ALL WEATHER REVISITED

Short - term optimization of a robust All Weather - inspired portfolio

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Abstract

The aim of this thesis is to study a pair of portfolio strategies that ideally could perform well during most economical environments. These environments are portrayed as periods of rising or falling market expectations of future growth, inflation and credit risk. The concepts behind these strategies are based on *Bridgewater Associates' All Weather* fund. Current expectations of future conditions are derived from market asset prices. Expectations are viewed as risk factors in the portfolio risk modeling that plays an essential part in the strategy.

In the first strategy the allocation will be decided by minimizing the conditional value-at-risk. The second one resembles a Risk Parity – strategy. Assets are categorized in which environments they perform well and are then allocated to sub-portfolios. The portfolio allocation is given when then the sub-portfolios have equality of a portfolio measure.

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1. Introduction

1. 1 Background

Following the financial crisis the demand for funds and assets with stable growth and high sharp ratios to allocate capital has increased. The number of mutual funds in Europe boosted significantly by 25% between 2004 and 2008 but then decreased 5 % after 2008 ([6], page 1). Investors such as pension funds have taken stronger approaches on funds due to the fear of sudden large investment losses. In bull markets optimism is high and people tend to invest less cautious to try and catch never-ending growth. Advanced quantitative strategies may appear as interesting top of the line investments with promising horizons. But what strategies have strong fundamental economical ideas and what are leveraged risk bets on bull market behavior or trends?

Economic cycles occur for which certain funds and their primary strategies may be more or less biased. Inflation, growth and creditworthiness are some of the most basic indicators and pricing variables in the economy. Building a stably growing fund that is unbiased to most environments and with a strategy that is transparent and fundamentally easy to understand would be a good candidate for risk adverse institutional investors. The experimental concept of this thesis is built on the fundamental ideas of Bridgewater Associates fund *All Weather*.

1.2 All Weather and the economical market views of Ray Dalio

Bridgewater Associates formed the hedge fund *All Weather* in 1996 ([1], page 5). Founder of Bridgewater Associates, *Ray Dalio*, where interested in putting together a portfolio that would perform well during most economical environments *" ... be it devaluation or something completely different ..."* ([1], page 1). Ray believed that assets where environmentally biased and that market shifts where based *"... on shifts in conditions relative to the conditions that are priced in "* ([1], page 1). He and his team broke down market movements as shifts in market expectations of future conditions of inflation and growth and categorized which market priced assets that where biased to what shifts [1]. A common fund strategy set up has been to allocate 60% of capital in stocks and 40% in less volatile assets, such as bonds ([1], page 3). If viewed through Bridgewater's perspective a majority of the portfolio risk is allocated in stocks, which makes it significantly exposed to changes in market expectations of future growth ([1], page 3).

The aim for Ray Dalio and his team was to create a portfolio that would not depend on predictions of when and in what direction market expectation shifted, but where the accumulated effect of this would be balance out by assets that are driven in different directions due to these shifts. They had earlier categorized which assets that where driven in what ways of certain shifts and by allocating 25% of the total risk in each of the four possible shift scenarios they had created the fundament of All Weather ([1], page 5).

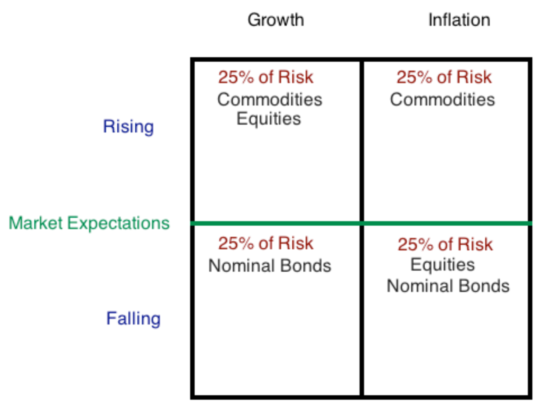


Fig 1.1 Example of how one may categorize assets to certain shifts in conditions in the economy and then allocate equal risk like in the All Weather portfolio [1]

As mentioned earlier Ray declared that certain assets where sensitive to shifts in the economical environments described by inflation and growth. In his paper "How the Economic Machine Works" he explains that economical growth is generally driven by three periodical components: productivity growth, the short-term debt cycle and the long-term debt cycle that repeats themselves [3]. Many scholars share the perception of repeating economical patterns and core causes of these have been largely researched and discussed. It is of importance to study these cycles to achieve an affective monetary policy [18] or to understand market behavior when investing.

The first component in Ray Dalio's framework is the productivity growth, which matters the most in the long run and doesn't fluctuate over time ([3], page 6).
The second component is normally referred to as the business cycle and is an effect of rapidly growing debt in the private sector that may lead to periods of recession ([3], page 3). The central bank sets the market interest rates, usually by purchasing short-term treasury bonds from the market along with other market actions, to stimulate or to lower the amount of credit in the economy, which leads to an increase or reduction in the purchasing of goods, assets and services. They do this mainly to keep the inflation healthy and stable in the economy ([3], page 3).
The third periodic component arises when the accumulated debt is too high in the economy as a result of credit growing faster then the reel productivity growth over a longer period of time ([3], page 3). When the debt leverage is too high central banks can´t stimulate the economy by lowering rates, since the debt cost is too high in the economy. This may lead to depression, deflation and followed by a time of *deleveraging* to lower reel debt and get the economy rising again [3]. After the financial crisis 2008 more unconventional methods where used by the central banks to recover the economy. Some banks used policies of *quantitive easing*, which means purchasing other fixed income assets than short-term government bonds to stimulate investment [19]. This way the banks would lower long-term interest rates and give credit to the private banks to sustain a positive inflation target forward.

Others have contributed to the discussion of what specifies and triggers these cycles. Jorda, Scumlarick and Taylor have made an extensive study of over 200 recessions in several advanced countries and in which they differentiate between a "normal" – recession and an often more severe "financial (systemic banking) crisis" – recession [17]. They emphasis the role of credit in the business cycle arguing, similarly as Dalio, that high credit leverage relative to GDP increases the probability, the downturn and the recovery time of an recession.
Borio shows two cycles in the economy; "the business cycle" and "the financial cycle" [18]. The financial cycle has lower frequency and at its peak it is often followed by a "financial crisis", i.e. systemic banking crisis. The writer states that credit to GDP and property prices are the two most solid economical variables to link the business cycle, the financial cycle and the financial crisis [18].

The economic cycles may perhaps not be static. Monetary and fiscal policy and a more global market may affect the duration of these cycles. Japan is an interesting example with a long period of nearly zero-inflation with hopes on its *Abenomics*. An important point to make about these cycles is that they are not very smooth, a burst is usually steeper than a period of inflating an economic bubble.

This section only points out economical cycles at a broad level. Furthermore, how economical conditions of inflation, growth and the amount of credit and creditworthiness may push the market.

1.2.1 Risk Parity

It is said in a Bridgewater article ([1], page 6) that the unconventional idea of focusing on allocating risk instead of capital between assets or strategies led to a generalized term for it called "Risk Parity" – portfolios, which may have been adopted by a consultant. A Risk Parity set up has won interest in research recently. Methods of balancing risk, defined as volatility, equally weighted risk (volatility) contribution [4] and Value-At-Risk has been introduced [7]. Depending on the risk measure, a benefit of risk budgeting is that one does not have to depend on forecasting expected returns [5], if this is not needed in the risk calculations.

To merge the theories of Modern Portfolio Theory introduced by Markowitz [8] and risk parity one may need to leverage less volatile assets to be able to take on equal risk between assets in the portfolio and still keep a high expected return [2]. Ray Dalio shows the benefits of leveraging low-volatile assets by achieving higher sharp ratios than non-leveraged efficient frontier portfolios, for the same amount of total risk (volatility). He points out that most assets have equal sharp ratios [2].
Initially solved by allowing long duration bonds in their portfolio to achieve higher volatility for bonds, that tended to go in opposite directions of stocks ([1], page 3), leveraging low-volatile assets is now a fundamental component in the building of the All Weather fund [2].

Increasing leverage of bonds could make a portfolio more sensitive to interest rates. After the financial crises risk parity portfolios may be exposed to historically low interest rates that could start to rise. Including bonds in a portfolio is still important for diversification during environments of rising rates if the reasons driving interest rates are understood [22]. If growth and inflation are the key factors driving rates, portfolio assets like equity and fixed-income may still manage to offset. If sovereign debt and central bank activities drives the rates or the market its more difficult to avoid losing money due to increasing interest rates [22].

1.3 Goal

The main goal of this thesis is to examine a group of purposed portfolio allocation algorithms that are inspired by the economical concepts of Ray Dalio and the All Weather fund. The research is aimed to see if it is possible to find a fundamentally based long-only portfolio strategy that is less sensitive to shifts in future market conditions as in the All Weather framework that performs well with low volatility and minor drawdowns. These conditions are future expectations of inflation, growth and also creditworthiness. The portfolio strategies will be based on trying to extract implicit quantitative measures of future market conditions that will then be used as variables in the asset pricing models. These variables can also be seen as risk factors in the calculations of the risk measurements used.

The first bit of the thesis is about showing how the implied future market conditions are derived. Secondly, the asset pricing models that have the implied future market conditions as variables are presented. Then follows a presentation and methodology for each of the strategies that will be examined.

When the methodologies have been presented, the strategies will be historically back tested with different assets from a chosen asset universe. The results will be presented and analyzed.

The last part of the thesis will discuss the results of the back tests, the biasness of the strategies and try to suggest future improvements and research.

Note: This thesis will only concern USD denominated assets to avoid FX risk.

2. Methodology

2.1 Implied scenario values

In this part, the derivation of the implied future market condition variables that are used in the market asset pricing models are presented.

It is important to point out that these implied values are not seen as predictions of real future conditions but as *indicators* of changes in market expectations of future conditions that are used in asset pricing functions. However, an indicator should follow the relative magnitude of the shift of the market expectations, to indicate the effect of certain events. In practice these indicators are asset specific and may depend on the asset's properties on more detailed levels. For example, two T- Notes with the same maturities and term structures but different coupon yields, may give different values of implied inflation because they have different marked asked yield quotes.

2.1.1 Implied inflation

Patwardhan and Devlin write that the government bond yield is priced of three components; the real yield, the expected inflation and the risk premium for shifts of the previous components ([9], page 7). Taking the yield spread between the nominal government bond and a benchmark inflation protected government bond derives the implied inflation.

In this thesis, the implied inflation will be the spread of US Treasury Nominal Yield Curve and the US Treasury Real Yield Curve. The real yield curve represents the real yield of TIPS, Treasury Inflation Protected Securities, at different maturities. These coupon bonds are similar to T-Notes and T-Bonds but are inflation protected in the way that the face value of the bonds are in after hand adjusted with respect *to realized CPI indices* to achieve the real yield that was quoted when bought. They are more or less inflation hedged. The spread is called the *"break even"* – inflation rate, and is the yield premium that bond investors demand to take on inflation risk ([9], page 7).

The relationship between the nominal U.S. Treasury yield curve (constructed of non-inflation protected bonds such as T-bills, T-Notes and T-Bonds), $\pi_{T-\mathrm{NomBond},M,t}$, and the real yield curve (constructed of TIPS), $\pi_{TIPS,M,t}$, at time t with different maturities M are here the following:

$$\pi_{T-\mathrm{NomBond},M,t} = \pi_{R,M,t} + \pi_{\mathrm{Imp}\,Inf,M,t} \tag{2.1}$$

$$\pi_{TIPS,M,t} = \pi_{R,M,t} \tag{2.2}$$

The parts constructing the nominal Treasury yield-to-maturity will only be the real yield, $\pi_{R,M,t}$, and the implied inflation ("break even" – inflation rate), $\pi_{\mathrm{Imp}\,Inf,M,t}$. The likelihood for a default payment is, in this paper, considered to be close to zero and a default risk premium is therefore not included.

