The Stability of Exomoons in the Habitable Zone

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Abstract

This is an analysis of all the planets at exoplanets.org, with the goal of finding out how many of these that could possibly have moons supporting life. I have first calculated the ranges of the habitable zones (HZ:s) for the stars in the archive, using the Runaway Greenhouse model for the inner limit, and the Maximum Greenhouse model for the outer limit, as presented by Kopparapu et al. (2013A). Then I investigate the stability of moons in systems with multiple planets, by inspecting the separation between the planets in terms of their the mutual Hill radii. I calculate how many habitable moons ($0.3M_{\oplus}$) individual planets can have, by first placing one at the edge of the planet's Roche lobe, and then another a variable number of mutual Hill radii further in, and then repeating until I reach the point where the moon or the planet fills its own Roche lobe. Finally, I attempt to fit hypothetical planets in the systems with 'gaps' in the habitable zone between discovered planets. A total of 19 confirmed planets were found in habitable zones, out of which 8 are in multiplanetary systems. Out of all 19, all except one were seemingly able to have at least one habitable moon, and a majority even able to have more than 10. Out of all confirmed planets, over 90% seemed to be able to have moons of habitable size, with almost 40% being able to have 10 or more. These numbers can be considered as upper limits, but even as such are quite optimistic as they rely on perfectly circular orbits, and are arrived at while ignoring destabilising phenomena such as mean-motion resonances.

Populärvetenskaplig Sammanfattning

Detta arbete handlar om möjligheterna att finna beboeliga månar som kretsar kring planeter runt andra stjärnor än vår egen sol - så kallade *exomånar*. Själva undersökningen är en analys av över 5000 planeter från onlinearkivet exoplanets.org, för att se hur många av dessa som skulle kunna ha månar som stödjer liv.

Även om majoriteten av fokuset i sökandet efter utomjordiskt liv har legat på exoplaneter än så länge, så finns det ingen uppenbar anledning till varför liv inte skulle kunna finnas på en mindre kropp, som kretsar kring en sådan planet. Dagens observationsmetoder är inte tillräckligt precisa för att upptäcka exomånar i de flesta fall, men de villkor som måste uppfyllas för att liv skulle kunna finnas är i princip identiska med vad som skulle krävas på en exoplanet.

De första av dessa villkor är att planeten eller månen inte kan ligga för nära eller för långt ifrån sin stjärna, då temperaturen måste vara lagom hög för att vatten ska kunna finnas i flytande form. Även om det inte är absolut säkert att vatten är ett krav för liv, så var denna undersökning fokuserad på endast detta scenario. Detta krav är nära sammankopplat till begreppet *beboelig zon*, vilket helt enkelt är det område runt en stjärna där temperaturen kan vara lagom hög för eventuellt liv. Zonens utsträckning beror på stjärnans ljusstyrka, och i mindre utsträckning på dess färg. Ju ljusare och rödare en stjärna är, desto längre ifrån den måste en kropp befinna sig för att kunna vara beboelig.

En andra faktor att ta i beaktning är att månen eller planeten behöver en atmosfär. Utan en sådan skulle kroppens yta vara mycket ogästvänlig, främst på grund av strålning, och det faktum att temperaturskillnader mellan dag och natt skulle kunna vara mycket stora. För att bibehålla en atmosfär måste kroppen vara tillräckligt massiv för att gravitationellt hålla kvar den, vilket i praktiken innebär att den måste vara betydligt större än alla månarna i solsystemet. Någon lägsta massa krävs också för att planeten eller månen ska vara geologiskt aktiv, vilket tros vara nödvändigt för liv då det bidrar med att återvinna material på kroppens yta.

Slutligen måste de ovan nämnda villkoren varit uppfyllda under en längre tid, så att eventuellt liv ska kunna ha fått en chans att uppstå. För att hålla sig stabil tillräckligt länge, får inte kroppens omloppsbana störas för mycket av andra objekt. Detta kan vara fallet om planeter eller månar ligger för nära varandra, vilket sätter gränser för hur tätt packat ett planetsystem kan vara, och för hur många månar individuella planeter kan ha.

Genom att ta dessa faktorer i beaktning, visade det sig att totalt 19 planeter från exoplanets.org verkar ligga i beboeliga zoner, varav 18 verkade kunna ha minst en beboelig måne var. 14 av dem verkade till och med kunna ha så mycket som 5 månar var. Dock gäller dessa siffror endast i det ideala fallet då alla månars omloppsbanor valts specifikt för att maximera stabiliteten i systemen, vilket säger väldigt lite om hur vanliga exomånar verkligen är. Till exempel visade undersökningen att planeterna i solsystemet skulle kunna ha många fler månar än de faktiskt har.

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Chapter 1

Introduction

In the investigation into the habitability of extrasolar objects, the main focus has long been on exoplanets, while exomoons have only been considered properly during the last few years. A reason for this is the obvious difficulty in detecting objects that do not primarily orbit a star, but rather a secondary object, while also being generally smaller than planets. At present time, the number of confirmed exoplanets are over 1000 (exoplanets.org), while only a few exomoon candidates exist (see, e.g., Bennet et al. 2014). Still, detection of extrasolar moons in the near future seems feasible, using current methods, and the transit method in particular (Hinkel & Kane 2011, Kipping et al. 2009).

The conditions necessary to sustain life are in essence the same regardless of whether it is a planet or moon that is being considered. Assuming liquid water to be a requirement, an atmosphere must be present, and limits are set for what ranges of temperature and pressure that are of interest. The constraints on the temperature in turn mean that the body must lie within a certain range of distances from the star, in the so-called Habitable Zone (HZ). The body must also be massive enough to gravitationally keep the gas of its atmosphere from escaping, and for it to have plate tectonics. The latter is believed to be a required mechanism for life, as it recycles materials on the surface of the body. These conditions must then be upheld for quite some time to allow potential life to arise, meaning the orbit of the body must be fairly stable.

In this investigation, all the objects at exoplanets.org (as of 2014-04-29) were evaluated to see which of these that could possibly have habitable moons, taking the issues presented above into account. The general approach was as follows:

First, it was determined what planets had orbits that lie in their host stars' habitable zones, using a procedure presented by Kopparapu et al. (2013A) to calculate the zone limits. Then all the systems with more than one discovered planet were investigated further to see if the separation between the bodies were large enough, in terms of their mutual Hill radii, in order for the planets to remain stable for long enough ($\sim$ 1Gyr). Hypothetical moons were then placed around all planets that had sufficient separation, and around all planets in systems with only one known planet. When placing moons, a variable requirement was used for the separation between the moons (in terms of their mutual Hill radii), to see how many could fit inside the planets' Roche lobes.

Also, it was attempted to add hypothetical planets to the exoplanet systems that had open 'gaps' in the habitable zone between discovered planets, while maintaining sufficient planet separations.

Lastly, the same analysis was carried out on the solar system, as a simple test of the reliability of the approach.

1.1 Habitable Zones

Strictly speaking, a habitable zone (HZ) is a region around a star, within which a planet with sufficient atmosphere would be able to maintain liquid water on its surface. The main determining factor here is the level of stellar flux incident on the planet, but the effective temperature of the star also plays a role. The latter is relevant because light of different wavelengths interact differently with the elements and compounds in the atmosphere, and as stars radiate like black bodies, the color of their light is highly temperature-dependant.

The first step in this investigation was to determine which planets at exoplanets.org that lie within the HZ of their respective stars. For this the work by Kopparapu et al. (2013A) was used, more specifically the inner limit set by the *runaway greenhouse* model, and the outer limit set by the *maximum greenhouse* model, first presented by Kasting (1988) and Kasting et al. (1993) respectively. These are both radiative-convective, 1-dimensional, cloud-free atmospheric models, relying on so called "inverse climate modelling". This means that an atmospheric composition and a surface temperature is specified first, after which the model is used to calculate the stellar flux required to sustain it. The runaway greenhouse limit is where the surface temperature of a planet with an Earth-like atmosphere (N_2-O_2-CO_2) has risen to such a level that the water vapor content of the atmosphere increases dramatically. This causes such a strong feedback to the greenhouse effect, that the temperature increases unhindered until all oceans are evaporated. The maximum greenhouse limit on the other hand, is where an N_2-atmosphere with up to a maximal CO_2 content (35 bars partial pressure) can maintain a temperature of 273 K.

Based on simulations with these models, using Earth-mass planets, Kopparapu et al. (2013A) present a formula for what levels of stellar flux S_{Eff} that correspond to the limits, as a function of the effective stellar temperature T_{Eff}:

$$S_{Eff} = S_{Eff,\odot} + aT_* + b(T_*)^2 + c(T_*)^3 + d(T_*)^4 \tag{1.1}$$

where $T_* = T_{Eff} - 5780K$, with both T_* and T_{Eff} in Kelvin. The function is normalized so that the solar flux incident on Earth is 1, meaning S_{Eff} and $S_{Eff,\odot}$ are dimensionless. The different parameters, a, b, c, and d are in units of powers of Kelvin to make the expression dimensionally consistent, K^{-1}, K^{-2}, K^{-3}, and K^{-4} respectively. They and $S_{Eff,\odot}$ take on different values in the different models, which can be seen in Table 1.1 below. The models are valid in the range $2600K \leqslant T_{Eff} \leqslant 7200K$.

Table 1.1: Coefficients for the Runaway and Maximum Greenhouse Models

Coefficient	Runaway Greenhouse	Maximum Greenhouse
$S_{Eff,\odot}$	1.0512	0.3438
a	$1.3242{\cdot}10^{-4}$	$5.8942{\cdot}10^{-5}$
b	$1.5418{\cdot}10^{-8}$	$1.6558{\cdot}10^{-9}$
c	$-7.9895{\cdot}10^{-12}$	$-3.0045\ {\cdot}10^{-12}$
d	$-1.8328\ {\cdot}10^{-15}$	$-5.2983\ {\cdot}10^{-16}$

A more illustrative display of the range of habitable levels of stellar flux can be seen in Figure 1.1. For a redder star, corresponding to lower stellar temperature, both the maximum and minimum levels are clearly lower than for a bluer star.

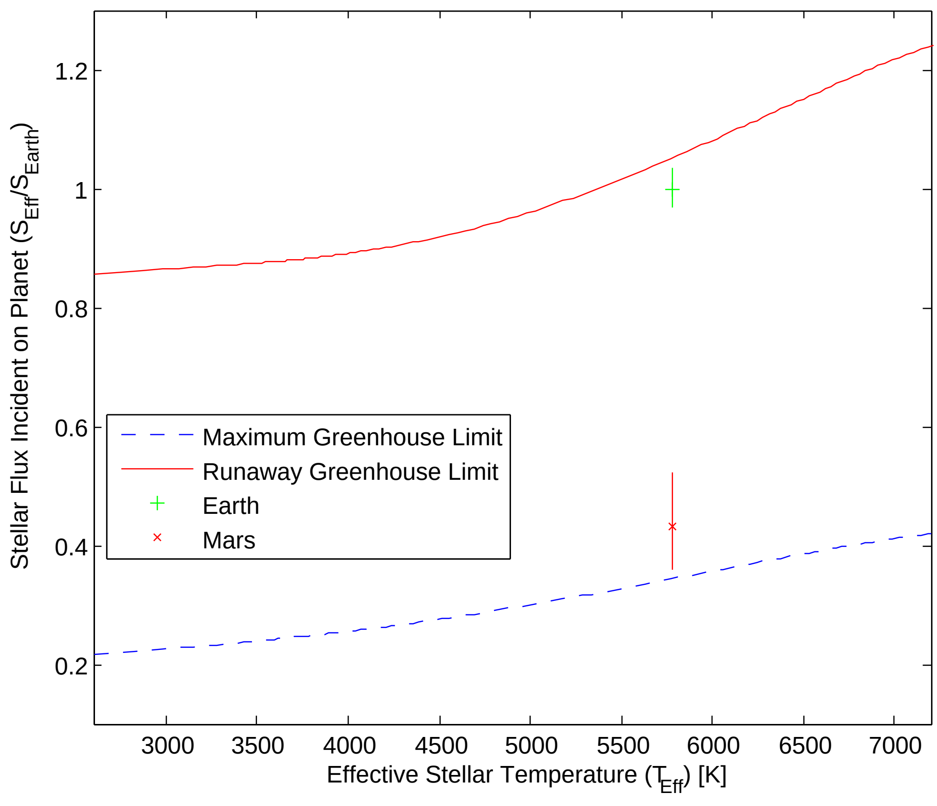

Figure 1.1: The range of stellar flux incident on a planet required for liquid water, as a function of effective stellar temperature. The upper and lower limits are set by the Runaway Greenhouse Model and the Maximum Greenhouse Model respectively. Earth and Mars have been added for illustration, both being inside the habitable zone. The vertical lines connected to the planets represents the variation in flux due to orbital eccentricity.

