Pricing and Hedging of swing options in the European electricity and gas markets

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Abstract

This report outlines a method to price and hedge a generalized swing option based on the European natural gas and electricity markets. The method is *model free* in that is does not assume a certain spot price dynamics. It only requires a *forward curve* and *European call options* prices that cover the delivery period of the swing option. The results is a lower bound derived from the price of forwards and call options on the market.

A method to approximate the *Greeks* of the swing option by evaluation on the lower bound is suggested. This approximation is based on a *Finite difference* method. Two hedges are constructed from the information. The first hedge is based on the weights of *forwards* and *call options* from the calculation of the lower bound. The second hedge is a *Greek* neutralizing hedge.

We conduct an empirical study on the lower bound. A Delta and Delta-Gamma Hedge is evaluated and we find the *Greeks* of the lower bound to be a blunt approximation of the swing option uncertainties. The evaluation reveals that the lower bound approximation effects the dynamics and hence the *Greeks* of the swing option. In addition the *Finite difference* method is unstable in its approximations, especially for finer granularity.

The lower bound is also compared to an existing Least Square Monte Carlo (LSMC) method. It is much faster than the LSMC and a price comparison give inconclusive results. Additional studies reveal a pricing defect lie within the much more complex LSMC. Finally a granularity study is implemented. Finer granularity increases the price of the lower bound slightly but the effects on the *Greeks* are more significant. Whether this effect on the *Greeks* is due to changes in the swing option or due to better approximations with the lower bound cannot be concluded.

Contents

Chapter 1

Introduction

The purpose of the thesis is to price a generalized swing option based on natural gas or electricity. It should also investigate methods for hedging the risks in a swing options, and evaluate how the specification of a Swing contract affect price and uncertainty. These specification include *strike price*, *Swing volume* and *granularity* in *exercise periods*. The thesis will also compare the results and performance to a Least Square Monte Carlo method[Kluge, 2006].

Due to the underlying supply and demand structure of natural gas and electricity consumer and producers are faced with volumetric risk in their undertakings. The stochastic nature of the demand structure in both natural gas and electricity means that agents want to have optionality in their volumes. The spot price market being dependent on the same demand structure experience price spikes in periods of low supply - high demand. Hence agents want to avoid being dependent on spot purchases in these time periods. At the same time the markets are becoming more dependent on seasonal production methods, such as wind power, while unseasonal production methods, fossil fuels and nuclear, is being phased out. The outlook is thus that the supply structure will be more effected by volumetric risk. Standard contracts in the markets, such as the forwards, fail to provide coverage for volumetric risk. Swing options has grown popular as a way of hedging these risk. The properties of the swing option is useful for hedging both price risk and volumetric risk over long time intervals.

The swing options is an exotic derivative which gives the holder the right to buy an underlying asset at a fixed *strike price* at several points in time. The holder also has the right to choose a dynamic volume at each point in time. In addition the holder is obligated to exercise within a predetermined interval of both continuous volume and cumulated volume. This interval is referred to as the *Swing* where the derivative draws its name from.

Commodities differs from the standard financial market in an important aspect. The underlying asset is a real physical asset which is produced, delivered and consumed. Most commodity market have recently been deregulated and as a result there are still few players and the products and market platform are non-standardized. The short term effect is seen on the spot price, such as jumps, seasonality etc. At the same time the price of energy, in its general form, is deduced from the world market price of crude oil. To comprehensively model electricity and natural gas prices complex dynamics and dependencies is required. Existing valuation techniques based on these dynamics rely on Monte Carlo simulation and deterministic numerical methods. In addition, the complex spot price dynamics rely on many parameter estimations to be performed. Simple spot dynamics fail to capture the characteristics of the market and consequently they are poor stepping stones in working our way up to more complex dynamics. Hence the first step from no model to a complex model is very big, increasing the risk for estimation errors.

This report uses an approach in [Keppo, 2004] to price and hedge swing options. Keppo assumes no specific spot price dynamics and proves that the swing options can be priced in terms of *forwards* and *european* call options. Further on he finds a lower boundary which only depends on a *Forward curve* at time t. This *forward curve* can be constructed using existing contracts in the market. This lower boundary is independent of spot price dynamics. The *forward* and *call options* can be used to hedge the swing option. Hence the induced complexity of a comprehensive spot price dynamics is eliminated. In addition, the method is solved using linear programming which is much faster than any simulation based approach.

Chapter 2 explains the characteristics of gas and electricity both as a physical asset (commodity) and on the market. Both gas and electricity exhibit many different characteristics which, if all should be incorporated in a comprehensive method, result in a large complex model. The chapter focuses on the problems this complexity causes and tries to emphasize the need for a simpler model. It goes on to outline existing models for pricing derivatives on both gas and electricity.

Chapter 3 defines the two main assumptions in this report. Further on it discusses and enforces these assumption in relation to practical markets. This chapter also outlines a set of definitions used throughout the report.

Chapter 4 outlines the Keppo framework for pricing swing options. The main advantage with the Keppo model is that is does not assume a certain price dynamics. We refer to it as model-free. By not assuming this price dynamics we alleviate any pricing and hedging from the high level of complexity in a model-based framework. First a method for pricing the swing option is briefly discussed. More detail is put into describing Keppo's lower bound which is the main topic of this report.

Chapter 5 derives a solution to Keppo's lower bound. It defines the two types of errors we expect to find in the lower bound approximation. Then it goes on to present a discretized solution to Keppo's lower bound which could be solved as *linear optimization problem*.

Chapter 6 present a methods for Risk Management using the lower bound. The classical *Greeks* are defined and a *Finite difference* estimation technique is derived. Two different hedges are presented. First assuming there is a complete set of standard contract in the market we show that up to three *Greeks* can be neutralized in a *Greek Hedge*. Then a *Replicating portfolio* is constructed which replicate the expected outcome of the lower bound.

Chapter 7 uses an Empirical Study to test the methods presented in the previous chapter. We test the *Finite difference* method, the *Replicating portfolio*, the *Greek Hedge* and the output of the *Linear Programming* solver. The study is divided into four sub studies, a daily and monthly evaluation of the *Greek Hedge*, a comparison study with a Least Square Monte Carlo method (LSMC), and a granulation study which looks at the effects of the number and length of the exercise rights in the swing option. The aim of the study is to the test the general stability, performance and output of the lower bound approximation. It also aims to find out specific questions, like the significance of the approximation errors.

1.1 Limitations

This report try to evaluate the swing option from the perspective in Keppo's framework. We will define limitations that form the scope of this report.

There is no standardized definition of a Swing Contract and many versions exists. To include all of the definition in to a comprehensive model would make our evaluation blunt, if it is even possible. We will limit ourselves to a swing option defined on fixed deterministic *strike*, cumulative and continuous constraints. In addition we limit ourselves to a penalty free swing option, i.e. the consumption constraints can not be violated by paying an additional penalty. It could be seen as a penalty based model where the penalties are set to infinity.

We will always assume that the holder wants to exercise optimally. In the Keppo model this optimal consumption is approximated. Hence we limit ourselves to a approximated optimal consumption strategy and do not discuss other exercise strategies. In a practical case we could for example have consumer who do not want to consume optimally since its internal demand is satisfied while the market do not offer resale opportunities.

In our evaluation of hedge strategies we will only make a suggestive discussion around a hedge strategy. No transaction cost will be assumed and hence we do not examine the trade-off between hedge rebalancing and transaction cost. As an effect the discussion regarding any rebalancing frequency will only be suggestive and not conclusive.

We will maintain focus on the Keppo framework in our evaluation. This means that when evaluating Keppo in relation to other models we will not focus on calibrating the other model perfectly to Keppo. Hence any conclusions regarding the loss or errors of the lower bound of Keppo is only suggestive and not conclusive.

Chapter 2

Markets

Natural gas and electricity are traded on several regional interconnected markets. Historically these markets where domestic and often regulated. In the last decades free trade agreements as well as infrastructural changes has softened the boundaries for these markets.

Natural gas is obtained by carriers, transnational pipelines and through domestic production. Most of the natural gas markets are based on trading hubs in this transporting network. Many of the bids taking place in these markets are virtual meaning that no actual physical commodity is delivered and the agents settle with a cash flow. Examples of markets are the NBP in the UK and TTF in the Netherlands. In this report we will use data from the NBP market and assume a swing option settlement on the UK market.

Electricity is different from natural gas in the sense that it is not a physical asset that can be stored and transported in the same sense as other commodities. However limitations in the infrastructure cause the markets to be divided geographically both internally and in between each other. One example is the Nordpool market which include all of the Nordic countries. The market is divided into 5 countries and a total of 13 geographical areas.

This chapter will describe the natural gas and electricity markets. It will also discuss implications of the characteristics in these markets on a spot price dynamics.

2.1 Electricity Market

Electricity is the most commonly used commodity in the whole world. It is used in almost every household, industry, transportation system etcetera. Infrastructure is on a global scale and is still being expanded and upgraded. The electricity grid needs to be upgraded to support growing production methods, such as wind, and to enable a more integrated market[1]. The cost of transporting electricity over vast areas are small and the transfer occurs instantaneously. On the other hand the storage capacity for electricity is almost non-existent and it must be consumed at the moment it is produced. However as future energy prices are expected to rise and more efficient technologies become available storage methods could become more wide-spread[2].

[1]Powering Europe: wind energy and the electricity grid, European Wind Energy Association

[2]`http://www.c2es.org/docUploads/10-50_Berry.pdf`

There are many different production methods for electricity, common ones are hydro power plants, nuclear power plants, gas turbines etc. Since electricity can be produced by consuming other energy commodities, e.g. gas and oil, the price of electricity in de-regulated markets is derived from the current most expensive production method. For example in the Nord Pool market, which is dominated by hydro and nuclear power production, the market price peak when extra gas and oil turbines have to be started to meet increasing demand.

2.1.1 Stylized facts: Electricity market

[Burger et al., 2007] and [Kluge, 2006] explain a number of characteristics of the electricity market. The characteristics are summarized below as stylized facts.

There are occurrences of price spikes in the markets. This is due to demand approaching maximum supply. As consumers of electricity are mostly unaware of the spot price the demand is very inelastic to prices. Hence most of the responsibility of preventing price spikes are put solely on supply. Efforts have been made to reduce this effect by so called smart grids. These are grids where the consumers are actively engaged in the spot price of electricity. For example, home monitors which warn consumers of rising electricity prices could make them turn of unnecessary equipment. Or electric devices could be connected to the internet so that they automatically turn off in times of high prices.

The demand seem to follow a seasonal pattern. This is mostly due to the cultural, infrastructural, climate factors. For example demand drops on Saturday-Sunday since industry is closed, people are on leave etc. In Israel, where the work week starts on Sundays the same effect could be seen on Friday-Saturdays. In northern countries the winters require internal heating of the houses and in southern countries the summer see rising demand due to air conditioning.

Electricity as a commodity cannot be stored efficiently. As an effect there are small buffering capacities for rising demand and risk of deficiency is higher. Since demand is seasonal and electricity cannot be stored the electricity production must follow demand. The implications is that the electricity price follow a seasonal pattern. There are also clusters of high volatility in the electricity markets. High electricity prices often occur due to high demands and therefore the risk of supply deficiency cause the volatility to rise. The spot price follow a mean-reverting pattern. Since other energy derivatives can be consumed to produce electricity the long term price of electricity would be expected to return to the production cost. In addition the spot price of electricity exhibit random size jumps. Due to non-store-ability of electricity and the random jump processes in the spot price the Market is automatically incomplete.

The Markets liquidity is bad; there are not many derivatives traded and the contracts cover small volumes. In addition the traded assets suffer from large bid-ask spreads. As an effect risk cannot be effectively hedged and any rebalancing of a portfolio is costly. This means that even though we can theoretically find a hedge to a derivative there might not be any traded assets to hedge it with.

The electricity price is long-term non-stationary. This is due to the inevitable depletion of fossil fuels. On longer time scales the electricity price should follow a general price increase since parts of electricity production is made by utilizing fossil fuels.

2.2 Natural gas market

Natural gas is a mixture of different hydrocarbon gases with methane being the most common. It is a commodity used mainly in electricity production, heating, households, and for production of industrial goods. Not as widespread as electricity it is still used around the whole globe. In the beginning of the 20th century natural gas was thought of as a by-product in oil production and due to technical and economic reasons the gas was often disposed straight away by burning. As energy prices became higher, technology evolved and environmental issues became a world wide topic natural gas has become more popular. Natural gas is considered more environmental friendly than other fossil fuels such as oil and coal since it emits less CO_2 and other pollutants.

Unlike electricity, natural gas can be stored with much smaller losses. This is most commonly done by a cooling process which converts the gas to a liquefied form, called Liquefied natural gas (LNG). Other forms exist such as Compressed natural gas (CNG). The infrastructure for natural gas transport mainly consist of carriers that ship across the oceans and pipelines over land. The cost for transporting natural gas far exceed the cost for electricity. Hence in gas markets a model often incorporate price and risk of freight and more focus is put on operational risk.

2.2.1 Stylized facts: Gas market

[Rodriguez, 2008] sums up a number of characteristics for prices of LNG. They are presented here as stylized facts.

The spot price of natural gas follow is mean reverting. This is due to the fact that the underlying supply-demand equilibrium price should fluctuate around a certain level. This level corresponds to the cost of producing natural gas. This mean reverting level is not constant and evolve with time. In addition it follows a stochastic behaviour and is often modelled as a stochastic process.

The spot price exhibit heteroscedasticity in form of clustering volatility. This is also connected to the underlying supply-demand structure. A common way to model such behaviour is by regime switching models such as Markov Chain Models. In addition the price also exhibit random jumps.

The price is leptokurtic meaning that extreme events are more common than in the normal distribution. This can partially be explained by the volatility clustering. Common ways to model such a behaviour is by using Stochastic Volatility models. Also as the natural gas markets are evolving and conditions change regularly it is more accurate to use implied volatility from the market than historical volatility. We use implied volatility to capture the markets expectations of future volatility. Relying solely on historical volatility would be more reasonable in mature markets where market development have reached more of a stand still.

As with the electricity markets the liquidity is bad; there are not many derivatives traded and the contracts cover small volumes. In addition the traded assets suffer from large bid-ask spreads. As an effect risk cannot be effectively hedged and any rebalancing a portfolio is costly. This means that even though we can theoretically find a hedge to a derivative their might not be any traded assets which we need to hedge it.

2.3 Spot price dynamics

The outlook of the stylized facts in Section 2.1.1 and 2.2.1 is pretty dim. The markets suffer from incompleteness and illiquidity and large unhedgeable risks. Adding mean-reversion, seasonality, spikes etc. and we end up with a models that has to incorporate a lot of different factors. The classical framework of a Geometric Brownian Motion (GBM) is not applicable and another modified version is needed.

The *Schwartz mean-reversion dynamics* [Schwartz, 1997] is often quoted as the basis for all commodity prices. It incorporates an Ornstein-Uhlenbeck mean reverting model where the mean reversion levels itself is modelled as a stochastic process. The mean reversion can be seen as the equilibrium of the underlying supply-demand process.

In [Kluge, 2006] the *Schwartz mean-reversion dynamics* is developed and a number of different electricity price stochastic processes are evaluated, emphasizing Equation 2.1. Kluges model is built up of components which incorporate each characteristics of the electricity spot. The big drawback is that a model on this level of complexity calls for numerical or simulation based methods even on the most basic derivative assets.

$$\begin{aligned} S_t &= \exp(f(t) + X_t + Y_t), \\ dX_t &= -\alpha X_t dt + \sigma dW_t, \\ dY_t &= -\beta Y_{t-} dt + J_t dN_t, \end{aligned} \tag{2.1}$$

where $f(t)$ is a deterministic periodic function for seasonality. N_t is a Poisson-process with intensity λ and J_t is an i.i.d. process representing the jump size. W_t, N_t and J_t are mutually independent.

Chapter 3

Model

To begin to describe our model we assume the usual probabilistic structure where we have a complete probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{0\leq t\leq T})$. Ω is a set of events, $\mathcal{F}$ is a σ-algebra, $\mathbb{P}$ is a probability measure and $(\mathcal{F}_t)_{0\leq t\leq T}$ is the natural filtration with respect to the process defined on the probability space. We also define Q as the risk neutral probability measure.

We denote $[0, \tau]$ as the *time horizon* for our market, not to be confused with τ_i (with subscript) used in Section 5. Financial contract on this time horizon with the spot $S(t)$ as underlying are traded continuously.

