Secular oscillations and Kozai cycles in planetary systems

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Populärvetenskaplig sammanfattning

Idag vet man att det finns planeter även utanför vårt solsystem, de snurrar runt sina stjärnor precis som vi snurrar runt Solen - vår stjärna. Man kallar dem ‘exoplaneter’, och hittills har man detekterat över tusen stycken. En del av alla dessa exoplaneter har till synes underliga egenskaper: Deras omloppsbanor är väldigt små, endast några hundradelar av Jordens omloppsbana, samtidigt som de är väldigt stora, i nivå med Saturnus och Jupiter. Att de är så nära sina stjärnor innebär att de mottar väldiga mängder värmande strålning, och således brukar de kallas ’hot Jupiters’. En del av dem befinner sig på omloppsbanor som lutar väldigt mycket relativt deras stjärnors rotationsaxlar.

Dessa ’hot Jupiters’ är problematiska av minst två skäl. För det första borde inte massiva planeter ha små omloppsbanor, och för det andra borde inte omloppsbanorna luta. En möjlig lösning på dessa problem är den så kallade Kozaieffekten, som när den kombineras med tidvattenskrafter och dynamik i stjärnhopar, kan transportera en Jupiterliknande planet från en stor omloppsbana till en liten och lutande omloppsbana.

I den här uppsatsen undersöks huruvida en sådan process är möjlig. Resultaten tyder på att den är det, men att den inte på egna ben kan vara förklaring till samtliga detekterade ’hot Jupiters’.

Abstract

The origin of the class of exoplanets typically referred to as *hot Jupiters* is to this day an unsettled matter. Some of the proposed formation channels predict certain values of the *spin-orbit misalignment* parameter, i.e. the angle between the stellar rotation axis, and the angular momentum vector of the planet orbit. One such formation channel is tidal capture following Kozai resonance (TCKR). This channel produces high misalignment, with some preference for angles around 39° and 141°. For single-planet systems, this channel is viable, but whether it functions in multi-planet systems or not is not yet clear.

This thesis primarily explores, by means of N-body simulations, what impact the secular oscillations between planets in multiplanetary systems have on TCKR. Secondarily, there is some investigation into if the outermost planets in multiplanetary systems can act as low-mass Kozai companions.

The primary results include that secular oscillations do in fact interact with the Kozai effect: for every multiplanetary system there appears to be a critical combination of the Kozai companion's orbit size and inclination beyond which the effect shuts down. Because of this, secular oscillations are generally detrimental to TCKR and the production of hot Jupiters. Lastly, it would appear that planetary Kozai companions are dysfunctional if their masses are roughly equal to or smaller than those of the system's inferior planets.

Contents

1 Introduction

1.1 Planets

A planet is, according to the most recent definition set by the IAU in 2006, a *celestial body*[1] that

1. orbits the Sun,
2. has a sufficiently large mass for self-gravity to make it roughly spherical in shape, and
3. has cleared its orbital neighbourhood.

The first condition appears reasonable at first glance. Giving it a second thought though, one realizes that would-be planets around other stars (*extra-solar* planets or *exoplanets*), are not planets, simply because they don't orbit our eminent Sun. Also, if any of the eight planets in the Solar System were to be ejected for some reason, it would cease to be a planet. One might think this a bit odd.

With the second condition, there might be problems when two equally massive objects are shaped differently due to unequal strengths of their constituent materials - one of the objects might then be shaped spherical by self-gravity, while the other is not. Such a situation is probably very rare, though.

The third condition is needed to exclude objects like Pluto, Eris and Sedna (*dwarf planets*) from the planet category. If excluding such objects is your wish, fair enough. The consensus in this is not quite total. As with the other conditions, there can be awkward grey-zone situations. If, for example, the Moon was a bit more massive, the barycenter of the Earth-Moon system might not be inside the Earth. In such a case, someone might think the Moon should cease to be a moon, and so the Earth-Moon system would instead be considered a *dwarf planet binary* since neither of them, on their own, fulfill condition 3.

As of today, there are eight known planets in our Solar System, of which half are terrestrial planets[2] (Mercury, Venus, Earth, Mars) and the rest are gas giants (Jupiter, Saturn) or ice giants (Neptune, Uranus). The existence of some of these has been known since ancient times (e.g. the evening/morning star - Venus), owing to their brilliance in the night sky. Some, on the other hand, were not discovered until fairly recently (e.g. Neptune, discovered 1846[3]).

For many years, humans had no idea of whether planets existed around other stars[4].

[1] This term is not clearly defined, it seems. Definitions vary. One of them is 'natural objects occuring outside Earth's atmosphere'. Obviously, Earth itself is not outside its own atmosphere, and can thus not be a planet. At the same time, Earth is listed by the IAU as one of the eight planets in the Solar System.

[2] Terrestrial planets are planets whose surfaces can be regarded solid. This vague definition is not particularly useful in practice other than to distinguish huge planets primarily made of gas from smaller planets whose mass is mainly liquid or solid.

[3] Neptune was discovered as a planet in 1846, but observed by Galileo already in 1612. He mistook it for a star.

[4] The answer is a definite no if one takes definitions seriously, but this thesis generally does not rigourously distinguish planets and exoplanets from this point on.

1.1.1 Exoplanets

Fortunately, the question of whether our Solar System is the sole harborer of planets is settled today, and the answer is a resounding no: According to the exoplanet database at exoplanets.org, the current number of confirmed extra-solar planets is 1516, along with 3359 unconfirmed Kepler candidates[5]. To the optimist this translates to almost 5000 discovered exoplanets.

There is a definition for exoplanets set by the IAU, which defines any object whose mass is below the deuterium fusion limit to be an exoplanet ($\lesssim 13$ Jupiter masses), given that it orbits a star or a stellar remnant (e.g. Murray and Dermott (1999)). One might wonder why it hasn't been changed to 'planet orbiting a star other than the Sun or a stellar remnant', which would naively make sense. A problem with this naive definition surfaces when the masses of would-be exoplanets approach or pass the minimum mass of stars. As a silly example, if the Sun encountered a binary star and, after the dynamical chaos, ended up with a binary companion... that companion would satisfy all the conditions for 'planet' status. At the same time, using the IAU definition, a large artificial construct, perhaps a derelict spaceship, would classify as an exoplanet if it was found in an orbit around a star or stellar remnant. Clearly there are pieces missing all over the place, but perhaps this is not much of an obstacle in practice.

Some exoplanets[6] have a remarkable characteristic that is foreign to planets: they are comparable to Jupiter in size but located on orbits significantly smaller than that of Mercury. They have been referred to as *hot Jupiters* for quite some time now.

1.2 Hot Jupiters

As a consequence of their proximity to their sun, these exoplanets are intensely irradiated and thus very warm (hence their name). One might imagine them to be well defined, but that does not seem to be the case.

Some examples of differing 'definitions':

- Raymond et al. (2005): Hot Jupiters are within 0.5 au of their sun, and have masses around 0.5 M_J.
- Naoz et al. (2011): Hot Jupiters are Jupiter-sized planets in very close proximity to their host star.
- Bayliss and Sackett (2011): Hot Jupiters are close-in, jovian mass planets.

[5] A Kepler candidate is a planet candidate discovered by the Kepler satellite - they're not strictly planets 'yet' since they might in reality be sunspots or some other phenomena that mimic a planet signature.

[6] According to Wright et al. (2012), 1.2 ± 0.38 % of Sun-like stars host hot Jupiters, while the HARPS (High Accuracy Radial velocity Planet Searcher, a spectrograph installed and running at VLT) survey found that 50 % of stars host at least one planet of any mass with period(s) below 200 days (Howard, 2013). Since planets evidently can have orbital periods longer than 200 days, this figure of 50 % is a lower limit.

- Showman and Guillot (2000): Exoplanets are dubbed 'hot Jupiters' if they are giant planets within 0.2 au and hotter than 1000 K.

There does not seem to exist a universally adopted definition in the astronomical community, but this will probably prove to be harmless in practise.

Where do hot Jupiters come from? Regrettably, there is as of yet no consensus. What is known is that hot Jupiters are probably not born into the circumstances they find themselves in today - any researcher on planet formation will probably tell you that planets of jovian mass must form relatively far from the host star for various reasons. These include, but are not limited to:

1. It is probably helpful, when forming a solid core, to have solid ice around. Preferably then, jovians would form beyond the ice line[7] (e.g. Lissauer (1993)).