The distances d from the star corresponding to the flux limits, which are the actual edges of the habitable zone, can then be calculated using

$$d = \left(\frac{L/L_{\odot}}{S_{eff}}\right)^{1/2} \mathrm{AU} \tag{1.2}$$

where L is the luminosity of the relevant planet's host star, and $L_{\odot}$ is the luminosity of the sun.

HZ ranges as a function of stellar mass can be seen in Figure 1.2, both for zero-age main-sequence (ZAMS) stars, and terminal main-sequence (TMS) stars. The radii and luminosities have been calculated as a function of stellar mass and metallicity, using the work of Tout et al. (1996) for ZAMS stars, and Hurley et al. (2000) for TMS stars. See Appendix A for details. The metallicity of a star is defined as the fraction of its total mass that is made up of elements other than Hydrogen and Helium.

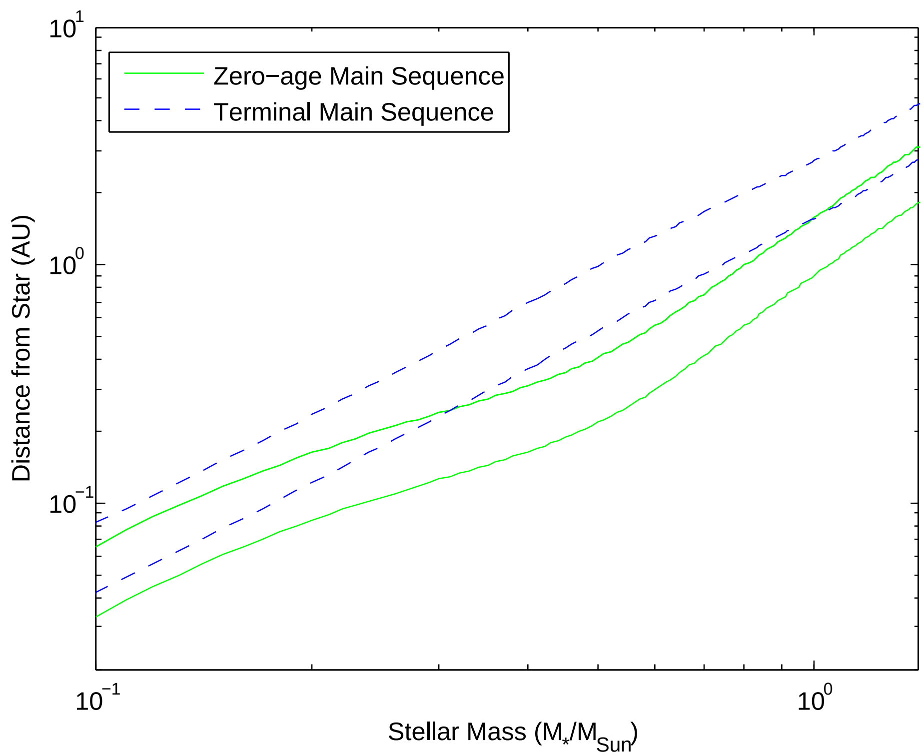

Figure 1.2: Habitable Zone ranges as a function of stellar mass, for zero-age main sequance stars, and terminal main-sequence stars. The luminosities and temperatures of the stars, as functions of mass and metallicity, were calculated using the formulae presented by Tout et al. (1996) and Hurley et al. (2000). The metallicity used was that of the sun, Z= 0.0122. The range of stellar masses only goes up to 1.4 solar masses, as the temperatures above this point are too high for the atmospheric models to be valid.

All exoplanets as exoplanets.org were investigated, to see if their orbits lay in the habitable zones of their stars, calculated using the procedure above. The largest and smallest distance between the individual planets and their respective stars, called apoapsis (r_{max}) and periapsis (r_{min}), were calculated from the semi-major axes a and eccentricities e of their orbits:

$$r_{max} = a(1+e) \quad r_{min} = a(1-e) \tag{1.3}$$

The requirement was for both of these to lie within the HZ.

1.2 Mass Limits for Habitable Bodies

When considering both the habitability and stability of hypothetical objects, their masses are highly relevant. The limits for the range of possible masses for a habitable object are largely unknown, with the only certainty being that Earth-mass ($M_\oplus$) bodies can support life.

As a lower limit, the mass of $0.3M_\oplus$ was used. This is based on the work by Williams et al. (1997), where the limit is discussed extensively, and several proposals are given. The value of $0.3M_\oplus$ is an estimate of the minimum mass required for the object to be geologically active, and have plate tectonics. This is thought to be a requirement for life, as it recycles material on the body's surface, and returns CO_2 to the atmosphere (Walker et al. 1981, Valencia et al. 2007). If CO_2 is not returned, it can not act as a buffer against temperature changes over long periods of time, as it is for example believed to do on Earth in the carbonate−silicate cycle.

An upper mass limit for a habitable object is of less importance when considering moons. The lower limit already presented is roughly 12 times as large as the mass of Ganymede - the heaviest moon in the solar system ($\sim 0.025M_\oplus$), while the upper limit must clearly be larger than the mass of the Earth.

In the attempts to place new, hypothetical moons in the systems at exoplanets.org, using bodies of mass equal to that of the lower limit was of greatest interest. This was because using a minimal mass would yield the maximum number of habitable moons, assuming they can be more numerous the lighter they are. For comparison, attempts were also made with the mass $0.0123M_\oplus$, corresponding to the mass of Earth's moon, and with 1 $M_\oplus$, the latter certainly being of habitable size. When attempting to place hypothetical 'gap' planets (See Section 1.3.3), the masses used were $0.3M_\oplus$, $1M_\oplus$, and $1M_{Jupiter}(\approx 318M_\oplus)$, the latter being relevant as a large planet would supposedly have a greater chance of having large moons.

1.3 Multiplanetary Systems

When looking into habitable bodies, the ones that are of interest are the ones that remain in the HZ for long enough for life to form. For this investigation, the required time was taken to be 1 Gyr, chosen as the only known instance of life; that on Earth, came about approximately this long after the formation of the planet (Schopf et al. 2007). The properties of the system must then be such that the orbit of a planet or moon remains stable and inside the HZ for this long.

1.3.1 Separations in Terms of Roche-, and Mutual Hill Radii

A highly relevant concept in these considerations, is that of a Roche lobe of a body. It can be described as the region around the body, inside which the gravitational pull of the object itself is greater than the pull from another body, in a system where both orbit around the common center of mass. More strictly, if a two body system, like a star and a planet, is regarded in a co-rotating (non-inertial) frame of reference, it is the region around the planet where the gravitational potential decreases as the planet is approached. If the rotational plane is viewed head-on, it is a rougly tear-drop shaped region centered on the planet, with the point directed towards the star. See Figure 1.3. Its precise shape is not easily calculatable, but it can be approximated as a sphere for practical purposes. If a small object is to orbit a planet like a moon, it must lie within the Roche lobe.

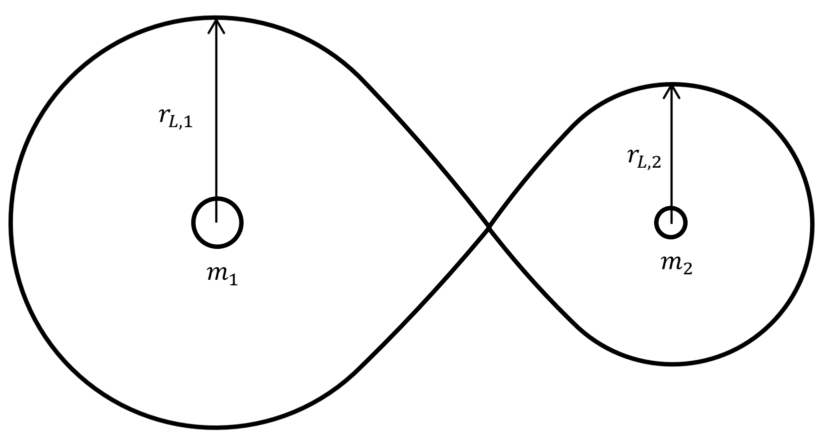

Figure 1.3: The Roche lobes around two bodies, for example a star and a planet. In the figure, m_1 is heavier than m_2, and therefore has a larger Roche lobe. The radii of the lobes, $r_{L,1}$ and $r_{L,2}$ have been drawn.

An evaluation of the radius of the spherical approximation of this region, r_L, is given by Eggleton (1983), to which he has made a fitting formula, accurate to better than 99%. The Roche lobe radius of a body of mass M_1, orbiting a body of mass M_2 at distance d, is

$$r_{L,1} = d \times \frac{0.49(M_1/M_2)^{2/3}}{0.6(M_1/M_2)^{2/3} + \ln(1 + (M_1/M_2)^{1/3})} \tag{1.4}$$

Inverting the mass ratio M_1/M_2 yields the Roche lobe radius of object 2 instead. In the case of an eccentric orbit the distance d varies, and therefore also the size of the Roche lobe. Its most relevant extent, considering the stability of possible moons, is when it is at its smallest, meaning at the planet's periapsis, when $d = r_{min}$.

When more than one body orbits a larger one, the stability can be threatened if the orbits lie too close to one another. A relevant measure of the separation here, is the so called mutual Hill radius of the two obiting bodies. This is in essence the average of the Hill sphere radii of the two planets, which are closely related to their Roche lobe radii. The mutual Hill radius of two bodies is given by (e.g., Chambers et al. 1996)

$$R_{Hill,1,2} = \left(\frac{M_1 + M_2}{3M_3}\right)^{1/3} \frac{a_1 + a_2}{2} \tag{1.5}$$

where M_1 and M_2 are the masses of the orbiting bodies, and M_3 is the mass of the central body. a_1 and a_2 are the semi-major axes of the orbiting bodies. Other definitions of the mutual Hill radius exist (see, e.g., Gladman, 1993).

1.3.2 Stability Timescales

The time it takes for a system to become unstable has been found to be a function of the the separation between planets divided by their mutual Hill radius; the so-called Δ-factor (Chambers et al. 1996):

$$\Delta_{1,2} = \frac{a_2 - a_1}{R_{Hill,1,2}} \tag{1.6}$$

where a_2 and a_1 are the semi-major axes of the outer and inner planet respectively.

