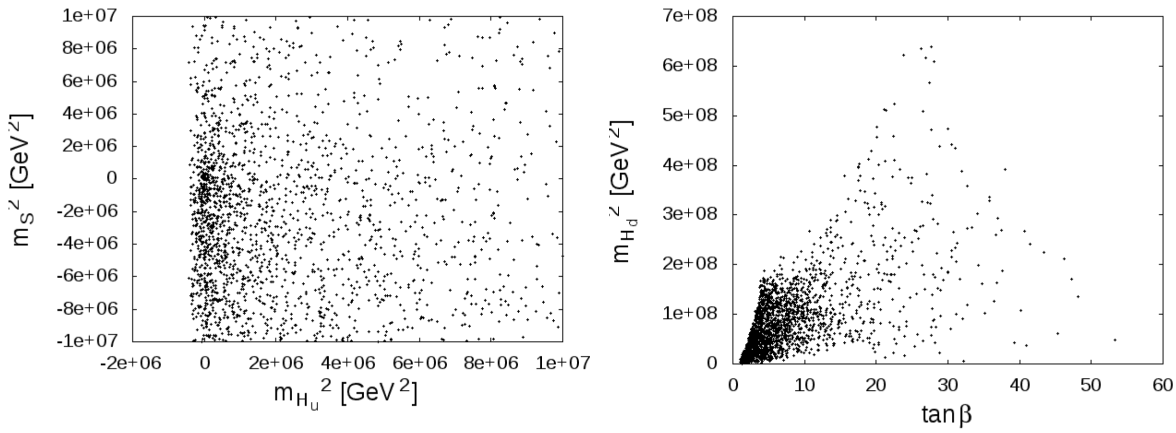

Figure 2: Illustration of limits in the parameter space (see text for details). To the left the accepted points in the $m_{H_u}^2$, m_S^2 plane are shown. To the right the $m_{H_d}^2$ as a function of tanβ

a supersymmetry breaking parameter and from the figure we see that it often gets out of the natural bounds of around 10 TeV^2. Thus there might be some interest to also examine a more constrained parameter set such that these derived masses also lies within the natural bounds. From the tadpole equations (65) we get a relation between $m_{H_u}^2$ and $m_{H_d}^2$,

$$m_{H_d}^2 = \tan^2\beta\, m_{H_u}^2 + (\tan^2\beta - 1)\Big(\frac{g_1^2 + g_2^2}{8}(v_u^2 - v_d^2) + \mu^2\Big) \tag{77}$$

From this we see that $m_{H_d}^2$ will be larger than $m_{H_u}^2$ by at least a factor of $\tan^2\beta$ and thus a natural constraint on $m_{H_u}^2$ still allows a larger than natural $m_{H_d}^2$. In addition the other terms are not insignificant for low $m_{H_u}^2$ and if we want $m_{H_d}^2$ within a natural value further constraints are needed on μ and tanβ.

Constraining the parameter space is not obvious, if we want $m_{H_d}^2$ within its natural bound one way is to limit the $m_{H_u}^2$ by a large amount. This on the other hand removes the possibility of a large $m_{H_u}^2$ in those cases when $\tan^2\beta$ is low. In addition the contribution $(\tan^2\beta - 1)\mu^2$ can easily give a large $m_{H_d}^2$ even with no $m_{H_u}^2$. Therefore we leave the above parameter choice alone in the first part of our study and let $m_{H_d}^2$ obtain larger values than what would at first be considered natural.

In addition we generate a more limited scan by using the above ranges for all parameters except $\tan^2\beta$ and $m_{H_u}^2$ for which the additional constraints

$$(\tan^2\beta - 1)\mu^2 < (2\text{TeV})^2, \quad \tan^2\beta\, m_{H_u}^2 < (2\text{TeV})^2 \tag{78}$$

are imposed in addition to the previous ones. This limits $m_{H_d}^2$ within natural bounds, but it also puts limits on $\tan^2\beta$. Finally we will compare this scan to the more unconstrained one.

For experimental constraints we at first use the program HiggsBounds version 4.0.0 [14] [15] [16] which contains all the current constraints from direct Higgs searches. It takes the number of charged and neutral Higgs bosons, their masses, widths, branching ratios and production cross sections as input. With this the program chooses the analysis which has the best expected exclusion limit and then tests it against the 95% confidence interval of that analysis with data from LEP, Tevatron and LHC.

There are also other constraints from direct searches for the supersymmetric partners, from cosmological constraints on dark matter and several low energy observables from flavor physics and the anomalous magnetic moment of the muon. As the gaugino masses are kept fixed in the scan we do not investigate relevant parts of the parameter space for dark matter candidates and thus we do not apply these constraints. For the low energy observables we will use the constraints for the anomalous magnetic moment of the muon, the loop mediated decay $b \to s\gamma$ and also the ρ_0 parameter, given by $\rho = \frac{m_W}{m_Z} \cos\theta_W$ that describes new sources of $\mathrm{SU}(2)_L$ symmetry breaking compared to the SM. The limits on the parameters are from [19],[20]. Finally as HiggsBounds only tests for the most likely channel for exclusion, we can get an accepted result from HiggsBounds by having the lightest Higgs avoiding detection without there existing any particle around 125 GeV. Thus we finally demand that there should be at least one Higgs in the 120-130 GeV mass range.

5 Parameter Searches

For the first part of the investigation we perform a broad parameter scan by varying the parameters within the ranges given by eq. 76. Most of the points generated are not allowed for theoretical reasons and give no output from SPheno. For the points that was allowed from SPheno we produced a total of 20000 points in both the CP violating and the CP conserving cases for comparison. We did also attempt some simulation with maximal CP phase. However they turned out to be rather uninteresting since, as we will see later, a larger CP phase limits the allowed ranges of λ and κ which the CP violating terms in the mass matrix also depend upon. Thus the results we obtained was essentially the same as with the varying CP phase.

Since the allowed ranges of $m_{H_d}^2$ and $m_{H_u}^2$ was already discussed in the previous section, the correlations between the other Higgs sector parameters: tanβ, λ, κ, μ and ϕ is shown for the CP conserving case in fig. 3 and CP violating one in fig. 4. The correlations between λ and κ arise from the demands of peturbativity [17]. The relatively low values for λ on the other hand is presumably from the tadpole equations as they are more often possible to solve for high v_s which favors a low λ for a given μ. In the CPV case, large κ and λ appears mostly for low ϕ as the CPV terms in the Higgs mass matrix are also proportional to $\lambda\kappa$. Thus the limits on the CPV terms, which arises from the lightest Higgs obtaining a negative mass squared, prevents them from obtaining large values for large CP violation. The μ and tanβ parameters on the other hand do not have large correlations with any of the other input parameters, but for tanβ lower values are favored and for μ there is a slight favor for higher values.

