



LUND UNIVERSITY

School of Economics and Management

Master in Economic Development and Growth

## Empirical Estimation of the Solow Growth Model: A Panel Approach

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*Abstract: This research examines the relevancy of Solow growth model in 20 OECD countries over the period 1971-2011. Following Mankiw-Romer-Weil (1992) and Islam (1995), I estimate both textbook and augmented Solow model. Along with OLS, estimation is carried out implementing both static panel and dynamic panel GMM carries out estimation. Results imply that appending human capital measure in augmented Solow model can better explain the international differences in levels of output per worker. Importantly, convergence rate is relatively stronger when growth is conditioned on workforce growth rate, rate of savings and human capital. Using efficient system GMM in dynamic panel, I have shown that speed of convergence is about 4.7 percent, which is twice than the standard consensus about convergence rate in literature.*

*Key words:* Growth, Solow model, Convergence, Dynamic panel GMM

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## 1. Introduction

In the recent years there is a growing number of empirical works on cross country growth and convergence. Within the theoretical and empirical growth literature, the Solow model (Solow, 1956) is being apprehended as the foundation of basic endogenous growth models. Another two noteworthy papers by Cass (1965) and Koopmans (1963) also provided a focus on the issue of convergence. Considering a set of assumptions, the main paradigm of Solow growth model asserted that long run rate of growth is exogenously determined. More explicitly, economies converge towards a steady state level of growth, which mainly depends on the rate of technological progress and work force growth.

There are several directions of research that empirically assess the validity of Solow's paradigm. First, one line of research developed the single cross-section regression. For example, the most cited and influential paper by Mankiw – Romer – Weil (Mankiw et al., 1992), hereafter MRW, examines the consistency of Solow's paradigm with international variation of living standard. Using a large set of cross-section data, they evidently confirm what the augmented Solow model predicts. Second, some researchers (Ding and Knight, 2009; Islam, 1995; Hoeffler, 2002) assumed parameter homogeneity across countries and used panel data econometric technique. Third, another group also used panel data but extends the parameter heterogeneity across countries (Quah, 1997, Temple, 1999).

In estimation issue, a notable paper by Caselli et al. (1996) do not find consistency of either textbook or the augmented version of Solow model in real life cross-country data. In this connection, Bond et al. (2001) underlines, due to weak instrument the first difference GMM estimator work poorly in Caselli et al. (1996). And they suggest applying more efficient GMM estimator, which accomplishes stationarity restriction.

Along with the above mentioned literature, several challenging studies have been produced in order to assess the relevancy of Solow's paradigm. While estimating panel countries data, parameter heterogeneity across countries and nonlinearity in growth process become an important issue. Many researchers also estimated the Solow growth model in cross-country context by holding the parameter homogeneity assumption.

In brief, though there is large empirical literature on assessing the validity of Solow model, very few (Bond et al., 2001; Ding and Knight, 2009) consider the dynamic panel estimation and include data of this recent decade. I attempt to fill this gap by applying sophisticated panel estimation on OECD countries using data over 41 years. For this dissertation, I implement advanced panel data techniques to deal with the empirical estimation. In fact, panel data approach is more appropriate than cross-section in many cases because it take into account the unobserved country effects. Considering MRW and Islam (1995) as base paper, this research examines how the empirical results vary with the adoption of both static and dynamic panel Generalized Method of Moments (GMM) techniques using the most updated data set. Firstly, I consider the homogeneity argument following the most influential paper by MRW and secondly, I address the parameter heterogeneity across panel. Following MRW, I estimate textbook and augmented Solow model by employing Ordinary Least Square (OLS), Secondly, I estimate static panel with unobserved country specific effect instead of cross-country differences in per worker output growth. Thirdly, I modify this regression equation into a dynamic Generalized Method of Moments (GMM) panel data framework to control for unobserved country effects and potential endogeneity problems. Finally, I tried to depict the nature of convergence predicted by Solow model.

The estimation of this research differs from the previous works. Most importantly, I implement both static and dynamic GMM panel estimation using long sample periods of 20 OECD countries. The dynamic panel GMM estimation is valid as Arellano and Bover (1995) reported constant correlation between independent variables and individual effect (i.e. country-specific effect) under the additional identification assumption.<sup>1</sup> I focus on this approach because; lagged differenced regressors provide a good instrument for current level, when there is high autocorrelation of explanatory variables in a panel set. Hence, while we consider first differenced GMM estimator in growth regression, it takes first difference to remove the effect of initial efficiency and lagged levels of explanatory variables are considered as an instrument in first difference equation. Bond et al. (2001) criticized first difference estimator in terms of bias and imprecision. They argued that first difference estimator has large downward finite sample bias. Because of the system difference GMM estimator has the original equation into the level, which enhance efficiency and significantly reduce finite sample bias. Consequently, this estimator is more efficient for panel approach; hence I consider system GMM techniques in this research.

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<sup>1</sup> As first differenced GMM estimator is subject to downward sample bias, for that reason Arellano and Bover (1995) and Bludell and Bond (1998) develop a dramatic improvement for this kind of sample bias through the

The remainder of this dissertation is organized as follows. Section 2 reviews the past contribution in Solow testing. Section 3 details the textbook Solow model and the augmented Solow specification with human capital. Section 4 provides empirical strategy in estimating 20 OECD countries panel data. Section 5 reports the result and discussion. Section 6 interprets endogenous growth convergence and Finally, I conclude the paper.

## 2. Literature Review

A prime observation of the major empirical growth literature has been the issue of convergence following by the Solow (1956) paradigm. Solow assumed diminishing marginal returns of capital, exogenous population growth and savings rate, no depreciation and technological progress. The model predicts that steady state level of income per capita is exogenously determined by savings and population growth rate, which lead to the view of convergence. In literature (Lee et al., 1997), there are three grounds in explaining the concept of convergence in Solow model. First, the *beta convergence*, which mainly implies the logarithm of output per capita moves towards its steady state level from some given initial condition. Second, *sigma convergence*, which mainly highlights the cross-country variance of output over time. More explicitly, theoretically there is a common equilibrium across countries, which is exogenously determined by global technologies and preferences, and the rate of convergence to steady state level is same across countries. On the other hand, the cross-country variance of output will reflect initial condition, country specific equilibria and adjustment rate within each country. Consequently, a single country can be converging to its own equilibria but cross-country equilibrium could be diverging. Third, logarithm of output per capita is treated as an integrated variable and different countries share a common deterministic and/or stochastic trend.

In testing Solow model, the theoretical and empirical framework provided by MRW has been very influential for the cross-section growth empirics in literature<sup>2</sup>. MRW estimated both textbook and augmented Solow model using cross-country growth regression. They found that the Solow model considering both human and physical capital accumulation provides a robust elucidation.

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<sup>2</sup> Islam (1995), Lee et al. (1997), Bond et al. (2001), Caselli et al. (1996) etc.

Islam (1995) criticized the methodological approach of MRW in two main grounds, Firstly, the single cross-section estimation of growth equation cannot deal with the country specific shock from the aggregate production function and hence raise the problem of omitted variable bias. Secondly the result obtained by Ordinary Least Square (OLS) ignores the important shocks by production technology, resource endowments and institution to the aggregate production function. Such assumption can violate the basic *orthogonality condition* i.e. shocks is very likely to be correlated with the explanatory variable which imply the OLS estimators are biased<sup>3</sup>. Therefore, along with the single cross-section Islam (1995) implemented a panel framework using the same sample as MRW. His research highlighted lower the value of output elasticity with respect to capital, the higher the rates of conditional convergence. After controlling for country specific effects in panel framework, he estimated fixed effect within group estimator. But while dealing with a dynamic panel set, the within group estimator may also be biased and inconsistent because the composite error term and the lagged dependent variable is not uncorrelated in finite time series.

Caselli et al. (1996) criticize MRW and argue that at least some explanatory variables may be endogenous and this problem may mislead the prediction of the convergence rate. To get rid of this problem, they suggested to use panel data instead of cross-section and importantly recommended generalized method of moments (GMM) to tackle the endogeneity and omitted variable bias. While considering lagged dependent variable and time invariant country specific effect, OLS and Fixed-effect (within group) estimate of the coefficients are likely to be upward biased and generate a correlation between the lagged dependent variable and the country specific effect. Consequently, the OLS and Fixed-effect estimator is no longer unbiased and consistent. Therefore, to solve these problems Caselli et al. (1996) suggested that the first differenced GMM could solve endogeneity problem and better explain the open economy version of neoclassical growth models. Using first differenced GMM, they found that income per capita converges to the steady state level at 10 percent per annum, which is largely contrasted with the recent consensus of 2-3 percent (MRW) convergence rate.

