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Valuation of Asian Options

-with Levy Approximation

Master thesis in Economics

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Abstract

Asian options are difficult to price analytically. Even though they have attracted much attention in recent years, there is still no closed-form solution available for pricing the arithmetic Asian options, because the distribution of the density function is unknown. However, various studies have attempted to solve this problem, Levy (1992) approximates the unknown density function using lognormal distribution by matching the first two moments. This paper investigates how accurate the Levy approach is by comparing values of Asian options from Levy's approach with Monte Carlo simulations. We find that Levy's analytic solution tends to over-estimate Asian option values when volatility is constant, but under-estimates under the scenario of having stochastic volatility.

Key words: Asian options, Monte Carlo simulation, constant volatility, stochastic volatility

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Table of Contents

1. Introduction

Historically, options were first only traded over-the-counter (OTC) and the terms were not standardized. The first exchange to have the listed options was the Chicago Board Options Exchange (CBOE) in 1973. During the 80s when London International Financial Futures Exchange (LIFFE) was established, option trading played an important role in financial markets. With the help of Black-Scholes (1973) breakthrough on the valuation of options, it became simple and affordable to price and hedge standard options (Perera, 2002).

As options became more frequently traded, there was a need for more complex options on the market. That is when exotic options came into existence; their structure was more complicated and attractive to many investors. Due to the increase of complexity and trading volume in exotic options, many simple exotic options are considered to be standard today (Clewlow & Strickland, 1997).

Exotic options, such as *path dependent options*, have a payoff that is determined by taking an average of the asset price during the whole period. Asian options, which are a kind of path dependent options, have a payoff that depends on either geometric or arithmetic average price of the underlying asset before maturity. Asian options are in general difficult to value since the distribution of the payoff is usually unknown. For geometric Asian options, the payoff is a product of normally distributed random variables, and they are easily priced with risk neutral expectations by having the underlying asset follows a geometric Brownian motion process. However, for arithmetic Asian options, its payoff is the sum of lognormal distributed random variables, for which there is no recognizable distribution function (Hull 2006).

There is no closed form solution for pricing arithmetic Asian options since the distribution is unknown. Nevertheless, many studies have tried to give an analytical approximation for valuation of Asian options. For instance, the binominal tree has been an efficient model used in pricing Asian options (Hull & White, 1993). Lower and upper bounds for option pricing has been introduced by Curran (1992), Roger and Shi (1992). There have been studies valuing Asian option under the assumption that the arithmetic average is log normally distributed. One such research was introduced by Levy (1992) as well as in Turnbull and Wakeman (1991). At last, Boyle (1977) and Hull and White (1987) introduced the Monte Carlo simulations to price Asian options. The Monte Carlo simulations provide us with the numerical solution for Asian options with stochastic diffusion processes (Milevsky & Posner, 1998).

The purpose of this paper is to evaluate the accuracy of Levy approximation formula, to see how it performs by comparing with simulations from Monte Carlo method with constant and stochastic volatilities. The option values from Monte Carlo simulation are assumed to be the real prices for Asian options, since the simulation methodology relies on the quality of its randomly number generator (Fu, Madan, & Wang, 1997). The simulations are as precise as real prices when the number of paths is large. In addition, for each volatility scenario, the comparison between models is studied when the option is Out of the money (OTM), At the money (ATM) and In the money (ITM).

The thesis is divided into different sections; the second chapter gives deeper background knowledge about exotic options and path dependent options. Later in the chapter we introduce Asian options, which is our main focus in this paper. The third chapter introduces different methods for pricing arithmetic Asian options, as well as our core method to value Asian options, the Levy approximation. Chapter four provides a literature review of past studies on pricing standard options as well as Asian options, even when volatility is stochastic.

The methodology used for this paper is described in chapter five, starting with an introduction of the Levy analytic approximation, and then followed by the method of Monte Carlo simulations under constant and stochastic volatility conditions. This leads to the approximation results in chapter six where the Levy approximation and Monte Carlo simulations have been divided into constant and stochastic volatility.

2. Options

In this part of the paper we intend to provide a theoretical background on exotic options in general. Path dependent options as well as Asian options are introduced with a final section about valuation of Asian Options.

2.1 Exotic Options

European and American call and put options are placed in the category of plain vanilla products. Brokers publish their prices and their implied volatilities often. They are standardized in their structure and traded frequently on the market. Through out the years, the complexity has increased and many nonstandard products are sold on a daily basis over-the-counter. These complex options are called exotic options and are more profitable compared with plain vanilla options. The reason for the existence of exotic options can be everything from tax, accounting, legal and regulatory reasons. They are also needed for hedging purposes. (Hull, 2006)

2.2 Path-dependent options

Path dependent option can be either European or American styled options, which have a different payoff than regular options. The payoff is determined by taking an average of the asset price during the whole period while regular options are only interested in the price at the maturity. These options commonly take commodities as the underlying assets, where the distance between the strike price and the average price is the payoff for options. Path dependent options can be divided into two groups, *weakly path dependent options and strongly path dependent options.* Weakly path dependent options are characterized via its payoff that depends on the asset price reaching a predetermined price level. Barrier options are weakly path dependent options with a predetermined barrier that can activate or terminate the option. Strongly path dependent options consist of a payoff that depends on the entire or part of the path of the asset price during the life of the option. Asian options are strongly path dependent options that take the average of the price of the underlying asset during the whole period (Jiang, 2005).

2.3 Asian Options

These path dependent options were first introduced on the Asian market in order to avoid the manipulation of prices on expiration date. This was a common problem in European options where speculators could drive up the prices before maturity. And through out time Asian Options have become popular for many different reasons. Several firms are affected by periodical payments in foreign currency and need to hedge their cash flows to reduce the exposure of the exchange rate. They are also frequently used in balance sheets, where investors seek to hedge their exposure via average rates rather than year-end rates. Another advantage of calculating the average of the price of the underlying asset during a certain interval is the lower volatility compared to European options. Averaging on prices of the underlying asset was commonly used in the 1970s with commodity-linked bond contracts. These contracts provided the holder an option with a bond that consisted of a commodity with an average value.

