Closing Time Effects on Derivative Pricing and Risk Measurement

Andreas Melin

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Abstract

Risk measures such as Value at Risk are highly dependent on a sample of daily returns. The daily return can be measured in different intervals and this thesis examines how the choice of the daily return interval affects derivative pricing, risk measures, fund performance and products with collateral agreements. It is shown that the VaR measure varies significantly with the choice of daily return interval and for the OMXS30, an optimal interval is presented. The daily return characteristics are examined using market microstructure theories and the concept of volatility is also handled for different time intervals. Collateral payments have a direct effect on a bank's result due to the funding costs of outstanding collateral. It is shown that the daily return interval is an important component in determining the payments and that the choice of return interval can be used to have an expected positive flow of collateral which leads to a positive revenue component for the bank.

Abstract

Riskmått som Value at Risk är väldigt beroende av ett sample av dagliga avkastningar. De dagliga avkastningarna kan mätas med olika tidsintervall och denna uppsats behandlar hur valet av tidsintervall påverkar prissättningen av derivatprodukter, riskmått, hur bra en fond ser ut att prestera samt produkter med collateralavtal. Det visas att Value at Risk-måttet varierar oerhört med valet av tidsintervall för de dagliga avkastningarna och det presenteras ett optimalt val av tidsintervall för OMXS30. Beteende hos de dagliga avkastningarna behandlas med teorier kring marknadens mikrostruktur och volatilitetsbegreppet i avseende på olika upplösning i tidsdimensionen diskuteras. Betalning av collateral har en direkt effekt på en banks resultat då det direkt slår mot bankens finansieringskostnader. Det visas att tidsintervallet för de dagliga avkastningarna är en viktig komponent för att beräkna collateralflödet och att valet av tidsintervall kan användas för att få ett förväntat positivt collateralflöde vilket blir en intäktskälla för banken.

Preface This thesis is written in conjunction with the Market Risk Control Department at Handelsbanken Capital Markets in Stockholm. I would like to thank the group for their never-ending support and especially for the intriguing discussions about financial risk modeling. I would also like to thank my supervisor at LTH, Erik Lindström, for valuable comments and powerful insights along the way.

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1 Background

Risk monitoring and asset pricing is essential in all fields of finance and is a important part of every serious investment banks operations. Risk measures in combination with risk limits are used to steer the company towards profitable risk taking, without taking risks that would be able to significantly damage operations. After the financial crisis in 2008, many of the risk measurement models used have been significantly criticized but many of them are still included in new regulatories such as Basel III as the recommended or the only accepted way to compute various risks. The risk measures are often constructed in such a way that they are easy to understand and that they answer a relevant question. The construction however leaves a couple of questions unanswered which makes the risk measures dangerous if they interpreted in the wrong way. Asset pricing is also a very important part of an investment banks operations. Without correct pricing, risk measures cannot be computed correctly and trading results will be erroneous. If the bank offers their costumers too low prices, the customers will buy as much as they can and the bank itself will not be able to hedge their risk without loosing money. If the prices are too high the costumers will buy from another market participant and the reputation of the bank will be damaged. Both asset pricing and risk measurement depends on high quality market data. A lot of companies that provides market data in various forms such as Bloomberg and Thomson Reuters, and investment banks depends severely on their products. By snapping data directly from the external source, the bank gets clean and validated data input to their asset pricing and risk measurement mechanisms. The data however is very expensive and a bank usually pays every time they download data. Market prices however are volatile and should be used directly when they are snapped, or seen in a theoretical perspective, snapped when they are needed. In order to save computational power and money, sometimes market data snapped at a certain time is used later on during the day. As an example, a stock option valued at 17.30 can use an asset price from 16.30 and a FX quote from 16.00. This can possibly yield bad prices even though the valuation model is correct. As for risk measurement, the risk figures can differ significant depending on the time the market data is snapped. For products with collateral agreements, different valuation times can lead to differences in valuation between the counterparts which possibly can yield large payments from one bank to another. This thesis will examine how sensitive different pricing techniques and risk measures are to market data snapped at different times, and how much a change in snapping time will affect the pricing and risk measurement. The focus will be on how to give an asset or fund an end of day price and how the VaR and CVAR measure is affected by changes in the data snapping time.

2 Relevant Literature

A good introduction to different asset types and the pricing of various assets is given in Hull [2011] and a bit more mathematical approach is given in Åberg [2010]. The risk neutral framework for valuation of derivatives is nicely covered in Björk [2009] and the mathematics needed can be found in Shreve [2008]. Pricing of interest rate products are covered in depth in Hull and White [1990] and especially swaps can be found in Rolapp [2006] with the theory of yield curve construction in Hagan and West [2007]. The estimation of data, especially volatility used to compute asset prices can be found in Becker et al. [2009] with some very interesting ideas on how to estimate intraday volatility from high frequency data in Zhang et al. [2005] and Chan and Karolyi [1991]. Some characteristics of the market micro structure and empirical properties of asset returns can be found in Cont [2001]. An introduction to various risk measures and especially Value at Risk is presented in Duffie and Pan [1997] and Linsmeier and Pearson [1999]. The more advanced reader will find both Uryasev [2000] and Sarykalin et al. [2008] as interesting approaches to Conditional Value at Risk and related optimization algorithms. In order to measure the performance of funds or separate assets, performance measures are used which are explained in Bailey [2012]. Stochastic analysis is used to model and simulate stock price movements, and Klebaner [1998] gives a thoroughly introduction to the subject. The Brownian motion itself is presented in detail in Morters and Peres [2008] and Westman and Hanson [2002] extends the framework to model stock price movements as jump diffusion processes.

3 Derivative Pricing

3.1 Fixed Income Markets

A fixed income security is defined as a financial obligation of an entity that promises to pay a specified, fixed amount of money at specified future dates. The most commonly traded fixed income products are bonds and preferred stocks. To price a bond, one must be able to compute the value today of a future fixed cash flow. The Net Present Value, NPV, of a future cash flow is computed by discounting the cash flow with an interest rate with the same maturity and which is regarded as risk free. The valuation process can be seen in the picture below.

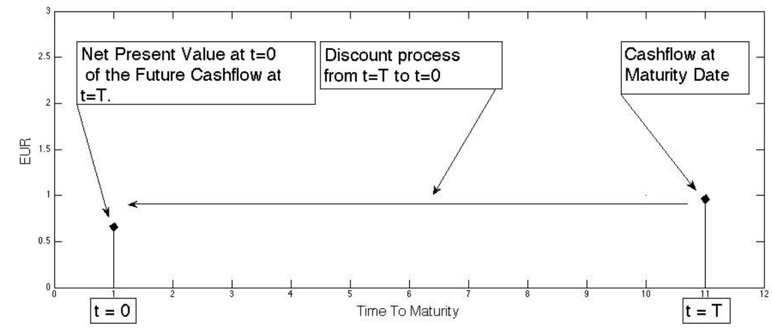

Figure 1: Valuation of a Zero Coupon Bond

To be able to price a cash flow when it does not exist a risk free rate with the same time to maturity, one must interpolate the discount rate points into a yield curve. With the yield curve, it is possible to compute the NPV for all cash flows that have a corresponding point on the yield curve. The usual representation of the yield curve, also used in Hagan and West [2007], is a function $F(t)$ defined on all future times t such that $F(t)$ represents the value today of receiving one unit of currency t years from now.

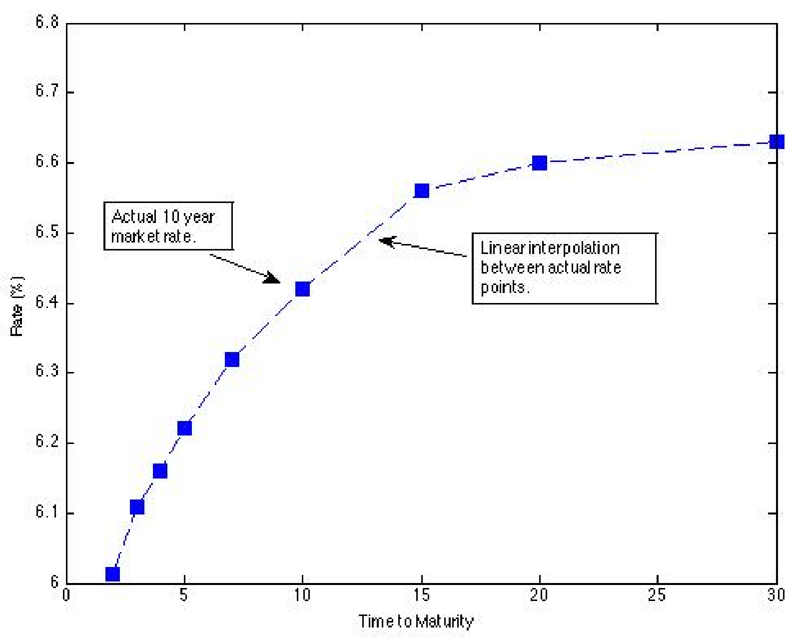

Figure 2: Actual interest rate points and an interpolated discount curve.

3.1.1 Construction of Discounting Curves

According to Rolapp [2006] it is market practice to choose the swap rate with the same currency as the contract that is to be valued and construct a set of swap rates with different maturities. From these points, there exists several ways of constructing a yield curve. Different interpolation methods yields different characteristics which largely can affect the pricing process in certain conditions according to Hagan and West [2006]. To examine these characteristics, three different interpolation methods is introduced and then empirically tested in 3.1.2. These methods are implemented as standard interpolation methods in Sungard's Front Arena Prime and hence are they commonly used among banks and funds.

Linear Interpolation Linear interpolation is the most simple method to construct a yield curve. It simply draws a straight line from one point to another. This method is robust against problems in the data set since one erroneously placed point does just affect the line to the points next to it. If the rates for a certain maturity is not used for valuation of the actual contract, the method will make a correct valuation even though a point is erroneously placed. As an example, lets compute the seven year rate using linear interpolation with the data set given in Table 1.

$$Y_i(t) = Y_i + \frac{t - T_i}{T_{i+1} - T_i}(Y_{i+1} - Y_i)Y_i(t) = Y_i + \frac{t - T_i}{T_{i+1} - T_i}(Y_{i+1} - Y_i) \quad (1)$$

Time to Maturity	Yield to maturity
0	2.00%
1	3.00%
3	4.00%
10	6.50%

Table 1: Constructed example data set for bond valuation.

