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Bachelor thesis

Chirally Symmetric Technicolor extension to the Standard Model

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Abstract

In this thesis a chirally symmetric technicolor model is investigated as an extension to the Standard Model. The extension is based on low-energy QCD, with a linear sigma model used to induce the additional degrees of freedom corresponding to the lightest particles of the new techni-sector. The main goal is to incorporate the Standard Model Higgs into the new model, as to explain its vacuum expectation value and origin within the framework of the new model. The new model is examined for two cases. First, introducing two new fermions under the new sector; this will be shown to give rise to a possible explanation of the Higgs vacuum expectation value's origin. Second, three fermions are introduced under the new sector, making for a possible identification of the Higgs as a composite object of the fermions in question.

Contents

1 Introduction

As of now, the current observational data from particle experiments can be fully covered within the framework of the Standard Model (SM). This includes the fairly recent discovery of the sought after Higgs particle. However, as the SM contains the existence of a Higgs particle, it does not offer any explanation of the origin to it. This thesis aims to extend the SM of particle physics as to incorporate, in first place, a deeper explanation of the origin of the Higgs boson. For this, a so called technicolor extension to the SM is considered. The technicolor extension is based on low-energy quantum chromodynamics (QCD), hence analogies between the two are drawn frequently.

The primary goal of this thesis is to reproduce and discuss the SM extension, presented in Ref. [1], which is a chirally symmetric technicolor (CSTC) model. The focus is to understand the basic theory underlying the model and to obtain the physical Lagrangians. Sect. 3 is devoted for that task. Further, the concept in the CSTC model will be additionally extended, as to consider a more extensive case; presented in Sect. 4. The study following in Sect. 4 will also be a reproduction of notes by R. Pasechnik, with the main goal being to follow the procedure done in Sect. 3 for the extended case.

As a start, the low-energy QCD theory, which the CSTC model is based upon, is generally outlined in Sect. 2. Several concepts therein are applied later on the CSTC model. In particular, the general model on which both QCD and the CSTC model is based on is called the linear sigma model (LσM), which is presented there for QCD.

As a brief review, an Appendix on basic group theory (Appendix A) and another on the essential theory of the SM (Appendix B) have been added. There some of the concepts, which are used in the thesis, are explained.

2 QCD: Chiral symmetry and Linear sigma model

The outlines of the low-energy theory for QCD is here presented, which will be the foundation of the CSTC model under study later. The main focus is on the LσM and its consequences considering spontaneous symmetry breaking. An additional comment is that the conventional units used are the natural units, where $\hbar = c = 1$ and whenever repeated indices are present summation over the indices is implied.

The QCD Lagrangian for the quarks can be written as

$$\mathcal{L}_{QCD} = i\overline{q}_L\gamma^\mu\mathcal{D}_\mu q_L + i\overline{q}_R\gamma^\mu\mathcal{D}_\mu q_R - \overline{q}_R\mathcal{M}q_L - \overline{q}_L\mathcal{M}q_R. \tag{1}$$

In (1), $\mathcal{M}$ denotes the matrix containing the masses of the different quarks and $\mathcal{D}_\mu$ is the covariant derivative. Here the case of quark flavours $n_F = 2$ is considered. Thus, we have

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \tag{2}$$

with straightforward generalization to arbitrary n_F. As is seen, the QCD Lagrangian (1) has in the chiral limit (i.e. for vanishing quark masses $\mathcal{M} = 0$) a so called chiral symmetry of the form $G_\chi = SU(2)_L \times SU(2)_R$ [2].

As is discussed in Ref. [5], in the low-energy region, the interaction term of the lightest mesons and the quarks can be taken to be of the form $\mathcal{L}_{int} = -\bar{q}\Phi q$. Here Φ is a set of scalar and pseudoscalar fields, corresponding to the mesons. In order to maintain chiral symmetry, the scalar field set must transform as

$$\Phi \to g_R \Phi g_L^\dagger, \tag{3}$$

under G_χ (where $g_R \in SU(2)_R$ and $g_L \in SU(2)_L$); Φ is said to be in the bi-fundamental representation of G_χ. As further derived in Ref. [5], one can write the set of mesons in the form $\Phi = (\mathbb{1}\sigma + i\tau_a\pi_a)$, where σ is a scalar field and π_a $(a = 1, 2, 3)$ is a triplet of pseudoscalar fields.[1] The name linear sigma model can now be apparent from the form of Φ, which is linear in the fields (particularly σ). A Lagrangian, with a typical form for a LσM, invariant under G_χ can now be set up for Φ as

$$\mathcal{L} = \frac{1}{4}|\partial_\mu \Phi|^2 - \frac{\mu^2}{2}|\Phi|^2 - \frac{\lambda}{4}|\Phi|^4. \tag{4}$$

The notation $|\Phi|^2 = \mathrm{Tr}(\Phi\Phi^\dagger)$ is used above [5].

The energy of the system is minimized when $|\Phi|$ is a constant, since then the kinetic term have no contribution to the energy. The potential of (4) is given by the last two terms, i.e.

$$V = \frac{\mu^2}{2}|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4. \tag{5}$$

The potential is minimized for $|\Phi|(\mu^2 + \lambda|\Phi|^2) = 0$, whereas to be bounded from below $\lambda > 0$. Hence, there are two cases for the minimum of the potential. Either $\mu^2 > 0$, which means the potential is minimized for $|\Phi| = 0$ and there is only one unique minimum point. Or $\mu^2 < 0$, such that the potential is minimized for $|\Phi|^2 = -\mu^2/\lambda = u^2$, $u = \sqrt{-\mu^2/\lambda}$. As $|\phi|^2 = \sigma^2 + \pi^2$ (where $\pi^2 = \pi_a\pi_a$), it implies there are degenerate minima on the 3-sphere $\sigma^2 + \pi^2 = u^2$. As a specific minimum is chosen to represent the ground state, interesting consequences arise [5].

The ground state can be chosen such that σ can be said to gain a vacuum expectation value (vev), hence $\langle\sigma\rangle = u$. Thus, $\Phi = \mathbb{1}(u + \sigma) + i\tau_a\pi_a$ and inserting this into the Lagrangian (4) yields

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2 - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 - \lambda u\sigma(\sigma^2 + \pi^2) - \lambda u^2\sigma^2 + \text{constants} \\ &= \frac{1}{4}|\partial_\mu\Phi|^2 - \frac{\mu^2}{2}|\Phi|^2 - \frac{\lambda}{4}|\Phi|^4 - \lambda u^2\sigma^2 + \text{constants}.\end{aligned} \tag{6}$$

As can be seen by examining (6), σ has obtained a mass term $m_\sigma^2 = 2\lambda u^2$ while the triplet π_a appears massless [5].

[1] Pseudoscalar fields are odd under parity transformation, while scalar fields are even.

The Lagrangian (6) is not invariant under the initial symmetry group G_χ, as a consequence of choosing a vev. The symmetry is said to be spontaneously broken. However, considering the subgroup $H_\chi = SU(2)_V$ of G_χ, where V stands for vector and treats left-handed (LH) and right-handed (RH) components equally (i.e. $g_L = g_R$), the Lagrangian appears invariant under it [3]. The invariance is due to that the transformation of Φ under $SU(2)_V$ reads $\Phi \to g_V \Phi g_V^\dagger = \mathbb{1}\sigma + iV\tau_a\pi_a V^\dagger$, so that σ is a singlet under $SU(2)_V$ and $\tau_a\pi_a$ transforms in the adjoint representation.

The (chiral) symmetry breaking can thus be written as $SU(2)_L \times SU(2)_R \to SU(2)_V$, with the consequence of yielding three massless particle modes. The massless particles arising due to the spontaneous symmetry breaking (SSB) are referred to as Goldstone bosons (GSBs), and correspond to a general phenomenon stated in the Goldstone theorem. The Goldstone theorem states that each generator, of a continuous symmetry group, which commutes with the Hamiltonian of a system but does not annihilate the ground state, gives rise to a massless GSB [4]. A generator satisfying the conditions stated in the Goldstone theorem is said to be broken.

For the LσM system considered in this section, the Lagrangian (or equivalently the Hamiltonian) is initially invariant under the chiral symmetry group, G_χ. However, since the ground state, chosen as $\langle|\Phi|\rangle = \mathbb{1}\langle\sigma\rangle = \mathbb{1}u$, is not invariant under any other transformation of the form (3) than the vector subgroup $SU(2)_V$ [3] where $g_L = g_R$, there are three broken generators from the initial symmetry group G_χ. Hence, Goldstone's theorem states that three massless modes should appear, as was seen in (6) (where the three π_a correspond to the GSBs).

In the CSTC extension of Ref. [1], to be treated in this thesis, the basic theory is built up from low-energy QCD. Hence, the LσM and the concepts presented above, will be used substantially when considering the CSTC extension. Further theory of LσMs and different versions of it in QCD can be found in Ref. [2][4]. Note also that LσMs do not exclusively deal with chiral symmetry, e.g. another important area of its application is the Higgs mechanism, briefly described in Appendix B.3 (as part of the SM summary).

3 Techni-QCD: Two techniquark case

In this section, the first part of the thesis is presented. The primary goal is to reproduce and discuss the theory of the chirally symmetric technicolor (CSTC) model treated in Ref. [1]. It explores an extension to the SM based on low-energy QCD, and hence analogies with QCD concepts (the main ones were discussed in Sect. 2) are made throughout the new proposed physics presented.

The starting point of the CSTC model is to introduce a new interaction sector, called the techni-sector or techni-QCD (T-QCD). Following in analogy to QCD, the new sector interacts through the so called technicolor charge under the group $SU(3)_{TC}$, which is propagated by technigluons (only interacting with the techni-sector). New additional fermions, henceforth referred to as techniquarks (or T-quarks), are introduced. They are in the fundamental representation of the $SU(3)_{TC}$ group. The theory is taken to initially maintain a global chiral symmetry, as in QCD, under $SU(n_F)_L \times SU(n_F)_R$, where n_F is the number of T-quarks considered in the theory.

To use analogy with low-energy QCD, the scale considered must be $\lesssim \Lambda_{TC}$, where Λ_{TC} is the confinement scale for T-QCD (as the confinement scale for QCD is Λ_{QCD}). An approximate lower limit as $\Lambda_{TC} \sim M_{EW}$, where $M_{EW} \sim 100$ GeV is the electroweak scale of the SM, is considered. Hence, regarding scales $\lesssim \Lambda_{TC}$, only the lightest techniparticle spectrum is relevant for study. And as was the basis of low-energy QCD, the spectrum of the lightest hadrons can be that resulting from a chiral symmetry breaking [3]. Thus, following the concept in QCD, one considers

$$SU(n_F)_L \times SU(n_F)_R \to SU(n_F)_V, \tag{7}$$

where $SU(n_F)_V$ is the vector-like subgroup of the initial chiral group $SU(n_F)_L \times SU(n_F)_R$. The chiral symmetry breaking of the techni-sector in (7) will be referred to as TChSB (techni-sector chiral symmetry breaking).

In Ref. [1], the simplest case of $n_F = 2$ is investigated. It corresponds to introducing two new fundamental fermions, T-quarks, under the new techni-sector. The T-quarks are set up as a doublet

$$\tilde{Q} = \begin{pmatrix} U \\ D \end{pmatrix}, \tag{8}$$

where the LH and RH components of $\tilde{Q}$ transform in the fundamental representation of $SU(2)_L$ and $SU(2)_R$. Thus, $\tilde{Q}_L \to g_L \tilde{Q}_L$ and $\tilde{Q}_R \to g_R \tilde{Q}_R$ respectively, where $g_L \in SU(2)_L$ and $g_R \in SU(2)_R$. Further, due to the TChSB as (7), three GSBs are obtained. Following the concept from Sect. 2, a LσM is used to describe the degrees of freedom corresponding to the TChSB. Thus, a set of scalar fields $\tilde{\Phi}$ is considered, where $\tilde{\Phi}$ transforms as $\tilde{\Phi} \to g_R \tilde{\Phi} g_L^\dagger$ (in analogy to (3)). The scalar fields are parametrized such that $\tilde{\Phi} = (\mathbb{1} S + i\tau_a P_a)/2$, $a = 1, 2, 3$. One thus have the scalar field S and the pseudoscalar fields P_a (corresponding to the GSBs). A Lagrangian concerning the techni-sector can then be set up as

$$\mathcal{L}_{2\sigma} = i\overline{\tilde{Q}}\gamma^\mu \partial_\mu \tilde{Q} + \mathrm{Tr}(\partial_\mu \tilde{\Phi} \partial^\mu \tilde{\Phi}^\dagger) + \mu_S^2 \mathrm{Tr}(\tilde{\Phi}\tilde{\Phi}^\dagger) - \lambda_{TC}(\mathrm{Tr}(\tilde{\Phi}\tilde{\Phi}^\dagger))^2 - 2g_{TC}\left(\overline{\tilde{Q}}_L \tilde{\Phi} \tilde{Q}_R + \overline{\tilde{Q}}_R \tilde{\Phi}^\dagger \tilde{Q}_L\right), \tag{9}$$

where the last term in (9) is a Yukawa-type term (allowed by symmetry) and the subscript 2σ denotes LσM for two T-quarks.

The TChSB can appear to be effectively induced by that S gains a vev, $\langle S \rangle = u$. The ground state is thus parametrized as $\mathbb{1}u/2$, being only invariant under the vector subgroup $SU(2)_V$. Alas, in the broken, vector-like, region (where $g_L = g_R$) the T-quark doublet transforms in the fundamental representation and $\tilde{\Phi}$ transforms in the adjoint representation. The scalar field σ does not transform in the broken, vector-like region; it is the triplet of pseudoscalar fields $\tau_a P_a$ that transforms, which hence is in the adjoint representation in the vector-like region (cf. Sect. 2).

In addition to the TChSB, the electroweak symmetry breaking (EWSB) of the SM is left unaltered, arising due to the vev obtained by the Higgs field (see Appendix B.3). Hence, the Higgs particle has the same status as in the SM, being a complex, $SU(2)_W$ doublet scalar field. In order to connect the new techni-sector with the SM, the techni-sector is gauged through the electroweak (EW) SM interactions. This means that the techni-sector is taken to interact with the EW gauge bosons of the SM. The gauging of the techni-sector is obtained by that, in the low-energy limit, one can on phenomenological grounds identify the vector group, $SU(2)_V$, from the TChSB with the weak isospin group, $SU(2)_W$, of the SM. A consequence of the identification $SU(2)_V \equiv SU(2)_W$ is that the Higgs particle and S can mix with the each other as they gain vevs, giving room for modified couplings in the SM. Also, as it is the vector group $SU(2)_V$ that is identified with $SU(2)_W$, the techni-sector interacts vector-like with the weak interaction, in contrast to the SM particles which interacts only through their LH components under $SU(2)_W$.

The techniparticles are classified differently under the EW group, $SU(2)_W \times U(1)_Y$, of the SM, such that they are in different representations of the group. As the T-quarks transform in the fundamental representation of $SU(2)_V$, they are also in the fundamental representation of $SU(2)_W$. Similarly, S is thus a singlet and $\tau_a P_a$ is in the adjoint representation of $SU(2)_W$. The couplings of the T-quarks with the EW gauge bosons are taken to be same as the corresponding quarks in QCD (i.e. the u and d quarks), so that the coupling constants g_1, g_2 are used for the $U(1)_Y, SU(2)_W$ respectively (as in the SM). Further, the T-quarks are assumed to have the same quantum numbers as the QCD quarks, meaning that the hypercharge of $\tilde{Q}$ is $Y_{\tilde{Q}} = Y_Q = 1/3$ and the weak isospin is taken to be $t_3^U = t_3^u = 1/2$ and $t_3^D = t_3^d = -1/2$.

Based on QCD, a non-zero value for the techniquark condensate,

$$\langle \bar{\tilde{Q}} \tilde{Q} \rangle \neq 0, \tag{10}$$

is taken as the main contributor for making the ground state non-invariant under any other subgroup than $SU(2)_V$, in turn giving rise to the TChSB as (7) [2]. As mentioned, the TChSB can appear to be effectively induced by that S gains a vev. The non-zero value of the techniquark condensate can then be seen as a source term for the vev, which will be explained further below. Additionally, it will also be seen that a non-zero techniquark condensate has the effect of giving rise to masses for the GSBs.

As the gauging of the techni-sector is performed, the Lagrangian (9) is modified as to incorporate the EW interactions of the techni-sector. The following covariant derivatives are thus introduced

$$\mathcal{D}_\mu = \left(\partial_\mu - \frac{iY}{2} g_1 B_\mu - \frac{i}{2} g_2 W_\mu^a \tau_a \right), \qquad \mathfrak{d}_\mu P_a = \partial_\mu P_a + g_2 \epsilon_{abc} W_\mu^b P_c, \tag{11}$$

based on the representations of the techniparticles under the EW group. Further, from symmetry arguments, a term mixing the Higgs doublet and the set of scalar fields $\tilde{\Phi}$ is allowed. Thus, in addition to the SM Lagrangian[2] the Lagrangian involving the new

[2]Here also considering the Higgs Lagrangian, see (132), which will be seen to contain mixed field terms as S and Higgs get vev.

techni-sector physics is

$$\mathcal{L}_{2\sigma} = \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mathfrak{d}_\mu P_a \mathfrak{d}^\mu P_a + (\mathcal{D}_\mu \mathcal{H})^\dagger (\mathcal{D}^\mu \mathcal{H}) + i\overline{\tilde{Q}}\gamma^\mu \mathcal{D}_\mu \tilde{Q} - g_{TC}\overline{\tilde{Q}}(S + i\gamma_5 \tau_a P_a)\tilde{Q} \\ + \frac{1}{2}\mu_S^2(S^2 + P^2) + \mu_H^2 \mathcal{H}^2 - \frac{1}{4}\lambda_{TC}(S^2 + P^2)^2 - \lambda_H \mathcal{H}^4 + \lambda \mathcal{H}^2(S^2 + P^2). \quad (12)$$

Note that in (12), the traces of (9) has been written out.

