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The Relative Performance of Conditional Volatility Models

- An Empirical Evaluation on the Nordic Equity Markets

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Keywords: Volatility components, forecasting, long-run volatility effects, explanatory power, conditional volatility

Abstract

By regressing volatility series of equity returns on macroeconomic variables using data from the Nordic countries (Denmark, Finland, Norway and Sweden), three conditional volatility models (GARCH(1,1), CGARCH and SV) are evaluated on their ability to capture effects of long-run volatility shocks. In addition, the same models' short-run forecasting performance is tested by employing a rolling window approach. The results suggest that none of the models are superior of capturing long-run volatility effects, and the same holds for the short-run forecasting performance. The Stochastic Volatility model has the worst performance on average, while the difference between the GARCH-type models are negligible.

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1. Introduction

Ever since Markowitz' Modern Portfolio Theory (1952) and the development of CAPM by Sharpe (1964) and Lintner (1965), the by far most important and widely used measure of risk has been the volatility (variance or standard deviation) of returns. In particular, asset allocation, risk management and asset pricing depend highly on volatility and the models used for estimation and forecasting it. From the past decades' increasing complexity of the financial markets, the ability to reduce (or optimize) risk exposure has become one of the most important merits for today's investors. Nowadays, it is even possible to trade derivative instruments for which volatility itself is the underlying asset (Poon and Granger (2003)), which further enables investors to achieve one's desired risk exposure.

Following the progression, a vast literature on the subject of volatility of financial assets has emerged. The arguably most important foundation for this branch of the literature is the work of Engle (1982), developer of the Autoregressive Conditional Heteroscedasticity (ARCH) model, and Bollerslev (1986), who extended the work by Engle into the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The basic idea is that the volatility of returns depends on past error terms (ARCH) as well as past variances (GARCH), implying that the volatility varies over time. Today, there are virtually countless variations and extensions of the most basic GARCH(1,1) model, all of which have different specifications in order to take various measures and events into account.

One must keep in mind that the effects of complex market structures and time varying volatilities are not isolated to individual financial markets, but are relevant for policy decisions at macro level as well. For instance, events such as the tech bubble in the early 2000's or the recent global subprime crisis have had great impacts on e.g. GDP growth and consumer behaviour. Thus, the research on volatility and volatility modelling remains highly topical and important, not only for investors, but for policy makers and consumers as well.

In excess of the important but simple ARCH model, Engle in collaboration with Lee (1999) introduced a model within a special branch of the GARCH-family that separates the total volatility into a long- and short-run component. This particular two-component model by Engle and Lee is called the Component GARCH (CGARCH) model, and it was developed with the purpose to better take long-run persistency of volatility shocks into account. Since then, several models have been developed based o the same principle (see e.g. Engle and Rangel (2008) and Cho and Elshahat (2014)). Since the financial markets are closely connected to macroeconomic factors, the long-run equity volatility is expected to partly be explained by variables such as GDP, inflation, interest rates etc. However, no consensus has been reached among researchers about which model has the best ability to capture these effects.

In excess of correctly describing contemporaneous states of the world, the probably most important attribute of a model is, as Guidolin et al. (2009) put forward, the ability to accurately depict future events by forecasting. For instance, investments are generally made on the basis of beliefs about the future, and thus the forecasting ability of e.g. volatility models are put to the ultimate test. Still, despite the extensive research on the subject of volatility forecasting, no model has yet proven to be superior for the purpose of accurately describing future volatility.

Based on the background above, the purpose of this thesis is to compare three conditional volatility models (GARCH(1,1), CGARCH and SV) on their ability to capture effects from persistent volatility shocks caused by macroeconomic factors, with particular interest in the long-run component of the CGARCH model. In addition, the short-run forecasting ability of the chosen models will be evaluated. The study is thus divided into two parts. The first part examines the explanatory power of macro variables on conditional volatility, as well as the ability of volatility models to capture effects of persistent volatility shocks. Three models with diverse properties are tested, namely the classic GARCH(1,1) model, the CGARCH model and a Stochastic Volatility (SV) model. The examination is performed by an OLS regression analysis. The second part evaluates the forecasting ability of the models. This is done by rolling window forecasts of the one-day-ahead conditional variance of the main stock indices in the Nordic countries (Denmark, Finland, Norway and Sweden). Several measures commonly seen in similar studies are used in order to evaluate the performances. The data set runs from 1993 to 2014, and includes periods of both economic distress as well as strong economic growth. This, in combination with the inclusion of four different markets provides robustness to the study.

The main results from this study tell us that the CGARCH model does not outperform the less sophisticated GARCH(1,1), neither for modelling long-run volatility nor for short-run forecasting, and that the SV model has the worst performance on average.

The remaining text is organized as follows: *Section 2* summarizes some of the most relevant previous research. *Section 3* provides a brief overview of the theoretical framework and presents the models of choice. In *Section 4*, the data is discussed, while *Section 5* describes the empirical methodology. *Section 6* presents the results, which are discussed and analyzed further in *Section 7*. Finally, *Section 8* concludes.

2. Literature Review

In this section, a brief summary over some of the most important and relevant studies previously made on the subject is provided.

Sharing similarities with the CGARCH model, Engle and Rangel (2008) introduces the Spline-GARCH in an attempt to find a model that allows for long-run volatility forecasts that are dependent on macroeconomic variables. They use a large data set covering nearly 50 countries, and the estimations are conducted using an unbalanced panel regression with various specifications. The authors' main findings suggest that the long-run volatility component of their model is relatively high when the volatility of macroeconomic variables such as GDP, inflation and interest rate is high and output is low.

Speight, McMillan and Gwilym (2000) use intra-day data from the UK FTSE-100 futures index to investigate the properties of the CGARCH model proposed by Engle and Lee (1999). Their findings support the component structure of the model, but the results also show that it is difficult to separate the long-term component from the total volatility for data at relatively low frequencies (at half-day frequency).

As a response to the findings of Engle and Lee (1999), Cho and Elshahat (2011) present their own version of the CGARCH model called the Modified Component GARCH (MC-GARCH). Their goal is to implement a model with a better ability to filter the long-run volatility in order to make it more distinguishable from the total conditional variance. The authors use various methods for evaluating the filtering performance of the MC-GARCH model, and conclude that their model outperforms the CGARCH model's filtering ability. Further studies of the properties of the MC-GARCH are conducted in Cho and Elshahat (2014). In their paper, they perform a comparison between the MC-GARCH model and the Spline-GARCH model introduced by Engle and Rangel (2008). The authors estimate the long-run and total variance on a daily basis and annualize the volatilities by average each year. The annualized volatilities are then modelled in a simple linear regression along with macroeconomic variables. Cho and Elshahat (2014) reach to the conclusion that macro variables better explain the long-run component of the MC-GARCH model than that of the Spline-GARCH model.

As for the forecasting abilities of ARCH/GARCH-family models, Poon and Granger (2003) provide an extensive review of most of the research made on the subject until the date of their article. They found 93 research papers on the matter, and conclude that financial volatility clearly is predictable. However, they cannot find any definite results suggesting that one volatility model, or class of volatility models, has a superior forecasting performance.

Yu (2002) performs a study on volatility forecasting in the New Zealand stock market with daily data. Nine different models are evaluated, including GARCH-family models and a Stochastic Volatility model, using four different measures such as RMSE and Theil's-U. One of their main conclusions is that the SV model exhibits superior forecasting performance according to three of the evaluation measures.

Goyal (2000) focuses entirely on GARCH models and their forecasting ability. The author employs a measure of actual volatility using daily data, and concludes that volatility forecasts explain very little of the actual volatility proxy. In addition, Goyal finds that a simple ARMA process has a better out-of-sample forecast ability than a more advanced GARCH-M model.

In a paper by Hansen and Lunde (2005), 330 ARCH-type models are compared by means of their ability to describe the conditional variance. The authors use the DM-USD exchange rate and IBM stock returns for evaluation. For the exchange rate analysis, Hansen and Lunde cannot show that a simple GARCH(1,1) model is outperformed by more sophisticated models. However, for IBM stock data, the GARCH(1,1) model is found to be inferior, and models that incorporate a leverage effect for individual stock returns are found to have better forecasting abilities.

Ding and Meade (2010) examine the forecasting ability of GARCH-, SV- and EWMA (Exponentially Weighted Moving Average) models under different simulated volatility scenarios. The authors find little difference between the models in the simulated experiments, but for real underlying data the EWMA model seems to be somewhat more reliable and accurate than the two other types of models.

3. Theoretical Framework

This section provides a short introduction to the theoretical background upon which this thesis is based. Section 3.1 introduces the concept of volatility clustering and conditional volatility. Section 3.2. describes the conditional volatility models in greater detail.

3.1. Constant-, and Conditional Volatility

It is often observed that the volatility in financial time series (of returns) is not constant over time[1]. That is, during some periods the volatility is relatively low, while for other periods the volatility is high. In addition, studies have found that periods with high (low) volatility tend to be followed by periods with high (low) volatility. Mandelbrot (1963) was one of the first researchers to come to this conclusion, and the phenomenon is known as volatility clustering.

In order to find a model that is able to capture the effect of volatility clustering, Engle (1982) developed the ARCH (Autoregressive Conditional Heteroscedasticity) model, in which the conditional variance depends on past squared returns. To illustrate, consider the following equations for describing the returns of a financial time series y_t:

$$y_t = \mu + u_t, \tag{1}$$

$$u_t = \varepsilon_t \sigma_t, \tag{2}$$

Where μ is the unconditional mean of y_t, and ε_t is an independent and identically distributed random variable with mean zero and unit variance, i.e. $\varepsilon_t \sim \text{iid}(0,1)$. *Equation (1)* is also known as the *mean equation*, *Equation (2)* is the GARCH process. Here, if the returns are homoscedastic, σ_t does not vary with time ($\sigma_t = \sigma$). However, if the series exhibit heteroscedasticity (or ARCH-effects), the conditional variance of the returns is given by:

[1] The term volatility is in this study synonymous with variance. The terms are used interchangeably throughout the text.

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2, \tag{3}$$

where $\omega > 0$ and $0 \leq \alpha < 1$ to ensure stationarity and non-negativity of σ_t^2. This equation of conditional volatility of some (G)ARCH model may include more lags of u^2, and can therefore more generally be written as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2. \tag{4}$$

Bollerslev (1986) generalized the ARCH model by Engle, so that the conditional variance depends both on past squared returns as well as past variances. The Generalized ARCH (GARCH) model is formulated as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \tag{5}$$

For ARCH and GARCH models, the time-t conditional variance is exogenously given and known at time t-1. Moreover, the estimation procedure is normally performed by Maximum Likelihood (ML).

Much research has been devoted to the question of what drives the volatility of financial returns. Most studies have focused on forecasting, either by studying the series of returns in isolation, or by measuring effects of news and announcements. However, only a fraction of the research made has considered general macroeconomic states as a factor for explaining the conditional volatility. It is a well known fact that volatility tends to be high during recessions, and low during periods of stable growth (Cho and Elshahat (2014)). A challenge faced by economists is thus to capture effects from macroeconomic factors, which are generally less frequently measured and less volatile than financial assets. Therefore, extensions of GARCH-type models that incorporate two volatility components have been developed, where one component captures long-run non-constant persistence and the other describes short-run volatility shocks. The purpose is to achieve more accurate estimates of total volatility.

3.2. Conditional Volatility Models

In this study, three main models are being evaluated on macroeconomic explanatory power, and short-run forecasting ability. Below follows a brief description of each model.

3.2.1. GARCH(1,1)

The most simple version of any GARCH(p,q) model is the GARCH(1,1). It is often found to be robust and useful for accurately describing and forecasting conditional variances of economic variables (see e.g. Bracker and Smith (1999) and Hansen and Lunde (2005)), and therefore it serves as a benchmark model in many evaluation studies. The equation for the conditional variance is specified as:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{6}$$

where ω is the intercept and $\omega > 0$, $\alpha > 0$, $\beta > 0$ and $(\alpha + \beta) < 1$ in order to ensure stationarity and non-negative values of σ_t^2.

3.2.2. CGARCH

The Component GARCH (CGARCH) model by Engle and Lee (1999) is a two-component GARCH model that captures "long memory" in the volatility of financial time series. The authors find that aggregate volatility is affected by shocks at different frequencies, but argue that the CGARCH model captures the effects of both short-run, and more persistent long-run volatility shocks. The long-run shocks can be caused by economic states and events, e.g. macroeconomic factors, while short-run volatility is typically caused by news and announcements. To see how the CGARCH model is specified, consider again *Equations (1)-(2)* above. In the CGARCH model, the conditional variance is given by:

$$\sigma_t^2 = q_t + \alpha(u_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1}), \tag{7}$$

$$q_t = \omega + \phi(u_{t-1}^2 - \sigma_{t-1}^2) + \rho q_{t-1}. \tag{8}$$

We can rewrite the expression above to obtain:

$$\sigma_t^2 = q_t + s_t, \tag{9}$$
$$s_t = \alpha(u_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1}), \tag{10}$$
$$q_t = \omega + \phi(u_{t-1}^2 - \sigma_{t-1}^2) + \rho q_{t-1}, \tag{11}$$

where s_t is the short-run, or transitory component, while q_t is the long-run time-varying component. σ_t^2 is stationary if $\rho < 1$ and $(\alpha + \beta) < 1$. Non-negativity is assured if $1 > \rho > (\alpha + \beta) > 0, \beta > \phi > 0, \alpha > 0, \beta > 0$ and $\omega > 0$. Moreover, by rewriting *Equations (9) - (11)*, a reduced form of the model can be expressed as:

$$\begin{aligned}\sigma_t^2 = \omega(1 - \alpha - \beta) + (\alpha + \phi)u_{t-1}^2 + [-\phi(\alpha + \beta) - \alpha\rho]u_{t-2}^2 \\ +(\rho + \beta - \phi)\sigma_{t-1}^2 + [\phi(\alpha + \beta) - \beta\rho]\sigma_{t-2}^2\end{aligned} \tag{12}$$

It is now apparent that the conditional volatility follows a restricted GARCH(2,2) process. The model reduces to a form of GARCH(1,1) if either $\alpha = \beta = 0$ or $\rho = \phi = 0$. Moreover, just as for the GARCH(1,1), the estimation for CGARCH is normally done by ML.

3.2.3. Stochastic Volatility

Unlike standard ARCH/GARCH models, the time-t conditional variance in a Stochastic Volatility (SV) model is not observable at time t-1 because the parameters are endogenously estimated. The foundation for SV models is found within the continuous time framework, but here the model is applied in a discrete time setting. Given the same mean equation and GARCH process as for previous models (see *Equations (1)-(2)*), we now assume that the logarithm of the conditional variance follows an AR(1) process:

$$ln(\sigma_t^2) = \omega + \phi ln(\sigma_{t-1}^2) + \eta_t. \tag{13}$$

However, since σ_t^2 is not observable at time t-1, the usual ML estimation is not feasible. To solve the problem, one can employ a Quasi-Maximum Likelihood (QML) estimation with the Kalman filtering procedure. To illustrate, let us first take squared logarithms and rewrite *Equations (1)-(2)* as:

$$ln(y_t^2) - E[ln(y_t^2)] = ln(\varepsilon_t^2) + ln(\sigma_t^2). \tag{14}$$

Transforming the above expression further by adding and subtracting $E[ln(\varepsilon_t^2)]$, we get:

$$ln(y_t^2) - E[ln(y_t^2)] = E[ln(\varepsilon_t^2)] + ln(\sigma_t^2) + \left[ln(\varepsilon_t^2) - E[ln(\varepsilon_t^2)]\right]. \qquad (15)$$

If we assume that $\varepsilon_t \sim \text{iid}(0,1)$, it is possible to show that $ln(\varepsilon_t^2) \sim \left(-1.27, \frac{\pi^2}{2}\right)$. To simplify, we rewrite *Equation (15)* as:

$$y_t^* = -1.27 + h_t + \xi_t, \qquad (16)$$

where $y_t^* = ln(y_t^2) - E[ln(y_t^2)]$, $h_t = ln(\sigma_t^2)$ and $\xi_t = ln(\varepsilon_t^2) - E[ln(\varepsilon_t^2)]$. The conditional variance (h_t) in *Equation (16)* is specified as:

$$h_t = \omega + \phi h_{t-1} + \eta_t. \qquad (17)$$

Although the true distribution of ξ_t in *Equation (16)* is unknown, we can obtain estimates of h_t by treating it as $\xi_t \sim NID\left(0, \frac{\pi^2}{2}\right)$. It is further assumed that $\eta_t \sim \text{NID}(0, \sigma_\eta^2)$ and $E[\xi_t \eta_t] = 0$. Now, we can estimate h_t by using Kalman filter based QML. The estimation procedure is explained below. First, we introduce a few notations in *Table 1*.