The 7 year YTM-rate is then computed by formula (1)

$$Y_3(7) = 4.00 + \frac{7 - 3}{10 - 3}(6.50 - 4.00) = 5.4286\% \quad (2)$$

Cubic Spline Interpolation A spline interpolation is an interpolation method where the interpolant is a type of piecewise polynomial which is called a spline. The cubic spline simply uses a third degree polynomial. The yield curve is represented by a number of functions $Y_x(t)$ each representing the yield curve between two rate points. According to Hagan and West [2006], the Cubic Spline method suffers from global errors if one rate point gets an erroneous value. If one point in the interest rate products that is building the yield curve is mispriced or there is an error in the conversion between the product prices and the yield curve, this error will change the whole resulting yield curve which will result in a faulty asset price. For the example data in Table 1, the following equations leads to the yield curve.

$$\begin{aligned}
Y_0(t) &= a_0 t^3 + b_0 t^2 + c_0 t + d_0 && T_0 \le t \le T_1 \\
Y_1(t) &= a_1 t^3 + b_1 t^2 + c_1 t + d_1 && T_1 \le t \le T_2 \\
Y_2(t) &= a_2 t^3 + b_2 t^2 + c_2 t + d_2 && T_2 \le t \le T_3
\end{aligned} \tag{3}$$

Every equation has 4 unknown variables, the coefficients a, b, c, and d which yields the following system of equations.

$$\begin{bmatrix}
T_0^3 & T_0^2 & T_0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
T_1^3 & T_1^2 & T_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & T_1^3 & T_1^2 & T_1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & T_2^3 & T_2^2 & T_2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_2^3 & T_2^2 & T_2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_3^3 & T_3^2 & T_3 & 1 \\
3T_1^2 & 2T_1 & 1 & 0 & -3T_1^2 & -2T_1 & -1 & 0 & 0 & 0 & 0 & 0 \\
6T_1 & 2 & 0 & 0 & -6T_1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3T_2^2 & 2T_2 & 1 & 0 & -3T_2^2 & -2T_2 & -1 & 0 \\
0 & 0 & 0 & 0 & 6T_2 & 2 & 0 & 0 & -6T_2 & -2 & 0 & 0 \\
6T_0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6T_3 & 2 & 0 & 0
\end{bmatrix}
\begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix}
=
\begin{bmatrix} 2.00 \\ 3.00 \\ 3.00 \\ 4.00 \\ 4.00 \\ 6.50 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The coefficients become

$$\begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix}
=
\begin{bmatrix} -0.1798 \\ 0 \\ 1.1798 \\ 2.0000 \\ 0.0143 \\ -0.1419 \\ 0.8811 \\ 2.2464 \\ 0.0006 \\ -0.0183 \\ 0.5104 \\ 2.6171 \end{bmatrix}$$

And the 7 year YTM-rate can then be computed by

$$Y_2(t) = a_2 * t^3 + b_2 * t^2 + c_2 * t + d_2 \Longrightarrow \tag{4}$$

$$Y_2(7) = 0.0006 * 7^3 - 0.0183 * 7^2 + 0.5104 * 7 + 2.6171 = 5.5019\%$$

Hermite Interpolation Hermite interpolation is method that interpolates data points as a polynomial function. The Hermite polynomial uses divided differences and is therefor closely related to the Newton polynomial according to Tobon [2011]. The difference is that the Hermite method needs more data to compute the polynomial but can on the other hand produce a polynomial of a higher degree than the Newton method. Contrary to the Cubic Spline method, big shifts in one rate point does just change the function locally. Let r be a vector $Y' = \{y_1, y_2, ..., y_n\}$ and

$$Y(t) = Y_i + m_i(t)(Y_{i+1} - Y_i) + m_i(t)(1 - m_i(t))g_i + m_i^2(t)(1 - m_i(t))c_i \quad (5)$$

where

$$\begin{aligned} m_i(t) &= \frac{t - T_i}{T_{i+1} - T_i} \\ g_i &= (T_{i+1} - T_i)y_i - (Y_{i+1} - Y_i) \\ c_i &= 2(Y_{i+1} - Y_i) - (T_{i+1} - T_i)(y_{i+1} + y_i) \end{aligned}$$

Then the vector Y' is computed by inserting $m_i(t)$, g_i, c_i in formula (5) which yields

$$y_i = \frac{1}{T_{i+1} - T_{i-1}} \left[\frac{(Y_i - Y_{i-1})(T_{i+1} - T_i)}{T_i - T_{i-1}} + \frac{(Y_{i+1} - Y_i)(T_i - T_{i-1})}{T_{i+1} - T_i} \right] \quad (6)$$

with the boundary conditions

$$\begin{aligned} y_1 &= \frac{1}{T_3 - T_1} \left[\frac{(Y_2 - Y_1)(T_3 + T_2 - 2T_1)}{T_2 - T_1} + \frac{(Y_3 - Y_2)(T_2 - T_1)}{T_3 - T_2} \right] \\ y_n &= \frac{1}{T_{n+1} - T_{n-1}} \left[\frac{(Y_n - Y_{n-1})(T_n - T_{n-1})}{T_{n-1} - T_{n-2}} + \frac{(Y_n - Y_{n-1})(2T_n - T_{n-1} - T_{n-2})}{T_n - T_{n-1}} \right] \end{aligned}$$

When the vector Y' is computed, it is possible to simply insert the desired t into the equation to obtain the interpolated interest rate.

3.1.2 Empirical Test: Sensitivity of Interpolation Methods

The goal is to examine the properties of the different interpolation methods and their effect on the theoretical price off an OTC interest rate swap. To see the models different characteristics, a mispricing in the underlying rates is simulated according to the first column in Table 3. Since the interpolation methods handles shifts in the interest rates point in different ways, the robustness concerning a mispricing in the underlying interest rates is shown in Table 3. The swap used for pricing is an interest rate swap that swaps a fixed rate for the 3 month STIBOR rate with start date in May 2009 and with maturity date in May 2024. To simulate the mispricing, the 1 year rate point for the underlying discount curve is shifted. The delta risk exposure of the IR-swap before the mispricing is

1D	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y	15Y
17	1910	708	8	-729	-1446	-2140	-2796	-5394	-11726	-162554	-4222

Table 2: Delta risks as a function of maturity for the IR-swap.

Since the swap has risk exposure for every maturity according to Table 2, it is shown in Table 3 that the Cubic Spline is most affected by the rate shift, especially for the downward shift. As the present value of the contract is around 31 Million SEK, the biggest difference (Cubic Spline with rate shift -0,15%) is below 0,005% of the present value. Since it is an OTC contract, it is not traded in the market and the present value of the contract is strictly theoretical. In that regard, the choice of discounting rates seems to affect the price far more than the actual interpolation methods since there is no market consensus on which discounting rates to use. However, the test confirms the hypothesis that the linear method handles a change in a rate point the best since it just adapts locally. The actual price changes are even so far too small to have any impact on general business.

Rate shifts in %	Cubic Spline	Linear	Hermite
0,15	849	518	959
0,06	200	236	475
0,03	17	142	313
0	0	0	0
-0,03	450	46	9
-0,06	666	140	171
-0,15	1532	423	655
Sum of differences	3714	1505	2582

Table 3: Price differences from the theoretical price as a function of shifts in the yield curve.

The over all price differences between the different interpolation methods is very small and since the contract is not traded in the market it is not possible to claim that one interpolation method is better than another. As long as the buyer accepted the price when the contract was sold, the counterpart just have to quote daily prices using the same method to discount to avoid jumps in the valuation.

3.2 Derivatives with Equity as Underlying

The most common derivative with equity as underlying are options. Options are traded both on exchanges and over-the-counter. There exists two types of options, a call option gives the holder the right to buy the underlying asset at a certain date at a certain price and a put option gives the right to sell the underlying asset at a certain date at a certain price. The predetermined price in the contract is known as the strike price and is denoted K. The date in the contract is known as the maturity and is denoted T. An option is a very versatile product that can be used in both speculative and conservative ways. This versatility however comes with the price that an option is a very complex security that can be extremely risky if used in certain ways. The payoff of an call option at maturity T is

$$max(S_T - K, 0)$$

and for the put options the payoff is

$$max(K - S_T, 0)$$

A common way to price options is to use the well known Black & Scholes formula according to Danthine and Donaldson [2005]. The Black-Scholes equation for european options is

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

The solution to this equation is the closed form solution for the price of an european option. For the call option, the closed form solution is

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)},$$

where

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma(T-t)}$$

and

$$d_2 = \frac{ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma(T-t)} = d_1 - \sigma\sqrt{T-t}$$

There exists several other ways the compute a theoretical price of an option. Stochastic differential equations with far more complex dynamics than the Black & Scholes model often yields far more accurate option prices. There is also a branch of simulation techniques which simulates the behavior of the underlying assets with an Monte Carlo algorithm. Another way the compute the option price is to discretisize the stochastic differential equations and numerically approximate the solution. An introduction to several option pricing techniques is given in Melin [2013].

3.3 Derivatives with Commodities as Underlying

The most common contract with commodities as underlying are futures and forwards. The contract is set between two parties to buy or sell a certain quantity of a specified asset to a fixed price at a fixed future date. A futures contract is a standardized contract that is actively traded on several exchanges. A forward contract is custom made contract between the two parties and does not trade in the markets. Unlike an option, both parties of a futures contract must fulfill the contract on delivery date. This feature makes the contract ideal for commodity producers such as farmers to fix the price for their harvest and thus reduce the uncertainty of their income. For commodities that exists in plentiful supply the contract can be priced via arbitrage arguments and this is typical for physical commodities. The price of a forward or future $F(t,T)$ at time t with delivery at T is the assets spot price $S(t)$ compounded by the risk free rate r during the contracts life $(T-t)$.

$$F(t,T) = S(t)(1+r)^{(T-t)}$$

with discrete compounding and

$$F(t,T) = S(t)e^{r(T-t)}$$

with continuos compounding.

3.4 Interest Rate Derivatives

To be able to compute prices for interest rate contracts one must be able to model the interest rate itself. There exists many models that tries to model the interest rate, and one of the more famous models is the Vasiciek model. The Vasiciek model assumes that interest rate movements only is driven by changes in market risk. The model specifies that the instantaneous interest rate follows the stochastic differential equation

$$dr_t = a(b - r_t)dt + \sigma dW_t \tag{7}$$

where b is the long term mean of the interest rate, a is the velocity at which the rate will reverse towards b and σ is the instantaneous volatility that enters the system. The model can be used to price various interest rate products such as caps, floors, bond options and swaptions and other even more complex contracts. The price of a bond in the Vasciek model can be calculated by

$$\begin{aligned} r_t &= e^{-at}\left[r_0 + \int_0^t abe^{au}du + \sigma\int_0^t e^{au}dW_u\right] \\ &= e^{-at}\left[r_0 + b(e^{at}-1) + \int_0^t \sigma e^{au}dW_u\right] \\ &= u_t + \sigma\int_0^t e^{a(u-t)}dW_u \end{aligned}$$

where u_t is a deterministic function and $E\left[r_t\right] = u_t$. The expected value $E\left[r_t\right]$ can be determined without solving the stochastic differential equation. By taking the integral of the dynamics in (7)

$$r_t = r_0 + \int_0^t (a(b - r_u)du + \sigma dW_u)$$

is obtained and this yields

$$u_t : E\left[r_t\right] = r_0 + \int_0^t a(b - E\left[r_u\right])du \Rightarrow$$

$$\frac{d}{dt}u_t = a(b - u_t)$$

This is a linear ordinary differential equation that can be solved using the integrating factor e^{at} .

$$u_t = E\left[r_t\right] = e^{-at}\left[r_0 + b(e^{at} - 1)\right]$$

Further, the volatility can be defined as

$$\sigma_t^2 = Var\left[r_t\right] = E\left[(\sigma e^{-at}\int_0^t dW_u)^2\right]$$

$$= \sigma^2 e^{-at} E\left[\int_0^t e^{2au}du\right]$$

$$= \sigma^2(\frac{1 - e^{-2at}}{2a})$$

Now using the risk neutral valuation framework, which is explained in detail in Björk [2009], the price of a zero coupon bond in the Vasiciek model can be written as

$$B(t,T) = E\left[exp(-\int_t^T r_u du) \mid F_t\right]$$

Another interest rate contract that is commonly traded is the interest rate swap. It simply exchanges the cash flow between two parties from two different interest rate products. An example is that Company A has loans with a floating rate and wants to buy an insurance against higher interest rates. One way to do this is to buy a interest swap and obtain a fixed rate instead of the floating rate. The value of the swap for the party that receives the fixed rate is

$$V_{swap} = B_{fix} - B_{float}$$

Where B_{fix} and B_{float} is the prices of a fixed rate bond and a float rate bond respectively. The price of the fixed rate bond is very intuitive and it can be expressed as the sum of the future cash flows discounted back to todays date. More formal it can be written as

$$B_{fix}(t,T) = \frac{N}{(1+r)^{(T-t)}} + \sum \frac{CF_t}{(1+r)^t}$$

The floating rate bond is a bit more complex to value. It can be valued using an argument that the bond is worth its notional value directly after an interest payment since it is the only “fair price” and there is no accrued interest. Suppose that the notional is N and that one interest payment is denoted k, the value of the bond just before an interest payment is $N + k$. Because the value of the bond is N directly after a payment, it can be regarded as an instrument providing just one cash flow at time t where t is the time to the next payment. Hence, the value of the future cash flow at time t is

$$(N + k)e^{-rt}$$

where r is the risk free rate.

