Dark Matter Subhalos Gravitational Dynamics and Evolution

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Abstract

In this thesis, I present a case where Dark Matter subhalos would annihilate at some stage in their lifetime. The implication of this scenario could potentially aid the Missing Satellites and the Cusp-Core density profile problem. The Dark Matter particles' annihilation process and effective "friction" lead to an energy and momentum loss into radiation and hence to the destabilization of the Dark Matter subhalo with subsequent collapse and total annihilation. Expected possible observational signature of this effect is a multi-GeV gamma-ray burst, which is potentially observable by high-energy gamma-ray detectors such as FERMI etc. The basic purpose here is to develop a simple analytic model for the collapse dynamics of such a subhalo. I present results that show how the particle departure rate of a given subhalo does not depend on initial parameters such as initial contraction velocity, the initial radius and the initial mass of a given subhalo. Only does it depend on fundamental constants and Dark Matter model parameters, namely its mass and couplings to ordinary matter particles in the Standard Model. This remarkable result potentially allows one to distinguish a whole class of catastrophic astrophysical events with universal characteristics independent on the formation history of such objects. These characteristics could then be used as constraints for the above-mentioned fundamental constants for a given Dark Matter model.

Populärvetenskaplplig beskrivning

Vardagliga ting, såsom detta papper, är sammanställt av vanlig materia. Det vi ser med våra ögon och genom teleskop när vi tittar upp mot stjärnorna är också sammanställt utav vanlig materia. Det visar sig att denna materia endast fyller vårt kända universum till 5 %. Resten kallas för Mörk Materia och Mörk Energi, där den föregående fyller universum med 27 % och den snarare till 68 %. Mörk Materia är postulerad till att existera dåden vanliga materians massa i galaxer inte räcker för att förklara galaxers rotationskurvor. För att ackommodera galaxers observerade rotationshastighet pytsas massa från en okänd källa in, s.k. Mörk Materia.

Nyligen har väldigt hög-energiska fotoner blivit detekterade från jordens rotationsbana av the Large Area Telescope (LAT) ombord Fermi satelliten. Detta stärker fallet för existensen av Mörk Materia dådet finns teorier om hur dessa fotoner kan komma till existens från kollisioner av Mörk Materia partiklar. Dessa fotoner skulle ha omkring 10,000 gånger energi relativt till någon annan kosmologisk händelse, såsom supernovor etc. Dessa höga energier kan troligtvis inte förklaras genom dagens ledande teori om vanlig materia, den s.k. Standard Model av partikelfysik och därfr borde modeller utanför denna modell studeras.

Idag tror de flesta astrofysiker att galaxer i universum är inbäddade i en svärm av sub halos som tillsammans gör ett halo av Mörk Materia. I det tidiga universumet förutsäger man att mörk och vanlig materia interagerade väldigt ofta. Vid ett senare tillfälle, d rum-tiden i universum expanderade, blev det för kallt för dem att interagera, den mörka materian frös där av ut. Dagens indirekt observerade mörka materia kan därför betraktas som relik från det förflutna. Något som de flesta astrofysiker inte tror och vad som är presenterat i denna uppsats är scenariot där sub halos har förmgan att, vid något tillfälle i sitt annars statiska liv, totalt kollapsa och stråla ut all dess materia påen väldigt kort tid (någon minut eller tvåfrån mina resultat). Det r från dessa utstrålningar man skulle kunna se de hög-energiska fotonerna.

Simulationer av universums och där av galaxers evolution visar en större mängd Mörk Materia än vi faktiskt har i universum. Ett sub halo som vid något tillfälle försvinner i form av strålningsprodukter skulle kunna förklara varför vi inte observerar fler sub halos.

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Chapter 1

Introduction

For some years now, the origin for some multi-GeV gamma-ray bursts remain unknown. As there is strong evidence supporting the existence of Dark Matter (DM) Bertone et al. (2005), models of self-annihilating DM structures (subhalos) could explain the source of these gamma-rays, Pasechnik et al. (2006), Berlin et al. (2014), Daylan et al. (2014), Belotsky et al. (2013) and Berezinsky et al. (2003).

Recently, an article by NASA covered this very topic, Reddy (2014). It is an important topic, as the particle species of DM is currently unknown and N-body simulations, which try to replicate the evolution of the universe from its early stages to today through computer simulations, rely on those DM parameter details. A DM subhalo that inevitably annihilates is an attractive scenario as it could explain two current problems of sophisticated N-body simulations namely, the Missing Satellite and Cusp-Core problem. They are discussed in sections 1.2 and 1.3.

What follows in Section 1.1 is a brief introduction to some ideas about DM. Chapter 2 explains the model for the dynamics of such a collapsing DM subhalo. It is derived from non-relativistic gravitating gas dynamics and in order to end up with analytical solutions, adiabatic approximations are used. If a reader is interested enough, those approximations can be developed further.

1.1 Dark Matter

Currently, the model that so far best describes the evolution of the universe is called the Λ Cold Dark Matter model (ΛCDM). It is sometimes called the Standard Model of Big Bang cosmology. Λ refers to the cosmological constant which is seen in Albert Einstein's field equation, Einstein (1916), and is associated with the existence of Dark

Energy. It was placed in his field equation because there needed to be something to "hold back gravity". It is the value for the energy density of the vacuum of space which accounts for the accelerating expansion of the universe, as discovered by Hubble (1929) who measured the distance of galaxies and their associated velocity at which they apparently move away from us i.e. their red shifts (as an object moves away from us, the observed wavelength of light is elongated i.e. Doppler shifted and appears more red in the gamma-ray spectrum). It turned out that the relationship between distance and velocity for cosmological objects is linear $v = H_0 d$ which shows that our universe has a cosmological constant (constant in space not in time) H_0. Riess et al. (1998) showed that not only is there a spatially constant relation between distance and velocity for cosmological objects but showed that in time, those objects move away from us at an accelerated pace. A radial factor of a receding object has a positive value of acceleration.

The term cold refers to the thermal property of the Dark Matter and in this sense, whatever the particle species of the CDM, it is non-relativistic (slow moving).