Table 1 - Notations for Kalman filter based QML

Notation	**Explanation**
ψ_t	Information set available at time t
$h_{t\vert t-1} = E[h_t \vert \psi_{t-1}]$	Estimate of h_t conditional on information available at time t-1
$P_{t\vert t-1} = E\left[\left(h_t - h_{t\vert t-1}\right)^2\right]$	Variance of h_t conditional on information available at time t-1
$h_{t\vert t} = E[h_t \vert \psi_t]$	Estimate of h_t conditional on information available at time t
$P_{t\vert t} = E\left[\left(h_t - h_{t\vert t}\right)^2\right]$	Variance of h_t conditional on information available at time t
$y_{t\vert t-1} = E[y_t \vert \psi_{t-1}]$	Forecast of y_t conditional on information at time t-1
$\zeta_{t\vert t-1} = y_t - y_{t\vert t-1}$	Forecast error
$R_{t\vert t-1} = E\left[\zeta_{t\vert t-1}^2\right]$	Conditional variance of the forecast error

The estimation process consists of two main steps: forecasting and updating. We start by calculating $h_{t|t-1}$ in order to obtain the forecast of y_t. Thus, we must set priors of σ_ξ^2 and σ_η^2 as well as for ω and ϕ in *Equation (17)* since no information is available at time 0.

From the priors, we get:

$$h_{t|t-1} = \omega + \phi h_{t-1|t-1}, \tag{18}$$

$$P_{t|t-1} = \phi^2 P_{t-1|t-1} + \sigma_\eta^2, \tag{19}$$

$$\zeta_{t|t-1} = y_t - y_{t|t-1} = h_t - h_{t|t-1} + \xi_t, \tag{20}$$

$$R_{t|t-1} = P_{t|t-1} + \sigma_\xi^2. \tag{21}$$

Now we update the values to make a more accurate inference about the conditional volatility. This is possible since the forecast error $\zeta_{t|t-1}$ contains new information about h_t. We have:

$$h_{t|t} = h_{t|t-1} + k_t \zeta_{t|t-1}, \tag{22}$$

$$P_{t|t} = P_{t|t-1} - k_t P_{t|t-1}, \tag{23}$$

where $k_t = \frac{P_{t|t-1}}{R_{t|t-1}}$. This is also known as the Kalman gain, which determines the weight assigned to new information about h_t contained in $\zeta_{t|t-1}$. The procedure described above is then repeated recursively until time t = T. The final log likelihood function is given by:

$$l(\theta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} \ln\left(P_{t|t-1} + \frac{\pi^2}{2}\right) - \frac{1}{2}\sum_{t=1}^{T} \frac{\left(y_t^* - h_{t|t-1} + 1.27\right)^2}{P_{t|t-1} + \frac{\pi^2}{2}}, \tag{24}$$

where $\theta = \{\omega, \phi, \sigma_\eta^2, \sigma_\xi^2\}$. For a more detailed explanation of the Kalman filtering based QML, see Kim and Nelson (2003).

4. Data

The equity data used in this study consists of daily MSCI index points ranging from January 1st 1993 to May 5th 2014[2]. The daily returns are calculated as:

$$y_t = ln\left(\frac{P_t}{P_{t-1}}\right), \tag{25}$$

where P_t is the closing index point at time t. Macroeconomic time series consist of quarterly observations ranging from 1993:Q1 to 2013:Q4. All data are obtained from DataStream for Denmark, Finland, Norway and Sweden.

Similar to Cho and Elshahat (2014), macroeconomic variables are chosen in accordance with previous research and/or economic theory. GDP growth is included to capture effects of the business cycles throughout the time series. The log of GDP and short-term (risk-free) interest rate are supposed to measure the uncertainty of fundamental macroeconomic factors. Lastly, inflation is used as a predictor for future states of the economy, since it is closely related to policy decisions. Total inflation is, however, separated into growth of money supply (M2) and CPI. The idea is that inflation consists of both monetary expansion as well as structural (non-monetary) inflation, caused by shifts in demand and supply curves of consumer goods. The two components can thus be separated and studied individually. Cho and Elshahat (2014) also include a proxy for the exchange rate, called Dollar Index. However, this index is not available for the Nordic countries, and is therefore not included here.

5. Empirical Method

Below follows a more detailed explanation of how the study is performed. Section 5.1.provides a brief overview of the methodology, while Section 5.2. describes the various measures employed in the forecasting performance evaluation.

5.1. Overview

In the first part of this thesis, where the extent macroeconomic variables' explanatory power on conditional variance is examined, I mainly follow the work done by Cho and Elshahat

[2] The MSCI index is a broad benchmark index provided by MSCI Inc.

(2014). This implies that the conditional volatilities for the three models under evaluation are regressed on macroeconomic variables by employing Ordinary Least Squares (OLS). As for the GARCH-type models (not the SV-model), I also investigate whether a normal-, or a student-t distribution best describes the data. This is because time series of financial returns are often found to exhibit excess kurtosis, or “fat tails”, and thus the commonly employed assumption of normality may not be adequate.

In the special case of estimating the SV model, the priors set as initial values are $\sigma_{\xi}^{2} = \frac{\pi^{2}}{2}$, $\sigma_{\eta}^{2} = 0.02$, $\phi = 0.95$ and $\omega = -1$, based upon previous experiments. A few other studies have found that the estimation is rather sensitive to the chosen priors, and therefore robustness checks are performed. However, when elaborating with other reasonable priors, the estimation results do not change notably.

In order to obtain series that match the quarterly macroeconomic data, the daily volatility series are averaged over each quarter. Moreover, following Engle and Rangel (2008) and Cho and Elshahat (2014), volatilities of macroeconomic variables are approximated by the absolute value of the residuals in an AR(1) process:

$$\Delta \ln(y_t) = \omega + u_t, \tag{26}$$

$$u_t = \alpha \mathrm{u}_{\mathrm{t}-1} + \varepsilon_{\mathrm{t}}, \tag{27}$$

$$\sigma_{y,t}^{2} = |\varepsilon_{\mathrm{t}}|. \tag{28}$$

The exception from the equations above is for the volatilities of interest rates, which are not calculated in log form. The reason for this is that interest rates are already expressed as "returns" or changes of invested capital over the period considered.

In the second part, I evaluate the short-term forecasting abilities of the three volatility models. This is performed by a static rolling window approach in order to forecast the one-day ahead conditional variance of each country's equity index. Several time periods with varying window sizes are studied. They are summarized in *Table 2*.

Table 2 - Forecasting periods

Early	Dates	Observations
In-sample	01/01/1993 – 01/02/1995	522
Out-of-sample	01/03/1995 – 01/01/1998	784
Mid		
In-sample	01/01/1993 – 01/03/2000	1869
Out-of-sample	01/04/2000 – 01/04/2004	1043
Late		
In-sample	01/01/2001 – 12/30/2005	1305
Out-of-sample	01/02/2006 – 01/03/2011	1306
Full		
In-sample	01/01/1993 – 12/20/1993	252
Out-of-sample	12/21/1993 – 05/07/2014	5317

The table shows the periods chosen for forecasting evaluation. The "In-sample" period is the initial window size ranging from time t-j to time t. The first observation in the "Out-of-sample" period at time t+1 is the first to be forecasted. The next forecasted observation is at time t+2, with a window ranging from time t-j+1 to t+1.

The in-sample and out-of-sample periods are chosen based on the general volatility pattern of each period. For instance, the initial in-sample of "Early" has relatively low volatility, while the out-of-sample covers a period of generally increasing volatility for all countries. From the choice of different periods with different volatility patterns, the aim is to analyze how the models perform under various scenarios which in turn provide robustness to the study.

5.2. Performance measures

In order to evaluate the forecasting performance, one needs a set of evaluation measures. Such commonly used measures in the literature are e.g. the Mean Square Error (MSE) and the Mean Absolute Error (MAE). These are also called "loss functions", and are based upon a comparison of the realized ("true") volatility and volatility forecast. The realized volatility serves as a proxy for the true volatility. A frequently identified problem in the literature is how to choose this proxy, since the true volatility is not directly observable. Following Pagan and Schwert (1990) and Pojarliev and Polasek (2001), the true volatility in this study is approximated by the squared daily equity returns. Andersen and Bollerslev (1998) showed that squared daily returns is an unbiased, albeit noisy proxy for the true volatility, and instead suggest that one should use data of high-frequency intra-day returns. However, due to data limitations, I use the classic, and simpler, squared daily returns approach. Below follows a description of the measures I have chosen to employ in this study.

5.2.1. Mean squared error

The MSE is the average squared difference between forecasted volatility and realized volatility at time t, and is defined as:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2, \tag{29}$$

where $\hat{\sigma}_t^2$ is the volatility forecast and σ_t^2 is the realized ("true") volatility.

5.2.2. Mean absolute error

The Mean Absolute Error (MAE) is the average difference between forecasted volatility and realized volatility in absolute terms.

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{\sigma}_t^2 - \sigma_t^2|. \tag{30}$$

5.2.3. Root mean squared error

The Root Mean Squared Error (RMSE) is defined as the square-root of the MSE:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}. \tag{31}$$

This measure is included since it is interesting to see whether the measured performance changes by letting the models be punished to a lesser extent by outliers.

5.2.4. Theil's-U

A less commonly used evaluation measure is the Thiel's-U statistic. It is defined as:

$$Theil's - U = \frac{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}{\frac{1}{T}\sum_{t=1}^{T}(\sigma_{t-1}^2 - \sigma_t^2)^2}. \tag{32}$$

In words, the statistic is the MSE of the forecasted volatility divided by the MSE of a random walk process, or a naïve forecast. Thus, the forecast error is standardized by the random walk error, which implies that values below one indicate that the volatility models provide better forecasts than a random walk. We expect the statistic to be below 1 for all models.

5.2.5. LINEX

Unlike the symmetric measures and statistics above, LINEX is asymmetric and defined as:

$$LINEX = \frac{1}{T}\sum_{t=1}^{T}\left[exp\left(-\alpha(\hat{\sigma}_t^2 - \sigma_t^2)\right) + \alpha(\hat{\sigma}_t^2 - \sigma_t^2) - 1\right], \tag{33}$$

where α is a given parameter. For this loss function, positive and negative forecast errors are weighted differently, depending on the sign of α. For $\alpha > 0$, negative errors ($\hat{\sigma}_t^2 - \sigma_t^2 < 0$) receive a larger weight than positive errors, and vice versa. Following Yu (2002), I use four different values of α: -20, -10, 10 and 20, where the negative values penalize over-predictions to a greater extent than under-predictions. The opposite holds for positive values.

In addition to the loss functions described above, I employ the same procedure as Pojarliev and Polasek (2001) by regressing the realized variance on a constant and the conditional variance forecast. The model is specified as:

$$\sigma_t^2 = \alpha + \beta\hat{\sigma}_t^2 + \varepsilon_t, \tag{34}$$

where σ_t^2 are the squared returns (realized variance) and $\hat{\sigma}_t^2$ is the volatility forecast at time t. In the regression model above, α should be close to zero and β close to one for the model not to be biased. The R^2-vaule from the regression serves as a measure of overall fit. That is, the higher the R^2, the better the forecast. For the purpose of maximal comparability between the models, I do not separate the long-run component from the CGARCH model since I am mainly interested in the forecasting ability of the total variance.

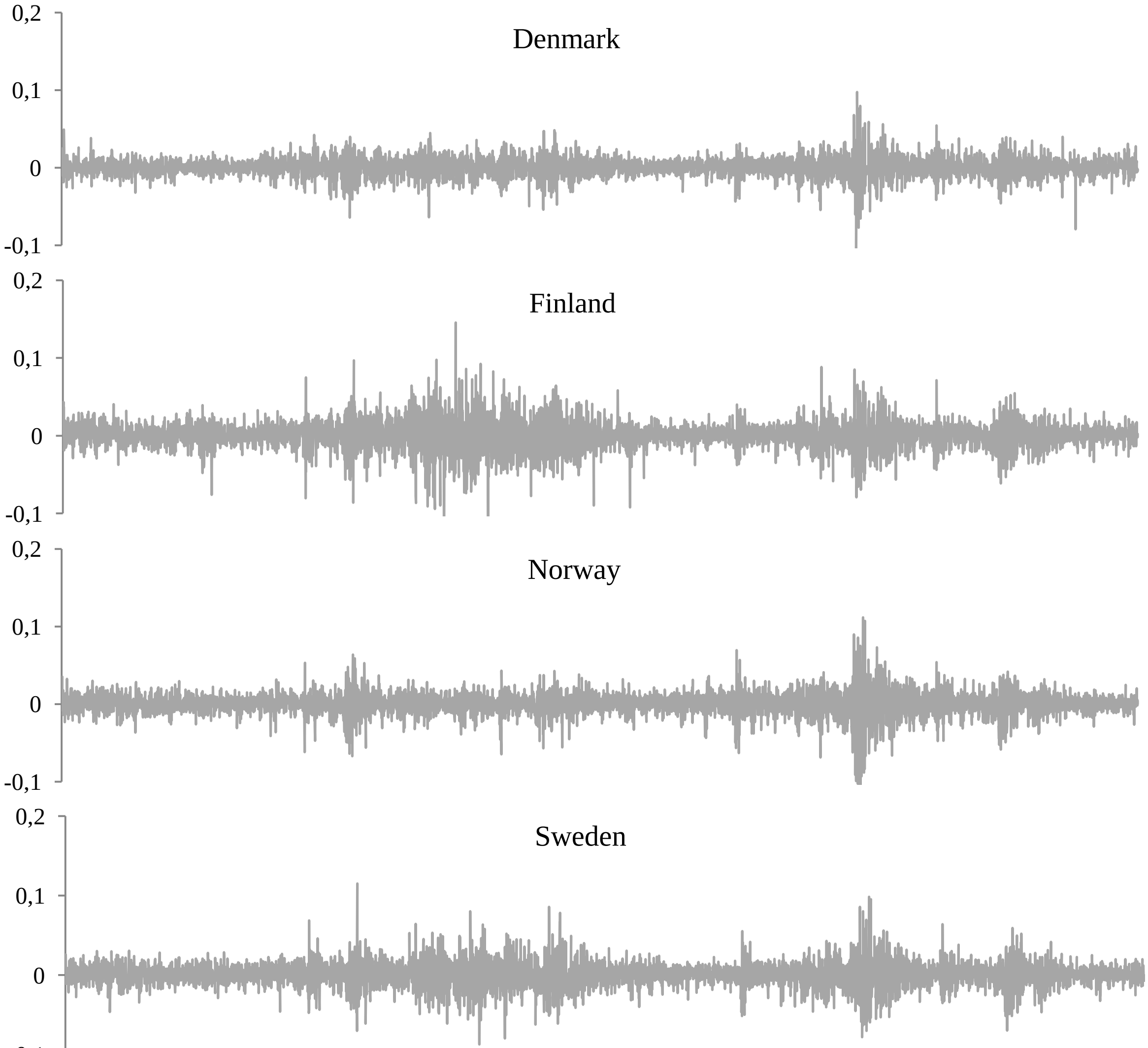
6. Results

Firstly, a preliminary analysis of descriptive statistics and other results of importance is conducted in Section 6.1. Section 6.2. provides the results from the conditional volatility model estimation. Section 6.3 presents the results from the regression analysis of the long-run volatility modelling, and lastly, Section 6.4.provides the short-run forecasting performance evaluation.