4 Pricing Discrepancies and Closing Times

4.1 Price Changes as a Function of Time

As proposed by Lunde and Timmermann [2004] a part of the stock price movements can be explained by observable factors which leads to a semi predictable market. Fama and French [2004] and Poterba and Summers [1989] provided empirical support for the existence of mean-reverting component in the stock price process and Campbell and Shiller [1988], Fama and French [1989], Ferson and Harvey [1991] showed that risk premiums, maturity premiums and expected earnings could be used to model and predict the stock price movements. The factors described can all be considered as a function of time since they all vary with time. The time itself might not be the reason to the price changes, but as time progresses the underlying factors changes which leads to changes in valuation. The information available, the investors will to take on risk and the expectations of the market does also change with time. This leads to that even fixed contracts such as bonds will have a fluctuation in their price during its life time. In Cont [2001] the stock price is explained as a continuos process that only can be observed during the time when the market is open. When the market is closed, the process is still ongoing but because the price cannot be observed, the price will make a jump when the market opens again. The plot below shows a price process which is unobservable but made visible inside the red dotted lines.

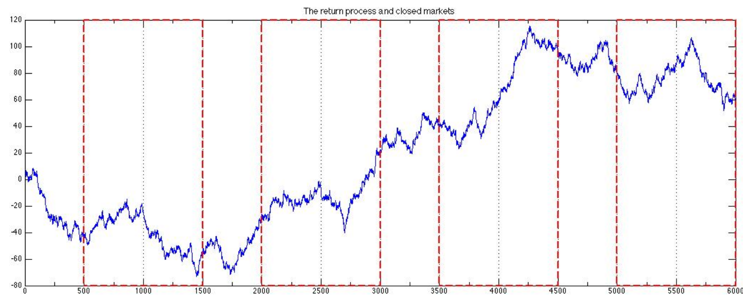

Figure 3: Accumulated returns of an asset and the markets opening hours.

4.2 Valuation Jumps Depending on Closing Times

Since the price movement is depending on time, the price will make large jumps if the time when the process is unobserved is long. If a portfolio consisting of equities is valued before market closing there will be a significant error in end of day pricing which will lead to a larger jump the following day when the market opens. In Table 4, it is shown how large the average mispricing component of an early closing is in regards to the asset price at closing time. For a single stock, the mean of the absolute price difference is well above 0,3% which states the obvious fact that a lot of available information for pricing is not being used if the end of day pricing is done before the markets close.

Stock name \ Closing time	16.00	16.15	16.30	16.45	17.00	17.15	17.30
ASTRA ZENECA	0,27%	0,24%	0,22%	0,20%	0,17%	0,13%	0,0%
ELECTROLUX B	0,35%	0,32%	0,29%	0,26%	0,22%	0,17%	0,0%
HM B	0,27%	0,25%	0,23%	0,21%	0,17%	0,17%	0,0%
SANDVIK	0,35%	0,31%	0,29%	0,25%	0,19%	0,16%	0,0%
SKANSKA	0,25%	0,24%	0,22%	0,19%	0,17%	0,13%	0,0%
SSAB A	0,42%	0,39%	0,36%	0,31%	0,27%	0,22%	0,0%
SWEDBANK A	0,35%	0,32%	0,29%	0,25%	0,21%	0,18%	0,0%
VOLVO B	0,32%	0,28%	0,24%	0,21%	0,19%	0,16%	0,0%

Table 4: Average absolute price difference from the actual closing price depending on the closing time.

To illustrate Table 4 even more clearly, the size of the valuation jumps are plotted in figure 4 and 5. The time between the when the price is snapped and market closing time is defined as τ. It can be seen that the size of the jumps gets significantly smaller when τ gets smaller. It can also be seen that the largest jump is the one from 17.15 until 17.30 which is interesting since in implies that the stock prices are more volatile at the very end of the day. This is supported by results in Cont [2001] and the results in Section 5.

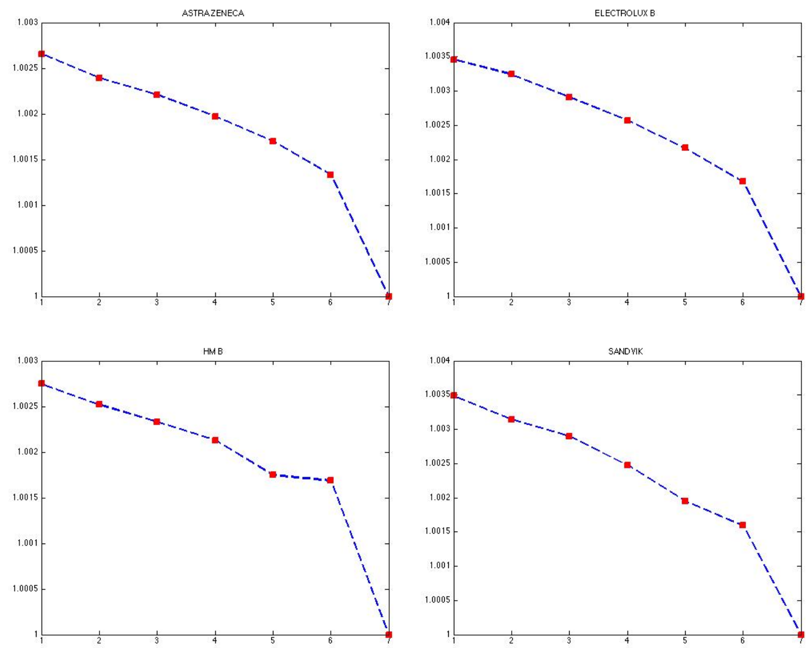

Figure 4: Average absolute price difference from the actual closing price depending on the closing time.

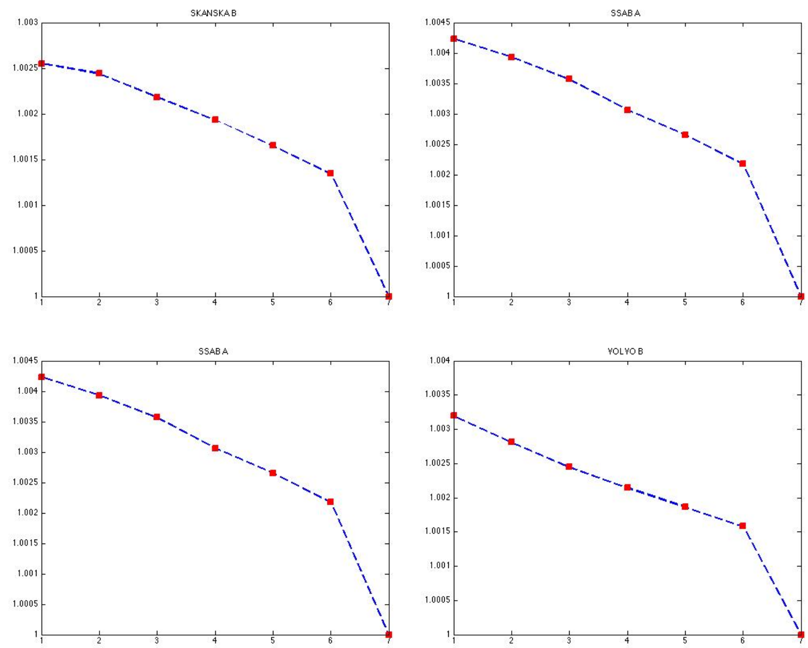

Figure 5: Average absolute price difference from the actual closing price depending on the closing time.

4.2.1 Distribution of Valuation Jumps

The expected value of a valuation jump between two time points is very dependent on the overall trend of the stock. If a stock has 10% positive return in one year, it is equal to a daily return of 0,04%. A valuation jump from 16.30 to 17.30 would then yield a return of 0,005%. These figures are far too small to work with and the volatility computed on a one year data set would be very large. Instead, the size of the valuation jump for an asset S between time t_1 and t_2, $J(S_{t_1}, S_{t_2})$ is computed by

$$J(S_{t_1}, S_{t_2}) = \mid S_{t_2} - S_{t_1} \mid$$

and it is normalized by S_{t_1} to be presented in percentage form. It is observed in Table 5 that the jump size from the actual closing time until the markets open the next day is dependent on the closing time. When the closing time goes towards market closing, the jump size gets smaller. Thus, it can be concluded that the time before closing holds valuable information about the stocks value.

Stock name \ Distribution	16.00	16.15	16.30	16.45	17.00	17.15
ASTRA ZENECA	0,27%	0,24%	0,22%	0,20%	0,17%	0,13%
ELECTROLUX B	0,35%	0,32%	0,29%	0,26%	0,22%	0,17%
HM B	0,27%	0,25%	0,23%	0,21%	0,17%	0,17%
SANDVIK	0,35%	0,31%	0,29%	0,25%	0,19%	0,16%
SKANSKA	0,25%	0,24%	0,22%	0,19%	0,17%	0,13%
SSAB A	0,42%	0,39%	0,36%	0,31%	0,27%	0,22%
SWEDBANK A	0,35%	0,32%	0,29%	0,25%	0,21%	0,18%
VOLVO B	0,32%	0,28%	0,24%	0,21%	0,19%	0,16%

Table 5: Expected value of valuation jump size depending on time.

The volatility of the jumps however is measurable and gives a hint about whether the last hour of trading gives a significant impact on a stocks closing price. As seen in Table 6, the volatility of the jumps is decreasing as τ is decreasing. In line with the results presented in Table 4, a large bit of the total volatility is obtained from 17.15 to 17.30. This yields that the volatility gets higher when market closing is getting closer which is in line with results in Cont [2001]. The volatility of the jumps between a fixed time and 17.30 is presented in Table 6. To further illustrate the distribution and difference of the calculated risk measures, they are plotted for every stock in Figure 6, 7, 8 and 9.

Stock name \ Distribution	16.00	16.15	16.30	16.45	17.00	17.15
ASTRA ZENECA	6,50%	5,86%	5,46%	4,78%	4,15%	3,40%
ELECTROLUX B	8,50%	7,98%	7,07%	6,27%	5,34%	4,14%
HM B	6,58%	6,06%	5,67%	5,14%	4,32%	4,16%
SANDVIK	8,69%	7,85%	7,19%	6,20%	4,92%	4,09%
SKANSKA	6,45%	6,01%	5,27%	4,68%	3,87%	3,31%
SSAB A	10,60%	9,72%	8,55%	7,34%	6,42%	5,22%
SWEDBANK A	8,99%	8,01%	7,28%	6,27%	5,21%	4,38%
VOLVO B	7,83%	6,91%	5,97%	5,40%	4,73%	3,85%

Table 6: Yearly volatility of valuation jumps depending on time.

4.3 VaR and CVaR Dependence on Closing Times

Value at Risk, or more commonly VaR, is by definition a lower α-percentile of some random variable X. VaR is often used in engineering areas involving uncertainties such as material construction, nuclear science and finance. Many finance regulations such as Basel I, Basel II and Basel III uses VaR as risk measure for portfolios with various assets. Since the measure uses the deviation of asset returns, it can be computed as easily both for equities, commodities or bonds. Portfolios that includes combinations of different asset types is just as easy as a single asset type portfolio since the total portfolio returns are used in the computations. The VaR measure is flexible in its definition and can be computed in several different ways. Some different ways will be introduced in this chapter.

