

# Use of Semidefinite Optimization in Emergency Power System Control

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UNIVERSITET

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MSc Thesis  
ISRN LUTFD2/TFRT--5927--SE  
ISSN 0280-5316

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Printed in Sweden by MediaTryck.  
Lund 2013

# Abstract

This thesis is a study of long-term voltage stability in power systems following a severe disturbance.

An optimization method for solving the optimal power flow problem is used on power system models. The method tries to optimize the generated active power while fulfilling the requirements from the consumers. Dynamic limitations of the generators are taken into account and an extension of the optimization routine which optimizes over several time steps is constructed.

The results from the optimization are then applied to a power system simulator during a scenario where the system is bound to collapse. Whether the stability of the system is improved or not is analysed and discussed.



# Acknowledgements

Anders Rantzer and Magnus Perninge at the Department of Automatic Control have supervised the work done for this thesis. A special thanks goes out to Lars Lindgren at the Department of Industrial Electrical Engineering and Automation for spending time explaining basic power system properties and teaching me how to operate the *ARISTO* simulator.



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# 1

## Introduction

This chapter starts with some historical background on the subject of power system failure and continues with a motivation why further research on the subject might be of interest. It then introduces the more specific goals with this thesis and presents the different software tools used. Finally, brief descriptions of the different chapters are presented.

### 1.1 Historical Background

In January 1996, the Swedish government deregulated the Swedish electricity market. This was following years of similar deregulations in other European countries and some people hoped that a competitive market would lower the electricity costs for the consumers<sup>1</sup>. The Swedish electricity market therefore fused with the Norwegian, who have had their market deregulated some years earlier, and was some years later also directly linked to the Finnish and Danish electricity markets when their deregulations took place. While the distribution of electricity is still monopolized, with the state-owned transmission system operator Svenska Kraftnät maintaining the main 400 kV grid, the power companies now have greater liberties when controlling the production of electricity.

The deregulated electricity production stresses the power grids even more than before [Sui et al., 2008]. The system operates closer to its maximal capacity and the safety margins are therefore smaller, making the event of system failure more likely. The increased use of alternative energy sources that produces power in a more irregular way, like wind power, might also add some extra strain on the system.

On September 23 2003, at 12:36, a large part of southern Sweden, as well as eastern Denmark, experienced a massive power failure [*Elavbrottet 23 september 2003 - händelser och åtgärder* 2003]. The blackout lasted up to 5 hours at some places and a total of 1,5 million people were affected. It was one of the greatest

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<sup>1</sup> This has not happened. On the contrary, the prices have increased since the deregulations. The increased prices might however be the results of other factors.

power failures ever experienced in northern Europe and the total societal cost was estimated to be around 500 million SEK.

It was a day with mild weather and the system was lightly loaded. Maybe because of this, it seemed like an appropriate day to temporarily disable many of the power lines for maintenance work. The lines on maintenance included, among others, the HVDC links to Germany and Poland. 4 of the 11 Swedish nuclear reactors available at that time were also out of service. Out of these 7 active reactors, only 4 were positioned in southern Sweden.

The first disturbance occurred at 12:30 when the only active nuclear reactor at the Oskarshamn power plant experienced a valve problem and had to shut down. Increased amounts of electricity had therefore to be imported from Finland, Denmark and southern Norway. The power transfers from northern to southern Sweden increased as well. This is not a tremendously rare occurrence and the power system should be able to handle it. However, only 5 minutes afterwards, at 12:35, a double busbar fault occurred at Horred in southwestern Sweden. This resulted in both significantly weakened transmission capabilities in western Sweden as well as the inability of two more nuclear reactors to deliver their power since they were connected to the busbars.

The total losses of 3000 MW nuclear produced power, as well as the line outages, became too much for the system to handle. After an additional minute of decreasing voltages the main 400 kV power grid shut down in Sörmland, south of Stockholm, and thus separated the two portions of the grid. The southern region had no way to balance its power generation with its consumption and the blackout therefore occurred there while the northern region returned to a stable state.

## **1.2 Motivation**

The magnitude of this blackout motivated the construction of additional power lines to strengthen the transfer capabilities to southern Sweden. The reopening and reconstruction of Öresundsverket in Malmö also serves to lessen the risk of another blackout since it increases the available power generation in southern Sweden.

The reason for the collapse was the systems inability to sustain low voltage values, a so-called voltage collapse. Granted that this scenario was a little bit extreme, with several disturbances occurring within an unfortunate short time span, one might still ask if the collapse could have been prevented by applying knowledge of voltage stability to control the system into a stable state. This can perhaps be done by rescheduling the remaining generation in an appropriate manner. And even if that turns out to be unsuccessful in this particular case, the method might be used to prevent other future blackouts. After all, this is not the only occurrence of a large-scale voltage collapse. This thesis will therefore focus on voltage stability and how the variables of a power system might be controlled in order to avoid a voltage collapse.

## 1.3 Goals

The goal of this thesis was to examine how an optimization method proposed in [Lavei et al., 2011] could be used on a power system in the event of some sort of large disturbance. More specifically, it involved both the implementation of the optimization method and the analysis of the results, as well as an effort to apply the results to control a more complex simulation model that experienced a disturbance.

## 1.4 Tools used

All the code was written in *MATLAB*. This is a list of the other software tools used.

### SeDuMi

*SeDuMi* is a software package for *MATLAB* that can be used to solve optimization problems over symmetric cones. In this thesis it will be used to solve semidefinite optimization problems. It is distributed freely and can be downloaded from <http://sedumi.ie.lehigh.edu/>.

### YALMIP

*YALMIP* is an optimization toolbox for *MATLAB* that provides an environment for high-level modelling [Löfberg, 2004]. It relies on external solvers for the actual computations (in this case, *SeDuMi* is used). In this thesis, it is used when implementing the previously mentioned optimization method. It is distributed freely and can be downloaded from <http://users.isy.liu.se/johanl/yalmip/>.

### ARISTO

*ARISTO* is a real-time power system simulator developed and owned by Svenska Kraftnät. Since 1993, it has been used both as an instrument in power system research and as a tool to teach power system operators to be better at handling disturbances. It has an implementation of Nordic32, a power system model resembling the Nordic power system, that will be used in this thesis.

### amcx

*amcx* is a *MATLAB* tool written by Lars Lindgren at the Department of Industrial Electrical Engineering and Automation in Lund. It works as a link between *MATLAB* and *ARISTO*, collecting data and enabling changes to the power system variables and reference values from *MATLAB*. It is used as a way to control the generators during the simulations in *ARISTO*.

## 1.5 Outline of the Report

The outline is as follows: Chapter 2 goes through the rudimentary power system theory needed for this thesis, along with how the system can be modelled. It also discusses stability in power systems. Chapter 3 mentions some of the system variables available to control and how changing them might affect the stability in the system. In chapter 4, the mathematical optimization theory needed for this thesis is presented along with how the desired optimization method works and how it is implemented. Chapter 5 presents some optimization results from simple test systems. Finally, in Chapter 6, the *Nordic32* test system is presented along with the simulation scenario. This is followed by discussions considering the results from several simulations.

# 2

## Power System Modelling and Stability

This chapter starts off with going through the rudimentary theory concerning power systems that is necessary in order to understand this report. The section after that covers power system modelling and how the loads behave in dynamic situations. The final sections introduces the concept of power system stability and discusses how the stability can be classified into different subcategories.

### 2.1 General Power System Properties

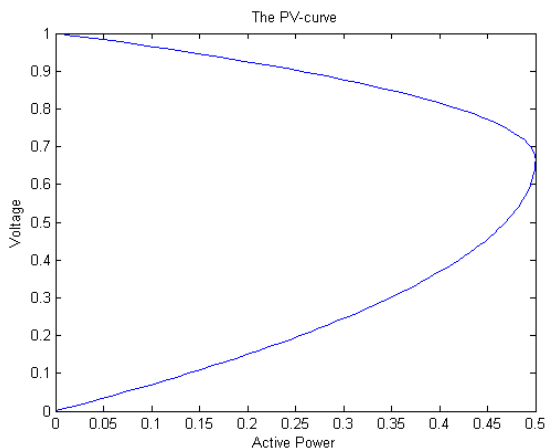
The reader is assumed to have at least a basic understanding of electric circuits and electrodynamics. That includes, among other things, phenomena such as complex voltages, active or reactive power and transformers. Some other general theory that is relevant will be presented here.

#### Shunt capacitors and shunt reactors

When discussing shunt capacitors and shunt reactors one refers to capacitors and reactors (inductors) that are connected between a power system node and ground. These connections will affect the reactive power in the node. For example, consider a capacitor with capacitance  $C_s$  connected between ground and a node with voltage  $V$ . The capacitance will create an impedance  $Z$  of  $\frac{1}{jC_s}$ . The change in complex power,  $S$ , in that node will be as follows.

$$S = VI^H = V\left(\frac{V-0}{Z}\right)^H = V\left(\frac{V}{\frac{1}{jC_s}}\right)^H = V(jC_s V)^H = -jC_s |V|^2 \quad (2.1)$$

Connecting a shunt capacitance to a power system node will decrease the reactive power output in that node. It can be shown analogously that connecting a shunt reactor will increase the reactive power. This will be of importance in the chapter



**Figure 2.1** A typical PV-curve

about Power System Control since connecting and disconnecting shunt capacitors and reactors are control options when working with power system stability.

## PV-diagrams

A convenient tool to use when dealing with voltage stability is the PV-curve. It can be described as the voltage in a node as a function of the acquired active power in that node. Every node has its own PV-curve. The curve is often presented tilted, with the active power on the x-axis. Figure 2.1 displays a PV-curve with its typical structure.

Several conclusions can be drawn from this figure. First of all, there is some kind of maximal value on the active power that can be acquired. Another conclusion is that, apart from the voltage value corresponding to the maximal active power value, there exists two different voltage values that will match every active power value. This will be of importance when discussing voltage stability.

## The per unit-system

When working with power systems, especially larger ones, it is common to normalize the different quantities. This is done by choosing a base power for the whole system and one or several base voltages for different regions of the system. The different regions are commonly connected to each other by transformers. The new per unit-values are acquired by taking the original values and dividing them by their chosen base value. These base voltages are typically chosen so that the value of the per unit-voltage is around 1. Instead of working with units like volt and ampere, one instead uses p.u. (per unit). How base currents ( $I_b$ ) and impedances ( $Z_b$ ) are calculated can be seen below ( $S_b$  is the chosen base power and  $V_b$  is a base voltage).

$$I_b = \frac{S_b}{\sqrt{3}V_b} \quad (2.2)$$

$$Z_b = \frac{V_b}{\sqrt{3}I_b} = \frac{V_b^2}{S_b} \quad (2.3)$$

There are plenty of reasons for doing this. For example, it makes analysis simpler. If one node experiences a voltage drop of 5 V it is of much greater importance if the base voltage is 10 V (drop of 0.5 p.u.) than if the base voltage is 400 V (drop of 0.0125 p.u.). It also makes it easier to declare constraints since one might limit every voltage to be within 0.9-1.1 p.u. regardless of their actual value in volt.