The mass spectrum for the Higgs sector is of special interest as there has to be a candidate that fits with the newly discovered boson at the LHC. As seen in fig. 5 we

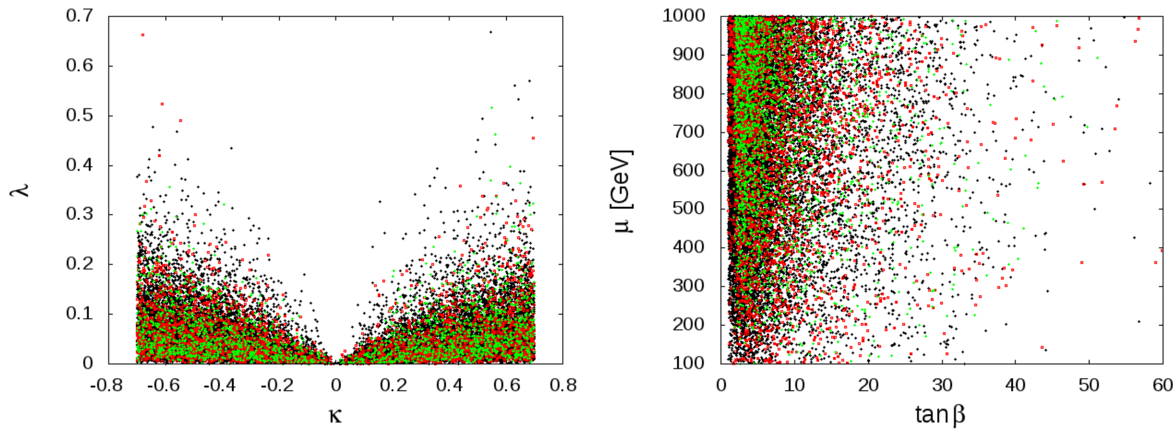

Figure 3: Correlations between input parameters in the CP conserving case. Points are plotted as follows: first black points are acceptable model points from SPheno but excluded by HiggsBounds, red points are excluded by the other constraints, see text for details, finally green are allowed by all constraints.

get accepted points in a mass range around 125 GeV. In addition, especially for the CP violating case we also get a decent number of points that are pseudoscalar-like with much weaker channel strengths which thus can have avoided detection. On the other hand, they are mostly excluded for other reasons, in particular as the next to lightest Higgs is far too massive to be the discovered boson as the mass we obtain is usually rather large, in the range of a few TeV. Finally the charged Higgs mass should have a mass scale comparable to the next to lightest Higgs mass and we see it also obtains large values. For comparison the squarks and sleptons all end up with masses around 1 TeV which is the M_{SUSY} we used to fix their mass matrices.

The observed Higgs has so far had nearly all of its properties measured to be close to what is expected in the standard model. One difference is the Higgs to two photon channel which in the ATLAS experiment is higher than expected with a measured signal strength of $1.55^{+0.33}_{-0.28}$ [21]. At the same time the CMS has not observed this excess with a signal strength of $0.77^{+0.27}_{-0.27}$ [22]. Therefore we compare the signal strength going to a final state xx, in this case taken to be $\gamma\gamma$ or $b\bar{b}$ for comparison, to what a standard model Higgs boson (H) with the same mass would have. Using that the dominating form of production is gluon-gluon fusion we get the signal strength

$$\begin{aligned} R^{h_1}_{ggxx} &= \frac{\sigma(gg \to h_1)_{NMSSM}}{\sigma(gg \to H)_{SM}} \frac{Br(h_1 \to xx)_{NMSSM}}{Br(H \to xx)_{SM}} \simeq \\ &\simeq \frac{\Gamma(gg \to h_1)_{NMSSM}}{\Gamma(gg \to H)_{SM}} \frac{Br(h_1 \to xx)_{NMSSM}}{Br(H \to xx)_{SM}} \end{aligned} \tag{79}$$

where in the second equality we have made the assumption that the differences in the radiative corrections for production and decay cancel in the ratio, following for example [23] [24].

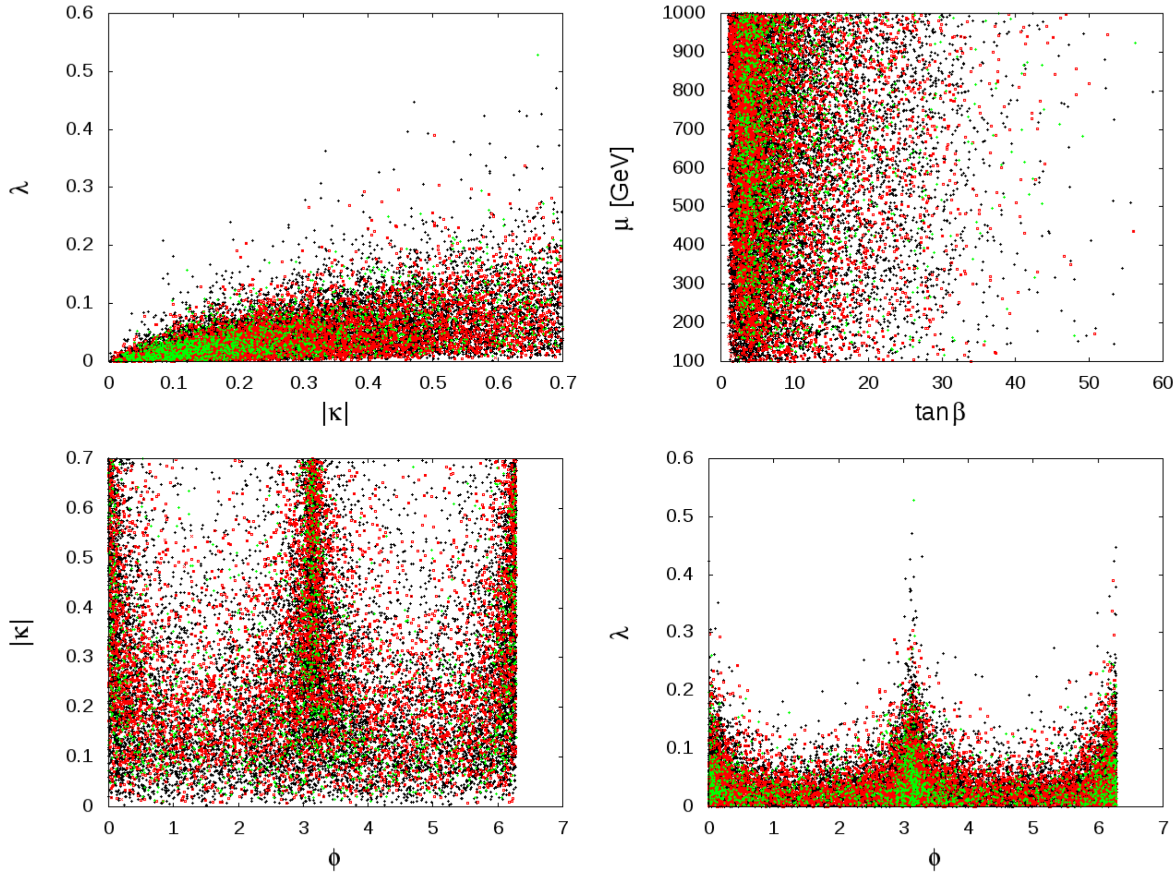

Figure 4: Correlations between input parameters of the model for the CP violating case, compared to fig. 3 we also have plots against the CP phase now included. The colour coding is the same as in fig. 3