In the text following, treating the CSTC model, the definitions

$$P^2 = \sum_{a=1}^{3} P_a P_a = \tilde{\pi}^0 \tilde{\pi}^0 + 2\tilde{\pi}^+ \tilde{\pi}^-. \quad (13)$$

and $\mathcal{H}^2 = \mathcal{H}\mathcal{H}^\dagger = \mathcal{H}_i^\dagger \mathcal{H}^i$ are used; $\mathcal{H}_i$, $i = 1, 2$, refers to the ith element of the Higgs doublet. The states $\tilde{\pi}^0, \tilde{\pi}^\pm$ are called technipions and are states of definite charge and definite mass, i.e. they correspond to physical techniparticles (mesons). Similar to the pions in QCD [8], the relation (13) is obtained through the linear combinations

$$\begin{cases} -P_1 + iP_2 = \sqrt{2}\tilde{\pi}^+ \\ -P_1 - iP_2 = \sqrt{2}\tilde{\pi}^- \\ P_3 = \tilde{\pi}^0 \end{cases} . \quad (14)$$

As the basic theory for the CSTC model is now presented, to start analysing what consequences the new techni-sector gives rise to, one has to induce the vevs of the Higgs and S. As mentioned, as an approximate lower limit $\Lambda_{TC} \sim M_{EW}$. This means that the vev of S is induced $\sim M_{EW}$ [3], which is the same scale as the Higgs vev is induced. Thus, the vevs of the Higgs and S can be induced at an approximate simultaneous energy scale. As in the SM (see Sect. B.3), the Higgs obtains a vev as

$$\mathcal{H} \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}, \quad (15)$$

where H is the expansion around the vev (v). Similarly,

$$S \to u + S. \quad (16)$$

The two equations above mean that $\langle \mathcal{H}_i \rangle = \langle \mathcal{H}_i^\dagger \rangle = v/\sqrt{2}$, for $i = 2$, and $\langle S \rangle = u$. As mentioned, since S and H mix as they gain vevs, they do not correspond to physical states, i.e. states with definite mass.

3.1 Vacuum stability

The potential part of the Lagrangian (12), denoted $\mathcal{L}_U$, is

$$\mathcal{L}_U = -g_{TC}\overline{\tilde{Q}}(S + i\gamma_5 \tau_a P_a)\tilde{Q} + \frac{1}{2}\mu_S^2(S^2 + P^2) + \mu_H^2 \mathcal{H}^2 \\ - \frac{1}{4}\lambda_{TC}(S^2 + P^2)^2 - \lambda_H \mathcal{H}^4 + \lambda \mathcal{H}^2(S^2 + P^2). \quad (17)$$

The minimum of $\mathcal{L}_U$ should be obtained when considering its vacuum expectation value. Hence, for that case, its first derivatives should be zero and the second derivatives should satisfy

$$\Big\langle\frac{\delta^2\mathcal{L}_U}{\delta(\mathcal{H}^i)^2}\Big\rangle\Big\langle\frac{\delta^2\mathcal{L}_U}{\delta S^2}\Big\rangle-\Big\langle\frac{\delta^2\mathcal{L}_U}{\delta\mathcal{H}^i\delta S}\Big\rangle^2>0 \tag{18}$$

and

$$-\Big\langle\frac{\delta^2\mathcal{L}_U}{\delta(\mathcal{H}^i)^2}\Big\rangle>0,\qquad -\Big\langle\frac{\delta^2\mathcal{L}_U}{\delta S^2}\Big\rangle>0. \tag{19}$$

so that the Hessian matrix is positive definite [16].

Considering the first derivatives of the potential, it is obtained that

$$\begin{aligned}\Big\langle\frac{\delta\mathcal{L}_U}{\delta\mathcal{H}^i}\Big\rangle&=\Big\langle\mu_H^2\mathcal{H}_i^\dagger-2\lambda_H(\mathcal{H}_i^\dagger)^2\mathcal{H}^i+\lambda\mathcal{H}_i^\dagger(S^2+P^2)\Big\rangle\\&=\Big\langle\mathcal{H}_i^\dagger\Big\rangle\Big\langle\mu_H^2-2\lambda_H\mathcal{H}_i^\dagger\mathcal{H}^i+\lambda(S^2+P^2)\Big\rangle=\sqrt{2}v(\mu_H^2-\lambda_H v^2+\lambda u^2)=0\end{aligned} \tag{20}$$

and

$$\begin{aligned}\Big\langle\frac{\delta\mathcal{L}_U}{\delta\mathcal{S}}\Big\rangle&=\Big\langle\mu_S^2S-\lambda_{TC}S(S^2+P^2)+2\lambda\mathcal{H}^2S-g_{TC}\langle\overline{\tilde{Q}}\tilde{Q}\rangle\Big\rangle\\&=\Big\langle S\Big\rangle\Big\langle\mu_S^2-\lambda_{TC}(S^2+P^2)+2\lambda\mathcal{H}^2-\frac{g_{TC}\langle\overline{\tilde{Q}}\tilde{Q}\rangle}{S}\Big\rangle\\&=u\left(\mu_S^2-\lambda_{TC}u^2+\lambda v^2-\frac{g_{TC}\langle\overline{\tilde{Q}}\tilde{Q}\rangle}{u}\right)=0\end{aligned} \tag{21}$$

must hold. As is evident from (20), the relation

$$\mu_H^2-\lambda_H v^2+\lambda u^2=0, \tag{22}$$

holds (since here we consider the degenerate minima case of the LσM potential, as in Sect. 2, implying $v\neq 0$).

As is derived later, the mass terms of the technipions is given by (31). Comparing with (21), where

$$\mu_S^2-\lambda_{TC}u^2+\lambda v^2-g_{TC}\langle\overline{\tilde{Q}}\tilde{Q}\rangle/u=0 \tag{23}$$

is obtained, yields

$$m_{\tilde{\pi}}^2=-\frac{g_{TC}\langle\overline{\tilde{Q}}\tilde{Q}\rangle}{u}. \tag{24}$$

Thus, as mentioned earlier, the non-zero value of the techniquark condensate is seen to give rise to the technipion mass. Note that $\langle\overline{\tilde{Q}}\tilde{Q}\rangle<0$ and $g_{TC}>0$, where the negative value for the quark condensate is a direct analogy taken from QCD [1].

By comparing (22) and (23), and also use (24), v^2 and u^2 can be uncoupled and expressed as

$$v^2=\frac{\mu_H^2\lambda_{TC}+\lambda(\mu_S^2+m_{\tilde{\pi}}^2)}{\lambda_H\lambda_{TC}-\lambda^2} \tag{25}$$

and

$$u^2 = \frac{\mu_H^2 \lambda + \lambda_H(\mu_S^2 + m_{\tilde{\pi}}^2)}{\lambda_H \lambda_{TC} - \lambda^2}. \tag{26}$$

Turning to the second derivatives and considering (19) yields

$$0 > \left\langle \frac{\delta^2 \mathcal{L}_U}{\delta(\mathcal{H}^i)^2} \right\rangle = \left\langle -2\lambda_H(\mathcal{H}_i^\dagger)^2 \right\rangle \Rightarrow -\lambda_H v^2 < 0 \Rightarrow \lambda_H > 0 \tag{27}$$

and

$$\begin{aligned} 0 > \left\langle \frac{\delta^2 \mathcal{L}_U}{\delta S^2} \right\rangle = \left\langle \mu_S^2 - \lambda_{TC}(3S^2 + P^2) + 2\lambda \mathcal{H}^2 \right\rangle &= \mu_S^2 - 3\lambda_{TC} u^2 + \lambda v^2 \\ &= -2\lambda_{TC} u^2 - m_{\tilde{\pi}}^2 \Rightarrow \lambda_{TC} > -\frac{m_{\tilde{\pi}}^2}{2u^2}, \end{aligned} \tag{28}$$

where $u, v > 0$ and (31) has been used. Further, considering that

$$\left\langle \frac{\delta^2 \mathcal{L}_U}{\delta \mathcal{H}^i \delta S} \right\rangle^2 = 4\lambda^2 u^2 v^2, \tag{29}$$

the vacuum stability relation (18) can be seen to equal $m_{\tilde{\sigma}}^2 m_h^2 > 0$, where the mass relations in (39) and (40), yet to be derived, have been used.

3.2 Physical states and masses

Inserting (15) and (16) into the Lagrangian (12), it is seen that mixed terms of H and S are obtained. The mixed terms mean that the states H and S are not states of definite mass, i.e. they do not describe the physical particle states. Collecting all terms with order two in any fields after insertion of the vevs in the Lagrangian, the following terms are obtained

$$\begin{aligned} \begin{pmatrix} H & S \end{pmatrix} \begin{pmatrix} \frac{1}{2}(\mu_H^2 - 3\lambda_H v^2 + \lambda u^2) & \lambda u v \\ \lambda u v & \frac{1}{2}(\mu_S^2 - 3\lambda_{TC} u^2 + \lambda v^2) \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix} \\ - \frac{1}{2}\big(P^2(\lambda_{TC} v^2 - \mu_S^2 - \lambda v^2)\big). \end{aligned} \tag{30}$$

As seen, and mentioned before, it is possible to identify

$$m_{\tilde{\pi}}^2 = \lambda_{TC} v^2 - \mu_S^2 - \lambda v^2. \tag{31}$$

Using (22) and (31), the matrix corresponding to the mixing of H and S can be written as

$$\begin{pmatrix} -\lambda_H v^2 & \lambda u v \\ \lambda u v & -\lambda_{TC} u^2 - m_{\tilde{\pi}}^2/2 \end{pmatrix}. \tag{32}$$

Diagonalising the matrix (32) gives states of definite mass, i.e. the physical states corresponding to linear combinations of S and H. The state corresponding to the physical

Higgs is to be denoted h and the other state corresponding to the physical scalar technimeson, as $\tilde{\sigma}$ (called the technisigma). Since (32) is a symmetric matrix, it can be diagonalised by an orthogonal matrix [17]. Thus

$$\begin{aligned}
&\begin{pmatrix} H & S \end{pmatrix} \begin{pmatrix} -\lambda_H v^2 & \lambda uv \\ \lambda uv & -\lambda_{TC} u^2 - m_{\tilde{\pi}}^2/2 \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix} \\
&\qquad = \begin{pmatrix} H & S \end{pmatrix} R^T R \begin{pmatrix} -\lambda_H v^2 & \lambda uv \\ \lambda uv & -\lambda_{TC} u^2 - m_{\tilde{\pi}}^2/2 \end{pmatrix} R^T R \begin{pmatrix} H \\ S \end{pmatrix} \\
&\qquad\qquad = \begin{pmatrix} h & \tilde{\sigma} \end{pmatrix} \begin{pmatrix} m_h^2 & \\ & m_{\tilde{\sigma}}^2 \end{pmatrix} \begin{pmatrix} h \\ \tilde{\sigma} \end{pmatrix}, \qquad (33)
\end{aligned}$$

where

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad (34)$$

and θ is determined such that the matrix (32) is diagonalised. Note that from (15), (16) and (33), inducing vevs for $\mathcal{H}$ and S yields

$$\begin{pmatrix} \mathcal{H}_2 \\ S \end{pmatrix} \to \begin{pmatrix} v + H \\ u + S \end{pmatrix} = \begin{pmatrix} v + hc_\theta - \tilde{\sigma} s_\theta \\ u + hs_\theta + \tilde{\sigma} c_\theta \end{pmatrix}, \qquad (35)$$

where c_θ, s_θ are $\cos\theta, \sin\theta$ respectively.

The mixing angle θ can be determined by inserting (35) into (30) and once again collect all terms with order two in any fields. Mixed terms of h and $\tilde{\sigma}$ is then obtained as

$$h\tilde{\sigma}\big(c_\theta s_\theta(-2\lambda_{TC}u^2 - m_{\tilde{\pi}}^2 + 2\lambda_H v^2) + 2\lambda uv(c_\theta^2 - s_\theta^2)\big). \qquad (36)$$

However, due to the definition of $\{h, \tilde{\sigma}\}$ as eigenstates of the matrix (32), the expression (36) must equal zero. Using that $2c_\theta s_\theta = \sin(2\theta)$ and $c_\theta^2 - s_\theta^2 = \cos(2\theta)$, the mixing angle can thus be expressed as

$$\tan(2\theta) = \frac{4\lambda uv}{2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 - 2\lambda_H v^2}. \qquad (37)$$

The masses of $\{h, \tilde{\sigma}\}$ can be obtained by diagonalising the matrix (32). Denoting the eigenvalues by χ, it is obtained that

$$\begin{aligned}
&\det \begin{pmatrix} -\lambda_H v^2 - \chi & \lambda uv \\ \lambda uv & -\lambda_{TC}u^2 - m_{\tilde{\pi}}^2/2 - \chi \end{pmatrix} = 0 \\
&\Rightarrow \chi = -\frac{1}{2}\left(\frac{1}{2}\Big(2\lambda_H v^2 + 2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 \pm \sqrt{16(\lambda uv)^2 + (2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 - 2\lambda_H v^2)^2}\Big)\right),
\end{aligned} \qquad (38)$$

so that the masses are

$$m_{\tilde{\sigma}}^2 = \frac{1}{2}\Big(2\lambda_H v^2 + 2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 + \sqrt{16(\lambda uv)^2 + (2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 - 2\lambda_H v^2)^2}\Big) \qquad (39)$$

and

$$m_h^2 = \frac{1}{2}\Big(2\lambda_H v^2 + 2\lambda_{TC} u^2 + m_{\tilde{\pi}}^2 - \sqrt{16(\lambda uv)^2 + (2\lambda_{TC}u^2 + m_{\tilde{\pi}}^2 - 2\lambda_H v^2)^2}\Big). \tag{40}$$

Note, by considering (37), (39) and (40), the parameters $\{\lambda_{TC}, \lambda_H, \lambda\}$ can be expressed in the parameters $\{m_{\tilde{\pi}}^2, m_{\tilde{\sigma}}^2, m_h^2\}$ as

$$\begin{cases} 2\lambda_{TC}u^2 = m_{\tilde{\sigma}}^2 c_\theta^2 + m_h^2 s_\theta^2 - m_{\tilde{\pi}}^2 \\ 2\lambda_H v^2 = m_{\tilde{\sigma}}^2 s_\theta^2 + m_h^2 c_\theta^2 \\ \lambda uv = \pm(m_{\tilde{\sigma}}^2 - m_h^2) c_\theta s_\theta \end{cases}. \tag{41}$$

Additional comments can be made on the possible values of the masses of the techniparticles. As a simple starting point, the masses of the techniparticles could be assumed to exhibit a similar mass hierarchy as the corresponding QCD particles. A scale factor could thus be applied to the masses of the QCD particles, as to get the masses for the corresponding techniparticles. As mentioned above, as an approximate lower limit, $\Lambda_{TC} \sim M_{EW} \sim 100$ GeV. Since $\Lambda_{QCD} \sim 100$ MeV, a scale factor of 1000 between the QCD particles and the techniparticles could be used. Of course the direct scale factor is a very hand wavy approximation, but could still be used to get a sense for the values of the different techniparticle masses considering a lower limit for the confinement scale (Λ_{TC}). With the direct scaling between QCD and T-QCD of a 1000, the masses of the techniparticles would be $m_{\tilde{Q}} \sim 300$ GeV, $m_{\tilde{\pi}} \sim 150$ GeV and $m_{\tilde{\sigma}} \sim 500$ GeV. If the technisigma vev, u, is assumed to be of the same order as the Higgs vev, v, one has $u \sim 100$ GeV, implying that $g_{TC} \sim 1$, by considering (44). From experimental measurements, $v \simeq 246$ GeV and $m_h \simeq 125$ GeV.

3.3 Physical Lagrangians

As the physical states of definite mass $\{\tilde{Q}, \tilde{\pi}^0, \tilde{\pi}^\pm, \tilde{\sigma}, h\}$ have been determined, the Lagrangian terms of (12) can be evaluated in these states to obtain the physical interaction terms. Starting of with the Yukawa-type interaction term

$$\begin{aligned} &- g_{TC}\overline{\tilde{Q}}(S + i\gamma_5\tau_a P_a)\tilde{Q} = -g_{TC}\begin{pmatrix}\overline{U} & \overline{D}\end{pmatrix}\begin{pmatrix} S + i\gamma_5 P_3 & i\gamma_5 P_1 + \gamma_5 P_2 \\ i\gamma_5 P_1 - \gamma_5 P_2 & S - i\gamma_5 P_3\end{pmatrix}\begin{pmatrix} U \\ D\end{pmatrix} \\ &= -g_{TC}\Big(\overline{U}\big((S + i\gamma_5 P_3)U + \gamma_5(iP_1 + P_2)D\big) + \overline{D}\big(\gamma_5(iP_1 - P_2)U + (S - i\gamma_5 P_3)D\big)\Big). \end{aligned} \tag{42}$$

Using that $S \to u + hs_\theta + \tilde{\sigma}c_\theta$ (from (35)) and (14), (42) can be written as

$$\begin{aligned} - g_{TC}\Big(&\big(u + hs_\theta + \tilde{\sigma}c_\theta\big)\big(\overline{U}U + \overline{D}D\big) + i\tilde{\pi}^0\big(\overline{U}\gamma_5 U - \overline{D}\gamma_5 D\big) \\ &- i\sqrt{2}\tilde{\pi}^+\overline{U}\gamma_5 D - i\sqrt{2}\tilde{\pi}^-\overline{D}\gamma_5 U\Big). \end{aligned} \tag{43}$$

The terms in (43) thus yield the three-vertex interaction terms between the T-quarks and the Higgs, technisigma and technipions. Mass terms for the T-quarks are also present in (43), corresponding to $-g_{TC}u\overline{U}U - g_{TC}u\overline{D}D$. Hence, the T-quarks have masses

$$m_{\tilde{Q}} = m_{U,D} = g_{TC}u. \tag{44}$$

The equal masses of the two T-quarks can be expected by considering symmetry arguments. In the vector-like region, a Lagrangian of the form presented for QCD in (1) is allowed as long as the masses are equal. Hence, as the T-quarks interact vector-like under $SU(2)_W$, equal mass terms follow.