The section below states the two main assumption used in this report. We want to emphasize already at this point that this report will not assume a specific spot price dynamic. By not making this assumption we avoid the complexity described in Chapter 2. This means that we choose a *model-free* pricing approach.

3.1 Assumptions

To be able to use the pricing scheme suggested in [Keppo, 2004] we make the following assumptions.

Assumption 3.1. *There exist European call options and forward contracts on the electricity price. The electricity derivative market is complete and there is no arbitrage.*

Assumption 3.2. *The stochastic process $g(t, \omega)$ is a right continuous semimartingale that can be continuous or discontinuous on the probability space $(\Omega,\mathcal{F}, \mathbb{P})$ along with standard filtration $\{\mathcal{F}_t : t \in [0, \tau]\}$. It satisfies,*

i $(t, \omega) \to g(t, \omega)$ *is Borel measure on* $[0, \tau] \times \mathbb{R}$.

ii $g(t, \omega)$ *is* $\mathcal{F}_t$*-adapted.*

Since the inability to store electricity makes the market incomplete, see [Kluge, 2006], the first assumption can be considered strong. Section 3.1.1 discusses Assumption 3.1.

3.1.1 More on completeness

Looking at the stylized facts in the Section 2.1.1 Assumption 3.1 may seem a bit weak. However it is a necessary condition if any conclusions is to be made in the Keppo framework. There are however fractions of the market which are traded competitively. The energy market can be described as in its infant years compared to other markets. Most of the energy markets where deregulated in the 90s and more will follow. At the same time the policy makers in the European Union push for expansion in transmission infrastructure and in the development of an integrated market[1].This suggest that Assumption 3.1 will become more valid as the market evolves and *forwards* and *call options* are traded more competitively.

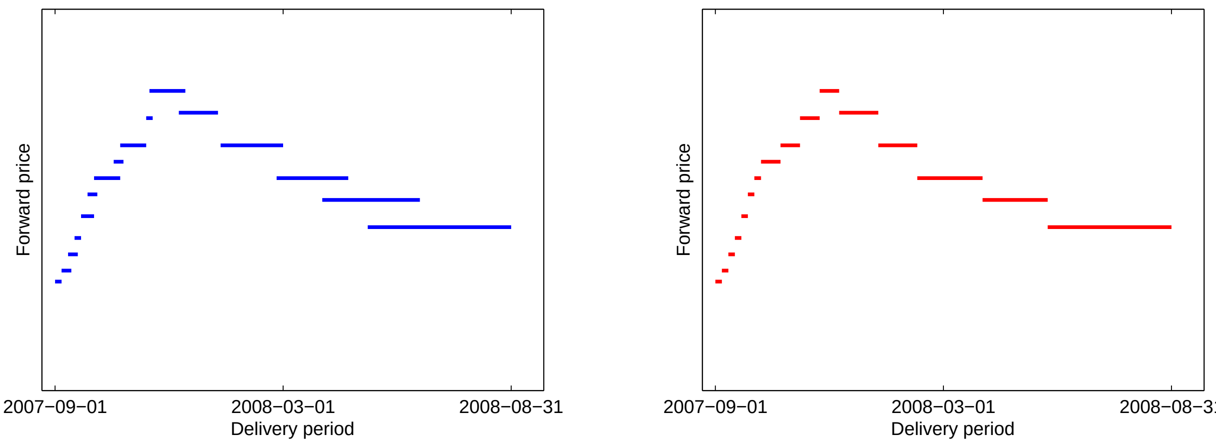

Figure 3.1: The figures shows examples of the completeness of forwards on a commodity market. The forward contracts have different delivery period and contract price. In the left figure the delivery periods overlap. In the right figure the delivery periods connect and do not overlap. Hence both examples have forwards that cover the entire delivery period 2007-09-01 to 2008-08-31.

To strengthen the weak point in the completeness assumption a number of measures can be taken. Firstly, the model could be used on markets which operate in the cutting edge of energy commodities, e.g. the Nordic Exchange Nordpool, TTF, NBP. This would lessen some of the effects described in the stylized facts section, e.g. illiquidity, large BID-ASK etc. Secondly, the model parameter could be configured to actual market contracts. An example of market data is shown in Figure 3.1. By defining the swing option such that delivery and exercise periods coincide with the market the replicating portfolio in Section 6.2.2 will consist of contracts on market. Hence the market completeness Assumption 3.1 is fortified.

3.2 Terminology and definition of standard contracts

The derivatives on a commodity market differ from the classical financial market. This section will distinguish the differences and define terminology and a standard set of products. From now on every *forward*, *call option*, *swing option* mentioned refers to the definitions in this section.
Firstly we assume that the time is t and define some basics that apply to all commodity derivatives,

[1]EU Energy Policy to 2050, Achieving 80-95% emissions reductions. European Wind Energy Association

- The *price* is defined as cost/amount. In this report the unit of *price* is €/MWh. Note that when referring to the swing option price this is not volume based, i.e. it is given in €.
- The *underlying* is the spot price $S(\cdot)$ of the commodity, i.e. electricity or natural gas.
- The *strike price* $K(\cdot)$ of a commodity is defined as fixed cost/amount during a time interval. In this report the unit of *strike price* is €/MWh. We should note that swing option *strike price* can be based on a stochastic market index, i.e. the *strike price* is floating and will not be fixed at time t. We limit ourselves to a fixed *strike price* in this report and do not include floating parts.
- The *delivery date* is a predetermined fixed future date $T > t$ when the commodity should be delivered. Since the commodity is a physical good and cannot be instantaneously delivered a *delivery period* $[T_0, T_1]$ can be defined during which the delivery occurs. Due to the delivery constraint of a physical commodity, derivatives are often settled by cash flow exchange at the delivery date.
- If the derivative is an option it has an *exercise period* which is a predetermined time interval $[\tau_i, \tau_{i+1}] \subset [T_0, T_1]$ during which the owner has the option to buy or sell. Each *exercise period* has a preceding fixed *exercise date*. The holder chooses to exercise or not at the *exercise date*. A derivative may contain several *exercise periods*. In the case of the swing option the *exercise periods* coincides with the exercise rights and add up to the *delivery period* $\sum[\tau_i, \tau_{i+1}] = [T_0, T_1]$.
- Every derivative has a predetermined fixed volume or fixed interval of volume.

Using the basic setting above we can now define the contract used in this report,

Definition 3.3. The *swing option* gives the holder a number of exercise rights to buy up to a upper bound volume at some strike price of a certain commodity at some predetermined set of exercise periods. The holder have the obligation to buy at least a lower bound volume at all of the exercise periods. This volume interval is referred to as the *Swing*. In addition, the holder have the obligation to exercise such that the cumulated volume is within a predetermined interval. The fair price of a *swing option* [Keppo, 2004] is;

$$z(t, T_0, T_1) = \sup_{p(\cdot)\in A} E^Q\left[\int_{t \bigvee T}^{T_1} e^{-r(y-t)} p(y)[S(y) - K(y)]dy|\mathcal{F}_t \right]$$

where $t \bigvee T_0 = \max[t, T_0]$, $z(t, T_0, T_1)$ is the arbitrage-free *swing option* price at time t for delivery period $[t \bigvee T_0, T_1]$, and A is an non-empty set of consumption processes that satisfy Assumption 3.2. $p(\cdot)$ is the consumption process of the holder of the swing option.

The swing options is basically an option to buy additional commodities but also an obligation to at least buy some. This notion might be confusing to some as the word option typically mean to you are not forced to do something. The lower bound restricts the optionally for the holder and as an effect decreases the value of the option. In an extreme case of high lower bounds and high Strike prices the swing option price can become negative. This fact must certainly cause some confusion when using the terminology "option".

Definition 3.4. The forward contract is an obligation to exchange a commodity at some future time T for strike $f(t, T)$. The strike is set such that the value at time t equals zero. The arbitrage free price is,

$$f(t, T) = E^Q[S_T|\mathcal{F}_t]$$

Note that $f(t, T)$ as a function of t is called the *forward curve*. Using methods similar to the one explained in [Fleten and Lemming, 2003] a continuous forward curve can be produced. The forward curve is basically an interpolation of market forwards expressed as a continuous curve. By integrating over the forward curve we get representations of forward contracts that are within the bid-ask spread of the actual market. We can also derive prices for forwards that are not present in the market by simply integrating the forward curve over the desired interval of time. These can be used as basis for pricing options on the specific market. We will use a forward curve to price the swing option in Section 5. Examples of a hourly forward curves and the integrated mean forward curve is shown in Figure A.2 and A.1.

Note that as the forward on the natural gas and electricity market are defined as swaps over finite time intervals, i.e. the actual forward price can represented as,

$$f(t, T_0, T_1) = E^Q\left[\int_{T_0}^{T_1} S(y)dy|\mathcal{F}_t\right] \tag{3.1}$$

Definition 3.5. The European call option is an option to buy a commodity at some future time T for strike K. The arbitrage free price is,

$$C(t, T, K) = E^Q[(S_T - K)^+|\mathcal{F}_t]$$

We emphasize that the *forwards* and *call options* are both defined on the spot at one point in time. A common way of defining the *call options* is on the swap as explained in Equation 3.1. The *call option* in this report is not defined on the swap and should not be confused in such a way.

Chapter 4

Pricing swing options

There are many different approaches to pricing swing options on the electricity and gas markets. As the swing option is a path dependent option subject to many constraints and the underlying spot dynamics is very complex no simple solution exists.

Pricing a derivative in this setting require the help of additional solvers. Other papers suggest pricing the Swing using simulation methods. [Dorr, 2003] uses a Least Squares Monte Carlo which incorporates a algorithm developed by [Longstaff and Schwartz, 2001] for pricing American Options. [Kluge, 2006] uses a Lattice Based trinomial forests to price the swing option. A simpler approach is assume *perfect foresight* on simulated paths, exercise perfectly and calculate the Swing Price as a mean of the discounted cash flows. By assuming *perfect foresight* the Swing Price will be overestimated, hence producing an upper bound. [Haarbrucker and Kuhn, 2009] choose a different method and develop a trinomial forest model and solve for a lower bound using linear programming. It is similar to the Keppo approach in that it solves a lower bound using linear programming. However it is not a *model-free* approach and is more complex than Keppo. All of the methods simulate paths and are very computationally intensive.

This chapter will introduce Keppo's approach to pricing swing options. Chapter 5 suggest a solution to Keppo's pricing approach.

4.1 Keppo's Corollary 1

We assume the holder wants to maximize profits and define the holders consumption process of the swing option as follows,

Definition 4.1.

$$p(t) = p_{low}(t) + p_S(t) + p_C(t)\mathbb{1}[S(t) \geq K], \quad \forall t \in [T_0, T_1]$$

where $p_S(\cdot)$, $p_C(\cdot) \in A_C(p_S)$, A_S is the class of positive stochastic processes that satisfy the conditions of 3.2. In addition they are subject to continuous and cumulative consumption constraints such that,

$$p_{low}(t) \leq p(t) \leq p_{up}(t)$$

$$e_{low} \leq \int_{T_0}^{T_1} p(y)dy \leq e_{up}$$

The dependence of $p_C(\cdot) \in A_C(p_S)$ is due to a stepwise decision process of the holder. In Section 4 the holder will first choose $p_S(t)$ and then choose $p_C(t)$.

The consumption process $p(t)$ is subject to both continuous and cumulative constraints. The continuous constraints $p_{low}(t)$ and p_{up} represent the continuous flow on consumed commodity that must be fulfilled. For the cumulative constraints e_{low} and e_{up} to have any effect on consumption they must be within the interval of the integrated processes $p_{low}(t)$ and p_{up} respectively. We require this to be true for the cumulative constraints and continue.

The intuition in this division of the consumption process comes from the definition of the Swing contract. The p_{low} volume is the obligation the holder has to continuously buy from the seller. As this is a deterministic function this part of the swing option could be replicated by buying forwards with volume $p_{low}(t)$ for the entire delivery period.

The $p_S(t)$ volume represents the obligation the holder has to consume the cumulative lower constraint volume but also the optionality to choose when to exercise. As the holder is assumed to exercise optimally and as the optimal choice path is stochastic this process must be stochastic. Assuming the holder tries to maximize profit while obligated to buy the accumulated volume of $p_S(t)$ the holder will use its optionality in time to choose to exercise periods that minimizes loss.

The final process $p_C(t)$ represent the optional volume between the lower and upper cumulative constraint. Assuming a maximize profit strategy the holder will choose to exercise such that profit is maximized.

It is sometimes possible to violate the constraints on the consumption process. This is done by adding penalties to the party that is responsible. We limit ourselves to contracts where it is not possible to violate the constrains in this report. This could be interpreted as infinite penalties in a penalty based model. Hence the sum of p_{low}, $p_S(t)$ and $p_C(t)$ should not exceed the continuous consumption constraints.

We now have three consumption processes where we know that at least one can be replicated using forward contract. However Keppo goes on to show that assuming this division on the consumption process the volumes $p_{low}(t)$, $p_S(t)$ and $p_C(t)$ can be replicated using a portfolio of forwards and European calls. In this way we have the entire Swing Contract defined in standardized contracts on the market. Using this pricing approach we can base the price of the Swing Contract on the implied information that lie in the market contracts.

A problem still remains in that these weights depend on the probability of the spot price being in the money during time t in the delivery period. Solving this probability would require numerical or simulation methods similar to the ones explained in the introduction to Chapter 4. Using these methods would take the Keppo model to a new level of complexity and the answer would not be analytical. In this way we would have lost all the advantages to the other models.

However as will be shown in Section 4.2 there is an analytical solution to a lower bound of the swing option Price.

4.2 Keppo's Corollary 2

In this section we will outline Keppo's Corollary 2. The main advantage with Corollary 2 is that we do not need to make any assumption of a spot price dynamics. Hence this approach to pricing swing option do not suffer from the same complexity issues as Corollary 1 and the other methods discussed in Chapter 4.

We start by making a Markov assumption on the consumption process in Definition 4.1. A Markov assumption implies that the stochastic process only depend on the information at time t and that the past information at $s < t$ does not affect the process. This means that the consumption process is only affected by the current spot price and how much is consumed continuously right now. It is also influenced by how much we have left to consume with respect to the continuous and cumulative constraints.

We will not evaluate the plausibility of this assumption to its fullest extent. However some remarks are made. To be a Markov Process the consumption process is indifferent to past spot price, including past volatile and in otherwise extreme behaviour. It is also indifferent to past consumption patterns and other past information. Considering the spot price could have made a sudden jump or have been in a period of high volatility the holder of the Swing would probably exercise the right to consume at a fixed price until the volatile period has surely ended. This would mean that the process in not a Markov process. However this past information could be reflected in the current implied volatility of standard contracts on the market and can hence be captured by a Markov Process. This reasoning can be extended to other parts of past and current information.

However Keppo shows that by making a Markov assumption the price of swing option can be stated as a linear optimisation problem. This assumption is a restriction of consumption process. Hence the Markov process B is a subset of the possible consumption processes of A as defined in Assumption 3.2. The restriction can be expressed as follows,

$$B \subset A \Longrightarrow \sup_{p(t)\in A} E^Q[\cdot] \geq \sup_{p(t)\in B} E^Q[\cdot],$$

where B is the set of possible Markov Consumption processes

Since we decrease the set of possible consumption processes in the supremum of Definition 3.3 the swing option value must be equal or lower. Hence the results is a lower bound which is presented in the following Corollary.