2. The mass contained between two radii grows with the radius itself in any disk of approximately uniform surface density. Further out (but not too far obviously, since the disk is finite), there will probably be more mass with which to form jovian planets.

3. Radiation pressure from the star will rarefy the inner parts of the disk first, the consequence of which is that the available time for gas accretion increases with orbit radius. (e.g. Boss (1995))

In spite of this, there are hot Jupiters. One answer as to their origins might be found in the *Kozai effect* or *Kozai mechanism*, a phenomenon that manifests in special three-or-more-body systems. This will be described in detail later, first we focus some more on hot Jupiters and their peculiarities.

1.2.1 Detection rate and bias

Owing to the particular properties of hot Jupiters and to some intrinsics of the two dominant planet detection methods used today i.e the *radial velocity* (RV) and *transit* methods, the detection rate of hot Jupiters is higher than what would be expected if all planets were equally detectable.

When using the radial velocity method, hot Jupiters are ideal because they induce Doppler shifts in the host stars' spectra with a high frequency (owing to their tight orbits i.e. short orbital periods) and high amplitude (owing to their considerable jovian mass and high orbital speeds). The high frequency permits measurements of multiple periods within reasonable timeframes (which improves precision), while the high amplitude is a necessary condition for detecting the planet's presence in the first place, and achieving higher signal-to-noise ratios (which also improves precision).

When using the transit method, hot Jupiters are ideal because planets (any orbiting object, really) on tight orbits have a higher probability of transiting its star:

[7] The ice line(s) is drawn along the heliocentric distance beyond which various elements condense into their solid forms.

$$p = \frac{R_* + R_p}{a_p\,(1 - e^2)}.$$

Here, p is the transit probability, R_* and R_p are the radii of the star and planet respectively, a_p is the semi-major axis, and e is the eccentricity. This equation was derived by Barnes (2007), and clearly shows that tighter orbits transit more often. To comprehend this geometrically, one may consider the planet's orbit. If the orbit is small, tilting the orbit will not have a large effect on the planet's movement in the plane perpendicular to the line of sight. But for large orbits, even the slightest tilt could move the planet's orbit out of the stellar disk.

Furthermore, owing to their jovian volumes, they attenuate the starlight by a few percent (the percentage is easily calculated from $\frac{R_p^2}{R_*^2}$), which is readily detectable by today's modern instruments. As is the case for the RV-method, the tight orbits of hot Jupiters i.e. their short periods allow for observation of multiple transits, which as mentioned improves precision.

These two methods are with today's technology the most successful for detecting exoplanets - primarily because they are viable out to the considerable absolute distances of many exoplanet systems (this applies more to the transit method - the RV method requires a lot of photons to fill the spectrum). Other methods like direct imaging and astrometry (the latter of which will be pioneered by the recently launched GAIA) are considerably affected, in a negative way, by increasing distance to the exoplanet(s).

Simply put, the RV- and transit methods love to detect hot Jupiters.

1.2.2 Spin-orbit misalignment

Hot Jupiters have another property foreign to our Solar System - their orbital inclinations (also called spin-orbit misalignments) as measured relative to the spin axes of their host stars (by means of the Rossiter-McLaughlin effect described below) do not seem to have a terribly preferred value. As of 2nd Jan 2014, 67.8 % of hot Jupiters at exoplanets.org have inclinations above 8°[8]. This is indeed foreign to our Solar System wherein no planet is inclined more than Earth i.e. roughly 7°.

When plotting the spin-orbit misalignments (they are actually *sky-projected* versions of the real three-dimensional angles, which makes them lower limits) of these hot Jupiters (fig. 1), it is clear that there is something going on apart from the clustering around 0°. This would be expected if the origin of hot Jupiters is (at least partly) dynamically governed. One candidate mechanism that might account for this is the Kozai mechanism mentioned earlier.

[8] This selection required a (temporary) definition of hot Jupiters, chosen to include planets within 0.5 au and masses above 0.25 M_J, but also that the planet had a measured spin-orbit misalignment. 56 hot Jupiters were found on that day.

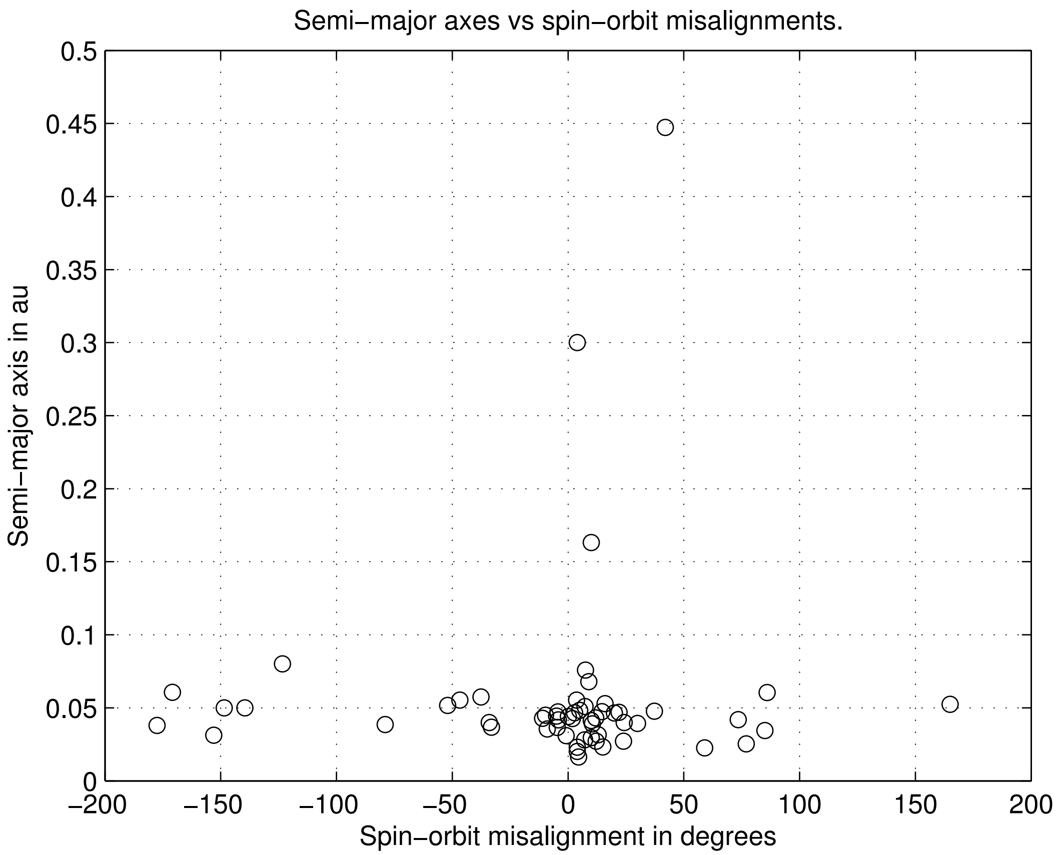

Figure 1: Projected spin-orbit misalignments for hot Jupiters, fetched from exoplanets.org on 2nd Jan 2014. The average misalignment is approximately -6° (rather than 0°), probably because 56 is a small sample of the total population of hot Jupiters, vulnerable to statistical fluctuations.

1.2.3 The Rossiter-McLaughlin effect

If one were to observe the average star with a spectrometer during a period of time within which some object, for whatever reason, transits the star, one would detect peculiarities in the star's spectrum. Strictly speaking, there are two conditions that need to be fulfilled for this to happen. The first is that the star rotates, and the second is that the transiting object does not transit perfectly along the rotation axis of the star.

Assuming that the star's intrinsic spectrum does not change appreciably from one point on its disk to another, the time-averaged spectrum that the observer detects is in principle a smeared out version of the stars 'true' spectrum. A 'true' spectrum is in this context defined as the spectrum that would be observed if its star did not rotate. The smearing is thus caused by the star's rotation (on the condition that the rotation axis is not perfectly aligned with our line of sight from Earth), and arises simply because half of the star's disk is rotating away from Earth, while the other half rotates towards Earth. This simultaneous red- and blueshift of the light manifests as a broadening or smearing of all spectral lines.