Gladman et al. (1993) found that a system with two planets on initially circular and coplanar orbits remains stable forever if $\Delta \geqslant 2\sqrt{3}$. For systems with more planets than that, no such simple rule exists. Chambers et al. (1996) investigated the time $t_{encounter}$ before a first close encounter, as a function of Δ for systems with a variable number of planets and mass ratios between the star and planets. Using numerical integrations, they found a trend for planets on initially near-coplanar, near-circular orbits, with identical Δ for all separations between neigbouring planets:

$$log(t_{encounter}/t_{orbit}) = b\Delta + c \tag{1.7}$$

where $t_{encounter}$ is the time at which a first close encounter occurs, t_{orbit} is the time of an orbital period (of the innermost planet), and b and c are constants depending on the number of planets and planet/star mass ratios. The timescale is thus a very sensitive function to variations in Δ. A close encounter was defined as when the separation between two planets decreases to less than their mutual Hill radius. For a mass ratio of 10^{-7}, they found the constants to take on the values presented in Table 1.2.

Table 1.2: Coefficients for the equation for the time $t_{encounter}$ (1.7) before a close encounter in planetary systems, with a mass ratio of $M_{planet}/M_{star} = 10^{-7}$, as presented by Chambers et al. (1996).

Number of Planets	a	b
3	1.176± 0.051	-1.663 ± 0.274
5	0.765± 0.030	-0.030 ± 0.192
10	0.756± 0.027	-0.358 ± 0.176
20	0.757± 0.038	-0.508 ± 0.244

The time before a close encounter decreases with the number of planets, but does not change much more when the number exceeds 5. For $t_{encounter} = 10^8$ orbits, Equation 1.7 yields the requirements $\Delta \gtrsim 7$ for a system with 3 planets, and $\Delta \gtrsim 10.5$ for a system with 5 planets. The latter of these is however an extrapolation of the results by Chambers et al. (1996), as they only carried out integrations up to $\Delta = 10$.

Marzari & Weidenschilling (2002) used a similar procedure of integration as Chambers et al. (1996), but with the greater mass ratio $M_{planet}/M_{star} = 10^{-3}$. The result of their work indicates a similar separation-stability timescale relationship as that found by Chambers et al. (1996), the one shown in Equation 1.7. No coefficients like those in Table 1.2 were presented, but the results imply that $a \approx 2.65$ and $b \approx -5.25$ for this mass ratio, in a system with 3 planets. This corresponds to a separation of $\Delta = 5$ yielding an instability time of $t_{encounter} \approx 10^8$ orbits.

Generally, the greater the difference between the masses of the star and the planets, the shorter the stability timescale becomes (Chambers et al. (1996), Davies et al (2013)).

The planet/star mass ratios of 10^{-7} and 10^{-3} are very similar to those between Mercury and the Sun, and between Jupiter and the Sun respectively. This would suggest that the results of Chambers et al. (1996) and Marzari & Weidenschilling (2002) are applicable to the solar system. In comparison, the mass ratios between Earth and its moon, and between Jupiter and its heaviest moon (Ganymede), are around 10^{-2} and 10^{-4} respectively. The mass ratio between Jupiter and a $0.3M_{\oplus}$-moon would be around 10^{-3}.

For all multiplanetary systems at exoplanets.org, the Δ-factors were calculated for the separation between all neighbouring planets, and was compared to the stability timescales just discussed. For all intents and purposes, all orbits were assumed to be coplanar. As the integrations by Chambers et al. (1996) and Marzari & Weidenschilling (2002) were performed on systems with circular orbits, the separations between the planets' orbits were taken to be the smallest possible distance between the planets; the difference between the outer planet's perihelion and the inner planet's aphelion. This was in order not to overestimate the stabilities of the systems. This means that the outer planet's semi-major axis, a_2, was replaced by $r_{2,min}$, and the inner planet's semi-major axis a_1 by $r_{1,max}$, in equations 1.5 and 1.6.

1.3.3 Adding Hypothetical 'Gap' Planets

For every multiplanetary system with space in the habitable zone between discovered planets, it was attempted to add a new, hypothetical planet. Meaning, this was done in all multiplanetary systems with (at least) one body already in the HZ, and in systems with known planets both on the inner and outer side of the HZ. To achieve the highest possible stability for such a 'gap' planet, its orbit was chosen to be circular, with a semi-major axis a_{hyp} such that the smaller of the two Δ-factors, with respect to the separation with the inner and outer neighbour respectively, was maximised. This principle is illustrated in Figure 1.4, with a gap planet between Earth and Mars.

The cases where discovered planets were only present on one of the sides of the HZ were considered too trivial, and were not investigated. This was partly because in such systems, trying to place a new planet in the HZ while only considering the separation relative to one neighbour, could almost always yield a stable orbit. The ease of doing this is illustated by the case in Figure 1.4, where a gap planet can be placed even though two planets were already present inside the HZ. The scenario of only having to consider one neighbour would then be even more trivial.

Also, in systems where the only discovered planets lie fairly close to the star, as is often the case, little is known about what actually lies further out. This is because both the common methods of discovering exoplanets, the transit and RV-methods, are better for finding planets in smaller orbits. See Appendix C for details.

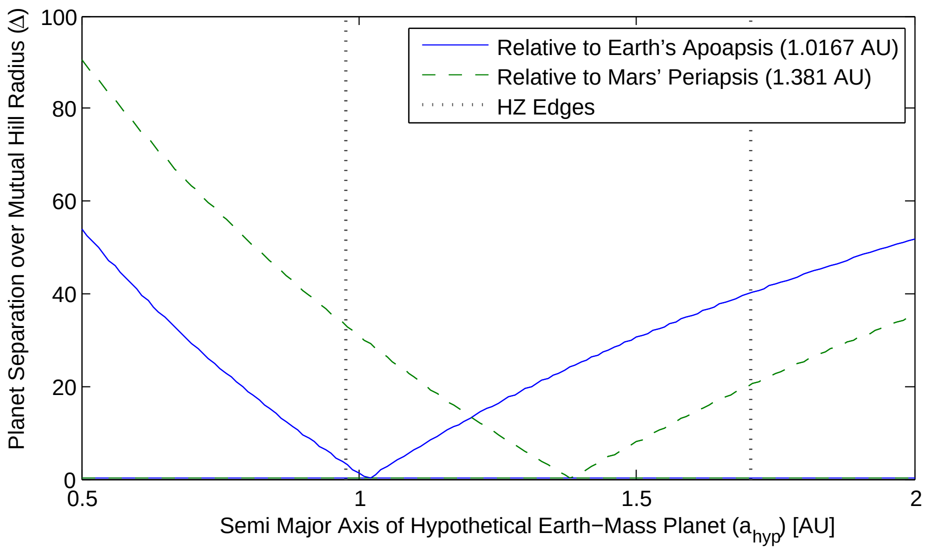

Figure 1.4: The Δ-factors for a hypothetical, Earth-mass planet in the solar system, relative to Earth and Mars as functions of the semi-major axis a_{hyp} of the planet. The ideal orbit, in terms of stability, would be that with a semi-major axis where the two lines intersect.

In cases where the orbit of highest stability does not lie inside the HZ (as is not the case in igure 1.4), the semi-major axis of the hypothetical planet was chosen to be at the edge of the HZ closest to this orbit.

Analytically, finding the point of intersection means solving the equation $\Delta_{1,hyp} = \Delta_{hyp,2}$ for a_{hyp}. The solution to this is arrived at through simple algebra, but becomes quite long. See Appendix B for the full expression. Note though, that not only the Δ-factors themselves, but also the semi-major axis of the optimal orbit is dependant on the masses of the neighbouring planets, and the mass of the gap planet itself.

Three different masses were used when attempting to add gap planets. These were, as mentioned in Section 1.2, $0.3M_\oplus$, $1M_\oplus$, and $1M_{Jupiter}$.

1.4 Systems With Moons

When determining whether a planet would be able to have moons, the basic assumption was that only moons remaining inside the planet's Roche lobe during their entire orbit, would be stable. It was also set as a requirement that the separation between the planet in question and its neighbours had to be adequate in order for the planet itself to be sufficiently stable, as discussed in the previous section. The limit of $\Delta \geqslant 5$ was primarily used, perhaps rather optimistically considering the stabillity timescales presented in Section 1.3.2. As long as this condition was fulfilled, the effects of other planets on the moons were ignored.

The scenario of a planet with multiple moons was treated in the same way as a system of a star with planets. The stability of such a system was considered with respect to the separations between the individual moons in terms of Δ-factors, much similar to what was discussed in Section 1.3. As mentioned there, the typical moon/planet mass ratios of the solar system are similar to the planet/star mass ratios, which supports the method of treating them the same way.

1.4.1 Inner and Outer Limits for Moon Orbits

The maximum semi-major axis of a moon would, in the ideal case of a circular orbit, be the Roche lobe radius minus the moon's own radius. To obtain their size, it was assumed that all moons were perfect spheres, with a mean density of that of Earth's moon, ρ_{moon} =3.346 g·cm^{-3}. The radius is then obtained through

$$\frac{M_{moon}}{\rho_{moon}} = \frac{4\pi R_{moon}^3}{3} \implies R_{moon} = \left(\frac{3M_{moon}}{4\pi\rho_{moon}}\right)^{1/3} \tag{1.8}$$

Using Equation 1.4 for the Roche lobe radius, and subtracting the moon radius, yields the outer limit

$$a_{moon,outer} = r_{min,planet} \times \frac{0.49(M_{planet}/M_{star})^{2/3}}{0.6(M_{planet}/M_{star})^{2/3} + \ln(1 + (M_{planet}/M_{star})^{1/3})} - R_{moon} \tag{1.9}$$

For an inner limit for how close a moon could be to its host planet, the Roche lobes in the two-body system consisting of the planet and its moon, was considered, temporarily ignoring their host star. If the planet and moon are brought closer and closer to each other, the sizes of their Roche lobes will decrease, but their shape will remain the same. Eventually, the sizes of the Roche lobes will decrease so much that the moon fills its entire Roche lobe. If the distance is decreased to less than this, the moon will reach outside its own Roche lobe, meaning its outer parts could be more strongly attracted to the planet than to itself, resulting in it breaking apart.

To find the point where this happens, the Roche lobe radius of the moon was set equal to the moon's radius;

$$a_{moon,inner} \times \frac{0.49(M_{moon}/M_{planet})^{2/3}}{0.6(M_{moon}/M_{planet})^{2/3} + \ln(1 + (M_{moon}/M_{planet})^{1/3})} = R_{moon} \implies$$
$$a_{moon,inner} = R_{moon}\left(\frac{0.49(M_{moon}/M_{planet})^{2/3}}{0.6(M_{moon}/M_{planet})^{2/3} + \ln(1 + (M_{moon}/M_{planet})^{1/3})}\right)^{-1} \tag{1.10}$$

In the case of the planet having very low density, it could occur that the planet fills its own Roche lobe before the moon does. For this reason, the lower limit $a_{moon,inner}$ was actually taken to be the maximum of the expression above (Equation 1.10) and the corresponding case where the planet's radius equals its Roche lobe radius. This simply means shifting $M_{planet} \leftrightarrow M_{moon}$ and $R_{planet} \leftrightarrow R_{moon}$ in Equation 1.10.

1.4.2 The Procedure of Placing Moons

It was attempted to place as many moons as possible between the limits described in the last subsection, while keeping the separation sufficient for stability. This was done by first placing a moon at the outer edge of the Roche lobe (given by Equation 1.9), and choosing its orbit to be circular. Then another moon was placed further in, at a new semi-major axis giving it a separation relative to the outer moon corresponding to a predetermined Δ-factor. For this, the expression for the Δ-factor was used, and then rearranged into an expression for the inner planet's semi-major axis:

$$\Delta_{k,k+1} = \frac{a_{k+1} - a_k}{R_{Hill,k,k+1}} = \frac{2(a_{k+1} - k_k)}{(a_{k+1} + r_k)\left(\frac{M_k + M_{k+1}}{3M_{planet}}\right)^{1/3}} \implies a_k = -a_{k+1}\left(\frac{D-1}{D+1}\right) \tag{1.11}$$

where

$$D = \left(\frac{M_k + M_{k+1}}{3M_{planet}}\right)^{1/3}\frac{\Delta_{k,k+1}}{2} \tag{1.12}$$

The index k indicates the outer moon of the pair, and $k+1$ signifies the inner moon. $k = 1$ then correspond to the first moon, placed at the edge of the Roche lobe. As the form of the equation indicates, it can be applied over and over to place new moons with a constant Δ-factor for all separations. This was done until additional moons would have to be inside the inner limit $a_{moon,inner}$.

