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A Chirally Symmetric Technifermion Sector

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Abstract

This thesis considers the low-energy effective field theory of a new technicolor extension. The extension preserves the Standard Model Higgs boson and introduces a chirally symmetric technifermion sector, in the framework of the gauged linear sigma model. In the nearly conformal limit, the considered extension leads to a common origin of the Higgs- and Technisigma vacuum expectation values, related to the technifermion condensate. In addition to this, the model stays within allowed boundaries of electroweak precision tests for the main part of the parameter space.

The results presented in this thesis are a reproduction of the paper "Chiral-Symmetric Technicolor with Standard Model Higgs boson", published 2013 by R. Pasechnik et al. [14]. The thesis may serve as a guide to the results obtained in the original paper, as well as being an audit of the decisions and assumptions made there.

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Contents

1 Introduction

This thesis investigates the low-energy effective field theory of a possible technicolor-extension to the Standard Model. In agreement with chiral perturbation theory in QCD, the model considered is chirally symmetric and retains the Standard Model Higgs.

When constructing an extension to the Standard Model, it is helpful to understand the features of a field theory itself. The equations of motion of a field theory is obtained from a so-called Lagrangian density, which format has certain requirements regarding gauge invariance and dimensionality. The Lagrangian density of the Standard Model is constructed such that it is locally gauge invariant under three internal symmetry groups, $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$. This section provides an introduction to the concepts of Lagrangian densities, gauge invariance, spontaneous symmetry breaking, chiral perturbation theory, the suggested extension and electroweak precision tests. A reader unfamiliar with group theory, is recommended to begin with Appendix A, containing the basics of Lie groups and Lie algebras.

1.1 The Lagrangian Density and the Equations of Motion

Lagrangians were originally introduced in classical mechanics, as an alternative method of obtaining the equations of motion for a system. Instead of calculating the resulting force at each moment of time, one can obtain the equations of motion for a system by simply identifying two scalar properties: the kinetic energy and the potential energy. The Lagrangian is defined as the kinetic energy minus the potential energy, $L(\mathbf{r},\dot{\mathbf{r}}) \equiv \frac{1}{2}m\mathbf{r}^2 - V(\mathbf{r})$, and yields the equations of motion when inserted into the Euler-Lagrange equation. The Euler-Lagrange equation is in its turn obtained by requiring that the particle trajectory yields an extremum of the action S [2].

Similar relations apply to the Lagrangian of a field theory, such as the Lagrangian of the Standard Model. The Lagrangian of a field theory is formally called a Lagrangian *density*, $\mathcal{L}$, but is referred to as simply a Lagrangian for the rest of this thesis. The Lagrangian of a re-normalizable theory is a function of a field and its first-order derivative, where the field is a function of space-time $\phi(x^\mu)$, $x^\mu = (x_o, \mathbf{x})$. The action S is then given by [3] (note however, that for a quantum field theory, in opposition to classical mechanics, there is more than one possible path):

$$S = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x)). \tag{1}$$

In order to keep the action dimensionless, the Lagrangian must have mass dimension 4, as apparent from equation 1. Scalar fields ϕ, vector fields B^μ, masses and the coupling constants μ, have dimension 1, while spinor fields ψ have dimension 3/2 and the coupling constants λ have dimension 0. Hence, a Lagrangian can only be combined in a limited amount of ways. Allowed combinations are of the sort:

$$\mu\phi^3, \;\; \mu^2\phi^2, \;\; \lambda\phi^4, \;\; \lambda^2\phi^2, \;\; \bar{\psi}\gamma^\mu\psi B_\mu, \;\; g\bar{\psi}\psi\phi, \;\; g\bar{\psi}\psi i\gamma^5\phi \quad \text{etc.} \tag{2}$$

where the the two last terms are the so-called Yukawa terms for a scalar field and a pseudoscalar field, respectively. Besides describing the interactions between (pseudo)scalar- and spinor fields, a Yukawa term also gives rise to the mass of fermions, as shown in Appendix C. The fifth term describes the interaction between fermions and gauge bosons, where the gauge field is introduced in the covariant derivative of the kinetic term, as shown in section 1.2.

The combinations of equation 2 will be used when constructing the Lagrangian of the new extension. Besides fulfilling the requirement of mass dimension 4, such terms must be invariant under the chosen symmetry.

1.2 Gauge Invariance

The Lagrangian of the Standard Model is invariant under a set of transformations U:

$$\psi_i(x) \to U_{ij}\psi_j(x),$$

where U is a group element in a Lie group $U(1)_Y$, $SU(2)_W$ or $SU(3)_C$ (where C denote color, W denote weak interactions and Y denote hypercharge). In the remainder of this thesis, the indices will be suppressed.

A group element U in one of these Lie groups is given by equation 110 (Appendix A), where ξ_i is the parameter and t_i are the generators of the group. The generators for SU(2) and SU(3) must be traceless Hermitian matrices, since a special unitary matrix requires the determinant to be one: $\det U = \det e^H = e^{TrH} = e^0 = 1$, where H is a traceless Hermitian matrix [4]. There are three generators of SU(2):

$$t_i = \frac{\tau_i}{2}, \quad [t_i, t_j] = \epsilon_{ijk} t_k \tag{3}$$

where τ_i are the three Pauli matrices. Hence, equation 110 tells us that each Lie Algebra element of SU(2) is a linear combination of three rotation matrices. Similarly, a group element in SU(3) is generated by:

$$t_a = \frac{\lambda_a}{2}, \quad [t_a, t_b] = \epsilon_{abc} t_c \tag{4}$$

where λ_a are the eight lambda matrices, also referred to as the Gell-Mann matrices. For U(1), on the other hand, there is only one generator:

$$t = \frac{Y}{2}, \tag{5}$$

where Y is the hypercharge.

The transformations are, for historical reasons, referred to as gauge transformations. A gauge transformation $\psi \to U\psi$ is *global* if the parameter x is simply a number, $\psi \to e^{-ix}\psi$, and *local* if the parameter if a function of space and time, $\psi \to e^{-ix(\vec{x_i},t)}\psi$. Since the Lagrangian of the Standard Model contains derivatives, a local gauge transformation, $U = e^{-ix(\vec{x_i},t)}$, will yield an extra term $\psi\partial^\mu U$:

$$\partial^\mu \psi \to U\partial^\mu \psi + \psi \partial^\mu U \quad \text{as} \quad \psi \to U\psi. \tag{6}$$

The theory can be made locally gauge invariant by introducing a so-called covariant derivative D^μ, defined such that the extra term is cancelled:

$$D^\mu \psi \to U D^\mu \psi, \tag{7}$$

To fulfill this requirement, the covariant derivative contains a new vector field N^a_μ, which transforms according to:

$$D^\mu = \partial^\mu - i g_n t_a N^a_\mu, \qquad N_\mu \to N'_\mu = -\frac{i}{g_n}(\partial_\mu U)U^\dagger + U N_\mu U^\dagger, \tag{8}$$

where N_μ denotes the product of the generator and the vector field, $t_a N^a_\mu$.

Our requirement of a locally gauge invariant theory has lead to the introduction of gauge fields (vector fields), where each field has a corresponding gauge boson. The eight generators of SU(3) correspond to the eight gluon fields of the Standard Model, the three generators of SU(2), correspond to the three gauge fields W^a_μ (a = 1, 2, 3), and the one generator of U(1) corresponds to the gauge field B_μ. As explained in Appendix B, the $W^3_\mu \equiv W^0_\mu$- and the B_μ-field mixes to the familiar gauge bosons Z and γ.

By using equation 8 and equations 3, 4 and 5, we may construct the complete covariant derivative of the Standard Model:

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\tau_i}{2} W_\mu^i - ig_3 \frac{\lambda_a}{2} G_\mu^a. \tag{9}$$

1.3 Spontaneous Symmetry Breaking

In the Standard Model, the massterm for gauge bosons, $\frac{1}{2} m_B^2 B_\mu B^\mu$, is not invariant under the field transformation in equation 8. Similarly, the fermionic massterms breaks the gauge invariance since the right-handed fields transform as singlets under $SU(2)_W$, while the left-handed fields transform as doublets. To maintain the local gauge invariance of the Lagrangian, all particles must therefore be considered massless. Masses are instead introduced via the so-called spontaneous symmetry breaking mechanism, also referred to as the Higgs Mechanism.

In this subsection we will demonstrate the symmetry breaking of a complex scalar field, $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$, under a U(1) symmetry. The Higgs field is in reality assigned to an SU(2) doublet, but the graphical representation of the U(1) scenario is superior for demonstrational purposes, such as this.

By including all scalar terms with mass dimension 4, that are invariant under the U(1) symmetry, the Lagrangian becomes:

$$\mathcal{L} = T - V = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \tag{10}$$

where the potential is given by the last two terms:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \tag{11}$$

If the potential is plotted as a function of ϕ_1 and ϕ_2 for a negative μ^2, it will obtain the shape of a mexican hat, as shown in figure 1. It is clear simply by looking at the plot, that the theory exhibits a global U(1) symmetry, since a phase shift correspond to rotations about the z-axis. By choosing a minimum value, somewhere along the degenerate circular minima, the global symmetry is broken. Excitations about the minimum point correspond to the physical spectrum, where excitations in the radial direction correspond to massive particle, while excitations in the bottom of the well does not cost any energy and therefore correspond to massless particles. The massless particles that do not annihilate the vacuum are referred to as Goldstone bosons, and according to the Goldstone Theorem there is one such particle for every broken generator of the symmetry. The Goldstone bosons does not appear in the physical spectrum. Instead, their degree of freedom is given to the gauge bosons, which by that obtain three polarization states, making them massive[1]. This phenomena is often referred to as the gauge bosons gaining their mass from eating the Goldstone bosons [4]. The fermions, on the other hand, gain their mass via the introduction of a Yukawa term, as shown in Appendix C.

[1] A massive vector boson has three polarization states, two transverse states, $m_s = \pm 1$ and one longitudinal state $m_s = 0$, while a massless vector boson only posses two transverse polarization states. This is due to the fact that massless particles travels at the speed of light, which means that its helicity is Lorentz invariant. Helicity refers to the direction of motion of a particle with respect to the direction of its spin, and it being Lorentz invariant refers to us not being able to reverse its direction of motion by travelling faster than it does. The absence of a third helicity state must correspond to the absence of $m_s = 0$ [7].

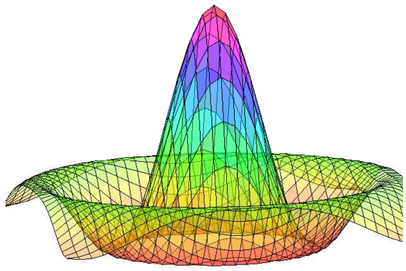
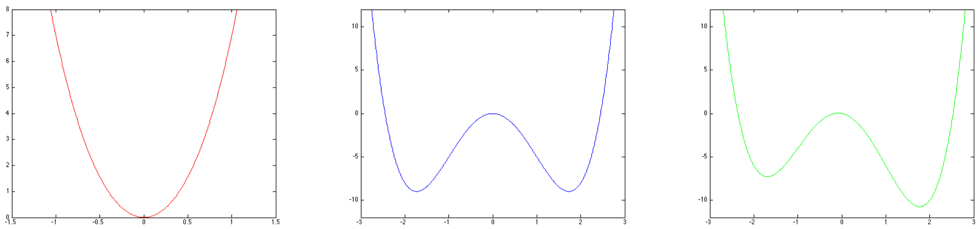


Figure 1: The mexican hat potential

The reason for choosing a negative μ^2 has now become clear. With a positive μ^2, we would only have massive excitations, as shown in figure 2a. Such a scenario would not provide masses for the gauge bosons, but instead introduce a whole spectrum of new, unwanted, massive particles.

Figure 2c displays the scenario of a linear term added to the potential. The slight tilt caused by the odd linear term, means that the Goldstone bosons will gain a small mass. The particles are thereby referred to as pseudo-Goldstone bosons, and will occur in the physical spectrum instead of giving their degree of freedom to the gauge bosons. The tilted potential is present in chiral perturbation theory, as covered in the following section. A linear term will also occur in the extension considered in this thesis. However, in the considered extension the linear term is globally invariant while in chiral perturbation theory, it is not. The considered extension therefore exhibits spontaneous symmetry breaking, while the chiral symmetry in QCD is broken explicitly.



(a) $y = 6x^2 + x^4$ (b) $y = -6x^2 + x^4$ (c) $y = -6x^2 + x^4 - x$

Figure 2: The potential part of the scalar Lagrangian for different signs of the μ-term (2a positive, 2b negative, 2c negative, and with a linear term).

1.4 Chiral Perturbation Theory in QCD

The incapacity of calculating QCD in the low energy limit, where the coupling constant enters non-perturbative magnitudes, lead to the development of chiral perturbation theory (ChPT). ChPT is a low-energy effective theory which considers the approximate chiral symmetry experienced by the lighest quarks, since their masses are negligible in relation to the confinement scale Λ_{QCD}. The technicolor extension considered in this thesis, which has its confinement scale at an even higher energy, will borrow this phenomenology. Note however that the chiral symmetry of the considered extension is an *exact* symmetry, which means that it will be broken spontaneously, instead of explicitly, as in ChPT.

As will be shown shortly, the chiral symmetry in QCD can only be considered an exact symmetry if quarks were massless. However, since the masses of the light quarks are negligible with respect to the energy scale of QCD, the linear term may be treated as a perturbation of an exact symmetry. For it to be considered a perturbation, we may only include the two, or three lightest quarks. In the technicolor extension treated in this paper, we have chosen to consider the two-flavor case, $q = u, d$.

That the chiral symmetry is an exact symmetry only in the massless scenario, can be seen from the QCD Lagrangian. The free-particle Lagrangian for quarks, or any other fermion for that matter, is given by the Dirac Lagrangian (constructed such that it yields the Dirac equation when inserted into the Euler-Langrange equation):

$$\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - \mathcal{M})q.$$

where the wave function q and the mass matrix $\mathcal{M}$, in the two flavor case, is given by:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{and} \quad \mathscr{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix},$$

and where the covariant derivative D_μ is introduced to preserve the local gauge invariance. By separating the quark fields into their left- and right handed parts[2], the Lagrangian becomes:

$$\mathcal{L} = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - \bar{q}_R \mathscr{M} q_L - \bar{q}_L \mathscr{M} q_R \tag{12}$$

where $\not{D} = \gamma^\mu D_\mu$. Besides being locally gauge invariant under rotations in color space, the two first terms of the Lagrangian also exhibit a global *chiral* symmetry. The massless Lagrangian remains invariant under separate transformations of left- and righthanded quarks; or in other words: it remains invariant under the set of linear transformations, g_L and g_R, belonging to the group $SU(2)_L \otimes SU(2)_R$:

$$\begin{aligned} q_R &\to g_R q_R, \quad g_R \in SU(2)_R \\ q_L &\to g_L q_L, \quad g_L \in SU(2)_L. \end{aligned}$$

The invariance of the first two terms of the Lagrangian is due to them only containing spinors of the same handedness. One of the spinors in each term transforms as the complex conjugate of the other. The infinitesimal rotations will therefore cancel each other out, $g_L g_L^\dagger = 1$. In the mass terms, on the other hand, the transformations do not cancel each other, since $g_L g_R^\dagger \neq 1$. Hence, the chiral symmetry in QCD is only exact in the massless scenario. Since the quarks are not massless, the chiral symmetry is explicitly broken. An explicitly broken symmetry refers to the fact that the Lagrangian contains terms which makes it non-invariant under the considered symmetry transformation.

To be precise, the massless Lagrangian is invariant under $SU(2)_L \otimes SU(2)_R \otimes U(1)_V \otimes U(1)_A$ [8]. However, we only need to consider $SU(2)_L \otimes SU(2)_R$, since $U(1)_V$ determines baryon number (not applicable for mesons), and since $U(1)_A$ has an anomaly [9].

At a certain energy scale ($\sim \Lambda_{\text{QCD}}$), the global chiral symmetry in QCD, $SU(2)_L \otimes SU(2)_R$, spontaneously breaks down to its vector subgroup:

$$SU(2)_L \otimes SU(2)_R \to SU(2)_{V=R+L}.$$

The unbroken symmetry is constructed from the subset of generators that annihilate the ground state. This is easily seen by requiring the vacuum to be invariant under linear transformations generated by T_i:

$$|0\rangle \to e^{i\epsilon(x)^i T_i} |0\rangle \simeq (\mathbb{1} + \epsilon(x)^a T_a) |0\rangle = |0\rangle,$$

[2] Using that

$$\bar{q}q = \bar{q}(P_L^2 + P_R^2)q = \bar{q}P_L P_L q + \bar{q}P_R P_R q = \bar{q}_R q_L + \bar{q}_L q_R$$

$$\text{and} \quad \begin{aligned} \bar{q}\gamma^\mu q &= \bar{q}(P_L^2 + P_R^2)\gamma^\mu(P_L^2 + P_R^2)q = \bar{q}P_L\gamma^\mu P_L q + \bar{q}P_R\gamma^\mu P_L q \\ &+ \bar{q}P_L\gamma^\mu P_R q + \bar{q}P_R\gamma^\mu P_R q = \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R \end{aligned}$$

where P_L and P_R are projection operators.

which is a condition only fulfilled if $T_a |0\rangle = 0$. By an equivalent argument, the generators of the broken subgroup (in our case the coset $SU(2)_{A=R-L}$), do not annihilate the ground state, $T_i |0\rangle \neq 0$.

There are several ways of explaining why the vector subgroup remains invariant under the chosen vacuum, while the axial vector subgroup does not. One version is to consider the conserved current and charge operators of the quantized version of Noether's theorem. Noether's theorem states that each continuous symmetry has a corresponding conserved charge. The charge is defined as the space integral over the zeroth component of the current:

$$Q^a(t) = \int d^3x J_0^a(\vec{x}, t)$$

and is time-independent if the current is conserved. The conserved current is defined as:

$$J^{\mu,a} = \frac{\partial \delta \mathcal{L}}{\partial \partial_\mu \Theta_a}, \quad \partial_\mu J^{\mu,a} = 0. \tag{13}$$

and the conserved charge, Q, is in fact the *generator* of the infinitesimal symmetry transformation [6]. It is important to emphasize that the symmetry transformation is infinitesimal. The charge operators Q shares the same Lie algebra as the generators of the global symmetry [10]:

$$[Q_L^a, Q_L^b] = i f_{abc} Q_L^c, \quad [Q_R^a, Q_R^b] = i f_{abc} Q_R^c$$
$$[Q_R^a, Q_L^b] = 0, \tag{14}$$

which means that the two Lie groups (the one generated by the charge operators, and the one generated by the generators of the global symmetry) are locally isomorphic, i.e. they may be considered identical under infinitesimal transformations about the identity. Further, we may investigate the commutator relations for Q_V^a and Q_A^a:

$$[Q_V^a, Q_V^b] = [Q_R^a + Q_L^a, Q_R^b + Q_L^b] = [Q_R^a, Q_R^b] + [Q_L^a, Q_L^b]$$
$$= i f_{abc} Q_R^c + i f_{abc} Q_L^c = i f_{abc} Q_V^c$$

and

$$[Q_A^a, Q_A^b] = [Q_R^a - Q_L^a, Q_R^b - Q_L^b] = [Q_R^a, Q_R^b] + [Q_L^a, Q_L^b] = i f_{abc} Q_V^c$$

where we see that the commutator of two axial vectors is not an axial vector. Since this kind of "parity doubling" (opposite-parity states with equal spin value) does not occur in the observed spectrum, we conclude that the axial generators must be the ones broken:

$$Q_A^a |0\rangle \neq 0, \ \ Q_V^a |0\rangle = 0.$$

Also, since the charge operator is constant in time (invariant), it commutes with the Hamiltonian, which means that the Goldstone theorem applies. The broken axial charge generators may therefore be considered to be the pseudo-Goldstone bosons.

