DYNAMIC PORTFOLIO STRATEGY
USING A MULTIVARIATE GARCH MODEL

MASTER THESIS IN FINANCE

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ABSTRACT

This paper examines if it is possible to achieve a higher cumulative and risk adjusted return through an active portfolio strategy compared to a passive portfolio strategy. This is done through a mean-variance framework in which the variance is forecasted using two different models. The results show that it is possible to achieve a higher cumulative and risk adjusted return by dynamically changing the weights of the assets in the portfolio. Especially if a simple market timing rule is used.

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The programing for the models used in this paper was made in VBA and MATLAB and are available upon request.

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1. Introduction

It has been widely debated if active portfolio management achieves higher returns, compared to the passive strategy of buy-and-hold. In the long run an investor should not gain excess return due to market efficiency. However an investor may achieve higher than market return due to anomalies in the market and pure chance. Some of the more well-known anomalies are the January effect, the smaller firm effect and the days of the week effect. Kiymaz and Berument (2003) found evidence of the days of the week effect on share price and volatility. In their study they found that the highest return and volatility, on the U.S. and Canadian market, occurs on Fridays. Thaler (1987) summarizes findings of seasonal patterns on the security market to provide evidence and discuss the January effect. He argues that it should be impossible to predict future returns and behaviour of stock prices based on publicly available information and historical prices due to market efficiency. Furthermore he discuss the findings of previous research which have shown that on average over one-third of the annual return occurred in January. Furthermore he points out that there is evidence of the anomaly deriving from the smaller firms included in the New York Stock Exchange. The smaller firm effect might be due to some pricing of risk by the investors. If the investors incorporate higher risk in firms with smaller market capitalization the expected return will be higher.

Fama and French (1993) argue that their three-factor model can capture anomalies on the stock market. Two of the factors in this model are the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks and the return of a portfolio of shares with a high book-to-market ratio in excess of the return on a portfolio of shares with low book-to-market ratio. By modelling the expected return with these factors it will include different risk levels depending on firm characteristics. Beating the market and getting a higher risk adjusted return may be argued to be the goal of all fund managers, private investors and investment firms.

In this study standard deviation will be used as the risk measure, derived from Modern portfolio theory. Modern portfolio theory has its root in the paper Portfolio Selection by Harry Markowitz (1952). Markowitz argues for the use of a well-diversified portfolio that maximizes the investor's utility dependent on the investor's preferences. Markowitz introduced what is known as the efficient frontier where the investor maximizes the return given a certain level of risk. It is possible for the investor to find the mean-variance portfolio, which gives the highest return with the lowest risk. This portfolio is found on what is known as Markowitz efficient frontier.[2] Using this classical mean-variance framework I investigate if it is possible to achieve a higher risk adjusted return by using a dynamic portfolio strategy on the Stockholm Stock Exchange between 2007-03-06 and 2013-12-30.

Three different portfolios will be examined and tested against two benchmarks. The portfolios that are tested against the benchmarks are:

- Portfolio one: The historical variance portfolio (HVP), which includes three assets, Ericsson B (ERIC-B), Scania B (SCV-B) and Skanska B (SKA-B). The weights of the assets in the portfolio are estimated through the historical mean of the assets variance.
- Portfolio two: The dynamic conditional correlation portfolio (DCCP), includes the same assets as the HVP but with the difference that the weights are estimated through a multivariate GARCH model i.e. the dynamic conditional correlation model (DCC).
- Portfolio three: The market timing portfolio (MTP) uses the same multivariate GARCH model as the DCCP but with the difference that it also uses a trigger for when to hold the portfolio and when to hold a risk-free asset.

The benchmark portfolios are the buy-and-hold portfolio of the three assets and the OMXS30 index.

[2] For the discussion of Markowitz portfolio theory and the efficient frontier see for example Investments by Mayo (2010)

When constructing a portfolio of risky assets the overall goal is to achieve the highest possible return with the lowest possible risk, i.e. for a risk averse investor to take on more risk the investor must be compensated with a higher expected return. Constructing a portfolio of assets with highly significant trade-offs between risk and return should in theory give a higher expected return when the volatility is high. The purpose of the study is to investigate, by a classical mean-variance framework, if dynamically changing optimal holdings in different mean-variance portfolios will yield higher cumulative and risk adjusted portfolio return compared to a basic buy-and-hold strategy.

The study is limited to only including three stocks in the portfolios due to the limitations of computer processing power and simplification of the study. The stocks are picked from the OMXS30 index to achieve the highest possible effectiveness since the index only includes the most traded stocks on the Stockholm Stock Exchange.

The results in the study shows that the Market Timing Portfolio outperforms all portfolios in all periods, except the index, both in cumulative return and risk adjusted return. Compared to the index the Market Timing Portfolio outperforms the index in the risk-adjusted return for all periods except the period directly after the financial crisis. The cumulative return for the Market Timing Portfolio is only greater compared to the index for the entire study period.

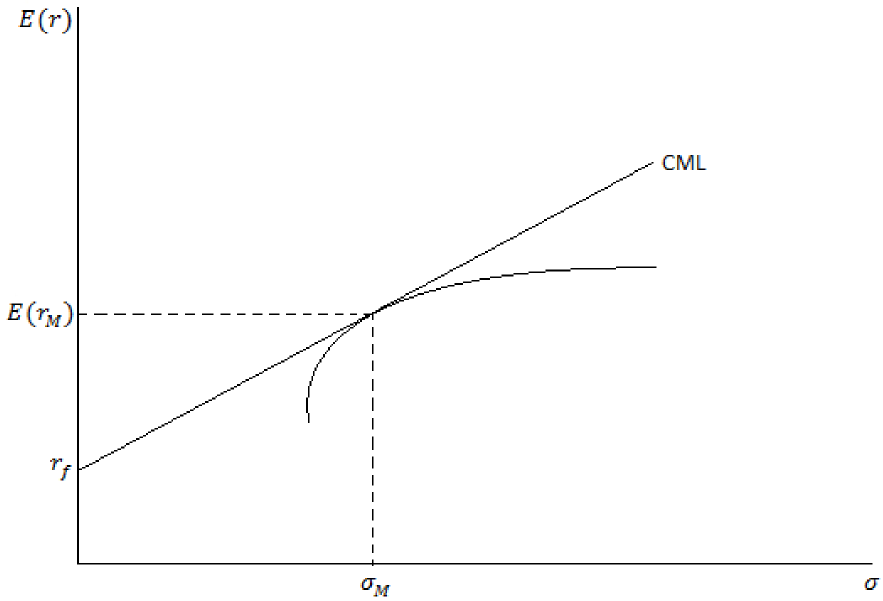
This paper is organized as follows. In the next section the background and theory are presented. In section three a discussion of the data used and the method for the study are provided. In the last section of the paper the result is presented and a conclusion of the paper is made.