Equation (2.1) is an approximated summation of the fundamental relationship between the nominal and the real yield:

$$1+\pi_{\mathrm{Nominal},M} = (1+\pi_{\mathrm{Real},M})(1+\pi_{\mathrm{Inflation},M}) \tag{2.3}$$

where $\pi_{\mathrm{Nominal},M}$ is the nominal yield, $\pi_{\mathrm{Real},M}$ is the real yield and $\pi_{\mathrm{Inflation},M}$ is the inflation over maturity M.

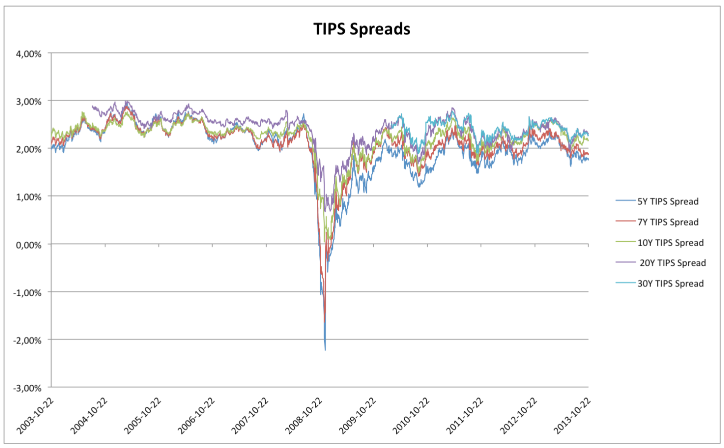


Fig 2.1 Showing the daily yield spread between the U.S. Treasury Nominal Yield and the TIPS Real Yield curve with the same maturities [10]

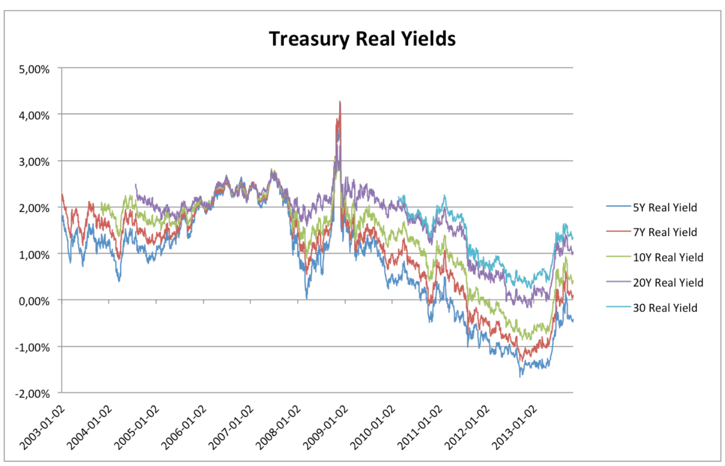


Fig 2.2 Showing the daily US Treasury real yields [10]

The figures in this section display some of the market shifts in these indicator values during the recent years. Fig 2.1 shows the implied inflation rate extracted as in equation (2.1) during the financial crisis. The market seems to have expected a strong deflation and in Fig 2.2 the rising of the U.S. Treasury real yields is seen. Comparing the later annualized CPI from quarterly data and the implied inflation rate in Fig 2.3, the market seems to have predicted this short-term course.

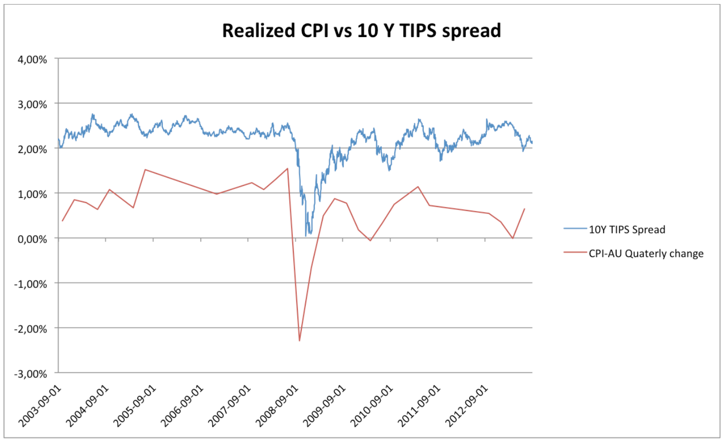


Fig 2.3 Showing the realized quarterly change from the CPIAUCSL index (Consumer Price Index for All Urban Consumers: All Items, Seasonally Adjusted, aggregation method: average) versus the daily 10Y TIPS spread [10]

In summary the implied inflation will work as an indicator of shifts in market expectations of future conditions. If it is a good estimator of future inflation at given horizons is another topic. Remember that T –notes and bonds pay semi-annual coupons and so the duration is not at the maturity date, in other words short-term fluctuations of expected future inflation might rapidly change the prices of long-term bonds. In times of recession firms and investors may perhaps seek safe investments with short duration, like T-bills, which also affects pricing.

2.1.1 Implied growth

Many companies perform well when the economy is growing and worse in times of recessions. Stocks are often fundamentally valued with the use of key values like P/E (Price-to–Earnings) ratios and PEG (Price-to-Earnings-Growth). Large stock indices reflect the general performance of stocks and tell investors' views of current market prices in relation to future conditions. Using the largest stock exchange indices and common stock valuation methods might be a good way to estimate implied future growth. A large stock index has the advantage to diversify a lot of specific branch or company risk.

Implied annual growth rate

Kajanoja suggests in an article that future GDP expectations can be extracted from the present market stock price by using a discounted cash flow model [11]. It states that one can estimate the short-term and long-term growth rate expectation of future dividends in a stock price. The long-term dividend growth rate expectations subtracted with expected long-term inflation have a linear relationship with the long term GDP growth in his framework [11].

In this thesis, the implied GDP annual growth rate, G_t, at time *t* will be calculated as:

$$S_t(D_t, G_t, \pi_{N,t}, N) = D_t \sum_{i=1}^{N} \frac{(1+G_t)^i}{(1+\pi_{N,t}(i))^i} \tag{2.4}$$

where D_t is the last 12-month paid dividends, S_t is current the stock price and $\pi_{N,t}(i)$ is the discount rate at year *i* in *1 x N* nominal yield vector $\pi_{N,t}$ that contains the nominal U.S. Treasury yields, which will be used as discount factors.

The yield curve will be constructed by using market yield data given at certain maturities. Yields in between the given maturities will be interpolated with a linear spline, and yields with greater maturities will be calculated by simple extrapolation. The extrapolation is done by increasing the treasury yield point with the greatest maturity by an arbitrary fixed factor *1 + x* for each following year. In Fig 2.4 is an example of a discount rate curve constructed with *N* = 70 and *x* = 0.01 which represents an increased yield of 1 % per year post 20 years of maturity.

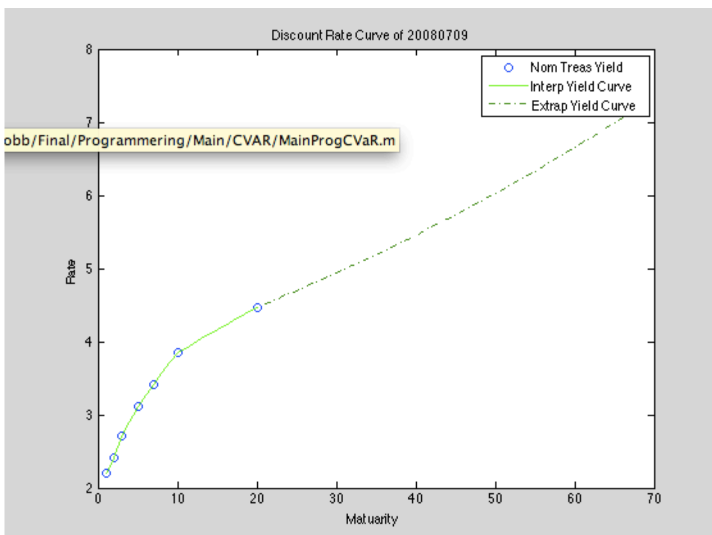


Fig 2.4 Showing the discount rate curve at 2008-07-09 constructed of nominal treasury yield points [10] that will be used to extract the implied growth rate of a large equity index.

The reason that *N* is limited is to try attaining a reasonable sensitivity of the market stock price to changes of the nominal yield points that are used to create the discount curve. A volatile long-term yield point could disturb the stock price modeling. Also, the duration of the stock index might be too far out, which perhaps does not reflect investors' investment horizons, especially during economic crises.

The stock price will also be sensitive to the implied inflation of the market, referring to $\pi_{N,t}(i) = \pi_{T\text{-NomBond},M,t} = \pi_{R,M,t} + \pi_{\text{Imp } Inf,M,t}$ from equation (2.1). Since company earnings and dividends might be affected by inflation, the implied growth rate G_t refers to the nominal annual growth rate. The annual dividend term D_t is the last 12 months of dividends instead of taking the most previous dividend and annualizing it, avoiding intra-year fluctuations due to effects of corporate taxes and such. Any implied growth rate is specific to the stock they are derived from.

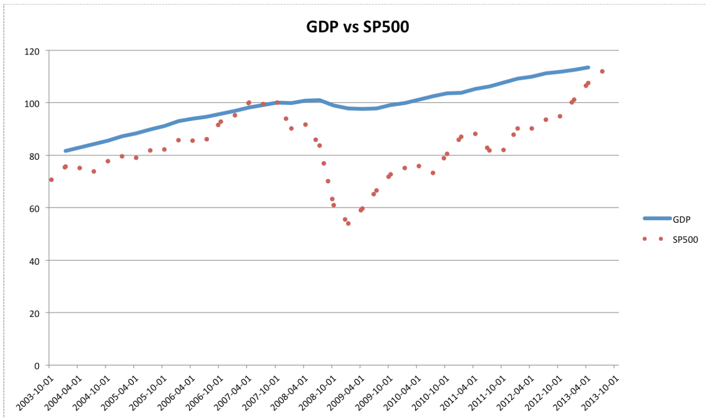


Fig 2.5 Showing the nominal GDP versus the SP500 with index = 100 at 2007/10/01 [10]

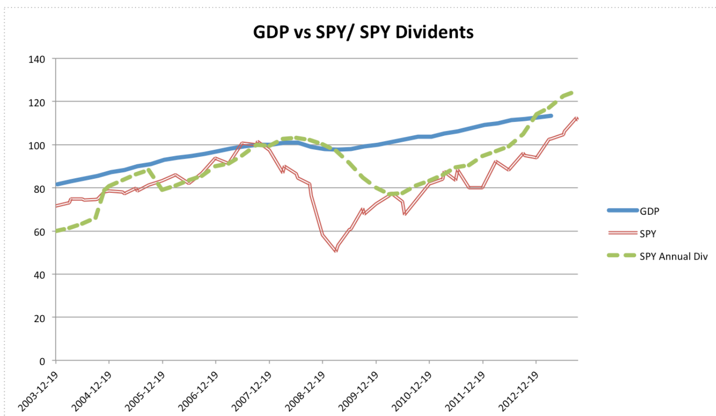
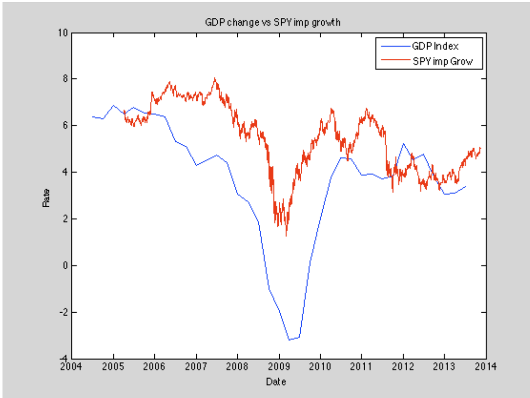


Fig 2.6 Showing the nominal GDP (index = 100 at 2007/10/01) versus the ETF SPY close price (index = 100 at 2007/09/21) and the realized total 12 month rolling dividends (index = 100 at 2007/09/21) [10] [16]

In Fig 2.5 the SP500 index follows the direction of the GDP the last 10 years. The decline of the index is heavier during the recession of 2008-2009 than of the GDP. This may be that stocks are often very leveraged consequently very sensitive. The rolling 12 month dividend of the SPDR® S&P 500® ETF (Exchange Traded Fund), which aims to replicate the return and dividend of SP500, weakened after the recession, see Fig 2.6.