The origin of the subscripts $V = R + L$ and $A = R - L$ can be shown from the conserved currents related to the QCD Lagrangian under an infinitesimal transformation:

$$\delta \mathcal{L}^0 = \bar{q}_R \left(\sum_a \partial_\mu \Theta_a^R \frac{\tau_a}{2} + \partial_\mu \Theta^R \right) \gamma^\mu q_R + \bar{q}_L \left(\sum_a \partial_\mu \Theta_a^L \frac{\tau_a}{2} + \partial_\mu \Theta^L \right) \gamma^\mu q_L.$$

Using equation 13, the corresponding conserved currents become:

$$L^{\mu,a} = \frac{\partial \delta \mathcal{L}^0}{\partial \partial_\mu \Theta_a^L} = \bar{q}_L \gamma^\mu \frac{\tau^a}{2} q_L, \quad R^{\mu,a} = \frac{\partial \delta \mathcal{L}^0}{\partial \partial_\mu \Theta_a^R} = \bar{q}_R \gamma^\mu \frac{\tau^a}{2} q_R,$$

which can be re-expressed in linear combinations defined as V and A:

$$V^{\mu,a} = R^{\mu,a} + L^{\mu,a} = \bar{q}\gamma^\mu \frac{\tau^a}{2} q, \quad A^{\mu,i} = R^{\mu,a} - L^{\mu,a} = \bar{q}\gamma^\mu \gamma^5 \frac{\tau^a}{2} q.$$

It is now apparent that the currents are named after how they transform under parity [10].

The chiral symmetry is explicitly broken via the introduction of a linear term in the scalar Lagrangian, and each broken generator correspond to a massive pseudo-Goldstone boson. In the two-flavor case, the three broken (axial) generators correspond to the three pions, $\pi^\pm$, π^0. In the three-flavor case, the eight broken (axial) generators correspond to the three pions, the eta, η, and the four kaons, K^0, $\bar{K^0}$, $K^\pm$ [9].

The spontaneous chiral symmetry breaking is related to a non-zero quark condensate:

$$\langle q\bar{q} \rangle \neq 0.$$

I will not include the details here, since it is a vast and complicated subject, but it can be shown that a condensate does indeed vanish whenever the ground state is invariant under chiral transformations [11]. The condensate can therefore be viewed as a the source of the symmetry breaking, which is a reasoning we will return to in the constructed extension.

1.5 The Gauged Linear Techni-Sigma Model

In analogy with QCD, the global chiral symmetry of the two lightest techniquarks $(\widetilde{U}, \widetilde{D})$, is spontaneously broken down to its vector subgroup: $SU(2)_L \otimes SU(2)_R \to SU(2)_V$[3]. The chosen scalar field responsible for the chiral symmetry breaking is the field used in the linear sigma model:

$$\Sigma = \frac{1}{2}\left(S + i\tau_a P_a\right) \tag{15}$$

where the sigma field S can be imagined as an expansion in the radial direction, in agreement with the Higgs boson, and where the three π-fields correspond to the three pseudo-Goldstone bosons of the theory.

After the chiral symmetry breaking, the remaining subgroup $SU(2)_V$ is gauged (i.e. made into a local symmetry), and identified with the Standard Model symmetry group $SU(2)_W$. That is, instead of introducing new gauge bosons, corresponding to the new symmetry groups required by a local gauge invariance, we choose the new gauge bosons to be the ones of the Standard Model $SU(2)_W$-group. Note that the identification is only valid in energies below the symmetry breaking-scale.

In our case, the identification $SU(2)_V = SU(2)_W$, leads to the technifermions having the same interactions as the fermions of the Standard Model, with exception for there being no distinction between left- and righthanded fields, so-called *vector-like* interactions. Since the technifermions couples to the gauge bosons of the Standard Model, they will contribute to their self-energy corrections. The contribution provides an opportunity to test whether our new model is plausible or not, as explained in section 1.6.

1.5.1 The CSTC Lagrangian

We will now begin constructing the Lagrangian of our chirally symmetric techni-color (CSTC) extension. The scalar Lagrangian responsible for the the chiral symmetry breaking, consists of all gauge invariant terms with dimension 4 that can be constructed with the Σ-field, namely:

$$\mathcal{L} = (\partial_\mu \Sigma)^\dagger (\partial^\mu \Sigma) - \mu^2 \Sigma^\dagger \Sigma - \lambda (\Sigma^\dagger \Sigma)^2$$

[3] We also assume the technifermion sector to be confined under $SU(3)_{TC}$, in analogy with $SU(3)_c$ in QCD.

$$= \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2} D_\mu P D^\mu P + \frac{1}{2}\mu_s^2 (S^2 + P^2) - \frac{1}{4}\lambda_{TC}(S^2 + P^2)^2 \tag{16}$$

using that

$$|\Sigma|^2 = \Sigma_{ij}\Sigma_{ji}^\dagger = \mathrm{tr}(\Sigma\Sigma^\dagger) = \frac{1}{4}\begin{pmatrix} S + iP^3 & iP^1 + P^2 \\ -iP^1 - P^2 & S - iP^3 \end{pmatrix}\begin{pmatrix} S - iP^3 & -iP^1 - P^2 \\ -iP^1 + P^2 & S + iP^3 \end{pmatrix}$$

$$= \frac{1}{4}\mathrm{tr}\begin{pmatrix} S^2 + (P^1)^2 + (P^2)^2 + (P^3)^2 & ... \\ ... & S^2 + (P^1)^2 + (P^2)^2 + (P^3)^2 \end{pmatrix} = \frac{1}{2}(S^2 + P^2).$$

and where the covariant derivative of the pion field is given by:

$$D_\mu P_a = \partial_\mu P_a + g\epsilon_{abc} W_\mu^b P_c, \tag{17}$$

We may include the SM Higgs field in the same scalar Lagrangian, in addition to all allowed mix terms between the Higgs- and the S- and P-fields. The potential part of the Lagrangian then becomes:

$$\mathcal{L}_{U,self} = \frac{1}{2}\mu_s^2(S^2 + P^2) + \frac{1}{2}\mu_H^2 \mathcal{H}^\dagger\mathcal{H} - \frac{1}{4}\lambda_{TC}(S^2 + P^2)^2 - \lambda_H(\mathcal{H}^\dagger\mathcal{H})^2 + \lambda\mathcal{H}^\dagger\mathcal{H}(S^2 + P^2). \tag{18}$$

Since the technifermions in our extension interact under the same symmetry group as the ordinary fermions of the Standard Model, they will possess the same kind of kinetic term:

$$\mathcal{L}_{kin} = i\bar{\widetilde{Q}}\gamma^\mu D_\mu \widetilde{Q}, \tag{19}$$

where D_μ is the covariant derivative of the Standard Model, as introduced in equation 9. The technifermions will however not gain their masses via the Higgs mechanism, since such a Yukawa term is not allowed. Both the left- and the right-handed technifermions are doublets under $SU(2)_W$, which means that they are written into one single Yukawa term $g_{\mathrm{TC}}\bar{\widetilde{Q}}\mathcal{H}\widetilde{Q}$, which would not have mass dimension 4. Instead, the Yukawa term includes the Σ-field:

$$\mathcal{L}_Y = -g_{\mathrm{TC}}\bar{\widetilde{Q}}(S + i\gamma_5\tau_a P_a)\widetilde{Q}, \tag{20}$$

which means that the technifermions gain their mass via the breaking of the chiral symmetry. The first term of equation 20 is a scalar Yukawa term $g\psi\phi\psi$, and the second a pseudoscalar Yukawa term $g\psi i\gamma_5\phi\psi$, both gauge invariant under the symmetry and with dimension 4. The Yukawa term is included in the potential part of the Lagrangian, after the expectation value has been taken on everything but the sigma field:

$$\mathcal{L}_{U,source} = -g_{\mathrm{TC}} S \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle. \tag{21}$$

Hence, the total CSTC-Lagrangian, is given by

$$\mathcal{L}^{CSTC} = \mathcal{L}_T + \mathcal{L}_{U,self} + \mathcal{L}_{U,source}$$

where the kinetic term is given by equation 19, and the first two terms in equation 16:

$$\mathcal{L}_T = \frac{1}{2}\partial_\mu S\partial^\mu S + \frac{1}{2}D_\mu P_a D^\mu P_a + i\bar{\widetilde{Q}}\widehat{D}\widetilde{Q} \tag{22}$$

Note that since the Yukawa term only can be constructed with the Σ-field, the technipions gain their mass when the chiral symmetry is already broken down to $SU(2)_V$. Since $g_L = g_R$ for the vector subgroup, and since the Σ-field transforms as:

$$\Sigma \to g_L \Sigma g_R^\dagger,$$

the mass terms are invariant, in oppose to the mass terms in QCD. Ergo, the symmetry is spontaneously broken, instead of being explicitly broken. The technipions become, as in QCD, massive pseudo-Goldstone bosons, due to the linear source term (21).

1.5.2 The mixing of the H- and S fields

There are several examples from the Standard Model (neutrino mixing, quark-flavour mixing, electroweak mixing), where the interaction eigenbasis of two or more fields with identical quantum numbers, differs from their mass eigenbasis. The phenomena is perhaps most clearly established in the case of electroweak mixing in the leptonic sector. Due to the identical quantum numbers of the B_μ- and W_μ^0 fields, they share common interactions with the leptonic sector, as shown in Appendix B. However, since it is known from experiments that the A_μ-field does not interact with neutrinos, the mass eigenbasis and interaction eigenbases clearly *must* differ. In the CSTC-extension as we have here, there is no such *demand* for the eigenbases to differ, although we must account for the possibility.

The system in the mass eigenbasis *must* be diagonal, since this is the basis in which we measure observables. Hence, the relation between the two bases needs to ensure a diagonalization of the system, when moving from the interaction eigenbasis to the mass eigenbasis. The diagonalization is ensured by a unitary matrix, since it is the equivalence to the matrix constructed of eigenvectors:

$$\begin{aligned}\begin{pmatrix} H' & S' \end{pmatrix} A \begin{pmatrix} H' \\ S' \end{pmatrix} &= \begin{pmatrix} H' & S' \end{pmatrix} U^{-1} D U \begin{pmatrix} H' \\ S' \end{pmatrix} \\ &= \begin{pmatrix} h & \widetilde{\sigma} \end{pmatrix} D \begin{pmatrix} h \\ \widetilde{\sigma} \end{pmatrix} = \lambda_{m,1} h^2 + \lambda_{m,2} \widetilde{\sigma}^2 \end{aligned} \tag{23}$$

where $\{H', S'\}$ is the interaction eigenbasis, $\{h, \widetilde{\sigma}\}$ is the mass eigenbasis, U is the matrix constructed with the eigenvectors as columns and is a unitary matrix, A is the mass matrix in the interaction eigenbasis:

$$A = \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix},$$

and D is the diagonalized mass matrix:

$$D = \begin{pmatrix} m_h^2 & 0 \\ 0 & m_{\widetilde{\sigma}}^2 \end{pmatrix}.$$

A unitary matrix is generally written as a complex rotation matrix. In the 2x2 scenario, the rotation matrix can be made real, since there are enough fields for absorbing all phases. Hence, equation 23 can be rewritten as:

$$\begin{pmatrix} H' & S' \end{pmatrix} A \begin{pmatrix} H' \\ S' \end{pmatrix} = \begin{pmatrix} H' & S' \end{pmatrix} R^{-1} D R \begin{pmatrix} H' \\ S' \end{pmatrix} = \begin{pmatrix} h & \widetilde{\sigma} \end{pmatrix} D \begin{pmatrix} h \\ \widetilde{\sigma} \end{pmatrix} \tag{24}$$

where R is a real 2x2 rotation matrix. The relation between the interaction eigenbasis, $\{H, S\}$ (hereby referred to as the gauge eigenbasis) and the mass eigenbasis $\{h, \widetilde{\sigma}\}$ is therefore described by:

$$\begin{pmatrix} H' \\ S' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \widetilde{\sigma} \end{pmatrix} \tag{25}$$

such that

$$\begin{cases} H' = h\cos\theta - \widetilde{\sigma}\sin\theta \\ S' = h\sin\theta + \widetilde{\sigma}\cos\theta \end{cases} \tag{26}$$

The mixing angle θ may possess any value inbetween zero and 2π. If the mixing angle turns out to be nonzero, which is rather likely, the gauge eigenbasis and mass eigenbasis differ from each other. When there, on the other hand, is no difference between the bases, the unitary matrix simply becomes the identity matrix.

In the mass eigenbasis, the SM Higgs- and the Technisigma fields are given by:

$$\mathcal{H} = v + hc_\theta - \widetilde{\sigma}s_\theta, \quad \mathcal{S} = u + hs_\theta + \widetilde{\sigma}c_\theta \tag{27}$$

where

$$\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H'(x) \end{pmatrix}, \quad \langle\mathcal{H}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\mathcal{S}\rangle = u, \quad \langle\mathcal{P}\rangle = 0 \tag{28}$$

Note that the Technisigma VEV is not necessarily larger than the Higgs VEV. Their relative hierachy will be explored in the result section, and turn out to be dependent on the $h\widetilde{\sigma}$-mixing angle.

1.5.3 The scale of the new dynamics

The confinement scale in QCD, $\Lambda_{QCD} \sim 200$ MeV, denotes the limit when the running coupling constant enters non-perturbative values. This scale is of the same order as the (explicit) chiral symmetry breaking [8]. By the same argument, the confinement scale of our extension is assumed to be in the order of $\Lambda_{TC} \sim u$, where $u \gtrsim v \sim 200$ GeV, which means that:

$$\Lambda_{TC}/\Lambda_{QCD} \gtrsim 1000. \tag{29}$$

In a naive estimate, the masses of the extension follows the same mass hierarchy as in QCD, where all masses are simply scaled up by a factor 1000:

$$m_{\widetilde{\pi}} \gtrsim 140 \text{ GeV}, \qquad M_{\widetilde{\sigma}} \gtrsim 500 \text{ GeV}$$

$$M_{\widetilde{Q}} \gtrsim 300 \text{ GeV}. \tag{30}$$

1.6 The Peskin–Takeuchi Parameters

The existence of new physics would alter, among other things, the self-energy contributions to W, Z and γ. Peskin and Takeuchi introduced three parameters, S, T and U, with well-known values and constraints from electroweak precision measurements [13]:

$$S = 0.00^{+0.11}_{-0.10}, \;\; T = 0.02^{+0.11}_{-0.12}, \;\; U = 0.08 \pm 0.11. \tag{31}$$

Hence, a deviation caused by new physics needs to stay within the boundary conditions in order to be plausible.

Note that the Peskin–Takeuchi (PT) parameters only consider *oblique* corrections. Oblique corrections refers to loop corrections where the loop propagators which does not couple directly to the external fermions [5]. Ergo, the S, T, U formalism assumes that the new particles only enter the weak interactions *indirectly*. Furthermore, the formalism assumes that new physics will not add an additional electroweak symmetry. Since each generator of a symmetry group corresponds to a gauge boson, the assumption states that there will be no other electroweak gauge bosons than W, Z and γ, which is fulfilled for our extension. Ergo, there are no additional anti-screening effects contributing to the PT-parameters, only screening effects. The S, T, U parameters are defined as:

$$\alpha S = 4s_W^2 c_W^2 \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2} - \frac{c_W^2 - s_W^2}{s_W c_W}\delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0)\right],$$

$$\alpha T = \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2},$$

$$\alpha U = 4s_W^2 \Big[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2} - c_W^2\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}$$
$$-2s_W c_W \delta\Pi'_{Z\gamma}(0) - s_W^2\delta\Pi'_{\gamma\gamma}(0)\Big] \qquad (32)$$

where Π_{XY} are the vacuum polarization functions to the gauge bosons W, Z and γ, and where $\delta\Pi'_{XY}(q^2)$ is defined as $d\delta\Pi/dq^2$.

Whenever the energy scale is large in comparison to the electroweak scale, $\Lambda_{TC} \gg M_{EW}$, we may work in the linear order in q^2 (where q^2 corresponds to the gauge bosons mass, since they are on-shell):

$$\frac{\delta\Pi_{XY}(q^2) - \delta\Pi_{XY}(0)}{q^2} = \delta\Pi'_{XY}(0) + \cancelto{0}{\mathcal{O}(q^4/\Lambda_{TC}^4)},$$

which also infers that the mass difference of Z and W is negligible:

$$\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2} = \frac{\delta\Pi_{WW}(M_Z^2) - \delta\Pi_{WW}(0)}{M_Z^2} + \cancelto{0}{\mathcal{O}(q^4/\Lambda_{TC}^4)}$$

Hence, in the linear order of q^2, the PT-parameters simplify to:

$$\alpha S = 4s_w^2 c_w^2 \left[\delta\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0)\right],$$

$$\alpha T = \delta\Pi'_{WW}(0) - \delta\Pi'_{ZZ}(0) = 0,$$

$$\alpha U = 4s_w^2\Big[\delta\Pi'_{WW}(0) - c_w^2\delta\Pi'_{ZZ}(0) - 2s_w c_w\delta\Pi'_{Z\gamma}(0) - s_w^2\delta\Pi'_{\gamma\gamma}(0)\Big]. \qquad (33)$$

where we note that T=0. The expression $\delta\Pi_{XY}(q^2)$ denotes the deviation of the vacuum polarization functions from SM results. The new physics-contributions come from the introduced particles (the technipions, the technisigma and the technifermions), in addition to the modified couplings of the SM-Higgs, minus its original contribution:

$$\delta\Pi_{XY}(q^2) = \Pi_{XY}^{new}(q^2) + \Pi_{XY}^{h}(q^2) - \Pi_{XY}^{SM,h}(q^2),$$

where

$$\Pi_{XY}^{new}(q^2) = \Pi_{XY}^{\tilde{\pi}}(q^2) + \Pi_{XY}^{\tilde{Q}}(q^2) + \Pi_{XY}^{\tilde{\sigma}}(q^2). \qquad (34)$$

The interpretation of the S, T, U-parameters and the vacuum polarization functions will be clarified when they are explicitly calculated in the result-section. Bear in mind that the S, T, U formalism is not applicable if the new physics violates the assumptions made.

1.6.1 Dimensional Regularization

In the case of tree-level diagrams, the momenta of the internal propagators are uniquely determined by the momenta of the external propagators (via the use of delta functions). When it comes to higher order corrections, such as the oblique corrections considered in this thesis, the diagrams involve integration over undefined loop momenta which may lead to divergences. Dimensional regularization is one of the methods to deal with these divergences.

Divergences are functions of the dimension of space-time, for example the integral (where D_1 and D_2 are the two loop propagators)

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{D_1 D_2} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\big(k^2 - m_1^2\big)\big((k-q)^2 - m_2^2\big)}$$

is finite in three dimensions, while being logarithmically divergent in four dimensions [6]. Hence, for a sufficiently small d, where d is a complex number, any loop integral will converge. After the integration is performed, one takes the limit $d \to 4 - 2\epsilon$, which should produce finite results for any physical quantity. The features of this regularization scheme will be displayed in further detail when used in the result section.

Dimensional regularization is a popular alternative to the Pauli-Villars- and Cut-off schemes. Manipulations in dimensional regularization are simple, and the scheme is gauge invariant (since gauge invariance is independent of the number of space-time dimensions).

2 Method

The results presented in this thesis are a reproduction of the paper "Chiral-Symmetric Technicolor with Standard Model Higgs boson", published 2013 by R. Pasechnik et al. [14]. However, since the paper mainly states its final results, it allowed for calculations being carried out in an independent and creative manner (in particular the loop evaluations).

All calculations were performed analytically, with exception from the final (numerical) evaluations of the loop diagrams and some of the vertex factors, obtained using Mathematica. The vertices were retrieved using a custom-made program named FeynArts [18].

3 Results

3.1 The Physical CSTC Lagrangian

This section includes the physical CSTC Lagrangian, i.e. the Lagrangian after the two spontaneous symmetry breakings. The section begins with relating the technipion mass, the higgs VEV and the technisigma VEV to the techniquark condensate, and continues with presenting the various parts of the physical Lagrangian. Mass terms, interaction terms and mixing angles are divided into separate subsections, for clarity.

