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Structural modeling of electricity spot prices, using a residual load and coal generation cost approach

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Abstract

This thesis aims at evaluating a couple of recently developed strategies for modeling the spot price of electricity. We use a structural modeling approach to model the fundamental behavior of total system load combined with separate models for wind- and solar energy infeed, connected to a number of supply functions to estimate the spot price from the system load. Furthermore we also evaluate the models performances when estimating prices of quarterly power futures and investigate the coal generation costs impact on spot prices over time, using amongst other things, an Artificial Neural Network approach.

Contents

1 Introduction

This thesis will look into the performance of a new model-class that is generally known as a structural modeling approach to model spot price of electricity in the German power market. The core elements of this approach is to model infeed from renewable sources, as well as finding and/or investigating what in previous research has been denoted as "Supply Functions", which relates to the structure of power supply in the power system and the merit order of fuels. The thesis will build on, combine and develop the works previously carried out by Wagner (2012) [1] and Yang et al. (2012) [2]. Since the different power markets differs a great deal from each other, the focused market in this thesis will be the German, as in the studies carried out by both Wagner and Yang. The German powermarket is one of the largest, most advanced and most liquid power markets in Europe, why they are an interesting and relevant subject to study. It is more and more realized in this field of study that the constant and significant fundamental changes in market structure are so significant (e.g. the fast and strong increase of renewables in Germany) that it is very hard to capture the price dynamics by more conventional modeling techniques. In this first section of the thesis, we attempt to give the reader an introduction to the market and its specific conditions and problems, and explain the key elements which will be addressed in this thesis. The German market will be addressed in section 2.1 in detail, with its particular settings and how they play a role in the spot price modeling of electricity in Germany.

1.1 Electricitymarket

The electricity- or power markets are a relatively new concept with it's earliest appearance in the 1980's in the United States. It came to grow during the 1990's and early 2000's in especially North America, Europe and a couple of countries in Asia. It was coupled in many places with the deregulation of the market for natural gas. In general, first the gas market was deregulated and the power market followed shortly after. The purpose of the deregulation was to increase the efficiency, transparency and the liquidity of the previously state controlled businesses. Electricity as a commodity has a number of idiosyncrasies that differs a lot from other commodities, which mainly derives from the non-storability of electricity. These idiosyncrasies are more extensively covered in 2.3.1, and are the main drivers of the complexity of the power markets. These factors and their geographical and physical constraints has led to a very uneven development of the markets, let alone the political factors. In many ways the deregulated power markets are still highly influenced by political decisions. This can be clearly observed in the German market where both increased subsidies for renewables and the politically imposed moratorium of nuclear energy after the Japanese Fukushima nuclear accident, has had significant impact of the foundation of the German power market. Furthermore, the increased installed capacity of renewables has the effect of increasing volatility in the market.

Despite the years that has passed since the start of the deregulations of the electricity markets, there is still a very limited understanding of the movements of spot prices.

Mainly in the case of the intra-day profile and the effect of the changing market conditions. The main problem when trading electricity as a commodity compared to other commodities is the non-storability of electricity, which yields very special characteristics. Where prices for other commodities takes into account the supply, demand, storage costs, historical price and expected future price of the commodity, the price of electricity is set mainly by supply and demand for the moment of when electricity is consumed. Since electricity cannot be stored, the historical and future prices has no impact on the spot price either, making also storage costs irrelevant.

Another factor that contributes to complicate the price estimation process of electricity is that the delivery process for electricity differs a lot from the delivery process of other traditionally traded commodities. For traditionally traded commodities there is a fixed delivery date, which the spot price is based upon. When it comes to electricity, there is instead a delivery period. Typically a forward contract of electricity is stated as a given amount of electricity over a period of a day, month, quarter etc. There are also contracts where the delivery period is defined as just certain times of the day or week, (e.g. peak- and off-peak hour contracts). This might not seem very complicated, although it changes fundamentally the conditions for electricity as a a commodity compared to other traditional commodities.

As a result of this, standard banking models for commodity spot prices or electricity derivatives has proven to simply not work for the electricity market. Hence, models has to be developed explicitly for the electricity market. To further complicate this process, the market is constantly evolving with changes in political regulations, market conditions, technological improvements etc, while the conditions for each individual power market are rather unique. This combined has caused a situation where there are no really "best practices" on the market, but all actors have more or less their own way of modeling the spot prices. Common models exists, but no consensus of "how it really is done". This yields great opportunities to make arbitrage gains, if a pricing model outperforms the counterparts pricing model, especially if they can better capture the affect on the spot price from renewables, as will be shown, has a great impact on the spot price of electricity.

As Carmona et al. (2012) briefly discusses, spot prices of electricity is highly dependent on the price of fuel for the power plants, most commonly coal and natural gas, since these power plants are often the price setting fuels on the power markets, according to the Merit Order (see section 2.6). Furthermore, in recent years there has been a significant increase of renewable energy sources, in the world in general, but in Europe in particular. Wagner (2012) [1] states that the considerable amount of renewable infeed to the electricity grid has a significant impact on the spot price, and he continues to prove that the price profile follows the profile of residual electricity load (Residual load is the total supply of electricity minus the infeed from renewable sources such as wind and solar, see section 2.5). Hence the structural models covered in this thesis will use residual load and fuel prices to model the spot price.

1.2 Purpose

The structural changes in the power market during the recent years, has given rise to a need of a new model-class which is able to take the structural changes into account in a forward looking perspective. This thesis aim is to evaluate and continue the works previously carried out by Wagner (2012) and Yang et al. (2012), which has contributed respectively by suggesting two different approaches for structural models. These models are, to date, in the front line of research in this kind of modeling in the power market. We use the residual load model from Wagner and compares the spot prices generated by both Wagner (2012) [1] and Yang et al. (2012) [2] with each other. We will extend the research by first investigating the Yang et al. (2012) [2] supply function in two parts, for peak- and off-peak hours, since the spot prices are assumed to have different behavior depending on the time and load during a day-cycle. Following, a more thorough investigation of the development of fuel and power prices and their correlation is conducted, to better explain the results. Thence we introduce an experimental Artificial Neural Network supply function approach to investigate and asses the potential model quality contribution of coal prices. Thereafter we want to investigate how these investigated supply functions (the supply functions from section 3, not the Neural Network estimated ones) performs when used to price future contracts. The future contracts investigated is quarterly futures for 2013. There are two main purposes of this research, where the primary purpose is to evaluate the models according to their Mean Squared Error (MSE), along with a visual and fundamental analysis. The secondary purpose is to estimate and compare the risk premiums yielded by the models when used to estimate future prices, and a more detailed analysis of the coal generation costs impact on spot prices of electricity.

This purpose is boiled down to the research question for the thesis, which is defined as following: "Does coal generation cost add any explanatory power to the new model-class "structural models", using a residual load modeling approach, and how well does a coal generation dependent supply function perform when used to estimate prices of future contracts?"

1.3 Result

The result from the analysis in section 5, is that the coal generation price does not contribute to a better fitting supply function, and that the fuel dependency of the power spot price needs to be modeled in a more complex way to yield satisfactory results. The structural changes in the power market is also affecting the correlation between coal generation cost and spot price of electricity, in a way the yields systematic errors in our estimations. We can observe that the dual-exponential Yang Heatrate function introduced by Yang et al. (2012) [2], provides a robust shape to explain a large part of the spot prices of electricity. However, it is clear that the extremes (both positive and negative ones) needs a more complex model. There are still some systematic variations in spot prices for normal levels of load that we had hoped that the model introduced by Yang et al. (2012) [2] would explain, using the coal generation cost. However, this seems to not be the case and these systematic variations needs another parameter for

explanation.

1.4 Disposition

The rest of this thesis is structured as following: section 2 provides some background information about the German power market, renewable energy, spot price, and it also introduces some concepts later used in the thesis. In section 3, we introduce the model parts and in section 4 we briefly discuss the data used. Furthermore section 5 contains the main results from the various analyses and the future price estimations. Finally section 6 contains the conclusions of the thesis and suggestions of extended research.

2 Background

2.1 The German Power Market

The German power market is one of the biggest and most liquid power markets in Europe, and it is in many aspects the front runner of market developments in Europe. The German market also has a well diversified mix of energy sources ranging from lignite and nuclear energy to hard coal and gas. During the last decade there has been a significant and almost unprecedented increase of installed capacity of renewable energy sources, and this development is expected to continue for decades to come. This development is mainly driven by wind- and solar energy.

There is also an increasing amount of energy production from biomass, which also is considered to be a renewable energy source. However, the production profile does not affect the power price in the same complicated fashion as e.g. wind and solar, since it has a flat baseline production profile. Another problem is that the changes in the market regarding new energy sources is not stationary. We need to be able to take into account a non-stationary constantly evolving market. This development dramatically changes the foundation of the energy market, and hence the need of a model able to capture some specific renewable-energy related phenomenons arise. The model needs to take into account both the effects on the price profile of electricity generated by the substantial amount of renewables, but also deal with the non-stationary development of installed renewable capacity.

In Germany, by September 2012, the total amount of installed capacity of solar and wind together represented about 30% of installed capacity. Individually that was about 29 GW of installed wind power and about 30 GW of solar power.

2.1.1 Power Exchanges

The energy in Germany is primarily traded on two exchanges.

- EPEX Spot Market

- EEX Forward Market

The normal energy consumer generally do not have a direct connection to the energy exchanges, as the actors are typically larger actors e.g.industrial wholesalers and producers of electricity. Large consumers like power intensive industries etc. are also present on the spot market. The distance between a large part of the consumers and the spot market, further strengthens the end consumers in-sensibility to spot price changes and increases the in-elasticity of spot prices on the power market. Which in general is mainly driven by their social behavior. Even industrial consumers that are active in the electricity trading market are inflexible and are mainly driven by their power need. Their trading aims at optimizing prices.

2.1.2 EPEX Spot Market

The EPEX Spot Market for electricity is situated in Paris, and facilitates spot trading of electricity for Germany, France, Austria, Switzerland, Belgium and the Netherlands.There is a time lag for the delivery of commodities traded on the spot market due to their physical nature. For the electricity spot market, bets are placed for delivery the following day. This means that in practice, the spot market for electricity really is a day ahead market. The EPEX also provides an intra-day market to satisfy the intra-day needs of selling or buying electricity for the actors on the market. This secondary market operates 24/7. In general EPEX is the exchange for the daily and day ahead market of electricity and due to official numbers from EPEX, roughly 35-40 % of the total electricity consumption in Germany is traded on this spot market. The EPEX Spot price also works as a reference index for the electricity contracts traded on the EEX Forward Market.

2.1.3 EEX Forward Market

The EEX Forward Market has a lot of similarities with the EPEX Spot Market. It is located in Leipzig, Germany and facilitates trading of electricity for Germany, France, Austria and Switzerland. On the EEX market, financial contracts with electricity as the underlying asset is traded. More precisely the underlying index is the EPEX spot. Futures contracts are either purely financial or combined with actual physical delivery and has a specified delivery period which can be months, quarters or years. These contracts on the EEX can range as far as 6 years in the future and has a physical delivery. The most commonly traded contract on this market is the Future contract, although a wide range of other derivatives are available as well. Other derivatives traded on the exchange is ranging from plain vanilla options to more exotic options, swings, swaps etc. Although the more complex ones are only traded "over the counter".

2.1.4 German Energy Mix

The German energy mix is a well diversified mix of most of the power sources available today in the world, as can be shown in figure 1. It consists of mainly:

- Nuclear Energy
- Hard Coal
- Brown Coal, or Lignite
- Natural Gas
- Renewables

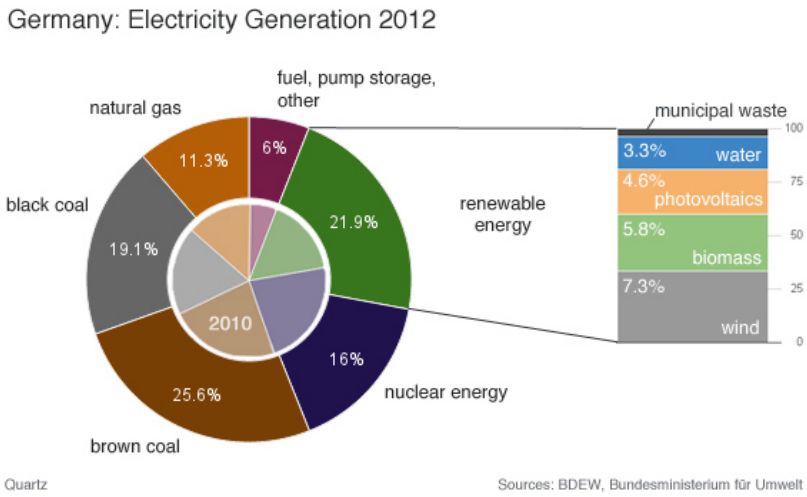
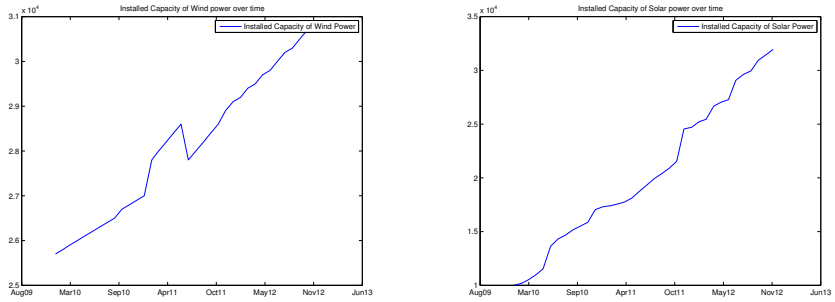


Figure 1: The German energy mix 2012, share of total amount of generated power over the year.

Each and every source has its own characteristic that needs to be taken into account when planning production and distribution of power. E.g. *Nuclear* and *Lignite* are very un-flexible in their production capacity and is best suited as base-load in the grid. This is due to the physical processes involved in the electricity production. For nuclear energy, the stability of the fission process requires a very stationary process to work efficiently and safe. Also the little margin that can be tweaked in nuclear electricity is very costly which makes the producers unwilling to do that unless the spot prices reaches very extreme levels. The inflexibility of Lignite, which is considered to be the lowest rank of coal, is derived from the low energy density and high moisture content. These factors makes lignite expensive and inefficient to transport and store, which results in a production cycle of Lignite power where the Lignite is mined and directly transported to the power plant to be burnt immediately without any storage. Hence a majority of the Lignite power plants are located in close proximity to the Lignite mines. This results in a situation where decreased power production of Lignite starts with a corresponding decrease in mining and the control time lag caused by the lead time from mining to power production, simply makes this process very inflexible. The complexity of especially these two power plant types makes the producers prepared to accept negative prices for a short time period in order to be able to continue with their production, in stead of the higher cost of regulation the production.



(a) Installed Capacities, Wind Power. (b) Installed Capacities, Solar Power.

Figure 2: Illustration of the development of installed capacities of renewables.

Hard Coal, *Natural Gas* and *Hydro electricity* are all more or less flexible. While Natural Gas and Hydro Electricity are fully flexible, the Hard Coal are only partially flexible. Natural Gas, however, is to large extent pushed out of the market due to the increased amount of lower-merit-order renewable infeed of electricity. This gives rise to a situation where gas power plants are only used in extreme events a few times of the year. This is also the reason of why Coal is the better choice for the fuel to use as the external fuel dependent input as Yang et al. (2012) motivates and which is replicated and discussed later in this thesis. This also gives rise to a situation where Natural Gas cannot be used to scale down electricity production on normal days, since it is only used in times of high demand. This leaves the task of controlling low demand and high renewable infeed to hydro electricity, which is only 3% of the market, and Hard Coal. In practice however, it is not very common to use the hydro electricity as regulatory power in Germany, and the large part of the regulation of power is done by using hard coal. Since the result of this is the lack of sufficient regulatory power or capability, the renewable infeed has a high impact on spot prices.

Another factor, very specific for the German market and energy mix during the later years is the German Nuclear Moratorium. After the Japanese nuclear accident in Fukushima following the earthquake and Tsunami 11:th of March 2011, the German government decided to close down several nuclear power plants and phase out the rest of them. Yang et al. (2012) [2] states numbers from Thomson Reuters (2012) [6] where 8.5GW out of 20.5GW were taken out of production. The German government is now following a plan to shut down all nuclear power plants until 2022. This change in the energy mix will change the conditions for the electricity market, especially in combination with the increased installed capacity of renewables, making the market even more sensitive to the stochasticity of renewable energy production. This has also increased the power prices' dependency of the coal price.

As can be seen in figure 2, there has been a significant increase of installed capacity for renewable energy in Germany the later years and this development is assumed to continue in a non-decreasing pace. However, using a structural model, one can take any shape of an increase, a stationarity or even a decrease of renewable energy into consideration. As opposed to the use of a reduced form

model (section 2.4).

2.2 Renewable Energy

The definition of renewable energy is energy from sources which are continually replenished, such as wind, solar, hydro, biomass etc. Hydro electricity is an extremely convenient component of the energy market, since it is very easy to regulate depending on positive and negative peaks of power in the grid and in the same time very cheap to use. In many places of the world it serves as a way of balancing the highly stochastic production from e.g. wind- and solar energy, since the production from these two energy sources are as unreliable as the weather. I.e. if it is windy, production of hydro electricity can be reduced, and when the wind dies, the production of hydroelectricity can be increased. This helps to even out the total load on the electricity grid, since big irregularities in the grid both has a bad impact physically on the grid and increases the volatility of electricity prices on the power markets. Since wind and solar energy are the renewable energy sources with the most significant impact on spot prices, Wagner (2012) [1], from here on they are the only sources for renewable energy to be considered in this thesis and will be referred to as "renewables" or "renewable infeed".

2.2.1 Physical

When dealing with renewables, it is important to bear in mind the difference of installed capacities for renewables, compared to the installed capacities of conventional energy (e.g. Coal, nuclear, lignite etc.). While conventional energy sources might be able to maximize the power plant, to utilize it's full installed capacity, renewables depends on weather factors and typically the actual infeed to the grid from renewable energy sources is only a fraction of the total installed capacity. Full infeed of the total installed renewable capacity is highly unlikely since that would require all wind turbines to have perfect wind conditions and all solar cells to have perfect solar conditions at the same time. The biggest problem with renewable energy is the inability to plan and schedule production ahead of time, but it produces energy if, and only if, the weather conditions are right. The share of installed capacity of renewable energy in the German market has dramatically increased during the last years, both due to heavy governmental subsidies, but also due to the nuclear moratorium after the Fukushima accident. The shutdown of several old nuclear reactors contributed to increase the overall market share of installed capacity for renewable energy, hence also worsening the sensibility towards renewable energy fluctuations.

2.2.1.1 Wind Since wind is very unpredictable and fast-changing, so is electricity production from wind. The problem with wind power in the grid is the timing. It is possible to predict on average a couple of days ahead whether it will be windy or not a specific day. But more precisely when it starts and the magnitude is uncertain and very difficult to assess. This is a big problem for power production since it can hardly be stored and needs to be balanced out

by the conventional power as soon as possible. This makes the infeed of wind power to the grid very spiky which has severe impact both on the grid itself and on the electricity prices. Typically they also have an upper and lower wind speed bound, where a minimum speed is required to make the rotors spin and a maximum bound where violations could seriously damage the constructions. Another significant physical factor for renewables in general, and wind energy in particular is the seasonal dependency. For wind the seasonality is not that strong, although there is generally more wind during the winter than the summer in Germany. This causes problems since the winter time (which will be shown in section 5) is very sensitive to changes in the supply, and wind power tend to cause an over supply of power while the consumption generally is lower. Wind power however, has the ability to generate power 24 hours of the day, although it's typically a little less windy during nighttime.

