

Migrating Dust Particles

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Abstract

Where the dust in a protoplanetary disk is as the disk evolves over time is essential to know for further studies of the planet formation process. A new star is created alongside a stellar nebula. The nebula contains rock, ice and gas which are not accreted to the star. Our nebula is constructed using the Minimum Mass Solar Nebula model and the column density is studied as the protoplanetary disk evolves over time. The Minimum Mass Solar Nebula model takes the mass of dust from the planets we see today in our solar system and spreads it out between the planets. To estimate the gas column density, simply multiply by about a hundred.

The dust in a protoplanetary disk will feel a head wind from the more slowly orbiting gas and start to drift towards the star. The ice particles sublime inside 2.7 AU as the temperature rises. A computer model is used here to simulate the changes in the column density over time. Micron-sized particles are simulated as they grow and drift towards the star. The ice-line at 2.7 AU results in a slight buildup of dust inside this radius. When a pebble grows it is due to how much dust it collides with and how much of it that sticks to the pebble. If the disk is less turbulent the pebbles grow slower due to fewer collisions and the column density changes more slowly. The same happens when the sticking factor is lowered. But with low turbulence ($\alpha = 10^{-4}$) the disk becomes thinner and more dense, which can increase the growth somewhat. When the pebbles grow slowly there is a tendency for dust buildup when the drift speed decreases with orbital distance (for $St < 1$, St is a measurement of how particles behave in fluids). When the pebble density is closely varied around 2 g cm^{-3} no big difference in the column density is shown. One can only see that the disk shrinks slightly faster with heavier pebbles.

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Populär sammanfattning

En nyfödd stjärna är omringad av stoft och gas. Efter hundratusentals år har stoftet drivit mot stjärnan där vi idag finner våra planeter i solsystemet. Den här processen kan simuleras i en enkel datormodell.

När en stjärna skapas omringas den av gas och stoft. Under stjärnbildningsprocessen tunnas gasen ut till en skiva för att bevara rörelsemängden, ungefär som när man kastar runt en pizzadeg. Denna skiva kallas för en protoplanetär skiva.

Turbulens i skivan gör det möjligt för stoft eller gruskorn att kollidera med varandra och växa sig större. Men när gruskornen växer sig större börjar de känna av en motvind från gasen som roterar runt solen långsammare än gruskornen på grund av gastryck. Gruskornen tappar fart och börjar falla mot stjärnan när de närmar sig centimeterstorlekar. Dessa gruskorn är byggstenarna för att planeter ska kunna skapas. Det är därför av stor vikt att veta hur grus och stoft utvecklas med tiden för att förstå hur planeter bildas.

Datormodeller har gjorts för att simulera dessa processer. Men för att kunna göra detta måste först information om den protoplanetära skivan samlas in. Saker som densiteten av partiklar, temperaturen, tjockleken på skivan och den så kallade ljushastigheten är alla viktiga egenskaper i skivan.

År 1981 kom den kände japanske astrofysikern Chushiro Hayashi med en modell med samband mellan dessa egenskaper, Minimum Mass Solar Nebula modellen. Denna modell används som grund för att förstå den protoplanetära diskens egenskaper. Modellen och några fysikresonemang är väsentligen allt som behövs för att komma igång med datormodellen. Resonemangen innefattar antaganden om turbulens i skivan, vilket är källan till kollisioner mellan gruspartiklarna och stoftet.

Kanske den mest intressanta egenskapen av den ovan nämnda modellen är kolumndensiteten. Den säger hur mycket materia det finns i skivan per areaenhet om man ser på skivan ovanför dess plan. Idag syns ingen fri gas i vårt solsystem. Tanken är att all gas antingen har absorberats av solen eller gasjättarna, eller dunstat bort på grund av solvindar. Stoftet som består av is och sten har däremot ganska effektivt skapat de planeter vi ser idag. Vi uppskattar den ursprungliga mängden gas till att ha varit hundra gånger så stor som mängden stoft.

Datorsimuleringen utförs på följande vis: en stoftpartikel börjar med en viss storlek på ett visst avstånd till stjärnan vid tiden noll. Sedan låter man partiklen ta ett litet steg i tiden, vilket betyder att den kolliderar med andra partiklar, blir större och börjar driva mot stjärnan. Nu har partiklen en ny position och ett nytt steg tas. Partiklens radie och dess position räknas ut igen. Detta fortsätter man att göra över så lång tid man är intresserad av att titta på.

Vi har nu information om hur den protoplanetära skivan och farför allt kolumndensiteten, utvecklas med tiden. Gruskornen kommer sedan att koncentreras i gasflödet och bilda kilometerstora planetesimaler. Dessa planetesimaler växer till storlekar där deras dragningskraft drar till sig mer material i skenande takt på sin väg att bli planeter.

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Chapter 1

Introduction

All planets in our solar system orbit the Sun in the same direction the Sun rotates about its own axis. It is therefore very likely that all matter in the Solar system comes from a primordial dust cloud, which later contracted. The contraction leads to very high densities in the center, where fusion ignited the Sun. The cloud also started to rotate due to the contraction, and the rotation flattened the cloud to a disk, a so called protoplanetary disk. This theory explains why all planets rotate around the Sun the same direction the Sun rotates around its own axis.

From the beginning there were no planets, just gas and dust around the Sun. The dust collided to create bigger particles, called pebbles. More and more dust collided with the pebbles; pebbles collided with pebbles, to create even bigger bodies. As the pebbles grew in size they drifted towards the Sun. This material was later accreted to planetesimals which are held together by self-gravity. As the planetesimals extend their gravitational range they have a run-away growth towards becoming planets. In this step the planetesimals grow more the larger they get. This is when even the gas cannot escape the path of the protoplanets, and it gets accreted as well. When protoplanets are dominating in the disk, they will perturb each others orbits, collide and eventually form a few planets.

The step from pebbles to km-sized objects is yet unknown in the planet formation process whilst the step from km-sized objects to protoplanets is well understood today. This is why the study of this report is focused on the drift and growth of pebbles.

A problem with pebble growth is that in some models the pebbles drift too rapidly to give planets a chance to form. Some unknown or uncertain parameters of the protoplanetary disk can be changed in a computer model to see how it affects the disk over time as the pebbles drift. One way to describe the disk profile is with the column density. It tells us how much material there is per unit area in the disk. The pebble drift changes the dust column density.

Dust and gas in other solar systems are very hard to study due to very faint protoplanetary disks. An easier way to study the development of the column density in protoplanetary disks is to do a computer simulation of the drift and growth of the particles. As the dust drifts towards the star, over time the dust distribution and the column density changes. This eventually results in different conditions for planet formation at different distances

to the star, at different times. With a computer simulation we can see how long the disk survives to create and feed planets.

In chapter 2 we will go through all properties of a protoplanetary disk. We start off with the Minimum Mass Solar Nebula in chapter 2.1, beginning with describing the column density (Σ) for both the gas and for the dust particles consisting of rock and ice. Then we derive the temperature (T) of the protoplanetary disk, which gives us the sound speed (c_s). The scale height (H) of the disk depends on the Keplerian frequency (Ω_K) and the sound speed. We derive the density (ρ) and show the definition of the scale height. We find the mid-plane density, which is of special interest for later simulations of growth in the mid-plane. These are the properties of a protoplanetary disk.

In chapter 2.2 we take a closer look at the physics behind radial drift. It is explained why the gas orbits the star with a sub-Keplerian speed. We define Δv as the difference between the Keplerian speed and the speed of the gas. We introduce the friction time τ_f and construct the stokes number ($St = \Omega_K \tau_f$) and write the radial drift speed v_r as a function of St and Δv . It is also shown that the maximal drift speed depends both on the radius of the pebble and its orbital distance.

Particle growth is examined in chapter 2.3 where we go through the physics behind it. First we derive a growth formula for a pebble. The relative dust speed δv and turbulence parameter α are introduced and it is shown how the dust scale height depends on the turbulence in the disk. Not all the dust that collides with a pebble is likely to stick, therefore a sticking factor ϵ is added to the pebble growth formula.

In chapter 3 we go through the method and how the computer model was constructed for the simulations. How a column density can be constructed using the growth and drift of pebbles is described in chapter 3.1. Every pebble represents a ring of dust, every ring has the same mass and the column density is calculated by counting the number of rings within a certain interval. In chapter 3.2 it is described how the simulation can be done using Euler-steps. Finally we want to freeze the time at certain points to examine the column density profile. A method to interpolate the positions of the pebbles at a certain time is presented in chapter 3.3.

Simulations with different values of some parameters are done in chapter 4 to see how these affect the evolution of the disk. A disk with or without an ice-evaporation line is simulated. Different turbulence parameters, sticking factors, initial sizes of the pebbles and pebble densities are also tested.

In chapter 5 we discuss the results and the problems with the Minimum Mass Solar Nebula.

Chapter 2

Theory

When a star is created, all the neighboring material does not go into the star. The leftovers create a protoplanetary disk around the star, consisting of gas and dust made of rock and ice. The amount of dust and gas in the disk is estimated by the Minimum Mass Solar Nebula, first constructed by Hayashi (1981). It is constructed by considering the orbits of the planets around the Sun today.

We assume that the dust in the solar nebula was efficiently incorporated into the planets, whilst most of the gas was accreted by the Sun or photo-evaporated. A protoplanetary disk has a mass of about 1 to 10 % of the stellar mass. The mass of the planets in our solar system is known and therefore the dust mass is known. It turns out that we get the primordial amount of gas in the solar nebula by multiplying the mass of the dust by about 100 to match current observations. When the mass of gas and dust is known, it is spread out evenly between the positions of the planets.

This means that we now know the density of gas and dust in our protoplanetary disk. From this, other properties of the protoplanetary disk can be derived, such as scale height, sound speed and temperature.

2.1 Minimum Mass Solar Nebula

From the reasoning above we get the column density of all mass in the protoplanetary disk if we know the disk mass and work backwards from (eq. 2.5) (Hayashi, 1981)

$$\Sigma(r) = 1700 \text{ g cm}^{-2} \left(\frac{r}{\text{AU}} \right)^{-3/2}. \quad (2.1)$$

The column density describes how much matter there is per unit area, at a distance r from the star, if the disk is looked at from above the plane of the protoplanetary disk.