The interaction terms between the EW gauge bosons of the SM and the technipions can be obtained by considering the kinetic term of the pions in (12) and the definition of their covariant derivative (11). Using the notation $\partial_\mu P_a = P_{a,\mu}$ and ignoring the propagation term, one has

$$\mathcal{L}_{\tilde{\pi};\tilde{\pi}V/VV} = \frac{1}{2}\big(g_2\epsilon_{abc}P_{a,\mu}W^{\mu,b}P_c + g_2\epsilon_{abc}W^b_\mu P_c P^\mu_a + g_2^2\epsilon_{abc}\epsilon_{ade}W^b_\mu P_c W^{\mu,d}P_e\big), \tag{45}$$

where V denotes any EW gauge boson $(A, Z, W^\pm)$. The pions can be expressed in their states of definite charge and mass as (14) and similarly, the W^i fields can be expressed as

$$\begin{cases} W^1 = -\frac{1}{\sqrt{2}}(W^+ + W^-) \\ W^2 = \frac{i}{\sqrt{2}}(W^- - W^+) \\ W^3 = Z\cos\theta_W + A\sin\theta_W \end{cases}, \tag{46}$$

where the right-hand side denotes states of definite charge and mass [8]. Noting that $P_{2,\mu}P_1 - P_{1,\mu}P_2 = 2i(\tilde{\pi}^-\tilde{\pi}^+_{,\mu} - \tilde{\pi}^+\tilde{\pi}^-_{,\mu})$ from (14) and using the properties of the Levi-Cevita tensor (ϵ_{abc}), (45) can be written as

$$\begin{aligned}\mathcal{L}_{\tilde{\pi};\tilde{\pi}V/VV} = ig_2\big(&-W^{\mu,+}(\tilde{\pi}^-\tilde{\pi}^0_{,\mu} - \tilde{\pi}^0\tilde{\pi}^-_{,\mu}) - W^{\mu,-}(\tilde{\pi}^0\tilde{\pi}^+_{,\mu} - \tilde{\pi}^+\tilde{\pi}^0_{,\mu}) \\ &+ (Z^\mu\cos\theta_W + A^\mu\sin\theta_W)(\tilde{\pi}^-\tilde{\pi}^+_{,\mu} - \tilde{\pi}^+\tilde{\pi}^-_{,\mu})\big) \\ &+ g_2^2\big(W^+_\mu W^{\mu,-}(\tilde{\pi}^0\tilde{\pi}^0 + \tilde{\pi}^+\tilde{\pi}^-) + (Z^\mu\cos\theta_W + A^\mu\sin\theta_W)^2\tilde{\pi}^+\tilde{\pi}^- \\ &- W^+_\mu(Z^\mu\cos\theta_W + A^\mu\sin\theta_W)\tilde{\pi}^-\tilde{\pi}^0 - W^-_\mu(Z^\mu\cos\theta_W + A^\mu\sin\theta_W)\tilde{\pi}^+\tilde{\pi}^0 \\ &- \frac{1}{2}W^-_\mu W^{\mu,-}\tilde{\pi}^+\tilde{\pi}^+ - \frac{1}{2}W^+_\mu W^{\mu,+}\tilde{\pi}^-\tilde{\pi}^-\big). \end{aligned} \tag{47}$$

The EW interactions for the T-quarks are obtained by considering the kinetic term in (12) and the definition of its covariant derivative (11). The gauge bosons have to be expressed in their states of definite charge and mass, given by (46), alongside with the corresponding relation for the B-field, $B = A\cos\theta_W - Z\sin\theta_W$ [8]. As mentioned, the hypercharge of the T-quark doublet is taken to be the same as that of the u, d quark doublet, i.e. $Y_{\tilde{Q}} = 1/3$. Similarly, the weak isospins were also taken to be the same as the corresponding quarks, i.e. $t_3^U = 1/2$ and $t_3^D = -1/2$. The electric charges of the T-quarks are then also the same as of the corresponding quarks, i.e. $q_U = q_u = 2/3$ and

$q_D = q_d = -1/3$, since $q_f = Y_{\tilde{Q}}/2 + t_3^f$ [8]. Thus, the EW interactions with the T-quarks are given by

$$\begin{aligned}
\mathcal{L}_{\bar{\tilde{Q}}\tilde{Q}V} &= \frac{1}{2}\bar{\tilde{Q}}\gamma^\mu(g_1 Y_{\tilde{Q}} B_\mu + g_2 W_\mu^a \tau_a)\tilde{Q} = -\frac{g_2}{\sqrt{2}}\overline{U}\gamma^\mu D W_\mu^+ - \frac{g_2}{\sqrt{2}}\overline{D}\gamma^\mu U W_\mu^- \\
&\quad + \left(\frac{1}{2}(g_1 Y_{\tilde{Q}} c_W + g_2 s_W)\overline{U}\gamma^\mu U + \frac{1}{2}(g_1 Y_{\tilde{Q}} c_W - g_2 s_W)\overline{D}\gamma^\mu D\right) A_\mu \\
&\quad + \left(\frac{1}{2}(-g_1 Y_{\tilde{Q}} s_W + g_2 c_W)\overline{U}\gamma^\mu U + \frac{1}{2}(-g_1 Y_{\tilde{Q}} s_W - g_2 c_W)\overline{D}\gamma^\mu D\right) Z_\mu \\
&= -\frac{g_2}{\sqrt{2}}\left(\overline{U}\gamma^\mu D W_\mu^+ + \overline{D}\gamma^\mu U W_\mu^-\right) + \sum_{f=U,D} e q_f \overline{f}\gamma^\mu f A_\mu + \sum_{f=U,D}\frac{g_2}{c_W}(t_3^f - q_f s_W^2)\overline{f}\gamma^\mu f Z_\mu,
\end{aligned} \tag{48}$$

where $g_2 s_W = e$, $g_1 c_W = e$ has been used and $c_W = \cos\theta_W$, $s_W = \sin\theta_W$ (θ_W is the Weinberg angle) [8].

The Higgs and technisigma couplings to the EW gauge bosons are obtained by considering the Lagrangian terms for the Higgs couplings in the SM case and then letting $H \to hc_\theta - \tilde{\sigma}s_\theta$ (see (35)). The relevant Lagrangian terms are given in Appendix B.3 by (139), yielding

$$\begin{aligned}
\mathcal{L}_{h/\tilde{\sigma};WW/ZZ} &= \frac{1}{2}\left(g_2^2 v W_\mu^+ W^{\mu,-} + v\frac{1}{2}(g_1^2 + g_2^2) Z_\mu Z^\mu\right)(hc_\theta - \tilde{\sigma}s_\theta) \\
&\quad + \frac{1}{4}\left(g_2^2 W_\mu^+ W^{\mu,-} + \frac{1}{2}(g_1^2 + g_2^2) Z_\mu Z^\mu\right)(hc_\theta - \tilde{\sigma}s_\theta)^2.
\end{aligned} \tag{49}$$

The Yukawa terms for the SM fermions will also be altered due to the Higgs-technisigma mixing. From Appendix B.3, specifically (143), the interaction term between Higgs and fermion is seen to be of the form $\mathcal{L}_{hff} = g_f \overline{f} f H/\sqrt{2}$. Using (144) and (140), one can obtain the relation $g_f/\sqrt{2} = g_2 m_f/2M_W$; leading to a Higgs-technisigma-fermion coupling (letting $H \to hc_\theta - \tilde{\sigma}s_\theta$) as

$$\mathcal{L}_{h/\tilde{\sigma},ff} = \frac{g_2 m_f}{2m_W}\overline{f} f(hc_\theta - \tilde{\sigma}s_\theta). \tag{50}$$

Finally, the Higgs and technisigma interaction terms with the technipions can be obtained from the potential terms of the Lagrangian, i.e. by considering (12); the relevant terms are $-\lambda_{TC}S^2P^2/2 + \lambda\mathcal{H}^2P^2$. By using (35), the following interaction terms are obtained

$$\begin{aligned}
\mathcal{L}_{h/\tilde{\sigma},\tilde{\pi}\tilde{\pi}} &= -\tilde{\sigma}(u\lambda_{TC}c_\theta + v\lambda s_\theta)P^2 + h(v\lambda c_\theta - u\lambda_{TC}s_\theta)P^2 \\
&= -\tilde{\sigma}\frac{g_{TC}}{2m_{\tilde{Q}}}(m_{\tilde{\sigma}}^2 - m_{\tilde{\pi}}^2)c_\theta(\tilde{\pi}^0\tilde{\pi}^0 + 2\tilde{\pi}^+\tilde{\pi}^-) - h\frac{g_{TC}}{2m_{\tilde{Q}}}(m_h^2 - m_{\tilde{\pi}}^2)s_\theta(\tilde{\pi}^0\tilde{\pi}^0 + 2\tilde{\pi}^+\tilde{\pi}^-),
\end{aligned} \tag{51}$$

where (41), (44) and (13) are also used in the second step.

3.4 Nearly conformal limit

As to restrict the CSTC model further by reducing the number of free parameters, the nearly conformal limit (NCL) can be investigated. Considering the theory of QCD in greater detail, it can be seen that QCD in the chiral limit (vanishing quark masses) is subject to a symmetry called conformal symmetry. The conformal symmetry in the chiral limit must also be respected by the LσM used in low-energy QCD, which has as a consequence that μ-terms are forbidden.[3] In reality, the quarks in QCD have non-zero mass and hence the conformal symmetry is broken. However, since the quark masses are small compared to the quark condensate $\langle\overline{q}q\rangle$ one could still imagine that the conformal symmetry is almost realised, implying that the μ-terms should be small (if existing).

By direct analogy with QCD, the above concept is applied to the techni-sector, where one then considers small T-quark masses compared to the T-quark condensate, or equivalently small T-quark masses compared to the technipion mass (since the technipion mass and the T-quark condensate are related through (24)). As μ-terms were forbidden by conformal symmetry, one considers the limit were both μ_S and μ_H, of the Lagrangian (12), vanish. The main consequence of the NCL is that the Higgs and technisigma vevs will be solely expressed through the T-quark condensate (or technipion mass); meaning that both vevs have the same origin. It can be seen explicitly by considering (25) and (26), while taking the limit $\mu_H, \mu_S \to 0$, yielding

$$\begin{cases} v^2 = \frac{\lambda m_{\tilde{\pi}}^2}{\delta} \\ u^2 = \frac{\lambda_H m_{\tilde{\pi}}^2}{\delta} \end{cases}, \tag{52}$$

where $\delta = \lambda_{TC}\lambda_H - \lambda^2$. Using (24), i.e. $m_{\tilde{\pi}}^2 = g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|$, it is obtained that

$$u^2 = \frac{\lambda_H g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|}{\delta u} \Rightarrow u = \left(\frac{g_{TC}\lambda_H}{\delta}\right)^{1/3} |\langle\overline{\tilde{Q}}\tilde{Q}\rangle|^{1/3}. \tag{53}$$

Hence, it is also obtained that

$$v^2 = \frac{\lambda g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|}{\delta u} = \frac{\lambda g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|}{\delta}\left(\frac{\lambda_H g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|}{\delta}\right)^{-1/3} = \frac{\lambda}{\lambda_H^{1/3}}\left(\frac{g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|}{\delta}\right)^{2/3}$$

$$\Rightarrow v = \left(\frac{\lambda}{\lambda_H}\right)^{1/2}\left(\frac{g_{TC}\lambda_H}{\delta}\right)^{1/3} |\langle\overline{\tilde{Q}}\tilde{Q}\rangle|^{1/3}. \tag{54}$$

Thus, it is evident that $u, v \propto |\langle\overline{\tilde{Q}}\tilde{Q}\rangle|^{1/3}$ and hence both originate from a non-zero T-quark condensate.

The parameters in the NCL can be plotted against the mass of the technisigma, in order to draw interesting conclusions about this scenario. The parameters of the CSTC model, expressed in the NCL are considered in Appendix C, whereas the plots are seen below in fig. 1 and 2. Note that the quantity $\overline{g}_{TC} = g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|$ has been defined and used.

[3]Conformal symmetry is not discussed any further here, only the fact that μ-terms are forbidden by it is what is relevant for the discussion.

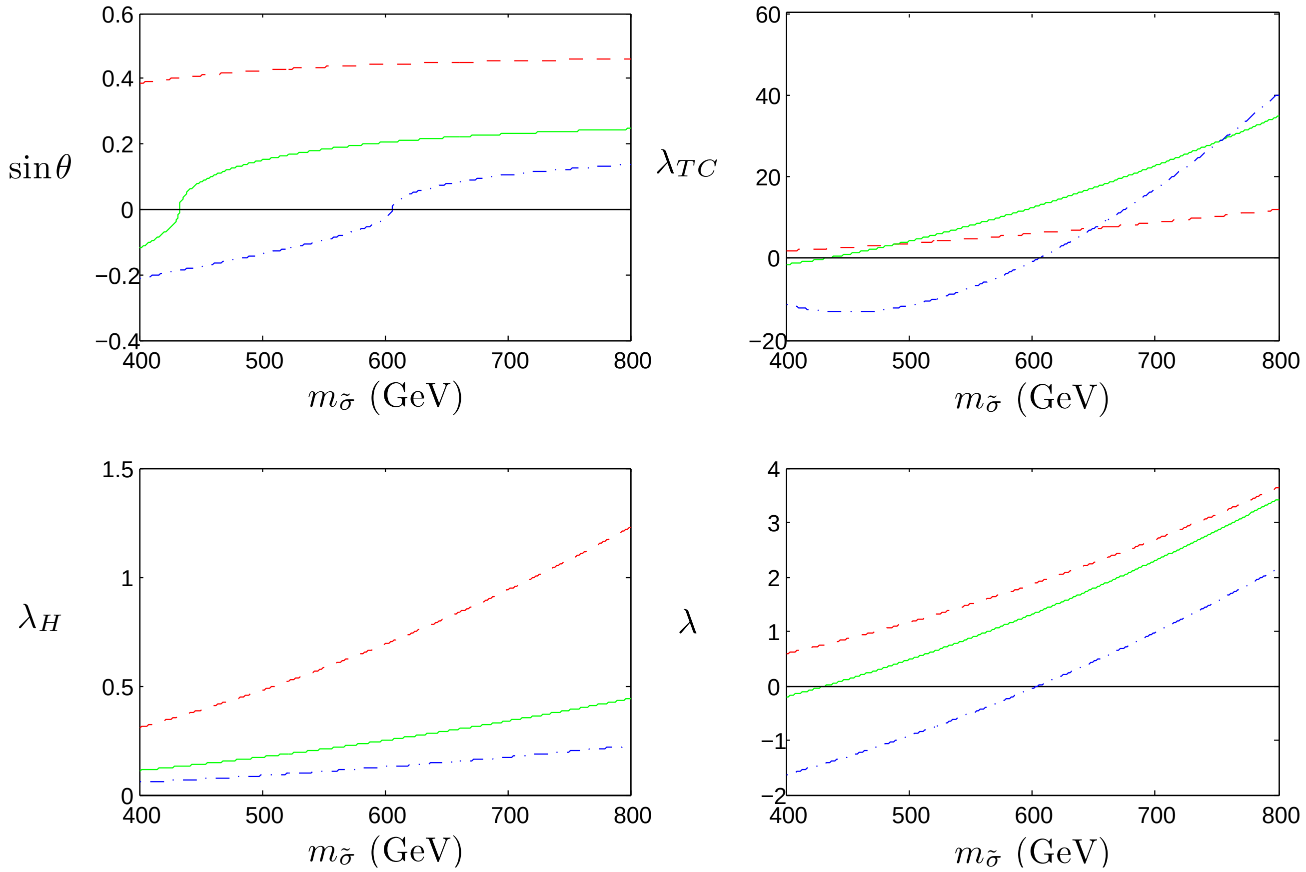

Figure 1: From top-bottom, left-right the plots of $\sin\theta$, λ_{TC}, λ_H and λ are presented as functions of the technisigma mass. The dashed (red), solid (green), dashed-dot (blue) lines correspond to a technipion mass of $150, 250, 350$ GeV respectively.

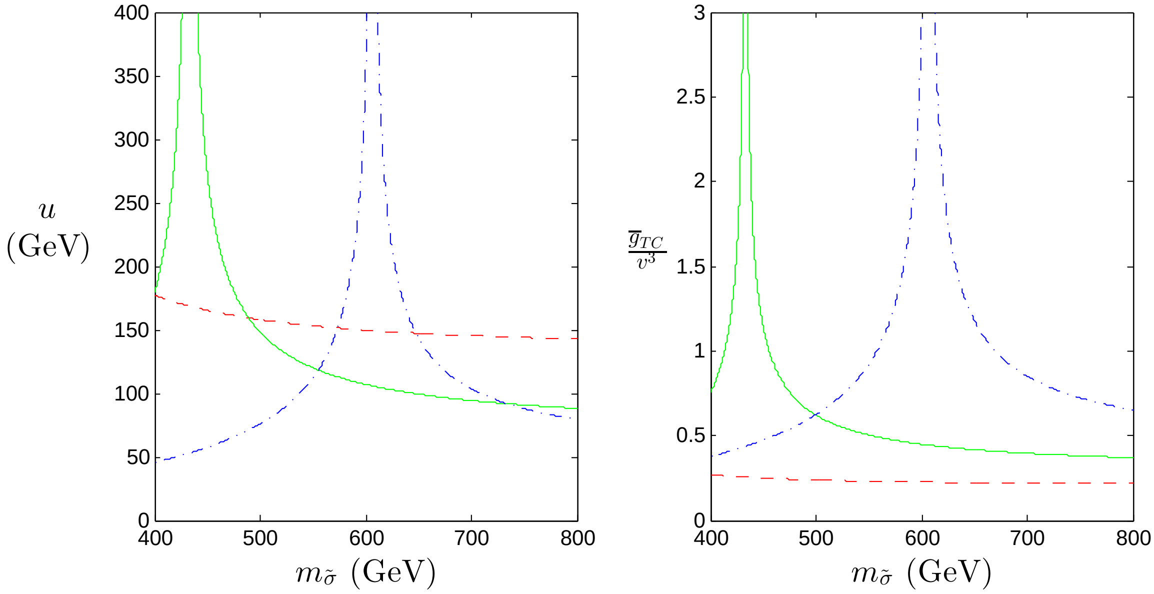

Figure 2: From left to right the plots of u and $\overline{g}_{TC}/v^3$ are presented as functions of the technisigma mass. The dashed (red), solid (green), dashed-dot (blue) lines correspond to a technipion mass of $150, 250, 350$ GeV respectively.