Definition 4.1 (Value at Risk) The VaR of X with confidence level $\alpha \in]0, 1[$ is

$$VaR_\alpha(X) = min\,\{z \mid F_X(z) \geq \alpha\}$$

For normally distributed random variables, the VaR is proportional to the standard deviation. If $X \sim N(\mu, \sigma^2)$and $F_X(z)$ is the cumulative distribution function of X, then according to Uryasev [2000] the VaR can be described as

$$VaR_\alpha(X) = F_X^{-1}(\alpha) = \mu + k(\alpha)\sigma$$

where $k(\alpha) = \sqrt{2}erf^{-1}(2\alpha - 1)$ and $erf(z) = (2/\sqrt{\pi}) \int_0^z e^{-t^2} dt$.

4.3.1 Estimating Current Volatility

To be able to compute the VaR measure with some methods and especially for normally distributed random variables, a good approximation of the standard deviation must first be obtained. The standard deviation can be interpreted as market volatility if the random variables are market returns. Two different ways to approximate the market volatility will be presented below.

Historical Volatility The easiest way to approximate the current market volatility is the compute the historical volatility of the market returns. This is made by simply treating every return as a random variable and compute the variance of the random variables. Often, a one year volatility measure is used but volatility in regards to another time scale can easily be transformed to yearly volatility. The standard formula for computing the variance of market returns is

$$\sigma_{t,T}^2 = \frac{1}{T-t} \textstyle\sum_{s=t+1}^{T} (R_s - \hat{\mu_{t,T}})^2$$

Black-Scholes Implied Volatility Another way to to approximate the current market volatility is to use the standardized Black & Scholes formula for option pricing. As we recall from section 3.2, the price of a call option by the Black & Scholes model is

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)},$$

where

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma(T-t)}$$

and

$$d_2 = \frac{ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma(T-t)} = d_1 - \sigma\sqrt{T-t}$$

If the options market price can be observed in the market, it can be used to solve the pricing formula for volatility instead. More exactly the price of an option in the Black & Scholes world is $C(S,T,t,K,r,\sigma^2)$ and if $C^*(S,t)$ is the observed market price of the same option, σ^2 can be found such that $C(S,T,t,K,r,\sigma^2) = C^*(S,t)$ and the volatility that solves this equation is the volatility that is implied by the market. This can then be done for several options with different strike prices to find a good approximate for the market volatility.

4.3.2 Methods for Calculating VaR

There exists several methods to calculate VaR. The methods differ on various points, but the biggest difference is the assumption on the distribution of the underlying data. The choice of method is up to the computational power available and the type of data. It is also possible to choose whether the VaR measure should be bounded by real returns (Historical Simulation) or if the returns should be interpreted as a part of unknown and approximated distribution (Monte Carlo Simulation).

Historical Simulation This is the easiest way to compute the VaR. It simply uses a time series of old returns which is sorted top-up. Sometimes weights are applied to the time series to let the more recent observations have more impact on the result. The return that corresponds to the α-th quantile is then the worst return with probability P defined as $P = 1 - \alpha$ and this return is then multiplied with the portfolio value to get the Value at Risk. If the returns are normally distributed this method should yield the same result as the Delta-Normal method.

Delta-Normal Method The Delta-Normal method assumes that all asset returns are normally distributed. Since a portfolio simply is a linear combination of assets, which are normally distributed, the portfolio return itself is also normally distributed. By computing the assets variances and their correlations the portfolio can be expressed as a linear combination of variances and covariances that is weighted on the exposure of the different assets.

Monte Carlo Simulation The distribution of the portfolio returns can be approximated using historical data. After the type of distribution has been chosen and the parameters has been approximated, the path of the portfolio value can be simulated by drawing returns from the return distribution. This is done for example one million times which yields a distribution of portfolio values at end of the time period. By choosing the portfolio value that corresponds to the α-quantile and subtracting that portfolio value from the starting portfolio value, the VaR is obtained.

4.4 VaR Calculations

Normally closing prices are used to compute the VaR to get an end of day VaR. If the prices are snapped before closing time, the sample that builds the VaR measure will be completely different. To show this effect, the VaR and CVaR is computed for eight different stocks for seven different times and the corresponding returns are presented in Table 7 and Table 8.

Stock name \ Distribution	16.00	16.15	16.30	16.45	17.00	17.15	17.30
ASTRA ZENECA	-3,13%	-2,97%	-3,14%	-3,11%	-3,34%	-3,70%	-3,35%
ELECTROLUX B	-3,45%	-3,33%	-3,80%	-3,52%	-3,48%	-3,53%	-3,58%
HM B	-3,72%	-3,43%	-3,82%	-3,67%	-3,67%	-3,73%	-3,88%
SANDVIK	-3,79%	-3,45%	-3,19%	-2,97%	-3,33%	-3,84%	-3,37%
SKANSKA	-5,90%	-5,48%	-5,13%	-5,04%	-4,95%	-5,12%	-4,70%
SSAB A	-4,84%	-5,10%	-4,48%	-4,48%	-4,51%	-4,67%	-4,59%
SWEDBANK A	-4,27%	-4,73%	-4,86%	-5,12%	-4,66%	-4,98%	-5,42%
VOLVO B	-4,01%	4,40%	-3,99%	-3,96%	-4,03%	-4,44%	-4,35%

Table 7: VaR dependence on closing times. Portfolio value 1 million.

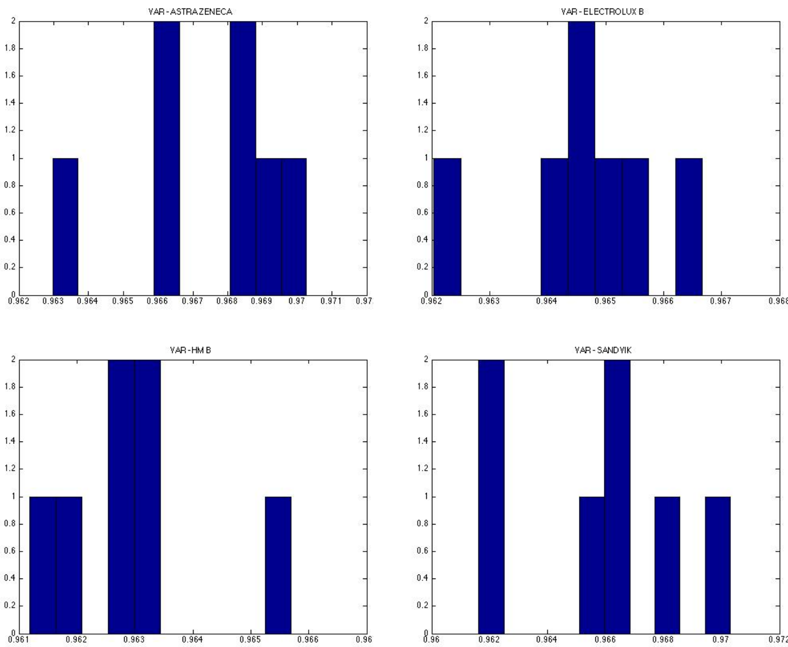

Figure 6: Distribution of VaR numbers with different closing times.

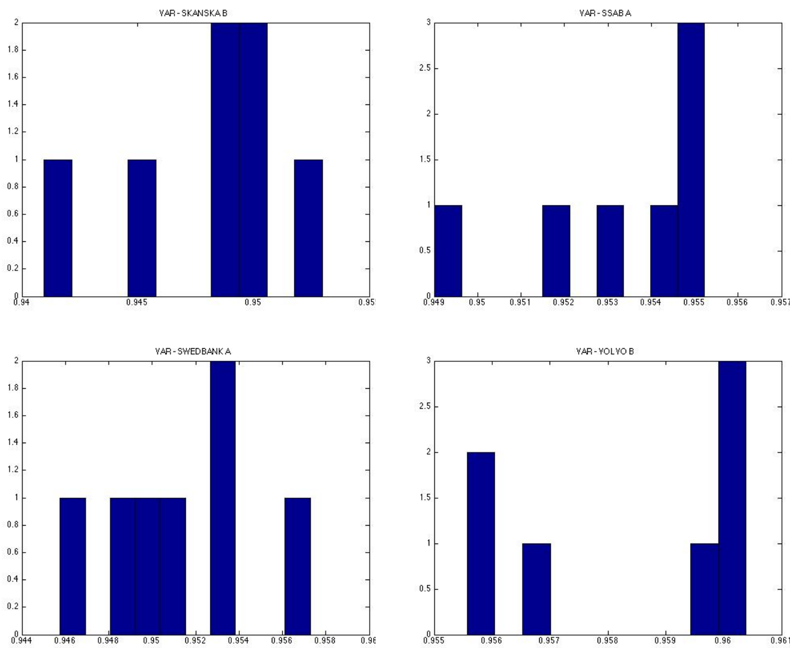

Figure 7: Distribution of VaR numbers with different closing times.

As can be seen in tables and the plots the VaR measure can vary a lot depending on the closing time. For some of the stocks the difference can be as big as 1% of the market price. This asymmetry in the histogram that is depending on the closing time makes portfolios where different snapping times are used very hard to model in a risk perspective. The table and plots with the CVaR measure shows a smaller asymmetry between the different snapping times. The risks are at the same time higher which results in higher risks with less uncertainty. That is interesting due to the capital required to be hold by a bank that is directly connected to the banks risk exposure. Table 8 presents the calculations for the CVaR measure and figure 8 and 9 shows the distribution of the CVaR measures.

Stock name \ Distribution	16.00	16.15	16.30	16.45	17.00	17.15	17.30
ASTRA ZENECA	-3,83%	-3,64%	-3,73%	-3,67%	-3,66%	-3,77%	-3,53%
ELECTROLUX B	-4,64%	-4,55%	-4,79%	-4,51%	-4,32%	-4,11%	-4,30%
HM B	-5,18%	-5,01%	-5,07	-5,01%	-5,01%	-4,99%	-5,10%
SANDVIK	-4,14%	-3,77%	-3,52%	-3,47%	-3,68%	-3,93%	-3,70%
SKANSKA B	-6,15%	-6,05%	-5,88%	-5,83%	-5,71%	-5,91%	-5,34%
SSAB A	-5,16%	-5,36%	-4,82%	-4,73%	-4,66%	-4,78%	-4,72%
SWEDBANK A	-6,74%	-7,27%	7,14%	-6,99%	-6,81%	-7,28%	-7,44%
VOLVO B	-4,07%	-4,40%	-4,12%	4,21%	-4,05%	-4,56%	-4,55%

Table 8: CVaR dependence on closing times.

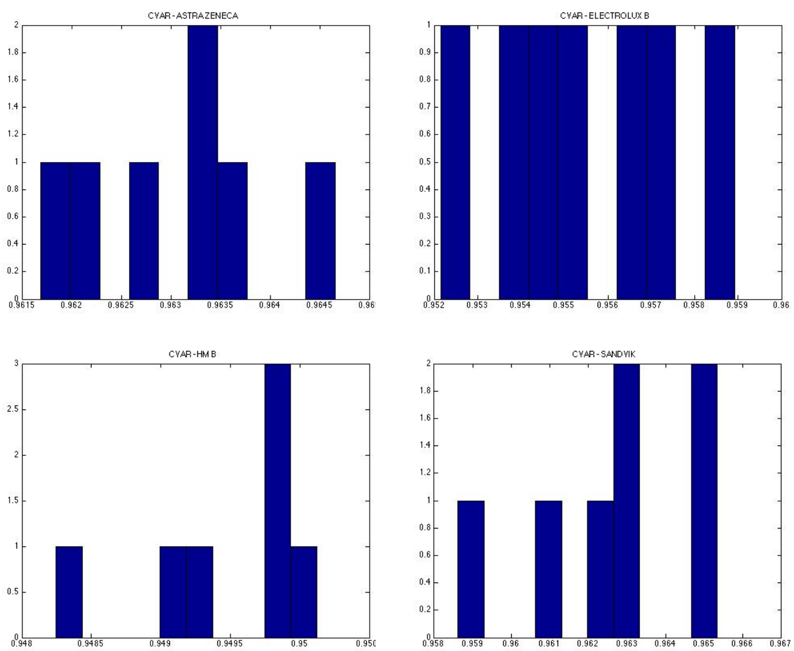

Figure 8: Distribution of CVaR numbers with different closing times.