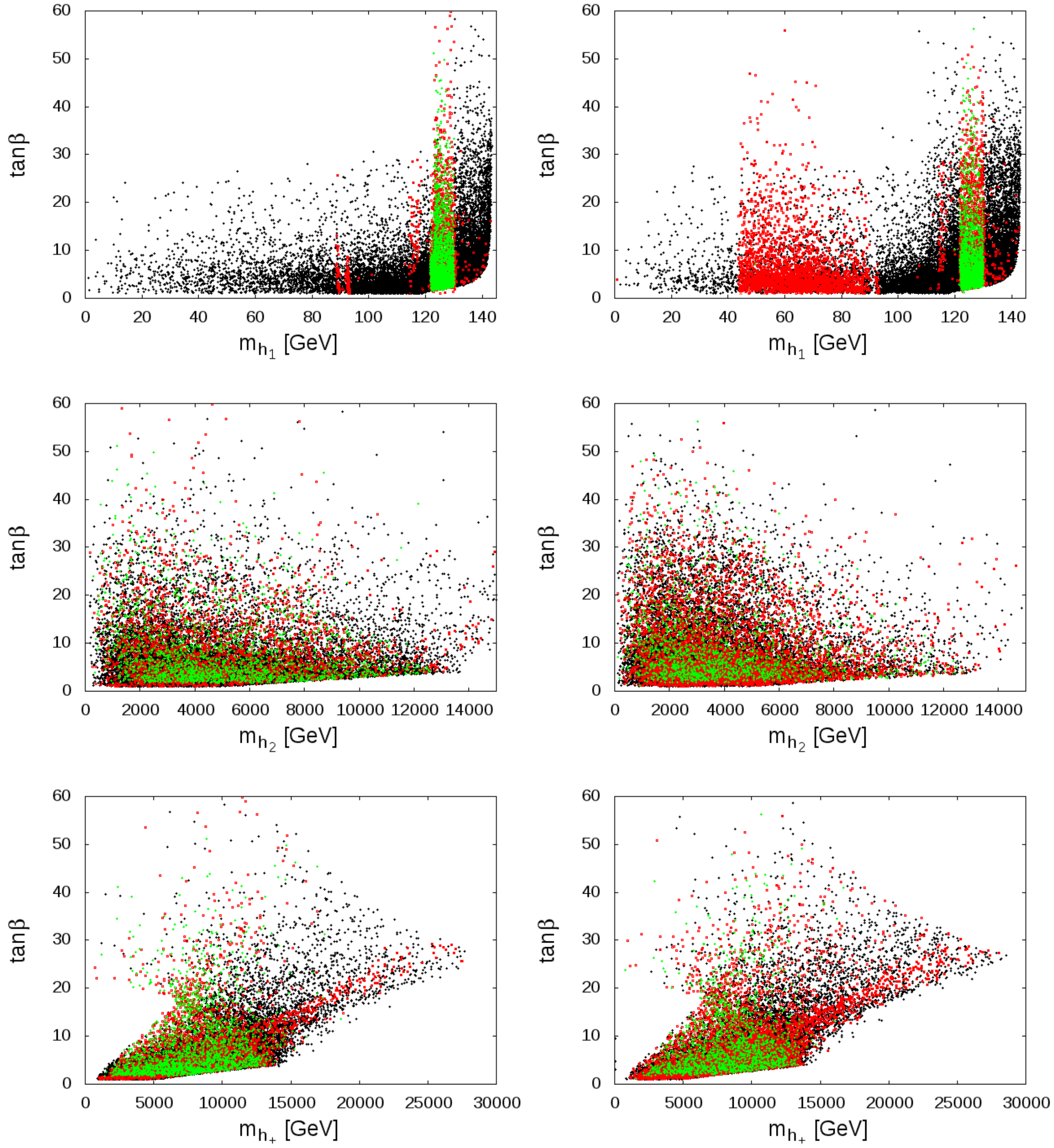

Figure 5: Masses for the two lightest Higgs bosons and the charged Higgs in the CP conserving case to the left and CP violating to the right. The same colour coding is used as in figure 3

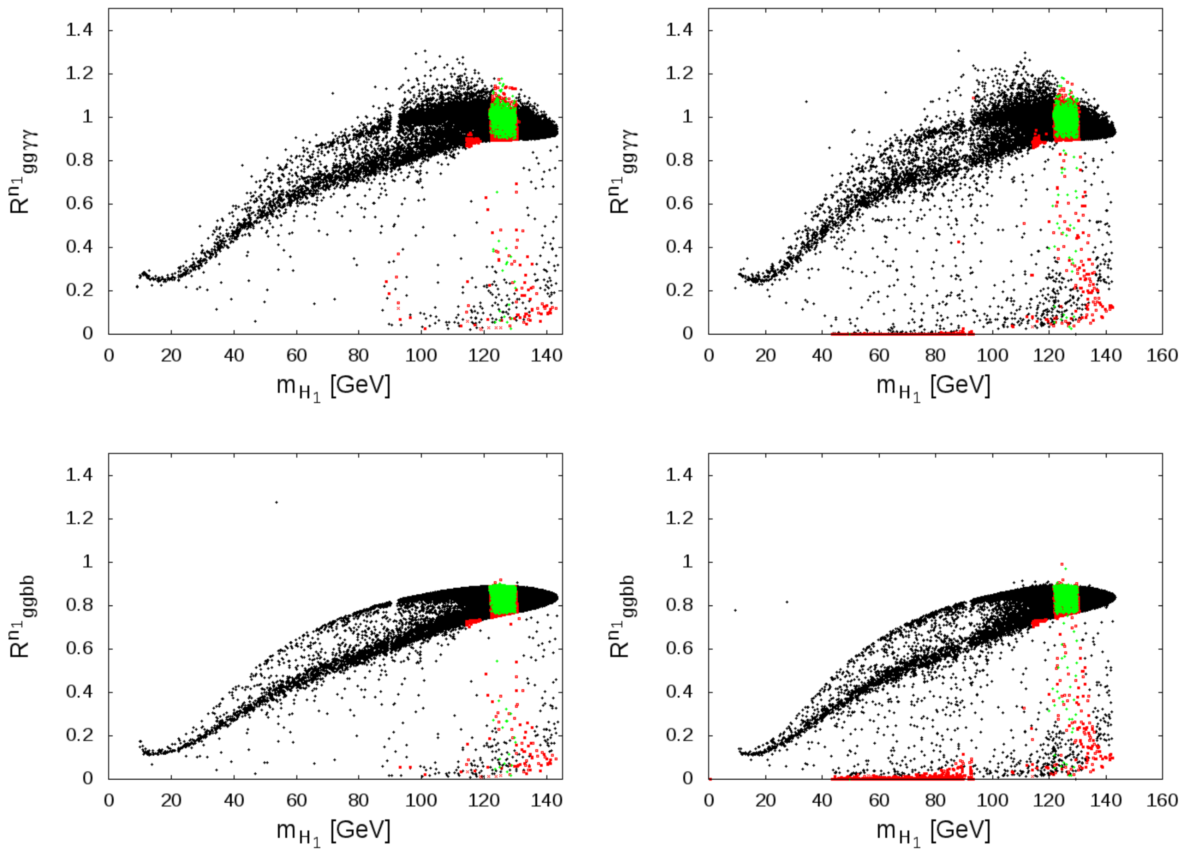

Figure 6: The plots show the signal strength to $\gamma\gamma$ and $b\bar{b}$ as a function of the Higgs mass. To the left the CP conserving case and to the right the CP violating case. The same colour coding is used as in figure 3.