6.1. Preliminary Analysis

Looking at time series plots of the equity returns in *Figure 1*, we immediately see that the volatility appears to be clustered.

Figure 1 - Equity returns



The figure shows the equity returns for each country plotted against time. The series run from January 1st 1993 to May 7th 2014.

As mentioned in *Section 2.1.*, this clustering phenomenon is a common finding in series of financial returns. We also see from the ARCH(1)-test in *Table 3* that the returns are heteroscedastic. Thus, our models for estimating conditional variance are likely to describe the volatility well.

Table 3 - ARCH test

	Denmark	**Finland**	**Norway**	**Sweden**
Obs*R^2	304,31***	106,49***	460,88***	181,52***
P-value	0,0000	0,0000	0,0000	0,0000

The ObsR^2 is a test statistic obtained by multiplying the R^2-value from an auxiliary regression of the squared residuals from an auxiliary regression of the returns, by the number of observations. This statistic is compared to the critical Chi-squared value, and the null is no presence of ARCH-effects.*

It is worth mentioning that the returns for Finland seem to reach a volatility-peak during the tech bubble in the early 2000's. Similar effects are apparent for Sweden, and the cause for this may be the large market cap of the two major telecom companies at the time: Nokia in Finland and Ericsson in Sweden. Moreover, for all countries, we see great volatility impacts from the global financial crisis in 2008/2009.

Table 4 provides the correlation coefficients between the macro variables for each country, while the descriptive statistics are presented in *Table 5*.

Table 4 - Correlations

					Denmark					
Correlation	GR. CPI	VOL. CPI	GR. GDP	LNGDP	VOL. GDP	GR. M2	GR. $M2_{t-1}$	VOL. RF	VOL. RF_{t-1}	VOL. RF_{t-2}
GR. CPI	1.0000									
VOL. CPI	0.1759	1.0000								
GR. GDP	0.0736	0.1582	1.0000							
LNGDP	0.0336	0.2012	-0.1954*	1.0000						
VOL. GDP	0.2042*	0.0621	0.1180	0.0131	1.0000					
GR. M2	0.0322	0.1392	0.0512	0.1262	0.2717**	1.0000				
GR. $M2_{t-1}$	-0.1018	0.0968	-0.0008	0.1088	0.1595	0.1999*	1.0000			
VOL. RF	-0.0535	0.0318	-0.2295**	-0.0015	0.1965*	-0.2302**	-0.0507	1.0000		
VOL. RF_{t-1}	-0.1052	0.0572	-0.0909	-0.1181	0.0675	-0.3156***	-0.1963*	0.5049***	1.0000	
VOL. RF_{t-2}	-0.1094	0.0087	-0.0397	-0.2889***	-0.1494	-0.2304**	-0.2041*	0.3405***	0.5743***	1.0000
Obs. 80										

Table 4 (continued)

					Finland					
Correlation	GR. CPI	VOL. CPI	GR. GDP	LNGDP	VOL. GDP	GR. M2	GR. $M2_{t-1}$	VOL. RF	VOL. RF_{t-1}	VOL. RF_{t-2}
GR. CPI	1.0000									
VOL. CPI	0.3252***	1.0000								
GR. GDP	0.2034*	-0.2119*	1.0000							
LNGDP	0.2050*	0.1412	-0.2696**	1.0000						
VOL. GDP	-0.0757	0.2280**	-0.3780***	0.1002	1.0000					
GR. M2	0.1126	-0.0159	0.0896	0.1913*	-0.0427	1.0000				
GR. $M2_{t-1}$	0.0435	0.1933*	-0.1726	0.1704	0.0977	-0.4133***	1.0000			
VOL. RF	-0.1044	0.1944*	-0.3877***	-0.1535	0.5195***	-0.0092	-0.0111	1.0000		
VOL. RF_{t-1}	0.0787	0.0904	-0.0543	-0.2769**	0.2529**	-0.2671**	0.0377	0.4393***	1.0000	
VOL. RF_{t-2}	-0.0440	0.1369	0.0098	-0.2859**	0.0572	-0.1033	-0.2629**	0.2137*	0.3168***	1.0000
Obs. 80										

Table 4 (continued)

					Norway					
Correlation	GR. CPI	VOL. CPI	GR. GDP	LNGDP	VOL. GDP	GR. M2	GR. M2 $_{t-1}$	VOL. RF	VOL. RF $_{t-1}$	VOL. RF $_{t-2}$
GR. CPI	1.0000									
VOL. CPI	-0.0244	1.0000								
GR. GDP	0.0093	-0.0804	1.0000							
LNGDP	-0.0318	0.1483	-0.1619	1.0000						
VOL. GDP	0.0466	-0.0278	-0.0286	-0.2726**	1.0000					
GR. M2	-0.1035	0.0659	0.0982	0.0422	-0.2924***	1.0000				
GR. M2 $_{t-1}$	0.0788	0.0978	-0.1151	0.0295	0.0572	-0.2687**	1.0000			
VOL. RF	0.0520	0.1370	-0.1575	-0.1161	0.2540**	-0.2520**	0.0623	1.0000		
VOL. RF $_{t-1}$	0.0124	-0.0148	-0.0699	-0.2677**	0.0376	-0.0921	-0.1228	0.4180***	1.0000	
VOL. RF $_{t-2}$	-0.0027	-0.2518**	0.1595	-0.2220**	-0.0612	-0.1536	-0.0779	0.0955	0.2200**	1.0000
Obs. 80										

Table 4 (continued)

					Sweden					
Correlation	GR. CPI	VOL. CPI	GR. GDP	LNGDP	VOL. GDP	GR. M2	GR. M2 $_{t-1}$	VOL. RF	VOL. RF $_{t-1}$	VOL. RF $_{t-2}$
GR. CPI	1.0000									
VOL. CPI	0.0972	1.0000								
GR. GDP	0.2215**	-0.1177	1.0000							
LNGDP	-0.1140	0.1113	-0.4712***	1.0000						
VOL. GDP	0.0426	0.0679	-0.1485	0.2356**	1.0000					
GR. M2	0.1867*	0.1108	0.0554	-0.0683	0.1173	1.0000				
GR. M2 $_{t-1}$	0.0399	0.0744	0.0139	0.0991	0.1102	0.22572**	1.0000			
VOL. RF	-0.1677	0.2502**	-0.4846***	0.4944***	-0.2324**	-0.0004	0.1324	1.0000		
VOL. RF $_{t-1}$	-0.2855**	0.1852	-0.4049***	0.2952***	-0.1263	-0.0662	0.0482	0.6421***	1.0000	
VOL. RF $_{t-2}$	-0.0274	-0.0421	-0.0309	-0.0918	-0.1332	-0.0357	-0.0619	0.1000	0.4249***	1.0000
Obs. 80										

*Table 5 presents the correlations between the macro variables used in the regression analysis for Denmark, Finland, Norway and Sweden. *, ** and *** denotes statistical significance at the 10%, 5% and 1% level, respectively.*

Table 5 - Despriptive statistics

Returns				
	Denmark	Finland	Norway	Sweden
Mean	0.0004	0.0003	0.0003	0.0004
Median	0.0002	0.0000	0.0001	0.0002
Max.	0.0975	0.1456	0.1144	0.1154
Min.	-0.1126	-0.1740	-0.2370	-0.0869
Std. Dev.	0.0115	0.0163	0.0151	0.0149
Skewness	-0.3989	-0.3848	-0.9383	0.0170
Kurtosis	9.8632	11.9367	18.5042	7.5952
J-B stat.	14188.7	23912.3	72489.8	6276.1
Prob.	0.0000	0.0000	0.0000	0.0000
Obs.	7133	7133	7133	7133

GR. CPI				
	Denmark	Finland	Norway	Sweden
Mean	0.0050	0.0040	0.0049	0.0031
Median	0.0041	0.0032	0.0049	0.0027
Max.	0.0138	0.0162	0.0246	0.0188
Min.	-0.0044	-0.0054	-0.0162	-0.0144
Std. Dev.	0.0045	0.0047	0.0054	0.0056
Skewness	0.1864	0.5381	-0.1653	0.1176
Kurtosis	2.1307	2.7813	7.0840	4.0618
J-B stat.	3.0191	4.1705	58.0593	4.0901
Prob.	0.2210	0.1243	0.0000	0.1294
Obs.	83	83	83	83

GR. GDP				
	Denmark	Finland	Norway	Sweden
Mean	0.0037	0.0059	0.0060	0.0065
Median	0.0030	0.0058	0.0061	0.0072
Max.	0.0382	0.0328	0.0353	0.0248
Min.	-0.0245	-0.0658	-0.0227	-0.0375
Std. Dev.	0.0122	0.0125	0.0124	0.0096
Skewness	0.1925	-2.2284	0.1877	-1.7753
Kurtosis	3.2163	14.7528	2.8031	8.5652
J-B stat.	0.66	546.38	0.62	150.71
Prob.	0.7196	0.0000	0.7330	0.0000
Obs.	83	83	83	83

GR. M2				
	Denmark	Finland	Norway	Sweden
Mean	0.0113	0.0118	0.0168	0.0128
Median	0.0106	0.0100	0.0191	0.0144
Max.	0.0900	0.0711	0.0557	0.0618
Min.	-0.0438	-0.0441	-0.0283	-0.0432
Std. Dev.	0.0241	0.0233	0.0184	0.0201
Skewness	0.4412	0.1734	-0.2077	0.0208
Kurtosis	3.5979	2.9228	2.2876	3.2149
J-B stat.	3.8340	0.4364	2.3516	0.1657
Prob.	0.1470	0.8040	0.3086	0.9205
Obs.	83	83	83	83

Table 5 (continued)

LNGDP				
	Denmark	Finland	Norway	Sweden
Mean	12.8071	10.4322	13.3099	13.4923
Median	12.8309	10.4679	13.3283	13.5076
Max.	12.9244	10.6476	13.4835	13.7285
Min.	12.5933	10.0867	12.9810	13.1878
Std. Dev.	0.0846	0.1706	0.1388	0.1634
Skewness	-0.7978	-0.6167	-0.7062	-0.3331
Kurtosis	2.6439	2.0749	2.4074	1.8138
J-B stat.	9.020	8.319	8.211	6.478
Prob.	0.0110	0.0156	0.0165	0.0392
Obs.	84	84	84	84

VOL. CPI				
	Denmark	Finland	Norway	Sweden
Mean	0.0038	0.0038	0.0036	0.0042
Median	0.0033	0.0029	0.0025	0.0034
Max.	0.0093	0.0118	0.0210	0.0172
Min.	0.0001	0.0004	0.0001	0.0000
Std. Dev.	0.0024	0.0028	0.0040	0.0036
Skewness	0.4426	1.0179	2.3604	1.5915
Kurtosis	2.2525	3.1629	9.5936	5.4252
J-B stat.	4.53	14.42	227.42	55.38
Prob.	0.1038	0.0007	0.0000	0.0000
Obs.	83	83	83	83

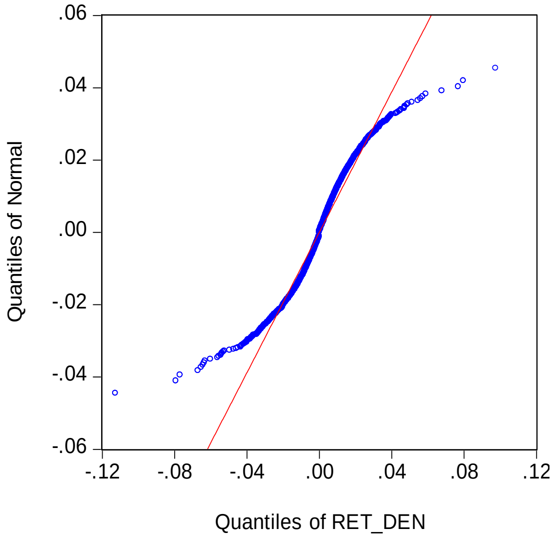
VOL. GDP				
	Denmark	Finland	Norway	Sweden
Mean	0.0088	0.0078	0.0091	0.0065
Median	0.0073	0.0058	0.0074	0.0051
Max.	0.0362	0.0675	0.0284	0.0423
Min.	0.0001	0.0000	0.0001	0.0001
Std. Dev.	0.0072	0.0089	0.0072	0.0068
Skewness	1.1855	3.9812	0.7623	2.6742
Kurtosis	4.4891	25.6329	2.6190	12.8079
J-B stat.	26.5	1990.8	8.5	431.6
Prob.	0.0000	0.0000	0.0140	0.0000
Obs.	83	83	83	83

VOL. RF				
	Denmark	Finland	Norway	Sweden
Mean	0.4286	0.3213	0.4417	0.3348
Median	0.3752	0.2409	0.3066	0.2414
Max.	1.4747	1.9987	2.3195	1.9512
Min.	0.0181	0.0010	0.0039	0.0068
Std. Dev.	0.3237	0.3139	0.4651	0.3214
Skewness	1.1800	2.1618	1.9851	2.3972
Kurtosis	4.3086	11.2095	7.6552	10.7638
J-B stat.	24.58	297.72	129.46	287.95
Prob.	0.0000	0.0000	0.0000	0.0000
Obs.	83	83	83	83

Table 5 presents the descriptive statistics for the equity retuns, growth rate of CPI, growth rate of GDP, growth rate of M2, log GDP, volatility of CPI, volatility of GDP and volatility of short-term interest rate.

From *Table 4*, we see that variations of GDP and the short-term interest rate in general exhibit significant correlations. Moreover, as is evident from the Jarque-Bera statistics in *Table 5* and the QQ-plots in *Figure 2*, none of the countries' equity returns follow a standard normal distribution. Instead they appear to exhibit excess kurtosis, or "fat tails", and therefore the t-distribution is expected to better describe the data.