While we use first differenced GMM estimator in growth regression, it takes first difference to remove the effect of initial efficiency and lagged levels of explanatory variables are considered as

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<sup>3</sup> In the presence of time invariant individual (country -specific) effects as well as lagged dependent variable implies OLS estimators are biased and inconsistent, in fact here the coefficient is potentially upward bias and correlated between lagged dependent variable and country specific effect (Hsiao, 1986).

an instrument in first difference equation.<sup>4</sup> However, Bond et al. (2001) criticized first difference estimator in terms of bias and imprecision. They argued that first difference estimator has large downward finite sample bias.<sup>5</sup> As Caselli et al. (1996) used first differenced GMM estimator in growth regression, it may act poorly due to weak instrument (Bond et al., 2001). And they suggest applying system GMM estimator instead of first difference. Because, the system difference GMM estimator has the original equation into the level which enhance efficiency and significantly reduce finite sample bias. Consequently, this estimator is more efficient for panel approach.

Accordingly, in a recent work using a large cross-country panel Hoeffler (2002) used OLS, fixed effect model, first-differenced GMM, system GMM and instrumental variable (IV) approach but found system GMM estimator better explain the augmented Solow model. Importantly, when unobserved country effects and endogeneity issue are controlled, augmented Solow model can completely account for Sub-Saharan Africa's poor growth performance.

Durlauf et al. (2001) used a general growth equation and found identical Cobb-Douglas production technology assumption is unsatisfactory in cross-country estimation. In relation to this assumption Duffy and Papageorgiou (2000) evidenced the alternative production function instead of standard Cobb-Douglas format. Durlauf et al. (2001) also found that parameter heterogeneity is strongest among poorer countries and nonlinearity in the growth process that may create omitted variable problem. In addition, estimation resulted in unstable physical capital coefficient, which shows the highest coefficient value is linked with highest income per capita countries. Finally, this research exhibits substantially lower values of growth rate when they differ intercept value, which implies the possibility of latent growth determinants of poor economies that is not captured by Solow specification.

Nevertheless, Murthy and Chien (1997) included a better measure of human capital and re-examine both the augmented Solow model and fully extended Solow model. They demonstrated that the fully extended Solow model (includes physical capital, human capital and technological advancement) exhibits a higher convergence rate when transitional dynamics are accounted for in OECD economic growth. In transitional dynamics, investment leading policy in human

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<sup>4</sup> See Ding and Knight (2009), p. 440

<sup>5</sup> When *sample period (t)* is small, the first difference results weak instruments for the subsequent first differences and cause downward finite sample bias.

capital, increased savings and trade policy will enable OECD economies to converge at higher rate.

Lee et al. (1997) comment on the panel data econometric approach of growth and convergence analysis. They mention that the panel approach can overcome the technical difficulties of cross-section estimation but in dynamic panel, there is the possibility of inconsistent parameter when growth effects and convergence speeds are heterogeneous.

From the above literature, it reveals that most of the studies analyzed cross-country growth regression and highlighted that the factor accumulation drives output per worker growth. However, several other researches emphasize the impact of total factor productivity (TFP) in explaining international differences in levels and growth of output per worker. Easterly and Levine (2001) and Hall and Jones (1999) followed this growth accounting. These researchers found a large variation in level of Solow residual and underline the importance of institutions, government policy and other social infrastructure determine the cross-country capital accumulation variation.

Accordingly, McQuinn and Whelan (2007) used a different approach to estimate conditional convergence. While most researchers used dynamics of per capita or per worker output, they employed capital-output ratio that yields a big difference in rate of convergence. While a major number of research found 2 percent convergence rate per annum, their estimation resulted in about 6 – 7 percent rates, without assuming the constant rate of technological advancement.

Lee et al (1997) considered a stochastic Solow growth model and investigated the properties of convergence rate. Their result reports that steady state varies significantly in cross country evidence and once this heterogeneity is entered in the estimates of beta, it shows substantially higher effect than the theoretical consensus.

Ding and Knight (2009) employed a panel data on 146 countries and specially emphasizes on China to test augmented Solow model. By employing system GMM for the cross-country panel analysis, they found that despite of restrictive assumptions, augmented Solow model with human capital and structural change indicate a significant international variation in economic growth. In particular, Ding and Knight (2009) examine and discover Chinese rapid economic growth is due

to a large volume of investment in physical capital, change in employment structure and output, conditional convergence gain and low population growth policy.

In fine, from the vast empirical literature survey on validating the Solow growth empirics, I found two main observations. First, most researchers used cross sectional data and second, very few used panel estimation with small sample period. Unlike other studies I attempt to fill this gap by utilizing a large sample period spanning 1971-2011 on 20 OECD countries to test both textbook and augmented Solow growth model.

### 3. Theoretical Background

#### 3.1 The Textbook Solow Model

Assuming diminishing marginal returns of capital, exogenous population growth and savings rate, no depreciation and technological progress, the model predicts that steady state per capita income is exogenously determined by savings and population growth rate. Starting with the textbook version of Solow model, I mainly notch the implication up in OECD countries. There are two factors: capital ( $K$ ) and labor ( $L$ ) which are paid by their marginal productivity. The production function in Cobb-Douglas framework at time  $t$  is given by:

$$Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha} \quad 0 < \alpha < 1 \quad (1)$$

$A$  is noted as the level of technology.  $L$  is assumed to be grow at exogenous population growth rate ( $n$ ) and  $A$  is assumed to grow at  $g$  which implies advancement of knowledge. Also it holds,

$$L(t) = L(0)e^{nt} \quad (2)$$

$$A(t) = A(0)e^{gt} \quad (3)$$

Solow assumes that a fixed portion of output, let call it  $s$  is saved and reinvested. Defining output per effective labour  $y = Y / AL$  and stock of capital per effective labour  $\kappa = K / AL$ , the transformation of  $\kappa$  takes the following form:

$$\begin{aligned} \dot{\kappa}(t) &= sy(t) - (n + g + \delta)\kappa(t) \\ &= s\kappa(t)^\alpha - (n + g + \delta)\kappa(t) \end{aligned} \quad (4)$$

Defining  $\delta$  as the rate of depreciation, equation (4) evidently converges to steady state level  $\kappa^*$

$$s\kappa^{*\alpha} = (n + g + \delta)\kappa^* \quad (5)$$

$$\therefore \kappa^* = \left[ \frac{s}{(n+g+\delta)} \right]^{\frac{1}{1-\alpha}}$$

Equation (5) clearly delineates that steady state value of  $\kappa^*$  corresponds positively with savings and negatively with rate of population growth and depreciation. Substituting equation (5) into Cobb-Douglas production function and solving for the equation yields the steady state per capita income.

$$\ln \left[ \frac{Y(t)}{L(t)} \right] = \ln A(0) + gt + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) \quad (6)$$

Thus, the central hypothesis of Solow version of neoclassical growth model concerns that the steady state level of income per capita is determined by savings, labour force growth rate and the technology parameter. More clearly, the growth rate of output per worker depend on initial output per worker  $y(0)$ , the preliminary technology  $A(0)$ , rate of technical advancement  $g$ , savings rate  $s$  (i.e.  $I/GDP$ ), growth rate of working age population  $n$ , depreciation rate  $\delta$  and capital share  $\alpha$ . Based on these determinants Solow predicts that a high savings rate increases the output per worker whereas a high workforce growth rate (anticipated with technical advancement and depreciation) reduces the growth of per worker output

### 3.2 Augmenting Human Capital in Solow Model

Following MRW a number of empirical investigations such as Audienis et al. (2001) and (Ding and Knight, 2009) have exerted the effect of human capital on growth process by augmented Solow model in cross country evidence. Here, the Cobb-Douglas production function is formulated as:

$$Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta} \quad 0 < \alpha + \beta < 1 \quad (7)$$

$H$  is defined as stock of human capital,  $\beta$  is the share of human capital in total output and all other variables are mentioned as before. The assumption of  $\alpha + \beta < 1$  indicates decreasing returns to scale. MRW remark the share of income invested in physical capital ( $s_k$ ) and share of income invested in human capital ( $s_h$ ) depreciate at a common rate  $\delta$ . Thus the natural progress of the economy is determined by

$$\dot{\kappa}(t) = s_k y(t) - (n+g+\delta)\kappa(t) = s_k \kappa(t)^\alpha h^\beta - (n+g+\delta)\kappa(t) \quad (8a)$$

$$\dot{h}(t) = s_h y(t) - (n+g+\delta)h(t) = s_h \kappa(t)^\alpha h^\beta - (n+g+\delta)h(t) \quad (8b)$$

Here  $y = Y / AL$ ,  $\kappa = K / AL$  and  $h = H / AL$  are quantities per effective unit of labour. The steady state value of physical capital and human capital come across by solving equation (8a) and (8b)

$$\kappa^* = \left[ \frac{s_{\kappa}^{1-\beta} s_h^{\beta}}{n+g+\delta} \right]^{\frac{1}{1-\alpha-\beta}} \quad \text{and} \quad h^* = \left[ \frac{s_{\kappa}^{\alpha} s_h^{1-\alpha}}{n+g+\delta} \right]^{\frac{1}{1-\alpha-\beta}} \quad (9)$$

Substituting (9) into the production function (7), rearranging by taking logs yields the steady state income per worker.

$$\ln \left[ \frac{Y(t)}{L(t)} \right] = \ln A(0) + gt - \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) + \frac{\alpha}{1 - \alpha - \beta} \ln(s_{\kappa}) + \frac{\beta}{1 - \alpha - \beta} \ln(s_h) \quad (10)$$

Equation (10) shows that income per capita is determined by population growth, physical capital and human capital. As theoretically, capital share ( $\alpha$ ) is 1/3, equation (10) implies that the elasticity of income per worker with respect to  $s$  and  $(n + g + \delta)$  is 0.5 and  $-0.5$  respectively.

