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Asymmetry in the dynamic conditional correlation of gold returns and stock returns

The U.S evidence of different-sized indices

Kittiporn Sinsukthavorn and Haukur Vidar Alfredsson

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Supervisor: Hossein Asgharian

Abstract

The purpose of this thesis is to explore the possibility of asymmetry in the dynamic conditional correlation of stock returns and gold returns. We hypothesize that this asymmetry might be different for large and small firms, as a result of size specific characteristics that may influence firm profitability and risk following negative market shocks. We investigate this on three different sized U.S. stock indices during a twenty two year long period by using the dynamic conditional correlation model and the asymmetric generalized dynamic conditional correlation model. Our results show that there is asymmetry in the dynamic conditional correlation of these stock indices and gold. Furthermore, we find that the asymmetric effect is not the same for large and small firms.

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1 Introduction

Gold has been the focus of a large amount of literature, both on stand alone bases and its relationship with other assets. A noticeable property of gold is its ability to hedge against inflation and currency risks along with very low and even negative correlation with the stock market. This makes gold an attractive investment option for investors, especially during downturns in the stock market. Adding the fact that the last few decades have seen a lot of turmoil in the financial markets, such as the Black Monday in 1987, the Asian financial crisis in 1997-1998, the dot.com bubble during the period of 1997 to 2001, the financial crisis of 2008 and the U.S. debt crisis in 2011, it should not be surprising that a substantial part of research on gold is focused on these market downturns. Even though the first conditional correlation models saw light of day well over a decade ago, the correlation between equity returns and gold returns have not been investigated in full. It is, therefore, our aim with this thesis to shed further light on this relationship.

When constructing portfolios, many investors want to minimize their exposure to risk for any given level of expected return. This is generally referred to as mean variance optimizing. In an attempt to do so, it has been shown that during periods of market shocks, investors tend to diversify their assets and move from stocks to bonds, called flight to quality, in order to avoid heavy losses (e.g. Gulko (2002), Hartmann et al. (2004)). In addition to a traditional portfolio asset, there are many studies that have focused more on other alternative assets such as commodities (e.g. Edwards & Caglayan (2001), Georgiev (2001), Erb & Campbell (2006), Chong & Miffre (2010), Tang & Xiong (2010)). Unlike other commodities, gold has durableness, global acceptance and easy authentication (Worthington & Pahlavani, 2007). Investors might make direct investment through purchasing gold bars or gold coins. Since holding gold bars and gold coins could have inconvenience and problem of storage, investors can indirectly invest through exchange-traded funds, which are investment funds invested in gold and gold stocks.

In a 2010 paper titled "*Is gold a safe haven? International evidence*" Baur and McDermott discuss the role of gold in the global financial system and its usage as an alternative quality asset to bonds or for diversification. They find evidence that gold is negatively correlated to stocks during periods of market turmoil and in some countries this negative correlation holds on average during all periods. Baur and McDermott define gold as a hedge if it is negatively correlated or uncorrelated on average with another asset/portfolio and as a safe haven if it has this property in certain periods only e.g. during the market downturn. Other studies, such as those of Chua, Sick & Woodword (1990) and Dempster & Artigas (2010), have also found gold to have low or negative correlation with stock markets, thus making it a good asset for diversification or hedging. A further property of gold is its ability to hedge both against inflation risk (e.g. Ghosh, Levin, Macmillan, & Wright (2004), Worthington & Pahlavani (2007)) and currency risk (e.g. Capie, Mills, & Wood (2005), Joy (2011)). It implies that gold returns have zero or negative correlation with the CPI index, which is a representative of inflation, and currency.

For any investor, private or institutional, risk seeking or risk averse, an assets return and variance are of great importance when deciding whether to invest in a given asset. When constructing a portfolio that consists of two or more different assets, another crucial factor needs to be considered; correlation between assets returns. Correlation is a measure of linear dependence between two variables that has been standardized to always fall between minus one and one (Verbeek, 2012). Since correlation coefficient closer to zero translates to a weaker linear dependence, assets with lower correlation coefficient are better for diversification. For a complete definition of diversification see chapter 2.1.

If the correlation coefficient is negative, the assets can be used to hedge away risk. This means that the returns of the assets are expect to move in opposite directions to a certain extent and thus countering each other's volatilities. How certain this is to happen depends on the strength of the negative linear relationship. Thus, assets displaying perfect negative correlation could be used to completely eliminate risk from a portfolio. From this it can be seen that the correlation of assets returns plays a vital role in optimal portfolio construction.

For the longest time correlation estimates were assumed to be constant over time or a rolling window estimation was used. Even as late as 1990 when Bollerslev introduced his model which allowed time varying variance and covariance, the correlation estimates were still assumed constant over time. However, in more recent years models, such as the dynamic conditional correlation model (DCC), allowing for time-varying correlation between assets have been put forth. Knowing how the correlations change over time has allowed for more optimal investment strategies as the investor will be better informed about the true relationship of his assets and therefore has more control over the amount of risk that his portfolio contains. Although models allowing for time-varying correlation estimates give investors better control over their investments, a further extension of these models has been made. Models with an asymmetric component in the time-varying correlation, such as the asymmetric generalized dynamic conditional correlation model (AG-DCC), have been introduced. These models are constructed such that negative and positive market shocks of the same size can have different effects on the correlation estimates, in which case there is said to be an asymmetric effect in the correlation estimates.

Should any asymmetry be in how the time-varying correlations develop, which is not accounted for in the model being used, it will lead to incorrect conclusions being drawn about the investor´s portfolio and possible investment opportunities. This will in turn lead to suboptimal investment strategies, regardless of the investors risk preferences. To elaborate on why this is of importance consider an investors who invests both in stocks and gold and makes his investment decisions based on models incorporating time-varying correlation. In some periods he has to adjust the proportion of each asset in order to meet his return requirement and acceptable risk level. However, if there is any asymmetry in the correlation between equity and gold which he is unaware of, the adjusted weights in his portfolio might deviate from the optimal weights. In doing so he could be over- or underexposing himself to risk without knowing it.

The aim of this thesis is to test whether there is an asymmetry in the conditional correlation between the returns of stock indices and gold. This will be done by investigating three different sized equity indices within the U.S, one for the large cap firms and two focusing particularly on small cap and mid cap firms. The research question we set out with is whether indices composed of different sized firms within the U.S. display the same symmetry or asymmetry in the correlation of their returns with those of gold.

We will focus on the possibility of asymmetry in the time-varying correlation between equity and gold with respect to positive and negative market shocks in equity and gold markets and how this affects portfolio construction. Previous studies, see Baur (2012), have found that there is asymmetry in the volatility of gold returns. This, however, does not indicate that there is necessarily asymmetry in the dynamic conditional correlation of gold and stocks as will be shown in chapter 2.4. We speculate that the asymmetry might not be the same for different sized indices due to general advantageous characteristics of large firms, such as having diverse capabilities, abilities to reap benefits from economies of scale and scope as well as standardization of procedures (Majumdar, 1997). Moreover, the price of small firms tends to be more sensitive to changes in the economy and the firms are less likely to survive in the bad economic conditions (Chan & Chen, 1991). This implies that large firms could have more effective operations, which leads to stronger capabilities to sustain profits during economic downturns. Therefore, we expect that negative market shocks will not affect the volatility of different sized stock indices in the same manner, which could lead to different asymmetry estimates in the conditional correlation. In order to investigate this, we will use three U.S. equity indices of different sizes. Our research is based on the DCC model proposed by Engle in 2002 and the AG-DCC model proposed by Cappiello, Engle & Sheppard in 2006.

This thesis contributes to the literature by investigating whether there is asymmetry in the dynamic conditional correlations of stock and gold returns, which to our knowledge has not been done before. We explore this correlation over a period of twenty two years from the early nineteen-nineties to the present day using daily observations. Our results confirm that there is asymmetry in the dynamic conditional correlation of gold returns and the returns of different sized stocks, but there are some differences between the large and small firm stocks. The empirical results show that the dynamic conditional correlation of large-cap stock returns and gold returns only displays an asymmetric change following a negative shock in the returns of the large-cap stocks. However, the dynamic conditional correlation of mid/small-cap stock returns and gold returns has an asymmetric change when there is a negative shock in either mid/small-cap stock or gold returns. We further show that there is no asymmetric volatility in gold returns following a negative shock in gold returns, which is inconsistent with the result of Baur (2012).

However, there are some noteworthy factors that might bias our results or reduce their applicability. The first point is that the results from the Ljung-Box test are inconsistent with some of the significant parameters in the DCC and AG-DCC models. Other points are related to the DCC and AG-DCC models. These models can be thought of as multivariate GARCH models. They are, however, always specified as GARCH (1,1) models without testing whether different specifications could give better fit with the data. Furthermore, there exists a published paper called "*Ten things you should know about the DCC*" by Caporin and McAleer (2013), which lists further disadvantages of the DCC model, mainly concerning possible problems with statistical properties of the DCC model.

The remainder of this thesis is divided into four parts. The first section introduces relevant theoretical background such as modern portfolio theory. The third chapter lists out the methodology implemented along with all associated tests. The fourth chapter contains the empirical results and the fifth chapter concludes the thesis.

2. Theoretical Background

In this chapter, the theories that are relevant with our study are described. First of all, the modern portfolio theory is explained in order to present the importance of correlation to portfolio management. Next, the relationship between correlation and diversification benefit is provided. Then, we give the explanation of flight to quality. Last of all, the theories regarding to asymmetry in the conditional volatility and correlation are presented.

2.1 Modern Portfolio Theory

Modern portfolio theory (MPT) by Markowitz in 1952 is one of the cornerstones of modern investment practice. The MPT suggests that investors attempt to maximize the expected return of a portfolio for a given portfolio risk level or minimize the risk for a given level of expected return. Therefore, investors carefully select which assets their portfolio should contain. Furthermore, the theory proposes the concept of diversification in investing. By choosing portfolio components, the investors' portfolio can have lower overall risk than any individual asset while maintaining the same amount of expected return. This is because the price or the return of different assets might move in different or opposite ways. For example, a portfolio consisting of both equity and gold could have a lower total risk than a portfolio constructed purely from either asset, as long as the two assets are not perfectly positively correlated.

The theory has been further developed under the name of post modern portfolio theory (PMPT) by Rom and Ferguson in 1993, in which some technical conditions such as an assumption of normal distribution of returns, stable asset correlation or iso-variance are relaxed in order to be applicable with market reality. Furthermore, changes to the underlying assumptions about investors have been changed to better reflect behavioral financial theories. Investors are no longer assumed to be rational and choosing portfolios with very stable returns, but investors rather have a minimum expected return, a benchmark, which they wish to obtain. They are concerned if the returns fall below this benchmark and consider anything below the benchmark as a loss to be avoided, but any potential upside a bonus. However, the original MPT is still of much importance in the portfolio theory.

According to the Capital Asset Pricing Model, there are two kinds of risk measures related to portfolio management; the portfolio's total risk and its systematic risk. The systematic risk is measured by the portfolio's beta while its total risk is a function of the correlation coefficients and variances of the assets in the portfolio. Since the non-systematic portion of a portfolios total risk can be diversified away, it is clear that the correlation between each asset in the portfolio is a significant factor in reducing the total risk. If a new asset, which has a sufficiently low correlation with the preexisting assets, is added to an existing portfolio, it is possible to lower the total risk of the portfolio without reducing the expected return. This is called "free lunch" by Markowitz. (Chua, Sick, & Woodward, 1990). It should be noted that unless the new asset has the same expected return as the portfolios preexisting assets then the free lunch cannot hold for a portfolio that aims to maximize expected return.

2.2 The gains from diversification

The expected return of a portfolio is the weighted average of the expected returns of assets in the portfolio, the variance of the portfolio is, however, generally smaller that the weighted average of the variance of each asset as a result of the assets being less than perfectly positively correlated. This is the gain from diversification.

Assuming two assets in the portfolio, the portfolio's variance is defined as

$$\sigma_p^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\sigma_1\sigma_2\rho_{1,2}$$

Where w_i is the weight of an asset i in the portfolio, $\sigma_1 and\ \sigma_2$ are standard deviation of asset 1 and asset 2 respectively, $\rho_{1,2}$ is the correlation of asset 1 and asset 2. If the two asset returns are perfectly positively correlated, they are identical so there is no gain from diversification. The portfolio's standard deviation is then just the weighted average of the standard deviations of the two assets. However, this is not usually the case. If the two asset returns are imperfectly correlated, the portfolio's standard deviation is smaller than if they are perfectly positively correlated. Generally, the lower the correlation estimates are, the lower portfolio's variance is. (Danthine & Donaldson, 2005)

2.3 Flight to quality

Following the occurrence of an unexpected event which leads investors to perceive an increase in risk in the financial system, they tend to move their asset holdings from the affected markets to more secure and liquid assets. This is called flight to quality. During those episodes, which often start with relatively small events, the financial markets can become unstable while the rests of the economy remains relatively unaffected. This happens as investors flock away from the affected markets, causing bottlenecks in the movement of capital from those markets. An U.S. example of such an event is from the fall of 1998 which began with the Russian default. The Russian default only eliminated a small fraction of the U.S. wealth, but it created circumstances which severely strained the financial sector. From there investors perceived an increased risk in the financial sector as losses crew in banks and hedge funds when prices of illiquid assets fell. As investors started to withdraw risk-capital from the affected markets, bottlenecks emerged in the movement of capital as parts of the financial sector were compromised while other parts of the economy were barely affected (Caballero & Krishnamurthy, 2005).