3.5 Precision tests

As the introduced techniparticles interact with SM particles, additional non-SM diagrams describing interactions will appear. In particular, higher-order corrections to the EW bosons (such as mass, decay, cross section modifications) should be present. As there have been substantial precision measurements on the electroweak sector of the SM, the new techni-sector must be able to be contained within those measurements. New physics corrections can be constrained by the so called Peskin-Takeuchi parameters, S, T, U [14]. As quoted in Ref. [1], they are at approximate values $S = 0.00^{+0.11}_{-0.10}, T = 0.02^{+0.11}_{-0.12}, U = 0.08^{+0.11}_{-0.11}$, where the sub- and superscripts denotes the regions of allowable deviation. Considering any mixing angle, degenerate T-quark masses and technipion and -sigma masses in regions around the ones obtained from the scaling of the QCD mesons, the CSTC model here under discussion can be seen to uphold the specific allowed regions for the S and U parameters. However, in order for the T parameter to be in accordance with measurements, the mixing angle has to be constrained $|s_\theta| \lesssim 0.55$. Further, as the mixing angle approaches zero, the T parameter vanishes completely; making the near-to-no-mixing limit (NNML) particularly interesting [1].

A reason for the construction of a chirally symmetric (vector-like) theory under the EW group $SU(2)_W \times U(1)_Y$, is that the S, T, U parameters of the model turn out to be within the allowed regions. Other technicolor models, without vector-like interactions do not necessarily imply such a property [15]. Hence, the vector-like interactions are a way to circumvent problems that have arisen for other, non-chirally symmetric technicolor models.

3.6 Discussion

Several remarks can be made on the constructed CSTC model and the conclusions it implies. As a start, the basic assumptions of the fundamental theory can be examined. Since no techniparticle modes has yet to be observed, assuming the naive mass scaling from the QCD particle spectrum, the techniparticles must seem to be suppressed such that a strong clear experimental signal is not visible. However, there is no definite argument stating that the mass scaling should be realised, even in an approximate sense. The CSTC model could still be realised even with heavier techniparticles than could be observed now (i.e. within the 1 TeV range). However, this means that the particle spectrum would not give rise to any signals, making it harder to confirm a possible techni-sector.

As was mentioned in the NCL scenario, both the Higgs and technisigma vevs are seen to have the same source, being the non-zero value of the T-quark condensate. As this scenario, if realised, offers a clear explanation of the Higgs vev within the model, the plausibility could be investigated. Using the adopted mass hierarchy of QCD, with the scale factor of 1000, the current T-quark masses are very small compared to the energy scale of the theory Λ_{TC} (of order ~ 0.01). In analogy with QCD [18], the T-quark condensate can be related to the confinement scale Λ_{TC}, very approximately, as $\langle\bar{\tilde{Q}}\tilde{Q}\rangle \sim \Lambda_{TC}^3$. As was required for QCD, low mass compared to the quark condensate implied a near conformal symmetry of the theory. It is hence, with the assumed mass scaling from QCD, possible that the NCL is indeed realised for the CSTC model.

The plots of the parameters of the NCL in fig. 1 and 2 should also be commented on, as to see what physical consequences are apparent. In order for the CSTC model to be realised, the current experimental results must be respected. As noted before, considering the S, T, U parameters, a constrained mixing angle had to be considered in order for the value of the T parameter to be consistent with experimental data. In fig. 1, it can be seen that as $s_\theta \to 0$ also $\lambda_{TC}, \lambda \to 0$, such that the techni-sector is completely disconnected from the Higgs sector (see (12) and recall that the μ-terms are put to zero in the NCL). At the same time, in the NNML, λ_H stays at a low value (< 1), as is favourable considering calculations (coupling constants cannot be arbitrarily large in order to be used in higher order calculations, as it leads to non-perturbative effects [7]). As was noted in Appendix C, the no-mixing limit corresponds to $m_{\tilde{\sigma}}^2 = 3m_{\tilde{\pi}}^2$, also visible in the figures; and is the reason why the dashed (red) line, corresponding to $m_{\tilde{\pi}} = 150$ GeV, has no no-mixing scenario (since it happens at a lower value of $m_{\tilde{\sigma}}$ considered on the present scale of the figures). Another consequence of the NNML is seen in fig. 2. There it is seen that as $\theta \to 0$ both u and $\overline{g}_{TC}$ tend to large values (in comparison to the EW scale). Hence, as a low mixing could be a probable case (as to not modify the SM too much), the confinement scale for the techni-sector would be a lot larger than the lower limit of $\sim M_{EW}$. Note however that independent of how large u (or equivalently Λ_{TC} or $\overline{g}_{TC}$) becomes, the Higgs vev can still be expressed in terms of the T-quark condensate , with subsequent explanation of the Higgs vev's origin.

A distinction between QCD and the CSTC model is that in QCD, masses for the GSBs are obtained by explicitly breaking the initial chiral symmetry in a perturbation manner, such that SSB is still induced [2]. However, in the CSTC model, the initial chiral symmetry is solely broken through SSB, where the mass term for the GSBs rest entirely on the non-zero value of the T-quark condensate. A reason to exclude an explicit symmetry breaking in the CSTC model can be considered that one of the goals of the model is to explain the EWSB within the framework of the model. And as was examined, the Higgs mechanism could be considered induced by a non-zero T-quark condensate (especially if the NCL scenario is approximately realised). As the EWSB is a SSB, an explicit symmetry breaking in the CSTC model might modify the theory such that the EWSB cannot be induced through the model spontaneously.

The choice of considering $\Lambda_{TC} \sim M_{EW}$ as an approximate lower limit can also be commented on. As a main focus of the CSTC model is to introduce an origin to the Higgs vev through the T-quark condensate, it could be argued that the scale of Λ_{TC} should be of the same scale as the Higgs vev v. From QCD, one expects the vev (u) breaking the chiral symmetry, to be related as $u \sim \Lambda_{QCD} \sim |\langle \bar{q}q \rangle|^{1/3}$ [3]. Thus, in analogy one could consider for the CSTC model (here u denotes the technisigma vev as before) $v \sim u \sim \Lambda_{TC} \sim |\langle \bar{q}q \rangle|^{1/3}$, as the T-quark condensate is desired to be the source of the Higgs vev.

4 Techni-QCD: Three techniquark case

In Sect. 3, the CSTC model extension to the SM of Ref. [1] was reproduced, where two T-quarks were considered (see (8)). A chirally symmeric model was there constructed under $SU(2)_L \times SU(2)_R$ with SSB to $SU(2)_V$, which was identified with the SM weak isospin group $SU(2)_W$. The SSB gave rise to a light technimeson particle spectrum, corresponding to technipions as GSBs. The GSBs were introduced via a LσM, also giving rise to the technisigma meson. In this section the basic theory of a three T-quark CSTC model shall be outlined. The study following is a reproduction of notes by R. Pasechnik.

Considering three T-quarks implies that the chiral symmetry group considered now is $SU(3)_L \times SU(3)_R$, with SSB to $SU(3)_V$. The two main reasons for introducing three T-quarks, instead of having only two, are:

- The technimeson particle spectrum due to the SSB will be larger, giving rise to more possible signals which could show up in further experimental precision measurements at particle colliders.
- The Higgs boson can be integrated into the technimeson particle spectrum, giving it a composite substructure of T-quarks. Hence, within the framework of the CSTC model of three T-quarks, the Higgs boson's origin can be explained.

As a start, the T-quarks' properties are examined. In analogy to QCD, one considers left-handed (LH) doublets (under $SU(2)_W$) and right-handed (RH) singlets (under $SU(2)_W$). The strong force in QCD have three color charges, i.e. QCD corresponds to an $SU(3)$ group. For the three T-quark CSTC model treated here, a two-technicolor charge force is considered (for simplicity), hence one has $SU(2)_{TC}$.[4] Even though the T-quarks' LH and RH components here initially are taken to be treated differently in $SU(2)_W$, chirally symmetric T-quarks can be constructed of the non-chirally symmetric ones, as is shown below. The implication is that the T-quarks interacting under $SU(2)_W$ will have vector-like interactions.

4.1 Constructing chirally symmetric techniquarks

As mentioned, in analogy to QCD, there are LH T-quarks which are in the fundamental representation of $SU(2)_W \times SU(2)_{TC}$ (henceforth referred to as LH bi-doublets) and RH T-quarks which are in fundamental representation of $SU(2)_{TC}$ (referred to as RH singlets). The $SU(2)_W$ index is denoted $a = 1, 2$ and the $SU(2)_{TC}$ index as $\alpha = 1, 2$. The hypercharges of the LH bi-doublets are zero and for the RH singlets they are of the same magnitude but of opposite sign for the generation members. Two generations, $A = 1, 2$, of T-quarks are considered, meaning that there are two LH bi-doublets and four RH singlets. Denoting the LH bi-doublet as Q_L and the corresponding RH singlets as U_R, D_R, one has the following infinitesimal transformations

$$Q^{a\alpha}_{L(A)} \to Q^{a\alpha}_{L(A)} + \frac{i}{2} g_2 \vartheta_k \tau_k^{ab} Q^{b\alpha}_{L(A)} + \frac{i}{2} g_{TC} \varphi_k \tau_k^{\alpha\beta} Q^{a\beta}_{L(A)}, \tag{55}$$

$$U^{\alpha}_{R(A)} \to U^{\alpha}_{R(A)} - \frac{i}{2} g_1 \theta U^{\alpha}_{R(A)} + \frac{i}{2} g_{TC} \varphi_k \tau_k^{\alpha\beta} U^{\beta}_{R(A)} \tag{56}$$

and

$$D^{\alpha}_{R(A)} \to D^{\alpha}_{R(A)} + \frac{i}{2} g_1 \theta D^{\alpha}_{R(A)} + \frac{i}{2} g_{TC} \varphi_k \tau_k^{\alpha\beta} D^{\beta}_{R(A)}. \tag{57}$$

Here g_1, g_2 and g_{TC} are the coupling constants of the $U(1)_Y, SU(2)_W$ and $SU(2)_{TC}$ respectively, with infinitesimal parameters θ, ϑ and φ. Note that in (56) and (57) the signs are different for the $U(1)_Y$ transformations and the magnitude of the hypercharges are set to unity.

The goal, as previously implied, is to construct three chirallly-symmetric T-quarks out of the ones in (55) - (57). It can be done by first taking $Q_{L(1)}$ and $D_{R(1)}$ as they are, and then consider the charge conjugation of $Q_{L(2)}$ and $U_{R(1)}$.[5] As charge conjugation flips the chirality of a chiral spinor, it is possible to define

$$Q^{a\alpha}_{R(2)} = \epsilon^{ab} \epsilon^{\alpha\beta} \mathcal{C} Q^{b\beta}_{L(2)}, \tag{58}$$

and

$$D^{\alpha}_{L(1)} = -\epsilon^{\alpha\beta} \mathcal{C} U^{\beta}_{R(1)}, \tag{59}$$

[4]Note that since the technicolor force acts in the fundamental representation on T-quarks, both LH and RH T-quarks are doublets in $SU(2)_{TC}$. Since the LH quarks also form a doublet in $SU(2)_W$ it is a bi-doublet.

[5]The second generation of the RH singlets is discarded for further use. Since two complete generations correspond to four techni-quarks, all of the members are not needed to construct a three T-quark model (and as is seen, to construct a chirally-symmetric three T-quark model one does not need $U_{R(2)}$ or $D_{R(2)}$). One could consider a chirally-symmetric four T-quark model, but to follow in the footsteps of QCD, the fourth T-quark is assumed to be a lot heavier than the three lightest [11].

where $\mathcal{C}$ denotes the charge conjugation operator. As is seen, (58) describes a RH bi-doublet and (59) describes a LH singlet. Investigating their transformation properties, one finds that (58) transforms like (55), and (59) transforms like (57). Hence, one can construct a chirally symmetric bi-doublet and singlet, as

$$Q^{a\alpha} = Q^{a\alpha}_{L(1)} + Q^{a\alpha}_{R(2)} \tag{60}$$

and

$$S = D^{\alpha}_{L(1)} + D^{\alpha}_{R(1)} \tag{61}$$

respectively.

The construction of the three chirally symmetric T-quarks can be seen in greater detail in Appendix D.1.

In addition to the SM Lagrangian, the extra terms for Q, S and the technigluon field $T_{\mu,n}$, with $n = 1, 2, 3$ is also present. Since Q (i.e. (60)) was constructed from objects (i.e. (55)) which transformed under $SU(2)_W \times SU(2)_{TC}$, Q will also transform under these groups. For S (i.e. (61)), (56) and (57) were used in its construction, and makes it transform under $SU(2)_{TC} \times U(1)_Y$. The allowed Lagrangian under the present symmetries is then

$$\mathcal{L}_{TC} = -\frac{1}{4}T^{n}_{\mu\nu}T^{\mu\nu,n} + i\overline{Q}\gamma^{\mu}D_{\mu,Q}Q - m_Q\overline{Q}Q + i\overline{S}\gamma^{\mu}D_{\mu,S}S - m_S\overline{S}S, \tag{62}$$

where

$$\begin{cases} D_{\mu,Q}Q = \left(\partial_\mu - \frac{i}{2}g_2 W^a_\mu \tau^a - \frac{i}{2}g_{TC}T^n_\mu \tau^n\right)Q \\ D_{\mu,S}S = \left(\partial_\mu + \frac{i}{2}g_1 B_\mu - \frac{i}{2}g_{TC}T^n_\mu \tau^n\right)S \end{cases}, \tag{63}$$

are the covariant derivatives for Q and S, and $T_{\mu\nu}$ is the field strength tensor of the technigluon field.

4.2 Parametrization

In Sect. 3 a CSTC model characterised by the SSB as (7) with $n_F = 2$ was considered. Here the extension to the case for $n_F = 3$ is examined. Thus, the following triplet is constructed

$$\hat{Q} = \begin{pmatrix} U \\ D \\ S \end{pmatrix} \tag{64}$$

with infinitesimal transformations under $SU(3)_L \times SU(3)_R$ as

$$\hat{Q}_L \to \left(1 + \frac{i}{2}\zeta_a\lambda_a\right)\hat{Q}_L \tag{65}$$

and

$$\hat{Q}_R \to \left(1 + \frac{i}{2}\xi_a\lambda_a\right)\hat{Q}_R. \tag{66}$$

By considering the Lagrangian (62), it is seen that (as was the case for the two T-quarks) the masses for the U and D are the same (m_Q). By simplicity and convenience the mass of

S is also considered to be approximately the same, such that $m_S \approx m_Q$. This is in contrast to QCD, where $m_s \gg m_d \approx m_u$ [11]. Considering the present masses for the T-quarks small compared to the confinement scale makes the chiral limit more (approximately) realised for the three T-quark case in T-QCD than for the three quark case in QCD.

As in the previous sections, the technimeson particle spectrum is obtained as the GSBs from the SSB of $SU(3)_L \times SU(3)_R$ to $SU(3)_V$, and is introduced via a LσM. Using the same notation for the technimeson as the mesons in QCD, one has

$$\pi^0, \pi^+, \pi^-; K^0, \overline{K^0}, K^+, K^-; \eta. \tag{67}$$

The pseudoscalar mesons above have a spin-parity 0^- [13]. To be completely general, their chiral partners with spin-parity 0^+, should also be considered. The chiral partners are here denoted as

$$a^0, a^+, a^-; H^0, \overline{H^0}, H^+, H^-; f. \tag{68}$$

The parametrization of the technimesons and their chiral partners can be done similarly to the case in Sect. 3, only here extending to higher dimension, thus

$$\begin{aligned}\hat{\Phi} = \frac{1}{\sqrt{2}} &\begin{pmatrix} \frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{6}}f + \frac{1}{\sqrt{3}}\sigma & a^+ & H^+ \\ a^- & -\frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{6}}f + \frac{1}{\sqrt{3}}\sigma & H^0 \\ H^- & \overline{H}^0 & -\sqrt{\frac{2}{3}}f + \frac{1}{\sqrt{3}}\sigma \end{pmatrix} \\ &- \frac{i}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}.\end{aligned} \tag{69}$$

Here σ is the scalar, $SU(3)_V$ singlet introduced as usual in a LσM and η_0 is its pseudoscalar chiral partner. Under $SU(3)_L \times SU(3)_R$, the matrix $\hat{\Phi}$ transforms infinitesimally as

$$\hat{\Phi}^\alpha_\beta \to \hat{\Phi}^\alpha_\beta + \frac{i}{2}\zeta_a(\lambda_a)^\alpha_\gamma \hat{\Phi}^\gamma_\beta - \frac{i}{2}\hat{\Phi}^\alpha_\delta(\lambda_a)^\delta_\beta \xi_a, \tag{70}$$

similarly to the cases in the previous sections (only here the infinitesimal transformations are considered explicitly).

The LσM Lagrangian allowed under $SU(3)_L \times SU(3)_R$, with transformations described by (65), (66) and (70) is

$$\begin{aligned}\mathcal{L}_{3\sigma} = i\overline{\hat{Q}}\gamma^\mu\partial_\mu\hat{Q} - \sqrt{6}\kappa\big(\overline{\hat{Q}}_L\hat{\Phi}\hat{Q}_R + \overline{\hat{Q}}_R\hat{\Phi}^\dagger\hat{Q}_L\big) + \mathrm{Tr}(\partial_\mu\hat{\Phi}^\dagger\partial^\mu\hat{\Phi}) + \mu^2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}) \\ - \lambda_1\big(\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi})\big)^2 - 3\lambda_2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}) + 2\sqrt{6}\Lambda_3\mathrm{Re}\big(\det(\hat{\Phi})\big).\end{aligned} \tag{71}$$

The second term of (71), i.e. $-\sqrt{6}\kappa\big(\overline{\hat{Q}}_L\hat{\Phi}\hat{Q}_R + \overline{\hat{Q}}_R\hat{\Phi}^\dagger\hat{Q}_L\big)$, is a Yukawa-type term between the T-quarks and the technimesons. The last term in the Lagrangian, $2\sqrt{6}\Lambda_3\mathrm{Re}\big(\det(\hat{\Phi})\big)$, is of more technical nature (not treated in this work); it can be seen to break certain symmetries present in the theory (being a desirable effect), but under $SU(3)_L \times SU(3)_R$ it can be shown to be invariant.