Since this thesis is dealing with numerically demanding computations, it might also be beneficiary to use the per unit-system because the quantities used will be around 1. To use a large realm of values in one numerical computation might lead to numerical issues and imprecise results.

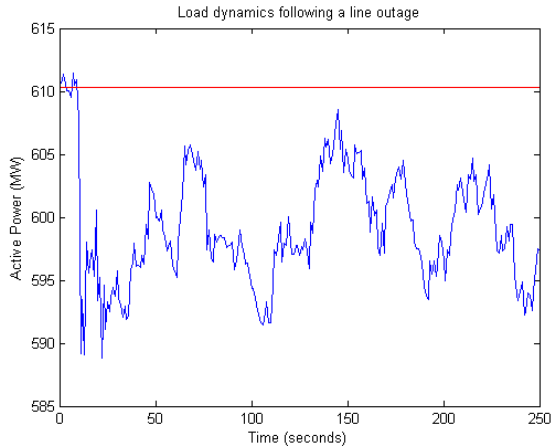
## 2.2 Power System Modelling

One significant portion of the work done for this thesis was to create mathematical representations of power systems that could be used with the optimization routine described in the optimization chapter. This includes modelling the load behaviour as well as creating a model of the overlaying system structure with the power flows and node voltages. Modelling of other specific components, such as generators and their dynamical behaviour, won't be covered by this thesis. Some texts that have been relevant when doing the modelling are [Arnborg et al., 1998],[Jóhannsson et al., 2013],[Navarro, 2005] and [Lavei et al., 2011].

### Modelling the Loads

In reality, the parameters concerning the loads (voltages, power outputs, etc.) won't be constant, especially not when the system is undergoing drastic changes. Figure 2.2 display the load behaviour following a line outage in an *ARISTO* simulation. Stable operations of a power system depends on its ability to always match the generation with the loads. When modelling a power system for stability analysis, one therefore has to pay special attention to how the load dynamics should be implemented. Load modelling is a complex issue and is, since the load dynamics is affected by many factors, generally based on considerable simplification. The modelling can be done in several ways. This thesis won't discuss them all in detail and will use an extremely simple load model when modelling power systems. This subsection will be more of a general overview of the subject.

First of all, how the load dynamics should be modelled depends on which time frame as well as which kind of power system stability that should be studied in the stability analysis. Further on in this text, only long-term voltage stability, which



**Figure 2.2** The red line is the consumed active power in the node prior to the line outage. The blue line is the actual power consumed in the node following a line outage after 10 seconds.

will be explained later in this chapter, will be considered. Load modelling might however be done in similar or identical ways when working with other kinds of stability.

The different load models are usually classified into two different categories, dynamic load models and static load models. A static load model presents the power and voltage in a node as directly dependent on each other, but only when considering their values at the same time. Dynamic load modelling, on the other hand, presents the relation as dependent not only on their present values but as a function of their whole history. A popular way to do static load modelling is by defining the active power  $P$  as an function of the voltage  $V$  to the power of some variable  $a$ .

$$P = P_0 \left( \frac{V}{V_0} \right)^a \quad (2.4)$$

$P_0$  and  $V_0$  are the initial values of the active power and the voltage. The reactive power can be modelled in an analogous way. This is called the exponential load model. Another common static load model is the ZIP-model which models the power as a polynomial.

$$P = P_0 \left( p_1 \left( \frac{V}{V_0} \right)^2 + p_2 \frac{V}{V_0} + p_3 \right) \quad (2.5)$$

All the  $p$ -variables are constants and the reactive power is modelled in an identical way. ZIP refers to impedance, current and power since the whole expression is made up of one term with constant impedance, one with constant current and one



with constant power. Finally, a dynamic load model with exponential recovery can have look like this:

$$T_p \frac{dP}{dt} + P = P_0 \left( \left( \frac{V}{V_0} \right)^{\alpha_s} - \left( \frac{V}{V_0} \right)^{\alpha_t} \right) \quad (2.6)$$

$T_p$ ,  $\alpha_s$  and  $\alpha_t$  are all constants.  $T_p$  is load recovery constant that covers the dynamic aspects of the load model.  $\alpha_t$  refers to the transient dependence of voltage while  $\alpha_s$  refers to the steady state dependence which will be dominating when working with long term analysis. This model might be implemented in many different ways depending on how one decides to handle the time-derivative.

## Modelling the Power System Flows

This subsection will go through how all the power flows in a power system can be modelled using one large system matrix. It will be a description of a DC power system but an AC power system can be modelled in a similar way.

Considering a DC transmission system with known line admittances, the flow from node  $i$  to node  $j$  can, using Kirchhoff's current law, be written as

$$I_{ij} = y_{ij}(V_i - V_j) \quad (2.7)$$

where  $y_{ij}$  is the line admittance between node  $i$  and  $j$ .  $V_i$  and  $V_j$  are the voltages at node  $i$  and  $j$  respectively. The total sum of all flows in and out of node  $i$  becomes

$$I_i = \sum_{j \neq i} y_{ij}(V_i - V_j) \quad (2.8)$$

This allows for a generalized matrix representation of the currents that looks like

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} \sum_{j \neq 1} y_{1j} & -y_{12} & \cdots & -y_{1n} \\ -y_{21} & \sum_{j \neq 2} y_{2j} & \cdots & -y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{n1} & -y_{n2} & \vdots & \sum_{j \neq n} y_{nj} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad (2.9)$$

The matrix containing all the admittances is called the bus admittance matrix [PSCS-course]. Since all the line admittances are positive, the matrix will have positive values on the diagonal and negative values everywhere else. The power  $P$  in node  $i$  is the product of  $V_i$  and  $I_i$ . The vector with voltages together with the bus admittance matrix may be used to calculate the total power loss in the whole system. Generator nodes might have positive values on  $P$  while consumer nodes will always have a negative value. By setting each row in the bus admittance matrix that is related to a consumer node to zero, the following expression is equivalent to the total generated power in the system.

$$V^T M_0 V \quad (2.10)$$

$V$  is a column vector containing the voltages in all nodes and  $M_0$  is the configured bus admittance matrix that only contains values different from zero on rows corresponding to generator nodes. This is the function that needs to be minimized in the optimization. That the function is quadratic and therefore nonlinear is obvious and its nonconvexity might be derived from the properties of the bus admittance matrix. In every node there is a constraint on the power that can be formulated as

$$V^T M_k V \leq P_k \quad (2.11)$$

where  $M_k$  is a zero matrix except for row  $k$  which is the  $k$ :th row from the bus admittance matrix.  $P_k$  might be a positive number if node  $k$  is a generator node and will always be negative if node  $k$  is a consumer. This is to assure that the generated power in every node is within acceptable limits while supplying the consumer nodes with their required power. Other constraints, such as limits on the acceptable voltages, might also be present.

Modelling an AC power system can be done in a similar way. The differences are that the voltage vector on the left side of the  $M$ -matrices has to be Hermitian transposed instead of regular transposed. The values in the  $M$ -matrices will also be complex since the line admittances are complex.

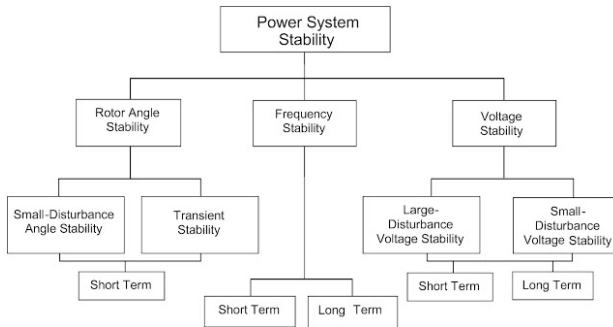
### 2.3 Power System Stability

A proposed definition of power system stability, taken from [Kundur et al., 2004], is as follows.

“Power system stability is the ability of an electric power system, for a given initial condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact.”

The definition considers the stability of the whole system but sometimes one only concerns oneself with the stability of a certain component, or a certain section of the grid. Nevertheless, in order for a system to be called stable every component has to maintain stability during a contingency. In several cases an unstable component will create cascading failures that forces other parts of the system into instability.

Real power systems in the nordic countries are always designed to be able to handle one large contingency, no matter where it is. That is, the system should always be able to maintain stability if any one transformer or power line is disabled at one time and can't therefore be overly dependent on one single component. This is called that the power system is fulfilling the  $n - 1$  criterium. [*Elavbrottet 23 september 2003 - händelser och åtgärder 2003*]



**Figure 2.3** Classification of power system stability

The disturbances might differ in magnitude. Some of them, such as a change of load demand, are smaller and occur frequently while a larger disturbance, such as a tripping of a line or a failing generator, happen more seldom. The instability might also occur at different time slots. Sometimes the system turns unstable a few seconds after the contingency and other times the system will appear stable for several minutes before a visible collapse occurs. Even though the stability criteria is the same in all these cases it is convenient to treat the problems separately since the causes for instability and the methods used to analyze and stabilize differ. This calls for a classification of different kinds of stability.

## 2.4 Different Kinds of Stability

The figure above displays all different types of stability in power systems. This thesis only deals with long-term voltage stability. For the sake of completion, the other kinds of stability will also be briefly mentioned.

### Rotor Angle Stability

The study of rotor angle stability deals with the ability of keeping the synchronous machines in the generator nodes in synchronism after a disturbance. The subsection of rotor angle stability that deals with large disturbances is often referred to as transient stability. The time frame of interest when studying transient stability is usually 3-5 seconds after the perturbation. Historically, stability problems of transient stability nature has been among the most occurring in the field of power system stability problems. Because of this, studies on transient stability answers for a large part of the research done on power system stability.

### Frequency Stability

Frequency stability refers to the ability of maintaining a nominal frequency during system operation when the balance between generation and load is severely

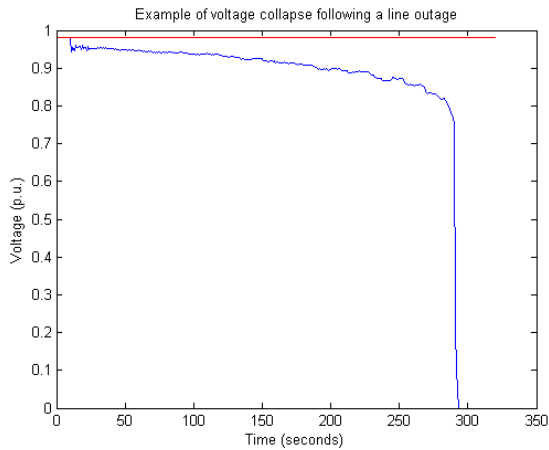
disturbed. The time frame of interest may vary since the disturbances causing frequency instability can be of very different character. For example, slow processes such as boiler dynamics might cause a long-term instability after several minutes. A short-term instability can happen when one or more power lines are disconnected and an isolated section of the power system is created. If this “island” has insufficient power generation to supply its power demands, the frequency will drop rapidly and the island will experience a blackout in seconds.