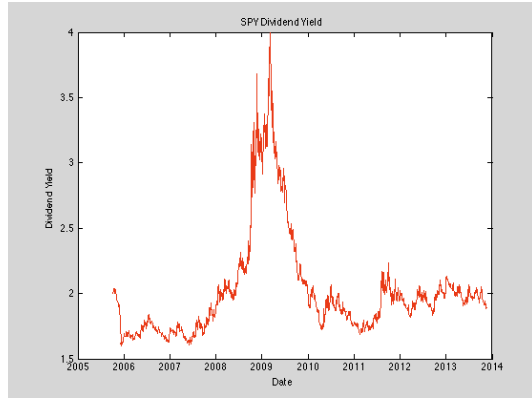


Fig 2.7 Showing the realized rolling one year nominal GDP change [10] versus the SPY implied growth rate calculated with N = 40 and x =0.01

Fig 2.8 Showing the estimated SPY dividend yield

The implied growth indicates changes in market expectation relative to the conditions they are priced in, which in the model is captured in the price relation to the discounting interest rates. Deriving the implied growth rate from SPY in Fig 2.7 we see that the implied growth moves slightly before the realized GDP.

Dividend yield

Another common key value used to valuate stocks is the D/P = Dividend/Price ratio. Investors want to be compensated for risk and thus require high expected returns for risky assets. The risk premium widens during pessimistic periods and the P/D ratio might go up. However, the level of the dividend yield depends greatly on the relative prices of other market assets, for instance if short-term interest rates are low, or if future growth look promising.

The dividend price ratio DP_t at time t will be calculated as the accumulated 12-month dividend D_t for a stock divided by the stock price S_t

$$DP_t = \frac{D_t}{S_t} \tag{2.5}$$

In Fig 2.8 one can see how the dividend yield went up during the financial crises but lowered after. Perhaps due to the low interest rates that led to a widened yield spread between treasury yield and SPY dividend yield.

2.1.1 Implied credit risk

The implied credit risk is in this thesis derived in the same way as the implied inflation. The yield spread between a nominal treasury bond and a corporate bond with the same maturities gives the implied credit risk in terms of yield for that corporation. Pricing due to other reasons such as corporate taxes will not be considered in the model.

A Treasury bond is viewed as if it where default risk free, so the corporate bond yield spread is the compensation investors demand for a certain probability of a defaulted bond payment. However, it is not of certainty that U.S. government bonds are free of default risk. The U.S. government has faced great challenges recently with its rising sovereign debt, fiscal policy and the altering debt ceiling leading to credit ratings being downgraded [20]. The long-term yield points probably have a higher default risk spread in the treasury yield curve. This does affect the fundamental concept behind the implied credit risk but the impact is subjectively regarded as small in this thesis.

The implied credit risk $\pi_{\text{Imp Cred Risk } A,M,t}$ for a corporate bond issued by corporation A is given by

$$\begin{aligned}\pi_{CorpBond\ A,M,t} &= \pi_{T-\text{NomBond},M,t} + \pi_{\text{Imp Cred Risk } A,M,t} \\ &= \pi_{TIPS,M,t} + \pi_{\text{Imp } Inf,M,t} + \pi_{\text{Imp Cred Risk } A,M,t}\end{aligned} \tag{2.6}$$

where $\pi_{CorpBond\ A,M,t}$ is the YTM yield for a certain corporate bond with maturity M at time t. It would be preferable for the benchmark bonds to have the same coupon rates and term structures but it is not essential in the framework.

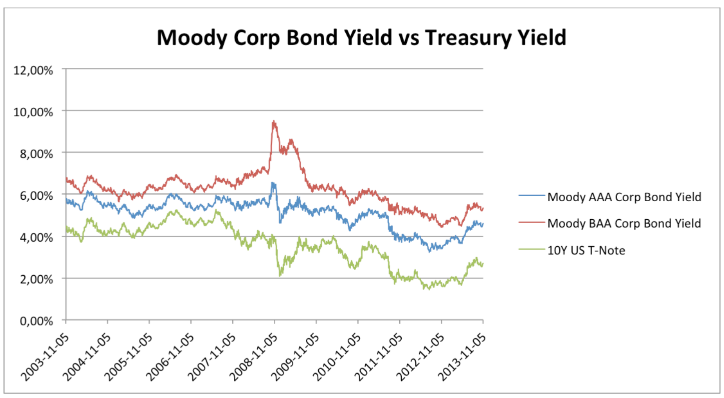


Fig 2.9 Showing Moody's Seasoned Aaa Corporate Bond Yield, Moody's Seasoned Baa Corporate Bond Yield and the US Treasury Nominal 10Y Note yield [10]

Looking through a historical perspective the corporate bond yield spread between higher and lower rated bonds widened for the Moody indices during the financial crisis, see Fig 2.9. The spread between the U.S. Treasury 10Y T-Note yield and Moody's high graded corporate bond yield also widened. During recessions the demand for safer assets increases and investors have a tendency to sell their riskier assets.

The implied credit risk might be an important building block in the portfolio to compensate when growth and inflation do not manage to offset each other, typically when low rates do not manage to stimulate growth and credit. Widening credit spreads may imply doubt in the market that could be correlated with stock movements.

2.2 Market asset valuation models

Following are the asset pricing models that will be used to calculate the profit and loss distribution in the risk modeling. Further simplifications will be done when modeling with more specific securities in the historical back testing, see section 3.1.1.

2.2.1 Bonds

The portfolio will contain three types of bonds; TIPS, nominal government bonds and corporate bonds. These bonds will be valued with the same pricing formula:

$$P(FV,\pi,c,T,N) = FV\left(\frac{c}{\pi}\left(1-\frac{1}{(1+\frac{\pi}{N})^{NT}}\right)+\frac{1}{(1+\frac{\pi}{N})^{NT}}\right) \tag{2.7}$$

where $P(FV,\pi,c,T,N)$ is the present bond price, π is the yield-to-maturity, c is the coupon rate, T is the time to maturity in years, FV is the face value and N the number of coupon payments made per year. However, the input YTM variable π will be differently constructed for each type of bond.

TIPS: In the real world, the face value of the bond is adjusted for inflation or deflation at a later time (after the realized CPI is presented). Here, the future change of the face value will not be considered in the present valuation/risk calculations, only the current bond *FV* will be used. The yield for pricing a TIPS bond with maturity *M* at time *t* will be the real rate $\pi_{R,M,t}$ mentioned in equation (2.2).

Nominal Government Bonds: The input yield in equation (2.6) for a nominal government bond, $\pi_{T-\mathrm{NomBond},M,t}$, is the sum the implied inflation $\pi_{\mathrm{Imp}\ Inf,M,t}$ and the real yield $\pi_{R,M,t}$ with maturity *M* as in equation (2.1).

Corporate bonds: The input yield in equation (2.6) for a corporate bond, $\pi_{CorpBond\ A,M,t}$, is the sum of a benchmark nominal government bond, $\pi_{T-\mathrm{NomBond},M,t}$ and the implied credit risk spread, $\pi_{\mathrm{Imp\ Cred\ Risk}\ A,M,t}$, with maturity *M* as in equation (2.5).

2.2.2 Stocks

Stocks will be priced as in the discounted cash flow model in equation (2.3) with the same input variables as mentioned in section 2.1.1. The input yield vector will be the nominal government yields $\pi_{T-\mathrm{NomBond},M,t}$ constructed of the real yields and implied inflation values.

2.3 Portfolio allocation strategies

2.3.1 Optimal CVaR

The first strategy is based on minimizing the risk measure *CVaR – Conditional Value-at-Risk*. First, an introduction of both *Value-at-Risk* and CVaR will follow. Then follows a description of the calculating and optimization of the risk measure.

Risk Measures

Value-at-Risk

The risk estimator *Value-at-Risk*, *VaR*, is often used in the financial industry. It tells that a future portfolio loss will be less then $\beta - VaR$ with probability β. This can be formally expressed as [14] (with modification from the original article)

$$\beta - VaR = \underset{l \in R}{\operatorname{argmin}} \left\{ P(L \leq l) \geq \beta \right\} \tag{2.8}$$

where L is a stochastic variable corresponding to the loss amount of the portfolio over a time horizon. It is the actually the β - quantile of the loss distribution for the portfolio. The VaR indicates that a loss greater than $\beta - VaR$ is expected to occur on average every $1/\beta$ -th time (if the parameters of the real world P/L distribution where static of course). VaR is not defined as a coherent risk measure because it does not fulfill the requirement of *sub-additivity,* which is

$$\rho(X+Z) \leq \rho(X) + \rho(Z) \tag{2.9}$$

where ρ is the function and *X*, *Z* are stochastic variables ([12], page 3).

It can be calculated in different ways by historical simulation, Monte Carlo simulation or fitting an appropriate probability distribution of the P/L and calculating the β - quantile calue [13]. The portfolio P/L can be expressed as a linear combination of the portfolio asset returns and allocation weights or by the portfolio's *risk factors*.
The risk factors are variables that impact the asset returns in the portfolio. Many assets in the portfolio can be sensitive to the same risk factors. For example, the asset price of a call option with a specific underlying stock will be sensitive to changes in both the stock price and the implied stock volatility of the option, which are two possible risk factors. Moreover, two call options that have the same underlying stock but different maturity dates will both be sensitive to the same stock price but different implied volatilities, which also could be considered as risk factors.
It is common to use a normal distribution to model the portfolio P/L. If and only if the stochastic P/L is modeled as a linear combination of risk factors from a multivariate Gaussian distribution it is then described by the mean vector and covariance matrix of the risk factors.

The method of linearizing non-linear relationships of assets returns and underlying variables in the risk calculations is called *a delta/gamma approach* ([13], page 105). This approach works best for smaller changes and is based on Taylor series approximations of assets pricing functions, like the Black-Scholes options formula. A linear relationship between an asset and a risk factor can be also statistically estimated instead of derived analytically.

The VaR has its attractions since the concept is easy to understand and can present the total accumulated portfolio risk nicely. It allows investors to compare the risk of several assets using the same static and it takes into account the correlations between the risk factors ([13], page 10).

A weakness of VaR is that the statistical modeling may not consider the whole range of factors from different sciences that affect the pricing of financial markets, like social psychology ([13], page 10). The VaR value does also not display the shape of the probability distribution and a larger tail in the loss distribution following $\beta - VaR$, can result in a big loss if it where greater than $\beta - VaR$.

Conditional VaR

The risk measure conditional value-at-risk, CVaR, goes by many names in financial mathematical literature; *tail conditional expectation* ([12], page 4), *expected tail loss*, *expected shortfall, tail VaR* among more ([13], page 32). It is closely related to VaR.
The measure gives the expected loss if the portfolio loss would exceed $\beta - VaR$ and can be formally written as [14] (with modification from original article)

$$\beta - CVaR = E[L \mid L \geq \beta - VaR] \qquad (2.10)$$

where $\beta - VaR$ is presented in equation (2.7).

The risk measure CVaR holds the benefit, compared to VaR, that it is sub-additive [12]. This property gives that the overall portfolio risk cannot exceed the accumulated risk of its sub-portfolios and therefore encourages diversification properly ([12], page 3). It is also suitable when it comes to linear optimization since sub-additivity is actually closely related to convexity ([12], page 3). This is beneficial when finding unique minimums, i.e. solutions, in problems of portfolio optimization ([12], page 3). It is one of the main reasons why the portfolio in this thesis will be optimized with respect to CVaR.