Corollary 4.2.

$$\begin{aligned} z_{low}(t, T_0, T_1) = &\int_{T_0 \vee t}^{T_1} e^{-r(y-t)}(f(t,y) - K(y))p_{low}(y)dy \\ &+ \int_{T_0 \vee t}^{T_1} e^{-r(y-t)}(f(t,y) - K(y))[p_{up}(y) - p_{low}(y)]\mathbb{1}\Big[y \in \Gamma_S^*(t)\Big]dy \\ &+ \int_{T_0 \vee t}^{T_1} C(t, y, K(y))[p_{up}(y) - p_{low}(y)]\mathbb{1}\Big[y \in \Gamma_C^*(t)\Big]dy \end{aligned} \tag{4.1}$$

where $p_{low}(\cdot) + [p_{up}(\cdot) - p_{low}(\cdot)][\mathbb{1}\left[y \in \Gamma_S^*(t)\right] + \mathbb{1}\left[y \in \Gamma_C^*(t)\right]] : [t \bigvee T_0, T_1] \to \mathbb{R}_+$ *is the optimal Markov consumption process at time t for the future time period* $[t \bigvee T_0, T_1], \Gamma_S^*(t)$ *and* $\Gamma_C^*(t)$ *are disjoint sets. These sets are obtained from the following linear optimization problem at time t*

$$\max_{\Gamma_S(t)\subset[t,\tau]} \left\{ \int_{T_0 \bigvee t}^{T_1} e^{-r(y-t)}(f(t,y) - K(y))[p_{up}(y) - p_{low}(y)]\mathbb{1}[y \in \Gamma_S]dy \right.$$
$$\left. + \max_{\Gamma_C(t)\subset[t,\tau]-\Gamma_S(t)} \int_{T_0 \bigvee t}^{T_1} C(t,y,K(y))[p_{up}(y) - p_{low}(y)]\mathbb{1}[y \in \Gamma_C(t)]dy \right\} \tag{4.2}$$

subject to,

$$\int_{T_0 \bigvee t}^{T_1} [p_{up}(y) - p_{low}(y)]\mathbb{1}[y \in \Gamma_S(t)]dy$$
$$= e_{low}(T_1 - T_0) - \int_{T_0}^{T_1} p_{low}(y)(t)dy - \int_{T_0}^{t \bigvee T_0} [p_{up} - p_{low}]\mathbb{1}\left[y \in \Gamma_S^*\right]dy(y$$
$$+ \int_{T_0 \bigvee t}^{T_1} [p_{up}(y) - p_{low}(y)]\mathbb{1}[y \in \Gamma_C(t)]\mathbb{1}[y \notin \Gamma_S]dy$$
$$= (e_{up} - e_{low})(T_1 - T_0) - \int_{T_0}^{T_1} p_{low}(y)(t)dy - \int_{T_0}^{t \bigvee T_0} [p_{up} - p_{low}]\mathbb{1}\left[y \in \Gamma_C^*\right]\mathbb{1}\left[y \notin \Gamma_S^*\right]dy. \tag{4.3}$$

Proof See *[Keppo, 2004, Corr. 2].*

Since the forward curve is real valued no two alternatives have the same value, i.e. there will not be two exercise dates where both choices has equal value. One implication of this is that a profit maximizing holder of the option will never choose to consume anything but p_{up} or p_{low}.

The Γ_S and Γ_C can be regarded as the the optimal consumption path for a long position in the swing option at time t, where Γ_S is the period where the holder must consume p_{up} and Γ_C is the period where the holder has the option to consume p_{up}, i.e. the holder only exercises the option when $S(t) > K$.

The effects of this lower bound approximation on the swing option is somewhat fussy. In Equation 4.2 we optimize over the optional parts of the swing option. Hence we find the set Forwards and Call Options that maximize the value of the swing option. If we compare to a Bermudan type option we could find an analogy to the swing option. The Bermudan option is a derivative which gives the holder the right to exercise one European Option at a discrete set of times up to the maturity date. It can be seen as something in between an American and European Option. The exercise rights in the swing option can be seen as a set of Bermudans with the exception that you cannot exercise multiple rights at the same date. Pricing a swing option by maximizing over a set of exercise periods as in Equation 4.2 would be the same as pricing a Bermudan as the max of the discrete set of possible European Options. Hence the lower bound approximation could be seen as giving up the Bermudan optionality in favour for the maximum European Option.

This loss of optionality will lower the price but also affect the sensitivity of the approximation. As the lower bound will fix parts of the weights in Forwards and Call Option to certain time periods it will less dynamic with respect to market changes. The swing optionality is more dynamic in such a comparison. Hence the lower bound approximation could only act as a fairly static hedge to the swing option. Small market changes could alter the optimum drastically and would require a rebalancing of the hedge to work effectively. This could cause jumps in an optimal hedge. As we will show in Section 7.3 these jumps do occur.

We have now found a analytical solution to price a lower bound of swing option. A discretized version of Corollary 4.2 is presented in Chapter 5.

Chapter 5

Lower bound of swing option

In this chapter we proceed to evaluate the lower bound in Corollary 4.2. We assume that the seller hedges the swing option by using a hedging portfolio and define a general hedge portfolio. Further on we find a numerical solution using standard linear programming. Chapter 6 outlines specific hedging strategies using this lower bound.

5.1 Estimation and error

The seller is assumed to hedge the swing option using the lower bound in Corollary 4.2. We extend the hedging portfolio representation in [Keppo, 2004] and define it as follows.

$$
\begin{aligned}
\int_t^{T_1} p^*(y)[S(y) - K]dy &= e^{-rt}z(t) + \int_t^{T_1} \phi_{Swing} dy \\
&= e^{-rt}z(t) + \int_t^{T_1} \phi_{Swing} \pm \phi_{LB} \pm \hat{\phi}_{LB} dy \\
&= e^{-rt}z(t) + \int_t^{T_1} (\phi_{Swing} - \phi_{LB}) + (\phi_{LB} - \hat{\phi}_{LB}) + \hat{\phi}_{LB})dy
\end{aligned}
$$

where $z(t)$ is the value of the hedging portfolio at time t, $p^*(\cdot)$ is the optimal consumption process of the swing option and it satisfies Assumption 3.2 and the consumption constraints. $\phi_{\cdot}$ is a martingale under Q-measure and it corresponds to the future gains and losses from the hedging strategy based on respective swing option estimate.

We define the following errors,

$$\varepsilon = \phi_{Swing} - \phi_{LB} \tag{5.1}$$

$$\eta = \phi_{LB} - \hat{\phi}_{LB} \tag{5.2}$$

The error ε is the difference between the value of the swing option and the lower bound estimate defined in Corollary 4.2. There are some methods to estimate this error. The lower bound estimate could be compared to other methods that give the fair price of the Swing. However, as all method fail to give an exact price of the Swing, bias of an estimate could distort the error.

The error η is the numerical error in calculating the lower bound (see Section 5.2). η is assumed to be small and we will assume it equal to 0 from now on for computational ease.

All of the other methods that price the value of the Swing should provide a value above this if the models are perfectly calibrated. Hence we should expect a positive ε.

The empirical study in Chapter 7 will compare the lower bound to a Least Square Monte Carlo method.

5.2 Discretization

To enable a numerical calculation of the optimal choice path described in Equation 4.2 the exercise period is discretized. We assume that the exercise period can be divided into N intervals specified in the vector τ. τ should not confused with the *time horizon* defined in Chapter 3. The intervals would typically correspond to time periods present in the market,e.g. hours, days, weeks. As the market is assumed to be complete τ should be chosen such that we can find forward and call prices with corresponding delivery periods.

We then define four set of variables for each exercise period ($4N$ in total). These variables will define the beginning and the end of Γ_S and Γ_C within each exercise period. A formal description is shown in Definition 5.1 and a graphical representation in Figure 5.1.

Definition 5.1.

$$\begin{aligned} T_0 = \tau_1 = S_S^1 \leq S_E^1 \leq C_S^1 \leq C_E^1 = \tau_2 = ... \\ = \tau_i = S_S^i \leq S_E^i \leq C_S^i \leq C_E^i = \tau_{i+1} = ... \\ = \tau_N = S_S^N \leq S_E^N \leq C_S^N \leq C_E^N = \tau_{N+1} = T_1 \end{aligned} \tag{5.3}$$

where $\sum_{i=1}^{N} [S_S^i, S_E^i] = \Gamma_S$ and $\sum_{i=1}^{N} [C_S^i, C_E^i] = \Gamma_C, \Gamma_S \bigcap \Gamma_C = \emptyset$

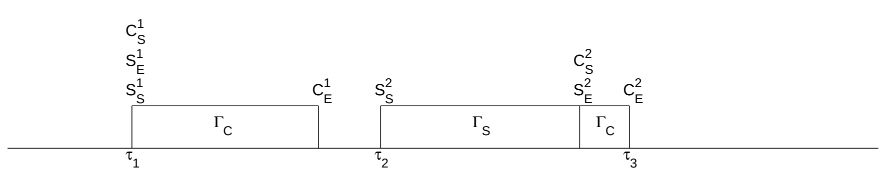

Figure 5.1: Figure shows an example of a division of $[T_0, T_1]$ into time steps of length $\Delta\tau$. As seen in the figure the values of $S_S^i, S_E^i, C_S^i, C_E^i$ determine the sets Γ_S, Γ_C.

We assume that the Swing Contracts are defined such that in each exercise period the *Strike* and continuous constraints are constant. Using Assumption 3.1 there should exist *Forward* and *Call option* contracts that match the intervals in τ. Hence $f(t,y)$ and $C(t,y,K(y))$ could be considered constant for each interval. Finally we assume that the delivery occurs instantaneously once within the interval. Settling virtually with cash-flows this would be possible, however dealing with physical commodity the assumption would lead to an approximation error. All these assumptions amount to the following for the discretized intervals.

Assumption 5.2.

$$e^{-r(y-t)} \sim e^{-r(z-t)}, \quad z \in [\tau_i, \tau_{i+1}] \quad fixed \tag{5.4}$$

$$f(t,y) \sim \overline{f}(t,\tau_i) \tag{5.5}$$

$$p_{up}(y) - p_{low}(y) \sim p_{up}(\tau_i) - p_{low}(\tau_i) \tag{5.6}$$

$$C(t,y,K(y)) \sim \overline{C}(t,\tau_i,\overline{K}(\tau_i)) \tag{5.7}$$

$$K(y) \sim \overline{K}(\tau_i) \tag{5.8}$$

$$\forall y \in [\tau_i, \tau_{i+1}]$$

With the exception of 5.4 these assumption are basically the discretized version of a Swing Contract specification. However as $e^{-r(y-t)}$ $(r > 0, \quad y > t)$ is monotone decreasing we can bound the discounting factor.

$$\int_{\tau_i}^{\tau_{i+1}} e^{-r(y-t)} dy > \int_{\tau_i}^{\tau_{i+1}} e^{-r(\tau_{i+1}-t)} dy = e^{-r(\tau_{i+1}-t)}(\tau_{i+1} - \tau_i)$$

By this approximation Equation 5.4 can be restricted from below and a lower bound is kept. We will assume this approximations including Assumption 5.2 for all calculation in Chapter 7.

We apply the discretization scheme to the optimization problem in Corollary 4.2. First divide the integral into a sum of sub integrals defined by τ. As the functions do not depend on y the results of each sub integral is simply the length of the interval. The lengths are derived as $S_E^i - S_S^i$ and $C_E^i - C_S^i$ respectively. The problem can now be solved as the following two step optimization problem.

$$1. \quad \max_{S_S^i, S_E^i \forall i \in [1,N]} \left\{ ... + [S_E^i - S_S^i] e^{-r(z-t)} [\overline{f}(t,\tau_i) - \overline{K}(\tau_i)][p_{up}(\tau_i) - p_{low}(\tau_i)] + ... \right\}$$

$$2. \quad \max_{C_S^i, C_E^i \forall i \in [1,N]} \left\{ ... + [C_E^i - C_S^i] \overline{C}(t,\tau_i,\overline{K}(\tau_i))[p_{up}(\tau_i) - p_{low}(\tau_i)] + ... \right\} \tag{5.9}$$

5.3 Linear solver

One way to solve the discretized optimization problem in Equation 5.9 is to use *Linear Programming*. The Linear Programming problem is a well known problem with many different solvers, including the *Simplex Method* explained in [Taha, 2007]. In a worst case scenario the Simplex solves a solution with precision $O(e^x)$ but in a practical case it is typically $O(x^2)$. The standard form of the linear programming problem is stated below.

$$
\begin{aligned}
&maximize && c^T x \\
&subject \quad to && Ax \leq b \\
& && A_{eq} = b_{eq} \\
& && x \geq C_{lowerbound} \\
& && x \leq C_{upperbound} \\
&and && x > 0
\end{aligned}
$$

We run the algorithm for a simple example and the results are shown in Figure 5.2.

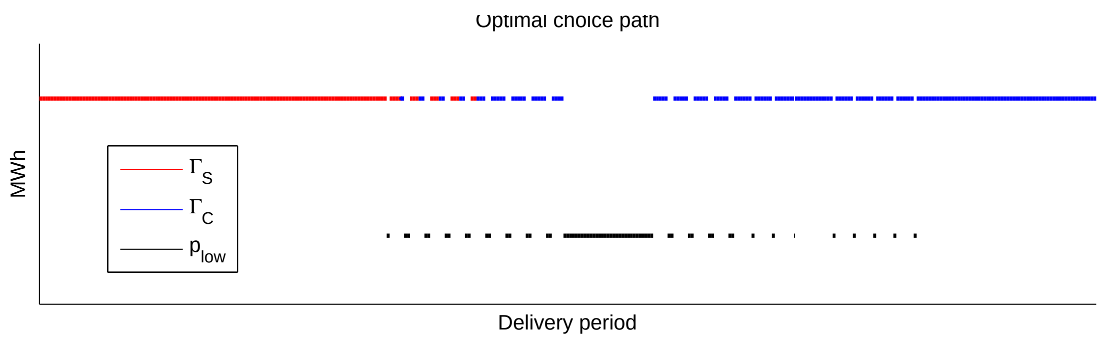

Figure 5.2: Figure shows an example of an optimal consumption path.

As $\overline{f}(t, \tau_i)$ and $\overline{C}(t, \tau_i, \overline{K}(\tau_i))$ are real valued functions the optimization will assign whole intervals in τ to Γ_S,Γ_C with one exception. The cumulative constraints in 4.3 will make the algorithm chose the linear programming constraint that has not been saturated with the largest marginal product as a subset of $[\tau(i), \tau(i+1)]$, i.e. Γ_S will consist of a number of whole intervals of τ and 1 partial interval. Γ_S will consist of a number of whole intervals of τ and up to 2. This is because Γ_S and Γ_C could share one partial interval and Γ_C could have one on its own.

This poses a problem in the model since there may not be contracts on the market for these partial interval. An approximate method would be to use a contract for the whole period and simple scale it linearly to the length of the partial interval. This would however mean that we consume less than $p_{up}(t)$ for the entire period in contrast to previous reasoning. On the other hand, by scaling in such a way, we can derive the value of the swing option from market products matching the intervals in τ. Since we can decide τ the contract can be calibrated to the market. We will assume this scaling interpretation for the rest of the report.

Note that due to the discounting factor being monotone the optimal solution is always to consume early in the interval. This means that the scaling error can be derived as solely a discounting error.

Chapter 6

Risk management

In this Chapter we will introduce the traditional financial *Greeks* as measurements of uncertainty in the value of a derivative. An approximation to the *Greeks* is suggested in the form of a *Finite difference* approximation. Further on suggest some hedging methods such as hedging the *Greeks* and the *replicating portfolio* derived from the weights in the Keppo modell.

6.1 Greeks sensitivity analysis

The *arbitrage free price* of an option depend on the input of certain market parameters. As these market parameters change the price of the option will also change. To measure the price sensitivity of the *replicating portfolio* with respect to a specific market parameter would indicate the level of risk in that parameter. The *Greeks* are defined the partial derivative of the value of a portfolio with respect to a certain parameter. Each *Greek* is given a letter from the Greek alphabet and are spelled phonetically, e.g. 'Γ' as Gamma, with the exception of ν which is spelled vega (common misconception as it looks like the Roman letter 'v'). Normally the *Greeks* are categorized as first or second order referring to the order of the partial derivative. The *Greeks* are defined below.

Definition 6.1. The Delta or Δ is the value change of the portfolio with respect to the underlying.

$$\Delta = \frac{\partial \Pi}{\partial F}$$

Definition 6.2. The Theta or Θ is the value change of the portfolio with respect to time.

$$\Theta = \frac{\partial \Pi}{\partial t}$$

Definition 6.3. The Gamma or Γ is the value change of the Δ with respect to the underlying.

$$\Gamma = \frac{\partial \Delta}{\partial F} = \frac{\partial^2 \Pi}{\partial F^2}$$

Definition 6.4. The Vega or ν is the value change of the portfolio with respect to the volatility.

$$\nu = \frac{\partial \Pi}{\partial \sigma}$$

Definition 6.5. The Rho or ρ is the value change of the portfolio with respect to the risk free rate.

$$\rho = \frac{\partial \Pi}{\partial r}$$

6.1.1 Finite difference estimator

Finding closed form expressions for the *Greeks* is not easy. Even though we have an analytical solution in Corollary 4.2 Γ_S and Γ_C are not differentiable. If we assume the consumption path to be indifferent to small changes it is possible to find a analytical solution. This would however not capture the large effects that a shift in the optimal consumption could have on the price. Hence it is not very useful and we need another way of finding the Greeks.