4.3 Pre-EWSB phase

In the two T-quark CSTC model of Sect. 3, vevs were given to the Higgs (see (15)) and the S field (see (16)). For the three T-quark model, the vevs will be given in two phases. First, only the σ meson acquires a vev as $\sigma \to \sigma + u$. It happens before the EWSB of the SM, hence it will be called pre-EWSB phase. Later, the Higgs doublet of the SM will be identified with some of the technimesons, and then it will obtain a vev in the usual sense (i.e. as in (15)). The phase were both σ and Higgs have acquired vevs will be called the post-EWSB phase. That the vev of σ and the vev of Higgs are given in two distinct energy regions is a consequence of considering $u^2 \gg v^2$. Further remarks on this are made in the discussion section.

Restricting to the pre-EWSB phase, as mentioned only σ aquires a vev, i.e. $\langle\sigma\rangle = u$. The potential part of the Lagrangian (71) is

$$U = \sqrt{6}\kappa\big(\overline{\hat{Q}}_L\hat{\Phi}\hat{Q}_R+\overline{\hat{Q}}_R\hat{\Phi}^\dagger\hat{Q}_L\big)-\mu^2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi})+\lambda_1\big(\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi})\big)^2+3\lambda_2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi})-2\sqrt{6}\Lambda_3\mathrm{Re}\big(\det(\hat{\Phi})\big), \tag{72}$$

which has a vacuum average (letting $\sigma \to u$ and the other fields to zero) reading

$$\langle U\rangle = \kappa u\langle\overline{\hat{Q}}\hat{Q}\rangle - \frac{1}{2}\mu^2u^2 + \frac{1}{4}\lambda_1u^4 + \frac{1}{4}\lambda_2u^4 - \frac{1}{3}\Lambda_3u^3. \tag{73}$$

Since (73) describes the vacuum, it is the minimum of the potential (72). Hence, $d\langle U\rangle/du = 0$ and $d^2\langle U\rangle/du^2 > 0$ must hold; leading to the relations

$$\frac{d\langle U\rangle}{du} = u\Big(\frac{\kappa\langle\overline{\hat{Q}}\hat{Q}\rangle}{u} - \mu^2 + \lambda_1u^2 + \lambda_2u^2 - \Lambda_3u\Big) = 0 \tag{74}$$

and

$$\frac{d^2\langle U\rangle}{du^2} = -\frac{\kappa\langle\overline{\hat{Q}}\hat{Q}\rangle}{u} + 2\lambda_1u^2 + 2\lambda_2u^2 - \Lambda_3u > 0, \tag{75}$$

where (74) has been used to write (75) on the form it is written.

The mass spectrum for the technimesons can be obtained by expanding each field around their vevs, which, for the pre-EWSB phase, means that $\sigma \to \sigma + u$ in $\hat{\Phi}$ (i.e. in (69)). The terms from the Lagrangian (71) contributing to the technimeson masses are

$$\mu^2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}) - \lambda_1\big(\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi})\big)^2 - 3\lambda_2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}) + 2\sqrt{6}\Lambda_3\mathrm{Re}\big(\det(\hat{\Phi})\big). \tag{76}$$

Evaluating the terms in (76) leads to

$$M^2_{\pi(0)} = M^2_{\eta(0)} = M^2_{K(0)} = -\mu^2 + (\lambda_1+\lambda_2)u^2 - \Lambda_3u = -\frac{\kappa\langle\overline{\hat{Q}}\hat{Q}\rangle}{u}, \tag{77}$$

where the subscript (0) denotes that the pre-EWSB phase is considered and the last step in (77) is obtained from (74). Further the mass terms

$$M^2_{a(0)} = M^2_{f(0)} = M^2_{H(0)} = -\mu^2 + (\lambda_1+\lambda_2)u^2 + \Lambda_3u = M^2_{\pi(0)} + 2\lambda_2u^2 + 2\Lambda_3u, \tag{78}$$

$$M^2_{\sigma(0)} = -\mu^2 + 3(\lambda_1+\lambda_2)u^2 - 2\Lambda_3u = M^2_{\pi(0)} + 2(\lambda_1+\lambda_2)u^2 - \Lambda_3u \tag{79}$$

and

$$M^2_{\eta_0(0)} = -\mu^2 + (\lambda_1 + \lambda_2)u^2 + 2\Lambda_3 u = M^2_{\pi(0)} + 3\Lambda_3 u. \tag{80}$$

are obtained. Note that using (75), (77) and (79) yields $M^2_{\sigma(0)} = d^2\langle U\rangle/du^2 > 0$.

From (77) - (80), the parameters of the Lagrangian (71) can be expressed in the technimeson masses as

$$\begin{cases} -\frac{\kappa\langle\overline{\hat{Q}}\hat{Q}\rangle}{u} = M^2_{\pi(0)} \\ \Lambda_3 u = \frac{1}{3}\big(M^2_{\eta_0(0)} - M^2_{\pi(0)}\big) \\ 2\lambda_2 u^2 = M^2_{H(0)} - \frac{1}{3}\big(2M^2_{\eta_0(0)} + M^2_{\pi(0)}\big) \\ 2\lambda_1 u^2 = M^2_{\sigma(0)} + M^2_{\eta_0(0)} - M^2_{\pi(0)} - M^2_{H(0)} \end{cases}. \tag{81}$$

Hence, the only parameter of the five independent ones of the Lagrangian (71) not expressible in technimeson masses is μ^2.

4.4 Post-EWSB phase

Due to the T-quark substructure of the technimesons in (69), they are classified differently in the EW group (see Appendix D.1 for additional explanation). Two $SU(2)_W$ doublets are present, in the forms

$$\mathcal{H} = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \qquad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}. \tag{82}$$

Both $\mathcal{H}$ and K have hypercharge 1. The SM Higgs doublet is here identified with $\mathcal{H}$, since it has matching properties with the Higgs (in the SM sector). Further the triplets of π_a and a_a are in the adjoint representation of $SU(2)_W$ with hypercharge 0. The rest of the technimesons are singlets under $SU(2)_W \times U(1)_Y$, thus not interacting with the EW gauge bosons of the SM.

The LσM Lagrangian (71) has to be modified by introducing covariant derivatives for the fields, due to their respective representations under the EW group $SU(2)_W \times U(1)_Y$ of the SM. For the T-quarks one has

$$i\overline{\hat{Q}}\gamma^\mu D_\mu \hat{Q} = i\overline{Q}\gamma^\mu D_{\mu,Q} Q + i\overline{S}\gamma^\mu D_{\mu,S} S, \tag{83}$$

where the right-hand side is given in (63). For the technimesons, the following covariant derivatives have to be introduced

$$\begin{cases} \partial_\mu \pi_a \to D_\mu \pi_a = \partial_\mu \pi_a + g_2 \epsilon_{abc} W^b_\mu \pi_c \\ \partial_\mu a_a \to D_\mu a_a = \partial_\mu a_a + g_2 \epsilon_{abc} W^b_\mu a_c \end{cases} \tag{84}$$

and

$$\begin{cases} \partial_\mu K \to D_\mu K = \Big(\partial_\mu - \frac{i}{2} g_1 B_\mu - \frac{i}{2} g_2 W^a_\mu \tau_a\Big) K \\ \partial_\mu \mathcal{H} \to D_\mu \mathcal{H} = \Big(\partial_\mu - \frac{i}{2} g_1 B_\mu - \frac{i}{2} g_2 W^a_\mu \tau_a\Big) \mathcal{H} \end{cases}. \tag{85}$$

Thus, the LσM Lagrangian reads

$$
\begin{aligned}
\mathcal{L}_{3\sigma} = i\overline{\hat{Q}}\gamma^\mu D_\mu \hat{Q} - \sqrt{6}\kappa\big(\overline{\hat{Q}}_L\hat{\Phi}\hat{Q}_R + \overline{\hat{Q}}_R\hat{\Phi}^\dagger\hat{Q}_L\big) + (D_\mu K)^\dagger D^\mu K + (D_\mu\mathcal{H})^\dagger D^\mu\mathcal{H} \\
+ \frac{1}{2}(D_\mu\pi_a D^\mu\pi_a + D_\mu a_a D^\mu a_a + \partial_\mu\sigma\partial^\mu\sigma + \partial_\mu f\partial^\mu f + \partial_\mu\eta_0\partial^\mu\eta_0 + \partial_\mu\eta\partial^\mu\eta) \\
+ \mu^2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}) - \lambda_1\big(\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi})\big)^2 - 3\lambda_2\mathrm{Tr}(\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}) + 2\sqrt{6}\Lambda_3\mathrm{Re}\big(\det(\hat{\Phi})\big). \quad (86)
\end{aligned}
$$

As mentioned, the phase were both σ and the SM Higgs doublet ($\mathcal{H}$) gain vevs would be referred to as the post-EWSB phase. The Higgs obtaining a vev, with subsequent expansion around it, is taken to be the same as in the SM, namely

$$
\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}. \quad (87)
$$

Since now that the SM Higgs has entered the theory and obtained a vev, it induces the EWSB (in the same way as in the SM). Since both σ and the Higgs obtain vevs, the vacuum average of the potential (73) is modified. Letting all fields in (69) go to zero except for σ and H^0 (and hence also $\overline{H^0}$) which go to u and $v/\sqrt{2}$ respectively, the vacuum average of the potential (72) is obtained to be

$$
\begin{aligned}
\langle U\rangle = \kappa u\langle\overline{\hat{Q}}\hat{Q}\rangle + \sqrt{\frac{3}{2}}\kappa v\langle\overline{S}D + \overline{D}S\rangle - \frac{1}{2}\mu^2(u^2+v^2) + \frac{1}{4}\lambda_1(u^2+v^2)^2 \\
+ \lambda_2\bigg(\frac{1}{4}(u^2+v^2)^2 + v^2\Big(u^2 + \frac{1}{8}v^2\Big)\bigg) - \Lambda_3 u\Big(\frac{1}{3}u^2 - \frac{1}{2}v^2\Big). \quad (88)
\end{aligned}
$$

To represent the minimum, (88) must have derivatives w.r.t. u and v equal to zero and satisfy (18). Hence, the following relations should hold

$$
\begin{cases}
\frac{\partial\langle U\rangle}{\partial u} = u\Big(\frac{\kappa\langle\overline{\hat{Q}}\hat{Q}\rangle}{u} - \mu^2 + \lambda_1(u^2+v^2) + \lambda_2(u^2+3v^2) - \Lambda_3\big(u - \frac{v^2}{2u}\big)\Big) = 0 \\
\frac{\partial\langle U\rangle}{\partial v} = v\Big(\sqrt{\frac{3}{2}}\frac{\kappa\langle\overline{S}D+\overline{D}S\rangle}{v} - \mu^2 + \lambda_1(u^2+v^2) + \lambda_2(3u^2 + \frac{3}{2}v^2) + \Lambda_3 u\Big) = 0
\end{cases}, \quad (89)
$$

and

$$
\left(\frac{\partial^2\langle U\rangle}{\partial u^2}\right)\left(\frac{\partial^2\langle U\rangle}{\partial v^2}\right) - \left(\frac{\partial^2\langle U\rangle}{\partial u\partial v}\right)^2 > 0, \quad (90)
$$

where

$$
\begin{cases}
\frac{\partial^2\langle U\rangle}{\partial u^2} = -\mu^2 + \lambda_1(3u^2+v^2) + 3\lambda_2(u^2+v^2) - 2\Lambda_3 u \\
\frac{\partial^2\langle U\rangle}{\partial v^2} = -\mu^2 + \lambda_1(u^2+3v^2) + \lambda_2(3u^2+\frac{9}{2}v^2) + \Lambda_3 u \\
\frac{\partial^2\langle U\rangle}{\partial u\partial v} = v\Big(2\lambda_1 u + 6\lambda_2 u + \Lambda_3\Big)
\end{cases}. \quad (91)
$$

Considering the post-EWSB phase, the Higgs has also obtained a vev (see (87)) in addition to σ. The mass terms for the technimesons are thus modified compared to the pre-EWSB phase. In particular, most of the technimesons will mix, such that they do not represent states of definite mass any longer. Only considering terms up to first order in v/u (since $u^2 \gg v^2$), there will be mixing between the neutral scalar fields $\{h, \sigma, f, a^0\}$, the charged pseudoscalar fields $\{\pi^\pm, K^\pm\}$ and the neutral pseudoscalar fields $\{K^0, \overline{K}^0, \eta_0, \eta, \pi^0\}$. The (mixed) mass terms are obtained by expanding each technimeson field around their vevs, implying that $\hat{\Phi}$ has the same appearance as in (69) but taking $\sigma \to \sigma + u$ and $\mathcal{H}$ as in (87). The Lagrangian terms contributing to the mass terms were stated in (76). Using the pre-EWSB mass relations (77) - (80), the relevant, mixed, mass terms are

$$M^2_{\pi(0)}(\pi^+\pi^- + K^+K^-) + \sqrt{\frac{3}{2}}v(\lambda_2 u + \Lambda_3)(\pi^+K^- + \pi^-K^+) \tag{92}$$

for the charged pseudoscalar fields,

$$\begin{aligned} M^2_{\pi(0)}(K^0\overline{K}^0 + \eta^2 + (\pi^0)^2) + M^2_{\eta_0(0)}\eta_0^2 + \sqrt{2}v(2\lambda_2 u - \Lambda_3)(K^0 + \overline{K}^0)\eta_0 \\ - \sqrt{3}v(\lambda_2 u + \Lambda_3)(K^0 + \overline{K}^0)\pi_0 - v(\lambda_2 u + \Lambda_3)(K^0 + \overline{K}^0)\eta \end{aligned} \tag{93}$$

for the neutral pseudoscalar fields and

$$\begin{aligned} M^2_{H(0)}(f^2 + h^2 + (a^0)^2) + M^2_{\sigma(0)}\sigma^2 + 2(2\lambda_1 uv + 6\lambda_2 uv + \Lambda_3 v)h\sigma \\ + \sqrt{2}(-3\lambda_2 uv + \Lambda_3 v)fh + \sqrt{6}(3\lambda_2 uv + \Lambda_3 v)ha_0 \end{aligned} \tag{94}$$

for the neutral scalar fields. In obtaining the expressions above, it has been used implicitly that any terms containing v^2 in combination with either λ_1 or λ_2 can be omitted. This follows from considering (81), where such terms can be seen to be of second order in v/u.

There lies some ambiguity as in how to set up the mass hierarchy of the pre-EWSB technimeson masses. In what follows, the conditions that $M^2_{\pi(0)} > M^2_{\eta_0(0)}$ and $M^2_{H(0)} > M^2_{\pi(0)}$ have been used. To get the states of definite mass the mixed terms must be diagonalised. The three relations (92) - (94) will thus be examined individually below, as to obtain the masses and the corresponding physical states.

4.4.1 Physical charged pseudoscalar states

The charged pseudoscalar fields $\{\pi^\pm, K^\pm\}$ with mass terms (92) are the simplest to decouple into definite mass states. The mixed mass terms can be set up as a 2×2 matrix problem. The linear combinations

$$\begin{aligned} \pi^\pm &= \tfrac{1}{\sqrt{2}}(\tilde{\pi}^\pm + \tilde{K}^\pm) \\ K^\pm &= \tfrac{1}{\sqrt{2}}(-\tilde{\pi}^\pm + \tilde{K}^\pm) \end{aligned} \tag{95}$$

can be shown to lead to states of definite mass for the new states $\{\tilde{\pi}^\pm, \tilde{K}^\pm\}$. The mass terms obtained, using (81) to express them in terms of the pre-EWSB masses, are

$$\begin{aligned} M^2_{\tilde{\pi}} &= M^2_{\pi(0)} - \sqrt{\tfrac{3}{8}}(M^2_{H(0)} - M^2_{\pi(0)})\tfrac{v}{u} \\ M^2_{\tilde{K}} &= M^2_{\pi(0)} + \sqrt{\tfrac{3}{8}}(M^2_{H(0)} - M^2_{\pi(0)})\tfrac{v}{u} \end{aligned} \; . \tag{96}$$

4.4.2 Physical neutral pseudoscalar states

As to start, finding the definite mass states of the neutral pseudoscalar fields $\{K^0, \overline{K}^0, \eta_0, \eta, \pi^0\}$ with mass terms as (93), can be made into a matrix problem of dimension 4×4. The fields K^0 and $\overline{K}^0$ can be rewritten into two pseudoscalar fields which are their own hermitian conjugate (as are the fields η_0, η and π^0). Examining (93), one sees that the linear combination $(K^0 + \overline{K}^0)$ appears in all the mixed terms. Further, considering the linear combination $i(K^0 - \overline{K}^0)$, the two new fields

$$\begin{aligned} \zeta &= \tfrac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \\ \xi &= \tfrac{i}{\sqrt{2}}(K^0 - \overline{K}^0) \end{aligned} \tag{97}$$

can be introduced to obtain the desired result. The mass problem in (93) can then be written in matrix form as

$$M^2_{\pi(0)}\xi^2 + \begin{pmatrix} \zeta & \pi^0 & \eta & \eta_0 \end{pmatrix} \begin{pmatrix} M^2_{\pi(0)} & -\frac{\sqrt{6}}{2}v(\lambda_2 u + \Lambda_3) & -\frac{\sqrt{2}}{2}v(\lambda_2 u + \Lambda_3) & v(2\lambda_2 u - \Lambda_3) \\ -\frac{\sqrt{6}}{2}v(\lambda_2 u + \Lambda_3) & M^2_{\pi(0)} & 0 & 0 \\ -\frac{\sqrt{2}}{2}v(\lambda_2 u + \Lambda_3) & 0 & M^2_{\pi(0)} & 0 \\ v(2\lambda_2 u - \Lambda_3) & 0 & 0 & M^2_{\eta_0(0)} \end{pmatrix} \begin{pmatrix} \zeta \\ \pi^0 \\ \eta \\ \eta_0 \end{pmatrix}. \tag{98}$$

One state of definite mass is immediately obtained, corresponding to the recently introduced ξ which has a mass $M^2_\xi = M^2_{\pi(0)}$. For the remaining four fields, the 4×4 matrix in (98) has to be diagonalised in order to obtain the masses (corresponding to the eigenvalues). The eigenvectors then correspond to the states of definite mass, which will be some linear combinations of the fields $\{\zeta, \pi^0, \eta, \eta_0\}$.