2. BACKGROUND AND THEORY

Markowitz (1952) introduced the portfolio selection model, which later was developed into the CAPM model by Sharpe (1964) and Lintner (1965). Since then, the CAPM model has been the cornerstone in modern financial theory.

The trade-off between risk and return for an efficient portfolio is shown with the capital market line (CML) [3], as in Figure 1

Figure 1. Capital Market Line



Note: Efficient frontier and the CML line where $\boldsymbol{E(r_M)}$ = expected return, $\boldsymbol{E(r_M)}$ = expected return of the market portfolio, $\boldsymbol{r_f}$ = risk-free rate and $\boldsymbol{\sigma}$ = standard deviation

As seen in the Figure 1 above the investor demands higher expected return as standard deviation (risk) of the efficient portfolio becomes higher i.e. further out on the x-axis. This is a standard result and well established in basic finance. In the figure above r_f is the risk-free rate and $E(r_M)$ is the expected return of the market portfolio. The figure above is also valid for other efficient portfolios and not only the market portfolio, in which case σ_M will be σ_p, which is the standard deviation of the other

[3] See for example Investments by Bondie, Kane and Marcus 9th Edition (2011)

efficient portfolios. We can derive the standard deviation of a portfolio with *n* assets by using matrix algebra:

$$Y_P = w_1 X_1 + w_2 X_2 + \cdots + w_n X_n \tag{1}$$

where Y_P is the portfolio, w_1 is the weight of asset X_1, w_2 is the weight of asset X_2 etc. Equation (1) can now be written in matrix form:

$$Y_P = (w_1, w_2, \cdots, w_n)\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} = w'\mathbb{X} \tag{2}$$

And calculating the variance of the portfolio gives us:

$$Var(Y_P) = Var(w'\mathbb{X}) = w' Cov(\mathbb{X}\mathbb{X}')w = w'Hw \tag{3}$$

In Equation (3) *H* is the variance-covariance matrix and *w* is the vector of weights and *w'* is the transpose of *w*. The standard deviation is given by the square root of Equation (3);

$$\sigma_P = (w'Hw)^{1/2} \tag{4}$$

In the case of a two-asset portfolio we get the following arrangement for the portfolio variance:

$$\sigma_p^2 = [w_1 \quad w_2]\begin{bmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_2^2 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{5}$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_1 w_2 \sigma_{2,1} + w_1 w_2 \sigma_{1,2} + w_2^2\sigma_2^2 \tag{6}$$

where $\sigma_{1,2} = \sigma_{2,1} = Cov(X_1, X_2)$. This gives the well-known two-asset portfolio standard deviation used in most of the basic financial literature:

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(r_1, r_2)} \tag{7}$$

It is now easy to show the trade-of between risk and return by using the mean-variance model. If the CML line is expressed as an ordinary straight line:

$$E(r_p) = r_f + s_p\sigma_p, \tag{8}$$

where s is the slope of the line. We can rewrite the expression to get the slope coefficient on the left hand side:

$$s_p = \frac{E(r_p) - r_f}{\sigma_p}, \tag{9}$$

This forms the Sharpe ratio by Sharpe (1964) used later in the study. It is easy to see and measure the trade-off between risk and return with the formula above. The goal for a rational investor would then be to maximize the slope coefficient to get as high return as possible given a level of risk, i.e. the higher the slope coefficient, the better off the investor is since he gets more return from every risk unit taken on, measured as standard deviation, in his portfolio.

One way of measuring the potential existence of this trade-off between risk and return is to regress the variance against the return. This has been done many times since Markowitz introduced his portfolio theory in 1952. One popular method to use is the so-called GARCH-M model. The GARCH-M models the variance to take care of some standard characteristics found on the financial market, more explicitly heteroscedasticity and autocorrelation, and then uses the modelled variance in the actual mean equation used to estimate the expected return. The trade-off between risk and return are used in this study when selecting the stocks to be included in the portfolios.

Active portfolio management is a widely discussed topic and researchers have found that there may be benefits in utilising this strategy. Boyd and Mercer (2010) present a bond management strategy in their study in which they reallocate bonds based on the business cycle and short-maturity interest rate cycle. They demonstrated that their allocation strategy easily dominates the two benchmarks, the U.S. Aggregate Index and the U.S. Credit Index based on risk-adjusted returns.

Alexander and Baptista (2010) propose in their study a strategy that lead to a selection of portfolios that are less risky by minimizing the “tracking error variance”, which is the difference between the portfolio’s return and the benchmark’s return. They found that by using their strategy portfolio managers could get a significant increase in the Sharpe ratio.

Cochrane (1999) discusses in his study that it is possible to obtain an increased Sharpe ratio by using a market-timing rule based on dividend price ratios. He also shows that it is possible to double the return over a five-year period by using a market timing strategy.

Pojarliev and Polasek (2001) found in their study that it is possible to achieve higher cumulative return and an increased Sharpe ratio by using active portfolio management. They proposed a market-timing rule in which they invest in a mean-variance portfolio built up of three indexes if the next period volatility, forecasted through a multivariate conditional variance model, is twice the size of the historical variance. If the variance is less they invest in a risk-free asset. Their results shows that the market timing portfolio performs very well compared to the benchmarks, both when it comes to total cumulative return and risk-adjusted return. They conclude that it is possible to exploit the trade-off between risk and return if the right combination of volatility modelling and portfolio strategy is used.

3. Data and Method

3.1 Data Description

The data used in the study is collected from the Nasdaq OMX Nordic webpage and covers a time horizon of ten years, 2004-01-02 to 2013-12-30, including 2507 trading days. The period is divided into different sub periods when conducting the study. The in-sample-period is the first 800 trading days, 2004-01-02 to 2007-03-06. The out-of-sample period is the remaining 1707 trading days, which will be used for the portfolio evaluation. The study will be conducted on daily data assuming 252 trading days per year.

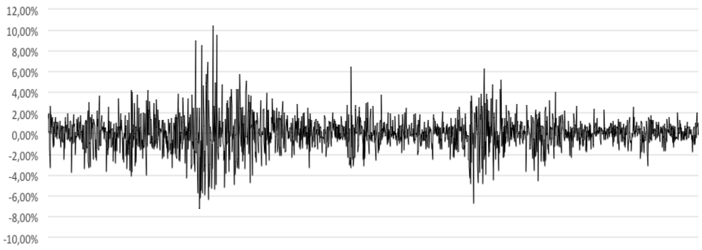
The daily closing prices collected from the Nasdaq OMX Nordic webpage are calculated as the log return, or the continuously compounded return as it may be known. Using first difference of the daily closing prices for the time series gives two major advantages in the study. Firstly, transforming closing prices to log returns transforms the series to become stationary, which is the foundation of time series analysis, and secondly, continuously compounded returns are time-additive, which will come in handy later in the study when the cumulative returns are calculated.[4] The formula for the log return can be expressed as follows;

$$r_t = ln\frac{P_t}{P_{t-1}} = ln(P_t) - ln(P_{t-1}) \qquad (10)$$

where r_t is the daily log return for the series at day t. Furthermore P_t and P_{t-1} are the closing price at day t and t-1, respectively. In Figure 2-5 below the log returns for all the time series are shown.