Asian options are traded over-the-counter and mainly on markets such as energy, oil and currency. They are provided either as European Asian options (Eurasian) or American Asian options (Amerasian), depending on if the holder wanted to exercise the option at maturity or on several occasions up to expiration. The disadvantage with American Asian options is that the investor will not be protected against manipulations of prices as in European Asian options (Lee & Lee, 2010).

The Asian call and put option has a payoff that is calculated with an average value of the underlying asset over a specific period. The Asian call and put options have the following payoffs:

$$\text{Asian Call Option} \qquad \max(\bar{S} - K, 0)$$

$$\text{Asian Put } Option \qquad max\ (K - \bar{S}, 0)$$

$$K = Strike\ price$$
$$\bar{S} = Average\ value\ of\ an\ underlying\ asset$$

Since Asian options are less expensive than their European counterparts, they are attractive to many different investors.

Apart from the regular Asian options there also exists Asian strike options. An Asian strike call option guarantees the holder that the average price of an underlying asset is not higher than the final price. The option will not be exercised if the average price of the underlying asset is greater than the final price. The holder of an Asian strike put option makes sure that the average price received for the underlying asset is not less than what the final price will provide. The following equations indicate the payoff for Asian strike options:

$$Asian\ strike\ call\ option \quad max(S_T - \bar{S}, 0)$$

$$Asian\ strike\ put\ option \quad max\ (\bar{S} - S_T, 0)$$

$$S_T = Value\ of\ an\ underlying\ at\ maturity$$
$$\bar{S} = Average\ value\ of\ an\ underlying\ asset$$

Asian options are divided into two different types when calculating the average, the geometric Asian option and the arithmetic Asian option. The most used Asian option is the arithmetic Asian option but these can be very difficult to price. The reason for this is because the distribution of the arithmetic average is unknown, and thus there is no closed-form solution for arithmetic average as long as the conventional assumption of a geometric diffusion is specified for the underlying asset (Hull, 2006).

$$The\ Arithemtic\ Asian\ option \quad A_T = \frac{1}{N}\sum_{t=1}^{N} S_{t_i}$$

$$The\ Geometric\ Asian\ option \quad G_T = \left(\prod_{i=1}^{N} S_{t_i}\right)^{1/N}$$

$$S_{t_i} = Asset\ price\ at\ dates\ t_i\ , for\ i = 1, \ldots , N$$

However, it is possible to derive a closed form solution for geometric Asian options with the help of risk-neutral expectations when the underlying asset follows a geometric Brownian motion process. The density function for the geometric average is assumed to be lognormal distributed. Even though the geometric Asian options are easily priced they are rarely used in practice. (Milevsky & Posner, 1998)

3. Pricing Options

In this section, methods used for pricing options are introduced. Starting with a presentation of the famous Black-Scholes option pricing formula, and then a brief outline for Monte Carlo simulations, which will be discussed in more detail in the methodology part of this paper. At last, the analytic approximation formula for Asian options from Levy (1992) is presented.

3.1 Pricing Asian options

In the early 1970s the Nobel Prize for economics went to Fischer Black, Myron Scholes and Robert Merton, who discovered what is today called the Black – Scholes model in their article "The Pricing of Options and Corporate Liabilities". This model has influenced the world of derivatives for many years and is even today used frequently to provide option prices. European call and put options are priced with help of the following basic Black-Scholes pricing model:

$$call\ option = S_o N(d_1) - \ Ke^{-rt} N(d_2)$$

$$put\ option = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln\left(\frac{S_0}{K}\right) + \ \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where

$$S_0 = stock\ price\ at\ time\ zero$$

$$K = strike\ price$$

$$r = continuously\ compunded\ risk - free\ rate$$

$$\sigma = stock\ volatiliy$$

$$T = time\ to\ maturity$$

Through Black-Scholes, an analytical closed form solution has been found for plain vanilla options with the assumption of no arbitrage conditions (Hull, 2006). By constructing a dynamic portfolio with proportions of the underlying stock and a risk free debt instrument, it is possible to create a replicating portfolio of the payoff from an option. The model assumes that under risk-neutral measure of Q, the underlying asset follows a lognormal diffusion process where the volatility is constant. Nevertheless, this gives an inaccurate representation that all options with varying strikes and maturities have the same implied volatility and that historical volatility is constant over time.

When closed form solutions are not available, Monte Carlo simulations is able to price complex *path dependent derivatives* with stochastic diffusion processes. The Monte Carlo method was first introduced by Boyle (1977), where a large number of simulations provide a high degree of accuracy within option pricing (London, 2005). Von Neumann first introduced the simulations during the Manhattan atomic project, where the simulations were seen as an imitation of the spins of the roulette wheels in Monte Carlo. The method is not only the most common used tool in pricing exotic options but also very applicable to plain vanilla options with a payoff that depends on the price at maturity (Raju, 2004).

Many researchers tried to improve and develop different models that could take the random path of the volatility with its own diffusion process with drift and diffusion parameters. Among many Heston (1993), Stein and Stein (1991), Scott (1987), Wiggins (1987) and Hull and White (1987) proved that stochastic volatility and stock prices are typically correlated with one another (Corrado & Su, 1998). A widely used and interesting approach was introduced by Hull and White (1987), who presented a great deal of flexibility and correlation between stochastic volatility and changes in stock price.

Hull and White (1987) first introduced the assumption that stochastic volatility is independent of the stock price. By comparing Black-Scholes and real option prices they were able to express the pricing error. When Black-Scholes was compared with the correct option prices it was shown that Black-Scholes is too low deep in and out of the money and too high at the money. Near or at the money indicates the largest price difference but the price error is quite small in relations to the correct option price. The assumption in the article is later weakened and numerical solutions are used for stock prices that are correlated with volatility. In positive correlation in the money options (ITM) were overpriced and out of the money (OTM) underpriced. (Hull & White, 1987).