Using a Finite Differences method such as a one described in [Glasserman, 2003] it would be possible to calculate a numerical approximation of the *Greeks*. We consider a setting where the partial derivative is calculated with respect to the parameter x for all times t, i.e. the value of the underlying is changed as $\Delta x(t) = h, \forall t$. For example calculating the Δ would be done by adding $h > 0$ to the entire forward curve. The change in a parameter is often due to some underlying factor which affect future assumptions, e.g. price fluctuations in the crude oil price due to changing macro economic factor which push the forward price. The reasoning is valid for time, forward price, volatility and therefore we assume this for all *Greeks* defined in Section 6.1.

First we want to calculate an approximation of a first order *Greek*. We start by doing two Taylor expansion on the value of the *Replicating Portfolio* with respect to an arbitrary parameter θ. All other parameters are considered constant and $h > 0$ is a very small number.

$$\begin{aligned}
\Pi(x+h) &= \Pi(x) + \frac{\partial \Pi(x)}{\partial \theta} h + O(h^2), \\
\Pi(x-h) &= \Pi(x) - \frac{\partial \Pi(x)}{\partial \theta} h + O(h^2), \\
\Leftrightarrow \frac{\partial \Pi(x)}{\partial \theta} &= \frac{\Pi(x+h) - \Pi(x-h)}{2h} + O(h^2)
\end{aligned} \tag{6.1}$$

Hence we have found an approximation of the first order *Greek*. Using the same procedure for higher order *Greeks* we get.

$$\begin{aligned}
\Pi(x+h) &= \Pi(x) + \frac{\partial \Pi(x)}{\partial \theta} h + \frac{1}{2}\frac{\partial^2 \Pi}{\partial \theta^2} h^2 + O(h^3), \\
\Pi(x-h) &= \Pi(x) - \frac{\partial \Pi(x)}{\partial \theta} h + \frac{1}{2}\frac{\partial^2 \Pi}{\partial \theta^2} h^2 + O(h^3), \\
\Leftrightarrow \frac{\partial^2 \Pi}{\partial \theta^2} &= \frac{\Pi(x+h) - 2\Pi(x) + \Pi(x-h)}{h^2} + O(h^2)
\end{aligned} \tag{6.2}$$

Both approximations will have an error term that will decay quadratically. Hence we have found a generalized way of computing the *Greeks* in any setting.

The $h > 0$ is a tuning parameter and must be set according to the pricing formula of the *Replicating portfolio*. In the setting in Chapter 5 where we use the algorithm a too small h can cause problems. The algorithm uses a built in tolerance parameter which tells the algorithm to stop if a the maximum value does not increase by more than the tolerance level ξ. A typical tolerance level is a very small value, e.g. 10^{-9}. If we want to calculate the *Greeks* using the approximations in Equation 6.1 and 6.2 and $h \leq \xi$ then the algorithm might calculate the same value for $\Pi(x) = \Pi(x + h)$. As seen in Figure 6.1 the approximation fails for very small h.

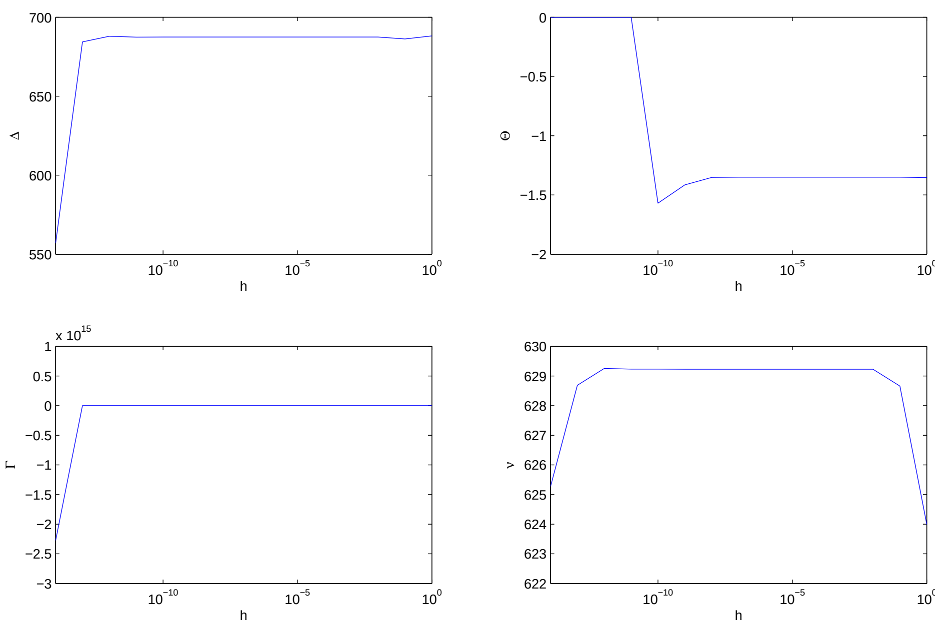

Figure 6.1: The *Greeks* calculated using the approximation in Equation 6.1 and 6.2 for different h. The x-scale is logarithmic. For small values of h the approximation fails (see value of the specific *Greek* becomes 0). The values of Γ for larger h is ~ 7, but it impossible to spot due to scaling.

From Figure 6.1 it is apparent that a value of h around 10^{-4} is suitable. Another way of choosing h is by doing it *percentage based*, i.e. h is equal to the current value of the input parameter times some percentage. This would make the *Finite difference* more stable to scaling differences. For example a small value h added to one input parameter could mean 5% increase while the same value to another would mean a 100% increase. This could cause stability problems in the estimation. However the percentage based approach could produce values closer to the threshold in the Linear Programming Algorithm. Since it is less apparent how large h actually is it is harder to calibrate a good percentage value for all cases.

When constructing a portfolio consisting of several contracts of different delivery dates the *Greeks* can be calculated separately for each month. This is done by shocking that particular period of time with the

Greek	Forward	Call Option	Bond
Δ	1	$e^{-r(T-t)}N(d_1)$	0
Γ	0	$\frac{N'(d_1)e^{-r(T-t)}}{S_0\sigma\sqrt{T-t}}$	0
Θ	0	$-S_0N'(d_1)\sigma e^{-r(T-t)}/(2\sqrt{T-t}) + rS_0N(d_1)e^{r(T-t)} - rKe^{-r(T-t)}N(d_2)$	$-re^{-r(T-t)}$
ν	0	$S_0\sqrt{T-t}N'(d_1)e^{-r(T-t)}$	0
ρ	0	$K(T-t)e^{-r(T-t)}N(d_2)$	$-(T-t)e^{-r(T-t)}$

Table 6.1: Greeks for the forwards, european call on assets that provide yield, and a risk free bond. As the forward is underlying it will have a delta of 1 and be neutral to all other greeks. The call option Greeks are analytical from [Hull, 2009]

certain sensitivity parameter, i.e. h is added to one period only. The *Greeks* are still calculated in the same fashion as previously. By using the method a more detailed sensitivity analysis can be made regarding how different periods of time and parameters effect the portfolio. In Section 7.3 and 6 we assume the discretization in Definition 5.1 and use this method to construct Greek-neutral hedge portfolios.

6.1.2 Alternative Greeks approximation

There are other methods of approximating the Greeks. One method could be to differentiate the analytical lower bound and try to approximate a derivative of Γ_C and Γ_S. The algorithm used for solving the linear optimization problem in Section 5.3 contain shadow prices. These shadow prices represent the marginal instantaneous change of the optimization problem with respect to a certain constraints. By using this shadow prices we could approximate a derivative of Γ_C and Γ_S.

Another thing that can be derived from algorithm is how much a certain constraint must change before the optimum is shifted. In this way we would be able to capture larger optimum changes that occur. These shifts do not affect the price heavily but it would effect the distribution of weights in the lower bound and hence the dynamics of a hedging portfolio.

6.2 Hedging

6.2.1 Hedging using the Greeks

The *Greeks* as defined in Section 6.1 are measurements of a financial contracts sensitivity to a certain parameter. Using a combination of different contract it would be possible to calculate a hedge portfolio which has no sensitivity to certain *Greeks*. The portfolio is then called neutral to the specific *Greek*, e.g. a very common hedge is the delta-neutral hedge.

Using Assumption 3.1 we have continuously traded forwards and European call options. There exist also a risk-free asset that evolves with the risk free rate r. By differentiation on Black 76 the analytical solution to the *Greeks* on a European Call are calculated. A complete set of the analytical Greeks are shown in Table 6.1.

If we construct a portfolio consisting of Forwards, Call Options, and Bonds we can eliminate up to 3 *Greek* sensitivities of the swing option. This is easily shown by the following linear equations. Lets look at one exercise period defined as in 5.1, we could construct the following relationship.

$$G = AX$$
$$G = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}^T$$
$$A = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}$$

where g_i is the *Greek* of the swing option and A_i is the column of Forward, Call Option and Bond in Table 6.1 with the corresponding greeks as rows, i.e. A_i is either a 3-by-1, 2-by-1, or 1-by-1 matrix. If we find an inverse of A there exists an unique solution to the weights. If the matrix is singular there exists infinite amount of solutions or none exists, [Sparr, 1994]. By looking at Table 6.1 it is apparent that Forwards and Bonds only have $\Delta \neq 0$ and $\Theta \neq 0$, $\rho \neq 0$ *Greeks* respectively. Therefore the matrix will be singular when hedging certain combinations. This implies that we cannot create a *Greek* neutral portfolio for all combination of *Greeks*.

6.2.2 Replicating Portfolio

Already in Keppo's Corollary 1 it was shown that the value of the swing option can be derived from a set of Forwards and Call Option. This suggests that by constructing a portfolio of Forwards and Call Option we can replicate the value of the swing option. The same reasoning hold for the lower bound approximation. By calculating the weights for the Forward and Call Options from the lower bound approximation we should get an approximation of a portfolio that replicates the value of the Swing. In this section we will prove that such a portfolio exist and henceforth we refer to it as the *Replicating portfolio*.

First assume the discretization in Definition 5.1. By the reasoning of scaling volume in Section 5.3 the output of the linear solver can be calibrated to the products in the market. We use this information to construct a portfolio consisting of products with delivery period equal to the intervals in τ. The forwards that replicate the swing option do not necessarily exist on the market. By introducing a bond we can always replicate these by forwards on the market (from forward curve) and a risk free. The result in presented as the following theorem.

Theorem 6.6. *Under Assumption 5.2 the swing option lower bound can be replicated using forwards contracts, call options and a risk free asset, with the following weights.*

$$\begin{aligned} x_{Forward} &= (p_{up} - p_{low})\Gamma_S + p_{low} \\ x_{call} &= p_{up} - p_{low}\Gamma_C \\ x_{bond} &= (F(t,T) - K)[(p_{up} - p_{low})\Gamma_S + p_{low}] \end{aligned}$$

Proof: *We discretize the swing option according to Definition 5.1 and prove the relation for one exercise period. The outcome for the two cases $S_{\tau_i} < K$ and $S_{\tau_i} \geq K$ is shown in Table 6.2*

Using this table we compute the total value of this portfolio for the two cases $S_{\tau_i} < K$ and $S_{\tau_i} \geq K$. By showing that the expected outcome of this portfolio equals the lower bound it can be derived that the

$y = t$	$t < y < \tau_i$	$y = \tau_i, \quad S_{\tau_i} < K$	$y = \tau_i, \quad S_{\tau_i} \geq K$
$\Pi_{forward}(t)x_{Forward} = 0$	$[\Pi_{forward}(y) - f(t,\tau_i)]x_{Forward}$	$(S_{\tau_i} - f(t,\tau_i)x_{Forward}$	$(S_{\tau_i} - f(t,\tau_i)x_{Forward}$
$C(t,\tau_i,K)x_{call}$	$C(y,\tau_i,K)x_{call}$	0	$(S_{\tau_i} - K)x_{call}$
$B(t,\tau_i)x_{bond}$	$B(y,\tau_i)x_{bond}$	$B(\tau_i,\tau_i)x_{bond}$	$B(\tau_i,\tau_i)x_{bond}$
(1)	-	(*)	(**)

Table 6.2: The table shows the outcome of two cases $S_{\tau_i} < K$ and $S_{\tau_i} \geq K$ for the *replicating portfolio*. $\Pi_{forward}(t)$ is the value of the Forward Contract. The value of the Forward contract is 0 when initialized and the Bond equals 1 at time τ_i.

portfolio and the lower bound must have equal value,

$$
\begin{aligned}
(*) &= (S_{\tau_i} - f(t,\tau_i))[(p_{up} - p_{low})\Gamma_S + p_{low}] + 0 + (F(t,T) - K)[(p_{up} - p_{low})\Gamma_S + p_{low}] \\
&= (S_{\tau_i} - f(t,\tau_i) + f(t,\tau_i) - K)((p_{up} - p_{low})\Gamma_S) + (S_{\tau_i} - f(t,\tau_i) + f(t,\tau_i) - K)p_{low} \\
&= (S_{\tau_i} - K)((p_{up} - p_{low})\Gamma_S) + (S_{\tau_i} - K)p_{low} \qquad (6.3) \\
(**) &= (S_{\tau_i} - f(t,\tau_i))[(p_{up} - p_{low})\Gamma_S + p_{low}] + (S_{\tau_i} - K)(p_{up} - p_{low})\Gamma_C \\
&+ (F(t,T) - K)[(p_{up} - p_{low})\Gamma_S + p_{low}] \\
&= (S_{\tau_i} - f(t,\tau_i) + f(t,\tau_i) - K)(p_{up} - p_{low})\Gamma_S \\
&+ (S_{\tau_i} - K)(p_{up} - p_{low})\Gamma_C + (S_{\tau_i} - f(t,\tau_i) + f(t,\tau_i) - K)p_{low} \\
&= (S_{\tau_i} - K)((p_{up} - p_{low})\Gamma_S) + (S_{\tau_i} - K)(p_{up} - p_{low})\Gamma_C + (S_{\tau_i} - K)p_{low} \qquad (6.4)
\end{aligned}
$$

As Γ_C and Γ_S represents the optimal consumption path they replicate the optimal behaviour of the holder of the swing option. Hence the values of 6.3 and 6.4 is the expected value of the swing option for the two cases $S_{\tau_i} < K$ and $S_{\tau_i} \geq K$. Thus (1) *in Table 6.2 must equal the value of the lower bound at time t. By this reasoning it can be determined that the the Forwards, Call Options and Bond with weights $x_{Forward}$, x_{Call} and x_{Bond} replicates the lower bound of the swing option.*
Q.E.D.

This replicating portfolio can be used by the seller of the Swing Contract as a hedging instrument. Since it consist of the Forwards and Call Option that give the holder similar consumption optionality and obligation as in the swing option it should react similarly to market changes over time. However it should be noted that the replicating portfolio is based on the lower bound approximation of the Swing. Hence it does not fully capture the instantaneous value of the swing option and also the lower bound approximation could distort the distribution of the weights.

6.2.3 Rebalancing and transaction costs

This report does not assume any transaction cost. However, some remarks will be made.

When evaluating a hedge portfolio on actual market data is important to model the transaction cost that occur when the buying and selling contracts on the market or over the counter (OTC). When managing a hedge portfolio the aim is to reduce the risk in holding this asset. By continuously monitoring and rebalancing a portfolio in the event of substantial changes an agent can reduce the risk. In theory we can always assume no transaction and rebalance our hedges without any loss. But in practise every transaction carries a cost with it. As it is not in the scope of this report to evaluate rebalancing frequencies we will not assume any transaction cost.

Chapter 7

Empirical study

This chapter introduces a number of studies which evaluate, discuss and compare the lower bound approximation of the Keppo framework. The examples are all based on a gas contract. The purpose of these studies is to test the performance and stability of the lower bound approximation, and to get a feeling of how large the error ϵ could be, and if the finite difference estimation is able to approximate the *Greeks* of the swing option.

The studies are comprised of a daily and monthly evaluation of the development of the price, Greeks and hedges of the Swing Contract. The lower bound approximation is also compared to a Least Square Monte Carlo model (LSMC). By doing this comparison we hope to find evidence that the lower bound error ϵ is small but also that our approximation of the *Greeks* are accurate. The final study will evaluate how the length of the exercise periods (granularity) will affect Greeks of the lower bound.

7.1 Environment

The main implementation of the empirical study is done in MATLAB. This includes MATLABs standard function for plotting, *plot* and *boxplot*. Gnu Linear Programming Kit[1] is used to solve the discretized linear optimization problem. It is linked to MATLAB through at Matlab Executable (MEX). This was done for reasons of stability and faster runtime when MATLABs built in function *linprog* failed to provide the same. Microsoft Excel is used as the Graphical User Interface for the swing option tool. It was linked to MATLAB through SpreadSheet Link Ex VBA macro. If no other reference is made all the calculations and plots refer to the tools above.