## 4. Methodology

### 4.1 Data and Variables

In this dissertation, I estimate both textbook Solow model and augmented Solow model with larger time period during 1971 to 2011. In the Solow model, per worker output growth depends on the initial value of per worker output, savings and work force growth rate (adjusted by depreciation rate,  $\delta$  and rate of technological advancement,  $g$ ). In the augmented Solow model, I augment the textbook Solow model by adding a measure of schooling. The variables I consider are as follows (see Table 1 and 2 for summary and descriptive statistics of these variables):

- *Output per worker (Y/L)*: I use output per worker (Y/L) instead of output per capita while testing Solow model. While testing the Solow's paradigm, one might ask to employ per capita or per worker variable. Since Solow started with a Cobb-Douglas production function it seems to be more appropriate to use per worker income and realistically every people of a country do not contribute to production. Additionally, population growth rate could be higher than average growth of labour force in some economies. MRW used per worker output whereas Islam (1995) and Caselli (1996) used per capital variable. Finally, for measuring Y/L, I divide real GDP<sup>6</sup> by the working age population.

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<sup>6</sup> GDP (US\$) constant in 2000

- *Workforce growth rate (n)*: I calculate  $n$  as the average growth rate of working age population in percentage points, where working age is defined as 15 to 64.
- *Rate of savings (s)*: I dividing real investment by real GDP to measure  $s$  which means actually the average share of real investment in real GDP.
- *Human capital (SCHOOL)*: I employed proxy for the human capital accumulation by the percentage of labour force that has secondary schooling. Rather than considering secondary school enrolment (as used by MRW, Caselli et al., 1996, Bond et al., 2001), I have used percentage of population aged 15-64 who have secondary schooling years. By this measurement, it rules out the potential bias from poor proxy argued by Gemmell (1996) and Temple (1999). I confine the focus on human capital investment measuring in the form of education and keeping aside investment in health and training

The empirical testing of economic growth theories using both cross-section and panel data are helpful to verify and implication in practical. I estimate the Solow growth model using panel data of 20 OECD countries over the period of 1971-2011<sup>7</sup>. Here, I particularly motivate to estimate the 20 OECD panel because this sample has a very high quality data i.e. it reduces the variation of omitted country-specific effects. I consider annual data of real *GDP* (GDP constant in 2000), investment (constant in 2000), working force (aged 16-64) and working people with secondary schooling of 20 OECD countries over 1971-2011.

I obtain data on Real GDP and Investment for the period 1971-2011 from Penn World Table 7.1<sup>8</sup>. Workforce and secondary schooling data are obtained from World Bank Development Indicator (2012). Based on the obtained workforce data, average workforce growth rate  $n$  is computed by taking the difference between the natural logarithms of total workforce at the end and beginning of each year and dividing by the number of years. I accumulate data on 20 OECD countries on which data are available. It is important to mention that I do not include the member countries that joined after 2000 in OECD. I drop Belgium, Czech Republic, Hungary, Ireland, Luxemburg, Poland and Turkey from the sample due to large missing data over the sample span.

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<sup>7</sup> The sample country include: Australia, Austria, Canada, Denmark, Finland, France, Germany, Iceland, Italy, Japan, South Korea, Netherland, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

<sup>8</sup> Available at [https://pwt.sas.upenn.edu/php\\_site/pwt71/pwt71\\_form.php](https://pwt.sas.upenn.edu/php_site/pwt71/pwt71_form.php)

Table 1 and Table 2 represent the data description, source and summary statistics of concerned variables. A list of panel OECD countries with examined time period is provided in appendix Table A1. Furthermore, the panel line plot of each variable is presented in appendix (*see* Figure A1 to A4). Importantly, the distribution of each data is depicted in appendix (*see* Figure A5).

Following MRW and Islam (1995) I assume  $g$  and  $\delta$  as fixed. Because,  $g$  is the advancement of knowledge and technology which is not country-specific. Also there is no available data on depreciation rate that ease the process of checking country-specific variation in  $\delta$ . Moreover, no one has shown any plausible explanation to expect great variation of depreciation across countries. The sum of depreciation rate ( $\delta$ ) and workforce growth rate ( $n$ ) is assumed to be 0.05. The natural logarithm of  $n$  and 0.05 is calculated by the variable  $\ln(n+g+\delta)$ .

## 4.2 Empirical Strategy

I calculate  $n$  as the average growth rate of working age population (15-64),  $s$  as the proportion of investment in GDP and  $Y/L$  as real income in 2000 divided by number of working people. More specifically, I want to examine whether real income is higher in countries with higher savings rates and lower in countries with higher population growth rate and depreciation rate. According to MRW I assume  $g$  and  $\delta$  as fixed. Because,  $g$  is the advancement of knowledge and technology which is not country-specific. Also there is no available data on depreciation rate, which ease the process of checking country-specific variation in  $\delta$ . Moreover, none have shown any plausible explanation to expect great variation of depreciation across countries. On the contrary, along with technology, the term  $A(0)$  also includes country's natural resource, weather and institutional quality etc. This assumption implies that  $\ln A(0) = a + \varepsilon$  where  $a$  is constant and  $\varepsilon$  is shocks.

Therefore, the income per capita in logarithmic form at an initial time becomes:

$$\ln \left[ \frac{Y}{L} \right]_{it} = a + \frac{\alpha}{1-\alpha} \ln(s_{it}) - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + \varepsilon_{it} \quad (11)$$

As I logically assume savings and population growth (corrected by  $g$  and  $\delta$ ) rate are independent of country specific shock  $\varepsilon$  i.e.  $[E(s_{it}\varepsilon_{it}) = 0, E(n_{it}\varepsilon_{it}) = 0]$ , methodology allows to estimate equation (11) by using Ordinary least Square (OLS).

In this case, researchers provide several justifications for maintaining the orthogonality condition. Firstly, this is not only Solow model, rather many standard empirical growth models

made the assumption of independence. In any model, population growth and savings rate can be endogenous but isoelasticity of preferences keeps  $s$  and  $n$  unaffected by country specific shock  $\varepsilon$ . Explicitly, if isoelasticity of utility holds, permanent differences in technology do not affect  $s$  and  $n$ . Secondly, Lucas (1988) suggested that cross-country variation in  $n$  cannot substantially influence income per capita in line with the Solow model. Thirdly, if the model is correctly specified, the elasticity of income per worker with respect to  $s$  and  $(n + g + \delta)$  are roughly 0.5 and  $-0.5$ . If OLS produces coefficients different from these values, we can reject the joint hypothesis that the assumption is correct.

For estimating an augmented Solow model, I focus on human capital investment measured in the form of education and keep aside investment in health and training. Although it is an exiguous setup, it encounters great difficulties in practice to measure human capital. Importantly, a large portion of investment in education takes the form of forgone labour earnings (Kendrick, 1994). Therefore, foregone earnings vary substantially with the level of human capital. Generally, a labourer with low human capital forgoes a higher wage and a labourer with high human capital forgoes a lower wage. Moreover, investment in education takes place both in government and private stages, which adds more complexity to measuring investment in education. Explicitly, not all investment in education yields human capital; it also broadens worker's mental development and experience. In sequel, I employed a proxy for human capital accumulation by the percentage of the labour force that has secondary schooling. I augment the textbook Solow regression by adding human capital measured by *SCHOOL* i.e. percentage of labour force who have secondary schooling. Explicitly, in an augmented Solow model, I estimate equation (11) by adding a measure of human capital (*SCHOOL*).

$$\ln \left[ \frac{Y}{L} \right]_{it} = a + \frac{\alpha}{1-\alpha} \ln(s_{it}) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta) + \eta_i + \varepsilon_{it} \quad (12)$$

As the OLS regression does not consider unobserved country specific effects in the model (eq. 11), therefore country heterogeneity can appear in the estimated parameters. Consequently, I estimate equation 11 with unobserved country specific effects ( $\eta_i$ ) eq. 12 using within-group Fixed Effect and GLS Random Effect approaches. Using unobserved country specific effects have several advantages e.g. it reduces measurement error. For testing the relevancy of unobservable country specific effects, I use Breusch and Pagan's LM test.