A way of probing cosmological parameters in the universe, such as its age and energy-density distribution, is through looking at anisotropies (fluctuations in temperature) in the first formed atoms in the universe. In the early universe, photons had a very short mean free path because spatially, it was so dense that the they continually scattered at a near vicinity. As the region was dense, it was hot too. When the universe spatially expanded, the particles could, in a sense, cool down and form atoms. The first photons from that era make up a picture of the early universe, it is called the Cosmic Microwave Background (CMB). The CMB contains the first radiation from the early universe. Measuring the frequencies of that radiations tells us about the matter distribution in the current universe, along with other parameters as well, some are listed in the Table 1.1.

Parameters	Numerical value
Dark Energy	$\Omega_\Lambda = 0.6825$
Dark Matter	$\Omega_d = 0.2671$
Baryonic Matter	$\Omega_b = 0.0490$
Universe's age	13.798 ± 0.037 Gy
Hubble constant	$H_0 = 67.80 \pm 0.77\,\mathrm{km\,Mpc^{-1}s^{-1}}$

Table 1.1: Parameters from the 15.5 month Planck mission Planck Collaboration et al. (2013).

1.1.1 The Neutralino as a Dark Matter particle candidate

A popular candidate for a DM particle is the supposed neutralino (χ), which is a heavy and stable particle predicted by supersymmetry (SUSY). In fact, SUSY partners are much heavier than all known particles in the Standard Model (SM). "Supersymmetry" means that for each fermionic and bosonic particle in the SM, there exists bosonic and fermionic counterparts. In other words, SUSY partners have exactly the same quantum numbers as the SM particles except for spin and mass, bosons are flipped into fermions and fermions are flipped into bosons. A specific mixture of SUSY partners of gauge bosons ("gauginos") and Higgs boson ("Higgsinios") is called neutralino. An asymmetric DM model i.e. where a DM particle does not have an anti-particle, would not fit a completely annihilating subhalo model.

The neutralino fits the criteria of being a Weakly Interactive Massive Particle (WIMP) which is important for a CDM scenario as a cold and thus slow-moving particle would suggestively be heavy and weakly interacting. A WIMP is also a natural candidate as its resulting mass density in the present Universe is of the order of the critical density ρ_c in the early Universe. The critical density is the density needed for space to remain flat, which is today's observed case. The WIMPs interact only via the weak force and gravitation. The weak W -boson coupling $\alpha_W = 1/30$ in the Standard Model is assumed here. The mass range of a WIMP particle could be as indicated in Fig. 1.1.

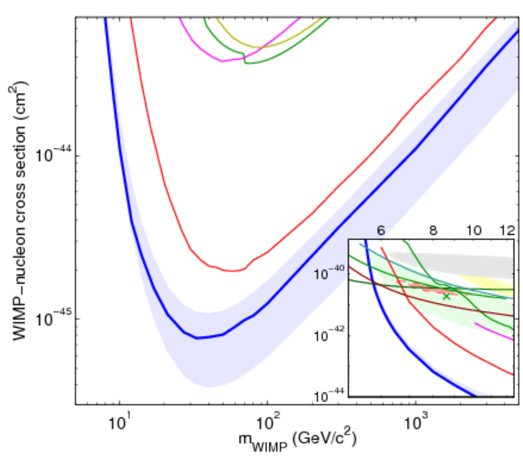

Figure 1.1: Large Underground Xenon (LUX) results with 90 % confidence level (blue solid line) on the exclusioin limit on the WIMP-nucleon elastic scattering cross-section as a function of WIMP mass Akerib et al. (2014).

Fig. 1.1 shows the exclusion limits on the DM scattering cross-sections. There is no DM interaction found above the blue solid line. The LUX experiment wants to observe interactions between DM and normal matter, this is done in a very pure liquid Xenon bath of 370 kg Aprile et al. (2012). This experiment is capable of searching for DM particles in the mass region 10-1000 GeV.

An attractive interaction model is the scenario where two DM particles annihilate into two SM particles, which in turn annihilate into gamma-rays. In a heavy Higgs boson model, the neutralino takes the following annihilation path (Gorbunov & Rubakov (2011) Eq. 9.54);

$$\chi\bar{\chi} \rightarrow f\bar{f}\,.$$

Here, f refers to a SM fermion. The gamma-rays are suggested to originate from the $f\bar{f}$ annihilation product:

$$f\bar{f} \rightarrow \gamma\gamma.$$

Another currently studied DM candidate is the Axion which is a hypothetical particle in the SM. An Axion would have a low mass ($m_A = 10^6 - 1\,\mathrm{eV}$). Theory predicts that the universe could be filled with very cold (Bose-Einstein condensate) Axions. If they exist, they would have decoupled from ordinary particle interaction sooner than the Neutralino and such WIMP candidates. There are many other suggestions but that goes beyond this thesis.

1.1.2 Detecting Dark Matter

As mentioned in the above section, DM can be directly detected via particle collision experiments. The XENON100 collaboration, Aprile et al. (2012), is one such experiment where one attempts to look for interactions via nuclear recoils in a liquid xenon target. They however reported that they found *no evidence for dark matter interactions.*

WIMPs could also show themselves in the next LHC upgrade which is planned to start running in Dec. 2014 allowing it to operate at 14 TeV, which is the supposed upper bound for the neutralino mass.

The density of baryonic and dark matter in clusters of galaxies can be determined by measuring their gravitational potential and thus their mass distribution. This method is called gravitational lensing and builds upon the fact that light from luminous objects residing behind a galaxy cluster gets bent by their gravitational field. This phenomena make luminous objects appear multiple times in a single image. In such studies, it turns out that there is not enough baryonic matter to account for the observed light distortion and thus the total observed mass distribution. This remaining non-visible i.e. non-electromagnetic interacting, mass distortion, is what we call DM.

Another method for detecting DM is by looking at the rotational curve for a given galaxy. In a simple model, one would extract the velocity from Newton's law

$$v(R) = \sqrt{\frac{GM(R)}{R}}\,,$$

where G is the gravitational constant. From this relation, it is seen that $v(R)$ is proportional to the mass $M(R)$. Thus, by observing the rotation of a galactic object, one could extract its mass. From the very famous plot, seen in Fig. 1.2, v(R) is constant even passing the periphery of the disk.