## Voltage Stability

Voltage stability refers to the ability of a power system to maintain bus voltages sufficiently near their nominal values following a contingency. Loads, voltage characteristics, and load dynamics are key components in the study of voltage stability and the general voltage stability problem depends on the systems ability to restore equilibrium between power demand and power supply. Voltage stability can, as can be seen in figure 2.3 above, be classified into several different subcategories depending on the time scale and the severity of the disturbance. Small disturbances, such as small incremental changes to power demand, are not studied in this thesis and all the power system models used are assumed to be small disturbance voltage stable. Short-term voltage stability has, just like transient stability, a time frame of interest of a few seconds and involves the dynamics of fast load components such as induction motors and HVDC links. However, short-term phenomena in general are also not of interest in this thesis and all the simulations will only cover scenarios where the system survives the first couple of seconds.

Long term voltage stability when the system is exposed to a severe disturbance is what this thesis is going to focus on. The time frame of interest extends to several minutes and the dynamics involve slow acting equipment such as tap-changing transformers and thermostatically controlled loads. The large disturbance might be a failing generator or a disconnected power transmission line. In most examples in this thesis, it will be the latter. When either one of these disturbances occur, the voltage in the load node will drop in order to make sure that a sufficient amount of power is delivered from the remaining accessible power sources into the load. As the load demand increases from the initial drop the voltages in the system has to somehow change in order to make sure that the new power demands are fulfilled. If the voltages in the nodes directly connected to the load node does not increase by a sufficient amount then the voltage in the load node has to decrease even more to deliver the demanded power to the node. This process of constantly decreasing node voltages might lead to a scenario where the system is unable to meet the demands from the loads and a partial or total blackout of the system will occur. This is what is often referred to as a voltage collapse. So, in order to prevent this chain of events one might want the voltages to not differ too much from their nominal value.

Figure 2.4 displays a voltage collapse following a large disturbance. The structural change of the system (in this case a disconnected power transmission line) can



**Figure 2.4** The red line is the stationary voltage value in a power system node prior to the line outage. The blue line is the actual development of the voltage value of the power system node following the line outage.

easily be detected by the voltage drop after about 10 seconds. The voltage then continues to drop during several minutes until the system is unable to sustain the low voltage and a blackout occurs after 290 seconds.

What action can be taken to prevent this from happening? There are actually numerous different methods that can be used to control the system and the next chapter will present some of them.

# 3

## Power System Control

The previous chapter presented some general properties of power systems along with some different types of power system stability problems and events that led to instability. This chapter will discuss some methods that can be used to control the system to keep it stable. Once again, the focus will be on long term voltage stability.

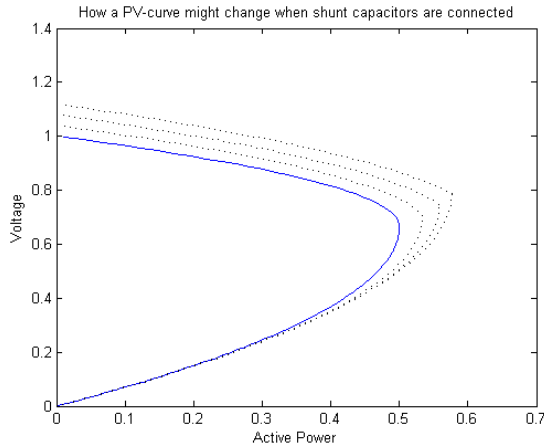
First, some of the different system variables available as control options will be mentioned. A brief introduction to Model Predictive Control, the general real-time control method used in this thesis, will follow in the subsequent section.

### 3.1 Introduction

The simplest way to control power systems to avoid voltage collapse would be to react to the basic signs of voltage instability like low voltage levels with direct actions like load shedding. That is, disconnecting parts of the system in order to save the rest of it from a larger failure. Load shedding can and will always be a control option but in several cases the whole system can be saved using less drastic control methods. In recent years the ability to measure system-wide voltage levels and power flow using PMUs (Phasor Measurement Units) has improved. The availability of that information without a considerable time delay, as well as increased computational capabilities to process that information, provides more advanced real-time approaches to power system control. [Glavic and Cutsem, 2009]

### 3.2 Controlling Shunts

One control option used in some papers [Glavic et al., 2011] is that of connecting shunt capacitors or shunt reactors to the nodes to relieve the system of the fluctuations in reactive power flow. Connecting a shunt capacitor to a node will change the PV-curve belonging to that node so that the node can be supplied with the same amount of active power as before but at a higher bus voltage. An example of how the PV-curve might change when one or several shunt capacitors are connected can be



**Figure 3.1** The blue line is the original PV-curve and the dotted ones display how it might change when one, two or three shunt capacitors are connected to the node.

seen in Figure 3.1 below. Not only does the voltage operating point for a fix power output change, but the maximal possible active power output is also increased. The control option of connecting shunt capacitors is therefore something one can apply to load nodes that are experiencing low voltage levels. The drawback is that the curve becomes steeper, making the system more sensitive to disturbances to the voltage values. Shunt reactors can in the same way be connected to nodes that are experiencing high voltage levels.

The control uses some sort of scheme to decide when to connect or disconnect the shunts. In some case the scheme is very simple, it might just be to connect the appropriate type of shunt as soon as a bus voltage leaves a predetermined set of acceptable values, and in those cases the control operation might be done manually by a system control engineer. However, more complex schemes that might require automated feedback are also being researched.

Shunt control can be used in symbiosis with other types of control but will not be part of the work done in this thesis.

### 3.3 Controlling Voltages or Active Power

Another control option available is to decide a sequence of reference values for the generator voltages. This control option is also used in several papers on the subject of power system stability and control [Glavic et al., 2011] [Larsson and Karlsson, 2003] and will be used in this thesis when working on the test systems. In some cases, the generator voltages in nodes close to the failing load nodes will increase after a contingency while the voltages in the failing load nodes are continuously

dropping. By using the information given by the prediction of the decreasing voltage values in the loads, an alternative and more optimal sequence of generator voltage values might be found. This sequence might prevent voltage collapse by forcing the system out of the situation with constantly decreasing load voltages.

The final control option that will be discussed is that of rescheduling active power outputs from the generator. This is the control method that will receive the most attention in this report since it will be used when controlling the Nordic32 power system model. It is similar to the previously mentioned control option in the sense that it involves the optimization of some sort of cost function in order to decide how the active power generation should change over time. Unlike controlling generator voltages, this might change more of the overall power flow structure of the system and can help relieve the nodes that will become more stressed after the system experiences the contingency. In order to make the method compliant with reality, some sort of restriction on how much the active power generation can be changed has to be imposed.

### 3.4 Model Predictive Control

The main idea behind *MPC* is that a system model modelling the dynamical behaviour of the real system is used in the real time control process to predict the future dynamics of the system during a contingency. The model might be updated during the control process to better fit the actual system state trajectories with those derived from the model and the new updated model might then be used in further control actions.

If the controller does not have access to the whole model structure, system state trajectories based on simulations in which no control actions are taken can be used as an approximation of the future system behaviour. These trajectories can then be used to determine the sequence of future control actions. Even in the somewhat unlikely case that the model should be absolutely accurate, the predicted dynamics will differ from the actual dynamics when the control is active. That is of course because the control actions taken will affect the system behaviour. Hopefully, the control actions taken will still prove meaningful and, if that is the case, save the system from a voltage collapse. That is the simple approach to *MPC* that will be used in the simulations later in this thesis. The sequence of control actions will only be declared once, no further real-time feedback in which more accurate system state trajectories are fed into the controller will be used.



# 4

## Optimization

This chapter starts of with going through some of the more basic optimization theory used in this thesis. The optimal power flow problem is then introduced and the method used to solve it, along with the modifications and assumptions made, are discussed in the final section.

### 4.1 General Optimization

Optimization in the way relevant for this thesis deals with finding the highest or lowest value of an objective function, in this case often denoted a cost function, while keeping the function parameters within acceptable bounds and fulfilling any number of other predetermined criteria.

Convexity is an important concept in optimization theory and this section will therefore start with some relevant definitions. [Böiers, 2010]

**Definition 1.** A set  $S$  in  $\mathbb{R}^n$  is called a convex set if

$$x_1, x_2 \in S \implies \lambda x_1 + (1 - \lambda)x_2 \in S \quad \forall \lambda \in [0, 1] \quad (4.1)$$

**Definition 2.** A function  $f$ , defined on a convex set  $S \subset \mathbb{R}^n$ , is called a convex function if

$$x_1, x_2 \in S \implies f(\lambda x_1 + (1 - \lambda)x_2) \leq \lambda f(x_1) + (1 - \lambda)f(x_2) \quad \forall \lambda \in [0, 1] \quad (4.2)$$

A convex function have the important property that any local minimum must also be the global minimum. This is very beneficial from an optimization standpoint since the search after the minimal value can be terminated once any local minimum is found. Optimization using a convex cost function, referred to as convex optimization, make up a significant subfield of optimization.

The field of convex optimization can also be categorised into several subcategories depending on, for example, the complexity of the objective function or the nature of the constraints. If the objective function is linear and the optimization

involves a constraint that an affine combination of symmetric matrices is positive semidefinite, then the optimization problem is within the field of semidefinite programming. The exact meaning of that constraint won't be discussed in great detail but the important aspect of semidefinite programming problems is that there exist well-performing methods for solving them and that they have a polynomial worst-case complexity.

## 4.2 The Optimal Power Flow Problem

The optimal power flow (OPF) problem is fundamental when studying power systems. It is an important part of many different appliances and will in this case be studied in the context of voltage or power control for improved stability of power systems. This section serves as an introduction to the subject, based primarily on [Lavei and Low, 2012]. The goal of solving the OPF problem is to find an optimal operating point that supplies each load with sufficient active and reactive power while keeping the generated reactive power and the bus voltages within certain bounds. Operating point in this context refers to a set of node voltages or a set of values for the power generated at the generators. Often, this means keeping the total power generation as close to the total power consumption as possible.

What the OPF problem seeks out to optimize may vary but in this thesis the aim is to minimize the sum of all the generated active power from all generator buses. Another cost function option that is sometimes used is a quadratic sum of all the active power generation. That kind of quadratic cost function can actually also be handled by a modified version of the optimization method introduced in the next chapter but will not be part of the work done for this thesis. The constraints used are restrictions on the magnitude of the node voltages, on the active and reactive power and on the power transmissions in each power line. A general arrangement of the constraints for a node  $k$  in a power system with  $n$  nodes looks like this.

$$P_k^{min} \leq P_k \leq P_k^{max} \quad (4.3)$$

$$Q_k^{min} \leq Q_k \leq Q_k^{max} \quad (4.4)$$

$$V_k^{min} \leq |V| \leq V_k^{max} \quad (4.5)$$

$$|V_k - V_i| \leq c_{ki} \quad \forall i \in [1..n] \quad (4.6)$$

The  $c_{ki}$  constraint is declared to limit the maximal power that can be lost in the line between node  $k$  and  $i$ . If node  $k$  and  $i$  are not directly connected to each other, this value can be seen as infinity and the constraint can be disregarded from in the optimization. Not all the other constraint might be used either. For a node defined as a load node (no generator connected to the node) the value of  $P_k^{max}$  will always be less than or equal to zero and the value of  $P_k^{min}$  will be disregarded from in all

the cases studied in this thesis. The same principles applies to both  $Q_k^{max}$  and  $Q_k^{min}$  for load nodes. For generator nodes on the other hand,  $P_k^{max}$  might be greater than or equal to zero. The value of  $P_k^{min}$  might not be defined in some nodes and might coincide with  $P_k^{max}$  in other nodes, those whose generator output is interchangeable. Other constraints might of course occur when working with OPF problems but these are all that will be implemented for the optimization routine used in this thesis.

