

# Spectrum Analysis of Heart Rate Variability (HRV)

Tiying Cui

Spetember , 2013

## Abstract

Heart Rate Variability (HRV) is the physiological phenomenon of variation in the time interval between heartbeats. High frequency (HF) HRV signals (0.12-0.4 Hz), especially, has been linked to parasympathetic nervous system (PSNS) activity. Activity in this range is associated with the respiratory sinus arrhythmia (RSA). In this thesis, we are interested in the differences between the power of HF-HRV in lying down position and standing up position, using analysis the HRV signal in frequency domain. Four non-parametric window spectrum methods are introduced for estimating the Power Spectra Density (PSD). Then the power of HRV is calculated using a traditional bandwidth [0.12-0.4] as well as a designed new bandwidth  $[f_0 \pm B]$ , where  $f_0$  is the corresponding peak frequency from RSA spectrum. From combining different methods with different bandwidths; the resulting estimated power are compared in two different positions in many ways. In the end, the hypothesis tests are constructed to check if there exists a significant difference in the mean and variance for different methods with for different bandwidths. Moreover, the robust methods are applied to analysis the HRV signal during some Yoga breathing exercises. All data in used are collected from a pre-designed experiment in February that held in IKDC in Lund.

# Contents

<b>CHAPTER 1 INTRODUCTION</b>	<b>1</b>
<b>1.1 BACKGROUND.....</b>	<b>1</b>
<b>1.2 Purpose.....</b>	<b>2</b>
<b>CHAPTER 2 SPECTRUM ANALYSIS</b>	<b>3</b>
<b>2.1 Power spectrum .....</b>	<b>3</b>
2.1.1 Fourier transform .....	3
2.1.2 PSD .....	4
<b>2.2 Non-paramatric spectrum methods.....</b>	<b>5</b>
2.2.1 Periodogram.....	5
2.2.2 Hanning .....	6
<b>2.3 Multiple window methods .....</b>	<b>6</b>
2.3.1 Thomson .....	7
2.3.2 Peak Matched Multiple Windows .....	7
<b>2.4 Bias and variance .....</b>	<b>8</b>
<b>CHAPTER 3 EVALUATION</b>	<b>9</b>
<b>3.1 Data.....</b>	<b>9</b>
3.1.1 Data Overview .....	9
3.1.2 Stationary process.....	11
3.1.3 Overlap and Non-overlap .....	11
<b>3.2 Results from spectrum analysis .....</b>	<b>11</b>
3.2.1 HRV and respiration spectrum .....	12
3.2.2 Estimation from four methods.....	12
<b>3.3 Power of HRV .....</b>	<b>14</b>
<b>3.4 Overlapped data with the length of 64 .....</b>	<b>17</b>
3.4.1 Compare bandwidths .....	17

3.4.2 Compare methods .....	18
3.4.3 Compare between up and down positions .....	18
<b>CHAPTER 4 HYPOTHESIS TEST</b>	<b>26</b>
4.1 Paired-sample t test.....	27
4.2 Two sample F test.....	29
<b>CHAPTER 6 CONCLUSION</b>	<b>31</b>

# Chapter 1

## Introduction

### 1.1 Background

Heart rate variability (HRV) is the physiological phenomenon of variation in the time interval between heartbeats. It is measured by the variation in the “RR interval” (where R is a point corresponding to the peak of the QRS complex of the ECG wave, and RR is the interval between successive Rs).

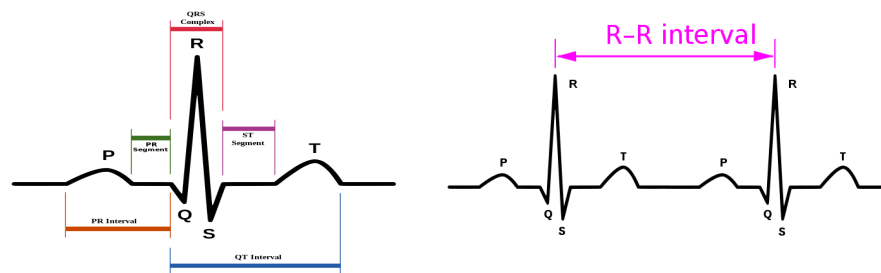


Figure 1.1 Representation of normal sinus rhythm and RR interval

HRV reflects cardiac automaticity under the control of the autonomic nervous system. The information in HRV is relevant to many cardiovascular and non-cardiovascular diseases such as myocardial infarction, diabetic neuropathy, high blood pressure, sudden cardiac death. HRV also related to emotional arousal in mental and social aspects.

There are many methods that are used to detect heartbeats, for example: ECG and blood pressure. ECG is considered the best method to detect HRV signals since it provides a clear waveform, which makes it easier to exclude heartbeats not originating in the sinoatrial (SA) node.

There are many methods that are used for the analysis of HRV signals. The most widely used methods can be grouped as time-domain and frequency-domain methods. The idea for the frequency domain methods is to transform the HRV signals, the time-series sequences to power spectral density (PSD) by using discrete Fourier transform (DFT). Therefore, the transformed data can be presented in a frequency scale, which has the dimensions of power per Hz. The most common classifications for HRV spectrum are: very low frequency (VLF:  $\leq 0.04\text{Hz}$ ), low frequency (LF:  $0.04 - 0.12\text{Hz}$ ) and high frequency (HF:  $0.12 - 0.4\text{Hz}$ ).

This master thesis mainly discusses the non-parametric spectral analysis methods in frequency-domain for HF-HRV.

High frequency HF-HRV signals (0.12 to 0.4 Hz), especially, has been linked to parasympathetic nervous system (PSNS) activity. Activity in this range is associated with the respiratory sinus arrhythmia (RSA).

Respiratory sinus arrhythmia (RSA) is a naturally occurring variation in heart rate that occurs during a breathing cycle: when we inhale the heart beats faster and when we exhale the heart beats slower.

In this master thesis, my supervisors and I used an experiment to collect both respiratory data and HRV data (from ECG) at the same time. The pairwise data are measured for 8 people (including myself) in two conditions: supine position (lying down) and upright position. Moreover, we also measured data during some yoga breathing exercises.

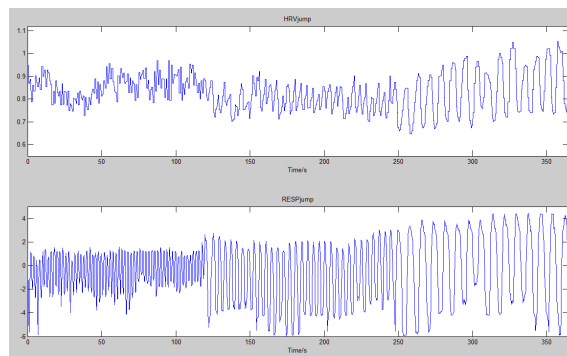


Figure 1.2 Example of HRV data and respiratory data

Figure 1.2 shows an example of what HRV data and respiratory data look like. The upper plot presents HRV data and the lower plot gives the corresponding respiratory data.

## 1.2 Purpose

Generally in parasympathetic cardiac regulation, HF-HRV is higher in supine position than in upright position. The purpose of this thesis is to study if the modern robust spectral analysis methods (such as multi-tapers) better differentiate between the two conditions compared to traditional spectral analysis. An appropriate method will give large differences between two positions.

## Chapter 2

### Spectrum Analysis

In statistical signal processing, the goal of spectral density estimation is to estimate the spectral density of a random signal from a sequence of time samples of the signal. The purpose of estimating the spectral density is to detect any periodicities in the data, by observing peaks at the frequencies corresponding to these periodicities.

Spectral estimation is a problem that is of great importance in many applications including data analysis, signal detection, classification and tracking etc.

The main approaches for spectrum analysis can be separated into two groups: parametric (such as AR, ARMA) and non-parametric methods (window methods). In this thesis, we focus on non-parametric methods.

#### 2.1 Power spectrum

##### 2.1.1 Fourier transform

The Fourier transform transfers a mathematical function of time into a new function, whose argument is frequency. The definition for discrete-time transform is showed below:

Let  $y(t)$  be a deterministic discrete-time signal. Assume  $\sum_{t=-\infty}^{\infty} |y(t)|^2 < \infty$ , then the discrete-time Fourier transform of the data sequence is:

$$Y(\omega) = \sum_{t=-\infty}^{\infty} y(t)e^{-i\omega t}, \omega \in [-\pi, \pi]$$

and the inverse discrete-time inverse Fourier transform is thus defined as:

$$y(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(\omega)e^{i\omega t} d\omega.$$

The frequency sampled Fourier transform, called the Discrete Fourier Transform (DFT) is obtained as

$$Y_k = \sum_{t=0}^{N-1} y(t)e^{-i\frac{2\pi}{N}kt}, k = 0, \dots, N-1$$

where  $y(0)...y(N-1)$  are the time series. This operation is useful in many fields but computing it directly from the definition is often too slow to be practical. The Fast Fourier Transform (FFT) is a way to compute the DFT more quickly since it reduces the computation cost. The formula below gives the definition of FFT:

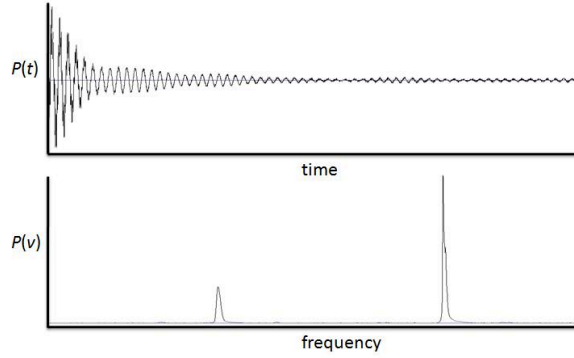


Figure 2.1 Signals review in time-domain and frequency-domain

Figure 2.1 presents the plots of a time scale signal (up) and the corresponding transferred frequency scale sequences (below) through FFT.

### 2.1.2 Power Spectral Density (PSD)

Let  $S(\omega) = |Y(\omega)|^2$  be the energy spectral density, then we got:

$$\sum_{t=-\infty}^{\infty} |y(t)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} S(\omega) d\omega \quad (\text{Parseval's theorem})$$

where,  $S(\omega)$  is the distribution of energy as a function of frequency.

According to the Wiener-Khinchin theorem, the power spectrum of a zero-mean stationary stochastic process  $y(t)$  can be calculated as the Fourier transform of its covariance function  $r(k)$ . Hence, some definitions are illustrated below.

The auto-covariance sequence (ACS) of  $y(t)$  is defined as:

$$r(k) = E\{y(t)y^*(t-k)\}$$

denoting  $E\{\}$  as the expected value and  $*$  as the complex conjugate.

First definition of Power Spectral Density (PSD):



$$\phi(\omega) = \sum_{k=-\infty}^{\infty} r(k) e^{-i\omega k}$$

where  $\phi(\omega)$  represents the distribution of signal power over frequency. From a given  $\phi(\omega)$ , the ACS  $r(k)$  can also be written as the inverse Fourier transform of the spectrum:

$$r(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi(\omega) e^{i\omega k} d\omega$$

Notice that  $r(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi(\omega) d\omega = E\{|y(t)|^2\}$  measures the (average) power of  $y(t)$ , so  $\phi(\omega)$  is non-negative ( $\phi(\omega) \geq 0$ ) and real-valued. Therefore,  $\phi(\omega) = \phi(-\omega)$ .