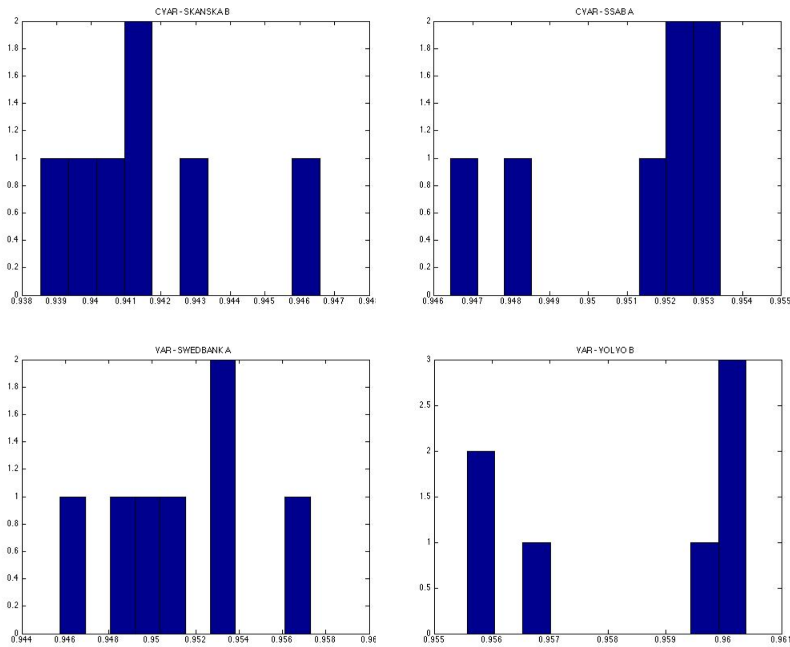

Figure 9: Distribution of CVaR numbers with different closing times.

To further illustrate the difference in the returns that yield the VaR figure, the maximal difference defined as

$$Maxdiff = \mid VaR_{Max} - VaR_{Min} \mid$$

is computed for both the VaR and the CVaR measure for eight stocks. It seems like the CVaR measure is a bit more compact than the VaR measure, and as described before would that incline that the CVAR measure yields larger risks with less uncertainty in regards to valuation time.

	Max. Difference VaR	Max. Difference CVaR
ASTRA ZENECA	0,73%	0,30%
ELECTROLUX	0,46%	0,68%
HM B	0,46%	0,19%
SANDVIK	0,87%	0,67%
SKANSKA	1,20%	0,80%
SSAB A	0,62%	0,70%
SWEDBANK	1,15%	0,71%
VOLVO B	0,48%	0,51%

Table 9: Maximal difference between computed VaR and CVaR values.

4.5 Theoretical Test: Is it Possible for a Fund to Have Significant Different Performance Measures due to the Closing Time

If two time series accumulates the same return over an interval, there can still be significant differences in the standard deviation of the series. The standard deviation is a critical component in many measures that are used to rank portfolios and funds. To illustrate how time series with the same amount of accumulated return can yield different performance measures, the following series has been constructed and the Sharpe ratio has been computed in accordance with Bailey [2012], thus

$$S_p = \frac{E\left[R_a - R_b\right]}{\sigma_p} = \frac{E\left[R_a - R_b\right]}{\sqrt{var\left[R_a - R_b\right]}}$$

	Day 1	Day 2	Acc. return	Annual std	Sharpe ratio
Serie 1	1,1%	-1%	12,16%	20,1%	0,497
Serie 2	0,8%	-0,705%	12,21%	14,40%	0,694
Serie 3	0,2%	0,11%	12,27%	2,9%	3,37
Serie 4	0,045%	0,045%	12,30%	$\sigma \to 0$	$S_r \to \infty$

Table 10: Sharpe ratio for four constructed return characteristics, all yielding an annual return of around 12,2%.

As seen in Table 10, the performance measures can differ depending on the structure of the underlying time series. If a portfolios value is measured at different times, the same portfolio could have different performance measures depending on the time for valuation. To illustrate this effect, Figure 10 shows three different time series all yielding 0% return over time. Every color has two daily return characteristics where one line is dashed. The intraday return measured at closing time is 0 for all time series but measured at 16.00, they will show positive or negative returns. Every other day the dashed line is the intraday return and every other day the full line will represent the intraday return. The blue series will will yield the highest standard deviation but still have a accumulated return around 0%. This phenomena gets interesting since many funds report to consumer web sites that measures and compares the funds performance, such as the swedish site www.morningstar.se. By sending data at another time than the funds own closing time, there will over time be large discrepancies between the measures indicated by the web site and the funds own figures. This effect could be both positive or negative, but it is not likely that a fund manager want to have more uncertainty regarding the performance of his or her fund.

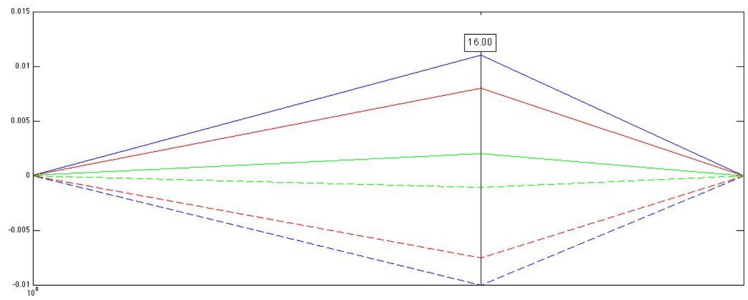

Figure 10: Representation of the different intra day return characteristics.

4.6 Empirical Test: Does it Exist a Difference in Performance Measures for a Fund Depending on the Closing Time

To examine whether there could be a significant difference in performance measures if the valuation time is changed, the Sharpe ratio has been computed for eight stocks and for the OMXS30. As a funds return is a linear combination of the underlying assets, the stocks represent a part of a fund return. As Table 11 shows, there is a difference in the Sharpe ratios that could be relevant when consumers are choosing in which funds to invest regarding to some performance measure.

	$\sigma_{17.30}$	$S_{17.30}$	$\sigma_{16.00}$	$S_{16.00}$	$S_{17.30} - S_{16.00}$	Abs. difference
OMXS30	16,6%	7,0490	16,9%	6,9509	0,0980	1,41%
ASTRA ZENECA	18,2%	5,1175	19,0%	4,8405	0,2770	5,72%
ELECTROLUX	27,6%	3,6039	26,9%	3,6506	-0,0473	1,31%
HM B	20,2%	3,8258	21,2%	3,9917	-0,1659	4,34%
SANDVIK	25,2%	3,9306	25,8%	3,7939	0,1368	3,60%
SKANSKA	21,1%	4,0566	21,7%	3,9176	0,1491	3,55%
SSAB A	33,0%	3,2581	33,0%	3,1784	0,7960	2,51%
SWEDBANK	29,0%	2,6783	28,9%	2,6729	0,0054	0,20%
VOLVO B	27,8%	3,4863	26,2%	3,6513	-0,1650	4,73%

Table 11: Sharpe ratios depending on closing time.

To see how sensitive consumer behavior is regarding to performance measures, swedish equity funds that is listed on www.morningstar.se by the 2014-04-25 has been sorted on standard deviation. This approach is encouraged by the site and since investors in general wants low standard deviations in the returns, it is more likely that consumers will choose the one with the lower standard deviation. This list also shows that the difference between being the one with lowest standard deviation and being the one at fourth place is just 0,5%, an effect that easily could be obtained by some bad luck during the choice of valuation time.

Fund	σ_P
Aktie-Ansvar Sverige	10,4%
Danske Invest Sweden A	10,4%
Nordea Swedish Stars utd	10,9%
PriorNilsson Sverige Aktiv	11,2%

Table 12: Top Swedish equity funds based on standard deviation according to morningstar.

5 Market Microstructure

5.1 Intraday Volatility Structure

The intraday volatility structure is computed by the technique introduced in Zhang et al. [2005] as the “Second best approach: Subsampling and averaging”. The method uses that even if the optimal sampling frequency is found, the data might not be used to its full extent. To ensure this, the variance estimator $[Y, Y]$ is averaged to $[Y, Y]^{(avg)}$ using K grids of average size $\bar{n}$. The following formula is for the equidistantly sampled case which has been used to compute the intraday volatility of OMXS30 displayed in Figure 11.

$$[Y,Y]_T^{avg} \approx \langle X,X\rangle_T + 2\bar{n}E\epsilon^2 + \left[4\frac{\bar{n}}{K}E\epsilon^4 + \frac{4T}{3\bar{n}}\int_0^T \sigma_t^4 dt\right]^{1/2} Z_{total}$$

For the full derivation and details of the formula, see Zhang et al. [2005].

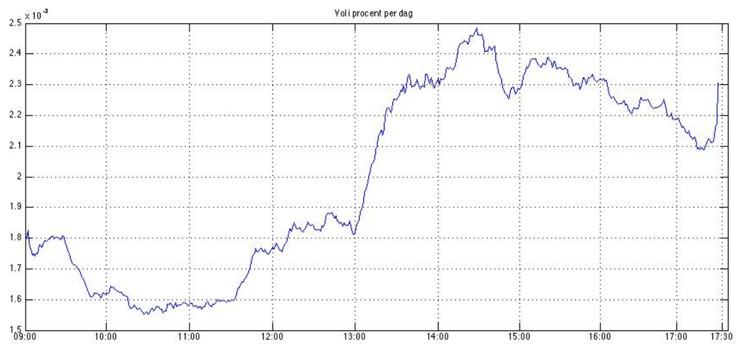

Figure 11: Intraday volatility for the OMXS30

5.2 Intraday Drift

If a stock has a yearly return of 10%, the intraday drift would be around 0,04%. This figure is very small and very hard to estimate. The underlying assumption that expected or historical yearly return is a good proxy and is scalable to a daily figure is also not considered a fact in financial theory. The variance of the estimate would also be so large that its confidence interval would probably cross the zero even for a positive yearly return, which is a huge contradiction. A small band moving average of daily drifts could just as easy be a good proxy for the daily drift. Since the computation, the reliability and the variance argues that intraday drift cannot be computed in a good way, with support of Lyons [1995], the intraday drift is assumed to be zero and is not investigated further.

5.3 VaR-dependence on Market Microstructure

To capture the effects of market micro structure on risk measures the Value at Risk and Conditional Value at Risk is computed with every minute of the trading day as stopping time. This means that for every minute between 09.00 and 17.30 a daily return is computed. This is repeated for 250 days which yields a data set containing 510 data series of 250 daily returns. When the VaR and CVaR is computed for every data series, it yields 510 VaR and CVaR figures for the OMXS30 respectively. These figures interpreted as a distribution of risk measures and the shape of the distribution shows how sensitive the measure is to the actual valuation time. One might argue that the closing prices at 17.30

is not the best prices seen from for example a liquidity perspective, and that it might be better to have 16.30 as the stopping time. Figure 12 and 13 shows that these type of decisions can change the measured risk in a portfolio to a great extent. The VaR is computed as a 99% measure and it is computed as,

$$VaR_{\alpha}(X) = min\{z \mid F_X(z) \geq \alpha\}$$

using the historical simulation technique described in section 4.3.2.