This is a static problem formulation concerning a steady state solution of the problem. It might also be of interest to consider a starting operating point of the system and analyse how one can move from that state into a more optimal one while fulfilling all these static constraints all the time. Constraints relevant to how quickly the system parameters can be changed, so called dynamic constraints, has also to be imposed and fulfilled. This will be discussed later in the next section.

One aspect of the OPF problem is that it is neither linear nor convex. This affects both computational speed and reliability of the optimization since only a locally optimal solution can be guaranteed. This thesis uses a method proposed in [Lavei et al., 2011] that transforms the OPF problem into another problem that is linear and therefore convex. The following section will describe this method.

### 4.3 Power Flow Optimization using Quadratic Programming

#### Static Case

This section is a description of the optimization method proposed in [Lavei et al., 2011] and how it is implemented in this thesis. It should be noted that although the following is a description of the method in a DC system, all actual optimizations done for this paper only deals with AC systems. Comments regarding the usage of this method in an AC system can be found in the end of this subsection. This method assumes a power system modelled as described in the chapter about power system modelling.

The proposed optimization method is as follows

**Proposition 1.** *Let  $M_0, \dots, M_K \in \mathbb{R}^{n \times n}$  be Metzler and  $b_1, \dots, b_K \in \mathbb{R}$ . Then*

$$\begin{aligned}
 \text{maximize} \quad & x^T M_0 x & = \text{maximize} \quad & \text{trace}(M_0 X) \\
 \text{subject to} \quad & x \in \mathbb{R}_+^n & \text{subject to} \quad & X \succeq 0 \\
 & x^T M_k x \geq b_k & & \text{trace}(M_k X) \geq b_k \\
 & k = 1, \dots, K & & k = 1, \dots, K
 \end{aligned} \tag{4.7}$$

A matrix is Metzler if all off-diagonal elements are non-negative. The  $\succeq$  symbol refers to the eigenvalues of the matrix and is in this particular case equivalent to the matrix being positive semi-definite (all eigenvalues greater than or equal to zero).

Minimizing  $M_0$  is the same operation as maximizing  $-M_0$  and it can easily be shown from the previous mentioned properties of the bus admittance matrix that the negative  $M$  is Metzler. All the matrices used in the power constraint formulations can also be shown to be Metzler since they are segments of the bus admittance matrix.

This new optimization problem is convex and can be solved exactly and efficiently using a solver for semi-definite programming problems. It can be shown that the solution will always be a matrix with rank one and equivalent to the outer product of the optimal solution to the original problem, i.e.,

$$x^T x = X \tag{4.8}$$

The proof of this is presented in [Lavei et al., 2011]. This means that the desired solution can always be easily extracted from the solution to the convex problem.

However, all of these properties are only guaranteed for a DC power system. In an AC power system the line admittances and voltages will be complex numbers and the matrices will no longer be Metzler. The same principles are still used but the feasibility of the extracted solution have to be controlled as well as the eigenvalues of the  $X$  matrix. If the second-largest eigenvalue differs significantly from zero the optimality of the solution may be questionable.

This subsection has been a description of the optimization method as it was originally proposed. The following subsections will discuss the modifications which was done for the sake of this thesis.

### Dynamic case

In the time-varying case a time dimension variable is included in the voltages and the number of variables in the OPF problem is therefore multiplied by the number of time steps. By sorting the voltages primarily by node number and secondarily by time, the variables can be put into a column vector. An example of this is the transformation shown below.

$$\begin{bmatrix} x_{1,1} & x_{2,1} & x_{3,1} \\ x_{1,2} & x_{2,2} & x_{3,2} \\ x_{1,3} & x_{2,3} & x_{3,3} \end{bmatrix} \mapsto \begin{bmatrix} x_{1,1} \\ x_{1,2} \\ x_{1,3} \\ x_{2,1} \\ x_{2,2} \\ x_{2,3} \\ x_{3,1} \\ x_{3,2} \\ x_{3,3} \end{bmatrix} \tag{4.9}$$

This vector is then used in an optimization routine very similar to the one in the static case. For example, in order to ensure that all the static constraints are fulfilled in the second time step the  $M_k$  matrix is copied into the 2,2-cell of a larger matrix and the constraint is declared as

$$V^T \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & M_k & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} V \leq C_k \quad (4.10)$$

where  $C_k$  is some general constraint belonging to the static system. If all of the  $M_k$  matrices are Metzler then all of the new matrices used will also be Metzler and an exact optimal solution will be found for the DC power system using the same optimization method as in the static case. Using an AC power system the feasibility conditions have to be confirmed on all different sets of voltages belonging to every single time step.

As already mentioned, the length of the voltage vector grows linearly as the number of time steps increases. This leads to the number of elements in the  $X$ -matrix growing quadratic as the number of time steps increases which will quickly lead to a very big matrix with potentially lots of variables used in the optimization. The next section will introduce dynamic constraints, i.e. constraints that connects the variables belonging to different time steps. Depending on the structure of these constraints some changes can be done to the overall structure of the problem which will speed up computational speed. These changes will be discussed in the section after that.

### Dynamic constraints

Real generators cannot change their output arbitrarily fast and because of this, dynamic constraints on the system has to be imposed. On the test systems, constraints on the variations of the voltage magnitudes were used as dynamic constraints. These constraints were originally declared as

$$V_{k,t} - dV_k \leq V_{k,t+dt} \leq V_{k,t} + dV_k \quad (4.11)$$

In other words the voltages, in this case the voltage in node  $k$ , were allowed to vary linearly in time with a predefined maximum slope  $dV$ . However, the nature of this optimization method limits the ways in which one might declare constraints since every constraint has to be on the form

$$x^H M_k x \leq C_k \quad (4.12)$$

Since the constraints have to be declared using only quadratic terms the original declaration can't be used. This issue can be circumvented by squaring each term and making  $dV_k$  adequately small. All values will be real since we are dealing with voltage magnitudes and it is assumed that  $V_{k,t} - dV_k$  is non-negative.

$$(V_{k,t} - dV_k)^2 \leq V_{k,t+dt}^2 \leq (V_{k,t} + dV_k)^2 \quad (4.13)$$

$$V_{k,t}^2 - 2V_{k,t}dV_k + dV_k^2 \leq V_{k,t+dt}^2 \leq V_{k,t}^2 + 2V_{k,t}dV_k + dV_k^2$$

If  $dV_k$  is a sufficiently small number, the term  $dV_k^2$  can be disregarded from.

$$\begin{aligned} V_{k,t}^2 - 2V_{k,t}dV_k &\leq V_{k,t+dt}^2 \leq V_{k,t}^2 + 2V_{k,t}dV_k \\ -2V_{k,t}dV_k &\leq V_{k,t+dt}^2 - V_{k,t}^2 \leq 2V_{k,t}dV_k \end{aligned} \quad (4.14)$$

Since  $dV_k$  is small the value of  $V_{k,t}$  won't change much (when using the p.u.-system, the value will always be close to one) and the term  $2V_{k,t}dV_k$  can therefore be treated as a constant for a given  $dV_k$ . The results will be something very similar to a linear constraint on  $V_{k,t}$ . This new constraint can be easily constructed with a matrix. This matrix will always be Metzler since both  $V_{k,t}$  and  $V_{k,t+dt}$  are on the diagonal. The dynamic solution in the DC power system case will therefore be exact.

Dynamic constraints on the voltage variations were however only used on the simple test systems. When this optimization technique was to be used on the more advanced *Nordic32* model the same type of constraints were not used. In order to make an optimization procedure similar and comparable to an optimization done in a previous paper [Capitanescu et al., 2009] the power outputs, not the voltages, of the generators had to be able to vary linearly in time. These constraints can be easily constructed using the required template.

$$\begin{aligned} dP_{k_{min}} &\leq P_{k,t+dt} - P_{k,t} &&\leq dP_{k_{max}} \\ \Leftrightarrow dP_{k_{min}} &\leq V^H M_{k,t+dt} V - V^H M_{k,t} V &&\leq dP_{k_{max}} \\ \Leftrightarrow dP_{k_{min}} &\leq V^H (M_{k,t+dt} - M_{k,t}) V &&\leq dP_{k_{max}} \\ \Leftrightarrow dP_{k_{min}} &\leq V^H M_{new} V &&\leq dP_{k_{max}} \end{aligned} \quad (4.15)$$

The  $M_{new}$  matrix used for this constraint will never be Metzler, not even in the DC case. The feasibility of the solution will therefore always need to be examined. These constraints only deal with the limitations present when controlling the outputs of the generators. The load dynamics, i.e. how the voltages and power outputs of the consumer nodes change with the rest of the values of the states of the system, is not covered by this. When doing the optimization, the power demand in the consumer nodes are assumed to follow predetermined trajectories which are input parameters to the optimization.

## Changes done for computational purposes

A constraint on the solution that will always have to be present is the fact that the  $X$  matrix has to be positive semidefinite. A problem that arises is that  $X$  will become very large when the number of time steps increases. This will limit the performance of the optimization quite severely.

One can observe that a lot of the elements in the  $X$  matrix are not involved in the optimization at all apart from the constraint that  $X$  should be positive semidefinite. In fact, if  $X_i$  denotes the  $n \times n$  matrix belonging to the state values at time step  $i$  in an  $n$ -node system,  $X$  can be described as the following block matrix.

$$X = \begin{bmatrix} X_1 & Y_{1,2} & Y_{1,3} & \cdots \\ Y_{2,1} & X_2 & Y_{2,3} & \cdots \\ Y_{3,1} & Y_{3,2} & X_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (4.16)$$

All of the elements in the off-diagonal matrix blocks, denoted  $Y$  above, will not be involved in the optimization apart from the constraint that  $X$  should be positive semidefinite. This follows from the fact that all the dynamical constraint only deal with variations of voltage and power state values and that predetermined load trajectories are used. Since these variables are not of any use, the constraint that  $X$  should be positive semidefinite is replaced with constraints that every single  $X_i$  matrix should be positive semidefinite.

$$X \succeq 0 \mapsto X_1 \succeq 0, X_2 \succeq 0, \dots \quad (4.17)$$

Since the off-diagonal elements are now not involved in optimization at all, their values can just be set arbitrarily to ensure that  $X$  is positive semidefinite which will always be possible if all the sub-matrices have that property.

This will increase computation speed and decrease memory usage since much fewer variables will be used. To further decrease memory usage, one might not even declare those variables at all.  $X$  will then not be a quadratic matrix any longer.