As the static panel regression does not allow us to estimate the possible dynamism of the model correspondingly I use the dynamic panel GMM estimators that were pioneered by Holtz-Eakin

et al. (1988), Arellano and Bover (1995), Blundell and Bond (1998) Bond et al (2001). The advantages of this method make possible to control for the unobserved individual (country-specific) effect by considering time invariant fixed effect and reducing its effects through a transformation in time-dimension. Another advantage of the dynamic panel model is eliminating the endogeneity problem through considering lags of independent variables as instruments. The panel consists of data from 20 OECD countries over the time period 1971-2011. Therefore dynamic panel data method is appropriate in this regard. In dynamic framework, equation can be written in following specifications;

$$(Y/L)_{i,t} = \alpha + \gamma_1 \ln[Y/L]_{i,t-1} + \beta' [X]_{i,t} + \eta_i + \varepsilon_{i,t} \quad (13)$$

Where  $(Y/L)_{i,t}$  is the difference of output per worker treated as a dependent variable,  $\ln[Y/L]_{i,t-1}$  is the log of lagged dependent variable and  $X$  represents the set of explanatory variables, which includes human capital accumulation (school), rate of technical advancement  $g$ , savings rate  $s$  (i.e.  $I/GDP$ ), growth rate of working age population  $n$ , depreciation rate  $\delta$ .  $\varepsilon_{i,t}$  is an iid (independently and independently distributed) error term with  $E[\varepsilon_{i,t}] = 0$  and the subscripts  $i$  and  $t$  denotes country and time period respectively.  $\eta_i$  is unobserved individual (country) specific effects that is not correlated with the error term ( $\varepsilon_{it}$ ).

For  $i=1, \dots, N$  and  $t=2, \dots, T$ , where  $(\eta_i + \varepsilon_{i,t})$  denotes as a standard error component structure;

For equation (13),  $E[\eta_i]=0$ ,  $E[\varepsilon_{i,t}]=0$ ,  $E[\varepsilon_{i,t}\eta_i]=0$  for  $i=1, \dots, N$  and  $t=2, \dots, T$

Considering the first difference to remove individual (country) specific effects,

$$\Delta[Y/L]_{i,t} = \alpha + \gamma_1 \Delta \ln[Y/L]_{i,t-1} + \beta' [\Delta X]_{i,t} + \eta_i + \Delta \varepsilon_{i,t} \quad (14)$$

Since the lagged dependent variable  $\Delta[Y/L]_{i,t}$  and are correlated with error term  $\Delta \varepsilon_{i,t}$ , which implies that the regressors are potentially endogenous and will not produce a consistent estimate of coefficient. Therefore, it is inevitable to use instruments to deal with equation (13). Here

$\Delta \varepsilon_{i,t}$  is not serially correlated and the explanatory variables are weakly exogenous<sup>9</sup>. Therefore, the dynamic panel GMM estimator employs the following moment conditions based on difference estimator for equation (13);

$$E[Y / L_{i,t-s} (\varepsilon_{i,t} - \varepsilon_{i,t-1})] = 0 \quad \text{for } t=3, \dots, T, \quad s \geq 2 \quad (15)$$

$$E[X_{i,t-s} (\varepsilon_{i,t} - \varepsilon_{i,t-1})] = 0 \quad \text{for } t=3, \dots, T, \quad s \geq 2 \quad (16)$$

Which can be written in following matrix form as;

$$M = \begin{pmatrix} y_{i1} & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & y_{i1} & y_{i2} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & y_{i1} & \dots & y_{i,T-2} \end{pmatrix}$$

Here,  $M$  is the instruments matrix corresponding to the endogenous variables, where  $\mathcal{Y}_{i,t-s}$  refers to  $[Y / L]_{i,t-s}$  for equation (14).

However, the efficiency and consistency of first differenced estimator is criticized in terms of bias and imprecision. Thus, to reduce potential biases and imprecision, Blundell and Bond (1998) suggest that, when explanatory variables have short time period, we can use a new estimator that combines a system in the difference estimator with the estimator in levels, which is named as system GMM. The difference operator in equation (14) uses the same instrument as above and the instruments for the levels are the lagged difference of the explanatory variables. The intuition here is that the difference in the explanatory variables and the country specific effects are uncorrelated (Das, 2013). Therefore the stationary properties are:

$$E[Y / L_{i,t+p} \eta_i] = E[Y / L_{i,t+q} \eta_i] \quad \text{and} \quad E[X_{i,t+p} \eta_i] = E[X_{i,t+q} \eta_i] \quad \forall p \text{ and } q$$

The additional moment conditions for the levels are

$$E[\Delta Y / L_{i,t-s} (\eta_i + \varepsilon_{i,t})] = 0 \quad \text{for } s=1 \quad (17)$$

$$E[\Delta X_{i,t-s} (\eta_i + \varepsilon_{i,t})] = 0 \quad \text{for } s=1 \quad (18)$$

---

<sup>9</sup> Assuming that the explanatory variables are not correlated with future  $\varepsilon_{i,t}$ .

Now I can use system GMM technique for both models to estimate consistent and efficient parameter by employing the moment conditions given in equation (15), (16), (17) and (18) to get more robustness of the result. Here, instruments use to overcome the potential endogeneity, which generates more consistent and efficient parameters.

Finally, to check the validity of the instruments in the system-GMM estimator, we implement two specification test, which is suggested by Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998). The *Sargan* and *Hansen* test of over-identification to check the validity of the instruments.

Table 1: Summary variable description

Variable name	Unit of measurement	Data source
GDP	US\$	PWT
Investment	US\$	PWT
Working population	Number	World Bank
Labour force with secondary schooling	Percent	World Bank

Table 2: summary statistics

Variable	Unit of measurement	Obs.	Mean	Std. Dev.	Min	Max
GDP	US\$(constant in 2000)	820	954855*	1842698*	2791*	11744219*
Investment	US\$(constant in 2000)	820	10465*	7660*	-29260*	803976*
Labour force with secondary schooling	Percent	820	38.30	15.90	2.9	80.97
Working population (16-64)	Number	820	27*	38*	0.12*	207*

Note: \* denotes number in million.

## 5. Result and Discussion

### 5.1 Estimating Textbook Solow Model

Following the econometric methodology, I estimate the equation (11) both with and without imposing the constraint that the coefficient of  $\ln(I / GDP)$  and  $\ln(n + g + \delta)$  are equal in size and opposite in sign. I assume that  $(g + \delta)$  is 0.05. Taking consideration of US data, the capital consumption allowance is about 10 percent of GDP and capital-output ratio is 3, which implies that  $\delta$  is close to 0.03. It also supports Romer (1989) that  $\delta$  is about 0.03 or 0.04 for cross-country sample. In addition the growth of income per working age population is about 1.7 percent, which suggests that  $g$  is about 0.02.

Table 3: Estimation of textbook Solow model

Variable	Coefficient	Standard error
Constant	7.980***	[0.429]
$\ln(I / GDP)$	0.093***	[0.009]
$\ln(n + g + \delta)$	-0.932***	[0.148]
$R^2$	0.24	
Observation	820	
Time	1971-2011	
Restricted Regression		
Constant	10.223***	[0.016]
$\ln(I / GDP)$	- 0.041***	[0.006]
$\ln(n + g + \delta)$		
$R^2$	0.05	
<i>Test of restriction:</i>		
Restricted $F$ value	5712.36***	
Observation	820	
Time	1971-2011	

Note: Dependent variable is Log of GDP per working age person

$(g + \delta)$  is assumed to be 0.05

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

The result of estimated textbook Solow model is reported in Table 3. First, the result depicts that the coefficient of  $\ln(I / GDP)$  and  $\ln(n + g + \delta)$  have the same sign predicted by Solow paradigm. The coefficients are statistically significant in 1 percent significance level. The magnitude of  $\ln(I / GDP)$  implies that 9 percent increase in savings bounce the output per

worker up by 1 percent. The coefficient (-0.932) of  $\ln(n + g + \delta)$  denotes higher population growth rate reduces the output per worker significantly. Second, the restricted test is statistically significant. Third, income per capita is importantly varying with the difference in savings and population growth rate. Importantly, the explanatory power of the regression i.e.  $R^2$  is 0.24. Therefore, these three plausible and persuasive findings from 20 OECD countries over 41 years evidently claim the correspondence of Solow model.

## 5.2 Estimating Augmented Solow Model

Table 4 illustrates the regression of log of income per working age people ( $Y/L$ ) on the log of investment rate ( $I/GDP$ ), log of  $(n + g + \delta)$  and log of the percentage of labour who have secondary education ( $SCHOOL$ ).