3.2 Levy approach on Asian options

The payoffs of Asian options depend on the average price of its underlying assets, where the average can be derived either geometrically or arithmetically. In real world, Asian options are commonly used on foreign currencies, interest rates, as well as commodities, for instance crude oil.

For the arithmetic Asian options, a closed form solution does not exist if the conventional assumption of a geometric diffusion is specified for the underlying asset, because the density function for arithmetic average is unknown, which unlike geometric average, is not lognormal distributed and thus has no explicit representation. Due to the attraction of Asian options as well as the pricing difficulties it has, recent studies have put their focus on finding a pricing formula for calculating the value of arithmetic Asian options, and has become a special discipline in computational finance (Potapchik & Boyle, 2008).

Levy (1992) had a straightforward approach, the so-called 'Wilkinson approximation', which is used to approximate the arithmetic density function by matching the first two moments. His approach has been claimed to be accurate and easily implemented for certain levels of volatility, which will be conducted in this study.

As in Cox and Ross (1976), applying the neutrality condition, with constant strike price K, the value of the arithmetic average Asian call option can be written as:

$$C[S(t), A(t), t] = e^{-r(T-t)} \mathbb{E}_t^Q Max[A(t) - K, 0]$$

where $\mathbb{E}_t^Q$ is an expectation operator conditioned on $[S(t),\ A(t)]$ at time t under the risk-adjusted density function.

However, pricing arithmetic Asian options using above equation is considered to be problematic for $K \neq 0$, because the approximation is not straightforward and thus requires knowledge of the distribution on the summation of lognormal distributed random variables. Even though the moment generating function for the sum of two lognormal distributed variables does exist, closed form expression for their density function is still not available. Levy (1992) assumes $M(t)$ as an undetermined component of the final arithmetic average, which is a sum of lognormal random variables.

$$M(t) = [A(t_N) - A(t)(m+1)/(N+1)]$$

where $A(t_N)$ represents the arithmetic average of $N+1$ prices of underlying assets, and $0 \leq m \leq N$. Studies have suggested that such sum of lognormal random variables can be very well approximated by another lognormal distribution.

Thus by accepting that $\ln M(t)$ follows a normal distribution with unknown mean $\alpha(t)$ and variance $v(t)$, the moment generating function $X(t) = lnM(t), \Psi_x(k)$ is used, given:

$$\Psi_x(k) = \mathbb{E}_t^Q[M(t)^k] = e^{k\alpha(t)+1/2k^2v(t)^2}$$

for $k = 1$ and $k = 2$, which yields the first and second moment for $lnM(t)$:

$$\alpha(t) = 2ln\mathbb{E}_t^Q[M(t)] - \frac{1}{2}ln\mathbb{E}_t^Q[M(t)^2],$$

$$v(t) = \sqrt{ln\mathbb{E}_t^Q[M(t)^2] - 2ln\mathbb{E}_t^Q[M(t)]}.$$

By assuming $M(t)$ is lognormal distributed, with mean $\alpha(t)$ and variance $v(t)$, the arithmetic call option is valued as:

$$C[S(t), A(t), t] = e^{-r(T-t)}\{\mathbb{E}_t^Q[M(t)]N(d_1)$$

$$-[K - A(t)(m+1)/(N+1)]N(d_2)\}$$

where

$$d_1 = \frac{\frac{1}{2}ln\mathbb{E}_t^Q[M(t)^2] - ln[K - A(t)(m+1)/(N+1)]}{v(t)},$$

$$d_2 = d_1 - v(t)$$

and $N(.)$ is the cumulative normal distribution function.

The main advantage of Levy's approach is that an approximation of closed form analytical solution for pricing arithmetic Asian options becomes possible within a certain range of volatility. When compared with other methods, this approach is less time consuming.

The payoff for the corresponding put option, $P[S(t), A(t), t]$, can be estimated by using the above expression for call options and follow the put call parity, which will not be discussed in this paper.

4. Literature Review

The breakthrough of Black-Scholes (1973) article has influenced the world of option valuation even to this day. Many researchers have tried to implement and improve the models to apply for different assumptions and terms.

Under the influence of Black-Scholes (1973), Cox and Ross (1976) published a paper on valuation of options based upon different jump and diffusion processes in order to solve difficulties with payouts and potential bankruptcies. They questioned the lognormal diffusion process that followed in the Black-Scholes and explained the importance of diffusion and jump processes in the stochastic process in continuous time

$$\frac{dS}{S} = \mu dt + (k-1)d\pi$$

where π is a continuous time Poisson process and $(k-1)$ is the jump amplitude. The article presents alternative jump and diffusion processes in order to give more insight in the option valuation (Cox & Ross, 1976).

A couple of years later, Heston (1993) resumed these phenomena of finding a closed form solution for European call options with stochastic volatility.

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S$$

With inspiration from previous researches Cox, Ingersoll, and Ross (1987) and their square-root process,

$$dv(t) = K[\theta - v(t)]dt + \sigma\sqrt{v(t)}dz_2(t)$$

Heston (1993) explained the correlation between the stochastic volatility and the asset price. This correlation gives incitements to explain the strike-price biases and skewness for the Black-Scholes model. With the help of stochastic interest rate he was able to apply the model to stock options, bond options and currency options (Heston, 1993).

The importance of pricing average options was very essential after their introduction during the late seventies since many investors sought out to protect themselves from movements in the commodity prices. In the article "A Pricing Method for Options based on Average asset values" Kemna and Vorst (1989) tried to find an analytical solution for the arithmetic average option before and during the final time interval but failed to do so. However, with the help of Monte Carlo simulations they were able to prove that average options have a lower value than a counterpart European option. Much consideration was put into the variance reduction technique used in Monte Carlo simulations to get a more accurate standard deviation result. This was done with the help of the geometric average option since it creates a lower bound for the arithmetic average option (Kemna & Vorst, 1990).