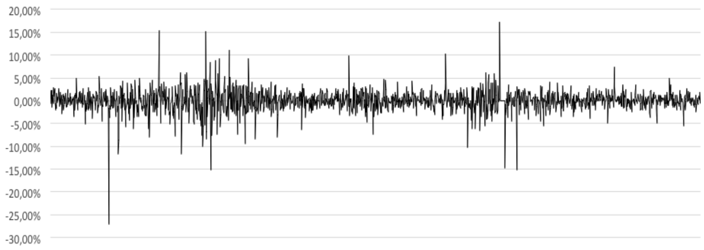
[4] See for example Analysis of Financial Time Series by Ruey S. Tsay 3rd Edition (2010)

Figure 2. OMXS30



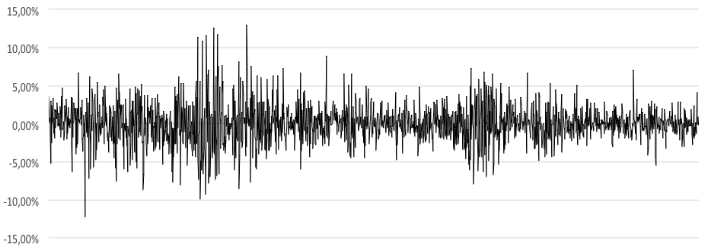
Note: Logarithmic return of the OMXS30 index from 3rd January 2007 until 30th December 2013

Figure 3. ERIC-B



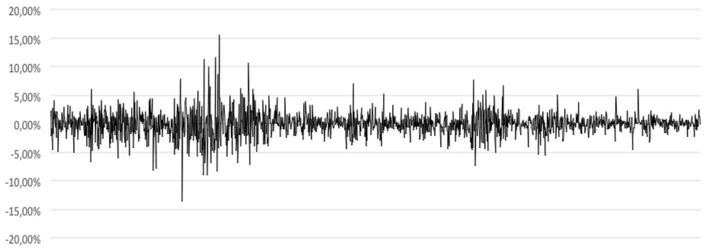
Note: Logarithmic return of the ERIC-B from 3rd January 2007 until 30th December 2013

Figure 4. SCV-B



Note: Logarithmic return of the SCV-B-B from 3rd January 2007 until 30th December 2013

Figure 5. SKA-B



Note: Logarithmic return of the SKA-B-B from 3rd January 2007 until 30th December 2013

It is fairly easy to see in Figure 2-5 above that the time series are subject to volatility clustering. In all the series we can see a period in the beginning of the series with higher than normal volatility. This is of course due to the financial crisis in the year of 2008 - 2009.

To test if the series are stationary, i.e. have constant mean, variance and autocovariance, a unit-root test is employed. The test used is the well-known Augmented Dickey-Fuller (ADF) unit-root test. The null hypothesis of the ADF test is that the series has a unit-root and hence rejecting the null indicates significant stationarity in the time series.[5] The descriptive statistics are shown in Table 1 below.

[5] See for example Analysis of Financial Time Series by Ruey S. Tsay 3rd Edition (2010)

Table 1: Descriptive Statistics

	ERIC-B	*SCV-B*	*SKA-B*	*OMXS30*
Mean	7.2E-05	0.0004	0.0003	0.0003
Median	0	0	0	0.0007
Standard Deviation	0.0230	0.0225	0.0194	0.0146
Kurtosis	17.7261	6.8187	8.9448	7.6122
Skewness	-0.7099	0.2103	0.1998	0.0258
Minimum	-0.2719	-0.1230	-0.1355	-0.0751
Maximum	0.1715	0.1297	0.1560	0.0987
Jarque-Bera	22853.75 (0.00)	1541.08 (0.00)	3706.89 (0.00)	2221.51 (0.00)
Unit Root Test				
ADF (Constant, no trend)	-40.72 (0.00)	-42.57 (0.00)	-41.96 (0.00)	-27.05 (0.00)

Note: Descriptive statistics of the OMXS30 index and the assets included in the portfolios. P-values in the parentheses.

Table 2: Correlation Matrix

	OMXS30	*ERIC-B*	*SCV-B*	*SKA-B*
OMXS30	1.00	-	-	-
ERIC-B	0.63	1.00	-	-
SCV-B	0.72	0.40	1.00	-
SKA-B	0.76	0.41	0.60	1.00

Note: Correlations between the assets and the OMXS30 index.

In the descriptive statistics table above we can see that the mean and median are very close to zero. The small positive number for the mean of the series show that the price of the assets has increased over time. Also note that the standard deviation, the risk of the series, is the lowest for the OMXS30 index. This is expected since the OMXS30 series is the only series, which is well diversified since it includes 30 stocks and the other series are individual assets. It can also be seen that the individual series are considerably more skewed than the index, indicating higher probabilities for the returns deviating from the mean. Also, the kurtosis is higher than 3 for all the series, which indicates non-normality in the return series i.e. all the series are leptokurtic which is confirmed with the highly significant Jarque-Bera test.

When it comes to the test for stationarity the ADF test is highly significant. The null hypothesis of a unit-root is therefore rejected and the conclusion is that all the series are stationary.

Furthermore, looking at the correlation matrix in Table 2 above shows that SKA-B and SCV-B have higher correlation with the index OMXS30 compared to ERIC-B. This is surprising since ERIC-B has a higher weight in the OMXS30 index but might be explained by the nature of the actual business models of the firms. SKA-B and SCV-B might have a total weight together with other stocks in the OMXS30 index that exceeds the weight of ERIC-B and which are more sensitive to fluctuations in the business cycle compared to ERIC-B.

3.2 Asset Selection

The trade-off between risk and return is a fundamental part of mean-variance analysis. Markowitz discussed in his ground-breaking paper from 1952, in which he presented the efficient frontier, that the concept of risk and yield should be “expected return” and “variance of return”. An investor will then demand a higher expected return if the variance of return is high, giving a positive relation between risk and expected return. The concept of the risk-return trade-off has been tested various times over the years and one of the more popular methods of testing for the risk-return trade-off is through the GARCH in mean model (GARCH-M). E.g. Narang and Bhalla (2011) used different versions of the GARCH-M model when testing the risk-return trade-off in the Indian capital market and found that the asymmetric models show evidence of the return being positively related to risk. Before the presentation of the GARCH-M model is conducted it is important to get an understanding of the basic GARCH model.