7.2 Data

For all the studies in this section we will use a forward curve data to derive our forward prices. The forward curves are derived from NBP[2] data using the methods explained in 3.2. Since the NBP is quoted in 0.01£/therm we assume a fixed exchange rate of 0.83 £/€and translate the contract in to €/MWh. The *Greeks* are calculated using a $h = 10^{-4}$ unless otherwise stated. When referring to the *lower bound* in this

[1] `http://www.gnu.org/software/glpk/`

[2] National Balancing Point

chapter it is referred to the solution to the Keppo's lower bound in Chapter 5.

The forward in the electricity and gas market is more similar to a Swap since it fixes the price over a period of time instead of a fixed date. The Black-76 model is a modification which handle commodity markets of the standard Black&Scholes. We will use the modified Black-76 shown in Equation 7.1 to approximate the price of the options in the empirical study.

$$
\begin{aligned}
&C(t,\tau_i,K) = e^{-r(\tau_i - t)}[f(t,\tau_i)N(d_1) - KN(d_2)],\\
&d_1 = \frac{\log(f(t,\tau_i)/K) + (\Sigma(t,\tau_i)^2/2)(\tau_i - t)}{\Sigma(t,\tau_i)\sqrt{\tau_i - t}},\\
&d_2 = d_1 - \Sigma(t,\tau_i)\sqrt{(\tau_i - t)},\\
&\Sigma(t,\tau_i) = \frac{1}{\tau_{i+1} - \tau_i}\int_{\tau_i}^{\tau_{i+1}} \sigma(t,u)du,\\
&N \text{is the standard Gaussian cumulative distribution function}
\end{aligned}
\tag{7.1}
$$

There is a lack of liquidity in the energy markets. As an effect there is little or no data to derive suitable volatilities from and therefore Assumption 7.1 and 7.2 are made. The report does not assume the effect of these assumption to be negligible, but due to the inherent problems of solving a more complex pricing issue these assumption are necessary. The reasons will be discussed in relation to each assumption separately.

Assumption 7.1. *There is no volatility smile or smirk, i.e. volatility is the same for in-the-money, at-the-money and out-of-the-money options.*

There is a lack of in-the-money or out-of-the-money options to derive a smirk from and if there are they suffer from large BID-ASK spreads. Hence we make this assumptions because there are scarce data to derive any basis on. The pricing discrepancy that occurs as an effect of this assumption would be most apparent when pricing deep in-the-money or out-of-the-money Call Options. Swing options are commonly initialized with the strike level set as the mean of the forward curve, hence the call options would be at-the-money in this case. However when pricing hedges that have not been rebalanced recently the mean level could have drifted far away from strike level, hence causing a larger price discrepancy. A low strike swing option is evaluated in Section 7.3.

This assumption actually ensures that we calculate a lower bound. Since volatility at-the-money is the lowest possible volatility assuming no smirk will decrease the value of the lower bound. This is due to that the Black-76 price increases with volatility. The reasoning would be true independent of Black-76 as the call option is insurance against rising spot prices. In a more volatile scenario the seller of insurance would always want a higher risk premium for the option and hence a higher price.

Assumption 7.2. *The volatility during the delivery period is fixed, i.e.*

$$\Sigma(t,y) = \Sigma(y)$$

where t is current time.

This assumption is made due to the time horizon of the derivative products on the market. The market typically covers 1-1,5 year a head of today's date with standard contracts. This restricts our ability to derive implied volatilities from the market on longer time horizons. Hence swing options with long delivery periods cannot be priced effectively long before the start of the delivery period, e.g. with a 1,5 year time horizon on standard products on the market and a 1 year (12 exercise months) swing option starting in 8 Months we cannot derive implied volatilities for the last two exercise periods.

Since the NBP market is still evolving the historical data is less representative than in a mature market. The information contained in prices imply the general expectations of the agents in the market. By using implied volatility instead of historical we utilize this implied information in our model. The volatility as a function of time is shown in Figure 7.1.

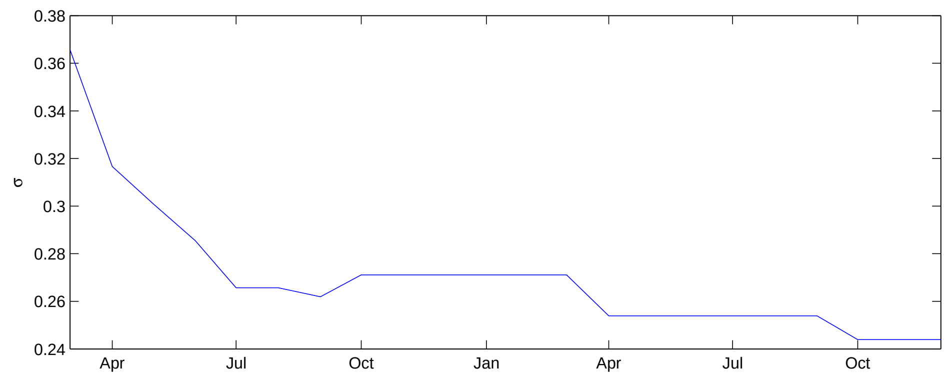

Figure 7.1: The Graph shows the volatility (yearly) as a function of time. The period is March 2012 to December 2013.

7.3 Results

This section will present the results of an empirical study on the lower bound. All studies will evaluate a swing option with monthly exercise periods. A daily and a monthly scenario will be presented and the hedges in Chapter 6 will be applied and discussed. A comparison study to a Least Squares Monte Carlo model will also be presented. Hopefully it will provide a feeling of how large the errors, as explained in Section 5.1, could be.

To make a more extensive evaluation of the rebalancing frequencies more cases could be included and longer time spans. However, a more fair evaluation would be to assume a model for the transaction cost of rebalancing the hedge. The model would be used to find an optimal rebalancing frequency and then compare on some cross validation data. This in not included in the scope of this report. Therefore this section evaluates and discusses the hedges and omits drawing any sharp conclusions concerning the rebalancing frequency.

As explained in Section 5.3 the optimization routine could produce partial intervals. But as we assume a scaling approximation in Section 5.3 the volume will be spread out to produce whole intervals. This will be seen as consuming in between p_{low} and p_{up} in the figures.

7.3.1 Daily evaluation

Using daily data for the entire month of March 2012 we evaluate a swing option price with delivery period 1 April 2012 - 31 March 2013. The set-up is explained in Section 7.1. The data consist of a daily *forward curve* which cover at least the delivery period. The evaluation period is 1 March - 30 March 2012. The forward prices are derived from the average of the *forward curve* as explained in Assumption 5.2 and the Call Option prices as explained in 7.2.

A swing option specification is constructed with daily constraints of [1, 2.2] MWh and cumulative constraints of [718,789.8] MWh. The initial forward curve is shown in Figure 7.2. The price is solved using the optimization routine. The results are presented in Figure 7.3 and 7.4.

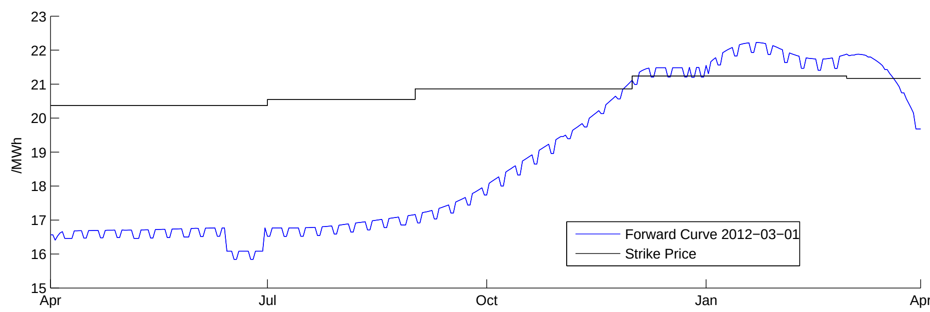

Figure 7.2: Daily study: The figure shows the result of the discretised solution to Keppo's lower bound for a swing option. The option is based on data from the NBP market. The delivery period is 1 April 2012 - 31 March 2013. The cumulative constraints are [718,789.8] MWh and the daily constraints are [1, 2.2] MWh. The Forward and Call Option prices correspond to the prices in the *replicating portfolio*.

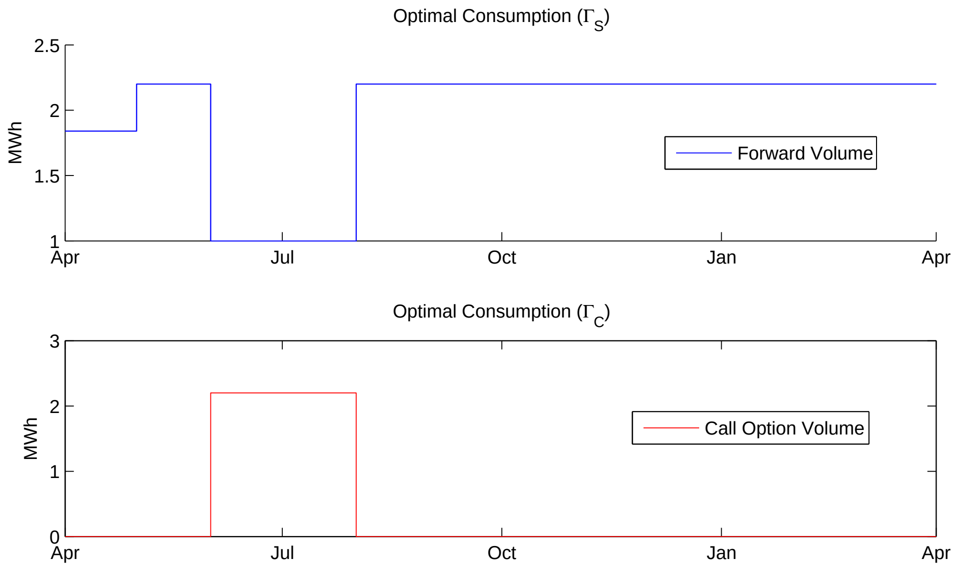

Figure 7.3: Daily study: The figure shows the result of the intial optimal consumption path of the lower bound.

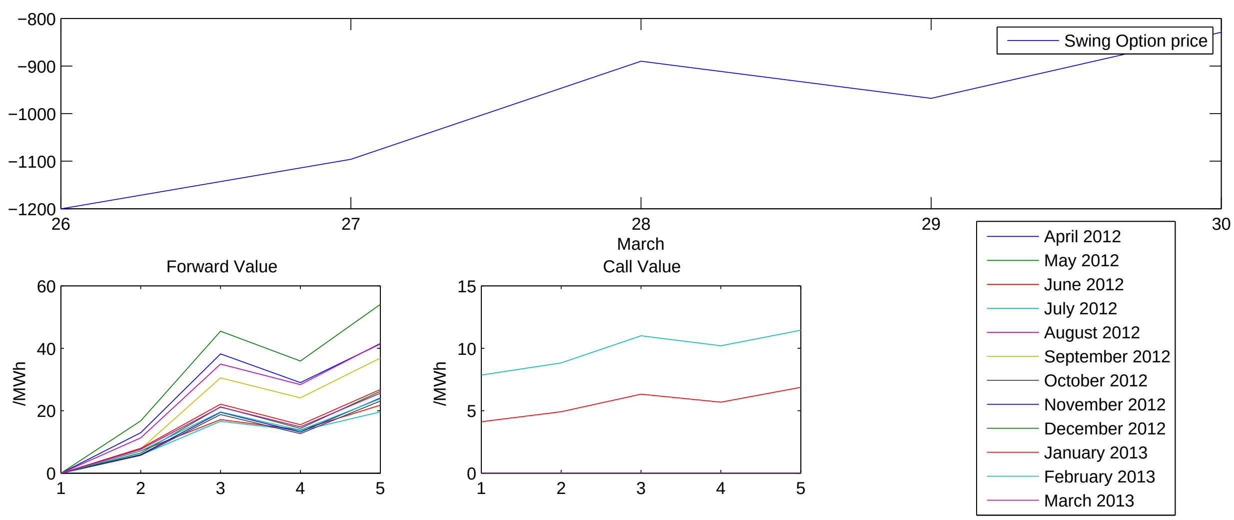

Figure 7.4: Daily study: The figure shows the value of the lower bound. The option is based on data from the NBP market. The delivery period is 1 April 2012 - 31 March 2013. The cumulative constraints are [718,789.8] MWh and the daily constraints are [1, 2.2] MWh. The Forward and Call Option prices correspond to the prices in the *replicating portfolio*.

The optimization routine finds June and July to be the two months were the swing option should be replicated by Call Options. This can be interpreted as that this is the period where the holder has the largest uncertainty in whether to exercise or not.

During the daily evaluation period we can follow how the price of the forwards and the call options in the *replicating* portfolio evolves. The prices in the *replicating portfolio* does not vary much. If the *replicating portfolio* is rebalanced every day the optimal choice path does not change at all. This can be interpreted as the holders expectations on where the exercise or not have not changed during the evaluation period. This is probably due to that the forward curve changes with shocks, i.e. the whole forward curve is raised or lowered and the shape does not vary.

To apply some risk management strategies to the evaluation we calculate the *Greeks* of the swing option. As seen in Figure 7.5 there is clear distinction between the months June, July 2012 and the rest of the delivery period. This can be connected to the results previously in Figure 7.3 where the *replicating portfolio* chose exactly June and July to be replicated by Call Options. It was reasoned that this was connected to the expectations of the holder of the swing option. In the *Greek* sensitivity perspective this reasoning could be extended. If the month July and June are the most uncertain months this would be visible in the *Greeks*, which Figure 7.5 clearly shows.

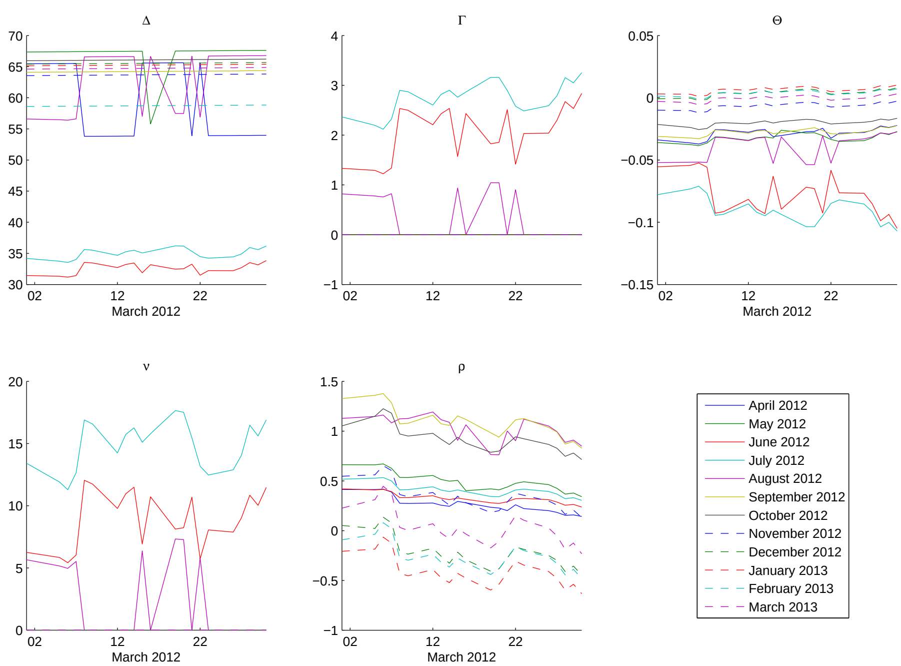

Figure 7.5: Daily study: Plot shows the daily Greeks of Swing Contract based on the NBP market. The period is 1 March - 30 March 2012. The Greeks sensitivies are calculated per month (12 total since contract is 1 year) using the finite difference method described in Section 6.1.1.

A static Delta and Delta-Gamma hedge are calculated to examine the effectiveness of these hedges on the swing option. The method is explained in more detail in Section 6.2.1. The results are presented in

Figure 7.6. During the evaluation there is a small drift in the Δ of both hedges. More apparent is the large jumps that occur in the Δ of months April, Maj and August of 2012. The shift is as big as $\sim$ 10 for the Δ and $\sim$ 1.2 for Γ. This is due to a shift in the optimal consumption path, see Figure A.3. The consumer changes his position in consumption to another month and hence the lower bound weights the forwards and call options differently. The Delta-Gamma hedge in comparison to the Delta hedge has no advantage in this shift. This is expected though since hedging with Greeks only neutralize small changes.