2.4 Asymmetry in conditional volatility and correlation

Stock volatilities and volatilities of other assets have been a subject of interest to many financial researchers for a long time. One of the things that has intrigued researchers is whether these volatilities change in the same way with negative and positive market shocks. Black (1976) noted that although he did not know how stock volatilities develop, it seems that when stocks go up, volatilities go down; and when stocks go down, volatilities go up. He suggested an explanation for this, known as the leverage effect. It states that when the value of a firm decreases, its leverage ratio increases. When the leverage ratio increases, the variance of the stocks is bound to increase, since the firms overall variance is expected to remain constant. This even happens for firms without leverage as when income falls, the firms costs will not decrease proportionally due to fixed costs. Thus, there is an increase in “operational leverage”. A further explanation by Black is that when a firms risk increases, due to changes in its environments, its expected return must increase for investors to want to hold the stocks. Thus, the stock prices will fall which will cause the firms current expected return to be higher in relation to the stock price.

Another theory, of a so called volatility feedback, has been put forth that explains the existence of asymmetry in volatilities. The theory, proposed by French, Schwert and Stambaugh in 1986, proposes that the expected market risk premium varies over time and is positively related to predictable volatility of stock returns. The unexpected stock returns are, however, negatively related to the unexpected change in the volatility of stock returns. They explain the effect of this relationship with the following "*If expected risk premiums are positively related to predictable volatility, then a positive unexpected change in volatility (and an upward revision in predicted volatility) increases future expected risk premiums and lowers current stock prices*".

Having asymmetry in the conditional volatilities of an asset is, however, not a sufficient condition, although it is a necessary condition, for the conditional correlation of that asset and another asset to have asymmetry. Therefore, the reasons for asymmetry in volatilities presented above may or may not apply for conditional asymmetries. To understand why this is let us consider an asset who's volatilities follow an asymmetric GARCH process. This can be shown as follows:

$$y_t = u_t$$

$$u_t = D_t \varepsilon_t$$

$$\varepsilon_t \sim (0, \sigma_t^2)$$

Here y_t is a vector of asset returns, u_t is a vector of residuals, ε_t is a vector of standardized residuals and D_t^2 is some asymmetric GARCH process. Note that D_t is the squared root of D_t^2. Then, the conditional correlation of y_t is (Engle, 2002):

$$\rho_{12,t} = \frac{E_{t-1}(y_{1,t} y_{2,t})}{\sqrt{E_{t-1}(y_{1,t}^2) E_{t-1}(y_{2,t}^2)}}$$

$$\rho_{12,t} = \frac{E_{t-1}((D_{1,t}\varepsilon_{1,t})(D_{2,t}\varepsilon_{2t}))}{\sqrt{E_{t-1}(D_{1,t}^2 \sigma_{1,t}^2) E_{t-1}(D_{2,t}^2 \sigma_{2,t}^2)}}$$

$$\rho_{12,t} = \frac{D_{1,t} D_{2,t} E_{t-1}(\varepsilon_{1,t}\varepsilon_{2t})}{D_{1,t} D_{2,t} \sqrt{E_{t-1}(\sigma_{1,t}^2) E_{t-1}(\sigma_{2,t}^2)}}$$

$$\rho_{12,t} = \frac{E_{t-1}(\varepsilon_{1,t}\varepsilon_{2t})}{\sqrt{E_{t-1}(\sigma_{1,t}^2)E_{t-1}(\sigma_{2,t}^2)}}$$

It can thus be seen that only an asymmetry in the volatility process of the standardized residuals can affect the conditional correlation of the asset returns, not asymmetry in the GARCH process. Knowing this, we can see that if there is a known asymmetry in the conditional correlation of some assets, then there is also asymmetry in the conditional variances. The opposite is, however, not always true. That is, knowing there is asymmetry in the conditional variances is not sufficient to say there is asymmetry in the conditional correlations.

There have been far fewer theories that explain the asymmetry in conditional correlations apart from those who address asset volatility. Cappiello, Engle and Sheppard (2006) propose an explanation for asymmetry in conditional correlations following a negative shock in the standardized residuals of both returns. If risk premiums are time varying, they suggest the following:

"*…a negative system shock will induce downward pressure on returns in any pair of stocks and will increase the variances of these securities in a CAPM-type world. If betas do not change, then covariances will increase. If idiosyncratic variances do not proportionally change, correlations will increase as well. Correlation may therefore be higher after a negative innovation than after a positive innovation of the same magnitude*".

3. Methodology

This chapter provides a thorough explanation of the research approach, the research hypothesis, model selection and the applied methodology employed in this paper. Also a description of how the data collection was performed along with any problems that have to be resolved.

3.1 Research Approach

As has been previously stated we will investigate whether asymmetry is to be found in the dynamic conditional correlation of the returns of equity indices and gold using the AG-DCC model proposed in 2006 by Cappiello, Engle and Sheppard. While previous researchers, such as Toyoshima, Tamakoshi and Hamori in 2012, have only used the AG-DCC model when investigating asymmetry in assets conditional correlation and drawn conclusions from that, we will also use Engle's DCC model from 2002. This will allows us to draw more robust conclusions about any asymmetry that may be detected by the AG-DCC model and evaluate if accounting for asymmetry truly improves the conditional correlation estimation. All calculations and estimation were performed in Eviews 8.0. The Eviews code is provided in appendix E.

As was stated in chapter one, the aim of this thesis is to investigate whether different sized U.S. indices share the same symmetry or asymmetry in the dynamic conditional correlation of their returns and gold returns. More formally we test the following three indices for asymmetry in their dynamic conditional correlation estimates: S&P500, MSCI US Mid cap 450, MSCI US Small cap 1750. Here S&P 500 is used to represent the large U.S. firms while the MSCI US Mid cap 450 and MSCI US Small cap 1750 focus on mid and small cap firms, respectively. For simplicity in the rest of this thesis we will shorten the name of the MSCI US Mid cap 450 and MSCI US Small cap 1750 as mid-cap and small-cap, respectively. For gold we use the London Gold Bullion daily prices in U.S. dollars. The three research hypotheses are as follows:

1. *H0: The dynamic conditional correlation of S&P500 daily returns and London Gold Bullion daily returns contains no asymmetry.*

   H1: The dynamic conditional correlation of S&P500 daily returns and London Gold Bullion daily returns contains asymmetry.

2. *H0: The dynamic conditional correlation of MSCI US Mid cap 450 daily returns and London Gold Bullion daily returns contains no asymmetry.*

   H1: The dynamic conditional correlation of MSCI US Mid cap 450 daily returns and London Gold Bullion daily returns contains asymmetry.

3. *H0: The dynamic conditional correlation of MSCI US Small cap 1750 daily returns and London Gold Bullion daily returns contains no asymmetry.*

   H1: The dynamic conditional correlation of MSCI US Small cap 1750 daily returns and London Gold Bullion daily returns contains asymmetry.

3.2 Data Collection

The data employed for this thesis consists of three equity indices; S&P500, MSCI US Small cap 1750 and MSCI US Mid cap 450, as well as London Gold Bullion daily price (U.S dollar per Troy ounce), all of which was collected from the Thompson Reuters Datastream database. The data spans the period from the first of June 1992 to the first of April 2014, which includes 5705 observations. The reason why this period was chosen is that data for the MSCI indices was only available from the beginning of June 1992. Although more data was available for the S&P500 index, we decided to use only the same period as for the MSCI indices to have a better comparison.

All the data used was obtained as daily index values (daily price for gold bullion) and was then transformed to daily returns using the following formula:

$$Return = ln\left(\frac{Index\ value_t}{Index\ value_{t-1}}\right)$$

A comprehensive summary of descriptive statistics for the series is provided in appendix A.

3.3 Models for conditional correlation

As this thesis focuses on dynamic conditional correlation estimates, it is important that they are accurate and unbiased. In order to achieve this we must carefully select which model we use to estimate the dynamic conditional correlations. It is good to remind the reader that the correlation estimate are a product of variance and covariance estimates. Since the prominent characteristic of financial time series is that the volatility changes over time, autoregressive conditional heteroscedastic (ARCH) model by Engle in 1982 is commonly used to describe and forecast the volatilities. The ARCH model assumes that the forecasted variance is predicted by past forecast errors. However, Bollerslev (1987) introduced a generalization of the ARCH model, so-called GARCH, which can capture the new information from the previous squared residual, the variance of the previous period, and a long-run average variance. The GARCH model is easy to estimate and has proven successful in predicting conditional variances (Engle, 2002). However, both the ARCH and GARCH models are not sufficient for our study because of lack of focus on co-movements of financial returns, which is the other crucial component for estimating the dynamic conditional correlation. Due to these limitations, multivariate GARCH models (MGARCH), such as the VECH model of Bollerslev, Engle, & Wooldridge (1988), the constant conditional correlation model of Bollerslev (1990), and the BEKK model of Engle & Kroner (1995), have been developed to incorporate conditional covariance. Even though the MGARCH model can investigate these conditional variance and covariance, more complexity of the model comes with costs that researchers should be aware of and take into consideration. The numbers of parameter rapidly increase with the number of assets as well as the model has to be imposed for positive definiteness of a matrix (see Silvennoinen & Terasvirta (2009) for reviewing different MGARCH models).

Allowing time-varying variance and covariance might imply that the correlation estimates should also change over time. However, there was not any paper that explicitly states about the correlation, except Bollerslev (1990) that assumes the correlation to remain constant. The assumption of constant correlation puts some constraints on our study and seems to be unrealistic with the true characteristics of financial times series of returns. Therefore, we consider a more flexible model of Engle (2002), which is called dynamic conditional correlation (DCC) model. In addition to having dynamic conditional correlation, the model can overcome the problem of increasing parameters in MGARCH models; therefore large correlation matrices can be estimated. However, the simpler model comes with a cost as well because the parameters in the original DCC model by Engle (2002) are in a scalar form, implying that the model assumes no asset-specific dynamics. Recent papers have extended the DCC model to be more generalized than the scalar form, such as Generalized Dynamic Conditional Correlation (GDCC) model by Hafner & Francese (2003) and Flexible Dynamic Conditional Correlation (FDCC) model by Billio, Caporin & Gobbo (2006).

As our thesis's objective is to investigate asymmetry in the dynamic conditional correlation of equity returns and gold returns, the DCC, GDCC and FDCC models are still insufficient. Thus, we searched for a better model that can capture the impact of negative shock in the market and explicitly indicate the asymmetry. The asymmetric generalized dynamic conditional correlation (AG-DCC) model by Cappiello, Engle & Sheppard (2006) is extended from the DCC model by Engle (2002). The model allows for conditional asymmetries both in volatility and correlation. Furthermore, the AG-DCC model allows for series-specific news impact and smoothing parameters, which is not the case for the DCC model. Additionally, the model has been used in recent studies. For instance, Yang, Zhou and Leung (2012) applied it to investigate the correlation of stock returns, corporate bonds, and the real estate assets such as REITS and CMBS. Toyoshima, Tamakoshi and Hamori (2012) use the model to study the conditional correlation between treasury and swap markets for different maturities. After we have reviewed all of the models that are previously mentioned, we believe that the AG-DCC model is the most suitable one for our study. However, we also use the DCC model in order to check for the robustness of the results from the AG-DCC model.

3.4 Model

This section presents our model by dividing it into three sub sections. The first part talks about how we specify the mean equation, which also includes the Jarque-Bera normality test. Then, the second part provides how we select the volatility models including the Ljung-box test. The last part shows the correlation models, which are the DCC and AG-DCC models.

3.4.1 Mean equation

In order to use the DCC and AG-DCC models we first have to specify the mean equation and conditional variance equation for each asset. This comes from the fact that the DCC and AG-DCC models use the standardized residuals of each asset as an input when calculating the conditional correlation. Let us begin by stating the mean equation of all the assets in general form:

$$y_t = \mu + u_t$$

Assuming we have *p* number of assets then y_t is a *p x 1* vector of asset returns at time *t*, μ is a *p x 1* vector of constants and u_t is a *p x 1* vector of error terms at time *t*. The error term vector itself is then made up of two parts:

$$u_t = D_t \varepsilon_t$$

$$u_t \sim N_p[0, H_t]$$

$$\varepsilon_t \sim i.i.d.\, N_p[0, I_p]$$

Where ε_t is a *p x 1* vector of the standardized residuals at time *t* and D_t is a *p x p* matrix containing conditional asset volatilities, $\sqrt{h_{i,t}}$, on the diagonal and zeroes on the off diagonal. Furthermore, we have that H_t is a *p x p* conditional variance covariance matrix of y_t:

$$Var(y_t|\varphi_{t-1}) = Var_{t-1}(y_t)$$

$$= Var_{t-1}(u_t) = D_t E_{t-1}(\varepsilon_t{}' \varepsilon_t) D_t$$

$$= H_t$$

Here $\varphi_{t-1}$is the information set at the previous period and $E_{t-1}(\varepsilon_t{}'\varepsilon_t)$ is a *p x p* matrix containing the time varying correlations of y_t. It will be denoted by R_t from here on. Rewriting the equation for the error term in terms of the standardized residual we have the following:

$$\varepsilon_t = D_t{}^{-1} u_t$$

It can then be seen that it is sufficient to know the return, the mean equation and the volatility process of each asset in order to extract the standardized residuals. How the standardized residuals are then used is explained in detail in the chapter of 3.4.3.1 and 3.4.3.2. (Liu (2014), Engle (2002))

As previously stated, the aim of this thesis is to investigate the dynamic conditional correlation of the return of equity indices and gold. In order to do so, we do not include any additional outside factors in the mean equation of these assets. This comes from the fact that the conditional correlation obtained from the DCC and AG-DCC models is in fact the conditional correlation of the standardized residuals. If no additional factors are included in the mean equation, the conditional correlation of the asset returns will be the same as those of the standardized residuals. However, if additional factors are included, this will no longer be the case and the conditional correlation parameters from the DCC can no longer be interpreted as the conditional correlation of the asset returns. The mean equation we start with for all assets is therefore:

$$y_t = \mu + u_t$$

This equation is, however, modified to some extent for all the assets. The reason for doing so comes from the fact that many of the returns have autocorrelation in the standardized residuals. This is described in detail in chapter 3.4.2 along with the mean equation for each asset.