Another additional limit for the number of moons was set, in that the sum of the mass of all the moons around a planet was not allowed to rise above one third of the mass of the planet itself. This supposedly very generous limit was mostly set so that the number of moons would not run away for planets far from their stars, for which Roche lobe radii would be large.

When using this procedure, the mass used for the moons were (as mentioned in Section 1.2) $0.0123M_\oplus$, $0.3M_\oplus$, and $1M_\oplus$.

Chapter 2

Analysis

2.1 Results

The full investigation was performed with all the planets at exoplanets.org, as of 2014-04-29. These were in total 5195 planets, out of which 1491 were confirmed, and most of the rest were candidates from the Kepler mission, so called Kepler Objects of Interest (KOI:s). They were distributed over 2329 systems (909 confirmed), which can be compared to the total number of stars that the Kepler Spacecraft monitors, 136000 (kepler.nasa.gov, 2014-05-23). While the unconfirmed planets were included in the full examination, the main focus was only on the confirmed ones. An illustration of the results, and how they were arrived at by excluding planets that did not qualify in different ways, can be seen in Figure 2.1.

The first step through which the planets had to qualify was:

1: Whether the radius and effective temperature of their respective stars were available, required for calculating the stellar luminosity, and therefore the ranges of the habitable zone. The luminosity was calculated using Stefan-Boltzmann's law, see Appendix A.

Then, the steps were different when trying to find planets with possible habitable moons, and when trying to find gap planets.

In the case of finding moons, the 'left path' in Figure 2.1, the steps were:

2: Whether the planet had an orbit partly in the calculated habitable zone.

3: Whether its orbit was completely in the calculated habitable zone.

4: Whether the mass of the planet and its host star was known (required for calculating Roche-, and mutual Hill radii).

5: Whether the planet could have habitable moons of mass $0.3M_{\oplus}$, considering whether it was sufficiently separated from its neighbours (here meaning if $\Delta \geqslant 5$), if it was massive enough, and if there was room inside the Roche lobe.

In the case of finding 'gap' planets, the 'right path', the steps were:

2: Whether the planet was in a multiplanetary system.

3: Whether it belonged to a pair of planets with a habitable gap in between (divided by two, so that what is counted are pairs).

4: Whether the gap was wide enough to support a $0.3M_{\oplus}$-planet.

Note that some planets can qualify into both paths, meaning there is some overlap between them. The number of planets qualifying in each step can be seen below, in Figure 2.1.

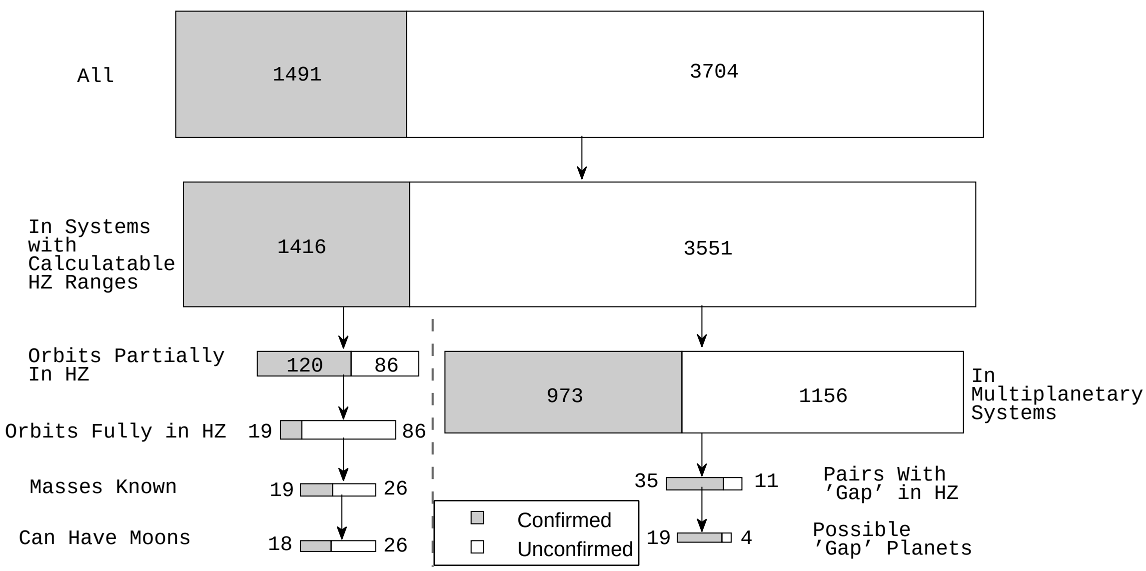

Figure 2.1: An overview of all the results obtained through the investigation, showing the different steps through which different planets were excluded from possibly having habitable moons (left path), or having a gap planet as a neighbour (right path). The area of the boxes are proportional to the number of planets having passed all the tests leading up to the box. These numbers are also displayed inside or just beside the boxes.

2.1.1 Planets in Habitable Zones

A total of 105 planets were found to lie in their respective stars' habitable zones, with 19 of these being confirmed planets. A Table of those is presented below (Table 2.1). The number of possible moons per planet was calculated for separations between moons corresponding to $\Delta = 5$. No system was found to contain more than 1 planet in the HZ. A figure of the same planets, as well as the ranges of the habitable zones of their stars, can be seen in Figure 2.2.

In total 19 planets, both confirmed and unconfirmed, were found in the habitable zones of multiplanetary systems. The configuration of the known planets in these systems are displayed in Figure 2.3.

Table 2.1: The 19 confirmed planets in habitable zones, together with some of their characteristics. The first 6 columns were taken directly from exoplanets.org, while the last two were calculated. The numbers of possible moons displayed are for the case of separations between the moons corresponding to $\Delta = 5$. The separation between mu Ara b and its neighbours was deemed too small ($\Delta < 5$) for it having moons. The (smallest) Δ-factors for all planets without known neighbours were marked with '-'. as there were no planet-planet separations to consider.

Planet's Name	Semi-Major Axis (AU)	Orbital Eccentricity	Mass of Star (M_{Sun})	Mass of Planet (M_{Earth})	Number of Discovered Planets in System	Smallest Δ-factor Relative to Neighbours	Number of Possible 0.3-$M_{\oplus}$ Moons
HD 10180 g	1.4	0	1.1	21	6	14	4
HD 10697 b	2.1	0.1	1.1	2000	1	-	21
HD 114729 b	2.1	0.2	1.0	300	1	-	11
HD 159868 b	2.3	0.01	1.2	700	2	7.1	15
HD 163607 c	2.4	0.1	1.1	730	2	11	16
HD 16760 b	1.1	0.07	0.78	4200	1	-	24
HD 188015 b	1.2	0.1	1.1	470	1	-	12
HD 23079 b	1.6	0.1	1.0	780	1	-	15
HD 28185 b	1.0	0.05	0.99	18	1	-	18
HD 7199 b	1.4	0.2	0.89	94	1	-	7
HD 73534 b	3.0	0.07	1.2	340	1	-	13
HD 99109 b	1.1	0.09	0.94	160	1	-	8
Kepler-16 b	0.65	0.007	0.69	110	1	-	7
Kepler-174 d	0.77		0.60	6.0	3	45	2
Kepler-186 f	0.34	0.01	0.48	1.4	5	54	1
Kepler-283 c	0.40		0.60	2.6	2	42	1
Kepler-62 f	0.72	0.09	0.69	35	5	6.3	5
mu Ara b	1.5	0.1	1.1	550	4	3.1	13 (0)
tau Gru b	2.5	0.07	1.2	390	1	-	13

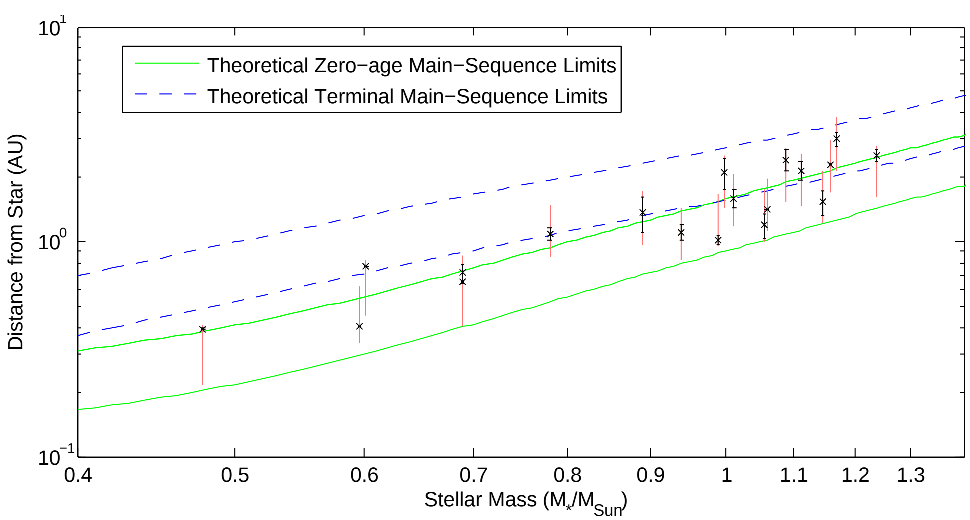

Figure 2.2: The 19 confirmed planets found in habitable zones. The black vertical lines correspond to the variation in star-planet separation due to orbital eccentricity, and the longer red vertical lines show the ranges of the habitable zone of each star. The theoretical limits of the habitable zone, as functions of stellar mass for Zero-Age-, and Terminal Main-Sequence stars are also shown, using the metallicity Z=0.0122. This is the same limits as shown in Figure 1.2.