3.1.1 The Vacuum Stability Equations

Vacuum stability refers to the requirement that the VEV is a minimum point of the potential. Using this requirement we may relate the technipion mass, the Higgs VEV and the technisigma VEV, to the techniquark condensate.

In order for the VEV to be an extremum, the derivative of the potential Lagrangian with respect to the VEV should be zero:

$$\begin{aligned} \left\langle \frac{\delta \mathcal{L}_U^{CSTC}}{\delta \mathcal{H}} \right\rangle = 0 \quad &\Leftrightarrow \quad \frac{\partial \left\langle \mathcal{L}_U^{CSTC} \right\rangle}{\partial v} = 0 \\ = \frac{\partial}{\partial v}\Big(\frac{1}{2}\mu_S^2 u^2 + \frac{1}{2}\mu_H^2 v^2 - \frac{1}{4}\lambda_{TC}u^4 - \frac{1}{4}\lambda_H v^4 &+ \frac{1}{2}\lambda v^2 u^2 - g_{\mathrm{TC}} u \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle \Big) \\ = v(\mu_H^2 - \lambda_H v^2 - \lambda u^2) &= 0 \end{aligned} \tag{35}$$

and similarly

$$\begin{aligned} \left\langle \frac{\delta \mathcal{L}_U^{CSTC}}{\delta \mathcal{S}} \right\rangle = 0 \quad &\Leftrightarrow \quad \frac{\partial \left\langle \mathcal{L}_U^{CSTC} \right\rangle}{\partial u} = 0 \\ = \frac{\partial}{\partial u}\Big(\frac{1}{2}\mu_S^2 u^2 + \frac{1}{2}\mu_H^2 v^2 - \frac{1}{4}\lambda_{TC}u^4 - \frac{1}{4}\lambda_H v^4 &+ \frac{1}{2}\lambda v^2 u^2 - g_{\mathrm{TC}} u \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle \Big) \\ = u\left(\mu_S^2 - \lambda_{TC}u^2 + \lambda v^2 - \frac{g_{\mathrm{TC}} \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle}{u}\right) &= 0 \end{aligned} \tag{36}$$

Combining this equation with the result for the technipion mass in equation 43, yields a relation between the technipion mass and the techniquark condensate:

$$m_{\widetilde{\pi}}^2 = -\frac{g_{\mathrm{TC}} \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle}{u} \tag{37}$$

where g_{TC} is positive and $\left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle$ defined as negative. Hence, equation 35 and 36 can be simplified to:

$$\begin{cases} \mu_S^2 = \lambda_{TC}u^2 - \lambda v^2 - m_{\widetilde{\pi}}^2 \\ \mu_H^2 = \lambda_H v^2 - \lambda u^2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} v^2 = \frac{\lambda_{TC}\mu_H^2 - \lambda(\mu_S^2 + m_{\widetilde{\pi}}^2)}{\lambda_{TC}\lambda_H - \lambda^2} \\ u^2 = \frac{\lambda_H(m_{\widetilde{\pi}}^2 + \mu_s^2) + \lambda\mu_H^2}{\lambda_H\lambda_{TC} - \lambda^2} \end{cases} \tag{38}$$

which displays the relation between the VEV:s and the techniquark condensate.

Furthermore, requiring the extremum to be a minimum point, poses conditions on the scalar self-couplings λ_{TC} and λ_H. The second derivative test for a bivariable function, such as $\mathcal{L}_U^{CSTC}$, is defined as

$$D(x,y) = \det(H(x,y)) = f_{xx}f_{yy} - f_{xy}^2, \quad H(x,y) = \begin{pmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{pmatrix}$$

where $H(x,y)$ is the so-called Hessian. The extreme value is a minimum point if both the determinant of the Hessian and f_{xx} are positive [20]. In our case, this scenario corresponds to:

$$\left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{S}^2} \right\rangle \left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{H}^2} \right\rangle - \left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{H} \delta \mathcal{S}} \right\rangle^2 > 0, \tag{39}$$

and

$$\left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{S}^2} \right\rangle < 0 \quad \text{or} \quad \left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{H}^2} \right\rangle < 0,$$

where the sign is flipped since $U = -\mathcal{L}_U^{CSTC}$. Using these conditions on the CSTC-Lagrangian yields:

$$\begin{aligned} \left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{S}^2} \right\rangle &= \frac{\partial}{\partial u}\left(u\mu_S^2 - \lambda_{TC}u^3 + u\lambda v^2 - g_{\mathrm{TC}} \left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle \right) \\ &= \mu_S^2 - 3\lambda_{TC}u^2 + \lambda v^2 < 0, \\ &\Leftrightarrow \quad \lambda_{TC} > -\frac{m_{\widetilde{\pi}}^2}{2u^2}, \end{aligned}$$

and

$$\begin{aligned} \left\langle \frac{\delta^2 \mathcal{L}_U^{CSTC}}{\delta \mathcal{H}^2} \right\rangle = \frac{\partial}{\partial v}\left(v\mu_H^2 - \lambda_H v^3 - v\lambda u^2 \right) = \mu_H^2 - 3\lambda_H v^2 - \lambda u^2 < 0 \\ \Leftrightarrow \quad \lambda_H > 0. \end{aligned}$$

using the conditions for the first order derivative in (36) and (35), respectively. That λ_H does not adopt negative values, which will display itself in the parameter space-plots in subsection 3.1.5.

3.1.2 The Mass Terms

3.1.2.1 The Mass of the Higgs and Technisigma

As opposed to gauge fields, where the masses are found through the kinetic term, the Higgs-type fields get their mass through the potential term. From equation 24, it is apparent that there are two methods of obtaining the masses of h and $\widetilde{\sigma}$. Either, one starts in the undiagonalized gauge basis $\{H', S'\}$, by inserting $H = v + H'$ and $S = u + S'$ into the potential CSTC Lagrangian of equation 18 (i.e. expanding about the VEV without introducing the mixing), or one starts in the automatically diagonalized mass basis $\{h, \widetilde{\sigma}\}$ by inserting $H = v + hc_\theta - \widetilde{\sigma}s_\theta$ and $S = u + hs_\theta + \widetilde{\sigma}c_\theta$ into said Lagrangian. After diagonalizing the first method, the two results are of course identical. For demonstrational purposes, I will show both methods in this section.

Starting in the gauge basis, and disregarding the P-field, which will not affect the higgs- and technisigma masses, the CSTC Lagrangian of equation 18 becomes:

$$\begin{aligned} \mathcal{L}_{U,self}^{CSTC} = \frac{1}{2}\mu_S^2(u+S')^2 + \frac{1}{2}\mu_H^2(v+H')^2 - \frac{1}{4}\lambda_{TC}(u+S')^4 - \frac{1}{4}\lambda_H(v+H')^4 \\ + \frac{1}{2}\lambda(v+H')^2(u+S')^2. \end{aligned}$$

Solving for all terms involving two fields (the mass terms), we may construct the mass matrix A:

$$A = \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix} = \begin{pmatrix} 2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2 & -2\lambda uv \\ -2\lambda uv & 2\lambda_H v^2 \end{pmatrix}. \tag{40}$$

where the off-diagonal terms are identical, $m_{12}^2 = m_{21}^2$ and corresponds to (1/2) of the $m_{12}^2 H'S'$-term, since the mass matrix is symmetric:

$$\begin{pmatrix} H' & S' \end{pmatrix} A \begin{pmatrix} H' \\ S' \end{pmatrix} = \begin{pmatrix} H' & S' \end{pmatrix} \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix} \begin{pmatrix} H' \\ S' \end{pmatrix} = m_{11}^2 H'^2 + 2m_{12}^2 H'S' + m_{22}^2 S'^2$$

The masses of h and $\widetilde{\sigma}$, are determined from diagonalizing the mass matrix A. The diagonalization of A is performed by determining the matrix eigenvalues, which will be the two diagonal entries in the diagonalized mass matrix:

$$\det \begin{pmatrix} m_{11}^2 - \lambda_m & m_{12}^2 \\ m_{21}^2 & m_{22}^2 - \lambda_m \end{pmatrix} = 0$$

$$\begin{aligned} \Leftrightarrow \ \lambda_m &= \frac{m_{11}^2 + m_{22}^2}{2} \pm \sqrt{m_{12}^2 m_{21}^2 - m_{11}^2 m_{22}^2 + \left(\frac{m_{11}^2 + m_{22}^2}{2}\right)^2} \\ &= \frac{1}{2}\Big[m_{11}^2 + m_{22}^2 \pm \sqrt{4m_{12}^2 m_{21}^2 + (m_{11}^2 - m_{22}^2)^2}\Big]. \end{aligned}$$

Introducing the entries in the mass matrix A, yields the eigenvalues:

$$\Rightarrow \quad \lambda_m = \frac{1}{2}\Big[2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2 + 2\lambda_H v^2 \pm \sqrt{(2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2 - 2\lambda_H v^2)^2 + 16\lambda^2 u^2 v^2}\Big] \tag{41}$$

where the negative solution correspond to the mass of the higgs, and the positive solution to the mass of the technisigma.

If we instead start off in the mass eigenbasis, i.e. including the mixing when expanding about the VEV:s, the potential Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_{U,self}^{CSTC} &= \frac{1}{2}\mu_S^2(u + h\sin\theta + \widetilde{\sigma}\cos\theta)^2 + \frac{1}{2}\mu_H^2(v + h\cos\theta - \widetilde{\sigma}\sin\theta)^2 \\ &- \frac{1}{4}\lambda_{TC}(u + h\sin\theta + \widetilde{\sigma}\cos\theta)^4 - \frac{1}{4}\lambda_H(v + h\cos\theta - \widetilde{\sigma}\sin\theta)^4 \\ &+ \frac{1}{2}\lambda(v + h\cos\theta - \widetilde{\sigma}\sin\theta)^2(u + h\sin\theta + \widetilde{\sigma}\cos\theta)^2. \end{aligned}$$

By identifying h^2, $h\widetilde{\sigma}$ and $\widetilde{\sigma}^2$-terms, the following mass equations are found (simplified using trigonometric identities, and the vacuum stability conditions from equation 38):

$$\begin{cases} m_{11}^2 = \sin^2\theta(2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2) + 2\lambda_H v^2\cos^2\theta - 4uv\lambda\sin\theta\cos\theta, \\ m_{22}^2 = \cos^2\theta(2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2) + 2\lambda_H v^2\sin^2\theta + 4uv\lambda\sin\theta\cos\theta, \\ m_{12}^2 = m_{21}^2 = \frac{1}{4}\Big[\sin 2\theta\big(2\lambda_H v^2 - 2\lambda_{TC}u^2 - m_{\widetilde{\pi}}^2\big) + 4\lambda uv\cos 2\theta\Big], \end{cases} \tag{42}$$

Since we are working in the mass eigenbasis, the mass matrix is already diagonalized. Ergo, $m_{12}^2 = 0$, $m_{11}^2 = m_h^2$ and $m_{22}^2 = m_{\widetilde{\sigma}}^2$, in the equation above. To get rid of the angles from the mass of the higgs and technisigma, one inserts the expression for the mixing angle of equation 49. The result is identical to equation 41 (found when diagonalizing the gauge basis). However, the simplifications are not shown here since the analytical calculation require a tedious amount of algebra.

3.1.2.2 The Mass of the Technipion

The mass of the technipion is found by keeping only the terms quadratic in P from the initial Lagrangian of equation 18. Ergo, the second and fourth term can be removed immediately, S^2 is removed from the first and fifth term, and from the third term and fifth term, only the VEV from S^2 and $\mathcal{H}$, respectively, are kept:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\mu_S^2 P^2 - \frac{1}{4}\lambda_{TC} 2u^2 P^2 + \frac{1}{2}\lambda v^2 P^2 \\ &= (\frac{1}{2}\mu_S^2 - \frac{1}{4}\lambda_{TC} 2u^2 + \frac{1}{2}\lambda v^2)(\widetilde{\pi}^0\widetilde{\pi}^0 + 2\widetilde{\pi}^+\widetilde{\pi}^-)\end{aligned}$$

where $P^2 = \sum_a P_a P_a = \widetilde{\pi}^0\widetilde{\pi}^0 + 2\widetilde{\pi}^+\widetilde{\pi}^-$. The massterm of a pseudo-scalar field is given by $-\frac{1}{2}m^2 P^2$ [4], hence:

$$m_{\widetilde{\pi}}^2 = \lambda_{TC} u^2 - \lambda v^2 - \mu_S^2 \tag{43}$$

By comparing this mass term to the tadpole equations, we note that the technipion mass would have been zero if it was not for the linear source term. This notion agrees with that fact that the Goldstone boson of a non-tilted potential should be zero (which the 3-dimensional Lagrangian would have corresponded to, if it was not for the source term).

3.1.2.3 The Mass of the Techniquark

The masses of techni-quarks are found through the Yukawa term in equation 20:

$$\begin{aligned}\mathcal{L}_Y^{CSTC} &= -g_{\text{TC}}\bar{\widetilde{Q}}(S\mathbb{1} + i\gamma_5\tau_a P_a)\widetilde{Q} \\ &= -g_{\text{TC}}\bar{\widetilde{Q}}u\mathbb{1}\widetilde{Q} - g_{\text{TC}}\bar{\widetilde{Q}}(hs_\theta + \widetilde{\sigma}c_\theta)\mathbb{1}\widetilde{Q} - ig_{\text{TC}}\bar{\widetilde{Q}}\gamma_5\tau_a P_a\widetilde{Q}\end{aligned} \tag{44}$$

using that $S = u + S' = u + hs_\theta + \widetilde{\sigma}c_\theta$. The masses of the techniquarks are given by the very first term of equation 44, while the latter terms explains their interactions with the higgs-, technisigma- and pion fields, and are treated in the following section:

$$\mathcal{L}_Y^{CSTC} = -g_{\text{TC}}\bar{\widetilde{Q}}u\mathbb{1}\widetilde{Q} = -g_{\text{TC}}\begin{pmatrix}\bar{\widetilde{U}} \\ \bar{\widetilde{D}}\end{pmatrix}\begin{pmatrix}u & 0 \\ 0 & u\end{pmatrix}\begin{pmatrix}\widetilde{U} \\ \widetilde{D}\end{pmatrix} = -g_{\text{TC}}u\big(\bar{\widetilde{U}}\widetilde{U} + \bar{\widetilde{D}}\widetilde{D}\big)$$

$$\Rightarrow \quad m_{\widetilde{U}} = m_{\widetilde{D}} = g_{\text{TC}}u \tag{45}$$

since the mass terms of fermions are defined as $-m^2\psi^\dagger\psi$. Note that the masses are degenerate. When comparing with the Yukawa term in the SM (C), we notice that the mass terms will always be degenerate for a chirally symmetric theory.

3.1.2.4 The Mass of the W- and Z bosons

The masses of the W- and Z bosons are already known from the Standard Model, but I chose to include the derivation here, since the mass terms are used for simplifying interaction terms in the following section.

The mass-terms of the gauge fields are determined by inserting the covariant derivative corresponding to the U(1) and SU(2) symmetries into the kinetic term of the Higgs Lagrangian:

$$\begin{aligned}(D_\mu\phi)^\dagger(D^\mu\phi) &= \phi^\dagger\left(ig_1\frac{Y}{2}B_\mu + ig_2\frac{\tau}{2}W_\mu\right)^\dagger\left(ig_1\frac{Y}{2}B^\mu + ig_2\frac{\tau}{2}W^\mu\right)\phi = ... \\ &= \left(\frac{1}{2}vg_2\right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2(g_1 B_\mu - g_2 W_\mu^0)^2,\end{aligned} \tag{46}$$

The first term of equation 46 is a mass-term for the charged W, since charged gauge bosons have mass-terms of the form $m^2 F_\mu^+ F^{-\mu}$. Hence:

$$M_W = \frac{vg_2}{2}. \tag{47}$$

The second term can be identified as the mass-term for Z_μ when comparing the equation with equation 125 (using $Y_L = -1$). The mass-term of a neutral field is given by $(m^2 F_\mu F^\mu)/2$, which means that:

$$M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}. \tag{48}$$

3.1.3 The Mixing Angle

The mixing angle is most conveniently found by starting in the mass eigenbasis, i.e. by expanding H and S in terms of h and $\widetilde{\sigma}$. Since the mass matrix is automatically diagonalized in the mass eigenbasis, the off-diagonal terms may be put to zero, from which an angle can be solved for. Putting equation m_{12}^2, from equation 42, to zero, and dividing each term with $\cos 2\theta$, yields the condition for the mixing angle θ:

$$m_{12}^2 = 0 \quad \Leftrightarrow \quad tan2\theta = \frac{4\lambda uv}{2\lambda_{TC}u^2 + m_{\widetilde{\pi}}^2 - 2\lambda_H v^2} \tag{49}$$

3.1.4 The Interaction Terms

3.1.4.1 The Vector-like Interactions of the Pion Field

The vector-like interactions of technipions, are given by the second term in equation 22:

$$\frac{1}{2} D_\mu P_a D^\mu P_a = \frac{1}{2}\big[D_\mu P_1 D^\mu P_1 + D_\mu P_2 D^\mu P_2 + D_\mu P_3 D^\mu P_3\big].$$

The covariant derivative can be expressed according to equation 17, where the Levi-Civita symbol gives a total of six possible permutations of a, b, c. Even permutations yield +1 and uneven -1, since the Levi-Civita symbol is completely antisymmetric in all indices:

$$\begin{aligned} \Rightarrow \quad \frac{1}{2} D_\mu P_a D^\mu P_a = \frac{1}{2}\big[&\partial_\mu P_1 + g_2(W_\mu^2 P_3 - W_\mu^3 P_2) + \partial_\mu P_2 + g_2(W_\mu^3 P_1 - W_\mu^1 P_3) \\ &+\partial_\mu P_3 + g_2(W_\mu^1 P_2 - W_\mu^2 P_1)\big]. \end{aligned} \tag{50}$$

After the $W_\mu^{1,2,3}$- and $P^{1,2,3}$-fields are expressed in terms of $W_\mu^{\pm,0}$ and $\widetilde{\pi}^{\pm,0}$, according to:

$$\begin{cases} W_\mu^3 = W_\mu^0 = \cos Z_\mu + \sin A_\mu \\ W_\mu^2 = i(W_\mu^- - W_\mu^+)/\sqrt{2} \\ W_\mu^1 = -(W_\mu^- + W_\mu^+)/\sqrt{2} \end{cases} \quad \text{and} \quad \begin{cases} P^3 = \widetilde{\pi}^0 \\ P^2 = i(\widetilde{\pi}^- - \widetilde{\pi}^+)/\sqrt{2} \\ P^1 = -(\widetilde{\pi}^- + \widetilde{\pi}^+)/\sqrt{2} \end{cases} . \tag{51}$$

and after all brackets are expanded, the vector-like interactions of the technipions can be identified. The vector-like interactions are simply all terms involving two pi-fields and one, or more, gauge field (the gauge field being either A_μ, Z_μ or $W^\pm$). Due to the extensive length of the fully expanded expression, and due its trivial nature, I have chosen to only include the final result, which is:

$$\begin{aligned} \mathcal{L}_{\widetilde{\pi}\widetilde{\pi}V}^{CSTC} &= ig_2 W_\mu^+ \big(\widetilde{\pi}^0 \partial_\mu \widetilde{\pi}^- - \widetilde{\pi}^- \partial_\mu \widetilde{\pi}^0\big) + ig_2 W_\mu^- \big(\widetilde{\pi}^+ \partial_\mu \widetilde{\pi}^0 - \widetilde{\pi}^0 \partial_\mu \widetilde{\pi}^+\big) \\ +ig_2 W_\mu^0 \big(\widetilde{\pi}^- \partial_\mu \widetilde{\pi}^+ - \widetilde{\pi}^+ \partial_\mu \widetilde{\pi}^-\big) &+ g_2^2 W_\mu^- W^{\mu+} \big((\widetilde{\pi}^0)^2 + \widetilde{\pi}^- \widetilde{\pi}^+\big) + g_2^2 (W_\mu^0)^2 \widetilde{\pi}^+ \widetilde{\pi}^- \\ &- g_2^2 W_\mu^0 W_\mu^- \widetilde{\pi}^0 \widetilde{\pi}^+ - g_2^2 W_\mu^0 W_\mu^+ \widetilde{\pi}^0 \widetilde{\pi}^- + \ldots \end{aligned} \tag{52}$$