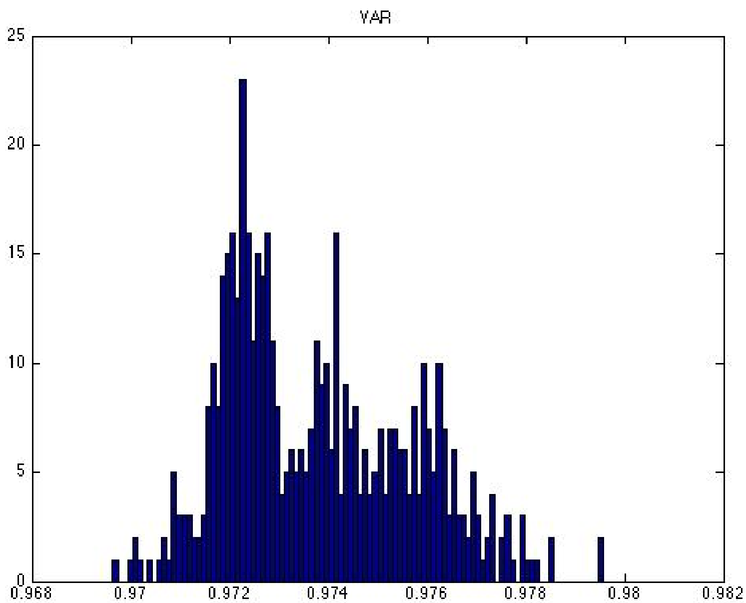

Figure 12: Distribution of VaR numbers for OMXS30 with different stopping times.

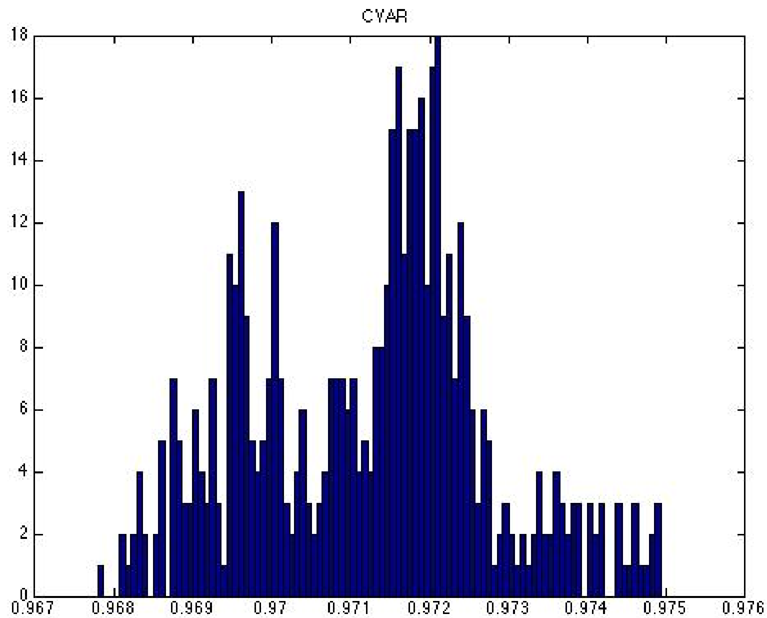

Figure 13: Distribution of CVaR numbers for OMXS30 with different stopping times.

As can be seen in Figure 12 and 13, the CVaR measure yields a higher risk and a more compact distribution. This means that risk described by the CVaR is more certain but it is also higher than the VaR. As they are defined slightly different, the differences are expected and it is up to risk managers to decide which method to use. However, the effects of the market micro structure is smaller for the CVaR measure as can be seen in the more compact distribution.

5.4 VaR Uncertainty and Time Dependence

Looking at Figure 12 and 13, one can argue that the shown distributions might consist of two different distributions. To try to find different characteristics in the VaR uncertainty, the total uncertainty measured as the distance between the smaller and the largest VaR value is computed for every half hour during the day. This yields that if there exists a point on the curve where the the uncertainty is significantly lower than the rest of the curve, it is highly probable that it would yield a tighter VaR distribution to use that time for valuation. As can be seen in Figure 14, 15:30 seems like a good time to snap prices in order to have a low uncertainty in the measured VaR.

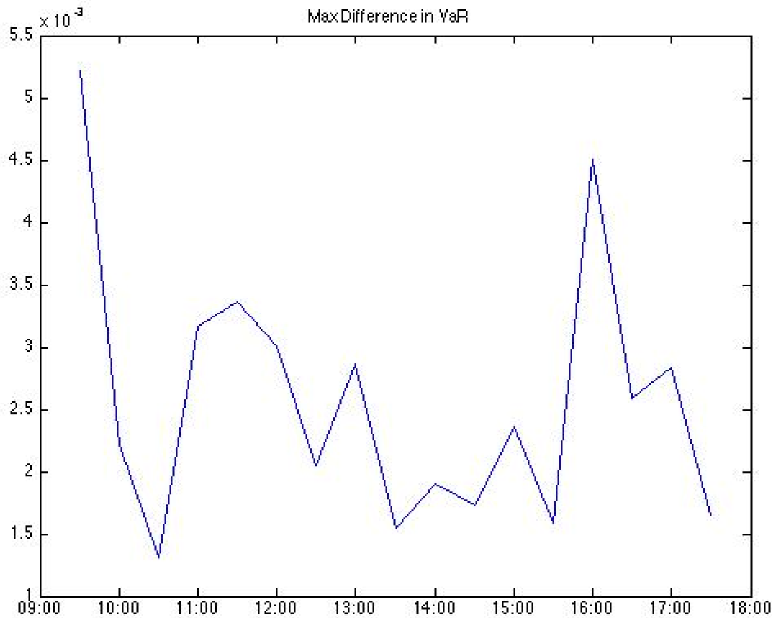

Figure 14: Max difference in VaR for every half hour for the OMXS30.

5.5 Empirical Test: VaR Computations on Equity Portfolios

Most equity portfolios are not as diversified as the OMXS30 indices. Since the diversification yields a lower standard deviation in the returns, the VaR and CVaR figures of an market index should be lower than the same figures for an equity portfolio. To see this, the same method described in section 5.3 is implemented to a equally weighted portfolio of eight swedish stocks. The results from the computations are presented in section 5.4.2.

5.5.1 Indices Returns as Portfolio Proxies

It is common to represent a equity portfolio with a linear combination of equity indices. This makes the data collection far more easy and the computations becomes less complex. However, as mentioned before the standard deviation of an market index is often far lower than a non fully diversified portfolio. By computing the VaR distribution for the constructed portfolio mentioned including the eight stocks and comparing it to the VaR distribution of the OMXS30, a difference in the shape of the distribution is likely be observed.

5.5.2 VaR Computations

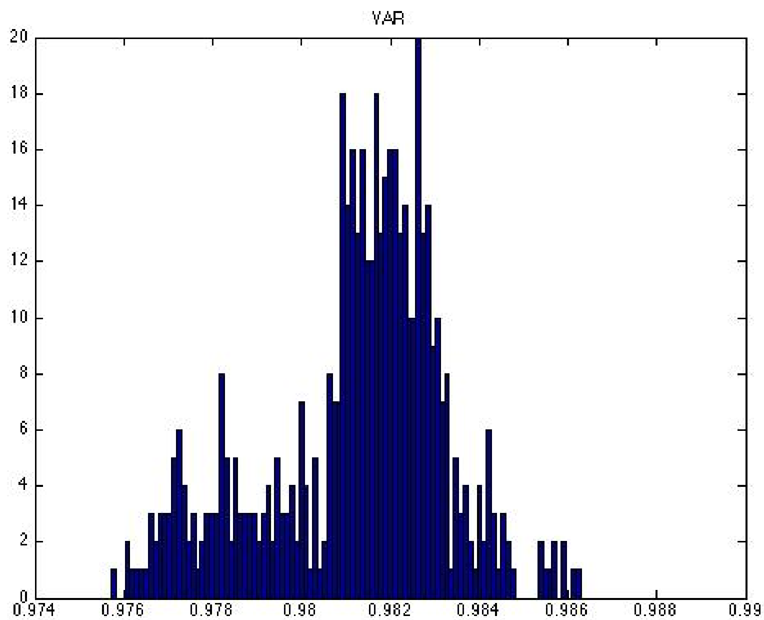

Figure 15: Distribution of VaR numbers for an equity portfolio with different stopping times.

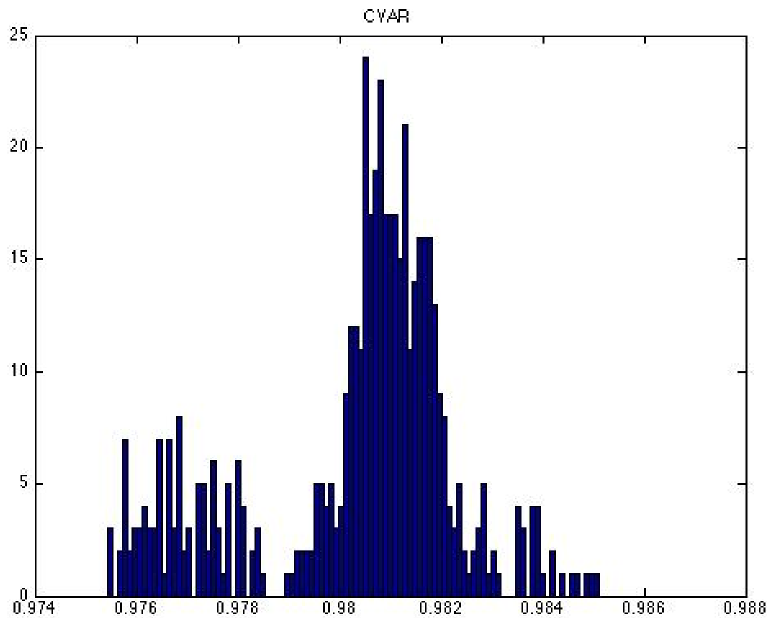

Figure 16: Distribution of CVaR numbers for an equity portfolio with different stopping times.

As proposed in Section 5.4.1 both the VaR and the CVaR for the equity portfolio is significantly broader then for the OMXS30. This concludes the assumption that a less diversified portfolio where returns are approximated by an equity index is likely to show a VaR or CVaR figure that is lower than it would actually be if computed on the actual portfolio instead of the proxies.

6 Time of Valuation for Products with Collateral Agreements

6.1 Time of Valuation

Collateral agreements are common between banks and other financial firms in order to minimize their respective credit risk. The agreement regulate how often and within which boundaries collateral should be payed by the counterpart. As an example, two banks (A and B) enters a one year forward contract on crude oil. At the time of signing the contract has zero value, but as soon as the crude oil price starts to move one bank will gain money and the other will lose money. Since the contract is one year long, the payment at maturity date could be very substantial if the crude oil price has increased or decreased a lot in combination with a significant contract size. If bank A has made a substantial profit, it cannot be completely sure that bank B can meet its obligations and pay bank A in cash. There is also no way for bank A to control if bank B has a hedge on the position or if bank B has several big, naked positions which has all gone in the wrong way. In that case, bank B could go bankrupt and bank A would not receive all or any of the money gained within the contract. To keep the risks down for both sides, a payment related to the daily price change is made every day. At a predetermined level, for example 3% of the contract value, the bank who has the short position will pay the excess value of the contract exceeding 3%. This is done every day and if the contract loses value, the bank who has the short position will receive the collateral payment.

The plot below shows a contract that has value zero at the start and where collateral has to be paid if the value of the contract exceeds the boundaries A and -A. One bank will value the contract after 420 minutes, that is 16.00. The other bank will value the contract at $420 + x$ minutes. The difference between the valuation times is set to the parameter τ.