3.2.1 Univariate GARCH Model

A linear regression’s “Best Linear Unbiased Estimator” (BLUE) is OLS when certain general assumptions are met.[6] One of the assumptions is homoscedasticity and nonautocorrelation in the disturbance, that is, the disturbance, ε_i, has the same variance and is uncorrelated with every other disturbance. Mathematically we can show this as:

[6] See for example Econometric Analysis by Greene (2012)

$$Var[\varepsilon_i|X] = \sigma^2 \ \forall i \tag{11}$$

$$Cov\left[\varepsilon_i, \varepsilon_j\middle|X\right] = 0 \tag{12}$$

These are strong assumptions and are usually not satisfied when it comes to financial time series. In particular, one of the major problems with financial time series is the presence of heteroscedasticity. The assumptions above will not hold in the presence of heteroscedasticity in the time series. Engle (1982) introduced a model called the ARCH model (autoregressive conditional heteroscedasticity) to take the violation of the non-independence of the disturbance in different periods into account. This is done by making the variance dependent on the residual's size from the previous period. He describes an ARCH(1) model, where the (1) is the number of lags, as follows;

$$\varepsilon_t = \epsilon_t \sigma_t \tag{13}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 \tag{14}$$

$$\epsilon_t \sim IID \ and \ N(0,1) \tag{15}$$

where ε_t is the residual (shock on return), ϵ_t is the disturbance or "white noise" and σ is the standard deviation. Equation (14) is the equation for the conditional variance, where ω is the constant and can be seen as the mean of the variance and α_1 is the previous squared residual's (ε_{t-1}^2) weight. He generalizes the model and expresses an ARCH(p) model as follows;

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{16}$$

where $\omega > 0$ and $\alpha_i \geq 0 \ \forall \ i = 1, \cdots, p$ to ensure that the variance will be positive.

Bollerslev (1986) introduced what is known as the GARCH model (generalized ARCH). In this model he also includes the lagged variance. A GARCH(1,1) model can be expressed as follows;

$$\varepsilon_t = \epsilon_t \sigma_t \tag{17}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{18}$$

$$\epsilon_t \sim IID \; and \; N(0,1) \tag{19}$$

where α is known as the ARCH term and β is the GARCH term. The generalized version of the GARCH(m,s) model is as follows;

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{20}$$

where $\omega > 0$, $\alpha_i \geq 0 \; \forall \; i = 1, \cdots, m$, $\beta_j \geq 0 \; \forall \; j = 1, \cdots, s$ and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_j) < 1$.

Engel (2001) describes the GARCH model as a good design to take into account the problem of volatility clustering on the financial markets. Volatility clustering is the phenomenon of high volatility today giving a higher probability of the volatility to be high tomorrow and vice versa if the volatility is low today.

One shortcoming of the GARCH model is that it does not take into account the leverage effect. The leverage effect is the phenomenon of negative shocks giving rise to higher volatility than positive shocks. To include this financial time series characteristic in the model, Nelson (1991) introduced the EGARCH-model (exponential GARCH). One unique feature of the EGARCH model is that the variance will always be positive thanks to the logarithmic value of σ_t^2. Nelson also included a leverage term in the model to account for the asymmetric effect. Ezzat (2012) expresses the EGARCH(1,1) model as follows;

$$ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{21}$$

The most interesting part of the model is the leverage term γ. If the term γ is significant and negative there exists an asymmetry during the estimation period, i.e. negative shocks give raise to higher volatility. In this study the EGARCH model will be used when estimating the GARCH in mean equation.

3.2.2 GARCH IN MEAN MODEL

This study follows Pojarliev and Wolfgang (2001) with the difference that instead of using three indexes in the portfolios this study will use individual stocks, which are picked from the market using the GARCH-M model. This is due to the risk-return trade-off discussed earlier. The idea of using GARCH-M models to find assets with significant risk-return trade-off have been done several times before.

As discussed earlier Narang and Bhalla (2011) investigated the risk-return trade-off on the Indian market, where they used the variance as a proxy for risk through the GARCH-M model. They found that expected return is positively related to risk on the Indian market. Other studies with the same result are for example the studies made by Campbell and Hentschel (1992) and Bansal and Lundblad (2002). However, all studies have not shown a positive, significant relationship between risk and return. For instance, Fama and Schwert (1977) and Nelson (1991) found a negative relationship between risk and return.

In the study by Narang and Bhalla (2001), one of their asymmetric GARCH-in-mean models is the EGARCH-M model. The model in this study will be a modified version of their mean equation and EGARCH equation, excluding a dummy variable in the equations, which they used to account for different periods, a pre-derivative period and a post-derivative period. The modified model is as follows;

$$r_t = a + f\sigma_t^{2\varphi} + bR_{t-1} + cR_{t-1}^{OMXS30} + \varepsilon_t \tag{22}$$

$$\varepsilon_t / I_{t-1} \sim N(0, \sigma_t) \tag{23}$$

$$ln(\sigma_t^2) = \omega + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \tag{24}$$

where φ = 1 and I_{t-1} is the information given up to time t-1. Using a simple selection rule of a positive and significant relationship between risk and return, where variance is used as a proxy for the risk, the three assets with the most significant and a positive

f term will be used in the portfolio. An EGARCH-M(1,1) model will be used in this study giving the expression for the conditional variance as follows;

$$ln(\sigma_t^2) = \omega + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 |\frac{\varepsilon_{t-1}}{\sigma_{t-1}}| + \gamma_k \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (25)$$

Table 3 includes the results from the EGARCH-M(1,1) model for all the stocks included in the OMXS30 index except NOKI-SEK and ATCO-B since these assets were included in the index much later than the other assets.

Table 3: EGARCH-M

	ABB	*ALFA*	*ASSA-B*	*ATCO-A*	*AZA*	*BOL*	*ELUX-B*	*ERIC-B*	*GETI-B*	*HM-B*
GARCH Term	1.95	4.11	2.22	2.25	1.91	-1.47	1.69	6.29	-1.76	14.63
Prob.	0.60	0.27	0.65	0.22	0.88	0.44	0.62	0.00	0.74	0.21

	MTG-B	NDA	SAND	SCA	SCV-B	SEB-B	SECU-B	SHB-A	SKA-B
GARCH Term	1.56	6.59	3.42	-8.80	11.75	2.65	0.02	-8.23	14.93
Prob.	0.70	0.34	0.51	0.47	0.01	0.72	0.99	0.26	0.01

	SWMA	TEL2-B	TLSN	VOLV-B	*LUPE*	SWED-A	*INVE-B*	SSAB-A	SKF-B
GARCH Term	6.33	4.11	-13.96	-15.93	2.18	0.79	12.26	-2.68	0.08
Prob.	0.50	0.55	0.23	0.12	0.44	0.91	0.05	0.40	0.99

Note: Regression results from the GARCH in-mean model.

As can be seen in the table above ERIC-B, SCV-B and SKA-B are the most significant assets with a positive risk term and therefore will be included in the portfolios since the chance of having random results will be minimized when using the assets with the most significant risk term.