Since the movement in prices become averaged the significance of the price at maturity decreases. Turnbull and Wakeman (1989) underline the problem with the pricing of averaging options by emphasizing that the binominal tree approach cannot be used when the number of nodes becomes too large to value the history of the asset price movements over the averaging period. Another concern of theirs was the speed of adjustment when the maturity of the option is less than the average period. They looked at the difference between the arithmetic and geometric average options prices. Their conclusions were that if the averaging period is shorter than the maturity of the option and the standard deviation is smaller it would lead to similar results. However, if the averaging period is larger than the option maturity there can be differences in the prices (Turnbull and Wakeman, 1991).

With the help of previous studies of Levy (1992) and Turnbull and Wakeman (1989), Curran (1994) explains in his article *"Valuing Asian and Portfolio Options by Conditioning on the Geometric Mean Price"* the difficulties with the Black-Scholes method and tries to solve the problem by presenting a method on conditioning on the geometric mean prices in order to calculate the option payoff. The article reaches out for a more accurate approximation of the average options and a faster and more accurate method for portfolio options than previous multinomial methods (Curran, 1992).

A couple of years later Milevsky and Posner (1998) tried to solve the enigma behind finding a closed form solution for arithmetic Asian options. Their aim was to derive the probability density function of the *infinite* sum of correlated lognormal random variable since the difficulty rises because the payoff depends on the *finite* sum of correlated lognormal variables. Focus was put into the valuation of the arithmetic Asian option where the density function was determined through the usage of the reciprocal gamma distribution. The cumulative density function of the gamma distribution *G(d)* had the same interpretation as the *N(d)* in the Black-Scholes equation. Through the gamma distribution the closed form analytical solution was found for the arithmetic Asian option (Milevsky & Posner, 1998).

5. Methodology

As studies have pointed out, the payoff of a path dependent option is rather difficult to calculate, since it depends on the path of the asset prices over time, rather than its final value. For path dependent Asian options, there is no closed form solution available, therefore, large amount of researches have been carried out in order to find an appropriate method to price these kinds of options. For our paper, we focus on two methodologies to price the arithmetic Asian options: an analytical solution as well as a simulation approach, which will be summarized in this section:

5.1 Analytic Approximation

The analytic approximation as a major approach has been widely used, and is also the most appealing method for pricing exotic or path dependent options, because comparing with other methods, it is less time consuming and thus easy to implement. The analytic approximation for pricing Asian options was first introduced by Turnbull and Wakeman (1991). Based on their study, Levy (1992) put forward another solution which is claimed to be more accurate. Therefore, in this paper, we will use Levy's approximation to value the arithmetic Asian option. (Boyle 1977)

As introduced in previous section, Levy in his paper approximated the distribution of arithmetic Asian option follows a lognormal distribution, which has the identical first two moments. However, Ju (2002) in more recent study states that Levy's approximation using lognormal density as the first-order true density only works for options with short maturities. Nevertheless, Levy (1992)'s approach still contributes to the development of analytical solution by avoiding time consuming procedures (Boyle, 1977).

Under the Black-Scholes setting, Levy (1992)'s approximation method for pricing arithmetic Asian options can be written as:

$$C_{Levy} \approx S_Z N(d_1) - K_Z e^{-rT_2} N(d_2)$$

where

$$d_1 = \frac{1}{\sqrt{v}}\left[\frac{\ln(L)}{2} - \ln(K_Z)\right], \qquad d_2 = d_1 - \sqrt{v}$$

$$S_Z = \frac{S}{(r-D)T}\left(e^{-DT_2} - e^{-rT_2}\right)$$

$$K_Z = K - S_{Avg}\frac{T - T_2}{T}$$

$$v = \ln(L) - 2[rT_2 + \ln(S_Z)]$$

$$L = \frac{M}{T^2}$$

$$M = \frac{2S^2}{(r-D)+\sigma^2}\left[\frac{e^{(2(r-D)+\sigma^2)T_2} - 1}{2(r-D)+\sigma^2} - \frac{e^{(r-D)T_2} - 1}{r-D}\right]$$

and

S = Spot price.

S_{Avg}=Average asset price.

X = Strike price.

r = Risk-free interest rate.

D = Dividend yield.

T = Time to maturity.

T_2= Time remaining until maturity.

σ = Observed volatility.

N(x) = Cumulative probability distribution function for a normal distribution.

5.2 Monte Carlo Simulation

Other than the analytic approximation, a major numerical approach for pricing derivatives is the Monte Carlo simulation, which is a stochastic process first introduced by Boyle (1977). This method is used for derivative valuation as well as hedging assets.

Moreover, Hull (2006) summarizes the procedure of Monte Carlo simulations with the assumption that the derivative depends solely on the underlying stock S, which yields a payoff at maturity T with constant volatility over time;

1. The very first step is to divide the time to maturity T into n equally spaced intervals, thus let $\Delta t = T/n$. Meanwhile assuming that the stock price follows Geometric Brownian Motion (GBM), so that one may sample a random number of ε with normal distribution and insert it into the equation to get the change in stock price: $\Delta S = \left(r - \frac{\sigma^2}{2}\right) * \Delta t + \varepsilon\sigma\sqrt{\Delta t}$, then add ΔS back to S which is the stock price for the next period. If the procedure repeats continuously, as a result, it forms a random path of S in a risk-neutral world.

2. Then the next step is to generate the payoff of the derivative at maturity T for all paths simulated.

3. Repeat step 1 and 2 to get a large number of sample values.

4. Calculate an average of the obtained sample values as an estimate of the expected payoff.

5. Since the Monte Carlo simulation is formed in a risk-neutral world, so that according to the risk-neutral measure, the price of a derivative is the discounted value of its future payoff, the final step of Monte Carlo simulation is to discount the expected payoff at the risk-free rate to get the payoff of the derivative.

5.2.1 Monte Carlo Simulation with Constant Volatility

The Monte Carlo simulation in recent studies has been implemented to solve more complex derivatives, for instance the path dependent options as well as some other exotic options. And the majority of these implementations have been carried out under the same conditions as the ones that apply in the Black-Sholes model with constant volatility. In this paper, we will follow the procedure of Monte Carlo simulation to estimate the payoff of a path dependent Asian option.