# 5

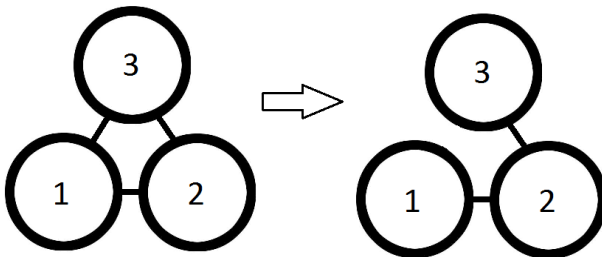
## Test Systems

In order to try out the optimization routine two very simple test systems were used. No real simulations were involved, only the results from the optimization were of interest. In all optimizations the cost function to be minimized was the sum of all the active power from all the generators.

### 5.1 Description of the Systems

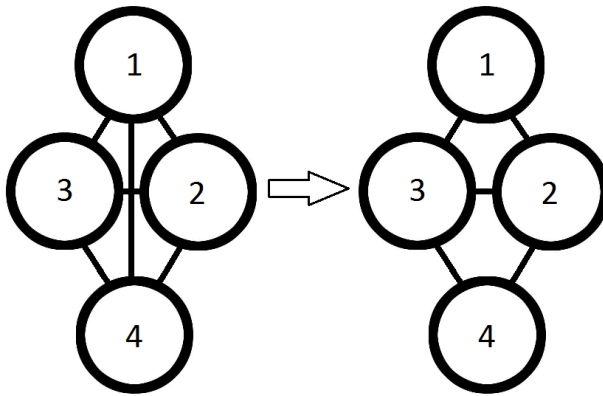
#### 3-node

Node 1 and 2 are generator nodes and node 3 is a load node. All nodes are connected to each other and all line impedances are equal with the reactance being 5 times larger than the resistance. The disconnected line is the one between node 1 and 3. The predetermined load trajectory is a large power drop followed by an exponential recovery. Linear constraints on how much the absolute values of the generator voltages can change in one time step are imposed. The static constraints on the system are maximum voltage values for both the generators with the disconnected generator (node 1) having a significantly lower maximum than the other



**Figure 5.1** A schematic overview over the 3-node test system, before and after the line outage.





**Figure 5.2** A schematic overview over the 4-node test system, before and after the line outage.

generator (node 2). Constraints on how much active power that can be lost in one line are also tried out.

### 4-node

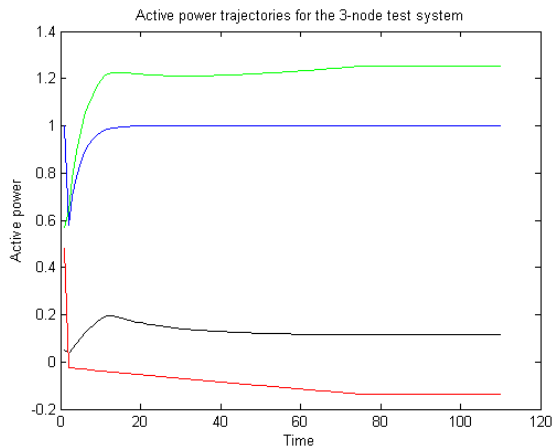
Node 1,2 and 3 are generator nodes and node 4 is a load node. The settings are identical to those of the 3-node system except that the disconnected line will be the one between node 1 and 4. The two remaining generators will not have the same dynamical constraint with one generator being much faster than the other. The fast generator will also have a somewhat larger maximum voltage value than the slower one.

## 5.2 Optimization Results

### 3-node

The following images display the optimal trajectories acquired from optimization routine.

Figure 5.3 display the acquired active power outputs from the generators as well as the predetermined load trajectory. The line outage will cause the generator in node 1 to be obsolete since it is not directly connected to the load node. A gradual increase in generated active power from generator 2 will follow. One thing to keep in mind is that the optimized cost function is the sum of the active power outputs from both generators. However, the active power output for the disconnected generator will be negative since it won't be able to provide the system with any power without the transmission line. This means that this generator will behave like a load node



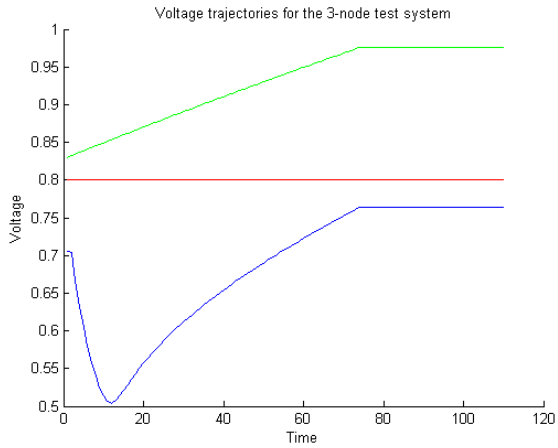
**Figure 5.3** The blue line is the predetermined load trajectory and the red and green lines are the acquired active power trajectories for generator 1 and 2. The black line represents the sum of all the active power outputs from all nodes and that is also being used as the cost function.

and it would perhaps be more preferable to only include the active power output from the other generator in the cost function. Another possible approach could be to define the cost function as the sum of all the active power losses in all transmission lines.

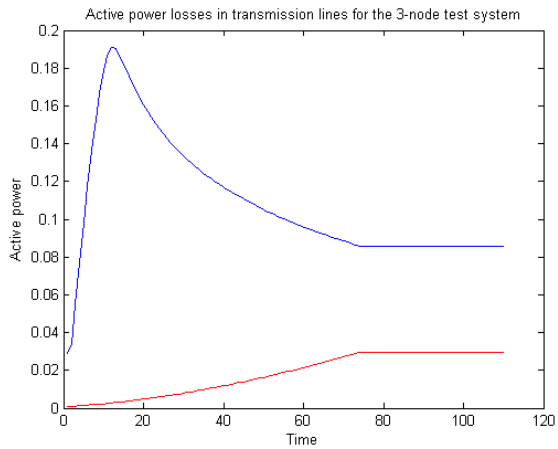
Figure 5.4 display how the voltages vary in the same time frame. In order to make sure that the load is supplied with sufficient power the voltage in the load node will experience an initial drop. Meanwhile, the voltage in generator 2 will increase as quickly as is permitted which will eventually lead to an increased load voltage as well. Initially, the voltage levels in both generators are close to each other since this will lead to a small active power loss in the line between the generators. The voltage in generator 1 will always be at its maximal value which is significantly smaller than the maximal voltage value of generator 2 and this will lead to increasing losses in the line between the generators when the voltage in generator 2 increases. Eventually the voltage level of generator 2 will settle for a value when the active power loss in the line between the generators exceeds the active power saved in the line between generator 2 and the load. This will be the optimal solution to the problem.

Figure 5.5 display the active power losses in the lines. The large peak is a result from the voltage drop in the load node. The constantly increasing loss in the line between the generators can also be seen although it will never come close to the initial losses in the other line.

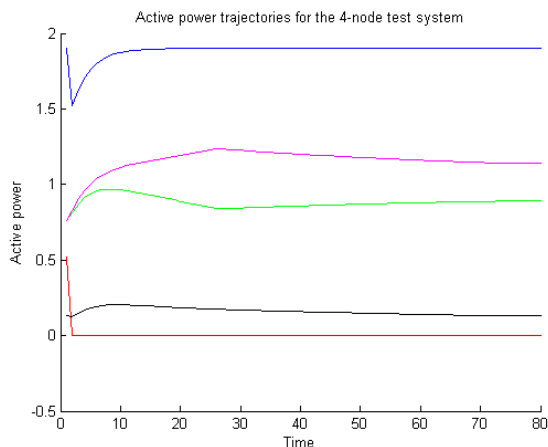
This test system can almost be considered to be trivial but was implemented in order to make sure that the optimization routine was working. In order to try to find



**Figure 5.4** The red and green line are the acquired voltage trajectories for generator 1 and 2 and the blue line is the voltage in the load node.



**Figure 5.5** The blue line is the active power lost in the line between generator 2 and the load. The red line is the active power lost in the line between the two generators.



**Figure 5.6** The blue line is the predetermined load trajectory and the red, green and purple lines are the acquired active power trajectories for generator 1, 2 and 3. The black line represents the sum of all the active power outputs from all nodes and that is also being used as the cost function.

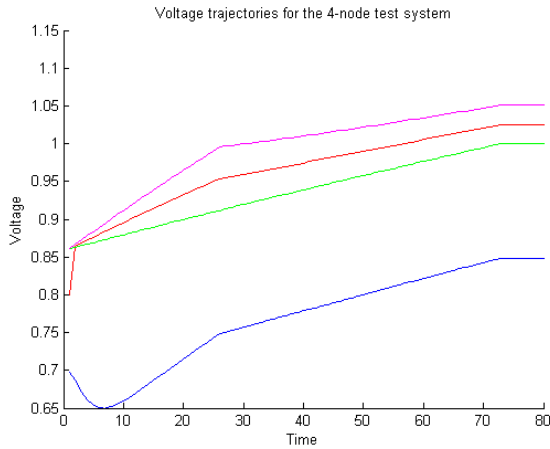
more interesting solutions the 4-node system was created.

## 4-node

The following images display the optimal trajectories acquired from optimization routine.

Figure 5.6 display the acquired active power outputs from the generators as well as the predetermined load trajectory. The big difference between this scenario and the previous 3-node one is that the generation will be rescheduled to two different generators instead of only one. Since one generator is much quicker than the other the generation will initially be very unevenly distributed with the fast generator taking care of a much larger portion of the generation. The disconnected generator will no longer act as a load node since it will just place its voltage value right between those of the two generator nodes that it is still connected to and its active power output will therefore always be zero.

Figure 5.7 display how the voltages vary in the same time frame. Initially, both active generators will increase their voltage as fast as possible while the disconnected generator keeps its voltage value in between. The voltage in the load node looks similar to that of the 3-node test system with an initial drop followed by an increase. The increasing differences between the voltage values for the two active generators will create increasing active power losses in the transmission lines connecting the two generators. This will lead to the optimal solution prompting the fast generator to slow down the acceleration of the voltage value increase for the fast



**Figure 5.7** The red, green and purple lines are the acquired voltage trajectories for generator 1,2 and 3 and the blue line is the voltage in the load node.

generator after about 25 time steps. The differences between the two voltage values will thereafter decrease and the generation will be partly rescheduled to the slower generator as could be seen in figure 5.6 before. Finally, the slower generator will reach its maximum voltage value and all node voltages will settle.

Unlike the previous example, this is a non-trivial solution which demonstrates the usefulness of the optimization method. It seems to provide a solution that seems to be optimal and does so even though all the theoretical requirements discussed in previous chapter are not satisfied. It does so in a pure theoretical context however, so in the next section it will be applied to a much more complex system model and also used together with a controller to control a simulation of a failing power system.

# 6

## Power System Simulations

This chapter will go through a large portion of the work done for this thesis. First, the system in which the simulations are made is introduced. This is followed by establishing the scenario which will be the center of the simulations. Finally, the results from the optimizations and simulations are presented.