3.3 Volatility Forecasts

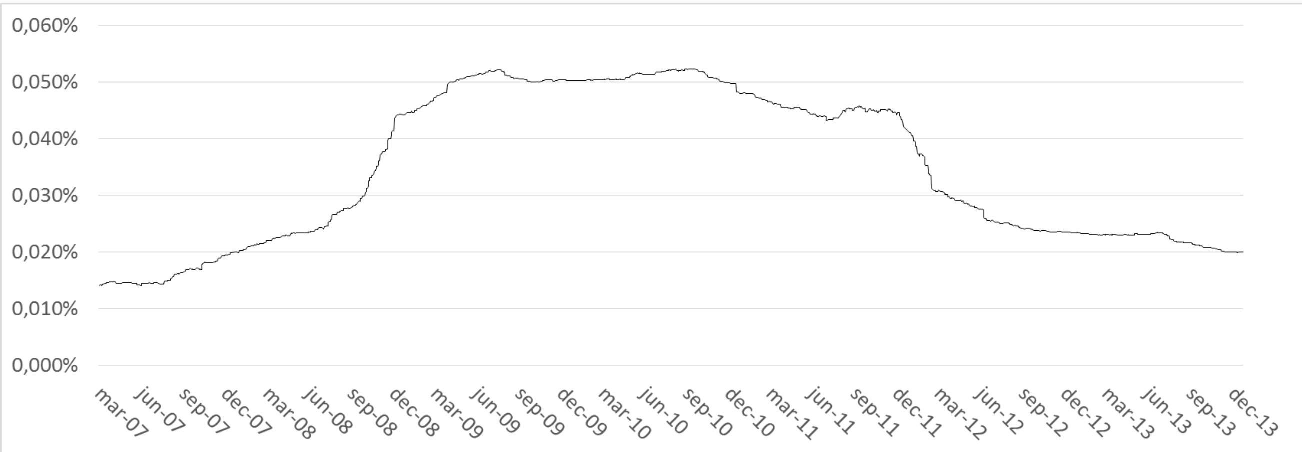
When calculating the optimal weights for the next day in the minimum variance portfolio the forecasted variance-covariance matrix (H matrix) has to be estimated. Two different ways of forecasting the variance-covariance matrix will be used proposed by Narang and Bhalla (2001). The first method of forecasting the H-matrix is through the Historical Variance Model and the second method is through a multivariate GARCH model.

3.3.1 Historical Variance Model

Narang and Bhalla (2001) proposed using a moving average of 800 trading days for the Historical Variance Model (HV). The forecast for period t+1 is thus only the variance of the last 800 trading days.

In Figure 6 it is easy to see that the HV model probably has a poor forecasting performance due to the smooth appearance, and we know that market volatility is not even close to being smooth. Furthermore the HV model uses a moving average of the last 800 observations including the financial crisis with high volatility, which will affect the forecasted variance under a long period, as can be seen in the figure. We can draw the conclusion that the HV model would probably perform better if a crisis period with high volatility was not included in the sample or if a smaller moving average period was used.

Figure 6. HV model



Note: One step ahead forecasted variance with the HV model.

3.3.2 Dynamic Conditional Correlation Model

A more sophisticated method of forecasting the variance is through a multivariate GARCH model. Narang and Bhalla (2001) used the BEKK (Baba-Engle-Kraft-Kroner) model defined by Engle and Kroner (1995) in their study. One of the contributions in this study will be to replace the BEKK model with the DCC model proposed by Engle and Sheppard (2001). The BEKK and DCC model are similar but with the difference that the BEKK model suffers from the “curse of dimensionality” whereas DCC does not as Caporin and McAleer (2012) writes it in their study about the differences between the BEKK and DCC model. Curse of dimensionality will make the number of parameters, which have to be estimated in the model, increase at a higher order than the number of added assets. This makes the estimation of the BEKK model more complex and time consuming.

The multivariate volatility model is very similar to the univariate volatility model described earlier (GARCH model), but with the significant difference that the former one simultaneously uses multiple time series when estimating time-varying correlation, i.e. it specifies how the covariance evolves over time between the assets in the portfolio. Bollerslev, Engle and Wooldridge (1988) introduced the first multivariate GARCH model named the VECH model. They expressed the VECH model as follows:

$$vech(H_t) = C + \sum_{i=1}^{q} A_i vech(\epsilon_{t-i}\epsilon'_{t-i}) + \sum_{j=1}^{p} B_j vech(H_{t-j}) \tag{26}$$

$$\epsilon_t | \psi_{t-1} \sim N(0, H_t) \tag{27}$$

where H_t is the conditional variance-covariance matrix, ϵ_t is the innovation (or disturbance) vector, C is a parameter vector, A and B are parameter matrices and ψ_{t-1} is the information set in time t-1. $vech(\cdot)$ denotes the column-stacking operator of the lower portion of the symmetric matrix. In the case of 2 time series and when $q = p = 1$ the VECH model will be defined as;

$$vech(H_t) = C + Avech(\epsilon_{t-1}\epsilon'_{t-1}) + Bvech(H_{t-1}) \tag{28}$$

In this case the variance-covariance matrix will have the form of:

$$H_t = \begin{bmatrix} \sigma_{11t} & \sigma_{12t} \\ \sigma_{21t} & \sigma_{22t} \end{bmatrix} \tag{29}$$

and the $vech$ operator takes the lower portion, excluding σ_{12t} which equals σ_{21t}, and stacks the element into a vector:

$$vech(H_t) = \begin{bmatrix} \sigma_{11t} \\ \sigma_{21t} \\ \sigma_{22t} \end{bmatrix} \tag{30}$$

Hence it is easy to see that σ_{11t} represents the conditional variance at time t for asset 1 and σ_{22t} represents the conditional variance for asset 2 at time t and σ_{21t} represent the conditional covariance between the two assets at time t.

Engle and Kroner (1995) discuss the major weakness of the VECH model and a new parameterization is proposed that impose two restrictions on the multivariate GARCH model that makes the H matrix positive definite for all values. The BEKK model is represented in Engle and Kroner (1995) as follows:

$$H_t = C_0^{*\prime} C_0^* + \sum_{k=1}^{K} C_{1k}^{*\prime} x_t x_t^{*\prime} C_{1k}^* + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ik}^{*\prime} \epsilon_{t-i} \epsilon_{t-i}^{\prime} A_{ik}^* + \sum_{k=1}^{K}\sum_{i=1}^{p} G_{ik}^{*\prime} H_{t-i} G_{ik}^* \tag{31}$$

where A_{ik}^* and G_{ik}^* are n x n parameter matrices and C_0^* is an triangular matrix of parameters. For a GARCH(1,1) model with 2 assets the BEKK model is represented by:

$$H_t = C'C + A'\epsilon_{t-1}\epsilon_{t-1}^{\prime} A + G' H_{t-1} G \tag{32}$$

It is now clear that the positive definiteness of the H matrix is ensured by the right hand side due to the outcome of multiplying a matrix and its transpose.