2.2.1.2 Solar Solar energy production has a more regular pattern compared to wind power. Both regarding yearly seasonality and daily. Even though the human eye might give a perception that the sunlight during wintertime is comparable to the sunlight during summertime, the intensity of the solar radiation decreases dramatically during wintertime, and during the darkest winter months in Germany hardly any solar power is produced. The intra-day power generation of solar energy follows the profile of a bell curve, with its peak at noon. The altitude of the curve shifts depending on seasonality and weather, while the tails of the bell curve changes due to yearly seasonality.

2.3 Spotprice

Commodities are usually traded as future/forward contracts, i.e. the commodity is purchased a specific day with a delivery date in the future stated in the contract. In traditional finance, forward/future contracts are priced according to the expected future value of the underlying stock. In the case of commodities, the equivalent to the value of the underlying stock, is the spot price. The spot price can be seen as the price of the commodity for "instant" delivery, in the same way the stock price is the price for a share at that particular instance of time. However, in practice, instant delivery is highly unrealistic for most commodities and different time lags for the spot price delivery exists for different markets and commodities.

2.3.1 Market and behavior for electricity as a commodity

Since renewable energy from solar and wind does not have to be fueled in any way, it only produces electricity when the conditions are right, the are practically no variable costs associated with the production of renewable energy. This causes the renewable power plants to always generate the maximal amount of energy possible and feed it directly into the grid. The unregulated maximum infeed of renewables is also a consequence of the subsidies scheme in Germany, called the "feed-in-tariff". This means that the renewable producer is not exposed to the spot market price of power, but is guaranteed a minimum price per

unit of electricity. This gives the renewable producers the incentive to always produce as much as possible, even when the prices are negative. The energy producers will therefore regulate the rest of the power of the grid and try to adapt the changes in the market caused by renewable energy. This, however, is a kind of tricky balancing act. Different energy sources offers different flexibility. Nuclear and lignite energy is in general very hard and/or costly to regulate, and often serves as the base-load in the electricity grid. Hard coal, gas and oil power plants however are easier to regulate, although especially gas and oil are relatively expensive fuels for electricity production and tends to imply higher electricity prices. Even though it is possible to try and counter the stochastic increase and decrease of renewables, there is naturally some time lags in the system. E.g. from the decrease of infeed of renewable energy is observed, to the point when energy production is increased from the other conventional power plants, or whole plants has had the time to be started. Solar also causes considerable volatility in the sense of little or much infeed, it influences the spot price. Since the infeed from solar power has a very regular shape and the power generated by solar power is quite predictive. The price volatility on the power market is to large extend driven by renewables in general, but since wind power in more unforeseeable, wind power is the largest contributor to uncertainty and is the single largest cause of downward spikes in the spot price.

In classical microeconomic theory, prices are set at the equilibrium where the supply and demand curve intersects. Hence, if supply decreases, the price should rise and the demand would decrease. In the real world, and especially in the power markets, these behaviors on the market are a little more complex. End-consumers rarely purchases their electricity from the spot market (apart from large industrial corporations perhaps), but they typically have long term contracts or subscriptions (in various forms) of electricity, with electricity retailers which in turn buy the electricity on the forward and the spot market. The price payed by the end consumer is often a fixed price stated in their contract with the retailer. This makes the consumer very un-sensitive to changes to the spot price of electricity, since he will not consume less power just because the spot price is high since he is affected by another price tariff. Due to the in-elasticity in the electricity spot price, demand on the market is not substantially changed with changes in supply and price. The only degree of freedom in this equation, where the energy shortages or peaks in the grid caused by the whimsicality of renewable energy can change, is the price. This will create a price spike that is either positive or negative depending on whether there is an over- or under supply of electricity in the grid. Price spikes was first observed on the electricity market during the Californian energy crisis in the beginning of the 2000's, and was the result of high consumption of energy in combination with outages of power plants. Today, price spikes are becoming increasingly common in the European market. The positive spikes still depends on an under-supply of power in the grid, but mainly due to a swift change in infeed of renewables to the grid. They can also appear when there is a peak in fuel prices, which causes the price of electricity from the price setting fuel on the market, to rise dramatically (See section 2.6 on pager 15). The negative spikes are becoming increasingly frequent and occurs when there is a sudden increase of renewable energy to the grid and the energy producers does not manage to regulate the residual load of the grid in time. Instead of shutting down inflexible and expensively regulated

power plants, the plant owner are prepared to accept a price lower than the production cost or sometimes even negative prices instead of the huge costs to regulate their production. Positive spikes now occur less frequent than a couple of years ago. Today they are mainly caused by cold weather and high gas prices. Negative spikes .

2.4 Spot-price modeling

An important thing to bear in mind when dealing with models of the electricity market is that the purpose of the models are not fundamentally *price forecasting*. In fact, all models implicitly estimates risks rather than actual prices, since a fundamental assumption in these markets is that prices can not be reliably and consistently forecasted, as stated by Wiley (2003) [8].

A basic example of why electricity spot prices can not be computed using standard financial models is that a *Net Present Value* can not be computed for electricity. For almost all other kinds of stocks or commodities it can, which is the present value of the flow of future dividends. For electricity however, this is not the case due to the non-storability. In quantitative finance for the commodity electricity, the extra dimension of complexity appears in form of that the relation between spot price and forward price is badly defined, or not non-existing. This is very different from an ordinary stock, commonly assumed in e.g. Black-Scholes theory. To solve this problem, some models Burger et al. (2002) [7] simply ignores the spot price and directly builds a model of the Futures price, since the future can be held over time, as opposed to electricity. Although these kinds of models does only provide information on the time-basis of the contract themselves (e.g. weekly, monthly, quarterly or yearly), and no information on hourly or daily basis.

There are several model types available in finance to model the spot price of commodities. They typically consists of standard blocks from mathematics and finance. Most of them use a Brownian Motion process for the stochastic part of the model, which is a very classic way of modeling stock prices in classical finance. ARMA-processes from classical time series analysis is also very common building blocks. To get an overview of the most common models, they can be split into 2 sub-categories:

2.4.1 Reduced Form Models

The Reduced Form models are a parametric approach, where every pattern and parameter of the process is solely derived from historical observed prices and not physical quantities, like solar radiation into photovoltaic cells or the amount of wind captured by wind turbines.

Even thought these models can capture historical behavior extremely well, they are unable to capture any kind of change of the market in a forward-looking perspective. This is especially a problem in power markets, which are not mature stationary markets, but experience a continuous non-linear change in the fundamental prerequisites. It also fails to capture changes due to world changing

events, like the 2008 Mortgage crisis, which fundamentally changed the demand of electricity in a way that still affects the markets to some extents. Since the Reduced form model only looks at historical data, it will take long before the model starts adapting to the new conditions. It is worth stated that Reduced Form models are widely used in the industry due to their simplicity and their robustness. There are a number of techniques to capture specific behaviors that can be used in any kind of modeling. Here they will be addressed as add-ons in this Reduced Form section, although they in practice can very well be used in a structural model as well. Since they are not used in this thesis they are only covered here in this section, since they are on a strictly "Nice-to-know" basis.

2.4.1.1 Complex model parts, (Jump-Diffusion Models or Regime-switching Models) The Jump diffusion models, used by e.g. Carmona et al. (2012) [3] consists of a *Brownian Motion* process for the standard movement of the process, spot prices in this case, and a *Poisson* process to capture the price spikes observed in the power markets. The intuition behind the Regime Switching models is that spot prices behaves differently depending on different price and/or load regimes. E.g. positive spikes are more likely to occur in times of high demand, while negative spikes on the other hand are triggered by low demand, but perhaps more importantly high renewable infeed. Furthermore, the prices can be said to behave differently direct after a spike, positive or negative. The selection of regimes is arbitrary, and the models are also a compromise between model resolution and complexity.

2.4.2 Structural Models

The Structural Models aims to split up the modeled process in several sub processes which can be individually modeled, to capture fundamental underlying behavior behind the main process, in this case the spot prices. In the case of this thesis, the deterministic seasonality and intra-day shape of the different types of load is modeled (except the intra-day shape of wind, which is not explicitly modeled). In the case of the power market, the structural Model is well suited since it allows us to understand and analyze the relationship between prices and underlying drivers more easily in this market than in most other markets. Using a structural model, the fundamental parameters can be controlled to account for fundamental changes in the market such as the increased installed capacity of renewables, or changes in fuel prices. Even though a structural model models underlying drivers of processes, it will never fully replicate the price setting mechanism of the spot market and thus a balance is always required between mathematical convenience and model realism.

The aim of the structural model is to find a strategy different from the pure price dependent approach - since this inevitably only makes us able too to look at historic prices. Instead we seek to explain the "structural quantities" which lead to price formation (supply/demand). A quantity like solar infeed and capacity is much more convenient in this case, because it makes us able to add already known differences between historic and future data to the approach. The big drawback of using the structural approach, is that it is distanced from the pure

price description domain. I.e. the model require extensive work to (re-)construct the price from the modeled structural quantities.

With this stated, we need to add that this model will also make use of a historical modeling too, e.g. in the load modeling, since we know that the power consumption behavior is not subject to a lot of structural changes.

2.5 Residual Load

Energy production from renewable sources introduces more uncertainty in the market, due to their volatile production profile. As energy production from renewable sources increases, so does the volatility caused by renewable infeed on the power market. Since the infeed from wind- and solar power is assumed to be uncorrelated, they will both be modeled individually and so will the total system load. Hence the residual load can be interpreted as:

$$\textit{Residual Load} = \textit{Total System Load} - \textit{Infeed from Wind power} - \textit{Infeed from Solar power}$$

Residual load could be said to represent the load of conventional energy in the grid. The electricity spot prices is highly correlated to the infeed of renewable energy and the current consumption level of power. This makes it very convenient to use a demand reducing approach, like Residual Load modeling, which allows for the individual adjustment of planned increases of installed capacities of renewable energy. This allows us to estimate and account for the traditionally irregular patterns, in which the installed capacities is increased. The Residual load is then used as input to our supply function (See section 3.6).

2.6 Merit Order

The Merit Order is a way of ranking the different sources of energy in electricity production in ascending order, according to the lowest marginal costs of production. Those plants/energy sources with the lowest marginal costs of production are the first ones brought to the market. In practice, this is regulated by a bidding system on the electricity markets, where all energy sources will bid at their marginal rate of production. I.e. at price zero for renewables, or the cost of fuel and operations for coal etc. In reality, subsidies can affect this mechanism on the market, such as the feed in system in Germany, although the Merit Order remains an efficient way of comparing marginal cost of production from different sources of energy and to explain market behavior.

2.7 Clean Dark Spread

Clean Dark Spread is the industry name for the net revenue of selling electricity from Coal fired energy plants, taking into account the cost of the fuel required (Coal) and the price of emissions. It is the traditional bench marking measure to asses the financial health and profitability of Coal fired power production. It is however a theoretical measure. In practice, the plant efficiency varies according to other factors as well.

$$CDS = S_{Electricity}(t) - P_{Coal}(t) - P_{Emissions}(t) \tag{1}$$

where
CDS is the Clean Dark Spread,
$S_{Electricity}(t)$ is the sport price of electricity at time t,
$P_{Coal}(t)$ is the cost of generating one unit of electricity at time t and,
$P_{Emissions}(t)$ is the cost of emissions to generate one unit of electricity at time t.

2.8 Artificial Neural Networks

Artificial Neural Networks (here after referred to as ANN or just NN) are a machine learning technique that uses a model that is inspired by how the central nervous system work in animals. It is commonly used for pattern recognition in large amounts of data, and is basically a series of simple mathematical models defining a function $f : X \to Y$, capable of approximating non-linear functions.

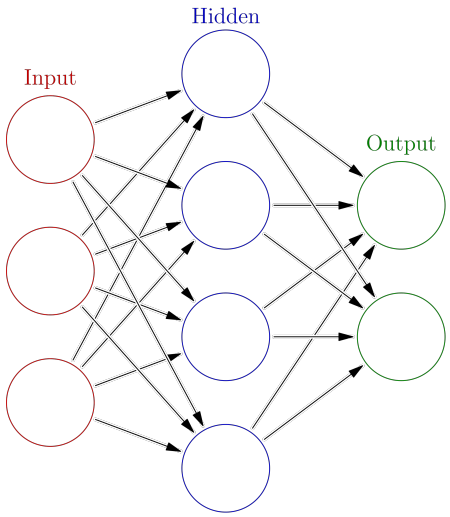


Figure 3: Concept illustration of an Artificial Neural Network.

Figure 3 show a concept drawing of an Artificial Neural Network. It generally consists of an input layer and an output layer, connected by a series of hidden layers of what is called "neurons". To explain how the ANN works in detail requires more time and effort than we want to take from the general reader. A detailed dissertation can be found in the works of Hassoun (1995) [9]. To explain this briefly for the uninitiated reader, the using of an ANN can be split up into two phases. The training phase and the execution phase. In the training phase an input is provided to the input layer, then propagated through the hidden layers connected by some mathematical models until it reaches the output layer. When it reaches the output layer, the result is returned and compared with the desired output. In an iterative process this is repeated with

adaptions made to the mathematical models connecting the neurons and layers for each iteration, to improve the results generated in the output layer. In the execution phase, the model is provided with a new set of inputs and the output of the ANN model can be observed. The validation phase does not include any adaptation of the models.

3 Method and primary results

In this section, the modeling strategy and methods used in this thesis will be covered and explained in details. Furthermore the result and performance of the models estimated will be displayed.

3.1 Overall Strategy, motivation

The overall modeling strategy for the load in this thesis is the same as introduced by the previous work of Wagner (2012) [1] and is extended mainly by combining the load modeling introduced by Wagner [1] with the supply function introduced by Yang et al. (2012) [2]. Last, a more detailed supply function combining the fuel dependent Supply function of Yang [2] with the intra-day peak- and off peak hour splitting strategy of Supply function modeling used by Wagner. These models and their performance will be evaluated and the differences and fitting properties will be examined.

An overview of the overall modeling strategy in this thesis can be described as following

1. Modeling of wind infeed
2. Modeling of solar infeed
3. Modeling of total system load
4. Calculating Residual Load
5. Estimating supply function

3.1.1 Modeling of renewable

Modeling of both wind and solar infeed are done in similar ways. Both are affected by an individual trend caused by the ever increasing installed capacities, and this has to be accounted for. To do this, and make it easy for our model to take an increasing installed capacity in the future into account, they are both modeled as the efficiency of installed capacity (could also be known as load factor or utilization rate) instead of their absolute infeed. See Figures 4 and 10. The definition of efficiency is shown in Definition 2 and the notations used is the same as introduced by Wagner (2012) [1].

Definition 1. *Efficiency*

$$E_t^{src} = \frac{AI_t^{src}}{IC_t^{src}} \tag{2}$$

where

$IC_t^{src} > 0$ denotes installed capacity and
$AI_t^{src} \in [0, IC_t^{src}]$ denotes the absolute infeed of energy from the source src at time t.

Since $0 < AI_t^{src} \leq IC_t^{src}$, $0 \leq AI_t^{src} \leq 1$, E_t^{src} can be viewed as the utility rate of installed capacity, or *efficiency*.

Subsequently the efficiency is transformed using the logit transform, which gives the data more normally distributed properties.

Definition 2. *Logit Transform*

$$logit : (0, 1) \to \mathbb{R} \tag{3}$$

The logit transformation transforms the variable $x \in (0, 1)$ as

$$logit(x) = log(\frac{x}{1-x}) \tag{4}$$

The logit-transformed efficiency is denoted as

$$E_t^{\tilde{s}rc} = logit(E_t^{src}) \tag{5}$$

where $E_t^{\tilde{s}rc}$ defined as

$$E_t^{\tilde{s}rc} = \eta_t^{src} + E_t^{\bar{s}rc} \tag{6}$$

where $E_t^{\bar{s}rc}$ is the random deseasonalized efficiency.

With the logit-transformation, an assumption has to be made. Since this transform will be undefined for the boundaries of the interval (i.e. 0 and 1), the assumption has to be made that there will never be zero infeed from renewable sources, nor will there never be full infeed from renewable energy. The assumption of the never occurring full efficiency of renewable energy production can be explained intuitively by that it is highly unlikely to have maximal wind power or solar power at all production locations at the same time. It can be viewed in the figure 5 and 12 that this assumption holds. When it comes to the lower limit, 0 however, we have observations of zero infeed of renewables in a few instances of wind power infeed, and for solar power we have a substantial amount of zero-infeed observations, since during the night time, no solar power is produced. The different models will deal with this problem in their own way. For wind power infeed, the number of zero-infeed occurrences are so few that we simple choose to ignore those values in the estimation of the model. For solar

power modeling however, this systematic occurrence of zero infeed of power is dealt with by modeling daily maximums instead of hourly data for the solar infeed, and the using a deterministic intra-day function to generate the hourly observations. See section 3.3. After the logit transformation, the deterministic seasonality is estimated using simple cosine functions, where details of the seasonality is stated in each subsection, 3.2and 3.3.

The deseasonalized logit transformed data will now follow close to a normal distribution, with a typical stationary mean reverting behavior, why this stochastic residual now can be modeled using an Ornstein Uhlenbeck process. This process is by definition stationary, Gaussian and Markovian and can be seen as a continuous-time analogue of the discrete AR(1)-process.

Definition 3. *Ornstein-Uhlenbeck Process*

$$d\bar{E}_t^{src} = \theta^{src}(\mu^{src} - \bar{E}_t^{src})dt + \sigma^{src}dW_t^{src}, \qquad \bar{E}_t^{src} = e_0 \tag{7}$$

where
e_0 denotes the initial value,
μ^{src} denotes the level of mean reversion,
θ^{src} is the speed of mean reversion for a particular source,
σ^{src} is the volatility, and
W_t^{src} is the stochasticity driving Brownian Motion process.

3.1.1.1 Renewable Modeling summary. To summarize the renewable modeling, the following steps is conducted in the given order:

1. Calculate the infeed *efficiency*.
2. Logit transform the efficiency
3. Estimate the seasonality
4. Calibrate the parameters for the Ornstein-Uhlenbeck process.

3.2 Modeling Wind

As described in the previous section 3.1.1 the efficiency is calculated, which can be observed in the figure 4.

3.2.0.2 Wind Seasonality To model the deterministic yearly seasonal component of wind infeed we choose to fit a seasonal function of the form

$$\eta_t^{wind} = a\cos(2\pi t + b) + c \tag{8}$$

to the data. The parameters are estimated using the Matlab function nlinfit, and can be observed in the figure 6. Both the logit transformed data and the

estimated seasonality function can be observed in the figure 7. Subsequently the next step is to fit the Ornstein-Uhlenbeck process to the de-seasonalized residual. Since the Ornstein-Uhlenbeck is a normally distributed process, the calibration data also needs to be normally distributed. As can be observed in the histogram and distribution plot of the residual in figure 8a and in the quantile plot in figure 8b, the residuals are close to normally distributed. The Jarque-Bera test in Matlab (jbtest) indicates that this is not the case. Since the distribution plots shows that the residuals are close to normally distributed, we continue anyway and the Ornstein-Uhlenbeck process can be estimated. The parameters can be found in figure 9.