Moreover,  $\phi(\omega)$  is periodic with the period  $2\pi$ . Since  $\omega \in [-\pi, \pi]$ , and  $f = \frac{\omega}{2\pi}$ , we have  $f \in [-\frac{1}{2}, \frac{1}{2}]$ .

Second definition of Power Spectral Density:

$$\phi(\omega) = \lim_{N \rightarrow \infty} E \left\{ \frac{1}{N} \left| \sum_{t=0}^{N-1} y(t) e^{-i\omega t} \right|^2 \right\}$$

The PSD describes how the power of a signal or time series is distributed with frequency. The power of the signal in a given frequency band can be calculated by integrating over the frequency values of the band.

## 2.2 Non-parametric spectrum methods

Non-parametric spectrum methods rely on the directly use of the available data. It begins by estimating the autocorrelation sequence from a given data. The power spectrum then is estimated via Fourier transform of an estimated autocorrelation sequence. A non-parametric spectrum method makes no assumption on the model and are therefore more robust, but often less precise than the parametric methods.

### 2.2.1 Periodogram

The periodogram spectral estimator is defined as:

$$\hat{\phi}_p(\omega) = \frac{1}{N} \left| \sum_{t=0}^{N-1} y(t) e^{-i\omega t} \right|^2$$

The periodogram is often computed from a finite-length digital sequence using the fast Fourier transform (FFT). This method is not a good spectral estimate because of spectral bias and the fact that the variance at a given frequency does not decrease as the number of samples used in the computation increases.

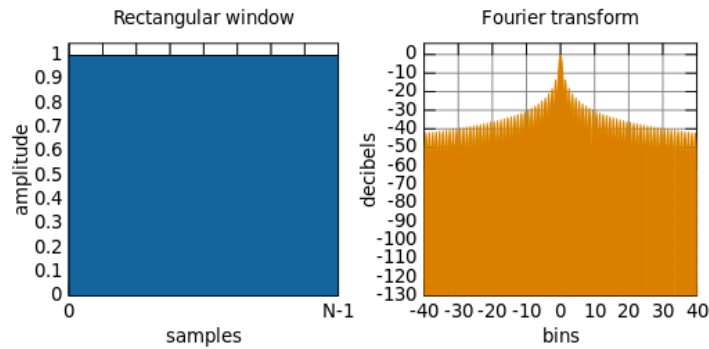


Figure 2.2 Form of window: Rectangle window

Figure 2.2 presents the form of the window for the periodogram. The plot to the right shows a sharp mainlobe and pretty high sidelobes.

## 2.2.2 Hanning

The Hanning window is defined as:

$$w(t) = 0.5(1 - \cos(\frac{2\pi t}{N-1}))$$

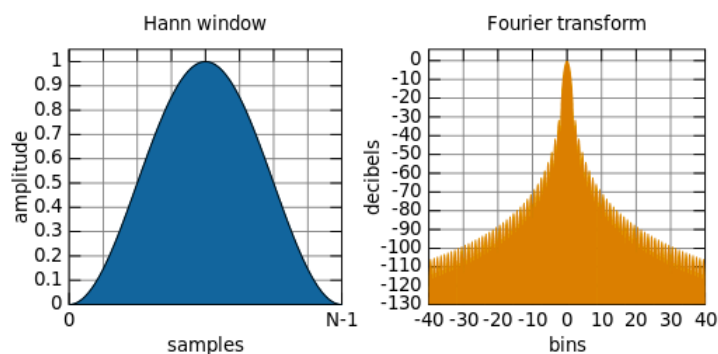


Figure 2.3 Form of window: Hanning window

The form of the Hanning window is shown in figure 2.3. From the picture, one can observe that the Hanning window have sharp main-lobe but lower side-lobes. This helps to reduce leakage compared to the periodogram.

## 2.3 Multiple window methods

### 2.3.1 Thomson

The estimated spectrum for the discrete-time random process  $y(t)$  is :

$$\hat{\phi}(\omega) = \sum_{k=1}^K \alpha_k \left| \sum_{t=0}^{N-1} y(t) h_k(t) e^{-i\omega t} \right|^2$$

The solution with respect to the window function  $h_k$  is the set of eigenvectors of the eigenvalue problem:

$$R_B q_k = \lambda_k q_k, \quad k = 1 \dots N.$$

where  $R_B$  is the  $(N \times N)$  Toeplitz covariance matrix and  $B$  is a predefined resolution bandwidth.

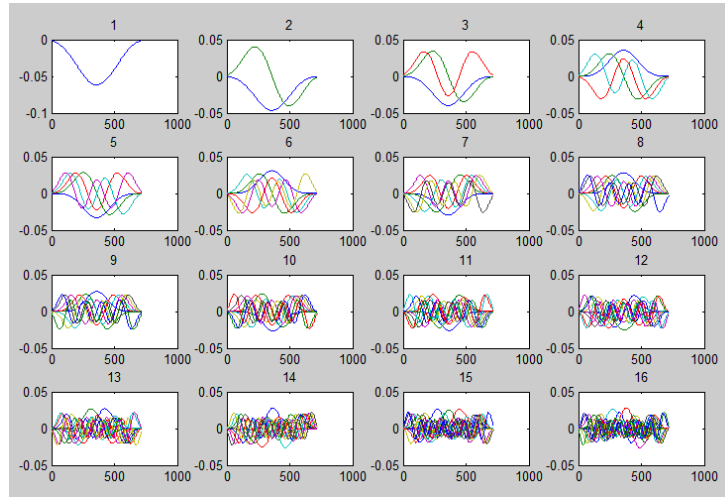


Figure 2.4 Form of windows: Thomson windows 1-16

Figure 2.4 shows the form of Thomson windows with number of window from one to sixteen. An estimated spectrum from Thomson method looks smooth. It has a box shaped main-lobe so it becomes more difficult if one want to look at a peaked spectrum.

### 2.3.2 Peak Matched Multiple Window (PM MW)

The window function  $h_k$  is the set of eigenvectors of the generalized eigenvalue problem:

$$R_B q_k = \lambda_k R_G q_k, \quad k = 1 \dots N.$$

where  $R_G$  is the  $(N \times N)$  Toeplitz covariance matrix that corresponds to a penalty frequency function.

The penalty frequency function is given by:

$$\phi_G(f) = \begin{cases} G & |f| > B/2 \\ 1 & |f| \leq B/2 \end{cases}$$

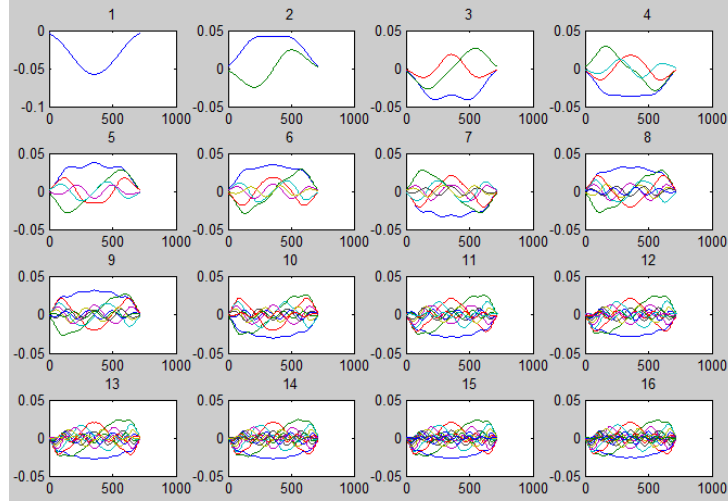


Figure 2.5 Form of windows: PM MW windows 1-16

Figure 2.5 plots the form of the windows with the number of windows from one to sixteen respectively. Peak Matched Multiple Method (PM MW) is good at catching a peaked spectrum. With the use of the penalty function, the leakage is suppressed outside the resolution width.

## 2.4 Bias and Variances

The spectral bias problem arises from a sharp truncation of the sequence. It can be reduced by multiplying the finite sequence by a window function which truncates the sequence gradually rather than abruptly.

The variance problem can be reduced by smoothing the periodogram. There are lots of techniques to reduce spectral bias and variance. One such technique is to solve the variance problem by averaging periodogram. The idea behind it is to divide the set of  $N$  samples into  $K$  sets of  $M$  samples, compute the DFT of each set, then square it to get the power spectral density and compute the average of all of them. Thus, it leads to a decrease in the standard deviation as  $\frac{1}{\sqrt{K}}$ . This reduction is also achieved from the multiple window methods using  $K$  windows.

## Chapter 3

### Evaluation

#### 3.1 Data

In February, my supervisors and I constructed an experiment to collect data that we are going to use in this thesis. There are 8 people (include myself) joined within this experiment but eventually we got the data from 6 persons since we rejected two of bad quality due to some unexpected disturbance.

The data are separated into four parts with respect to four different experiments:

- (1) Lying down (breathing frequency: 0.2Hz)
- (2) Standing up (breathing frequency: 0.2Hz)
- (3) Diaphragmatic breathing (breathing frequency: free)
- (4) Alternate nostril breathing (breathing frequency: free)

Each experiment lasted 5 minutes that makes each sequence contains approximate 1200 data samples since the sample frequency is 4Hz. In the first two experiments, we asked the participators to follow an organized tone to breath in and out. The rate for this tone is 0.2 Hz. The last two experiments conducted two classical Yoga breathing methods: diaphragmatic breathing and nostril breathing. Both of the yoga breathings required complete breathing, which is to breath as much as you can but not rapid. Instead, the aim is to breath smoothly and slowly in and out. In this thesis, I will focus on the analysis of the results from the first two experiments.

##### 3.1.1 Data overview

The original data contains the information from 6 persons, which also include the data from myself. For each person, we got the HRV sequences and the respiration sequences. Experiment (1) gives the two sequences in lying down position, and experiment (2) gives the data in standing up position.

Let  $y_{h_i} \{i = 1..n\}$  present the HRV data and  $y_{r_i} \{i = 1..n\}$  present the respiration data, where  $n = 1200$ . More particularly, let  $y_{h_{down}}$  present the HRV data in lying down position and  $y_{h_{up}}$  the HRV data in upright position. Similarly for the respiratory sequences  $y_{r_{down}}$  and  $y_{r_{up}}$ .

Figure 3.1-a plots 12 sequences (6 HRV sequences on the left and the 6 respiration sequences on the right) for 6 people in lying down position in blue color. Figure 3.1-b presents the data in standing up position in red color.