When it comes to the estimation process, we use the three stage estimation employed by the AG-DCC model. It allows for any type of univariate GARCH process to be used when modeling the volatility of the asset as long as it has covariance stationarity. Furthermore, it allows for the returns of the indices to be assumed stationary even though they may not be. If the returns are not truly normally distributed, a quasi maximum likelihood function should be used when estimating the parameters in the DCC and AG-DCC models, rather than the normal log likelihood function (Engle, 2002). With this in mind we conducted a Jarque-Bera normality test on the returns. The results of the tests, which can be seen in appendix A, were that the null hypothesis of normality was rejected for all the returns at the five percent level. Thus, we conclude that the returns are not normally distributed and we use quasi maximum likelihood estimation.

3.4.2 Volatility models

In order to estimate the conditional correlations in the DCC and AG-DCC models the volatility process of each asset must be known. The AG-DCC model uses a three step estimation approach, described in chapter 3.4.3.3, and as a result of that the correlation estimates will only be consistent if the univariate volatility models are correctly specified (Cappiello, Engle & Sheppard (2006)). It follows that knowledge of the correct conditional variance model, D_t^2, implies that the correct volatility model is also known. We, therefore, conduct a rigorous search for the correct variance model and the correct model specification for each asset. The models we tested are the following:

(1) Autoregressive Conditional Heteroskedastic model (ARCH)
(2) Generalized Autoregressive Conditional Heteroskedastic model (GARCH)
(3) Threshold Generalized Autoregressive Conditional Heteroskedastic model (TGARCH)
(4) Exponential Generalized Autoregressive Conditional Heteroskedastic model (EGARCH)
(5) Asymmetric Power Autoregressive Conditional Heteroskedastic model (APARCH)

When determining what model and what specification fit each asset best, we first make sure all the coefficients are significant at the 5% level. Next, we perform a Ljung-Box test on both the standardized residuals and the standardized residuals squared. This is necessary to see if the mean equation and volatility models we have chosen are adequate, where the test on the standardized residuals is used to see if the mean equation is adequate and the test on the standardized residuals squared is used to see if the volatility model is adequate. The Ljung-box test examines if there is autocorrelation in the standardized residuals (squared), which is not accounted for in the model being used. The formula it uses is the following:

$$Q_k = T(T+2)\sum_{k=1}^{K}\frac{1}{T-k}r_k^2$$

Here K is the number of lags included, T is the number of observations, r_k are the estimated autocorrelation coefficients of the standardized residuals (squared) and Q_k is the test statistic. Q_k is approximately Chi-squared distributed with K-p-q degrees of freedom. The null hypothesis is that there is no autocorrelation detected (Verbeek, 2012). A problem with the implementation of the Ljung-Box test is that if few lags are chosen, we might miss autocorrelation in higher lags. At the same time if many lags are chosen, the test may have low power since significant autocorrelation at one lag might be diluted by insignificant autocorrelation of the other lags (Brooks, 2008). In an attempt to bypass this problem, we decide to do the tests for multiple sets of lags, ranging from one to twenty five lags. The mean equation and volatility model chosen for each asset can be seen in the table one while the Q statistics and the corresponding probability values for those models are given in the appendix D.

In order to account for autocorrelation in the standardized residuals, we added a lag of the dependent variable to the mean equation. Adding an outside factor to the mean equation would limit us from generalizing the correlation between the standardized residuals of the asset returns to the asset returns themselves. This is, however, not so when the factor is a lag of the dependent variable itself, that is when the factor added is the return of the asset in a previous period. This comes from the fact that last periods return is known in this period and is therefore not an unknown stochastic process.

While all autocorrelation is accounted for in standardized residuals of all the series, there is autocorrelation in the standardized residuals squared of the gold returns that could not be taken care of. After rigorous testing of different models, it appears that using an EGARCH(9,0,0) model can account for all autocorrelation from the first nine lags, but not for ten lags or more. The implication of this is that we expect to see significant parameters in the DCC and AG-DCC models that affect the variance of the standardized residual of the gold returns. This is because the DCC and AG-DCC models include a lag of the variance from the previous period in the calculation of this period's variance. For further detail see chapter 3.4.3. However, since the volatility models of the stock indices do not reject the null hypothesis of no autocorrelation on the standardized residuals squared, we expect that the parameters, which govern the volatility of the standardized residuals of the stocks in the DCC and AG-DCC models, to be insignificant.

Last of all, we use the Schwarz Bayesian information criterion (BIC) to select the best model. Another information criterion available is the Akaike information criterion (AIC). However, when the sample size goes towards infinity, the BIC will almost always select the true model while AIC tends to favor overparameterized models (Verbeek, 2012). Our sample size consisted of over six thousand observations so we choose to use BIC. The model chosen for each asset and its specification can be seen in table one while a detailed description of each volatility model is given in appendix B.

Table 1: Selected model and mean equation

	Index	Selected Model	Mean Equation
1	Gold Bullion LBM U$/Troy Ounce	EGARCH(9,0,0)	$y_t = \mu + y_{t-12} + u_t$
2	S&P500	EGARCH(2,1,2)	$y_t = \mu + y_{t-12} + y_{t-13} + u_t$
3	MSCI U.S. mid cap 450	TARCH (2,1,1)	$y_t = \mu + y_{t-1} + u_t$
4	MSCI U.S. small cap 1750	EGARCH(2,2,2)	$y_t = \mu + y_{t-1} + u_t$

As it is shown in table one, the GARCH models that are selected for the returns of all stock indices include asymmetric terms. Therefore, they show all have asymmetric effects in their volatilities. However, the GARCH model specification selected for the gold returns does not include an asymmetric term, indicating that the volatilities of the gold returns are symmetric. Our result is contradicted with the result of Baur (2012), which presents that the coefficient of asymmetric effects in the volatility process of gold returns is negative and highly significant. The reason for the different findings could be attributed to the fact that we use a different period from Baur. However, as this is not the main subject of the thesis, we will not speculate on this further. Detailed information on each series mean and variance equation along with p-values of the coefficients as well as information criterions are provided in appendix C.

3.4.3 Correlation models

This section presents the detailed descriptions of the DCC and AG-DCC models. Moreover, the three stage estimation procedure is provided. Finally, some necessary modification of the DCC and AG-DCC models is presented.

3.4.3.1 Dynamic Conditional Correlation (DCC) by Engle 2002

The DCC model was developed from the constant conditional correlation model (CCC) of Bollerslev (1990), in which the time-varying variance covariance matrix of returns, H_t, is composed of an unconditional correlation matrix (R) and a conditional variance matrix (D_t^2):

$$H_t = D_t R D_t$$

However, in 2002 Engle proposed the dynamic conditional correlation model, which allows R to change over time and the conditional covariance matrix of returns, H_t, is constructed as:

$$H_t = D_t R_t D_t$$

Here D_t is diagonal matrix of volatilities, $\sqrt{h_{i,t}}$, and R_t is time-varying correlation matrix. . Each D_t^2 can be defined by any type of a univariate GARCH process, as discussed in chapter 3.4.1. R_t is generated from the conditional variances and covariance of the standardized residuals ,ε_t. The paper of Engle (2002) shows that the conditional correlation between two asset returns, $y_{1,t}y_{2,t}$, is based on information from the previous period and lies between -1 and 1. It is defined as:

$$\rho_{12,t} = \frac{E_{t-1}(y_{1,t}y_{2,t})}{\sqrt{E_{t-1}(y_{1,t}^2)E_{t-1}(y_{2,t}^2)}}$$

If we assume that the mean equation from chapter 3.4.1 has a zero mean as well as the standardized residuals are normally distributed with a mean of zero and a variance of one, the return can be written as the multiplication of the conditional standard deviation and the standardized residual:

$$y_{i,t} = \sqrt{h_{i,t}}\,\varepsilon_{i,t} \text{ where i=1,2,…,n}$$

Thus,

$$\rho_{12,t} = \frac{E_{t-1}(\varepsilon_{1,t}\varepsilon_{2,t})}{\sqrt{E_{t-1}(\varepsilon_{1,t}^2)E_{t-1}(\varepsilon_{2,t}^2)}} = E_{t-1}(\varepsilon_{1,t}\varepsilon_{2,t})$$

Therefore, the correlation of two asset returns is the covariance of their standardized residuals. Engle (2002) proposed that R_t should be obtained by using the series of standardized residuals as:

$$R_t = Q_t^{*-0,5} Q_t Q_t^{*-0,5}$$

Where Q_t is the conditional covariance matrix of standardized residuals, which follows a bivariate GARCH process, Q_t^* is a diagonal matrix with the square root of the ith diagonal element of Q_t. Therefore Q_t^* contains only the standard deviation of the ith standardized residual. The standard DCC model of Engle (2002) is shown as

$$Q_t = (1 - a - b)\bar{Q} + a\varepsilon_{t-1}\varepsilon'_{t-1} + bQ_{t-1}$$

Where $\bar{Q}$ is unconditional covariance matrix of standardized residual, a and b are scalars such that a+b < 1. By using the scalars, it implies the assumption of common dynamics among the assets used in the DCC model (Billio, Caporin, & Gobbo, 2006). However, the DCC model can be written in matrix form (Ding & Engle, 2001), which is more generalized as

$$Q_t = (II' - A - B) \circ \bar{Q} + A \circ \varepsilon_{t-1}\varepsilon'_{t-1} + B \circ Q_{t-1}$$

Where I is a vector of ones and " $\circ$ " is the Hadamard product of two identically sized matrices. If A, B, and $(II' - A - B)$ are positive semi definite, Q_t will also be semi definite. If any one of A, B, and $(II' - A - B)$ is positive definite, Q_t will also be.

In the estimation, it is essential that Q_t is positive definite in order to guarantee that R_t is a correlation matrix that has ones on the diagonal and all other elements are in the interval [-1,1]. For the two assets, R_t is shown as:

$$R_t = \begin{bmatrix} 1 & \frac{q_{12,t}}{\sqrt{q_{11,t}}\sqrt{q_{22,t}}} \\ \frac{q_{12,t}}{\sqrt{q_{11,t}}\sqrt{q_{22,t}}} & 1 \end{bmatrix}$$

The conditional correlation between equity returns and gold returns is the thus equal to $\frac{q_{12,t}}{\sqrt{q_{11,t}}\sqrt{q_{22,t}}}$.

3.4.3.2 Asymmetric Generalized Dynamic Conditional Correlation (AG-DCC) by Cappiello, Engle and Sheppard (2006)

In their 2006 paper Cappiello, Engle and Sheppard claimed that the standard DCC model *"does not allow for asset-specific news and smoothing parameters or asymmetries"*. They modified the model to factor in asymmetric correlation, in a so called asymmetric generalized DCC (AG-DCC) as:

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'n_{t-1}n'_{t-1}G + B'Q_{t-1}B$$

Where Q_t is the conditional covariance matrix of standardized residuals, $\bar{Q}$ is the unconditional covariance matrix of standardized residuals and A, B, and G are p× p parameter matrices, $\bar{N}$ is defined as $\bar{N} = E[n_t n'_t]$ and $n_t = I[\epsilon_t < 0] \circ \epsilon_t$. Here $I[.]$ is a p×1 indicator function that will equal to 1 if the $\epsilon_t < 0$ or 0 otherwise, while " ∘ " is the Hadamard product. The additional n_t term indicates asymmetries when there is negative shock.

For Q_t to be positive definite, the intercept $(\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G)$ needs to be positive semi-definite and Q_0 is positive definite. If the A, B, and G matrices are replaced by scalars, the model is a so-called asymmetric DCC (A-DCC) and is written as:

$$Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - g^2\bar{N}) + a^2\varepsilon_{t-1}\varepsilon'_{t-1} + g^2 n_{t-1}n'_{t-1} + b^2 Q_{t-1}$$

For Q_t to be positive definite, the intercept $(\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - g^2\bar{N})$ needs to be positive semi-definite, which is the same as for AG-DCC. A necessary and sufficient condition for this to hold is that $a^2 + b^2 + \delta g^2 < 1$, where δ is maximum eigenvalue $\left[\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}\right]^5$. Drawbacks of the AG-DCC model are large amount of parameters and complexity. Assuming p is the number of assets, p^2 parameters are required for each correlation term. Even though diagonal matrices are used, the numbers of parameters still linearly increase with the number of assets. Therefore, the scalar version is preferred when there are many assets. Moreover, using scalar has inflexible assumption of having common dynamic among the assets.