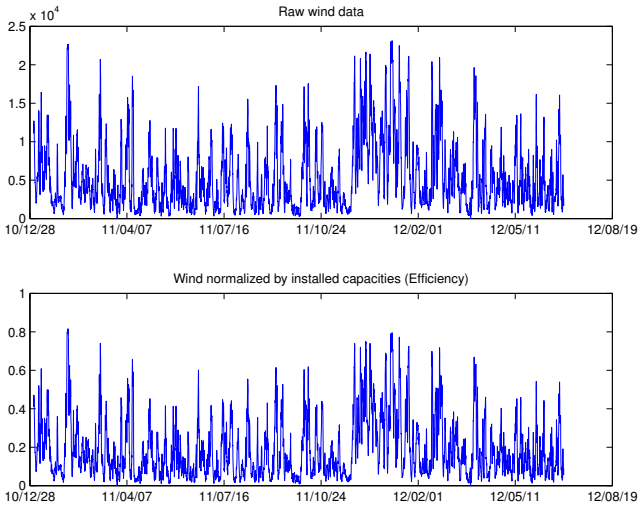


Figure 4: Illustration of wind data.

	Wind Efficiency properties
Max	0.82
Min	0.00
Mean	0.19

Figure 5: Table of wind efficiency properties

	a	**b**	**c**
Seasonality Parameters	0.44	-0.04	-1.79

Figure 6: Table of wind seasonal parameters.

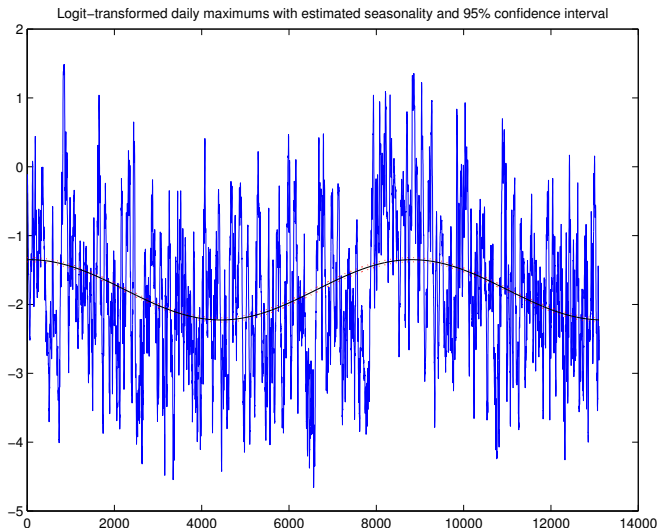
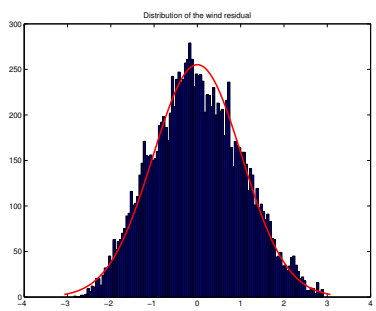
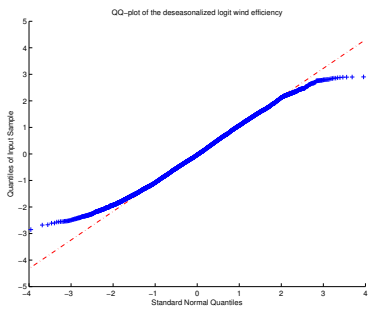


Figure 7: Illustration of the wind seasonality.



(a) Distribution of wind residuals



(b) QQ plot of wind residuals

Figure 8: Wind residual Gaussianity test.

	Mu	**Sigma**	**Lambda**	**Half life**
Ornstein-Uhlenbeck Parameters	0.00	9.95	47.39	-0.01

Figure 9: Table of wind estimated Ornstein-Uhlenbeck parameters for wind

3.3 Modeling Solar

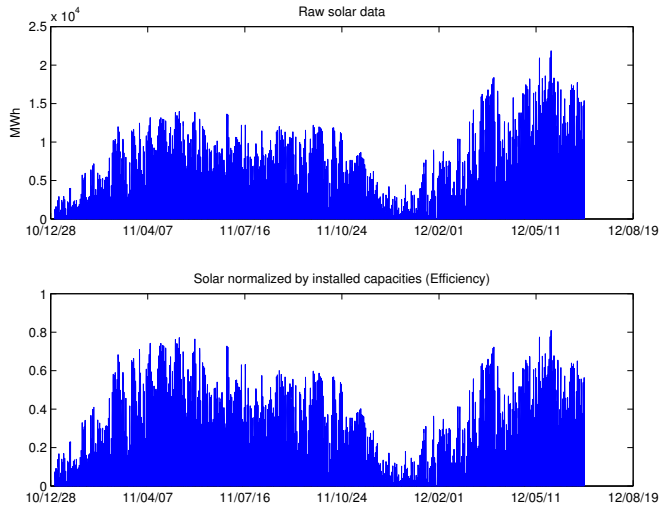


Figure 10: Illustration of the raw solar infeed data of the modeling period, and calculated efficiency.

In the solar efficiency plot in figure 10, it becomes clear that the solar infeed to the grid during wintertime is substantially lower, compared to the summertime.

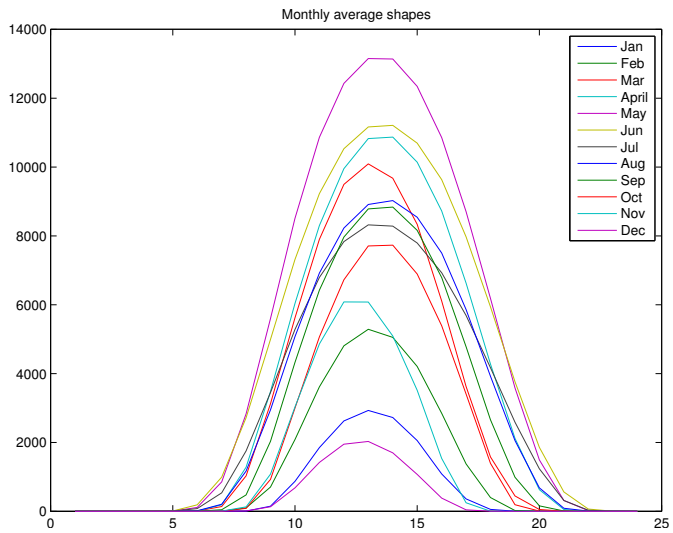


Figure 11: Average daily solar energy production profile per month.

Looking at figure 11 it also becomes clear that infeed starts and ends at different times of the day, during different times of the year, let alone the fact that the maximum production capacity is dramatically different during the winter time, compared to summertime. This is however trivial, since the sun rises and sets

during different hours of the day, depending on the season. Note that the intra-day distribution is very similar in shape for periods close to each other. This makes it reasonable to assume a deterministic model for the daily distribution of solar power infeed, in combination with a stochastic model with daily resolution to model only the daily maximums. By modeling the daily maximum infeed of solar power, the seasonality and distribution of the solar power infeed is preserved and for each month we estimate hourly parameters to transform the daily maximum infeed to an hourly distribution. This function is defined as following:

Definition 4. *Daily maximum to intra-day shape transformation*

$$h_i^{month} = c_i^{month} M_t \tag{9}$$

where

h_i^{month} denotes the absolute infeed hour $i \in [0, 23]$ for a specific month of the year,

c_i^{month} denotes the proportion of the daily maximum fed into the system hour i, for a given month. $c_i^{month} \in [0, 1]$ and

M_t denotes the daily maximum at time t.

	Solar Efficiency properties
Max	0.81
Min	0.00
Mean	0.12

Figure 12: Table of solar efficiency properties

Subsequently the daily maximum process of solar infeed of electricity is defined as following for each day:

Definition 5. *Daily maximum to intra-day shape transformation*

$$\tilde{M}_t^{solar} = logit\left(\max_{(t:d_t=i)}\left(E_t^{solar}\right)\right), i = 0, 1 \ldots, d_T \tag{10}$$

The deterministic seasonal component for the logit transformed daily maximums is defined as

$$\eta_t^{solar} = a_1 \cos(2\pi t + b_1) + a_2 \cos(4\pi t + b_2) + c \tag{11}$$

and is able to capture two peaks per year. The physical explanation of this is not properly examined, but if the data would follow closer to a one-peak

seasonality, the estimator would estimate the parameters for the incorrect peak close to zero and make them insignificant. This is however not the case, which can be observed in figure 13. The parameters for the solar seasonality are also estimated using the Matlab function nlinfit and the estimated parameters can be found in figure 15.

Subsequently the normal distribution properties of the de-seasonalized logit transformed daily maximums is plotted in the subfigures if figure 14. Looking at the histogram in figure 14a, conclusions can be made that the residual are not as close to normally distributed as the residual in the wind modeling in section 3.2. The normality of the distribution is also rejected by the Jacque-Bera test. As Wagner (2012) [1] hints in his paper, there could be advisable to model this stochastic part by an higher order ARMA-process. To be consistent with Wagners (2012) [1] strategy, this is only noted and added to the suggestions of extended research in section 6.1. Concluding that the distribution of the residual is close enough to a normal distribution, the modeling is continued by estimating the parameters for the Ornestein-Uhlenbeck process for the solar daily maximums. The estimated parameters are found in figure 16.

3.4 Modeling Load

A very common approach for the modeling of total system load is using a deterministic function and a stochastic function in a similar way as used in the modeling of wind- and solar energy. However, the load model is strongly driven by two parts. Yearly temperature fluctuations and social behavior. Decreasing temperatures during the winter months increases the energy consumption due to the increased energy used in heating of buildings, and the social behavior of society affects total power consumption on both a weakly and intra-day period. The weakly behavior of load demand is highly affected by the difference

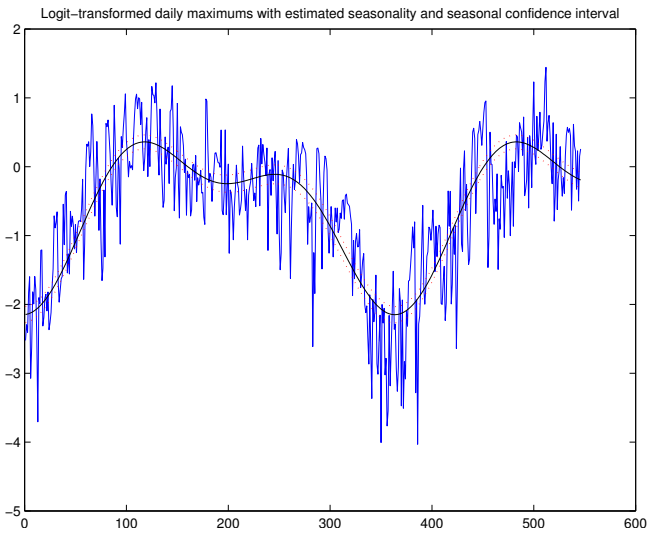
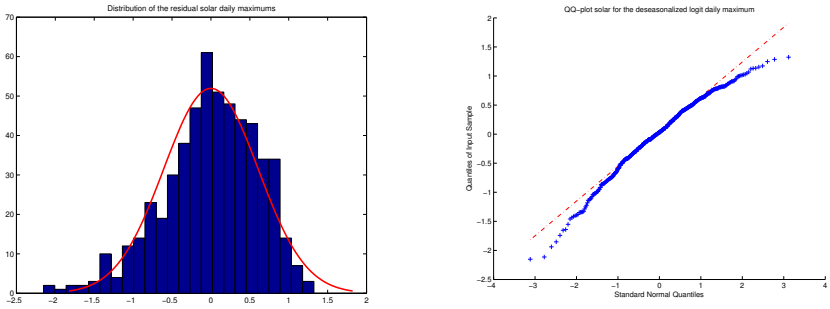


Figure 13: Illustration of the solar seasonality.



(a) Distribution of solar residuals. (b) QQ plot of solar residuals.

Figure 14: Solar residual Gaussianity test.

	a1	**b1**	**a2**	**b2**	**c**
Seasonality Parameters	-0.99	0.21	-0.55	-25.24	-0.63

Figure 15: Table of solar seasonal parameters

	Mu	**Sigma**	**Lambda**	**Half life**
Ornstein-Uhlenbeck Parameters	0.00	14.43	282.68	-0.00

Figure 16: Table of estimated Ornstein-Uhlenbeck parameters for the solar model.

between normal weekdays and weekends. Weekday demand is highly affected by the the industrial production, which is mainly carried out during regular weekdays, and the fact that people are at work. The consumption pattern is significantly different for weekends where the industrial production is lower and people in general have a different daily energy consumption pattern due to the increased ratio of people free from work. This pattern is also observable when it comes to national holidays and similar. The days before a national holiday will be denoted as bridge days, and there is a significant correlation between the consumption pattern of Saturdays and Bridgedays, and Sundays and holidays respectively. Therefore the notation W_1 and W_2 is introduced to denote the disjunct group of holidays, where W_1 denotes holidays and Sundays, and will be referred to as "holidays" and W_2 denotes Bridgedays and Saturdays and will be referred to as "Bridgedays". To account for these deviations from the normal weekdays, the model used is first introduced by Smeers et al. (2010) [4] and also used by Wagner (2012) [1].Similar to solar load, the consumption load process is decomposed into a stochastic time series part (for the daily Peak average load) and deterministic profiles (generating 24 hourly load values, given a daily Peak value) for each day type (weekday, W_1 or W_2). The load model will be defined as following:

Definition 6. *Load Model*

$$L_t = \delta^L_{d_t}(t, \psi_t + l_t) \tag{12}$$

where

$\delta^L_{d_t}$ is the intra-day, time dependent load curve,
ψ_t is a deterministic seasonal part and
l_t is an Ornstein-Uhlenbeck process

The Ornstein Uhlenbeck process is defined as

$$d_t^{load} = \theta^{load}(\mu^{load} - l_t^{src})dt + \sigma^{load}dW_t^{load}, \qquad lt^{src} = l_0. \tag{13}$$

First of all the seasonal part of the total system load is estimated using the Matlab command nlinfit. The seasonal component is assumed to have a yearly period of the shape:

$$y_t^{load} = a\cos(2\pi t + b) + c \tag{14}$$

and is estimated only from the business days. Since the seasonal part has to take the non-business days into consideration a function is defined using the two groups of non-businessdays and rescales the daily peakhour averages for the non business days as following:

$$\psi_t = (1 - \omega_1 \mathbb{1}_{t\in W_1} - \omega_1 \mathbb{1}_{t\in W_2})y_t \tag{15}$$

where

ω_i is the weight of the weekday mean for holidays and bridge days respectively.

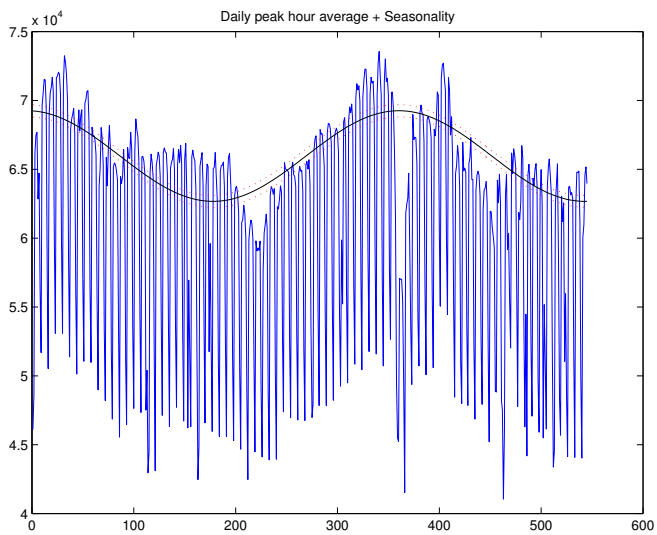
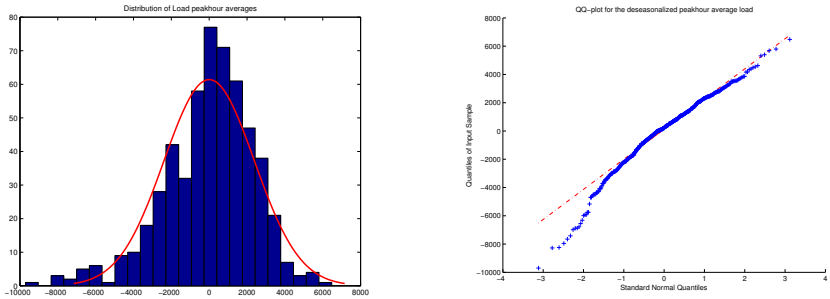


Figure 17: Illustration of the Load seasonality

The time series of daily peakhour averages and the estimated seasonality for non-business days can be observed in figure 17 and the estimated seasonal parameters for the non business days can be found in figure 18. Subsequently

	a	b	c
Seasonality Parameters	-3291.20	-2742.53	65960.23

Figure 18: Table of load seasonal parameters



(a) Distribution of the Load residuals (b) QQ plot of the Load residuals

Figure 19: Load residual Gaussianity test.

the Ornstein-Uhlenbeck process for the de-seasonalized daily peak hour averages is estimated.

Looking at the normal distribution properties of the residual in figure 19 the conclusion can be draw that the residual are not quite normally distributed (which also is indicated by Matlabs Jaque-Bera test, jbtest) , but close enough to be sufficient. As well as for solar and perhaps also wind modeling, a future extention of this model could be done using higher order processes to try and catch more information and thus yielding a more normal distribution. Thence the Ornstein-Uhlenbeck parameters for the de-seasonalized peakhour averages is estimated and can be found in figure 20.

	Mu	Sigma	Lambda	Half life
Seasonality Parameters	0.00	44075.39	171.72	-0.00

Figure 20: Table of estimated Ornstein-Uhlenbeck parameters for the Load model.

3.5 Residual Load

Using the estimated models in previous sections, a formal definition of the Residual Load if formed as following.

Definition 7. *Residual Load*

$$R_t = L_t - IC_t^{wind} \cdot E_t^{wind} - IC_t^{solar} \cdot E_t^{solar} \tag{16}$$

3.6 Supply Functions

3.6.1 Approach for Supply Functions

Motivated by Barlow (2010) [5] and previously used by Wagner (2012) [1] the concept of the Supply Function is to map the Residual Load to a spot price, such that:

$$S_t = f(R_t)$$

where S_t is the spot price of time t and, R_t is the Residual Load at time t.

In this thesis, mainly three supply functions will be covered. Starting from the first basic one.

3.6.1.1 Wagner Split Peak function as defined in Wagner (2012) [1], is only depending on the Residual Load for peak- and off peak hours respectively. The function is defined as

Definition 8. *Wagner Supply Function*

For Peak hours, the supply function is

$$S^{peak}(t) = \begin{cases} \min\left(p_{cap}, a + \frac{b}{x_{cap}-x(t)}\right) & \text{if } x(t) < x_{cap} \\ p_{cap} & \text{if } x(t) \geq x_{cap} \end{cases} \tag{17}$$

and for off peak hours

$$S^{offpeak}(t) = \begin{cases} \max\left(p_{floor}, a + \frac{b}{x(t)-x_{floor}} + cx(t)\right) & \text{if } x(t) > x_{floor} \\ p_{floor} & \text{if } x(t) \leq x_{floor} \end{cases} \tag{18}$$

where $x(t)$ is the Residual Load at time t, the x_{cap} and x_{floor} is the load boundaries motivated in Wagner (2012) [1] and p_{cap} and p_{floor} is the price boundaries set by the exchange. These are set to

$x_{cap} = 85GW$
$x_{floor} = 10GW$
$p_{cap} = 3000€$
$p_{floor} = -3000€$.

The estimated parameters can be found in section 5.1.