During the evaluation period the Δ and Γ of the hedges drifts to approximately 2 to 0.5 respectively. The change in Δ is small considering the same *Greeks* in Figure 7.5 lie around 35. The Γ however changes by 1 and has a value around 2.5 in the *Replicating portfolio*.

To insure Δ and Γ neutrality the hedges would need to be continuously rebalanced. Assuming there are transaction cost this would not be possible and a trade off structure between cost and Delta-Gamma neutrality is needed. From the results previously it is apparent that daily rebalancing for both Δ and Γ would only reduce small discrepancies. However if the hedges where rebalanced in the event of a these large shifts in the optimal consumption the hedges would be fairly neutral. This would mean we suggest a dynamic rebalancing frequency in favour of a static fixed interval rebalancing frequency.

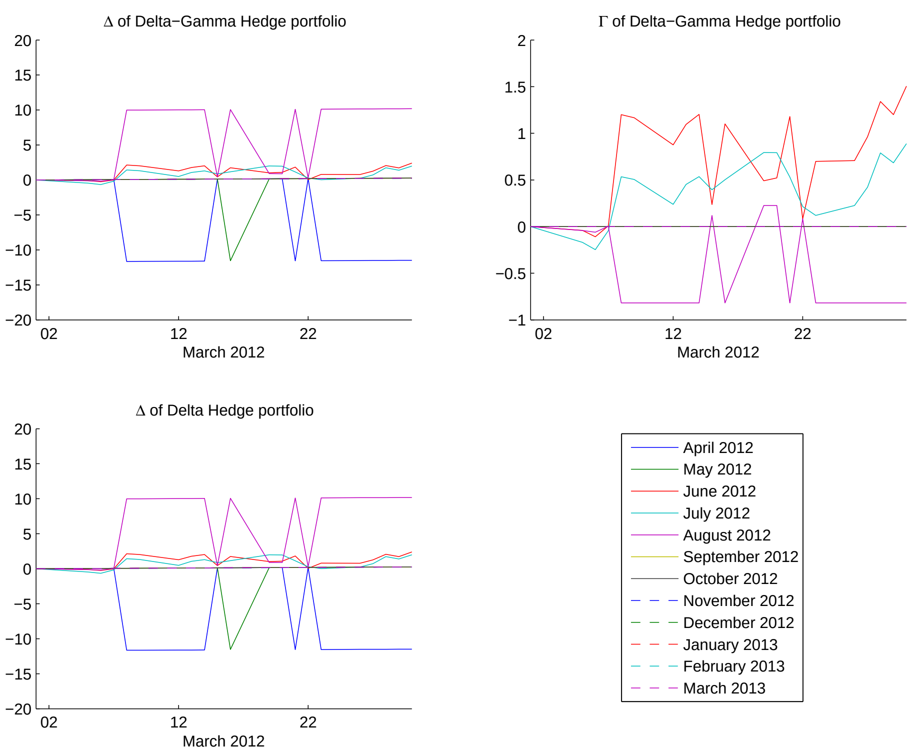

Figure 7.6: Daily study: Plot shows the Greeks of Delta and Delta-Gamma Hedge. The period is 1 March - 30 March 2012 and the hedges were rebalanced on the 26 March. The Greeks sensitivies are calculated as explained in Section 6.2.1.

7.3.2 Monthly Evaluation

Using data for 7 months in a row we evaluate a swing option with delivery period 1 April 2012 - 31 March 2013. The set-up is explained in Section 7.1 and is exactly the same as in Section 7.3.1. The evaluation period is 16 August 2012 - 15 February 2013 and consist of one forward curve for each month which cover at least the *delivery period*. The forward prices are derived from the average of the *forward curve* as explained in Assumption 5.2 and the Call Option prices are explained in Section 7.2.

The same swing option contract specification as in the daily evaluation is constructed with daily constraints of [1, 2.2] MWh and cumulative constraints of [718,789.8] MWh. The initial forward curve is shown in Figure 7.7. The price is solved for using the optimization routine. The results are presented in Figure 7.8 and 7.9.

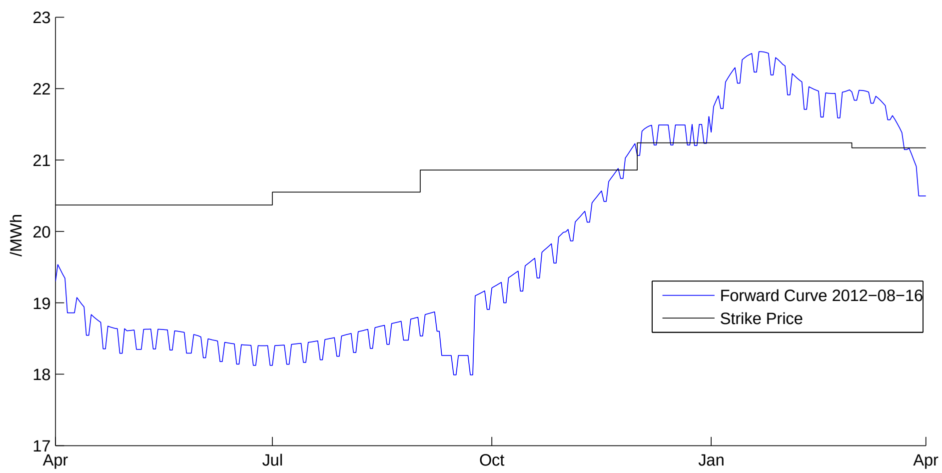

Figure 7.7: Monthly study: The figure shows the result of the discretised solution to Keppo's lower bound for a swing option. The option is based on data from the NBP market. The delivery period is 1 April 2012 - 31 March 2013. The cumulative constraints are [718,789.8] MWh and the daily constraints are [1, 2.2] MWh. The Forward and Call Option prices correspond to the prices in the *replicating portfolio*.

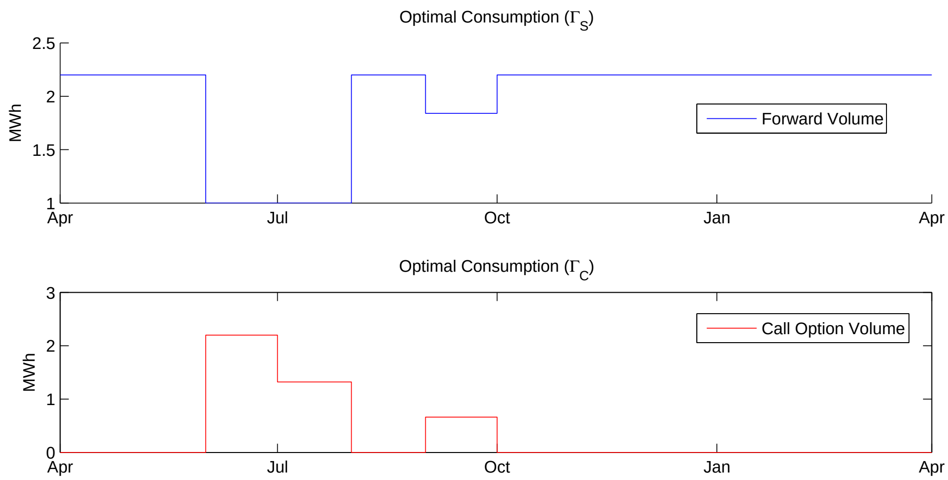

Figure 7.8: Monthly study: The figure shows the result of the intial optimal consumption path of the lower bound.

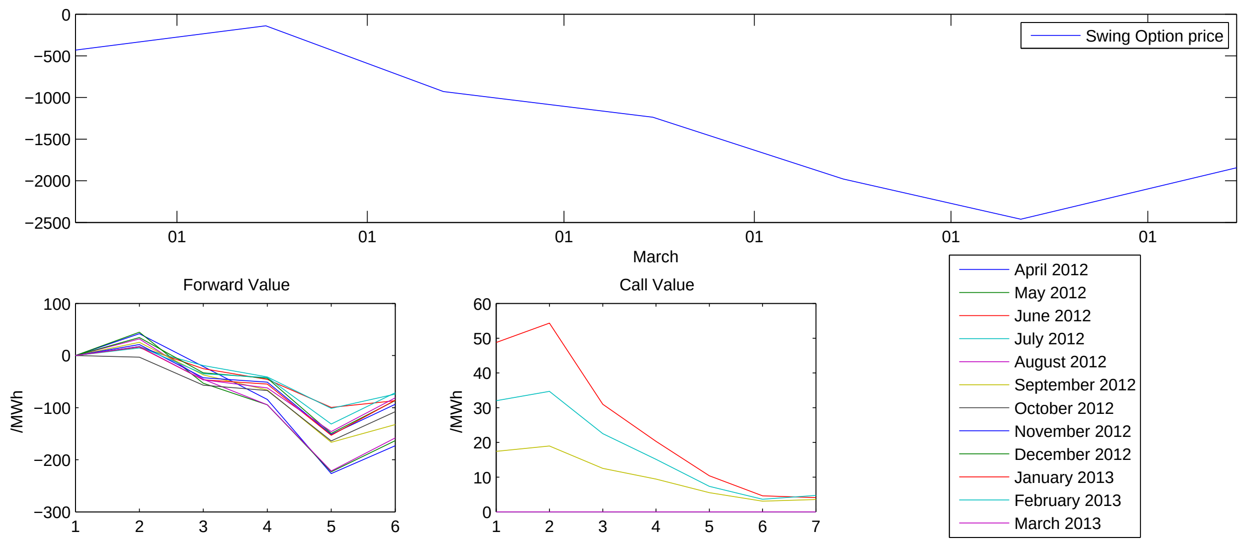

Figure 7.9: Monthly study: The figure shows the value of the lower bound. The option is based on data from the NBP market. The delivery period is 1 April 2012 - 31 March 2013. The cumulative constraints are [718,789.8] MWh and the daily constraints are [1, 2.2] MWh. The Forward and Call Option prices correspond to the prices in the *replicating portfolio*.

The *Greeks* are calculated for each month and are presented in Figure 7.10. Compared to the daily evaluation study the values fluctuate much more which is reasonable since it is over longer periods of time. Comparing the values that seem to fluctuate the most to the results in Figure 7.8 it becomes apparent that the month that are replicated by call options are the once that have the most volatile *Greeks*. An intuitive reasoning to this is that in these periods the holder is the most uncertain whether to consume or not, hence it replicated by call options and the *Greeks* fluctuate the most. In contrast most of the exercise periods which are replicated by forwards are close to constant or only fluctuate slightly.

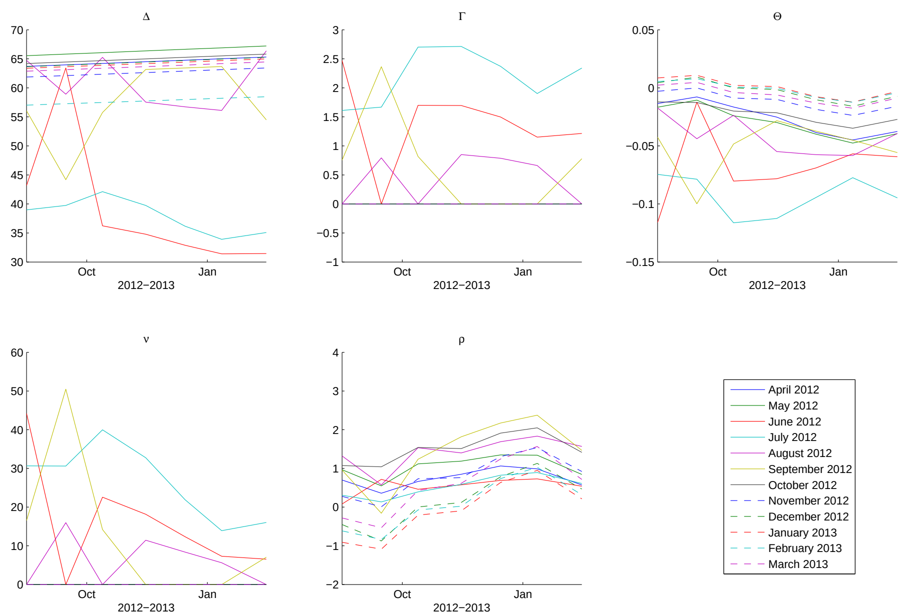

Figure 7.10: Monthly study:Plot shows the monthly Greeks of Keppo's lower bound for a Swing Contract. The option is based on data from the NBP market. The period is 16 August 2011 - 15 February 2012. The Greeks sensitivies are calculated per month (12 total since contract is 1 year) using the finite difference method described in Section 6.1.1.

A static Delta and Delta-Gamma hedge is calculated and are presented in Figure 7.11.

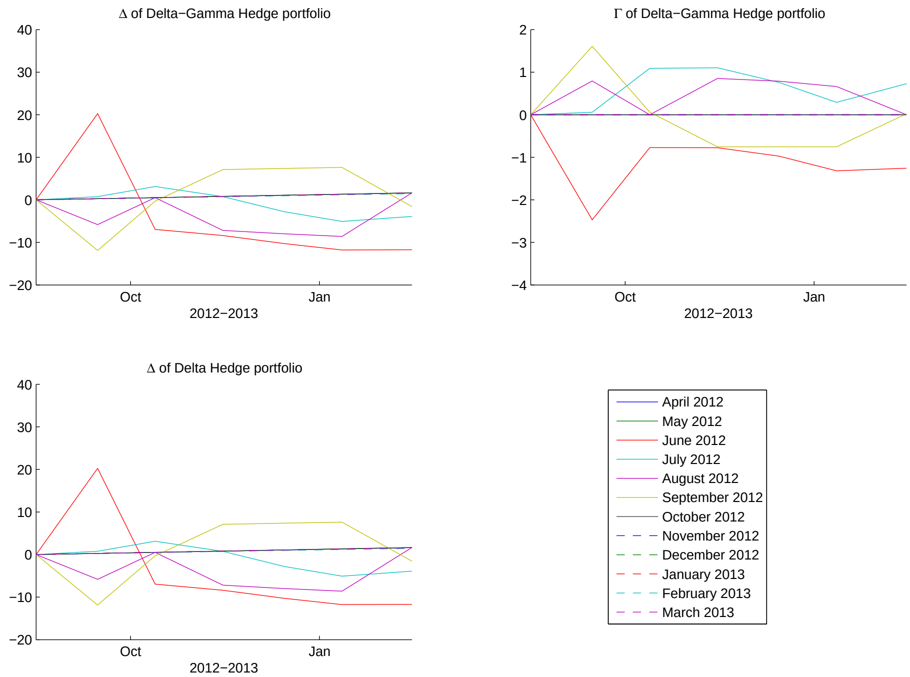

Figure 7.11: Monthly study:Plot shows the Greeks of Delta and Delta-Gamma Hedge. The period is 16 August 2011 - 15 February 2012 and the hedges were rebalanced on the 16 August. The Greeks sensitivies are calculated as explained in Section 6.2.1.

As seen in both the Delta-Gamma hedge and the Delta hedge the Δ and Γ both change substantially from each measure point. The Δ drift to as much as ~ 20 and Γ to ~ 2.5. By measuring at frequency of 1/Month we obviously miss the points of action in time where the portfolio makes the jumps we saw in the daily study. The plot suggest the the jumps occur somewhere in between but due to the interpolation we have no real suggestion. However the monthly study still gives us a notion of how far a static hedge can drift.

7.3.3 Comparison study

To compare the performance and results of Keppo model we construct 4 cases and compare to a Least Squares Monte Carlo model (LSMC). The study will compare the price, *Greeks* and performance of the two model. The expectation of this comparison study is to get a hint of how big the *lower bound* error in Section 5.1 is and to validate the finite difference *Greek* estimation method in Keppo. We also expect the Keppo model to be much faster than the LSMC, as it does not utilize a Monte Carlo simulation.

The 4 cases cover a 1 year swing option in the period 2012-03-01 to 2012-03-31. The volatility is specified in Figure 7.1 and. The 4 cases is combinations of a high/low cumulative lower bound and fixed high/low monthly strike price and are explained in Table 7.1. The high strike price is based on the average of the forward curve for the *delivery period* (21.16).