3.1.4.2 The Vector-like Interactions of the Techniquarks

The interactions between techniquarks and the gauge bosons of the Standard Model, is given by the third term in the kinetic CSTC-Langrangian of equation 22. The term is similar to the Standard Model interaction between fermions and gauge bosons, and hence the physical Lagrangians are identical as well (apart from there being no distinction between right and left, there only being two flavours of quarks and no leptons, and there being no SU(3) symmetry group). The physical Lagrangian of the Standard Model-scenario is given in Appendix B, and translates to:

$$\begin{aligned}\mathcal{L} = &\sum_{f=\widetilde{U},\widetilde{D}} eQ_f\bar{f}\gamma^\mu f A^\mu \\ +\frac{g_2}{c_W} &\sum_{f=\widetilde{U},\widetilde{D}} \bar{f}\gamma^\mu f(T_f^3 - Q_f s_W^2)Z_\mu \\ &+\frac{g_2}{\sqrt{2}}(\bar{\widetilde{U}}\gamma^\mu \widetilde{D}W_\mu^+ + h.c.)\end{aligned} \tag{53}$$

3.1.4.3 The Vector-like Interactions of the Higgs and Technisigma

The interactions between the gauge bosons of the Standard Model, and the Higgs field, are altered, due to H' now being a linear combination of h and $\widetilde{\sigma}$. The kinetic term of the Higgs Lagrangian then becomes:

$$\begin{aligned}\mathcal{L} = (D_\mu\mathcal{H})^\dagger(D^\mu\mathcal{H}) &= \mathcal{H}^\dagger\left(ig_1\frac{Y}{2}B_\mu + ig_2\frac{\tau_a}{2}W_{\mu a}\right)^\dagger\left(ig_1\frac{Y}{2}B^\mu + ig_2\frac{\tau_a}{2}W_a^\mu\right)\mathcal{H} \\ &= \frac{1}{8}\left|\begin{pmatrix} g_1B_\mu + g_2W_\mu^0 & -g_2\sqrt{2}W_\mu^+ \\ -g_2W_\mu^- & g_1B_\mu - g_2W_\mu^0\end{pmatrix}\begin{pmatrix}0 \\ v + hc_\theta - \widetilde{\sigma}s_\theta\end{pmatrix}\right|^2 \\ &= \frac{1}{4}g_2^2W_\mu^+W^{\mu-}(v + hc_\theta - \widetilde{\sigma}s_\theta)^2 + \frac{1}{8}(g_1B_\mu - g_2W_\mu^0)^2(v + hc_\theta - \widetilde{\sigma}s_\theta)^2\end{aligned}$$

where the $(g_1B_\mu - g_2W_\mu^0)$ -term can be identified as $(-Z_\mu\sqrt{g_1^2 + g_2^2})$ from equation 125 in Appendix B (using $Y_L = -1$):

$$\Rightarrow \mathcal{L} = \frac{1}{4}g_2^2W_\mu^+W^{\mu-}(v + hc_\theta - \widetilde{\sigma}s_\theta)^2 + \frac{1}{8}Z_\mu Z^\mu(g_1^2 + g_2^2)(v + hc_\theta - \widetilde{\sigma}s_\theta)^2. \tag{54}$$

By expanding this Lagrangian, we get the interaction between $\widetilde{\sigma}$ and the gauge bosons (using the mass of the W-boson found in equation 47, and the mass of the Z-boson found in equation 48):

$$\begin{aligned}\mathcal{L}_{\widetilde{\sigma}WW} + \mathcal{L}_{\widetilde{\sigma}ZZ} &= -\frac{g_2^2}{2}W_\mu^+W^{\mu-}v\widetilde{\sigma}s_\theta - \frac{1}{4}Z_\mu Z^\mu(g_1^2 + g_2^2)v\widetilde{\sigma}s_\theta \\ &= -g_2M_WW_\mu^+W^{\mu-}\widetilde{\sigma}s_\theta - \frac{1}{2}Z_\mu Z^\mu\sqrt{g_1^2 + g_2^2}M_Z\widetilde{\sigma}s_\theta,\end{aligned} \tag{55}$$

the interaction between h and the gauge bosons (simplified in the same manner):

$$\mathcal{L}_{hWW} + \mathcal{L}_{hZZ} = g_2M_Wc_\theta hW_\mu^+W^{\mu-} + \frac{1}{2}Z_\mu Z^\mu\sqrt{g_1^2 + g_2^2}M_Zhc_\theta, \tag{56}$$

and the quartic terms, h^2VV, $h\widetilde{\sigma}VV$ and $\widetilde{\sigma}^2VV$ (easily read off from equation 54):

$$\mathcal{L}_{\widetilde{\sigma}^2VV} + \mathcal{L}_{h^2VV} + \mathcal{L}_{h\widetilde{\sigma}VV} = (hc_\theta - \widetilde{\sigma}s_\theta)^2\Big(\frac{1}{4}g_2^2W_\mu^+W^{\mu-} + \frac{1}{8}Z_\mu Z^\mu(g_1^2 + g_2^2)\Big),$$

3.1.4.4 The Interactions between the technipion and the Higgs and Technisigma

Interactions of type $h\widetilde{\pi}\widetilde{\pi}$ and $\widetilde{\sigma}\widetilde{\pi}\widetilde{\pi}$ are found from keeping the terms linear in h and $\widetilde{\sigma}$, while quadratic in P, from equation 18. Ergo, the first, second and fourth term are immediately disregarded, in addition to only keeping the terms linear in h from $\mathcal{H}^2$ and the terms linear in $\widetilde{\sigma}$ from S^2:

$$\mathcal{L} = -\frac{1}{4}\lambda_{TC}(2uhs_\theta + 2u\widetilde{\sigma}c_\theta + P^2)^2 + \frac{1}{2}\lambda(2vhc_\theta - 2v\widetilde{\sigma}s_\theta)P^2$$

where yet some more terms can be disregarded from the expansion of the first term. This yields:

$$\mathcal{L} = -\frac{1}{4}\lambda_{TC}(4uhs_\theta + 4u\widetilde{\sigma}c_\theta)P^2 + \frac{1}{2}\lambda(2vhc_\theta - 2v\widetilde{\sigma}s_\theta)P^2.$$

which can be divided into the separate results for the technisigma and the Higgs field:

$$\mathcal{L}_{h\widetilde{\pi}\widetilde{\pi}} = -(\lambda_{TC}uhs_\theta - \lambda vhc_\theta)P^2,$$

and

$$\mathcal{L}_{\widetilde{\sigma}\widetilde{\pi}\widetilde{\pi}} = -(\lambda_{TC}u\widetilde{\sigma}c_\theta + \lambda v\widetilde{\sigma}s_\theta)P^2.$$

3.1.4.5 Techniquark Interactions from the Yukawa Term

The interactions of techni-quarks with h, $\widetilde{\sigma}$ and $\widetilde{\pi}$ are found through the expansion of last two terms of the Yukawa-term of equation 44, where:

$$\begin{aligned}\tau_a P_a = \tau_1 P_1 + \tau_2 P_2 + \tau_3 P_3 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} P_1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} P_2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} P_3 \\ &= \begin{pmatrix} P_3 & P_1 - iP_3 \\ P_1 + iP_2 & -P_3 \end{pmatrix} = \begin{pmatrix} \pi^0 & -\sqrt{2}\pi^+ \\ -\sqrt{2}\pi^- & -\pi^0 \end{pmatrix}\end{aligned}$$

using the linear combinations of equation 51 for performing the last simplication. Hence, the Lagrangian becomes:

$$\begin{aligned}\mathcal{L}_Y^{CSTC} &= -g_{\text{TC}} \begin{pmatrix} \bar{\widetilde{U}} \\ \bar{\widetilde{D}} \end{pmatrix} \begin{pmatrix} hs_\theta + \widetilde{\sigma}c_\theta & 0 \\ 0 & hs_\theta + \widetilde{\sigma}c_\theta \end{pmatrix} \begin{pmatrix} \widetilde{U} \\ \widetilde{D} \end{pmatrix} \\ &\quad -ig_{\text{TC}} \begin{pmatrix} \bar{\widetilde{U}} \\ \bar{\widetilde{D}} \end{pmatrix} \gamma_5 \begin{pmatrix} \pi^0 & -\sqrt{2}\pi^+ \\ -\sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \begin{pmatrix} \widetilde{U} \\ \widetilde{D} \end{pmatrix}\end{aligned}$$

$$= -g_{\text{TC}}\big(\widetilde{\sigma}c_\theta + hs_\theta\big)\big(\bar{\widetilde{U}}\widetilde{U} + \bar{\widetilde{D}}\widetilde{D}\big) - ig_{\text{TC}}\big(\bar{\widetilde{U}}\gamma_5\widetilde{U}\widetilde{\pi}^0 - \sqrt{2}\bar{\widetilde{U}}\gamma_5\widetilde{D}\widetilde{\pi}^+ - \sqrt{2}\bar{\widetilde{D}}\gamma_5\widetilde{U}\widetilde{\pi}^- - \bar{\widetilde{D}}\gamma_5\widetilde{D}\widetilde{\pi}^0\big)$$

$$\begin{aligned}= -g_{\text{TC}}\big(\widetilde{\sigma}c_\theta + hs_\theta\big)\big(\bar{\widetilde{U}}\widetilde{U} + \bar{\widetilde{D}}\widetilde{D}\big) - ig_{\text{TC}}\widetilde{\pi}^0\big(\bar{\widetilde{U}}\gamma_5\widetilde{U} - \bar{\widetilde{D}}\gamma_5\widetilde{D}\big) + i\sqrt{2}g_{\text{TC}}\widetilde{\pi}^+\bar{\widetilde{U}}\gamma_5\widetilde{D} \\ +i\sqrt{2}g_{\text{TC}}\widetilde{\pi}^-\bar{\widetilde{D}}\gamma_5\widetilde{U}\end{aligned}$$

3.1.4.6 Fermion Interactions from the Yukawa Term

The Yukawa term in Appendix C, equation 129 (which describes the fermion interactions of the Standard Model) will be altered, now that there is a new expression for the Higgs field:

$$H' = hc_\theta - \widetilde{\sigma} s_\theta \quad \Rightarrow \quad \mathcal{L} = -\frac{m_f}{v}\bar{f}fH' = -\frac{m_f}{v}\bar{f}f(hc_\theta - \widetilde{\sigma}s_\theta).$$

Using the expression for the mass of the W-bosons in equation 47, the interaction term can be rewritten as:

$$\mathcal{L} = -\frac{g_2 m_f}{2M_W}\bar{f}f(hc_\theta - \widetilde{\sigma}s_\theta).$$

3.1.5 The Parameter Space

From the mass terms of the higgs and technisigma in equation 41, and from the mixing angle in equation 49, we can find expressions for the coupling constants as functions of the masses, the coupling constant g_{TC} and the mixing angle alone:

$$\lambda_{TC} = \frac{1}{2u^2}\big(-m_{\widetilde{\pi}}^2 + M_{\widetilde{\sigma}}^2 c_\theta^2 + M_h^2 s_\theta^2\big) = \frac{g_{\mathrm{TC}}^2}{2M_{\widetilde{Q}}^2}\big(-m_{\widetilde{\pi}}^2 + M_{\widetilde{\sigma}}^2(1-s_\theta^2) + M_h^2 s_\theta^2\big), \tag{57}$$

$$\lambda = \pm\frac{1}{2uv}\big(M_{\widetilde{\sigma}}^2 - M_h^2\big)c_\theta s_\theta = \pm\frac{g_{\mathrm{TC}}}{2M_{\widetilde{Q}}v}\big(M_{\widetilde{\sigma}}^2 - M_h^2\big)c_\theta s_\theta, \tag{58}$$

$$\lambda_H = \frac{1}{2v^2}\big(M_{\widetilde{\sigma}}^2 s_\theta^2 + M_h^2(1-s_\theta^2)\big). \tag{59}$$

where the Sigma VEV was re-expressed using equation 45. Note that by doing this, we go from explaining our theory in terms of the seven parameters:

$$\lambda, \quad \lambda_{TC}, \quad \lambda_H, \quad g_{\mathrm{TC}}, \quad \langle\bar{\widetilde{Q}}\widetilde{Q}\rangle, \quad \mu_H^2 \quad \text{and} \quad \mu_S^2$$

to another, equivalent, set of seven parameters:

$$M_{\widetilde{\sigma}}, \quad m_{\widetilde{\pi}}, \quad M_{\widetilde{Q}}, \quad g_{\mathrm{TC}}, \quad s_\theta, \quad v \quad \text{and} \quad M_h$$

where $M_{\widetilde{Q}}$ and g_{TC} constitute the technisigma VEV. Since the mass of the Higgs and its VEV are known, there are only five parameters left to vary. Besides decreasing the amount of variables, the change is preferable, since the parameter space is now expressed in terms of measurable quantities.

The equations can be used to plot the coupling constants as a function of $\sin\theta$. The results for λ_{TC}, λ and λ_H, are shown in figure 3, 4 and 5, respectively. The parameter space is explored by varying one parameter at a time, in the ranges:

$$g_{\mathrm{TC}} = 1,\ 1.5,\ 2, \quad M_{\widetilde{\sigma}} = 400,\ 500,\ 700\ \text{ GeV}$$

$$M_{\widetilde{Q}} = 300,\ 400,\ 500\ \text{ GeV} \quad \text{and} \quad m_{\widetilde{\pi}} = 150,\ 250,\ 350\ \text{ GeV}.$$

The ranges are constructed using the constraints on the masses found in (30), and the varying is performed such that the mass hierarchy is preserved in each plot. g_{TC} is varied in agreement to equation 45, using that the Sigma VEV should be larger than the Higgs VEV: $u \gtrsim v \simeq 246$ GeV. Note that s_θ is plotted from 0 to 1, even though it is allowed to vary from -1 to 1. The reason for this is that such a plot would not add any information, after informing that λ_{TC} and λ_H is symmetric with respect to $s_\theta \to -s_\theta$, while λ is antisymmetric with respect to the same thing.

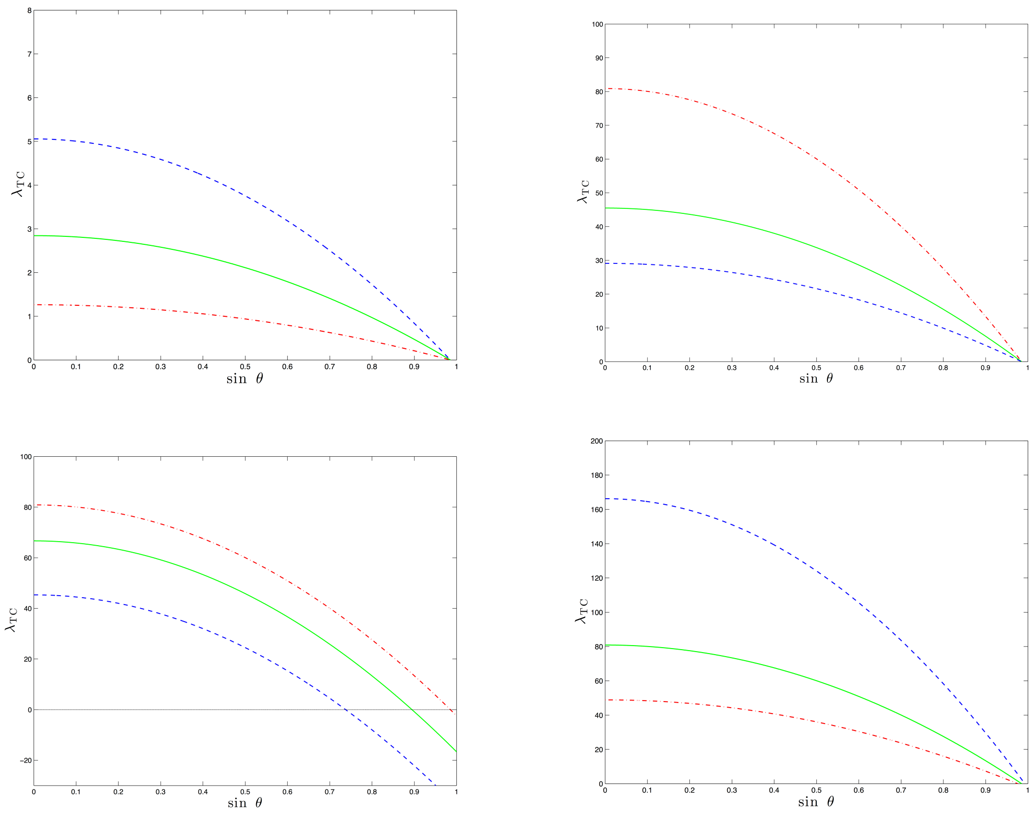


Figure 3: The coupling constant λ_{TC} as a function of the mixing angle $\sin\theta$. The dashed-dotted (red), solid (green) and dashed (blue) lines in each plot correspond to the variation of one parameter at a time: (1) $g_{\rm TC} = 1,\ 1.5,\ 2$, $M_{\widetilde{Q}} = 300$ GeV, $m_{\widetilde{\pi}} = 150$ GeV and $M_{\widetilde{\sigma}} = 500$ GeV. (2) $g_{\rm TC} = 2$, $M_{\widetilde{Q}} = 300,\ 400,\ 500$ GeV, $m_{\widetilde{\pi}} = 150$ GeV and $M_{\widetilde{\sigma}} = 500$ GeV. (3) $g_{\rm TC} = 2$, $M_{\widetilde{Q}} = 300$ GeV, $m_{\widetilde{\pi}} = 150$ GeV and $M_{\widetilde{\sigma}} = 400,\ 500,\ 700$ GeV. (4) $g_{\rm TC} = 2$, $M_{\widetilde{Q}} = 300$ GeV, $m_{\widetilde{\pi}} = 150,\ 250,\ 350$ GeV and $M_{\widetilde{\sigma}} = 500$ GeV.

3.2 The Nearly Conformal Limit

As we saw in the introductory section, spontaneous symmetry breaking requires negative (quadratic) μ-terms. There is however an alternative interpretation of the Higgs Mechanism. In such a theory, the QCD Lagrangian possess a so-called conformal symmetry, which is exact in the chiral limit ($m_q \to 0$), and which forbids μ-terms. The theory behind these statements are beyond the level of this thesis, here we will simply consider it to be a limit where $\mu_{S,H} \ll m_{\widetilde{\pi}}$.

3.2.1 A Common Origin of the Higgs and Technisigma VEV

In the limit $\mu_{S,H} \ll m_{\widetilde{\pi}}$, the expressions in equation 38 simplify to:

$$\begin{cases} v^2 = \frac{\lambda m_{\widetilde{\pi}}^2}{\lambda_{TC}\lambda_H - \lambda^2} \\ u^2 = \frac{\lambda_H m_{\widetilde{\pi}}^2}{\lambda_{TC}\lambda_H - \lambda^2} \end{cases} . \tag{60}$$

where $m_{\widetilde{\pi}}^2$ in both expressions is proportional to the technifermion condensate, as shown in equation 37. Hence, according to this model, the higgs and the technisigma VEV:s have a common origin, the technifermion condensate.