A protoplanetary disk consists of rock, ice and gas. Close to the star the ice particles sublime due to an increase in temperature. This happens at about 2.7 AU where the gas temperature is about 170 K. The effects can be seen in the column density of the dust,

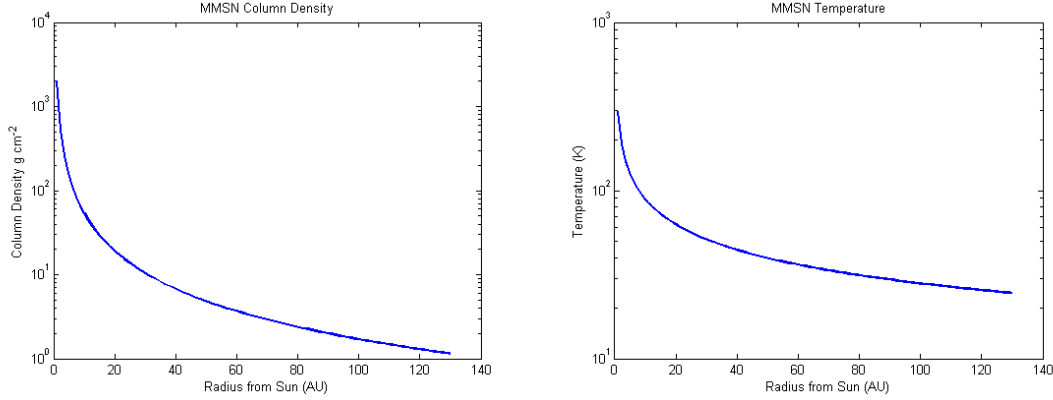


Figure 2.1: The plot to the left shows how the gas column density, in g cm^{-2} , decreases with the radius, in astronomical units (AU), from the star. The same can be seen with the temperature, measured in Kelvin, which is at its highest closest to the star.

$$\Sigma_d(r) = \Sigma_{\text{rock}}(r) = 7 \text{ g cm}^{-2} \left(\frac{r}{\text{AU}} \right)^{-3/2} \text{ for } r < 2.7 \text{ AU}, \quad (2.2)$$

$$\Sigma_d(r) = \Sigma_{\text{rock+ice}}(r) = 30 \text{ g cm}^{-2} \left(\frac{r}{\text{AU}} \right)^{-3/2} \text{ for } r > 2.7 \text{ AU}. \quad (2.3)$$

These two equations will later be used as the column dust density Σ_d , depending on the distance r to the star. The disk consists almost exclusively of gas, hence the gas column density is

$$\Sigma_{\text{gas}} \approx \Sigma(r). \quad (2.4)$$

as in figure 2.1. Hayashi (1981) used a nebula mass of $0.013M_{\odot}$ between 0.35 and 36 AU, which you get from equation (2.5). The column density times the area $2\pi r dr$ gives the mass at distance r . The total mass of the disk is therefore given by

$$M_{\text{disk}} = \int_{r_0}^{r_1} 2\pi r \Sigma(r) dr. \quad (2.5)$$

In this study the nebula is extended to 100 AU, and the mass is changed accordingly, with the same mass distribution in the inner region. After integrating the column density from 0.35 AU to 100 AU we get a disk-mass of $0.222M_{\odot}$ which is later used to calculate the developing dust column density.

The temperature in the disk is harder to infer. For the simplest case we have a optically thin disk, so that every dust-particle is hit by the radiation directly from the star. The amount of energy per unit time per area that the Sun radiates is the flux

$$F_{\odot} = \frac{L_{\odot}}{4\pi r^2} \quad (2.6)$$

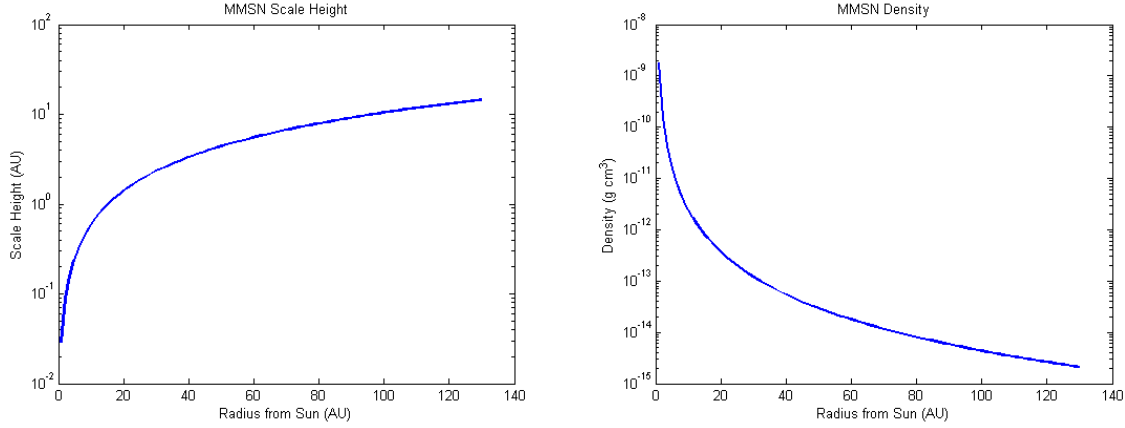


Figure 2.2: To the left we see how the scale-height of the disk increases with radius. Both the height and radius are given in AU. The density of dust and gas in the disk, given in g cm^{-3} , decreases when the radius to the star increases.

where r is the distance to the Sun and L_{\odot} is the solar luminosity.

When the particles in the disk are in thermal equilibrium they radiate the same amount of energy as they absorb. The dust radiates like a blackbody in all directions. The total power radiated from a particle with radius R is therefore

$$P_{\text{out}} = 4\pi R^2 \sigma_{\text{SB}} T_{\text{eff}}^4 \quad (2.7)$$

where σ_{SB} is Stefan Boltzmann's constant and T_{eff} is the effective temperature of the particle.

The same particle only gets hit by the rays from the Sun on one side, with the total effective area of a circle with radius R . The total power that hits the particle is

$$P_{\text{in}} = \pi R^2 F_{\odot}. \quad (2.8)$$

There is equilibrium when the two powers are equal, $P_{\text{in}} = P_{\text{out}}$. After solving for T_{eff} we get

$$T_{\text{eff}} = \left(\frac{F_{\odot}}{4\sigma_{\text{SB}}} \right)^{1/4}. \quad (2.9)$$

Inserting equation 2.6 yields

$$T_{\text{eff}}(r) = \left(\frac{L_{\odot}}{16\pi r^2 \sigma_{\text{SB}}} \right)^{1/4}. \quad (2.10)$$

And finally, after inserting the values for the luminosity of the Sun and Boltzmann's constant, we get

$$T(r) = 280 \text{ K} \left(\frac{r}{\text{AU}} \right)^{-1/2}. \quad (2.11)$$

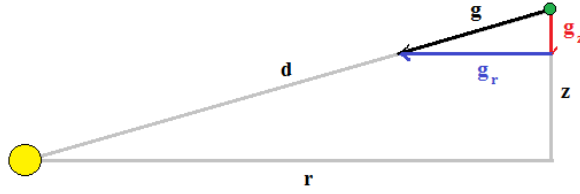


Figure 2.3: This is a sketch of a particle above the plane of the disk, feeling the gravitational force of the star.

The sound speed depends on the temperature T the following way (Hayashi, 1981)

$$c_s = 9.9 \times 10^4 \text{ cm s}^{-1} \left(\frac{2.34}{\mu} \frac{T}{280 \text{ K}} \right)^{-1/2}. \quad (2.12)$$

and $\mu = 2.34$ is the mean molecular weight of the matter in the disk. The temperature $T(r)$ can be seen in figure 2.1.

The scale height, which is a measure of the thickness of the disk at a certain distance, is given by (Hayashi, 1981)

$$H(r) = \frac{c_s}{\Omega_K} \quad (2.13)$$

where $\Omega_K = \sqrt{GM/r^3}$ is the Keplerian frequency at the given radius, G is the gravitational constant and M is the mass of the star (Hayashi used the half-thickness which is a factor $\sqrt{2}$ bigger than the scale height). See left figure 2.2 where the scale height is plotted as a function of the semi-major axis r .

The density of dust and gas in the disk depends both on the radius to the sun and the height above the mid-plane. It is proportional to the column density divided by the scale height. The density has a Gaussian distribution about the mid-plane.

$$\rho(r) = \frac{\Sigma(r)}{\sqrt{2\pi}H(r)} \exp \left[-\frac{z^2}{2H(r)^2} \right]. \quad (2.14)$$

To show this, we begin with a hydrostatic equilibrium condition. As seen in figure 2.3 $g_z/g = z/d$. This gives the acceleration due to gravity in the vertical direction

$$g_z = g \frac{z}{d} = -\frac{GM}{d^2} \frac{z}{d} \approx -\frac{GM}{r^3} z = -\Omega_K^2 z. \quad (2.15)$$

The acceleration due to the gas pressure is given by

$$a_{\text{gas}} = -\frac{1}{\rho} \frac{dP}{dz}. \quad (2.16)$$

For constant temperature T we can write $P = nkT = (\rho/\mu)kT = c_s^2 \rho$ so that

$$a_{\text{gas}} = -c_s^2 \frac{1}{\rho} \frac{d\rho}{dz} = -c_s^2 \frac{d \ln \rho}{dz} \quad (2.17)$$

A gas-particle in equilibrium does not change its velocity v_z in the vertical direction, hence $a_z = g_z + a_{\text{gas}} = 0$

$$\frac{dv_z}{dt} = -\Omega_K^2 z - c_s^2 \frac{d \ln \rho}{dz} = 0 \quad (2.18)$$

Recalling the scale height equation $H(r) = c_s/\Omega_K$ and insert the expression for H yields

$$\frac{d \ln \rho}{dz} = -\frac{\Omega_K^2}{c_s^2} z = -\frac{z}{H^2}. \quad (2.19)$$

This is the derivation and definition of the scale height.