To fully understand the results a small explanation of the LSMC model will follow. The method is 3-

factor simulation model similar to the one explained in [Kluge, 2006]. The number of simulation paths are set to 500 for this comparison. In a live case it would be suitable to use a larger set to decrease the variance of the estimate. The method derives its volatilities from historical data. This is in contrast to our lower bound model that is based on implied volatilities. It should be noted that the historical volatilities are several volatilities based on, e.g. spot, long-term factors etc. This means that volatilities are not directly transferable to the lower bound model. Hence we are forced to use these different volatilities and note that the models must be calibrated according to each other. Only perfectly calibrated models can be used to draw conclusive results. Calibrating the LSMC model perfectly to the lower bound is not in the scope of this report. We aim to get a hint of how the performance and results of the lower bound relate to other models. The Greeks in the LSMC model is calculated using a *percentage based Finite Difference* method.

Case	**Monthly Strike**	**Cumulative lower bound**
1	$K = 21.16$	$e_{low} = 718$
2	$K = 21.16$	$e_{low} = 500$
3	$K = 10$	$e_{low} = 718$
4	$K = 10$	$e_{low} = 500$

Table 7.1: Table shows the difference in the setup of the case study.

With the mean value of the LSMC it would be possible to do a pointwise comparison to the Lower bound. This could give an approximation of how large the error of the Keppo lower bound is. Using the confidence interval of the LSMC the discrepancies could be compared to the uncertainty of the LSMC. The 4 cases are executed in both the lower bound and LSMC and the results are shown in Figure 7.12. The static hedge refers to a swing option with a delta neutral hedge, i.e. on the initial date we sell forward contracts corresponding to the estimated Greeks of the Swing Contract. Since the forward contract have value zero at initial date the effect is that the expected value is kept while the confidence interval is reduced.

We calculate the 4 cases for the LSMC and the lower bound. The computational time for the LSMC is around 2-3 minutes while the lower bound finish in 4-5 seconds. As expected the lower bound is much faster. Considering that the LSMC also will increase in computational time for larger sample sizes (only 500 in this example) the lower bound is much faster when comparing performance.

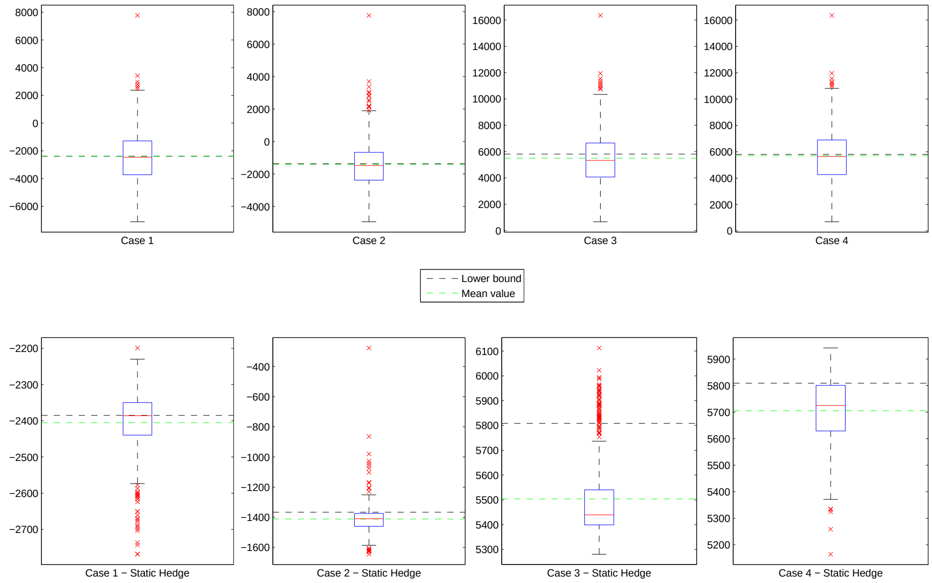

Figure 7.12: Figure shows a comparison of the 4 cases in Table 7.1 between the LSMC and the Lower bound. The boxplot is calculated from the output from the LSMC using MATLABs built in function *boxplot*. The black and green dotted line refers to the lower bound and mean value of LSMC respectively. The boxplot is defined as the default valued boxplot in MATLAB
3

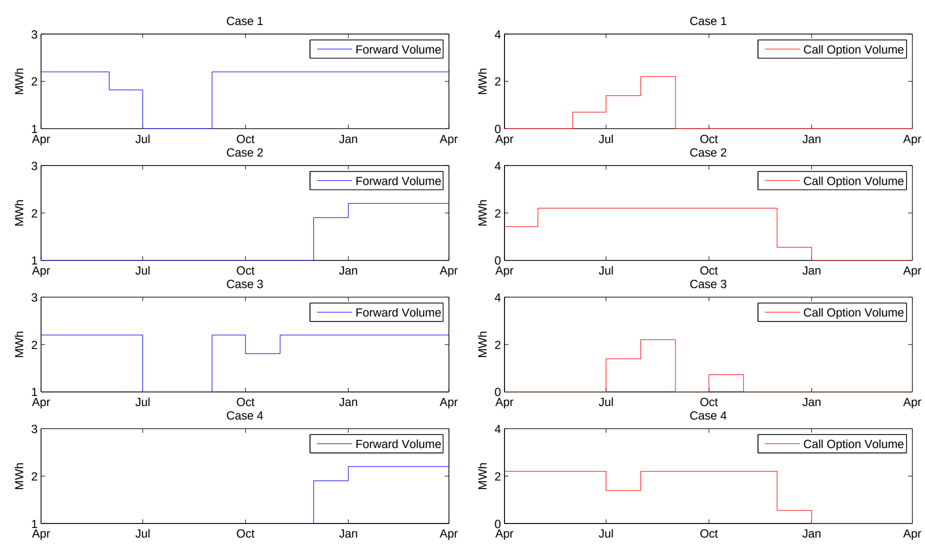

Figure 7.13: Figure shows the optimal consumption path for the 4 cases in Table 7.1.

	Lower bound (Daily)			Lower bound (Monthly)			LSMC
Case	**Bond**	**Call**	**Price**	**Bond**	**Call**	**Price**	**Price**
1	-2389	4	-2385	-2391	5	-2386	-2405
2	-1409	43	-1366	-1410	49	-1361	-1411
3	5402	407	5808	5384	406	5790	5504
4	3995	1813	5809	3984	1807	5790	5705

Table 7.2: Table shows the results of the case study split into the representation of the *replicating portfolio*. The columns Bond and Call represent the replicating value of the swing options in terms of a *bond* and *call options*. Since the *forwards* are initialized on the valuation date the value of the forwards are per definition equal to zero. Daily or monthly refers to the lenghts of the exercise periods.

As seen in Figure 7.12 the lower bound is explicitly above both the median and average value of the LSMC. The discrepancy is larger for the two cases with lower strike level. The lower cumulative bound also seem to increase the discrepancy between the lower bound and the LSMC, however it is not as significant effect.

The results are inconclusive regarding the errors in Section 5.1. The lower bound value explicitly end up above the mean and median value of the LSMC. What we expected was the error ϵ to be positive, when comparing the lower bound to LSMC we find a negative ϵ. There could be several explanations to this. The error could be due to a bias in the estimate of the LSMC or it could be an effect of the parameters in both models. As the lower bound is model free while the LSMC is not, i.e. the lower bound does not assume a certain spot price dynamics, there are several parameters that should be cross-calibrated between the models. The LSMC is a much more complex model than the lower bound and contain lots of parameters that do not have a direct counterpart in the lower bound. Discrepancies in this cross-calibration could produce a negative ϵ.

The strike level cause larger errors between the LSMC and lower bound. Intuitively a very low strike level such as in Case 3 and 4 the holder of the swing option would, with high probability, choose to consume e_{up}. The additional optionality in e_{low} of Case 4 implies that the value must be higher than Case 3. As is the case with both the lower bound and LSMC. The lower bound does not add much value to this additional optionality. This can be the effect of a low strike level where the holder rarely wants to consume less. In the LSMC however this effect is much larger suggesting that the probability of such events would occur more often. This imply inaccuracy in the cross-calibration between the LSMC and lower bound. It could also be due to that ϵ scale with the strike level.

By testing the LSMC tool with simplified pricing cases we found that the implied volatility in the lower bound should be somewhat smaller than in our empirical study. At the same time the volatility in the lower bound only affects the pricing of the call options. Since the call options are replicated with the Swing interval $[e_{low}, e_{up}]$ errors in volatility can only affect parts of the lower bound price, especially in Case 1 and 3 where the Swing is narrow. If we exclude the value of the call options and compare with LSMC we still find values of the lower bound which exceed the LSMC, see Table 7.2. This implies that there are more calibration errors in between the models than just in the volatility. By trying more cases, e.g. the extreme $e_{low} = e_{up}$ and more mid level strike swing option, the models could be more satisfactory cross-calibrated. However since the LSMC is the most complex model and contain more parameters that type of study would mainly focus on adjusting the LSMC parameters. This is not in the scope of this report and will not be investigated.

Additional studies of the output of the LSMC revealed that its consumption levels where unreasonable low. The LSMC suggested we consume just above the cumulative lower bounds in all cases. Especially the low strike case studies 3 and 4 where we would assume the holder to consume all of the volume. This suggest either an extreme behaviour due to inappropriate calibration or some kind of error in the LSMC.

Using the finite difference approximation in the LSMC we compare the Δ values of the two models. The results are shown in Figure 7.14 and 7.15.

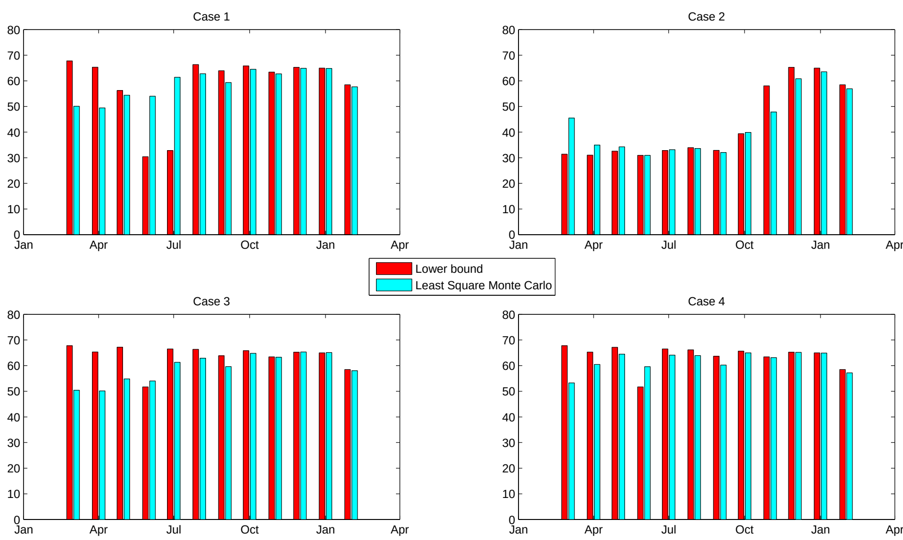

Figure 7.14: The plot shows a comparison between the Δ values of the lower bound and LSMC.

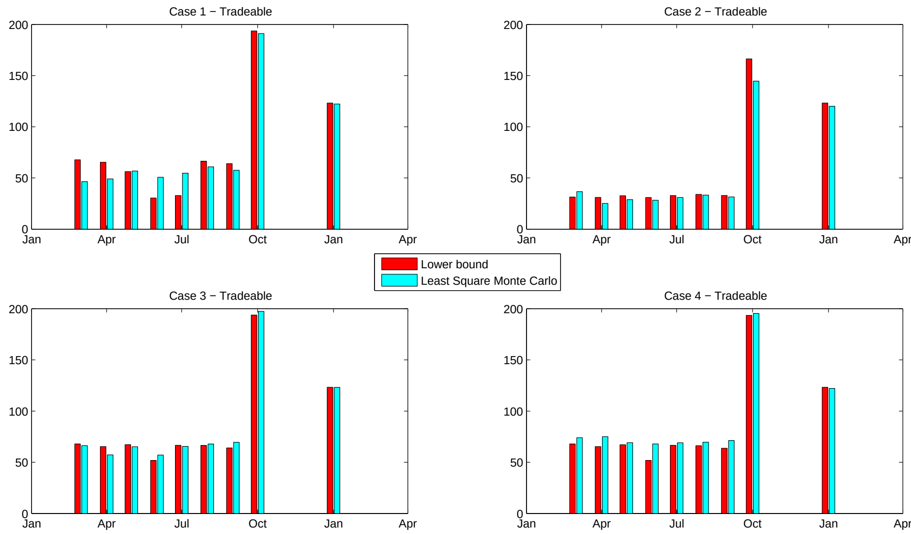

Figure 7.15: The plot shows a comparison between the Δ values of the lower bound and LSMC in tradeable contract. Tradeable refers to a discretization where the exercise periods match available contracts in the market.

Differences between Δ in the LSMC and the lower bound are mostly seen in the earlier months. The lower bound approximates a higher Δ for the early months and much lower in June and July. As seen in Figure 7.13 June-Aug are the months most often included in Γ_C. The intuition in this is that holder is most uncertain whether to consume or not in these months. Therefore it is replicated with Call Options. The biggest difference between LSMC and the lower bound is also seen in Case 1.

7.3.4 Granularity study

The swing option holder chooses time and volume to consume at a predetermined strike price. In the theoretical case we can assume a continuous time nomination, however in practise the interval is divided into time segments. These could be days, weeks, months and do not have to be of the same length. We will refer to the length of these time periods as granularity.

Intuitively if the holder has the right to nominate shorter time periods there should be more optionality in the swing option. Hence a fine granularity should raise the price of the swing option. As explained in Section 6.2.2 the lower bound of the swing option could be replicated by forwards and call options. Since the call options and forward have different Greeks the granularity of the swing option effectively spreads out sensitivity to different time periods of the year. Hence by altering the granularity we would expect to see changes in the sensitivity of the swing option. This study will compare daily and weekly granularity effect on the Greeks of the lower bound.

In Table 7.2 the daily portfolio has a higher value than its respective monthly portfolio. In Section 5.2 we approximated the discounting factor by a fixed discounting at the end of each exercise period. A fine granularity would therefore increase this discrepancy and vice versa. Since we are observing a modest discrepancy it is implied that the discounting error is small.

Assuming identical spacing in the granularity the discounting approximation would have the same effect on all exercise periods. Hence this approximation should have little effect on the Greek approximation. We will assume this effect to be negligible.

We use the same setup as in Section 7.3.1 and calculate the Greeks using the finite difference method explained in Section 6.1.1. The shock value h are still applied according to the daily study granularity (months). In such a way we can produce monthly greeks with finer granularity that could be comparable.

At first the fine granularity caused the finite difference method to become more unstable when using the suggested h. By applying a percentage based h which derived its value based on the size of the specific input we were able to stabilize the output. Two fixed spacing granularities of weeks and days where implemented. The perturbation h was set to 5% which proved to be a somewhat stable value. The results are shown in Figure 7.16 and 7.17. The calculation times for weekly and daily granularity was 36 and 145 seconds respectively. Using hourly granularity the method became too unstable and the calculation times increased manifold. Therefore we will not include hourly granularity. As the daily study uses the same setup the results of Figure 7.5 is comparable as monthly granularity. We assume that the different calibration of h in the *Finite Difference* method of Figure 7.5 do not distort this comparison.

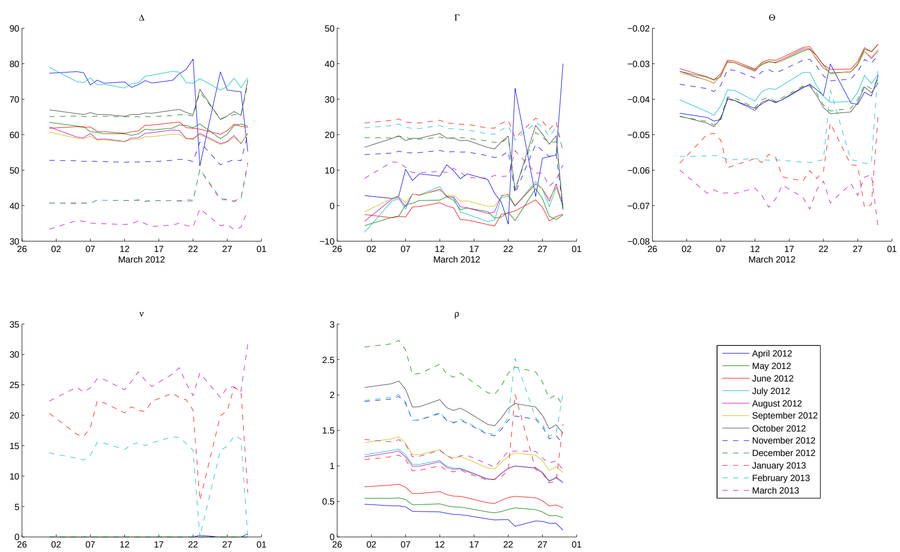

Figure 7.16: Plot shows the monthly Greeks of Keppo's lower bound for a Swing Contract. The option is based on data from the NBP market. The period is 16 August 2011 - 15 February 2012. The Greeks sensitivies are calculated per month (12 total since contract is 1 year) using the finite difference method described in Section 6.1.1. The granularity is weekly.