Using (81) and only considering terms of first order in v/u, the eigenvalues of the matrix in (98) are

$$M^2_{\pi(0)},\ M^2_{\eta_0(0)},\ M^2_{\pi(0)} + \frac{M^2_{H(0)} - M^2_{\pi(0)}}{\sqrt{2}}\frac{v}{u},\ M^2_{\pi(0)} - \frac{M^2_{H(0)} - M^2_{\pi(0)}}{\sqrt{2}}\frac{v}{u}. \tag{99}$$

The corresponding eigenvectors, i.e. the physical states are

$$\begin{aligned} \tilde{\pi}^0 &= -\tfrac{1}{\sqrt{3}}\pi^0 + \eta \\ \tilde{\eta}_0 &= -\tfrac{\mathfrak{k}v}{\mathfrak{m}u}\zeta + \eta_0 \\ \tilde{\psi} &= \left(\tfrac{\mathfrak{m}u}{\mathfrak{k}v} + \tfrac{\mathfrak{n}}{\sqrt{2}\mathfrak{k}}\right)\zeta + \left(-\tfrac{\sqrt{3}\mathfrak{m}u}{2\mathfrak{k}v} + \tfrac{\sqrt{3}(\mathfrak{n}+\mathfrak{k})\mathfrak{m}}{2\sqrt{2}\mathfrak{n}\mathfrak{k}}\right)\pi^0 + \left(-\tfrac{\mathfrak{m}u}{2\mathfrak{k}v} + \tfrac{(\mathfrak{n}+\mathfrak{k})\mathfrak{m}}{2\sqrt{2}\mathfrak{n}\mathfrak{k}}\right)\eta + \eta_0 \\ \tilde{\chi} &= \left(\tfrac{\mathfrak{m}u}{\mathfrak{k}v} - \tfrac{\mathfrak{n}}{\sqrt{2}\mathfrak{k}}\right)\zeta + \left(\tfrac{\sqrt{3}\mathfrak{m}u}{2\mathfrak{k}v} + \tfrac{\sqrt{3}(\mathfrak{n}+\mathfrak{k})\mathfrak{m}}{2\sqrt{2}\mathfrak{n}\mathfrak{k}}\right)\pi^0 + \left(\tfrac{\mathfrak{m}u}{2\mathfrak{k}v} + \tfrac{(\mathfrak{n}+\mathfrak{k})\mathfrak{m}}{2\sqrt{2}\mathfrak{n}\mathfrak{k}}\right)\eta + \eta_0 \end{aligned}, \tag{100}$$

where

$$\begin{aligned} \mathfrak{m} &= M^2_{\pi(0)} - M^2_{\eta_0(0)} \\ \mathfrak{n} &= M^2_{H(0)} - M^2_{\pi(0)} \\ \mathfrak{k} &= M^2_{H(0)} - M^2_{\eta_0(0)} \end{aligned}, \tag{101}$$

have been defined. The masses of the physical states are thus

$$M_{\tilde{\pi}^0}^2 = M_{\pi(0)}^2, \;\; M_{\tilde{\eta}_0}^2 = M_{\eta_0(0)}^2, \;\; M_{\tilde{\psi}}^2 = M_{\pi(0)}^2 + \frac{\mathfrak{n}}{\sqrt{2}}\frac{v}{u}, \;\; M_{\tilde{\chi}}^2 = M_{\pi(0)}^2 - \frac{\mathfrak{n}}{\sqrt{2}}\frac{v}{u}. \tag{102}$$

Note that when deriving the eigenvectors (100), only the leading and next-to-leading order terms in u have been considered. This means that for $\tilde{\pi}^0$ and $\tilde{\eta}_0$, terms of order $1/u^2$ have been omitted and for $\tilde{\psi}$ and $\tilde{\chi}$, terms of order $1/u$ have been omitted.

4.4.3 Physical neutral scalar states

The mass terms (94) for the neutral scalar states $\{h, \sigma, f, a^0\}$ can directly be written in a similar matrix form as (98). Diagonalising the matrix gives the definite masses of the physical states, which are

$$M_{H(0)}^2, \;\; M_{\pi(0)}^2 + \frac{\mathfrak{M}}{3\sqrt{2}}\frac{v}{u}, \;\; M_{\pi(0)}^2 - \frac{\mathfrak{M}}{3\sqrt{2}}\frac{v}{u}, \;\; M_{\sigma(0)}^2, \tag{103}$$

where $\mathfrak{M}$ is defined as

$$\mathfrak{M} = \sqrt{81M_{H(0)}^4 + 19M_{\pi(0)}^4 + 28M_{\eta_0(0)}^4 + 34M_{\pi(0)}^2M_{\eta_0(0)}^2 - 18M_{H(0)}^2\left(4M_{\pi(0)}^2 + 5M_{\eta_0(0)}^2\right)}. \tag{104}$$

The scalar states of definite mass are obtained by considering the eigenvectors corresponding to the masses (eigenvalues). In terms of the mixed, non-physical states they are

$$\begin{gathered}
\tilde{h} = \tfrac{\mathfrak{J}}{\sqrt{3}\mathfrak{L}}a^0 + f \\
\tilde{\varrho} = \left(\tfrac{3u\mathfrak{K}}{v\mathfrak{N}} - \tfrac{\mathfrak{M}}{\sqrt{2}\mathfrak{N}}\right)h + \left(-\tfrac{3\sqrt{3}u\mathfrak{K}\mathfrak{L}}{2v\mathfrak{N}\mathfrak{M}} + \sqrt{\tfrac{3}{2}}\tfrac{\mathfrak{L}\left(1-\frac{\mathfrak{N}^2}{\mathfrak{M}^2}\right)}{2\mathfrak{N}}\right)a^0 + \left(\tfrac{3\mathfrak{K}\mathfrak{J}u}{2v\mathfrak{N}\mathfrak{M}} - \tfrac{3\mathfrak{J}\mathfrak{P}^2}{2\sqrt{2}\mathfrak{N}\mathfrak{M}^2}\right)f + \sigma \\
\tilde{\varsigma} = \left(\tfrac{3u\mathfrak{K}}{v\mathfrak{N}} + \tfrac{\mathfrak{M}}{\sqrt{2}\mathfrak{N}}\right)h + \left(\tfrac{3\sqrt{3}u\mathfrak{K}\mathfrak{L}}{2v\mathfrak{N}\mathfrak{M}} + \sqrt{\tfrac{3}{2}}\tfrac{\mathfrak{L}\left(1-\frac{\mathfrak{N}^2}{\mathfrak{M}^2}\right)}{2\mathfrak{N}}\right)a^0 + \left(-\tfrac{3\mathfrak{K}\mathfrak{J}u}{2v\mathfrak{N}\mathfrak{M}} - \tfrac{3\mathfrak{J}\mathfrak{P}^2}{2\sqrt{2}\mathfrak{N}\mathfrak{M}^2}\right)f + \sigma \\
\tilde{\sigma} = -\tfrac{v\mathfrak{N}}{3u\mathfrak{K}}h + \sigma
\end{gathered} \tag{105}$$

where

$$\begin{aligned}
&\mathfrak{N} = 6M_{H(0)}^2 - 7M_{\pi(0)}^2 + 3M_{\pi(0)}^2 - 2M_{\eta_0(0)}^2 \\
&\mathfrak{K} = M_{H(0)}^2 - M_{\sigma(0)}^2 \\
&\mathfrak{L} = 9M_{H(0)}^2 - 5M_{\pi(0)}^2 - 4M_{\eta_0(0)}^2 \\
&\mathfrak{J} = 9M_{H(0)}^2 - M_{\pi(0)}^2 - 8M_{\eta_0(0)}^2 \\
&\mathfrak{P} = \Big[15M_{H(0)}^4 - 10M_{\pi(0)}^4 - 3M_{\sigma(0)}^4 + 2M_{H(0)}^2\left(2M_{\pi(0)}^2 - 6M_{\pi(0)}^2 - 11M_{\eta_0(0)}^2\right) \\
&\qquad +4M_{\sigma(0)}^2M_{\eta_0(0)}^2 + 8M_{\eta_0(0)}^4 + 2M_{\pi(0)}^2\left(7M_{\sigma(0)}^2 + M_{\eta_0(0)}^2\right)\Big]^{1/2}
\end{aligned} \tag{106}$$

are defined. Identifying the masses with the corresponding states, one obtains

$$M_{\tilde{h}}^2 = M_{H(0)}^2, \;\; M_{\tilde{\varrho}}^2 = M_{\pi(0)}^2 + \frac{\mathfrak{M}}{3\sqrt{2}}\frac{v}{u}, \;\; M_{\tilde{\varsigma}}^2 = M_{\pi(0)}^2 - \frac{\mathfrak{M}}{3\sqrt{2}}\frac{v}{u}, \;\; M_{\tilde{\sigma}}^2 = M_{\sigma(0)}^2. \tag{107}$$

Note that, just as for the neutral pseudoscalar eigenvectors (see (100)), only the leading and next-to-leading order terms in u have been considered for the scalar eigenvectors (105).

4.5 Discussion

The confinement group for the technicolor force is taken to be an $SU(2)$ group, instead of an $SU(3)$ group, as is the case in QCD and the two T-quark CSTC model. The motivation can in first case be due to simplicity, such that an $SU(3)$ confinement group could also later be investigated. Further, a possible Dark Matter candidate of the techniparticle spectrum cannot be realised in an $SU(n_F)_{TC}$ group as long as n_F is odd, by comparison to experimental data and assuming conservation of technibaryon number (the techni-sector analogy to baryon number) [21]. Thus, $SU(2)_{TC}$ is here chosen to be the confinement group in consideration.

In the two T-quark case (Sect. 3) a mass scaling from QCD was applied to the CSTC model, as to get an approximate sense for probable mass values of the techniparticles. For the three T-quark model, such a mass scaling cannot be done if the Higgs particle wants to be incorporated in the techniparticle spectrum. As the Higgs is taken to be part of the chiral partners (68) of the pseudoscalar mesons (67), if one applied a direct scaling such as in the two T-quark case with a factor of 1000, the Higgs would be a lot heavier than what is observed. The chiral partners of 0^- pseudoscalar fields considered in QCD is a complex matter [20]; however, the masses of them are still considered $\gtrsim 500$ MeV which is not possible to directly scale to the Higgs mass of $m_h \simeq 125$ GeV [19][20], by considering similar arguments of the two T-quark model of a techni-confinement scale close to the EW scale. Thus, if the Higgs particle is desired to be incorporated into the three T-quark theory, a scaling of the QCD particles seem inapplicable for the model.

The choice of using $u^2 \gg v^2$ can follow from considering the elaborated two T-quark CSTC model of Ref. [1], treated in Sect. 3. There it was noted that for the theory to be in accordance with EW precision measurements, the mixing angle had to be restricted (see Sect. 3.5). As a consequence of approaching the no-mixing limit, the technisigma vev, u, grew large (fig. 2). A similar approach for the three T-quark case could also be applied, as to motivate the consideration of $u^2 \gg v^2$.[6]

As the Post-EWSB phase is investigated, the non-diagonal T-quark condensate, $\langle \overline{D}S + \overline{S}D \rangle$, appears automatically as the vacuum of the potential is considered (see (88)). In the two T-quark model, only the diagonal T-quark condensate was present, and the non-diagonal T-quark condensate is a consequence of the extended number of T-quarks and the identification of the Higgs as (82).

When considering the mass terms for the technimesons in the post-EWSB phase (Sect. 4.4), the initial expressions (92) - (94) were simplified to first order in v/u. As the three techniquark model is only outlined in this thesis, future elaboration of the model should include higher order terms in these expressions from the start. Thus, the mass terms obtained in this thesis, i.e. (96), (102) and (107), must be checked such that they are consistent with the results obtained from including higher orders.

Further comments on the post-EWSB phase consider the expansion of the Higgs, which

[6] Note that further arguments for $u^2 \gg v^2$ exist, however they are not treated in this thesis.

was taken as (87), i.e. as in the SM. In the SM, the additional dofs corresponding to the fields set to zero in (87) become the extra polarization parts of the EW gauge bosons as to create the massive $W^{\pm}$ and Z^0 [8]. With the Higgs appearing in the CSTC model, considering terms to first order in v/u and using an expansion of the Higgs as $\mathcal{H} \to (H^+, v+h)$ (and similar for complex conjugate), mixed terms between $H^{\pm}$ and $a^{\pm}$ (similar to the charged pseudo scalar states $K^{\pm}, \pi^{\pm}$) will appear. As the above model is further developed, this should also be included and considered further as to investigate the coupling to the EW gauge bosons; and was excluded in the text above as the main focus (with the time at hand) was to consider the scalar sector involving the Higgs (h) mixing.

5 Summary and conclusions

A CSTC model has in this thesis been considered as an extension to the SM. The CSTC model is based on QCD, which was briefly examined in Sect. 2. The two T-quark extension was reviewed in Sect. 3 and the three T-quark model was investigated in Sect. 4.

As a consequence of the chiral symmetry breaking in the two T-quark model, it could be seen that the Higgs and the technisigma vev could be expressed in terms of the non-zero T-quark condensate. Considering the NCL, both vevs were exclusively dependent on the T-quark condensate, causing both vevs to have the same origin. In addition, the particle spectrum of the techni-sector should also consist of three technipions, corresponding to the GSBs from the chiral symmetry breaking. Further, the model seemed to be in consistency with EW precision tests, such that the values of the S, T, U parameters for the model did not violate the experimental limits. However, for the T parameter to be within the correct region, the mixing angle had to be restricted; whereas the mixing angle was reduced, the T parameter approached zero (recall the experimental limits $T = 0.02^{+0.11}_{-0.12}$).

In the three T-quark case, the concept for the two T-quark case was extended, giving rise to a greater particle spectrum related to the SSB of the chiral symmetry of the model. From these particles, arising from the symmetry breaking, the SM Higgs particle could be identified; such that within the framework of the model, the Higgs particle appears naturally. However, the final step treated in this thesis was obtaining the physical states with definite masses for the technimesons (post-EWSB phase), so the three techniquark model has just been outlined here. In further studies of the model, higher order terms in v/u when considering the technimeson masses should be included from the start. Hence, the terms in (92) - (94) and the subsequent calculations in Sects. 4.4.1 - 4.4.3 need to be checked against the case when including higher orders. As a next step the physical Lagrangian should be obtained and there after the model should undergo a detailed study of possible interactions and parameter values. The S, T, U parameters must be tested as for the two T-quark model, so as to see that the three T-quark model is consistent with the allowed values. Also, as the mass hierarchy is not as defined as in the two T-quark case (by the considered scaling from QCD), alternative mass hierarchies could also be examined, to see if possible desirable effects arise.

References

[1] Roman Pasechnik, Vitaly Beylin, Vladimir Kuksa, Grigory Vereshkov. *Chiral-Symmetric Technicolor with Standard Model Higgs boson*. Phys.Rev. D88 (2013) 075009. arXiv:1304.2081v3[hep-ph].

[2] Johan Bijnens. *Chiral Perturbation Theory Beyond One Loop*. Prog.Part.Nucl.Phys. 58 (2007) 521-586. arXiv:hep-ph/0604043v2.

[3] Matthew Schwartz. *Quantum Field Theory and the Standard Model*. 2013. ISBN: 1107034736.

[4] Stefan Scherer. *Introduction to Chiral Perturbation Theory*. Adv.Nucl.Phys. 27 (2003) 277. arXiv:hep-ph/0210398v1.

[5] Franz Gross. *Relativistic Quantum Mechanics and Field Theory*. 1999. ISBN-10: 0471353868.

[6] Ashok Das. *Lectures on quantum field theory*. 2008. ISBN: 13 978-981-283-285-6.

[7] Michael E. Peskin, Dan V. Schroeder. *An Introduction To Quantum Field Theory*. 1995. ISBN-10: 0201503972.

[8] Gordon Kane. *Modern Elementary Particle Physics; Updated Edition*. 1993. ISBN: 0-201-62460-5.

[9] Johan Bijnens. Lecture notes: *Groups and representations*. http://home.thep.lu.se/~bijnens/fytn04/groups.pdf.

[10] Jean-Pierre Derendinger. *Lecture Notes on the Standrad Model of Elementary Particle Physics*. http://www.derendinger.itp.unibe.ch/Documents,_publications,_lecture_notes_files/JPD_St_Model.pdf.

[11] David Griffiths. *Introduction to Elementary Particles; Second, Revised Edition*. 2008. ISBN: 978-3-527-40601-2.

[12] Johan Bijnens. Lecture notes: *A non-Abelian field strength and kinetic term*. http://home.thep.lu.se/~bijnens/fytn04/nonabelian.pdf.

[13] B. R. Martin, G. Shaw. *Particle Physics; 3rd Edition*. 2009. ISBN: 978-0-470-03293-0.

[14] Michael E. Peskin, Tatsu Takeuchi. *Estimation of oblique electroweak corrections*. Phys.Rev. D46 (1992) 381-409. prd.aps.org/abstract/PRD/v46/i1/p381_1.

[15] Tatsu Takeuchi. *Estimation of oblique electroweak corrections*. 1992. SLAC-PUB-5730. http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-5730.pdf.

[16] Robert A. Adams, Christopher Essex. *Calculus a complete course; 7th edition* . 2009. ISBN: 0321549287.

[17] Ron Larson, David C. Falvo. *Elementary Linear Algebra; 6th edition*. 2009. ISBN-13: 978-0-618-78376-2.

[18] Stanley J. Brodsky, Robert Shrock. *Condensates in Quantum Chromodynamics and the Cosmological Constant*. Proc.Nat.Acad.Sci. 108 (2011) 45-50. arXiv:0905.1151v2[hep-th].