This ordinary differential equation (2.19) can be solved for $\ln \rho$

$$\ln \rho = \ln \rho_0 - \frac{z^2}{2H^2}. \quad (2.20)$$

Which gives us the solution for ρ

$$\rho(z) = \rho_0 \exp \left[-\frac{z^2}{2H^2} \right]. \quad (2.21)$$

We still do not know the density in the mid-plane. But we know the connection between the column density and the density in form of an integral. If we integrate the density over the height of the disk we should get the column density,

$$\Sigma = \int_{-\infty}^{\infty} \rho(z) dz = \rho_0 \int_{-\infty}^{\infty} \exp \left[-\frac{z^2}{2H^2} \right] (z) dz. \quad (2.22)$$

With a substitution of variables this can be identified as a standard integral,

$$\Sigma = \sqrt{2}H\rho_0 \int_{-\infty}^{\infty} \exp(\xi^2) d\xi = \sqrt{2\pi}H\rho_0. \quad (2.23)$$

The mid-plane density therefore is

$$\rho_0 = \frac{\Sigma}{\sqrt{2\pi}H}. \quad (2.24)$$

This finally gives us equation (2.14). The mid-plane density increases with the column density. When the scale height increases, on the other hand, the mid-plane density decreases.

As a last thing we also have the relation of the aspect ratio of the disk,

$$H/r = \frac{c_s}{v_K} = 0.033 \left(\frac{r}{\text{AU}} \right)^{1/4} \quad (2.25)$$

to calculate the scale height

2.2 Radial drift

A protoplanetary disk consists mainly of three kinds of particles: gas, ice and rock. The gas in the disk is radially pressure-supported, as a consequence of the high temperature and density close to the star. This makes the gas orbit the star at a sub-Keplerian speed v_{gas} . The difference between the Keplerian speed $v_{\text{K}} = \sqrt{GM/r}$ and v_{gas} (G being the gravitational constant and M being the mass of the star), is $v_{\text{K}} - v_{\text{gas}} = \Delta v$. In the midplane ($z = 0$) the parameter Δv is also (Nakagawa et al., 1986)

$$\Delta v = -\frac{1}{2} \left(\frac{H}{r} \right)^2 \frac{\partial \ln P}{\partial \ln r} v_{\text{K}}. \quad (2.26)$$

The gas velocity is derived from the famous formula $F = ma$, where m is the mass of the gas-particle. The centripetal force must be equal and opposite to the gravitational force and the force from the gas pressure to have a stable orbit, $0 = F_c + F_g + F_P$. This gives

$$0 = \frac{mv_{\text{gas}}^2}{r} - G \frac{mM}{r^2} - m \frac{1}{\rho} \frac{\partial P}{\partial r}. \quad (2.27)$$

Solving for v_{gas} yields

$$v_{\text{gas}} = \sqrt{v_{\text{K}}^2 + \frac{r}{\rho} \frac{\partial P}{\partial r}} = \sqrt{1 + \frac{H^2}{r^2} \frac{r}{P} \frac{\partial P}{\partial r}} v_{\text{K}} \quad (2.28)$$

where we have used that the pressure can be written as $P = c_s^2 \rho$ and the scale height $H = c_s r / v_{\text{K}}$. We get Δv to be

$$\Delta v = v_{\text{K}} - v_{\text{gas}} = \left(1 - \sqrt{1 + \frac{H^2}{r^2} \frac{\partial \ln P}{\partial \ln r}} \right) v_{\text{K}}. \quad (2.29)$$

A Taylor expansion of the expression in the parenthesis to the first order gives equation (2.26).

To show that $\partial \ln P / \partial \ln r = -3.25$ we use the ideal gas law

$$P = nkT = \frac{\rho}{\mu} kT = \rho(r) T(r) \frac{k}{\mu} \quad (2.30)$$

and $k/\mu = \text{constant}$. Note that from the mid-plane density in equation (2.14) we have

$$\rho(r) = \frac{\Sigma(r)}{\sqrt{2\pi} H(r)} \propto \frac{r^{-3/2}}{r^{5/4}} \propto r^{-11/4} \quad (2.31)$$

and equation (2.11) for the temperature

$$T(r) \propto r^{-1/2}. \quad (2.32)$$

Inserting (2.31) and (2.32) into (2.30) we get

$$P \propto r^{-13/4} = A r^{-13/4} \quad (2.33)$$

where A is just a constant. Taking the derivative of the pressure with respect to r yields

$$\frac{\partial P}{\partial r} = (-13/4)A r^{-13/4-1}. \quad (2.34)$$

This eventually leads to

$$\frac{\partial \ln P}{\partial \ln r} = \frac{r}{P} \frac{\partial P}{\partial r} = \frac{r}{A r^{-13/4}} (-13/4)A r^{-13/4-1} = (-13/4) = -3.25 \quad (2.35)$$

This means that equation (2.26) can be rewritten as

$$\Delta v = -\frac{1}{2} \left(\frac{H}{r} \right)^2 (-3.25)v_K. \quad (2.36)$$

This speed difference between the Keplerian speed and the gas rotational speed is actually constant at about 50 m/s, since $(H/r)^2 \propto r^{1/2}$ and $v_K \propto r^{-1/2}$. This difference in speed results in a head-wind for the ice and rock-particles. They lose speed and start to fall towards the star with a speed (Weidenshilling, 1977)

$$v_r = -\frac{2\Delta v}{St + St^{-1}} \quad (2.37)$$

where $St = \Omega_K \tau_f$ and τ_f is the friction time. The friction time is

$$\tau_f = \frac{R\rho_\bullet}{c_s\rho_g}, \quad (2.38)$$

where ρ_\bullet is the pebble mass-density, taken to be $\rho_\bullet = 2 \text{ g cm}^{-3}$, and $\rho_g \approx \rho$ is the gas density. The friction time contains information about how the particle interacts with the gas flow.

The Stokes number describes how particles behave in fluids. A small number means that the particles couple to the flow of the fluid while a large number corresponds to particles with larger momenta, continuing on their initial path.

In equation (2.37) and figure 2.4 we can note that for very large St , St^{-1} becomes negligible and the drift speed v_r approaches zero. The same is for very small St where St^{-1} becomes large and v_r again approaches zero. The highest radial drift speed is achieved at $St=1$. As the radius of the dust increases, the Stokes number increases. The Stokes number decreases as the semi-major axis r decreases.

To get maximum radial drift speed for the pebbles their Stokes number should be unity. This occurs for different pebble sizes at different distances to the star. This can be seen in figure 2.5. In this plot St is fixed at 1 and the pebble radius is drawn as a function of the semi major axis. The closer to the star a pebble is the bigger it must be to maintain its drift speed. This means that particles that do not grow can pile up simply due to their new position which lowers their Stokes number and their drift speed.

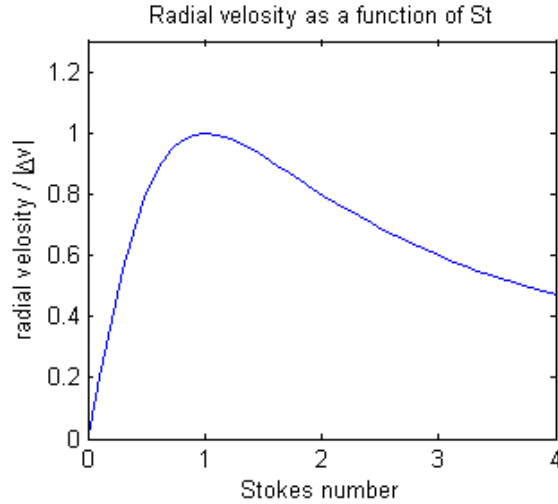


Figure 2.4: This shows that the maximum radial drift speed in equation (2.37) is achieved at $St=1$. Smaller particles (smaller St) go along with the gas and does not drift as much, while bigger particles (bigger St) does not get affected as much by the head wind from the gas.

2.3 Particle growth

To simulate the particle growth the following model is used. The growth depends on the area of the pebble, the density of dust-particles colliding with it, and a turbulence factor which measures how often it collides with other dust. The mass growth can be written as

$$\dot{m} = 4\pi R^2 \rho_d \delta v \quad (2.39)$$

Here R is the pebble radius, ρ_d is the dust density and δv is the difference in speed between the pebble and the dust. The mass of a single pebble depends on its volume and its density ρ_\bullet . If we have spherical particles the mass is

$$m = \frac{4\pi R^3}{3} \rho_\bullet \quad (2.40)$$

We can take the derivative with respect to time on equation (2.40) to get

$$\dot{m} = 4\pi \dot{R} R^2 \rho_\bullet \quad (2.41)$$

Combining equation (2.39) and (2.41) yields

$$\dot{R} = \frac{\delta v \rho_d}{\rho_\bullet} \quad (2.42)$$

The relative speed δv of particles colliding in a turbulent gas is set to

$$\delta v = \sqrt{St \cdot \alpha} \cdot c_s \quad (2.43)$$

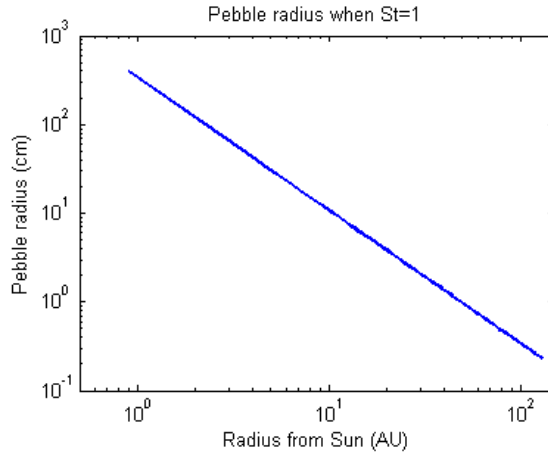


Figure 2.5: $St=1$ is kept fixed to see at which pebble radius we have maximum drift speed at a certain orbital distance. The pebble radius is drawn as a function on the semi major axis. For maximum drift speed the pebble radius increases as the semi major axis decreases. This is due to a higher density closer to the star, which the pebbles try to decouple from. For pebble radii either bigger or smaller the drift speed will be submaximal.

for $St < 1$ (Weidenschilling, 1984). The turbulence parameter α takes values between 10^{-2} and 10^{-4} (Johansen et al., 2014). For this report a value of $\alpha = 10^{-2}$ is primarily chosen. The turbulent rms speed of the gas is $v_t = \sqrt{\alpha} \cdot c_s$, so $\delta v = \sqrt{St} \cdot v_t$ is always smaller than v_t for $St < 1$. The equation is invalid for $St > 1$.