It assumes that the underlying asset follows a Geometric Brownian motion where in a risk-neutral world, the drift term is equal to the risk-free interest rate. In a continuous time notation, the stock price is:

$$dS = rSdt + \sigma Sdz$$

where S is the stock price, r is the risk-free interest rate, σ is the volatility which for now is assumed to be a constant, and dz is a Wiener process, which means that $\Delta z = \varepsilon\sqrt{\Delta t}$, and ε follows the standard normal distribution with mean zero and variance of one. Therefore, for a discrete time system, a change in stock price becomes:

$$\Delta S = rS\Delta t + \sigma S\Delta z = rS\Delta t + \sigma S\varepsilon\sqrt{\Delta t}$$

thus $\frac{\Delta S}{S} = r\Delta t + \sigma\varepsilon\sqrt{\Delta t}$, which also follows a normal distribution, and represents a percentage change in stock return over a short time period Δt.

We then follow the procedure for Monte Carlo simulation as mentioned above, and divide the lifespan of the stock into n short intervals with length of Δt, then by applying Itô's Lemma to the stock price process, we get:

$$dln(S) = \left(\frac{1}{S}rS + 0 + \frac{1}{2}\left(-\frac{1}{S^2}\right)\sigma^2 S^2\right)dt + \frac{1}{S}\sigma S dz$$

$$= \left(r - \frac{1}{2}\sigma^2\right)dt + \sigma dz$$

Apply to discrete time notation, the above formula becomes:

$$\Delta \ln(S) = \left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

Since Δt represents short time interval, the change in stock price becomes:

$$\ln(S_{t+\Delta t}) - \ln(S_t) = \left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

which eventually gives the path generating formula for the stock price by applying Monte Carlo simulation:

$$S_{t+\Delta t} = S_t e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}}$$

where S_t denotes the value of the stock at time t, ε represents a number randomly sampled from a normal distribution with zero mean and standard deviation equals to one. And when the volatility is constant, the parameters μ and σ are also constant, which makes the above equation the true value of stocks, instead of an approximation. For this paper, the formula is encoded into MATLAB to create a series of random paths following Geometric Brownian motion, which is then used to value the price of an arithmetic Asian option using Monte Carlo simulation.

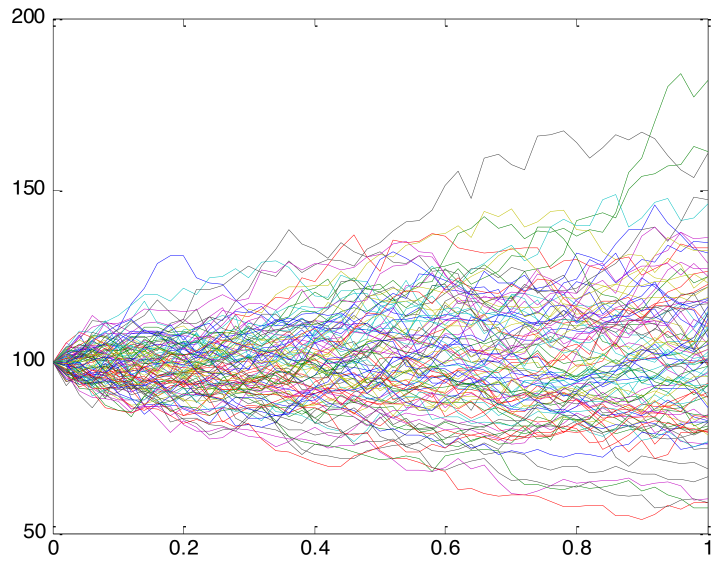

Figure 1 shows 100 randomly generated paths of the stock price in one year with fixed initial value of 100.

Before moving forward, there are two aspects we must declare when implementing Monte Carlo simulation. One thing is the Random Number Generator (RNG), which is a computational device designed to generate random sequence without any pattern. The RNG needs to be clearly defined in order to produce independently and identically distributed ε. In our case, ε is assumed to be normal distributed with (0,1). Moreover, the initial value of the underlying asset must be fixed to a certain number when generating the random path. With different initial values, the RNG will generate different paths every time, even though we run the same simulation. And using fixed initial numbers also ensures us that we are able to compare our simulations with results implemented by the analytic method.

The Monte Carlo simulation in general is a good way to price the value of options mainly due to its advantages compared to other methods: First of all, Monte Carlo simulation is suitable when the value of options depends either on the path or the final value of the underlying asset. The Asian option for example depends on the average price of the underlying asset, whereas the lookback option depends on the maximum or the minimum price of the underlying, and both of these two options can be priced using the Monte Carlo simulation.

On the other hand, the Monte Carlo simulation is recommended because one can actually control the accuracy of the results generated. Previous studies have shown that the accuracy of the method relies on the number of simulations, which can be indicated by the value of the standard error: $\frac{\sigma}{\sqrt{N}}$

Moreover, the confidence interval for option prices can also indicate the goodness of the estimation, which a 95% confidence interval can be presented as follow:

$$\mu - \frac{1.96\sigma}{\sqrt{N}} < P < \mu + \frac{1.96\sigma}{\sqrt{N}}$$

Where σ is the standard deviation, μ is the mean price and N is the number of simulations to be chosen.

5.2.2 Monte Carlo Simulation with Stochastic Volatility

As in Black-Sholes and other models used for pricing options, the volatility is assumed to be constant over time. However, in real financial markets, volatility changes dramatically from time to time. Therefore, in order to make our analysis more close to reality, we will also apply Monte Carlo simulation with stochastic volatility and see how the results generated differ from constant volatility.

Among all studies that incorporate stochastic volatilities, the most famous is the one introduced by Hull and White (1987), which is followed in this paper. The same assumptions are used for the underlying stock as described in the previous section but with one exception, the volatility σ is not constant, but rather follows a stochastic process in a risk-neutral world:

$$dS = rSdt + \sigma Sdz$$

$$dV = \mu Vdt + \xi Vdw$$

where $V = \sigma^2$, ψ and ξ are the so called instantaneous drift and standard deviation of the variance. In general, dz and dw as being two Wiener process are assumed to have a correlation ρ.