But what happens then if the transiting object obscures a region of the stellar disk? Given that the object does not transit perfectly along the star's rotational axis, it must be that one half of the star is being obscured more than the other at all times except the one instant during which the object crosses the axis (note that this does not necessarily happen). The effect of this situation on the detected spectrum is a slight shift. If the transiting object is obscuring a region of the stellar disk that is rotating towards the observer for example, a portion of the blue-shifted light from the star is removed, and the spectrum as a whole moves towards the red. Interpreting this as actual change in the radial velocity of the star is of course physically erroneous.

What would a graph of radial velocity against time look like as the object transits? The answer is that it depends, amongst other things, on the angle between the star's rotation axis, and the trajectory of the transiting object. If this were a right angle for example, the amplitude of the graph would be at a maximum, while if the angle approached zero, the amplitude would as well. This effect is called the Rossiter-McLaughlin effect, named after R. A. Rossiter and Dean Benjamin McLaughlin, who published papers on the subject to ApJ simultaneously in 1924 (McLaughlin, 1924; Rossiter, 1924). In practice, it permits determination of the *projected* spin-orbit misalignment angle. The angle is typically referred to as a spin-orbit misalignment as the transiting object is usually a binary companion or a planet.

1.3 The Kozai effect

The idea that birthed this thesis was that a dynamical peculiarity arising in some three-or-more-body systems, called the Kozai effect, might account for said spin-orbit misalignment and the close orbits of hot Jupiters. The Kozai effect was first noted by Lidov (1962) and Kozai (1962), and is consequently sometimes referred to as the Lidov-Kozai mechanism/effect.

When does the Kozai effect manifest itself? This is a rather complex question to answer, so let us begin by restricting ourselves to the simplest case. Consider a binary star, the primary of which hosts a planet. For the Kozai mechanism to operate in such a system, there is at least one condition that must be fulfilled, and that condition is that the mutual inclination between the orbits of the planet and the binary companion is larger than 39.23°[9] (e.g. Innanen et al. (1997)). Additional conditions may be introduced if the complexity of the system increases. Examples might include severely bloated stars, multiple planets in exotic resonances, external torques, etc.

How does the Kozai effect manifest itself? This also is a rather complex question, to which the answer is a function of circumstance. In the simplest system introduced in the previous paragraph, the Kozai effect manifests itself as long-period oscillations in the eccentricity and inclination of the planet (see figures 2 and 3). These oscillations are sometimes referred to as *Kozai cycles.* The eccentricity will oscillate between the planet's initial eccentricty and some maximum value e_{max} determined by the initial mutual inclination i_0 between the Kozai companion and the planet (Innanen et al., 1997)[10]:

$$e_{max} = \sqrt{1 - (5/3)\cos^2(i_0)}. \tag{1}$$

The mutual inclination will oscillate between its initial value and the critical angle mentioned above, 39.23°[11]. At the critical angle, the eccentricity is at its maximum, e_{max}. The timescale (time elapsed before reaching the first eccentricity maximum) can be estimated with another equation (Innanen et al., 1997):

$$\tau = 0.42 \frac{\ln(1/e_0)}{\sqrt{\sin(i_0)^2 - 0.04}} \frac{b^3}{GM} n. \tag{2}$$

Here, e_0 is the initial eccentricity of the planet, i_0 is the initial mutual inclination, b is the semi-minor axis of the Kozai companion, M is the mass of the Kozai companion, G is the gravitational constant, and $n = \frac{2\pi}{P}$ is the mean motion of the planet (P is its orbital period).

As the complexity of the system increases, so does the complexity of the Kozai effect manifestation. Two planets whose orbits are resonating for example, may very well alter the type of outcome described in the simplest case. Beyond

[9]This condition is probably not fulfilled if the planet and stars were formed together - in such a situation, the mutual inclinations should tend to zero as the orbits align with the total angular momentum vector. The critical value of the mutual inclination given assumes circular orbits, it changes with eccentricitiy.

[10]This equation is valid only if the initial eccentricity is close to zero.

[11]Or 140.77° for retrograde orbits.

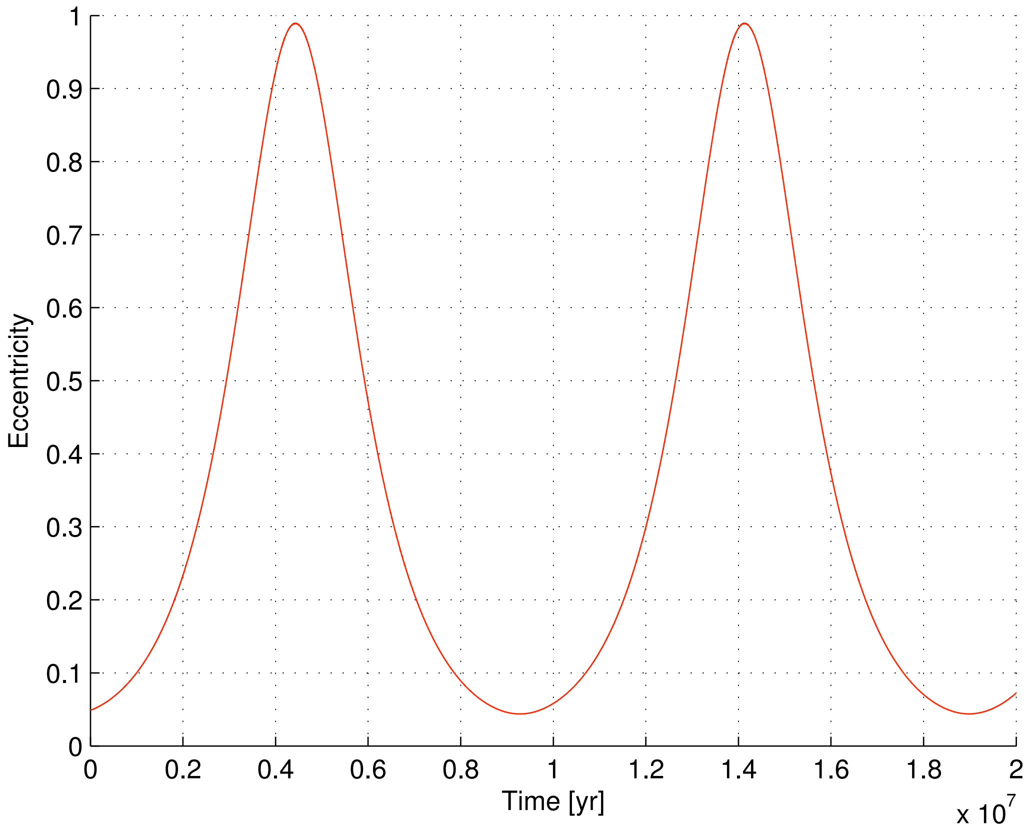

Figure 2: An example of a simple Kozai effect manifestation. In terms of Jupiter's eccentricity, this is what will happen to Jupiter if the Sun had a Kozai companion, and Jupiter were the sole planet of the Solar system. The Kozai companion's orbit is circular and has a semi-major axis of 500 au, while the initial mutual inclination between the Kozai companion and Jupiter is $\sim 87°$.