The dust in the disk will fall towards the mid-plane, while the gas will have a wider spread or height. This will lead to a Gaussian dust distribution with scale height

$$\frac{H_d}{H_g} = \sqrt{\frac{\alpha}{St + \alpha}}. \quad (2.44)$$

This is an equilibrium between the dust sedimentation and the turbulent diffusion, a battle between the dust wanting to fall towards the mid-plane and the turbulence, mixing up the dust again (Johansen et al., 2014).

For small particles we have a small Stokes number. Then the scale height of the dust approaches the scale height of the gas. But for large Stokes numbers, the dust scale height approaches zero; the particles end up in the mid plane of the disk. The particle density ρ_d gets very high, since

$$\rho_d(r) = \frac{\Sigma_d}{\sqrt{2\pi}H_d} \quad (2.45)$$

where $\Sigma_d(r)$ is the dust column density. ρ_d goes to infinity as H_d goes to zero.

All dust that collides with the pebble is not likely to stick to it. Sometimes all the dust sticks, and sometimes a collision only leads to fragmentation. Most likely seems the scenario when a dust particle collides with the pebble and transfers some of its mass upon

impact. To account for this, a sticking factor ϵ is added to the \dot{R} equation (2.42),

$$\dot{R} = \epsilon \frac{\delta v \rho_d}{\rho_\bullet}. \quad (2.46)$$

The value of ϵ is typically used to be 10 % in this study, but other values are examined as well.

Chapter 3

Method

3.1 The computer model

The physics behind dust accretion was covered in chapter 2. We can use this to construct a computer model for the growth and position of a pebble in a protoplanetary disk. Pebbles starting at different orbital distances are simulated and their positions and sizes are followed as a function of time.

All pebbles that have the same initial size and are at the same orbital distance will grow similarly. Therefore a simulation of a single pebble can be seen as a simulation of a ring of pebbles, growing and drifting closer to the star. A simulation is done with several rings at different initial orbital distances, equidistantly separated. We let these rings represent all the dust in the nebula. The mass is evenly distributed between the rings, giving them the same mass. To calculate the column density you simply count how many rings N you have in a pre-determined interval, multiply by the mass m of a ring, and divide by the area of the considered ring-segment, figure (3.1),

$$\Sigma_i = \left[\frac{\text{mass}}{\text{area}} \right] = \frac{mN}{2\pi r_i \Delta r}. \tag{3.1}$$

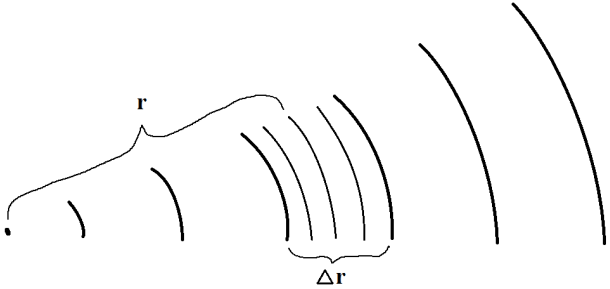


Figure 3.1: This is a sketch of a slice of the protoplanetary disk. The thin lines represents the rings while the thicker lines just separates the different segments where the column density is calculated, according to equation (3.1).

This is done for all i intervals of binned data, to get the column density at every distance from the star.

3.2 The simulation

The simulations were done with MATLAB. At every location (distance from the star) the column density, temperature, sound speed, scale height, density, Keplerian frequency, friction time and Stokes number are calculated. From this information the drift speed and accretion rate of the pebble is calculated. The new position r , the time t as it was at that position and its radius R at that time, is calculated. This was done using simple Euler-steps. The semi major axis r is changed as

$$r = r + v_r \cdot dt. \quad (3.2)$$

Here, v_r is the radial velocity and dt is length of the time-step.

The particle size R changes

$$R = R + \dot{R} \cdot dt \quad (3.3)$$

where and \dot{R} is the radial growth with respect to time. Finally the time changes as

$$t = t + dt. \quad (3.4)$$

The length of the time-step depends on how fast the pebble increases in size and how fast it is drifting. The smallest of the following ratios determine the step-size

$$dt = f \cdot \min\left(\frac{r}{v_r}, \frac{R}{\dot{R}}\right). \quad (3.5)$$

Here f is taken as a constant. As the pebble grows faster, or falls towards the star faster, the time-step decreases. This procedure is used to minimize both the numbers of steps taken (computation time) and minimize the error. It is accomplished by taking larger time steps when barely any growth or drift occurs and taking smaller time steps when the change in position or size happens more rapidly. Figure 3.2 shows how particles grow and approach the star.

In Appendix B a 4th order Runge-Kutta method is tested against the Euler method to see if there are any significant differences in the result.

3.3 Time interpolation

After the simulation of the particles is done we want to look at a specific point in time to see what the disk looks like then. The main problem in doing so is that we do not have information on the disk for all points in time. Each pebble has its position and size defined at time t_i which are not the same for each pebble. We therefore have to make an interpolation.

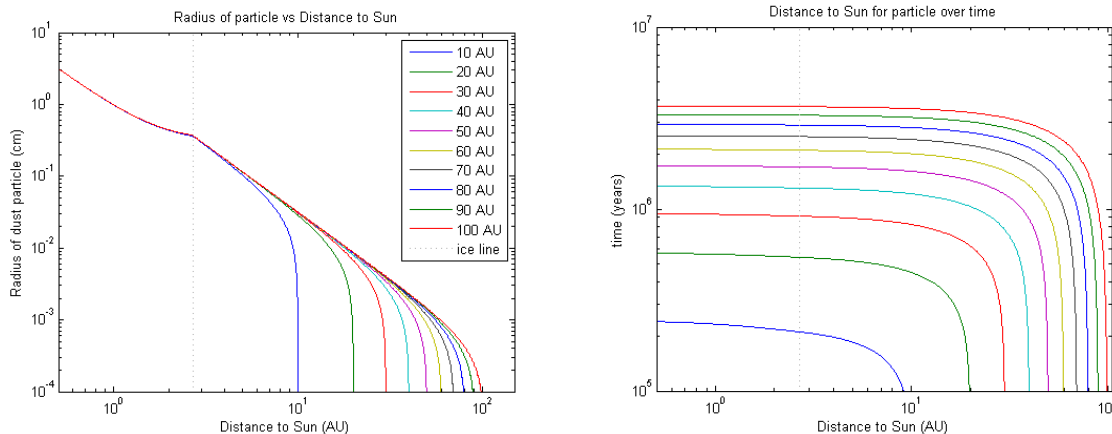


Figure 3.2: The box in the top-right corner tells the initial position of the pebbles in the simulation. The pebbles start at the bottom of the plot and grow upwards as time and their radius increase. The left plot shows the particle size as a function of orbital distance. One can clearly see the ice line at 2.7 AU, where the pebble does not grow as rapidly as before due to less dust. In the plot to the right time is plotted as a function of orbital distance. Micron-sized particles drift towards the star as they grow in size and start to feel the head wind from the gas.

An easy way to see how it can be done is the following. A line is drawn horizontally in the right plot in 3.2 at the time of interest. Then the intersections for each pebble with the horizontal line tell the position of the pebbles at that time. This is a linear interpolation and is the simplest kind of interpolation.

To do the linear interpolation I look at the two points in time, one before (t_1) and one after (t_2) the one of interest. I also look at their corresponding radius, r_1 and r_2 , to the star at these points in time. Then a line is drawn between the two points so that there now is a linear correlation between the time and radius to the star. Now the radius to the star is a linear function of time for this small interval. To extract the value at the wanted time simply insert the time in the function of time, to get the value of the function at that time.

In equation form we have the following equation system:

$$\begin{cases} r_1 = a_1 t_1 + a_0 \\ r_2 = a_1 t_2 + a_0 \end{cases} \quad (3.6)$$

valid between $t_1 < t < t_2$. This is solved for a_1 and a_0 and then insert in

$$r(t) = a_1 t + a_0 \quad (3.7)$$

to get the semi major axis r of a certain pebble at time t .

This was done at different times to see the development of the disk, and in particular the column density.

3.4 A new column density

When all this data is collected a new column density can be calculated according to equation (3.1). This is done by examining where the rings are at the times of interest and re-calculate the column density at the given times.

This new column density is not re-entered into the program, so the column density used for the particle growth and drift is a static one which does not develop over time. Gas accretion to the star and photo-evaporation of the gas are also effects not included in the model. To get a more realistic model this should also be incorporated. But the current model is a good and simple start to get a feel for how a protoplanetary disk could develop.

Chapter 4

Results

A computer model based on all the things covered in chapter 2 and 3 was created, and can be viewed in its whole in the Appendix A. Now we can see how particles grow and drift in the protoplanetary disk. We can also see how the drift of the pebbles (figure 3.2b) would affect the column density (figure 4.1). When the pebble radius increases and brings St closer to unity the drift speed increases due to the head wind from the slower orbiting gas. The effects of the drift can be seen in figure 3.2a where the pebble radius is drawn as a function of the semi major axis r . First the particles grow very slowly, especially in the outer regions where the dust density is low. It takes a long time for the pebbles to grow to sizes when they start to decouple from the gas. When the pebbles get closer to the center the dust density increases and the pebbles grow faster. As the pebbles reach the ice-line at 2.7 AU the dust density decreases and one can see a small plateau in the growth.