For the conditional variance-covariance matrix Engle and Sheppard (2001) proposed the model;

$$H_t = D_t R_t D_t \tag{33}$$

where D_t is a diagonal matrix of time varying standard deviations, $\sqrt{\sigma_{it}^2}$, from a univariate GARCH model:

$$\sigma_{it}^2 = \omega_i + \sum_{m=1}^{M_i} \alpha_{im}\varepsilon_{it-m}^2 + \sum_{s=1}^{S_i} \beta_{is}\sigma_{it-s}^2 \tag{34}$$

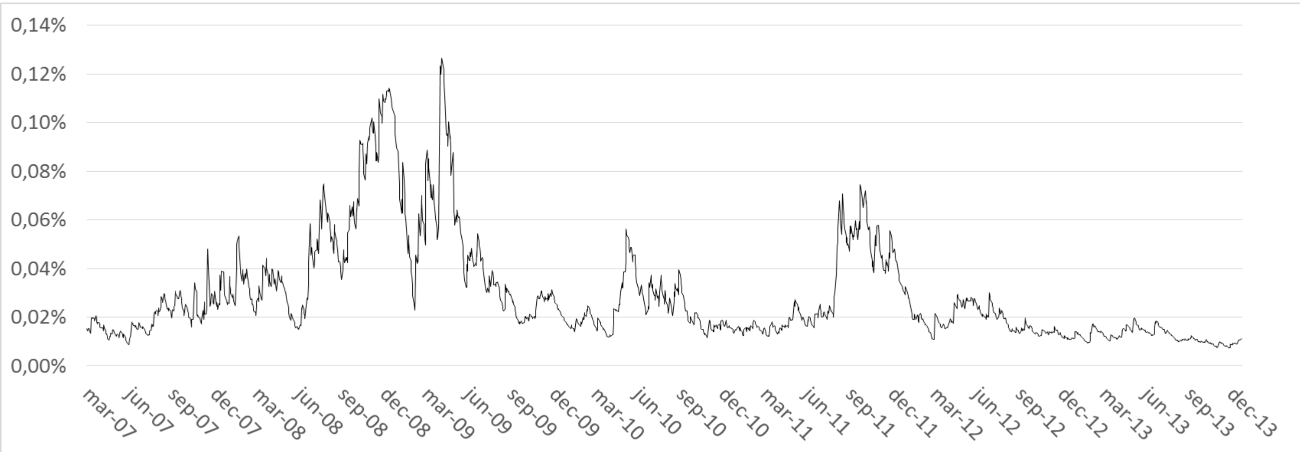
which is the same GARCH model as the GARCH(m,s) model presented earlier in the study, and R_t it the time varying correlation matrix which satisfies:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{35}$$

$$Q_t = \left(1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n\right)\overline{Q} + \sum_{m=1}^{M} \alpha_m \left(\epsilon_{t-m}\epsilon_{t-m}^{'}\right) + \sum_{n=1}^{N} \beta_n Q_{t-n} \tag{36}$$

where ϵ_t is a vector with elements $\epsilon_t = a_{it}/\sqrt{\sigma_{ii,t}}$, $\overline{Q}$ is the unconditional covariance matrix and α and β are nonnegative scalars. The one step ahead forecast from the DCC(1,1) model can be seen in Figure 7.

Figure 7. DCC model



Note: One step ahead forecasted variance with the DCC(1,1) model.

From looking on the graphs it is easy to see that the DCC model has better forecasting properties compared to the historical variance model in Figure 6. Both forecasting models are formally tested with the auxiliary regression model proposed by Pagan and Schwert (1990). Pojarliev and Polasek (2001) used this auxiliary regression model in their study to formally test the forecasting performance of their models. They express the forecasting performance model as follows:

$$r_{i,t}^2 = \alpha + \beta \hat{\sigma}_{i,t}^2 + \varepsilon_t \tag{37}$$

where $r_{i,t}^2$ are the squared returns or "realized volatility" of the portfolios and $\hat{\sigma}_{i,t}^2$ are the variance forecasts give by the HV and DCC model. The results from the auxiliary regressions are shown in Table 4.

Table 4: Forecasting Performance

	α (Prob.)	β*(Prob.)*	R^2
HV	0.0003 (0.00)	0.0917 (0.59)	0.0001
DCC	-8.82E-05 (0.04)	1.6116 (0.00)	0.0956

The measure of overall fitness, R^2, is lower for the HV model and β, the measure of the bias in the model, is highly insignificant. For the DCC model R^2 is higher and β highly significant. We can therefore conclude, as expected, that the HV model performs poorly compared to the DCC model. The last coefficient, α, is also a measure of bias in the model and is significant in both models. For the models not to be biased α should be close to 0 and β close to 1. The conclusion is that the DCC model should be the favoured model. As can be seen from β in the table for the DCC model there is a bias in the model. Comparing this with the results of Pojarliev and Polasek (2001) gives us the same conclusion. In their case the multivariate volatility model outperforms the historical variance model in all aspects. The highest R^2 they obtained for the multivariate volatility model they used (the BEKK model) is 0.11 compared with 0.095 for the DCC model in this study.

3.4 Portfolio Construction

As discussed in section Background and Theory investors will only accept more risk if the expected return is higher. All risk averse investors will therefore hold a portfolio on the efficient frontier. Where on the efficient frontier depends on the investors' preferences. Pojarliev and Polasek (2001) discuss that in the absence of a target for the portfolio return, due to no investor preferences, and the lack of a risk-free asset the portfolio is the global minimum-variance portfolio found the furthest to the left on the frontier in Figure 1, i.e., it is the portfolio, which gives the highest amount of expected return given the minimum amount of variance in a mean-variance portfolio. The minimum-variance portfolio is found by minimizing the portfolio variance given in Equation (3).

$$\min_{w} w'Hw \;\; s.t. \;\; w'\iota = 1 \tag{38}$$

Using the Lagrangian for the minimization problem

$$\mathcal{L} = w'Hw - \lambda(w'\iota - 1) \tag{39}$$

First order derivation gives

$$\frac{\partial \mathcal{L}}{\partial w} = 2Hw - \lambda\iota = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = w'\iota - 1 = 0 \tag{41}$$

By solving for the weights vector *w* we can find the weight of each asset in the portfolio, which will minimize the variance of the portfolio. Solving Equation (42) for *w* gives us:

$$w = \frac{1}{2}\lambda H^{-1}\iota \tag{42}$$

To solve for λ we multiply with the transpose of ι which give us:

$$\iota' w = \frac{1}{2}\lambda\iota' H^{-1}\iota \tag{43}$$

And since $w'\iota = 1$ we have that $\iota' w = 1$ and can now solve for λ:

$$\lambda = 2/(\iota' H^{-1}\iota) \tag{44}$$

Substituting λ in Equation (44) gives:

$$w = H^{-1}\iota/(\iota' H^{-1}\iota) \tag{45}$$

And now we rewrite the equation:

$$w = \frac{1}{C} H^{-1} \iota \tag{46}$$

where C is:

$$C = \iota' H^{-1} \iota \tag{47}$$

In the derivation above H^{-1} is the inverse variance-covariance matrix of the return and ι is a vector of ones. The portfolio weights in all the portfolios will only depend on the H^{-1} matrix, which is also called the precision matrix. The weights vector w is calculated without a target, i.e., no pre-specified portfolio return in all the portfolios below.