[19] Muneyuki Ishida, Shin Ishida.*Existence of $\sigma(600)/\kappa(900)$-Particle and New Chiral Scalar Nonet Chiralons*. In *Upton 1997, Hadron spectroscopy* 381-384. arXiv:hep-ph/9712231v1.

[20] Particle data group. *NOTE ON SCALAR MESONS BELOW 2 GEV*. 2013. http://pdg.lbl.gov/2013/reviews/rpp2013-rev-scalar-mesons.pdf.

[21] Roman Pasechnik, Vitaly Beylin, Vladimir Kuksa, Grigory Vereshkov. *Vector-like technineutron Dark Matter: is a QCD-type Technicolor ruled out by XENON100?*. Eur.Phys.J. C74 (2014) 2728. arXiv:1308.6625v2[hep-ph].

A Basic group theory

Group theory is an important tool in physics due to the fact that groups can be used to describe symmetries. These symmetries then in turn dictate the building blocks of e.g. the SM and possible extensions to it (as is the topic of this thesis). Hence, the basic concepts of group theory and the basic groups of the SM are presented here.

Formally, four conditions are required to form a group. Denoting a group as G, it needs to fulfil:

1. There exists a group multiplication $*$, where $\forall g_1, g_2 \in G : g_1 * g_2 = g_3 \in G$.
2. There exists an identity element $\mathcal{I} \in G$, where $\forall g \in G : g * \mathcal{I} = \mathcal{I} * g = g$.
3. Every element has a unique inverse g^{-1}, so $\forall g \in G : g * g^{-1} = g^{-1} * g = \mathcal{I}$.
4. The group multiplication is associative, such that $\forall g_1, g_2, g_3 \in G : (g_1 * g_2) * g_3 = g_1 * (g_2 * g_3)$.

If all group elements commute, the group is known as an Abelian group, otherwise it is called a non-Abelian group [8].

Lie groups are groups whose elements can be written as

$$g(\theta_1, \theta_2, ..., \theta_n) = \exp\left(\sum_{j=1}^{n} i\theta_j T_j\right), \tag{108}$$

where $\theta_j \in \mathbb{R}$ are the parameters of the group and T_j, which are Hermitian matrices, are called the generators (of the group). The space of $T = \sum_{j=1}^{n} \theta_j T_j$ is called the Lie algebra of a certain Lie group. The commutation relation of the generators, $[T_a, T_b]$, define the lie algebra of a lie group in question. The groups considered in this thesis are all Lie groups [9][10].

A group, G, can be considered in different representations. A representation is when one has a set of objects, S, and the group elements, g, (in the representation f_g) act on an object, s, of this set such that the resulting object is still in S. Further, the action of the group elements on an object $s \in S$ should follow the procedure of the group multiplication. The representation conditions can be written as

$$\forall g \in G : \exists f_g : \forall s \in S : f_g(s) \in S \tag{109}$$

and

$$\forall g_1, g_2 \in G : \forall s \in S : f_{g_1 * g_2}(s) = f_{g_1}(f_{g_2}(s)) \tag{110}$$

respectively [9].

The groups considered in the SM are the ones called $SU(3)$, $SU(2)$ and $U(1)$, all of which are lie groups. The group $U(1)$ is the set of all complex phase factors with standard multiplication as the group multiplication. If a group element is denoted U, one then has

$U = e^{i\epsilon}$. Comparing with (108), it is seen that the generator is in this case a real number. Also $U(1)$ is unitary $U^\dagger U = 1$ and the elements have magnitude of unity. The $U(1)$ group is an Abelian group [8].

The other two groups are of the same type. Generally, $SU(N)$ is the group of $N \times N$ matrices which are unitary ($U^\dagger U = \mathcal{I}$) and have determinant equal to unity ($\det(U) = 1$). The group multiplication is matrix multiplication, hence $SU(N)$ are non-Abelian groups. Due to the constraints of unitarity and determinant equal to unity, $SU(N)$ has $N^2 - 1$ parameters [8]. This means that, comparing with (108), one also has $N^2 - 1$ generators. Hence, for $SU(2)$ there are three generators, which can be chosen to be the Pauli matrices (τ_i, $i = 1, 2, 3$), and for $SU(3)$ there are 8, and can be chosen as the Gell-Mann matrices (λ_a, $a = 1, \ldots, 8$). The Pauli matrices satisfies the commutaion relation $[\tau_i, \tau_j] = 2i\epsilon_{ijk}\tau_k$, where the structure constants, ϵ_{ijk}, are the Levi-Civita symbol [8]. The Gell-Mann matrices satisfies $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$, where f_{abc} are the structure constants (given by $f_{abc} = \mathrm{Tr}([\lambda_a, \lambda_b]\lambda_c)/4i$) [4].

A specific type of representation is the fundamental representation. In the fundamental representation, the representation of the group elements are themselves, i.e. (with previous notation from (109) and (110)) $f_g = g$. For an $SU(N)$ group, the objects on which the elements (i.e. the matrices) act on are column vectors of size N. Hence, in the fundamental representation, denoting an $SU(N)$ group element (matrix) as M and the object it acts on (column vector) as c, the action of the group is

$$c \to Mc \text{ [9].} \tag{111}$$

Another representation is the adjoint representation, which is the Lie algebra of a Lie group $T = \sum_{j=1}^{n} \theta_j T_j$. The action of the group on the adjoint representation is

$$T \to MTM^\dagger \text{ [9].} \tag{112}$$

B The Standard Model

The Standard model (SM) is here briefly presented. The elementary particles and their interactions are considered, by first investigating the fermions and their properties. Then the gauge bosons are treated, which are the force mediators and hence interactions are introduced. Finally, the Higgs particle is considered, which is the field necessary to introduce masses of the particles in the SM.

B.1 Fermions

Fermions are spin-$\frac{1}{2}$ particles and in the SM they are divided into leptons and quarks. The equation of motion for fermions is called the Dirac equation. It is obtained from the Lagrangian

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi, \tag{113}$$

where ψ is a fermionic field (a four-component complex spinor function) with mass m.[7] Using the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = 0, \tag{114}$$

on (113), yields the equation of motion for the fermions, i.e. the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \ [6]. \tag{115}$$

There is no unique representation of the γ-matrices, and a specific representation is the chiral representation, where

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{116}$$

Note that the γ-matrices in (116) are 2×2 block-matrices (they are really 4×4 matrices). It can thus be expected that the fermion field can be written as

$$\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}, \tag{117}$$

where χ_R and χ_L are two component spinors. Given the two component form of ψ, projection operators can be defined as

$$P_R = \frac{1 + \gamma^5}{2}, \qquad P_L = \frac{1 - \gamma^5}{2}. \tag{118}$$

They can be seen to satisfy the standard projection operator rules

$$\begin{cases} P_L^2 = P_L \\ P_R^2 = P_R \\ P_R + P_L = 1 \\ P_R P_L = 0 \end{cases}. \tag{119}$$

The projection operators have the effect

$$P_R \psi = P_R \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} = \psi_R \tag{120}$$

and, similarly,

$$P_L \psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} = \psi_L. \tag{121}$$

The components ψ_L and ψ_R are referred to as chiral states, where ψ_L is called the left-handed (LH) and ψ_R the right-handed (RH) component of ψ respectively [8].

[7] To be precise, the Lagrangian of (113) is not Hermitian. Hence a Lagrangian written as $\mathcal{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \overline{\psi}(i\gamma^\mu \overleftarrow{\partial}_\mu + m)\psi$, which is Hermitian, is more extensive than (113). However, in the present text, it is sufficient to use the simpler (113) as the Lagrangian of the fermions, since the only difference between this one and the Hermitian one is a total divergence which does not affect the action [6].

The meaning of left- and right-handedness can be further understood by considering the massless limit, where they also correspond to eigenstates of helicity. To see this, consider a free fermion; its space-time dependence is of the form $\mathrm{e}^{-ip_\mu x^\mu}$. The Dirac equation, (115), using (116), (117) and taking the limit $m \to 0$ then yields

$$\vec{\sigma} \cdot \hat{p} \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_R \\ -\chi_L \end{pmatrix}. \tag{122}$$

Applying P_L respectively P_R, yields the two relations

$$\begin{matrix} (\vec{\sigma} \cdot \hat{p})\psi_L = -\psi_L \\ (\vec{\sigma} \cdot \hat{p})\psi_R = \psi_R \end{matrix}. \tag{123}$$

Since $(\vec{\sigma} \cdot \hat{p})$ is the helicity operator (i.e. it measures if the spin and momentum is parallel or antiparallel), one hence see that in the massless limit, a chiral LH state corresponds to a left-handed helicity state (negative helicity) and a chiral RH state corresponds to a right-handed state (positive helicity) [8][4].

The chiral states are very important in the SM, since they are treated differently in the weak interaction. Further, LH and RH decomposition is the basis of symmetries considered in low-energy QCD, which in turn is the basis of the technicolor extension considered in this thesis [8][2][1].

B.2 Interactions

Interactions in the SM occur through the propagation of so called gauge bosons, which in contrast to fermions, are spin-1 particles. The interactions, and hence the gauge bosons, arise from the various symmetry groups considered in the SM. As mentioned in Appendix A, the groups considered in the SM are $SU(3) \times SU(2) \times U(1)$, or with a more careful notation

$$SU(3)_C \times SU(2)_W \times U(1)_Y, \tag{124}$$

where the subscripts denote the specific spaces of the groups, since there could e.g. be other $SU(3)$ groups as well [8].

For the groups (124) to define a symmetry of the SM, the Lagrangian should be invariant under transformations of each group individually. Since the groups (124) are Lie groups, they correspond to a phase transformation (see (108)) or gauge transformation (which is the term used by historical convention). Considering the fermionic field ψ in the fundamental representation and denoting a general (unitary) group element as U, the requirement that $\mathcal{L} \to \mathcal{L}$ as $\psi \to U\psi$ does not in general hold for the fermionic Lagrangian (113). The non-invariance is due to that $\partial_\mu(U\psi) \neq U\partial_\mu\psi$ if $U = U(x)$, i.e. if U is a local symmetry (has a space-time dependence). If U does not have a space-time dependence it is called a global symmetry, and one would automatically have an invariant theory; but in the SM, local symmetry (under the groups in (124)) is a demand [8][11].

The Lagrangian can be made invariant by exchanging the derivative in (113) to a so called covariant derivative, $\partial_\mu \to \mathcal{D}_\mu$. It is defined to be in the adjoint representation of the group, so that it transforms as $\mathcal{D}_\mu \to U\mathcal{D}_\mu U^\dagger$, and hence leaves the fermionic Lagrangian (113) invariant. The form of the covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu - \frac{ig}{2} T^a F^a_\mu, \tag{125}$$

where $g \in \mathbb{R}$ (called the coupling strength or constant), T^a are the generators of the concerned group and F^a_μ are new fields, called gauge fields (i.e. the fields of the gauge bosons) [8]. For gauge invariance of the Lagrangian (113), i.e. $\mathcal{D}_\mu\psi \to U\mathcal{D}_\mu\psi$, it is found that the following transformation must hold

$$T^a F^a_\mu \to -\frac{i}{g}(\partial_\mu U)U^\dagger + UT^a F^a_\mu U^\dagger \text{ [11].} \tag{126}$$

Taking $U = \exp\left(i\theta^a(x)T^a\right)$ and considering an infinitesimal transformation, the field alone transforms as

$$F^a_\mu \to F^a_\mu + \frac{1}{g}\partial_\mu\theta^a(x) - f^{abc}\theta^b(x)F^c_\mu, \tag{127}$$

where f^{abc} are the structure constants of the lie algebra in consideration [8]. Note that since the gauge bosons are bosons and not fermions, they are not described by the Dirac equation. Instead, the Lagrangian term describing their propagation is given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\big(F_{\mu\nu}F^{\mu\nu}\big), \tag{128}$$

where

$$F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu - ig[F_\mu, F_\nu] \text{ [12].} \tag{129}$$

The full covariant derivative of the SM groups (124) reads

$$\mathcal{D}^f_\mu = \partial_\mu - ig_1\frac{Y^f}{2}B_\mu - ig_2\frac{\tau^i}{2}W^i_\mu - ig_3\frac{\lambda^a}{2}G^a_\mu, \tag{130}$$

where $i = 1, 2, 3$, $a = 1, 2, \ldots, 8$ and f is fermion flavour. The second term of (130) corresponds to the $U(1)_Y$ symmetry. The generator of $U(1)_Y$ is Y_f, which is a constant (as mentioned in Appendix A) and where the subscript f indicates that it could be different depending on the fermion flavour. The value of Y_f for fermion f is called the hypercharge of f. All fermions are in the fundamental representation of $U(1)_Y$. The third term of (130) corresponds to the $SU(2)_W$ symmetry, which is called the weak isospin space (and the $SU(2)_W$ interaction is called the weak interaction). It acts differently on LH and RH fermions, where LH fermions are put in $SU(2)_W$ doublets (i.e. in the fundamental representation) and RH fermions are singlets (hence they do not interact by the weak interaction). The leptons and quarks are put in each respective doublet, with the lepton doublet as $L_{(A)}$ and quark doublet as $Q_{L,(A)}$, where $A = 1, 2, 3$ denotes the number of generations. The corresponding right-handed singlets are $e_{R,(A)}, d_{R,(A)}, u_{R,(A)}$ (it is not known if right-handed neutrinos exist). Finally, the fourth term of (130) corresponds to the $SU(3)_C$ group, called the color space. Only the quark carry color charge, i.e. they are the only fermions in the fundamental representation of $SU(3)_C$, as triplets. The leptons are color singlets and hence do not interact by $SU(3)_C$ [8].

The Lagrangian for the SM fermions can now be written as

$$\mathcal{L} = \sum_{f=L,e_R,Q_L,u_R,d_R} \overline{f} i\gamma^\mu \mathcal{D}^f_\mu f, \tag{131}$$

where only the first generation has been considered, but for the other two the expression is the same. Note that if a term in (130) acts on a fermion state, f, which is a singlet in that space it gives zero by definition [8].

The Lagrangian (131) does not contain any mass terms, which is due to the fact that mass terms cannot be introduced without breaking the gauge invariance.[8] However, there is a way of introducing mass terms without breaking the symmetries explicitly (i.e. by adding a symmetry breaking term), but instead break them spontaneously, through the so called Higgs mechanism, see Appendix B.3 [8].

B.3 The Higgs mechanism

Masses can be obtained by interaction with a complex scalar field, called the Higgs field ($\mathcal{H}(x)$), via the subsequent SSB that occurs for its system; the process is called the Higgs mechanism. In the SM, masses are present for the observable gauge bosons $W^\pm$ and Z^0 and the fermions. The $W^\pm$ and Z^0 are part of $SU(2)_W \times U(1)_Y$, hence the Higgs field must have a Lagrangian (or equivalently a Hamiltonian) which is invariant under these groups.[9] The Higgs mechanism follows a typical LσM, so the procedure is similar to Sect. 2, only that here the symmetry group $SU(2)_W \times U(1)_Y$ is considered instead of the chiral group $SU(2)_L \times SU(2)_R$ [8].

The Lagrangian (cf. (4)) considered is

$$\mathcal{L} = (\mathcal{D}_\mu \mathcal{H})^\dagger (\mathcal{D}^\mu \mathcal{H}) - \mu^2 \mathcal{H}^\dagger \mathcal{H} - \lambda(\mathcal{H}^\dagger \mathcal{H})^2, \tag{132}$$

where

$$\mathcal{H} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{133}$$

and

$$\mathcal{D}_\mu = \partial_\mu - ig_1 \frac{Y_H}{2} B_\mu - ig_2 \frac{\tau^i}{2} W^i_\mu. \tag{134}$$

Following Sect. 2 and minimizing the potential, $V(\mathcal{H}) = \mu^2 \mathcal{H}^\dagger \mathcal{H} + \lambda(\mathcal{H}^\dagger \mathcal{H})^2$, degenerate minima are obtained along the 3-sphere $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = (2\mathcal{H}^\dagger \mathcal{H} =) v^2$. A ground state

[8] Consider the mass term of the fermion, which is of the form $m\overline{\psi}\psi$. Using (119), that $\overline{\psi}_L = \overline{\psi} P_R$ and $\overline{\psi}_R = \overline{\psi} P_L$ one obtains $m\overline{\psi}\psi = m\overline{\psi}_R \psi_L + m\overline{\psi}_L \psi_R$. Since ψ_R is an $SU(2)_W$ singlet and ψ_L a doublet, the fermion mass terms are not $SU(2)_W$ gauge invariant and can hence not be added to the SM Lagrangian. Similarly, mass terms for the gauge fields cannot either be added to the SM Lagrangian, since they are of the form $mF_\mu F^\mu$ and will not transform invariantly for any of the SM groups, since they transform as (127) [8].

[9] $SU(3)_C$ is not considered because its gauge bosons, the gluons, are massless, so they do not interact with the Higgs field (i.e. Higgs is an $SU(3)_C$ singlet) [8].

can then be chosen, for simplicity, as

$$\langle \mathcal{H} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{135}$$

where $\phi_3 = v$ and $\phi_1 = \phi_2 = \phi_4 = 0$ [8].

Considering the Goldstone theorem, there is a Hamiltonian (Lagrangian (132)) invariant under $SU(2)_W \times U(1)_Y$ with a ground state (135) that is not annihilated by their generators (since (135) is not invariant under $SU(2)_W \times U(1)_Y$). Hence four GSBs are expected to arise. More careful analysis shows that the ground state (135) actually is invariant under a new $U(1)$ group, with generator $Q = t_3 + Y_H/2$, where t_3 is the weak isospin. The new $U(1)$ group is denoted $U(1)_{EM}$ and corresponds to the electromagnetic interaction, propagated by the photon (and Q is thus the electric charge). Hence, the SSB that occurs under the Higgs mechanism has the form $SU(2)_W \times U(1)_Y \to U(1)_{EM}$ and is referred to as the electroweak (spontaneous) symmetry breaking (EWSB) [8].