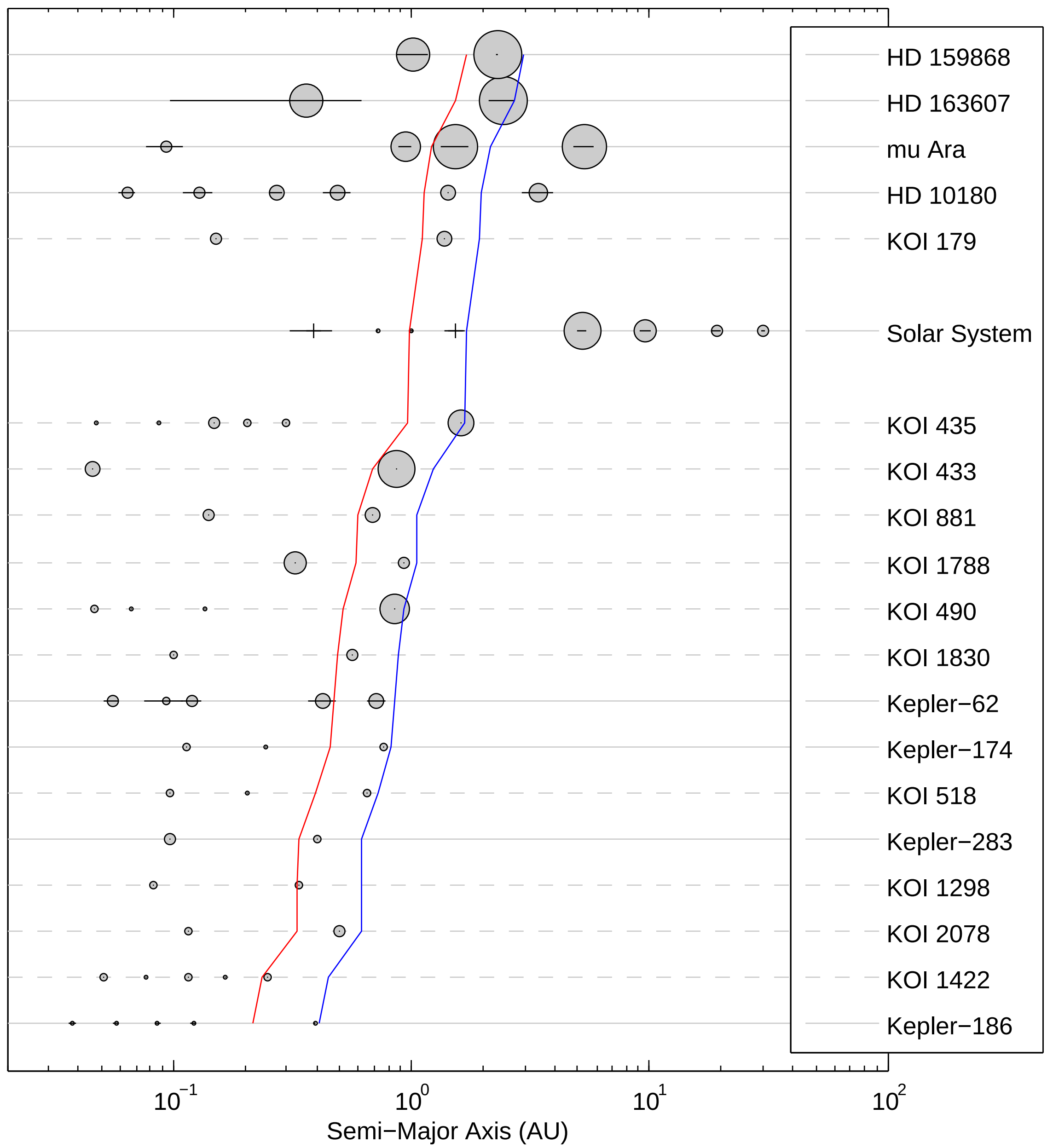

Figure 2.3: The 19 multiplanetary systems with planets in the habitable zone, with the solar system added for comparison. The solid grey lines represent systems where the planet in the HZ is confirmed, while the dashed lines represent systems where it is unconfirmed. The solid black lines connected to each planet represent the variation in star-planet separation due to orbital eccentricity. The red and blue lines represent the edges of the HZ. As can be seen, no exoplanet system has more than 1 planet in the HZ. The radii of the planets are proportional to the cube root of their masses, meaning the ratios of their sizes would be accurate, had they had equal densities. Mercury and Mars were too light to be seen in the figure, and have therefore been marked with '+'. The systems without known neighbours to the planet in the HZ, are not shown here.

2.1.2 Gap Planets

A total of 22 systems seemed to have room for an additional, Earth-mass planet in the habitable zone when the applied requirement of planet separation was $\Delta \geqslant 5$. A plot of the configurations of those systems, with optimal semi major axes of the gap planets, can be seen in Figure 2.4. In Figure 2.5, the number of possible gap planets are shown as a function of the required Δ-factor for the planet-planet separation.

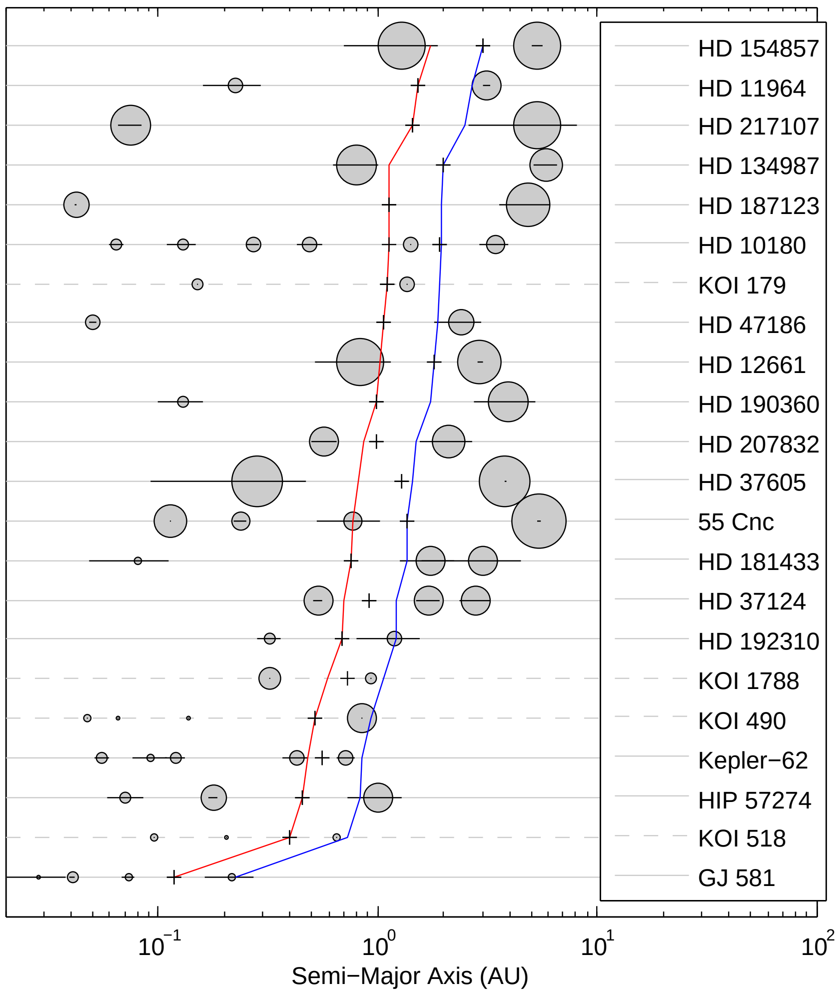

Figure 2.4: The 22 exoplanet systems with possible gap planets in the habitable zone, marked by '+'. The solid grey lines represent systems where the planets are confirmed, while the dashed lines represent systems with unconfirmed planets. The solid black lines connected to each planet represent the variation in star-planet separation due to orbital eccentricity. The red and blue lines represent the edges of the HZ. The radii of the planets are proportional to the cube root of their masses, meaning the ratios of their sizes would be accurate, had they had equal densities. Note that several of the systems also have confirmed planets in the HZ, and are therefore also shown in Figure 2.3. Note also that two gap planets could be fitted into HD 10180.

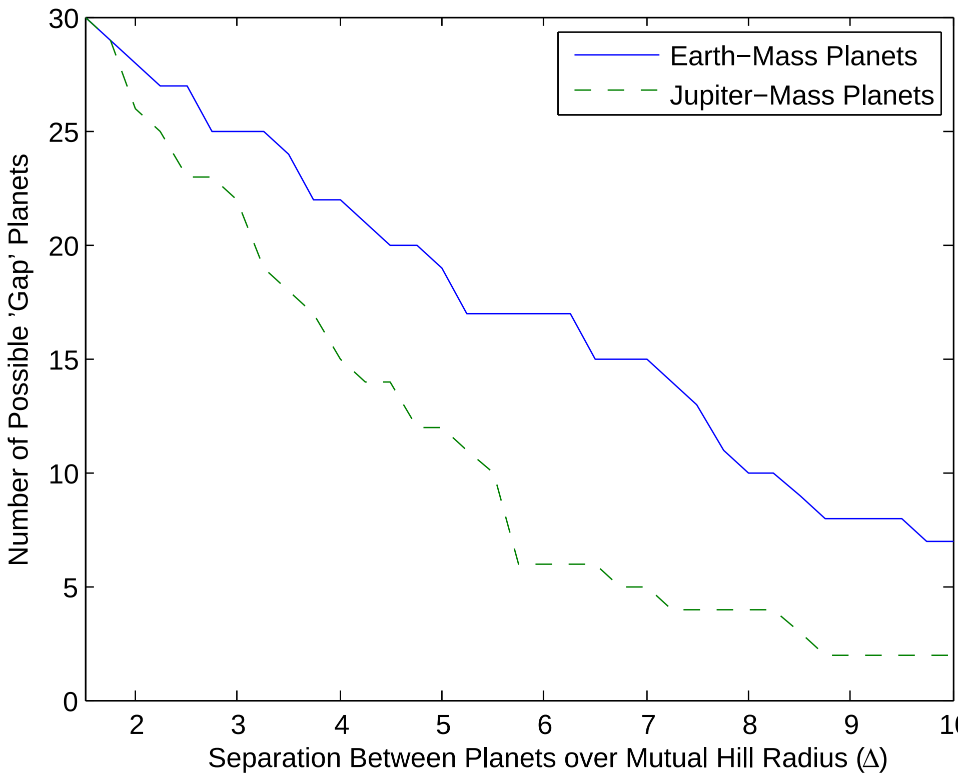

Figure 2.5: The total number of possible gap planets in systems with confirmed planets, as functions of the required Δ-factor for planet-planet separation. Lines are shown both for Earth-mass and Jupiter-mass gap planets. No line is shown for $0.3M_{\oplus}$-planets, as the numbers were identical to the case with $1M_{\oplus}$. The total number of gaps found in habitable zones were 32.

2.1.3 Multiple Moons

Many of the planets seemed to be able to support several moons, which for example could be seen earlier in Table 2.1. The total number of habitable moons, when using Δ=5, was 193, all orbiting around a total of only 18 planets (as can be seen in Table 2.1). However, the number of moons were clearly dependant on the requirement on the separation between moons. In Figures 2.6, 2.7 and 2.8, the fraction of all confirmed planets that could support 1, 2, 3, 5, and 10 moons is displayed, as a function of the required Δ-factor for the separation between the moons.

When placing moons of mass $0.3M_{\oplus}$, all moon/planet mass ratios were larger than 10^{-5}. The highest occuring ratio was 1/3, as this was the limit of what was allowed.

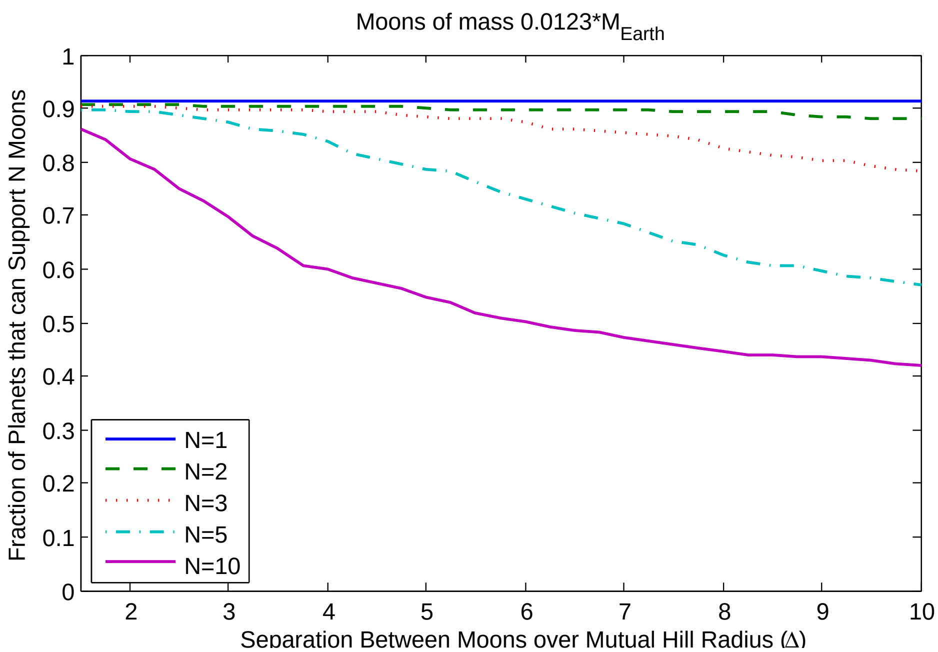

Figure 2.6: The fraction of all confirmed (1491) planets that can have N moons of mass 0.0123 $M_{\oplus}$, as a function of the required Δ-factor for the separation between the moons. All planets with $\Delta \leqslant 5$ relative to any neighbour were considered unable to have moons.