The parameters used in the simulations are,

Parameter	Value
ϵ	0.10
α	10^{-2}
R_{initial}	1 μm
Number of pebbles	5000
Δr	1 AU
ρ_{\bullet}	2 g cm ⁻³

unless stated otherwise. A simulation of the dust column density with these parameters can be seen in figure 4.1. We can always return to this figure later when we compare the effects of changing the value of a parameter. We are interested to see what differences in the evolution of the dust column density the changes lead to.

At the start we have 50 rings per interval. When the pebbles start to drift we will see some fluctuations in the column density due to this low amount of rings per interval. When a ring from a neighboring interval i gets inside interval $i - 1$ before the ring at the other edge has left interval $i - 1$ and before a new ring from interval $i + 1$ has reached interval i , we get a slightly higher column density in interval $i - 1$ and a slightly lower column density in interval i .

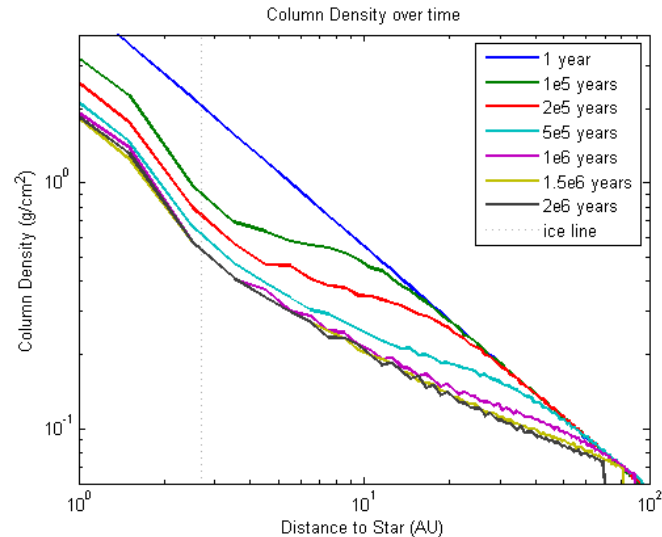


Figure 4.1: Here we can see how the dust column density develops over time. The blue line represents the density at the start, when no drift has occurred. The particles closest to the star grow faster due to a higher density of dust and start to fall towards the star, causing a lower column density where they used to be. Over time we can see how the particles further out also grow and start to fall towards the star, causing the column density to drop there as well.

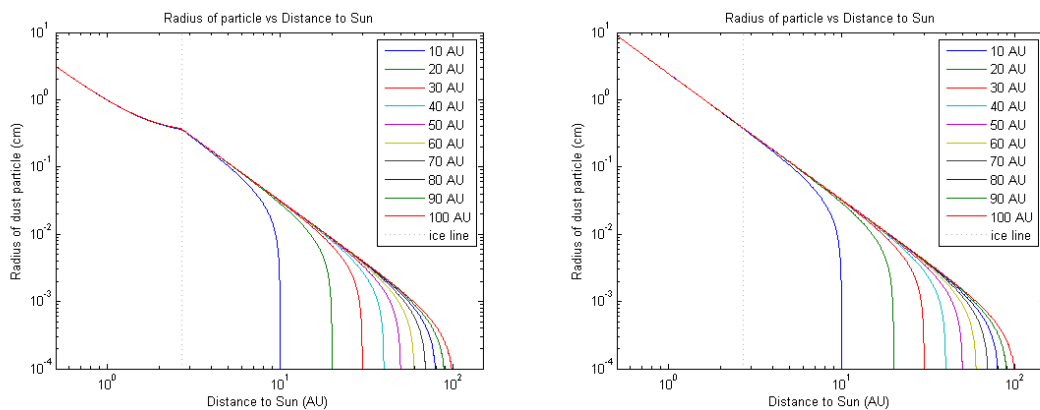


Figure 4.2: The box in the top-right corner tells the initial position of the pebbles in the simulation. The figure to the left shows a simulation with no ice in the dust inside 2.7 AU. This lowers the number of particles for the pebbles to collide with, and the growth slows down. In the figure to the right there is ice dust all the way through the protoplanetary disk and the pebble radius continue to increase as a power law inside 2.7 AU.

4.1 Ice sublimation

Comparing simulations with ice-particles in the entire solar nebula, to simulations with an ice-sublimation line clearly shows a difference. The pebbles grow slower when there are

fewer particles to collide with (figure 4.2), and they do not fall as fast towards the star as they would have with the ice present. In figure 4.3 we see how this affects the column density. As expected we no longer see the buildup of dust inside 2.7 AU when the ice line is removed.

The ice particles in the dust disappear inside 2.7 AU for the pebbles to collide with. But the ice already stuck to the pebbles still stay on the pebbles after they have breached the ice-line. Subtracting the mass of the ice from the pebbles would be an improvement for further studies, especially if the central part of a solar system is the area of interest. I did not have time to incorporate this into the program.

Since the dust column density consist of about $3/4$ ice and $1/4$ rock the pebbles would lose most of their mass. This would lower the column density a lot if all ice would be removed from the pebbles and the ring mass. But then the Stokes number would fall as well and we might see an even bigger buildup of dust, compensating the lower pebble mass.

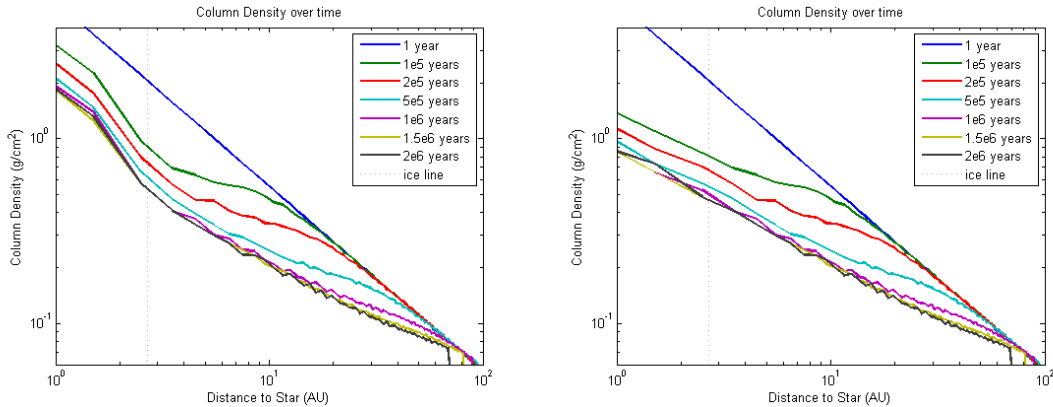


Figure 4.3: A comparison between simulations with or without ice-sublimation within 2.7 AU is shown. The plot to the left shows the original simulation where there is no ice inside 2.7 AU to collide with the dust. The plot to the right shows how the column density develops when there is ice all the way through the disk. The pebbles grow slower where there are no ice particles in the dust. It appears as if the slower growth, in the simulation with an ice-line, accumulates particles. This increases the dust column density, compared to the simulation without an ice-line.

4.2 Turbulence parameter

The turbulence parameter α determines how often the pebbles will collide with the dust. It does this in two ways. A higher alpha increases the collision speed δv , but it also increases the dust scale height, resulting in a lower dust density in the mid-plane. The original simulation (figure 4.1) had an $\alpha = 10^{-2}$. In figure 4.4 we can see two simulations, first with $\alpha = 10^{-3}$ and the second with $\alpha = 10^{-4}$.

One can see a tendency to a buildup of dust with low turbulence e.g. $\alpha = 10^{-4}$ at some orbital distances and times e.g. about 5 AU at $2 \cdot 10^5$ years. With low turbulence the pebbles grow slower due to fewer collisions. The drift speed v_r depends on Δv and St which depends on the semi major axis r . This leads to a decrease in drift speed as the particles drift towards the star, if they do not simultaneously grow, as discussed in figure 2.5.

The smaller St slightly closer to the star in combination with slow growth at the start leads to the bumps in the column density. Around a bump the pebbles a bit closer to the star will not drift because their Stokes number is low, whilst just outside St is slightly higher and the particles drift a bit just to lower their St and start to build up. The bump in the column density stays until the particles reach higher speeds and start to spread out again. It can be compared with a group of cyclists in a peloton. When it goes fast the cyclists are more spread out compared to when it goes more slowly.

The growth speed seem to differ more between positions r of the pebbles when the turbulence is low, at least around a bump.

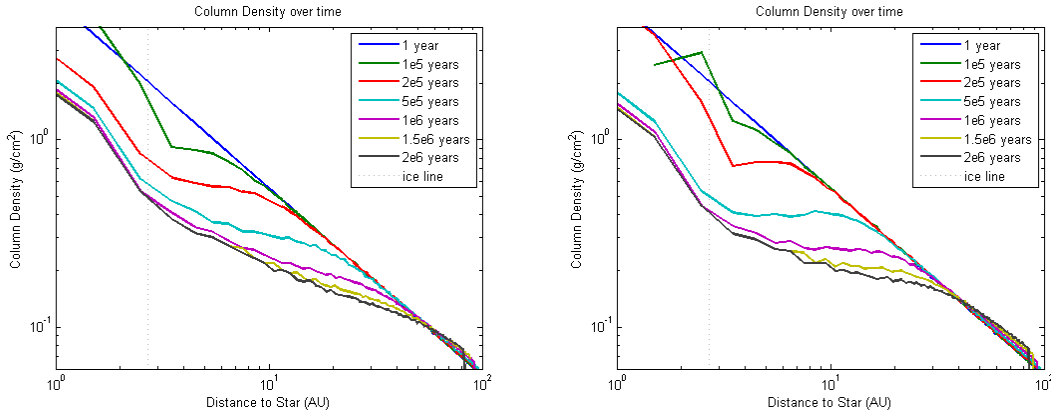


Figure 4.4: Simulations are done here with different turbulence parameter α . The left plot has $\alpha = 10^{-3}$, while the plot to the right has $\alpha = 10^{-4}$. The lower turbulence the more we can see a tendency for dust buildup as discussed in figure 2.5, the pebbles slow down as they approach the star when they do not grow.