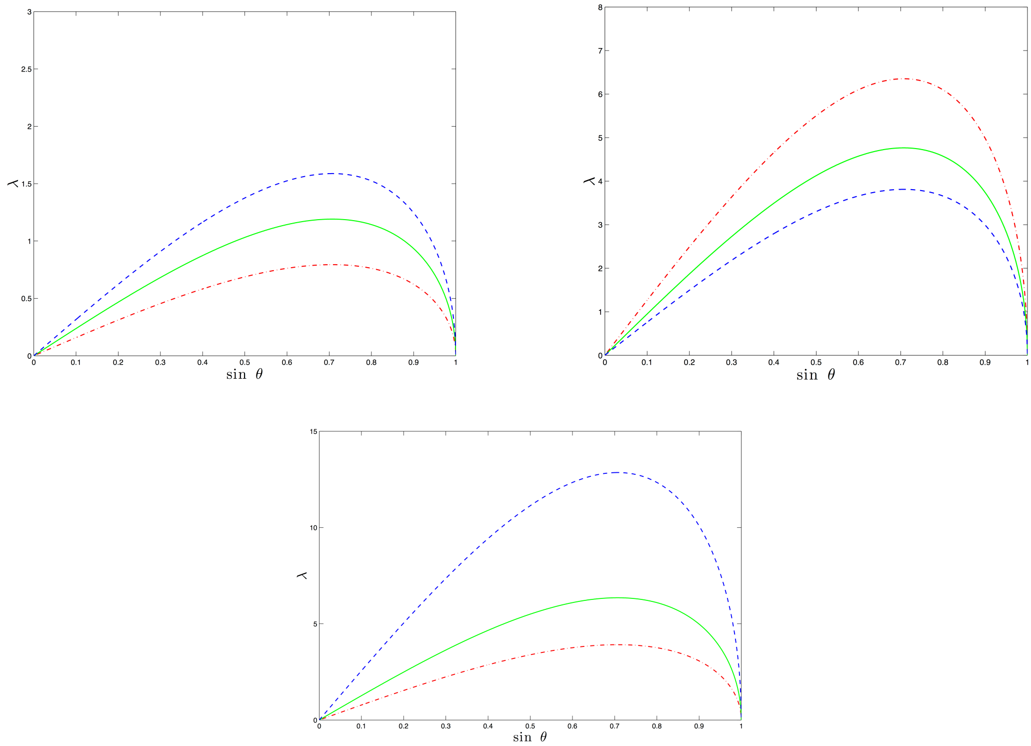


Figure 4: The coupling constant λ as a function of the mixing angle $\sin\theta$. The dashed-dotted (red), solid (green) and dashed (blue) lines in each plot correspond to the variation of one parameter at a time: (1) $g_{\mathrm{TC}} = 1,\ 1.5,\ 2,\ M_{\widetilde{Q}} = 300$ GeV and $M_{\widetilde{\sigma}} = 500$ GeV. (2) $g_{\mathrm{TC}} = 2$, $M_{\widetilde{Q}} = 300,\ 400,\ 500$ GeV and $M_{\widetilde{\sigma}} = 500$ GeV. (3) $g_{\mathrm{TC}} = 2$, $M_{\widetilde{Q}} = 300$ GeV and $M_{\widetilde{\sigma}} = 400,\ 500,\ 700$ GeV.

By introducing $\bar{g}_{\mathrm{TC}} = g_{\mathrm{TC}} \left|\left\langle \bar{\widetilde{Q}}\widetilde{Q} \right\rangle\right|$, $\beta = \lambda_{TC}\lambda_H - \lambda^2$ and $\xi\lambda = |\lambda|$, the VEV:s can be re-expressed as:

$$u^2 = \frac{\lambda_H m_{\widetilde{\pi}}^2}{\beta} = \frac{\lambda_H \bar{g}_{TC}}{u\beta} \quad \Leftrightarrow \quad u = \left(\frac{\lambda_H}{\beta}\right)^{1/3} \bar{g}_{TC}^{1/3}$$

$$v^2 = \frac{\lambda m_{\widetilde{\pi}}^2}{\beta} = \frac{\lambda \bar{g}_{\mathrm{TC}}}{u\beta} = \frac{\lambda \bar{g}_{\mathrm{TC}} \beta^{1/3}}{\bar{g}_{\mathrm{TC}}^{1/3} \lambda_H^{1/3} \beta} \quad \Leftrightarrow \quad v = \left(\frac{\xi\lambda}{\lambda_H}\right)^{1/2} \left(\frac{\lambda_H}{\beta}\right)^{1/3} \bar{g}_{\mathrm{TC}}^{1/3} \tag{61}$$

where the positive value of λ is ensured by defining ξ as $\mathrm{sgn}(M_{\widetilde{\sigma}}^2 - 3m_{\widetilde{\pi}}^2)$.

Hence, the VEVs are related accordingly:

$$u = v \left(\frac{\lambda_H}{\xi\lambda}\right)^{1/2}, \tag{62}$$

and the coupling constant can be expressed as:

$$\bar{g}_{\mathrm{TC}}^{1/3} = v \left(\frac{\lambda_H}{\xi\lambda}\right)^{1/2} \left(\frac{\beta}{\lambda_H}\right)^{1/3} \quad \Leftrightarrow \quad \bar{g}_{\mathrm{TC}} = v^3 \left(\frac{\lambda_H \lambda_{\mathrm{TC}}}{\lambda} - \lambda\right) \left(\frac{\lambda_H}{\xi\lambda}\right)^{1/2}$$

starting from the expression for v from equation 61.

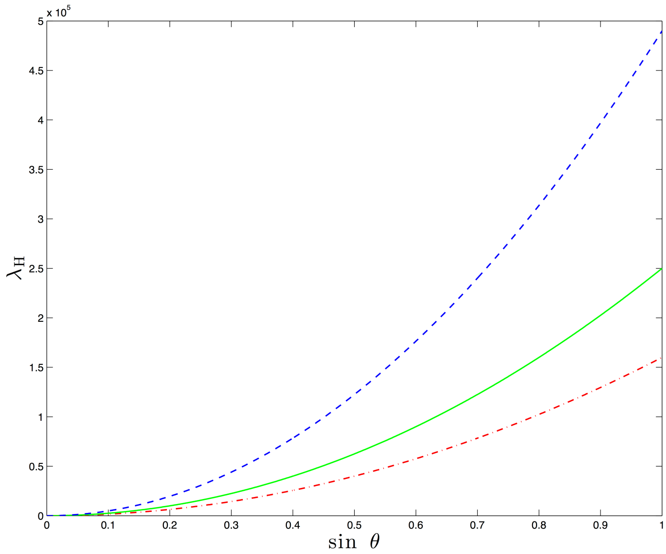


Figure 5: The coupling constant λ_H as a function of the mixing angle $\sin\theta$. The dashed-dotted (red), solid (green) and dashed (blue) lines in the plot corresponding to the variation of the technisigma mass: $M_{\widetilde{\sigma}} = 400,\ 500,\ 700$ GeV.

3.2.2 The Higgs and Technisigma Masses

The alteration of the h- and $\widetilde{\sigma}$ masses in the nearly-conformal limit, is found by re-expressing the original mass matrix of equation 40 (gauge eigenbasis), using equation 60 (top) and 62, and yields:

$$M = \begin{pmatrix} 3m_{\widetilde{\pi}}^2 + 2\lambda v^2 & -2v^2\sqrt{\xi\lambda\lambda_H} \\ -2v^2\sqrt{\xi\lambda\lambda_H} & 2\lambda_H v^2 \end{pmatrix} \tag{63}$$

where the masses are found from diagonalizing said mass matrix:

$$\Longrightarrow \lambda_m = \frac{1}{2}\Big[m_{11}^2 + m_{22}^2 \pm \sqrt{4m_{12}^2 m_{21}^2 + (m_{11}^2 - m_{22}^2)^2}\Big]$$
$$= \frac{1}{2}v^2\left[3\frac{m_{\widetilde{\pi}}^2}{v^2} + 2\lambda + 2\lambda_H \pm \sqrt{\left(3\frac{m_{\widetilde{\pi}}^2}{v^2} + 2\lambda - 2\lambda_H\right)^2 + 16\lambda\lambda_H}\right]$$

or by simply re-expressing the eigenvalues of equation 41, right away.

3.2.3 The Mixing Angle

The expression for the mixing angle in the nearly-conformal limit is found by re-expressing equation 49 (mass eigenbasis), using equation 60 (top) and 62:

$$\tan 2\theta = \frac{4v^2\sqrt{\xi\lambda\lambda_H}}{3m_{\widetilde{\pi}}^2 + 2\lambda v^2 - 2\lambda_H v^2}.$$

or, equivalently:

$$\sin\theta = \sin\left[\frac{1}{2}\arctan\left(\frac{4v^2\sqrt{\xi\lambda\lambda_H}}{3m_{\widetilde{\pi}}^2 + 2\lambda v^2 - 2\lambda_H v^2}\right)\right]. \tag{64}$$

3.2.4 The Parameter Space

We may once again re-express the scalar self-couplings in terms of the technipion- technisigma and Higgs mass:

$$\lambda = \frac{3m_{\widetilde{\pi}}^2(M_h^2 + M_{\widetilde{\sigma}}^2) - M_h^2 M_{\widetilde{\sigma}}^2 - 9m_{\widetilde{\pi}}^4}{6v^2 m_{\widetilde{\pi}}^2}, \tag{65}$$

$$\lambda_{TC} = \frac{\lambda}{\lambda_H}\left(\lambda + \frac{m_{\widetilde{\pi}}^2}{v^2}\right), \quad \lambda_H = \frac{M_h^2 M_{\widetilde{\sigma}}^2}{6v^2 m_{\widetilde{\pi}}^2}, \tag{66}$$

where the techniquark mass is implicitly included via the relation to λ_H. Since the mass of the Higgs is known, there are only three parameters in the nearly conformal limit, $m_{\widetilde{\pi}}$, $M_{\widetilde{\sigma}}$ and $M_{\widetilde{Q}}$. The scalar self-couplings, the mixing angle of equation 64 and the technisigma VEV, are plotted as functions of the technisigma mass, for a variation of technipion masses, as shown in figure 6.

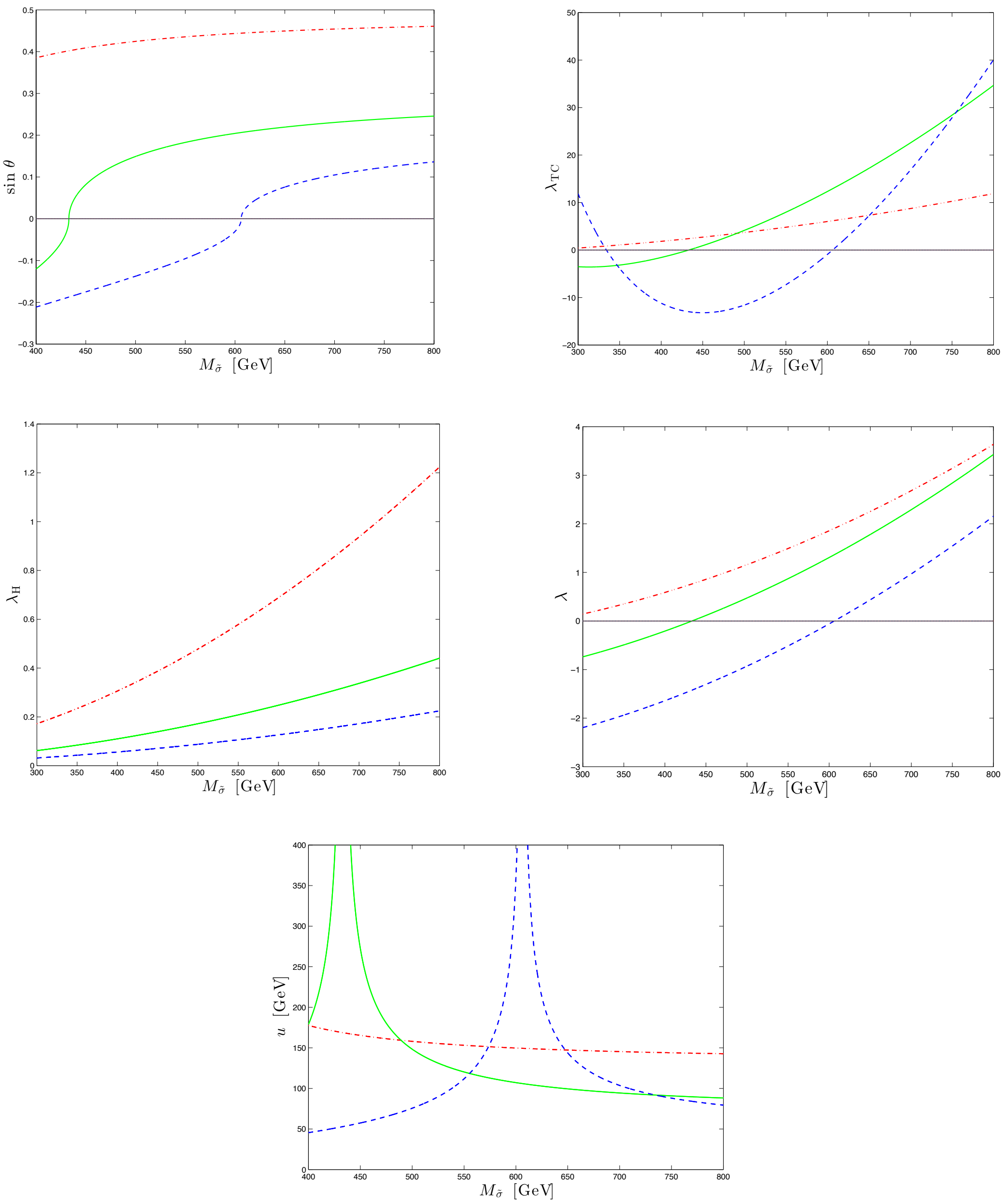


Figure 6: The mixing angle, scalar self-couplings and technisigma VEV, plotted as functions of the technisigma mass, for a variation of technipion masses. The dashed-dotted (red), solid (green) and dashed (blue) lines in each plot correspond to $m_{\widetilde{\pi}} = 150$,250 and 350 GeV, respectively.

3.3 The PT-parameters in the scenario of no scalar contribution

The corrections to the vacuum polarization functions of equation 34 may be re-divided into of scalar and non-scalar contributions:

$$\delta\Pi_{XY}(q^2) = \delta\Pi^{\rm sc}_{XY}(q^2) + \Pi^{\widetilde{\pi}}_{XY}(q^2, m^2_{\widetilde{\pi}}) + \Pi^{\widetilde{Q}}_{XY}(q^2, M^2_{\widetilde{Q}}), \tag{67}$$

where the scalar contribution is given by:

$$\delta\Pi^{\rm sc}_{XY}(q^2) = \Pi^{\widetilde{\sigma}}_{XY}(q^2, M^2_{\widetilde{\sigma}}) + \Pi^{h}_{XY}(q^2, M^2_h) - \Pi^{\rm SM,h}_{XY}(q^2, M^2_h). \tag{68}$$

It so happens that the scalar contribution can be expressed solely in terms of $\Pi_{XY}^{SM,h}$; the Higgs- and technisigma gauge interactions are related via:

$$\Pi_{XY}^{\widetilde{\sigma}}(q^2, M_{\widetilde{\sigma}}^2) = \frac{s_\theta^2}{c_\theta^2}\Pi_{XY}^{h}(q^2, M_h^2) = s_\theta^2 \Pi_{XY}^{SM,h}(q^2, M_h^2),$$

as seen from comparing their Lagrangians (equation 55 and 56), and by using that the modified Higgs contribution is equal to the SM result when multiplied with c_θ:

$$\Pi_{XY}^{h}(q^2, M_h^2) = c_\theta \Pi_{XY}^{SM,h}(q^2, M_h^2),$$

Hence the scalar contribution of equation 68 can be rewritten as:

$$\delta\Pi_{XY}^{\mathrm{sc}}(q^2) = s_\theta^2 \Pi_{XY}^{\mathrm{SM,h}}(q^2, M_{\widetilde{\sigma}}^2) - s_\theta^2 \Pi_{XY}^{\mathrm{SM,h}}(q^2, M_h^2) \tag{69}$$

That is, the scalar contribution is zero in the no-mixing limit, as well as in the case of degenerate Higgs and Technisigma masses. This is the scenario which will be considered in the analytical calculations in this thesis. This section begins with producing a list of integrals needed for the loop calculations, continues with evaluating the contributions from non-scalar particles, and finishes with calculating the corresponding PT-parameters.

3.3.1 List of Integrals

As will become apparent in the following section, all loop contributions considered in this thesis may be re-expressed in terms of two common integrals:

$$B_0(q^2, m_1^2, m_2^2) = \int \frac{1}{D_1 D_2}, \qquad A_0(m^2) = \int \frac{1}{D}, \tag{70}$$

where $D_1 = k^2 - m_1^2$ and $D_2 = (k-q)^2 - m_2^2$ are the two loop propagators. Note how $A_0(m^2)$ represent both $A_0(m_1^2)$ and $A_0(m_2^2)$. The denominator $D = k^2 - m^2$ can correspond to both D_1 and D_2, using $m = m_1$, or $m = m_2$ with the variable substitution $k = k + q$, respectively.

The loop contributions considered in this thesis, belong to the special case where the two propagators have degenerate masses, $m_1 = m_2 = m$. This section will however, to a start, treat the general case $m_1 \neq m_2$. The integrals A_0 and B_0, are pre-evaluated in this section, in order to provide a list of integrals to refer back to when the actual loop calculations begin. This section also includes how to re-express the commonly occuring integrals

$$B^\mu = \int \frac{k^\mu}{D_1 D_2}, \qquad B^{\mu\nu} = \int \frac{k^\mu k^\nu}{D_1 D_2} \tag{71}$$

in terms of A_0 and B_0. For clarity, definitions are presented in boxed equations, while the final form of the integrals are presented after a "therefore" sign ($\therefore$).