As mentioned earlier the shape of the β-tail of the loss distribution is not being described in VaR and optimizing on this value might be quite risky. The CVaR is sensitive to the shape of the β-tail and penalizes allocations for probabilities that are far out, which is another reason why it was chosen to be the measure to minimize.

Estimating the loss distribution

The loss distribution for each asset will be estimated by Monte Carlo simulating changes of implied scenario values and then putting them into the asset pricing functions, presented in section 2.2, to get simulated losses or profits.

The implied scenario variable changes will be simulated either by a multivariate normal distribution or by re-sampling historical data like in non-parametric bootstrap. When simulating, the number of historical data points is fixed to re-estimate the parameters for the normal distribution and also the empirical distribution for the non-parametric bootstrapping. The reason for this is to quickly capture the current volatility and correlation of the implied scenario values.

When bootstrapping, the process is picking a random day from the historical data set and re-pricing the assets as all the implied scenario variables moved that historical day. This is repeated a large number of times to calculate a simulated empirical loss distribution.

The changes of the implied scenario variables are absolute and not relative. Fixed-income assets are very sensitive to absolute changes in yield, but the absolute changes are steadier compared to relative changes when rates are low, and therefore fit the probability distribution better.
The Monte Carlo – approach, which allows for estimating the P/L – distribution of non-linear asset pricing functions without linear approximations of sensitivities, seems preferable when having a large group of fixed income assets.

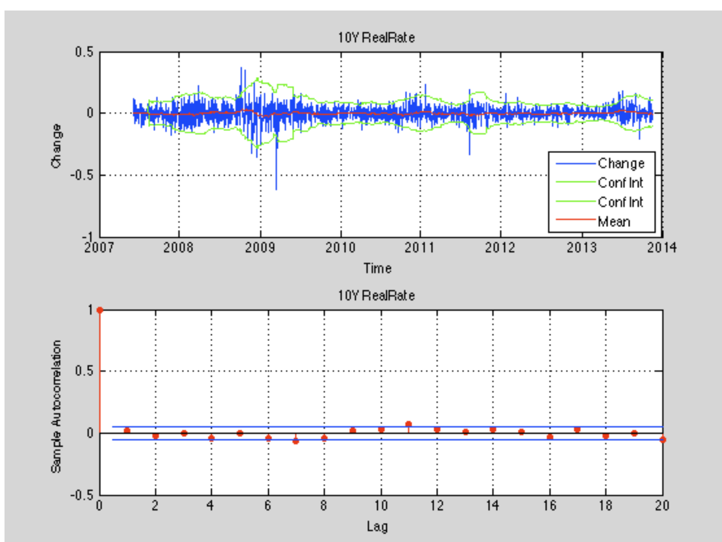


Fig 2.10 Showing the daily absolute changes (1 unit y – axis = 100 Basis points) of the 10Y TIPS real rate point [10]

Fig 2.10 displays the daily absolute changes of the 10Y real rate together with the rolling estimated normal distribution. The green lines shows the 95% - confidence interval of the estimated distribution and the red line is the mean value. The number of historical data points is 40 in the picture. The second subplot is the autocorrelation plot. There was almost no autocorrelation for the daily changes of any of the other pricing variables. This could be because a day is a short time horizon and the autocorrelation estimation does not capture medium-term trend for the variables.

Optimization routine

The idea of the strategy is to let movements due to shifts in expected future economic conditions offset each other like in the All Weather framework. The implied scenario variables are there to indicate the future expectations in the market. By minimizing the risk measure after simulating asset losses this balance will be achieved. The CVaR of the portfolio depicts the attributions due to shifts in the implied scenario variables. This is the reason why the minimum CVaR at the confidence level β gives the allocation.

The implied scenario values are the only random variables in the assets pricing models in this thesis. The assets are chosen to try and diversify many other specific risk factors, like branch risks. In portfolios with larger universes of assets, other risk factors might be needed to be included or the pricing models should perhaps be different.

In an article by Rockafellar and Uryasev, minimizing CVaR can become a problem of linear optimization thanks to a convex approximation of this risk measure [14]. Start to define the loss function of the portfolio as $f(\mathbf{x},\mathbf{y})$. The input variable $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is the allocation vector in the portfolio for each asset *1 ,2 ,..., n* and the second variable $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ is a vector of stochastic variables that determines the loss or gain for each asset. Given an allocation **x,** the $\beta - VaR$ is defined as $\alpha_\beta(\mathbf{x})$ and $\beta - CVaR$ as

$$\phi_\beta(\mathbf{x}) = (1-\beta)^{-1} \int_{f(\mathbf{x},\mathbf{y}) \geq \alpha_\beta(\mathbf{x})} f(\mathbf{x},\mathbf{y}) p(\mathbf{y}) d(\mathbf{y}) \tag{2.11}$$

The minimization is based on the convex function $F_\beta(\mathbf{x},\alpha)$ that is an expression of both $\phi_\beta(\mathbf{x})$ and $\alpha_\beta(\mathbf{x})$ as

$$F_\beta(\mathbf{x},\alpha) = \alpha + (1-\beta)^{-1} \int_{\mathbf{y} \in R^n} [f(\mathbf{x},\mathbf{y}) - \alpha]^+ p(\mathbf{y}) d(\mathbf{y}) \tag{2.12}$$

If function $F_\beta(\mathbf{x},\alpha)$ is minimized with respect only to α it returns $\phi_\beta(\mathbf{x})$ [14]. This gives us

$$\min_{\mathbf{x}} \phi_\beta(\mathbf{x}) = \min_{(\mathbf{x},\alpha)} F_\beta(\mathbf{x},\alpha) \tag{2.13}$$

The stochastic function $F_\beta(\mathbf{x},\alpha)$ can be estimated by drawing sample vectors $\mathbf{y_1}$, $\mathbf{y_2}$, … , $\mathbf{y_q}$ and using these to calculate the estimator function $\tilde{F}_\beta(\mathbf{x},\alpha)$, similar to the method of *basic Monte Carlo integration* [15], which is [14]

$$\tilde{F}_{\beta}(\mathbf{x},\alpha) = \alpha + \frac{1}{q(1-\beta)}\sum_{i=1}^{q}[f(\mathbf{x},\mathbf{y}_i) - \alpha]^{+} \tag{2.14}$$

The estimator should converge depending on the sample quantity. If equation (2.14) is optimized the suggested allocation **x** for the portfolio is given.

2.3.2 The fundamental Risk Parity Method

The second allocation strategy method that will be tried out is a Risk Parity approach. Here, the assets in the portfolio will be fundamentally categorized to different sub-portfolios that may be biased to different economical environments. The asset allocation in each sub-portfolio will be decided so that all the sub-portfolios have an equal value of a chosen measure. The allocation will be decided to attain one of the three terms; *equal volatility*, *equal volatility contribution* or *equal capital* between the sub-portfolios. The sub-portfolios should hopefully offset each other so that the portfolio is not more exposed to certain environments. In a risk parity set up the allocation is determined by the asset risk [5]. These measures could be viewed as trivial risk measures.

The term equal volatility means that the linear combination of asset volatilities and asset weights in each sub-portfolio are the same for all sub-portfolios. If $w_{k,j}$ is the portfolio weight for asset *j* in sub-portfolio *k* the relationship can be written as

$$\sum_{j=1}^{N} w_{k,j}\sigma_j = \sum_{j=1}^{N} w_{l,j}\sigma_j \tag{2.15}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} w_{i,j} = 1 \tag{2.16}$$

where σ_j is the volatility for the return of asset *j* and $\mathbf{w} = (w_{1,1}, w_{1,2}, \ldots, w_{M,N})$ is the portfolio asset allocation. The total number of assets are *N* and the total number of sub-portfolios are *M*. By using this measure one does not depend on the correlation between the assets. To attain equal volatility one can minimize the expression

$$\Omega = \sum_{i=1}^{M}\sum_{j=1}^{N} w_{i,j}\sigma_j \tag{2.17}$$

$$\min_{\mathbf{w}} \sum_{i=1}^{M}\left(\frac{\Omega}{M} - \sum_{j=1}^{N} w_{i,j}\sigma_j\right)^2 \tag{2.18}$$

with the function `fmincon` in MATLAB®.

The second way to choose the allocation is to have equal volatility contribution to the total volatility of the portfolio return. A sub-portfolio *m* contributes $\tilde{\sigma}_m^2$ to the total volatility of the portfolio σ_p^2 in the way

$$\tilde{\sigma}_m^2 = \sum_{j=1}^{N} w_{m,j} \sum_{i=1}^{M} \sum_{k=1}^{N} w_{i,k} \sigma_{k,j} \tag{2.19}$$

$$\sigma_p^2 = \sum_{m=1}^{M} \tilde{\sigma}_m^2 \tag{2.20}$$

These equations have the same variable as in equation (2.15) but also include the covariance $\sigma_{k,j}$ of the returns of assets *k* and *j*. Using this measure, when allocating, also considers the covariance between the assets in the portfolio. The allocation can be calculated by minimizing

$$\min_{\mathbf{w}} \sum_{m=1}^{M} \left(\frac{\sigma_p^2}{M} - \tilde{\sigma}_m^2\right)^2 \tag{2.21}$$

with the function `fmincon` in MATLAB®.

The volatilities and covariances of the asset return are estimated from simulated asset prices. This is done in the exact same way as when estimating the loss distribution from Monte Carlo simulated implied scenario variables, see part **Estimating the loss distribution** in section 2.3.1.

The way that the above optimization routines are set up are similar to a numerical solution for risk parity portfolios suggested by Maillard, Roncalli and Teiletche [4]. They define a *equal-weighted risk contribution portfolio* where all the assets achieve equal risk contribution to the portfolio as below

$$w^* = \left\{ w \in [0,1]^n : \sum w_i = 1, \quad w_i \times \partial_{w_i} \sigma(w) = w_j \times \partial_{w_j} \sigma(w) \quad \text{for} \quad \text{all} \quad i,j \right\} \tag{2.22}$$

with w_i being the portfolio weight for asset *i*. Note that the variable w_i refers to individual assets and not sub-portfolios. They suggest a numerical solution for the optimization routine that is

$$\min_{w^*} = \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i (\Sigma w)_i - w_j (\Sigma w)_j)^2 \tag{2.23}$$

where Σ is the asset covariance matrix, $(\Sigma w)_i$ denotes row i in the vector of Σw and that could be solved using a *Sequential Quadratic Programming* algorithm. The benefit with constructing the optimization like in equation (2.23) is that is does not include non-linear inequality constraints [4]. This regards the optimization problems in equations 2.18 and 2.21. In the paper they write that they came upon examples where an optimization was tricky to find with the method and suggest some modification.

The portfolios that where simulated in this thesis did not lead to any problems with attaining balanced sub-portfolios. The portfolio may have had marginally different asset allocation for each historical simulation due to the covariance matrix being estimated from simulated data. However, if the weight variables where multiplied with 1000 the optimization converged and equality of a chosen measure between the sub-portfolios where attained during the simulation. When `fmincon` is used in the simulation the function automatically switches to an `active-set algorithm`. In figures 2.11 and 2.12 the distribution between the sub-portfolios of a simulated example portfolio during the year 2009 is seen. The sub-portfolios are categorized as “Growth Upp” and “ Growth Down” and contain two different assets in each sub-portfolio. Fig 2.12 reveals a stable equality of 50% of the different measures in each of the sub-portfolios. Notice that the benchmark portfolio has the allocation 60/40 between bonds and stocks.