4.3 Sticking factor

The sticking factor ϵ affects how much dust attaches to the pebble when a collision occurs. The original simulation (figure 4.1) had $\epsilon = 10\%$. In figure 4.5 simulations with sticking factors of 100% and 1% are compared. With a high sticking factor the pebbles grow very rapidly and fall fast towards the star, as they start to feel the head wind from the gas. With a sticking factor of 100% all the pebbles have fallen into the star after a million years. With a low sticking factor it takes longer time before a change can be seen in the column density profile. The particles in the outer regions beyond 30 AU barely move, while the

inner pebbles grow faster due to a higher density closer to the star. This gives a lower density in the inner regions as the pebbles fall towards the star, compared with the initial column density. But in the outer regions the pebbles fall slower the closer to the star they are, resulting in a buildup at the edge of the disk. The buildup lasts until the pebbles grow to bigger sizes to get closer to the line in figure 2.5 where the drift speed is at its maximum.

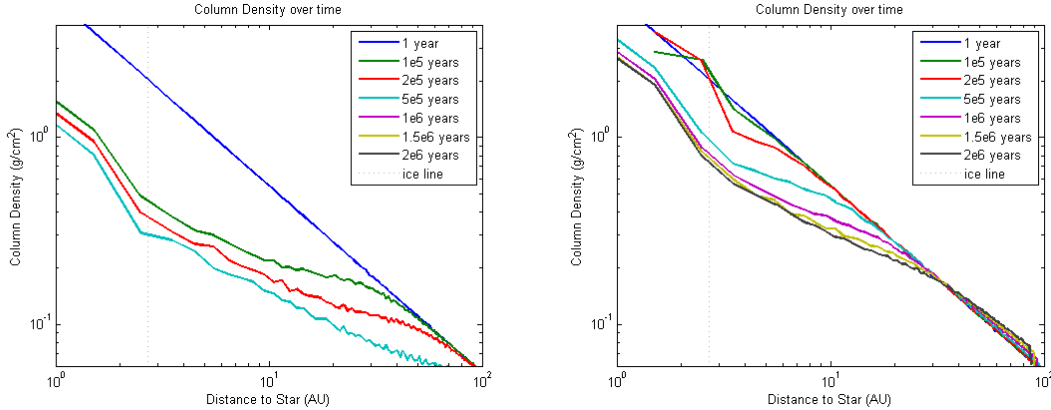


Figure 4.5: Two different sticking factors are tested in these two simulations. To the left all the dust that collides with the pebble sticks, $\epsilon = 1$. To the right only 1 % sticks, $\epsilon = 0.01$. This clearly affects how fast the pebbles grow and their drift speed. After one million years no dust is left in the protoplanetary disk where $\epsilon = 1$, whilst the column density after two million years with $\epsilon = 0.01$ resembles the column density at 0.1 million years with $\epsilon = 1$, except for the buildup of dust in the outer regions of the protoplanetary disk with $\epsilon = 0.01$.

4.4 Initial size

In the original simulation (figure 4.1) the pebbles had the size $1 \mu\text{m}$. Initial sizes of 0.1 and $5 \mu\text{m}$ are here compared in figure 4.6. With smaller particles the evolution of the disk looks practically the same as in figure 4.1. But with bigger pebbles the pebbles in the outer part of the protoplanetary disk accumulates and actually rises the column density temporarily as the edge protoplanetary disk moves closer to the star. It takes a lower pebble radius to achieve a high drift speed in the outer regions of the protoplanetary disk as discussed in figure 2.5. This leads to a buildup of dust when the pebbles do not grow fast enough to maintain their Stokes number and drift speed. This is seen especially in the outer regions where the dust density is low and the pebbles grow slower than in the inner regions.

4.5 Pebble density

The pebble density does not seem to affect the column density much when changed around 2 g cm^{-3} , which was the original density used (figure 4.1). It affects the friction time which

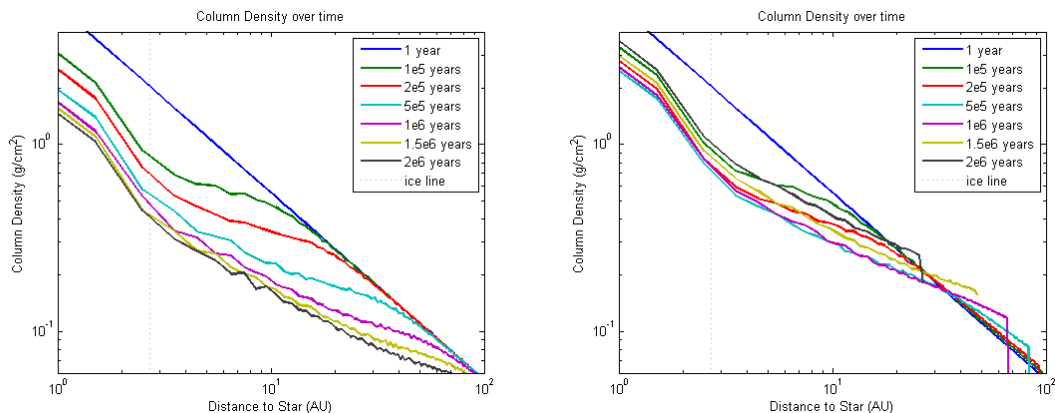


Figure 4.6: Here are two simulations with two different initial sizes for all the pebbles. To the left the pebbles start with a radius of $0.1 \mu\text{m}$. To the right the radius is $5 \mu\text{m}$. The plot with the smaller dust to the left very much resembles the original simulation (figure 4.1). In the simulation with the bigger pebbles a more rapid drift is seen in the outer regions of the protoplanetary disk. The disk shrinks much more rapidly. The pebbles drift rapidly until they do not grow enough to maintain their drift speed, then a buildup of dust starts to show in a locally higher column density, as seen at the edge of the disk.

affects the drift speed, but the column density profile looks almost the same in figure 4.7 where densities of 1 g cm^{-3} and 3 g cm^{-3} are tested. One difference is that the edge of the protoplanetary disk has moved closer to the star after two million years with the heavier pebbles compared with the lighter ones.

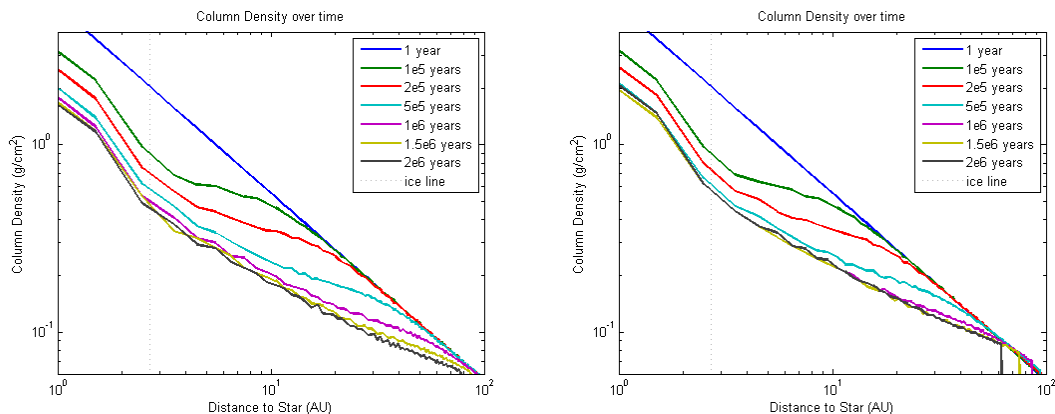


Figure 4.7: Here are two simulations with different pebble densities. To the left we have a density of 1 g cm^{-3} and the density is 3 g cm^{-3} to the right. The effects shows only in the long run where the edge of the protoplanetary disk moves closer to the star fast when the pebble density is higher.

4.6 Runge-Kutta

In Appendix B a 4th order Runge-Kutta method is tested against the Euler method to see if there are any significant differences in the result.

Chapter 5

Discussion

5.1 The true column density

If we were to follow the column density from the Minimum Mass Solar Nebula model (Hayashi, 1981), the dust column density should be $\propto r^{-3/2}$ and not $\propto r^{-1}$ as we see in equation (3.1) and figure 4.1 at the initial time (slope = -1). But we can still study how a protoplanetary disk develops over time. Figure 4.1 would still look about the same, the dust column density would just have a steeper slope, a slope of $-3/2$ instead of -1 .

The idea behind how planets form in a protoplanetary disk revolves around the fact that dust from the outer regions drift inwards. Some of the dust gets accreted by larger bodies that later can form planets. The Minimum Mass Solar Nebula model is constructed by spreading out the dust from the planets where we see them today and construct a density from that. But we do not think that the dust at a certain radius created the planets there, so this assumption of the initial density is likely to be wrong. This means that the column density in the Minimum Mass Solar Nebula can be wrong, and we should not despair just because our column density examined in this model does not have the same profile.

5.2 Choices of parameters

In doing a model of a protoplanetary disk some freedom is given in parameters which is not fully determined yet, or can vary in different solar nebulas.

In this simulation there was the following "free" parameters $\alpha = 10^{-2}$, $\rho_{\bullet} = 2 \text{ g cm}^{-3}$, sticking factor ϵ , initial sizes R of the pebbles and the dust mass in the protoplanetary disk $M_{\text{disk}} = 0.222M_{\odot}$. The mass of the protoplanetary disk just shows up as a scaling factor, and does not affect the profile of the disk. All the other parameters and a simulation with or without an ice-line at 2.7 were covered in the results.

The model was somewhat simplified by only simulating dust in the mid-plane of the disk. One could also try simulations with initial positions with the height $z \neq 0$ for future studies. This will require some additional computing power since e.g. the density now depends on the orbital distance r as well as the height z above the midplane, and the same

is for all simulated particles. But since the density is the highest in the midplane it seems more likely that planets would form in this region. This can be seen in our solar system today.

There are objects like Pluto and other trans-Neptunian objects, which do not orbit the sun in this plane. To know how these objects formed one might have to study particle growth above the mid-plane as well. Even though one thought is that Neptune perturbed Pluto's orbit a long time ago to give Pluto its highly elliptical orbit we see today.