3.6.1.2 Yang Heatrate Function Yang et al. (2012) [2] introduces the fuel dependent Heatrate Supply function. The function takes coal and emissions prices into consideration by adding the Coal generation cost as a parameter to its supply function. Yang proves that coal prices has a better explanatory power than e.g. gas which is higher in the merit order, why coal generation cost is used. Intuitively this could be explained by the fact that coal is the price setting fuel of the market during a majority of the time. Gas is only the price setting fuel during periods of high demand, which would effect only the extreme values. (As a side note, it will be shown that the extremes will be the part of the prices that the models has a hard time catching). The Yang heatrate function is defined as following:

Definition 9. *Yang Heatrate Supply Function*

$$S(t) = P^{fuel}(t) \cdot \frac{e^{\frac{x(t)-a}{b}} - e^{-\frac{x(t)-c}{d}}}{2} + h \tag{19}$$

where $P^{fuel}(t)$ is the so called Coal generation cost i.e. the cost of generating power from coal

$$P^{fuel}(t) = P^{coal}(t) + 0.3831 \cdot P^{CO^2}(t) \tag{20}$$

where in turn

$P^{CO^2}(t)$ is the price of one metric-ton of CO^2,
0.3831 is the factor of how many metric tons of emissions that is generated by burning the amount of coal required to generate 1 MWh of electricity. (Unit metric-ton/MWh) and

$$P^{coal}(t) = API2/(FX \times 6.69) \tag{21}$$

where
$API2$ is the cost in dollar of a metric ton of a specific kind of coal(API 2) used as a reference,
FX is the daily €/$ spot exchange rate, and
6.69 is a factor describing the cost of generating i MWh electricity from coal, with a fictive efficiency of 100%.

The estimated parameters for the function can be found in section 5.2.

3.6.1.3 Split Peak Heatrate Function Due to the difference of electricity consumption pattern and prices in peak and off peak hours, it could also be reasonable to do the same for the fuel dependent Heatrate function. Therefore this thesis introduces the Split Peak Heatrate Function. The definition of the Split Peak Heatrate function is the same as in equations 19 and 20, although the modeling data is split in peak and of peak hours and two separate sets of

coefficients are estimated for each subgroup. The estimated parameters can be found in section 5.3.

3.6.2 Fitting Problematics

As previously stated, the Matlab function nlinfit was used to calibrate all the supply functions used in this thesis. However, the results proved to depend highly on the choice of initial conditions to the estimator. Due to the size of the parameters , the choice of 1 as initial conditions for the parameters proved to be unable to find any reasonable estimations. Instead the curve fitting tool on Matlab was used in combination of a classical "trail-and-error" technique of different starting conditions. Using this technique, parameters for more or less straight lines were found for the different supply functions, and those parameters were in turn used as initial values for the nlinfit command. The parameters are stated in section 3.6.3.

3.6.3 Initial Conditions

	a	b	c	d	h
Yang Heatrate Model	38130	8076	134900	24560	64.39
Split Peak Heatrate Model, Peak Hour	67130	3606	94270	35330	4.889
Split Peak Heatrate Model, Off Peak Hour	67130	3606	94270	35330	4.889

4 Data

4.1 Introduction to the data

The data for the estimations and validations of the models in this thesis, has been gathered from the German power exchanges, The European Network of Transmission System Operators for Electricity (ENTSO-E) and E.ON.

4.2 Load data

The load data used, corresponds to the German total consumption of electricity. There is also a small part of power exported to Germany's neighboring countries, e.g. France during some conditions and a small part of power imported during some other conditions. Modeling these cross country transmissions is extremely complex and the structure of these transmissions more or less has the shape of white noise. The complexity and whimsicality of this phenomenon makes it convenient to neglect this effect and hope for an average out effect.x

The load data consists of hourly total system load for the German electricity market, starting in 2011-01-01, and ranging to 2013-03-28. The unit for electricity used through out this thesis is MWh, unless anything else is stated. A time series plot of the load data used in this thesis can be observed in figure 21. The load data has a clear sign of a yearly seasonality, but also has a strong weekly season as well.

4.3 Wind Power

The wind power infeed consists of hourly observations in MWh, and ranges from 2011-01-01 to 2013-03-28. The time series of the data can be observed in figure 22. It show clear signs of a yearly seasonality, and a very spiky character.

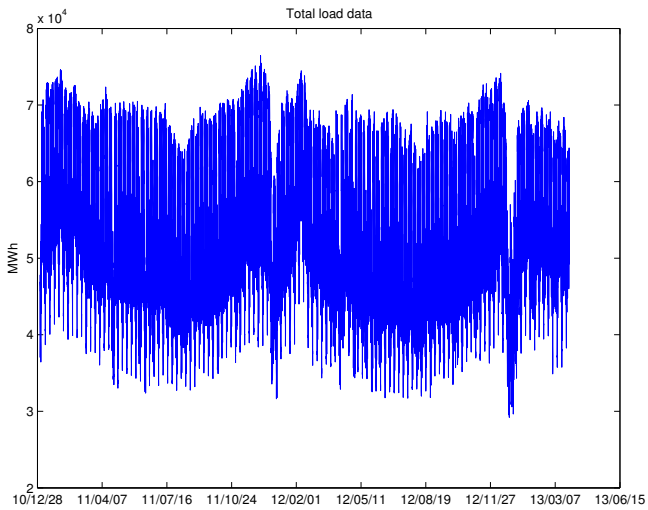


Figure 21: Load Data.

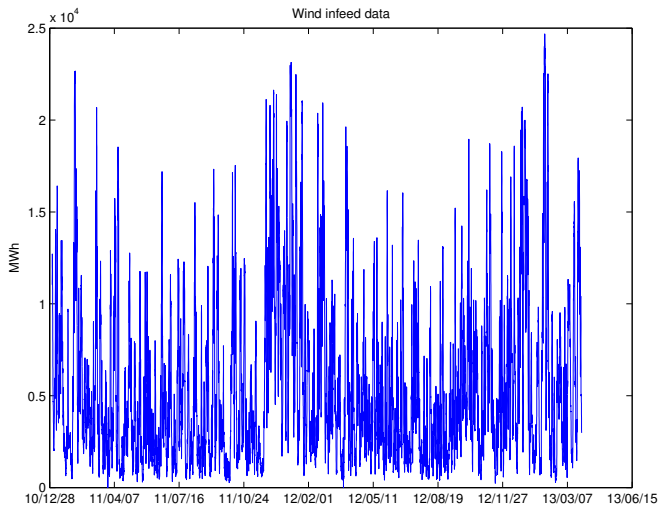


Figure 22: Wind Data.

4.4 Solar Power

The solar power infeed consists of hourly observations in MWh, and ranges from 2011-01-01 to 2013-03-28. The time series of the data can be observed in figure 23. The solar data has a very clear seasonal behavior, with a very little amount of infeed during the winter months. It also has a clear daily seasonality as well, which can be observed in figure 11 in section 3.3.

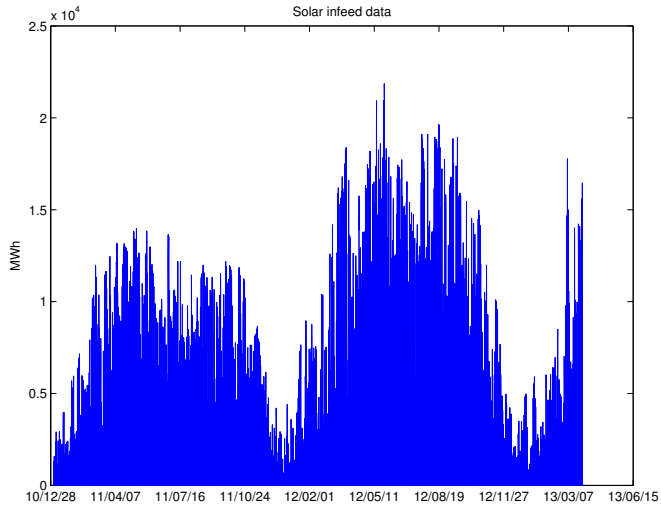


Figure 23: Solar Data.

4.5 Installed Capacities

The installed capacities of renewable energy used in this thesis, is given on a monthly basis. They can be splitted in two sub-categories, the absolute historic installed capacities and the future estimated capacities.

4.5.0.1 Historic installed capacities This data is gathered from E.ON and used to estimate the separate models of wind and solar infeed to the grid. The period of historic installed capacities ranges from January 2011 to March 2013.

4.5.0.2 Future installed capacities This data from E.ON is the future estimates of installed capacities of renewables, according to the market conditions and the planned future production of new plants. The period of future installed capacities ranges from April 2013 to December 2013.

4.6 Price data

In the modeling process covered in this thesis, price data is used both for spot price of electricity, but also the spot prices of fuels of power plants and emissions. The fuels and emissions prices are significant components in spot prices, according to Yang et al. (2012) [2], Carmona et a.l (2012) [3].

4.6.1 Historical Spot price

The spot price data of electricity consists of a time series of historic hourly spot prices from the EPEX Spot market, ranging from 2011-01-01 to 2013-03-28.

4.6.2 Historic Fuel Price

To calibrate models with regards to fuel prices, historical spot price of fuels are used, since they are assumed to provide a good explanatory power of peaks and patterns in the electricity spot market. The price setting fuel on the EPEX Spot Market according to the Merit Order curve for the majority of the time, is the coal generation price.

Together with the price of natural gas, Coal generation price is the primary price setting fuel on the power market.

Coal generation means the cost of generating 1 MWh of energy from coal, including the cost of emissions. Yang et al. (2012) [2] concludes that coal generation price has a better explanatory power than gas prices, when using only a single fuel. This is reasonable and consistent with the argument that increased power production from renewables pushes the gas generation plants even more out of the market which intuitively would lead to an even smaller explanatory power of gas prices. Furthermore Yang et al. (2012) [2] claims that coal generation prices and prices of natural gas is highly correlated. This will be investigated more closely in the section 5.5. The Historic fuel prices ranges from 2011-01-01 to 2013-03-28.

4.6.3 Future Fuel Price

When estimating the price of quarterly futures for 2013 in section 5.7.2, forward curves are used with an hourly resolution, as seen on the estimation day, 29:th of June 2012. Fuelprices are given in the unit $ / ton. A time series of historical spot price exchange rates is used to convert the units into €/ton and finally into €/generated MWh. For calculations, see section 5.2.

4.7 Modeling and validation Data

The data is split in two sets. A modeling set upon which the model parameters is estimated, and a validation set which is used as an unbiased validation of the estimated model. The modeling set is set from 1:st January 2011 to 30:th June 2012, and the validation set from the 1:st of July 2012 to the 28:th of March

2013. The end date of 28:th of March is set simply because of lack of newer data. As concluded in the introduction, the electricity market is not stationary, but has a rather fast pace of change of the fundamental factors (which is the main reason of why to pursue the structural approach). This makes it desirable not to use "too old" data. On the other hand, in general modeling theory more data is said in general to produce a better model. In this case a compromise has to be made, and the choice of periods reflects this compromise.

5 Main Results

For each of the models, the performance both in- and out of sample is examined. The mean squared error (MSE) is used as a measure of the fitting quality fit between the modeled price trajectories and the real ones. The price trajectories are also plotted in comparison to the real price to give a visual perception of the performance of the model. The price/load plot will also be provided to give an additional perception of the fitting properties of the models. From both the time series plots and the price/load plots we will make a visual analysis as well, to determine the quality of the models and possibly identify the models weaknesses. The MSE is defined as:

Definition 10. *Mean Squared Error MSE*

$$MSE = \frac{1}{n} \sum_{0 \leq i = 1}^{n} (\hat{Y} - Y)^2 \tag{22}$$

where

$\hat{Y}$ is a vector of the estimated values, and
Y is a vector the true values.

The performance of the models can be found in figure 24 and the details of the fits will be examined in each subsection below.

	Modeling Set	**Validation Set**
Wagner Split Peak Model	95.18	3774.24
Yang Heatrate Model	105.53	274.30
Split Peak Heatrate Model	103.80	270.14

Figure 24: Table of model MSE

What can be observed directly in the price clouds, in any of the sections below, is the fundamental difference in the shape of the clouds between the modeling and validation period. This strengthens the argument that fundamental conditions of the electricity market indeed is constantly changing. This difference will dramatically decrease the performance of any model, since the foundations upon which the model is estimated simply has changed until the validation period.

In figure 25, the time series of spot price data of electricity is plotted for the whole data set. Especially interesting is the period around Christmas, new years eve and the first weeks of any year. These periods will be referred to as *Winter periods*, since they occur around the Christmas and winter period each year. This is a period where the normal social patterns of power consumption is broken due to the holidays etc, where industrial production is decreasing and the normal pattern of everyday life of people is temporarily changed. This often coincides with cold weather which makes this period very vulnerable to anomalies or disturbance in the power market. These periods will be studied more in detail for each model below and their individual ability to capture the events these periods. In the section 6 it will also be covered in general and analyzed. To be able to better understand and analyze the peculiarities in the winter periods, some information about what abnormal events that took place in the power market during these times might be in place.

5.0.0.1 Winter period 2011/2012 The winter period 2011/2012 had substantially lower temperatures than usual, which led to increased power consumption, mainly to heat buildings. The demand for power was not only driven by the German need of heating. This cold-streak also affected France, which has a high degree of direct electrical heating. Due to the cross-integrated power grids in Europe, the increasing French demand led to power export from Germany to France, which in turn lowered the supply of electricity for the German market and contributed to raise the spot price. Another coinciding event, which perhaps had a larger affect on the high spot price, was a dramatic increase of the price of natural gas during the same period. Since gas is placed in the top of the merit order curve, it is mostly used as complementary power in times of high demand, and it is the price setting energy source on the spot market when used. Hence the increased demand in combination with the high gas price can explain the bulk of this event.

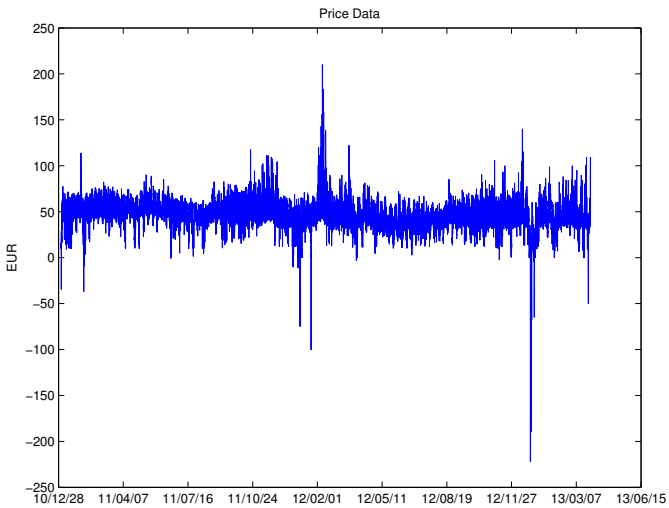


Figure 25: price Data.

5.0.0.2 Winter Period 2012/2013 The winter period of 2013 was not characterized by any temperature extremes, but the power consumption in general was very low. Instead this period was extremely windy, which generated a high infeed of wind power to the grid. Since the overall consumption was low and due to the typical low level of industrial production, the supply of energy on the market was relatively high. The producers tried to regulate their production to compensate for the high amount of wind power but had difficulties to reduce production in the extent needed to keep spot prices reasonable. Especially lignite plants were unable to regulate their production and accepted large negative prices on the spot market for their energy, to keep production running. These events gave rise to large negative price spikes.

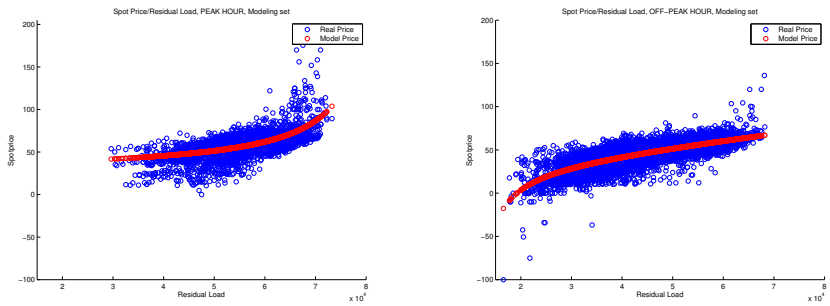
It is very interesting to observe phenomenons so fundamentally different for similar parts of the year as the ones described above. Especially on a market which to some extent has fundamental factors that follows clear seasonal pattern, such as the yearly seasonality of power consumption, or the electricity generated by solar power. These two events also seem to indicate some kind of a threshold behavior, where under certain conditions the extreme price spikes becomes more than just temporary spikes, but can become a temporary period with a spiky behavior. Both kinds of events can be observed on the power market.

5.1 Wagner Split Peak Model

	a	b	c
Peak Hour	25.01	-925001.54	0.00
Off Peak hour	25.20	-357607.20	0.00

Figure 26: Table of Wagner Split Peak Model.

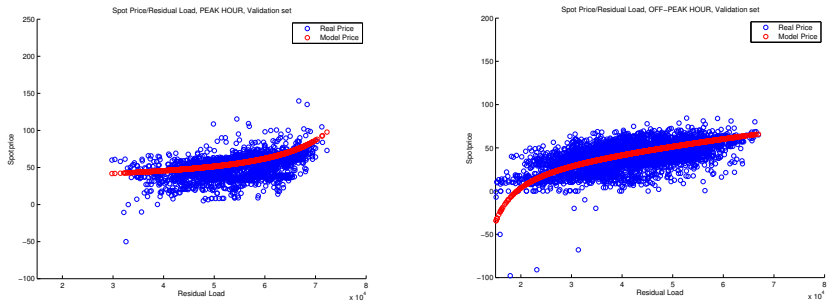
In figure 27, the Wagner Model performance on the modeling data can be observed. The "cloudy" shape of the real price makes it impossible for this one dimensional model to capture all variations in prices. The peak hour part has difficulties capturing extreme values both in the upper and lower price range,



(a) Wagner model applied on the peak hour modeling data

(b) Wagner model applied on the off peak hour modeling data

Figure 27: Price cloud fitting of the Wagner model, Modeling data



(a) Wagner model applied on the peak hour validation data

(b) Wagner model applied on the off peak hour validation data

Figure 28: Price cloud fitting of the Wagner model, validation data

while the off peak hour part captures the negative extremes quite well. This gives rise to a situation where the positive extremes are the ones most significantly deviating from the real price and hardest to catch. Hence a first attempt to improve this model could be to make the model able to capture the positive peak hour extremes better. On the other hand, the validation set does not have the same proportion of extreme outliers for the high residual load peak hour prices. This might instead imply that the solution of this problem is not to model outliers, but to first understand the fundamental factors behind the electricity spot price and the electricity market. This can be viewed as a proof of what is stated in the introduction in section 1, that the market is constantly changing over time and such changes are extremely hard to catch. Looking at the function parameters in figure 26, we make an interesting observation. The c-parameter for the Off Peak Hour is insignificant, which clearly contradicts Wagners claim of the model design. Since Wagner (2012) [1] motivates his selection of Supply Function as an empirical model, this is a sign of that the selected-one was developed specifically after market conditions valid most significantly during his modeling period (01/08/2010 to 31/07/2011). This can be viewed as the first evidence in the hazard of choosing Supply Functions that models temporary behavior and does not perform robustly over a longer period of time due to the rapid market changes.

Bearing in mind that the validation set is roughly half the size of the validation set, there still seem to be a substantially lower rate of positive extreme values in the validation set than in the modeling set. A vague tendency can also be imagined in a little wider spread of the data for the validation set than the modeling set. These two factors could help to further prove the statement that the foundations of the market has changed (gradually of course) between the two sets of data. To get a perception about the performance over time, the time series of the real and model estimated price is plotted both for the modeling and the validation data. Since the Wagner Split Peak function is a function of residual load, the time series of the residual load is plotted beneath the price function, to investigate e.g. price peaks and whether they occur due to a low amount of residual load or if there is another reason behind it.