For the AG-DCC model, we adjust the general model to be a bivariate model as we have only two assets in our study and we want to capture asset specific dynamics. Since we have few assets, the problem of too many parameters in the estimation is not an issue for us. The model is shown as:

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'n_{t-1}n'_{t-1}G + B'Q_{t-1}B$$

Where A, B, and G matrices are of the dimension 2× 2. Then, the Q_t matrix is constructed as follows:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} = \left(\begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}' \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} - \right.$$

$$\left. \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}' \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} - \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix}' \begin{bmatrix} n_{11} & n_{12} \\ n_{12} & n_{22} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} \right) +$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix}' \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix}' \begin{bmatrix} n_{1,t-1} \\ n_{2,t-1} \end{bmatrix} \begin{bmatrix} n_{1,t-1} \\ n_{2,t-1} \end{bmatrix}' \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} +$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}' \begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

For modeling the DCC model, we use the same approach as for the AG-DCC model except that we set the matrix G equal to zero. Therefore, the Q_t matrix of the DCC model is constructed as follows:

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'Q_{t-1}B$$

Then, the Q_t matrix is written as follows:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} = \left(\begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}' \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} - \right.$$

$$\left. \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}' \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \right) + \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix}' \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} +$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}' \begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

3.4.3.3 Estimation

After the model is well specified, the parameters are estimated by maximizing the quasi log likelihood function, since the returns were not normally distributed as was discussed in chapter 3.4.1. Based on Engle (2002), the quasi log likelihood function is defined as

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\,log|D_t| + u'_t D_t^{-1} D_t^{-1} u_t - \varepsilon'_t \varepsilon_t + log|R_t| + \varepsilon'_t R_t^{-1} \varepsilon_t)$$

The log likelihood can separated to the sum of a volatility part and a correlation part:

$$\mathrm{L}(\theta, \emptyset) = L_V(\theta) + L_C(\theta, \emptyset)$$

Where θ is a set of parameters in D_t and $\emptyset$ is the parameters in R_t in equation $H_t = D_t R_t D_t$. Therefore, the volatility part is

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2log|D_t| + u'_t D_t^{-2} u_t)$$

And the correlation part is

$$L_C(\theta,\emptyset) = -\frac{1}{2}\sum_{t=1}^{T}(log|R_t| + \varepsilon'_t R_t^{-1}\varepsilon_t - \varepsilon'_t\varepsilon_t)$$

Based on Cappiello, Engle and Sheppard (2006), there are three stages in the estimation process. They describe these steps in the following way:

"In the first stage, univariate volatility models are fit for each of the assets, and estimates of hit are obtained. In the second stage, asset returns, transformed by their estimated standard deviations, are used to estimate the intercept parameters of the conditional correlation. Finally, the third stage conditions on the correlation intercept parameters to estimate the coefficients governing the dynamics of correlation."

3.4.3.4 Problems and remedies of the DCC model

The DCC model presented by Engle in 2002, and by extension the AG-DCC model of Cappiello, Engle and Sheppard in 2006, has inherent structural problems. Two main problems are of concern, which need to be addressed in order to use the model. The first one is the assumption that the standardized residuals, ε_t, are normally distributed with a mean zero and variance of one. Since the covariance matrix Q_t is the covariance matrix of the standardized residuals, it cannot be that they have both a variance of Q_t as well as a variance of one simultaneously. Due to a suggestion from Caporin and McAleer (2013), we assume that the standardized residuals are normally distributed with a mean of zero and variance of Q_t; $\varepsilon_t \sim N(0, Q_t)$.

The second problem is that the DCC model assumes that $E_{t-1}[\varepsilon_t{}'\varepsilon_t] = R_t$, where R_t is the correlation matrix of the standardized residuals (which is also the correlation matrix of the asset returns, as was shown previously in chapter 2.4), when in fact $E_{t-1}[\varepsilon_t{}'\varepsilon_t] = Q_t$. Although it can be shown that $R_t = Q_t$ only if $\varepsilon_t \sim N(0,1)$, we know, as previously mentioned, that this is not true. Therefore, we modify the covariance matrix of y_t, H_t, by calculating it as $H_t = D_t Q_t D_t$, instead of $H_t = D_t R_t D_t$.

Using these two remedies it still holds that the correlation matrix of the standardized residuals is identical to the correlation matrix of y_t, as well as all other assumptions of the model hold and calculations can be done as normal. The only exception to this is that the covariance matrix of the standardized residuals, Q_t, will not be equal to the correlation matrix of the standardized residuals, R_t. This however does not pose a problem since we can still calculate the correlation of the standardized residuals using the normal correlation formula.

4 Empirical Results

This chapter provides the empirical results along with discussion and interpretation. We will begin by presenting the results from the DCC and AG-DCC models along with an in depth comparison and discussion. Next, we take a close look at the asymmetric parameters in the AG-DCC model and consider how negative and positive shocks in the gold and stock markets affect the dynamic conditional correlation estimates. Finally, some qualitative economic reasons, which might lend support to our findings, are presented.

4.1 Comparing the AG-DCC and DCC

In order to determine whether the AG-DCC model outperforms the DCC model, we rely on the Schwarz Bayesian information criterion and the maximized log likelihood value. Furthermore, we calculated the Akaike information criterion for robustness in the results. We find that all three information criterions suggest that the AG-DCC model outperforms the DCC model for all three of our dynamic conditional correlation estimates. The results are shown in detail in table two.

To further compare the models we look at the parameter estimates of the Q matrix of both models for all three equity indices, presented in table three and four. Overall the DCC model has fewer insignificant parameters at the five percent level for the S&P500 and small-cap indices. The opposite is true for the mid-cap index. However, the AG-DCC model includes three more parameters than the DCC model, which account for the asymmetry. Looking only at the parameters that the models have in common, we see that the S&P500 index has one insignificant parameter under both models while for the other two indices the results are the same as before.

Since not all of the parameters that govern autocorrelation in the variance of the standardized residuals of the stock indices, b11 and b22, are insignificant, the results contradict with what we expected to find from the Ljung-Box test on the standardized residuals squared. There are two possible reasons why this could happen. The first one is that the Ljung Box test is not sufficiently powerful to detect the autocorrelation while the DCC/AG-DCC model can detect the autocorrelation. We then end up not rejecting the null hypothesis of the Ljung-box test of no autocorrelation when we should reject it. A second reason has to do with that the p-values, which Eviews calculates for the Ljung-Box test of the standardized residuals, may not be correct. This is a technical problem which we could not bypass. It should, however, be noted that even if the p-values are incorrect, we still choose the best mean equation and volatility model based on the information criterions mentioned previously.

Since the information criterions used will prefer models with better explanatory power and at the same time penalize for including variables that do not add to the explanatory power of the model, the results from table one are even more decisively in favor of AG-DCC for the S&P500 and MSCI US Small cap 1750 indices.

Table 2: Log-likelihood value and information criterions

This table presents the log-likelihood value and information criterions (AIC and BIC) of each stock return series under both the DCC and AG-DCC models. These criterions are then used to select which models fits the data better and should be used to estimate the dynamic conditional covariances. For the Log-Likelihood value a higher score indicates a better fit while for the AIC and BIC a lower score indicates a better fit

Model	Log-Likelihood value	AIC	BIC
DCC			
S&P500	-7.421	0.005	0.012
MSCI US Mid cap 450	62.558	-0.020	-0.013
MSCI US Small cap 1750	64.585	-0.021	-0.014
AG-DCC			
S&P500	6.282	0.001	0.011
MSCI US Mid cap 450	86.753	-0.027	-0.017
MSCI US Small cap 1750	80.015	-0.025	-0.014

Table 3: Parameter estimates of the DCC model

This table presents the coefficient of the parameters in the conditional covariance matrix of the standardized residuals, based on the DCC model. Furthermore, the table shows which coefficients are significant at the 5% confidence level. The "a" coefficients govern how much the standardized residuals of the previous period affect the variance/covariance of the standardized residuals this period. The "b" coefficients govern how much the variance/covariance of the standardized residuals from the previous period affect the variance/covariance of the standardized residuals this period. Parameters marked "11" govern the variance of the standardized residuals of the gold returns, while parameters marked "22" govern the variance of the standardized residuals of the stock returns. For further detail see chapter 3.4.3.2.

DCC			
Index against gold returns	**S&P500**	**MSCI US Mid cap 450**	**MSCI US Small cap 1750**
a11	0.238	-0.084	0.137
a12	0.011*	0.213	0.075
a22	0.125	-0.016*	0.118
b11	-0.479	0.958	-0.450
b12	-0.820	0.081*	0.201
b22	0.521	-0.313*	0.961

*** Insignificance at the 5% level**

Table 4: Parameter estimates of the AG-DCC model

This table presents the coefficient of the parameters in the conditional covariance matrix of the standardized residuals, based on the AG-DCC model. Furthermore, the table shows which coefficients are significant at the 5% confidence level. The "a" coefficients govern how much the standardized residuals of the previous period affect the variance/covariance of the standardized residuals this period. The "b" coefficients govern how much the variance/covariance of the standardized residuals from the previous period affect the variance/covariance of the standardized residuals this period. The "g" coefficients govern how large of an asymmetric effect from a negative shock in the standardized residual of the stock and/or gold return from last period have on the variance/covariance of the standardized residuals this period. Parameters marked "11" govern the variance of the standardized residuals of the gold returns, while parameters marked "22" govern the variance of the standardized residuals of the stock returns. For further detail see chapter 3.4.3.2.

AG-DCC			
Index against gold returns	**S&P500**	**MSCI US Mid cap 450**	**MSCI US Small cap 1750**
a11	0.068	0.151	0.176
a12	0.151	0.031*	0.023*
a22	0.023*	0.118	0.132
b11	0.576	-0.635	-0.550
b12	0.791	0.741	0.804
b22	-0.580	0.634	0.552
g11	-0.053*	-0.014*	0.011*
g12	-0.020*	0.191	0.167
g22	-0.250	-0.070	-0.083

*** Insignificance at the 5% level**

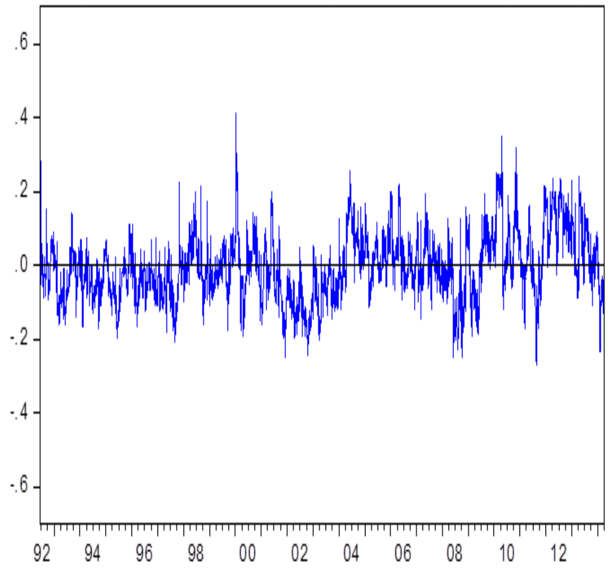
4.1.1 Correlation estimation of the DCC and AG-DCC models.

To get a full representation of how the dynamic conditional correlation estimates differ between the AG-DCC model and the DCC model, one needs to consider a relatively long period of observations. The reason is that the covariance matrix of todays' standardized residuals, Q_t, which is used to calculate the dynamic conditional correlation, depends on the shocks in yesterdays' standardized residuals as well as the covariance matrix of yesterdays standardized residuals, Q_{t-1}. It can then be seen that the Q_{t-1} matrix also depended on the Q_{t-2} matrix, and so on. Thus, the difference in the dynamic conditional correlation estimates between the AG-DCC and the DCC can accumulate over time.

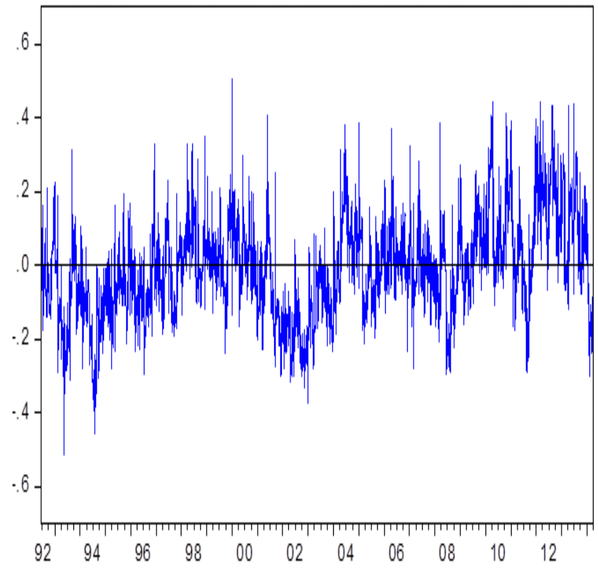
Graphs one to six present how the dynamic conditional correlations develop under both models for each return series, while graphs seven to nine show the difference in the dynamic conditional correlation estimates of the AG-DCC and DCC models. When calculating the difference, the DCC model was used as a baseline. Thus, the graph shows how much the correlation from the AG-DCC model deviates from the correlation in the DCC model. It should be noted that the y axis on graphs seven to nine has different scale. This is because there are some large outliers in the graphs, which are different among the three graphs. Therefore, when we use the same scale for every graph, they end up being harder to read.

The results in table five show that the conditional correlation of the stock and gold returns is on average very low and slightly different among the indices. For the S&P500 and small-cap, the mean correlation estimates are negative, implying that gold has a hedging property against the stock index on average. However, the correlation of mid-cap and gold returns is positive on average, indicating that gold can only help to diversify the risk of the investment portfolio rather than hedge the risk.