Based on large number of numerical procedures, Hull and White (1987) concludes that with stochastic volatility, Monte Carlo simulation can be efficiently used to derive option prices by assuming that the above two Wiener processes are not correlated, where $\rho = 0$. Following the same idea as earlier by generating the stock price under the constant volatility, with ξ assumed to be constant.

$$dlnV = \frac{1}{V}dV - \frac{1}{2}\frac{1}{V^2}(dV)^2$$

$$= \left(\mu - \frac{\xi^2}{2}\right)dt + \xi dw$$

so that $$\Delta lnV = \left(\mu - \frac{\xi^2}{2}\right)\Delta t + \xi \Delta w$$

$$lnV_{t+\Delta t} = lnV_t + \left(\mu - \frac{\xi^2}{2}\right)\Delta t + \xi(w_{t+\Delta t} - w_t)$$

Which gives us the volatility at each point in time by a stochastic process as:

$$V_{t+\Delta t} = V_t e^{\left(\mu - \frac{\xi^2}{2}\right)\Delta t + \xi\varepsilon\sqrt{\Delta t}}$$

similarly, ε is a random sample from a standardized normal distribution with mean zero and variance of one.

When applying the Monte Carlo simulations with stochastic volatility, and assuming the volatility is uncorrelated with the stock price, but allow parameters μ and ξ to depend on σ and t, which means that the instantaneous variance follows a so called mean-reverting process that is:

$$\mu = \alpha(\sigma^* - \sigma)$$

and ξ and α and σ^* are constants. If μ is constant instead, the volatility would have a drift but not follow a mean-reverting process.

The instantaneous variance can be reformed as follow:

$$dV = \alpha(\sigma^* - \sigma)Vdt + \xi Vdw$$

where α is the speed of mean-reversion; σ^* is the volatility in the long-run; and ξ is the volatility of volatility. This mean-reverting process is applied building on the assumption that return of the underlying stock is uncorrelated with the volatility of the option.

Furthermore, a series of stochastic variances will be generated which are independently and identically distributed. And each variance is calculated by adding the value from the previous period with a random number. Then the variance will be used to generate the price of underlying stock as:

$$S_{t+\Delta t} = S_t e^{\left(\mu - \frac{V_t}{2}\right)\Delta t + \sqrt{V_t}\varepsilon\sqrt{\Delta t}}$$

In order to generate the path of the price of the underlying stock, the above equations will be coded into MATLAB, and the generated path will then be used to price our Asian options.

6. Approximation Results

This section of our paper illustrates how accurate the Levy analytic approximation is. The examination of accuracy is done under two volatility scenarios: constant and stochastic volatilities. Furthermore, for each volatility setting, the comparison between models for pricing arithmetic Asian options is done when the option is In the money (ITM), At the money (ATM) and Out of Money (OTM).

6.1 Accuracy of Estimation

Asian option values obtained from Levy's approach is compared with Monte Carlo simulations using 100,000 paths. In which the pricing error – computed as the deviation from the value of Monte Carlo simulation divided by the Monte Carlo value – is calculated to indicate the accuracy.

In addition, the accuracy of each Monte Carlo simulation is indicated by its standard error. The Monte Carlo simulation of option prices, in our case, is carried out by simulating 100,000 paths for the underlying stock price and by taking an arithmetic average of the simulated prices, given the value of the arithmetic Asian option:

$$f_{Asian} = \frac{1}{n}\sum_{i=1}^{n} f_i$$

By assuming the simulations are statistically independent, thus the variance of the simulations can be written as:

$$Var(f_{Asian}) = \frac{1}{n^2}\sum_{i=1}^{n} Var(f_i) = \frac{Var(f)}{n}$$

where f is the option value, and thus the standard error is obtained by taking the square root of the variance:

$$SE(f_{Asian}) = \sqrt{\frac{Var(f)}{n}}$$

which gives an idea that the accuracy of Monte Carlo simulation improves as the number of simulation increases.

Throughout this paper, the standard error from the Monte Carlo simulations is used to form a 95% Confidence Interval, which also illustrates the accuracy of Levy's analytic approach.

6.2 Constant Volatility

In this section, the simulated values of the arithmetic Asian options are illustrated when the volatility is considered to be constant. The estimated values from Levy's approach are compared with values obtained from the Monte Carlo simulation, where both of these two methods assume that the underlying stock follows the Brownian motion process with drift. Then the accuracy of Levy's approach is analyzed when the option is Out of the money, At the money and In the money, with increasing volatilities.

6.2.1 Out of Money

A call option is said to be 'Out of money' when the price of the underlying stock is lower than the strike price of the option, i.e. $S_t < K$.

Table 4.1

Valuation of arithmetic Asian options with OTM: S =90, K =100,T =1 year, r =7%, Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)

				95% Confidence Interval		
Volatility	*Levy Approximation*	*Monte Carlo Simulation*	*Standard Error*	*Lower bound*	*Upper bound*	*Pricing Error (%)*
10%	0,2917	0,2976	0,0037	0,2904	0,3048	-1,9825
20%	1,7663	1,7779	0,0138	1,7509	1,8049	-0,6525
30%	3,6760	3,6313	0,0255	3,6261	3,7260	1,2310
40%	5,6215	5,5758	0,0377	5,5020	5,6497	0,8196
50%	7,6233	7,5306	0,0515	7,4297	7,6315	1,8949
60%	9,7664	9,6623	0,0667	9,5317	9,7930	1,0774

The table above represents the results when the volatility is constant and the Asian option is OTM. As can be seen, with increasing volatility, the option value rises for both estimations from Levy and from Monte Carlo.

Furthermore, the pricing error of Levy approximation is different at different volatility levels. With low volatilities i.e. 10%, the Levy approach is more likely to under-estimate the values of Asian options by 1.9825%. Where in contrast, with rather high volatility, i.e. 60%, the Levy approach tends to over-estimate by 1.0774% compared to the results obtained from Monte Carlo simulation. All option values from the Levy approach are covered by the 95% confidence interval, so in general, Levy has an outstanding performance for pricing arithmetic Asian options when the options are OTM with constant volatility.