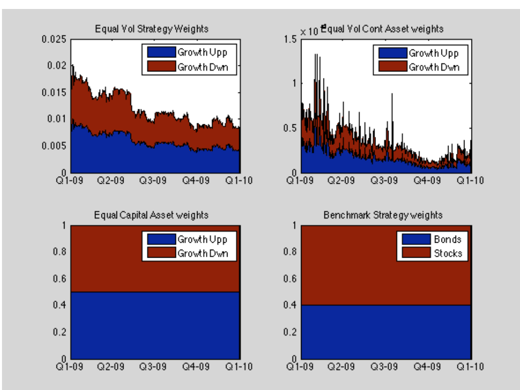


Fig 2.11 Showing the distribution and total value of a certain measure between the sub –portfolios of an example portfolio

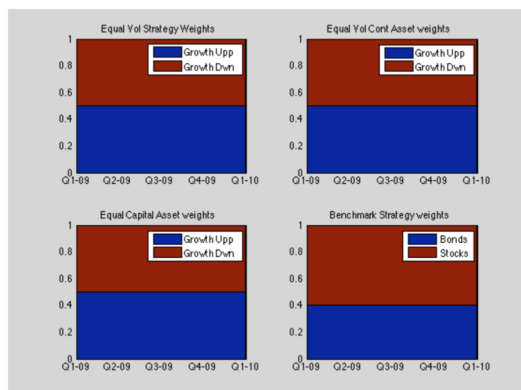


Fig 2.12 Showing the normalized distribution of a certain measure between the sub-portfolios of an example portfolio

The last method will be to allocate equal amount of capital for each sub-portfolio, therefore the term equal capital. The assets in the sub – portfolio will be distributed equal capital among them. No statistical measure of risk or forecasts of returns are used when allocating capital, but relies only on the fundamental analyses when categorizing assets. This is to see if the use of estimated risk balancing makes the strategy better.

The sub – portfolio categorization

The focus is to balance out the biasness of certain economical environments. The sub – portfolios will therefore contain assets that perform well during changes of future economical conditions of: inflation, growth and credit risk. The assets are assumed to be biased in the following way

Stocks: Goes up when economical growth increases. Nominal stock profits might increase with inflation, if the rate is at an appropriate level. The real debt should decrease for companies if the inflation increases which is favorable. Having deflation is bad for overall economical growth since profits decrease and salaries take time to lower.

Nominal treasury bonds: Treasury bonds do not contain much default payment risk and so during low growth and high credit risk these assets are very attractive to many buyers. But during bull markets with high-expected growth and low credit risk the yield might be too low for investors and so the prices fall. Rising inflation causes the yields to rise, to compensate for the inflation, and deflation causes the yields to lower.

TIPS: Since U.S. Treasury is the issuer; these securities usually have the highest credit rate. During times of rising credit risk and falling growth the prices of TIPS go up.
High inflation attracts investors to buys these securities and deflation repels them.

Corporate bonds: During times of high credit risk the corporate yield spreads benchmarked to government bonds should widen and decrease during times of low credit risk. Falling growth is bad for companies which lead to rising corporate bond yields, especially for high-yielding corporate bonds. Rising growth might have the opposite affect. High inflation will increase the yield of nominal treasury bonds and so the corporate yields should increase. In times of deflation nominal treasury yields decrease, which might lower the corporate bond yields at first. Companies may take a hit when profits decline and the reel debts might increase, which should lead to larger corporate bond yield spreads.

These are just guesses how assets perform during the economical scenarios. The scenarios are probably no even distinct. High growth can lead to, or occur, at the same time as high inflation, and the opposite situation for falling growth (if monetary policy is not efficient). It may also be a question of money flows between securities if the relative prices are low. If both nominal rates and growth are low but stable, maybe corporate bond yields spreads are low since they have the highest yield or that it could be an affect of fiscal policy.

3. Empirical results

3.1 The simulated portfolio

3.1.1 The Asset Universe

The portfolio will consist of several ETFs that are presented in Table 3.1. They are chosen to represent different asset classes that might be biased to changes in economic conditions. Each ETF is a bucket of a specific asset class, similar to a typical asset index. Using ETFs is good for demonstration when constructing a simple portfolio that is indirectly built of many securities to lower specific security risk.

Name	***Description***	***Pricing variables***
TIP - iShares TIPS Bond ETF	Bond ETF containing TIPS	*Real rate point, ETF yield spread*
IEF - iShares 7-10 Year Treasury Bond ETF	Bond ETF containing U.S. Treasury bonds with maturities 7-10 years	*Real rate point, Imp inf point, ETF imp YTM spread*
TLT - iShares 20+ Year Treasury Bond ETF	Bond ETF containing long – term U.S. Treasury bonds	*Real rate point, Imp inf point, ETF imp YTM spread*
LQD - iShares iBoxx Investment Grade Corporate Bond ETF	Bond ETF containing low yield corporate bonds	*Real rate point, Imp inf point, ETF imp YTM spread*
HYG - iShares iBoxx High Yield Corporate Bond ETF	Bond ETF containing high yield corporate bonds	*Real rate point, Imp inf point, ETF imp YTM spread*
SPY - SPDR® S&P® 500 ETF	Stock index ETF targets to replicate the price and yield of S&P500	*Real rate curve, Imp inf curve, ETF Imp growth rate OR the dividend yield*

Table 3.1 Showing the different ETF that are included in the simulated portfolio

The implied yield of a bond ETF is calculated by solving π_t in equation (2.7) having all other variables known. This is done with the function `fsolve` in MATLAB®. The known variables are set up in the following way; the present value P_t is the daily estimated clean close price, the face value *FV* is fixed to 100, the coupon rate c_t is the accumulated 12 month dividend, the coupon payment frequency *N* is equal to the dividend payment frequency, the maturity *T* is arbitrary to match the duration.

The daily estimated clean close price is the daily close price Y_t of the bond ETF subtracted by the accrued dividend payment that is the rolling accumulated 12-month dividend times the number of years since last payment

$$P_t = Y_t - c_t \frac{NbrOfDays}{360} \tag{3.1}$$

To attain the implied yield spread one only calculates the spread between the benchmark nominal yield point and the implied ytm of the ETF.

The implied annual growth rate of **SPY** is extracted similarly by solving G_t in equation (2.4) and with all other variables known. Again, this is done with the function `fsolve` in MATLAB® and the way to choose the known variables is presented in the section 2.1.1.

The implied dividend yield DP_t of **SPY** is extracted by calculating equation (2.5) and knowing the stock price P_t and the rolling accumulated 12-month dividend D_t.

The estimated indices of the accumulated returns with re-invested dividends for the ETFs are seen in Fig. 3.1. The approximation is simplified with no dividend taxes considered.

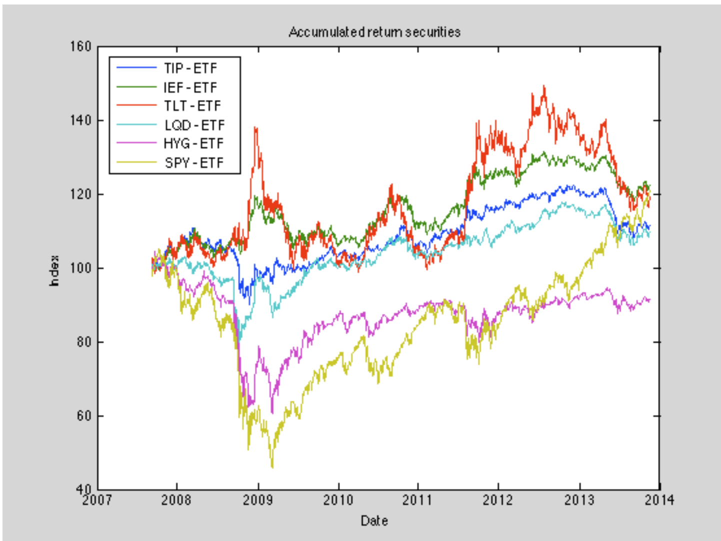


Fig 3.1 The estimated accumulated return indices with re-invested dividends for all ETFs with index = 100 at start date = 2007-04-11 [16]

3.1.2 The implied scenario variables

The possible implied scenario values that can be used in the portfolio simulation are presented in Table 3.2.

Name	*Description*
U.S. Treasury yield points (1Y, 2Y & 3Y)	The yield points from the U.S. Treasury yield curve
Real rate yield points (5Y, 7Y, 10Y, 20Y & 30Y)	The yield points from the U.S. TIPS yield curve
Implied inflation rate points (5Y, 7Y, 10Y, 20Y & 30Y)	The spread points between the U.S. Treasury yield curve and the U.S. TIPS yield curve
TIP - spread	The spread between a benchmark U.S. TIPS yield point and the implied YTM of TIP
IEF - spread	The spread between a benchmark U.S. Treasury yield point and the implied YTM of IEF
TLT - spread	The spread between a benchmark U.S. Treasury yield point and the implied YTM of TLT
LQD - spread	The spread between a benchmark U.S. Treasury yield point and the implied YTM of LQD
HYG - spread	The spread between a benchmark U.S. Treasury yield point and the implied YTM of HYG
SPY – implied growth rate	**The implied growth rate of SPY**
SPY – dividend yield	**The dividend yield rate of SPY**

Table 3.2 The implied scenario variables that could be used when simulation different portfolios with the asset universe in Table 3.1

Which variables that are used in the modeling are free of choice. SPY can either be priced with the dividend yield or the implied growth rate.

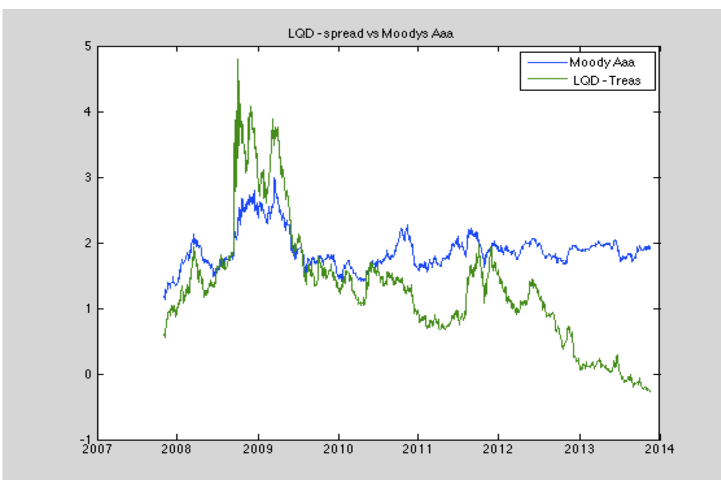


Fig 3.2 The spread between the nominal 10Y U.S. Treasury yield for Moody Aaa yield index and the calculated implied yield of bond ETF LQD [10] [16]

The ETFs are chosen to reflect buckets of different asset classes that in turn are biased to different economical conditions. The implied scenario values from these ETFs are pricing variables but in some way also indicators of economical conditions since the assets they are derived from are assumed to be biased to economical conditions. For example a sharply raising implied HYG - spread would signal increasing market expectations of growing future credit risk. Probably the interaction between implied scenario variable tell more of market expectations than variables solo.

Fig 3.4 shows the 50 data points rolling correlation between the daily absolute changes of three of the implied pricing variables and the HYG – treas spread. The benchmark yield spread points are the 10Y U.S. Treasury yield for both HYG and LQD. The SPY implied growth is negatively and the LQD – treasury spread positively correlated with the HYG – spread. It makes sense since higher credit risk should reduce optimism of future growth.

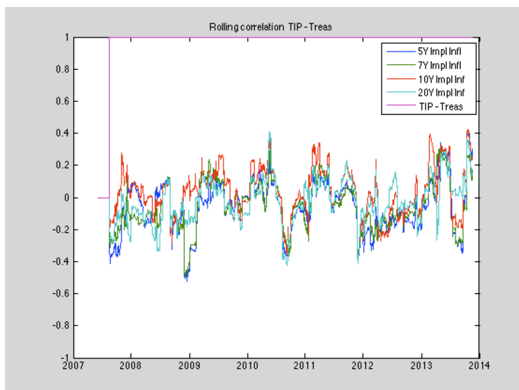


Fig 3.3 Shows the rolling correlation of the implied inflation rates and the TIP – Treas spread

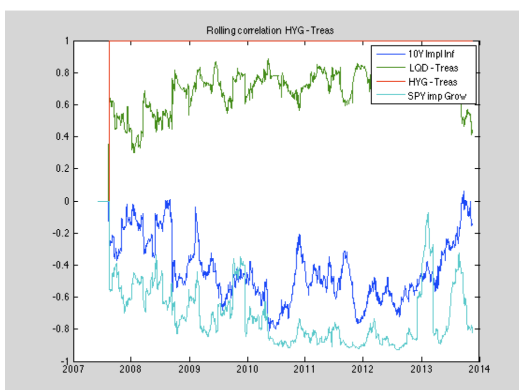


Fig 3.4 Shows the rolling correlation of the HYG –Treas spread and the 10Y implied inflation rate, the SPY implied growth rate and the LQD - Treas spread

Unfortunately the implied yield spread between TIP and the treasury 10Y yield point, called **TIP – Treas,** is not very correlated with the implied inflation rate points, see Fig 3.3.