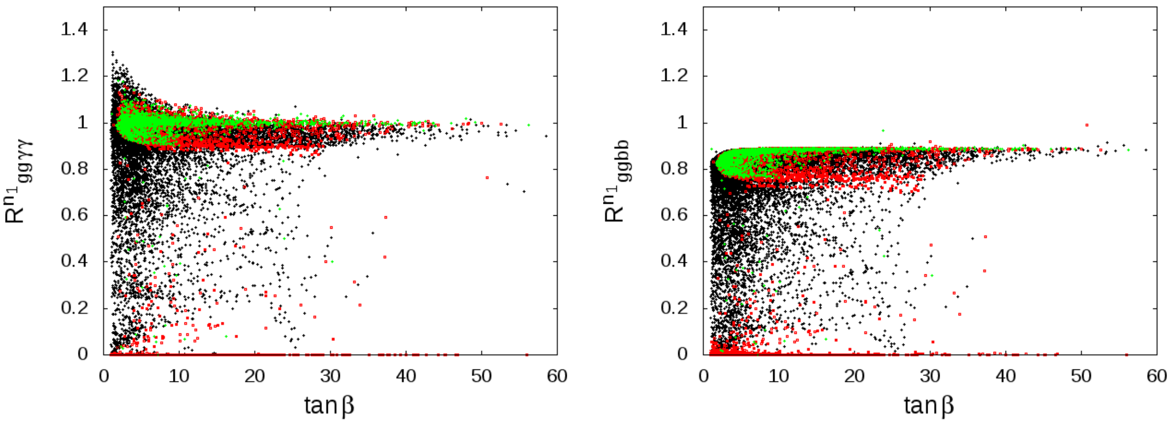

Figure 7: The signal strength to $\gamma\gamma$ and $b\bar{b}$ as a function of tanβ with both figures in the CP violating case. The same colour coding is used as in figure 3.

The signal strengths for $\gamma\gamma$ and $b\bar{b}$ in different models are shown in fig. 6. As can be seen from the figure there is not much spread in the results, fixed squark and slepton mass scale means there is little change from these particles in the loop corrections. Similarly, the in general high mass of the other Higgs bosons means loop corrections with these particles will be suppressed. Still the results are in agreement compared with observation. There may also be a small enhancement in the $\gamma\gamma$ channel compared to the SM and in addition the bb channel is also slightly weaker than the SM. The largest signal strength to $\gamma\gamma$ was found for low tanβ as shown in fig. 7, while the low tanβ gave no enhancement to $b\bar{b}$. Finally as seen in fig. 6 there appears to be barely no difference between the CP violating and the CP conserving cases.

To single out the effect of the CP phase we would like some benchmark scenarios and vary the CP phase. Here we use the m_h^{max} and no-mix scenarios as discussed in section 4 as benchmarks. Values obtained for the A_t parameter are plotted against the lightest Higgs mass in fig. 8. Here we see that the Higgs mass reaches its maximum for around $A_t = \sqrt{6}M_Q$ and also has a local minimum for $A_t = 0$. We also used more colours for the constraints as the different exclusion regions are clearly shown here. The anomalous magnetic moment of the muon excludes next to no points, the ρ-parameter excludes large $|A_t|$ and the loop mediated decay $b \to s\gamma$ exclude points with negative A_t. Finally the points with positive A_t and low mass that passes the HiggsBounds check are only excluded as we cannot find a next to lightest Higgs in the correct mass range. We also see the upper limit for $|A_t|$, as the lightest Higgs gets a negative mass for large values of $|A_t|$. Now typical benchmark points can be selected for the m_h^{max} and no mix scenarios.

Before we move over to the benchmark points we will consider some additional constraints on the model. As mentioned at the end of the constraint section we discussed the $m_{H_d}^2$ parameter and natural constraints on this. Thus we also investigated a more constrained version of the model. We generated another set of points by using the additional constraints discussed at the end of the section on parameter choices. As before, we produced a total number of 20000 points that was allowed from SPheno in this con-

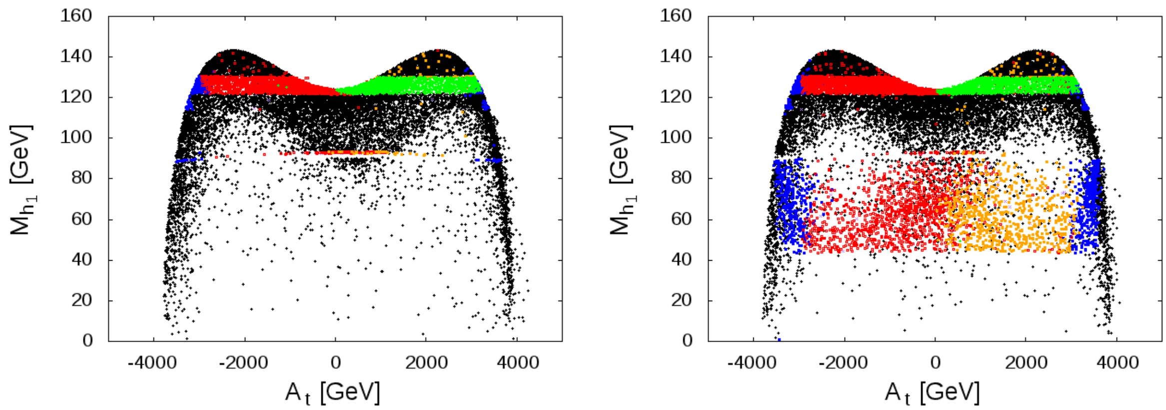

Figure 8: The lightest Higgs mass as a function of A_t, the CP conserving case to the left and CP violating to the right. The points excluded by HiggsBound are colored black and the accepted ones green as in the other plots but the additional constraints are shown as different colours. Points excluded by the anomalous magnetic moment of the muon are colored purple, by the ρ parameter blue, by $b \to s\gamma$ decay are red and finally these which do not have any Higgs in the range of 120-130 GeV are colored orange.

strained model, but as we are interested in the differences to the previous result we only did it for the CP violating case.