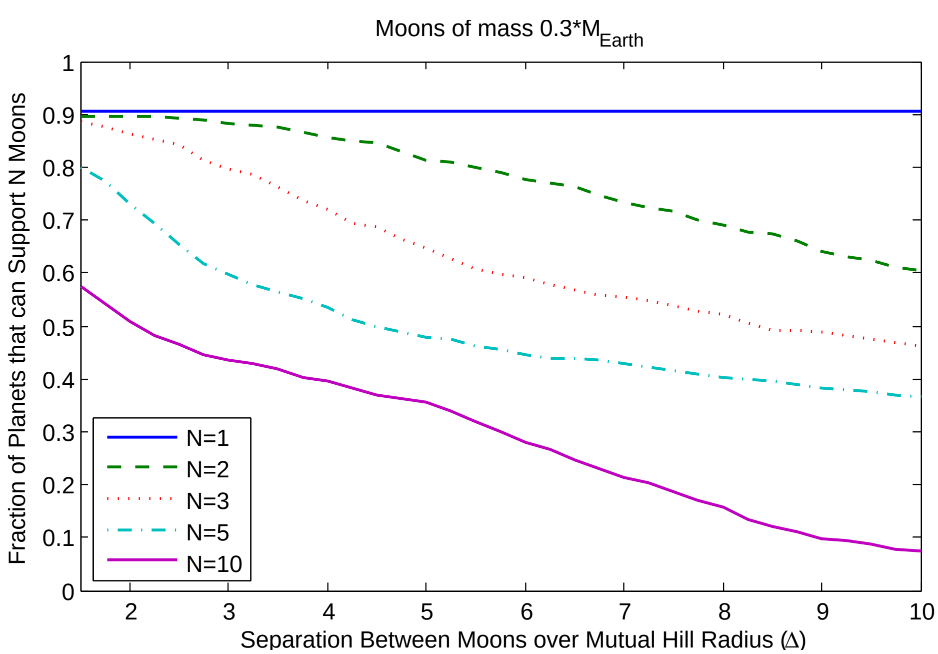

Figure 2.7: The fraction of all confirmed (1491) planets that can have N moons of mass 0.3 $M_{\oplus}$, as a function of the required Δ-factor for the separation between the moons. All planets with $\Delta \leqslant 5$ relative to any neighbour were considered unable to have moons.

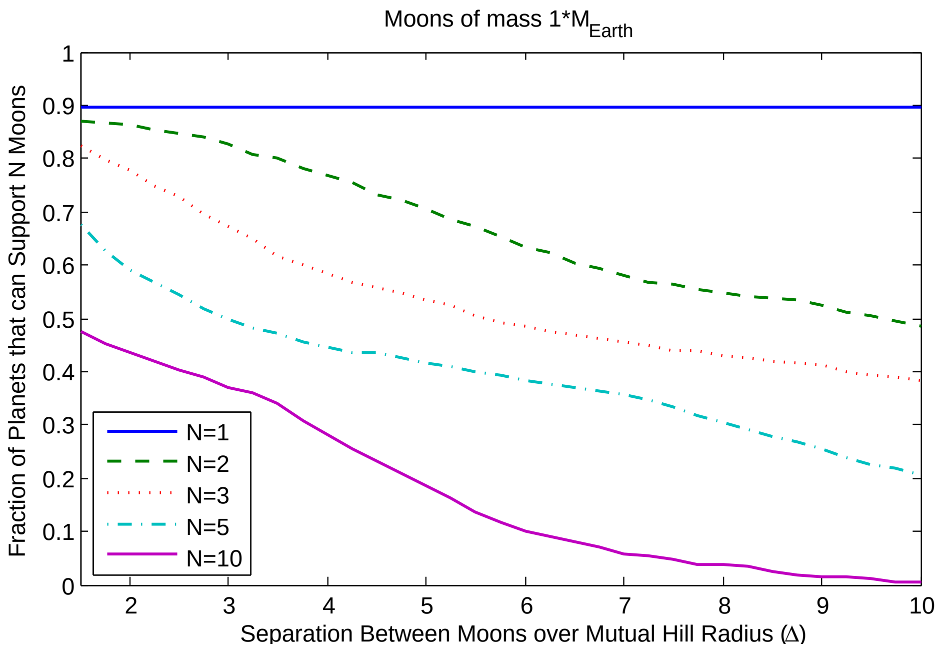

Figure 2.8: The fraction of all confirmed (1491) planets that can have N moons of mass 1 $M_{\oplus}$, as a function of the required Δ-factor for the separation between the moons. All planets with $\Delta \leqslant 5$ relative to any neighbour were considered unable to have moons.

2.1.4 Applying the Approach to the Solar System

When applying the same approach to the Solar System, as has been done to the exoplanet systems, Earth and Mars were found to be in the HZ, which stretched from 0.99 to 1.67 AU. A hypothetical gap planet of $1M_{\oplus}$ could fit between Earth and Mars. Specifically, an $0.3M_{\oplus}$-planet would have an optimal orbit with semi-major axis of 1.22 AU, yielding $\Delta = 16.69$, and a $1M_{\oplus}$-planet would have an optimal semi-major axis of 1.20 AU yielding $\Delta = 13.33$. A Jupiter-mass object could maximally get a Δ of 2.24. A table of the results, with possible number of moons for the different planets, can be seen in Table 2.2.

Table 2.2: The results when applying the approach to the solar system. The first four columns were taken from `http://solarsystem.nasa.gov/` at 2014-02-27, and the last four were calculated. The number of possible moons displayed are for the case of separations between the moons corresponding to $\Delta = 5$.

Planet	Semi-Major Axis (AU)	Orbital Eccentricity	Mass of Planet ($M_\oplus$)	Smallest Δ-factor Relative to Neighbours	Number of Possible $0.0123 M_\oplus$ Moons	Number of Possible $0.3 M_\oplus$-Moons	Number of Possible $1 M_\oplus$-Moons
Mercury	0.387098	0.20563	0.055292949	47.48473331	1	0	0
Venus	0.723327	0.0067	0.815044397	24.26545798	4	0	0
Earth	1.00000261	0.01671123	1	24.26545798	4	1	0
Mars	1.523679	0.093315	0.107473138	14.30073172	1	0	0
Jupiter	5.204267	0.048775	317.9068315	6.5182043	40	14	9
Saturn	9.5820172	0.055723219	95.18451355	6.5182043	29	10	7
Uranus	19.22941195	0.044405586	14.53570633	11.97313834	18	6	3
Neptune	30.10366151	0.011214269	17.1511623	12.40165246	20	6	4

2.2 Discussion

Out of all of the results, the one that can be considered most interesting is that 18 confirmed planets were found to possibly have habitable moons. These were all except one of the 19 confirmed planets found in habitable zones. Out of all 1491 confirmed planets, over 90 % seemed to be able to have at least one moon of habitable size, which can be seen in Figure 2.7. With the requirement of $\Delta \geqslant 5$, about 50 % of planets even seemed to be able to have 5 or more.

This can immediately be determined to be very optimistic numbers, mainly by comparing with the results obtained when applying the same approach to the Solar system. It does for example show that Jupiter would be able to support 40 moons of mass equal to that of Earth's moon, while in reality it has four of roughly that mass. Similarly, the results show that Earth would be able to have 4 moons of the same mass as the single one it has.

This illustrates that what has been found says very little of how common exomoons actually are. This is especially true for heavier moons, of masses $0.3M_\oplus$ and $1M_\oplus$, of which the solar system has none, while the analysis shows that it could have 37 and 23 respectively. What has been found in this investigation should in the end only be considered a very generous upper limit for the number of moons.

Also, the number of gap planets is also fairly optimistic, being an upper limit. Not much comparison can be made with the solar system in this case, but one hypothetical planet could be fitted into the HZ between Earth and Mars, which of course is not there.

2.2.1 Underestimations of Required Separations

The main requirement used for separations, both between planets and moons, was $\Delta \geqslant 5$. For a mass ratio of 10^{-3}, which turned out to be typical for $0.3M_{\oplus}$-moons in exoplanet systems, this yields a stability timescale of 10^8 orbits. This is quite short for many of the hypothetical moons. For the confirmed planets in habitable zones, the orbital periods of the moons at the edge of the Roche lobes ranged from around one month to one year, while moons further in could have orbital periods down to around 10^{-2} days.

With a period of one year, 10^9 orbits would be required before reaching 1 Gyr, the assumed required time for life to arise. A period of one day would similarly require $\sim 3 \cdot 10^{11}$ orbits, and a period of 10^{-2} days would require as much as $\sim 3 \cdot 10^{13}$ orbits. For the latter of these, a Δ-factor of around 7 would be required to remain stable for this long (using Equation 1.7 and the coefficients derived from the work by Marzari & Weidenschilling (2002)). For smaller moon/planet mass ratios, such as 10^{-5}, even larger Δ-factors would be required ($\Delta \gtrsim 10$, Davies et al. 2013). However, the stability timescales for Δ-factors larger than 6 are only extrapolations of the results of Marzari & Weidenschilling (2002) and Davies et al. 2013).

In any case, it can safely be said that using a Δ-factor of 5 has yielded an overestimation of the number of moons. From Figures 2.6, 2.7, and 2.8, it can be seen how the number of moons goes down with increasing requirement on the Δ-factors. However, even using the much more conservative requirement $\Delta \geqslant 10$ still yields that a majority ($\sim 60\%$) of all confirmed planets can have two or more moons of habitable size each. Also, Jupiter could still have 20 moons of the mass of Earth's moon, showing that the numbers still are much higher then in reality.

The fraction of all planets being able to have 1 moon is of course independant of the requirement on moon-moon separations.

When calculating Δ-factors for neighbouring planets at exoplanets.org, these were found to range from negative values (corresponding to overlapping orbits), to above 50. No clear 'cutoff' limit was found, below which the number of planets would expectedly drop due to orbital instability. For example, around 30% of all confirmed planets in multiplanetary systems had Δ-factors smaller than 5. This indicate that the separations in terms of mutual Hill radii is not the only determining factor for whether orbits can be stable. More details such as eccentricity and inclination must be taken into account properly. A small number of planets would be expected to have orbits with separations such that any Δ-factors would be small, but as the time before a close encounter is such a strong function of Δ (Equation 1.7), these would be expected to become unstable so quickly that only very few should be around at any given time.

2.2.2 Capacities of Planets to have Massive Moons

The very generous limit of allowing the moons around a planet to have a combined mass of up to 1/3 of the mass of the planet itself, meant that this was rarely a limiting factor for the number of moons. However, in reality it must surely play a role. Out of the planets in the solar system, Earth is the one with the most massive collection of moons, relative to its own mass (even if it is just 1 moon). In this case, the mass of the moon is 0.0123 of that of the planet itself. In comparison, the combined mass of Jupiter's moons is around $2 \cdot 10^{-4}$ of that of Jupiter. This would suggest some more strict limit as to how massive moons can be relative to their planets. The physical reason for this could be some destabilizing phenomenon occuring either during or after planet formation, which primarily effects heavy moons. However, even if this is a limiting factor, the mass ratio of the Earth and its moon would be similar to Jupiter and a $3M_{\oplus}$-moon, meaning moons of habitable sizes could still be very possible.