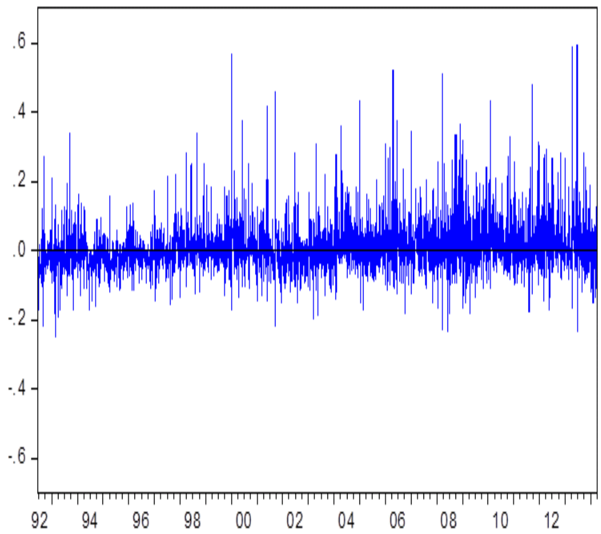
Moreover, the different average correlation estimates from the two models shows that taking the asymmetry into consideration has some impact to the estimation. Even though the sign of the correlation is in the same direction for each index, the correlation value is still somewhat different. This becomes more apparent when we look at the average difference of the dynamic conditional correlation estimate in absolute values. It can be seen that although the mean correlation estimates of the AG-DCC and the DCC are relatively similar, then the AG-DCC model estimates the correlation to be between 0.065 and 0.077 higher in absolute value on average. This indicates that the DCC model underestimates the strength of the correlation whether it is positive or negative.



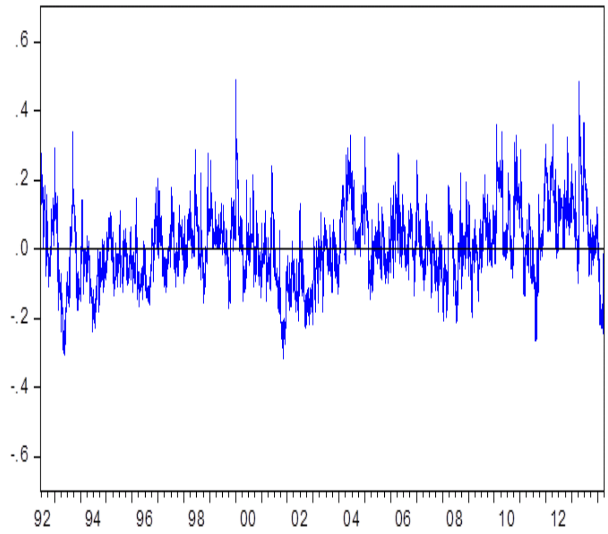
Graph 1: DCC of S&P500 and gold returns



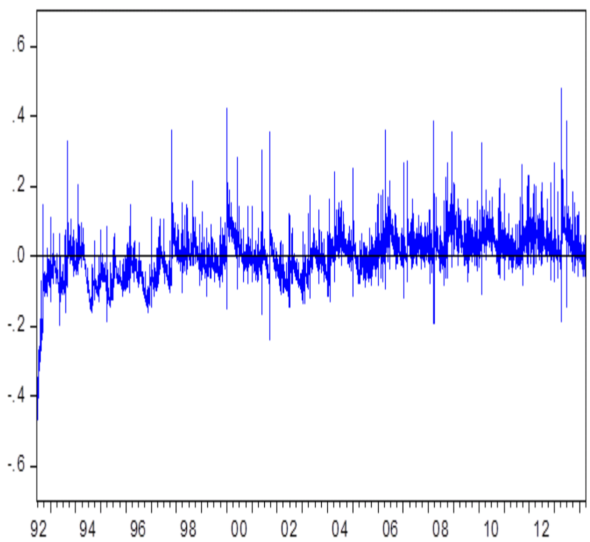
Graph 2: AG-DCC of S&P500 & gold returns



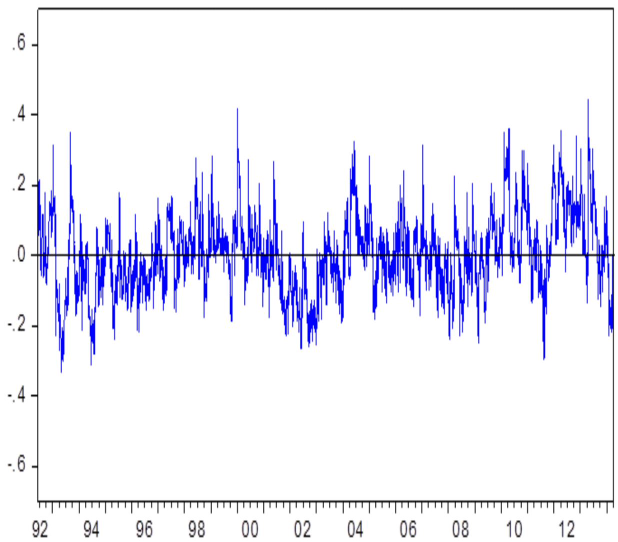
Graph 3: DCC of Mid-cap & gold returns



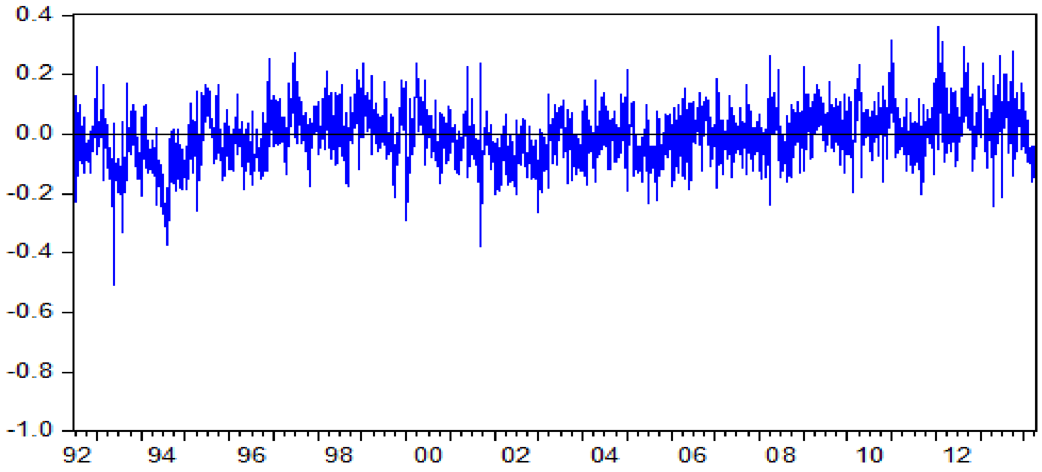
Graph 4: AG-DCC of Mid-cap & gold return



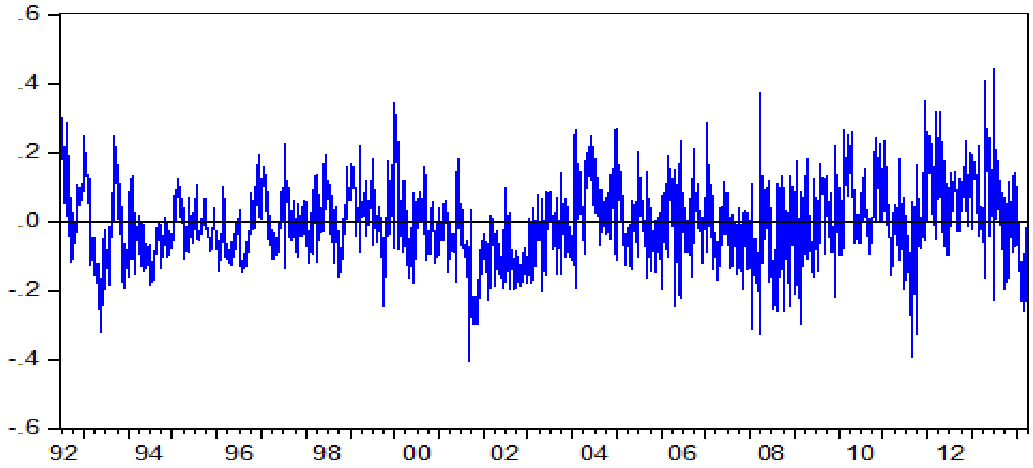
Graph 5: DCC of Small-cap & gold returns



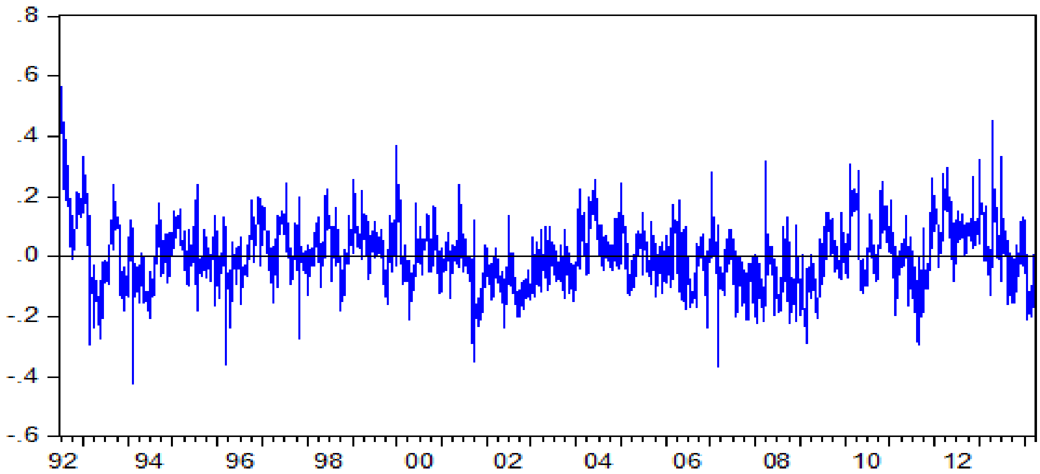
Graph 6: AG-DCC of Small-cap & gold returns



Graph 7: Differences (DCC&AGDCC)-S&P500



Graph 8: Differences (DCC&AGDCC)-Midcap



Graph 9: Differences (DCC&AGDCC)-Smallcap

In terms of volatility, the correlation estimate from the AG-DCC model is more volatile than the estimate from the DCC model. It indicates that the correlation of the stock returns and gold return changes more dramatically than that is expected from the DCC model. For example, by using the DCC model the negative correlation of S&P500 and gold return during the financial crisis in 2008 is underestimated when it is compared to the estimate from the AG-DCC model. However, when the stock market recovers in 2012, the correlation turns to be positive. The DCC model still suggests too low correlation than that of the AG-DCC model. This means that when portfolio weights need to be adjusted due to the change in the dynamic conditional correlation between assets in the portfolio, the adjustment in the weight might be too low if investors use the DCC model.

We realize that it is subjective to conclude how large of a difference in the correlation estimates there should be to cause a significant loss or unnecessary risk taking to an investor due to sub-optimal portfolio structure. We can however conclude from our results with certainty that investors will be faced with sub-optimal portfolio weights if they allocate their assets without considering this asymmetry.

Table 5: Mean and standard deviation of conditional correlation estimates

This table presents the mean and the standard deviation of the dynamic conditional correlation of gold returns and different indices returns, which is generated from the DCC and AG-DCC model. The bottom row of this table presents the mean of differences in the dynamic conditional correlation of the DCC model and AG-DCC model. It should be noted that the differences were calculated in absolute values and then averaged.

	S&P500	**MSCI US Mid cap 450**	**MSCI US Small cap 1750**
DCC			
Mean	-0.007	0.006	-0.002
Standard deviation	0.090	0.061	0.069
AG-DCC			
Mean	-0.013	0.005	-0.002
Standard deviation	0.133	0.104	0.109
Differences (AG-DCC and DCC)			
Mean	0.065	0.077	0.077

4.2 Effect of market shocks on correlation asymmetry

As was shown previously, the asymmetry in the dynamic conditional correlations of the AG-DCC model is incorporated in the covariance matrix of the standardized residuals, Q_t, which is then used to calculate the dynamic conditional correlations. Therefore, the asymmetry can be captured in the variances and the covariance of the standardized residuals. Thus, the asymmetric terms may affect the dynamic conditional correlation estimate in different manners, depending on whether there is a negative shock in only one of the standardized residuals or both. Furthermore, a negative shock in only one asset's standardized residuals may have different effect from a negative shock in only the other asset's standardized residuals. It can be seen from table four that not all of the asymmetric parameters were statistically significant for any given assets at the five percent level. In fact, the parameter g11 was never statistically significant, while g22 was always statistically significant. The parameter g12 was statistically significant for the Q_t matrix dynamics of the small-cap and gold as well as for mid-cap and gold. Knowing this allowed us to see how different shocks alter the asymmetry in the dynamic conditional correlations in different ways.

The effects that different negative shocks in the standardized residuals have on the variances and covariance of the standardized residuals can be seen in table six and seven. It should be noted that "no effect", "increase", "decrease", which are stated in table six and seven, are additional asymmetric effects that occur when either or both of the standardized residuals of gold and stock returns have a negative shock, compared to when both the standardized residuals have positive shocks. The scenarios that we consider are:

Scenario 1: A negative shock in the standardized residual of the gold returns, implies that there is a positive shock in the standardized residual of stock return that occurs at the same time. The result will then compare with the situation of a joint positive shock in the standardized residuals of gold returns and stock returns.

Scenario 2: A negative shock in the standardized residual of the stock returns, implies that there is a positive shock in the standardized residual of the gold returns that occurs at the same time. The result will then compare with the situation of a joint positive shock in the standardized residuals of gold returns and stock returns..

Scenario 3: The result under a joint negative shock in the standardized residuals of gold returns and stock returns. The result will then compare with the situation of a joint positive shock in the standardized residuals of gold returns and stock returns.