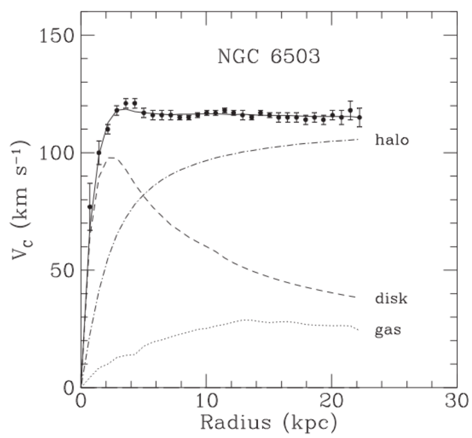

Figure 1.2: The rotational velocity of the galaxy NGS 6503, Begeman et al. (1991). The three labeled lines show the supposed different dependencies of the rotational velocity whilst the overall result is shown in the top curve.

This remarkable result is a strong argument for the existence of DM as there needs to be something accounting for the "extra" observed rotational velocity, which is in turn proportional to the mass.

1.1.3 Structure formation of Dark Matter subhalos

If the early universe was hot then DM particles were in kinetic equilibrium with ordinary (baryonic) matter. That means that the DM particles annihilated into SM particles and vice versa.

$$\text{SM} \leftrightarrow \text{DM}$$

As the universe cooled (due to its spatial expansion) there came a point in time at which the temperature, denoted T_d, became too cold to maintain the kinetic equilibrium state of the DM and SM particles, they decoupled.

$$\text{SM} \not\leftrightarrow \text{DM}$$

If the decoupling temperature T_d was much smaller than the mass of the DM particle m_χ, that is to say $T_d\,(1\,\text{K} = 8.6 \cdot 10^{-14}\,\text{GeV}) << m_\chi = 100\,\text{GeV}$, then one effectively ends up in a cold, i.e., non-relativistic, DM environment because its surrounding empty space is very cold. In the WIMP scenario, the DM interacts weakly (especially at long distances) and will as such start to isolate it self further. Primordial perturbations from the early universe provided over-dense DM regions, which in turn attracted more DM. Once such an over-density is large enough, DM structures became gravitationally bound. In this way, structures of DM are allowed form, Diemand & Moore (2011).

Initially, the newly bound DM structures were relatively small. At a later stage, they would collapse from self-gravitational instability (Jeans instability principle) and merge with another DM substructure. This continued hierarchically until massive objects were formed. This created potential wells for the baryons to "fall into". In this way, baryonic structures were formed such as stars, galaxies etc. This era is called the structure formation epoch. Today, DM structures or subhalos exist as a relic in and around galaxies as survivors from the (bottom-up) hierarchy formation, extending the bound of a galaxy.

1.2 Missing satellite problem (MSP)

Kauffmann et al. (1993) were the first to show that, when simulating the evolution of galaxies, there ought to be ~ 100 DM subhalos which would potentially be able to host observable satellite galaxies ($L > 10^6 L_\odot$). This prediction shows an overabundance of DM subhalos given that there are only ~ 10 satellites brighter than this around the Milky Way.

Bullock (2010) showed, from existing models, that the number of subhalos can be expressed as an analytical power-law-like equation. It estimated, for a Milky Way size host halo, that a total of $\sim 10^{11} - 10^{17}$ subhalos would exist within 400kpcs of the Sun.

The following is an excerpt from the letter:

"The most extreme way one might characterize the MSP is to say that CDM predicts $> 10^{11}$ *subhalos while we observe only* $\sim 10^2$ *satellite galaxies. While this statement is true, it should not worry anyone because the vast majority of those CDM subhalos are less massive than the Sun. No one expects a satellite galaxy to exist within a dark matter halo that is less massive than a single star."*

If it turns out that no such dim satellites exist then a self-collapsing DM subhalo could be a solution. As it would follow that the DM subhalos would have already annihilated before the satellite was even formed, or, that an already formed satellite would, in some way, cease its existence as the number of DM subhalos got lower in count by annihilating, destabilizing the baryonic content and ending the satellite's life.

1.3 Cusp-Core Problem

Observations have lead astronomers to believe that the mass-density distribution of DM towards the center of a galaxy is constant as the rotational velocity of DM grows linearly with the radius (right part of Fig. 1.3). This behaviour can as such be described to behave as a central core, constant density ρ. The same galaxy formation and evolution simulations mentioned in section 1.2 also showed a power-law like behaviour of the DM's mass-density profile (left part of Fig. 1.3). As that result contradicts observations it is named: "the cusp-core problem". Just like in MSP, simulations predict an over-abundance of DM subhalos.

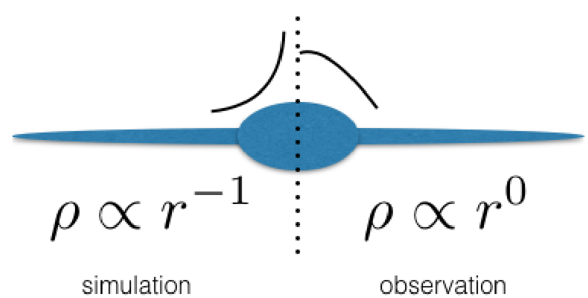

Figure 1.3: A galaxy (blue disk) with two different DM mass-density profiles ρ as a function of radius r. Left profile comes from prediction, right from observation. The solid lines are to indicate the DM mass-density as a function of radius in a galaxy.

The mass-density of a DM subhalo is proportional to its radius, $\rho \sim r^{\alpha}$, where α determines the slope of a power-law curve ($\alpha = 0$ for linear core profile). An analytic solution to the simulations, Navarro et al. (1996), found the DM mass-density distribution to be $\alpha = -1$ towards the center of a galaxy and an $\alpha = -3$ towards the edge. The following relation was to be called the "universal density profile" but is also known as the Navarro-Frenk-White (NFW) profile:

$$\rho_{NFW}(r) = \frac{\rho_0}{(r/R_s)\,(1 + r/R_s)^2}$$

where ρ_0 and R_s vary from halo to halo.

The cusp-core problem is taken seriously as its simulations are based upon the ΛCDM paradigm, the leading theory of Big Bang cosmology. As such, one cannot dismiss its conclusion, which disagrees with observation.