The Wagner Split Peak Function has an over all good fit for the modeling set.

It seem to capture the seasonal fluctuations fairly good. Some spikes are also caught, the clearest example would be the negative spike around the beginning of February 2012. Also the small spike in the beginning of 2011 is captured. Both of these spikes arise from an individual significant drop in residual load, that can be identified in the residual load plot below. There are one relatively large negative price spike not captured, which is the one in January of 2012. An explanation would be that this spike does not arise from a sudden drop in residual load as the other two, since no such thing easily can be identified in the residual load section. Since the Wagner model only captures spikes using the residual load, this one is missed since it is obviously due to other factors. The extreme peak-period around the winter period 2011/2012 is hardly caught at all. Given the background information about this peaking period stated in the section 5, the Wagner model lack the ability to account for the sudden increase of gas prices. Regarding the increased demand, the residual demand plot shows that residual demand during this period was in its higher region, but not extremely large, which motivates the result from the model as prices in a high price range relative to the rest of the year, but not extremely large. Since the Wagner model captures the spikes using the divergence term in the denominator, the residual load would have to be closer to the x_{cap} to yield more extreme spikes.

The unbiased performance on the validation set gives rise to some more peculiar event, than the modeling set. The general level of prices is still captured quite nicely by the model, although in detail there are still quite significant differences. The most interesting events, and perhaps the only significantly deviating, are the ones during the winter period 2012/2013. The negative spikes are caused by a significantly low residual load, caused by the reasons stated earlier in section 5. First of all, there are three really extreme spikes on -3000€each. They arise from the fact that the residual load drops below the x_{floor} limit during three hours of off peak hours, resulting in the Wagner model yielding the spot price of p_{floor}

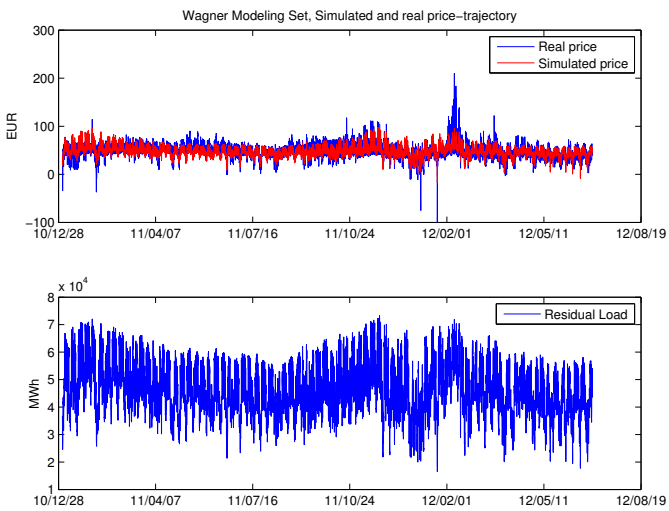


Figure 29: Time series of real and estimated price, modeling period.

(figure 30 has rescaled axis, where the full extent of these extreme spikes are not shown to get a more detailed view of the time series plot). Since the real spot rice is no where near the magnitude of the models generated prices around this period, these spikes significantly decreases the performance of the model. Around the same period is also a number of other significantly low, but not as extreme negative spikes. They are also generated due to the low residual load, but does not hit the x_{floor} threshold. As can be seen in the figure 31, the capturing of the spikes in the extreme winter period 2012/2013 are not very consistent. In a number of occasions, the model yields negative spikes when there in fact were none, and in at least one instance a real spike is missed by Wagners model.

This would imply that the residual load does not provide enough evidence alone to explain the spikes, although it certainly plays a big part. A number of reasons can be assumed to explain the issues of capturing the extreme events in the validation period. First of all it seem to be very sensitive to quantities. If the residual load in a given time period is lowering even a small bit below a "low" lever, it might yield a serious oversupply of power in the market. Due to the in-elasticity of the power market this relative small change in supply can give rise to very serious consequences on the spot market, since no imbalance is allowed in the system. The model also fails to show the real physical differences, in a model depending only on residual load. E.g. lignite plants might still be running, even for load levels or times of the year when they are typically not running, due to slow responsiveness of power regulation control or scheduling problems. If this coincides with low levels of residual load, the sensitivity of the market might cause abnormal or unpredictable behavior of the spot price. The comparison between the spikes yielded by the model and the real spikes also indicates that the Wagner Split Peak model's way of dealing with spikes, the divergence term in the denominator, works somewhat satisfying for smaller deviations in residual load, but fails to capture the level of spikes caused by

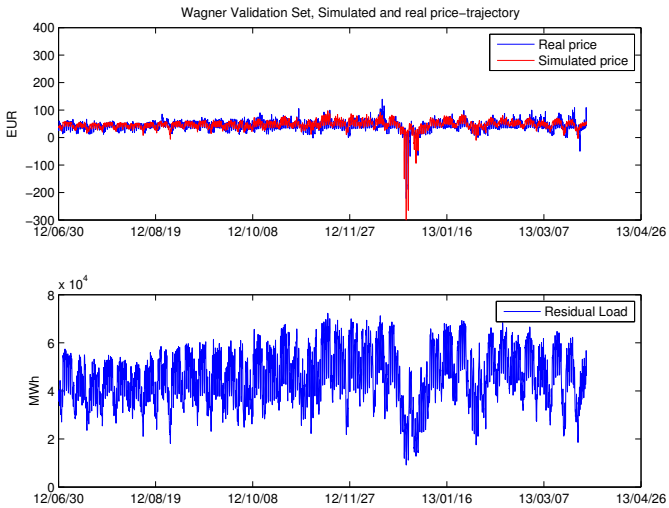


Figure 30: Time series of real and estimated price, validation period.

residual load. It is also indicated that spikes seem to occur for other reasons than the level of residual load which the model naturally fails to capture. Hence a better way of capture the level of the spikes than Wagners model is needed.

5.2 Yang Heatrate Function

As previously addressed in several instances of this thesis, the spot price of electricity is assumed to depend to some extent on fuel prices. To which extent and how has been examined before by e.g. Yang et al. (2012) [2]. A more detailed analysis of the fuel prices impacts on power prices, and behavior will be conducted later on in the Analysis section 5.5, here the performance of the actual model is first examined. The estimated model parameters can be found in figure 32.

In the price cloud plot in figure 33, the price/residual load data is plotted with the performance of the three-dimensional Heatrate function in red. The green line is the Heatrate function, without the fuel parameter (the coal generation cost). This is plotted as a reference. Looking at the price cloud plots in figure 33, the Yang Heatrate Function transforms the supply line into a cloud, covering a larger portions of the observations in the Price/ Residual Load plot. We expect this behavior to add increasing explanatory power to the model, and reduce the MSE compared to the two-dimensional Wagner Split Peak Model. Interestingly enough, the model seems to get a deviant behavior due to the fuel part, especially

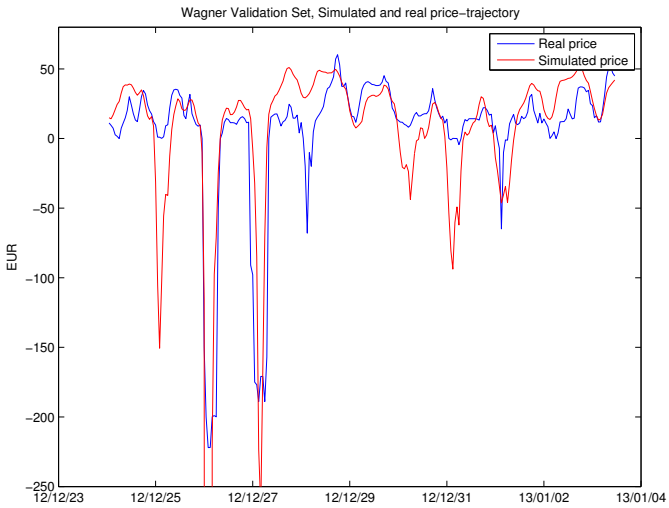
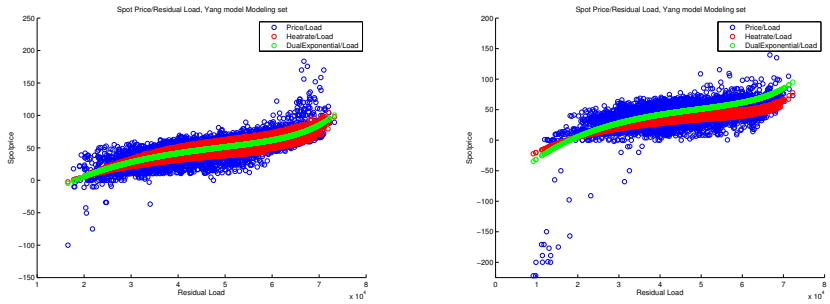


Figure 31: Time series of real and estimated price, extreme period, December 2012.

	a	b	c	d	h
Parameters	64674.19	5107.25	67687.51	23284.40	4.32

Figure 32: Table of the yang Heatrate Function.



(a) Yang Model applied on the modeling data

(b) Yang Model applied on the validation data

Figure 33: Yang Model Performance

for higher values of residual load. Observing the time series plots of the Yang Heatrate model in figures 34 and 35, offers some more explanation of why this is the case.

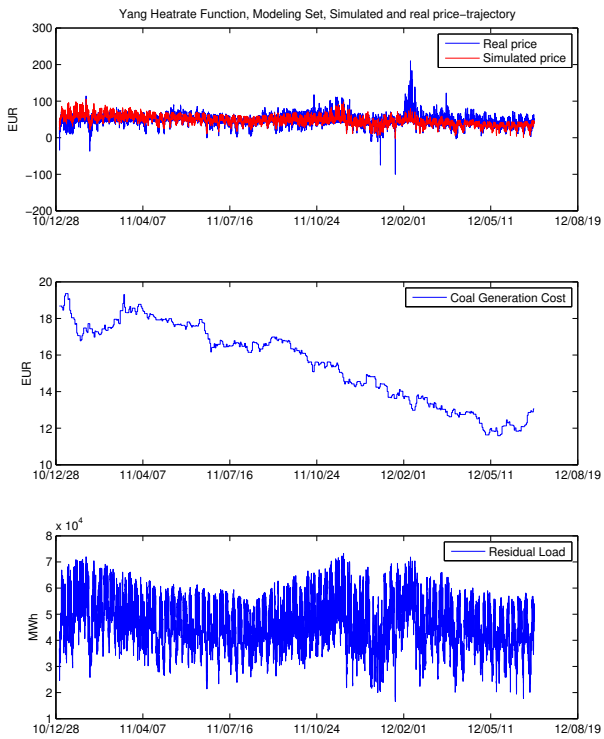


Figure 34: Time series of real and simulated price, modeling period.

In the beginning of this modeling period, the Yang Heatrate model seems to perform well. However as the fuel prices are declining over time, so is the estimated price generated by the Yang Heatrate model. The real prices however is not following this declining trend. In the end of the modeling period, there

is a significant offset between real prices and the modeled ones. The fuel part seemed to give the Yang Heatrate model additional information and explanatory power of the movement of power spot prices when looking at the "cloudy" shape of the Yang Heatrate function in the subplots of figure 33. As opposed to the straight line generated by the Wagner Split Peak model in figures 27 and 28. This would lead us to believe that this model would perform better than the Wagner split peak one. This is however not the case, at least not in the modeling period (since the Yang Heatrate Function performs marginally worse than the Wagner split peak function the modeling set but not on the validation set). For the Validation period the source of the big error of the Wagner Split Peak model is identified as being the overestimation of the negative spikes and it is difficult to say anything about the models performance without these sources of errors, ceteris paribus.

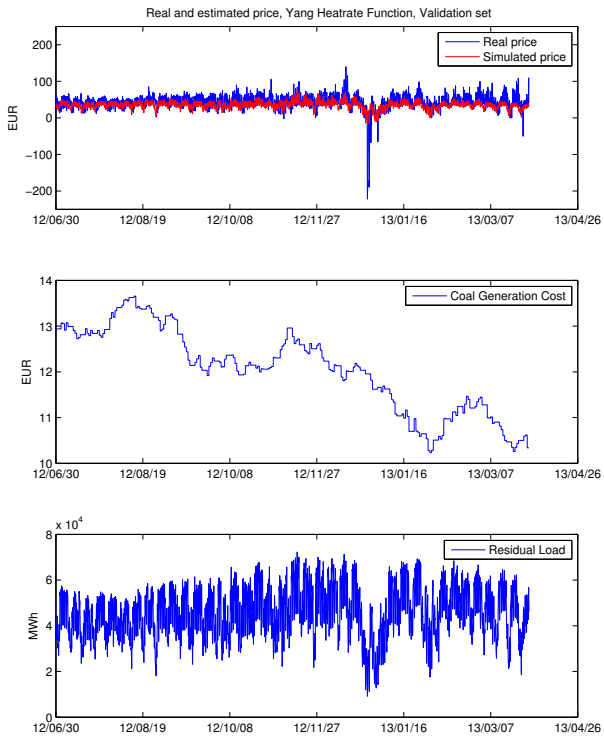


Figure 35: Time series of real and simulated price, modeling period.

Continuing the evaluation, leaving the theory about the artificial explanatory power of Coal Generation cost at the side, the model fails to capture most spikes in the modeling set, including the extreme period around winter period 2011/2012. Looking at the shape of the Heatrate function in figure 33a it becomes clear that the slope of the Heatrate function is too low to capture most extreme events. There are two spikes in the real price that can be interesting to study closer, and that is the two negative spikes between October 2011 and February 2012. The later spike has a corresponding significant decrease in residual load, which is the reason of why this spike is captured. When looking

at the residual load for the time of the first spike, no corresponding spike in residual load can be observed, and this is also the reason of why that peak is not captured. This indicates that residual load is a reason of why spikes occur, but not the only one. There are more hidden factors that this model lack the ability to capture.

Looking at the validation set in figure 35, it can be observed that the Yang Heatrate function captures the spot price fairly good for lower Residual Loads, but fails to capture the full magnitude of the prices for higher Residual Loads. Due to the continuing decline of fuel prices, this is not very surprising, since it is the same behavior as discussed for the modeling period, meaning the decreasing model performance due to decreasing fuelprices. Regarding the extreme period during the winter period 2012/2013, this model performs badly in capturing the extreme negative prices observed in this time period. The model yields marginally lower prices for this period, but not even close to the real observed ones. Looking back at the figures 33a and 33b, the shape of the Supply function makes it impossible for this model to reproduce the extreme negative prices during this period and even though the result is not very satisfying, it is at least explained and the performance is still more robust than the one of the Wagner Split Peak Model.

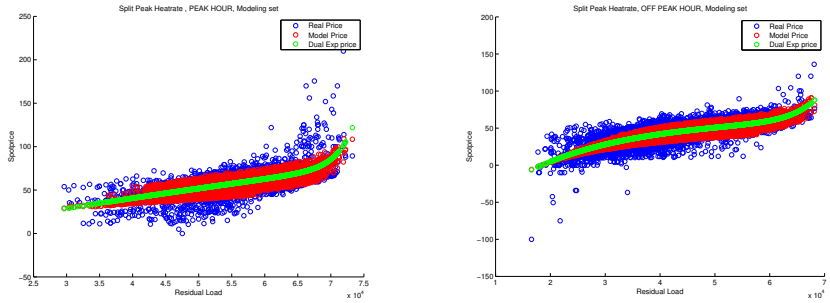
5.3 Combined Split Peak, Heatrate Function

	a	**b**	**c**	**d**	**h**
Peak Hour	68227.43	2870.18	707525.49	216479.46	13.81
Off Peak hour	62529.07	4250.13	55534.98	18201.43	3.84

Figure 36: Table of the Split Peak Heatrate function parameters.

Since the Combined Split Peak Heatrate Function uses the same function and input as the regular Yang Heatrate function, this model is expected to suffer from the same drawbacks. However, if there are substantial differences between peak and off peak hours, this strategy should lead to an increase in performance and hopefully be able to capture the peak hour values a little bit better. This due to the peak and off peak hour split, since the regular Yang Heatrate function failed mainly in capturing the peak values.

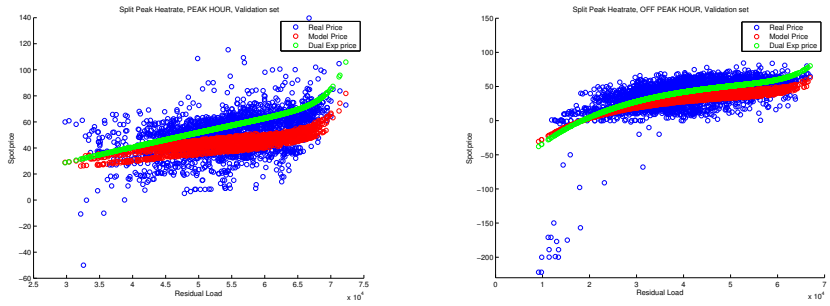
By observing the plots in figure 37, a slight increase of the slope for the price of high Residual Loads can be observed, mainly for the peak hour data. The lower part of the peak hour supply function both for the validation set in 37a and 38a hardly has any slope of their lower exponential tail. This causes the Split Peak Heatrate function for peakhours to behave like a single exponential function rather than a double one for these data sets. The off peak hour has a shape very similar to the regular Yang Heatrate function. The green line in these plots are still the simple dual exponential, non fuel dependent supply function, used as a reference. For the validation set plots, conclusions can be made once again that the fuel part is actually decreasing the models performance in this model as well. This is however not surprising, given the similarities to the previous Yang Heatrate Model.



(a) Wagner Model applied on the peak hour modeling data

(b) Split Peak Heatrate Model applied on the off peak hour modeling data

Figure 37: Split Peak Heatrate function, Modeling data model performance



(a) Split Peak Heatrate Model applied on the peak hour validation data

(b) Split Peak Heatrate Model applied on the off peak hour validation data

Figure 38: Split Peak Heatrate function, Validation data model performance

In the time series domain, the result looks very similar to the one from the Yang Heatrate Model. Given the relatively small difference in MSE between these two models, the insignificant difference in the time series data is understandable. This implies that no better model explanation is offered by splitting the data sets in in peak and off peak hours, at least not for this kind of model. But also that there are more explanatory parameters needed to explain the variations in fuel price, which should provide better accuracy to take in to account before it makes sense to try and split the loads in peak and off peak hours. Like the previous Yang Model, the Split Peak Heatrate Model starts off in a relatively good price level in the beginning of 2011, but experience a level drop in comparison to the real price in the end of the modeling period. In general, this Split Peak Heatrate Model captures spikes marginally better than the Yang Heatrate Model, but in most cases it takes a larger resolution of the plot to be able to actually observe it. E.g. the peak in the end of 2012 is estimated to -30.00 €/MWh using the Split Peak Heatrate Model, but to just -22.00 €/MWh with the Yang heatrate Model.