Table 6: The asymmetric effects of a negative shock in standardized residuals on the variances and covariance of standardized residuals of S&P500 and gold returns

S&P500 index	Negative shock in the standardized residuals of gold return – (1)	Negative shock in the standardized residuals of stock return – (2)	Negative shock in both the standardized residuals of gold return and stock return – (3)
variance of the standardized residuals of gold returns (q11)	no effect	no effect	no effect
variance of the standardized residuals of stock returns (q22)	no effect	increase	Increase
covariance of the standardized residuals of gold returns and stock returns (q12)	no effect	no effect	no effect

Table 7: The asymmetric effects of a negative shock in standardized residuals on the variances and covariance of standardized residuals of mid-cap/small-cap index and gold returns

Mid-cap and Small-cap index	Negative shock in the standardized residuals of gold return – (1)	Negative shock in the standardized residuals of stock return – (2)	Negative shock in both the standardized residuals of gold return and stock return – (3)
variance of the standardized residuals of gold returns (q11)	no effect	increase	increase
variance of the standardized residuals of stock returns (q22)	increase	increase	no effect/increase
covariance of the standardized residuals of gold returns and stock returns (q12)	no effect	decrease	inconclusive

4.2.1 Asymmetric correlation: S&P500 & Gold

The discussion in this chapter is based on the results in table four and six. For the dynamic conditional correlation of the S&P500 and gold indices, only a negative shock in the standardized residuals of the S&P500 index return will cause asymmetry. This is a result of only one of the asymmetric parameters, g22, being statistically significant. A negative shock in the standardized residual of the S&P500 index return will increase the variance of the S&P500 index return in the next period. Therefore, the dynamic conditional correlation will be lower than otherwise. These findings of no asymmetric effect in the variance of the standardized residuals of the gold returns following a negative shock in the standardized residuals of the gold returns coincide well with the EGARCH(9,0,0) model used to capture the other part of the volatility of the gold returns, since no asymmetric terms were included in the EGARCH specification. Likewise, the findings of an asymmetric effect in the volatility of the standardized residuals of the S&P500 index returns following a negative shock in the standardized residuals of the S&P500 index returns is consistent with the EGARCH(2,1,2) specification. The findings of no asymmetry in the volatility of gold returns are, however, inconsistent with the findings of Baur from 2012.

4.2.2 Asymmetric correlation: Small Cap/Mid Cap & Gold

The discussion in this chapter is based on the results in table four and seven. Since the same asymmetric parameter terms, g12 and g22, are statistically significant in the Q_t matrix dynamics when using the small-cap and the mid-cap indices, they will be discussed together.

If there is only a negative shock in the standardized residual of gold, the variance of the small cap index will increase, which result in a lower correlation in absolute value. The variance of the small cap index will likewise increase if there is only a negative shock in the standardized residuals of the small cap index. However, if there is a negative shock in both the standardized residual of gold and the small cap index, the shocks will work against each other and cause an overall lower increase in the variance of the small cap index, and a lower reduction in the correlation. The variance of the gold is, however, unaffected by a negative shock in the standardized residuals of the gold index while a shock in the standardized residuals of the small cap index will increase the variance of the gold, which lowers the correlation in absolute value. As with the S&P500 index, these findings coincide well with the GARCH models chosen for the stock indices returns and the gold returns.

For the covariance of the standardized residuals of the small cap index and gold, a negative shock in the standardized residuals of gold will have no affect on its own. A negative shock in the standardized residuals of the small cap will on the other hand always reduce the covariance of the standardized residuals of small cap and gold, and thus lower the correlation. However, if there is a negative shock in both standardized residuals, the shock in the standardized residual of gold will increase the covariance while the shock in the standardized residual of the small cap index will decrease the covariance. It will then depend on the size of each shock whether the covariance increases, decreases or stays constant and thus the overall affect on the correlation is uncertain.

4.3 Asymmetric correlation relating to mean-variance framework

When comparing the AG-DCC and the DCC models one might consider constructing a mean-variance optimizing portfolios from the stock and gold indices to see whether the AG-DCC can give better performance than the DCC model. One can see that this will be problematic as the premise of the thesis is that the DCC model does not capture asymmetric effects in the dynamics of the dynamic conditional correlation. This appears to be true from our findings. Since both portfolios would be constructed from the same assets, we know that by definition there can be only one optimal portfolio for any given risk level and only one portfolio that maximizes the Sharpe ratio. Since we believe that the AG-DCC model captures the dynamics of the Q_t correctly, but not the DCC model, we know that the AG-DCC model will be the one that can find this optimal portfolio. The DCC model will under or overestimate components of the Q_t matrix and although it will give us some estimate of optimal portfolio weights, we will know these are not the true optimal weights.

To illustrate this further, consider the case where we want to find the portfolio that can give us the lowest risk with expected return equal to μ. The DCC model will find a portfolio to do this and the AG-DCC model will find another. A possible outcome would be that both models, according to their estimations, find a portfolio with the same risk and expected return equal to μ. Since the DCC model has the wrong asset variances and covariance, it cannot truly minimize the portfolio's risk, expect by chance. Therefore, the DCC portfolio is not the optimal portfolio and in reality it will have a higher risk than the DCC estimation suggests. Similarly the DCC model might tell us that it can give us a portfolio with an equal or higher

Sharpe ratio than the AG-DCC model can. This will once again not be true as the DCC model does not accurately calculate the portfolio risk, thus it should only be able to match the maximum Sharpe ratio suggested by the AG-DCC model by chance and otherwise suggest a sub-optimal maximized Sharpe ratio.

How much the DCC model will deviate from the true optimal portfolio will depend on current and previous market shocks and therefore there is little gain in modeling any period as an example. It is sufficient to realize that according to our findings, the AG-DCC model will outperform the DCC model, except in a few random instances where the result will be the same for a short time.

4.4 Economic reasoning

Here we will present some economic reasoning in support of our findings on asymmetry in the variances and covariance of the standardized residuals from the Q_t matrix, whose elements are used to estimate the dynamic conditional correlation. In order to save notation and to make the discussion easier to follow, the standardized residual of gold returns will be referred to as ε_{Gold} , the standardized residual of the S&P500 returns will be denoted as $\varepsilon_{S\&P500}$ and the standardized residual of the Mid-cap/Small-cap returns will be defined as $\varepsilon_{MC/SC}$. The discussion will center around the three scenarios, labelled as (1), (2) and (3) in tables six and seven; A negative shock in ε_{Gold} and a positive shock in $\varepsilon_{S\&P500}$ or $\varepsilon_{MC/SC}$, a negative shock in $\varepsilon_{S\&P500}$ or $\varepsilon_{MC/SC}$ and a positive shock in ε_{Gold} and finally a joint negative shock in both ε_{Gold} and $\varepsilon_{S\&P500}/\varepsilon_{MC/SC}$. It should be noted that we will simultaneously discuss the asymmetry in the Q_t matrix when using on one hand the S&P500 and gold indices and on the other hand the mid-cap/small-cap and gold indices. Finally, the reader should be aware that this is not a conclusive list of possible reasons for our findings.

4.4.1 Scenario one

A negative shock in ε_{Gold} and a positive shock in $\varepsilon_{S\&P500}$ /$\varepsilon_{MC/SC}$ shows that a negative shock in ε_{Gold} does not cause any asymmetric effect in the variance of ε_{Gold}. We believe that it is a natural result since gold does not have any leverage or operations, which incur fixed costs. Therefore, gold is not subject to Black's (1976) leverage effect, which states that a firm whose value declines will have a larger increase in its stocks variance compared to when stock prices increase. This comes from the fact that fixed costs, from debt and operations, do not decrease proportionally with the loss of income.

However, it can be seen that following a negative shock in ε_{Gold} there is no asymmetric effect in the variance of $\varepsilon_{S\&P500}$ while there is an asymmetric effect in $\varepsilon_{MC/SC}$, which causes an increase in next periods variance. This must be a result of some size related firm characteristics. For instance, gold price decreases when interest rates are expected to increase (Fortune, 1987) and interest rates tend to be raised in an effort by the central bank to reduce spending in the economy during boom periods. Since smaller firms tend to have higher leverage than large firms as well as less efficient operations (Chan and Chen, 1991), it is reasonable to assume that when interest rates increase, smaller firms will suffer worse. Based on the post modern portfolio theory, investors might be concerned that the returns of smaller firms will fall below the investors' benchmark for minimum expected return. They, however, do not expect returns to fall below this benchmark for larger firms. According to the post modern portfolio theory, investors value potential upside as a bonus, but not to the same extent as they fear a loss. This causes investors to sell their small and mid cap stocks on a larger scale following an increase in interest rates than they will buy them at following a decrease in interest rates. This in turn causes an asymmetric increase in the variance of $\varepsilon_{MC/SC}$ following an increase in interest rates. Prices will then be expected to fall as investors leave the market, but not enough to offset the increase in prices due to the advantageous business environment of the boom period. This comes from the fact that we are considering scenario one where the $\varepsilon_{MC/SC}$ is positive.

The covariance will not be expected to display any asymmetry since this asymmetric increase in the variance of $\varepsilon_{MC/SC}$ is idiosyncratic. It means that the non-systematic risk is not shared between the gold and mid-cap/small-cap stocks. Then, the asymmetric variance $\varepsilon_{MC/SC}$ does not affect the covariance of ε_{Gold} and $\varepsilon_{MC/SC}$.

4.4.2 Scenario two

A negative shock in $\varepsilon_{S\&P500}$ /$\varepsilon_{MC/SC}$ and a positive shock in ε_{Gold} presents an asymmetric increase in the variance of both $\varepsilon_{S\&P500}$ and $\varepsilon_{MC/SC}$. This asymmetric increase in variance can be explained by Black's leverage effect, which is discussed in scenario one. However, for the variance of ε_{Gold} there is only an asymmetric increase when there is a negative shock in

$\varepsilon_{MC/SC}$, not when there is a negative shock in $\varepsilon_{S\&P500}$. This might be explained by the post modern portfolio theory. When there is a negative shock in $\varepsilon_{MC/SC}$, some investors might perceive that the expected return of small and mid cap firms has a higher probability of falling below the investors' benchmark return than for large firms. This is due to the lower capabilities of smaller firms to survive during bad economic conditions as a result of higher leverage and less efficient operations (Chan and Chen, 1991). Therefore, these investors may not consider small and mid cap stocks as a viable investment asset and instead move to other safer asset such as gold, which is called flight to quality. Thus, there is an asymmetric increase in the variance of ε_{Gold} when there is a negative shock in $\varepsilon_{MC/SC}$, but not when there is a negative shock in $\varepsilon_{S\&P500}$.

Last of all an asymmetric decrease in the covariance of ε_{Gold} and $\varepsilon_{MC/SC}$ is found following a negative shock in $\varepsilon_{MC/SC}$ whereas the covariance of ε_{Gold} and $\varepsilon_{S\&P500}$ displays no asymmetric effect following a negative shock in $\varepsilon_{S\&P500}$. Due to asymmetric effects in variances of ε_{Gold} and $\varepsilon_{MC/SC}$, there is an increased demand for gold at the same time as there is a lower demand for mid and small cap stocks, so it should not be surprising that their prices will move in opposite directions. This leads to an asymmetric decline in the covariance of ε_{Gold} and $\varepsilon_{MC/SC}$. However, the covariance of ε_{Gold} and $\varepsilon_{S\&P500}$ does not display any such asymmetry. This is because the asymmetric increase in the variance of $\varepsilon_{S\&P500}$ is idiosyncratic and will not affect the covariance (as previously mentioned in scenario one).

4.4.3 Scenario three

A joint negative shock in both ε_{Gold} and $\varepsilon_{S\&P500}/\varepsilon_{MC/SC}$ shows an asymmetric increase in the variance of $\varepsilon_{S\&P500}$ while the variance of $\varepsilon_{MC/SC}$ can either increase asymmetrically or have no asymmetric effect. The asymmetric increase in the variance of $\varepsilon_{S\&P500}$ is applicable to Black's leverage effect in the same way as previously stated in scenario two. For $\varepsilon_{MC/SC}$ there are two factors that asymmetrically increase its variance. The first can be attributed to Black's leverage effect as it results from the negative shock in $\varepsilon_{MC/SC}$ while the second one comes as a result of the negative shock in ε_{Gold}. These two effects then work to cancel each other out. Looking back to scenario one where there is a negative shock in ε_{Gold}, we suggest that the asymmetric increase in the variance of $\varepsilon_{MC/SC}$ is caused by an increase in interest rates which could also be the case here. An increase in interest rates could even be the sole cause of the negative shock in both ε_{Gold} and $\varepsilon_{MC/SC}$ if the increase in interest rates is large enough. However, it seems quite counter intuitive that the effect of the increased interest rates and the leverage effect will reduce each others effect instead of adding to each other. Therefore, we speculate that when a joint negative shock in ε_{Gold} and $\varepsilon_{MC/SC}$ takes place, there is some omitted factor that brings positive news for mid and small cap firms bot not for large cap firms. This causes the variance of $\varepsilon_{MC/SC}$ to range from having an increasing asymmetric effect to having no asymmetric effect, depending on the size of this omitted factor, while there is always an increasing asymmetric effect in the variance of $\varepsilon_{S\&P500}$.

Scenario three further shows an asymmetric increase in the variance of ε_{Gold} when there is a negative shock in $\varepsilon_{MC/SC}$, but not when there is a negative shock in $\varepsilon_{S\&P500}$. It is interesting to point out that this is exactly the same result as in scenario two, even though there is now a negative shock in ε_{Gold} rather than a positive one. We start by noting that under most circumstances there will be some asymmetric increase in the variance of $\varepsilon_{MC/SC}$. Then, it is reasonable to assume that the attractiveness of mid and small cap stocks will be the same as it was reasoned to be according to the post modern portfolio theory in scenario two. Thus, we speculate that the asymmetric increase in the variance of ε_{Gold} following a negative shock in $\varepsilon_{MC/SC}$ comes from investors who are interested in the asset specific properties of gold, such as its inflation and currency hedging capabilities, rather than the change in value of gold during the previous period.