3.2 Historical back testing

The historical back testing means simulating the historical result as if the strategies would have been implemented back in the days. The reason why the historical back testing period is only 2007 to late 2013 is for a couple of reasons; the financial crisis led to a severe period of low growth and clear deflation which puts the portfolio biasness to the test, the original All Weather portfolio was said to be relatively successful during this time [2] and the data for many of the ETFs are not very old.

The portfolio parameters are adjusted for each simulation portfolio to display their impact. All tests will be benchmarked to a 60/40 – portfolio that has 40% of capital allocated in the bond ETFs and 60% in the stock ETF SPY. The accumulated return plots are represented as an index that equal 100 at the simulation start date.

The parameters that are adjusted for the simulated portfolios are:

Nbr of data points in estimation:
The number of historical data points of changes in the implied scenario variables when estimating the Monte Carlo simulation distribution.

Beta:
The β-confidence level in the CVaR optimization, see equation 2.10. It is only of interest for the CVaR strategy.

Sim dist of variables:
Monte Carlo simulation distribution, bootstrap or normal distribution

Holding period:
How often the re-allocation occurs.

In the historical back tests all bond ETFs, except TLT, where modeled with a maturity of ten years and benchmarked to the same nominal treasury yield point to extract the implied YTM spread. The bond ETF where modeled and benchmarked with a maturity of 20 years. When SPY was modeled with the implied growth rate the discount horizon was $N = 40$.

3.1.1 Optimal CVaR

The simulated portfolios 1-7, see Table 3.3 and Table 3.4, are all based on the strategy *optimal CVaR*. What differentiates simulated portfolios 1-5 from each other is that the parameters, presented in the previous section of *3.2 Historical back testing*, are set differently with the ETF **SPY** modeled with the implied growth rate. For the case of the simulated portfolios 6-7, modeling the ETF SPY with the dividend yield sets up these portfolios with different parameters between them. The reason for the different simulated portfolios is to see the concepts sensitivity to the parameter settings.

The reason that the impact of the modeling of SPY needs to be studied is because it is the only stock ETF in the asset universe. If stocks and bonds should offset, is it easier to model the connection with bond yield points being the dividend discount curve or does a simpler dividend yield model capture the correlation to bonds?

Modeling SPY with implied growth rate

In these first back tests the price of SPY where modeled with the implied growth rate. The different simulated portfolios and some results are seen in Table 3.3.

	Simulated Portfolio 1	**Simulated Portfolio 2**	**Simulated Portfolio 3**	**Simulated Portfolio 4**	**Simulated Portfolio 5**	**Benchmark Portfolio**
Parameter settings & Description	*Nbr of data points in estimation=* ***50***, *Beta =* ***90%***, *Holding period =* ***1 day*** *Sim dist of variables =* ***Normal dist***	*Nbr of data points in estimation =* ***50***, *Beta =* ***90%***, *Holding period =* ***1 day*** *Sim dist of variables =* ***Bootstrap***	*Nbr of data points in estimation =* ***40***, *Beta =* ***80%***, *Holding period =* ***1 day*** *Sim dist of variables =* ***Bootstrap***	*Nbr of data points in estimation =* ***30***, *Beta =* ***80%***, *Holding period =* ***4 days*** *Sim dist of variables =* ***Bootstrap***	*Nbr of data points in estimation =* ***40***, *Beta =* ***90%***, *Holding period =* ***1 day*** *Sim dist of variables =* ***Bootstrap***, *The simulated 20Y US nominal yield point is not included when simulating the price of SPY*	*The Benchmark Portfolio is daily re-allocated so that 60% of the capital is in SPY and 40% is equally shared between the bond ETFs*
Max Drawdown	*-22.5 %*	*-20.5 %*	*-20.4 %*	*-31.8 %*	*-16.7 %*	*- 44.0 %*
Amount of losses exceeding the daily est. VaR	*7.1 %*	*7.2 %*	*13.1 %*	*16.1 %*	*20.8 %*	
Daily volatility	*0.4 %*	*0.4 %*	*0.4 %*	*0.5 %*	*0.4 %*	*0.9 %*
Total Return	*- 8.1 %*	*14.8 %*	*6.2 %*	*-22.6 %*	*12.5 %*	*6.4 %*

Table 3.3 The set ups and results of the CVaR optimized simulated portfolios that had SPY modeled with the implied growth rate

Below is the simulated portfolio 1 along with the result of the benchmark portfolio. The fall during the financial crisis was worse for the benchmark portfolio and the volatility was greater during most periods, see Fig 3.6. Measuring the volatility over the whole time period the CVaR optimization achieved much lower volatility for all simulated portfolios 1-5.

Fig 3.5 The result indices of simulated portfolio 1 and the benchmark portfolio

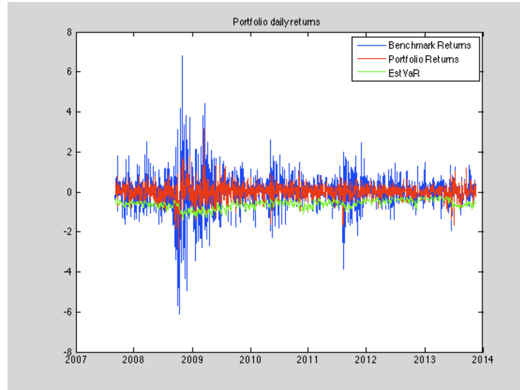


Fig. 3.6 The daily returns (in %) of simulated portfolio 1 and the benchmark portfolio, along with the estimated daily VaR (90%)

Simulating the implied scenario variables with bootstrap instead of using a fitted normal distribution resulted in a better portfolio performance. It also suggested a more steady allocation, see Fig 3.7 .

One idea why the allocation from the normal distribution simulation is much more aggressive is that the shape off the tails do not change a lot in the in non-parametric bootstrap data set. Looking at Fig 3.7 the passive allocation term is about the same size as historical data points in the estimation, the re-sampling data set. The quantiles of the empirical distribution do not change frequently. The estimated normal distribution is taking into concern the whole data set and hence, the tail distribution changes more often. The tails should have an impact in the CVaR minimization. If the number of data points in the estimation were greater, the large tails of real historical financial data would have had a higher weight when estimating the normal distribution and probably led to a more smooth allocation. The trade-off would be that rising volatility and correlation would not be as quickly captured. The reason why the number of data points in estimation was around 30-50 was that it seamed to capture quick changes in correlation and volatility when trying out appropriate levels. In periods of crisis or market crashes the volatility rises quickly and holds on a time period until it becomes more stable, see Fig 2.10.

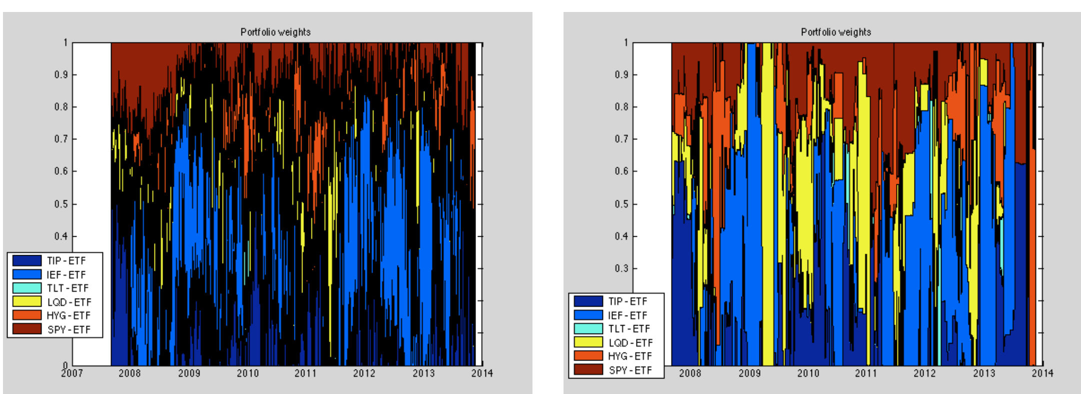


Fig 3.7. The asset allocation for simulated portfolio 1 (left) and simulated portfolio 2 (right)

All the simulated portfolio performances can be seen in Fig. 3.8 . The decline in the bond ETFs during 2013 had a tougher impact in the simulated strategy portfolios than the benchmark portfolio.

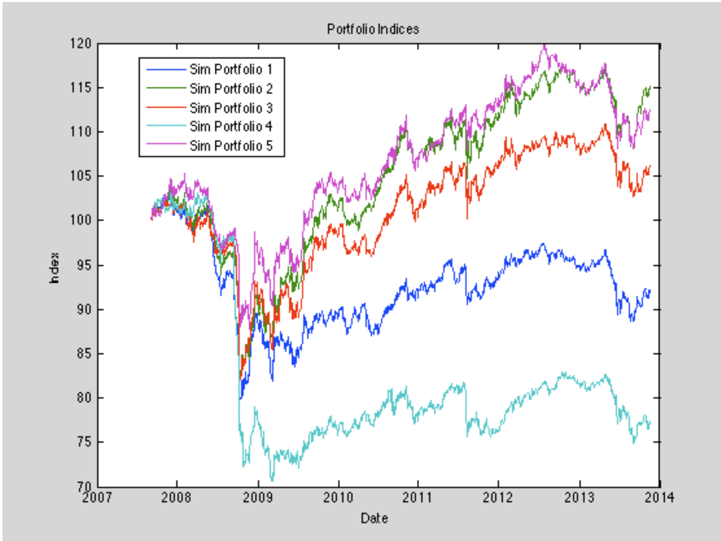


Fig 3.8. The result indices for simulated portfolio 1-5

Modeling SPY with dividend yield

Below is the simulated portfolio set ups and results when SPY was modeled with the dividend yield as the implied scenario value, see Table 3.4.

	Simulated Portfolio 6	**Simulated Portfolio 7**
Parameter settings & Description	*Nbr of data points in estimation = **50**, Beta = **95%**, Holding period = **1 day** Sim dist of variables = **Bootstrap***	*Nbr of data points in estimation= **50**, Beta = **95%**, Holding period = **1 day** Sim dist of variables = **Normal dist***
Max Drawdown	*-20.6 %*	*-21.0 %*
Amount of losses exceeding the daily est. VaR	*6.6 %*	*6.4 %*
Daily volatility	*0.4 %*	*0.4 %*
Total Return	*8.3 %*	*-1.2 %*

Table 3.4 The set ups and results of the CVaR optimized simulated portfolios that had SPY modeled with the dividend yield

The results did not improve much comparing to modeling with the implied growth rate. The allocation pattern over time where similar to when having SPY modeled with the implied growth rate. Using normal distributed simulated variables led to a more active re – allocation compared to when non-parametric bootstrap simulating them.