2.2.3 Comparison of Results for Confirmed and Unconfirmed Planets

When compairing the Figures 2.3 and 2.4, some interesting differences can be seen. They show multiplanetary systems, with the discovered planets in the HZ, and the hypothetical gap planets in the HZ respectively. However, the first mainly contains unconfirmed (KOI) planets, while the second primarily contains confirmed ones. Also, the systems with unconfirmed planets have almost no planets further out than the HZ, while many of the systems with confirmed ones do.

Similarly in Figure 2.1, the ratio between the numbers of confirmed and unconfirmed planets remain roughly the same in most of the steps of the investigation, but not all. From this it is clear that something differs between those two groups of planets, more than just their status as confirmed and unconfirmed.

That unconfirmed planets seem to be underrepresented in Figure 2.4, among planets with habitable gaps in between them, can be explained by considering the detection method used to find them. In order for a system to be able to have a gap in the habitable zone, at least one discovered planet has to lie in the HZ or further out. Finding planets far from a star is fairly difficult when using the transit method, which is what has been used for the vast majority of the unconfirmed planets. See Appendix C for details on the method. It is because of this that relatively few gap planets has been found between unconfirmed planets.

On the other hand, when just looking for discovered planets in the habitable zone, there is no requirement of having any additional discovered body any further out than the planet in question. This then allows many more of the systems with unconfirmed planets to qualify.

One other instance where unconfirmed planets are clearly overrepresented, is in the group of planets having orbits partially in the HZ (on the 'left path' in Figure 2.1). This is because the unconfirmed planets were all missing values for their eccentricities, which is then taken to be zero. For such orbits, laying partially in the HZ means laying inside it completely, as the star-planet distance does not vary. As can then be expected, the ratio between confirmed and unconfirmed planets is shifted back when moving on to the next step, where only planets with orbits entirely in the HZ remain.

2.2.4 Accuracy of Habitable Zone Ranges, and Comparison with The Habitable Zone Gallery

The list of confirmed planets in habitable zones (Table 2.1) was compared to a similar list at The Habitable Zone Gallery (`http://www.hzgallery.org/`, 2014-05-21). The HZ ranges presented at the site are based the Runaway Greenhouse model and Maximum Greenhouse model as presented by Kopparapu et al. (2013A), just like in this investigation. It was found that the lists were similar, but differing by three planets. Two bodies, GJ 667 C d and GJ 667 C e are on the list at hzgallery.org, while they were not found to lie in the habitable zone in this investigation. The reason for this is missing data at exoplanets.org, more thoroughly discussed under Section 2.2.5. More specifically, the radius of the star GJ 667 C was missing.

Similarly, the planet HD 603607 c is not found in the list at hzgallery.org, while it was found to lie in the HZ of its star in this investigation. This can be explained by the fact that the entries at hzgallery.org are calculated using updated coefficients for the equation presented by Kopparapu et al. (2013A);

$$S_{Eff} = S_{Eff,\odot} + aT_* + b(T_*)^2 + c(T_*)^3 + d(T_*)^4 \tag{2.1}$$

The updated coeficients were presented in an erratum to the original paper (Kopparapu et al. 2013B), and lead to HD 603607 c being outside the HZ. However, it also results in Kepler-186 f lying outside the HZ, which it should not according to hzgallery.org. The reason for this is probably some inaccuracy in the value for the eccentricity of that planet's orbit, as the entry at exoplanets.org only had one significant digit as opposed to two or more like most entries.

In any case, the limits of the habitable zones from the work by Kopparapu et al. (2013A) should not be considered to be exact, and were arrived at by considering only Earth-mass planets. Since then, Kopparapu et al. (2014) has investigated the dependence of habitable zone ranges on the mass of a planet, finding that lighter planets generally have zone limits further from the star. The highest possible stellar flux for a habitable planet around a Sun-like star, differs by more than 10% between a $0.1M_{Earth}$-planet and a $1M_{Earth}$-planet.

In their original paper, Kopparapu et al. (2013A) also mention the possibility of liquid water having been present on both Mars and Venus in the past, based on which they also present two empirical models that are much more generous than the Runaway Greenhouse model and the Maximum Greenhouse model. These were modified for the specific purpose of putting the orbits of Mars and Venus, as they were in the past when they might have held water, inside the HZ. While the accuracy of these models can be questioned, their existence makes it clear that the actual limits of habitable zones may be more or less different from what is yielded by the Runaway-, and Maximum Greenhouse models.

2.2.5 Missing Data

When working with the archive at exoplanets.org, missing data was sometimes an issue. A large number of planets, including almost all KOI:s, were missing eccentricity. This was no direct obstacle in calculations as it was then assumed to be zero, but most probably contributed to an overestimation of the number of planets in the habitable zone. Out of the confirmed planets, a total of 101 planets, in addition to the 19 presented, had parts but not all of their orbits in the HZ, illustrating that taking eccentricity into account is important. In the end however, this investigation was focused mostly on the confirmed planets, where this was only a minor issue. Still, as can be seen in Table 2.1, two confirmed planets found to lie in habitable zones were missing eccentricity, and a third had the entry 0, which is most probably not exact.

More important data, such as stellar radius and stellar effective temperature, was also missing for some systems. Excluding those systems was the first step of the investigation, as can be seen in Figure 2.1. It affected a total of 228 planets, and amongst them excluded GJ 667 C d and GJ 667 C e mentioned in the previous section.

2.2.6 Lower Mass Limit for Habitable Bodies

The lower limit for the required mass of a habitable body is very uncertain, as discussed in Section 1.2. That plate tectonics has to be present is strictly speaking not known, and disregarding this issue leads to lower limits. Williams, Kasting & Wade (1997) present the limit of $0.07M_{\oplus}$, taking only loss of gas by means of thermal escape into consideration. Thermal escape is where the Maxwell-Boltzmann distribution of velocities in the gas leads to a small but significant fraction of the particles in the atmosphere receiving a high enough velocity to escape the planet.

At the same time, Valencia et al. (2007) suggest that for plate tectonics to actually take place, the mass of the object would almost have to be as high as that of the Earth. Meaning, that if plate tectonics is required, Earth itself may be in the lower end of the range of allowed masses. These widely different limits illustrate the uncertainty of what actually qualifies as a habitable planet, or moon.

2.2.7 Moons in very Small or Large Orbits

The inner and outer limits that were used for the semi-major axes of moons around planets should probably be considered quite optimistic. As discussed earlier in Section 2.2.1, the short orbital periods of moons on very small orbits could lead to instabilities, unless the separations between the moons correspond to Δ-factors much larger than 5. Mean-motion resonances between moons could also have greater effect for smaller orbits, leading to the innermost moon receiving an eccentric orbit. This results in greater tidal forces on an inner moon as it is pulled between the planet and the outer moons. This in turn can cause its surface to become unstable and therefore less suited for life. In less severe cases, it can simply make the moon more geologically active, and therefore warmer. This is the case of Io around Jupiter. It is in a 2:1 mean-motion resonance with the moon Europa, and a 4:1 mean-motion resonance with the moon Ganymede, and has thus obtained a highly eccentric orbit and become the most vulcanically active body in the Solar System. The heating due to tidal forces on such a body could mean that the habitable zone would lie further from the star. At the same time, planet eclipses shadowing the moon would be more common for moons in small orbits, contributing to a colder climate (Forgan & Kipping, 2013).

Also, the magnetic field of the host planet can erode away an atmosphere of moons in small orbits (Kaltenegger, 2000), which would rule out life.

Very large orbits could also be unfavourable, as variations in star-moon distance could cause greater temperature variations. For example, the largest possible orbit around Jupiter, that at the edge of the Roche lobe, would have a semi-major axis of around 0.24 AU. Also, Barnes & O'Brien (2002) indicate that orbits around planets only can be stable out to 1/3 of the Hill radius, which is closely related to the Roche lobe radius used as an outer limit here.

It could also be the case that the Roche lobe of a planet would reach outside the HZ, even if the planet itself remains inside it, meaning moons orbiting at the edge would themselves leave the HZ periodically. However, this did not occur for any of the moons in this analysis.

Chapter 3

Conclusions

Based on the results of my investigation, exomoons seems to be fairly common, as almost all of the considered planets were able to support at least one. The total number of possible habitable moons was found to be 193, however these were all distributed amongst just 18 planets. Out of all 19 confirmed planets in habitable zones, only one was deemed unable to have moons.

The largest limiting factor for where potential moons could be, was the number of discovered planets in habitable zones around which to place them. The considerations regarding the minimum mass for a habitable body, and the required separation between planets and moons, made relatively little difference in the end as most planets could have moons. This remained true even for moons of the mass $1M_{\oplus}$, and when the requirement on the separations beteween bodies was as strict as $\Delta \geqslant 10$.

When applying the approach to the solar system, it was clearly seen that it yields a number of moons much larger than what is actually present. This clearly shows that what has been found can only be considered an upper limit for how many moons can fit in a system, but how valid it is as such is also questionable.

What has been considered in this investigation is in many ways an ideal case, where all moon orbits were specifically selected for maximal stability. Several factors were not taken into account, which could have set additional constraints on the orbits of the moons. Mean-motion resonances could potentially destabilise moons in small orbits, where the magnetic field of the host planet could also strip the moon of its atmosphere. Similarly, moons in large orbits could also become unstable, and the variation in star-moon distance would lead to large temperature variations that could be hazardous to life. Also, it was found that separations between moons would in many cases have to be larger than what was assumed here.

Another relevant limitation could be that the combined mass of the moons around a planet should not be allowed to be as high as 1/3 of the mass of the planet itself, as it was allowed in this investigation. It can for example be seen that in the solar system, no moons exist that are nearly this massive relative to their host planets.

How common exomoons actually are is still an open question. To answer it, more details such as the ones mentioned above should be taken into account when detemining where moons could have stable orbits. However, a more interesting approach would involve looking at the formation of planetary systems, which could show how moons actually end up where they are. Of special interest is how common it is for heavy moons to form, and for them to avoid collisions and go into stable orbits.

In any case, with recent advances in the methods of detecting extrasolar objects, actual discoveries of exomoons could soon be a reality.

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Appendix A

Luminosities and Radii of Zero-age-, and Terminal Main-Sequence Stars

For zero-age main-sequence stars, formulae for the luminosity L and radii R are presented by Tout et al. (1996). These are based on model simulations of stellar evolution, for which analytic expressions involving stellar mass M and stellar metallicity Z were fitted to the results. All variables (L, R, M) are in solar units.

$$L_{ZAMS} = \frac{a_1 M^{5.5} + a_2 M^{11}}{a_3 + M^3 + a_4 M^5 + a_5 M^7 + a_6 M^8 + a_7 M^{9.5}} \tag{A.1}$$

$$R_{ZAMS} = \frac{a_8 M^{2.5} + a_9 M^{6.5} + a_{10} M^{11} + a_{11} M^{19} + a_{12} M^{19.5}}{a_{13} + a_{14} M^2 + a_{15} M^{8.5} + M^{18.5} + a_{16} M^{19.5}} \tag{A.2}$$

Similarly, formulae for the luminosity and radii of terminal main-sequence stars were presented by Hurley et al. (2000), also based on stellar evolution model simulations.

$$L_{TMS} = \frac{a_{17} M^3 + a_{18} M^4 + a_{19} M^{a_{22}+1.8}}{a_{20} + a_{21} M^5 + M^{a_{22}}} \tag{A.3}$$

$$R_{TMS} = \frac{a_{24} + a_{25} M^{a_{27}}}{a_{26} + M^{a_{28}}} \qquad M \leqslant a_{23} \tag{A.4}$$

$$R_{TMS} = \frac{c_1 M^3 + a_{29} M^{a_{32}} + a_{30} M^{a_{32}+1.5}}{a_{31} + M^5} \qquad M \geqslant a_{23} + 0.1 \tag{A.5}$$

A linerar interpolation should be used to connect eqs A.4 and A.5. All a_n are polynomials of the logarithm (of base 10) of the metallicity, up to eighth degree, and c_1 is a constant. The values they take on for a star of metellicity equal to that of the sun, Z=0.0122, can be seen in Table A.1. See Table 1 of Tout et al. (1996), and Appendix A of Hurley et al. (2000) for details on the coefficients of the polynomials for a_n (Note however that the coefficients' names have been changed here). The radius and luminosity are plotted as functions of stellar mass in Figure A.1.