The covariance of ε_{Gold} and $\varepsilon_{S\&P500}$ displays no asymmetric effect as the asymmetric increase in the variance of $\varepsilon_{S\&P500}$ is idiosyncratic and does thus not affect the covariance. The covariance of ε_{Gold} and $\varepsilon_{MC/SC}$, however, has an asymmetric effect whose direction is unclear. This comes from the fact that there may or may not be any asymmetric increase in the variance of $\varepsilon_{MC/SC}$. Even if there is an asymmetric increase in the variance of $\varepsilon_{MC/SC}$, it is not certain what factor is causing the asymmetric increase. Therefore, it is difficult to determine the direction of asymmetric effect in the covariance of ε_{Gold} and $\varepsilon_{MC/SC}$.

5. Conclusion

The findings show that there is an asymmetry in the dynamic conditional correlation of different sized indices and gold returns, allowing us to reject the null hypothesis of no asymmetry in the dynamic conditional correlations. Additionally, we find that the dynamic conditional correlation of large-cap stock returns and gold returns only displays an asymmetric change following a negative shock in the returns of the large-cap stocks. The dynamic conditional correlation of mid/small-cap stock returns and gold returns, however, has an asymmetric change when there is a negative shock in either mid/small-cap stock or gold returns.

Another interesting finding is that the variance of gold returns does not appear to have asymmetry following a negative shock in gold returns, which is inconsistent with Baur's 2012 findings of asymmetric volatility in gold returns.

Since our results indicate an asymmetry in the dynamic conditional correlation of stock returns and gold returns, we highly recommend that investors should consider the asymmetric effect in their investment strategy, in order to avoid sub-optimal portfolio investment.

An interesting subject for future research is to investigate further what economic factors cause the asymmetry in the dynamic conditional correlation estimates. Moreover, future studies could research whether this asymmetry is the same for the stock indices of other countries. Similar studies using other commodities could also be of interest.

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7. Appendices

A. Descriptive statistics for the series and Jarque-Bera normality test on the returns

	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera	Probability
S&P500	0.000266	0.011563	-0.249162	12.16679	20005.59	0.000000
MSCI US Mid cap 450	0.000358	0.012552	-0.467925	11.66376	18050.76	0.000000
MSCI US Small cap 1750	0.000371	0.013001	-0.427548	10.07982	12088.68	0.000000
Gold Bullion LBM U$/Troy Ounce	0.000234	0.010145	-0.327591	10.69690	14164.54	0.000000

All series show the daily positive returns on average, but the smaller firm size index presents higher returns. It also comes with the higher volatility. Moreover, the zero value of a probability in the Jargue-Bera test indicates that all of the series' distribution is non-normal.

B. Description of each volatility model

Here a detailed description of each volatility model used in the thesis is given. It should be noted that the notation of the models may have been altered slightly from the papers in which they were published. This is done for ease of comparison between volatility models and to keep consistency with other parts of the thesis, which makes it easier for the reader to connect different parts and models within the thesis together.

Autoregressive conditional heteroscedasticity (ARCH)

The ARCH model was introduced by Engle in 1982. It allows for the estimation of nonconstant conditional one period variance based on the past, while having a constant unconditional variance. The conditional variance equation for an ARCH(p) process is as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2$$

Generalized ARCH (GARCH)

The GARCH model is an extension of the ARCH model. It was developed by Bollerslev in 1986. It allows past conditional variances to affect the current conditional variance by including them in the variance equation. The GARCH(p,q) process is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

Threshold GARCH (TGARCH)

This model was proposed by Glosten, Jagannathan and Runkle in 1993. By using a dummy variable, I_{t-k}, that takes the value of one when the residual of the y series is larger than zero and value of zero otherwise, the model allows negative and positive shocks in the residual to affect the variance differently. The TGARCH (p,q,r) process is defined as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{j=k}^{p} \gamma_k I_{t-k} u_{t-k}^2$$

Exponential GARCH (EGARCH)

In 1991 Nelson introduced the EGARCH model. It is expressed at the log of the variance, σ_t^2, rather than directly in terms of the variance as is done for most GARCH processes. The model differs a bit from the traditional GARCH process, most noticeably by having two terms that use a past value of a residual, u_t. One of these terms is a absolute value, which assures that there is asymmetry in the log of the conditional variance as long as the parameter γ is not equal to zero. The EGARCH(p,q,r) is expressed as follows:

$$Log\,(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \frac{|u_{t-i}|}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2) + \sum_{j=k}^{p} \gamma_k \frac{u_{t-k}}{\sigma_{t-k}}$$

Asymmetric power GARCH (APARCH)

The APARCH model was proposed by Ding, Granger and Engle in 1993. It allows for the estimation of the power parameter δ rather than imposing it. Furthermore it allows for asymmetry in the estimates through the estimation of the parameter γ_i. The APARCH model is presented as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i(|u_{t-i}| - \gamma_i u_{t-i}) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta$$

Here $|\gamma_i| \leq 1$ and there cannot be more asymmetric terms then there are ARCH(p) terms.

C. Detailed information on each series mean and variance equation

Dependent Variable: SP500RETURN			
Variable	**Coefficient**	**Prob.**	
C	0.000181	0.0868	
SP500RETURN(-12)	0.028509	0.0271	
SP500RETURN(-13)	0.046454	0.0005	
Variance Equation	**Coefficient**	**Prob.**	
ω	-0.263020	0.0000	
α1	-0.094574	0.0099	
α2	0.214735	0.0000	
β1	-0.234653	0.0000	
γ1	0.117842	0.0000	
γ2	0.981632	0.0000	
R-squared	0.000246	Mean dependent var	0.000272
Adjusted R-squared	-0.000106	S.D. dependent var	0.011572
S.E. of regression	0.011572	Akaike info criterion	-6.580423
Sum squared resid	0.760807	Schwarz criterion	-6.569901
Log likelihood	18710.56	Hannan-Quinn criter.	-6.576759
Durbin-Watson stat	2.135539		

Dependent Variable: MIDCAPRETURN			
Variable	**Coefficient**	**Prob.**	
C	0.000288	0.0107	
MIDCAPRETURN(-1)	0.074118	0.0000	
Variance Equation	**Coefficient**	**Prob.**	
ω	1.89E-06	0.0000	
α1	-0.065762	0.0004	
α2	0.074276	0.0002	
β1	0.899490	0.0000	
γ1	0.149301	0.0000	
R-squared	-0.005490	Mean dependent var	0.000357
Adjusted R-squared	-0.005667	S.D. dependent var	0.012553
S.E. of regression	0.012588	Akaike info criterion	-6.442728
Sum squared resid	0.903555	Schwarz criterion	-6.434569
Log likelihood	18381.66	Hannan-Quinn criter.	-6.439888
Durbin-Watson stat	2.142646		

Dependent Variable: SMALLCAPRETURN			
Variable	Coefficient	Prob.	
C	0.000408	0.0003	
SMALLCAPRETURN(-1)	0.087560	0.0000	
Variance Equation	**Coefficient**	**Prob.**	
ω	-0.011099	0.0000	
α1	0.110014	0.0000	
α2	-0.101931	0.0000	
β1	-0.175943	0.0000	
β2	0.173817	0.0000	
γ1	1.894254	0.0000	
γ2	-0.894788	0.0000	
R-squared	-0.009806	Mean dependent var	0.000370
Adjusted R-squared	-0.009983	S.D. dependent var	0.013002
S.E. of regression	0.013067	Akaike info criterion	-6.378819
Sum squared resid	0.973571	Schwarz criterion	-6.368328
Log likelihood	18201.39	Hannan-Quinn criter.	-6.375166
Durbin-Watson stat	2.196641		

Dependent Variable: GOLDRETURN			
Variable	Coefficient	Prob.	
C	5.37E-05	0.6254	
GOLDRETURN(-12)	-0.058778	0.0001	
Variance Equation	**Coefficient**	**Prob.**	
ω	-10.89809	0.0000	
α1	0.256159	0.0000	
α2	0.258917	0.0000	
α3	0.270155	0.0000	
α4	0.295778	0.0000	
α5	0.269943	0.0000	
α6	0.222673	0.0000	
α7	0.213191	0.0000	
α8	0.199020	0.0000	
α9	0.174998	0.0005	
R-squared	0.000454	Mean dependent var	0.000232
Adjusted R-squared	0.000278	S.D. dependent var	0.010154
S.E. of regression	0.010153	Akaike info criterion	-6.553823
Sum squared resid	0.585818	Schwarz criterion	-6.539795
Log likelihood	18641.24	Hannan-Quinn criter.	-6.548938
Durbin-Watson stat	2.001451		

D. the Q statistics and the corresponding probability values for those models

Ljung-Box test for autocorrelation in standardized residuals					
Series	**Nr of Lags**	**Q Statistic***	**P-value**	**Q Statistic****	**P-value**
Gold return	1	0.6202	0.431	1.8625	0.172
	5	3.9939	0.550	2.8248	0.727
	10	13.841	0.180	40.079	0.000
	15	18.221	0.251	112.92	0.000
	20	28.127	0.106	141.12	0.000
	25	36.164	0.069	237.74	0.000
S&P500 return	1	1.8478	0.174	0.0377	0.846
	5	9.8394	0.080	2.8205	0.728
	10	17.956	0.056	6.3390	0.786
	15	19.971	0.173	9.5977	0.844
	20	26.492	0.150	10.382	0.961
	25	29.309	0.251	11.272	0.992
Mid Cap return	1	0.0001	0.992	0.3642	0.546
	5	4.6583	0.459	2.7930	0.732
	10	9.4369	0.491	5.2749	0.872
	15	19.519	0.191	8.3950	0.907
	20	24.017	0.242	10.614	0.956
	25	26.369	0.388	13.087	0.975
Small Cap return	1	0.0860	0.769	0.1108	0.739
	5	1.4443	0.919	3.8794	0.567
	10	5.4303	0.861	6.0030	0.815
	15	18.718	0.227	7.7858	0.932
	20	22.103	0.335	10.859	0.950
	25	26.924	0.360	12.239	0.985
*Standardized Residual					
**Standardized Residual Squared					