The time series validation data plot in figure 40 does not offer any great surprises. Just as for Yang Heatrate Model, the price level of the Split Peak Heatrate Model initiates the validation period with a small offset, which is increasing with time throughout the period. As expected this model captures the extremes a little better than the Yang Heatrate Model, but still just marginally, why we conclude that this strategy does not offer any explanatory power for

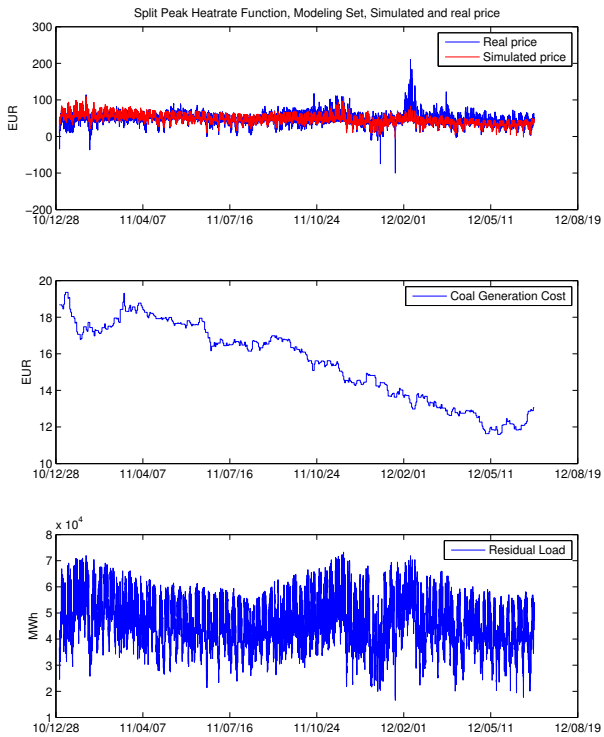


Figure 39: Time series of real and estimated price, modeling period.

this shape of models.

5.4 Conclusions, Supply Functions

After examining these supply functions, conclusions can be made that the spot price dependency of the coal generation price as Yang et al. (2012) [2] claims is not as significant as expected. In fact, even though the Dual Exponential function mapping residual load to price seems to be a more robust choice than the Wagner empirical Split Peak model, the fuel part seemed to give rise to a systemic error of the performance of the fuel dependent models. Especially on the validation set. The theory behind the fuel dependent component were that it would yield a better fitting model, due to the merit order argument. Although in this case it does not provide any evidence of that being the case. Wagner showed a good responsiveness to extreme events in the modeling set, but dramatically overstated the extreme events in the validation set. This could indicate the need of a better way of mapping residual load to spot price than such kind of empirical models. It also indicates that even though a model can be fitted to catch temporary relations or perhaps even pseudo relations in the modeling set, the rapid evolvement of the power market makes these connections hard to model using a static model like these. This implies a need of a further sectioning and explicit modeling of the complex fuel price to electricity spot

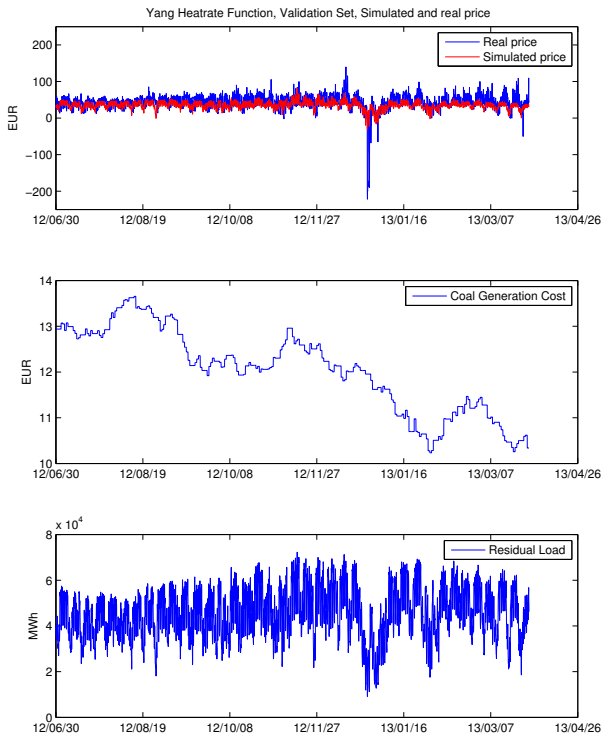


Figure 40: Time series of real and estimated price, validation period.

price relations.

5.5 Fuel Price Analysis

The result from the investigations of the supply functions performance gives rise to a need of looking closer on the history and effect of fuel prices.

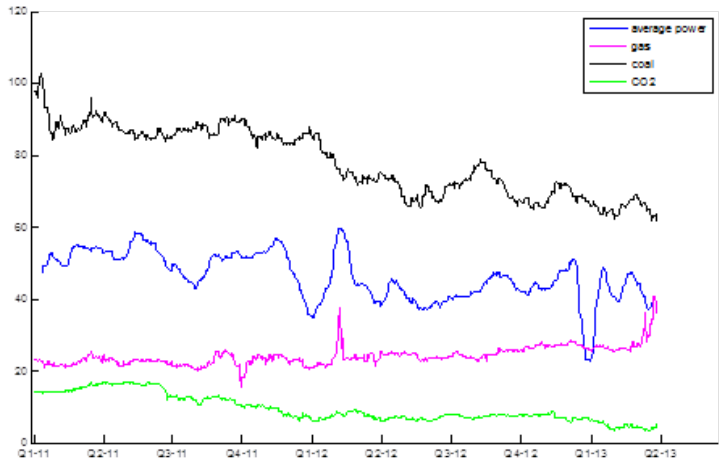


Figure 41: Comparison, fuel prices and average power price.

Observing the price history of electricity and some of the fuels from Q1, 2011 to Q2, 2013 brings some interesting insights. The gas price does not seem to have any significant impact on the average electricity price at any time, except the extreme winter period of 2011/2012. The explanation for this is, as discussed in the introduction, is that gas plants are only used to produce power during times of very high demand, typically peak hours and cold periods. But even during these periods, power generation from gas plants becomes more and more rare, since the increased amount of installed capacity of renewable energy pushes the gas power plants even higher in the merit order. For the extreme period in the beginning of 2012 however, the high demand called for the use of gas power plants and during that extended period of time, gas prices drove the high spot price of electricity. Looking at the figure 41, evidence of this can be seen.

The figure 41 also raises some questions about some market behavior stated by other writers in the field. E.g. this figure is contradicting the claim from Yang et al. (2012) [2] that the prices of coal and natural gas is highly correlated. For this particular period of time it turns out not to be the case. Hence it highlights the whimsicality in temporary, perhaps random correlations that can be observed in different market conditions. Intuitively this can be explained in large parts by the market consensus that the coal prices are mainly driven by worldwide factors, where the development in China is the single largest factor. Whereas the price of natural gas is driven by much more local factors in Europe, since the export of natural gas is much more local than coal. In the case of the models examined in this thesis, there are other problems to correct (e.g. the coal based fuel part which is not working satisfying) before trying to adopt a model to the much more rarely affecting gas prices. However, if pursuing such

strategy it might be hard to try and fit a supply function to the thin cloud of gas generated Residual Load/Price observations.

Looking at the coal prices in comparison to the average spotprice of electricity offers a great deal of explanation of what happened in the coal price dependent supply functions earlier in this section. One clearly sees that the overall trend for both electricity spot prices and coal prices is downwards, which could motivate a long term correlation between them. However, what can be observed in figure 41, but perhaps better in the figure 42, is the increasing spread between the two. Comparing the spread early in the investigated period with the spread in the end shows that when the spread is changing, so is the correlation between coal prices and electricity prices. This could explain why the fuel dependent supply function fails to capture the right level of spot prices. Since the models assumes a tighter correlation between declining coal prices and electricity prices, they simply fail when the fundamental market conditions change. Figure 42 also shows a tendency of an assymetric dependence between power prices and coal generation price. When coal generation cost is increasing, so is the spot price of electricity, but when coal generation prices are decreasing, the spot price of electricity show little or no effect of also decreasing because of it.

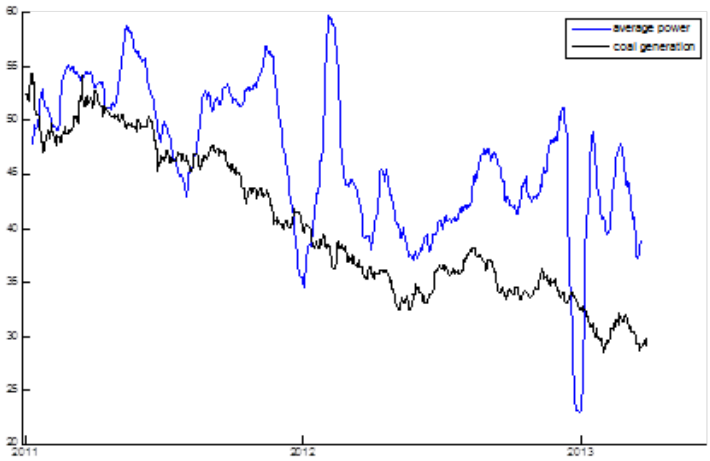


Figure 42: Comparison, Coal generation cost and average power price.

As shown in the figure 43, coal prices has a long term impact on spot prices of electricity, but with an alternating dynamic. Both the different models and the fuel plots has failed to produce any evidence of coal prices affecting the spot price of electricity in the short term. Though this is not very surprising, since coal is mostly traded on future contracts , and the spot price of coal is really a synthetic price of coal with delivery 3 months ahead of the current day. When trying to explain spikes and extreme prices, residual load and gas prices are the factors with the most explanatory power.

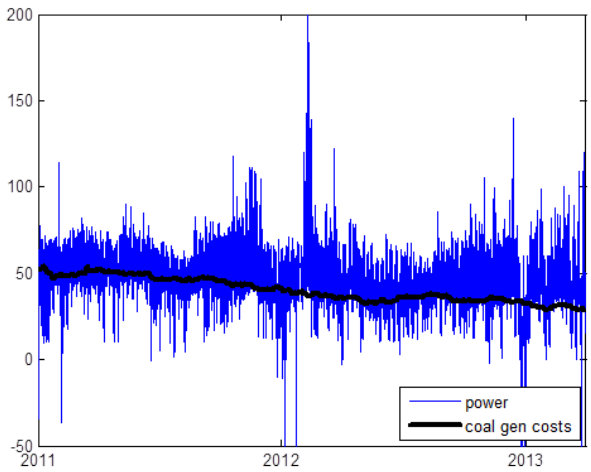


Figure 43: Trend comparison, Coal generation cost and average power price.

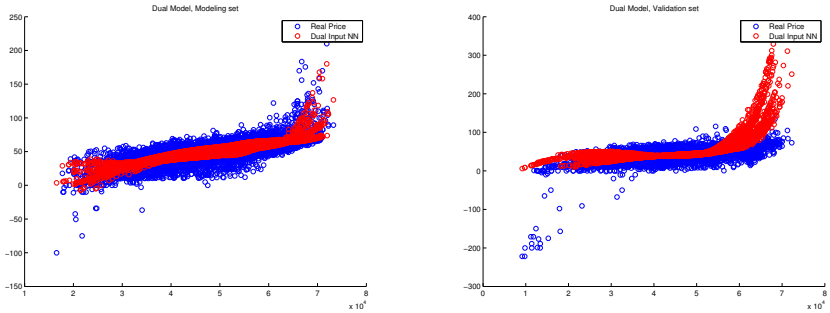
5.6 An Alternative Supply Function based on Artificial Neural Networks

Subsequently, with the previously stated evidence of the "punishing" effect of Fuel Prices, when using the previously discussed Supply Functions, we continue by suggesting a comparison of the performance of two new kinds of Supply functions. The comparison will be a regression analysis of fitting an Artificial Neural Network to the data. Two models will be used, the first one with Residual Load and Fuel (Coal Generation Cost) as input and the second one with only Residual Load as input. The aim is to see if the Coal Generation Cost generally adds any information to the models. Just as in the previous analysis of the Supply Functions, the models will be estimated on the Modeling Set and validated on the Validation set. This is interesting, since Artificial Neural Network is not bound as strictly to a specific shape as the other supply functions are. However this is a semi-scientific approach since the Neural Networks will yield somewhat different results each time they are estimated, as a difference to the deterministic functions for the other supply functions. Perhaps a more scientific approach would be to do calculate a MSE average over e.g. 10.000 simulated nets to give a better perception to what error level the nets converge. This would however require a lot of CPU time, since each net estimation is a relatively computational intensive operation. Instead, the nets presented is deliberately chosen to represent the empirical results by repeatedly simulating these nets with various set ups. Deviating estimations yielding huge errors are common when repeatedly estimating these nets, although the purpose of this section is to examine the impact of fuel and reasonable shapes for the supply functions to provide more clues in how to design better supply functions rather than suggesting to use Neural Networks as a supply functions. The estimated nets consists of a Matlab standard of 10 Neurons, which empirically has proven to yield satisfying results. The same kind of "cloud" plots for the Price/ Residual Load and time series plots are provided as for the analytic Supply Functions to compare the results.

	Modeling Set	**Validation Set**
Dual Input	69.58	965.88
Single Input	93.34	224.37

Figure 44: Table of the MSE's for the estimated Neural Networks

A typical result of the Neural Network estimations with the above stated preconditions are given in the figure 44. Even though the estimated Neural Networks does not perform better than the analytic Supply Functions every time, this result clearly shows that it is possible, even though it is not with an impressive difference. A more interesting fact however is that the Dual Input always, without exceptions thus far performs significantly better in sample than any of the other models. But at the same time it also always performs significantly worse than the Single Input Net on the Validation Set. Consistently throughout this investigation, the Dual Input Net has performed better than the Single Input one on the Modeling Set, while the Single Input Net consistently has out performed the Dual Input Net on the Validation Set.



(a) The Dual Input model performance on the Modeling set

(b) The Dual Input model performance on the Validation set

Figure 45: Price/Residual Load Plot of NN Dual Input Model

This could be yet another evidence of how the increased Clean Dark spread is degrading the out of sample performance of models. Since it is an important factor in the electricity spot price, but with a complex and ever changing correlation, it gives raise to the need of a more complex modeling of this relationship.

Looking at the shape of the Dual Input NN Model in figure 45, the shape differs quite significantly from the shape of the previous fuel dependent models (Yang

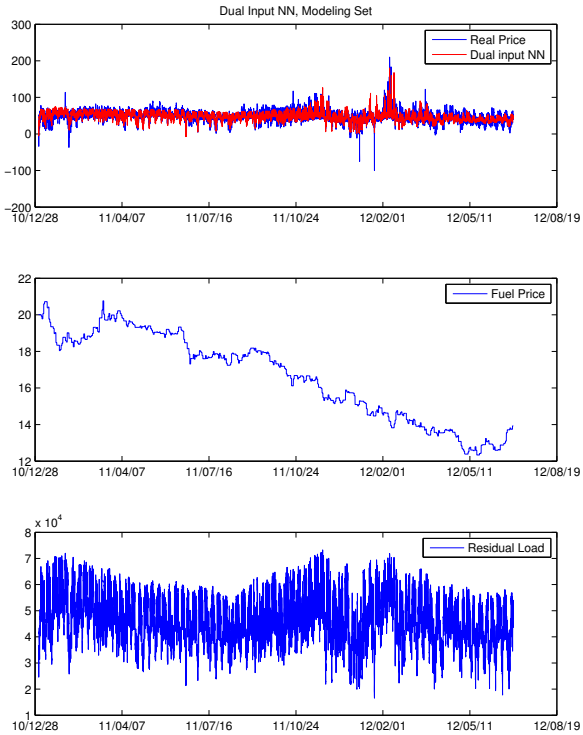


Figure 46: Time series of real and estimated price, modeling period.

Heatrate Function and the Split Peak Heatrate Function). This model has a rather narrow spread in the middle of the cloud and the tails/extremes are captured with a greater spread. The MSE and this price cloud plot indicates that the fuel part input nicely helps to explain the extremes, both positive and negative. Continuing by looking at the models performance on the Validation set however, brings fourth a different picture. For the validation set, the spread of the model cloud is very narrow in the middle implying that the fuel part mostly contributes in affecting the extremes and not so much the normal fluctuations. The most negative observations are not captured by the model, but on the other hand deviations of the magnitude as in the Validation set was not present in the Modeling set so it is reasonable that the model lack the ability to deal with that. Apart from that, the model seem to capture the lower end of the observations quite nicely. The most extreme deviations and perhaps the most interesting part of the models performance on the Validation set is in the part of the positive extremes. The model yields a high number of positive extremes and with a significant magnitude as well, even though the Validation set in itself actually contains less positive extremes than the Modeling set. This can easily be identified as the source of the large Validation MSE for the Dual Input NN model. Clearly the changing underlying dynamic in the market and e.g. the increased Clean Dark Spread has a devastating impact on this model, which turned out to get a good result when relying heavily on the fuel prices in the modeling set, but lack the ability of satisfyingly estimate the spot price for the

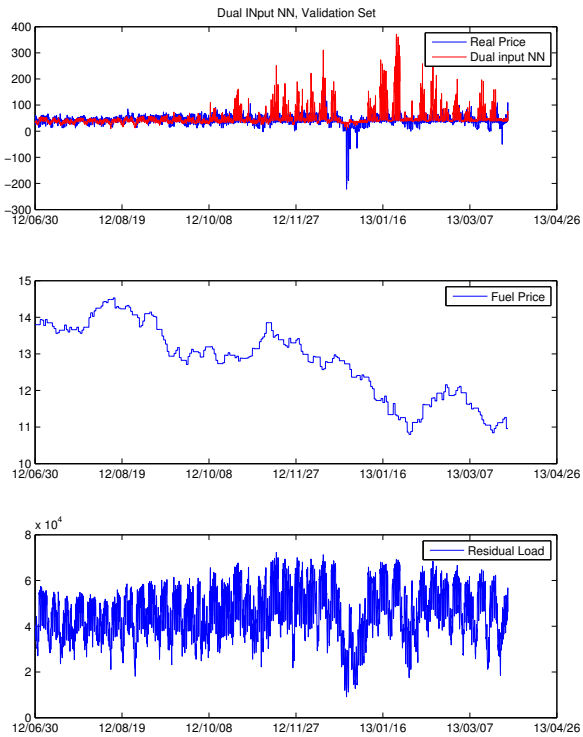
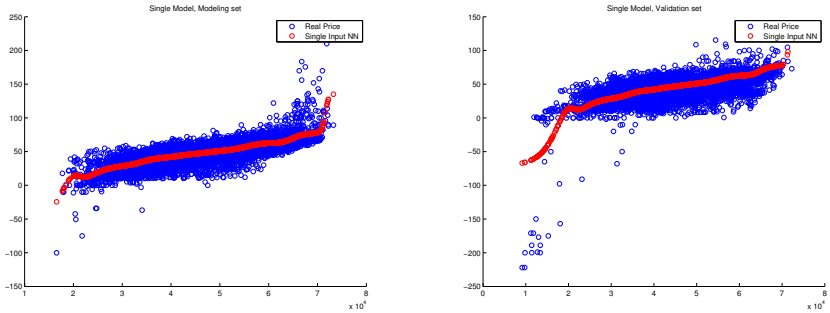


Figure 47: Time series of real and estimated price, validation period.



(a) The Single Input model performance on the Modeling set

(b) The Single Input model performance on the Validation set

Figure 48: Price/Residual Load Plot of NN Single Input Model

Validation set with the changing conditions.

The time series model perfomance plot in figure 46 shows a very nicely reproduced time series of the price for the Dual Input NN model. Overall it has a nice fit and even manages to reproduce the extremes in the Winter Period 2011/2012 with the right price level of the spikes. Still has some problems in fully capture the extent of the negative prices but in general a very nice fit. On the Validation set however the situation is dramatically different. First of all there can be noted that the model does not seem to be able to capture the negative spikes at all. But perhaps most significant of all, is the extremely overstated price spikes in the later period of the time period. It clearly indicates that some relationship present in the modeling period has changed dramatically between the two periods. One explanation to the models bad performance on the Validation set could be that the spikiness in combination with the decreasing coal generation cost during the Modeling period causes the model to generate spikes as the coal generation cost decreases further.