Figure 2 - QQ-plots of equity returns



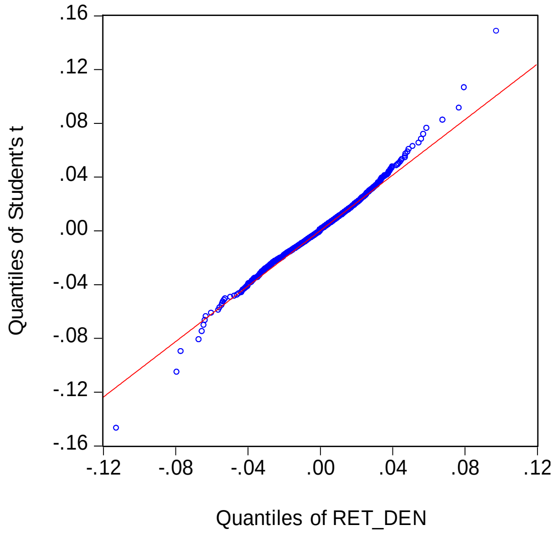
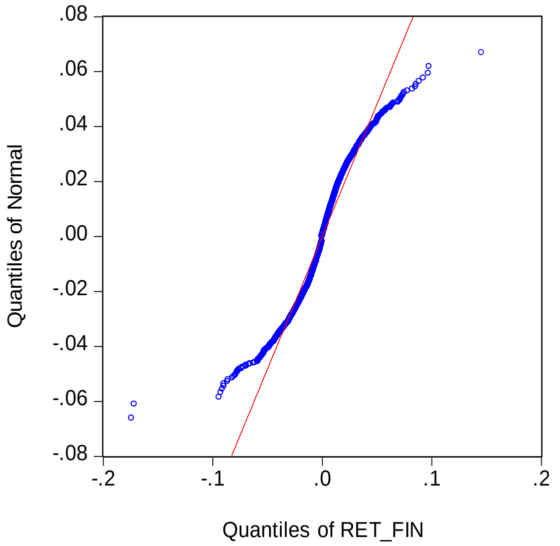


Figure 2 (continued)



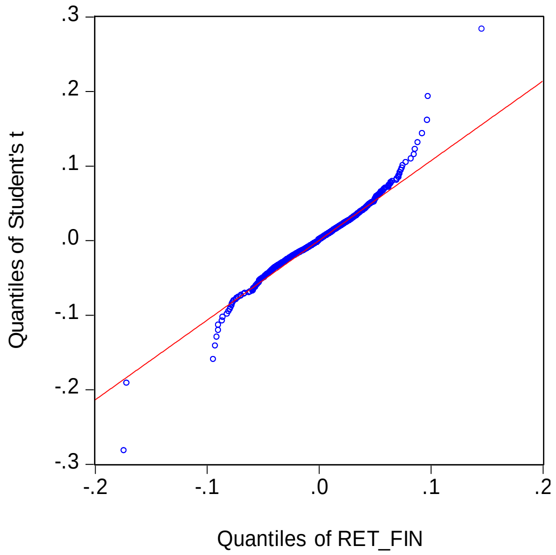
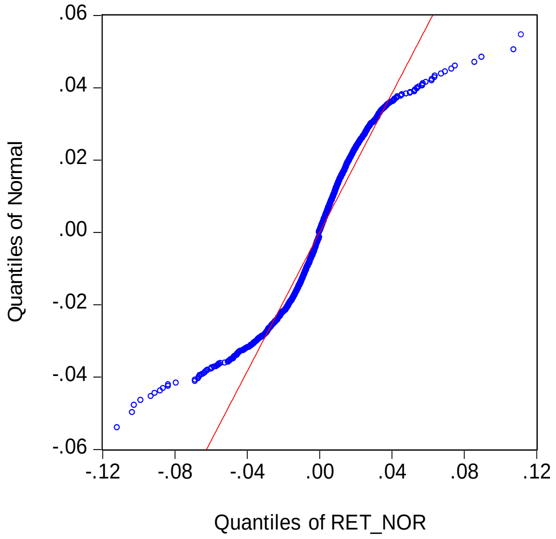


Figure 2 (continued)



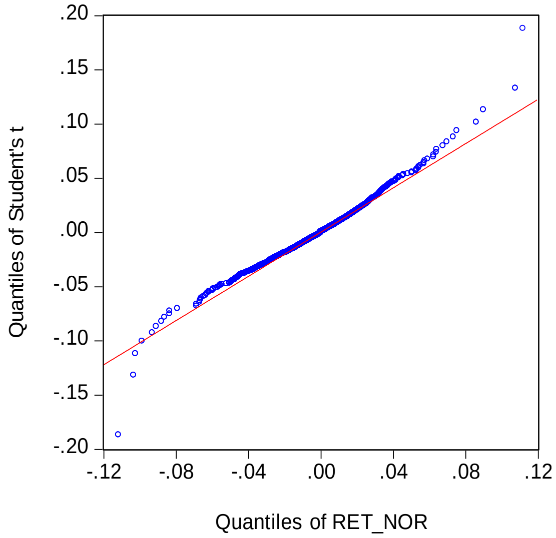
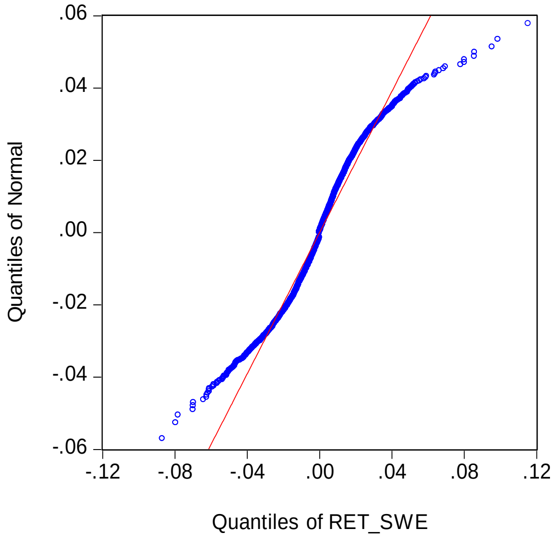


Figure 2 (continued)



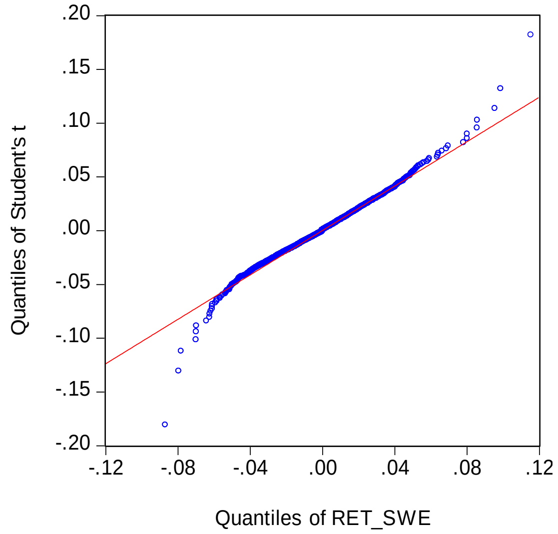


Figure 2 shows QQ-plots of the equity returns, where RET_DEN is the returns for Denmark, RET_FIN is the returns for Finland, RET_NOR is the returns for Norway and RET_SWE is the returns for Sweden,

6.2. Model Estimations

Table 6 present the coefficients from the conditional variance estimations for all models. Note that the estimations of the SV model differ quite significantly from the GARCH-type models, and thus the coefficients are not interpreted in the same way.

Table 6 - Coefficients from volatility estimation

Denmark					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
Parameter			Value		
ω	0.0001*** (0.0000)	0.0001*** (0.0000)	0.0000*** (0.0000)	0.0000*** (0.0000)	-0.1170*** (0.0298)
α	0.0843*** (0.0091)	0.0864*** (0.0130)	0.0780*** (0.0039)	0.0881*** (0.0082)	
β	0.8310*** (0.0200)	0.8283*** (0.0288)	0.9045*** (0.0046)	0.8979*** (0.0085)	
ρ	0.9965*** (0.0008)	0.9969*** (0.0014)			
ϕ	0.0199*** (0.0032)	0.0255*** (0.0064)			0.9873*** (0.0032)
η					-4.1482*** (0.2321)

Finland					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
Parameter			Value		
ω	0.0003** (0.0001)	0.0005 (0.0006)	0.0000*** (0.0000)	0.0000*** (0.0000)	-0.0349*** (0.0126)
α	0.0566*** (0.0071)	0.0419*** (0.0112)	0.0534*** (0.0024)	0.0606*** (0.0060)	
β	0.8491*** (0.0237)	0.8934*** (0.0349)	0.9434*** (0.0021)	0.9366*** (0.0058)	
ρ	0.9986*** (0.0006)	0.9989*** (0.0016)			
ϕ	0.0314*** (0.0031)	0.0378*** (0.0076)			0.9959*** (0.0014)
η					-4.9803*** (0.2292)

Table 6 (continued)

Norway					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
Parameter			Value		
ω	0.0001*** (0.0000)	0.0001*** (0.0000)	0.0000*** (0.0000)	0.0000*** (0.0000)	-0.0822*** (0.0248)
α	0.0186 (0.0132)	0.0787*** (0.0129)	0.0984*** (0.0059)	0.0957*** (0.0087)	
β	-0.5528 (0.3980)	0.8565*** (0.0247)	0.8801*** (0.0068)	0.8875*** (0.0097)	
ρ	0.9799*** (0.0040)	0.9952*** (0.0019)			
ϕ	0.0948*** (0.0063)	0.0309*** (0.0096)			0.9907*** (0.0027)
η					-4.3660*** (0.2380)

Sweden					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
Parameter			Value		
ω	0.0002*** (0.0000)	0.0002** (0.0001)	0.0000*** (0.0000)	0.0000*** (0.0000)	-0.0595*** (0.0170)
α	-0.0220** (0.0112)	-0.0218 (0.0161)	0.0743*** (0.0048)	0.0715*** (0.0067)	
β	-0.3431 (0.4200)	-0.2468 (0.7005)	0.9178*** (0.0052)	0.9229*** (0.0068)	
ρ	0.9918*** (0.0024)	0.9941*** (0.0032)			
ϕ	0.0769*** (0.0048)	0.0744*** (0.0069)			0.9932*** (0.0019)
η					-4.6116*** (0.2155)

*Table 6 presents the values of the coefficients of the conditional variance for Denmark, Finland, Norway and Sweden. Standard errors within parenthesis. *,** and *** denotes statistical significance at 10%, 5% and 1% level, respectively.*

From *Table 6* above, we see that the α and β coefficients in the CGARCH models for Sweden are not statistically different from 0, even at the 10% level. This implies that the long-run component cannot be distinguished from the total conditional volatility. Thus the models are reduced to a form of GARCH(1,1). The same holds for the normally distributed CGARCH with Norwegian data. This is illustrated in *Figure 3* in the *appendix*, where we clearly see that the long-run component is indistinguishable from the total variance for the above mentioned cases. Moreover, from *Table 6*, we see that many of the GARCH-type estimations are near-unit root processes, which is expected and particularly common for CGARCH models (see Speight, McMillan and Gwilym (2000) and Engle and Lee (1999)).

6.3. Long-Run Volatility

All variables in the regression analysis are tested for unit root by Augmented Dickey-Fuller (Fuller (1976)) (ADF) tests. If one or more series are unit root processes, the results from a regression analysis are difficult to interpret because the variables exhibit (theoretical) infinite variance. In such cases, nonsense-causality (so-called spurious regressions) may arise, for which the results are unreliable. Lag length for the ADF test is set by Akaike (1974) Information Criterion (AIC). Every variable is tested with three different ADF specifications (1. no intercept; 2. including intercept; 3. including both intercept and trend). The results are not presented here to save space, but are available upon request. Variables for which the null hypothesis of a unit root is not rejected at the 10% level for at least one of the three specifications are transformed by first differencing or by detrending. Detrending is performed using the Hodrick-Prescott (HP) filter, with $\lambda = 1600$ as suggested by Hodrick and Prescott (1997) when using quarterly data. All variables are shown to be stationary after transformation.

The results from the regressions are presented in *Table 7*. Here, CGARCH is the total variance of the CGARCH model, while LCGARCH is the long-run component.

Table 7 - Regression results

Denmark							
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables				Coefficient			
C	0.0001**	0.0001**	0.0001**	0.0001**	0.0001**	0.0001**	0.0001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GR. CPI $^{\Delta}$	-0.0019	-0.0021	-0.0026	-0.0029	0.0002	0.0000	-0.0014
	(0.0016)	(0.0017)	(0.0018)	(0.0019)	(0.0011)	(0.0011)	(0.0013)
GR. GDP	-0.0041	-0.0044	-0.0046	-0.0049	-0.0025*	-0.0030*	-0.0032*
	(0.0025)	(0.0027)	(0.0029)	(0.0032)	(0.0015)	(0.0018)	(0.0018)
GR. M2	0.0012	0.0013	0.0014	0.0015	0.0005	0.0007	0.0006
	(0.0012)	(0.0013)	(0.0014)	(0.0014)	(0.0007)	(0.0008)	(0.0008)
GR. $M2_{t-1}$	-0.0002	-0.0002	-0.0003	-0.0003	0.0001	0.0001	-0.0001
	(0.0005)	(0.0005)	(0.0006)	(0.0007)	(0.0003)	(0.0004)	(0.0004)
VOL. RF	0.0001*	0.0001*	0.0001*	0.0001*	0.0001**	0.0001**	0.0001**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0000)
VOL. RF_{t-1}	0.0001	0.0001	0.0000	0.0000	0.0001*	0.0001*	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VOL. CPI $^{\Delta}$	0.0009	0.0009	0.0016	0.0017	-0.0005	-0.0005	0.0010
	(0.0031)	(0.0032)	(0.0037)	(0.0039)	(0.0017)	(0.002)	(0.0024)
VOL. GDP $^{\Delta}$	0.0024	0.0025	0.0030	0.0032	0.0007	0.0009	0.0020
	(0.0020)	(0.0021)	(0.0022)	(0.0024)	(0.0012)	(0.0014)	(0.0015)
$LNGDP^{d}$	-0.0002	-0.0001	0.0004	0.0005	-0.0011**	-0.0010	0.0002
	(0.0007)	(0.0007)	(0.0007)	(0.0008)	(0.0006)	(0.0006)	(0.0006)
R^2	0.3109	0.3083	0.2805	0.2806	0.4005	0.3819	0.2974
Obs.	80	80	80	80	80	80	80

Table 7 (continued)

Finland							
Dependent variable	CGARCH $^{\Delta}$ N-dist	CGARCH $^{\Delta}$ T-dist	GARCH $^{\Delta}$ N-dist	GARCH $^{\Delta}$ T-dist	LCGARCH $^{\Delta}$ N-dist	LCGARCH $^{\Delta}$ T-dist	SV $^{\Delta}$
Variables				Coefficient			
C	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
GR. CPI	-0.0008	-0.0008	-0.0011	-0.0009	-0.0018	-0.0017	-0.0035
	(0.0052)	(0.0069)	(0.0075)	(0.0077)	(0.0054)	(0.0059)	(0.0074)
GR. GDP	0.0002	0.0003	0.0004	0.0005	-0.0003	-0.0002	-0.0005
	(0.0016)	(0.0023)	(0.0025)	(0.0026)	(0.0017)	(0.0019)	(0.0016)
GR. M2 $^{\Delta}$	0.0014	0.0016	0.0017	0.0018	0.0008	0.0010	0.0005
	(0.0008)	(0.0010)	(0.0011)	(0.0011)	(0.0007)	(0.0008)	(0.0008)
GR. M2 $^{\Delta}_{t-1}$	0.0007	0.0008	0.0008	0.0008	0.0004	0.0005	-0.0001
	(0.0008)	(0.0009)	(0.0019)	(0.0011)	(0.0007)	(0.0007)	(0.0007)
VOL. RF	0.0000	0.0000	-0.0001	-0.0001	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-2}	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. CPI $^{\Delta}$	0.0034	0.0036	0.0036	0.0037	0.0027	0.0029	0.0020
	(0.0052)	(0.0038)	(0.0041)	(0.0043)	(0.0031)	(0.0032)	(0.0032)
VOL. GDP $^{\Delta}$	-0.0003	-0.0004	-0.0004	-0.0005	0.0001	0.0000	-0.0015
	(0.0016)	(0.0018)	(0.0020)	(0.0021)	(0.0012)	(0.0014)	(0.0017)
$LNGDP^{d}$	0.0026**	0.0027**	0.0028**	0.0028*	0.0025***	0.0026**	0.0025**
	(0.0011)	(0.0013)	(0.0014)	(0.0015)	(0.0009)	(0.0016)	(0.0012)
R^2	0.2002	0.2002	0.1921	0.1965	0.1742	0.1783	0.0909
Obs.	80	80	80	80	80	80	80

Table 7 (continued)