3.3.1.1 Evaluating $B_0(q^2, m_1^2, m_2^2)$ and the special case $B_0(q^2, m^2, m^2)$

The first integral of equation 70 is explicitly written as:

$$B_0(q^2, m_1^2, m_2^2) = \int \frac{d^d k}{(2\pi)^d}\frac{1}{D_1 D_2} = \int \frac{d^d k}{(2\pi)^d} \cdot \frac{1}{(k^2 - m_1^2 + i\epsilon)((k-q)^2 - m_2^2 + i\epsilon)},$$

where the denominators can be transformed into exponentials using α-parameterization, which is defined as:

$$\boxed{\frac{1}{(\Omega + i\epsilon)^\lambda} = \frac{i^{-\lambda}}{\Gamma(\lambda)}\int_0^\infty d\alpha \ \alpha^{\lambda-1} e^{i\Omega\alpha}}\ . \tag{72}$$

Using the parameterization twice - the first one with $\Omega = k^2 - m_1^2$ and $\lambda = 1$, and the second one with $\Omega = (k-q)^2 - m_2^2$ and $\lambda = 1$ - the integral becomes:

$$B_0(q^2, m_1^2, m_2^2) = i^{-2} \int_0^\infty d\alpha d\beta \int \frac{d^d k}{(2\pi)^d} \; e^{i(k^2 - m_1^2)\alpha} e^{i((k-q)^2 - m_2^2)\beta},$$

where the off-diagonal terms can be removed by a variable substitution:

$$k = k + \frac{q\beta}{\alpha+\beta} \quad \Rightarrow \quad B_0(p^2, m_1^2, m_2^2) = i^{-2} \int_0^\infty d\alpha d\beta \; e^{\frac{iq^2\beta\alpha}{\alpha+\beta} - im_1^2\alpha - im_2^2\beta} \int \frac{d^d k}{(2\pi)^d} \; e^{ik^2(\alpha+\beta)}.$$

The integral over momentum k is simply a Gaussian integral, generally expressed as:

$$\boxed{\int \frac{d^d k}{(2\pi)^d} e^{i(\alpha k^2 - 2qk)} = \frac{i^{1-d/2}}{(4\pi)^{d/2}} e^{-iq^2/\alpha} \frac{1}{\alpha^{d/2}}} \tag{73}$$

with $q = 0$ and $\alpha = \alpha + \beta$ in our case. The integral is then given by:

$$B_0(q^2, m_1^2, m_2^2) = \frac{i^{-1-d/2}}{(4\pi)^{d/2}} \int_0^\infty d\alpha d\beta \; \frac{1}{(\alpha+\beta)^{d/2}} e^{\frac{iq^2\beta\alpha}{\alpha+\beta} - im_1^2\alpha - im_2^2\beta}.$$

The final integration is performed using Feynman parameters:

$$\boxed{\begin{array}{l} \alpha = Lx, \quad \beta = L\bar{x} = L(1-x) \\ \text{where } \; L \in (0, \infty), \quad x \in (0,1) \end{array}} \tag{74}$$

where the shift from $d\alpha d\beta$ to $dLdx$ involves a Jacobian of value L:

$$J = \frac{\partial\alpha\partial\beta}{\partial L \partial x} = \begin{vmatrix} \frac{\partial\alpha}{\partial x} & \frac{\partial\beta}{\partial x} \\ \frac{\partial\alpha}{\partial L} & \frac{\partial\beta}{\partial L} \end{vmatrix} = L(1-x) + Lx = L.$$

Ergo, the integral may be rewritten as:

$$B_0(q^2, m_1^2, m_2^2) = \frac{i^{-1-d/2}}{(4\pi)^{d/2}} \int_0^1 dx \int_0^\infty dL \;\; L^{1-d/2} e^{iL(x\bar{x}q^2 - m_1^2 x - m_2^2 \bar{x})}.$$

where the integral over L is performed through another α-parameterization, using $\alpha = L$, $A = q^2 x\bar{x} - m_1^2 x - m_2^2 \bar{x}$ and $\lambda = 2 - d/2$ in (72):

$$B_0(q^2, m_1^2, m_2^2) = \frac{i^{1-d/2}\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 dx \frac{1}{(q^2 x\bar{x} - m_1^2 x - m_2^2 \bar{x})^{2-d/2}} \; .$$

The integral can be re-written in a manner preferable for numerical evaluation, using that $i^d = (-1)^{2-d/2}$:

$$\therefore \; B_0(q^2, m_1^2, m_2^2) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{D_1 D_2} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 dx \frac{1}{(-q^2 x\bar{x} + m_1^2 x + m_2^2 \bar{x})^{2-d/2}} \; .$$

In the special case of equal masses, $m_1 = m_2 = m$, the integral simplifies to:

$$\therefore \; B_0(q^2, m^2, m^2) = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 dx \frac{1}{(-q^2 x\bar{x} + m^2)^{2-d/2}}, \tag{75}$$

using that $\bar{x} = 1 - x$.

$B_0(q^2, m_1^2, m_2^2)$ and $B_0(q^2, m^2, m^2)$ can be evaluated by returning to four dimensions, $d = 4 - 2\epsilon$:

$$\Rightarrow B_0(q^2, m_1^2, m_2^2) = \frac{i\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}} \int_0^1 dx \frac{1}{(-q^2 x\bar{x} + m_1^2 x + m_2^2 \bar{x})^\epsilon}$$

and

$$B_0(q^2, m^2, m^2) = \frac{i\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}} \int_0^1 dx \frac{1}{(-q^2 x\bar{x} + m^2)^\epsilon}$$

The evaluation of $B_0(q^2, m_1^2, m_2^2)$ and $B_0(q^2, m^2, m^2)$ hence depend on the values of the momentum and mass of the propagators. For example, if $q = 0$, $B_0(q^2, m^2, m^2)$ becomes:

$$B_0(0, m^2, m^2) = \frac{i\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}(m^2)^\epsilon}$$

which can be evaluated as a Taylor expansion. In order to obtain a dimensionless logarithm in the expansion of this expression, and to further simplify the obtained result, one may use the $\overline{MS}$-scheme. The $\overline{MS}$-scheme involves multiplying $B_0(q^2, m_1^2, m_2^2)$ with:

$$(\mu^2)^\epsilon f(\epsilon),$$

where

$$f(\epsilon) = \frac{e^{\gamma_E \epsilon}}{(4\pi)^\epsilon}.$$

The symbol γ_E is the Euler–Mascheroni constant (~ 0.577), the factor $(\mu^2)^\epsilon$ ensures the mass dimension zero, and $(1/4\pi)^\epsilon$ cancels the $(1/4\pi)^{-\epsilon}$ from $B_0(q^2, m_1^2, m_2^2)$:

$$\Rightarrow B_0(0, m^2, m^2) = \frac{i}{(4\pi)^2 \Gamma(\epsilon) e^{\gamma_E \epsilon}} \left(\frac{m^2}{\mu^2}\right)^{-\epsilon} \simeq \frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon} - ln\left(\frac{m^2}{\mu^2}\right)\right)$$

where the expression has been Taylor expanded using Mathematica, around zero, to the first order.

The divergent term $1/\epsilon$ will cancel automatically in all diagrams considered in this thesis. If it does not, it can often be removed using re-scaling parameters (a method which is not covered here). Divergences are not allowed in physical quantities.

3.3.1.2 Evaluating $A_0(m^2)$

The second integral of equation 70 is explicitly written as:

$$A_0(m^2) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{D} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i\epsilon)}.$$

The general case

$$A_0^n(m^2) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{D^n} = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i\epsilon)^n},$$

is considered in Appendix D. However, for the loop diagrams treated in this thesis, only the special case $n = 1$ occurs.

$A_0(m^2)$ is evaluated in a manner equivalent to the previous integral, $B_0(q^2, m_1^2, m_2^2)$. The denominator is once again re-expressed as an exponential using α-parameterization (72), with $\lambda = 1$ and $\Omega = k^2 - m^2$:

$$\Rightarrow \quad A_0 = i^{-1} \int \frac{d^d k}{(2\pi)^d} \int_0^\infty d\alpha \;\; e^{i(k^2 - m^2)\alpha},$$

after which $e^{ik^2\alpha}$, integrated over momentum, is identified as a Gaussian integral (73), with $\alpha = \alpha$ and $q = 0$. Hence, the integral may be rewritten as:

$$A_0(m^2) = \frac{i^{-d/2}}{(4\pi)^{d/2}} \int_0^\infty d\alpha \;\; \alpha^{-d/2} e^{-im^2\alpha}.$$

Since there is only one D-term in the denominator, there is no β-parameter, and hence the Feynman parameterization is not required. Instead, the integral is evaluated by doing another α-parameterization right away, using $\Omega = -m^2$ and $\lambda = 1 - d/2$ in (72), producing the final result:

$$\therefore \; A_0(m^2) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{D} = \frac{i^{1-d}\Gamma(1-d/2)}{(4\pi)^{d/2}(-m^2)^{1-d/2}} = \frac{(-i)\Gamma(1-d/2)}{(4\pi)^{d/2}(m^2)^{1-d/2}}.$$

$A_0(m^2)$ is evaluated by changing from d to 4 dimensions, $d = 4 - 2\epsilon$, which yields:

$$A_0(m^2) = \frac{(-i)\Gamma(-1+\epsilon)}{(4\pi)^{2-\epsilon}(m^2)^{-1+\epsilon}}$$

Using the $\overline{MS}$-scheme, $A_0(m^2) \to A_0(m^2) \cdot (\mu^2)^\epsilon f(\epsilon)$, yields:

$$\Rightarrow A_0(m^2) = \frac{(-i)m^2}{(4\pi)^2}\Gamma(-1+\epsilon)\left(\frac{\mu^2}{m^2}\right)^\epsilon e^{\gamma_E \epsilon} \;\; \simeq \;\; \frac{im^2}{(4\pi)^2}\left(\frac{1}{\epsilon} + 1 - ln\left(\frac{m^2}{\mu^2}\right)\right)$$

where the expression has been Taylor expanded in Mathematica, around zero, to the first order. Comparing with the result for $B_0(0, m^2, m^2)$, we find the relation:

$$A_0(m^2) = \big(B_0(0, m^2, m^2) + 1\big)m^2$$

3.3.1.3 Expressing B^μ in terms of $A_0(m^2)$ and $B_0(q^2, m_1^2, m_2^2)$ or the special case $B_0(q^2, m^2, m^2)$

The explicit form of $B\mu$ is:

$$B^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{D_1 D_2} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{(k^2 - m_1^2 + i\epsilon)((k-q)^2 - m_2^2 + i\epsilon)},$$

which is an easily solved integral after turning the numerator into a scalar, i.e. after multiplying the numerator by q_μ. This multiplication may be performed by identifying that the integral is equal to a function which must contain a vector. The only choice of vector is the momentum q, since mass is a scalar, hence:

$$B^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{D_1 D_2} = q^\mu B_1(q^2, m_1^2, m_2^2). \tag{76}$$

Now both sides may be multiplied with q_μ:

$$q_\mu B^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{kq}{D_1 D_2} = q^2 B_1(q^2, m_1^2, m_2^2),$$

where the scalar integral is easily simplified by expressing the numerator in terms of D_1 and D_2:

$$kq = \frac{1}{2}(D_1 - D_2 + q^2 - m_2^2 + m_1^2) \tag{77}$$

$$\Rightarrow q_\mu B^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{kq}{D_1 D_2} = \frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \left(\frac{1}{D_2} - \frac{1}{D_1} + \frac{q^2}{D_1 D_2} - \frac{m_2^2}{D_1 D_2} + \frac{m_1^2}{D_1 D_2}\right)$$

$$= \frac{1}{2}\big(A(m_1^2) - A(m_2^2) + (q^2 - m_2^2 + m_1^2)B(q^2, m_1^2, m_2^2)\big) = q^2 B_1(q^2, m_1^2, m_2^2)$$

$$\Leftrightarrow \ \therefore \ B_1(q^2, m_1^2, m_2^2) = q_\nu B^\mu / q^2 = \frac{1}{2q^2}\big(A(m_1^2) - A(m_2^2) + (q^2 - m_2^2 + m_1^2)B_0(q^2, m_1^2, m_2^2)\big). \quad (78)$$

where B_1 is inserted into (76) for the final result. In the equal masses scenario, the result simplifies to:

$$B_1(q^2, m^2, m^2) = \frac{B_0(q^2, m^2, m^2)}{2}$$

$$\Leftrightarrow \ \therefore \ B^\mu = \frac{q^\mu}{2} B_0(q^2, m^2, m^2). \quad (79)$$

3.3.1.4 Expressing $B^{\mu\nu}$ in terms of A_0 and B_0

The explicit form of $B^{\mu\nu}$ is given by:

$$B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{D_1 D_2}$$

which in accordance with the previous subsection may be re-expressed as:

$$B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{D_1 D_2} = g^{\mu\nu} B_{00} + q^\mu q^\nu B_{11}. \quad (80)$$

The integral can be solved in two ways, either by multiplying both sides with q_μ:

$$q_\mu B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{(kq)k^\nu}{D_1 D_2} = q^\mu (B_{00} + q^2 B_{11}), \quad (81)$$

or by multiplying both sides with $g_{\nu\mu}$:

$$g_{\nu\mu} B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{k^2}{D_1 D_2} = q^2 B_{11} + d B_{00} \quad (82)$$

where d is the number of dimensions, since $g^{\nu\mu} g_{\mu\nu} = tr(\delta^\mu_\mu) = d$. The two possible solutions gives us two equations, (81) and (82) to solve for our two unknowns, B_{00} and B_{11}. We start by evaluating (81), where (kq) is given by equation 77:

$$\Rightarrow \ q_\mu B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{(kq)k^\nu}{D_1 D_2} = \frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \frac{(D_1 - D_2 + q^2 + m_1^2 - m_2^2)k^\nu}{D_1 D_2}$$

$$= \frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \left(\cancelto{0}{\frac{k^\nu}{D_1}} - \underbrace{\frac{q^\nu}{D_1}}_{q^\nu A_0} - \cancelto{0}{\frac{k^\nu}{D_1}} + \underbrace{\frac{k^\nu}{D_1 D_2}}_{B^\nu = q^\nu B_1} (q^2 + m_1^2 - m_2^2) \right)$$

$$\Leftrightarrow \ \therefore \ q_\mu B^{\mu\nu} = \frac{q^\nu}{2}\Big[- A_0(m_1^2) - (q^2 + m_1^2 - m_2^2) B_1(q^2, m_1^2, m_2^2)\Big] = q^\mu (B_{00} + q^2 B_{11}) \quad (83)$$

where the variable substitution $k = k - q$ was used on the first term k^ν/D_2. The integral $A^\nu = k^\nu/D_1$ is zero since:

$$A^\nu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\nu}{D_1} = q^\nu A_1$$

$$\Leftrightarrow q_\nu A^\nu = \int \frac{d^dk}{(2\pi)^d}\frac{kq}{D_1} = -\frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\left(\cancelto{0}{1} - \frac{D_2}{D_1} + \underbrace{\frac{1}{D_1}}_{A_0(m_1^2)}(q^2+m_1^2-m_2^2)\right) = 0 = q^2 A_1$$

using that

$$\int \frac{d^dk}{(2\pi)^d}\frac{D_2}{D_1} = \int \frac{d^dk}{(2\pi)^d}\frac{k^2 - \cancelto{0}{2kq} + q^2 - m_2^2}{D_1} = \int \frac{d^dk}{(2\pi)^d}\left(\cancelto{0}{1} + \frac{q^2-m_2^2+m_1^2}{D_1}\right)$$
$$= A(m_1^2)\big[q^2+m_1^2-m_2^2\big].$$

The 1 is removed by using the fact that the integral of any constant in dimensional regularization is equal to zero.

The second equation for determining B_{00} and B_{11} is obtained from (82):

$$\therefore\ g_{\nu\mu}B^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d}\frac{k^2}{D_1D_2} = \int \frac{d^dk}{(2\pi)^d}\left(\frac{1}{D_2} + \frac{m_1^2}{D_1D_2}\right)$$
$$= A_0(m_2^2) + m_1^2 B_0(q^2,m_1^2,m_2^2) = q^2B_{11} + dB_{00}. \tag{84}$$

With (83) and (84), we now have two equations for solving for B_{00} and B_{11} via Gaussian elimination. In the degenerate mass scenario, the equations simplify to:

$$\begin{cases} q_\mu B^{\mu\nu} = \frac{q^\nu}{2}\Big[-A_0(m^2) - q^2B_1(q^2,m^2,m^2)\Big] = q^\mu(B_{00}+q^2B_{11}) \\ g_{\nu\mu}B^{\mu\nu} = A_0(m^2) + m^2B_0(q^2,m_1^2,m_2^2) = q^2B_{11} + dB_{00}, \end{cases} \tag{85}$$

where the Gaussian elimination yields:

$$B_{11}(q^2,m^2,m^2) = \frac{1}{18q^2}(q^2-6m^2) + \frac{1}{3q^2}A_0(m^2) + \frac{1}{3q^2}(q^2-m^2)B_0(q^2,m^2,m^2)$$
$$B_{00}(q^2,m^2,m^2) = -\frac{1}{18}(q^2-6m^2) + \frac{1}{6}A_0(m^2) - \frac{1}{12}(q^2-4m^2)B_0(q^2,m^2,m^2) \tag{86}$$

The expression for B_{11} and B_{00} are then inserted into equation 80, for the final result of $B^{\mu\nu}$.

In equation 86, we used that the divergent part[4] of B_{00} (for equal masses, taken from literature) is:

$$\mathrm{div}(B_{00}) = -\frac{1}{4}\left(\frac{1}{3}q^2 - m_1^2 - m_2^2\right)$$
$$\Rightarrow\ dB_{00} = (4-2\epsilon)B_{00} = 4B_{00} + \frac{1}{6}\left(q^2-6m^2\right).$$

[4]The divergent part is the expression times $1/\epsilon$ in the Taylor expansion, i.e. m^2 for $A_0(q^2)$, or 1 for $B_0(0)$.

3.3.2 Loop evaluations

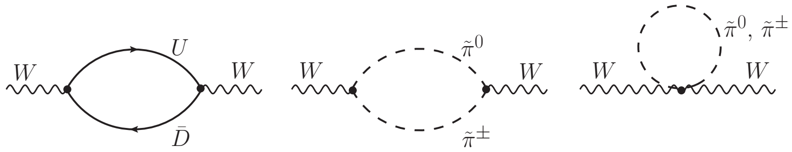


Figure 7: *Self-energy contributions from techniquarks and technipions with W-legs.*

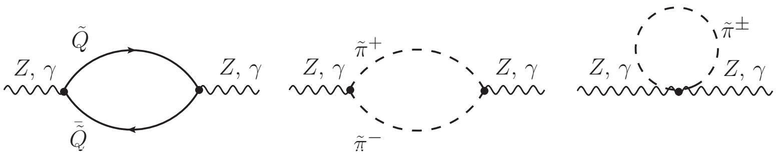


Figure 8: *Self-energy contributions from techniquarks and technipions with Z- and γ legs.*

3.3.2.1 The Technifermion Contribution

The self-energy contribution from technifermions, comes from the left images of figure 7 and 8. Such a two-propagator fermion loop is given by:

$$\begin{aligned}\Pi^{\mu\nu}_{\text{ferm}} &= b_{XY} \int \frac{d^4k}{(2\pi)^4} (i\gamma^\nu_{kj}) \frac{i(\not{k}+m)_{ji}}{k^2-m^2+i\epsilon} (i\gamma^\mu_{il}) \frac{i((\not{k}-\not{q})+m)_{lk}}{(k-q)^2-m^2+i\epsilon} \\ &= b_{XY} \int \frac{d^4k}{(2\pi)^4} \frac{tr(\gamma^\nu(\not{k}+m)\gamma^\mu((\not{k}-\not{q})+m))}{D_1 D_2} \end{aligned} \tag{87}$$

where $(i\gamma^\mu)$ comes from the vertices, and where b_{XY} denotes the rest of the vertex factor, unique to each diagram:

$$b_{\gamma\gamma} = e^2 q_f^2, \quad b_{\gamma Z} = e^2 q_f^2 \frac{c_w}{s_w} \quad b_{ZZ} = e^2 q_f^2 \frac{c_w^2}{s_w^2}, \quad b_{WW} = \frac{e^2}{2s_w^2}.$$

The vertex factors can be read straight off the Lagrangian in equation 53 (adding a $i\gamma^\mu$-factor), and are also verified using the Mathematica program FeynArts. The subscript w denotes the Weinberg angle and $\not{k} = \gamma^\alpha k_\alpha$. The technifermion charge for zero hypercharge is given by $Q = T_3 + Y_Q/2 = T_3 = \pm 1/2$.