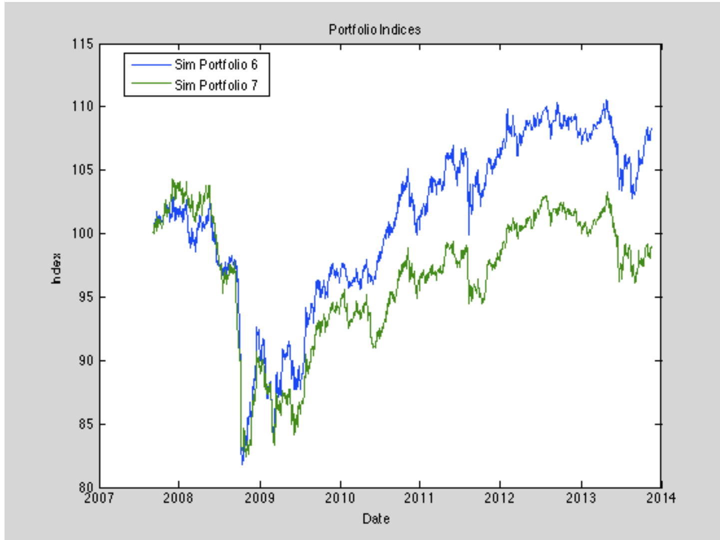


Fig 3.9. The result indices for simulated portfolio 6-7

3.1.2 The fundamental risk parity method

Two similar portfolios with different sub-portfolio set ups where historically back-tested. The two simulated risk parity portfolios, *RP Portfolio I* and *RP Portfolio II*, set ups can be seen in Table 3.5 and Table 3.6.

RP Portfolio I

Sub - portfolios	Growth *Rising*	Growth *Falling*	Inflation *Rising*	Inflation *Falling*
Assets	• SPY • LQD • HYG	• IEF • TLT • TIP	• TIP • SPY	• IEF • TLT • LQD, • HYG

Table 3.5. The sub-portfolios and categorization for RP Portfolio I

RP Portfolio II

Sub - portfolios	Growth *Rising*	Growth *Falling*	Inflation *Rising*	Inflation *Falling*	Credit Risk *Rising*	Credit Risk *Falling*
Assets	• SPY	• IEF, • TLT	• TIP	• IEF, • TLT	• IEF, • TLT • TIP	• LQD, • HYG

Table 3.6. The sub-portfolios and categorization for RP Portfolio II

When back-testing the risk parity strategies, **the bootstrap simulation** was used consistently, the number of data points in the estimation was set to **50**, the holding period was one day and SPY was modeled with **the dividend yield**. The parameter set up was chosen to resemble **Simulated Portfolio 2**, the best CVaR optimized portfolio. The parameters where fixed, since focus was on the categorization and choice of measure to balance.

In Fig 3.11 and Fig 3.15, one sees the result of using the different allocation methods. Equal volatility contribution performed the worst for both simulated RP Portfolios I and II. This might have been an effect of the correlation estimation that suggested less stable allocation than the equal volatility property. The equal volatility and equal capital performed similarly which can be seen in the allocation plot in Fig 3.10. and Fig 3.13 .

The attribution plots, see Fig 3.12. and Fig 3.15 , show the individual indices of the accumulated return for each strategy with respect to the absolute allocation in the portfolio. Absolute allocation means, in this context, that the allocation was not normalized when calculating these indices. Having this fundamental categorization of the assets' biasness, these figures shows the large attribution of rising growth and rising inflation in the benchmark portfolio as suggested by Bridgewater ([1], page 3). Even though the large downfall during the financial crisis, the attribution of the different strategies where more balanced when allocating with the risk parity methods.

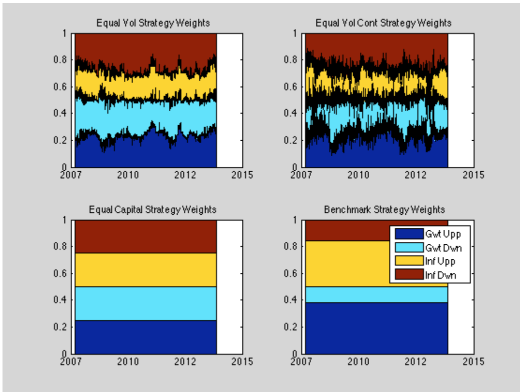


Fig 3.10. The sub-portfolio capital allocation for RP Portfolio I and the benchmark portfolio

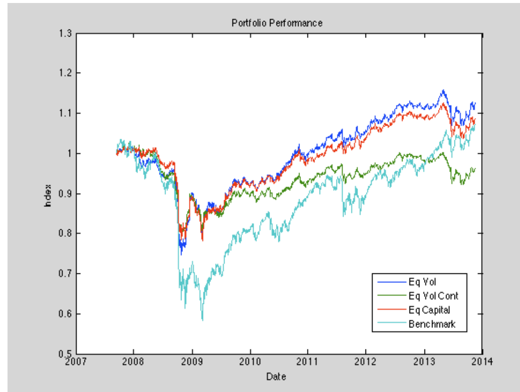


Fig 3.11 The result indices for the benchmark portfolio and RP Portfolio I using the different allocation methods

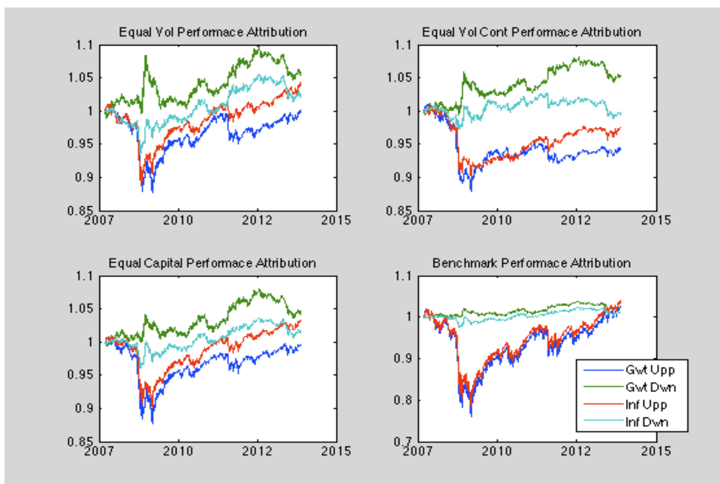


Fig 3.12. Showing the performance of the sub-portfolios of RP Portfolio I and the benchmark portfolio

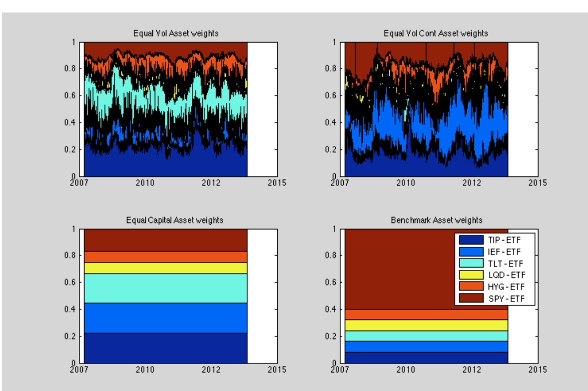


Fig 3.13 The asset allocation for RP Portfolio II and the benchmark portfolio

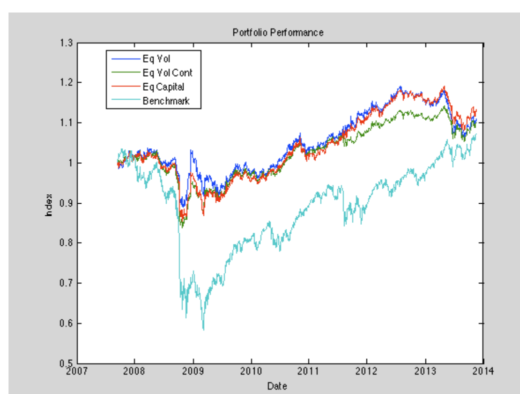


Fig 3.14 The result indices for the benchmark portfolio and RP Portfolio II using the different allocation methods

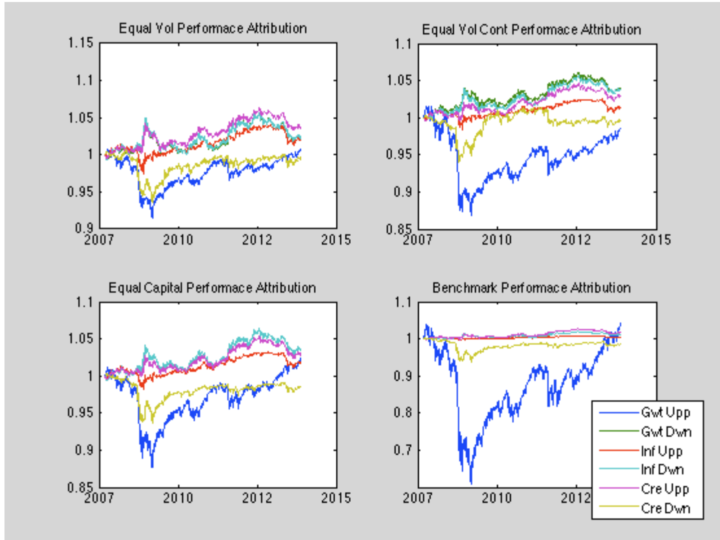


Fig 3.15. Showing the performance of the sub-portfolios
of RP Portfolio II and the benchmark portfolio

3.3 Analysis of results

CVaR Optimization

For the CVaR strategies the use of bootstrap simulation resulted in much better performance than with the use of a fitted normal distribution. This might have been expected since it led to heavier tails better imitating real data. Another reason for this could have been that the empirical cumulative distributions are slightly tilted, depending on the latest market movements. Even though the normal distribution includes the estimated expected return it perhaps does not capture the current direction of an asset. It furthermore seems to be a momentum for SPY, compared to TLT, which benefits the equity heavy benchmark portfolio. The momentum resembles the business cycles where the phase of a growing economy is usually slower than the decline, which happens in periods of recession. Using the normal distribution led to a less stable allocation that could be relativity costly due to transaction fees.

The volatility of the CVaR optimized portfolios where overall lower than for the benchmark portfolio. This was expected to be one of the pros of these portfolios based on the idea that the risk measurement and asset modeling was efficient. The daily losses did not exceed the VaR generally during the simulation but clusters of losses exceeding the value did occur, see Fig. 3.6. . This happened during the financial crises and a period of 2013 when many of bond ETFs prices dropped. A poor risk modeling wrecking the strategy where not shown in the results. The suggested allocation did show less volatility than the benchmark portfolio allocation and the amount of losses exceeding the estimated VaR seemed sufficient.

The point of trying a holding period of longer than one day was to perchance capture longer - term correlation between the assets or the momentum of an asset. Choosing a passive allocation period of 4 business days resulted in a poor performance. Maybe because of the large downfall during the start of the financial crisis which it did not quickly re-allocate during.

Since the duration of SPY is far out, when discounting with the low nominal U.S. Treasury yield curve, the asset price model is most sensitive to long – term yield points. To avoid over-sensitivity to the 20Y yield point, this pricing variable was fixed daily when simulating SPY losses for simulated portfolio 5. The portfolio performed well and had the shortest time to recovery during the financial crisis. The risk measure did not perform efficiently and with a VaR confidence level of 90% the amount of losses exceeding the estimated VaR-threshold was almost 21%, which brings doubt to the risk modeling.

When modeling with the dividend yield of SPY the results became fairly similar to when using the implied growth rate. The performance was better for the bootstrap simulated portfolio, the volatility was lower than the benchmark portfolio and the amount of losses exceeding VaR where fair.

The simulated portfolio results depended a lot on the parameter set up. That is, the number of historical days in the data set for the bootstrap simulation and the confidence level of CVaR. This seems to be a bad indicator for the strategy since the concept is the same but made a difference of almost 10% in accumulated return between the portfolios.

The fundamental risk parity method

The risk parity methods worked overall well with a modest accumulated return. They became bond ETF heavy that resulted in a large drawdown in 2013. These portfolios did not decline as much as the benchmark portfolio during the crisis. The rolling portfolio volatilities where however generally small compared to the benchmark portfolio.

The sub-portfolio attributions did not offset each other as efficient as hoped for. But some sub - portfolios showed an expected biasness. This can be seen in the growth falling sub – portfolio, see Fig 3.12. and Fig 3.15. . As mentioned in *Background*, see section 1.2, this categorization displays how the benchmark portfolio would be betting on rising growth.