1.4 The Fermi-LAT gamma-ray excess

High-energy gamma-rays have been detected by the Large Area Telescope (LAT) on board the Fermi satellite. The mission was launched June 11, 2008 and went on for 195 weeks. The LAT observes 20% of the sky at any given moment and surveys the sky every 3 hours at low earth orbit. The detector is set up such that when a gamma-ray enters, it has 16 tungsten sheets to interact with. The material is chosen such that it can capture events at 8 keV - 300 GeV. The gamma-rays can produce an electron and an anti-electron pair which gets its energy measured at the last part of the detector namely the calorimeter. Most of the gathered data is associated with already known astrophysical events. However, unassociated (excess) lines were found, like the monochromatic gamma-ray line at an energy of $E_\gamma \approx 130$ GeV, Nolan et al. (2012).

It is claimed by Tempel et al. (2012) that this line has a statistical significance of 4.5σ and originates from the central region of a galaxy (Weniger (2012)). This could however just be "*an upward statistical fluctuation of the background*" - Boyarsky et al. (2013) and therefore it is concluded that more statistics need to be gathered. The following figure is taken from Tempel et al. (2012).

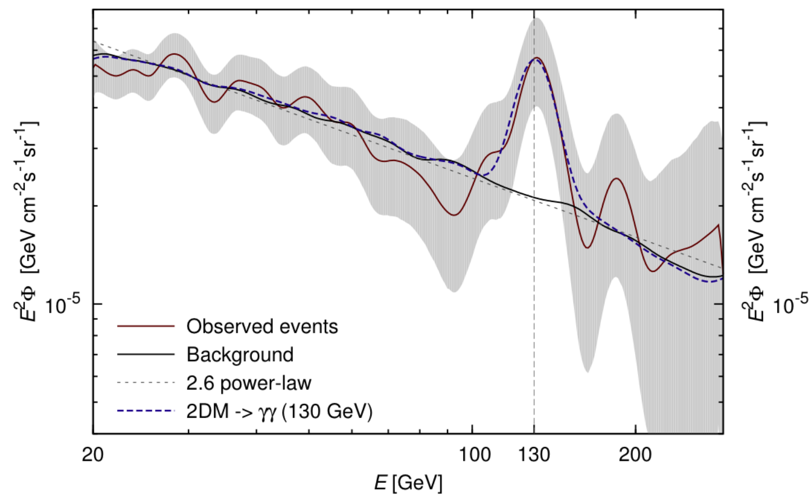

Figure 1.4: Estimated high-energy gamma-ray spectrum (dark red solid line with peak at 130 GeV) with a 95% confidence level (grey area) (Weniger (2012)). The solid black line and the dotted line running through the middle is the data fitted background and its respective predicted 2.6 power-law spectrum. The dark blue dashed line represents a simple DM annihilation into SM gamma-rays model, Cirelli et al. (2011).

Fig. 1.4 shows a clear peak at $E_\gamma \approx 130$ GeV as the dark red line indicates the gamma-ray spectrum from this unknown event. A compelling observation in figure 1.4 is the dashed dark blue line which represents the simple scenario of a 2 DM $\rightarrow$ 2 $\gamma\gamma$ event. It is indeed interesting that the simple model seems to fit best at the excess peak. So far, a strong peak with a high statistical significance that comes from the center of a galaxy exists and seems to agree with a simple DM particle annihilation model. This is very compelling for a DM subhalo annihilation scenario.

There are however less optimistic opinions about the $E_\gamma = 130$GeV gamma-ray line. One of them, Shakya (2014), finds the excess line "*inconsistent with neutralino annihilation into two photons*" (but does go on to approve another DM model, *albeit with parameters stretched to their limits*).

Chapter 2

Modelling the collapse

2.1 Preliminary remarks

The following model is constructed for an adiabatic collapse such that the following approximations would be valid:

- spherically-symmetric, isotropic and spatially-homogeneous collapse
- instantaneous departure of annihilation products from the clump
- zero angular momentum of Dark Matter particles
- pure Dark Matter (no baryonic component)
- the Dark Matter is symmetric WIMP-like, namely, is capable of its complete self-annihilation (asymmetric DM models would not apply here).

The first approximation is reasonable and based upon the fact that during the collapse process the denser regions of DM annihilate faster then less dense ones, thus all the initial inhomogeneities in mass distribution are relatively quickly washed out. Other approximations are made for simplicity of the analysis and can be discarded later on if necessary.

Such a simplified dynamics can be analysed fully analytically as an exactly solvable problem of mathematical physics.

2.2 The model

The model is formulated based upon the theory of non-relativistic gravitating gas dynamics. The energy balance equation

$$\underbrace{\frac{\partial}{\partial t}\int \rho_\chi \,\mathrm{d}V}_{\text{rate of energy increase}} = \underbrace{-\int \rho_\chi v_i \,\mathrm{d}S_i}_{\text{energy crossing a surface element}} \underbrace{-\int \frac{2(\sigma v)_{ann}\rho_\chi^2}{m_\chi}\,\mathrm{d}V}_{\text{energy loss in annihilation}} \tag{2.1}$$

determines real time dynamics of the energy loss of the subhalo during its evolution. Here, ρ_χ is the DM subhalo energy density for which perfect sphericity, independent of spatial coordinates, is assumed; $\rho_\chi(t) = 3M(t)/4\pi R(t)^3$. v_i $(i = 1, 2, 3)$ are the spatial components of the radial velocity of a DM particle crossing the surface element S_i pointing outwards of the volume. $(\sigma v)_{ann} \simeq \text{const}(t)$ is the kinetic DM annihilation cross section and $m_\chi \sim 0.1 - 1$ TeV is the typical WIMP DM particle mass. Differentially, Eq. (2.1) represents the energy conservation in a given volume element $\mathrm{d}V$ of the clump. Namely, the rate of increase of its energy $\partial\rho_\chi/\partial t\,\mathrm{d}V$ is given by an amount of energy crossing the area surrounding the volume, $-\rho_\chi v_i\,\mathrm{d}S_i$, and the total annihilation rate in that volume which describes the dissipation of energy, $-2(\sigma v)_{ann}\rho_\chi^2/M_\chi\,\mathrm{d}V$.