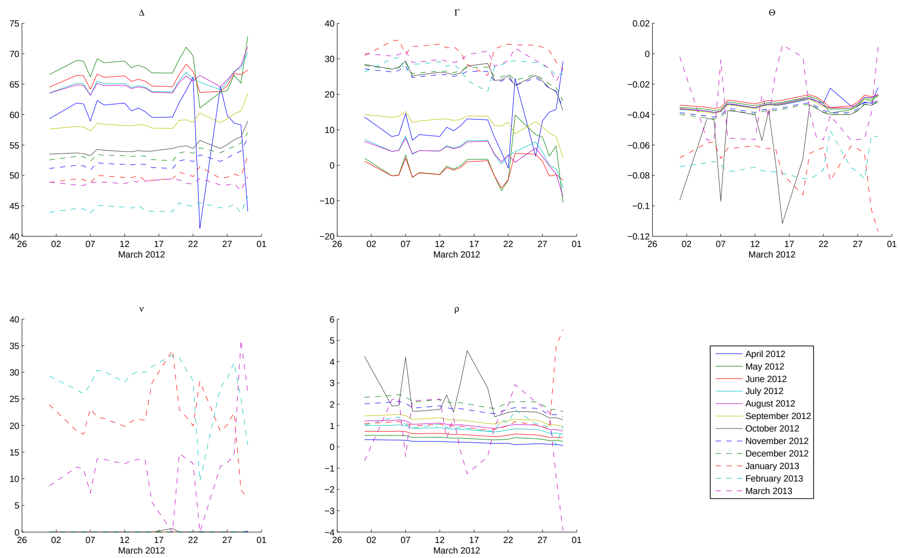

Figure 7.17: Plot shows the Greeks of Delta and Delta-Gamma Hedge. The period is 16 August 2011 - 15 February 2012 and the hedges were rebalanced on the 16 August. The Greeks sensitivies are calculated as explained in Section 6.2.1. The granularity is daily.

From the results it is obvious that granularity has a large effect on the *Greek* sensitivities of a swing option. The effect is most visible when going from monthly (see Section 7.3.1) to weekly granularity. The Δ jumps much less compared to the monthly granularity. This is probably due to that the shift in optimum in the linear programming algorithm does not cause such a large shift in volume between the months. In the monthly study a shift in optimum could cause the entire optional volume of a month to disappear. The Γ goes from three months in the range 1-3 to twelve months in the range (-8)-30. The ν is roughly double in its range. However only in three months are $\nu \neq 0$ while all twelve months have the other $Greeks \neq 0$. This suggest that something maybe wrong in the *Finite difference* method for ν.

When going from weekly to daily granularity we also see an effect. The range of Γ is increased to (-10)-35. The Δ still moves more smoothly than compared to the monthly study.

By using a finer granularity the lower bound now spreads out Γ_C more evenly over the whole period. The intuition is that the holder of the Swing is more uncertain where it is profitable to exercise. I good example are the weekends when prices are generally lower. On a daily granularity it would be possible to choose p_{low} every weekend and switch that volume to more profitable weekdays. In the monthly granularity case this is not possible.

The effect of going from weekly to daily granularity is not as substantial. The largest effect is seen in ρ where months October 2012, January 2013, March 2013 differ substantially. The *Greeks* generally lie in the same range and follow the same day to day changes. The intuition in this case is somewhat harder to determine. Maybe the smaller granularity do not add much optionality to the swing option. Maybe an even smaller granularity is needed to have optionality in intra day trading.

7.4 Analysis

The *Greeks* estimates in Section 7.3 show some interesting results. The *Finite difference* method is able to calculate some of the *Greeks*. However we saw some evidence of faulty approximations of ν. It is somewhat unstable when calculating on finer granularity and fails to provide any values for intra-day granularity. At the same time calculating on finer granularity is much more computationally expensive and calculation times increase substantially. It is also apparent that the *Greeks* jumps due to shifts in the optimal solution in the linear programming algorithm. In reality these are probably not jumps but occur due to the frequency of forward curves. We only have daily forward curves. This suggest that some sensitivity analysis on the proximity of shifts would be suitable. Another approach would be to use the Delta or Delta-Gamma hedge and rebalance when a jump occurs.

Looking at the Markov assumption that produces the lower bound we can provide some reasoning around these jumps. The Markov assumption lowers the optionality of the swing option and hence lowers the price. By maximizing over the expectations in the forward curve the holder chooses to nominate the lower cumulative obligated volume in the time periods where it is most certain not to make any loss. This is the volume decided by Γ_S and is replicated as forwards. Compared to the swing option the lower bound is more static in these volumes. Hence the lower bound fails to follow the finer granularity dynamics of the swing option and will make these jumps when shifting volume in Γ_S over time. As an effect the *Greek* approximation will also jump on finer granularity and the Delta and Delta-Gamma hedge fail to follow the swing option dynamically. In the results section we can see that these jumps are significant. In addition Γ neutrality, in the Delta-Gamma hedge, fail to capture the changes in Δ. Hence we must conclude that using the *Finite difference* method on the lower bound is a blunt tool for hedging the swing option.

The comparison study between the lower bound and the LSMC gives inconclusive results. The LSMC provides values below the lower bound and hence there must be a calibration error. Disregarding the value dependent on volatility of the lower bound, i.e. the call options, we still find values of the LSMC below. The conclusion is thus that the error lie with the LSMC. Additional studies found that the LSMC consumed close to the lower cumulated level even though the strike was extremely low. This implies that the model is defect.

When comparing the Δ of the LSMC and the lower bound we spot the biggest difference in the months of Γ_C. As Γ_C and Γ_S are disjoint sets and a Call Option has lower Δ than a Forward, see Table 6.1 ($e^{-r(T-t)}N(d_1) < 1$, these months will have a lower Δ. Whether this Δ difference can be seen as a loss of properties in the lower bound approximation or due to better *Greek* approximation cannot be concluded.

As finer granularity in a Swing Contract means that the holder has more optionality we would expect to see a higher price. The granulation study confirms this and also shows that finer granularity have a larger impact on the *Greeks* of the lower bound. As reasoned finer granularity would mean that the holder could take advantage of the higher frequency seasonality. The distribution of Γ_C and Γ_S would therefore be more spread out and hence the same effect would be seen in the *Greeks* of the lower bound.

By using this finer granularity *Greeks* we lessen the effects of the jumps and get higher Γ values. Maybe using this Δ and Γ in Delta-Gamma hedge could prove to more effective then the rougher granularity *Greeks*.

This method would increase the scaling error discussed in Section 5.3. And as the optionality increases we can no longer ensure a lower bound. However by increasing the optionality we would hope to mitigate the loss of dynamics in the price of the swing option compared to the lower bound. This would mean that the *Greeks* of this finer granularity lower bound approximate the swing option much better.

The *Finite Difference* method became unstable for very fine granularity (intra-day) and hence prevented any such study.

Chapter 8

Conclusion and Outlook

8.1 Conclusion

In this report we formulate a solution to the lower bound of the value of a swing option in the gas and Energy market. The solution is based on a *model-free* approach meaning that we do not assume a certain spot price dynamics. In addition all assumptions on the solution minimize the result ensuring that we calculate a lower bound.

The problem is solved using a linear programming algorithm that calculates the price of a standard 1 year monthly contract in 4-5 seconds. The final result is a model that only depends on the prices of forwards and call options. Hence we provide a stable, fast and simple method for bounding the price of the swing option.

The lower bound approximation is due to a loss of optionality in the swing option. We argue that this loss of optionality decreases the price and changes the dynamics of the swing option with respect to the underlying. The effects of this is above all seen as jumps in the *Greeks* with respect to time.

Using the lower bound we produce two different hedges. First a *replicating portfolio* is derived. It is constructed from the weights of the *forwards* and *call options* in the solution to the swing option.

The other hedge is based on neutralizing the *Greeks*. Using a *Finite Difference* method on the lower bound we are able to neutralize up to three different *Greeks*. A Delta and a Delta-Gamma hedge is evaluated using a monthly and daily study. The change in dynamics from using a lower bound is seen in both hedges in forms of large jumps. While the method is fairly successful in eliminating small price changes it is a blunt tool when hedging larger price shifts.

The effects of granularity is examined on the lower bound. By comparing price and *Greeks* on similar contracts with monthly, weekly and daily granularity we show that both the price increases and the *Greeks* change substantially. The effects is mostly seen in Γ where the finer granularity seem to capture price seasonality on higher frequencies.

Finally the method is compared to a Least Square Monte Carlo method (LSMC). The results around the lower bound are inconclusive in that the LSMC price is lower. Hence we cannot conclude anything about the lower bound errors. However using the lower bound we have thus found an error in the more complex LSMC. The simplicity of the lower bound has proven to be useful support for a complex model.

8.2 Outlook

A method to improve the Hedging instruments could be to use the sensitivity analysis discussed in Section 6.1.2. This could make a *Greek* approximation more stable as we do not have to configure a small perturbation h. At the same time the sensitivity analysis could provide information of how close we are to a change of optimum and hence predict the jumps we saw in the results. Finally using the shadow prices could improve the performance of the method on finer granularity.

Using this alternative approach a new Delta and Delta-Gamma hedge could be evaluated using a finer granularity. This would test if the finer granularity lower bound is a better approximation of the *Greeks* in the swing option. As it is now the *Finite Difference* method becomes computationally intensive for finer granularity and decreases the performance of the method substantially.

Another hedge could be constructed using the mean squared error between the lower bound and a set of market derivatives. This hedge could capture the larger price movement that could occur in the price of the swing option. To construct this hedge we could assume a distribution on the underlying and simulate the proximity of the lower bound using Monte Carlo methods. The method can be summarized as following,

$$\begin{aligned}
&\min(E^Q\Big[(\hat{\varphi}_{LB} - \sum_i \beta_i \Pi_i)^2 | \mathcal{F}_t\Big]) \\
&= \min(\int (\hat{\varphi}_{LB} - \sum_i \beta_i \Pi_i)^2 w(x) dx) \\
&\approx \min(\frac{1}{N} \sum (\hat{\varphi}_{LB} - \sum_i \beta_i \Pi_i)^2)
\end{aligned}$$

The last step is equal to minimizing the residual sum of squares and can be solved using the normal equation of linear regression. A visual example of this is shown in Figure 8.1. By simulating on the proximity of the lower bound we achieve better estimates of a hedge resistant to larger price changes. The trade-off lie in the loss of resolution as we calculate an average. A Delta Hedge would therefore be better when hedging smaller price changes. The Delta would be an approximation of the point wise derivative in the Figure. If we include other derivatives we could construct portfolios that capture more than linear trends, e.g. the right plot in Figure 8.1.

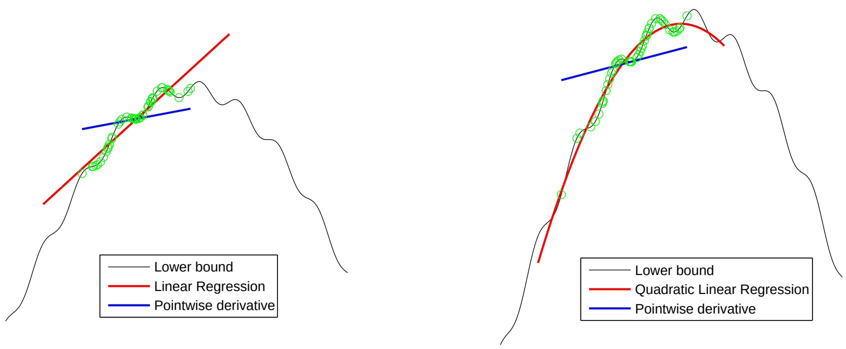

Figure 8.1: The figure shows an example of simulation the proximity of the lower bound. The density is assumed to be normal distributed and the number of particles is 100. Left figures shows a linear regression with $y_i = \beta_0 + \beta_1 x_i$. Right figure shows a linear regression with $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$.

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Appendix A

First Appendix

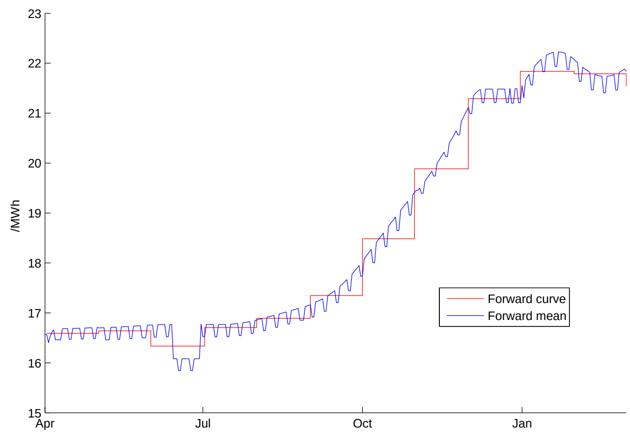

Figure A.1: Plot shows example of mean forward curve compared to the actual forward curve.

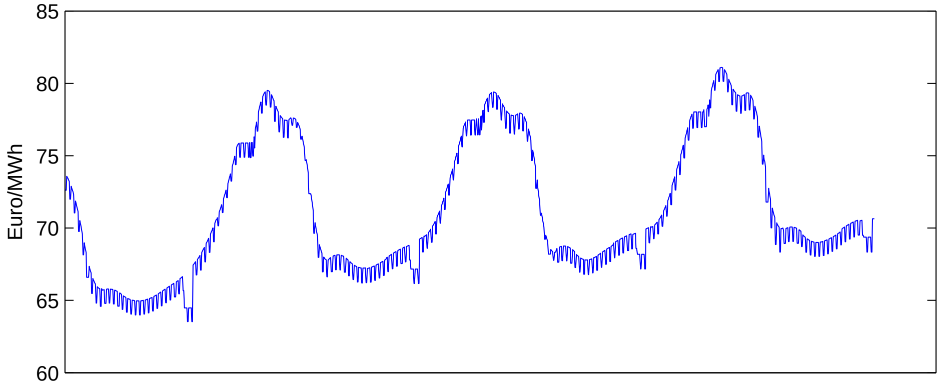

Figure A.2: A sample of a forward curve. The curve follows a sinusodial pattern where the lowest frequency is on yearly scale. The top of the curve represent the winter and there are

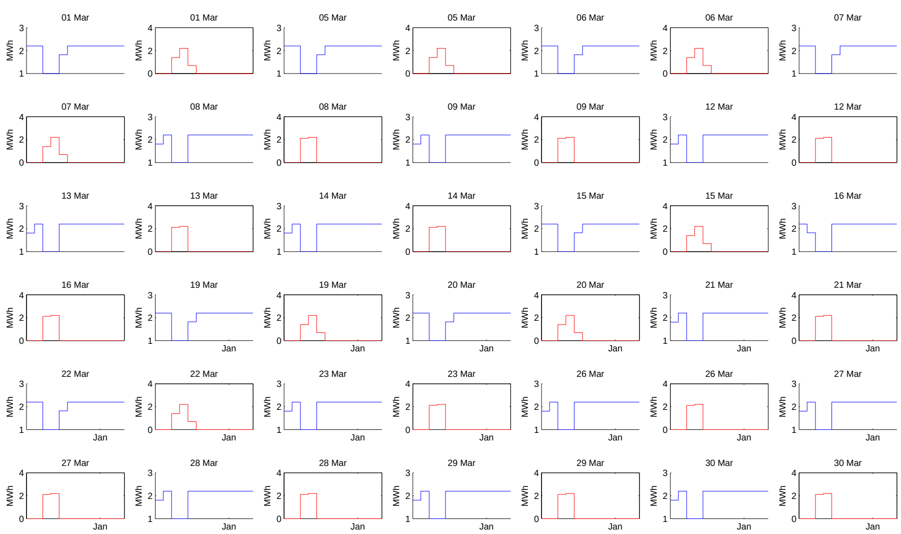

Figure A.3: Plot shows the daily Optimal consumption paths of Swing Contract based on the NBP market. The period is 1 March - 30 March 2012.