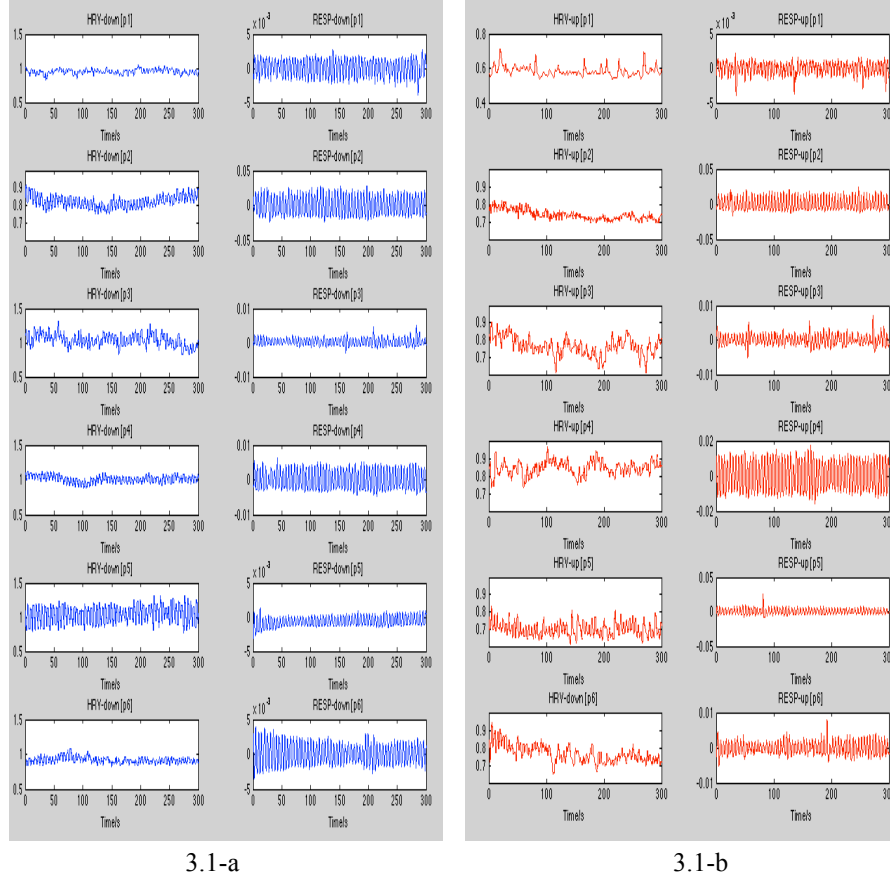


Figure 3.1 The original HRV and respiratory data in down (blue) & up (red) positions from 6 persons

The data looks pretty different for different individuals. We can see that there exists a big difference in amplitude for the respiration data. It is probably because the tightness differed when putting on the respiration detector devices to the chest. Most of the HRV data seems stable; there is no obvious linear trend in them.

mean	HRV-DOWN	HRV-UP
person1	0.9522	0.5828
person2	0.8227	0.74
person3	1.04	0.7617
person4	1.0127	0.8445
person5	1.0452	0.6979
person6	0.9163	0.7662

Table 3.1 Mean of the original HRV in Down & Up positions from 6 persons

For the mean of HRV sequences, it is already well known that the heart beat is slower in lying down position than in standing up position. That leads to a larger distance between RR-intervals, which give bigger HRV sequences. Compared to standing up

position, we can see in Table 3.1, that the mean of HRV in lying down position are bigger.

### 3.1.2 Stationary process

As we know, before doing further analysis, one should make sure that the data is a stationary process since one of the assumptions in non-parametric spectrum estimation is that the data should be a zero-mean stationary stochastic process. Therefore, the mean and the variance should not change over time and shouldn't follow any trends.

In order to make sure the stability, first take away the differences of mean for all data:

$$y_{h_i} = y_{h_i} - \frac{1}{n} \sum_{i=1}^n y_{h_i}, \quad n = 1200$$

$$y_{r_i} = y_{r_i} - \frac{1}{n} \sum_{i=1}^n y_{r_i}, \quad n = 1200$$

Further more, if the data shows any linear trends, these should be subtracted as well. Also, filter techniques (High/Low pass filter) can be implemented when the data contain noise. After dealing with mean and linear trends, the original data finally become nearly stationary stochastic sequences as the input signals in the following analysis.

### 3.1.3 Overlap and Non-overlap

Dividing data into several short sequences will give more information on analysis. It is also good for checking the stability and reliability of each method when comparing the power of HRV between two positions. The total 1200 data (for each person) can be divided into two groups: Overlapped and Non-overlapped groups. In Non-overlapped groups, I choose the data length  $l=64$  and  $128$  respectively. The corresponding numbers of sequences are 18 for  $l=64$  and 9 for  $l=128$ . Similarly, in Overlapped groups, I also take the same data length, both of the lengths with 50% overlapped rate. Therefore, the data is divided into 36 sequences for  $l=64$  and 17 sequences for  $l=128$ . Table 3.2 below shows the plan for grouping of data with respect to each person.

	length of sequences	no. of sequences
<b>Non-overlap</b>	64	18
	128	9
<b>Overlap 50%</b>	64	36
	128	17

Table 3.2 Grouping of data for each person

### 3.2 Results from Spectrum Analysis

The analysis of the HRV signal is made in the frequency domain by using single window methods (Periodogram and Hanning) as well as multiple window methods (Thomson and PM MW). The estimated PSD  $\hat{S}(f)$  are compared for the different methods.

#### 3.2.1 HRV and RSA spectrum

Take the data from person 1 with full length as an example. Figure 3.2-a plots the estimated spectrum for HRV and respiration in two positions. The method used for estimation is the periodogram.

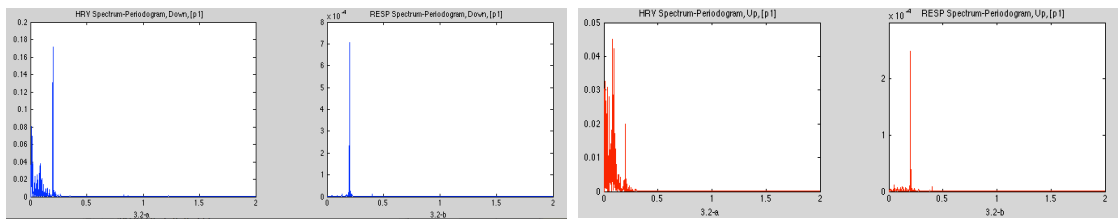


Figure 3.2-a Estimated spectra for HRV and respiratory data using periodogram in down (blue) & up (red) position of person 1 with full length.

Let  $\hat{S}_{\text{periodogram}}^{\text{hrv-down}}$ ,  $\hat{S}_{\text{periodogram}}^{\text{resp-down}}$ ,  $\hat{S}_{\text{periodogram}}^{\text{hrv-up}}$  and  $\hat{S}_{\text{periodogram}}^{\text{resp-up}}$  denote the estimated spectra using periodogram for HRV and respiration in two positions respectively. The estimated HRV  $\hat{S}_{\text{periodogram}}^{\text{hrv-down}}$  and respiration spectrum  $\hat{S}_{\text{periodogram}}^{\text{resp-down}}$  in lying down position are shown on the left side of the plots in figure 3.2-a with blue color. The estimated spectra in upright position are shown on the right side of the plot in red.

The peak of the respiration spectrum for both positions lies in the high-frequency (HF [0.12-0.4 Hz]) HRV band. It usually matches to the peak of HF-HRV spectrum. This is because the information in high-frequency (HF) band mainly reflects parasympathetic influences on the respiratory activity. Moreover, the corresponding frequency gives the peak value of respiration spectrum should around 0.2 Hz, which is the breathing rate we used in the experiments.

#### 3.2.2 Estimation from four methods

Applying the other three methods to the dataset, we get  $\hat{S}_{\text{hanning}}$ ,  $\hat{S}_{\text{thomson}}$  and  $\hat{S}_{\text{pmmw}}$  separately.



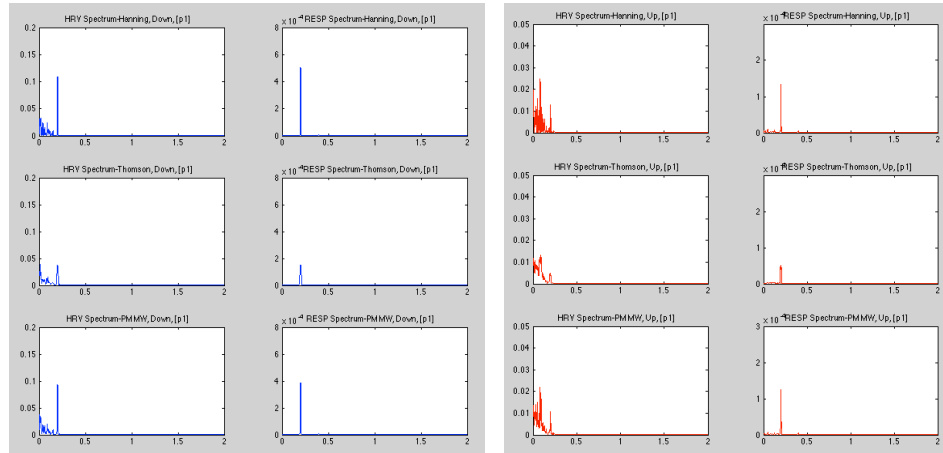


Figure 3.2-b Estimated spectra for HRV and respiration using Hanning/Thomson (now=4) /PM MW (now=4) in Down (blue) & Up (red) position of person 1 with full length

In figure 3.2-b, on the left side in blue color, presented the HRV (to the left) and respiration (to the right) spectra in lying down position. The estimated spectra in upright position are shown in the right side in figure 3.2-b with red color. For both sides, the two plots on top in the first row are estimated by Hanning window. Then comes the result from Thomson and the last two plots at the bottom are estimated by Peak Matched Multiple Windows (PM MW). The number of windows (now) is 4 for both the Thomson and the PM MW methods.

Under the same length of axes of coordinates with respect to two positions, there seems exists a quite large difference between them.

Down	peak of HF-HRV	peak of RESP	peak frequency of RESP
Periodogram	0.1711	0.0007	0.1997
Hanning	0.1095	0.0005	0.1997
Thomson( now=4)	0.0378	0.0001	0.1978
PM MW (now=4)	0.0932	0.0003	0.1997
Up	peak of HF-HRV	peak of RESP	peak frequency of RESP
Periodogram	0.0199	0.00024	0.2002
Hanning	0.0128	0.00013	0.2002
Thomson( now=4)	0.0049	0.00005	0.1982
PM MW (now=4)	0.0106	0.00012	0.2002

Table 3.3 Peak of HRV and respiratory data in Down & Up positions of person 1 with full length

Table 3.3 presents the values at the peak for all estimated spectra as well as the frequency corresponding to the peak location of respiration spectra.

For the HRV spectrum, the peak values are different both between methods and positions. The difference between positions is more obvious compared to the difference between methods. For the respiration spectrum, all of the peak spectra are very small, and they are pretty close to 0, which is also true for all other groups of sequences (Overlap/Non-overlap).

For the peak frequency of the respiration spectrum, all the figures are very close to 0.2 Hz, but it is not true for some groups of sequences (for example: overlap 64 and Non-overlap 64). The reason for this differences is, on one hand, lack of information in limited length of sequences; on the other hand, caused by the method itself, for example: Thomson window, which is bad for catching the peak.