Continuing with the loop evaluation, the trace in equation 87, may be simplified to:

$$\begin{aligned}\text{tr}(\gamma^\nu(\gamma^\alpha k_\alpha + m)\gamma^\mu((\gamma^\beta k_\beta - \gamma^\sigma q_\sigma) + m)) &= \text{tr}(\gamma^\nu\gamma^\alpha\gamma^\mu\gamma^\beta)k_\alpha k_\beta - \text{tr}(\gamma^\nu\gamma^\alpha\gamma^\mu\gamma^\sigma)k_\alpha q_\sigma \\ +m^2\text{tr}(\gamma^\nu\gamma^\mu) &= 4(2k^\nu k^\mu - g^{\nu\mu}k^2) - 4(k^\nu q^\mu - g^{\nu\mu}kq + k^\mu q^\nu) + 4g^{\nu\mu}m^2\end{aligned}$$

using that the trace of any odd number of gamma matrices is equal to zero, and that $\text{tr}(\gamma^\nu\gamma^\alpha\gamma^\mu\gamma^\beta) = 4(g^{\nu\alpha}g^{\mu\beta} - g^{\nu\mu}g^{\alpha\beta} + g^{\nu\beta}g^{\alpha\mu})$. Hence, the integral becomes (switching to d-dimensions):

$$\begin{aligned}\Pi^{\mu\nu}_{\text{ferm}} &= 4e^2 q_f^2 \int \frac{d^dk}{(2\pi)^d} \cdot \frac{g^{\mu\nu}(m^2-k^2+kq) + 2k^\nu k^\mu - k^\nu q^\mu - k^\mu q^\nu}{D_1 D_2} \\ &= 4e^2 q_f^2 \int \frac{d^dk}{(2\pi)^d} \left[g^{\mu\nu}\left(- \underbrace{\frac{\cancel{D_1}}{\cancel{D_1} D_2}}_{A_0} + q_\mu \underbrace{\frac{k^\mu}{D_1 D_2}}_{B^\mu} \right) + 2 \underbrace{\frac{k^\nu k^\mu}{D_1 D_2}}_{B^{\mu\nu}} - q^\mu \underbrace{\frac{k^\nu}{D_1 D_2}}_{B^\nu} - q^\nu \underbrace{\frac{k^\mu}{D_1 D_2}}_{B^\mu} \right]\end{aligned}$$

where B^μ and $B^{\mu\nu}$ can be re-expressed in terms of A_0 and B_0, using equation 79 and 80:

$$\Rightarrow \therefore\ \Pi^{\mu\nu}_{\text{ferm}} = 4e^2 q_f^2 \Big[g^{\mu\nu} \left(-A_0 + q^2 \frac{B_0}{2} + 2B_{00} \right) + q^\mu q\nu \Big(2B_{11} - B_0 \Big) \Big] \tag{88}$$

where B_{00} and B_{11} are defined in equation 86. We now wish to obtain the scalar expression, Π_{ferm}, using the gauge invariance:

$$\Pi^{\mu\nu}_{\text{ferm}} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_{\text{ferm}}. \tag{89}$$

By re-writing the expression in (88):

$$\Pi^{\mu\nu}_{\text{ferm}} = g^{\mu\nu} \left[-A_0 + q^2 \frac{B_0}{2} + 2B_{00} \right] - \frac{q^\mu q^\nu}{q^2} \left[q^2 B_0 - 2q^2 B_{11} \right],$$

we see that the two brackets must be *equal* to satisfy (89), and that both of them in fact are $\Pi_{\gamma\gamma}$:

$$\Pi_{\text{ferm}} = -A_0 + q^2 \frac{B_0}{2} + 2B_{00} = q^2 B_0 - 2q^2 B_{11}$$

$$\Leftrightarrow\ \Pi_{\text{ferm}} = \frac{1}{3} \left(-2A_0 - \frac{1}{3} q^2 + 2m_{\tilde{\sigma}}^2 + B_0 (q^2 + 2m_{\tilde{Q}}^2) \right) \equiv \frac{1}{3} F_{\tilde{Q}} \tag{90}$$

where 1/3 has been taken out of the expression for aesthetic reasons.

The result for all four technifermion contributions may be gathered in a general expression:

$$\Pi^{\tilde{Q}}_{XY} \propto g_1^2 N_{\text{TC}} K_{XY} F_{\tilde{Q}} \tag{91}$$

where $g_1 = e/s_w$, $N_{\text{TC}} = 3$ (number of technicolors), and where K_{XY} is given by:

$$K_{\gamma\gamma} = s_w^2, \quad K_{\gamma Z} = c_w s_w, \quad K_{ZZ} = c_w^2 \quad \text{and} \quad K_{WW} = 1 \tag{92}$$

Normalization constants and other factors may be ignored as long as they are identical for the techniquark- and the technipion contribution, for reasons that will become apparent when calculating the PT-parameters.

3.3.2.2 The Technipion Contribution

The technipion contribution appear as diagrams with both one and two propagators, as shown in the middle- and right images of figure 7 and 8. The one-propagator diagrams are given by:

$$\Pi^{\mu\nu}_{XY,1} = i c_{XY} g^{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \frac{i}{(k^2 - m^2)} = -c_{XY} g^{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \frac{i}{D} = -c_{XY} g^{\mu\nu} A_0 \tag{93}$$

where i/D is the propagator for pseudoscalars (and scalars). The factor $ig^{\mu\nu}$ has been taken out of the vertices, obtained in FeynArts. The remaining part of the vertex factors, specific to each diagram, is given by c_{XY}:

$$c^{\tilde{\pi}^0}_{WW} = \frac{e^2}{s_w^2}, \quad c^{\tilde{\pi}^+,\tilde{\pi}^-}_{WW} = \frac{e^2}{s_w^2}, \quad c^{\tilde{\pi}^+,\tilde{\pi}^-}_{ZZ} = \frac{2c_w^2 e^2}{s_w^2},$$

$$c^{\tilde{\pi}^+,\tilde{\pi}^-}_{\gamma Z} = \frac{2c_w e^2}{s_w} \quad \text{and} \quad c^{\tilde{\pi}^+,\tilde{\pi}^-}_{\gamma\gamma} = 2e^2. \tag{94}$$

Before recovering the scalar version of the form factor, we must add the two different tensor contributions, $\Pi^{\mu\nu}_{XY} = \Pi^{\mu\nu}_{XY,1} + \Pi^{\mu\nu}_{XY,2}$, where subscript 1 denotes the one-propagator diagrams, and 2 denotes the two-propagator diagrams. The two-propagator diagrams are given by:

$$\Pi^{\mu\nu}_{XY,2} = a_{XY} \int \frac{d^dk}{(2\pi)^d} \frac{i}{(k^2 - m^2)} \frac{i}{((k-q)^2 - m^2)} (2k^\mu - q^\mu)(2k^\nu - q^\nu)$$

$$= -a_{XY} \int \frac{d^dk}{(2\pi)^d} \left(4 \underbrace{\frac{k^\mu k^\nu}{D_1 D_2}}_{B^{\mu\nu}} - 2q^\nu \underbrace{\frac{k^\mu}{D_1 D_2}}_{B^\mu} - 2q^\mu \underbrace{\frac{k^\nu}{D_1 D_2}}_{B^\nu} + q^\mu q^\nu \underbrace{\frac{1}{D_1 D_2}}_{B_0} \right)$$

$$= -a_{XY} 4 g^{\mu\nu} B_{00}$$

since all terms involving $q^\mu q^\nu$ are zero for on-shell gauge bosons, $q_\mu e^\mu = q_\nu e^\nu = 0$, where e^μ, e^ν are polarization vectors. The factor $(2k^\mu - q^\mu)(2k^\nu - q^\nu)$ comes from the vertices obtained in FeynArts. The rest of each vertex is unique for each diagram, and given by the constant a:

$$a^{\widetilde{\pi}^0\widetilde{\pi}^+,\widetilde{\pi}^0\widetilde{\pi}^-}_{WW} = -\frac{e^2}{s_w^2}, \quad a^{\widetilde{\pi}^+,\widetilde{\pi}^-}_{ZZ} = -\frac{e^2 c_w^2}{s_w^2},$$

$$a^{\widetilde{\pi}^+,\widetilde{\pi}^-}_{\gamma Z} = -\frac{e^2 c_w}{s_w} \quad \text{and} \quad a^{\widetilde{\pi}^+,\widetilde{\pi}^-}_{\gamma\gamma} = -e^2. \tag{95}$$

Adding the two tensor contributions for each type of legs:

$$\Pi_{WW} = \Pi_{XY,1} \left(c^{\widetilde{\pi}^+,\widetilde{\pi}^-}_{WW} + c^{\widetilde{\pi}^0}_{WW} \right) + \Pi_{XY,2}\, a^{\widetilde{\pi}^0\widetilde{\pi}^+,\widetilde{\pi}^0\widetilde{\pi}^-}_{WW}$$

etc. yields the same expression for any type of legs, namely:

$$\Pi^{\mu\nu}_{XY,2} = \Pi^{\mu\nu}_{XY,1} + \Pi^{\mu\nu}_{XY,2} \propto A_0 - 2B_{00} = \frac{2}{3} A_0 + \frac{1}{9}(q^2 - 6m^2_{\widetilde{\pi}}) + \frac{1}{6}(p^2 - 4m^2_{\widetilde{\pi}}) B_0$$

$$\propto 2A_0 + \frac{1}{3} q^2 - 2m^2_{\widetilde{\pi}} + \frac{1}{2}(p^2 - 4m^2_{\widetilde{\pi}}) B_0 \equiv F_{\widetilde{\pi}} \tag{96}$$

The general expression for the technipion contribution is therefore given by:

$$\Pi^{\widetilde{\pi}}_{XY} \propto g_1^2 K_{XY} F_{\widetilde{\pi}}, \tag{97}$$

where $F_{\widetilde{\pi}}$ is given in (96) and where K_{XY} is given by the same expressions as for the techniquark contribution, seen in (92).

3.3.3 Evaluation of the PT-parameters

The S, T, U-parameters can be evaluated using the loop contributions of equation 91 and 97. For linear order in q^2 (33), *all* terms are of the form

$$\delta\Pi'^{\widetilde{Q}+\widetilde{\pi}}_{XY}(0) = \frac{d}{dq^2}\left(\Pi^{\widetilde{Q}}_{XY}(0) + \Pi^{\widetilde{\pi}}_{XY}(0) \right) = \frac{d}{dq^2}\left[g_1^2 K_{XY} \left(3F_{\widetilde{Q}}(0) + F_{\widetilde{\pi}}(0) \right) \right]$$

$$= g_1^2 K_{XY} \left(\frac{7}{2} B_0 - \frac{2}{3} \right) \equiv g_1^2 K_{XY} \hat{\Sigma} \tag{98}$$

which results in the S and U parameters being zero (the T parameter is zero from the start in linear order in q^2):

$$\frac{\alpha S}{4 s_W^2 c_W^2} = \delta\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0)$$

$$= g_1^2 \hat{\Sigma} \left[K_{ZZ} - \frac{c_W^2 - s_W^2}{s_W c_W} K_{Z\gamma} - K_{\gamma\gamma} \right] = g_1^2 \hat{\Sigma} \left[c_w^2 - \frac{c_W^2 - s_W^2}{s_W c_W} c_w s_w - s_w^2 \right] = 0, \tag{99}$$

and

$$\frac{\alpha U}{4s_w^2} = \Big[\delta\Pi'_{WW}(0) - c_w^2\delta\Pi'_{ZZ}(0) - 2s_W c_W \delta\Pi'_{Z\gamma}(0) - s_W^2\delta\Pi'_{\gamma\gamma}(0)\Big]$$

$$= g_1^2\hat{\Sigma}\left[K_{WW} - K_{ZZ} - 2s_w c_w K_{Z\gamma} - s_w^2 K_{\gamma\gamma}\right] = g_1^2\hat{\Sigma}\left[1 - c_w^4 - 2s_w^2 c_w^2 - s_w^4\right] = 0. \qquad (100)$$

4 Discussion

4.1 The Gauged Linear-Sigma Model

The considered technicolor extension borrows its low-energy effective field theory from ChPT in QCD. To my understanding, this is a commonly occuring procedure among technicolor extensions. The assumption that the two phenomenologies would correspond perfectly is perhaps a bit naive, but very convenient. It allow us to directly borrow the results developed for QCD, such as the mass spectrum and the gauging and identification of symmetry groups.

What separates our extension from many other technicolor extensions, is that there is no distinction between left- and righthanded techniquarks, which leads to degenerate masses and a spontaneously broken symmetry. Note that the chiral symmetry of our scalar Lagrangian may be broken (and constructed) without including a source term. Including the source term is a conscious decision, in order to yield a common origin of the Higgs- and the technisigma VEV in the nearly conformal limit. However, the source term is allowed under the symmetry.

Besides choosing our scalar field to be $\Sigma = \frac{1}{2}(S + i\tau_a P_a)$ (which was the field commonly used in QCD before the non-linear sigma model was introduced), and besides including the source term, we have not made any assumptions when constructing the Lagrangian. The Lagrangian simply consists of all allowed combinations, i.e. gauge invariant terms with mass dimension 4.

4.2 The Parameter Space

4.2.1 Before Introducing the Nearly-Conformal Limit

The parameters $M_{\widetilde{Q}}$, $M_{\widetilde{\sigma}}$ and $m_{\widetilde{\pi}}$ (figure 3, 4 and 5), are varied in a range similar to the original article. They cannot be varied to a much greater extent assuming there being a factor 1000 difference between the confinement scales, and that the particles possess the same mass hierarchy as in QCD. The scale may of course be larger than this. However, we are not too interested in such scenario, since if the masses are too large, they cannot be found at the LHC anyway. Apart from that consideration, there are, to my knowledge, no limitations apart from that the techniquark mass should be of order Λ_{TC}, since it is proportional to u (45), and that the technipion should be small with respect to the confinement scale (in order to be considered a pseudo-Goldstone boson).

The variation of g_{TC}, on the other hand, is chosen differently from the one in the original paper. The original paper chose values ranging from 2 to 8 (which are values often considered in QCD), while I chose values ranging from 1 to 2. As argued for previously, there should be no risk for larger values of g_{TC} than 2 for $M_{\widetilde{Q}} = 300$ GeV, since it is given by $M_{\widetilde{Q}}/u$, where $u \gtrsim v \simeq 246$ GeV.

The shape of the plots can easily be verified by equations 57, 58 and 59. For example, the zero-crossing for subfigure 3.1 is given by (for $m_{\widetilde{\pi}} = 150$ GeV, $M_{\widetilde{\sigma}} = 500$ GeV and $M_h = 126$ GeV):

$$\lambda_{TC} = \left(-150^2 + 500^2(1 - s_\theta^2) + 126^2 s_\theta^2\right) = 0 \quad \Leftrightarrow \quad s_\theta \simeq \pm 0.986$$

according to (57), which also agrees with the plot. Subfigure 3.3 looks a bit different from the others, since the variation of the technipion mass causes the zero-crossing to shift.

The plots displays how the coupling constant λ vanishes both in the no- and the maximal $h\widetilde{\sigma}$-mixing limit, while λ_H and λ_{TC} only vanishes in the maximal mixing limit (for any value of $M_{\widetilde{\sigma}}$, $M_{\widetilde{Q}}$ and g_{TC}, and for low values of $m_{\widetilde{\pi}}$). λ and λ_H remains at rather constrained values for any mixing angle, while λ_{TC} has the potential of becoming very large for low values of the $h\widetilde{\sigma}$-mixing. It becomes especially large for high values of $M_{\widetilde{\sigma}}$, which is a rather unconstrained parameter. This could pose a problem since field theory treats interactions as perturbations. There are no exactly solvable interactive field theories beyond two spacetime dimensions known to man [5].

4.2.2 In the Nearly-Conformal Limit

The parameter space of figure 6, displays the variation of $\sin\theta$, λ_{TC}, λ_H, λ, u och $\bar{g}_{TC}/v^3$ as a function of $M_{\widetilde{\sigma}}$, for different values of $m_{\widetilde{\pi}}$ (where $M_{\widetilde{\sigma}}$ and $m_{\widetilde{\pi}}$ are the only free parameters in the nearly conformal limit). We see that λ_{TC} and λ vanishes in the no-mixing limit, while u diverges, and λ_H remains unaffected.

Once again λ_H and λ, are rather constrained, while λ_{TC} is stretching towards the unperturbative region, and even more so when considering larger values of $M_{\widetilde{\sigma}}$, which is not improbable.

The no-mixing limit would be the scenario if LHC does not find any kind of deviations in the Higgs boson properties. In such a case $u \gg v$, as shown in the bottom right image of figure 6, which means that the energy scale of the technifermion sector is way above the electroweak scale. In such a case, λ_{TC} is way out is the perturbative regime. Note that this does not make the theory less probable, it just makes it uncalculable. In addition, the techniquark mass would be of order Λ_{TC}, since it is directly proportional to the technisigma VEV (45), while the masses of the technipion, and the technisigma could remain at the electroweak energy scale.

A discussion whether the nearly conformal limit is a plausible limit or not, is beyond the scope of this thesis, and will not be included.

4.3 Loop contributions

The Peskin–Takeuchi parameters were calculated in the scenario of no scalar contributions, which occur in either the no-mixing limit, or in the case of degenerate masses of technisigma and Higgs. Degenerate masses are not very likely, since the Technisigma is estimated to be 500 GeV or larger, as shown in an earlier section. There is of course a possibility that the sigma does not follow the mass hierarchy, and that it has not been discovered due to its large width. However, it does not seem very likely.

The no-mixing limit has less reason to be ruled out. As we saw in the plots of the parameter space, the no-mixing limit does correspond to a divergent technisigma VEV in the conformal limit, but the found S, T, U-parameter would also be valid for negligible contributions from the scalar sector. For $u \gg v$, we would expect minimal deviations from expected data of the Higgs (or anything measurable for that matter), which is a safe assumption unless deviations are indeed found. However, as u increases, Λ_{TC} rapidly increases, which could infer a non-perturbative regime, even for very small deviations from the no-mixing limit (when the mixing is exactly zero, $\lambda_{TC} = 0$). As mentioned previously, this does not make the theory less likely, but it rules out the possibility of calculable scalar self-interactions. The S, T, U-parameters would be valid in any case, since they do not involve scalar self-interactions.

When it comes to a theory where the electroweak scale is not negligible in comparison ($M_{\mathrm{EW}} \sim$ 80 GeV, $\Lambda_{\mathrm{TC}} \gtrsim 200GeV$), one should also include the V, W, X-parameters, in addition to using the S, T, U-parameter *beyond* linear order in q^2. However, since the no-mixing limit implies that $u \gg v$ (neglecting the possibility of degenerate masses of the Higgs and Technisigma), the linear order S, T, U-parameters should be adequate in this case. We may also note that the calculations are valid no matter if we are in the conformal limit or not, since the μ-terms are only involved in (pseudo)scalar self-couplings, as apparent from the Lagrangian.

The scenario of no scalar contributions and hypercharge zero, was chosen due to its simplicity. The scenario involved the least amount of loop diagrams, and for hypercharge zero, all contributions canceled each other. Putting the hypercharge to zero, means that the charge of the fermions become equal to their isospin:

$$Q = T^3 - Y_{\widetilde{Q}}/2 = T^3 = \pm\frac{1}{2}.$$

Hence, the hypercharge changes the vertices for any diagrams involving γ legs (which involves the techniquark charge), or Z legs (which involves the techniquark charge and the isospin), as seen from the Lagrangian involving vector-like interactions between technifermions and gauge bosons.

The next step would have been to consider a general case involving any mixing angle, and hypercharge $Y = 1/3$, as in the Standard Model. This scenario has been covered in the original article, and shows that the model stays within the allowed boundaries for most variations of the parameter space. The strongest constraints appearently come from the T-parameter.

Even if the calculation is a special case scenario, it displays how the S-parameter may vanish when using the linear sigma model. Many other technicolor extensions suffers from the S parameter being a rather large constant, no matter how high the energy scale is taken [19].

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Appendices

A Group Theory Elements

The electromagnetic-, weak- and strong interactions between all known particles can be described using three internal symmetry groups; U(1), SU(2) and SU(3). Before discovering the consequence of this statement, it is essential to understand what a group is.

This section provides the basic features of group theory, including an introduction to Lie groups and Lie algebras. Lie groups are of importance since all the groups of the Standard Model, U(1), SU(2) and SU(3), are Lie groups, and Lie algebras can be used to relate groups.

A group is an algebraic structure consisting of a set G and a binary[5] operation *, fulfilling four group axioms [15]:

1. Closure. The set must be closed under the * operation: $\exists * : \forall U_1, U_2 \in G : U_1 * U_2 = U_3 \in G$.
2. Associative. Combining three (or more) elements in the group differently will not change the outcome: $\exists * : \forall U_1, U_2, U_3 \in G : (U_1 * U_2) * U_3 = U_1 * (U_2 * U_3)$.
3. Identity. The must exist an identity element, that does not effect another element under the * operation: $\exists e : \forall U \in G : U * e = e * U = U$.
4. Inverse. Every element has an inverse, which gives the identity when combined with its non-inverse. $U^{-1} \in G : U * U^{-1} = U^{-1} * U = e$.

For example, the set of all integer numbers form a group under addition. ($\mathbb{Z}$, +) fulfills all axioms, with the identity being zero and the inverse of U being $-U$.