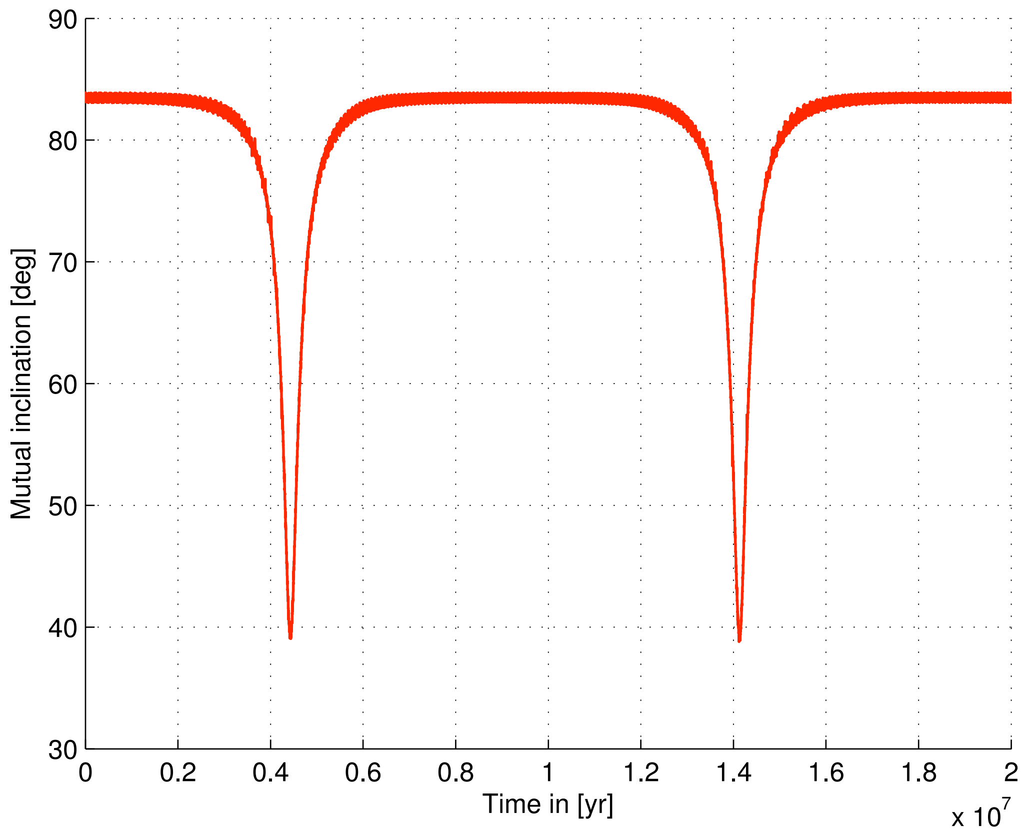

Figure 3: Just like figure 2, but in terms of the mutual inclination between Jupiter and the Kozai companion. Note how the minimum here coincides with the maximum in figure 2.

this simplest case, predicting particular manifestations of the Kozai effect in a particular system is not at all trivial. Simulations will therefore be paramount in developing an understanding of the Kozai effect and what it can do to realistic planetary systems.

1.4 Tidal interactions

As the separation between objects (celestial bodies) becomes comparable to the sizes of the bodies themselves, it could happen that the gravitational force gradient across one or both bodies is significant enough to have appreciable consequences. Very eccentric planets, for example, may find themselves in this situation as they pass through their periastra.

Tidal locking is probably the most well-known consequence of tidal interactions, owing to the fact that the Moon is tidally locked to the Earth, and that Mercury is almost tidally locked to the Sun. It forces the spin periods of one or more orbiting bodies to equal their respective orbital periods, i.e. they show the same face to whatever they are orbiting, at all times.

For the purpose of this thesis, the most important consequence of tidal interactions is circularization of the orbit, also referred to as tidal capture. As the name implies, this is a phenomenon that slowly transforms an eccentric orbit into a more circular orbit the radius of which will be about the size of the original orbit's periastron distance (e.g. Mardling (1996)). This might be a production channel for hot Jupiters: a jovian planet is formed, undergoes Kozai cycles, obtains a very high eccentricity, and is tidally captured by its sun. This process will be referred to as tidal capture by Kozai resonance (TCKR).

1.5 Open clusters

What is the ideal environment for such a chain of events? The first step is to form a large planet - this probably requires high metallicity (Jupiter's central regions probably consist of up to 30 Earth masses worth of $Z > 2$ material[12], e.g. de Pater and Lissauer, 2001), and so globular clusters are erased from the list of candidate environments, leaving open clusters and lower density star-forming regions.

The second step is for the Kozai effect to operate - this requires a binary companion unless the jovian-hosting star already had one[13]. Acquiring or losing a binary companion is a dynamic process the frequency of which obviously benefits from a high stellar density. Globular clusters are probably the densest stellar regions, but globular clusters, as mentioned, typically have low metallicities (e.g. Harris et al. (1992)), and run the risk of forming very few planets of jovian size (Buchhave et al. (2012)). Thus, open clusters emerge as the most promising candidate environment.

[12] Z is the atomic number, equal to the number of protons in the nucleus of an element.
[13] Actually, an outer planet on an inclined orbit could fill this role also, see section 1.6.

1.6 Potential TCKR processes

In an open cluster, the process that might produce hot Jupiters by means of the Kozai effect would go something like this:

1. A single protostellar disk forms a star and a jovian planet.

2. This system exchanges into a binary in the cluster[14], allowing for the Kozai effect to operate, and tidal capture ensues.

3. At some point during or after the capture, our planet-hosting star is relieved of its companion in a second encounter with some star in the cluster. What remains as the cluster disintegrates is a single star (later referred to as an *in-and-out binary*, owing to its history) with a hot Jupiter.

This process is a bit contrived in that it requires rather unlikely events to occur (the binary exchanges).

Note that the last step is not necessary for the hot Jupiter production itself, but hot Jupiters have been observed around single stars, and this must be accounted for somehow.

An alternative process, which might or might not work, would go something like this:

1. A single protostellar disk forms a star and a jovian planet, plus a second planet further out.

2. This system undergoes a close encounter with another cluster star, which excites the outer planet's orbit to a high inclination.

3. This outermost planet takes the role of a Kozai companion, imposing Kozai cycles on the inner jovian, leading to TCKR and a hot Jupiter.

This version is promising in the sense that it is not as contrived as the previous one. Close encounters are necessarily more common than binary exchanges. The problem here though is that the planet mass might be too small to have a strong enough influence on the inferior planet. This will be investigated to some extent.

1.7 Secular oscillations

It would be useful to know if multiplanetary systems can undergo the TCKR processes laid out above without changing the outcomes. One might think at first glance that the presence of multiple planets would simply impose the Kozai cycles on all planets simultaneously, leading to chaos until only a single planet remains (a potential hot Jupiter). Alas, planets in a multiplanetary system often undergo *secular oscillations* (e.g. Murray and Dermott (1999)). These are long-period oscillations in inclination and/or eccentricty of two or more

[14] Exchanging into a binary means taking one of the binary's stars' place, sending a previously bound star out of the potential well.

planets arising from the mutual torque between their orbits. They are stronger between planets of high mass and small orbital separations, since gravity is the force at play.

The Kozai effect itself can be regarded a resonant secular oscillation of sorts, so it is conceivable that secular oscillations interfere significantly with the Kozai effect. It is for example possible that they suppress the Kozai cycles that a single planet would have experienced, either partly or completely.

A primary objective of this thesis is to investigate how secular oscillations in multiplanetary systems change the Kozai cycles - are they a hindrance to TCKR, or perhaps beneficial?

A couple of example plots of what secular oscillations might look like can be found in figures 4 and 5.

1.8 Previous Research

The idea that the Kozai effect might have a role to play in the histories of planetary systems is not new. There are many articles written since the original papers that explore the Kozai effect. Some examples include Fabrycky and Tremaine (2007), who investigated if Kozai oscillations were to blame for the fact that very tight stellar binaries tend to have a third companion further out; Innanen et al. (1997) who explored the stability of planetary systems within a binary star; Naoz et al. (2012) who synthesized a population of hot Jupiters from jovian planets inside binaries, and many more.

1.9 Summary

Planets exist, and not only around the Sun. If a planet is orbiting a star other than the Sun, they are *exoplanets,* and a subset of these are *hot Jupiters.* Hot Jupiters are exoplanets of jovian mass on very tight orbits (typically a few percent of an au), and their existence is difficult to explain with contemporary planet formation theory alone.

The Kozai effect is a three-or-more-body phenomenon that may periodically boost the eccentricity of one or more planets up to ~1, and the implied meager periastra distances can allow for tidal capture, which might result in a hot Jupiter. This thesis aims to investigate, by means of N-body simulations, how secular oscillations between planets alters the Kozai effect's ability to produce hot Jupiters.

Section 2 describes some details about and around the simulations, section 3 presents investigated systems and the respective results and then finally there are conclusions in section 4.

2 Simulations

2.1 Simulation software

> A hybrid symplectic integrator that permits close encounters between massive bodies.

These are the words that John E. Chambers, the author of the program MERCURY, chose as title for his 1999 paper (Chambers, 1999), in which he describes the program in detail. Its main distinguishing feature is its ability to integrate systems with two different algorithms at the same time - it uses symplectic maps for everything except close encounters, during which a conventional integrator takes over. This is useful because sympletic integrators are unique in their ability to conserve energy over long integration periods, but they don't handle close encounters particularly well, so for those situations some other integrator e.g. Bulirsch-Stoer is employed until the encounter is over. When it is, the symplectic integrator resumes. The naive result is an integrator that is faster (symplectic integrators are generally very fast) and just as good as but probably better than conventional integrators at conserving energy (at least over long integration periods).