### 6.1 An Overview of the Nordic32 Test System

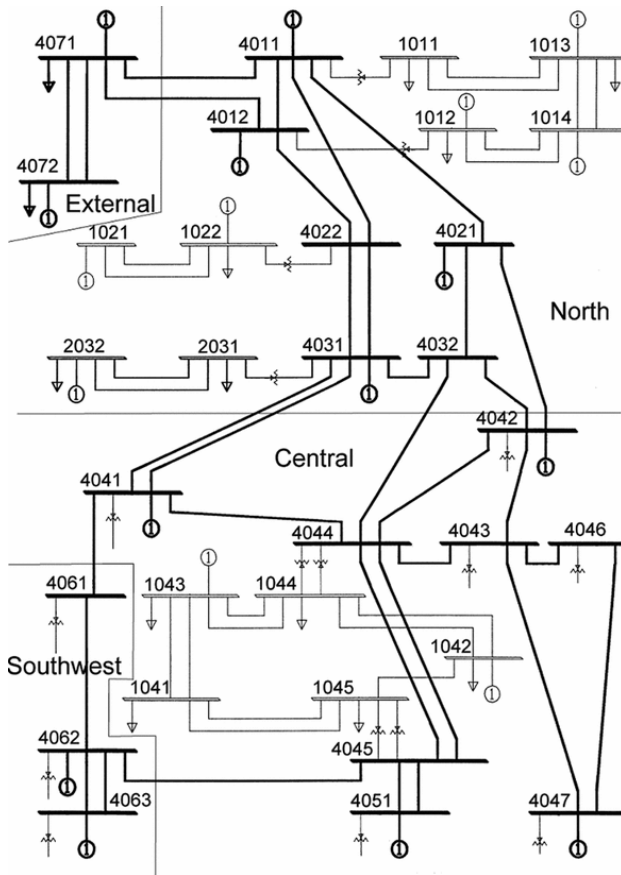
This section will go over the general properties of the *Nordic32* test system.

The power system model used in this thesis is the *Nordic32* test system as defined in [Long Term Dynamics Phase II 1995]. This will be referred to as the original documentation of the power system as there exist modified models and additional load flows not declared in that article. It is designed to resemble the actual Swedish and Nordic power system in terms of dynamical properties and behaviour and it is used in numerous articles on the subject of voltage stability [Larsson and Karlsson, 2003] [Capitanescu et al., 2009] [Glavic et al., 2011]. Just like in the real system, the desired nominal frequency is 50 Hz.

The system contains 32 nodes which are divided into one main transmission system with 400 kV base voltage and 4 regional subsystems with base voltages at 130 and 220 kV. The main transmission system contains 19 of the 32 nodes and consists of 4 major parts; north, central, southwest and external. The southwest<sup>1</sup> and external parts are almost self-contained in terms of power generation and consumption while there are major load flows coming from the north into the central region. This resembles the actual Swedish power system where there are lots of hydro generation in the north while most of the power consumption is taking place in central and southern Sweden. The original documentation declares the required active and reactive power for all the loads in the system.

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<sup>1</sup> In reality, the southwest region is no longer self-contained since the closure of the Barsebäck nuclear power plant in southern Sweden. However, this model was originally constructed in 1995 when the power plant was still active.



**Figure 6.1** A schematic overview of the *Nordic32* power system model. All 32 power system nodes are shown. The circles connected to the nodes are generators and the arrows are load. Arrows between two different nodes symbolises transformers. The shunt capacitors and reactors are not shown.

The system contains 20 generator nodes, 22 loads and 11 shunts of either type. A generator node might have one or none loads connected to it, Figure 6.1 provides an overview of the system. The original documentation present two different load flow scenarios in which some of the generator capacities differ slightly. Two (or three, depending on which load flow model is used) of the generator nodes contains two separate generator machines but they are treated as one in the optimization. The active and reactive power outputs of each machine are simply added together.

Each generator node has constraints on the reactive power output and a predefined value on the active power output. The values might differ depending on which load flow is used. In this thesis, the load flow denoted "Basic load flow" in the original documentation is used. It is the somewhat more stressed scenario of the two available.

## The Optimization Model

This section will go over how the *Nordic32* test system is implemented in the optimization routine.

With the original intent to achieve results comparable to those of an earlier publication [Capitanescu et al., 2009], the dynamical constraints imposed in the optimization routine involved the active power outputs from the generators. After the line outage the active power outputs of the generators are assumed to vary linearly in time. The time constants used to declare those constraints were derived from the parameters declared as MBASE in the original documentation. The generators are assumed to be able to increase or decrease half of their MBASE value in one hour. A table showing the MBASE values as well as the maximum increase or decrease in one second for all the generators is shown in [referens]. In the generator nodes containing several machines the MBASE values of all the machines were added together.

In the original documentation, nine of the loads are modelled as buses connected to the lower side of transformers on the main grid. These loads are modelled as separate nodes in the optimization and the total number of nodes used is therefore 41.

In order to ensure stability the voltage in each node is allowed to vary in the interval [0.95 p.u., 1.05 p.u.]. No dynamic constraints are imposed on the voltages.

The optimization routine always run over a time span of 5 minutes with time steps of 1 minute, making the total number of time steps 6 (including the pre-set stationary initial values). The resulting active power trajectories from the optimization is then used as reference values to control the generators during the simulations. After 5 minutes, no further changes to the references for the active power generation will be made.



## The Simulation Model

This section will comment on how the *Nordic32* test system is implemented in the *ARISTO*.

No load flow model equivalent to the the desired "Basic load flow" was found to be already implemented in *ARISTO*. Because of this, several parameters had to be manually changed. Starting out with a load flow model with high power transfers, the values of loads and shunts were configured. The transformer ratios were also not equivalent with those in the original documentation. Since changing the values of those in *ARISTO* would be cumbersome, they were instead changed in the optimization model so that both models were equivalent in that aspect.

All generators in the simulation model will have upper limits on the active power generation. This is contrary to the original documentation used for the optimization routine, where no such constraints were clearly defined.

It should be noted that although the optimization is done with the original *Nordic32* model the model used in the simulations in *ARISTO* is the updated *Nordic32 – svedala*. The latter has an additional number of subsystems with lower base voltage levels but the behaviour in the higher voltage levels should not differ much from the original model. For the sake of simplicity, the lower voltage subsystems were disconnected from the rest of the system during the simulations. Only one subsystem, where the generation of one node was relocated and done on that lower voltage level, was still connected to the system. The generation in some of the nodes are also divided up into several separate machines. This differs from the original definition from [*Long Term Dynamics Phase II 1995*] where the generation in most nodes was done by one single generator. In order to make sure that this will affect the behaviour of the system as little as possible, the generation was rescheduled prior to the simulations so that each generator in each node was working at the same percentage of its maximum capacity. For example, consider a case where the total generation in one node was to be 500 MW and two generator units were available with maximum generation capacities of 600 MW and 400 MW. In that case, the first generator unit would have to supply the system with 300 MW while the second supplied 200 MW so that both operated at 50 % of their maximum capacity. In the same way the generator rescheduling during the simulations was redistributed so that 60 % of the increase or decrease affected the first generator unit while the second took care of the remaining 40 %.

## 6.2 The Scenario

The disturbance to be studied was decided to be a line outage. The particular line to fail was decided to be the one between node 4041-4044, see Figure 6.1. This line outage scenario was chosen for several reason.

First of all, it is a generally important line connecting the northern section of the grid with the central and southern part. Since there are large power transfers

going from north to south, taking out that line will severely stress the system and put significant pressure on the remaining lines connecting the northern and central area (for example, 4042-4044).

This scenario was also chosen because it had already been studied in a previous article [Capitanescu et al., 2009]. However, since the load flow used in that article was not specified any direct conclusion concerning the effectiveness of either method compared to each other can't be drawn.

### **Simulation Outcome without any control**

If the *ARISTO* simulation is running and there is no user interference following the line outage, a voltage collapse will occur after 280 seconds. The low voltage levels will quickly spread throughout the system during a period of 6-7 seconds with some nodes collapsing a few seconds after the others.

This scenario will not make the whole system collapse. Out of the 32 nodes in the system 19 still be operational after 10 minutes (the simulation length). Since the voltage levels in the remaining nodes are within normal bounds and seemingly unchanging, it is assumed that the remaining system is stable. Those nodes that survive will in great extent be those in the northern part of the system that are supplied with abundant power from the northern generators and not dependent of any power transfers from the central and south western areas.

The collapsing nodes are; the whole south western area (4061-4063) and most of the central area (4042-4047,1041,1043-1045). The only node in the central area that manages to survive the surrounding collapse while still supplying its load with sufficient power is 4041. Both 1042 and 4051 will still be operational, although not connected to the rest of the grid, but this is due to automatic emergency load shedding and since both nodes have generators capable of supplying some portion of the demanded power. This is made possible since *ARISTO* partitioned the total node load into several loads connected to the node.

## **6.3 Optimization Results**

This section and the next will go through some of the optimization and simulations results when applying the method to the scenario.

### **1 - Optimization without generator upper limits and without load dynamics**

In the first optimization no further changes to the data is made. That means no maximal values for the active power generation and no changed generator speeds. The load trajectories acquired from the collapsing scenario without control are not used in this optimization, instead the system is assumed to be able to achieve stationarity in one minute (the length of the time step). The optimization set-up can be seen in Table 6.1.

Generator	Initial (MW)	Maximal Change (MW/min)
1012	600	6.67
1013	300	5.00
1014	550	5.83
1021	400	5.00
1022	200	2.08
1042	360	3.33
1043	180	1.67
2032	750	7.08
4011	669	8.33
4012	600	6.67
4021	250	2.50
4031	310	2.92
4041	0	2.50
4042	630	5.83
4047	1080	10.00
4051	600	5.83
4062	530	5.00
4063	1060	10.00
4071	300	4.17
4072	2000	37.50

**Table 6.1** Table showing the optimization set-up with the original *MBASE*-parameters as base for the generators speed.

The resulting active power trajectories are shown in Table 6.2. Several of the generators reschedule their output as fast as they can given the dynamic constraints. All eight generators in the central and south western region will be given directions to increase their output. Eight out of the twelve generators in the northern and external region will have to decrease theirs.

## 2 - Optimization with generator upper limits and without load dynamics

Since the initial values of the active power generation were close to the maximal values in the simulation model, upper limits were imposed on 6 of the 20 generators in the optimization. One of the generators (4041) is actually disconnected and a total of 5 of them operated initially at their maximal capacity. The affected generators were 1042,1043,4041,4042,4047 and 4063. This is 6 out of the 8 generators in the central and south western region. The only generator out of these that will still be able to increase its output is 4063 with its initial generation of 1060 MW and maximal generation of 1080 MW.