	Norway						
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables	Coefficient						
C	0.0002***	0.0002***	0.0002***	0.0002***	0.0002***	0.0001***	0.0001***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)
GR. CPI	0.0028	0.0023	0.0028	0.0029	0.0028	0.0002	0.0005
	(0.0034)	(0.0033)	(0.0034)	(0.0035)	(0.0034)	(0.0023)	(0.0024)
GR. GDP	-0.0052*	-0.0053*	-0.0052*	-0.0054*	-0.0052*	-0.0037*	-0.0046**
	(0.0030)	(0.0030)	(0.0031)	(0.0031)	(0.0032)	(0.0022)	(0.0023)
GR. M2 $^{\Delta}$	0.0001	0.0002	0.0001	0.0001	0.0001	0.0003	0.0000
	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0007)	(0.0007)
GR. M2 $^{\Delta}{}_{t-1}$	-0.0013**	-0.0012*	-0.0013**	-0.0014**	-0.0013**	-0.0005	-0.0008*
	(0.0007)	(0.0006)	(0.0007)	(0.0007)	(0.0007)	(0.0005)	(0.0005)
VOL. RF	0.0001*	0.0002*	0.0001*	0.0001*	0.0001*	0.0002	0.0001*
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)
VOL. RF_{t-2}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000*	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VOL. CPI	-0.0002	-0.0003	-0.0002	-0.0003	-0.0003	-0.0010	0.0004
	(0.0037)	(0.0041)	(0.0037)	(0.0039)	(0.0037)	(0.0039)	(0.0036)
VOL. GDP	-0.0065	-0.0068	-0.0064	-0.0067	-0.0064	-0.0059	-0.0057
	(0.0048)	(0.0048)	(0.0047)	(0.0049)	(0.0047)	(0.0036)	(0.0035)
$LNGDP^{d}$	0.0024	0.0021	0.0024	0.0025	0.0024	0.0007	0.0018
	(0.0023)	(0.0024)	(0.0023)	(0.0024)	(0.0023)	(0.0018)	(0.0019)
R^2	0.1293	0.1496	0.1287	0.1292	0.1291	0.2155	0.2245
Obs.	80	80	80	80	80	80	80

Table 7 (continued)

Sweden							
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables	Coefficient						
C	0.0002*** (0.0001)	0.0002*** (0.0001)	0.0002*** (0.0001)	0.0002*** (0.0001)	0.0002*** (0.0001)	0.0002*** (0.0001)	0.0002*** (0.0001)
GR. CPI	0.0026 (0.0036)	0.0027 (0.0037)	0.0026 (0.0036)	0.0027 (0.0037)	0.0026 (0.0036)	0.0027 (0.0037)	0.0036 (0.0037)
GR. GDP	-0.0103*** (0.0033)	-0.0105*** (0.0034)	-0.0103*** (0.0033)	-0.0105*** (0.0034)	-0.0103*** (0.0033)	-0.0105*** (0.0034)	-0.0066*** (0.0024)
GR. M2	-0.0004 (0.0011)	-0.0004 (0.0011)	-0.0004 (0.0011)	-0.0004 (0.0011)	-0.0004 (0.0011)	-0.0004 (0.0011)	-0.0009 (0.0009)
GR. $M2_{t-1}$	-0.0005 (0.0011)	-0.0005 (0.0012)	-0.0005 (0.0011)	-0.0005 (0.0012)	-0.0005 (0.0011)	-0.0005 (0.0012)	-0.0004 (0.0009)
VOL. RF	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)
VOL. RF_{t-1}	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)
VOL. RF_{t-2}	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)	0.0000 (0.0001)
VOL. CPI	0.0055 (0.0044)	0.0058 (0.0045)	0.0056 (0.0044)	0.0059 (0.0045)	0.0055 (0.0044)	0.0058 (0.0045)	0.0055 (0.0036)
VOL. GDP $^{\Delta}$	0.0057* (0.0032)	0.0058* (0.0033)	0.0057* (0.0032)	0.0057* (0.0033)	0.0057* (0.0032)	0.0058* (0.0033)	0.0033 (0.0023)
$LNGDP^{d}$	-0.0008 (0.0018)	-0.0008 (0.0019)	-0.0008 (0.0018)	-0.0008 (0.0019)	-0.0008 (0.0018)	-0.0008 (0.0019)	-0.0005 (0.0016)
R^2	0.3139	0.3148	0.3140	0.3150	0.3132	0.3141	0.2179
Obs.	80	80	80	80	80	80	80

*Table 7 presents the regression analysis results with estimated volatility series as dependent variables for Denmark, Finland, Norway and Sweden. Newey-West robust standard errors within parenthesis. *, ** and *** indicates significance level at 10%, 5% and 1% respectively. "GR." denotes "Growth rate of", "VOL." denotes "Volatility of" and "RF" denotes short-term (risk-free) interest rate.*
$^{\Delta}$ denotes that the variables are regressed in first difference due to non-stationarity.
^{d} denotes that the variables are detrended due to non-stationarity.

From the regression results in *Table 7,* it is difficult to find any general patterns or tendencies. In most cases however, the coefficients have the expected sign, but they tend to be statistically insignificant. For Norway and Sweden, the growth rate of GDP seems to partially explain the estimated volatility, although the coefficient is only marginally significant in the case of Norway. We also find weak significance of the volatility of the short-term interest rate for Denmark and Norway.

As we saw from the coefficients in the CGARCH models with Swedish data in *Table 7*, the models reduce to a form of GARCH(1,1) for both distributions. Therefore, in the case of Sweden, there are almost no differences between the CGARCH, LCGARCH and GARCH(1,1) models. We observe the same effect for the normally distributed CGARCH for Norway.

Moreover, for Finland and Norway, we obtain low R^2-values, indicating that the macroeconomic variables are not suitable for explaining the conditional volatility (as estimated by our models) for these markets. The R^2-values are somewhat higher for Denmark and Sweden, albeit considerably lower than those obtained by Cho and Elshahat (2014). The normally distributed LCGARCH for Denmark obtains the highest R^2-value of 0.4005. The same model also has the greatest number of significant variables. For Sweden and Finland, the models with SV as dependent variable appear to be the worst fitted. Macroeconomic volatilities are generally insignificant, and do not have any impact on the quarterly averaged conditional equity volatility.

Variations of the regression models above are presented in the *appendix*, for which lags of variables currently regressed in levels are added. This is done in order to achieve robustness and to investigate whether variables other than those currently regressed in lags have potential (counter-) cyclical behaviour or forecasting power on equity volatility. However, the results from these regressions are very similar to the ones presented above, and do not add much to the analysis.

6.4. Short-Run Forecasting

As mentioned in *Section 5.1.*, the forecasting ability of the conditional variance models are evaluated by several measures. *Table 8* provide the results for the MSE, MAE, RMSE, Thiel's-U and LINEX measures for the chosen sample periods. Values of α (see *Equation (33)*) are sorted in an increasing order. Hence, LINEX1 is calculated with $\alpha = -20$ and LINEX4 with $\alpha = 20$.

Table 8 - Evaluation measures

Early

In-sample: 01/01/1993 – 01/02/1995

Out-of-sample: 01/03/1995 – 01/01/1998

Denmark								
Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	*0.0134*	0.6814	*0.1157*	*0.7407*	*0.2682*	*0.0670*	*0.0668*	*0.2667*
CGARCH T-dist	0.0133	0.6824	0.1154	0.7390	0.2669	0.0666	0.0664	0.2654
GARCH N-dist	0.0132	0.6796	0.1150	0.7362	0.2650	0.0661	0.0660	0.2635
GARCH T-dist	0.0132	**0.6773**	0.1150	0.7362	0.2649	0.0661	0.0660	0.2635
SV	**0.0130**	*0.7051*	**0.1140**	**0.7300**	**0.2603**	**0.0650**	**0.0648**	**0.2590**

Finland								
Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.1659	1.6202	0.4073	0.9010	3.3916	0.8380	0.8190	3.2392
CGARCH T-dist	0.1658	1.6206	0.4071	0.9007	3.3909	0.8376	0.8181	3.2350
GARCH N-dist	**0.1636**	1.6164	**0.4045**	**0.8949**	**3.3458**	**0.8267**	**0.8079**	**3.1955**
GARCH T-dist	0.1666	**1.6126**	0.4081	0.9029	3.4089	0.8419	0.8220	3.2497
SV	*0.1956*	*1.6991*	*0.4423*	*0.9784*	*4.0213*	*0.9909*	*0.9631*	*3.7993*

Norway								
Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	**0.0443**	0.9515	**0.2105**	**0.8350**	**0.8972**	**0.2228**	**0.2198**	**0.8734**
CGARCH T-dist	0.0476	**0.9506**	0.2182	0.8655	0.9645	0.2394	0.2361	0.9377
GARCH N-dist	0.0459	0.9568	0.2142	0.8496	0.9289	0.2306	0.2276	0.9043
GARCH T-dist	0.0463	0.9555	0.2153	0.8539	0.9384	0.2330	0.2298	0.9131
SV	*0.0491*	*0.9610*	*0.2216*	*0.8789*	*0.9951*	*0.2469*	*0.2433*	*0.9664*

Sweden								
Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.0667	1.1976	0.2583	0.8116	1.3578	0.3373	0.3331	1.3240
CGARCH T-dist	0.0672	1.2008	0.2592	0.8072	1.3594	0.3376	0.3332	1.3243
GARCH N-dist	0.0658	1.1899	0.2566	0.7990	1.3325	0.3309	0.3265	1.2976
GARCH T-dist	**0.0655**	**1.1845**	**0.2560**	**0.7972**	**1.3266**	**0.3294**	**0.3250**	**1.2914**
SV	*0.0686*	*1.2897*	*0.2620*	*0.8160*	*1.3917*	*0.3453*	*0.3403*	*1.3512*

Table 8 (continued)

Mid

In-sample: 01/01/1993 – 01/03/2000

Out-of-sample: 01/04/2000 – 01/04/2004

Denmark

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	*0.0003*	*0.1202*	*0.0167*	0.7519	2.0173	0.5020	0.4975	1.9812
CGARCH T-dist	*0.0003*	0.1194	0.0165	0.7497	2.0058	0.4991	0.4946	1.9697
GARCH N-dist	**0.0002**	**0.1064**	**0.0150**	**0.5989**	**0.0045**	**0.0011**	**0.0011**	**0.0045**
GARCH T-dist	**0.0002**	0.1094	0.0154	0.7541	2.0285	0.5049	0.5004	1.9931
SV	**0.0002**	0.1084	0.0156	*0.7564*	*2.0444*	*0.5084*	*0.5032*	*2.0026*

Finland

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	*3.5023*	8.2835	*1.8715*	*0.7321*	*79.3816*	*18.5985*	*16.5599*	*62.8903*
CGARCH T-dist	3.5004	*8.3224*	1.8709	0.7319	79.3064	18.5846	16.5540	62.8793
GARCH N-dist	3.4786	8.2339	1.8651	0.7295	79.0207	18.4928	16.4298	62.3310
GARCH T-dist	3.4909	8.1463	1.8684	0.7308	79.2813	18.5562	16.4902	62.5671
SV	**3.4189**	**7.6447**	**1.8490**	**0.7228**	**77.8967**	**18.2039**	**16.1210**	**61.0499**

Norway

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	**0.0887**	1.5704	**0.2978**	**0.8031**	**1.7926**	**0.4458**	**0.4413**	**1.7562**
CGARCH T-dist	0.0894	1.5898	0.2991	0.8065	1.8070	0.4495	0.4450	1.7714
GARCH N-dist	0.0901	1.5810	0.3001	0.8093	1.8200	0.4526	0.4480	1.7831
GARCH T-dist	0.0900	*1.5984*	0.3001	0.8093	1.8190	0.4525	0.4480	1.7834
SV	*0.0943*	**1.5418**	*0.3071*	*0.8284*	*1.9095*	*0.4745*	*0.4688*	*1.8640*

Sweden

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	**0.4108**	**3.7514**	**0.6410**	**0.6445**	**8.3599**	**2.0719**	**2.0369**	**8.0802**
CGARCH T-dist	0.4141	3.7745	0.6435	0.6470	8.4267	2.0884	2.0533	8.1451
GARCH N-dist	0.4309	3.8232	0.6564	0.6600	8.7710	2.1735	2.1362	8.4732
GARCH T-dist	0.4424	3.8446	0.6651	0.6687	9.0110	2.2320	2.1921	8.6913
SV	*0.5056*	*3.9249*	*0.7111*	*0.7150*	*10.3368*	*2.5558*	*2.5011*	*9.8994*

Table 8 (continued)

Late

In-sample: 01/01/2001 – 12/30/2005

Out-of-sample: 01/02/2006 – 01/03/2011

Denmark

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	*0.0057*	0.2863	*0.0752*	0.7270	7.8202	1.9190	1.8520	7.2832
CGARCH T-dist	0.0053	*0.2989*	0.0729	0.7239	7.7539	1.9027	1.8362	7.2211
GARCH N-dist	0.0026	0.2305	0.0507	**0.7227**	**7.7277**	**1.8963**	**1.8301**	**7.1975**
GARCH T-dist	0.0017	0.2169	0.0418	0.7231	7.7308	1.8977	1.8327	7.2102
SV	**0.0006**	**0.1348**	**0.0249**	*0.7599*	*8.5983*	*2.1033*	*2.0171*	*7.9079*

Finland

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.3459	2.7212	0.5881	0.7260	7.0742	1.7487	1.7107	6.7700
CGARCH T-dist	0.3436	*2.7433*	0.5862	0.7236	7.0245	1.7368	1.6997	6.7278
GARCH N-dist	**0.3392**	2.6979	**0.5824**	**0.7189**	**6.9375**	**1.7149**	**1.6774**	**6.6380**
GARCH T-dist	0.3395	2.7217	0.5826	0.7192	6.9419	1.7162	1.6791	6.6456
SV	*0.3518*	**2.6247**	*0.5931*	*0.7321*	*7.2113*	*1.7806*	*1.7379*	*6.8697*

Norway

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	1.0009	4.3373	1.0004	0.7127	20.5870	5.0738	4.9379	19.4984
CGARCH T-dist	1.0020	*4.3502*	1.0010	0.7131	20.5982	5.0781	4.9449	19.5312
GARCH N-dist	0.9937	4.3090	0.9968	0.7102	20.4568	5.0396	4.9004	19.3416
GARCH T-dist	**0.9924**	4.3160	**0.9962**	**0.7097**	**20.4139**	**5.0310**	**4.8959**	**19.3314**
SV	*1.0778*	**4.2204**	*1.0382*	*0.7396*	*22.4779*	*5.5017*	*5.2806*	*20.7075*

Sweden

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.4462	3.1303	0.6680	0.7152	9.1422	2.2579	2.2053	8.7212
CGARCH T-dist	0.4491	3.1648	0.6701	0.7175	9.1912	2.2711	2.2203	8.7849
GARCH N-dist	**0.4438**	3.1350	**0.6662**	**0.7133**	**9.0903**	**2.2454**	**2.1937**	**8.6766**
GARCH T-dist	0.4454	*3.1723*	0.6674	0.7146	9.1149	2.2525	2.2027	8.7161
SV	*0.4622*	**2.9903**	*0.6799*	*0.7280*	*9.5157*	*2.3445*	*2.2789*	*8.9907*

Table 8 (continued)

Full

In-sample: 01/01/1993 - 12/20/1993

Out-of-sample: 12/21/1993 – 05/07/2014

Denmark

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	**0.1430**	1.5594	**0.3782**	**0.7384**	**2.9358**	**0.7244**	**0.7065**	**2.7930**
CGARCH T-dist	0.1469	*1.5812*	0.3833	0.7482	3.0120	0.7435	0.7259	2.8709
GARCH N-dist	0.1436	1.5629	0.3790	0.7398	2.9465	0.7272	0.7095	2.8052
GARCH T-dist	0.1440	1.5745	0.3794	0.7407	2.9556	0.7291	0.7109	2.8094
SV	*0.1490*	**1.4727**	*0.3860*	*0.7536*	*3.0829*	*0.7576*	*0.7332*	*2.8869*

Finland

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.8822	*3.5364*	0.9392	0.7304	19.4777	4.6253	4.2218	16.2161
CGARCH T-dist	*0.8915*	3.4989	*0.9442*	*0.7342*	*19.7093*	*4.6771*	*4.2643*	*16.3724*
GARCH N-dist	**0.8779**	3.4868	**0.9369**	**0.7286**	**19.4335**	**4.6085**	**4.1961**	**16.0997**
GARCH T-dist	0.8786	3.4743	0.9373	0.7289	19.4490	4.6122	4.1997	16.1139
SV	0.8830	**3.2653**	0.9397	0.7307	19.6400	4.6466	4.2109	16.1182

Norway

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	0.3259	*2.2587*	0.5709	0.7345	6.6557	1.6462	1.6134	6.3925
CGARCH T-dist	**0.3170**	2.2404	**0.5630**	**0.7244**	**6.4827**	**1.6023**	**1.5681**	**6.2090**
GARCH N-dist	0.3213	2.2337	0.5669	0.7294	6.5600	1.6229	1.5911	6.3052
GARCH T-dist	0.3220	2.2448	0.5674	0.7301	6.5733	1.6262	1.5945	6.3188
SV	*0.3348*	**2.1397**	*0.5786*	*0.7445*	*6.9399*	*1.7039*	*1.6454*	*6.4713*

Sweden

Model	MSE[4]	MAE[2]	RMSE[1]	Theil's-U	LINEX1[3]	LINEX2[3]	LINEX3[3]	LINEX4[3]
CGARCH N-dist	*0.3182*	2.5608	*0.5641*	*0.7510*	*6.5142*	*1.6094*	*1.5733*	*6.2252*
CGARCH T-dist	0.3163	*2.5681*	0.5624	0.7488	6.4760	1.5998	1.5636	6.1865
GARCH N-dist	0.3054	2.5409	0.5526	0.7358	6.2567	1.5452	1.5093	5.9695
GARCH T-dist	0.3056	2.5452	0.5528	0.7360	6.2616	1.5463	1.5103	5.9730
SV	**0.3022**	**2.4218**	**0.5498**	**0.7320**	**6.2245**	**1.5332**	**1.4900**	**5.8786**

The table presents the results from five evaluation measures for the "Early", "Mid", "Late" and "Full" sample. Bold numbers indicate best value and italic numbers indicate the worst value.[1] = Values multiplied by 10^3, [2] = Values multiplied by 10^4, [3] = Values multiplied by 10^5, [4] = Values multiplied by 10^6.