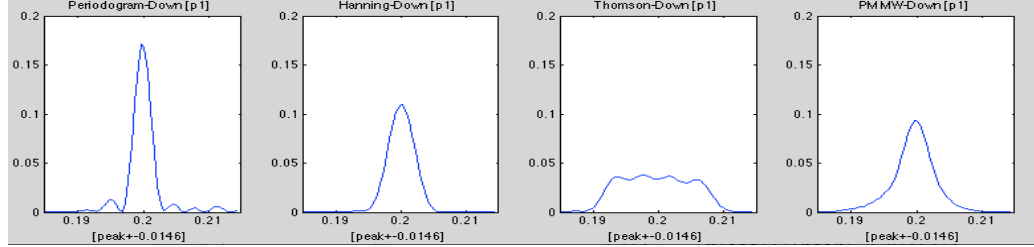


Figure 3.3 Estimated spectra for HRV in lying down position around peak area of person 1 with full length

Figure 3.3 demonstrates the HRV spectrum for the data with full length in a very close distance in the frequency band [0.185, 0.215] Hz. From the plots, one can observe that the shape of the estimated HRV spectrum very clearly. The differences between the four methods correspond to the different shapes of the window itself from the definition.

A little more on this part, compare the peak spectrum for HRV to respiration; one can see that even the values of the peak spectrum of respiration are pretty small, they varies along with the values of the peak spectrum of HRV. That is, a larger value in respiration also has a larger value in HRV.

### 3.3 Power of HRV

This section focuses on estimation of the power of HRV from the HRV spectrum with respect to different bandwidths:

$$\hat{P} = \sum_{f=0.12}^{0.4} \hat{S}(f) \quad (a)$$

or

$$\hat{P} = \sum_{f=f_0-B}^{f_0+B} \hat{S}(f) \quad (b)$$

where  $\hat{P}$  is the estimated power of HRV,  $f_0$  is the peak frequency of the respiration spectrum.  $2B$  is the bandwidth, which can be varied.

Method (a) is the so called traditional method since it just simply sums up all the estimated HRV powers inside the whole high-frequency (HF) band 0.12-0.4 Hz. Method (b) , the new method, change the bandwidths by different values of B. Notice that the new method with frequency bandwidth:  $[f_0 - B, f_0 + B]$  should not jump out of the predefined high-frequency (HF) band [0.12,0.4]. The new bandwidth is always limited to be inside this band:

$$0.12 \leq f_0 - B < f_0 + B \leq 0.4$$

In this thesis, I chose three different bandwidths for B: 0.08, 0.05 and 0.02 respectively. The corresponding new bandwidth are  $f_0 \pm 0.08$  ,  $f_0 \pm 0.05$  and  $f_0 \pm 0.02$  . Here comes the result for one person:

		Power of HRV [0.12-0.4]
<b>Periodogram</b>	DOWN	1.7217
	UP	0.4476
	<b>Down/Up</b>	<b>3.8468</b>
		Power of HRV [0.12-0.4]
<b>Hanning</b>	DOWN	1.5222
	UP	0.3679
	<b>Down/Up</b>	<b>4.1380</b>
		Power of HRV [0.12-0.4]
<b>Thomas (now=4)</b>	DOWN	1.7103
	UP	0.4196
	<b>Down/Up</b>	<b>4.0755</b>
		Power of HRV [0.12-0.4]
<b>PM MW (now=4)</b>	DOWN	1.7272
	UP	0.4289
	<b>Down/Up</b>	<b>4.0273</b>

Table 3.4-a Estimated power of HRV in Down & Up positions in traditional band of person 1 with full length

		Power of HRV -new	frequency band $\pm 0.08$
<b>Periodogram</b>	DOWN	1.6477	[0.1197, 0.2797]
	UP	0.4255	[0.1202, 0.2802]
	<b>Down/Up</b>	<b>3.8728</b>	
		Power of HRV -new	frequency band $\pm 0.08$
<b>Hanning</b>	DOWN	1.4633	[0.1197, 0.2797]
	UP	0.3540	[0.1202, 0.2802]
	<b>Down/Up</b>	<b>4.1337</b>	
		Power of HRV -new	frequency band $\pm 0.08$
<b>Thomas (now=4)</b>	DOWN	1.6524	[0.1178, 0.2778]
	UP	0.4094	[0.1182, 0.2782]
	<b>Down/Up</b>	<b>4.0362</b>	
		Power of HRV -new	frequency band $\pm 0.08$
<b>PM MW (now=4)</b>	DOWN	1.6534	[0.1197, 0.2797]
	UP	0.4075	[0.1202, 0.2802]
	<b>Down/Up</b>	<b>4.0573</b>	

Table 3.4-b Estimated power of HRV in Down & Up positions in band  $f_0 \pm 0.08$  of person 1 with full length

		Power of HRV -new	frequency band $\pm 0.05$
<b>Periodogram</b>	DOWN	1.4328	[0.1497, 0.2497]
	UP	0.2907	[0.1502, 0.2502]
	<b>Down/Up</b>	<b>4.9285</b>	
		Power of HRV -new	frequency band $\pm 0.05$
<b>Hanning</b>	DOWN	1.2925	[0.1497, 0.2497]
	UP	0.2499	[0.1502, 0.2502]
	<b>Down/Up</b>	<b>5.1754</b>	
		Power of HRV -new	frequency band $\pm 0.05$
<b>Thomas (now=4)</b>	DOWN	1.4216	[0.1478, 0.2478]
	UP	0.2696	[0.1482, 0.2482]
	<b>Down/Up</b>	<b>5.2734</b>	
		Power of HRV -new	frequency band $\pm 0.05$
<b>PM MW (now=4)</b>	DOWN	1.4301	[0.1497, 0.2497]
	UP	0.2789	[0.1502, 0.2502]
	<b>Down/Up</b>	<b>5.1277</b>	

Table 3.4-c Estimated power of HRV in Down & Up position in band  $f_0 \pm 0.05$  of person 1 with full length

		Power of HRV -new	frequency band $\pm 0.02$
<b>Periodogram</b>	DOWN	1.2257	[0.1797, 0.2197]
	UP	0.2043	[0.1802, 0.2202]
	<b>Down/Up</b>	<b>5.9990</b>	
		Power of HRV -new	frequency band $\pm 0.02$
<b>Hanning</b>	DOWN	1.1436	[0.1797, 0.2197]
	UP	0.1675	[0.1802, 0.2202]
	<b>Down/Up</b>	<b>6.8258</b>	
		Power of HRV -new	frequency band $\pm 0.02$
<b>Thomas (now=4)</b>	DOWN	1.2301	[0.1778, 0.2178]
	UP	0.1879	[0.1782, 0.2182]
	<b>Down/Up</b>	<b>6.5475</b>	
		Power of HRV -new	frequency band $\pm 0.02$
<b>PM MW (now=4)</b>	DOWN	1.2369	[0.1797, 0.2197]
	UP	0.1918	[0.1802, 0.2202]
	<b>Down/Up</b>	<b>6.4483</b>	

Table 3.4-d Estimated power of HRV in Down & Up positions in band  $f_0 \pm 0.02$  of person 1 with full length

Table 3.4 lists estimated power of HRV with full data length in four tables with respect to the four different bandwidths. As the bandwidth becomes more narrow, the values of  $\hat{P}$  becomes smaller (from around 1.7 in [0.12-0.4] to around 1.2 in  $[f_0 \pm 0.02]$ ) since the number of data that within the band becomes smaller. But from taking the ratio  $\frac{\hat{P}^{down}}{\hat{P}^{up}}$  of these two positions, it can be shown that the ratio increases when narrowing the bandwidth (jump from around 4 to around 6). This may indicate that a narrow bandwidth will better differentiate between the two positions.

So far, I go over all the steps for calculating power of HRV using the whole length of data from person 1. Now, let's discuss the power of HRV with divided groups of sequences. Take the overlapped sequences with the length of 64 for example.

### 3.4 Overlapped sequences with the length of 64.

The data is separated into 36 sequences. Similarly as above, the powers of HRV are calculated for each sequence according to the formula in (a), traditional method and (b), new method,

$$\hat{P}_i = \sum_{f=0.12}^{0.4} \hat{S}(f), i = 1 \dots 36$$

or

$$\hat{P}_i = \sum_{f=f_0-B}^{f_0+B} \hat{S}(f), B = 0.08 / 0.05 / 0.02, i = 1 \dots 36$$

Hence, 36 points are calculated. They represent the power of HRV for 36 overlapped sequences. These points could make up a line. Let's just call it the **Power line**  $\hat{P}$ .

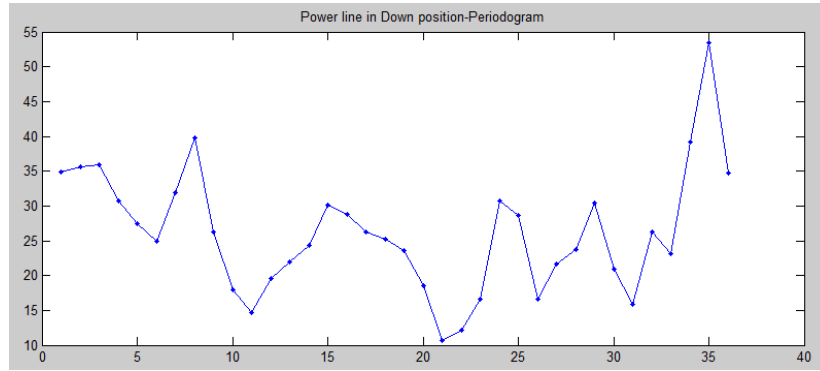


Figure 3.4 Estimated power line in lying down position using periodogram with traditional bandwidth from person 5.

The power line in figure 3.4 shows how the power of HRV varies with time.

#### 3.4.1 Compare bandwidth (fix one method)

Since there are four different bandwidths ( $0.12 - 0.4$ ;  $f_0 \pm 0.08$ ;  $f_0 \pm 0.05$ ;  $f_0 \pm 0.02$ ) for calculation of power of HRV, we will get four power lines  $\hat{P}_{[0.12-0.4]}$   $\hat{P}_{[f_0 \pm 0.08]}$   $\hat{P}_{[f_0 \pm 0.05]}$   $\hat{P}_{[f_0 \pm 0.02]}$  with respect to those four bandwidths. For a fixed spectrum method, the power lines are compared for the different bandwidth. The results are shown in figure 3.5 below.

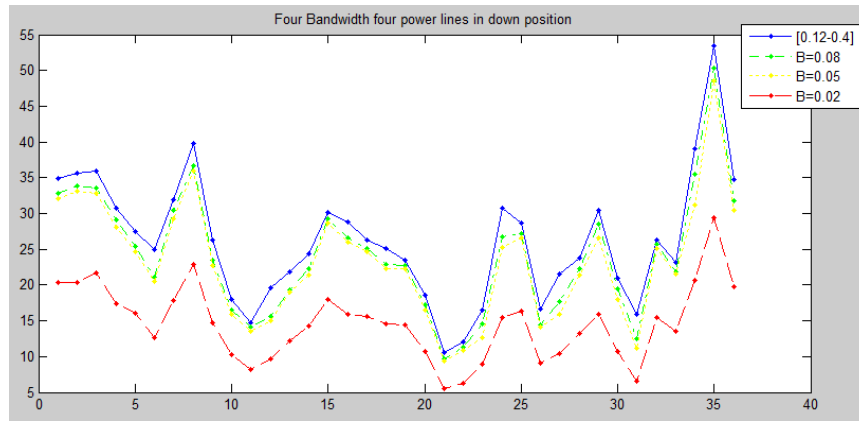


Figure 3.5 Estimated power line in lying down position using periodogram with four bandwidths from person 5.