The notable differences are shown in fig 9. The values obtained for λ are in general a bit higher, closer to the theoretical limit. In addition the constraint on tanβ is dependent on the μ parameter, but as we select the μ parameter first there is not a bias towards lower μ. The lowering of tanβ does remove the top end of the lightest Higgs mass as this gives the largest loop corrections. In particular it removes the possibility of the no-mix scenario with the observed Higgs being the lightest one. It does, on the other hand, restrain the heavier Higgs masses and the charged Higgs mass. Still, masses of a few TeV are favored so the masses are still rather large. For the other plots we made, apart from the already shown constraints, the more constrained model gives no noticeable changes.

To study the effects of the CP phase, benchmark points were chosen as in table 3. As seen in fig. 8, in the no-mix scenario the Higgs boson can barely reach the observed mass range and as such the available parameter space is very limited, with large tanβ needed to reach the observed mass range. In fact the point chosen was the only one accepted by all constraints unless one wants to go a bit further away from $A_t = 0$. For the m_h^{max} scenario we choose two points, the first (1) in the observed range and the second (2) with a higher mass for the lightest Higgs than observed as the mixing will reduce its mass and thus get it back into the observed mass range.

For the m_h^{max} scenarios, in the two example points, as we can see in fig. 10 and 11 respectively that the lightest Higgs mass decreases with more CP violation as expected with more mixing of the states. In the first example there is also a jump as the mass of

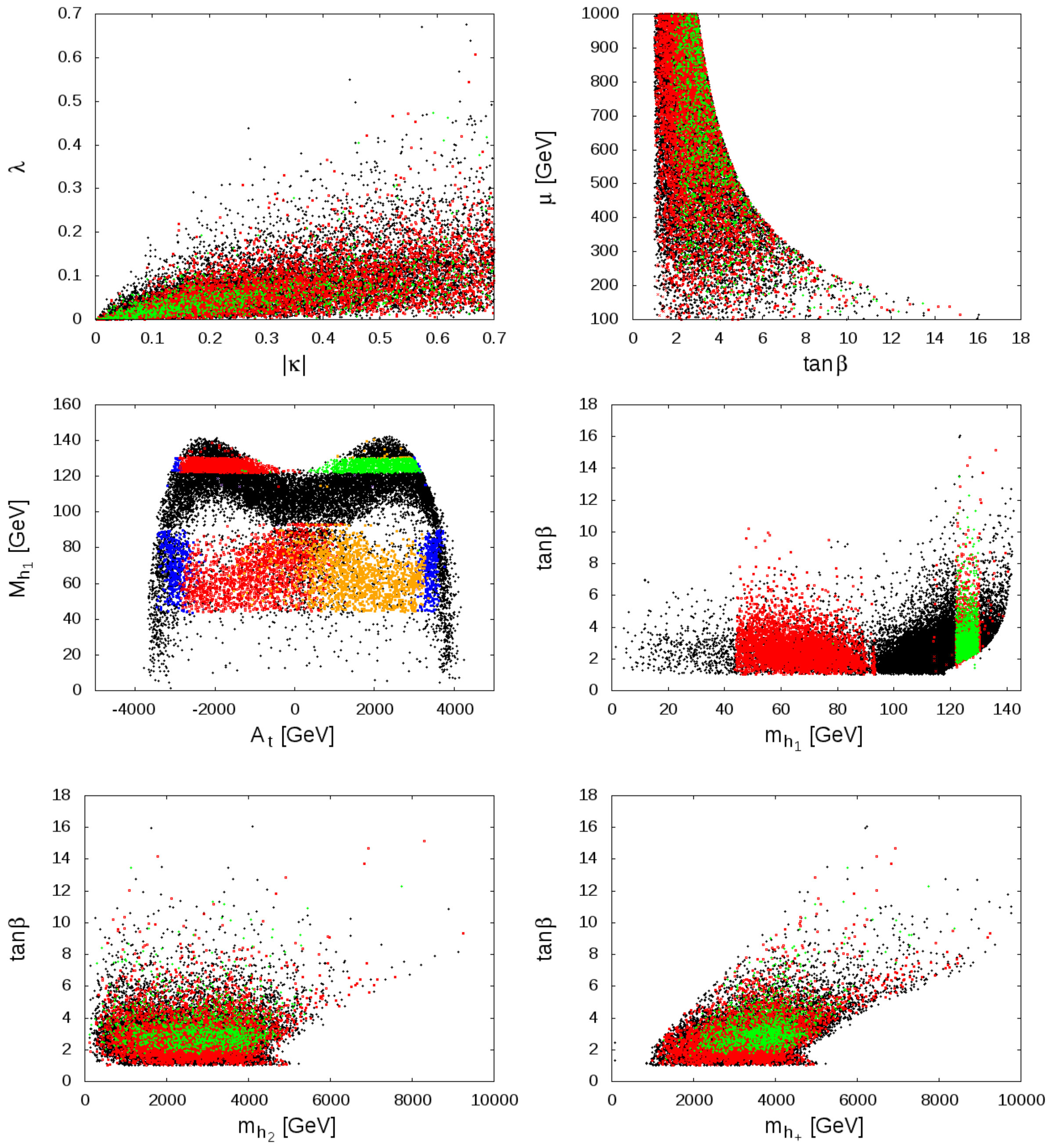

Figure 9: Results from the more constrained parameter scan. The first two plots should be compared with the corresponding plots in figure 3. The following plots shows the results corresponding to the results of figure 5 and 8. The same colour coding is used as the corresponding image in the non constrained model.

Model	tanβ	$\|\lambda\|$	$\|\kappa\|$	μ [GeV]	$m^2_{H_u}$[GeV^2]	m^2_S[GeV^2]	A_t [GeV]
$m_h^{max}(1)$	1.758	0.004956	0.1022	480.5	$7.815 \cdot 10^6$	$9.696 \cdot 10^4$	2462
$m_h^{max}(2)$	10.84	0.1303	0.2088	848.5	$-1.328 \cdot 10^5$	$3.815 \cdot 10^4$	2429
no-mix	36.23	0.05048	0.1603	490.6	$-9.752 \cdot 10^4$	$9.720 \cdot 10^3$	19.35

Table 3: Values for the points chosen for the two m_h^{max} and the no-mix scenarios. All values given to four significant digits.

the lightest Higgs boson turns mostly pseudoscalar while the second one does not. We do not know precisely why this happens but we tried a few other points and both cases occur frequently. The fact that the second one is much sharper comes from the higher λ and κ which makes a change in CP phase have larger effect on the CP violating mass term as $M_{CPV} \propto \lambda\kappa$. Here we also see the limits for large CP phase together with high λ and κ

The more massive Higgs bosons also change their masses although the changes does differ as examplified by the next to lightest Higgs in fig. 10 and 11. At the same time the charged mass changes similar to the next to lightest Higgs mass in the first scenario. As mentioned previously the charged mass should be unaffected by a change in the CP phase at tree level. Thus the changes we see in the more massive Higgs might at least in part come from 1-loop corrections. In addition to this, precisely how the heavy Higgs masses change depends on which Higgs state it is close to in the CP conserving case. This can be seen in the different behavior for the next to lightest Higgs in fig. 10 and 11. The similar change in the first scenario compared to the charged Higgs suggests that it in this case is close to the next to lightest CP even Higgs as its mass is close to that of the charged one. For the second scenario, as it does not follow the change in the charged one it is presumably closer to the lightest CP odd. Also as the CP violating terms are larger in this scenario we get a much larger effect from the CP violation.