Analogically, the linear momentum balance equation

$$\begin{aligned}\underbrace{\frac{\partial}{\partial t}\int \rho_\chi v_i \,\mathrm{d}V}_{\text{rate of momentum increase}} &= \underbrace{-\int (\rho_\chi v_i v_k + P_{ik})\,\mathrm{d}S_k}_{\text{momentum crossing a surface element}} \underbrace{-\int \rho_\chi \nabla_i\phi \,\mathrm{d}V}_{\text{gravitational interaction}} \\ &\quad \underbrace{-\int \nu_{dis}\rho_\chi v_i\,\mathrm{d}V}_{\text{friction term}} \underbrace{-\int \frac{2(\sigma v)_{ann}\rho_\chi^2}{m_\chi} v_i\,\mathrm{d}V}_{\text{momentum loss in annihilation}}\end{aligned} \tag{2.2}$$

describes a change in the total linear momentum associated with a radial motion of DM particles in a given volume of the collapsing subhalo. Here, P_{ik} is the pressure density tensor of the DM inside the subhalo which vanishes in the homogeneous approximation, ν_{dis} is the frequency of collisions of collapsing DM particles with outgoing annihilation products and ϕ is the non-relativistic gravitational potential. In the limit of zero angular momentum, this equation follows from the linear (radial) momentum conservation. Namely, the rate of increase of the linear momentum in a volume element $\partial(\rho_\chi v_i)/\partial t\,\mathrm{d}V$ is given by the total linear momentum crossed the surrounding surface, $-\rho_\chi v_i v_k\,\mathrm{d}S_k$, the net DM momentum loss in the volume element due to the gravitational interactions, $-\rho_\chi \nabla_i \phi \, \mathrm{d}V$, and the elastic re-scattering of falling DM particles off escaping products of their annihilation (DM particles transfer a part of their linear momentum into relativistic radiation such that this momentum instantaneously leaves the clump with the radiation), $-\nu_{dis}\rho_\chi v_i \, \mathrm{d}V$, and the last piece, $-2(\sigma v)_{ann}\rho_\chi^2 v_i/M_\chi \, \mathrm{d}V$, describes the momentum loss due to DM annihilation into relativistic products.

At last, the non-relativistic gravitational potential created by the DM density ρ_χ is defined by the General Relativity (Poisson's-like) equation

$$\Delta\phi = 4\pi G \rho_\chi \,, \tag{2.3}$$

where $G = M_{PL}^{-2}$ is the gravitational constant in natural units expressed in terms of the Planck mass $M_{PL} \simeq 1.2 \cdot 10^{19}$ GeV.

The last terms in the energy and momentum balance equations describe the instantaneous departure of annihilation products from the collapsing subhalo. The momentum dissipation term containing ν_{dis} has a direct analogy in classical mechanics, namely, the "friction". It also has the feature where it is inversely proportional to the collision time scale; $\nu_{dis} = \tau_{dis}^{-1}$. The frequency of collisions and density of annihilation products can be estimated as follows (Gorbunov & Rubakov (2011) Eq. 9.7):

$$\nu_{dis} = n_1 (\sigma v)_S, \qquad n_1 \sim \frac{(\sigma v)_A n_\chi^2}{\nu_{dis}}, \qquad n_\chi = \frac{\rho_\chi}{m_\chi}. \tag{2.4}$$

Here, n_1 is the number density of SM particles created during the annihilation processes and (σv) represents particle cross sections where v is the relative velocity of the two annihilating particles and σ the annihilation cross-section. The term $(\sigma v)_S$ more precisely represents the kinetic cross-section and $(\sigma v)_A$ the annihilation cross section. Combining the above formulas one obtains

$$\nu_{dis} \sim \sqrt{(\sigma v)_A (\sigma v)_S}\, n_\chi.$$

It turns out, for this case, that the annihilation and elastic scattering cross sections are energetically at the same order of magnitude; $(\sigma v)_S \sim (\sigma v)_A$. Thus, using the last term in Eq. (2.4)

$$\nu_{dis} = b\, \frac{(\sigma v)_A \rho_\chi}{m_\chi}, \tag{2.5}$$

where b is a numerical coefficient close to unity, $b \sim 1$, meaning that it would be just as likely for the DM to elastically collide with the SM particles as it would be for the DM to annihilate into SM particles.

2.3 Analytical analysis

To aid the solving of some equations, a few relations are derived in the beginning. Since an isotropic, non-rotational and homogeneous subhalo collapse model is used, a vector element x_i will change only with time; $x_i = x_i(t)$. Based on those assumptions, there are some trivial definitions to be made, namely

$$|x_i| \equiv R(t) = \sqrt{x_1^2 + x_2^2 + x_3^2}\,, \qquad \dot{x}_i \equiv v_i\,, \qquad H(t) \equiv \frac{\dot{a}}{a}, \tag{2.6}$$

where the scale factor $a(t) = R(t)/R_0$ is introduced as a dimensionless ratio of the subhalo radius $R(t)$ and its associated initial value $R_0 = R(t = 0)$. $H(t)$ then becomes the parameter for a given subhalos' annihilation rate.

As $x_i = n_i R(t)$, using the terms in Eq. (2.6) yields the useful result

$$v_i = H(t) x_i. \tag{2.7}$$

Next, the functions containing the ∇ operator are simplified using Eq. (2.7);

$$\begin{aligned} (v_k \nabla_k) v_i &= \left[\frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3}\right] v_i = H(t) v_i, \\ \nabla_k v_k &= \left[\frac{\partial}{\partial x_1} H(t) x_1 + \frac{\partial}{\partial x_2} H(t) x_2 + \frac{\partial}{\partial x_3} H(t) x_3\right] = 3H(t). \end{aligned} \tag{2.8}$$

Lastly, a parameter ξ links the non-relativistic gravitational potential with the above discussed vector element x_i;

$$\begin{aligned} \nabla_i \phi &= \xi x_i \\ \nabla_i \nabla_i \phi &= \nabla_i \xi x_i \\ \Delta \phi &= 3\xi \end{aligned} \tag{2.9}$$

now inserting the Poisson's-like equation from Eq. (2.3) which finally yields an expression for $\nabla_i \phi$, namely

$$\nabla_i \phi = \frac{4\pi G \rho_\chi}{3} x_i\,, \qquad \xi = \frac{4\pi G \rho_\chi}{3} \tag{2.10}$$