Similarly, from the plot we can see that the narrow bandwidth gives the lowest curve.

### 3.4.2 Compare Methods (fix one bandwidth)

For a fixed bandwidth; consider how will the power curve looks like using periodogram, Hanning, Thomson and Peak Matched Multiple Window methods. The results are shown in figure 3.6 below.

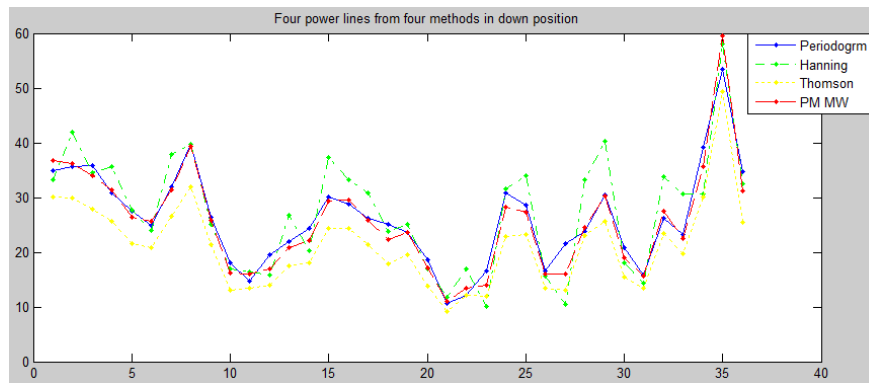


Figure 3.6 Estimated power line in lying down position using four methods with traditional bandwidth from person 5.

### 3.4.3 Compare between Up and Down positions

Before the comparison the logarithm is applied to all the sequences so that the data become more comparable.



Figure 3.7 Estimated power line using periodogram with traditional bandwidth from person 5.

Figure 3.7 compares the power line estimated by the periodogram with traditional bandwidth in lying down (the upper line) position to the line in standing up (the lower line) position. When lying down, the heart beats slower. That leads to a higher power of the HRV. It makes the power line in lying down position higher than the power line in upright position.

The power line can be estimated using different methods. Let's fix a bandwidth, then compare the results between up and down positions. The results are shown in figure 3.8.

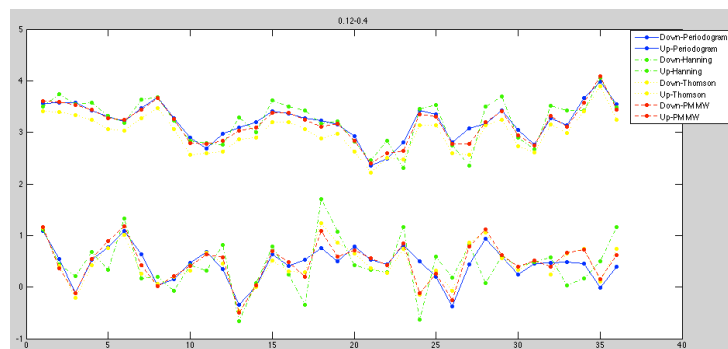


Figure 3.8 Estimated power line using four methods with traditional bandwidth from person 5.

Person 5		mean	variance
DOWN	Periodogram	3.2102	0.119
	Hanning	3.228	0.1814
	Thomson	2.9939	0.1266
	PM MW	3.1774	0.1286
UP	Periodogram	0.4462	0.1162
	Hanning	0.4449	0.2554
	Thomson	0.4498	0.1513
	PM MW	0.4808	0.153

Table 3.5 Statistics for power lines from four methods with traditional bandwidth in Down & Up positions of person 5

Table 3.5 presents the statistical information for each power line. From the figures, we can see that in lying down position, the mean values for the four methods are around 3.1. More specifically, the mean values for periodogram, Hanning and Thomson are very close to each other. The mean value for Thomson is slightly smaller compared to the other three. In standing up position, the differences are even smaller. For the variances, we can see that Hanning window seems to have the largest variance among these four methods.

Hence, it is hard to tell which method better differentiate the power of HRV between the two positions since the values are pretty much the same. Therefore the power lines are compared using the new method with  $f_0 \pm 0.08$ ;  $f_0 \pm 0.05$ ;  $f_0 \pm 0.02$ .

Note that from the previous result we know that a smaller bandwidth will lead to smaller value for the power line. Therefore a smaller bandwidth will shrink the distance between two positions. But, in other sense, changing of the bandwidth can make the differences between different methods to become more obvious!

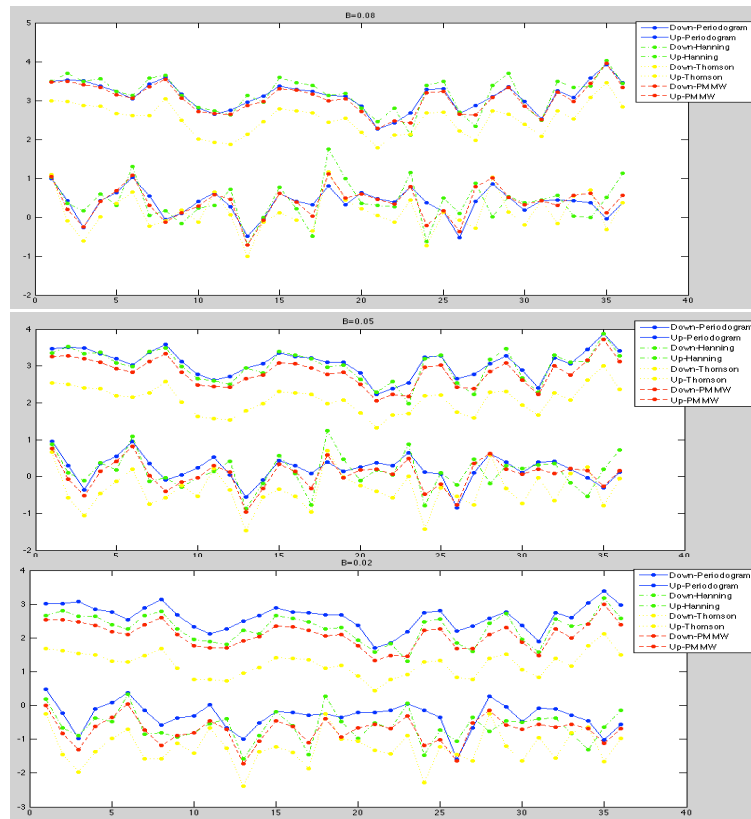


Figure 3.9 Estimated power line using four methods with new bandwidths from person 5.

When compared with traditional bandwidth, although the distance seems clear, it gives the similar results for all spectrum methods. So one can not decide which method is the better. But when using the new bandwidths, the distance shrinks and there shows a



clearly difference between different spectrum methods! Figure 3.9 above demonstrates the results.

【B=0.08】		mean	variance
DOWN	Periodogram	3.1206	0.1326
	Hanning	3.1711	0.1956
	Thomson	2.5243	0.1528
	PM MW	3.0512	0.1381
UP	Periodogram	0.365	0.1252
	Hanning	0.3896	0.2722
	Thomson	0.1126	0.2333
	PM MW	0.3655	0.1687
【B=0.05】		mean	variance
DOWN	Periodogram	3.077	0.1393
	Hanning	2.9994	0.1859
	Thomson	2.0837	0.1376
	PM MW	2.8222	0.1407
UP	Periodogram	0.2063	0.1319
	Hanning	0.1335	0.2352
	Thomson	-0.354	0.2591
	PM MW	0.0575	0.156
【B=0.02】		mean	variance
DOWN	Periodogram	2.6145	0.1439
	Hanning	2.2858	0.1774
	Thomson	1.2209	0.1311
	PM MW	2.0873	0.1408
UP	Periodogram	-0.3103	0.1654
	Hanning	-0.6188	0.2068
	Thomson	-1.2442	0.2572
	PM MW	-0.7302	0.155

Table 3.5 Statistics for power lines from four methods with new bandwidths in Down & Up positions of person 5

From table 3.5, we can see the lying down position periodogram and Hanning give the higher values in mean, for all the three bandwidths, then comes the PM MW, which is just slightly smaller than the other two methods. Thomson gives the smallest mean. In standing up position, for B=0.08, Thomson gives the smallest mean which is 0.11, where the means for the other three methods are all around 0.36. For B=0.05, Thomson still get the smallest mean, -0.35 and the range between the all mean values is around 0.5. For B=0.02, the range between the mean values has increased to 0.9.

Table 3.6 below states the difference between positions with respect to every method with the four bandwidths.

DOWN-UP	0.12-0.4	B=0.08	B=0.05	B=0.02
Periodogram	2.764	2.7555	2.8707	2.9248
Hanning	2.7832	2.7815	2.8659	2.9046
Thomson	2.5441	2.4116	2.4377	2.4651
PM MW	2.6965	2.6757	2.7647	2.8175

Table 3.6 Differences in Down & Up positions from four bandwidths of person 5

The difference between two positions is slowly increasing when narrowing the bandwidth.

We are interested in finding the differences between the different methods. Let's compare two methods with all possible bandwidths at a time. Here, 16 pairs are constructed into 4 groups between the methods for each person:

A: Periodogram vs Hanning  $\log(\hat{P}_{hanning}) - \log(\hat{P}_{periodogram})$

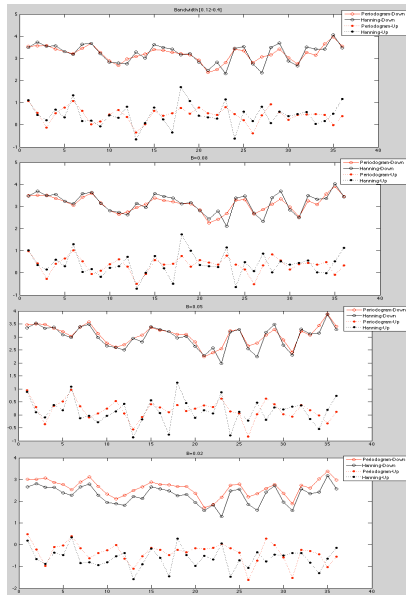
B: Periodogram vs Thomson  $\log(\hat{P}_{thomson}) - \log(\hat{P}_{periodogram})$

C: Periodogram vs PM MW  $\log(\hat{P}_{PMMW}) - \log(\hat{P}_{periodogram})$

D: Thomson vs PM MW  $\log(\hat{P}_{thomson}) - \log(\hat{P}_{PMMW})$

Each group contains 4 pairs with respect to the four kinds of frequency bandwidths. Still, we take the data from person 5 as an example. The results are shown in table 3.7 below.