With the observations in mind from the Dual Input NN model, we continue by comparing the properties and plots discussed above with the corresponding ones for the Single Input NN model.

The two dimensional Single Input (or Residual Load Input) NN model estimates the Supply Function to the largest part as a straight line with sharp increase/decrease at each end, as can be observed in figure 48. Naturally the hardest part for the models to capture is the extremes, and the need of additional information to explain them is obvious, due to the irregularities.

Looking at the modeling set in figure 49 a rather good overall performance without any significant errors can be observed. The extremes are not captured to its full extent, but that is consistent with the previously results showing that any of the supply function investigated in this thesis has problem of capturing the extent of extreme values in a satisfying way.

In the Validation period in figure 50, no extremes in the same fashion as for the Dual Input Model can be observed, further strengthening the argument of that the estimated correlation to the coal generation yielding the problems for

that model. Overall the performance is satisfying for the simplicity of this kind of model. which leads us in to a discussion of the explanatory power of the coal generation cost.

From this Neural Network Investigations of the Coal generation dependency, we may draw a number of conclusions. We already know from the section 5.5 that there indeed is a correlation of coal generation cost and power prices, although in a longer perspective. There is also a shorter perspective correlation, as indicated by the good fit of the Dual Input model in the Modeling period, but this correlations is proven to change a lot over time due to, i.e. increased Clean

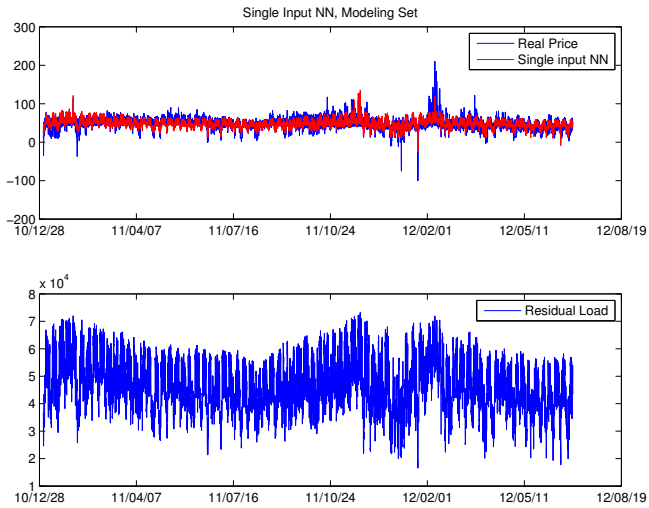


Figure 49: Time series of real and estimated price, modeling period.

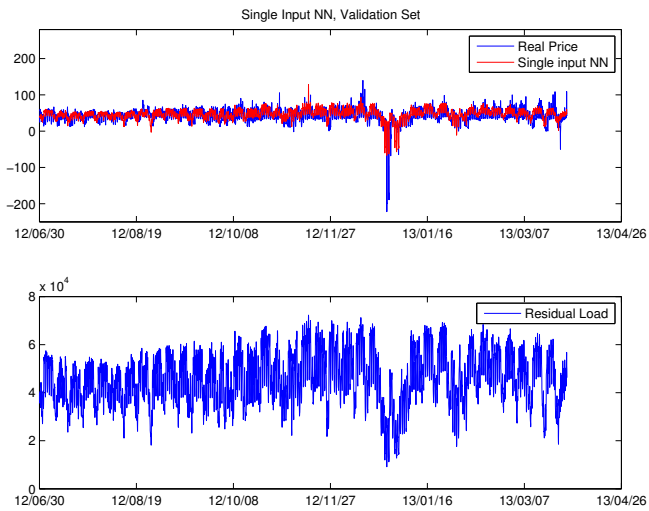


Figure 50: Time series of real and estimated price, validation period.

Dark Spread. Another further complicating factor is that the power prices are more sensitive to increases in coal generation cost than in decreases, which is not taken into account in any of the Supply Functions investigated. A vague relation between coal generation cost and power prices, as previously concluded in this thesis is a reasonable assumption. Hence we state that the correlation between coal generation cost and power prices is a rather complex one and needs to be researched further. Furthermore the very linear relationship between Residual Load and Spot prices of electricity to the largest part is very interesting and it's rather clear the the coal generation cost is not sufficient to explain the nice and even distribution of real spot prices of electricity around this estimated line. Reasonably this deviation should be to some extent explained by the merit order of electricity sources in combination with perhaps other factors as well. E.g. gas prices may play a role as well for the fuel dependent part, but they are relatively infrequently used in power production and are also increasingly rare due to the extended use of renewables that pushes the gas plants out of the markets. All this boils down to the need of a more complex power generation cost model that might increase the explanatory power of the fuels, their correlation and impact on electricity spot prices.

5.7 Estimation of Risk-Premiums

Like Wagner (2012) [1], we want to apply our different Supply Functions to estimate the price of future contracts and investigate how well the complete model performs in comparison to the forward contracts currently traded on the market. This makes it possible to also investigate possible seasonal deviations and behavior. Whilst doing this, it is important to be aware of the simplifications and limitations of the models investigated. E.g. the only risk captured by the model is the one related to wind or solar energy, or load fluctuations. The models lack the ability to account for changes in the supply structure for any other power source than renewable energy, like emissions prices, power plant outages etc. Let alone the fact that the models has flaws in it's performance already identified. The aim is to investigate if the models capture the average prices for each quarter equally in a similar way, or if the models performance has significant performance differences between different quarters. Note that in this section it is not possible to distinguish between model shortcomings and risk premium, since the risk premium is defined as the difference between the estimated average prices and the real traded ones.

5.7.1 Execution

To do this, 5000 trajectories of Residual Load is generated for the year 2013, taking into account the estimated installed capacities from section 4.5. Each of these residual load trajectories are mapped to spot prices, using the Supply Functions from section 3.6. This yields three sets of price trajectories for each residual load trajectory. Before mapping these Residua Load trajectories to spot prices, the shape of the Residual Load is examined to asses the models perfomance in reproducing the residual Load. To do this we calculate the first four moments for the Residual Load data set along with the maximum and

minimum value. The result can be found in figure 51. The shape of the average simulated residual load and the real residual load data used in the model are not expected to be exactly identical for a number of reasons. First of all since the estimated increase in installed capacity of renewables are taken in to account, which will affect the distribution of residual load. Second of all, as concluded in sections 3.2 to 3.4, the residuals from which we estimate the Ornstein-Uhlenbeck processes are deviating in various extent from normal distributions which also affects the simulated average residual load in comparison to the real one.

	Mean	**Var**	**Std**	**Skewness**	**Kurtosis**	**Min**	**Max**
Real wind Data	5430.06	19898711.65	4460.80	1.43	4.91	0.00	24690.45
Simulated Wind Data	6055.01	22143996.23	4650.84	1.25	4.41	285.80	24932.05
Real Solar Data	2555.02	16206364.84	4025.71	1.72	5.28	0.00	21861.88
Simulated Solar Data	4152.45	39895446.04	6313.90	1.58	4.62	0.00	30609.20
Real Load Data	54488.86	105219250.59	10257.64	-0.08	1.88	29201.00	76431.00
Average Load Data	54749.63	103305717.58	10163.53	-0.03	1.84	30666.93	76625.85
Real Res-Load Data	46503.13	109546231.31	10466.43	-0.08	2.48	9166.00	73263.95
Average Res-Load Data	44542.17	108189970.22	10393.58	-0.09	2.73	7357.06	72777.16

Figure 51: Table of the moments for the data series

5.7.2 Risk Premiums

To make proper comparison to real market value of the future contracts, a couple of assumptions has to be made. A date of estimation must be chosen, i.e. a date from where the future prices are gathered to reference the estimated ones, but also the estimated fuel future prices for the investigated period. Since future prices are moving each day, a selection of such day is crucial. For this investigation we put ourselves in the position of a trader on the 29:th of June, 2012 and from that date the fuel and power futures prices are gathered. Furthermore we use the models discussed in this thesis in the section 3.6, to estimate the quarterly base load future contracts for 2013 and compares the performance to the real quarterly base load futures traded on the market the very same day. For each quarter, the corresponding simulated spot price data for all trajectories and each model are gathered. From this data, the average is calculated for each quarter and model, yielding the fundamental quarterly future prices. The future prices estimated is a future with instantaneous fulfillment defined as following:

Definition 11. *The value of a forward contract with instant fulfillment.*

$$F_t^{\mathbb{Q}}(T_1, T_2) = \frac{r}{e^{r(T_2-T_1)} - 1} \int_{T_1}^{T_2} e^{r(T_2-T)} \mathbb{E}^{\mathbb{Q}}[S_T|\mathcal{F}_t]dT \tag{23}$$

where $\mathbb{Q}$ is the risk neutral market measure, as opposed to the real-world, or the market measure $\mathbb{P}$. For the uninitiated reader, the difference can be explained as following; Since these future prices calculated above reflects only the fundamental factors of the market such as installed capacities of renewables, consumption patterns, and to some extent the merit order curve, this calculated price only reflects these factors as well. i.e. risk free, or fundamental factors. We assume an

interest rate of 0%. On the real market however, the future prices also depends on market expectations, planned outages and many more uncertain factors, in combination with the premium of plain uncertainty of the future (traditionally referred to as the risk premium, which might make this a bit confusing), yielding the need of a total risk premium as as an addition to the fundamental price.

	Q1	**Q1-STD**	**Q2**	**Q2-STD**	**Q3**	**Q3-STD**	**Q4**	**Q4-STD**
Real	51.96	0.00	44.66	0.00	45.50	0.00	52.91	0.00
Wagner Spit Peak Model	49.06	12.73	42.15	19.18	44.92	12.46	47.56	12.64
Yang Heatrate Model	42.28	7.97	38.34	8.27	40.35	7.90	43.22	9.13
Split Peak Heatrate Model	42.19	8.27	38.59	8.66	40.58	8.28	43.15	9.46

Figure 52: Table of Future Prices

In figure 52 the estimated quarterly future prices and their deviation can be found along with the corresponding real future prices of the particular period. Unsurprisingly, the prices estimated by the Yang heatrate and the Heatrate Split Peak Function are almost identical. Once again, it can be observed that the Wagner Model captures the level of prices better than the other two models. Looking at the standard deviation however, gives the impression that the Wagner model contains a lot of uncertainty. A possible and likely explanation for this high standard deviation is that spikes in the same fashion as in the Winter period on the validation set, is produced occasionally during the simulated trajectories which yields a high level of standard deviation. Looking back at the Validation set performances for the different supply functions in section 3.6, the higher tendency to spike for the Wagner model in comparison to the rest of the models would serve as a reasonable explanation for this phenomenon. Thence we continue by investigating the level of premiums yielded by the different models. The premiums can be found in figure 53. First of all the premiums of the Yang Heatrate and the Split Peak Heatrate function is substantially lower than the Wagner model, which was already known. Since we already know that these models are suffering in performance due to the too high correlation to fuel prices assumed by the models, we might say that the premium between the real future prices and estimated ones for these models also consist of a *Model Error Premium* as following;

$Premium = Risk\ Premium + Model\ Error\ Premium$

	Q1	**Q2**	**Q3**	**Q4**
Wagner Spit Peak Model	0.06	0.06	0.01	0.10
Yang Heatrate Model	0.19	0.14	0.11	0.18
Split Peak Heatrate Model	0.19	0.14	0.11	0.18

Figure 53: Table of Future Premiums

	Q1	Q1-STD	Q2	Q2-STD	Q3	Q3-STD	Q4	Q4-STD
Wagner Peak Risk Premium	0.20	1.49	0.12	0.64	0.12	0.72	0.21	1.40
Wagner Off Peak Hour Risk Premium	0.09	1.64	0.17	3.66	0.15	3.36	0.13	2.08

Figure 54: Table of Future Premiums estimated in the paper "Residual Demand Modeling and Application to Electricity Pricing", Wagner (2012) [1]

The risk premiums are stated as percentages in decimal form, and the standard deviation of the Wagner futures are the deviation of the price estimation in his paper. Comparing the Futures estimation result with the results from Wagner (2012) [1], a significant difference in premium estimation can be observed. Yet again this is a proof of the un-robustness over time for Wagners Supply function. If the model would be consistent over time, the premium estimates should at least be close to each other. This is however not the case in this comparison.

5.7.3 Market Conditions and Futures Development

Subsequently we want to investigate the history and future of the futures contracts as they carries a lot of information regarding the expectations of the markets.

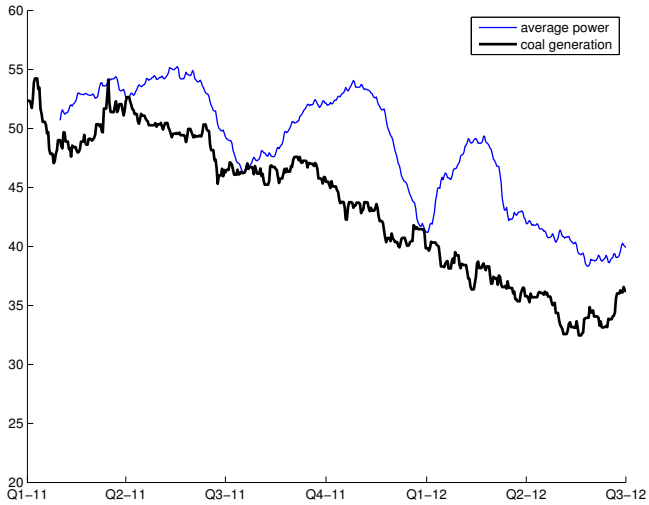


Figure 55: Moving average figure of futures of electricity and coal generation

The figure 55 shows a moving average of the historical plot of future prices for electricity and coal generation cost. In general this plot is very similar to the corresponding plot of the spot prices in figure 42, showing that the spread for the futures of both power and coal generation also has increased over time. It also seem to show signs of the ability of the power price to be able to increase independently of what happens with the coal generation price, but must increase in times of increasing coal generation cost. This could be explained in part by an increased price of base load in the grid following higher coal generation prices. However, for longer periods (e.g. the "humps" in the figure) the power price could depend on other sources of energy higher up in the merit order, being the price setting ones. This would be an asymmetric, disproportional dependence of coal. But it is also worth mentioning that in the end, nobody can explain exactly why the market trades a power forward contract at a certain amount higher than the coal generation cost. There are both technical and psychological aspects to it, but it is in the nature of trading. One physical factor could be

that one producer eagerly wants to sell large amounts of power on a forward contract, so that will have a decreasing effect on the power price.

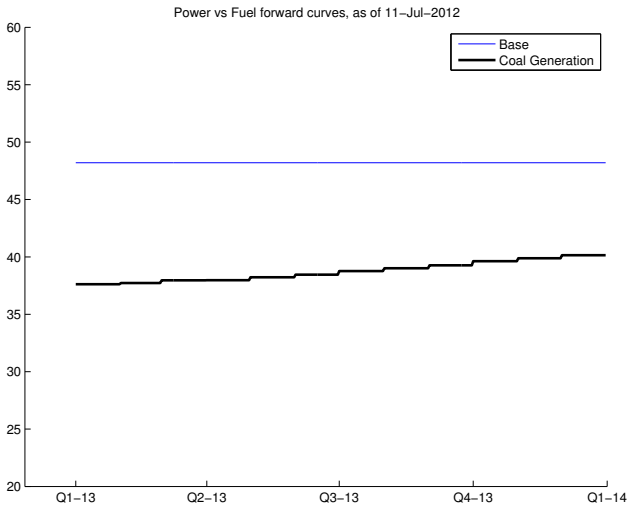


Figure 56: Forward curves of Base load and coal generation forwards for 2013, for the 29:th of June 2012.

The rather simple plot in figure 56 shows the forward curves for 2013 on the day 29:th of June 2012, the reference day of the analysis. The plot shows that the market anticipates a substantially higher power price for the year 2013, but in a steady level. It also shows an expected small increase for the price of coal generation for 2013 with a large Clean Dark Spread between them. This shows that the markets anticipates an increasing Clean Dark Spread for 2013, significantly wider than observed earlier in history. This is further evidence of what has already been observed in the fuel dependent models out turns on the Validation sets, the changing conditions of the market, which makes the Heatrate, fuel dependent Supply Function approach insufficient.

6 Conclusions

In this paper we investigate the performance of coal generation cost dependent structural spot price models on the German power market, and compare the performance of the structural modeling approach combined with three different supply functions to estimate the price of power future contracts. Previous researchers such as Wagner (2012) [1] have concluded that the Residual Load offer more explanation of the intra-day profile of the spot price of electricity than than Total Load of the system This is also something we could see evidence of e.g. in the section 1.1. The ever increasing occurrence of price spikes (both positive and negative ones) shows a threshold like behavior, meaning that the size of the spikes, in the case of a spike appears to be very arbitrary. Both the Wagner split peak model and the various versions of the fuel dependent Heatrate function tries to capture these spikes, but when Wagner overstates the size of the spikes, The Heatrate function understates it. Quite significantly. Even though we have concluded the models to have their individual flaws, they have at least captured the level of prices during the normal periods fairly good. The estimation error for spikes was found to be much larger than the general estimation error in calmer periods of the year. This might call for a more thorough investigation of the behavior of extreme values in electricity pricing. The difficulties in the spot price modeling seem to mainly depend on the Residual Load to spot price connection rather than the supply modeling. Even though there would be problems in the fundamental supply modeling, this would be disguised in the bad systematic performance of the Supply Functions. Furthermore, the analysis' in sections 5 and 5.6 shows indications of the need for a regime switching model for the extreme values. Both in the lower and upper level of residual loads, where the behavior of the Supply functions has clear breakpoints of where the behavior is changed. This would not substitute the need of a better modeling of the fuel dependency, but rather serve as a complement.

The Wagner Split Peak model show on the performance set that the use of a divergence term manages to capture the occurrence of spikes, but fails in general to capture the levels, since the market mechanisms that produce spikes evidently occur due to more sophisticated processes. The Dual Input NN model showed a very nice performance in sample, this indicates that fuel prices might offer a significant part of the explanation in the short-term perspective. Although the problem of asymmetric dependence between power generation prices and power prices must first be properly modeled, along with the difference in both short-term and long-term trend dependency. Afterwards we would be able to utilize the explanatory power that fuel prices might have on electricity spot prices. Especially if the goal is a model that is robust over time, which we have identified as the main problem in structural modeling. Changing market conditions like .e.g. the increasing Clean Dark Spread must also be properly modeled and dealt with to avoid model problems as the one showed on the Validation set, both regarding spike overestimation and fuel dependency, both short and long term. What is clear from this investigation is that to be able to produce better models on the Supply Function form, these complex correlations must be modeled in a more general way to offer proper robust explanatory power over time. It is however unclear if this is possible, since the change of the market might be of a more stochastic nature and lack the ability to be fundamentally

modeled. This is yet to be discovered. All the investigations ranging from the Wagner Split Peak Model, the various fuel dependent Heatrate models to the investigation of the Fuel/Power price correlation, shows the danger of using empirical models based solely on historic data. It also shows the pace of changes in the market. This gives rise to the need of more complex structural models, modeling the underlying driving factors, and their correlations to create robust structural models that yields satisfactory results over time. Although the risk of sudden, unanticipated changes, such as the nuclear moratorium in Germany after the Fukushima accident, will always be present in the energy market.