Since several of the most increasing generators in test 1 are now constrained

Generator	Initial (MW)	After 5 min (MW)
1012	600	566.67
1013	300	275.00
1014	550	520.83
1021	400	375.00
1022	200	210.42
1042	360	376.67
1043	180	188.33
2032	750	785.42
4011	666	624.57
4012	600	566.67
4021	250	262.50
4031	310	324.58
4041	0	21.04
4042	630	659.17
4047	1080	1131.43
4051	600	631.84
4062	530	555.00
4063	1060	1111.05
4071	300	279.17
4072	2000	1879.80

**Table 6.2** Table showing the acquired active power trajectories for test 1

the active power trajectories will be very different. They can be seen in Table 6.3. The increased output from the generators in the central and south western region is much more limited and the decreased output from the generators in the northern and external region is therefore also much smaller. Still, 7 out of the 12 generators in the northern and external region will decrease their output. Generator 4011 is the only generator changing the direction of its variation, now increasing instead of decreasing.

Numerical uncertainties will make some of the values exceed their maximum value but the magnitude of the overshoot was deemed to be insignificant. Note that the load trajectories were not used in this test either.

### 3 - Optimization with generator upper limits and with load dynamics

The third test uses almost the same set-up as the second with the difference that the load trajectories are now used. This creates a bit more uneven generator trajectories since the desired active power inputs in the loads vary. The result after 5 minutes can be seen in Table 6.4. This does of course not show how the trajectory varies during the whole time span but that will be more notable among the simulation results.

Generator	Initial (MW)	After 5 min (MW)
1012	600	566.67
1013	300	275.00
1014	550	520.83
1021	400	375.00
1022	200	210.42
1042	360	360.00
1043	180	180.00
2032	750	785.42
4011	666	707.90
4012	600	566.66
4021	250	262.50
4031	310	324.58
4041	0	5.47
4042	630	630.00
4047	1080	1081.67
4051	600	631.35
4062	530	555.00
4063	1060	1081.02
4071	300	279.17
4072	2000	1979.11

**Table 6.3** Table showing the acquired active power trajectories for test 2

Most of the values after 5 minutes don't differ significantly from the ones acquired in test 2. Generator 4011 will now decrease its output again, just like in test 1.

It should be noted that this optimization created a solution with some of the stationary solutions being infeasible. The varying load values seems to have created a more numerically demanding problem. This resulted in a solution that sometimes supplied the loads with insufficient active or reactive power or allowed the reactive power supplied by some generator to exceed the limits of that generator.

#### **4 - Optimization with generator upper limits, without load dynamics and with altered generator speeds**

The fourth and final optimization presented in this report uses the maximal active power values from test 2 and no load trajectories. The simulations indicated that some of the generators could change their output much faster than the *MBASE*-value suggested. Because of this, the speed of both generator 4051 and generator 4062 have been doubled. The results are shown in Table 6.5. A significant portion of the generation have now been rescheduled to generator 4051 and generator 4062. Generator 4011 will increase its production just like in test 2.

Generator	Initial (MW)	After 5 min (MW)
1012	600	580.01
1013	300	285.00
1014	550	532.50
1021	400	385.00
1022	200	206.25
1042	360	376.67
1043	180	180.00
2032	750	771.25
4011	666	661.27
4012	600	579.99
4021	250	262.50
4031	310	324.58
4041	0	5.59
4042	630	630.00
4047	1080	1081.67
4051	600	631.50
4062	530	555.00
4063	1060	1081.03
4071	300	287.50
4072	2000	1958.15

**Table 6.4** Table showing the acquired active power trajectories for test 3

## 6.4 Simulation Results

This section will go through the results when applying the optimized trajectories to the simulations. Reference values for the active power output of every generator will be updated once every minute starting one minute after the line outage until 5 minutes have passed.

The development of every single generator won't be discussed but the overall ability to follow the reference values, as well as the improved life length of the system, will be mentioned.

The impending collapse won't be avoided in any of these simulations. However, it can be delayed by some amount of time and that time will differ depending on which optimization result that is used.

### 1 - Optimization without generator upper limits and without load dynamics

Using the optimization data from test 1, the system will experience a voltage collapse after approximately 555 seconds, or around 9 minutes and 15 seconds. The same area will be affected as in the scenario with no control.

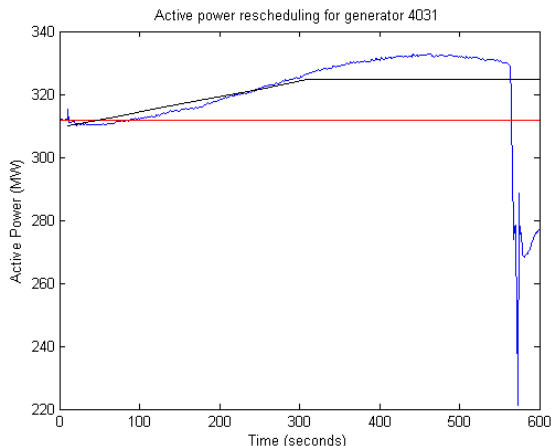
Generator	Initial (MW)	After 5 min (MW)
1012	600	566.67
1013	300	275.00
1014	550	520.83
1021	400	375.00
1022	200	210.42
1042	360	360.00
1043	180	180.00
2032	750	785.42
4011	666	666.36
4012	600	566.66
4021	250	262.50
4031	310	324.58
4041	0	6.02
4042	630	630.00
4047	1080	1081.66
4051	600	660.74
4062	530	580.00
4063	1060	1081.02
4071	300	279.17
4072	2000	1952.00

**Table 6.5** Table showing the acquired active power trajectories for test 4

The active power output from generator 4031 can be seen in Figure 6.2. This is one of the southernmost generators in the northern part of the system and the optimization routine decided that an increase in generation from it would be beneficiary.

It manages to follow the reference trajectory fairly well during the initial increase, implying that the dynamic constraints concerning that particular generator weren't too lenient. This is followed by an overshoot and the output will remain above the reference until the collapse occurs after about 9 minutes and 15 seconds.

Another generator whose output was intended to be increased is 1042. This is a generator in the central 130 kV subsystem in which almost half of the total central area power consumption is taking place. The total generation in this subsystem is initially only covering about 20 % of its consumption and this particular generator is taking care of 2/3 of that generation. To reroute generation here in the occurrence of a disturbance would seem like a logical decision to make. However, this optimization didn't take the maximal active power outputs from the generators in the simulation model into account. Both 1042 and 1043 (the other generator in that subsystem, about half as productive as 1042) are initially operating very close to their maximal capacity and a desired increase could possibly be rejected. Figure 6.3 displays the active power trajectory in generator 1042 during the simulations.



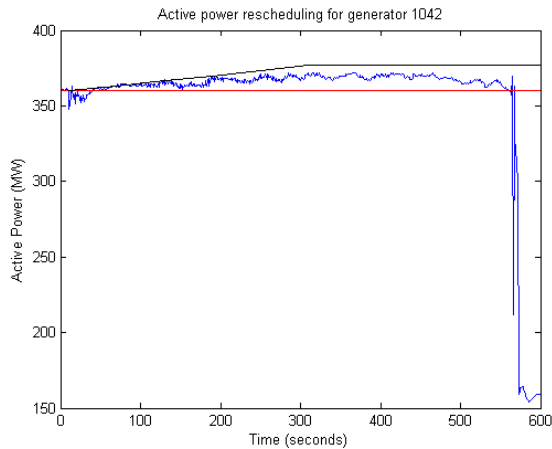
**Figure 6.2** The development of the active power output from generator 4031 following the line outage in test 1. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.

While an increase in its output can definitely be observed, the generator has problems maintaining its output and won't be able to access to desired higher output levels at all. Generator 1043 won't be able to increase its active power generation by any amount.

As mentioned in the optimization section, eight generators in the northern and external region will be given directions to decrease their generation. One of the generator whose output will have to decrease is 1012, one of the most productive generators in the northernmost region. In Figure 6.4, the development can be seen. The generation is decreasing, although not following the reference value very well and not making any visible changes at all during the first couple of minutes. One possible reason for this may be that the dynamical constraints imposed on this generator are not strict enough, assuming faster rescheduling than what is possible. It may also be that preparing for a substantial output change takes preparation, which would explain the initial lack of response.

Another possible explanation takes frequency stability into account. As mentioned in the chapter about power system stability, the system will need to maintain some kind of balance between generation and consumption in order to maintain its frequency of 50 Hz. When the optimization prompted for this decrease in generation, it assumed that the increasing generators would be able to follow their reference value adequately good. When they don't, a decrease in generation might cause the system to be frequency unstable. The simulation has a built-in frequency controller and when a variation in frequency is detected it will override any other controller in order to keep the system frequency stable. This might explain the de-





**Figure 6.3** The development of the active power output from generator 1042 following the line outage in test 1. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.

layed decrease since the generator will wait to change its output until the increasing generator have come close to their new desired outputs.

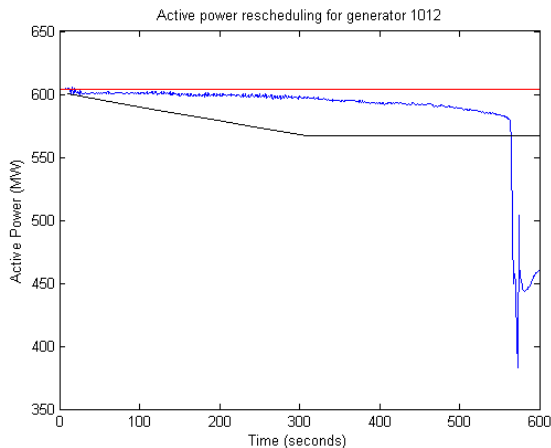
## 2 - Optimization with generator upper limits and without load dynamics

Using the optimization data from test 2, the system will experience a voltage collapse after approximately 450 seconds, or around 7,5 minutes. The same area will be affected as in the scenario with no control.

Adding upper limits on the active power generation made it impossible to increase the output from many of the generators close to the critical area. Instead, the decrease in generation from the northern and external region had to be less extensive. This made the power transfers going north to south larger than in the previous test and more like the ones in the scenario where no control actions were taken. This might explain why the collapse occurred earlier than in the previous test.

One problem is that it really isn't possible to increase the generation in the central and south western region more than this under these circumstances. The generation increased as much as it could even in the previous test. The reason why it survived longer in that case might just be because some of the generators could temporarily overshoot their maximal generation values. Given that aspect of it, the earlier collapse might not be that strange.

A phenomena that becomes apparent in this test is that of the time-delayed increased generation. This can be observed in Figure 6.5 which shows the output from generator 4031, the same as the one shown in the earlier test. The desired total in-



**Figure 6.4** The development of the active power output from generator 1012 following the line outage in test 1. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.

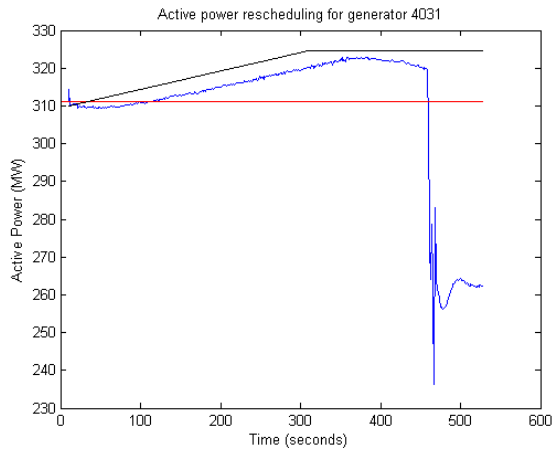
crease is approximately 15 MW, the same as in test 1. In that case, the generation followed the reference values fairly well apart from the overshoot. Here it will initiate the increase much later and not quite be able to reach its stationary value until the collapse occurs. This behaviour might be at least partially avoided by updating the reference value more often (here, the system is basically on its own during the first minute) but that does not explain why this behaviour occur now and not in the previous test.