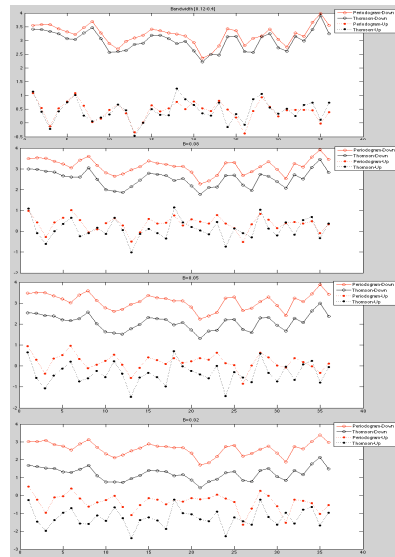
#### Group A: Periodogram vs Hanning



0.12-0.4	Down	Up	var(down)/var(up)
	Hanning-Periodogram	Hanning-Periodogram	
mean	0.0178	-0.0014	
var	0.048	0.208	0.2308
B=0.08	Down	Up	var(down)/var(up)
	Hanning-Periodogram	Hanning-Periodogram	
mean	0.0505	0.029	
var	0.0405	0.216	0.1877
B=0.05	Down	Up	var(down)/var(up)
	Hanning-Periodogram	Hanning-Periodogram	
mean	-0.0776	-0.052	
var	0.0261	0.1777	0.1466
B=0.02	Down	Up	var(down)/var(up)
	Hanning-Periodogram	Hanning-Periodogram	
mean	-0.3288	-0.248	
var	0.0311	0.2538	0.1226

Table 3.7-a Differences for Group A in Down & Up positions from four bandwidths of person 5

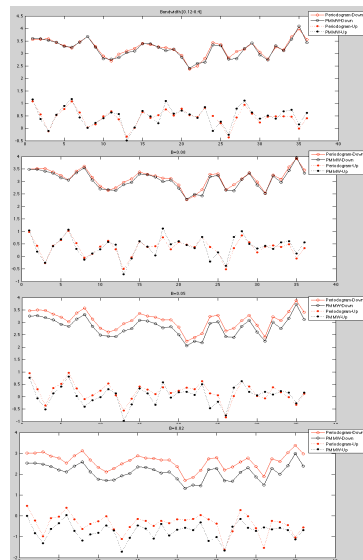
## Group B: Periodogram-Thomson



0.12-0.4	Down	Up	var(down)/var(up)
	Thomson-Periodogram	Thomson-Periodogram	
mean	-0.2163	0.0036	
var	0.0097	0.0514	0.1886
B=0.08	Down	Up	var(down)/var(up)
	Thomson-Periodogram	Thomson-Periodogram	
mean	-0.5963	-0.248	
var	0.0179	0.1226	0.1458
B=0.05	Down	Up	var(down)/var(up)
	Thomson-Periodogram	Thomson-Periodogram	
mean	-0.9933	-0.5395	
var	0.0142	0.1756	0.0811
B=0.02	Down	Up	var(down)/var(up)
	Thomson-Periodogram	Thomson-Periodogram	
mean	-1.3936	-0.8733	
var	0.0193	0.2439	0.079

Table 3.7-b Differences for Group B in Down & Up positions from four bandwidths of person 5

## Group C: Periodogram-PM MW



0.12-0.4	Down	Up	var(down)/var(up)
	PM MW-Periodogram	PM MW-Periodogram	
mean	-0.0329	0.0346	
var	0.0065	0.0327	0.1997
B=0.08	Down	Up	var(down)/var(up)
	PM MW-Periodogram	PM MW-Periodogram	
mean	-0.0693	0.0149	
var	0.0046	0.0366	0.1254
B=0.05	Down	Up	var(down)/var(up)
	PM MW-Periodogram	PM MW-Periodogram	
mean	-0.2549	-0.1281	
var	0.0032	0.041	0.0775
B=0.02	Down	Up	var(down)/var(up)
	PM MW-Periodogram	PM MW-Periodogram	
mean	-0.5272	-0.3593	
var	0.0073	0.1046	0.0703

Table 3.7-c Differences for Group C in Down & Up positions from four bandwidths of person 5

## Group D: Thomson-PM MW

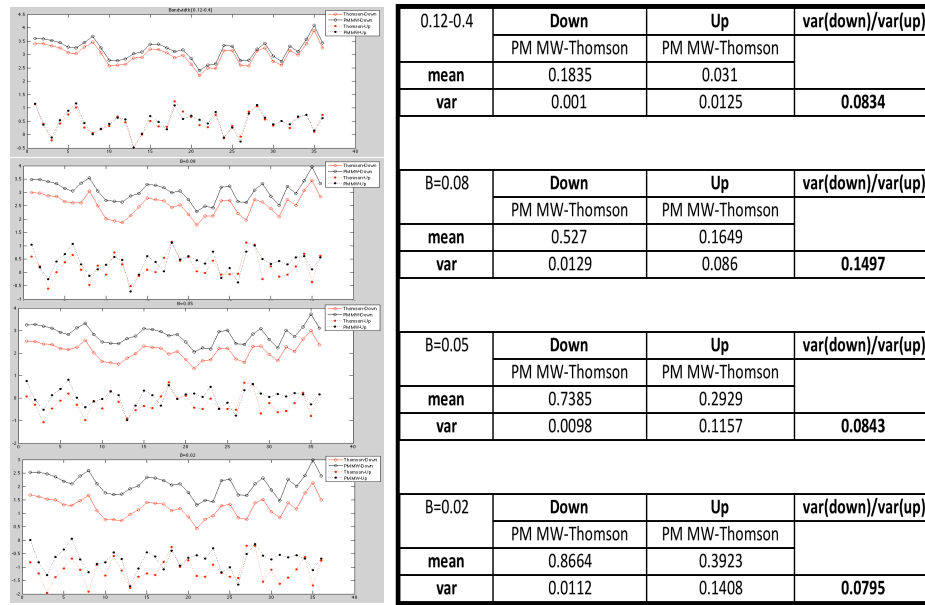


Table 3.7-d Differences for Group D in Down & Up positions from four bandwidths of person 5

Look at one of these pairs, for example Group A with traditional methods. Figure 3.10 present the corresponding groups.

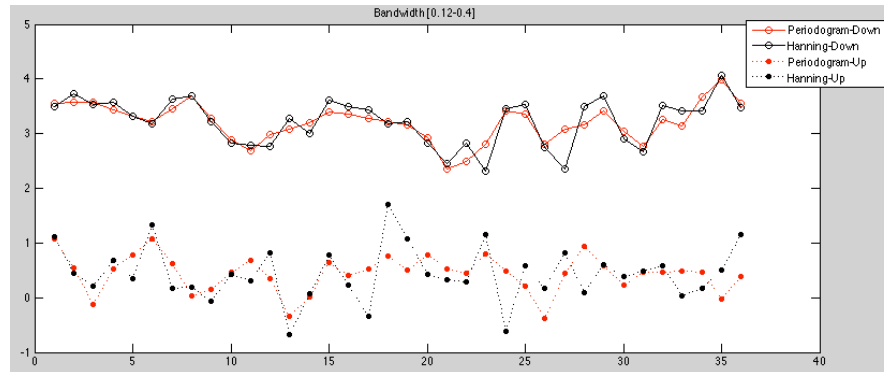


Figure 3.10 The pairs in Group A with traditional bandwidth in Down & Up positions of person 5

Check the difference between methods in two positions:  $\log(\hat{P}_{\text{hanning}}) - \log(\hat{P}_{\text{periodogram}})$

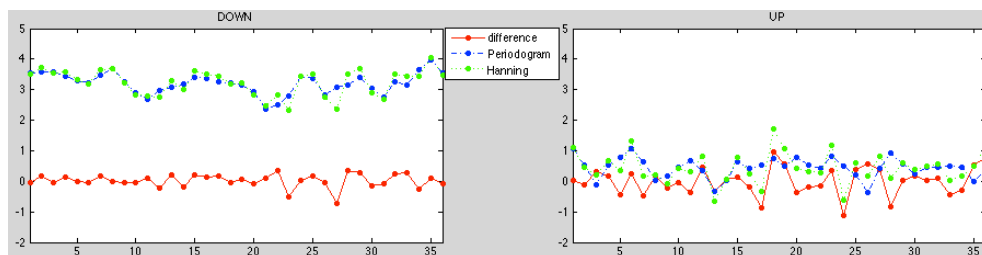


Figure 3.11 Differences between methods for this pair in Down & Up positions of person 5

Figure 3.11 plots the differences between methods in lying down position (to the left) and standing up position (to the right). Let  $\mathbf{d}$  be the difference sequences between the two methods. Let  $d_{down} = \log(\hat{P}_{hanning}^{down}) - \log(\hat{P}_{periodogram}^{down})$  and  $d_{up} = \log(\hat{P}_{hanning}^{up}) - \log(\hat{P}_{periodogram}^{up})$ . Therefore, we get two sequences that only present the differences between certain methods.

In the end of this chapter, we combined two factors: spectrum methods as well as different bandwidths for comparison of power lines in two positions. In order to decide a good method with an appropriate bandwidth, hypothesis tests are implemented for testing that if there is a significant difference in mean/variance between pairs.

## Chapter 4

### Hypothesis Tests

Statistical hypothesis test is a method of making decisions using data from a scientific study. From the hypothesis tests, one can determine what outcomes of a study would lead to a rejection of the null hypothesis for a pre-specified level of significance. Therefore, one can decide whether results contain enough information to cast doubt on conventional wisdom, given that conventional wisdom has been used to establish the null hypothesis.

A result is called statistically significant if it has been predicted as unlikely to have occurred by chance alone, according to a significance level. The phrase ‘test of significance’ was coined by statistician Ronald Fisher. Yet hypothesis testing is a dominant approach to data analysis in many fields of science.

The process of a hypothesis testing usually containing several steps as below:

- 1) State the relevant null and alternative hypotheses.
- 2) Consider the statistical assumptions being made about the sample in doing the test.
- 3) Decide which test is appropriate, and find a relevant test statistic  $T$ .
- 4) Derive the distribution of the test statistic under the null hypothesis from the assumptions. For example the test statistic might follow a Student’s  $t$  distribution or a  $F$ -distribution.
- 5) Select a significance level  $\alpha$ , a probability threshold below which the null hypothesis will be rejected. Common values are 5% and 1%.
- 6) Compute from the observations the observed value  $t_{obs}$  of the test statistic  $T$ . Check if it is in the critical region or not. Here, the probability of the critical region is  $\alpha$ .
- 7) Decide to either reject the null hypothesis or not. The decision rule is to reject the null hypothesis  $H_0$  if the observed value  $t_{obs}$  is in the critical region. Otherwise, accept “fail to reject”.

In the end, the testing procedure forces us to whether we can accept the null hypothesis  $H_0$ , or reject  $H_0$  and accept the alternative hypothesis  $H_1$ . And one can conclude that the research hypothesis is supported by the data.

Statistical hypothesis testing acts as a filter of statistical conclusions. It helps to justify conclusions in a statistical point of view. Hypothesis testing plays an important role in the whole of statistics and in statistical inference and it has been the favored statistical tool in some experimental social sciences.

In this thesis, I choose to use Paired-sample  $t$ -test as well as Two-sample  $F$ -test on our results in Chapter 3. I will compare two samples at one time, from checking the basic statistic: mean

and variance, which from different combination of spectrum methods and calculation bandwidths. The aim is to show if there is a better method with a certain bandwidth which leads to our target question.