6.2.2 At the Money

A call option is said to be 'At the money' when the price of underlying stock is equal to the strike price of the option, i.e. $S_t = K$.

Table 4.2

Valuation of arithmetic Asian options with OTM: S =100, K =100,T =1 year, r =7%, Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)

				95% Confidence Interval		
Volatility	*Levy Approximation*	*Monte Carlo Simulation*	*Standard Error*	*Lower bound*	*Upper bound*	*Pricing Error (%)*
10%	4,2669	4,2457	0,0141	4,2181	4,2733	0,4993
20%	6,2849	6,2485	0,0260	6,1976	6,2994	0,5825
30%	8,4351	8,3504	0,0387	8,2744	8,4263	1,0143
40%	10,6288	10,5397	0,0523	10,4371	10,6422	0,8454
50%	12,8501	12,6059	0,0670	12,4746	12,7371	1,9372
60%	15,0957	14,9141	0,0841	14,7493	15,0789	1,2176

The outcomes from table above indicate the results for the ATM options with constant volatility. Comparing to *Table 4.1*, the Asian options are more expensive when the price of underling stock is equal to the strike price, which in our case is S = K= 100. By looking at the pricing errors, Levy approximation over-estimates the option value at all levels of volatility with the highest pricing error of 1.9372% at 50% volatility level. Consequently, when the volatility is around 50% and 60%, the option values composed by Levy fall outside the 95% Confidence Interval, which is regarded as 'unreasonable' in the critical level.

6.2.3 In the Money

A call option is said to be 'In the money' when the price of underlying stock is greater than the strike price of the option, i.e. $S_t > K$.

Table 4.3

Valuation of arithmetic Asian options with OTM: S =110, K =100,T =1 year, r =7%, Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)

Volatility	Levy Approximation	Monte Carlo Simulation	Standard Error	95% Confidence Interval Lower bound	95% Confidence Interval Upper bound	Pricing Error (%)
10%	13,0242	13,0206	0,0195	12,9824	13,0588	0,0276
20%	13,7660	13,6731	0,0359	13,6028	13,7434	0,6794
30%	15,3063	15,2142	0,0510	15,1143	15,3141	0,6054
40%	17,2062	17,0652	0,0664	16,9352	17,1953	0,8262
50%	19,2842	18,9587	0,0831	18,7958	19,1217	1,7169
60%	21,4678	21,0627	0,1008	20,8651	21,2604	1,9233

The table above exemplifies the results when the volatility is constant and the Asian option is ITM. Again, the option values are increasing with higher volatilities, when comparing with options that are OTM and ATM.

Under the scenario of constant volatility and options that are ITM, the Levy approximation over-estimates the option values regardless of the volatility level. However, with volatilities of 20%, 40%, 50% and 60%, the pricing error of Levy is relatively high, and the approximated values fall outside the 95% confidence interval, which are considered to be unreliable.

To summarize, the option value increases with increasing volatility, which yields a higher standard error as well. And with constant volatility Levy gives an over-estimation of option values when options are OTM, ATM and ITM; and it only under-estimates during OTM with relatively low volatilities. The pricing error overall is greater for volatilities at high levels, which are very rare in reality.

6.3 Stochastic Volatility

When the volatility is stochastic, the variance used for Monte Carlo simulation is assumed to follow the mean-reverting process. Whereas for the Levy approximation, in order to get consistent results, the volatility used is the arithmetic mean of the simulated volatility is the so-called mean variance.

6.3.1 Out of The Money

In occurrences of the strike price being above the stock price with stochastic volatility similar outcomes are discovered.

Similarly to the outcomes with constant volatility, it can be underlined that Levy approximations under-estimate option values to a greater extent, as shown by the table below, when the volatility is around 10% to 40%. The pricing error on the other hand is much bigger under stochastic volatility than with constant volatility for OTM options. And the most important is that the approximation from Levy falls into the 95% confidence interval only when the volatility is at 50%, with the pricing error of 1.2375%. Therefore, we can conclude that the estimation from Levy is not significant for OTM options with stochastic volatility.

Table 4.4

Valuation of arithmetic Asian options with OTM: S =90, K =100,T =1 year, r =7%,

Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)

Speed of mean reversion: α=10, long run mean:σ^*=15%, volatility of vol.: ξ=30%

				95% Confidence Interval		
*Volatility**	*Levy Approximation*	*Monte Carlo Simulation*	*Standard Error*	*Lower bound*	*Upper bound*	*Pricing Error (%)*
10%	0,2727	0,2963	0,0037	0,2890	0,3035	-7,9649
20%	1,7226	1,7796	0,0138	1,7525	1,8066	-3,2030
30%	3,5032	3,6451	0,0254	3,5953	3,6948	-3,8929
40%	5,4798	5,5971	0,0380	5,5227	5,6716	-2,0957
50%	7,7148	7,6205	0,0515	7,5195	7,7215	1,2375
60%	9,7326	9,4914	0,0656	9,3629	9,6199	2,5412

*Note, the volatility above is the starting value for estimating stochastic volatility

6.3.2 At the Money

With options that are ATM i.e. $S_t = K$, and volatility is stochastic, the values of the Asian options are once again under-estimated by the Levy approximation.