5.3 Conclusions

The rise in the column density inside 2.7 AU in figure 4.1 is related to the lack of ice particles in the surrounding dust, which results in a slower growth of dust particles. After comparing simulations with or without an ice-line (figure 4.3) one can see that this is the case, since there is no buildup of dust inside 2.7 when there is no ice-evaporation line.

The thing that seems to have the greatest impact on how fast the protoplanetary disk evolves is how much dust sticks to a pebble when a collision occurs. The entire disk can be accreted into the star within a million years with maximum sticking efficiency, while it takes several million years with a sticking factor of 10 %. It seems very important to know the details in pebble growth to be able to predict the evolution of a protoplanetary disk. To get a complete picture of it one must consider fragmentation as well when two equal sizes pebbles collide.

Different pebble densities between that of water and rock do not seem to affect the drift of the pebbles and the column density much. Only a slightly faster decrease in the protoplanetary disk radius can be seen, as the outer pebbles drift a little bit faster when they are heavier.

When a pebble drifts a lot compared to its growth we see tendencies to dust buildup. This happen most clearly in the simulation with large initial pebble-sizes at the outer regions of the protoplanetary disk. It is also seen at some distances when the turbulence in the disk is low and the collision rate goes down.

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Appendix A

The Code

This is the simulation of the pebbles in the protoplanetary disk. In this case the nr of particle x is set to 5000 particles equidistantly separated up to 100 AU. When all particles are inside 0.1 AU the simulation stops. Three text documents are created to store the time t , orbital distance r and pebble radius R .

A.1 Particle drift

```
clear;

AU=1.5*10^13;
year=31536000;

x=5000; %nr of particles
for j=1:x
R(j)=0.0001; %cm
r(j)=100*j/x; %AU
tid(j)=0;
end

f=100000; %stepsize-related
ff=100000; %stepsize-related

epsilon=0.10; % Sticking factor

s=50000; %nr of steps

w=2; %step-counter
for i=1:1:s
```

```

%Calculating vr and Rdot in function MMSN
[vr,Rdot,Rdotr]=MMSN( r(i,:),R(i,:));

dt=min(-ff*r(i,:)./vr/(1.5*10^13)*year,f*R(i,:)./Rdot/(AU)*year); %time step in seco

k1=vr;
k_1=Rdot;

rSteg=dt.*k1;
RSteg=dt.*k_1;
RRSteg=0.23333*RSteg;

%Ice-sublimation line

for k=1:1:x
    if r(i,k)>2.7
        R(i+1,k)=R(i,k)+epsilon*RSteg(k);
    elseif r(i,k)<=2.7
        R(i+1,k)=R(i,k)+epsilon* RRSteg(k);
    end
end

end

%R(i+1,:)=R(i,:)+epsilon*RSteg;

r(i+1,:)=r(i,:)+rSteg;
R(i+1,:)=R(i,:)+epsilon*RSteg;
tid(i+1,:)=tid(i,:)+dt;
%Radius=R(i+1,:);

time=max(tid)/(year); %converts to years)

%if the outer ring fall onto the star, stop
if r(i,x)<.1
    break
end
w=w+1;
end

time; %time of simulation in years%
radius=min(r); %Radius from the sun at the end of our simulation
tid=tid./(year);%the time

```

%Creating text documents for time (years), radius (AU) and Radius (cm) for later plots a

```

format = '';
for n=1:x
format=[format,'%6.5f '];
end
format = [format,'\n'];

cat = [tid];
fid = fopen('tid01100.txt','w');
fprintf(fid,format ,cat(:,1:x).');
fclose(fid);

dog = [r];
fed = fopen('radius01100.txt','w');
fprintf(fed,format,dog(:,1:x).');
fclose(fed);

wolf = [R];
fad = fopen('Rad01100.txt','w');
fprintf(fad,format,wolf(:,1:x).');
fclose(fad);

Timefinder(w,x);

```

The new radial drift and pebble radius is calculated in the function MMSN.

```

function [ vr,Rdot,Rdotr ] = MMSN(r,R)

pr=1; %pebble density
AU=1.5*10^13;
rG=2.5834*10^(-4);%sqrt of gravitational constant
rMs=4.46*10^16;%sqrt of sun mass
alpha=1e-2;%turbulence parameter

T=280*r.^(-.5);%temperature
c=9.9*10^(4)*(T/280).^(-0.5);%gas sound speed
H=0.033*AU*r.^(1.25);%Scale height

```

```

S=1700*r.^(-1.5);%column density
Sp=S*0.0176;%dust column density

vk=rG*rMs/sqrt(AU).*r.^(-.5);%keplerian speed
dv=-0.5*(H./r).^2/(AU)^2*(-3.25).*vk;%headwind
p=S./H/sqrt(2*pi);%gas density

tf=R.*pr./c./p; %friction time
kF=vk./r; %keplerian frequency
St=tf.*kF/1.5e13;%Stokes number

Hp=H.*sqrt(alpha./(St+alpha));%dust scale height
pp=Sp/sqrt(2*pi)./Hp; %dust density in midplane

%"random motion"
delv=sqrt(St.*alpha).*c;

%pebble radial growth
Rdot=0.0176*delv.*pp/pr;
Rdotr=0.00412*delv.*pp/pr;

vr=-2*dv./(1./St+St)/(AU); %radial drift

end

```

A.2 Time interpolation

The following function takes in two values: the number of particles y and the number of steps w the simulation took. It creates a new text document which tells the position of the pebbles at the times of interest. This information is later used in the function Bin to calculate the dust column density.

```

function [ ] = Timefinder(w,y)

%finding radius and R at time 'time'

x=y; %number of particles/rings from Dust.m
s=w; %number of steps taken (w from Dust.m)

load tid01100.txt;
load radius01100.txt;

```

```
load Rad01100.txt;

X=[1, 0 ; 1, 0];

y=[0 0]';
z=[0 0]';

%finds Radius and radius at time 'time' via linear interpolation
%and puts it in a text file. radius(AU) on top and Radius(cm) below.

time=1;

for k=1:x
    M(k)=0;

end
for i=1:x

for j=1:s

    if tid01100(j,i)>=time;
        X(2,2)=tid01100(j,i);
        y(2,1)=radius01100(j,i);
        z(2,1)=Rad01100(j,i);
        X(1,2)=tid01100(j-1,i);
        y(1,1)=radius01100(j-1,i);
        z(1,1)=Rad01100(j-1,i);

        a=X\y;
        radius1=time*a(2)+a(1);
        M(1,i)=radius1;

        b=X\z;
        Rad1=time*b(2)+b(1);
        M(2,i)=Rad1;

        break
    end
end
end
end
.
```

```
.  
.   
.   
.   
.   
M=0;  
  
X=[1, 0 ; 1, 0];  
y=[0 0]';  
z=[0 0]';  
  
time=2000000;  
  
for k=1:x  
    M(k)=0;  
  
end  
for i=1:x  
for j=1:s  
    if tid01100(j,i)>=time;  
        X(2,2)=tid01100(j,i);  
        y(2,1)=radius01100(j,i);  
        z(2,1)=Rad01100(j,i);  
        X(1,2)=tid01100(j-1,i);  
        y(1,1)=radius01100(j-1,i);  
        z(1,1)=Rad01100(j-1,i);  
  
        a=X\y;  
        radius1=time*a(2)+a(1);  
        M(1,i)=radius1;  
  
        b=X\z;  
        Rad1=time*b(2)+b(1);  
        M(2,i)=Rad1;  
  
        break  
    end  
end  
end  
M; % 2 times x matrix with the interpolated data  
  
%Creating text document (radius;Radius).
```

```
cat = [M];
fid = fopen('t2e6alph4.txt','w');
fprintf(fid,format ',cat(:,1:x).');
fclose(fid);

%Plots the Column Density
Bin(x); %Plots the column density in function Bin
```

A.3 Binning the data and Column Density

Here all text files created in the previous simulation are imported. The resolution dr determines how big the bin will be when we bin the data to construct our new column density. See figure (3.1). Here the mass of the disk is determined as well. The function takes in the number of particles x and plots the dust column density at different times determined in function Timefinder.

```
function [ ] = Bin(x)

%bin stuff
s=x;
dr=1; %must be on the form 1/n, n integer)
k=100/dr;

mS=1.9891e33; %in grams
AU=1.5*10^13; % in cm

m=0.0222*mS; %mass of the disk (between 10% and 0.01%)

fdust=30/1700; %ratio dust to gas

mRing=fdust*m/s; % in grams

%the radius for each interval
for j=1:k
    rbin(j)=0.5*dr+(j-1)*dr;
end

load t1alph4.txt;
load t1e5alph4.txt;
load t2e5alph4.txt
load t5e5alph4.txt;
```



```

load t1e6alph4.txt;
load t15e5alph4.txt;
load t2e6alph4.txt;

bin1(k+1)=0;
bin2(k+1)=0;
bin3(k+1)=0;
bin4(k+1)=0;
bin5(k+1)=0;
bin6(k+1)=0;
bin7(k+1)=0;

for j=1:s

    if t1alph4(1,j)<0.001
        bin1(k+1)=bin1(k+1)+1;
    else
nr=floor(1+k*(t1alph4(1,j)/100));
bin1(nr)=bin1(nr)+1;
    end
end
.
.
.
.
.
for j=1:s

    if t2e6alph4(1,j)<0.001
        bin7(k+1)=bin7(k+1)+1;
    else
nr=floor(1+k*(t2e6alph4(1,j)/100));
bin7(nr)=bin7(nr)+1;
    end
end

bin1=bin1(1:k);
bin2=bin2(1:k);
bin3=bin3(1:k);
bin4=bin4(1:k);
bin5=bin5(1:k);
bin6=bin6(1:k);
bin7=bin7(1:k);