An interesting observation is that the equal volatility and equal capital property portfolios had fairly similar allocations and very similar results. In the second Risk Parity portfolio the equal volatility contribution strategy had a bit different allocation than the other two risk parity portfolios but the results where similar.

The equal volatility contribution led to the most dynamic allocation. This could have been that the weights are squared in the equality equation (2.19) for the sub -portfolio. A point to make is that the correlations between the assets are not empirically estimated from close prices but estimated from the simulated asset price changes. The price changes are extra sensitive to high volatility in the implied scenario variables since the pricing functions are convex.

4. Conclusion

4.1 Summary

The purpose of this thesis was to try out, and hopefully find, a short-term allocation strategy based on the concepts of *Bridgewater's All Weather Fund* that would perform well during most economical conditions. These conditions referred to periods of rising and falling market expectations of future growth, inflation and credit risk. The strategy was supposed to be long - only and the assets in the portfolio would be balanced in such a way that the impact of changing market expectations of future economical conditions would offset.

Current market expectations of future conditions where extracted from market asset prices. Market expectations where pricing variables in the used asset pricing models. These quantitative values of market expectation where called implied scenario variables in the thesis. They where used as risk factors when modeling and estimating the portfolio and asset risk.

The allocation was given by two different methods. The first method was to try and minimize the affect of changes in the implied scenario variables by minimizing the conditional value-at-risk, having only the implied scenario variables as stochastic variables in the model. The conditional value-at-risk was estimated by Monte Carlo simulating asset losses.
The second method was to categorize different assets to what economical conditions they where biased to. The assets where then allocated to different sub-portfolios. The asset allocation would be chosen so that the sub-portfolios had equivalence of a measure, similar to a risk parity strategy.

The allocation strategies did not manage to create a portfolio that performed well during all environments when historically back testing different portfolios. Both the financial crisis and the decline of the Bonds ETFs in 2013 led to large losses in the simulated portfolios.

The performances shifted a lot due to different parameter set ups. This revealed a great weakness in the general methodology of the CVaR optimization strategy, since the hypothetical theory was the same for all the simulated portfolios.
The fundamental risk parity approach showed positive signs of the use categorization as the sub-portfolios at moments moved correctly. However, the sub-portfolios did not manage to off-set suitably.

4.2 Discussion

A portfolio strategy of this type probably needs to perform well and stable most of the time to achieve credibility that it would work in most environments. If long-term investors where to invest in the portfolio large single drawdowns could repel them. Therefore, the pre-launch stress tests needs to be reliable. The strategy is not a type of arbitrage strategy but a risk balancing strategy that, depending on the efficiency, pays off in the long run like many assets.

Transaction costs where not used in the simulations, which probably would have led to other results. They might have lowered the accumulated result, but if they where considered in the risk estimation they would also penalized transaction, which may have led to a different allocation. The dynamic re-allocation shown in the back - testing of the CVaR method with normal distribution simulated pricing variables could be costly for a fund.

A general problem with the fundamental risk parity modeling is to take into account the correlation between economical conditions. High growth may lead to decreased credit risk but also to high inflation, which affects the corporate bonds in both ways.

Some asset models are maybe a bit non - robust. The implied yield spread between TIP and a nominal U.S. treasury yield point was supposed to correlate with the implied inflation rate to encourage allocation to TIP, when expectations of future deflation or inflation where shifting in the market. The difficult asset price modeling of a mixed bucket of TIPS with different maturities and adjusted principals, perhaps led to a poor risk modeling of this asset.

A DCF model used for SPY may perhaps not reflect and capture the actual relationship between short-term nominal yields and stocks.

Modeling the assets with more than only the implied scenario variables could have been beneficial when using the CVaR optimization routine. In the methodology, the point was to minimize the potential negative attribution of shifts in the implied scenario variables. Since these where the only variables pricing the assets, the optimization might have been inefficient of doing this. It could have been better to model the asset with more risk factors, and only simulate changes of implied scenario variables to find an allocation that minimized their effect.

It has not been possible to attain performance data of Bridgewater's All Weather for this thesis. The closest data to compare is a hypothetical All Weather asset mix simulated during a short period around the financial crisis of 2008 that Ray Dalio presents in his paper *Engineering targeted returns & Risks* (page 9, [2]). In the paper he points out the importance of leveraging less volatile asset to achieve high return. The simulated portfolio is implemented a bit earlier in time than the simulated portfolios in this thesis and the cumulative total return of July 2007- April 2010 is also higher, +18.6%. However, the maximal drawdown seems to be larger. These two differentiating results could be because of leveraging. An interesting observation is that the simulated portfolios when using the optimal CVaR strategy is that the portfolios manage to perform similarly as the 60/40 benchmark portfolio with half the risk (volatility). This is one of the benefits Dalio points with the All Weather strategy [2].

Referring to informal sources in a news article by Gandel the All Weather fund was said to have taken a large downfall in 2013 due to declines in the bond prices [22] just like the simulated portfolios in this thesis. The writer equals All Weather's prominent reputation as a result of a leveraged bond portfolio strategy with the past years falling interest rates leading to stable performance. In the article, Gandel also makes an argument that the fundamental strategy may become weaker in the future since:

> *"…Dalio said Bridgewater had back-tested All-Weather and found that it would have done fine in, say, the late 1970s, and other periods of rising interest rates. But here's the flaw. In the 1970s, interest rates were much higher than they are now. So any money a fund would have lost on falling bond prices would have been more than offset by high interest rates. "*

However, the strategies investigated in this thesis where not aimed to specifically beat the hedge fund All Weather but where inspired by the concepts behind the fund.

4.3 Suggestions for future improvements and research

The portfolio strategy set up performed good enough but did poor during the financial crises 2008. It may however laid ground for future ideas in the area for a combined fundamental and quantitative strategy. The attribution plots in section 3.1.2 do reveal some accuracy in the fundamental categorization. A deeper analysis and further historical back testing could be good for future research. One way could be to not always allocate equal between the sub-portfolios but to have the majority of the allocation in a sub-portfolio depending on the magnitude of the implied scenario variables. For example if implied growth weakened one would allocate more to the growth down portfolio.

It would also be good if the historical level could be incorporated in the model in some way. Now the interest rates are extremely low in a historical perspective. The levels should therefore not decline much more one might assume.

Market expectations of future conditions could perhaps be extracted through derivatives like futures, instead. The implied market expectation could then be used to tilt the allocation in a fundamental risk parity strategy.

During turbulent market times could have been good to include long-only T-Bill positions. High volatility may come in waves and during large stock crashes assets behave irregular and risky. Including derivatives could otherwise allow for a more definite hedging to avoid a large drawdown.

Finally, a definition and an indicator of turbulent markets could be incorporated when the strategy is working poorly. For example, when the **VIX** index hits a decided level the algorithm could transform to buy some T-bills or hedge positions instead of just balancing.

5. Bibliography

4.1 Literature references

[1] Bridgewater Associates, "The All Weather Story", *Bridgewater Associates*, http://www.bwater.com/Uploads/FileManager/research/All-Weather/All-Weather-Story.pdf, [Collected: September 2013]

[2] Bridgewater Associates, "Engineering Targeted Returns and Risks", *Bridgewater Associates*, http://www.pionline.com/assets/docs/CO7012569.PDF, [Collected: September 2013]

[3] Bridgewater Associates, "How the Economic Machine Works", *Bridgewater Associates*, http://www.bwater.com/Uploads/FileManager/research/how-the-economic-machine-works/ray_dalio__how_the_economic_machine_works__leveragings_and_deleveragings.pdf, [Collected: October 2013]

[4] S. Maillard, T. Roncalli, J. Teiletche, "On the properties of equally-weighted risk contributions portfolios", *Journal of Portfolio Management*, Vol. 36, No. 4, pp. 60-70, 2010

[5] P. Rappoport, N. Nottebohm "Improving on Risk Parity: Hedging forecast uncertainty", *J.P. Morgan*, [Online], JP Morgan Institutional, http://www.jpmorganinstitutional.com/blobcontent/785/189/1323345640070_WP_Improving_on_Risk_Parity.pdf, [Collected: October 2013]

[6] B. Bruder, T. Roncalli, "Managing Risk Exposures using the Risk Parity Approach", working paper, January 2012 [Online], SSRN, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2009778, [Collected: October 2013]

[7] S. Darolles, C. Gouriéroux, E. Jay, "Portfolio Allocation with Budget and Risk Contribution Restrictions", November 2012 [Online], http://www.qminitiative.org/UserFiles/files/New_Indexing_july2012.pdf, [Collected: October 2013]

[8] H. Markowitz, "Portfolio Selection", *The Journal of Finance*, Vol. 7, No. 1, pp. 77-91, March, 1952

[9] W. Dewlin, D. Patwardhan, "Measuring market inflation expectations", Macroeconomic Policy Division, *the Australian Treasury*, [Online] http://www.treasury.gov.au/~/media/Treasury/Publications%20and%20Media/Publications/2012/Economic%20Roundup%20Issue%202/Downloads/01_Measuring_market_inflation_exp.ashx, [Collected: October 2013]

[11] L. Kajanoja, "Extracting Growth and Inflation Expectations from Financial Market Data", *Bank For International Settlements*, BIS Working Papers No 22, April 2005

[12] C. Acerbi, D. Tasche, "Expected Shortfall: a natural coherent alternative to Value at Risk ", *Economic Notes*, Vol 31, Issue 2, pages 379–388, July 2002

[13] K. Dowd, Measuring Market Risk. West Sussex, England: John Wiley & Sons Ltd, 2002

[14] R.T. Rockafellar, S. Uryasev, "Optimization of Conditional Value-at-Risk", *The Journal of Risk,* Vol. 2, No. 3, pp. 21-41, 2000

[15] M. Sköld, "Computer Intensive Statistical Methods", Lund University*,* Lecture Notes for FMS091/MAS221*,* 2nd print, August 2006

[17] O. Jorda, M. Schularick, A. Taylor, "When Credit Bites Back: Leverage, Business Cycles, and Crises", *Federal Reserve Bank of San Francisco*, Working Paper Series, October 2012, [Online], http://www.frbsf.org/publications/economics/papers/2011/wp11-27bk.pdf, [Collected: March 2014]

[18] C. Borio, "The financial cycle and macroeconomics: What have we learnt?", Monetary and Economic Department, *Bank For International Settlements*, BIS Working Papers No 395, December 2012

[19] M. A. S. Joyce, A. Lasaosa, I. Stevens, M. Tong, " The Finacial Market Impact of Quantitative Easing in the United Kingdom", *International Journal of Central Banking,* Vol. 7, No. 3, pp. 113-161, 2011

[20] Standard & Poor's, "United States of America Long – Term Rating Lowered To 'AA+' Due To Political Risks, Rising Debt Burden; Outlook Negative", *Standard & Poor's*, August 2011 [Online], http://www.standardandpoors.com/ratings/articles/en/us/?assetID=1245316529563, [Collected: March 2014]

[21] S. Gandel, "Ray Dalio's 'All-Weather' fund goes cold", *Fortune Magazine*, January 2014 [Online], http://finance.fortune.cnn.com/2014/01/09/ray-dalios-all-weather-fund-goes-cold/, [Collected: March 2014]

[22] K. Hogan, P. Hodges, "Will rising rates sink Risk Parity?", *Blackrock, Inc* , September 2013 [Online], https://www.blackrock.com/institutions/en-us/literature/whitepaper/market-advantage-will-rising-rates-sink-risk-parity.pdf, [Collected: March 2014]

4.2 Data references

[10] Data from Federal Reserve Bank of St Louis, *Federal Reserve Bank of St Louis*, Updated daily [Online], http://research.stlouisfed.org/fred2/, [Collected: October 2013]

[16] Data from Yahoo finance, *Yahoo!*, Updated daily [Online], http://finance.yahoo.com, [Collected: October 2013]