Table 4: Estimation of augmented Solow model

Dependent Variable: Log of GDP per working age person		
Variable	Coefficient	Standard error
Constant	7.428***	[0.381]
$\ln(I / GDP)$	0.069***	[0.008]
$\ln(n + g + \delta)$	-0.593***	[0.134]
$\ln(SCHOOL)$	0.398***	[0.033]
$R^2$	0.41	
Observation	820	
Time	1971-2011	
Restricted Regression		
Constant	6.970***	[0.127]
$\ln(I / GDP) - \ln(n + g + \delta)$	0.026***	[0.004]
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.511***	[0.199]
$R^2$	0.47	
<i>Test of restriction:</i>		
Restricted F value	732***	
Observation	820	
Time	1971-2011	

Note:  $(g + \delta)$  is assumed to be 0.05; SCHOOL is the percentage of working age population who have secondary education.

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Here, I augment the textbook Solow regression by adding human capital measured by *SCHOOL*. Appending human capital has a significant influence on the overall result. It considerably reduces (from 9 percent to 6 percent) the magnitude of the coefficient of physical capital investment and upgrades the fitness of regression. The coefficient (0.398) of human capital ( $\ln(SCHOOL)$ ) correctly describes the international variation in output per capita captured by differences in secondary schooling skill. The countries whose labour force has more schooling and thus skill, the output per worker grow faster. Now, it explains about 41 percent of the variation of income per worker in cross country sample. Hence, the result in Table 4 strongly supports the existence of augmented Solow model.

### **5.3 Measuring Unobserved Country Specific Effect**

When I run OLS, I ignore the unobserved country specific effects for equation (11) both in textbook and augmented Solow model. Therefore, there is possibility to appear heterogeneity of countries in the estimated parameters. Hence, I estimate the Solow model which encounter unobserved country specific effects by Fixed Effect (FE) – Within and Random Effect (RE) – GLS regression.

However, estimating country specific effects has a number of methodological benefits e.g. it allows accounting for specific effects. After that, I employ Breusch and Pagan's LM test for examining the relevancy of unobservable country specific effects. This test helps to decide between the acceptance of RE-GLS and OLS. If I can reject the null hypothesis<sup>10</sup>, OLS is not the appropriate technique for estimation and vice versa. Importantly, I use Hausman test to investigate the relationship between regressors and unobserved country effects. Hausman test<sup>11</sup> allows testing for the misspecification between FE and RE estimation.

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<sup>10</sup>  $H_0$ : Irrelevance of unobserved country specific effects and  $H_A$ : Relevance of unobserved country specific effects.

<sup>11</sup>  $H_0$ : No correlation exists between regressors and unobserved country specific effects and  $H_A$ : Correlation exists between regressors and unobserved country specific effects.

Table 5: Fixed effect and random effect

	Fixed Effect-Within	Random Effect –GLS
Textbook Solow Model		
Constant	7.446*** [0.308]	7.447*** [0.316]
$\ln(I / GDP)$	0.073*** [0.004]	0.073*** [0.004]
$\ln(n + g + \delta)$	-1.089*** [0.105]	-1.089*** [0.105]
$R^2$	0.23	0.24
LM Test		4189.62***
Hausman Test ( <i>p-value</i> )	0.9054	
Observations	820	820
Augmented Solow Model		
Constant	6.690*** [0.293]	6.680*** [0.299]
$\ln(I / GDP)$	0.054*** [0.004]	0.054*** [0.004]
$\ln(n + g + \delta)$	-0.966*** [0.098]	-0.960*** [0.097]
$\ln(SCHOOL)$	0.288*** [0.030]	0.295*** [0.029]
$R^2$	0.39	0.39
LM Test		4370.71***
Hausman Test ( <i>p-value</i> )	0.7584	
No. of Country	20(OECD)	20(OECD)
Observations	820	820

Note: Dependent Variable is Log difference of GDP per working age person  
Standard errors in parentheses.  
\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

Since, OLS do not control for country specific effects, I carry out FE-Within and RE-GLS regression for both textbook and augmented Solow hypothesis. Results are represented in Table 5. Additionally, to examine the relevancy of country specific effects, LM statistics refers that I can reject the null hypothesis, implying OLS is not appropriate technique to show the relation between output growth and  $\ln(s)$ ,  $\ln(n + g + \delta)$  and  $\ln(SCHOOL)$ . Using FE-Within estimation I reject the null at 1 percent level that implies  $\ln(I / GDP)$  and  $\ln(SCHOOL)$  have significant positive effect while  $\ln(n + g + \delta)$  has significant negative effect on dependent variable. Under Hausman, testing null hypothesis that the preferred model is random effect vs. the alternative the fixed effects, I cannot reject the null hypothesis with *p-value* 0.9054 and 0.7584. Therefore, it runs out fixed effect and random effect appears to be appropriate for those models. In RE-GLS, two-tail *p-values* test the hypothesis that each coefficient is different from 0. Since, the *p-value* is less than 0.001 I can say that  $\ln(I / GDP)$  and  $\ln(SCHOOL)$  have significant

positive effect on income per worker while  $\ln(n + g + \delta)$  has significant negative impact on income per worker. Economically, it correctly matches with the prediction that Solow made.

#### 5.4 Dynamic Panel GMM estimation: Text book Solow model

This section shows the dynamic panel estimation results. Firstly, run regression using Fixed Effect-within group specification, later on estimate eq. (12) on the dataset described above by using difference and system GMM panel techniques. Subsequently, I also run both *Hansen* and *Sargan* tests to check the validity of using these specifications.

Table 6: Dynamic panel estimate of textbook Solow model

	Fixed Effect-Within	Dif GMM	System GMM
$\ln(Y / L)_{t-1}$	-0.0328*** (0.0065)	-0.525* (0.317)	-0.00971* (0.0721)
$\ln(I / GDP)$	0.039** (0.009)	0.0525* (0.002)	0.0446* (0.004)
$\ln(n + g + \delta)$	-0.0576*** (0.0187)	-0.0674** (0.0344)	-0.0419* (0.0552)
Constant	0.192** (0.0865)	-0.162 (0.0995)	0.000661 (0.856)
Implied $\square$	0.008	0.023	0.069
Observations	820	780	820
R-squared	0.295		
Hansen test (p-value)			0.593
Sargan test (p-value)			0.239
Number of country	20(OECD)	20(OECD)	20(OECD)

Note: Dependent Variable is Change in GDP per working age person

Standard errors in parentheses.

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

According to the econometric assumptions, we know that the pooled OLS estimation is upward biased and the fixed effects model is downward biased (Baltagi, 2008). Thus, I consider difference GMM and system GMM techniques as an efficient estimator. Even though, using Monte Carlo experiments by Blundell and Bond (1998), (Blundell and Bond, 2000) demonstrates that the difference GMM estimators of the lagged dependent variable are strongly downward biased. Thus, they suggests for the system GMM estimation, which is set between the upper bound of pooled OLS estimation and lower bound of fixed and difference GMM estimation. Thus, I consider both difference GMM and system GMM in the following specifications.

Moreover, in each estimation, I check the validity of the additional instruments and moment restrictions in the system GMM model compare to the difference GMM estimation.

The dynamic panel estimation result is presented in Table 6, which reports the result using Fixed Effect-Within, difference GMM and system GMM estimators. These specifications consider change in output per worker ( $\Delta Y/L$ ) as a dependent variable with a one year lagged output per worker and a set of other explanatory variables (equation 13). The coefficients of the lagged output per worker show the significance of including this variable in all specifications. The negative sign of the coefficient confirms that there is clear evidence of conditional convergence.

Column (1) depicts the fixed effect- within estimator allowing the parameter homogeneity across countries. The coefficients of rate of investment and  $\ln(n+g+\delta)$  have the expected positive and negative sign respectively with statistical significance. Column (2) and (3) represent difference-GMM and system-GMM estimator respectively. The coefficient of investment rate has significantly positive impact in explaining variation in output per worker even after controlling for unobserved individual (country) specific effect. Similarly, I also identified a significantly negative impact of  $\ln(n+g+\delta)$  as expected as well which implies that the specifications are correct. Therefore, the dynamic panel GMM results also support the findings of MRW and Islam (1995) for the textbook Solow model. However, to test the validity of the estimating result I use both Sargan and Hansen test for system-GMM specification. Under the both tests, I cannot reject the null hypothesis which implies that the first difference instrumental variables are not correlated with error term. Hence the instruments are valid for the estimation.

To sum up, I get the expected results, which capture the same sign and magnitude in advanced panel approach. In particular this approach provides more significant result and importantly it provides about 4.7 percent rate of convergence, which is about twice than the standard consensus rate (2-3 percent, MRW) in literature.