```

A.3. BINNING THE DATA AND COLUMN DENSITY APPENDIX A. THE CODE

```
for m=1:k
    hej(m)=(m-0.5)*dr;
end
xdat=hej;

%column density
% 2pi*r*dr=A=sigma

mbin1=mRing*bin1;
cdbin1=mbin1./(2*pi*dr.*rbin*AU*AU);
mbin2=mRing*bin2;
cdbin2=mbin2./(2*pi*dr.*rbin*AU*AU);
mbin3=mRing*bin3;
cdbin3=mbin3./(2*pi*dr.*rbin*AU*AU);
mbin4=mRing*bin4;
cdbin4=mbin4./(2*pi*dr.*rbin*AU*AU);
mbin5=mRing*bin5;
cdbin5=mbin5./(2*pi*dr.*rbin*AU*AU);
mbin6=mRing*bin6;
cdbin6=mbin6./(2*pi*dr.*rbin*AU*AU);
mbin7=mRing*bin7;
cdbin7=mbin7./(2*pi*dr.*rbin*AU*AU);
%nr rings/area

loglog(xdat, cdbin1, xdat, cdbin2, xdat, cdbin3, xdat, cdbin4, xdat, cdbin5, xdat, cdbin6,
xdat, cdbin7, 'LineWidth', 1.5)
%adding ice-sublimation line
hx = graph2d.constantline(2.7, 'LineStyle', ':', 'Color', [.7 .7 .7]);
changedependvar(hx, 'x');

xlabel('Distance to Star (AU)')
ylabel('Column Density (g/cm^2)')
title('Column Density over time')

axis([ 1 100 .06 4])

legend('1 year', '1e5 years', '2e5 years', '5e5 years', '1e6 years', '1.5e6 years',
'2e6 years', 'ice line')
```

Appendix B

Runge-Kutta 4th Order

B.1 The Code

To examine how well suited the Euler method was in this case the same program was tested with a different stepping method, Runge-Kutta of the 4th order. The code here is the same as the one in appendix A except for the changes in the loop where the steps are taken. The time interpolation and binning was done the same way as before.

```
clear;
.
.
.

for i=1:1:s

    %Calculating vr and Rdot in function MMSN
    [vr,Rdot,Rdotr]=MMSN( r(i,:),R(i,:));

    dt=min(-ff*r(i,)./vr/(1.5*10^13)*year,f*R(i,)./Rdot/(AU)*year); %time step in seconds

    %Runge-Kutta 4th order
    k1=vr;
    k_1=Rdot;
    [vr,Rdot,Rdotr]=MMSN( r(i,.)+k1.*dt/2,R(i,.)+k_1.*dt/2);
    k2=vr;
    k_2=Rdot;
    [vr,Rdot,Rdotr]=MMSN( r(i,.)+k2.*dt/2,R(i,.)+k_2.*dt/2);
    k3=vr;
    k_3=Rdot;
    [vr,Rdot,Rdotr]=MMSN( r(i,.)+k3.*dt,R(i,.)+k_3.*dt);
    k4=vr;
```

```
k_4=Rdot;

rSteg=dt.*(k1+2*k2+2*k3+k4)/6;
RSteg=dt.*(k_1+2*k_2+2*k_3+k_4)/6;
RRSteg=0.23333*RSteg;

%Ice-sublimation line
for k=1:1:x
    if r(i,k)>2.7
        R(i+1,k)=R(i,k)+epsilon*RSteg(k);
    elseif r(i,k)<=2.7
        R(i+1,k)=R(i,k)+epsilon* RRSteg(k);
    end
end

r(i+1,:)=r(i,:)+rSteg;
tid(i+1,:)=tid(i,:)+dt;

time=max(tid)/(year); %converts to years

%if the outer ring fall onto the star, stop
if r(i,x)<.1
    break
end
w=w+1;
end
.
.
.

Timefinder(w,x);
```

B.2 Plots

Here will follow four comparisons between Euler plots and Runge-Kutta plots.

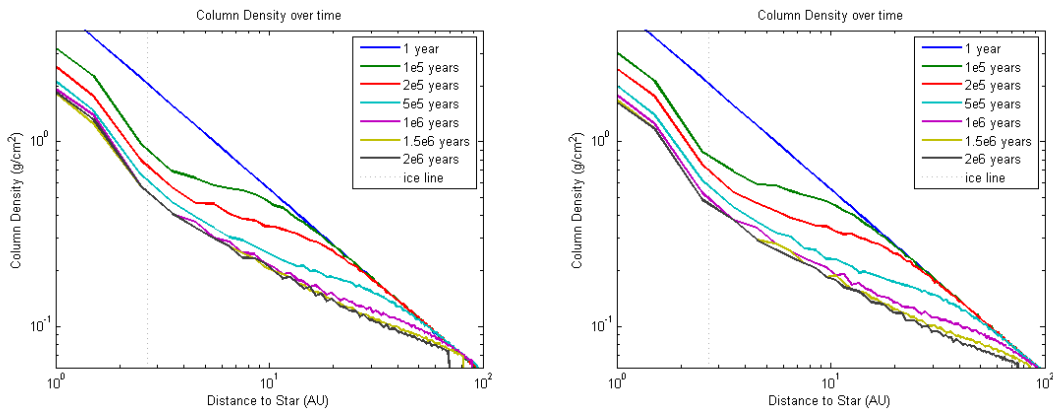


Figure B.1: The dust column density. The plot to the right is with the 4th order Runge-Kutta method. The plot to the left is the result of the original Euler method.

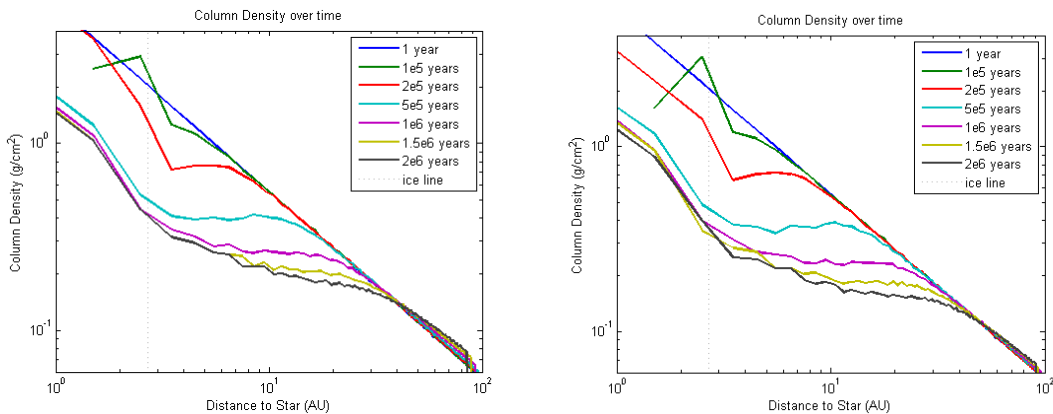


Figure B.2: Low turbulence ($\alpha = 10^{-4}$) and the bump at 50 AU after $2 \cdot 10^5$ years still remains. The plot to the right is with the Runge-Kutta 4th order method.

The conclusion I draw from this is that the conclusions from using the Euler method are still valid since it did not differ much between the Euler method and the 4th order Runge-Kutta in this case.

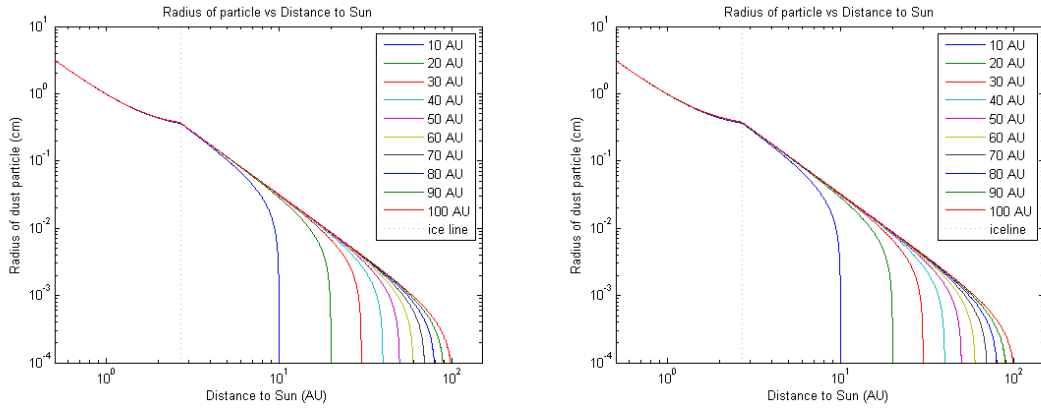


Figure B.3: Pebble growth. The plot to the right is with the Runge-Kutta 4th order method.

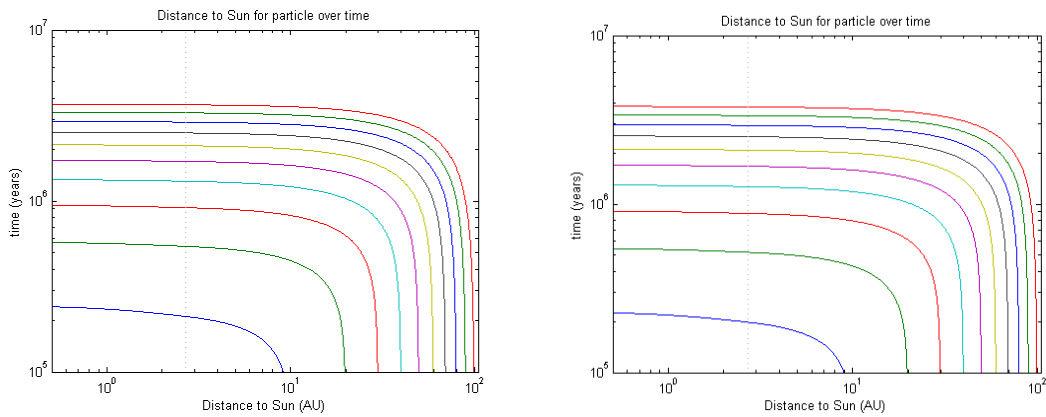


Figure B.4: Pebble drift. The plot to the right is with the Runge-Kutta 4th order method.