A quick look at *Table 8* above shows that the results are mixed. However, the same model generally tends to have the best performance through all measures, with some exceptions for MAE. Moreover, the SV model has the worst forecasting performance more often than the

GARCH-type models. Interestingly however, we find that SV, despite its otherwise poor performance, often has the best forecasts according to MAE. This is true for all samples except for the "Early" one. Also, the normally distributed GARCH-type models tend to perform better overall than the t-distributed counterparts. For instance, in three out of four samples with data from Finland, the normally distributed GARCH(1,1) model appears to have the best forecasting ability. We have seen from the J-B statistics and the QQ-plots that the t-distribution, rather than the standard normal, is expected to better describe the data. However, this does not seem to be the general case here. Moreover, in the "Late" sample, where the out-of-sample covers the turbulence from the recent global crisis, GARCH(1,1) appears to provide the best forecasts. Furthermore, we see that the forecasting errors for Finland tend to be larger than for the other countries, with exception of the "Late" sample.

For further analysis, *Table 9* below present the results from the auxiliary regressions where realized volatility is regressed on a constant and the volatility forecast for each model. The results are mixed in this evaluation as well. Nevertheless, we notice that the models with the highest R^2-value for each country and sample tend to be the same models that achieve the best results in *Table 8*. In fact, this is true for 13 out of 16 scenarios. Moreover, we see that in the "Late" sample, the highest R^2-values are obtained exclusively for the GARCH(1,1) models. In general however, we observe very low R^2-values from the regressions. This finding is similar to the results obtained by Pojarliev and Polasek (2001), and is probably dependent on the noisiness of our proxy for actual volatility. A discussion of this seemingly poor forecasting performance can be found in e.g. Andersen and Bollerslev (1998).

In the "Mid" sample for Norway, the β-coefficient is quite far from one, suggesting that the models exhibit some form of bias. In addition, the SV model is the worst predictor for the actual volatility more often than any other model, which is in line with the results presented in *Table 8*.

Table 9 - Auxiliary regressions results

Early

In-sample: 01/01/1993 – 01/02/1995

Out-of-sample: 01/03/1995 – 01/01/1998

Denmark						**Finland**					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV	Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	α	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
β	0.7442*** (0.1973)	0.7740*** (0.2052)	0.8301*** (0.2208)	0.8255*** (0.2210)	0.8691*** (0.2005)	β	1.0693*** (0.2969)	1.1477*** (0.3144)	1.1593*** (0.3226)	1.2174*** (0.3413)	0.8631*** (0.2562)
R^2	*0.0497*	0.0532	0.0579	0.0580	**0.0778**	R^2	0.1699	0.1727	**0.1837**	0.1714	*0.0194*
Obs.	784	784	784	784	784	Obs.	784	784	784	784	784
Norway						**Sweden**					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV	Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000** (0.0000)	α	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
β	1.1740*** (0.3911)	0.7073** (0.2864)	0.9497** (0.3791)	0.9744** (0.4133)	0.8620** (0.3507)	β	0.8098** (0.3175)	0.8450*** (0.3180)	0.9555*** (0.3618)	1.0298*** (0.3804)	0.8772*** (0.2770)
R^2	**0.0871**	*0.0209*	0.0530	0.0434	0.0512	R^2	0.0811	0.0789	0.0952	**0.0995**	*0.0573*
Obs.	784	784	784	784	784	Obs.	784	784	784	784	784

Table 9 (continued)

Mid

In-sample: 01/01/1993 – 01/03/2000

Out-of-sample: 01/04/2000 – 01/04/2004

Denmark						**Finland**					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV	Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000	0.0000	0.0000	0.0000	0.0000	α	0.0004***	0.0004***	0.0004***	0.0004***	0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)		(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
β	0.8551***	0.9050***	0.8007***	0.7782***	1.1819***	β	0.3719***	0.3769***	0.4079***	0.3845***	0.6480***
	(0.1906)	(0.2010)	(0.1871)	(0.1847)	(0.2890)		(0.1159)	(0.1161)	(0.1240)	(0.1243)	(0.1645)
R^2	0.0770	**0.0811**	0.0705	0.0756	*0.0687*	R^2	0.0102	0.0105	0.0108	*0.0101*	**0.0134**
Obs.	1043	1043	1043	1043	1043	Obs.	1043	1043	1043	1043	1043

Norway						**Sweden**					
Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV	Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000	0.0000	0.0000	0.0000*	0.0000*	α	0.0001***	0.0001***	0.0001***	0.0001***	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.9071***	0.8132***	0.8235***	0.7836***	0.7765***	β	1.6363***	1.5978***	1.4181***	1.4367***	1.1698***
	(0.1159)	(0.1161)	(0.1240)	(0.1243)	(0.1645)		(0.1682)	(0.1810)	(0.1549)	(0.1636)	(0.1465)
R^2	**0.0825**	0.0782	0.0709	0.0736	*0.0273*	R^2	**0.3054**	0.2945	0.2442	0.2231	*0.0924*
Obs.	1043	1043	1043	1043	1043	Obs.	1043	1043	1043	1043	1043

Table 9 (continued)

Late
In-sample: 01/01/2001 – 12/30/2005
Out-of-sample: 01/02/2006 – 01/03/2011

Denmark

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.9510***	0.9566***	0.9641***	0.9313***	1.5260***
	(0.1240)	(0.1183)	(0.1147)	(0.1119)	(0.3062)
R^2	0.2148	0.2211	**0.2240**	0.2237	*0.1685*
Obs.	1306	1306	1306	1306	1306

Finland

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000**	0.0000**	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.8562***	0.8378***	0.9045***	0.8863***	1.1280***
	(0.1112)	(0.1016)	(0.1118)	(0.1147)	(0.1680)
R^2	0.1091	0.1159	**0.1247**	0.1243	*0.0967*
Obs.	1306	1306	1306	1306	1306

Norway

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000**	0.0000**	0.0000*	0.0000*	0.0000*
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.9215***	0.9112***	0.9517***	0.9376***	1.5362***
	(0.0952)	(0.0968)	(0.0967)	(0.0959)	(0.2116)
R^2	0.2541	0.2537	0.2585	**0.2598**	*0.2286*
Obs.	1306	1306	1306	1306	1306

Sweden

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000*	0.0000**	0.0000*	0.0000*	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.8841***	0.8337***	0.8972***	0.8548***	1.3974***
	(0.1089)	(0.0986)	(0.1086)	(0.1037)	(0.2204)
R^2	0.1547	0.1525	**0.1587**	0.1579	*0.1409*
Obs.	1306	1306	1306	1306	1306

Table 9 (continued)

Full

In-sample: 01/01/1993 - 12/20/1993

Out-of-sample: 12/21/1993 – 05/07/2014

Denmark

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000***	0.0000***	0.0000***	0.0000***	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.7684***	0.7325***	0.7767***	0.7863***	1.4374***
	(0.0602)	(0.0636)	(0.0643)	(0.0747)	(0.2557)
R^2	**0.1892**	0.1728	0.1839	0.1802	*0.1550*
Obs.	5317	5317	5317	5317	5317

Finland

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000***	0.0000***	0.0000***	0.0000***	0.0000**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.7282***	0.7040***	0.7650***	0.7608***	1.0049***
	(0.0548)	(0.0489)	(0.0572)	(0.0472)	(0.0841)
R^2	0.0885	0.0804	**0.0889**	0.0885	*0.0773*
Obs.	5317	5317	5317	5317	5317

Norway

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000***	0.0000***	0.0000***	0.0000**	-0.0000*
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.8007***	0.8401***	0.8244***	0.8204***	1.4407***
	(0.0845)	(0.0767)	(0.0753)	(0.0752)	(0.1932)
R^2	0.2453	**0.2606**	0.2523	0.2514	*0.2329*
Obs.	5317	5317	5317	5317	5317

Sweden

Model	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	SV
α	0.0000***	0.0000***	0.0000***	0.0000***	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β	0.6472***	0.6635***	0.7495***	0.7516***	1.1442***
	(0.0795)	(0.0798)	(0.0578)	(0.0578)	(0.1082)
R^2	*0.1029*	0.1050	**0.1236**	0.1225	0.1226
Obs.	5317	5317	5317	5317	5317

*Table 9 presents the results from auxiliary regression, where squared daily returns are regressed on a constant and conditional variance for the "Early", "Mid", "Late" and "Full" sample. Newey-West robust standard errors within parenthesis. *, ** and *** denotes statistical significance at the 10%, 5% and 1% level, respectively.*

7. Analysis and Discussion

The first area of focus has been to evaluate which conditional volatility models that are able to capture effects of persistent volatility shocks. This has also served the purpose of examining to which extent macroeconomic variables have explanatory power on the long-run conditional equity volatility.

After performing appropriate transformations of non-stationary series and running OLS regression analyzes, we find that the results are mixed. Despite having the expected sign in most cases, the majority of the coefficients of the macroeconomic variables are insignificant. This differs considerably from e.g. Cho and Elshahat (2014) and Engle and Rangel (2008), whose findings show that several macroeconomic variables included in this study have explanatory power on the conditional long-run volatility of equity returns. They do, however, use slightly different model specifications (particularly Engel and Rangel (2008), see *Section 2*), which may partly explain the diverse outcomes. Also, since the Nordic countries are small, rather homogenous and very dependent on the economic state on a global level, it may be the case that the long-run volatility of these equity markets are explained by domestic macroeconomic factors to a lesser extent than e.g. the U.S. counterpart.

Moreover, Cho and Elshahat (2014) use annual data ranging from 1960 to 2009. In the analysis made here, using annual data could possibly separate the models further from each other, since we would generally expect larger movements of variables at annual frequencies than on quarterly basis. Longer horizons/frequencies could thus, in turn, be beneficial for the long-run component of the CGARCH model, making it more distinctly different from the other models. In addition, it is possible that the macro variables in the period before the 1990's or 2000's explain a large part of their conditional volatility, which may also explain the differences between our results. However, given the scope, time frame and data availability, experiments with different frequencies or time periods have not been conducted here.

Nevertheless, the hypothesis was that the long-run component of the CGARCH model would be superior for estimating macro-related persistent volatility shocks, but the differences between the models have proven to be small. Furthermore, in some cases, the long-run component of the CGARCH has shown to be inseparable from the total volatility (because α and β could not be statistically separated from 0, see *Equation 12*), which has reduced the model to a special form of GARCH(1,1). Similar findings are discussed in Cho and Elshahat (2011). Only with data from Denmark do we see some indication that the long-run component may outperform the other models' estimation ability.

The SV model appears to be the least able of taking long-run volatility into account. Yet again, the differences between the models are not overwhelming. Taken together, the CGARCH is generally not better than the relatively simple GARCH(1,1) for modelling persistent macroeconomic volatility effects on equity returns on quarterly basis, despite its two-component structure.

In the second part, the short-run forecasting ability has been evaluated by estimating the one-day ahead total conditional variance. Here, the long-run component is not separated from the total volatility of the CGARCH model for maximal comparability. Several sample periods based on the general volatility patterns that can be distinguished from the equity return series have been chosen for evaluation. First, the performance has been evaluated by so-called loss functions. Similar to the results from the long-run volatility regressions, the results from this analysis are mixed as well. The normally distributed GARCH(1,1) model have the best forecasting ability for Finland in most cases. Also, in the "Late" sample period, the GARCH(1,1) appears to generally have the best performance. The forecasts for this sample cover the recent global subprime crisis, which suggests that the GARCH(1,1) is more able than the other models to accurately describe future short-term volatility in periods of very high turbulence. However, no general pattern can be found for the rest of the countries or samples. Over all scenarios, the SV model has the highest number of worst performances.

In the auxiliary regression analysis of the forecasting performance, the results are similar to those obtained from the loss functions. Again, the SV model generally achieves the worst results. Moreover, we notice that the R^2-values are generally low or very low, and the models for Finland in the "Mid" sample appear to suffer from some form of bias. This bias and low R^2-values are likely dependent on the proxy for true volatility (i.e. the dependent variable in each regression).

What is interesting however, is that the models obtaining the highest R^2-value for each country and sample period, tend to be the same models that have the best performance according to the loss functions. This suggests that the results are not completely random, although it is difficult to come to any general conclusions regarding the superiority of any particular model for any particular volatility scenario. The exception is GARCH(1,1) in the "Late" sample, which, according to both the loss functions and the auxiliary regressions, achieves the best result in this period of highly turbulent and volatile markets. In addition, the GARCH(1,1) generally provides the best forecasts for Finland, which appears to have the most volatile equity market throughout the sample (see *Table 4* and *Figure 1*). One could thus come to the conclusion that the GARCH(1,1) is relatively more accurate than the other models in periods of financial turmoil. However, this conclusion is not supported by the results from the "Mid" sample, which covers the highly volatile tech bubble in the early 2000's, and for which it cannot be shown that the GARCH(1,1) provides the best forecasts.

Furthermore, normally distributed rather than t-distributed GARCH-type models have generally the best forecasting performance, which is rather surprising given the results from the J-B statistics and QQ-plots (see *Section 6.1*). A possible explanation is that the subsamples have less fat-tailed distributions than the full sample. This has, however, not been investigated.

Nevertheless, from the forecast evaluation we may conclude that the differences between the GARCH(1,1) and the CGARCH models generally are small or negligible. Furthermore, the SV model appears to be relatively inaccurate and unreliable, and behaves slightly different from the GARCH-type models. This result regarding the SV model is in line with the findings of Dunis et. al (2001), but contradicts the conclusions from e.g. Hansson and Hördahl (2005) and Yu (2002), whose studies show that SV-type models are preferred to GARCH counterparts. Also, according to Adrian and Rosenberg (2008), two-component models are often found to outperform one-component specifications for explaining equity market volatility. However, this is not the case here.