### **5.5 Dynamic Panel GMM estimation: augmented Solow model**

The role of schooling also determines the output per worker as MRW and other influential papers have claimed that increasing schooling has positive contribution. Accordingly, Table 7 shows the result of augmented Solow model considering human capital. Following equation (10), I incorporate the log of secondary schooling as a measure of human capital along with rest right hand side variables of the textbook Solow model. The coefficient on lagged output per worker,

rate of savings,  $\ln(n + g + \delta)$  and  $\ln(SCHOOL)$  have expected sign and significant effects in all three specifications indicating strong evidence of faster convergence after including a measure of human capital. I find that system GMM estimator yields the consistent estimate which is well accepted over the fixed effect –within group and difference GMM estimator. Likewise other previous estimation results, the augmented Solow model also supports MRW and Islam (1995) and others. Hence the role of both initial stock and consecutive growth rate of human capital will foster output per capita growth in OECD countries.

Table 7: Dynamic Panel GMM estimation: Augmented Solow model with human capital

	Fixed Effect-Within	Dif GMM	System GMM
$\ln(I / GDP)_{t-1}$	-0.0344***	-0.199	0.00148
	-0.00605	-0.312	-0.0727
$\ln(I / GDP)$	0.000197	-0.000172	-0.00109
	-0.000949	-0.00247	-0.00454
$\ln(n + g + \delta)$	-0.0574***	-0.0926***	-0.0409
	-0.0187	-0.0328	-0.063
$\ln(SCHOOL)$	0.0358**	0.0821***	0.0929*
	-0.00292	-0.00285	-0.0619
Constant	0.195**	-0.214**	-0.151
	-0.0866	-0.0868	-0.82
Implied $\square$	0.063	0.029	0.047
Observations	820	780	820
R-squared	0.35		
Hansen test (p-value)			0.614
Sargan test (p-value)			0.291
Number of country	20(OECD)	20(OECD)	20(OECD)

Note: Dependent Variable is Change in GDP per working age person

Standard errors in parentheses.

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

Compared with textbook Solow model, inclusion of *SCHOOL* variable in the regression leads to several important changes in augmented Solow model. Firstly, data says that, the coefficient of savings rate becomes statistical insignificant while we add human capital measure. And coefficient of *SCHOOL* variable now explains a major portion of cross country differences in per worker output in OECD economies. Secondly, the inclusion of human capital measure

increased the explanatory power of regressors. Now, it can explain about 36 percent variation in OECD country's differences in output per worker.

## 6. Endogenous Growth and Convergence

The estimated model with physical and human capital will be endogenous if the sum of factor share equates 1 (i.e.  $\alpha + \beta = 1$ ). This implies that countries that save more grow faster for an indefinite period and those countries need not converge in per capita income even if they have same preference and technology. In this section, I reinvestigate the evidence on convergence to assess whether it supports Solow model or not.

### 6.1 Theory

While estimating the textbook and augmented Solow model, international variation in income per capita is explained mostly by physical capital, human capital and population growth rate. But it explains better when I add human capital later on. I assumed countries were in steady state level in 1971, therefore, result of Table 1 and Table 3 only forecast that income per capita of a country converge to that country's steady state value. Consequently, Solow predicts convergence after controlling for steady state determinants. Additionally, Solow judges the speed of convergence by taking the deviation of per capita income from an initial period.

$$\frac{d \ln(y(t))}{dt} = \lambda[\ln(y^*) - \ln(y(t))] \quad (19)$$

Where,  $\lambda = (n + g + \delta)(1 - \alpha - \beta)$

The model advices a natural regression for investigating convergence speed. Equation (19) refers that

$$\ln(y(t)) = (1 - e^{-\lambda t}) \ln(y^*) + e^{-\lambda t} \ln(y(0)) \quad (20)$$

Where  $y(0)$  is income per working age population in 1971. Taking the difference, I get

$$\ln(y(t)) - \ln(y(0)) = (1 - e^{-\lambda t}) \ln(y^*) - (1 - e^{-\lambda t}) \ln(y(0)) \quad (21)$$

Following up and solving for  $y^*$ , it yields

$$\ln(y(t)) - \ln(y(0)) = (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln(s_k) + (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln(s_h) \quad (22)$$

$$-(1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) - (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln(y(0))$$

The last expression tells that income per worker is a function of initial income and determinants of steady state.

## 6.2 Estimation of Conditional Convergence

Table 6 includes the log difference of income per working age person as dependent variable. The log of income per worker is the only explanatory variable in right hand side. The coefficient of  $\ln(Y70)$  is statistically significant and negative. This result is supported with Dowrick and Nguyen (1989) who found OECD countries converged since 1950, the post World War II period. Table 8 and Table 9 represent the estimation of equation (22) along with the test of restriction. I impose restriction that the sum of coefficients of  $\ln(I / GDP)$  and  $\ln(n + g + \delta)$  is zero (Table 8). Again, restriction that the sum of coefficients of  $\ln(I / GDP)$ ,  $\ln(n + g + \delta)$  and  $\ln(SCHOOL)$  is zero later. The restricted  $F$  value is statistically significant.

Following Romer (1987) the conditional and unconditional convergence of Solow model can be represented by the Figure 1. Panel (a) illustrates the scatter diagram of growth rate of income per capita and log of income per capita from 1971 to 2011. Panel (b) of the diagram partials out the  $\ln(I / GDP)$  and  $\ln(n + g + \delta)$  from the income level and growth. Panel (c) partials out  $\ln(SCHOOL)$  along with  $\ln(I / GDP)$  and  $\ln(n + g + \delta)$ . It is evident that the convergence trend is relatively stronger in panel (c) compared to panel (a) and panel (b).

Table 8: Unconditional convergence

Variable	Coefficient	Standard error
Constant	4.672***	[0.948]
$\ln(Y70)$	-0.398***	[0.096]
$R^2$	0.49	
<i>Test of restriction:</i>		
Restricted $F$ value	86.28***	
Observation	20 (OECD)	
Time	1971-2011	

Note: Dependent Variable is Log difference of GDP per working age person

$Y70$  is the GDP per working age people in 1971

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

Table 9: Conditional convergence on savings and population growth

Variable	Coefficient	Standard error
Constant	0.961	[2.689]
$\ln(Y70)$	-0.076	[0.285]
$\ln(I / GDP)$	0.023*	[0.010]
$\ln(n + g + \delta)$	-0.251	[0.263]
$R^2$	0.30	
<i>Test of restriction:</i>		
Restricted $F$ value	7.44**	
Observation	20 (OECD)	
Time	1971-2011	

Note: Dependent Variable is Log difference of GDP per working age person

$Y70$  is the GDP per working age people in 1971

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

Table 10: Conditional convergence on savings, population growth and human capital

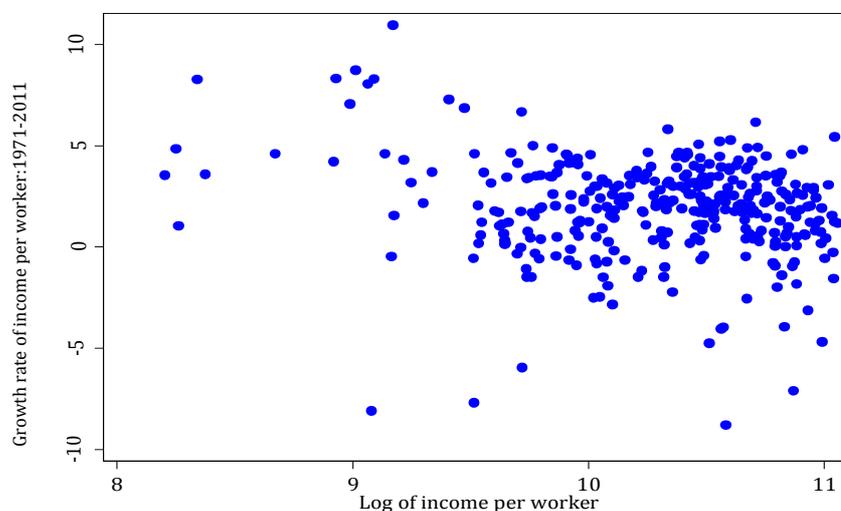
Variable	Coefficient	Standard error
Constant	-0.315	[0.948]
$\ln(Y70)$	0.003	[0.264]
$\ln(I / GDP)$	0.036*	[0.018]
$\ln(n + g + \delta)$	-0.310	[0.242]
$\ln(SCHOOL)$	0.156*	[0.074]
$R^2$	0.18	
<i>Test of restriction:</i>		
Restricted $F$ value	8.22**	
Observation	20 (OECD)	
Time	1971-2011	