After taking the necessary steps – transforming the surface integrals to volume integrals with Gauss's theorem, applying the chain rule, using the result from Eq. (2.5) and $P_{ik} = 0$ – equations (2.1) and (2.2) can be represented in the following differential forms

$$\begin{gathered}\frac{\partial \rho_\chi}{\partial t} + \nabla_i \rho_\chi v_i = -\frac{2(\sigma v)_A \rho_\chi^2}{m_\chi}\,, \\ \frac{\partial}{\partial t}(\rho_\chi v_i) + \rho_\chi \nabla_k (v_i v_k) = -\rho_\chi \nabla_i \phi - b\,\frac{(\sigma v)_A \rho_\chi}{m_\chi} v_i - \frac{2(\sigma v)_A \rho_\chi^2}{m_\chi} v_i\,, \\ \Delta \phi = 4\pi G \rho_\chi\,,\end{gathered} \tag{2.11}$$

where

$$\frac{\partial \rho_\chi}{\partial t} = \frac{d\rho_\chi}{dt} \equiv \dot{\rho}_\chi\,, \qquad \frac{\partial v_i}{\partial t} = \dot{v}_i - v_k(\nabla_k v_i)\,, \tag{2.12}$$

An intermediate step is shown for convenience where the second term in Eq. (2.11) is further simplified using Eqns. (2.7), (2.8), (2.10) and the chain rule

$$\frac{\partial \rho_\chi}{\partial t} v_i + \frac{\partial v_i}{\partial t} \rho_\chi + \rho_\chi \left[(v_k \nabla_k) v_i + v_i(\nabla_k v_k)\right] = -\rho_\chi \frac{4\pi G \rho_\chi}{3} x_i - b\,\frac{(\sigma v)_A \rho_\chi}{m_\chi} v_i - \frac{2(\sigma v)_A \rho_\chi^2}{m_\chi} v_i. \tag{2.13}$$

Now the first term in Eq. (2.11) is used in Eq. (2.13) along with Eq. (2.12) to obtain the first simplified resulting differential equations

$$\begin{gathered}\dot{\rho_\chi} + 3H\rho_\chi = -\frac{2(\sigma v)_A \rho_\chi^2}{m_\chi}\,, \\ \dot{H} + H^2 = -\frac{4\pi G \rho_\chi}{3} - \frac{H}{\tau_{dis}}\,, \qquad \tau_{dis}^{-1} = b\,\frac{(\sigma v)_A \rho_\chi}{m_\chi}.\end{gathered} \tag{2.14}$$

where τ_{dis}^{-1}, as mentioned earlier in the previous section, is equivalent to the collision frequency term ν_{dis} (the "friction" term). Since the latter grows linearly with the clump density $\rho_\chi(t)$, the "friction" becomes increasingly important over late stages of the collapse. It is equally important as the energy dissipation rate due to annihilation and should thus not be neglected in studies for the DM subhalo dynamics.

When multiplying a factor of $8\pi G/3$ to the first term in Eqns. (2.14), a factor of $2H$ to the second and transforming them together leaves the following expression

$$\frac{d}{dt}\left(H^2 - \frac{8\pi G \rho_\chi}{3}\right) + 2H\left(H^2 - \frac{8\pi G \rho_\chi}{3}\right) + \frac{2(\sigma v)_A \rho_\chi}{m_\chi}\left(bH^2 - \frac{8\pi G \rho_\chi}{3}\right) = 0\,. \tag{2.15}$$

At this stage, it is clear that if $b = 1$, the system becomes completely integrable which generates a new system of equations with initial conditions. If a function f is chosen such that $f = H^2 - 8\pi G\rho_\chi/3$ then Eq. (2.15) becomes

$$\frac{\dot{f}}{f} = -2H - \frac{2(\sigma v)_{ann}}{m_\chi}\rho_\chi \,. \tag{2.16}$$

After dividing the first term Eq. (2.14) by a factor of ρ_χ it is substituted into Eq. (2.16) in order to obtain

$$\frac{\dot{f}}{f} = H + \frac{\dot{\rho}_\chi}{\rho_\chi} \tag{2.17}$$

which is integrated over time from an initial time t_0 to any time t

$$\begin{gathered} \int_{t_0}^{t} \frac{\dot{f}}{f}\,\mathrm{d}t = \int_{t_0}^{t} \frac{\dot{R}}{R}\,\mathrm{d}t + \int_{t_0}^{t} \frac{\dot{\rho}_\chi}{\rho_\chi}\,\mathrm{d}t \\ \frac{f}{f_0} = \frac{R_0}{R}\frac{\rho_{\chi 0}}{\rho_\chi} \end{gathered} \tag{2.18}$$

here, R_0 is the initial radius of the subhalo, $\rho_{\chi 0}$ the initial energy-density and f_0 is expressed through those terms. The result in Eq. (2.18) is combined with the definition of f in order to obtain the latter of the following important relations where the former came from transforming the first term in Eq. (2.14) in terms of the subhalo's radius R(t) and mass M(t);

$$\boxed{\frac{dM}{dt} = -\frac{3(\sigma v)_A}{2\pi m_\chi}\cdot\frac{M^2}{R^3}}\,, \qquad \boxed{\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R}\left(1 - \beta^2\frac{R}{R_0}\right)}\,. \tag{2.19}$$

Here $\beta^2 \leq 1$ is a constant fixed by initial subhalo contraction conditions which can be found from the relation $\dot{R}_0^2 = 2GM_0(1-\beta^2)/R_0$. From the last term in Eq. (2.19) it is evident that the subhalo will contract faster as its size decreases, leading to an aggressive annihilation; $\dot{R} \sim R^{-1/2}$, $\dot{R}(t) \to \infty$ as $R(t) \to 0$.