#### 4.1 Paired-sample t-test

Using t-tests are appropriate for comparing means under relaxed conditions. Paired-sample t-tests are appropriate for comparing two samples where it is impossible to control important variables. The difference between the members becomes the sample. Typically the mean of the differences is then compared to zero.

Statistics:

$$t_{obs} = \frac{\bar{d} - 0}{(s_d / \sqrt{n})}, \quad df = n - 1$$

$\bar{d}$  is the sample mean of differences.

$s_d$  is the standard deviation of differences.

$n$  is the sample size.

Assumption: Normal population of differences or  $n > 30$  and  $\sigma$  unknown or small sample size  $n < 30$

Hence, construct a test for our case:

$$H_0 : \mu_d = 0$$

$$H_1 : \mu_d \neq 0 \quad \text{where } df = n - 1, n = 36.$$

Select the significant level  $\alpha = 0.05$ . The result of the  $t_{obs}$  for each paired-samples are listed in Table 4.1 below.

<b>T static</b>				
<b>person1 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-0.3155	1.7001	-6.0555	-9.2336
Thomson-Periodogram	8.8327	-0.2795	-3.2535	-7.3009
PM MW-Periodogram	13.4481	9.9423	-16.724	-35.0422
Thomson-PM MW	-13.6995	1.4876	2.1397	3.66
<b>T static</b>				
<b>person2 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	1.0848	3.8662	-15.6354	-32.59
Thomson-Periodogram	-1.5272	-16.0005	-33.7708	-38.6177
PM MW-Periodogram	-3.756	-10.1342	-51.0619	-56.7654
Thomson-PM MW	-0.4449	18.6543	30.0142	38.44
<b>T static</b>				
<b>person3 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	4.5706	7.8128	-4.2596	19.59
Thomson-Periodogram	1.273	-2.0055	-7.5889	-11.4446
PM MW-Periodogram	-1.5607	-4.9734	-35.0595	-55.2444
Thomson-PM MW	-5.864	2.0488	5.5699	7.4913
<b>T static</b>				
<b>person4 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	1.4077	5.9161	-23.0752	-57.3826
Thomson-Periodogram	-2.5765	-10.5909	-23.5453	-37.487
PM MW-Periodogram	4.2331	-17.3219	-123.7706	-119.0408
Thomson-PM MW	4.5376	10.1062	14.6536	19.1866
<b>T static</b>				
<b>person5 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	2.1939	7.3768	-17.5896	-62.5468
Thomson-Periodogram	-131.9225	-197.0815	-413.8339	-427.1839
PM MW-Periodogram	-29.9	-89.1	-471.3	-427.3
Thomson-PM MW	1085.6	241.7	445.8	457.7
<b>T static</b>				
<b>person6 DOWN</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-0.1198	1.4839	-13.6801	-34.6751
Thomson-Periodogram	2.8949	-8.5094	-14.877	-24.6884
PM MW-Periodogram	10.4916	2.6757	-58.6703	-77.9679
Thomson-PM MW	1.0788	9.9498	17.4769	23.2894
<b>T static</b>				
<b>person1 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-1.0045	-0.9386	-1.5621	-2.0721
Thomson-Periodogram	11.1135	4.4966	3.8921	3.2166
PM MW-Periodogram	8.2671	7.3228	4.5213	0.8433
Thomson-PM MW	-19.4366	-4.512	-3.0751	-2.762
<b>T static</b>				
<b>person2 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-4.2906	-3.484	-3.4808	-2.6966
Thomson-Periodogram	12.0234	2.8128	0.5617	-1.0509
PM MW-Periodogram	3.225	4.985	-2.9337	-4.1749
Thomson-PM MW	-12.4093	-4.8019	-2.9229	-1.7941
<b>T static</b>				
<b>person3 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-1.6729	0.6348	-0.5186	-2.5397
Thomson-Periodogram	7.121	2.9651	2.1469	0.1839
PM MW-Periodogram	5.5492	3.6159	-3.1135	-7.7625
Thomson-PM MW	-13.5495	-2.8171	-0.8913	-0.2594
<b>T static</b>				
<b>person4 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-3.6892	-3.0914	-4.263	-5.4891
Thomson-Periodogram	11.2381	5.3855	0.4987	-2.9895
PM MW-Periodogram	3.5587	3.5562	-7.0859	-14.3814
Thomson-PM MW	-17.9863	-3.3612	-0.5144	1.0453
<b>T static</b>				
<b>person5 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-0.0398	0.7943	-1.7312	-5.7809
Thomson-Periodogram	0.4144	-11.9673	-18.1761	-21.1829
PM MW-Periodogram	6.3	2.4	-18.5	-20.3
Thomson-PM MW	14.7	11.3	15	16.5
<b>T static</b>				
<b>person6 UP</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	-3.6038	-2.29	-8.7764	-12.6455
Thomson-Periodogram	0.7233	3.4495	0.5894	-1.9271
PM MW-Periodogram	4.3452	3.3753	-6.5516	-11.2459
Thomson-PM MW	-11.821	-2.6657	-0.1667	1.543

Table 4.1 Results from t-test in two positions for all 6 people

The two tails  $t(0.05, 35) = 2.0301$ . Compare the results to 2.0301. If  $t_{obs} > 2.0301$ , then reject the null hypothesis  $H_0$ , thus the mean for the  $d_{down}$  or  $d_{up}$  not equals to zero. It is indicated that there exists a significant difference between the methods when estimating the power of HRV. Moreover, we can tell how they differ from the mean of  $d_{down}$  or  $d_{up}$ . If  $t_{obs} > 2.0301$ , reject the null hypothesis  $H_0$ . If  $t_{obs} < 2.0301$ , do not reject the null hypothesis  $H_0$ .

For example: Table 3.8 presents the mean of  $d_{down}$  and  $d_{up}$ . If the value of the mean is positive, then the level of the power line using one method is higher than the power line using the other method. In our case, the mean of  $d_{down}$  is 0.0178, and it is indicated that the power of HRV using the Hanning method is higher compared to using the periodogram in lying down position. If the value of the mean is negative, then the results should be interpreted the other way around. From checking the difference in mean, we will know about how the power line is located using different methods so that we could find out a method that better differentiate between the two positions.



Person 5	Down	Up
0.12-0.4	Hanning-Periodogram	Hanning-Periodogram
mean	0.0178	-0.0014
var	0.048	0.208

Table 4.2 Statistics for  $d_{down}$  and  $d_{up}$  in Group A of person 5

## 4.2 Two-sample F test

F-tests, also known as analysis of variance, are commonly used when deciding whether grouping of data by category are meaningful. If the variance of test scores of the left-handed in a class is much smaller than the variance of the whole class, then it may be useful to study lefties as a group. The null hypothesis is that two variances are the same—so the proposed grouping is not meaningful.

Statistics:

$$F = \frac{s_1^2}{s_2^2}$$

$s_1$  is the standard deviation for sample 1.

$s_2$  is the standard deviation for sample 2.

Assumption: Normal populations. Arranged so  $s_1^2 \geq s_2^2$ , and reject  $H_0$  for  $F > F(\alpha/2, n_1 - 1, n_2 - 1)$ .

In our case,  $n_1 = n_2 = 36$ . Let the significant level  $\alpha = 0.05$ . The results are for each pair of sequences with respect to 6 persons that are listed in table 4.2 below.

<b>F statistics</b>				
<b>person1</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	5.3442	6.7715	11.6576	5.4542
Thomson-Periodogram	2.9603	3.2362	2.5714	2.151
PM MW-Periodogram	3.9062	5.963	6.812	4.5413
Thomson-PM MW	3.7327	2.5994	2.5445	2.6178
<b>F statistics</b>				
<b>person2</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	5.5371	6.4392	10.1833	10.7411
Thomson-Periodogram	3.0248	6.8587	6.3371	5.4437
PM MW-Periodogram	2.265	3.6846	5.0251	6.7935
Thomson-PM MW	5.9137	7.3206	8.3056	9.8232
<b>F statistics</b>				
<b>person3</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	3.125	9.3897	9.3897	5.9032
Thomson-Periodogram	5.3967	2.6947	2.4079	2.3036
PM MW-Periodogram	3.1676	3.7807	4.2644	4.4287
Thomson-PM MW	6.2035	2.1418	2.2297	2.1944
<b>F statistics</b>				
<b>person4</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	8.5616	8.4746	15.1515	13.1752
Thomson-Periodogram	3.1466	2.0859	2.2512	3.0066
PM MW-Periodogram	9.0171	9.6712	8.547	5.2966
Thomson-PM MW	2.05	1.7507	1.9168	2.2568
<b>F statistics</b>				
<b>person5</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	4.3328	5.3277	6.8213	8.1566
Thomson-Periodogram	5.3022	6.8587	12.3305	12.6582
PM MW-Periodogram	5.0075	7.9745	12.9032	14.2248
Thomson-PM MW	11.9904	6.68	11.8624	12.5786
<b>F statistics</b>				
<b>person6</b>	0.12-0.4	B=0.08	B=0.05	B=0.02
Hanning-Periodogram	2.5543	2.8547	2.9922	3.0788
Thomson-Periodogram	4.9456	3.8432	3.6166	3.864
PM MW-Periodogram	6.3412	5.9172	6.079	5.711
Thomson-PM MW	3.126	2.8401	2.9386	3.1536

Table 4.3 Results from F test for all 6 people

The results in table 4.3 will be compared to  $F(0.025, 35, 35) = 1.76$ . If  $F < 1.76$ , it indicates that there is no significant differences between the two samples. In our case this means  $d_{down}$  estimated by these two methods with a certain bandwidth at the same level with  $d_{up}$ . It shows that the two methods are pretty stable for the estimated power of HRV in standing up and lying down positions.

Otherwise, if  $F > 1.76$ , as the most results that shown in table 4.3, it indicates that there exists a significant differences between the two samples. In other words, when using these two methods to estimate the power of HRV with certain bandwidth,  $d_{down}$  has a significant difference from  $d_{up}$ . We might say that in most cases, (even from the plots, we see one power line using one method seems to follow another power line than using another method, but still), there exists a difference between lying down and standing up position when applying different methods.

## Chapter 5

### Conclusion

In this thesis, we studied the analysis for HRV signals in frequency domain. The Power Spectral Densities (PSD) for HRV signals and respiratory signals are estimated based on Fast Fourier Transform (FFT). The periodogram, Hanning, Thomson and Peak Matched Multiple Windows (PM MW) methods are implemented for estimation of HRV spectrum as well as the peak frequency from respiratory spectrum.

The periodogram and Peak Matched Multiple Windows (PM MW) methods present more accurate results on the respiratory signals. The estimated respiratory spectrum using these two methods showed that most of the peak frequencies are around 0.2 Hz, which is close to the respiratory rate that we used in the experiments. The Hanning and Thomson methods sometimes overestimated the peak frequency especially for short length of data.

The estimated power of HRV varies when changing the frequency bandwidths. It is smaller in narrow bandwidth compare to the wider ones. But a more narrow bandwidth can show a larger ratio when comparing the power of HRV in lying down and standing up positions.