Looking at the sum of the Higgs masses $\sum_{i=1}^{5} m^2_{h_i}$ in fig. 10 and 11, which at tree level corresponds to the trace of the mass matrix and therefore should be unchanged, we see that this is not the case, thus supporting that the change comes from loop corrections to the masses.

Finally the signal strength to $\gamma\gamma$ shows a slight increase with some CP violation compared to the $b\bar{b}$ which does not. Still, as seen in fig. 6, the $\gamma\gamma$ channel does increase and reaches a maximum at a slightly lower mass range and the effect is probably due to the changing mass. Thus the changes in signal strength might be an entirely indirect effect as the CP phase affects the Higgs mass. The jump in mass is reflected in the jump in coupling as it is in one case discontinuous while in the other case a gradual change.

The no-mix case proved to be much less interesting for the lightest Higgs as seen in fig 12. The mass and with it the channel strengths barely change at all with the CP phase. On the other hand the masses of the heavier Higgs bosons does experience far more change than in the m_h^{max} scenario. The large change in the charged Higgs boson indicates that the changes are mostly from loop corrections also in the non charged case and the sum of the Higgs masses seems to confirm this.

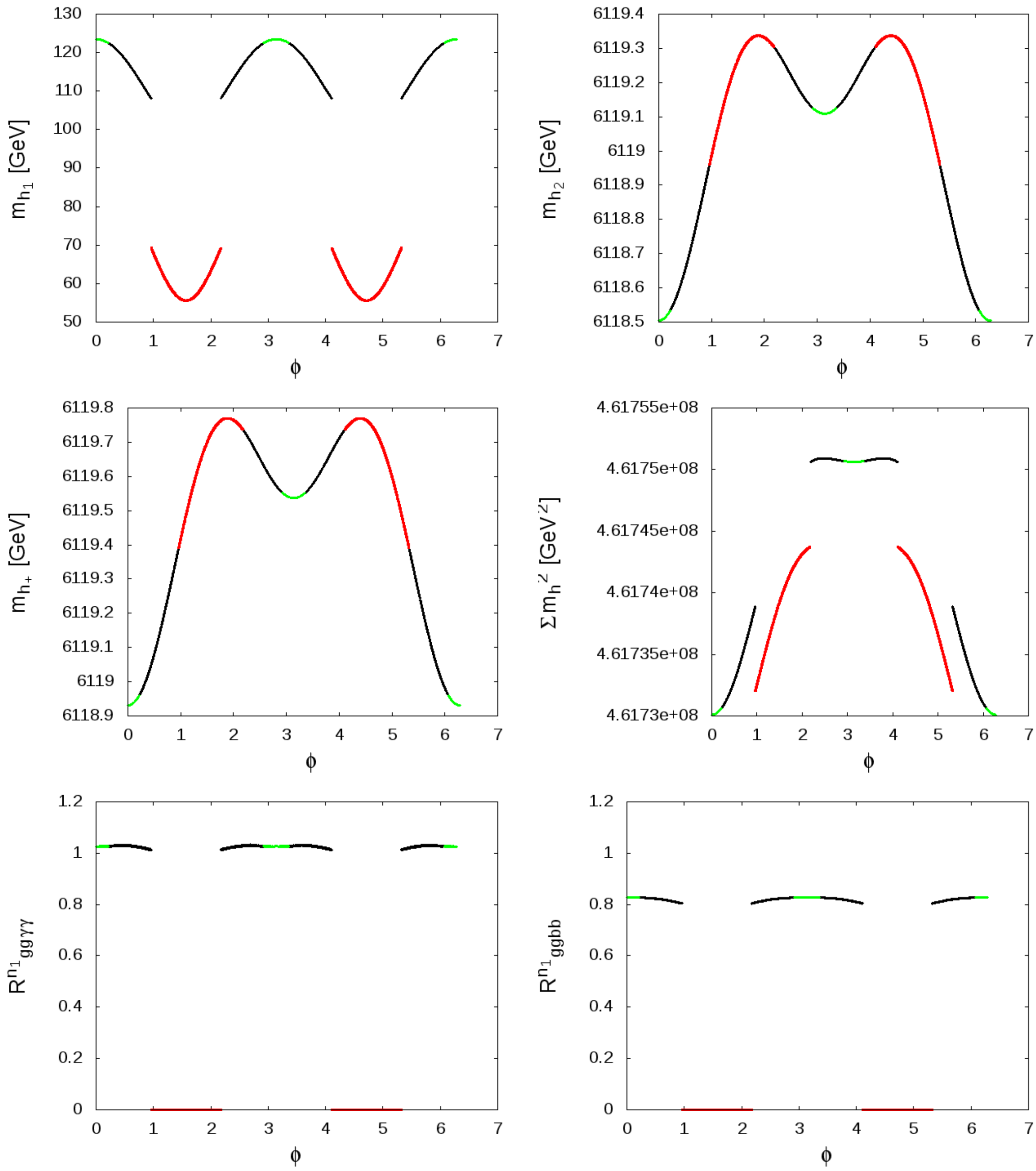

Figure 10: First is shown the lightest, next to lightest and the charged Higgs boson mass dependence on the CP violating phase in the first (1) m_h^{max} scenario. The fourth picture is the sum of the 5 uncharged Higgs bosons mass squared. The last two plots show the CP dependence for the signal strengths to $\gamma\gamma$ and $b\bar{b}$. The same colour coding is used as in figure 3.