8. Conclusions and Suggestions for Further Research

This study has evaluated three different models (GARCH(1,1), CGARCH and SV) on their ability to describe and forecast conditional variance of the main Nordic equity indices. The first evaluation had the purpose of examining how well the above mentioned models capture effects of long-run persistent volatility shocks from macroeconomic variables, and to see to which extent these macro variables can explain stock market fluctuations. The initial hypothesis was that the long-run component of the CGARCH model would provide the best estimates, but the results have shown to be mixed. We conclude that none of the models are superior for modeling persistent volatility. However, we see tendencies suggesting that the SV model is the least appropriate for this purpose.

The second evaluation is based upon short-run forecasting ability. This is examined by measuring errors from a rolling window one-day ahead conditional variance forecast, and by an auxiliary regression analysis. Despite mixed results, we notice that GARCH-type models following a normal distribution tend to obtain the best ranking according to the various measures. This is surprising, since the distribution of the (complete) return series has shown to be far from normal. Furthermore, we have seen that the SV model have the highest number of worst forecasting performances over all time periods and countries. The general conclusion is that the SV model is the least able to accurately describe both long- and short-run conditional volatility. We also conclude that the GARCH(1,1) model is not inferior compared to the more advanced CGARCH, and may even be superior in periods of high volatility. Thus, employing the simpler GARCH(1,1) when estimating and forecasting conditional volatility on the Nordic equity markets is probably the better choice.

For future research, it would be interesting to investigate whether at GED distribution or similar would be better at describing the data used in this study. Furthermore, incorporating the macroeconomic variables into the actual models could provide a deeper understanding and more interpretable results than those obtained here. Allowing for asymmetry in the models might also improve the general performance.

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Appendix

Figure 3 shows the volatility plots of total volatility (CGARCH) and the long-run component (LCGARCH) in the CGARCH model.

Figure 3

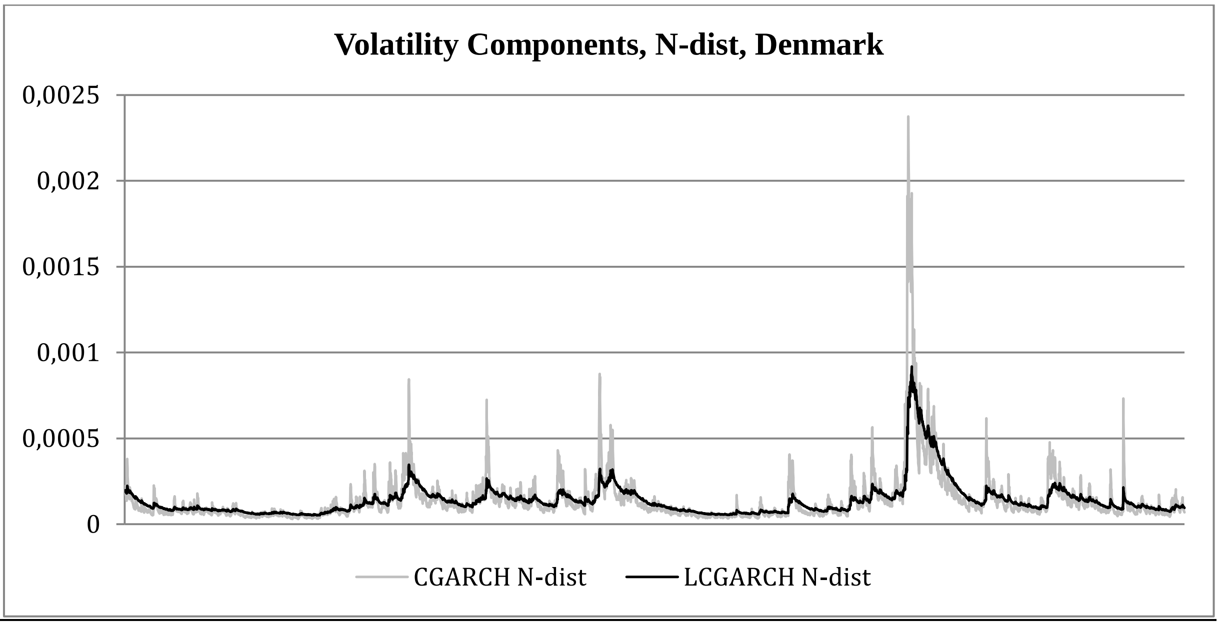

Figure 3 (continued)

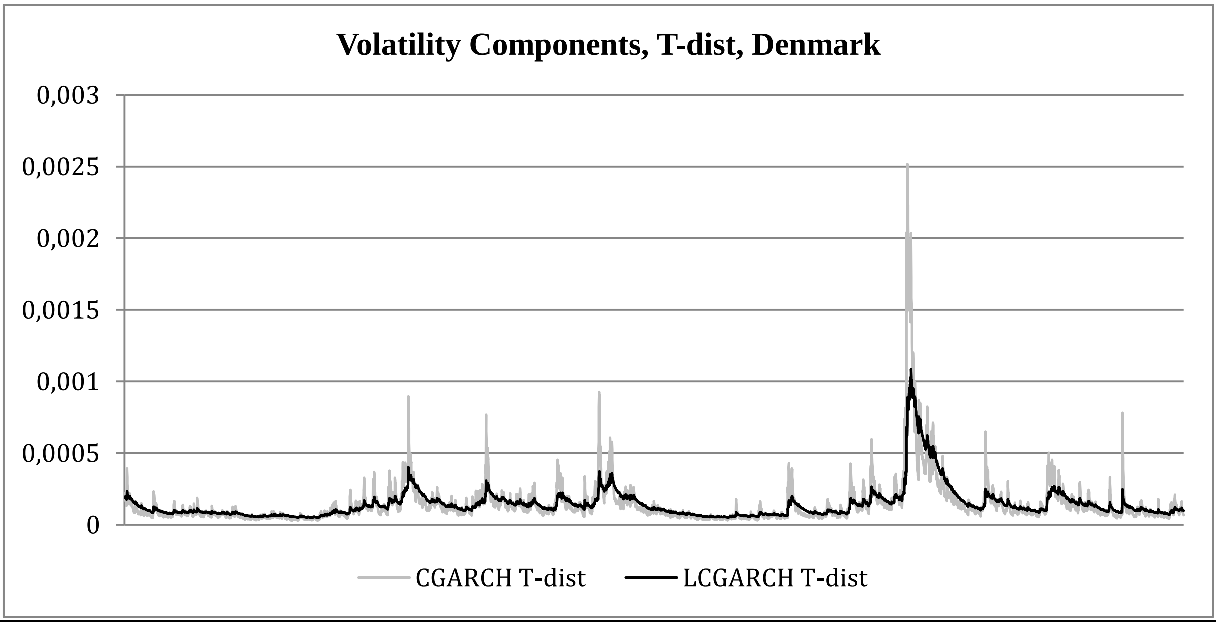

Figure 3 (continued)

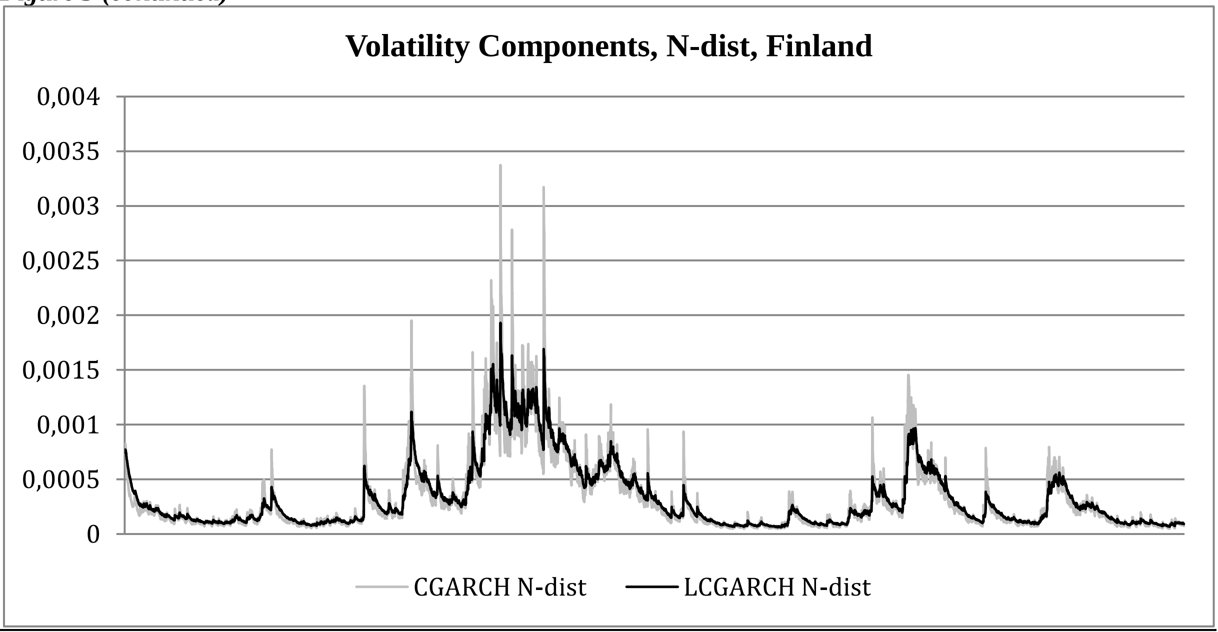

Figure 3 (continued)

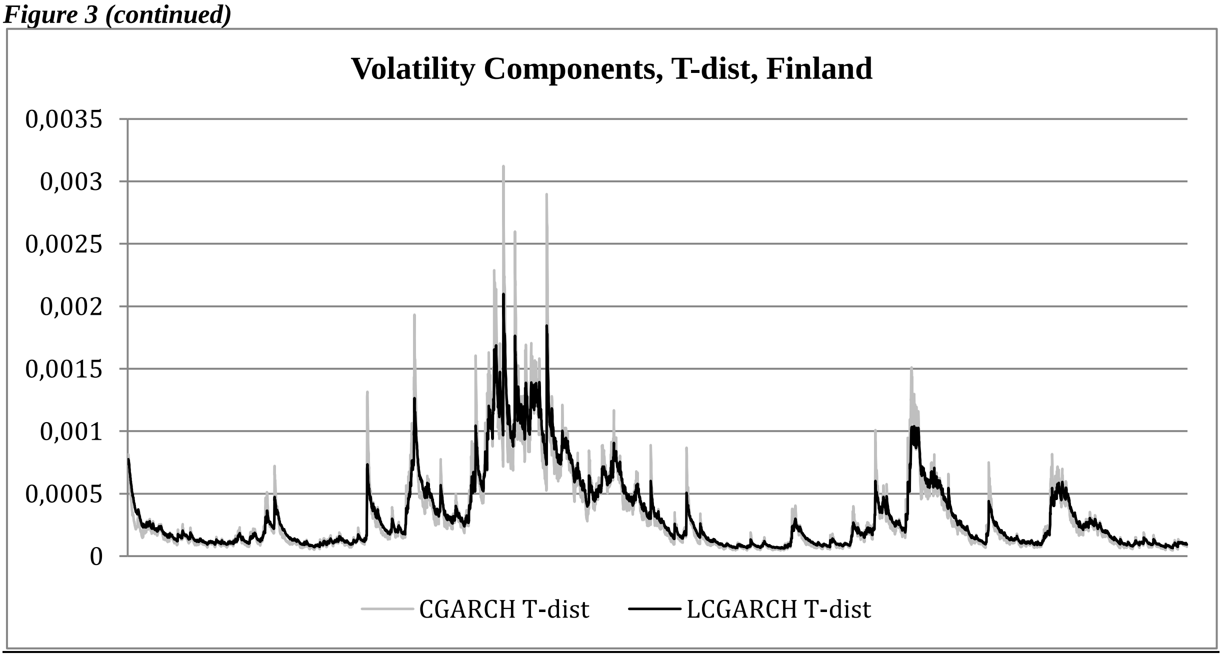

Figure 3 (continued)

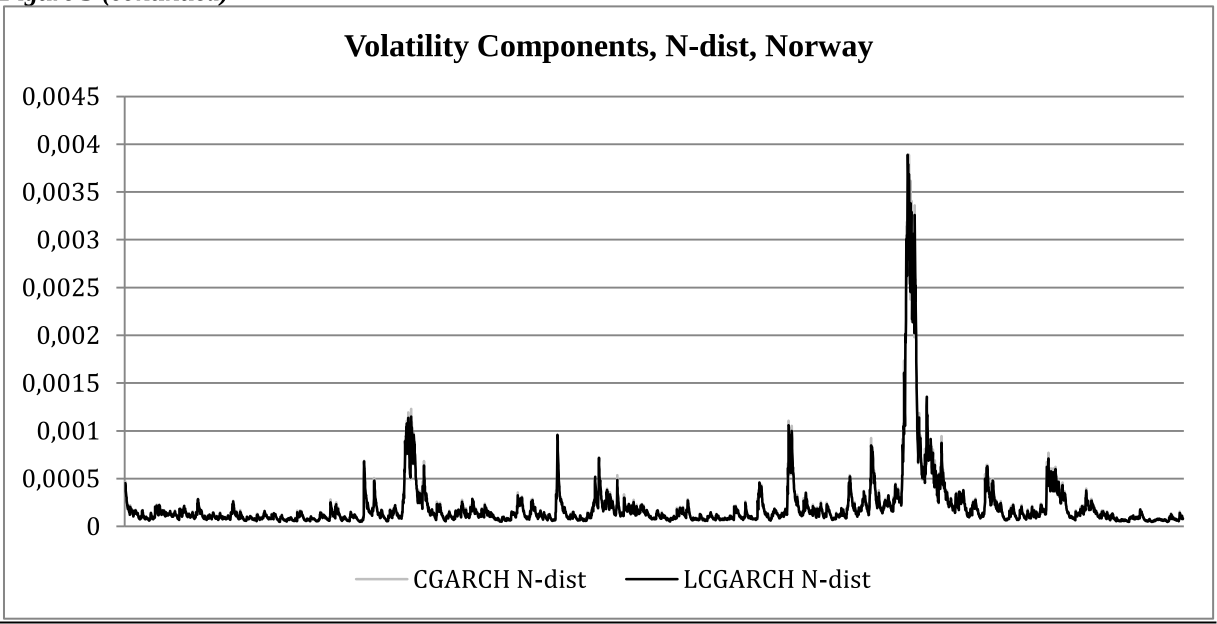

Figure 3 (continued)

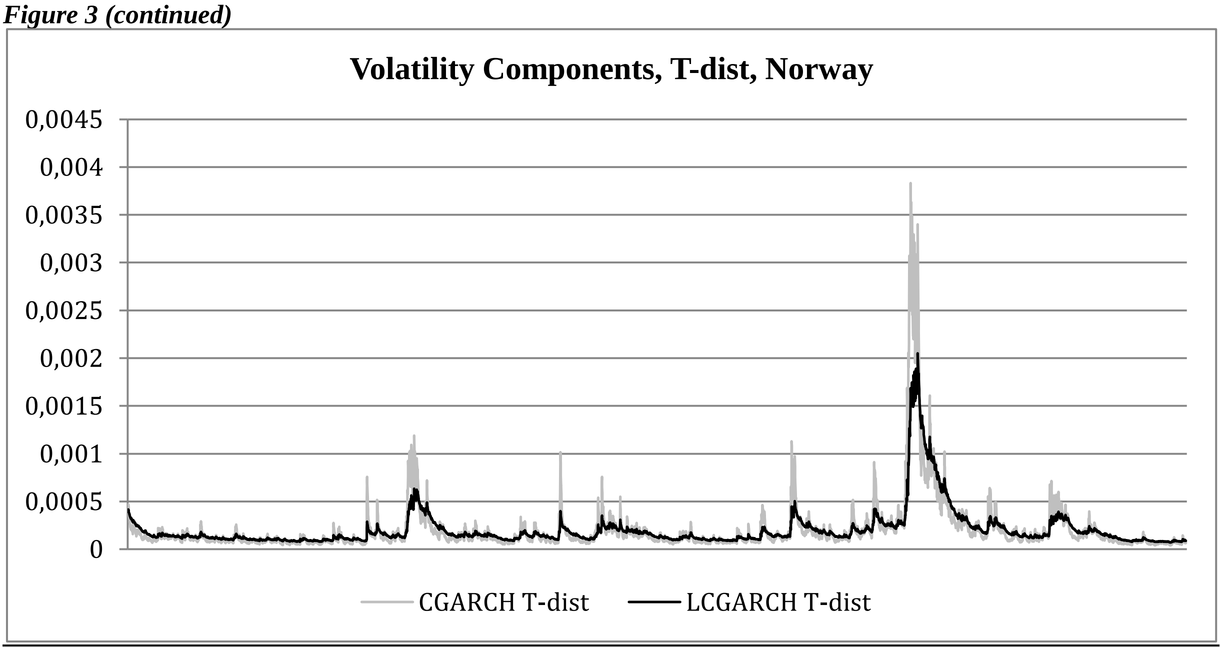

Figure 3 (continued)