Results from hypothesis test indicated that most pairs we construct for comparison exists a significant difference in mean value. For those pairs, which did not show a significant difference in mean, we could also check the stability. From the F test, we could say that for some paired sample of differences, there exists a significant difference for the variances between lying down and standing up positions when estimating the HRV power.

## Bibliography

### Books:

- [1] P. Stoica and R. Moses. *Spectral Analysis of Signals*. Prentice Hall, 2005, ch2, ch5.
- [2] M. Standsten, *Time-Frequency Analysis of Non-stationary Processes*, Lund Univ. (2013), 11-19.

### Articles:

- [3] M. Hansson, "Optimization of weighting factors in the peak matched multiple window method," *IEEE international conf. ICASSP'97*, vol. 5, Munich, Germany, April 21-24, 1997.
- [4] M. Hansson and G. Salomonsson, "A multiple window method for estimation of peaked spectra," *IEEE Trans. on Signal Processing*, vol. 45, no. 3, pp. 778-781, Mar. 1997.
- [5] M. Hansson, "Optimized weighted averaging of peak matched multiple window spectrum estimates," *IEEE Trans. on Signal Processing*, vol. 47, no. 4, pp. 1141-1146, Apr. 1999.
- [6] M. Hansson and P. Jönsson, "Estimation of HRV spectrogram using multiple window methods focusing on the high frequency power," *Medical Engineering & Physics*, vol. 28, no. 8, pp.749-761, Oct. 2006.
- [7] M. Hansson-Sandsten and P. Jönsson, "Multiple window correlation analysis of HRV power and respiratory frequency," *IEEE Trans. on Biomedical Engineering*, vol. 54, no. 10, pp. 1770-1779, Oct. 2007.
- [8] P. Jönsson and M. Hansson-Sandsten, "Respiratory sinus arrhythmia in response to fear-relevant and fear-irrelevant stimuli," *Scandinavian Journal of Psychology*, vol. 49, no. 2, pp. 123-131, 2008.
- [9] S. Kodituwakku, S.W. Lazar, P. Indic, C. Zhen, E.N. Brown and R. Barbieri, "Point process time-frequency analysis of dynamic respiratory patterns during meditation practice," *Med Biol Eng Comput*, vol. 50, no. 3, pp. 261-275, Mar. 2012.
- [10] P. Venkatakrishnan, R. Sukanesh and S. Sangeetha, "Detection of quadratic phase coupling from human EEG signals using higher order statistics and spectra," *SIViP*, vol. 5, no. 2, pp. 217-229, Jun. 2011.
- [11] O. Kiss, B. Schumann, A. Kluttig, K. Halina and J. Haerting, "Time domain parameters can be estimated with less statistical error than frequency domain parameters in the analysis of heart rate variability," *J Electrocardiol*, vol. 41, no. 4, pp. 287-291, Jul-Aug. 2008.

[12] G. F. Lewis, S. A. Furman, M. F. McCool and S. W. Porges, “Statistical strategies to quantify respiratory sinus arrhythmia: Are commonly used metrics equivalent?” *Biol Psychol*, vol. 89, no. 2, pp. 349-364, Feb. 2012.

[13] E. Jovanov, “On spectral analysis of heart rate variability during very slow yogic breathing,” *Conf Proc IEEE Eng Med Biol Soc, ol Psychol*, vol. 3, pp. 2467-2470, Shanghai, China, September 1-4, 2005.

[14] W. Buqing and W. Weidong, “Research progress of the methods for heart rate variability analysis,” *Beijing Biomedical Engineering*, vol. 26, no. 5, pp. 551-554, Oct. 2007.

Internet:

HRV: [https://en.wikipedia.org/wiki/Heart\\_rate\\_variability](https://en.wikipedia.org/wiki/Heart_rate_variability)

RSA: [https://en.wikipedia.org/wiki/Respiratory\\_sinus\\_arrhythmia](https://en.wikipedia.org/wiki/Respiratory_sinus_arrhythmia)

## Appendix A

### Matlab code

```
clear; clc;
load results7.mat

%data for one person two positon: up and down
fs=4;
hrv=downhrv7;
resp=downresp7;
hrv1=uphrv7;
resp1=upresp7;

%take mean and linear trend away
h=detrend(hrv);
r=detrend(resp);
h1=detrend(hrv1);
r1=detrend(resp1);

% Overlap l=64,50% : 36 sequences
l=64;
p=0.5;
n=length(h);
L=floor((n-l)/(l*p))+1;
Oh64down=zeros(l,L);
for i=1:L
    Oh64down(:,i)=h(((l*p*(i-1))+1):(l*p*(i-1)+l));
end
Or64down=zeros(l,L);
for i=1:L
    Or64down(:,i)=r(((l*p*(i-1))+1):(l*p*(i-1)+l));
end
Oh64up=zeros(l,L);
for i=1:L
    Oh64up(:,i)=h1(((l*p*(i-1))+1):(l*p*(i-1)+l));
end
Or64up=zeros(l,L);
for i=1:L
    Or64up(:,i)=r1(((l*p*(i-1))+1):(l*p*(i-1)+l));
end

% set FFTL and the corresponding frequency length for x asix.
FFTL=4096;
fy=[0:FFTL/2-1]*fs/FFTL;

%% Spectrum
% Periodogram
% for i=1:L
%     sh(:,i)=(abs(fft(Oh64down(:,i),FFTL)).^2)/l;
%     sr(:,i)=(abs(fft(Or64down(:,i),FFTL)).^2)/l;
%     sh1(:,i)=(abs(fft(Oh64up(:,i),FFTL)).^2)/l;
%     sr1(:,i)=(abs(fft(Or64up(:,i),FFTL)).^2)/l;
% end

%Hanning
% han=hanning(l);
% w=han/sqrt(han'*han);
```

```

% % Thomson
now=4;
w=thomson(1,now);

% PM MW
%now=4;
%w=multipeakwind(1,now);

for i=1:L
    sh(:,i)=mtspectrum(Oh64down(:,i),w,fs,FFTL);
    sr(:,i)=mtspectrum(Or64down(:,i),w,fs,FFTL);
    sh1(:,i)=mtspectrum(Oh64up(:,i),w,fs,FFTL);
    sr1(:,i)=mtspectrum(Or64up(:,i),w,fs,FFTL);
end

%% power of hrv
% find peak for resp
pf=zeros(L,1);
pfl=zeros(L,1);
index=zeros(L,1);
index1=zeros(L,1);
for i=1:L
    [value,index(i)] = max(sr(:,i));
    [value,index1(i)] = max(sr1(:,i));
    pf(i)=fy(index(i));
    pfl(i)=fy(index1(i));
end

pf
pfl
% for result 2
% % Thomson: i=3 pf=0 pfl=0 index=1 index1=1
% % i=4 pf=0.0752 index=78
% % Thomson, k=0.08,i=[1,2,5:36]
% % k=0.05/0.02, i=[1,2,4:36]

% for result 3
% % Thomson: i=12 pfl=0.0762 index1=79
% % i=13 pfl=0.0674 index1=70
% % i=20 pfl=0.0527 index1=55
% % i=23 pfl=0.0781 index1=81
% % i=27 pfl=0.0742 index1=77
% % i=31 pfl=0.0527 index1=55
% % Thomson, k=0.08,i=[1:11,14:19,21,22,24:26,28:30,32:36]

% for result 7
% % Thomson: i=6 pfl=0.0781 index1=81
% % i=14 pfl=0.0742 index1=77
% % i=30 pf=0.0732 index=76
% % Thomson, k=0.08,i=[1:5,7:13,15:29,31:36]

% traditional method
dp=zeros(L,1);
up=zeros(L,1);
for i=1:L
    shdown=sh(:,i);

```

```

        dp(i)=sum(shdown(124:410));
        shup=sh1(:,i);
        up(i)=sum(shup(124:410));
end
dp
up
ratio=dp./up

md=mean(dp)
sd=std(dp)
mu=mean(up)
su=std(up)

figure(3)
plot(dp,'o:')
hold on
plot(up,'r.:')
hold off
legend('Down','Up')
title('PM MW Power of HRV [0.12,0.4]')

% new method try bandwidth 0.02/0.05/0.08

dpn=zeros(L,3);
upn=zeros(L,3);
B=[0.02 0.05 0.08];
k=floor(FFTL/4*B);
for i=1:L;
    shdown=sh(:,i);
    shup=sh1(:,i);
    for j=1:3;
        dpn(i,j)=sum(shdown(index(i)-k(j):index(i)+k(j)));
        upn(i,j)=sum(shup(index1(i)-k(j):index1(i)+k(j)));
    end
end

% dpn=zeros(L,1);
% upn=zeros(L,1);
% k=floor(FFTL/4*0.08);
% for i=[1:16,19:36];
%     % Hanning: i=17 pf1= 0.0605 index1=63
%     %             i=21 pf1= 0 index1=1
%     %             i=36 pf=0.0615 index=64
%     % Hanning, k=0.08, i=[1:16,18:20,22:35]
%     %             k=0.05/0.02, i=[1:20,22:36]
%     % Thomson: i=5 pf1=0.0703 index1=73
%     %             i=17/18 pf1=0 index1=1
%     %             i=21 pf=0.0781 index=81
%     %             i=32 pf1=0.0625 index1=65
%     %             i=33 pf=0.0771 index==80
%     %             i=36 pf=0.0625 index=65
%     % Thomson, k=0.08,i=[1:4,6:16,19:20,22:31,34,35]
%     %             k=0.05/0.02, i=[1:16,19:36]
%     shdown=sh(:,i);
%     shup=sh1(:,i);
%     dpn(i)=sum(shdown(index(i)-k:index(i)+k));
%     % else
%     %     dpnh(i)=sum(shdown(1:index+k));
%     upn(i)=sum(shup(index1(i)-k:index1(i)+k));
% end

```



```

dpn
upn
dpn./upn

% dpn=dpn(find(dpn));
% mdn=mean(dpn)
% sdn=std(dpn)
% upn=upn(find(upn));
% mun=mean(upn)
% sun=std(upn)
%
% figure(3)
% plot(dpn,'o:')
% hold on
% plot(upn,'r.:')
% hold off
% legend('Down','Up')
% title('Thomson Power of HRV with B=0.02')

mdn=mean(dpn)
sdn=std(dpn)
mun=mean(upn)
sun=std(upn)

i=1
figure(3)
plot(dpn(:,i),'o:')
hold on
plot(upn(:,i),'r.:')
hold off
legend('Down','Up')
title('PM MW Power of HRV with B=0.02')

%% plot
% plots of cutted sequences
figure(1)
for i=1:L
subplot(6,6,i)
% plot(Oh64down(:,i))
% plot(Or64down(:,i))
% plot(Oh64up(:,i))
plot(Or64up(:,i))
title(num2str(i))
end

% plots of Periodogram/Hanning/Thomson/PM MW spectrum
figure(2)
for i=1:L
%     shdown=sh(:,i);
%     shdown=sr(:,i);
%     shdown=sh1(:,i);
%     shdown=sr1(:,i);
subplot(6,6,i)
plot(fy,shdown(1:FFTL/2))
title(num2str(i))
end

```