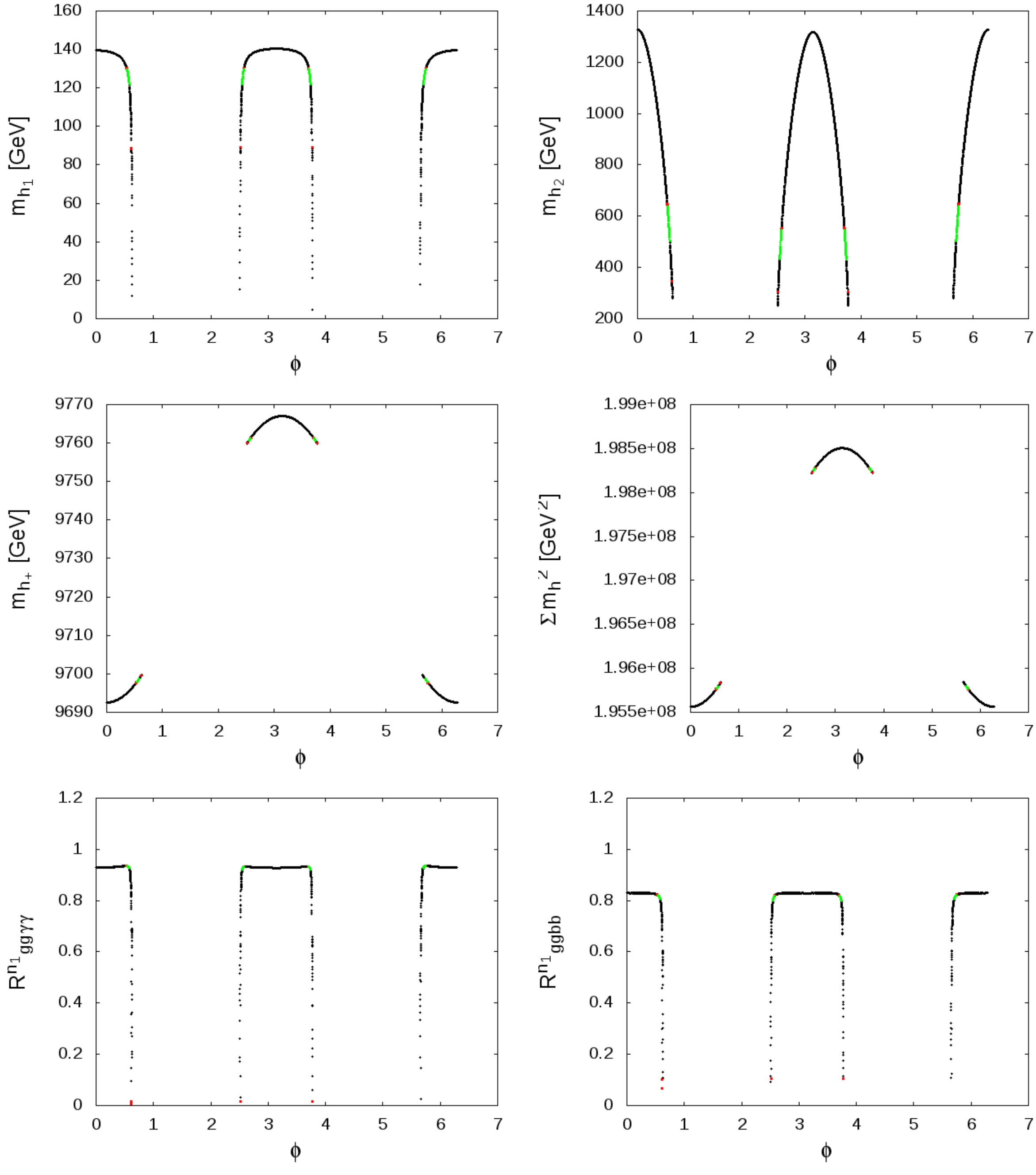

Figure 11: Same plots as in figure 10 but for the second (2) m_h^{max} scenario. The same colour coding is used as in figure 3.

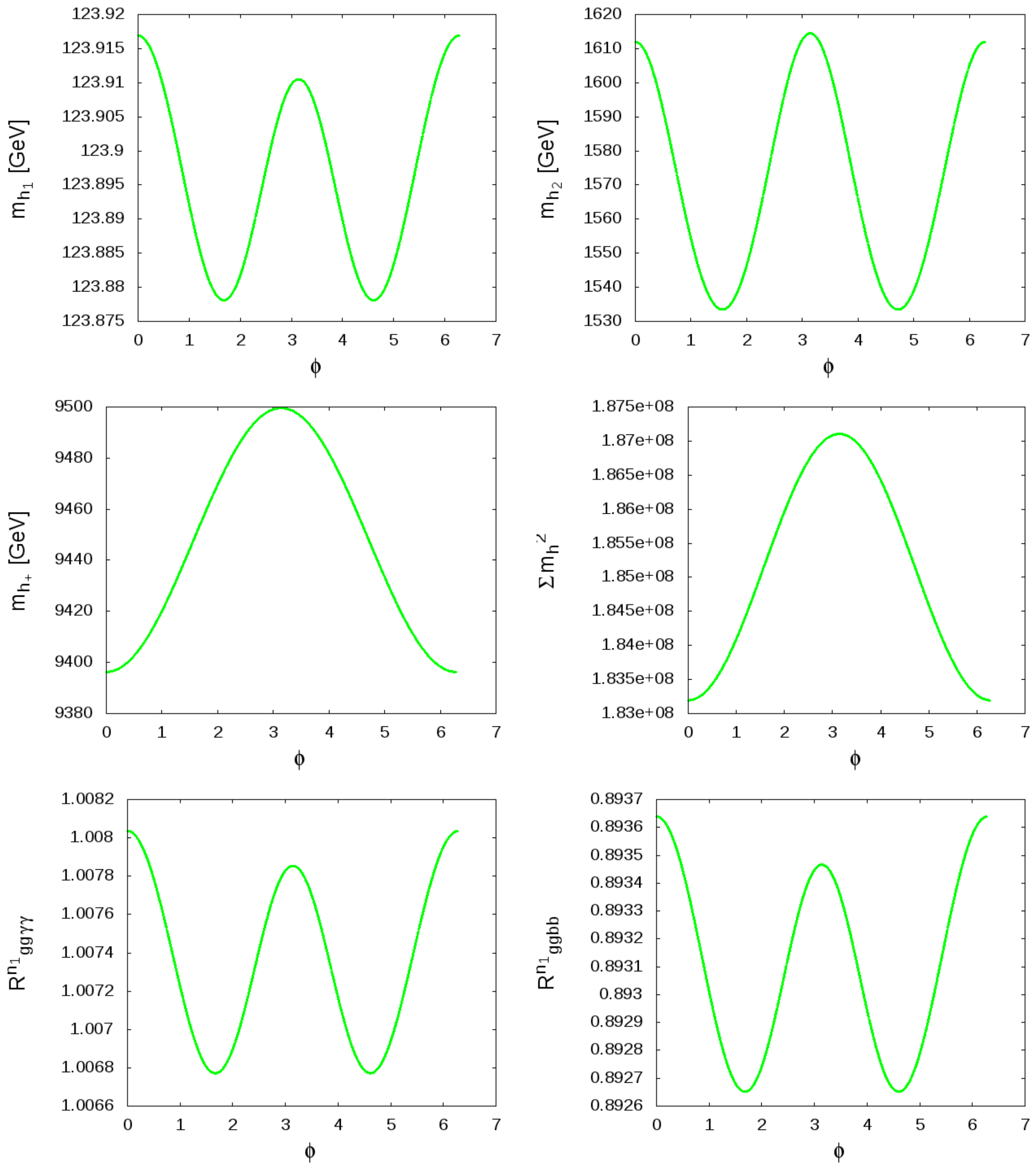

Figure 12: Same as figure 10 but for the no-mix scenario. Starting from top left we show how the lightest, next to lightest and the charged Higgs boson mass depend on the CP violating phase in the no-mix scenario. The fourth plot is the sum of the 5 uncharged Higgs bosons mass squared. The last two plots show the dependence of the CP phase for the signal strengths to $\gamma\gamma$ and $b\bar{b}$. The same colour coding is used as in figure 3.