For the purpose of this thesis, MERCURY is the N-body solver that will be used when investigating the Kozai mechanism. It might be of interest to know that the integration algorithm used in practice for this thesis was 'BS2', a Bulirsch-Stoer algorithm.

2.2 Simulation hardware

The vast majority of simulations were performed with the Platon system, one of several supercomputers at the LUNARC facility in Lund, Sweden. The NOTA research group at Lund Observatory has access to 64 dedicated processors there, all of which were used for the simulations done as part of this thesis. Simulation time varied considerably, depending primarily on the time-resolution of the MERCURY output, time required for the integrator to finish ('integration time'), and the 'density' of the planetary systems.

2.3 Simulation approach

The objective is to probe how different planetary systems respond to the presence of a Kozai companion. To do this, the approach was to, for a given system and Kozai configuration, integrate the system 360 times. The first run set the initial mean anomaly of the Kozai companion to $1°$, the second run set it to $2°$, and so on. The idea is to change the system slightly so that statistics can be gathered (the N-body problem is chaotic in general), and the mean anomaly was deemed sufficient for this purpose.

Determining which configurations to simulate was done on the fly, as results were amassed. It was always the case however, that the initial inclination of the

Kozai companion was either 49°, 59°, 69°, 79°, or 89°[15].

Preparing directories and files for the supercomputer was primarily done with simple Fortran 77 programs (along with a few bash scripts) written specifically for this purpose. Analyzing the results was done with programs written in both Fortran 77 and Matlab. Plotting was done with Matlab.

2.4 Estimating the hot Jupiter production with TCKR

To estimate the number of produced hot Jupiters in an open cluster, the reference cluster in Malmberg et al. (2007) will be the starting point. It consists of 700 stars, of which 232 stars (20 %) are initially in binary systems. After $9 \cdot 10^8$ years, 21 stars (3 %) were in-and-out binaries, while 49 % (344) of stars had undergone close encounters without exchanging into or out of binaries.

Assuming that close encounters, regardless of distance at closest approach, always incline the outermost planet but affect nothing else, the hot Jupiter production estimate from the simpler TCKR process is quite straightforward:

1. Choose a planetary system and an inclination (‘configuration´) for its outermost planet
2. Simulate this system 360 times (using the initial mean anomaly to wiggle around the initial conditions, see section 2.3)
3. Note the hot Jupiter candidate frequency[16]
4. Repeat for as many different configurations as practically possible

From here, what’s left to do for each planetary system is this sum:

$$F = \frac{344}{700} \sum_{i=1}^{n} \frac{I_i}{180°} \frac{f_i}{360}$$

F is the estimated fraction of stars in the cluster that house a hot Jupiter candidate. The sum runs over the n simulated configurations: I_i is the width of the inclination span that configuration i is assumed to represent, and f_i gives that configuration’s number of produced hot Jupiter candidates. In this thesis, it was always the case that five configurations were tested, with inclinations equal to 49°, 59°, 69°, 79°, and 89°. These were assumed to represent inclinations according to this table (inclinations below 44° were assumed to not produce hot Jupiter candidates):

[15] It was the authors belief, during the early stages of this thesis, that peculiar/unphysical numerical effects might spring from a perfectly right angle. So 89° was instead used as the maximum inclination, which propagated downwards when keeping the spacing at 10°. In retrospect, this might have been unnecessary.

[16] A definition of ‘hot Jupiter candidate’ is needed in practice.

Inclination	Represented inclination span
49°	$[44, 54)^\circ$
59°	$[54, 64)^\circ$
69°	$[64, 74)^\circ$
79°	$[74, 84)^\circ$
89°	$[84, 90]^\circ$

The estimate is of course unrealistic, since all those 344 stars that underwent close encounters probably did not host the same planetary system, if they hosted planets at all. Still, it's an upper limit order-of-magnitude estimate of the hot Jupiter production in an open cluster.

For the contrived TCKR process, the estimation is a bit trickier, because the outcome is sensitive to the (binary) Kozai companion's semi-major axis and inclination, which both follow statistical distributions. The probability of exchanging into a binary with a semi-major axis a is proportional to a (Davies et al., 1993), so the probability of exchanging into and then out of a binary is taken to be proportional to a^2. The primordial distribution of binaries in the cluster is proportional to a^{-1}, so the total proportionality is just a. Meanwhile, the probability of exchanging into a binary with inclination i is proportional to $\sin i$. The limits of integration are $[1, 1000]$ au and $[0, 180]^\circ$[17], and integrating over both variables should yield 0.03 (21 stars out of 700), which gives the normalization constant. At this point it is 'known´ how common a particular Kozai configuration (now representing the semi-major axis and inclination *combination*) is. What then follows are these steps (for a unique planetary system):

1. Choose the mass and eccentricity of the Kozai companion. These values were never changed throughout this thesis project. The mass was set to 0.8 $M_\odot$, and the eccentricity to 0.001.

2. Choose a Kozai configuration

3. Simulate the chosen system and configuration 360 times (again using the initial mean anomaly to wiggle around the initial conditions)

4. Note the hot Jupiter candidate frequency of each configuration

5. Repeat for as many different configurations as practically possible

What's left is to perform the double Riemann integral:

$$F = N \iint a \cdot \sin(i) \cdot f(a, i)\, da di$$

F is the fraction of stars in the cluster that house a hot Jupiter candidate. N is the normalization constant mentioned earlier, a and i are the semi-major axis

[17] The semi-major axis limits are imposed by the definitions of a binary star used by Malmberg et al. (2007), while the inclination limits are implied by the fact that the Kozai cycles are assumed to be insensitive to whether the companion is pro- or retrograde.

and inclination respectively of the in-and-out binary, and f is the step-function the values of which are obtained from step 3 above.

When constructing f, each inclination is assumed valid as per the table above. It is moreover assumed that retrograde and prograde companion orbits are equivalent. As an example, 89° represents both $[84, 90]°$ and $(90, 96]°$.

Each semi-major axis, for the purpose of constructing f, is assumed valid between the half-way points of its adjacent configurations. For example, if all you have are three configurations with 75, 100, and 150 au, they are assumed valid between $[1, 87.5]$, $[87.5, 125]$, and $[125, 1000]$ au. The integration limits are always 1 and 1000 au, so that space must be filled. That is why the 75 and 150 au configurations, in this example, were assumed valid across a much larger space than the 100 au configuration is.

This contrived TCKR process suffers from the same problem as the simpler one, namely that all those 21 out of 700 stars are unlikely to host a particular planetary system, if they host planets at all (the estimate is thus an upper limit).

The estimates probably also suffer from the finite configuration space resolution. It would be better to test, say, 100 different semi-major axes and at least 10 different inclinations, but that would mean 360 times 1000 simulation runs to probe a single planetary system's susceptibility to TCKR. The resources available to the author of this thesis did not permit that.

3 Results

Here follows a walkthrough of the simulation results, one planetary system at a time. For the first two systems, detailed results are described after a brief introduction to the system and a summary of its results.

3.1 Jupiter and Saturn

The Solar System's two most massive planets are known to undergo secular oscillations with each other, as shown in figures 4 and 5[18]. As the two planets pull on each other with their mutual gravity, they exchange angular momentum. In other words, they oscillate secularly, the period of which is roughly 40,000 years. The eccentricity amplitude is about 0.015 and 0.035 for Jupiter and Saturn respectively, while the inclination amplitudes are about 1.0° and 2.3°, respectively.

The fact that Jupiter and Saturn oscillate secularly makes it an interesting system within the scope of this thesis since examining the behaviour of secularly oscillating systems in the presence of a Kozai companion is among the thesis objectives. That said, this simple system might not occur in nature, which could be regarded a downside. Even so it shall serve as a starting point owing to its simplicity (two planets only) and the fact that the accuracy to which we know its parameters is relatively fantastic[19]. The integration time was 10^8 years.

The most significant results extracted from the simulation runs include:

1. There is a critical Kozai configuration, possibly many. The $[200, 69]$ au/° configuration is stable, while the $[200, 79]$ au/° configuration is not. Somewhere between these two configurations, the secular oscillations overpower the Kozai companion's influence.

2. Saturn is in the majority of simulations the planet most likely to be removed.

3. Ejections and collisions with the Sun are about equally responsible for removing planets from the system.

4. The collisions with the Sun are necessarily caused and preceded by very eccentric orbits ($e \sim 1$), but their lifespans are almost always shorter than 10^4 years, and always shorter than 10^5 years. This is probably not a viable way of producing hot Jupiters.