E. Eviews code for the DCC and AG-DCC model

The code that we used is based on a code for an A-DCC model, which was retrieved from a thread called "Dynamic conditional correlation multivariate GARCH" and was posted in 2011. We then modified that code from a scalar model to a bivariate model.

```
'DCC Model by Cappiello et al (2006) Bivariate (A AG-DCC model without the G terms).

'Load workfile and measure length of series
series obscount = 1
scalar obslength = @sum(obscount)

'Specify the return series
series y1 = goldreturn
series y2 = smallcapreturn

'Specify the number of iterations in the MLE (Engle & Sheppard (2001) used just one iteration)
!itermle = 1000
```

```
'Specify the initial values of the parameters
coef(1) a11
coef(1) a12
coef(1) a22
coef(1) b11
coef(1) b12
coef(1) b22

'Values may vary depending on which will result in the highest Likelihood value
a11(1) = 0.35
a12(1) = 0.35
a22(1) = 0.35
b11(1) = 0.1
b12(1) = 0.1
b22(1) = 0.1

'Setting the sample
sample s0    @first+13 @last
sample s1    @first+14 @last
sample sf    @first+15 @last
sample sf_alt   @first+15 @last

'Initialization at sample s0
smpl s0

'Each return series is modeled with their respective GARCH specifications:
'Use Bollerslev-Wooldridge QML
'Standard errors

equation eq1.arch(9,0,asy=0, egarch,m=1000,c=1e-5,h) y1 c y1(-12) 'res_s11_1000 @ res_s11_1000
equation eq2.arch(2,2,asy=2, egarch,m=1000,c=1e-5,h) y2 c y2(-1) 'res_s22_1000 @ res_s22_1000

'Make residual series   -- This is Ut
eq1.makeresids e1
eq2.makeresids e2

'Make a garch series from the univariate estimates
eq1.makegarch h11
eq2.makegarch h22

'Normalizing the residuals from e to e* (named as "e1n" and "e2n")   -- Here we change from Ut to epsilon t.
series e1n = e1/h11^0.5
series e2n = e2/h22^0.5

'Make residual series for asymmetries in DCC model
series n1n = @recode(e1n<0,e1n*e1n,0)
series n2n = @recode(e2n<0,e2n*e2n,0)

'The Q_bar=E[en*en'] components and its sample equivalent
series qbar11 = @mean(e1n*e1n)
series qbar12 = @mean(e1n*e2n)
series qbar21 = @mean(e2n*e1n)
series qbar22 = @mean(e2n*e2n)

'The N_bar
series nbar11 = @mean(n1n*n1n)
series nbar12 = @mean(n1n*n2n)
series nbar21 = @mean(n2n*n1n)
series nbar22 = @mean(n2n*n2n)

'Initialize the elements of Qt for variance targeting
series q11 = @var(e1)
series q12 = @cov(e1,e2)
series q21 = @cov(e2,e1)
series q22 = @var(e2)

'*********************************************************************************************
```

```
'Declare a Loglikelihood object
logl dcc
dcc.append @logl logl

'The elements of matrix Qt
dcc.append  q11 = qbar11- ((a11(1)*qbar11 + a12(1)*qbar12)*a11(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a12(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b11(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b12(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a11(1)+e2n(-1)*a12(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b11(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b12(1))
dcc.append  q12 = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1))+ (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
dcc.append  q21 = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
dcc.append  q22 = qbar22- ((a12(1)*qbar11 + a22(1)*qbar12)*a12(1) + (a12(1)*qbar12 +a22(1)*qbar22)*a22(1)) - ((b12(1)*qbar11 + b22(1)*qbar12)*b12(1) + (b12(1)*qbar12 +b22(1)*qbar22)*b22(1)) + (a12(1)*e1n(-1) + a22(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b12(1)*q11 + b22(1)*q12(-1))*b12(1) + (b12(1)*q12(-1) +b22(1)*q22(-1))*b22(1))

'As input to detQQQ
dcc.append  q12n = (qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1))+ (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1)))/((abs(q11)^0.5)*(abs(q22)^0.5))
dcc.append  q21n = (qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1))+ (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))) /((abs(q22)^0.5)*(abs(q11)^0.5))

'Setting up the Loglikelihood function, assuming that resid ~ N(0,H)

'The Loglikelihood function is L' = -0.5*{summation from t=1 to T of ([log of determinant of inverse(diag[Qt])*Qt*inverse(diag[Qt])] + [en'*inverse(diag[Qt])*Qt*inverse(diag[Qt])*en])

'Taking the adjoint{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  detQQQ = 1 - q12n*q21n

'Tomando el adjoint{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  cofact11 =   1*1
dcc.append  cofact12 =(-1)*q21n
dcc.append  cofact21 =(-1)*q12n
dcc.append  cofact22 =   1*1

'Taking the inverse{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  invQQQ11 = cofact11/detQQQ
dcc.append  invQQQ12 = cofact12/detQQQ
dcc.append  invQQQ21 = cofact21/detQQQ
dcc.append  invQQQ22 = cofact22/detQQQ

'Taking the en'*inverse{inverse(diag[Qt])*Qt*inverse(diag[Qt])}*en
dcc.append  enQQQen11 = e1n*invQQQ11*e1n
dcc.append  enQQQen12 = e1n*invQQQ12*e2n
dcc.append  enQQQen21 = e2n*invQQQ21*e1n
dcc.append  enQQQen22 = e2n*invQQQ22*e2n

'Append the loglikelihood function
'Instead of log(detQQQ) use log(abs(detQQQ))
dcc.append logl = -0.5*(log(abs(detQQQ)) + (enQQQen11+enQQQen21+ enQQQen12+enQQQen22)  -(e1n^2 + e2n^2 + 2*e1n*e2n))

'Specifies the sample data where the estimation will be made
smpl sf
```

```
'Estimates the parameters now using BHHH algorithm
dcc.ml(b, showopts, m=!itermle, c=1e-5)

'A detQQQnpd = 0 indicates detQQQ is positive definite
series count = (detQQQ<=0)
scalar detQQQnpd = @sum(count)

'Display the estimated parameters
show dcc.output

'Forecast the q's by initializing them
series  q11f = 0
series  q12f = 0
series  q21f = 0
series  q22f = 0

'Specify the sample period to be forecasted
smpl sf
series  q11f = qbar11- ((a11(1)*qbar11 + a12(1)*qbar12)*a11(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a12(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b11(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b12(1)) + (a11(1)*e1n(-1) +
a12(1)*e2n(-1))*(e1n(-1)*a11(1)+e2n(-1)*a12(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b11(1) + (b11(1)*q12(-1)
+b12(1)*q22(-1))*b12(1))
series  q12f = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1))+ (a11(1)*e1n(-1) +
a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1)
+b12(1)*q22(-1))*b22(1))
series  q21f = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1))+ (a11(1)*e1n(-1) +
a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1)
+b12(1)*q22(-1))*b22(1))
series  q22f = qbar22- ((a12(1)*qbar11 + a22(1)*qbar12)*a12(1) + (a12(1)*qbar12 +a22(1)*qbar22)*a22(1)) -
((b12(1)*qbar11 + b22(1)*qbar12)*b12(1) + (b12(1)*qbar12 +b22(1)*qbar22)*b22(1)) + (a12(1)*e1n(-1) +
a22(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + ((b12(1)*q11 + b22(1)*q12(-1))*b12(1) + (b12(1)*q12(-1)
+b22(1)*q22(-1))*b22(1))

'To plot the time-varying conditional correlation
smpl sf_alt
series rt = q12f/(@sqr(q11f)*@sqr(q22f))
graph rtgraph.line rt
rtgraph.axis(1) range(-0.7,0.7)
show rtgraph

delete e1n e2n
delete q12n q21n
delete n1n n2n
delete qbar*
delete xbar*
delete cofact*
delete enqqqen*
delete invqqq*
delete enqqqen*
delete eigenvect*
delete invqqq*

show detqqqnpd

series eigmins
scalar eigmin



'AG-DCC Model by Cappiello et al (2006) Bivariate

'Load workfile and measure length of series
series obscount = 1
scalar obslength = @sum(obscount)
```

```
'Specify the return series
series y1 = goldreturn
series y2 = smallcapreturn

'Specify the number of iterations in the MLE (Engle & Sheppard (2001) used just one iteration)
!itermle = 1000

'Specify the initial values of the parameters
coef(1) a11
coef(1) a12
coef(1) a22
coef(1) b11
coef(1) b12
coef(1) b22
coef(1) g11
coef(1) g12
coef(1) g22

'Values may vary depending on which will result in the highest Likelihood value
a11(1) = 0.35
a12(1) = 0.35
a22(1) = 0.35
b11(1) = 0.1
b12(1) = 0.1
b22(1) = 0.1
g11(1) = 0.1
g12(1) = 0.1
g22(1) = 0.1


'Setting the sample
sample s0    @first+13 @last
sample s1    @first+14 @last
sample sf    @first+15 @last
sample sf_alt   @first+15 @last

'Initialization at sample s0
smpl s0

'Each return series is modeled with their respective GARCH specifications:
'Use Bollerslev-Wooldridge QML
'Standard errors

equation eq1.arch(9,0,asy=0, egarch,m=1000,c=1e-5,h) y1 c y1(-12) 'res_s11_1000 @ res_s11_1000
equation eq2.arch(2,2,asy=2, egarch,m=1000,c=1e-5,h) y2 c y2(-1) 'res_s22_1000 @ res_s22_1000

'Make residual series   -- This is Ut
eq1.makeresids e1
eq2.makeresids e2

'Make a garch series from the univariate estimates
eq1.makegarch h11
eq2.makegarch h22

'Normalizing the residuals from e to e* (named as "e1n" and "e2n")   -- Here we change from Ut to epsilon t.
series e1n = e1/h11^0.5
series e2n = e2/h22^0.5

'Make residual series for asymmetries in DCC model
series n1n = @recode(e1n<0,e1n,0)
series n2n = @recode(e2n<0,e2n,0)

'The Q_bar=E[en*en'] components and its sample equivalent
series qbar11 = @mean(e1n*e1n)
series qbar12 = @mean(e1n*e2n)
series qbar21 = @mean(e2n*e1n)
series qbar22 = @mean(e2n*e2n)
```

```
'The N_bar
series nbar11 = @mean(n1n*n1n)
series nbar12 = @mean(n1n*n2n)
series nbar21 = @mean(n2n*n1n)
series nbar22 = @mean(n2n*n2n)

'Initialize the elements of Qt for variance targeting
series q11 = @var(e1)
series q12 = @cov(e1,e2)
series q21 = @cov(e2,e1)
series q22 = @var(e2)

'**********************************************************************************************

'Declare a Loglikelihood object
logl dcc
dcc.append @logl logl

'The elements of matrix Qt
dcc.append  q11 = qbar11- ((a11(1)*qbar11 + a12(1)*qbar12)*a11(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a12(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b11(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b12(1)) - ((g11(1)*nbar11 + g12(1)*nbar12)*g11(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g12(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a11(1)+e2n(-1)*a12(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g11(1)+n2n(-1)*g12(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b11(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b12(1))
dcc.append  q12 = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 + g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
dcc.append  q21 = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 + g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
dcc.append  q22 = qbar22- ((a12(1)*qbar11 + a22(1)*qbar12)*a12(1) + (a12(1)*qbar12 +a22(1)*qbar22)*a22(1)) - ((b12(1)*qbar11 + b22(1)*qbar12)*b12(1) + (b12(1)*qbar12 +b22(1)*qbar22)*b22(1)) - ((g12(1)*nbar11 + g22(1)*nbar12)*g12(1) + (g12(1)*nbar12 +g22(1)*nbar22)*g22(1)) + (a12(1)*e1n(-1) + a22(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + (g12(1)*n1n(-1) + g22(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b12(1)*q11 + b22(1)*q12(-1))*b12(1) + (b12(1)*q12(-1) +b22(1)*q22(-1))*b22(1))

'As input to detQQQ
dcc.append  q12n = (qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 + g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1)))/((abs(q11)^0.5)*(abs(q22)^0.5))
dcc.append  q21n = (qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) - ((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 + g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1)))/((abs(q22)^0.5)*(abs(q11)^0.5))

'Setting up the Loglikelihood function, assuming that resid ~ N(0,H)

'The Loglikelihood function is L' = -0.5*{summation from t=1 to T of ([log of determinant of inverse(diag[Qt])*Qt*inverse(diag[Qt])] + [en'*inverse(diag[Qt])*Qt*inverse(diag[Qt])*en])

'Taking the adjoint{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  detQQQ = 1 - q12n*q21n

'Tomando el adjoint{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  cofact11 =   1*1
dcc.append  cofact12 =(-1)*q21n
dcc.append  cofact21 =(-1)*q12n
```

```
dcc.append  cofact22 =   1*1

'Taking the inverse{inverse(diag[Qt])*Qt*inverse(diag[Qt])}
dcc.append  invQQQ11 = cofact11/detQQQ
dcc.append  invQQQ12 = cofact12/detQQQ
dcc.append  invQQQ21 = cofact21/detQQQ
dcc.append  invQQQ22 = cofact22/detQQQ

'Taking the en'*inverse{inverse(diag[Qt])*Qt*inverse(diag[Qt])}*en
dcc.append  enQQQen11 = e1n*invQQQ11*e1n
dcc.append  enQQQen12 = e1n*invQQQ12*e2n
dcc.append  enQQQen21 = e2n*invQQQ21*e1n
dcc.append  enQQQen22 = e2n*invQQQ22*e2n

'Append the loglikelihood function
'Instead of log(detQQQ) use log(abs(detQQQ))
dcc.append logl = -0.5*(log(abs(detQQQ)) + (enQQQen11+enQQQen21+ enQQQen12+enQQQen22)  -(e1n^2 +
e2n^2 + 2*e1n*e2n))

'Specifies the sample data where the estimation will be made
smpl sf

'Estimates the parameters now using BHHH algorithm
dcc.ml(b, showopts, m=!itermle, c=1e-5)

'A detQQQnpd = 0 indicates detQQQ is positive definite
series count = (detQQQ<=0)
scalar detQQQnpd = @sum(count)

'Display the estimated parameters
show dcc.output

'Forecast the q's by initializing them
series  q11f = 0
series  q12f = 0
series  q21f = 0
series  q22f = 0

'Specify the sample period to be forecasted
smpl sf
series  q11f = qbar11- ((a11(1)*qbar11 + a12(1)*qbar12)*a11(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a12(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b11(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b12(1)) - ((g11(1)*nbar11 +
g12(1)*nbar12)*g11(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g12(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-
1)*a11(1)+e2n(-1)*a12(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g11(1)+n2n(-1)*g12(1)) + ((b11(1)*q11(-
1) + b12(1)*q12(-1))*b11(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b12(1))
series  q12f = qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 +
g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-
1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-
1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
series  q21f =qbar12- ((a11(1)*qbar11 + a12(1)*qbar12)*a12(1) + (a11(1)*qbar12 +a12(1)*qbar22)*a22(1)) -
((b11(1)*qbar11 + b12(1)*qbar12)*b12(1) + (b11(1)*qbar12 +b12(1)*qbar22)*b22(1)) - ((g11(1)*nbar11 +
g12(1)*nbar12)*g12(1) + (g11(1)*nbar12 +g12(1)*nbar22)*g22(1)) + (a11(1)*e1n(-1) + a12(1)*e2n(-1))*(e1n(-
1)*a12(1)+e2n(-1)*a22(1)) + (g11(1)*n1n(-1) + g12(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b11(1)*q11(-
1) + b12(1)*q12(-1))*b12(1) + (b11(1)*q12(-1) +b12(1)*q22(-1))*b22(1))
series  q22f = qbar22- ((a12(1)*qbar11 + a22(1)*qbar12)*a12(1) + (a12(1)*qbar12 +a22(1)*qbar22)*a22(1)) -
((b12(1)*qbar11 + b22(1)*qbar12)*b12(1) + (b12(1)*qbar12 +b22(1)*qbar22)*b22(1)) - ((g12(1)*nbar11 +
g22(1)*nbar12)*g12(1) + (g12(1)*nbar12 +g22(1)*nbar22)*g22(1)) + (a12(1)*e1n(-1) + a22(1)*e2n(-1))*(e1n(-
1)*a12(1)+e2n(-1)*a22(1)) + (g12(1)*n1n(-1) + g22(1)*n2n(-1))*(n1n(-1)*g12(1)+n2n(-1)*g22(1)) + ((b12(1)*q11 +
b22(1)*q12(-1))*b12(1) + (b12(1)*q12(-1) +b22(1)*q22(-1))*b22(1))

'To plot the time-varying conditional correlation
smpl sf_alt
series rt = q12f/(@sqr(q11f)*@sqr(q22f))
graph rtgraph.line rt
rtgraph.axis(1) range(-0.7,0.7)
show rtgraph
```

```
delete e1n e2n
delete q12n q21n
delete n1n n2n
delete qbar*
delete xbar*
delete cofact*
delete enqqqen*
delete invqqq*
delete enqqqen*
delete eigenvect*
delete invqqq*

show detqqqnpd
```