It turns out that the system of equations in Eq. (2.19) does not have a simple analytical solution for $R(t), M(t)$ and then t itself. That is why a parametric representation of those parameters as functions through ζ is needed. $dR/d\zeta$ was conveniently chosen to solve the following differential equation

$$\frac{dR}{dt} = \frac{dR}{d\zeta}\frac{d\zeta}{dt}\,. \tag{2.20}$$

The results are presented below

$$R(\zeta) = R_0(\zeta^2 + \beta^2)^{-1},$$

$$M(\zeta) = M_0 \left[1 + \frac{(\sigma v)_A}{m_\chi} \left(\frac{3\rho_{\chi 0}}{2\pi G}\right)^{1/2} \left(\frac{\zeta^3}{3} + \beta^2 \zeta\right)\right]^{-2},$$

$$\left(\frac{8\pi G \rho_{\chi 0}}{3}\right)^{1/2} t(\zeta) = \frac{1}{\beta^2}\left(\frac{\zeta}{\zeta^2+\beta^2} - (1-\beta^2)^{1/2}\right) + \frac{1}{\beta^3}\left(\arctan\frac{\zeta}{\beta} - \arctan\frac{(1-\beta^2)^{1/2}}{\beta}\right) +$$
$$+ \frac{(\sigma v)_A}{3m_\chi}\left(\frac{3\rho_{\chi 0}}{2\pi G}\right)^{1/2}\left(\ln(\zeta^2+\beta^2) + \frac{2\beta^2(\zeta^2+\beta^2-1)}{\zeta^2+\beta^2}\right) \tag{2.21}$$

where $M_0 \sim M_\odot = 2 \cdot 10^{33}\,\text{g}$, $\rho_{\chi 0} \sim 10^{-4}\,\text{g/cm}^3$ could be realistic initial values of the mass and density of a Solar-type DM subhalo. By looking at Eq. (2.21) it is evident that the collapsing subhalo will be completely annihilated at sufficiently large time scales, i.e. $M(t) \to 0$, $R(t) \to 0$ at $t \to \infty$. The time dependence of such a subhalo's annihilation rate is given by the following generic expression

$$-\frac{dM}{dt} = M_0 \rho_{\chi 0} \cdot \frac{2(\sigma v)_A}{m_\chi} \cdot \frac{(\zeta^2+\beta^2)^3}{\left[1 + \frac{(\sigma v)_A}{m_\chi}\left(\frac{3\rho_{\chi 0}}{2\pi G}\right)^{1/2}\left(\frac{\zeta^3}{3} + \beta^2\zeta\right)\right]^4}. \tag{2.22}$$

Since a subhalo has such a static life before its supposedly inevitable collapse, plots are only presented in the vicinity of such a collapse, where most of the mass is radiated away. The following section will contain models for analysing the dynamics of the subhalo in the vicinity of the annihilation process.

In dimensionless variables $\mu(t) = M(t)/M_0$ and $a(t) = R(t)/R_0$, the terms in Eq. (2.19) take the form

$$\dot{\mu} + \lambda\frac{\mu^2}{a^3} = 0, \qquad H^2 = \frac{8\pi G}{3}\rho_\chi(1-\beta^2 a), \qquad H = \frac{\dot{a}}{a}, \qquad \lambda = \frac{2(\sigma v)_A}{m_\chi}\rho_{\chi 0}. \tag{2.23}$$

For a late stage subhalo, a scenario where its density is very high and $a(t) \ll 1$ is considered. The solution of the first term in Eq. (2.23) together with the expression in Eq. (2.22) gives the dimensionless radiation intensity rate $|\dot{\mu}|$ which is represented as a function of the scale factor $a = a(t)$ as follows:

$$|\dot{\mu}|(a) = \frac{12\pi G m_\chi g^2}{(\sigma v)_A} \cdot \frac{a^3}{(g + (1-g)a^{3/2})^4}, \qquad g = \frac{(\sigma v)_A}{m_\chi}\left(\frac{3\rho_{\chi 0}}{2\pi G}\right)^{1/2} < 1. \tag{2.24}$$

The solution for the real time as a function of a is then given by

$$t(a) = \frac{(\sigma v)_A}{4\pi G m_\chi g}\left[\frac{2}{3}(1 - a^{3/2})(1-g) - g\ln a\right]. \tag{2.25}$$

where a typical weak-induced WIMP annihilation cross section would be $(\sigma v)_A \sim \alpha_W^2/8\pi m_\chi^2$, $\alpha_W \sim 1/30$. The position of the peak can be found from the extremum condition for the function $\mu(a(t))$ as

$$\frac{d\mu(a(t))}{dt} = \frac{d\mu(a)}{da}\cdot \dot{a} = 0 \quad \rightarrow \quad \frac{d\mu(a)}{da} = 0\,, \qquad \dot{a} \neq 0\,. \tag{2.26}$$

Solving this equation leads to the peaked value of the scale factor a, namely

$$a^* = a(t = t^*) = \left(\frac{g}{1-g}\right)^{2/3}, \tag{2.27}$$

where t^* is the time of maximum in the intensity of radiation counted from the initial moment t_0. Dimensionless time η is introduced along with other convenient parameters

$$y = \left(\frac{a}{a^*}\right)^{3/2}, \qquad \eta = \frac{t - t^*}{\tau_0}\,, \qquad \tau_0 = \frac{(\sigma v)_A}{3\pi G m_\chi}\,, \tag{2.28}$$

with $\tau_0 = 15\,\text{seconds}$ if $m_\chi = 100\,\text{GeV}$. Now, the radiation intensity rate $|\dot{\mu}|$ can be represented in the simple form

$$\left|\frac{d\mu}{d\eta}\right| = \frac{4}{(1-g)^2}\cdot\frac{y^2}{(1+y)^4}\,, \qquad \eta(y) = \frac{1}{2}(1 - y - \ln y)\,. \tag{2.29}$$

In a small vicinity of the peak, $|d\mu/d\eta|$ takes the form

$$\left|\frac{d\mu}{d\eta}\right|_{t\sim t^*} \simeq \frac{1}{4(1-g)^2}\left(1 - \frac{\eta^2}{2}\right) \simeq \frac{1}{4}\left(1 - \frac{(t-t^*)^2}{2\tau_0^2}\right)\,. \tag{2.30}$$

From this relation it is seen that the time dependence of the annihilation rate, in a vicinity of the peak, depends only on fundamental DM particle constants.

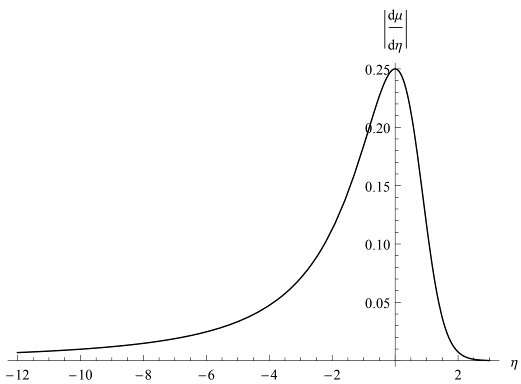

Figure 2.1: The dimensionless mass change rate $|d\mu/d\eta|$ as a function of dimensionless time η.