Note: Dependent Variable is Log difference of GDP per working age person

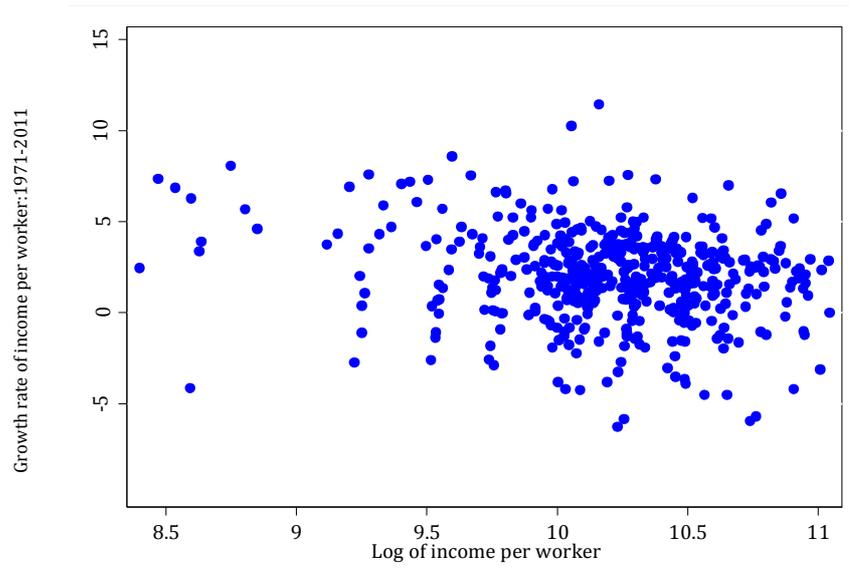
$Y70$  is the  $C$  **Figure 1: Unconditional and conditional convergence of Solow model**

\*\*\*  $p < 0.01$ , \*

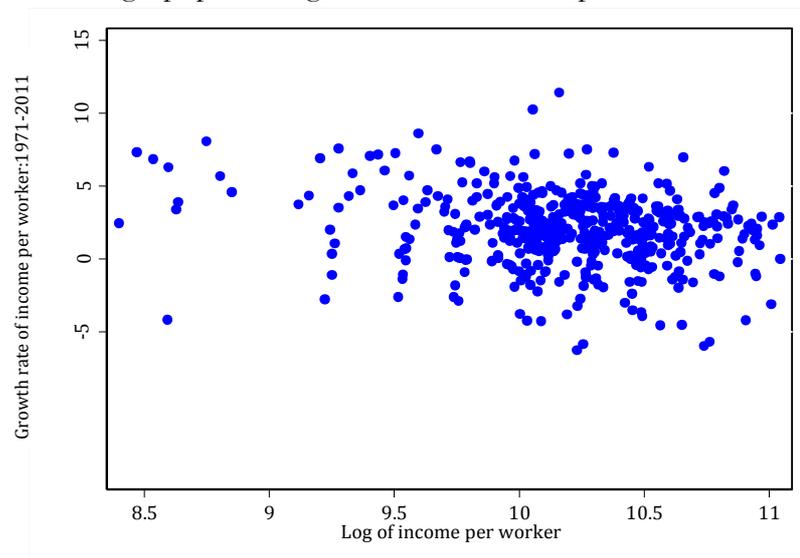
(a) Unconditional



(b) Conditional on savings and population growth



(c) Conditional on savings, population growth and human capital



## 7. Conclusion

In this study, I have examined the role of textbook and augmented Solow model in explaining the growth process of 20 OECD countries over 1971-2011. Following MRW and Islam (1995), I extended their cross-section analysis into large panel analysis using a robust and consistent dynamic panel GMM technique (system GMM). I have shown that the incorporation of human capital measure can better explain the international variation in the level of output per worker and dynamic panel estimation produce more efficient estimator for explaining it.

The estimation is carried out under the assumption of independence of savings and population growth rate from unobserved country effects in several grounds of strong argument. Relaxing this assumption of homogeneity of country effects and incorporating unobserved country shock. I estimate the unconditional and conditional convergence of growth process towards steady state level. Both the textbook and augmented Solow model is verified by using these three sets of techniques.

Firstly, the estimation of textbook Solow model demonstrates that higher workforce growth rate significantly reduces the output per worker and about 9 percent increase in savings bounce the output per worker up by 1 percent.

Secondly, when I append human capital measure and estimate augmented Solow model, the effect of physical capital in explaining cross-country variation of output per worker considerably reduces from 9 percent to 6 percent. Importantly, about 3.9 percent contribution is exerted by human capital measure i.e. secondary schooling skill. In real context, it implies that the countries whose labour force has more schooling and thus skill, the output per worker grow faster. Thirdly, the conditional convergence rate is higher than unconditional convergence when it is conditioned on workforce growth rate, rate of savings and measure of human capital. Fourthly, the efficient system GMM estimator provides more significant result. It provides about 4.7 percent rate of convergence that is about twice than the standard consensus rate (2-3 percent , MRW) in literature.

The result suggests that cross-country variation in income per worker is better explained by augmented Solow growth model. In this model, the income per worker is determined from rate of savings, physical capital and human capital. The result implies that higher savings rate leads to higher income in steady state which again increases the level of human capital even if the rate of human capital accumulation is constant. The growth rate of working age population has a significant impact on income per worker. It substantially declines the per capita income which is resulted from decreasing marginal productivity of labour. The speed of convergence of growth process is much better when I added human capital.

More generally, the results indicate the Solow model is consistent in cross-country variation with both physical capital and human capital. Rate of savings, growth rate of working age population, physical capital and human capital do explain the much variation in cross-country evidence.

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## Appendix

Table A1: List of panel countries and time period

Country	Year
Australia	1971-2011
Austria	1971-2011
Canada	1971-2011
Denmark	1971-2011
Finland	1971-2011
France	1971-2011
Germany	1971-2011
Iceland	1971-2011
Italy	1971-2011
Japan	1971-2011
Korea	1971-2011
Netherland	1971-2011
New Zealand	1971-2011
Norway	1971-2011
Portugal	1971-2011
Spain	1971-2011
Sweden	1971-2011
Switzerland	1971-2011
United Kingdom	1971-2011
USA	1971-2011