Table A.1: Coefficients for the equations A.1, A.2, A.3, A.4, and A.5, to use for calculating the luminosity and radius of stars with metallicities equal to that of the sun, Z=0.0122. The coefficients are in reality up to eighth-degree polynomiels of the logarithm (of base 10) of the stellar metallicity.

Coefficient	Value	Coefficient	Value	Coefficient	Value
a_1	0.4802	a_2	16.0139	a_3	0.0005
a_4	7.7668	a_5	8.4969	a_6	1.6758
a_7	0.0111	a_8	1.5500	a_9	6.2306
a_{10}	10.3855	a_{11}	0.9422	a_{12}	0.0728
a_{13}	0.0108	a_{14}	2.8036	a_{15}	17.5685
a_{16}	0.0008	a_{17}	4593.6	a_{18}	3986.2
a_{19}	786.6681	a_{20}	3330.7	a_{21}	233.6317
a_{22}	7.0928	a_{23}	1.3999	a_{24}	7.1487
a_{25}	18.0594	a_{26}	13.5779	a_{27}	1.9620
a_{28}	3.8768	a_{29}	2.3983	a_{30}	0.0074
a_{31}	1.1206	a_{32}	5.5209	c_1	$-8.672073 \cdot 10^{-2}$

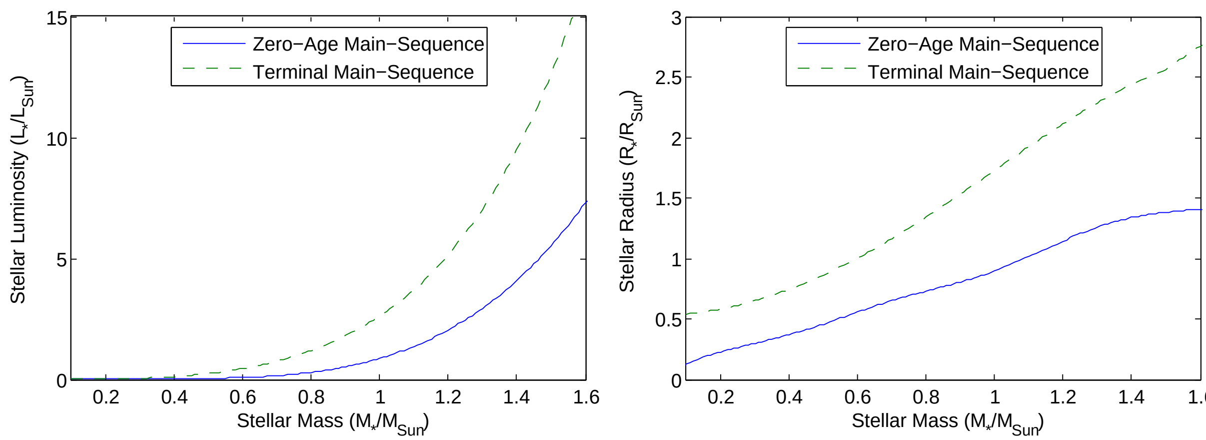

Figure A.1: Luminosity and Radius of stars with metellicity equal to that of the sun, Z=0.0122, as functions of stellar mass. Results are shown for both Zero-Age Main-Sequance and Terminal Main-Sequence stars, obtained through equations A.1, A.2, A.3, A.4, and A.5.

With the radius and luminosity obtained, the effective temperature of a star T_{Eff} can be calculated, using Stefan-Boltzmanns law:

$$L = 4\pi R^2 \sigma T_{Eff}^4 \implies T_{Eff} = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4} \tag{A.6}$$

where σ is the Stefan-Boltzmann constant, $5.670373 \cdot 10^{-8}$ W m^{-2} K^{-4}, T_{Eff} is in K, L is in W, and R is in m. In Figure A.2 the effective temperature T_{Eff} is shown as a function of stellar mass.

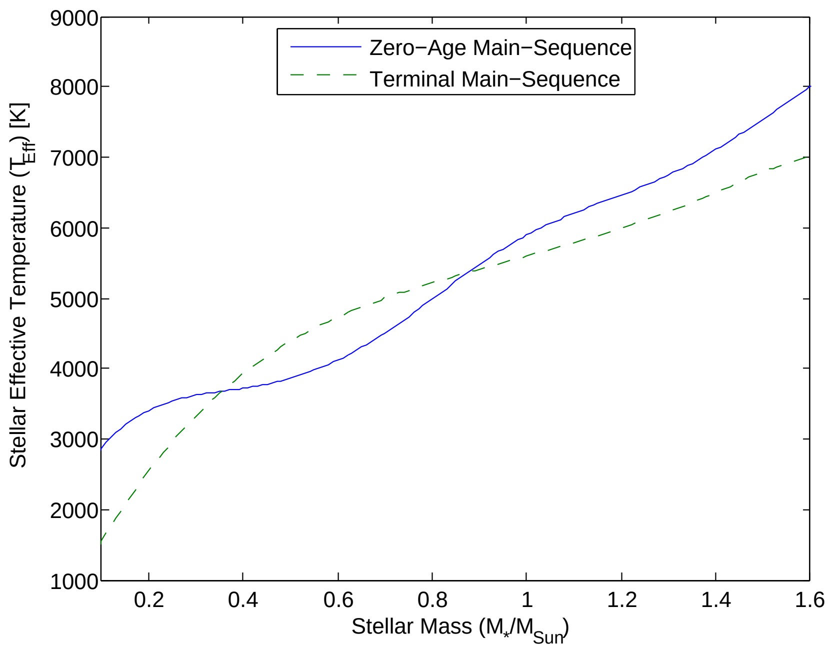

Figure A.2: Effective temperature T_{Eff} of stars with metellicity equal to that of the sun, Z=0.0122, as a function of stellar mass. Results are shown for both Zero-Age Main-Sequance and Terminal Main-Sequence stars, obtained through equations A.1, A.2, A.3, A.4, A.5, and then A.6.

Then, with the effective temperature and luminosity obtained as functions of stellar mass, the habitable zone ranges can be calculated as a function of stellar mass, using Equations 1.1 and 1.2. This is what is plotted in Figure 1.2.

Appendix B

Analytic Solution for the Optimal Semi-Major Axis of Hypothetical 'Gap' Planets

When placing a hypothetical 'gap' planet between two known objects in a system, the orbit of highest possible stability is the one of greatest interest. This is achieved when the orbit is circular, and the semi-major axis a_{hyp} is such that the Δ-factors for the separation with the inner and outer neighbour respectively, are equal. The equation to be solved for the optimal semi-major axis is then as follows:

$$\Delta_{1,hyp} = \Delta_{hyp,2} \implies \frac{a_{hyp} - r_{1,max}}{R_{Hill,1,hyp}} = \frac{r_{2,min} - a_{hyp}}{R_{Hill,hyp,2}} \tag{B.1}$$

It has an exact, analytic solution, but one that is rather long and cumbersome to obtain. However, writing out the definition of the mutual Hill radii, and rearranging finally yields

$$\frac{a_{hyp} - r_{1,max}}{R_{Hill,1,hyp}} = \frac{r_{2,min} - a_{hyp}}{R_{Hill,hyp,2}} \implies \frac{2(a_{hyp} - r_{1,max})}{(a_{hyp} + r_{1,max})\left(\frac{M_1+M_{hyp}}{3M_{star}}\right)^{1/3}} = \frac{2(r_{2,min} - a_{hyp})}{(r_{2,min} + a_{hyp})\left(\frac{M_{hyp}+M_2}{3M_{star}}\right)^{1/3}}$$

$$\begin{aligned} \implies a_{hyp} = \Bigg(\bigg((r_{2,min} + r_{1,max})^2 \tilde{M}_1^2 + (-2\tilde{M}_2 r_{2,min}^2 + 12 r_{1,max}\tilde{M}_2 r_{2,min} - 2r_{1,max}^2\tilde{M}_2)\tilde{M}_1 + \\ + (r_{2,min} + r_{1,max})^2\tilde{M}_2^2\bigg)^{1/2} + \\ + (r_{1,max} - r_{2,min})\tilde{M}_1 + \tilde{M}_2 r_{2,min} - r_{1,max}\tilde{M}_2\Bigg) \Bigg/ (2\tilde{M}_1 + 2\tilde{M}_2) \end{aligned} \tag{B.2}$$

where

$$\tilde{M}_1 = \left(\frac{M_1+M_{hyp}}{3M_{star}}\right)^{1/3} \quad \tilde{M}_2 = \left(\frac{M_{hyp}+M_2}{3M_{star}}\right)^{1/3} \tag{B.3}$$

Appendix C

Methods for Discovering Exoplanets

When discovering exoplanets, there are mainly two methods that are used. These are the Radial Velocity method (RV-method), and the Transit method.

The RV-Method

In a system with a star and planets, all bodies will move around the common center of mass. Due to it being much heavier, the star itself will often move much less than any of the other objects. Still, this motion can be observed, which is what is used in the RV-method. The radial velocity of a star, relative to an observer on Earth, is measured by considering the displacement of absorption lines in the spectrum of the star, due to the doppler effect. The heavier the planet, and the closer it is to its star, the more the star itself will move. Thus, the method is best for finding such planets. Until a few years ago, the RV-method was the method responsible for the largest number of discovered exoplanets. It can detect planets in all types of orbits except those that are in a plane perpendicular to the line of sight from the observer. This is because for such orbits, the radial velocity of the star is zero. A disadvantage however, is that when considering the shift of an individual line in a spectrum, only a small portion of the light can be used, which in practice means that the level of noise has to be fairly low. Therefore, the method is mostly used for nearby stars.

The Transit Method

The transit method is based on observing a star during a longer period of time, to detect if its brightness decreases periodically. If such a thing occurs, it can be bacause of a planet passing in front of the star, as it moves in its orbit. The method has recently become responsible for a very large amount of discovered planets, since the launch of the Kepler Spacecraft. It continually monitors 136000 stars (kepler.nasa.gov, 2014-05-23), and after having observed three transits in a system, the discovery is counted as an unconfirmed Kepler Object of Interest (KOI). Usually, an RV-observation is required for confirmation.

The main disadvantage of the method is that it can only discover planets with orbits in a plane almost perfectly aligned with the observer's line of sight. The further the planet is from the star, the less inclined the plane can be in order for the planet to pass in front of the star. Also, as planets further from their stars have longer orbital periods, the star has to be monitored for longer to detect them.

The Kepler Mission has not been going on for long enough to find exoplanets with orbital periods much longer than a year. The semi-major axis of such a planet, orbiting a star of $1M_{Sun}$, would be 1 AU, so for sun-like stars few planets have been found much further out than this.

Unlike in the RV-method, all of the light from a star can be used for a detection. Therefore it is used at much greater distances.