Although the inclusion of maximal values didn't improve the performance of this simulation, they were considered to be essential in order to correlate the optimization model and the simulation model as much as possible. Therefore, they will be part of all the other tests in this report.

### 3 - Optimization with generator upper limits and with load dynamics

Using the optimization data from test 3, the system will experience a voltage collapse after approximately 440 seconds, or almost 7,5 minutes. The same area will be affected as in the scenario with no control.

Since the load demands are no longer constant, the optimal active power trajectories will have more diverse structures. In the previous two test, the trajectories were in most cases linear increases or decreases at maximum speed during the 5 minute time frame. The central and south western generators operating at maximal capacity were left untouched. An example of how this test differs from that can be seen when looking at generator 4047. This is one of the generators in the central

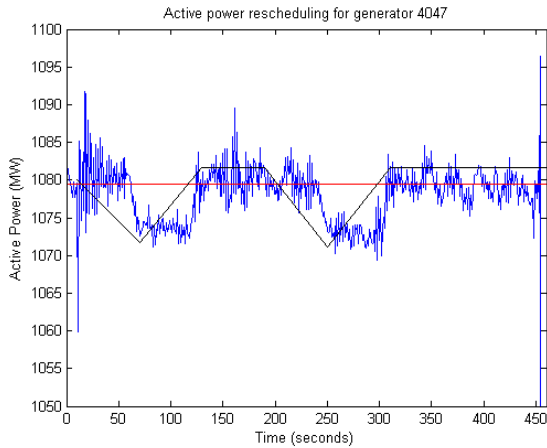


**Figure 6.5** The development of the active power output from generator 4031 following the line outage in test 2. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.

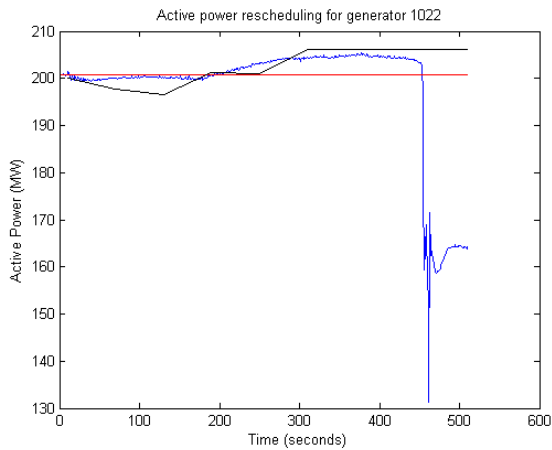
region that are initially operating at maximal capacity (1080 MW). In the previous two test, the output was constantly 1080 MW until the collapse. Figure 6.6 display the generation during test 3. The optimization calls for a small initial decrease, followed by a return to its maximal capacity. This is repeated once more before the collapse. The fluctuations are less than 10 MW and the generator seem to be able to follow the trajectory quite well, although the time delay caused by the sparse updating of the reference value becomes apparent.

Not all generators are able to follow their new reference values as well as generator 4047. Figure 6.7 display the generation from generator 1022, one of the northern generators whose output are set to increase. In this case, the sudden shifts in desired generation becomes useless since the generator won't be able to keep up with the fast changes. Using the load trajectories as feedback would most likely require a more thorough configuration of the dynamical aspects of all the generators. Since this test didn't deliver improved results, the use of load trajectories wasn't part of the fourth and final optimization routine.

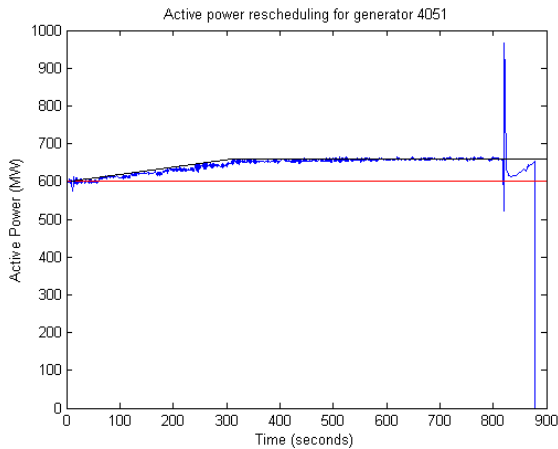
Like mentioned before, 6 out of the 8 generators in the central and south western area are already operation at or very close to their maximal capacity. On the other hand, the other two (4051 and 4062) are not even close to theirs. Since they didn't seem to have any problems following their reference values during these three tests, their speed were doubled in the fourth and final optimization routine.



**Figure 6.6** The development of the active power output from generator 4047 following the line outage in test 3. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.



**Figure 6.7** The development of the active power output from generator 1022 following the line outage in test 3. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.



**Figure 6.8** The development of the active power output from generator 4051 following the line outage in test 4. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output.

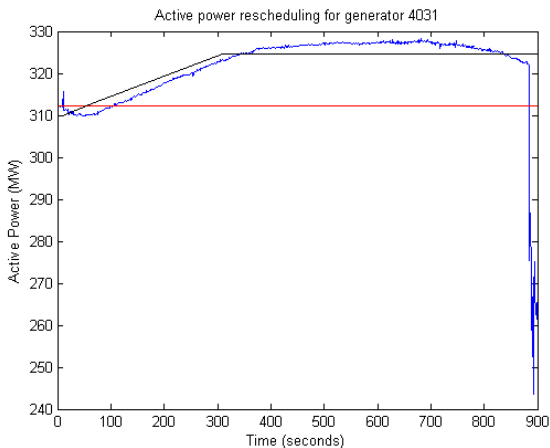
#### 4 - Optimization with generator upper limits, without load dynamics and with altered generator speeds

Using the optimization data from test 4, the system will experience a voltage collapse after approximately 810 seconds, or around 13,5 minutes. The region facing voltage collapse won't be identical to the non-control one in this case since node 4051 also will experience a collapse. This brings the total number of collapsing nodes to 14 out of 32.

Both generator 4051 and generator 4062 manages to follow their desired trajectories without any notable problems. Figure 6.8 displays the generation from generator 4051. The additional access to active power prolongs the system's life length by the largest amount of time so far. Node 4051 will for some reason also collapse, the reason for this is not clear but it would seem reasonable that it has something to do with the increased output from that node.

This test was all that was considered the most successful yet in terms of how well the generators were able to follow their reference values. Several of the generators whose output was set to be increased experienced the same behaviour as mentioned in test 2, that of the time-delayed generation (As in Figure 6.5). This was most noticeable in the generators in the northern region. As this was perhaps partially due to the fact that the reference values were updated one minute into the simulations, one final simulation was done.

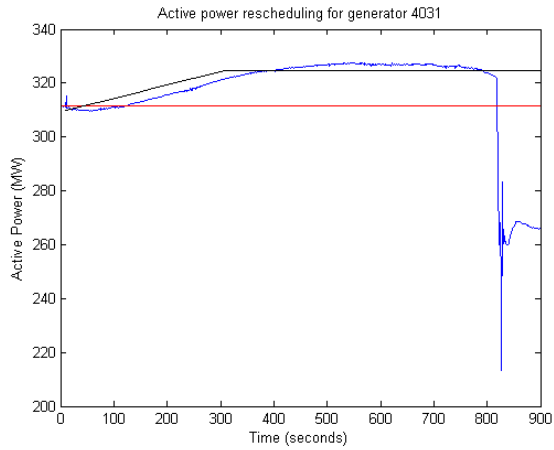
Because of this, one final simulation was made. In this simulation, the same optimized trajectories were used as before but the reference values were sent into the simulation 45 seconds earlier. This resulted in a system lasting approximately



**Figure 6.9** The development of the active power output from generator 4031 following the line outage in test 4. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output. This was a simulation using an earlier update of reference value.

885 seconds, or almost 15 minutes.

The time-delayed generation is still noticeable but its magnitude is definitely somewhat less significant. Figure 6.9 display the generation from generator 4031, the same development without the earlier input can be seen in Figure 6.10.



**Figure 6.10** The development of the active power output from generator 4031 following the line outage in test 4. The black line is the desired reference value, the red line is the initial value and the blue line is the actual output. This was a simulation using the normal update of reference value.

# 7

## Discussion

The simulations resulted in giving the system a prolonged lifespan without saving it. What can be learned and what can be improved?

One of the most noticeable aspects of the simulations were that the voltages were never close to the values that they should have according to the model used for the optimization. In fact, the optimization considered a quite narrow acceptable span of values, 0.95-1.05 p.u., while some of the voltages in the simulation were already close to those limits prior to the line outage and significantly lower afterwards. This implies that there were some kind of substantial disparity between the optimization model in *MATLAB* and the simulation model in *ARISTO*. Granted that the values of the loads, transformers and shunts already had been configured in either model it seems likely that some other parameter, which did not get examined, also would need to have been changed. Perhaps the line impedances? Improving the correlation between the two models would be vital if one desires to improve the results.

Another thing to keep in mind is that the only thing that the optimization sat out to accomplish was to minimize the generated power. There is no guarantee that this is the wisest thing to do when the real objective is to enhance stability and prevent a collapse. Another cost function, specifically designed for the stability problem, might have been a better choice. With this cost function, the results from the optimizations became almost trivial. The results were always a maximal increase in nearby generation and a corresponding decrease in long-distance generation which is a pretty obvious solution when the objective is to minimize the losses. Constraints on the power transfers in each line are also absent, imposing such limitations might perhaps help.

### 7.1 Future Work

There are some different paths to choose from if one decides to continue the work done in this thesis.

Apart from making sure that all the static variables in the optimization and simulation model really are identical, one might also want to model the generators in



a more accurate way in the optimization. This would mean stepping away from the linear trajectories and imposing more advanced dynamic constraints. This might be difficult considering the limitations of the optimization method but could, if done right, possibly improve performance quite a lot.

Most simulations didn't really make use of the load modelling and in the one that did, the results didn't improve because of the inexact generator models. If the generators were modelled more accurately the load modelling could possibly be more useful. One might even want to try to implement the more advanced load models, instead of the simple one-time feedback ones used in this thesis.

Another potential extension of this work could be to implement a *MPC* controller with continuous feedback. Instead of just letting the simulated system try to follow predetermined trajectories the optimization would need to be repeated several times during the simulations. Data would be collected from the simulations in real-time and used to update the optimization parameters in order to make the optimization results more relevant. This could be done with or without the previously mentioned improvements. A problem that needs to receive special attention if doing this is the computational time of the optimization. The implementation of the optimization used in this thesis took more than one minute to finish and would therefore need to be much quicker in order to be sufficient for a real-time controller.

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