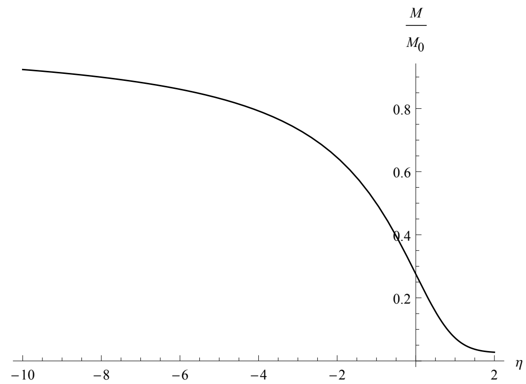

Figure 2.2: The dimensionless mass $\mu(t) = M(t)/M_0$ as a function of dimensionless time η.

The annihilation rate $|d\mu/d\eta|$ is shown in Fig. 2.1. From Fig. 2.2 it is noticed that about 80% of the subhalo mass is lost during the time interval

$$t_A = 6\tau_0 = \frac{2(\sigma v)_A}{\pi G m_\chi} = \frac{\alpha_W^2}{4\pi^2 G m_\chi^3}\,. \tag{2.31}$$

The effectively intensive annihilation happens during $t_A^{(-)} = 4\tau_0$ before maximum and $t_A^{(+)} = 2\tau_0$ after maximum. The other physical quantities – the radius $a(t) = R(t)/R_0$ and the density $r(t) = \rho_\chi(t)/\rho_{\chi 0}$ of the DM subhalo – are shown in Figs. 2.4 and 2.3 respectively.

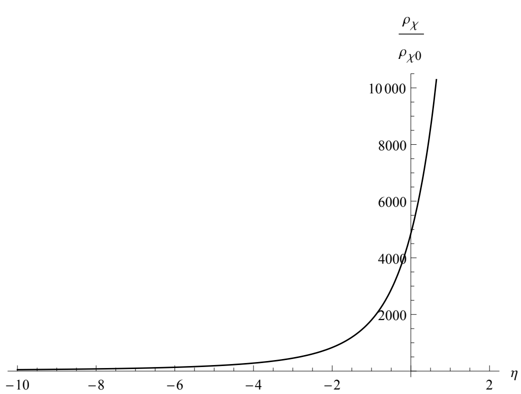

Figure 2.3: The dimensionless energy-density $r(\eta) = \rho_\chi(\eta)/\rho_{\chi 0}$ as a function of dimensionless time η.

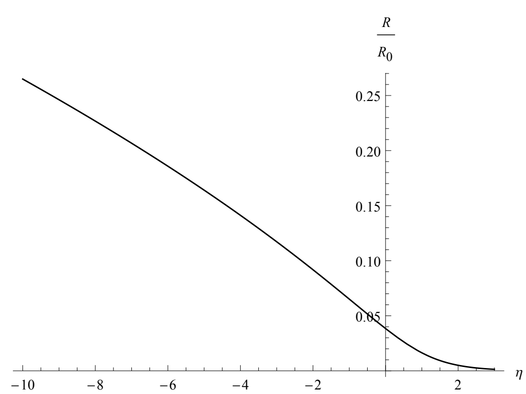

Figure 2.4: The dimensionless radius $a(\eta) = R(\eta)/R_0$ as a function of dimensionless time η.

Chapter 3

Conclusions

In this thesis I have proposed an exactly solvable model for total annihilation of DM subhalos. Such a subhalo can live for billions of years but will, supposedly, eventually annihilate. If DM particles would annihilate into SM fermions, multi-GeV gamma ray bursts are expected to be observed in the short time interval of ~ 90 seconds. Two important parameters for the search of such gamma-ray signatures (peak burst time and energy rate) can be determined with DM parameters only. This result potentially allows one to distinguish a whole class of catastrophic astrophysical events with universal characteristics independent of the formation history of such objects. These characteristics can then be used to constrain the most important DM properties like WIMP mass and DM couplings to ordinary matter particles in the Standard Model.

Fig. 2.1 shows an anticipated rapid annihilation as predicted by Eq. (2.29). The time intervals ($t_A^{(-)}$ and $t_A^{(+)}$) could in principle be extracted from observational data. A characteristic feature of this function is its strong asymmetry with respect to the maximum which again, could be extracted from data. They are determined by the fundamental constants only and does not depend on initial density and the initial contraction rate of the DM subhalo. The shape of the curve after the peak could be understood by the trivial fact that the subhalo simply looses particles as it annihilates off in the collapse process.

Fig. 2.2 shows the shape of the mass distribution of the collapsing subhalo. The rapid loss in mass is understood through the rather steep slope towards the end. The steep slope is in congruence with what was discussed after Eq. (2.19) which was the ever increasing contraction rate of a given subhalo. This is shown by the the change in radius curve in Fig. 2.4. To emphasize the point further, the growth in density seen in Fig. 2.3 shows how fast the annihilation process occurs. The density grows by an order of magnitude during the accelerating contraction. The curve strongly depends on the initial energy-density of the subhalo.

In this model, the DM particles were assumed to have zero angular momentum. This allows the subhalo to collapse into a singularity, as no particles would rotate around the central mass. Due to the effective "friction" term in Eq. (2.2) the momentum of falling DM particles are converted into radiation, in this way, angular momentum is not conserved and one arrives at an annihilating subhalo scenario. If angular momentum is conserved, then the subhalo would never had collapsed into a singularity. In that scenario, it would just form a DM core of fixed radius at the end of the collapse. However, this does not happen due to an expected loss of the total angular momentum by radiation due to small, but finite, DM self-interaction and self-annihilation rates.

What could be done next is to add a baryonic core to a subhalo such that it could resemble reality more. It would interesting to see what happens if the angular momentum is added. One could study other DM candidates and see how their parameters, such as mass and annihilation cross-section compares with observed gamma-ray signatures. On that note, it would be great to search for the signature time scales as well as the energies of the gamma-ray bursts with Fermi.

Lastly, the effective "friction" term is often overlooked but, as mentioned, the linear dependence ν_{dis} has on the density ρ_χ, makes it increasingly important as the subhalo contracts and should thus not be neglected in future models of collapsing DM subhalos.

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