In all portfolios short-selling will be allowed and the assumption of no transaction costs will be made.

3.4.1 The Historical Variance Portfolio

This portfolio is the simplest one where the forecasted variance-covariance matrix, as discussed earlier, is only the variance of the last 800 trading days for the three assets in the portfolio. When the variance-covariance matrix is calculated the inverse of the matrix is calculated to get the H^{-1} matrix, which is used in the formulas above to calculate the weights vector. The weights vector will then be the forecasted weights of the assets to be held until the next trading day until the new forecasted weights are calculated through the new forecasted variance-covariance matrix.

3.4.2 The Dynamic Conditional Correlation Portfolio

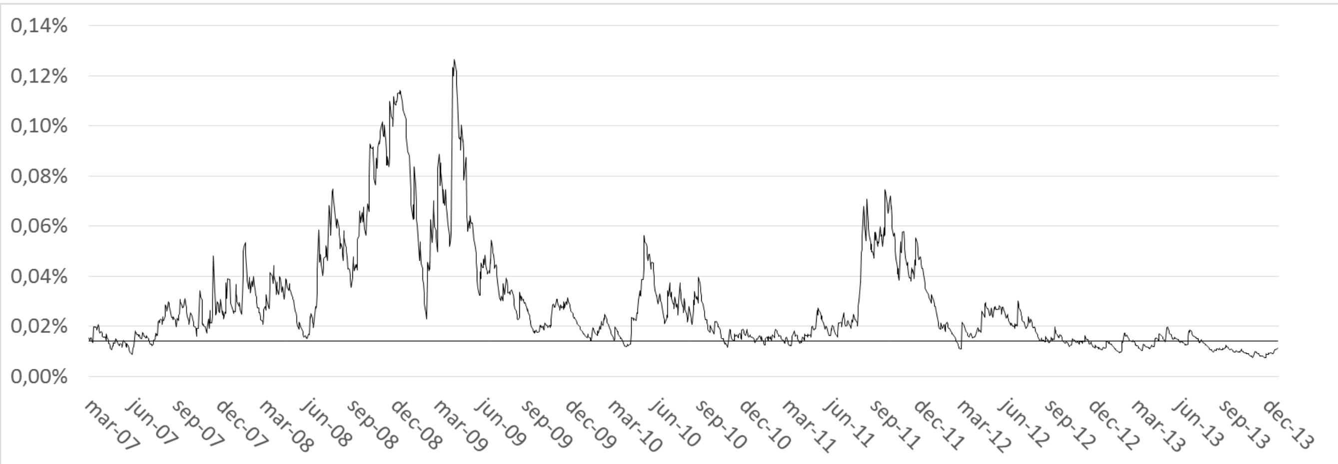
The weights in the DCC portfolio are calculated by using the forecasted variance-covariance matrix from the DCC model discussed in Section 3.3.2. instead of the simple historical variance model. Except from that, the calculations are made exactly as for The Historical Variance Portfolio above.

3.4.3 The Market Timing Portfolio

The Market Timing Portfolio also uses the DCC model to estimate the variance-covariance matrix to be used in the calculation of the weights vector. But in this case a market-timing rule will be used inspired by Pojarliev and Polasek (2001). The rule they imposed was that if the forecasted variance is twice as high as the historical volatility, they invested in the market, otherwise in a risk-free asset. In this study the market-timing rule will not be as aggressive as their rule. My proposed timing rule is; if the forecasted volatility is higher than the historical volatility, the investment will be in the market and otherwise in a risk-free asset. The reason for this is the risk-return trade-off discussed earlier. If the expected variance is higher than normal for the next period the expected return should be higher as well, giving higher return in the long run, and investing in the risk-free asset when the forecasted variance is lower than normal will lower the total portfolio risk. This means that if the volatility is higher than normal the investment will be in the portfolio and, thus, it still is an aggressive timing rule and the investment in the stocks will be made when the volatility in the portfolio is high and in the risk-free asset when the portfolio volatility is stable. The 6-month STIBOR rate will be used as a proxy for the risk-free rate, which as of writing this paper is approximately 1%.

Figure 8 illustrates the market timing strategy. The lower horizontal line is the historical variance with a value of 0.014% and the investment in the DCC portfolio will be executed when the indicator of the forecasted variance is above the line. When the indicator is below the line the investment will be made in the risk-free asset.

Figure 8. Trigger for the market timing portfolio



Note: One step ahead forecasted variance with the DCC(1,1) model and a trigger for investment in the mean-variance portfolio or the risk-free asset

3.5 Testing the Portfolios

The portfolios' performance is compared with two benchmark portfolios and their performances. The benchmark portfolios are two buy-and-hold portfolios i.e. using a passive portfolio strategy. The first portfolio is created using the same assets as in the previous portfolios but the portfolio will be made up by 1/3 of each stock, giving the assets the same weight in the portfolio. The simple buy-and-hold strategy is, buying them at the start of the out-of-sample-period and selling them at the end of the out-of-sample-period. This portfolio is called the buy-and-hold portfolio, BHP. The second portfolio is the OMXS30 index. Investing in the index portfolio at the beginning of the out-of-sample-period and selling at the end of the period. This portfolio will simply be called OMSX30. The out-of-sample-period will also be divided into sub periods when the comparison is made to test the performance during different periods. This is done since the sample period includes a non-normal market event, the financial crisis, and it will also show if the results are consistent regardless of when the strategy is implemented. All criteria used for the comparison of the portfolio performance are as follows;

- Total cumulative return
- Annual total rate of return
- After crisis cumulative return
- Annual after crisis rate of return
- Two years cumulative return
- Annual two years rate of return
- One year cumulative return
- Annualized standard deviation for all the periods
- Sharpe ratios for all the periods

where the Sharpe ratio, as discussed before, is expressed as;

$$s_p = \frac{\overline{r_p} - r_f}{\sigma_p}$$

where $\overline{r_p}$ is the average return of portfolio P and σ_p is the standard deviation of portfolio P, and r_f is the risk-free rate (STIBOR).

4. RESULTS

Table 5 below includes the results from all the portfolios in the study. Starting by looking at the holding period for the entire out-of-sample-period shows us that the total cumulative return for the MTP is much higher compared to the returns of the other portfolios. Also, the Sharpe ratio is about three times as high compared to the other portfolios. Figures 9 through 13 illustrate the cumulative return for the entire out-of-sample-period of the mean-variance portfolios and the benchmark portfolios.