5. The accumulated time that single survivors spent on orbits the periastra of which were smaller than 0.05 au is typically millions of years, and does not seem to correlate with configuration.

[18]Both of these figures were generated with data from a MERCURY simulation. Obviously it's impossible (during a human lifetime) to actually measure several periods of this oscillation, given its length.

[19]Physical quantities used to integrate this system, such as planet mass, orbital elements and ephemeris, etc., were fetched with the HORIZONS ephemeris tool, supplied by NASA and available on the web. 2000-01-01 was the reference date.

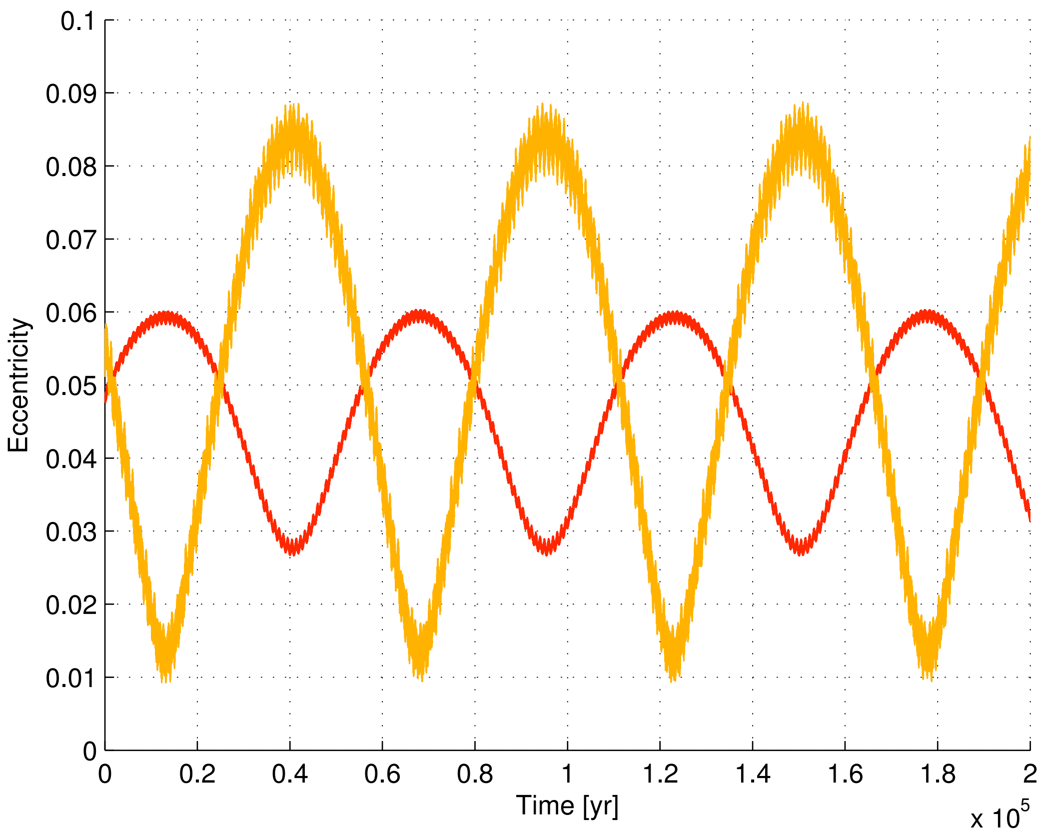

Figure 4: The secular oscillations between Jupiter and Saturn involves an exchange of eccentricity. Jupiter in red, Saturn in orange.

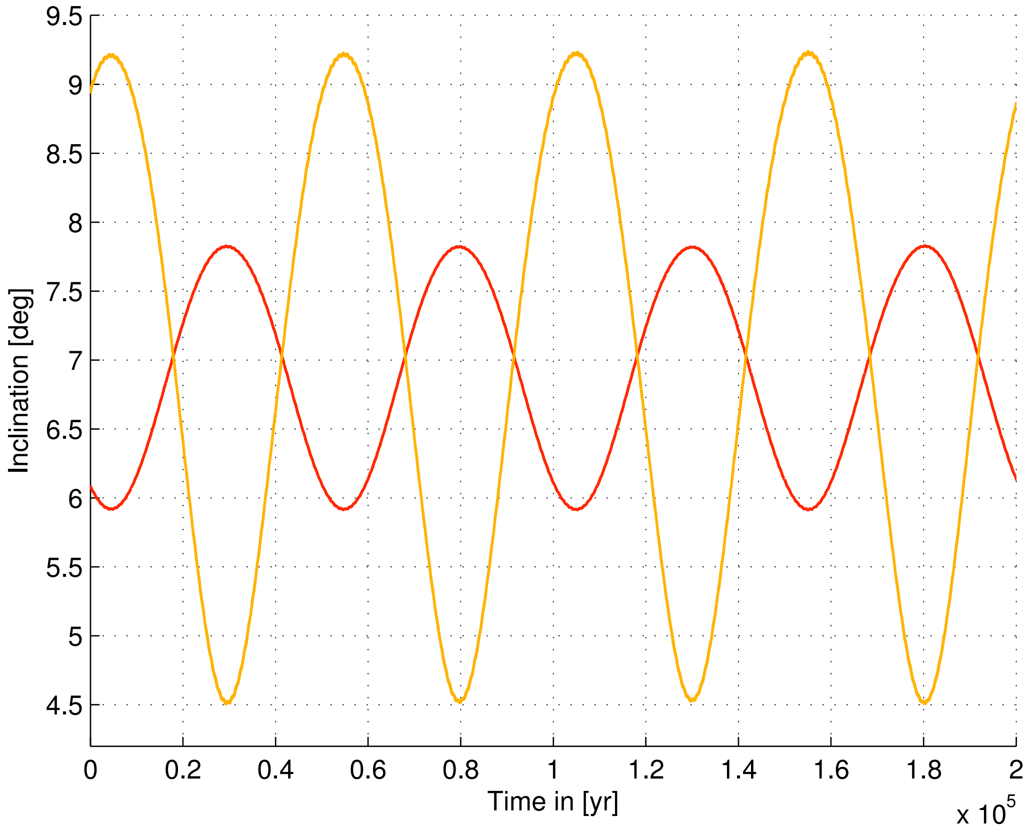

Figure 5: The secular oscillations between Jupiter and Saturn also involves an exchange of inclination. Jupiter in red, Saturn in orange.

6. This system is less likely to produce hot Jupiters compared to single-planet systems.

3.1.1 Result #1 - there is a critical Kozai configuration

Figure 6 is a plot showing which configurations were stable and which were not. Stability in this context is achieved if none of the 360 simulations removed a planet, while instability is achieved if one or more simulation runs removed a planet. Empty markers in figure 6 represent unstable configurations, while blue markers represent stable configurations.

The Kozai timescale as calculated with eq. 2 gives $\sim 10^5$ years for all Kozai configurations with a semi-major axis of 200 au. Bearing in mind that the secular oscillation period for Jupiter and Saturn is $\sim 4 \cdot 10^4$ years, there is no apparent 'coincidence' here.

The right-hand table in table 1 (which actually contains two tables) shows exactly how unstable or stable each configuration was. The $[150, 49]$ au/° and $[200, 79]$ au/° configurations are very nearly stable, implying that there are several critical configurations. It is not unthinkable that there is a critical *curve* in figure 6, beyond which (beyond as in wider orbit and less inclination) the Kozai effect shuts down.

3.1.2 Result #2 - Saturn is the less lucky one

The left-hand table in table 1 contains the frequencies of Saturn's removal[20]. It is apparent that Saturn was the less lucky planet. This is to be expected since Saturn's orbit is larger and so not as tightly bound as Jupiter is.

Looking at the tables, one notices a break in the trend - at the highest inclinations, it is suddenly the other way around - Jupiter is more likely to bite the dust, although admittedly both planets lead dangerous lives there. This is somewhat expected, given that Jupiter's predicted periastron distance in those configurations is about 0.001 au (using eq. 1), considerably smaller than the radius of the Sun (0.005 au). So even if Saturn was removed swiftly, Jupiter would risk collision with the Sun. This is consistent with the fact that those configurations in which Jupiter and Saturn were equally likely to be removed, were also the configurations that were most likely to remove *both* planets.