Returning to following the procedure of Sect. 2, a field h (i.e. ϕ_3 in (133)) can be considered to have obtained the vev (135) yielding

$$\mathcal{H}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{136}$$

Inserting (136) into the Lagrangian (132) and taking $Y_H = 1$ yields

$$\frac{1}{8} \left| \begin{pmatrix} 2\partial_\mu - ig_1 B_\mu - ig_2 W^3_\mu & -ig_2(W^1_\mu - iW^2_\mu) \\ -ig_2(W^1_\mu + iW^2_\mu) & 2\partial_\mu - ig_1 B_\mu + ig_2 W^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \right|^2. \tag{137}$$

Multiplying it all out, the following expression is obtained

$$\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{8}\left[g_2^2 \Big((W^1_\mu)^2 + (W^2_\mu)^2 \Big)(v+h)^2 + \Big(g_2 W^3_\mu - g_1 B_\mu \Big)^2 (v+h)^2 \right] \text{ [8].} \tag{138}$$

The initial symmetry, $SU(2)_W \times U(1)_Y$, started with the four fields B_μ, W^i_μ. From (138), the states $\big((W^1_\mu)^2 + (W^2_\mu)^2\big)$ and $\big(g_2 W^3_\mu - g_1 B_\mu\big)$ have definite masses and can be identified with three fields as $\big((W^1_\mu)^2 + (W^2_\mu)^2\big) = 2W^+_\mu W^{-\mu}$ and $\big(g_2 W^3_\mu - g_1 B_\mu\big) = \sqrt{g_1^2 + g_2^2} Z_\mu$. It is now possible to express (138) as

$$\begin{aligned} \frac{1}{2}(\partial_\mu h)^2 &+ \left(\frac{1}{2} g_2 v\right)^2 W^+_\mu W^{-\mu} + \left(\frac{1}{2} g_2\right)^2 W^+_\mu W^{-\mu} h^2 \\ &+ \frac{1}{2} g_2^2 v W^+_\mu W^{-\mu} h + \frac{1}{2}\left(\frac{1}{2} v \sqrt{g_1^2 + g_2^2}\right)^2 Z_\mu Z^\mu \\ &+ \frac{1}{2}\left(\frac{1}{2}\sqrt{g_1^2 + g_2^2}\right)^2 Z_\mu Z^\mu h^2 + v\left(\frac{1}{2}\sqrt{g_1^2 + g_2^2}\right)^2 Z_\mu Z^\mu h, \end{aligned} \tag{139}$$

giving rise to the mass terms

$$M_W = \frac{g_2 v}{2} 2, \qquad M_Z = \frac{v\sqrt{g_1^2 + g_2^2}}{2}. \tag{140}$$

As noted, only three gauge fields with definite masses were observed after the EWSB. There is thus one field missing comparing the four initial ones B_μ, W_μ^i with the three final ones $Z_\mu, W_\mu^\pm$. From the linear combinations of $Z_\mu, W_\mu^\pm$ in terms of B_μ, W_μ^i, the final field is the combination orthogonal to Z_μ, i.e. $\left(g_2 W_\mu^3 + g_1 B_\mu\right)/\sqrt{g_1^2+g_2^2} = A_\mu$. Since A_μ has no mass term it corresponds to the gauge boson of the $U(1)_{EM}$ symmetry (i.e. it is the electromagnetic field) [8].[10]

Fermions will also gain mass due to interactions with the Higgs field, through the so called Yukawa interaction. Having the Higgs as a doublet one can construct terms as [11]

$$g_d \overline{Q}_L \mathcal{H} d_R + g_u \overline{Q}_L \mathcal{H}_C u_R + g_d \mathcal{H}^\dagger \overline{d}_R Q_L + g_u \mathcal{H}_C^\dagger \overline{u}_R Q_L, \tag{141}$$

where g_d, g_u are coupling constants and

$$\mathcal{H}_C = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} \tag{142}$$

is introduced.[12] Note that in (141), the two last terms are the complex conjugate of the two first terms [8].

Letting $\mathcal{H}$ go to (136), and similar for $\mathcal{H}_C$, the Yukawa terms (141) become

$$\frac{g_d v}{\sqrt{2}}\overline{d}d + \frac{g_d}{\sqrt{2}}\overline{d}dh - \frac{g_u v}{\sqrt{2}}\overline{u}u - \frac{g_u}{\sqrt{2}}\overline{u}uh. \tag{143}$$

The masses of the fermions can be identified as

$$m_d = \frac{g_d v}{\sqrt{2}}, \qquad m_u = \frac{g_u v}{\sqrt{2}} \text{ [8]}. \tag{144}$$

C Calculations: Two techniquark case

Nearly conformal limit parameters

The parameters in the two T-quark CSTC model are here evaluated in the NCL. Considering (53), (54) and defining $\overline{g}_{TC} = g_{TC}|\langle\overline{\tilde{Q}}\tilde{Q}\rangle|$, we can write

$$u = \left(\frac{\lambda_H \overline{g}_{TC}}{\delta}\right)^{1/3}. \tag{145}$$

Hence, it is also obtained that

$$v = \left(\frac{\lambda}{\lambda_H}\right)^{1/2}\left(\frac{\lambda_H \overline{g}_{TC}}{\delta}\right)^{1/3}. \tag{146}$$

[10]The unbroken symmetries $SU(3)_C \times U(1)_{EM}$ have massless gauge bosons (which propagate the interactions), since mass terms for gauge fields are not allowed for unbroken symmetry groups [8].

[11]Here considering the first generation of quarks, but the process is the same for the other generations; and leptons. However, for leptons the right-handed neutrino might not exist so if it should be included or not is a question in itself [8].

[12]$\mathcal{H}_C$ needs to be introduced to obtain mass terms for both the u and d quarks [8].

From (27) , $\lambda_H > 0$ and hence for v to be real $\lambda > 0$ must hold; thus, one takes $\lambda \to \xi\lambda = |\lambda| > 0$, where $\xi = \text{sign}(m_{\tilde{\sigma}}^2 - 3m_{\tilde{\pi}}^2)$. From (145) and (146) it is obtained that

$$u = \left(\frac{\lambda_H}{\xi\lambda}\right)^{1/2} v. \tag{147}$$

and

$$\bar{g}_{TC} = v^3\left(\frac{\lambda_{TC}\lambda_H - \lambda^2}{\lambda}\right)\left(\frac{\lambda_H}{\xi\lambda}\right)^{1/2}. \tag{148}$$

Since $m_{\tilde{\pi}}^2 = \bar{g}_{TC}/u$ (see (24)), using (147) and (148), we have

$$m_{\tilde{\pi}}^2 = v^2\left(\frac{\lambda_{TC}\lambda_H - \lambda^2}{\lambda}\right). \tag{149}$$

The mixing angle θ and the masses for technisigma and Higgs are also altered when considering the NCL. The mass matrix (32) can be expressed in the NCL, using (147) and (149), as

$$-\frac{1}{2}\begin{pmatrix} 2\lambda_H v^2 & -2v^2\sqrt{\xi\lambda\lambda_H} \\ -2v^2\sqrt{\xi\lambda\lambda_H} & 2\lambda v^2 + 3m_{\tilde{\pi}}^2/2 \end{pmatrix}. \tag{150}$$

As before, (150) can be diagonalised to give the masses for the physical fields h and $\tilde{\sigma}$, which now are

$$\begin{cases} m_h^2 = \frac{1}{2}\left(2\lambda_H v^2 + 2\lambda v^2 + 3m_{\tilde{\pi}}^2 - \sqrt{16\lambda\lambda_H v^4 + (2\lambda v^2 + 3m_{\tilde{\pi}}^2 - 2\lambda_H v^2)^2}\right) \\ m_{\tilde{\sigma}}^2 = \frac{1}{2}\left(2\lambda_H v^2 + 2\lambda v^2 + 3m_{\tilde{\pi}}^2 + \sqrt{16\lambda\lambda_H v^4 + (2\lambda v^2 + 3m_{\tilde{\pi}}^2 - 2\lambda_H v^2)^2}\right) \end{cases}. \tag{151}$$

The mixing angle in the NCL is given by using (147) on (37), i.e. by

$$\tan(2\theta) = \frac{4v^2\sqrt{\xi\lambda\lambda_H}}{2\lambda v^2 + 3m_{\tilde{\pi}}^2 - 2\lambda_H v^2}. \tag{152}$$

As in the general case (see (41)), the parameters $\{\lambda_{TC}, \lambda_H, \lambda\}$ can be expressed in the parameters $\{m_{\tilde{\pi}}^2, m_{\tilde{\sigma}}^2, m_h^2\}$ as

$$\begin{cases} \lambda_{TC} = \frac{\lambda}{\lambda_H}\left(\lambda + \frac{m_{\tilde{\pi}}^2}{v^2}\right) \\ \lambda_H = \frac{m_{\tilde{\sigma}}^2 m_{h^2}}{6m_{\tilde{\pi}}^2 v^2} \\ \lambda = \frac{3m_{\tilde{\pi}}^2(m_{\tilde{\sigma}}^2 + m_h^2) - m_{\tilde{\sigma}}^2 m_h^2 - 9m_{\tilde{\pi}}^4}{6m_{\tilde{\pi}}^2 v^2} \end{cases}, \tag{153}$$

obtained by considering (149) and (151).

As a final, interesting scenario, the no-mixing limit (where $\theta = 0$) can be considered. From (152), one has for $\theta = 0$ that $\lambda\lambda_H = 0$ should hold. Using (153) yields

$$0 = \lambda\lambda_H = \frac{m_{\tilde{\sigma}}^2 m_h^2(m_h^2 - 3m_{\tilde{\pi}}^2)}{36v^2 m_{\tilde{\pi}}^4}(3m_{\tilde{\pi}}^2 - m_{\tilde{\sigma}}^2), \tag{154}$$

for $\theta = 0$. As $m_h^2 \simeq m_{\tilde{\pi}}^2$, considering the mass scaling from QCD, the term where $\theta = 0$ due to $(m_h^2 - 3m_{\tilde{\pi}}^2) = 0$ is not as probable. In order for this scenario to be true, the technipion mass should be around $m_{\tilde{\pi}}^2 \approx 40$ GeV. However, the case where $(3m_{\tilde{\pi}}^2 - m_{\tilde{\sigma}}^2) = 0 \Rightarrow \theta = 0$ is not constrained in the same sense and is thus of more interest.

D Calculations: Three techniquark case

D.1 Construction of chirally symmetric techniquarks

As was stated in Sect. 4.1, three chirally symmetric T-quarks could be constructed by considering the two LH bi-doublets $Q^{a\alpha}_{L(1)}, Q^{a\alpha}_{L(2)}$ and the two RH singlets $U^{\alpha}_{R(1)}, D^{\alpha}_{R(1)}$. In this section the chirally symmetric, or vector-like, bi-doublet, seen in (60), is constructed, while the vector-like singlet (61) follows in a similar manner.

The objects used in construction of the vector-like bi-doublet are the two LH bi-doublets $Q^{a\alpha}_{L(1)}$ and $Q^{a\alpha}_{L(2)}$. The bi-doublet for the first generation is left as it is, while the one for the second generation is altered. Consider first the charge conjugation of the bi-doublet under transformation. From (55), it is obtained that

$$\begin{aligned}\mathcal{C}Q^{a\alpha}_{L(2)} \to \mathcal{C}\left(Q^{a\alpha}_{L(2)} + \frac{i}{2}g_2\vartheta_k\tau^{ab}_k Q^{b\alpha}_{L(2)} + \frac{i}{2}g_{TC}\varphi_k\tau^{\alpha\beta}_k Q^{a\beta}_{L(2)}\right) \\ = \mathcal{C}Q^{a\alpha}_{L(2)} - \frac{i}{2}g_2\vartheta_k(\tau^{ab}_k)^*\mathcal{C}Q^{b\alpha}_{L(2)} - \frac{i}{2}g_{TC}\varphi_k(\tau^{\alpha\beta}_k)^*\mathcal{C}Q^{a\beta}_{L(2)}, \quad (155)\end{aligned}$$

where $\mathcal{C}$ denotes the charge conjugation operator. When charge conjugation is applied to a chiral spinor, it flips the chirality. Thus it is possible to define (see (58)) a RH bi-doublet as

$$Q^{a\alpha}_{R(2)} = \epsilon^{ab}\epsilon^{\alpha\beta}\mathcal{C}Q^{b\beta}_{L(2)}, \quad (156)$$

where ϵ^{ij} denotes the two-dimensional Levi-Civita tensor. Multiplying (155) with $\epsilon^{ca}\epsilon^{\gamma\alpha}$ thus yields

$$Q^{c\gamma}_{R(2)} = \epsilon^{ca}\epsilon^{\gamma\alpha}\mathcal{C}Q^{a\alpha}_{L(2)} \to \epsilon^{ca}\epsilon^{\gamma\alpha}\mathcal{C}Q^{a\alpha}_{L(2)} - \frac{i}{2}g_2\vartheta_k\epsilon^{ca}(\tau^{ab}_k)^*\epsilon^{\gamma\alpha}\mathcal{C}Q^{b\alpha}_{L(2)} - \frac{i}{2}g_{TC}\varphi_k\epsilon^{\gamma\alpha}(\tau^{\alpha\beta}_k)^*\epsilon^{ca}\mathcal{C}Q^{a\beta}_{L(2)}. \quad (157)$$

In the expression above, we have terms of the form $\epsilon^{ki}(\tau^{ij}_s)^*Q^j$ (omitting all indices except for the ones which the Pauli matrix applies to). Considering the anti-symmetry of the Levi-Civita tensor, it is possible to write $\delta^{ij} = \epsilon^{ik}\epsilon^{jk} = -\epsilon^{ik}\epsilon^{kj}$. Thus, (157) contains terms of the form

$$\epsilon^{ki}(\tau^{ij}_s)^*Q^j = \epsilon^{ki}(\tau^{ij}_s)^*\delta^{jl}Q^l = -\epsilon^{ki}(\tau^{ij}_s)^*\epsilon^{jm}\epsilon^{ml}Q^l. \quad (158)$$

The property $\epsilon\tau^*_s\epsilon = \tau_s$ can be seen to hold, which considering indices translates to $\epsilon^{ki}(\tau^{ij}_s)^*\epsilon^{jm} = \tau^{km}_s$. Taken the recent stated properties into account, (157) can be written as

$$Q^{c\gamma}_{R(2)} \to Q^{c\gamma}_{R(2)} + \frac{i}{2}g_2\vartheta_k\tau^{cb}_k Q^{b\gamma}_{R(2)} + \frac{i}{2}g_{TC}\varphi_k\tau^{\gamma\beta}_k Q^{c\beta}_{R(2)}. \quad (159)$$

The expression obtained in (159) is seen to transform identically to $Q^{a\alpha}_{L(1)}$, by considering (55). Hence, one can construct a vector-like bi-doublet by adding $Q^{a\alpha}_{L(1)}$ and $Q^{a\alpha}_{R(2)}$, resulting in

$$Q^{a\alpha} = Q^{a\alpha}_{L(1)} + Q^{a\alpha}_{R(2)}. \quad (160)$$

As by construction, $Q^{a\alpha}$ is a doublet under both $SU(2)_{TC}$ and $SU(2)_W$ and transforms as

$$Q^{a\alpha} \to Q^{a\alpha} + \frac{i}{2}g_2\vartheta_k\tau^{ab}_k Q^{b\alpha} + \frac{i}{2}g_{TC}\varphi_k\tau^{\alpha\beta}_k Q^{a\beta}. \quad (161)$$

D.2 Technimeson substructure

Even though the technimesons are treated as fundamental particles in the low-energy theory that is considered, they should in reality be built up by T-quarks. The pseudoscalar technimesons, (67), can be considered to have a T-quark substructure as

$$P_a = \overline{\hat{Q}}\gamma_5\lambda_a\hat{Q} \tag{162}$$

and the η_0 a substructure as

$$\eta_0 = \overline{\hat{Q}}\gamma_5\hat{Q}, \tag{163}$$

where $\hat{Q}$ is given in (64). Further, their chiral partners, (68), can be taken to have substructure

$$S_a = \overline{\hat{Q}}\lambda_a\hat{Q} \tag{164}$$

and the σ a substructure as

$$\sigma = \overline{\hat{Q}}\hat{Q}. \tag{165}$$

The above substructures can be used by considering the substructures of the corresponding mesons in QCD. Examining the quark structure of the pseudoscalar fields (67) in QCD, it can be seen to correspond to the structure presented in (162) [13]. The concept is then extended to the chiral partners. Note however that in (162) and the other substructures above, not the physical states are obtained, but one has to take linear combinations as in (14) in order to get the physical, charged states. The linear combinations that should be taken become clear when expanding the parametrization of the mesons as in (69) through the substructures of them, (162) - (165); given by

$$\hat{\Phi}^i_j = \frac{1}{\sqrt{6}}\left(\sigma - i\eta_0\right)\delta^i_j + \frac{1}{2}\left(S_a - iP_a\right)(\lambda_a)^i_j. \tag{166}$$

The T-quark substructure of the technimesons as (162) - (165) determine the transformation properties of the technimesons in the EW groups $SU(2)_W \times U(1)_Y$ of the SM. As mentioned, $\hat{Q}$ is given by (64), where $Q = (U, D)$ transforms as a doublet under $SU(2)_W$ and S as a singlet. Further, it was mentioned that the hypercharge of Q were $Y_Q = 0$, while S had hypercharge with negative sign, and can be taken to be $Y_S = -1$. The technimesons will thus be in different representations of $SU(2)_W \times U(1)_Y$ depending on their T-quark substructure.

As an example, it can be checked by considering (164) that the Higgs doublet identified as in (82) have a T-quark substructure of $\overline{S}Q$. As S is an $SU(2)_W$ singlet and Q an $SU(2)_W$ doublet, the quantity $\overline{S}Q$ is in the fundamental representation of $SU(2)_W$. Further, as the hypercharges were $Y_Q = 0$ and $Y_S = -1$, it means that $Y_{\overline{S}Q} = 1$, since $Y_{\overline{S}} = 1$ (hypercharge sign flips when considering antiparticles [13]). Thus, it is seen that the properties of $\overline{S}Q$ under $SU(2)_W \times U(1)_Y$ coincide with that of the SM Higgs, justifying the identification in (82).