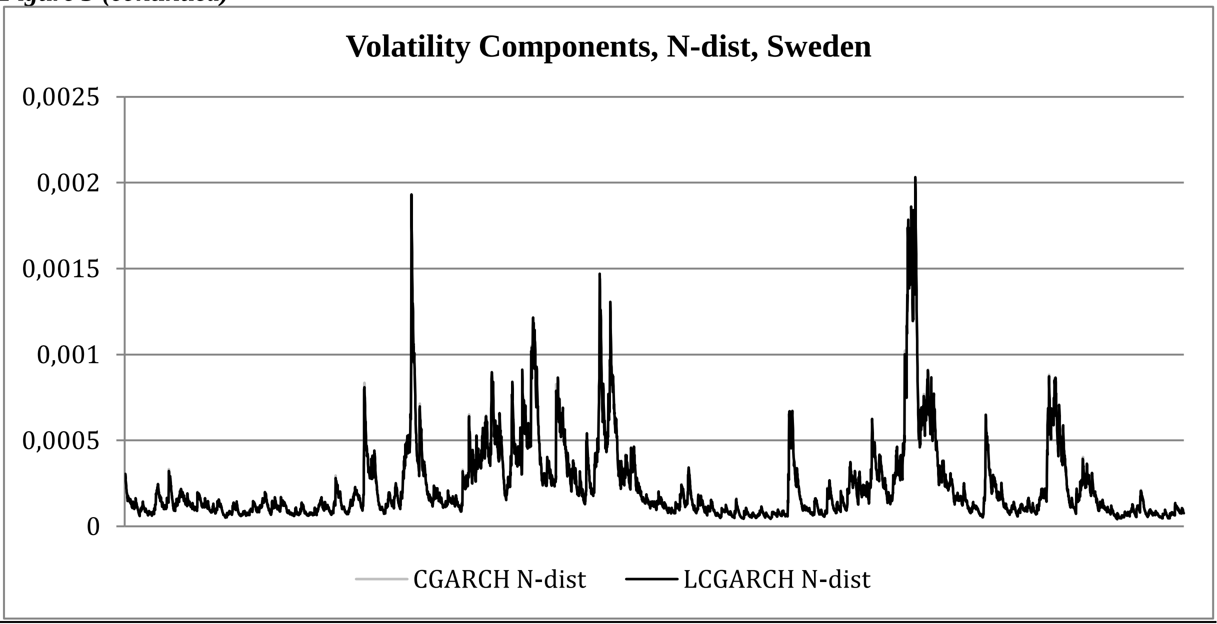

Figure 3 (continued)

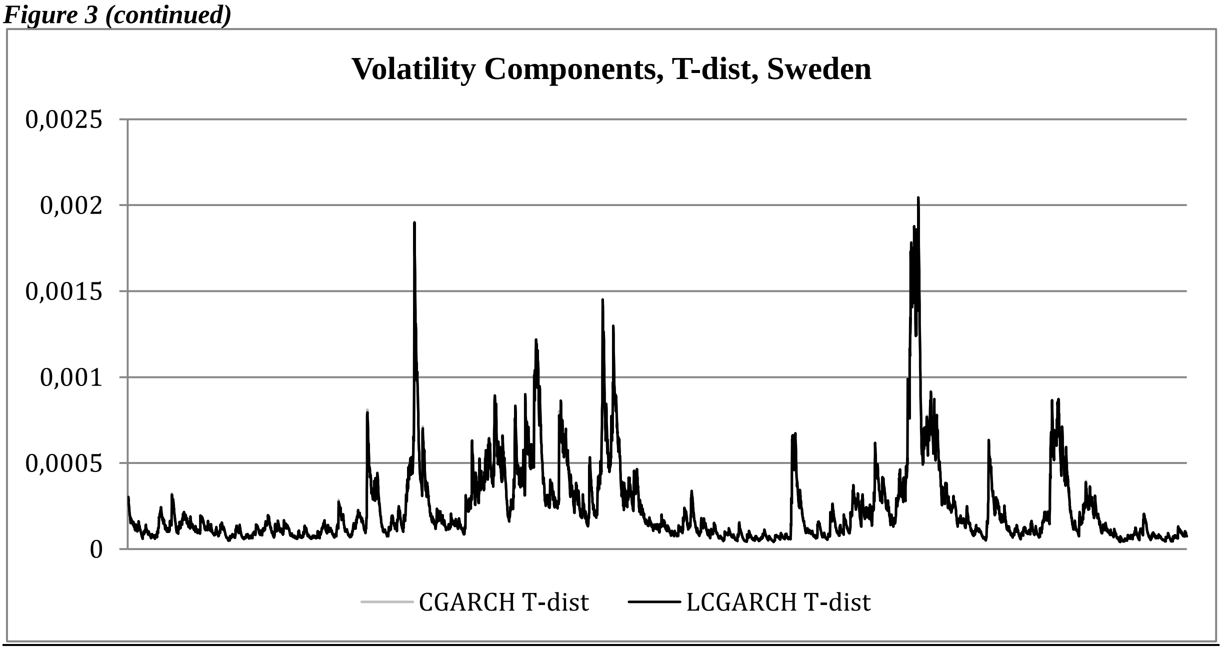

Table 10 presents the results from the additional regression analysis of the long-run volatility. Lags of variables previously regressed in levels have been added.

Table 10 - Regression results,Additional lags

			Denmark				
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables				Coefficient			
C	0.0001	0.0001	0.0001*	0.0001	0.0001**	0.0001*	0.0001**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GR. CPI $^{\Delta}{}_{t-1}$	-0.0046	-0.0049	-0.0056	-0.0059	-0.0026	-0.0032	-0.0030
	(0.0039)	(0.0041)	(0.0045)	(0.0048)	(0.0022)	(0.0026)	(0.0026)
GR. GDP_{t-1}	-0.0037**	-0.0038**	-0.0035**	-0.0037**	-0.0028**	-0.0032**	-0.0021*
	(0.0017)	(0.0018)	(0.0017)	(0.0018)	(0.0014)	(0.0016)	(0.0012)
GR. M2	0.0013	0.0014	0.0016	0.0018	0.0004	0.0005	0.0007
	(0.0014)	(0.0015)	(0.0016)	(0.0017)	(0.0008)	(0.0009)	(0.001)
GR. $M2_{t-1}$	-0.0003	-0.0003	-0.0005	-0.0005	0.0001	0.0001	-0.0002
	(0.0007)	(0.0007)	(0.0008)	(0.0008)	(0.0004)	(0.0005)	(0.0005)
VOL. RF	0.0001**	0.0001**	0.0001*	0.0001*	0.0001**	0.0001**	0.0001**
	(0.0001)	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
VOL. RF_{t-2}	0.0000	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
VOL. CPI $^{\Delta}{}_{t-1}$	-0.0025	-0.0026	-0.0030	-0.0032	-0.0014	-0.0017	-0.0028
	(0.0041)	(0.0043)	(0.0047)	(0.0050)	(0.0025)	(0.0029)	(0.0033)
VOL. GDP $^{\Delta}{}_{t-1}$	0.0032	0.0034	0.0035	0.0037	0.0021	0.0024	0.0021
	(0.0021)	(0.0022)	(0.0023)	(0.0024)	(0.0014)	(0.0017)	(0.0014)
$LNGDP^{d}{}_{t-1}$	0.0015	0.0016	0.0020	0.0021	0.0003	0.0006	0.0013
	(0.0013)	(0.0013)	(0.0014)	(0.0015)	(0.0007)	(0.0008)	(0.0011)
R^2	0.3349	0.3342	0.3094	0.3091	0.4001	0.3890	0.2924
Obs.	80	80	80	80	80	80	80

Table 10 (continued)

			Finland				
Dependent variable	CGARCH $^{\Delta}$ N-dist	CGARCH $^{\Delta}$ T-dist	GARCH $^{\Delta}$ N-dist	GARCH $^{\Delta}$ T-dist	LCGARCH $^{\Delta}$ N-dist	LCGARCH $^{\Delta}$ T-dist	SV $^{\Delta}$
Variables				Coefficient			
C	-0.0001	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
GR. CPI_{t-1}	0.0053	0.0057	0.0061	0.0064	0.0037	0.0042	-0.0004
	(0.0047)	(0.0040)	(0.0044)	(0.0045)	(0.0038)	(0.0038)	(0.0044)
GR. GDP_{t-1}	0.0026	0.0027	0.0026	0.0027	0.0022	0.0024	0.0019
	(0.0017)	(0.0027)	(0.0029)	(0.0030)	(0.0020)	(0.0022)	(0.0025)
GR. M2 $^{\Delta}$	0.0018**	0.0020**	0.0022**	0.0023**	0.0012**	0.0014**	0.0009
	(0.0007)	(0.0008)	(0.0009)	(0.0009)	(0.0006)	(0.0006)	(0.0008)
GR. $M2^{1}_{t-1}$	0.0009	0.0010	0.0010	0.0011	0.0007	0.0007	0.0003
	(0.0007)	(0.0008)	(0.0009)	(0.0009)	(0.0006)	(0.0007)	(0.0008)
VOL. RF	0.0000	0.0000	-0.0001	-0.0001	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-2}	0.0000	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. CPI^{Δ}_{t-1}	-0.0073	-0.0077	-0.0074	-0.0079	-0.0036	-0.0044	-0.0009
	(0.0050)	(0.0076)	(0.0085)	(0.0088)	(0.0051)	(0.0057)	(0.0090)
VOL. GDP^{Δ}_{t-1}	0.0022	0.0024	0.0025	0.0027	0.0013	0.0015	0.0025
	(0.0016)	(0.0018)	(0.0019)	(0.0020)	(0.0011)	(0.0012)	(0.002)
$LNGDP^{d}_{t-1}$	0.0011	0.0011	0.0011	0.0011	0.0014	0.0013	0.0016*
	(0.0011)	(0.0013)	(0.0015)	(0.0015)	(0.0011)	(0.0011)	(0.0009)
R^2	0.2458	0.2433	0.2264	0.2318	0.2094	0.2149	0.1120
Obs.	80	80	80	80	80	80	80

Table 10 (continued)

	Norway						
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables	Coefficient						
C	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001**	0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
GR. CPI_{t-1}	0.0141	0.0135	0.0140	0.0145	0.0141	0.0083	0.0082
	(0.0116)	(0.0112)	(0.0116)	(0.012)	(0.0116)	(0.0073)	(0.0073)
GR. GDP_{t-1}	-0.0043	-0.0045	-0.0043	-0.0044	-0.0043	-0.0035*	-0.0036
	(0.0030)	(0.0030)	(0.0030)	(0.0031)	(0.0030)	(0.0021)	(0.0022)
GR. $M2^{\Delta}$	0.0015	0.0016	0.0015	0.0015	0.0015	0.0013	0.0011
	(0.0018)	(0.0018)	(0.0018)	(0.0019)	(0.0018)	(0.0013)	(0.0012)
GR. $M2^{\Delta}{}_{t-1}$	0.0000	0.0001	0.0000	0.0000	0.0000	0.0004	0.0002
	(0.0009)	(0.0010)	(0.0009)	(0.0010)	(0.0009)	(0.0008)	(0.0008)
VOL. RF	0.0001**	0.0001**	0.0001**	0.0001**	0.0001**	0.0001	0.0001*
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. RF_{t-2}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
VOL. CPI_{t-1}	0.0142	0.0131	0.0141	0.0146	0.0142	0.0075	0.0099
	(0.0172)	(0.0165)	(0.0171)	(0.0177)	(0.0172)	(0.0104)	(0.0108)
VOL. GDP_{t-1}	-0.0049	-0.0055	-0.0049	-0.0051	-0.0049	-0.0048	-0.0037
	(0.0040)	(0.0043)	(0.0040)	(0.0042)	(0.0040)	(0.0036)	(0.0033)
$LNGDP^{d}{}_{t-1}$	0.0040	0.0038	0.0040	0.0041	0.0040	0.0022	0.0033
	(0.0036)	(0.0036)	(0.0036)	(0.0037)	(0.0036)	(0.0026)	(0.0027)
R^2	0.2217	0.2345	0.2214	0.2216	0.2216	0.2669	0.2743
Obs.	80	80	80	80	80	80	80

Table 10 (continued)

Sweden							
Dependent variable	CGARCH N-dist	CGARCH T-dist	GARCH N-dist	GARCH T-dist	LCGARCH N-dist	LCGARCH T-dist	SV
Variables	Coefficient						
C	0.0002***	0.0002***	0.0002***	0.0002***	0.0002***	0.0002***	0.0002***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
GR. CPI_{t-1}	0.0010	0.0010	0.0010	0.0010	0.0011	0.0010	0.0013
	(0.0039)	(0.0040)	(0.0039)	(0.0040)	(0.0039)	(0.0040)	(0.0031)
GR. GDP_{t-1}	-0.0043	-0.0044	-0.0043	-0.0044	-0.0043	-0.0044	-0.0039
	(0.0030)	(0.0031)	(0.0030)	(0.0031)	(0.0030)	(0.0031)	(0.0025)
GR. M2	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0008
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0008)
GR. $M2_{t-1}$	-0.0013	-0.0013	-0.0013	-0.0013	-0.0013	-0.0013	-0.0010
	(0.0013)	(0.0013)	(0.0013)	(0.0013)	(0.0013)	(0.0013)	(0.0010)
VOL. RF	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)
VOL. RF_{t-1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0001)
VOL. RF_{t-2}	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
VOL. CPI_{t-1}	0.0116***	0.0118***	0.0116***	0.0117***	0.0117***	0.0118***	0.0088***
	(0.0041)	(0.0042)	(0.0041)	(0.0042)	(0.0041)	(0.0042)	(0.0031)
VOL. GDP^{Δ}_{t-1}	-0.0008	-0.0008	-0.0008	-0.0007	-0.0008	-0.0008	-0.0012
	(0.0022)	(0.0023)	(0.0022)	(0.0023)	(0.0022)	(0.0023)	(0.0014)
$LNGDP^{d}_{t-1}$	0.0015	0.0016	0.0015	0.0016	0.0015	0.0016	0.0013
	(0.0015)	(0.0015)	(0.0015)	(0.0015)	(0.0015)	(0.0015)	(0.0012)
R^2	0.1641	0.1644	0.1648	0.1653	0.1645	0.1648	0.1493
Obs.	80	80	80	80	80	80	80

*Table 10 presents the regression analysis results with estimated volatility series as dependent variables for Denmark, Finland, Norway and Sweden. Newey-West robust standard errors within parenthesis. *, ** and *** indicates significance level at 10%, 5% and 1% respectively. "GR." denotes "Growth rate of", "VOL." denotes "Volatility of" and "RF" denotes short-term (risk-free) interest rate.*
$^{\Delta}$ denotes that the variables are regressed in first difference due to non-stationarity.
^{d} denotes that the variables are detrended due to non-stationarity.