3.1.3 Result #3 - Ejections and Solar collisions

Table 2 contains the collision frequencies of all simulated configurations, calculated as collisions per planet removal. The ejection frequency is equal to the difference between the collision frequency and the number one. The point to take home from this table is that the collision frequency increases with initial inclination. This is perhaps not so surprising, since the Kozai cycle eccentricity peak increases with initial inclination (eq. 1), and higher eccentricity should lead to both an increase in orbit-crossing i.e. planet-planet scattering and a decrease

[20] Removals occur either by a Solar collision or ejection.

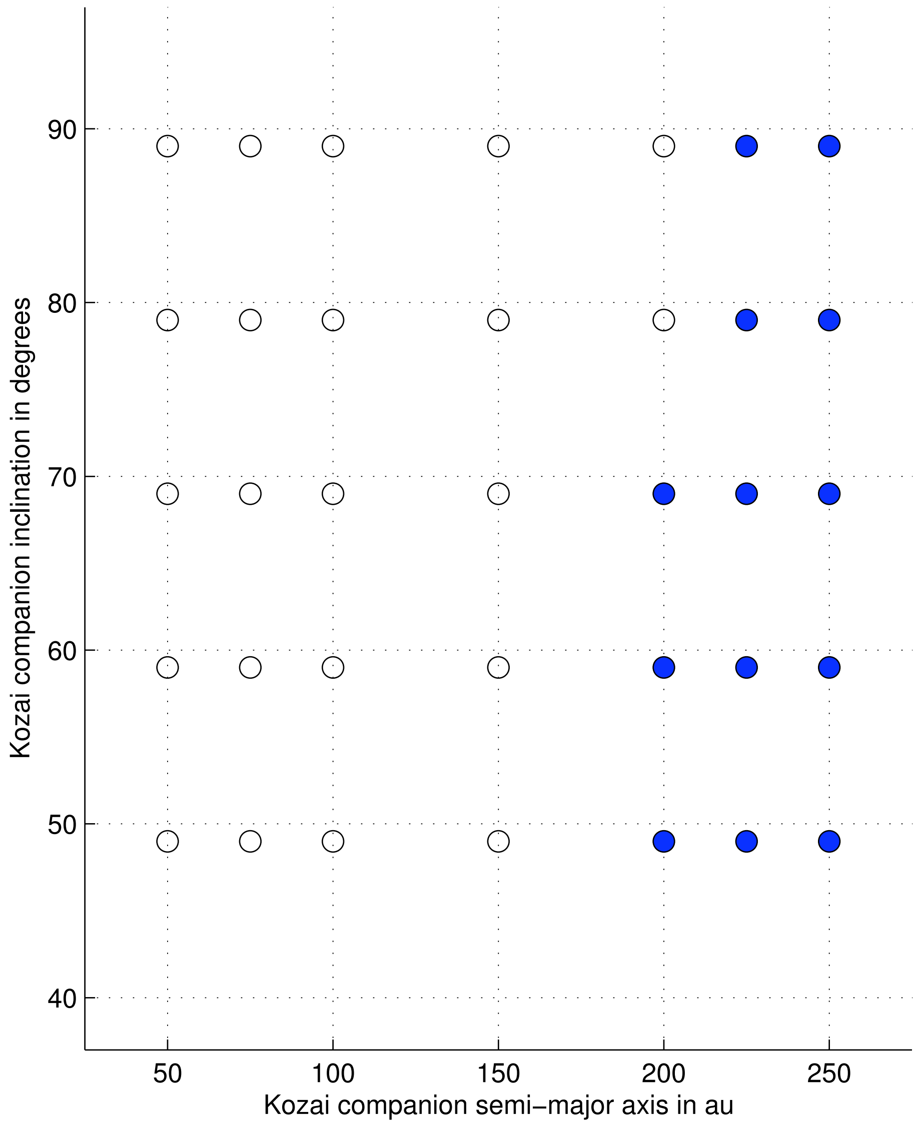

Figure 6: In this plot, each marker represents the 360 simulation runs of a particular configuration. On the horizontal axis is the initial semi-major axis of the Kozai companion orbit, while the initial inclination of the Kozai companion is on the vertical axis. If the marker is empty, at least one of the 360 runs was unstable at some point during the integration. If the marker is filled, all of the 360 runs were stable throughout the entire integration time of 10^8 years.

	50	75	100	150	200	225	250
49	0.997	1	0.997	1	-	-	-
59	0.997	1	0.994	1	-	-	-
69	1	1	0.991	0.997	-	-	-
79	0.803	0.810	0.695	0.826	0.785	-	-
89	0.450	0.471	0.475	0.429	0.533	-	-

	50	75	100	150	200	225	250
49	360	360	360	7	0	0	0
59	360	360	360	360	0	0	0
69	360	360	362	360	0	0	0
79	361	364	361	363	14	0	0
89	631	611	623	631	515	0	0

Table 1: The leftmost columns give the initial inclinations (°), and the top rows give the semi-major axes (au). In the left table are the frequencies of Saturn's removal, as fractions of the total number of planets removed in all 360 runs of each configuration, which are in the table on the right.

	50	75	100	150	200	225	250
49	0.258	0.338	0.405	0	-	-	-
59	0.325	0.463	0.638	0.380	-	-	-
69	0.516	0.627	0.707	0.519	-	-	-
79	0.581	0.818	0.797	0.688	0.571	-	-
89	0.808	0.888	0.911	1	0.817	-	-

Table 2: Collision frequencies, i.e. the fractions of planet removals caused by collisions with the Sun. The leftmost column and top row are as in table 1.

in the number of scatter events needed to end up on a collision course. There is another noteworthy fact however - the frequency peaks between 100 and 150 au. This is interesting, because while there was no 'coincidence' between the critical configuration and the period of the secular oscillations between Jupiter and Saturn, there is such a coincidence here. The Kozai timescales (calculated with eq. 2), are between 10^4 and $4 \cdot 10^4$ years for the peaking configurations (shorter for smaller semi-major axes and lower inclinations), which is eerily similar to the secular oscillation period between Jupiter and Saturn at $4 \cdot 10^4$ years.

It is interesting to know which of the two planets is experiencing the collisions. Table 3 gives the fractions of Solar collisions that Saturn is responsible for. The trend is fairly obvious - higher inclinations increase the probability of collision, while also evening out odds between the planets. At the highest inclinations, the distribution is more or less 1:1, if one ignores the $[50, 89]$ au/° configuration.

3.1.4 Result #4 - Solar collision timescales are probably too short

With so many Solar collisions, one might wonder if the very eccentric orbits preceding the collisions might be enough to produce hot Jupiters. Table 4 lists in its left column the lifetime of those orbits that ended up colliding the planet into the Sun. The lifetime is defined as the time spent with a periastron smaller than 0.05 au. In the right column is listed the total number of collisions that followed orbits that had lifetimes according to the left column. Unfortunately,

	50	75	100	150	200	225	250
49	1	1	1	-	-	-	-
59	1	1	1	1	-	-	-
69	1	1	0.988	0.994	-	-	-
79	0.661	0.781	0.621	0.768	0.75	-	-
89	0.850	0.565	0.536	0.429	0.617	-	-

Table 3: In each cell is the fraction of Solar collisions that Saturn was responsible for. The leftmost column (°) and top row (au) specify what configuration each cell stems from.

Orbit lifetime (years)	Number of collisions
$< 10^4$	5685
$10^4 < 2 \cdot 10^4$	13
$2 \cdot 10^4 < 3 \cdot 10^4$	2
$3 \cdot 10^4 < 4 \cdot 10^4$	1
$7 \cdot 10^4 < 8 \cdot 10^4$	1

Table 4: On the left are the lifetimes of the pre-collisional orbits, and on the right is their frequency.

probably none of these pre-collisional orbits have a chance at significant TCKR before the planet is destroyed. Typical timescales for tidal interactions are millions of years (Jackson et al. (2008)), rather than thousands as for the pre-collisional orbits.

The orbit lifetimes are expressed the way they are because the output resolution of the simulations was 10^4 years.

3.1.5 Result #5 - Accumulated time at eccentricity peaks

In most of the simulation runs, the typical outcome was that one planet survived and one was removed (e.g. table 1). These surviving planets, ‘single survivors’, are easily affected by the Kozai companion, and so enter a Kozai cycle. At the eccentricity peaks, a subset of the single survivors’ periastra are smaller than 0.05 au. If one calculates the accumulated time spent on these orbits, the typical answer is a few million years (figure 7). This does not appear to correlate strongly with configuration.