We also attempted to find a suitable point where the next to lightest Higgs was the observed one. The unconstrained parameters we used meant very few next to lightest Higgs of masses close to that of the observed Higgs even in our more constrained run. The result was not good as we did not find a single Higgs with both the right mass and couplings close to the SM. Thus we could not conclude anything more on the scenario with the next to lightest Higgs as the observed one.

6 Conclusion

In this thesis we have investigated the Next to Minimal Supersymmetric Standard Model (NMSSM). The extended Higgs sector in these models can lead to large phenomenological differences compared to the standard model and we confronted the model with the newest constraints from the properties of the newly discovered Higgs boson. In particular the effects of a tree level CP violating phase, which is possible in the NMSSM, was investigated and how this effects the phenomenology.

A supersymmetric theory was investigated as it solves the hierarchy problem for the Higgs boson, apart from providing interesting possibilities for new physics such as a dark matter candidate in the lightest supersymmetric partner. The NMMSM was chosen over the Minimal SuperSymmetric Model (MSSM) as the MSSM has the so called μ-problem where the μ parameter naturally has a value of order of the Planck mass and cancellations down to the EW scale would be required to obtain the measured Z mass. The NMSSM on the other hand provides a natural way to obtain a small value for this parameter. In addition the MSSM has a more constrained Higgs sector and it can barely reach the observed Higgs mass via large loop corrections while the NMSSM can much more easily do so.

Since we used the rather unusual choice of $m_{H_u}^2$ and m_S^2 as free parameters we had the difficult task of constraining these parameters as they are not directly constrained by experiments. We tried to apply some natural bounds but this was not obvious as this also gives limits on the available parameter space for other parameters.

For the results we saw that there is still plenty of available points in the parameter space even when demanding that the lightest Higgs is in the mass range of the measured Higgs. On the other hand the heavier Higgs had high masses, up to tens of TeV, mainly due to the unconstrained parameters we used. In addition we also investigated the signal strengths to see if they matched the SM prediction and how large any differences were. In general they did differ a bit especially the decay to $b\bar{b}$ which always was slightly lower than expected but the majority of points are within experimental limits from the LHC. Even the possible excess of di-photon events in the ATLAS experiment could be consistent with our model, especially for scenarios with a low tanβ. The changes in couplings between the different parameter points was not so large as the loop induced coupling to $\gamma\gamma$, where large effects can be seen, needs lighter new particles, such as the charged Higgs contributing in the loop to give a noticeable effect.

When it comes to constraining the parameter space, we also did simulations where we demanded that the derived parameter $m_{H_d}^2$ should also lie within the naturalness bound we imposed on $m_{H_u}^2$. This did limit the high masses for the other Higgs bosons but they still typically have masses of a few TeV. We did on the other hand not take into account the other derived parameters such as the trilinear scalar couplings A_λ and A_κ on which one could also impose a bound and check what limits this gives. On the other hand, as it is commonly done to use parameters such as m_{h_+} as a free parameter there might also be some interest to compare the differences between such a bound and naturalness bounds we used here and see what limits they put on each other and on measurable properties.

In addition we varied the top squark mixing A_t as this gives loop corrections to the lightest Higgs boson mass. With this we used the m_h^{max} and the no-mix scenarios to pick typical points to investigate the effects of the CP violation. We could get large changes in the masses when varying the CP violating phase for the Higgs bosons that become the lightest CP-odd and CP-even when there is no CP violation present. The masses of the other Higgs bosons had fairly small changes which also appeared to mostly come from 1-loop effects. The changes in couplings came rather fast with the mixing of the lightest Higgs making it more pseudoscalar like. Otherwise there was not a lot of changes in the couplings.

The large masses typically found for the heavier Higgs bosons also limits the case for the discovered Higgs boson being the next to lightest one. While we did find parameter points for which the lightest Higgs boson avoided detection, especially in the CP violating scenarios, we did not find a suitable next to lightest Higgs boson to match up with the observed one. This was mostly due to the relatively unconstrained parameters which did not provide statistics in this particular region. On the other hand, as the scenarios with CP violation had a much larger number of undetected light Higgs, CP violation in the case of the next to lightest Higgs as the observed one might be of interest for further investigation.

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7 Appendix

Four vectors are represented by Greek indicies $\mu, \nu, ... = 0, 1, 2, 3$ and we have

$$x^\mu = (t, \overrightarrow{x}), \quad p^\mu = (E, \overrightarrow{p}), \quad \partial^\mu = (\partial/\partial t, -\overrightarrow{\nabla}) \tag{80}$$

with the spacetime metric as

$$g_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \tag{81}$$

In addition we will use the two component Weyl spinor notation instead of four component Dirac or Majorana spinors. For this we use the representation for the γ^μ matrices as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{82}$$

where

$$\begin{aligned} \sigma^0 = \bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma^2 = -\bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned} \tag{83}$$

The 4-component Dirac spinors can be decomposed into two two-component anticommutating objects ξ_α and $\chi^{\dagger\dot{\alpha}}$ whith spinor indicies $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} \tag{84}$$

The ξ_α is called a left handed Weyl spinor and $\chi^{\dagger\dot{\alpha}}$ a right handed Weyl spinor. This comes naturally as if we act with a right or left handed projection operator on the Dirac spinor we get

$$P_L\Psi = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \quad P_R\Psi = \begin{pmatrix} 0 \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} \tag{85}$$

thus justifying their name. The hermitian conjugate of a left handed spinor (undotted indicies) is a right handed one (dotted indicies) and vice versa. This is why we write $\chi^{\dagger\dot{\alpha}}$ as a right handed spinor and we will use daggers to denote right handed spinors throughout this paper. The spinor indices are raised and lowered by the totally antisymmetric ϵ tensor where

$$\epsilon^{12} = -\epsilon^{21} = -\epsilon_{12} = \epsilon_{21} = 1 \tag{86}$$

In general we suppress the spinor indicies when we contract over them

$$\xi\chi = \xi_\alpha\chi^\alpha = \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta, \quad \xi\sigma^\mu\chi^\dagger = \xi_\alpha(\sigma^\mu)^{\alpha\dot{\beta}}\chi^\dagger_{\dot{\beta}} \tag{87}$$

which we do throughout the paper.

Finally there are many useful identities for these objects, the one used here several times is the Fierz identity

$$\xi_\alpha(\chi\eta) + \chi_\alpha(\eta\xi) + \eta_\alpha(\xi\chi) = 0 \tag{88}$$