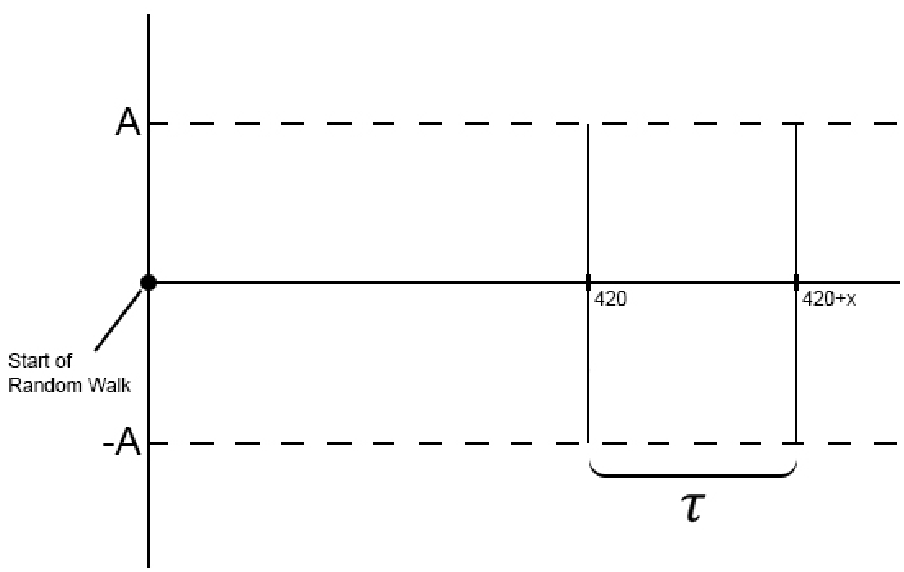

Figure 17: Visual representation of the collateral mechanics.

For a contract thats pays collateral, both counterparts must choose a time when they value the contract and pay collateral to the counterpart if the current valuation and the collateral agreement stipulates so. The time bank A choose to value the contract is defined as the banks stopping time, in this case t_A. This section will show that the chosen time is important in regards to how much collateral one side will have to pay. The paid collateral will always be even when the contract matures, but there can be a huge difference in how much money one side has to pay and how many payments they have to make. Since the bank that during the life of the contract has the money over night gets an over night interest rate and the one who needs to borrow the money has to pay the same rate it is crucial to pay as little as possible and at as few times as possible. This is where the choice of stopping time gets very important.

6.1.1 Simulation of Aggregated Results

SHB uses a third party software to handle collateral payments. The system makes it possible to see every daily payment to each counterpart but it is not possible to see how much a certain contract or portfolio contributes to the payment. That is because the total payment is a netted sum of individual payments. In regards to the valuation time, every portfolio might have its own valuation time and the assets included can be valued with prices that differs from the valuation time. As shown in Figure 18, the system can be seen as a black box that yields a single payment per counterpart each day. The payment is dependent on the sum of multiple asset values which values either depend on the movement of some underlying assets or the asset value depend on the asset itself. If the system is observed during a long period such as a year, it is possible according to Björk [2009] to approximate the underlying assets with a random walk with zero drift. This yields that every portfolio has expected return zero and the the sum of the portfolios also has expected value zero. The collateral payments also has expected value zero with variance dependent on the total number of assets. This means that on average there should be the same amount of outgoing and ingoing payments for the bank and that the total payments should sum to zero. If that is not the case, the full specifications of the box cannot be modeled with a random walk that implies there exist some characteristics in the box which cant be interpreted. This could be discrepancies in the valuation time, difference between how the banks collateral team works or a factor that is unidentified.

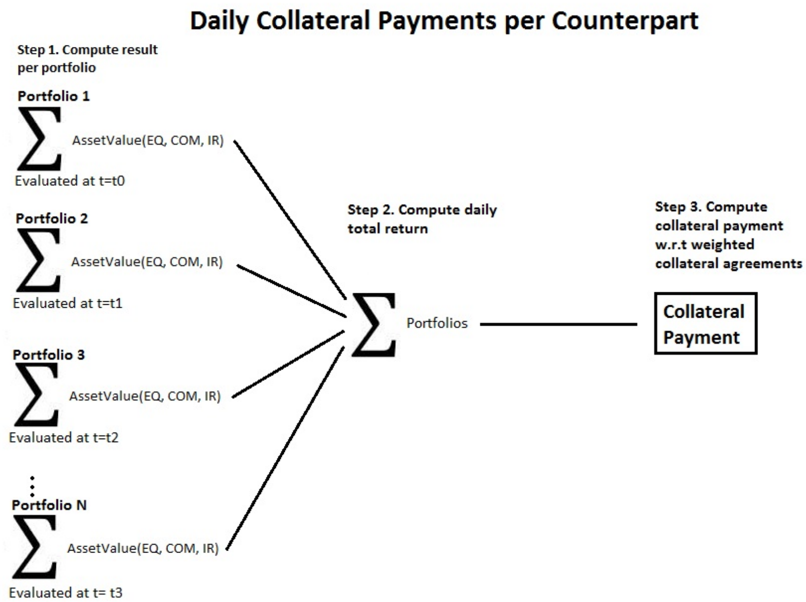

Figure 18: The process of computing the daily collateral payment for a single counterpart.

6.1.2 Brownian Motions and Random Walks

The Brownian motion is described by the Wiener process, named after Norbert Wiener. The Wiener process is a stochastic process with stationary independent increments. The Brownian motion X_t is characterized by four facts:

Definition 6.1: The Brownian Motion

1. $X_0 = 0$ almost surely
2. X_t is almost surely continuous
3. X_t has independent increments
4. $X_t - X_s \sim N(0, t-s)$ for $0 \leq s \leq t$

The standard Brownian motion has several interesting properties, some of them are:

- The expectation is zero, $E\left[X_t\right] = 0$
- The variance is t, $Var(X_t) = E\left[X_t^2\right] - E^2\left[X_t\right] = E\left[X_t^2\right] - 0 = t$
- The covariance is, $Cov(X_s, X_t) = min(s, t)$

Quadratic Variation For a Brownian motion or any other stochastic process, it is possible to define its variation or Quadratic variation even though it is a stochastic variable. A limit must then be introduced, and convergence in probability $\delta_n = max_i(t_{i+1}^n - t_i)$ is chosen since it is the weakest one and the easiest one to work with according to Kurtz [2007]. The Quadratic variation of a Brownian motion X_t can then be defined as

$$T_n = \sum_{i=1}^{n} \left| B(t_i^n) - B(t_{i-1}^n) \right|^2$$

with the expectation

$$E\left[T_n\right] = T_n = \sum_{i=1}^{n} \left| B(t_i^n) - B(t_{i-1}^n) \right|^2 = \sum_{i=1}^{n} (t_i - t_{i-1}) = t$$

Martingale Property for Brownian Motion The Martingale property is very important to understand the Brownian motion. It basically states that the best guess of the state of the Brownian motion in the next time step is its current state. To understand the definition of the property, the concept of filtration must be explained. A filtration $\mathcal{F}_t$ is simply all the information about the states of the Brownian motion from zero up to the state at time t. Since every increment of the Brownian motion is independent and normal distributed, the process from t to $t+s$ is a new Brownian motion which fulfills all of the criterias in Definition 6.1. It is now possible to define the Martingale property.

Definition 6.2: Martingale Property for the Brownian Motion A stochastic process $\{X(t), t \geq o\}$ is martingale if for any t it is integrable, $E\left|X(t)\right| < \infty$, and for any $s > 0$

$$E\left[X(t+s)\left|\mathcal{F}_t\right.\right] = X(t) \qquad a.s.,$$

where $\mathcal{F}_t$ is the information about the process up to time t.

Symmetry of the Brownian Motion To prove that the brownian motion is symmetric it must be shown that $X_t := -W_t$ is a Brownian motion if W_t is a Brownian motion. According to Definition 6.1, it must be shown that all four properties of the Brownian motion still is valid for X_t.

1. $X_0 = -W_0 = 0$
2. Since $X_{t_j} - X_{t_{j-1}} = -(W_{t_j} - W_{t_{j-1}})$ and we know that $W_{t_n} - W_{t_{n-1}}, W_{t_{n-1}} - W_{t_{n-2}}, ..., W_{t_1} - W_{t_0}$ are independent, we obtain that the random variables $X_{t_n} - X_{t_{n-1}}, X_{t_{n-1}} - X_{t_{n-2}}, ..., X_{t_1} - X_{t_0}$ are independent.
3. $X_t - X_s = -(W_t - W_s) \sim -N(0, t-s) = N(0, t-s)$ since the normal distribution is symmetric.

4. Clearly $t \to X_t(w) = -W_t(w)$ is continuous, since $t \to W_t(w)$

Now it can be concluded that the Brownian motion behaves in the same way in both positive and negative directions since it is symmetric.

Stopping Times Stopping times will be used to define certain times when the state of a Brownian motion is measured. Since those times will be known, a simple definition of stopping times will suffice.

Definition 6.3 (Stopping times) A random time T is called a stopping time for $B(t)$, $t \geq 0$,if for any t it is possible to decide whether T has occurred or not by observing $B(s)$, $0 \leq s \leq t$.

6.1.3 Theoretical Approach to Determine an Effective Valuation Time

If the market is sampled every minute, from the market opening at 09.00 there is 420 minutes until 16.00 which is then used as benchmark for the first closing time in this section. Due to symmetry, the probability of the random walk being larger than A or smaller than -A, at a certain time t, is equal and does not depend on the variance of the Brownian motion. Since the variance is positive, the following proposition holds if $x \geq 0$ and X_t is a Brownian motion due to the definition of quadratic variation.

$$P(X_{420} \leq A) = P(\frac{X_{420}}{\sqrt{A}} \leq 1) \leq P(X_{420+x} \leq A) = P(\frac{X_{420+x}}{\sqrt{A}} \leq 1) \qquad (8)$$

and this is true due to the fact that

$$Var(X_{420}) = \sqrt{420}\sigma = \sqrt{420} \leq Var(X_{420+x}) = \sqrt{420+x}\sigma = \sqrt{420+x} \qquad x \geq 0$$

For the extra steps x it is true that $P(X_x \leq -2A) \geq 0$ for all $x \geq 0$. If $x \to 0$ then $P(X_x \leq -2A) \to 0$ since the quadratic variation of the Brownian Motion goes towards zero. This means that it is possible that one random walk yields two collateral payments if $x \geq 0$. If $x = 0$, then there can only be one collateral payment for each Random walk. This is why the bank B will always will pay more collateral than bank A.

The probability of a random walk after x steps to be larger (smaller) than A (-A) is equal due to the symmetry of the Brownian motion.

6.2 Simulation Test: Is the Number of Unnecessary Collateral Payments Dependent on the Stopping Time?

To simulate a daily return of a financial contract a random walk is used. Due to the fact that an intraday price change is modeled, the drift μ is expected to be zero according to Björk [2009]. The random walk simulates the price of the contract and the two boundaries A and $-A$ simulates the price level of the contract where collateral has to be paid. The simulation is conducted by letting the random walk start from zero and then measure its value on the first stopping time t_1 which is located after 420 steps, one step for each minute between 09.00 and 16.00. If the value of the random walk is larger than the boundary A at time t_A company A has to pay collateral. The random walk then continues with extra steps according to Table 5 and checks the value of the random walk at time t_B. If the value of the random walk is smaller than $-A$ at t_B company B

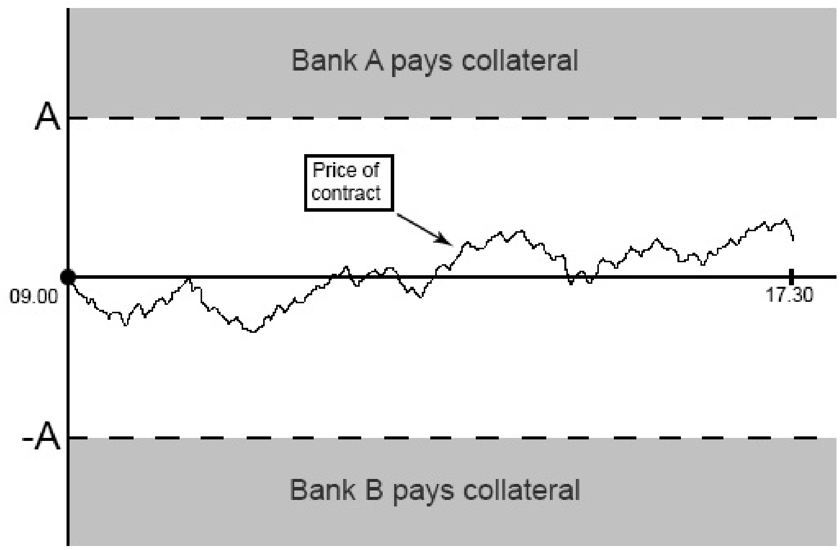

Figure 19: If the price of the contract reaches the grey level, there will be a collateral payment.