To summarize the results of the investigation of the fuel dependent Heatrate functions, one may conclude that the shape of the Heatrate Function is more general than the Wagner Split Peak Function. This is because it does not seem to carry the same tendency to overestimate spikes, or suffer from performance inconsistency between the Modeling and Validation sets. The Fuel dependent part of course had a bad impact on the models performance over time, but that is considered to be a flaw in the connection to the fuel prices as previously been extensively discussed, rather that a flaw of the general modeling strategy itself. It also showed that using this kind of robust shape of the supply function, hardly any additional information is captured or increase in performance is noted due to the Heatrate Split Peak strategy. Thus we conclude that rather than trying different models for Peak and Off Peak hours, the Merit Order fuel dependency is a better explanation to the price variation during the day, and since the difference in power demand during a day yields different price setting power sources due to the Merit Order, this would be a better explanation of the price setting mechanisms during a day.

As discussed thoroughly in the section 5.5 and 5.5, the increased Clean Dark Spread has a bad impact on the fuel dependent models due to the changing behavior over time. Although it mostly affects the Peak hour prices. This is due to the fact that during Off Peak hours, the price setting fuel is mostly lignite which makes them somewhat insensitive to changes in the coal price. An evidence of this is the small spread of the fuel dependent Heatrate functions for the lower Residual Loads. However, this further indicates the importance to capture the relations between the price of different fuels, Residual Load and spot power prices. This because its proven that the relations are not sufficiently captured using the Peak and Off Peak hour split or the Coal generation cost. The varying Clean Dark Spread and the asymmetric dependence between Coal Generation cost and power spot prices, seem to be just the first challenge to overcome when using a fuel dependent Residual Load/Supply Function approach to model power spot prices. This since coal still is the most general price-setting fuel (despite the problems addressed here).

To expand this approach the even more complex relation, mainly to prices higher up in the Merit Order needs to be modeled, since we have clearly observed flaws in capturing (except for the negative spikes) the higher end of power prices. E.g. Gas prices could be reasonable to include, and also to model WHEN they are used as the price setting fuel and only use them for the most extreme values of Residual Load. The need of modeling prices lower in the merit order does not seem to add very much explanatory power since the main flaws of the models covered in this thesis is experienced in the higher end of the price

range. Also, there are no market for lignite, which makes this hard, which would be the fuel below hard coal in the merit order. Overall we conclude that the non-stationarity of the power spot market is a big problem in the spot price modeling of electricity and the complex relations between fuels, Residual Load and spot prices still needs extensive research. What we have identified here as the problems can serve as the first steps to expand this strategy of modeling further. The constant market evolvement and non stationarity does not show any signs of declining any time soon.

In the Introduction section (1) we state that a structural model require extensive work to (re-)construct the price from the modeled structural quantities. The different Supply Functions is our approach of doing this. However, it turns out that overall this is a very difficult process that require more extensive work to function properly.

Even though we have evaluated some strategies to model fundamental factors of the electricity spot price market, and found some more leads into what might provide more information and explanatory power, there are still obvious signs that it might not only be the explanatory power of the fuel prices that is missing from the models. Nothing indicates that other factors e.g. changes in the demand structure, does not affect the spot prices. It is important to bear in mind that there are still a lot of research to be done to fully understand what is happening on these markets. What we have noted and whats kind of a side note to the rest of the analysis, is the extreme sensitivity in the power spot price to unexpected changes during the winter periods. Between the winter periods of 2011/2012 and 2012/2013 there in an unprecedented inconsistency in model behavior. The winter 2011/2012 had significant positive spikes and the winter after, 2012/1013 had significant negative spikes. These sudden and extreme changes are evidence of the winter period being a very sensitive period for disturbances and discrepancies in the power supply and demand. Partly this is because the increased capacity of wind power, where the effect of a very windy winter, as observed in the winter 2012/2013 , is only expected to increase as the capacity increases.

6.1 Suggestions of Extended Research

6.1.1 Extensive fuel price correlation modeling

A lot of focus in this thesis has been on the correlations between price-setting fuel prices, their inter correlation and their correlation to the spot price of electricity. Even though this thesis concludes that this probably explains a big part of the power spot price movements that we have not been able to capture, we have only concluded that our way of modeling it is not sufficient. Many arguments for a more complex modeling of fuel price relations to power spot price has been presented in this thesis and this is the authors primary suggestion of continuing research.

- Short term fuel dependence part
- Long term fuel dependence part

6.1.2 Higher order ARMA-process modeling of renewables and total load

As Wagner (2012) [1] declared in his paper, there is evidence of the need of a more complex higher order ARMA process for both the renewables parts and the total load part. Although the problems in this thesis has mainly focused on the Supply Functions, since this has been identified as the largest source of error, there is still clear evidence from the section 3 that this would be able to improve the model.

6.1.3 Generalised Hyperbolical Distributions

Since there were clear indication that the residuals in the load modeling section was not normally distributed, there might be a good approach to assume a Generalised Hyperbolical Distribution (GHD) for the residuals instead. The benefit of this approach would be that the GHD is more flexible and controlled by more parameters than the standard normal distribution. This would also require to use a Generalised Hyperbolical Distributed stochastic process to drive the Ornstein-Uhlenbeck process, instead of the normally distributed Browninan Motion used in this thesis.

6.1.4 Additional measures of quality

One can argue that the MSE is not sufficient to use as a quality measure alone, but should be combined with other measures to asses the quality of the models. Other potential measures of quality could be Least Squares, R^2 Peak & Off Peak spread size, price moments etc.

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1 Appendix

1.1 Week Samples

To give the reader a better perception of how the models are performing throughout the investigated period, this subsection is intended to provide a couple of snapshots of what is happening during 4 weeks for different parts of the investigated period. Overall, this section does not provide extensive information for the conclusions or final analysis of the thesis, hence this section wont be extensively commented. Since the events displayed are present due to illustrative reasons and does not have any significant impact on the analysis itself, the first sample week wille commented briefly, while the rest of the weeks will be left to the reader to draw conclusions and form his own opinion regarding how the different input parameters affects what is happening on the electricity market.

1.1.1 February 2011

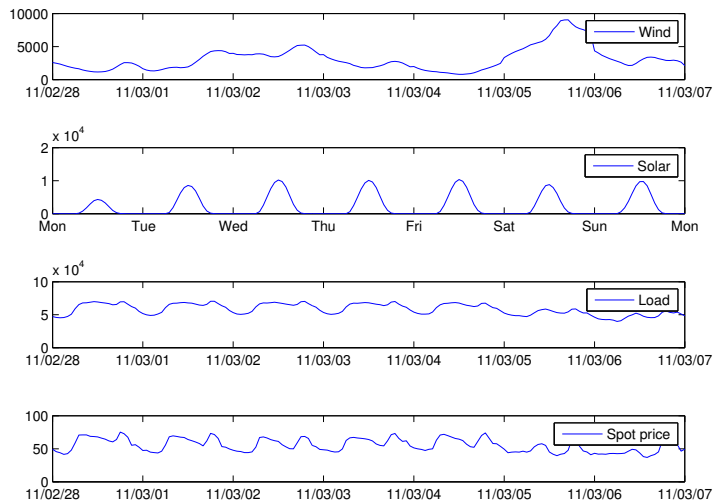


Figure 57: Data Input Sample Week, February 2011

In figure 57, the different inputs are stacked in graphs on top of each other, for the reader to get a perception of how the different factors move during this sample week, how they relate to each other and what the result is, in terms of spot price.

In 58 the real spot price is plotted along side with the spot prices generated by the different models. In the selection proces of sample weeks, weeks were deliberately chosen as weeks where the Wagner Split Peak Model performed somewhat worse or equally good than the other two models. The motivation for this is that it would be easier to compare the actual detailed shape of the Supply Functions to each other and the real spot price. Also the systematic deviation due to decreasing fuel prices is already something that is known in this stage

and does not need further highlighting. The week stretches from Monday to Sunday.

In figure 59, the plots of the normalized real price correlates with the normalized residual load. This plot generally strengthens the initial argument of that the shape of spot prices is formed by the Residual Load rather than the total load. In the title of each subplot, the correlation between the un-normalized Spot Price and Residual/Total Load can be found. From here on and throughout this section 1.1 follows three more sets of periods with their respective data in the same way as shown this far.

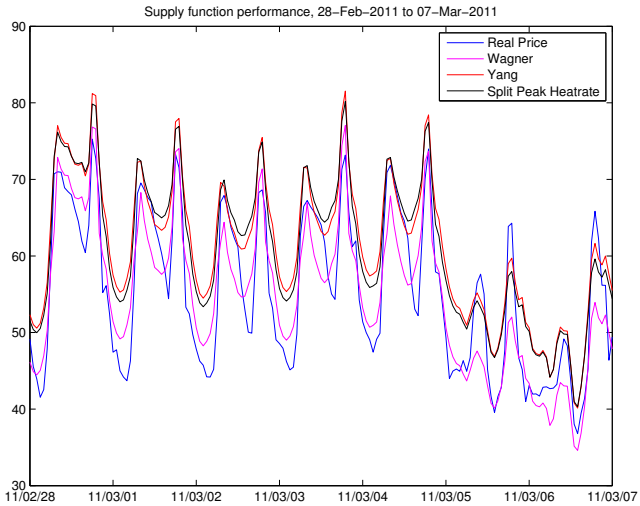
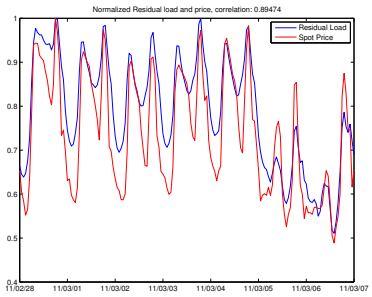
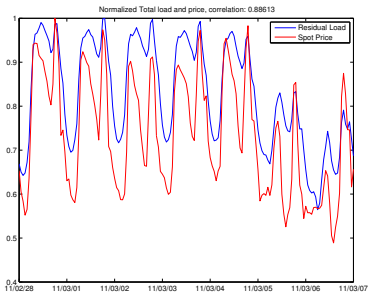


Figure 58: Supply Functions Performance Sample Week, February 2011



(a) Normalized Residual Load and Price



(b) Normalized Total Load and Price

Figure 59: Correlations Between Spot Price And Load, February 2011

1.1.2 September 2011

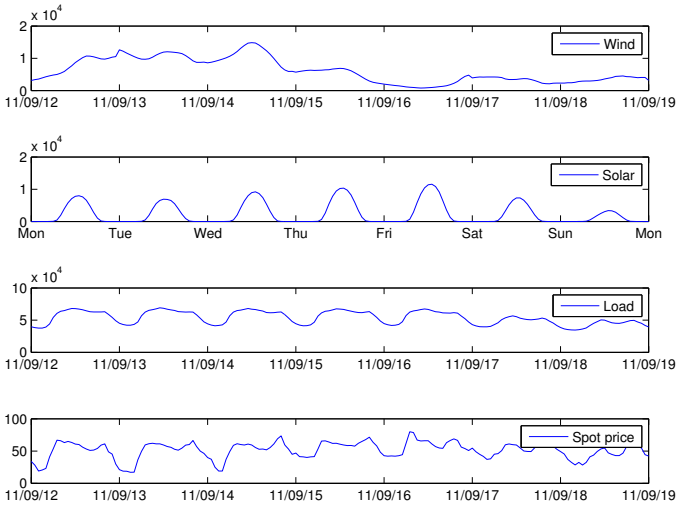


Figure 60: Data Input Sample Week, September 2011

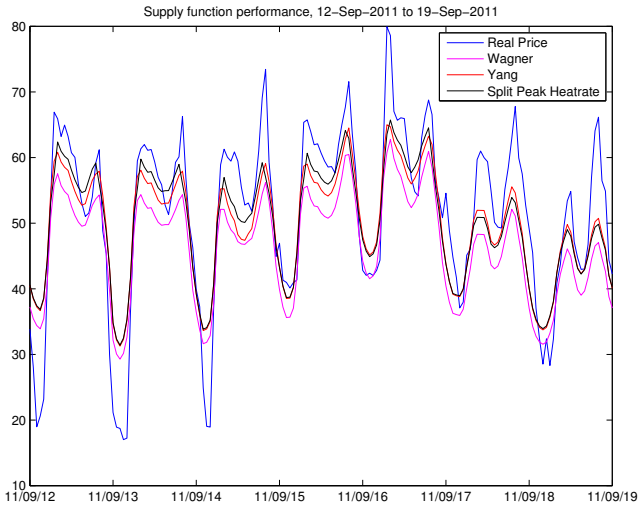
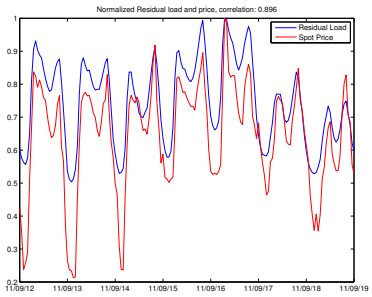
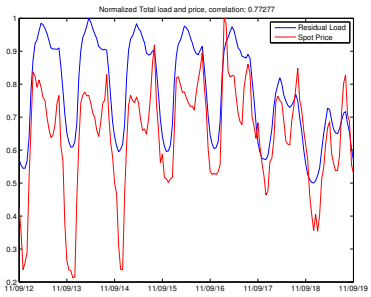


Figure 61: Supply Functions Performance, September 2011



(a) Normalized Residual Load and Price



(b) Normalized Total Load and Price

Figure 62: Correlations Between Spot Price And Load, September 2011

1.1.3 January 2012

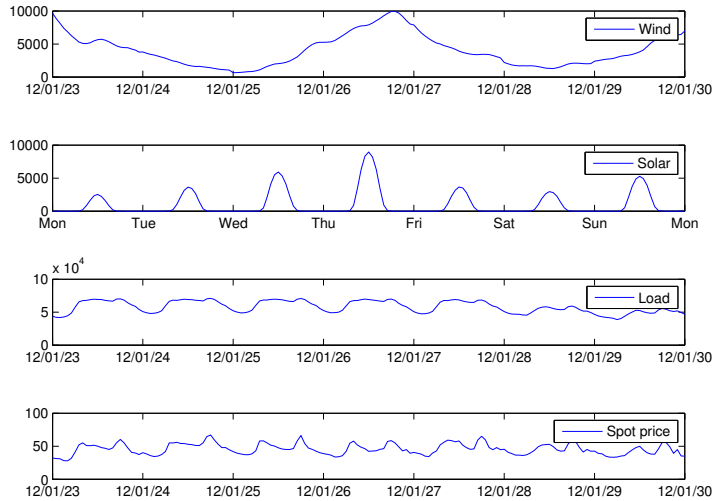


Figure 63: Data Input Sample Week, January 2012

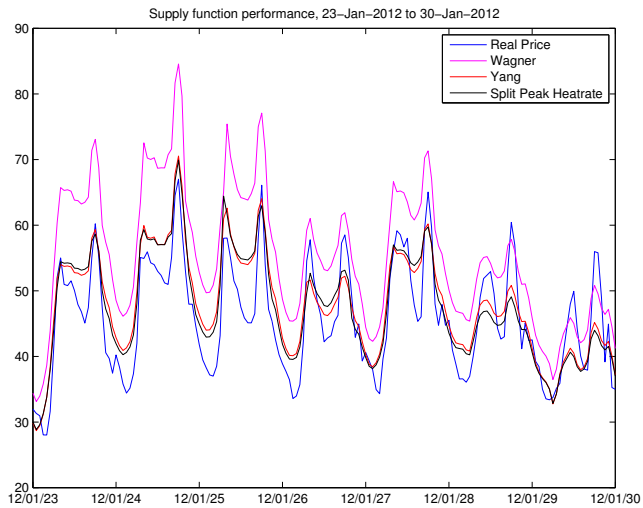
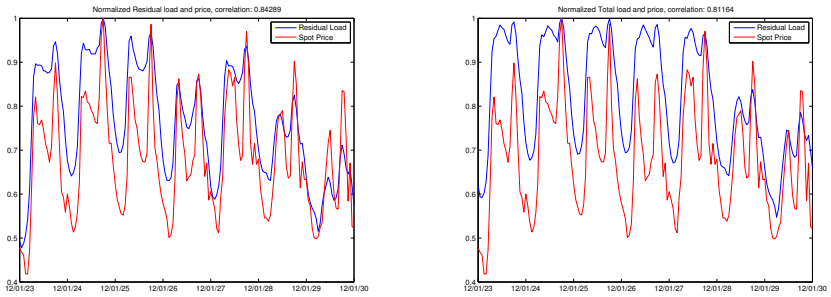


Figure 64: Supply Functions Performance, January 2012



(a) Normalized residual load and price (b) Normalized total load and price

Figure 65: Correlations Between Spot Price And Load, January 2012

1.1.4 October 2012

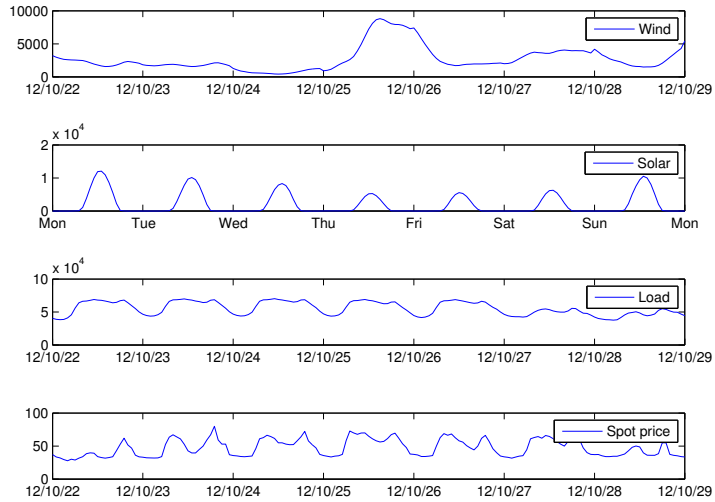


Figure 66: Data Input Sample Week, October 2012

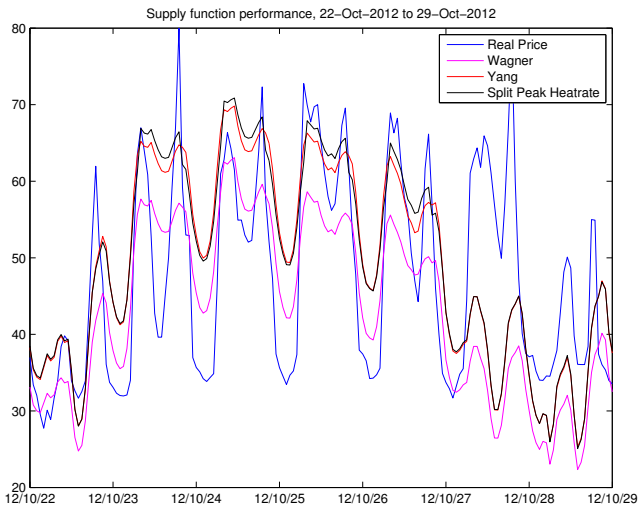
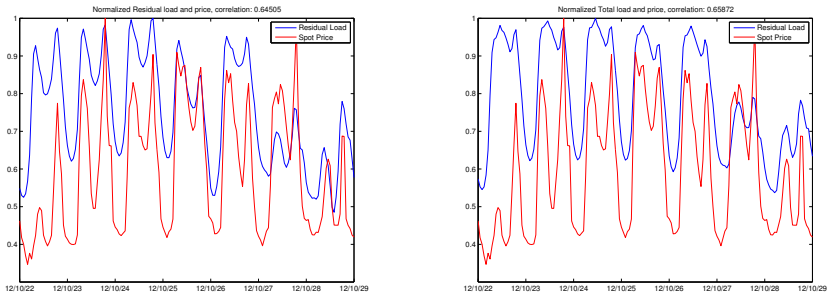


Figure 67: Supply Functions Performance, October 2012



(a) Normalized residual load and price (b) Normalized total load and price

Figure 68: Correlations Between Spot Price And Load, October 2012