Figure A1: GDP of OECD countries from 1971 to 2011

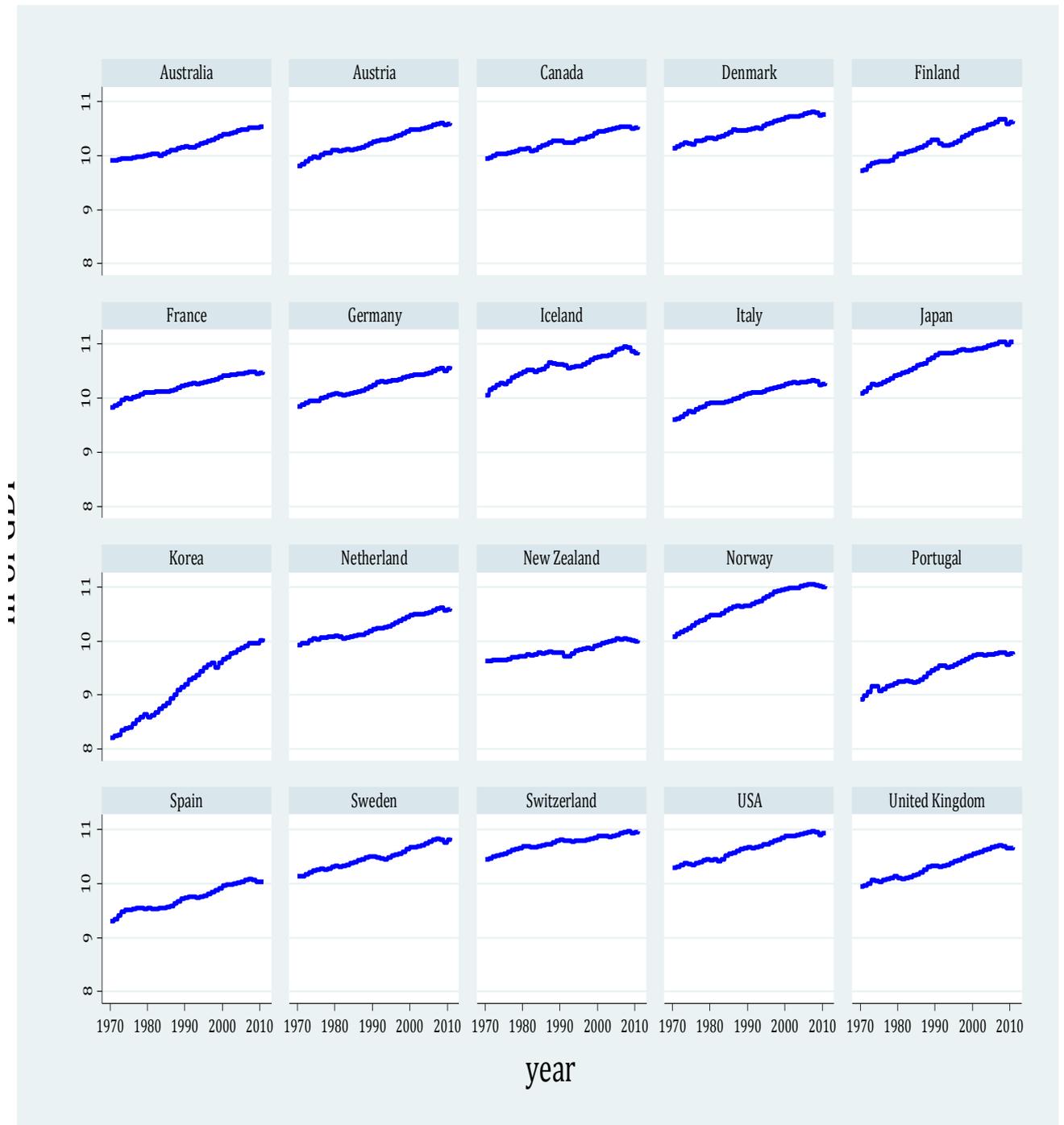


Figure A2: Rate of savings of OECD countries from 1971 to 2011

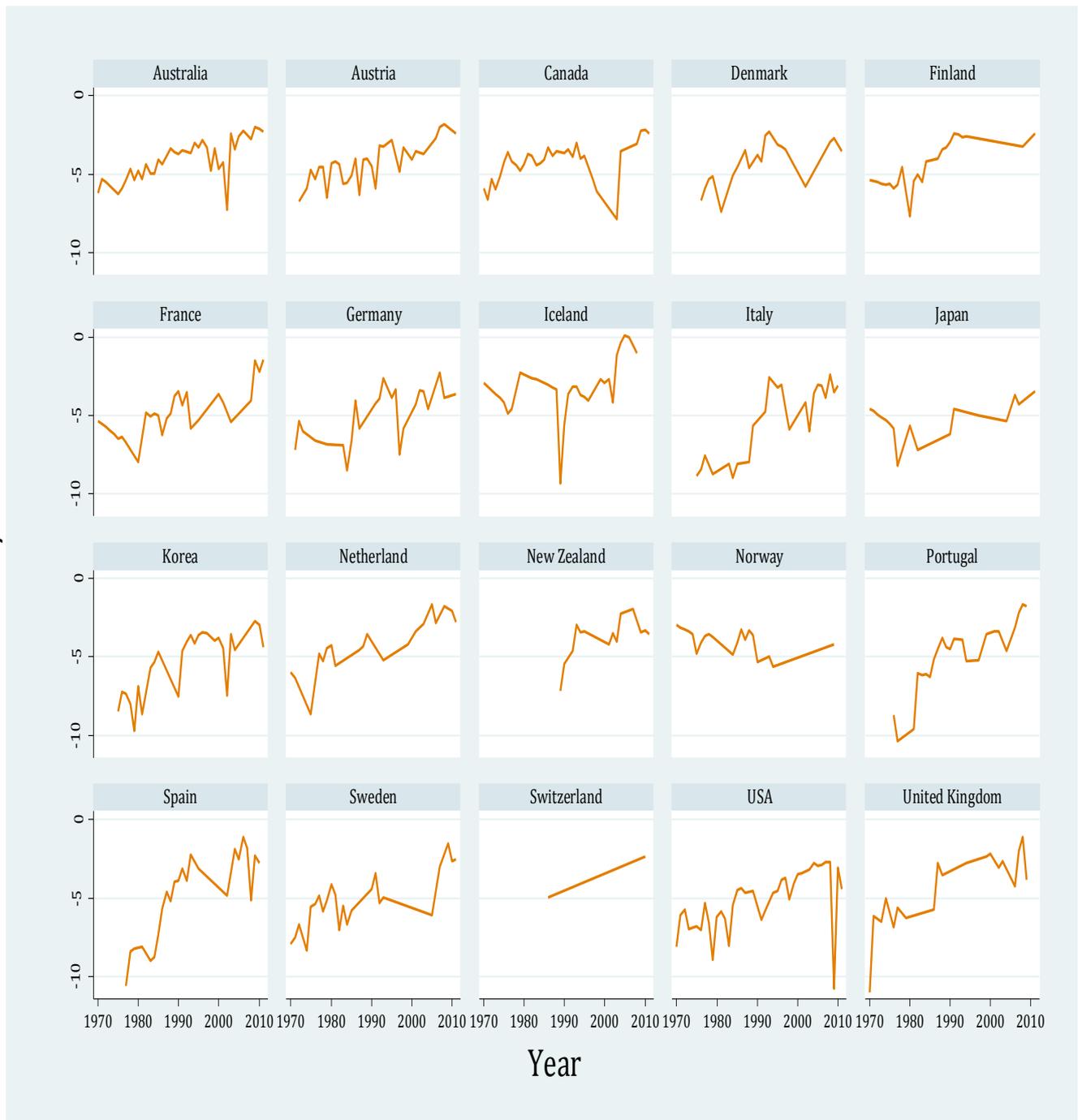


Figure A3: Growth rate of working age population in OECD countries: 1971-2011



Figure A4: Workforce with secondary schooling in OECD countries

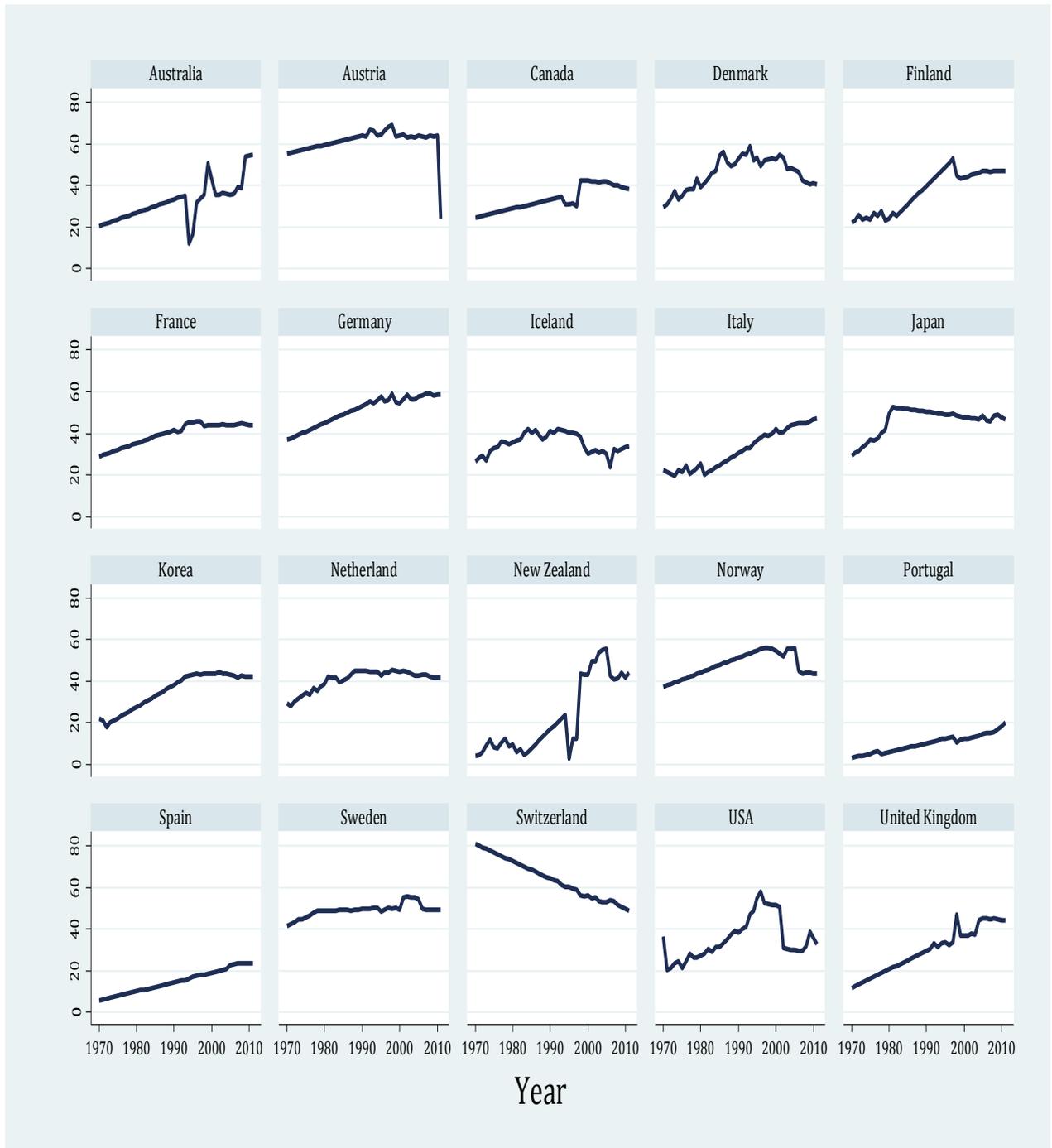


Figure A5: Density function of the variables