By shifting the number of extra steps x, it is shown how a bigger $|t_A - t_B| = \tau$ will give extra collateral payments for company B. This procedure is repeated one million times and then it is possible to compare the ratio of collateral payments by bank A and payments made by bank B. Because of the symmetry of the Brownian motion, it is not necessary to fit the Brownian motions parameters to real market data. As long as the problem is symmetric, the ratio between the collateral payments will be correct. The pseudo code for the simulation process can be seen below.

Algorithm 1 Collateral Simulation Process

var1=0
var2=0
Repeat one million times

- Let the Random walk X_t take 510 steps
- if $X_{420} > 50$ then var1 = var1+1
- if $X_{510} <$ -50 then var2 = var2+1

var1 equals the number of collateral payments for company A
var2 equals the number of collateral payments for company B

6.2.1 Simulation with an Ito Process

The simulated process is $dX_t = \mu dt + \sigma dW_t$ where $\mu = 0$ due to the intraday time period. The zero drift for intraday periods is also described by Becker et al. [2009]. This yields a simple zero mean stochastic process $X \sim N(0, 1)$ which is repeated in every time step like $Y_t = Y_{t-1} + X$ where $X \sim N(0, 1)$ according to the definition of the Brownian motion. For one million simulations, the flow of collateral payments can be seen in Table 5.

Extra steps	Payments from A	Payments from B	Extra payments from B
20	7450	8580	15,1%
30	7529	9196	22,1%
40	7445	9896	32,9%
60	7217	11181	54,9%
90	7311	13327	82,2%

Table 13: Simulated collateral payments using an Ito process.

6.2.2 Simulation with a Jump Process

The price process of the market could also be interpreted as a stochastic process with a possibility to make jumps. The assumption that the process could take jumps is empirically validated in Westman and Hanson [2002] and could easily be explained by political statements, terrorist actions or natural disasters. The jump process is defined as $dX_t = \mu dt + \sigma dW_t + \delta dQ$ where dQ is a random variable defined as

$$dQ = \left\{ \begin{array}{ll} X \sim N(0,1) & prob = 0,008 \\ 0 & otherwise \end{array} \right\}$$

Since the problem i symmetric the size of δ just alters the number of total payments. The ratio between the payments from A and B will be the same for every δ. With that in mind, δ is set to 6 as it yields about two times the payments as the Ito process for the interval and due to the symmetry it does not have to be estimated with market data in order to give valid results. The same simulation technique as in 6.2.1 is then used to find the number of payments for both counterparts.

Extra steps	Payments from A	Payments from B	Extra payments from B
20	16104	18306	13,7%
30	16023	19402	21,0%
40	16138	20590	27,6%
60	16129	22850	41,7%
90	16264	25971	59,7%

Table 14: Simulated collateral payments using a Jump process

As the variance is higher for the Jump process than the Ito process, the percentage of extra payments is larger. As seen in table 15, the total sum of payments is also larger which implies that the total number of payments and the percentage of extra payments could be interpreted as a function of the process variance. This result yields that if the market can be modeled by a process with jump characteristics, then the choice of stopping time is extra important.

Extra steps	Ito process	Jump process	Total extra payments
20	16030	18597	16,0%
30	16725	19358	15,7%
40	17341	19823	14,3%
60	18398	21763	18,2%
90	20638	23979	16,1%

Table 15: Total number of Collateral Payments

Table 15 compares the total number of payments generated by the Ito process and the Jump process. As stated, the jump process yields more payments since the variance increases as the jumps are introduced.

6.3 Empirical Test: Does Collateral Payments depend on the Stopping Time?

The problem regarding collateral payments is not strictly mathematical. If a contract has moved in such way that it should yield a collateral payment, the receiving part must send a request of payment. The counterpart could then argue about valuations discrepancies and it is not sure that a payment will take place. This makes it very important to have qualified and competent staff in the collateral department. Since an outgoing payment yields a direct cost or a direct loss of income, a bank or financial institution will try to minimize its collateral flow without losing counterparts, costumers or market reputation. The effect regarding the effectiveness of the collateral staff is hard to quantify and analyze in a mathematical fashion. Since factors such as stopping times, competence and ability within the collateral department and their backup from valuation groups, it all boils down to payments into the bank and out from the bank, it is very hard to isolate the factors and analyze them separately. In a mathematical point of view without the other effects, the extra collateral payments for the one has the latest closing time is because of the positive probability that the value of the contract goes from receiving collateral to paying collateral. In the introduced framework it is equivalent with the random walk going from 0 to $-2A$ in the interval τ which is illustrated in Figure 18.

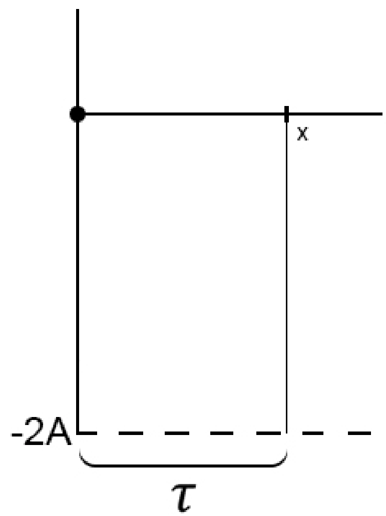

Figure 20: The probability for a random walk to reach in -2A in τ is larger than zero.

6.3.1 What is the Real Cost of a Non Optimal Closing Time?

As described in section 6.3, the money transferred due to collateral payments is not just due to the stopping time. As stated before, a contracts price assumed to follow a random walk with zero drift intraday, thus its price is normal distributed. When several normal distributed contract returns are summarized to a portfolio, the portfolio return is still normal distributed with zero as expected value. With many portfolios summarized to a result for one counterpart, the result per day has still zero mean and the variance is lower than the sum of the individual variances. A sum of n of those daily results for one counterpart should sum to zero, with an expected value that goes to zero when n becomes big. By taking real collateral data and assuming that the contracts follow the random walk, the closing times and valuation models are identical and that the different collateral teams are equally effective the collateral payments should sum to zero. The number of payments, the number of days with positive cash flow and the total paid interest should also be equal for both counterparts. The tables below shows real data concerning the bank and its counterparts. The period is one year.

Counterpart	Received money	Transferred money	Excess payout
Bank A	X	1,33*X	33%
Bank B	Y	1,16*Y	16%
Bank C	Z	1,35*Z	35%

Table 16: Total amount of received and transferred collateral payments. Masked digits.

Table 16 shows that SHB pays out more collateral than it receives during the period of measurement and for the three mentioned counterparts.

Counterpart	Number of incoming payments	Number of outgoing payments
Bank A	119	117
Bank B	132	159
Bank C	121	101
Total	372	377

Table 17: Total number of ingoing and outgoing collateral payments.

Table 17 shows that the number of ingoing and outgoing payments are about the same if measured as the total sum of the individual counterparts.

Counterpart	Days with positive cash flow	Days with negative cash flow
Bank A	89	144
Bank B	94	197
Bank C	87	135

Table 18: Total number of days with positive and negative cash flow.

Table 18 shows that the total number of days with negative cash flow is significantly larger for SHB than for its counterparts.

Counterpart	Interest cost for the counterpart	Interest cost for SHB	Extra interest
Bank A	X	1,40*X	40%
Bank B	Y	2,09*Y	110%
Bank C	Z	9,52*Z	952%

Table 19: Total interest rate cost for SHB and its counterparts. Masked digits.

Table 19 shows that SHB pays significant more interest than its counterparts for the measured period.

7 Discussion and Conclusions

The valuation of derivatives does not invite to significant challenges. For the interest rate products, the construction of the yield curve is essential and a lot of effort should be put in to choosing the right underlying interest rate points for the curve. As proven in 3.1.2, different interpolation methods does not yield a significant price change for a bond and the valuation mechanism seems fairly robust to mispriced interest rate points. For the other types of derivatives, the challenging part is to obtain high quality input parameters to the valuation methods. As for the volatility, several methods are introduced in 4.3 and 5.1. The analysis of valuation jumps from an early closing time to market close shows significant results that the closing time affect the size of the valuation jump which is made at market opening the following day.

The VaR and CVaR are both very sensitive to changes in the valuation time. The CVaR shows a more compact distribution than the VaR, but shows at the same time higher total risk. From the banks point of view, if the risk figures are lower, less money has to be hold to make up for the risks. This fact make the banks choose the VaR approach even though the distribution is broader and the uncertainty is larger. When risk limits for trades are expressed in VaR figures, the measure becomes more relative than absolute since a change in the closing time would yield a new correct but completely different VaR figure. VaR can be a good measure of risk if it is compared with the same measure from the previous days. To just have one VaR figure can be very illusive and bring a false confidence of knowing the risk exposure. If just one figure is presented, it is impossible to know where in the risk distribution the figure is located and as shown in 4.4 the difference can be 1% for equity indices such as OMXS30 and even larger for equity portfolios with poor diversification. It is shown that for the Swedish stock market, it would yield the lowest uncertainty to use 15:30 as valuation time.

In 4.5 it is shown that a fund or equity portfolio can have severely different risk measures depending on when the portfolio is valued. To counter this effect, some performance measures are computed over a period as long as ten years. If one year measures are used, which is common for new funds, there is a very large dependence between closing times and the standard deviation. It is proven that for the same yearly return, the standard deviation can go from zero to infinity all depending on the closing time. This is tested empirically in 4.6 and it shows that a portfolio composed with either luck or bad luck can show very low or very high standard deviation depending on the time of valuation.

The intraday volatility structure is introduced and estimated in section 5. The intraday drift is disregarded because of the assumption that the stock returns follows a brownian motion with such a low intraday drift that it can be regarded as zero. The technique of proxying equity portfolios with indices when computing risks is introduced and it is shown that this method often underestimates the underlying risks.

Contracts with collateral agreements are introduced in section 6. The assumption that all contract prices follows a random walk makes it possible to analyze the closing time dependence both analytical and by simulation. The structure of the collateral payments are examined, and it shows that the infrastructure in place to make the computations and payments easy also makes it hard to evaluate the performance of the collateral group and to separate the effects from various identified sources of error. The goal was to separate the closing time as the source for additional payments and to show that a changed closing time would solve the problem. Instead, it has been showed that there are indications that SHB pays to much in collateral and several possible reasons for this has been mentioned. To move the closing time backwards is however a good idea not depending on the possible reasons for the additional payments. The figures of the total cost of collateral interest for every counterpart is masked in this presentation, but the sum is however big enough for the bank to start their own investigation of possible factors that leads to additional collateral payments.

8 Proposal for Further Research

The quantitative parts of this thesis is all limited by the used data set. Since high frequency data stock market data is very expensive, only one year of minute data was available for the indices and the eight different stocks used. The results would have more impact if the period was extended and if the tests were conducted with sub periods to check if the results were valid in both up trending and down trending markets. Further, it would be interesting to extend the VaR analysis to leveraged derivatives to see if the skewness in the measure depending on the closing time would increase with the leverage. From the banks perspective, the collateral mechanism is interesting to investigate further. The impacts from the various sources is hard to separate but the results still implies that SHB is paying significantly more collateral than it statistically should. This indicates that there are improvements to be made in the choice of closing time and in the soft areas that cannot be quantified.

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