Table 4.5

Valuation of arithmetic Asian options with OTM: S =100, K =100,T =1 year, r =7%,

Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)

Speed of mean reversion: k=10, long run mean:σ^*=15%, volatility of vol.: ξ=30%

				95% Confidence Interval		
*Volatility**	*Levy Approximation*	*Monte Carlo Simulation*	*Standard Error*	*Lower bound*	*Upper bound*	*Pricing Error (%)*
10%	4,2284	4,2424	0,0141	4,2148	4,2700	-0,3300
20%	6,1667	6,2164	0,0259	6,1657	6,2670	-0,7995
30%	8,4481	8,4649	0,0387	8,3889	8,5408	-0,1985
40%	10,4257	10,4517	0,0523	10,3493	10,5542	-0,2488
50%	12,5867	12,6200	0,0668	12,4890	12,7509	-0,2639
60%	14,5793	14,7149	0,0838	14,5506	14,8792	-0,9215

*Note, the volatility above is the starting value for estimating stochastic volatility

The above table represents the estimated option values when the option is ATM with stochastic volatility. ATM options are more expensive than OTM options, which is coherent with the results of constant volatility. As can be seen from the above table, the pricing errors for Levy approximation at all volatility levels are negative, which means that Levy in general under-estimates ATM options when having mean-reverting stochastic volatilities. Even so, the option values from Levy are considered to be significant, since they are not rejected by the 95% confidence interval.

6.3.3 In the Money

Under the condition of stochastic volatility, ITM option outcomes are analogous with options that are OTM.

Table 4.6

Valuation of arithmetic Asian options with OTM: S =110, K =100,T =1 year, r =7%,
Dividend D=0, Number of observation N=260 (assuming 260 trading days a year)
Speed of mean reversion: k=10, long run mean:σ^*=15%, volatility of vol.: ξ=30%

				95% Confidence Interval		
*Volatility**	*Levy Approximation*	*Monte Carlo Simulation*	*Standard Error*	*Lower bound*	*Upper bound*	*Pricing Error (%)*
10%	13,0209	13,0276	0,0982	12,9895	13,0657	-0,0514
20%	13,6906	13,7460	0,0360	13,6755	13,8165	-0,4030
30%	15,1730	15,1737	0,0508	15,0741	15,2733	-0,0046
40%	16,9369	16,9657	0,0663	16,8357	17,0957	-0,1698
50%	19,3249	18,9610	0,0828	18,7987	19,1232	1,9192
60%	21,5012	21,2120	0,1013	21,0135	21,4106	1,3634

*Note, the volatility above is the starting value for estimating stochastic volatility

The difference between having constant and stochastic volatility when options are ITM, is that Levy over estimates option values for all levels of constant volatility. While under stochastic volatility, Levy over-estimates option values only with high volatilities, which in our case is when volatility is at a level of 50% and 60%. Nevertheless, the over-estimated option values are beyond the 95% confidence interval, and consequently are considered to be insignificant and therefore should not be taken into consideration. Overall speaking, the Levy approximation under-estimates option values when they are ITM with stochastic volatility.

To sum it up, Asian options with stochastic volatility, which follow a mean-reverting process, Levy approximation tends to under-estimate option values. In particular, for options ATM, Levy under-estimates at all level of volatilities, whereas for ITM options, it only happens with lower volatility up to 40%. However, when Asian options are OTM, the outcomes from the Levy approximation are not reliable, since the option values are not significant under the 95% confidence interval at almost all levels of volatility, with an exception of volatility level at 50%.

7. Conclusion

The main purpose of this thesis is to test how accurate the Levy approximation is when pricing arithmetic Asian options with constant and stochastic volatility. In addition, for each volatility scenario, the analysis on Levy approximation is examined for Asian options that are OTM, ATM and ITM.

Levy approximation formula altogether gives good estimation for Asian option values. The pricing error is relatively small (less than 1%) for option prices that are statistically significant, which in our case are option prices not rejected under the 95% confidence interval. However, Levy approximation tends to over-estimate Asian option values when the volatility is constant, with an exception of OTM Asian options where Levy gives an under-estimation of option values when volatility is at 10%-20%. With stochastic volatilities, Levy in contrast is inclined to give under-estimated option values; the reason is that with stochastic volatility, using mean variance does not seem to capture all the impact that the stochastic volatility has on option prices.

In addition, when volatility is high around 60%, the option values estimated by Levy´s formula are more likely to generate insignificant results regardless of the volatility being constant or stochastic. This is because high variance makes the Levy approximation more sensitive to volatility changes; and can also be explained by having pricing error increases with increasing volatility. With further research, studies could be done by having more focus on the moneyness conditions with a reasonable low volatility, for instance, how Levy approximation performs when Asian options are deep in the money. In addition, different variance reduction techniques could be applied to Monte Carlo simulations to have more accurate benchmarks with lower standard error.

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Appendix

Matlab code for Monte Carlo simulation with constant volatility

```
function f = Asian_Constant(S,r,sigma,T,nsteps,Npaths)

dt=T/nsteps;
mu=(r-0.5*sigma^2)*dt;
sig=sigma*sqrt(dt);
b=exp(-r*T);
path=zeros(Npaths,nsteps+1); path(:,1)=S;

for i=1:Npaths
    for j=1:nsteps
        path(i,j+1)=path(i,j)*exp(mu+sig*randn);
    end
end

Payoff=path;
Each_run_mean=sum(Payoff,2)/(nsteps+1);
Dispayoff=b*max(Each_run_mean-K,0);
Asian_constant=sum(Dispayoff)/Npaths;
f=Asian_constant
```

Matlab code for Monte Carlo simulation with stochastic volatility

```
function f = Sto_price(S,K,r,sigma,T,nsteps,Npaths,k,Q,z)

dt=T/nsteps;
mu=(r-0.5*sigma^2)*dt;
sig=sigma*sqrt(dt);
b=exp(-r*T);
path=zeros(Npaths,nsteps+1);
path(:,1)=S;

for i=1:Npaths
    for j=1:nsteps

sigam_v=sigma*exp((k*(Q-sqrt(sigma))-z^2*0.5)*dt+z*sqrt(dt)*randn);
        mu=(r-0.5*sigma^2)*dt;
        sig=sigma*sqrt(dt);
        path(i,j+1)=path(i,j)*exp(mu+sig*randn);
        sigam=sigam_v;
    end
```

```
end

Payoff=path;
Each_run_mean=sum(Payoff,2)/(nsteps+1);
w=Each_run_mean-K;
Disp=b*max(Each_run_mean-K,0);
asian_stochastic=sum(Disp)/Npaths;
f= asian_stochastic;
```