Another example, more suited for our applications, is the general linear group GL(n). GL(n), short for GL(n,C), is the set of non-singular linear transformations in a n-dimensional complex space. Or, equivalently, GL(n) is the set of all nxn complex matrices with a non-zero determinant [17]. GL(n) forms a group under matrix multiplication, and its identity element is the identity matrix. In fact, the identity matrix is the identity element, is a property shared amoung all groups consisting of square matrices.

U(n) is a subgroup of GL(n), with the additional condition that the nxn complex matrices are unitary, $UU^\dagger = 1$. U(1), in particular, is the set of complex 1x1-matrices with absolute value 1[6]. The U(1) group is *Abelian*, i.e. commutative, since a 1x1-matrix is simply a (complex) number, and numbers always commute. SU(n) is in its turn a subgroup of U(n):

$$GL(n) \supset U(n) \supset SU(n),$$

with the additional condition that the complex, unitary, nxn-matrices have determinant 1. The symbol $\supset$ denotes "contains a subset". The SU(2) and SU(3) group of the Standard Model are *non-Abelian*, since matrices in general do not commute.

A.1 Lie groups

U(1), SU(2) and SU(3) are Lie groups. Consider a continuous group with one real parameter, α, in a one-dimensional linear vector space[7]. A continuous group is a group where all group elements can be obtained by a continuous variation of a real parameter [17]. For example, a continuous

[5]A binary operation on a set is an operation combining two elements of the set and returning another element of the set.

[6]The absolute value is equal to one, since: $|e^{i\alpha}| = \sqrt{e^{-i\alpha} \cdot e^{i\alpha}} = \sqrt{1} = 1$.

[7]The definition of a linear vector space can be found in the following section "Lie Algebras". The definition is not included here, since I have chosen to emphasize some other features.

group can be the group of all rotations in a plane, where the parameter is the angle of rotation [7]. The group elements, $U \in G$, for different values of the parameter $\alpha = \xi_1, \xi_2, \xi_3 ...$, must satisfy the four group axioms. For example, the identity axiom requires:

$$U(\xi) * U(\xi^{-1}) = U(\xi^{-1}) * U(\xi) = U(e),$$

and the closure axiom requires:

$$U(\xi_1) * U(\xi_2) = U(\xi_3) \in G.$$

Since the relation must hold for all elements of G, the parameter ξ_3 can be seen as a function of ξ_1 and ξ_2:

$$\xi_3 = f(\xi_1, \xi_2).$$

If ξ_3 is an analytical function of ξ_1 and ξ_2, and if ξ_1 is an analytical function of ξ_1^{-1}, the group G is a Lie group. An analytical function is a function which is locally given by a convergent power series, hence it can be differentiated an infinite amount of times [17].

The group elements of a Lie group can be reached continuously from the identity transformation, when using a certain set of operators. Such operators are called the generators of the Lie group. For example, the SU(2)-group of the Standard Model is generated by the three Pauli matrices, while SU(3) is generated by the eight Gell-Mann matrices. The generators are found by considering an infinitesimal transformation. Let us start out with a simple transformation of a one-parameter Lie-group:

$$x \to x' = \xi x, \tag{101}$$

where ξ is the parameter, and where the homogeneous linear transformation is carried out in a one-dimensional vector space. For future purposes, x' will be expressed as a function of ξ and x:

$$x' = f(x, \xi). \tag{102}$$

If the transformation ξ is chosen to be the identity transformation e, the transformation will obviously leave the system unchanged:

$$x' = f(x, e) = x. \tag{103}$$

An infinitesimal transformation, i.e. a transformation varying infinitesimal from the identity, can be expressed as $da \equiv \epsilon$:

$$x' = f(x, e + d\xi) = x + dx. \tag{104}$$

Expanding the expression in the lowest order in $d\xi$, yields:

$$x + dx = f(x, e) + \left[\frac{df(x,\xi)}{d\xi}\right]_{\xi=e} d\xi = f(x, e) + \frac{df(x,e)}{d\xi} d\xi, \tag{105}$$

where the insertion of equation 103 gives us:

$$dx = \frac{df(x,e)}{d\xi} d\xi \equiv u(x) d\xi. \tag{106}$$

The infinitesimal change in x of an arbitrary function F(x) is then given by:

$$dF = \frac{dF}{dx} dx = \frac{dF}{dx} u(x) d\xi = \left[u(x)\frac{d}{dx}\right] F d\xi \equiv tF d\xi$$

$$\Leftrightarrow \; d \equiv t d\xi \tag{107}$$

where the operator $t \equiv \left[u(x)\frac{d}{dx}\right]$ is the generator of the group.

Now, consider a continuous group G. The infinitesimal transformation of a group element U, is found by using equation 107:

$$U(e + d\xi) = U(e) + dU(e) = U(e) + td\xi U(e) = (\mathbb{1} + td\xi)U(e). \tag{108}$$

Equation 108 shows us how applying the operator $(\mathbb{1} + d\xi U)$ on the identity element, yields a new element $U(e + d\xi)$. Similarly, applying the operator N times yields the expression:

$$U(e + Nd\xi) = (\mathbb{1} + td\xi)^N U(e), \tag{109}$$

where $\lim_{N\to\infty}$ gives:

$$U(\xi) = \exp(\xi^a t_a). \tag{110}$$

The infinitesimal transformation of U is given by [17]:

$$U(\xi) = \mathbb{1} + \xi^a t_a + \mathcal{O}(\xi^2). \tag{111}$$

A similar derivation can be made for the generators of SU(2) and SU(3). For SO(2), the three generators are given by:

$$\begin{aligned} t_1 &= i\left[x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}\right] \\ t_2 &= y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y} \\ t_3 &= i\left[y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right], \end{aligned} \tag{112}$$

with a matrix representation corresponding to the three Pauli matrices. Similarly, SU(3) has 8 generators with a matrix representation corresponding to the eight Gell-Mann matrices.

The commutator of any two generators of a Lie group can be expressed as a linear combination of its generators:

$$[t_a, t_b] = f^c_{ab} t_c, \tag{113}$$

or, without using the summation convention:

$$[t_a, t_b] = \sum_c f^c_{ab} t_c. \tag{114}$$

The constants f^c_{ab} are so-called structure constants. The structure constants form a certain representation of the generators of the group, the so-called *adjoint representation*. The structure constants are completely anti-symmetric. For example, exchanging the order of the lower indices, changes the sign:

$$f^c_{ab} = -f^c_{ba}. \tag{115}$$

This property can be shown easily from equation 113 by using that:

$$[t_a, t_b] = t_a t_b - t_b t_a = -(t_b t_a - t_a t_b) = -[t_b, t_a]. \tag{116}$$

Another property of structure constants are that they satisfy the Jacobi relation:

$$[t_a, [t_b, t_c]] + [t_b, [t_c, t_a]] + [t_c, [t_a, t_b]] = 0. \tag{117}$$

Using that $[t_b, t_c] = f^d_{bc} t_d$ etc., equation 117 may be rewritten as:

$$\left(f^d_{bc} f^e_{ad} + f^d_{ca} f^e_{bd} + f^d_{ab} f^e_{cd}\right) t_e = 0, \tag{118}$$

where all coefficients must be zero since the generators are linearly independent:

$$f^d_{bc}f^e_{ad} + f^d_{ca}f^e_{bd} + f^d_{ab}f^e_{cd} = 0. \tag{119}$$

In fact, it can be proven that whenever the conditions of equation 115 and 119 are fulfilled, there exist generators which satisfies the commutation relation of equation 113.

A.2 Lie Algebras

A Lie algebra is a special case of a *linear* algebra. In order to define a linear algebra, we must first define the properties of a field and a linear vector space. The definition of a field does, in fact, contain the definition of a group; a field is an algebraic structure consisting of a set A and two binary operations, addition and multiplication, where:

1. the set forms an Abelian group under addition,
2. the set forms an Abelian group under multiplication, when excluding the additive identity (zero),
3. and for all $a, b, c \in A$: $a(b + c) = ab + ac$, $(a + b)c = ac + bc$[8].

Now that a field is defined, we may define a linear vector space. A set of elements V form a vector space over a field F, if V forms an Abelian group under addition. Also, the elements of the field and the set should be combined under scalar multiplication according to [16]:

1. $ax_1 \in V$
2. $a(bx_1) = (ab)x_1$
3. $(a + b)x_1 = ax_1 + bx_1$
4. $a(x_1 + x_2) = ax_1 + ax_2$
5. $1x_1 = x_1$

where a, b are element in F and x_1, x_2 elements in V. Note that the scalar multiplication used differ from the scalar multiplication defined as the dot product of vectors.

The elements of a linear vector space V over a field F, form a *linear algebra*, if the elements fulfill the following criteria under an additional binary operation:

1. The set is closed under the binary operation: $x_1, x_2 \in V : x_1x_2 = x_3 \in V$.
2. $x_1, x_2 \in V$, $a \in F : ax_1x_2 = x_1ax_2$.

Consider a vector space V spanned with the linear combinations, $\alpha, \beta, ...$, of the generators of a Lie group. Using equation 113, we identify that the commutator $[\alpha, \beta]$ is also an element of the vector space (since it is also a linear combination of the generators of the Lie group). If commutation is chosen as the "additional" binary operation, the linear algebra formed by the vector space is called a *Lie algebra.*

Lie algebras are useful since two locally isomorphic[9] Lie groups have the same Lie algebra. Hence, by only considering infinitesimal rotations about the identity, we may study a single Lie algebra instead of several Lie groups.

[8]The third condition might appear trivial, but that is just due to the fact that all mathematics up to university level is built on the theory of fields. For example, solving the equation $ax = bc$ for x by multiplying by the inverse of a, is something which can not always be taken for granted.

[9]A group is *locally* isomorphic if it is isomorphic in the neighborhood of the identity. Isomorphism corresponds to a one-to-one mapping between two groups, see Fraleigh for a proper definition.

B The mixing of the A_μ- and B_μ fields

The first term in the Lagrangian $\bar{\psi} i\gamma^\mu D_\mu \psi$ will yield interaction terms with two fermion fields ψ and one gauge field (included in the covariant derivative). It can be shown that such an interaction term will in fact correspond to one fermion turning in to another fermion by the emission of a gauge boson. The procedure is demonstrated below for the covariant derivative of SU(2) acting on leptons doublets:

$$\begin{aligned} \bar{L} i\gamma^\mu D_\mu L &= \begin{pmatrix} \bar{\nu_L} & \bar{e_L} \end{pmatrix} i\gamma^\mu \left[i g_2 \frac{\tau^i}{2} W^i_\mu \right] \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \ldots \\ = -\frac{g_2}{2} &\left[\bar{\nu_L}\gamma^\mu \nu_L W^0_\mu - \sqrt{2}\bar{\nu_L}\gamma^\mu e_L W^+_\mu - \sqrt{2}\bar{e_L}\gamma^\mu \nu_L W^-_\mu - \bar{e_L}\gamma^\mu e_L W^0_\mu \right] \end{aligned} \tag{120}$$

We may easily identify from equation 120, that the diagonal Pauli matrix yields the neutral current processes, while two other Pauli matrices with off-diagonal entries yield charged current processes. This result is quite intuitive, since off-diagonal entries of a 2x2-matrix causes rotations of a two-entries column vector, while a diagonal matrix does not.

The second and third term of equation 120, correspond perfectly to the terms in the final interaction Lagrangian shown in Appendix D. The first and fourth term, on the other hand, does not appear in the final Lagrangian. They do not appear since the W^0_μ-field of SU(2) mixes with the B_μ field of U(1). The mixing occur since the interaction Lagrangian for U(1) contains terms identical to the first and fourth term of equation 120:

$$\begin{aligned} \mathcal{L}_{int} &= \bar{L} i\gamma^\mu \left[i g_1 \frac{Y_L}{2} B_\mu \right] L = \bar{e_R} i\gamma^\mu \left[i g_1 \frac{Y_R}{2} B_\mu \right] e_L \\ &= \frac{g_1}{2} \left[Y_L (\bar{\nu_L}\gamma^\mu \nu_L + \bar{e_L}\gamma^\mu e_L) + Y_R (\bar{e_R}\gamma^\mu e_R) \right] B_\mu . \end{aligned} \tag{121}$$

The first two terms of equation 121 correspond to the first and fourth term of equation 120. The mixings of the W^0_μ- and the B_μ, can be determined by using the fact that the electromagnetic field A_μ does not interact with neutrinos. Hence we can gather the neutrino terms from equations 120 and 121, which will correspond to Z_μ.

$$\left(-\frac{g_1}{2} Y_L B_\mu - \frac{g_2}{2} W^0_\mu \right) \bar{\nu_L}\gamma^\mu \nu_L \Rightarrow Z_\mu \propto g_1 Y_L B_\mu + g_2 W^0_\mu \tag{122}$$

The electromagnetic field A_μ is then the field orthogonal to the comination of Z_μ:

$$A_\mu \propto g_2 B_\mu - g_1 Y_L W^0_\mu . \tag{123}$$

When normalized, the expressions for Z_μ and A_μ are:

$$A_\mu = \frac{g_2 B_\mu - g_1 Y_L W^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \tag{124}$$

$$Z_\mu = \frac{g_1 Y_L B_\mu + g_2 W^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \tag{125}$$

After a bit of algebra, and adding the quark interactions, we arrive at:

$$\begin{aligned} \mathcal{L} = &\sum_{f=l,q} e Q_f (\bar{f}\gamma^\mu f) A^\mu \\ &+ \frac{g_2}{\cos\theta_W} \sum_f [\bar{f_L}\gamma^\mu f_L (T^3_f - Q_f \sin^2\theta_W) + \bar{f_R}\gamma^\mu f_R(-Q_f \sin^2\theta_W)] Z_\mu \\ &+ \frac{g_2}{\sqrt{2}} \sum_{q,l} [(\bar{q^+_L}\gamma^\mu q^-_L + \bar{l^+_L}\gamma^\mu l^-_L) W^+_\mu + h.c.] \\ &+ \frac{g_3}{2} \sum_q \bar{q_\alpha}\gamma^\mu \lambda^a_{\alpha\beta} q_\beta G^a_\mu . \end{aligned}$$

where f stands for all fermions, q for all quarks and l for all leptons. The full derivation can be found in Kane, Chapter 7 [4].

C Yukawa interactions of the Standard Model

The interactions between fermions and the Higgs field are found through an added Yukawa term to the Standard Model Lagrangian. The added term is (as found in Kane, p.109 [4]) for the quarks:

$$\mathcal{L} = g_d \bar{Q}_L \mathcal{H} d_R + g_u \bar{Q}_L \mathcal{H}_c u_R + g_d \bar{d}_R \mathcal{H}^\dagger Q_L + g_u \bar{u}_R \mathcal{H}_c^\dagger Q_L$$

where the doublet f_L, the doublet $\mathcal{H}^\dagger$ and the singlet f_R, are ordered such that the Lagrangian is a singlet. g_u, g_d are the Yukawa coupling constants of the up-quark and the down-quark, respectively. Exchanging $\mathcal{H}$ and $\mathcal{H}_c$ for:

$$\mathcal{H} \to \begin{pmatrix} 0 \\ \frac{v+H'}{\sqrt{2}} \end{pmatrix}, \quad \mathcal{H}_c \to \begin{pmatrix} \frac{-v-H'}{\sqrt{2}} \\ 0 \end{pmatrix}, \tag{126}$$

yields four terms; two mass terms and two interaction terms:

$$\mathcal{L} = g_d \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} 0 \\ \frac{v+H'}{\sqrt{2}} \end{pmatrix} d_R + g_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} \frac{-v-H'}{\sqrt{2}} \\ 0 \end{pmatrix} u_R + h.c.$$

$$= g_d \bar{d}_L \left(\frac{v+H'}{\sqrt{2}} \right) d_R + g_u \bar{u}_L \left(\frac{-v-H'}{\sqrt{2}} \right) u_R + g_d \bar{d}_R \left(\frac{v+H'}{\sqrt{2}} \right) d_L + g_u \bar{u}_R \left(\frac{-v-H'}{\sqrt{2}} \right) u_L$$

$$\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R = \bar{\psi}\psi \quad \Rightarrow \quad \mathcal{L}_{int} = g_d \bar{d} d \left(\frac{v+H'}{\sqrt{2}} \right) d_R + g_u \bar{u} u \left(\frac{-v-H'}{\sqrt{2}} \right)$$

$$= \frac{g_d \bar{d} d v}{\sqrt{2}} - \frac{g_u \bar{u} u v}{\sqrt{2}} + \frac{g_d \bar{d} d H'}{\sqrt{2}} - \frac{g_u \bar{u} u H'}{\sqrt{2}}.$$

Hence, the so-called Dirac mass of the quarks are given by $m_d = -\frac{g_d v}{\sqrt{2}}$ and $m_u = \frac{g_u v}{\sqrt{2}}$, and we can rewrite the interaction terms as:

$$\mathcal{L}_{int} = -\frac{m_d}{v} \bar{d} d H' - \frac{m_u}{v} \bar{u} u H'. \tag{127}$$

The mass- and interaction terms of leptons are obtained in an equivalent manner:

$$\mathcal{L} = g_e \bar{L} \mathcal{H} e_R^- + h.c = ... = \frac{g_e v}{\sqrt{2}} \bar{e} e + \frac{g_e}{\sqrt{2}} \bar{e} e H',$$

$$\Rightarrow \quad \mathcal{L}_{int} = -\frac{m_e}{v} \bar{e} e H'. \tag{128}$$

Note that the there is no term giving mass to the neutrinos, since it would require the inclusion of a right-handed neutrino[10]. Here we may also note that the coupling strength is proportional to the mass of the fermion $g_f \propto \sqrt{2} m_f / v$, hence the Higgs couples stronger to heavier fermions.

By comparing the quarks in equation 127 and the leptons 128, we find a general expression for the interaction terms of fermions:

$$\mathcal{L}_{int} = -\frac{m_f}{v} \bar{f} f H'. \tag{129}$$

[10] The right-handed neutrino is not included since we do not know whether it exists. It only interacts via gravity. It cannot interact via the strong- or electromagnetic force (it possesses neither charge nor colour charge), nor may interact weakly since it is right-handed. However, it is known from neutrino oscillations that neutrinos must possess mass.

D The general scenario A^n

The integral

$$A^n = \int \frac{d^dk}{(2\pi)^d} \frac{1}{D^n} = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i\epsilon)^n},$$

is evaluated in a manner very similar to the integral B_0 of section 3.3.1.1. The denominator is once again re-expressed as an exponential using α-parameterization (72), with $\lambda = n$ and $\Omega = k^2 - m^2$:

$$\Rightarrow \quad A^n = \frac{i^{-n}}{\Gamma(n)} \int \frac{d^dk}{(2\pi)^d} \int_0^\infty d\alpha \;\; \alpha^{n-1} e^{i(k^2-m^2)\alpha},$$

after which $e^{ik^2\alpha}$, integrated over momentum, is identified as a Gaussian integral (73), with $\alpha = \alpha$ and $q = 0$. Hence, the integral may be rewritten as:

$$A^n = \frac{i^{1-n-d/2}}{(4\pi)^{d/2}\Gamma(n)} \int_0^\infty d\alpha \;\; \alpha^{n-1-d/2} e^{-im^2\alpha}.$$

Since there was only one term in the denominator, there is no β-parameter, and hence the Feynman parameterization is not required. Instead, the integral is evaluated by doing another α-parameterization right away, using $\Omega = -m^2$ and $\lambda = n - d/2$ in (72), producing the final result:

$$\therefore \; A^n = \int \frac{d^dk}{(2\pi)^d} \frac{1}{D^n} = \frac{i^{1-d}\Gamma(n - d/2)}{(4\pi)^{d/2}\Gamma(n)(-m^2)^{n-d/2}} = \frac{i(-1)^n\Gamma(n - d/2)}{(4\pi)^{d/2}\Gamma(n)(m^2)^{2-d/2}}.$$

using that $(-m^2)^{n-d/2} = (-1)^n i^{-d} (m^2)^{n-d/2}$.