Table 5. Results

	HVP	DCCP	MTP	BHP	OMXS30
Total Cumulative Return	17,19%	16,18%	37%	-15,47%	14,47%
Daily st. dev.	1,90%	1,95%	1,88%	1,74%	1,63%
Average Annual Rate of Return	2,37%	2,24%	4,78%	-2,45%	2,02%
Annual st. dev.	30,23%	30,99%	29,91%	27,58%	25,92%
Sharpe Total Period	0,045	0,040	0,126	N/A	0,039
Total After Crisis Return	60,36%	59,62%	84,01%	49%	92,13%
Annual After Crisis Return	9,99%	9,89%	13,08%	4,73%	14,07%
After Crisis st. dev.	24,80%	25,54%	24,07%	34%	23%
Sharpe After Crisis	0,36	0,35	0,50	0,11	0,58
2 Year Return	26%	26,81%	32%	16%	33,37%
Annual 2 year return	12%	13%	15%	8%	15%
2 year st. dev.	18%	18,77%	15%	41%	16%
Sharpe 2 year	0,61	0,62	0,95	0,16	0,88
1 Year Return	18%	16,27%	14%	10%	20,16%
1 year st. dev.	15%	16,03%	8%	32%	13%
Sharpe 1 year	1,07	0,95	1,65	0,29	1,46

Note: Results from the mean-variance portfolios and the benchmark portfolios

Figure 9. OMXS30

Note: Cumulative return of the OMXS30 index for the entire out-of-sample period

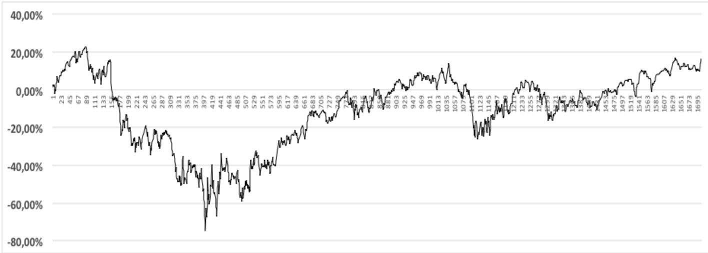
Figure 10. BH portfolio

Note: Cumulative return of the BH portfolio for the entire out-of-sample period

Figure 11. HV portfolio

Note: Cumulative return of the HV portfolio for the entire out-of-sample period

Figure 12. DCC portfolio



Note: Cumulative return of the DCC portfolio for the entire out-of-sample period

Figure 13. MT portfolio

Note: Cumulative return of the MT portfolio for the entire out-of-sample period

It is easy to see that the portfolios follow the same trend over time. Looking at the period around the financial crisis we can see that the portfolios based on the three stocks decrease to almost negative 80 per cent and OMXS30 only declines to a negative 60 per cent. This might be one of the reasons why OMXS30 outperforms all portfolios in the after crisis period. An important thing to note about the after crisis period is that the investor must enter the market at its bottom when it just starts to turn around. This is highly unlikely and mostly due to pure luck and therefore the after crisis period is the least important period when comparing the performance of the portfolios.

In all other periods where the market entry is in a normal market state the MTP outperforms all other portfolios when looking only on the Sharpe ratio. However, when it comes to the cumulative return the MTP is not that impressive in the sub periods after the crisis. One interesting point to notice is that the BHP portfolio would have a negative 15.47 per cent cumulative return if the investor bought the portfolio in the beginning of the out-of-sample-period and the MTP had a positive 37 per cent cumulative return under the same period. The same result can be found for all the periods, the MTP outperforms the BHP and so do the rest of the dynamic portfolios without the market-timing rule.

Overall the MTP performs well and it appears as if the MTP is the strategy to prefer if a financial crisis will occur during the holding period. But since this is a rare event the most interesting periods are the two and one year horizon. Focusing on these periods we can see in Table 5 that the HVP, DCCP and OMXS30 outperform the MTP in the one-year period if we only focus on cumulative return. When it comes to the risk adjusted performance we can see that the MTP is more satisfying with its higher Sharpe ratio. In the two-year return period the only portfolio to outperform the MTP, when it comes to cumulative return, is the OMXS30. But the standard deviation for the MTP is lower thanks to the investments in the risk-free assets, giving an increased Sharpe ratio. Comparing this with the results of Pojarliev and Polasek (2001) the results are not that impressive. They found that their market-timing portfolio outperformed all portfolios in their study. This might be due to the nature of their portfolios. In their MT portfolio they used three indexes instead of individual shares and compared the results with a world index. Therefore a fairer benchmark in this study is the BHP and it seems that the MTP performs well compared to the BHP, both in Sharpe ratio and cumulative return.

The lower cumulative return for the MTP in the last year compared to the HVP and DCCP is probably due to the trigger of the MTP that only invests in the market when the forecasted variance is higher than the historical variance. As seen in Figure 8, the market seems to be more stable in the last year, which results in the trigger for the investment in the market to be above the forecasted variance and hence most of the investment during the period is made in the risk-free asset instead of the market. Resulting in a lower cumulative return but a higher Sharpe ratio due to the lower risk in the portfolio, measured as standard deviation.

5. CONCLUSION

This study uses the EGARCH-M model to find stocks included in the OMXS30 index with significant trade-off between risk and return to be used in a dynamic portfolio strategy. Two models are used to forecast the variance-covariance matrix for the portfolios, a historically based volatility model, and a multivariate GARCH model, the DCC model, which are used in the calculation of the weights vector for the portfolios.

The results indicate that it is difficult to gain a higher than market return when using active portfolio management for a portfolio with few assets without using some sort of market timing rule. What can be seen is that using active portfolio management on a non-diversified portfolio gives higher return and higher Sharpe ratio compared to a passive strategy of just buying and holding the assets. The result also indicates that it might be a good idea to use a more advanced market timing strategy that dynamically changes the trigger depending on the overall market trend. A new market timing rule could be that if the market has a low volatility during a certain time period the trigger will automatically be set lower and vice versa, i.e., if the variance is lower during a few days the trigger might be set at 0.5 of the historical variance and if the variance is higher over the same length of period then the trigger might be set at 2.

To summarise, the MTP is a good strategy compared to just buying and holding the assets in the portfolio. Also, other dynamic portfolio strategies outperform the buy-and-hold strategy as well. Nevertheless, it seems as if the OMXS30 still is a good buy-and-hold portfolio compared to the dynamic strategies without the market-timing rule. This market timing strategy can hence be used by all investors not holding the market index or investors looking for a portfolio that gives higher return per risk unit compared to the market index. An interesting study would be to use all the stocks included in the OMXS30 index in the MTP portfolio or use three or more indexes on the Stockholm Stock Exchange and compare the results against the OMXSPI index, which includes all the shares on the stock exchange in Sweden.

It is also worth mentioning the forecasting properties of the DCC model. As shown in Table 4 the DCC model is a reliable multivariate GARCH model with highly significant forecasting properties. We can therefore conclude that using the DCC model with portfolio strategies that make use of the DCC models properties will give the investor opportunities to exploit the trade-off between risk and return as discussed in the section Theory and Background.

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