It should be noted that the configurations in which the Kozai companion’s semi-major axis is small have faster Kozai cycles in general, and so the risk is high that the period of time in which the single survivor orbits with a periastron smaller than 0.05 au is not resolved. If it is not resolved by the output timestep, 10^4 years, the accumulated time will be zero. This is not a catastrophe if one assumes that the runs that successfully accumulate time are somewhat representative of their configuration. Either way, it is clearly the case that a fraction of single survivors spend enough time with small periastra for tidal interaction to be significant.

Figure 7 features 56 plot markers. Each marker represents a single survivor's successful accumulation of time spent on an orbit whose periastra was smaller than 0.05 au during the Kozai cycle eccentricity peak. Out of these 56 markers, 46 stem from the $[200, 89]$ au/° configuration, while the ten leftmost ones stem from $[\leq 150, \geq 79]$ au/° configurations. The fact that only $\geq 79°$ configurations contribute is probably due to the fact that inclinations closer to 90° are more likely to produce eccentricities high enough for a periastron smaller than 0.05 au (e.g. equation 1), while the fact that the $[200, 89]$ au/° configuration is dominant is probably due to its large semi-major axis. This stretches the Kozai cycle in time (see equation 2), increasing the likelihood for resolving the eccentricity peak and thus successful accumulation of time spent there.

The horizontal axis in figure 7 is 'almost meaningless' because its only function is to separate the markers and in a somewhat clumsy way correlate with the semi-major axis of configurations. As was implied above, the semi-major axis increases to the right. The leftmost marker, for example, is from the $[50, 89]$ au/° configuration. While the spread in accumulated time increases to the left, it is still of the same magnitude (if one ignores the outliers near the bottom).

3.1.6 Result #6 - Hot Jupiter candidate production

The number of hot Jupiter candidates produced is counted in two different ways, one optimistic, and one conservative. The optimistic method is to assume that all single survivors (planets that remained in the simulation after all other planets had been removed) that at some point reached periastra smaller than 0.05 au during their Kozai cycle eccentricity peaks become hot Jupiters. The conservative method is much the same, but it requires in addition that the accumulated time spent at periastra smaller than 0.05 au is 'countable'. It is countable only if each Kozai cycle peak at which the periastron is smaller than 0.05 au, lasts longer than the output timestep i.e. 10^4 years. This might still be a fairly optimistic approach.

Following the contrived TCKR procedure outlined in section 2.4, table 5 is filled with the resulting numbers. The 'cluster fraction' is the fraction of stars within the cluster that produced a hot Jupiter through the contrived TCKR process. The 'in-and-out binary fraction' is the fraction of in-and-out binaries that produced a hot Jupiter (the only difference between the two fractions is thus a factor of $0.03 \cdot 700 = 21$). These numbers are more interesting when put into perspective, so in table 6 is listed the theoretically predicted hot Jupiter productions of some imaginary single-planet systems. In the leftmost column is listed which planet is considered, while the other columns are as in table 5. The numbers presented here are as mentioned theoretical, i.e. not the result of any simulations. The fraction is determined by assuming that all configurations in which the estimated Kozai cycle peak gives a periastron of less than 0.05 produce a hot Jupiter, always. In this sense it's perhaps best compared to the optimistic approach.

If one can trust the numbers, it is clear that the secular oscillations between

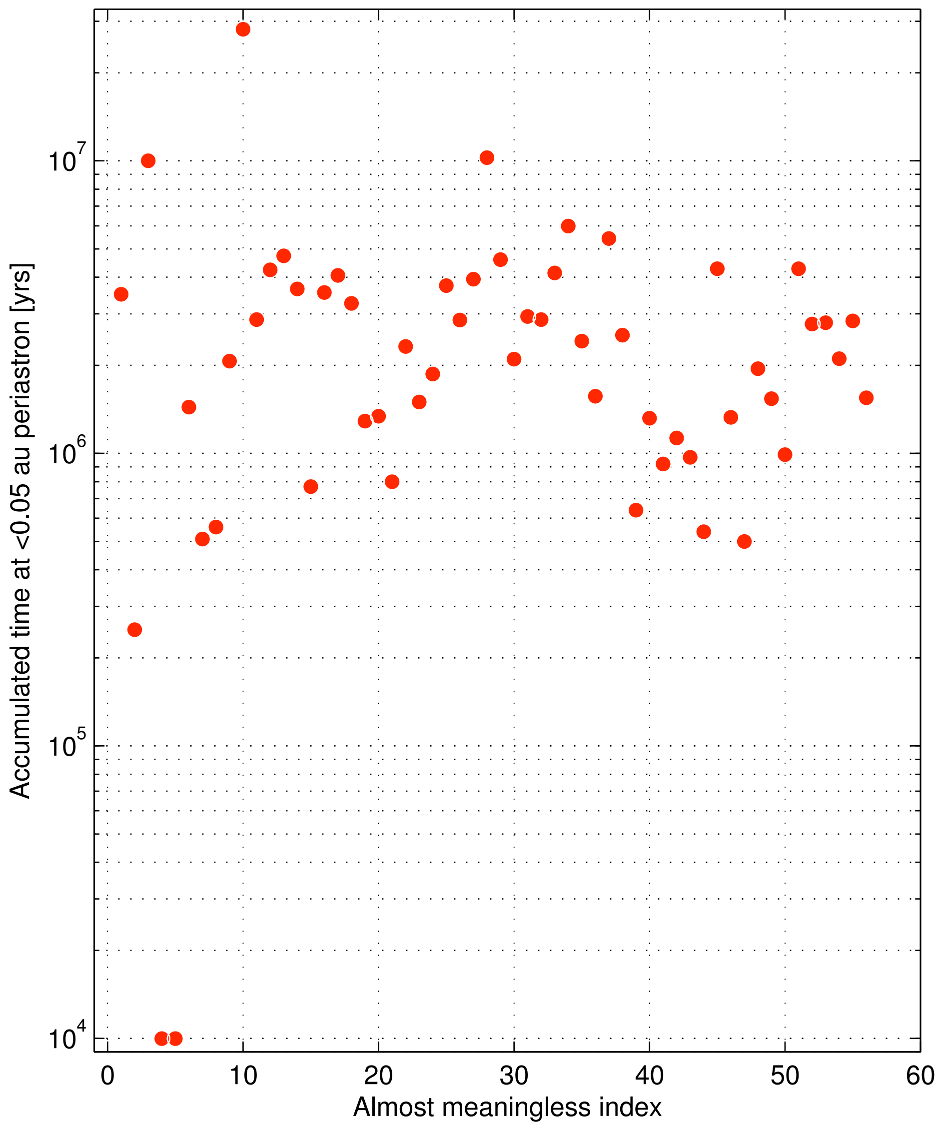

Figure 7: Each marker represents a single simulation run's successful accumulation of time spent on orbits whose periastra were smaller than 0.05 au during the Kozai cycle eccentricity peaks. The total amount of markers is 56, of which the 46 rightmost ones come from the [200, 89] au/° configuration. The ten leftmost ones come from all other configurations with either 89° (8) or 79° (2) initial inclination.

	Cluster fraction	In-and-out binary fraction
Conservative	0.000006	0.000207
Optimistic	0.000033	0.001119

Table 5: In this table are the conservative and optimistic hot Jupiter production fractions in the whole cluster (middle column), and in the in-and-out binary subset (right column), that the Jupiter and Saturn system produced.

	Cluster fraction	In-and-out binary fraction
Jupiter	0.002069	0.068973
Saturn	0.001540	0.051334
Uranus	0.001093	0.036449
Neptune	0.000843	0.028103

Table 6: The hot Jupiter production fractions for four imaginary single-planet systems, with columns as in table 5.

Jupiter and Saturn are detrimental to the production of hot Jupiters. Perhaps mainly because the secular oscillations stop the Kozai cycles from occuring at and beyond ~200 au.

3.2 Solar System giants

The system of giant planets in the Solar System, i.e. Jupiter, Saturn, Uranus, and Neptune, is a natural stepping stone from the simplistic Jupiter and Saturn system. By adding Uranus and Neptune, the system becomes more realistic, and it is of interest to see if the two outer giants interfere significantly with the secular oscillations between Jupiter and Saturn. Integration settings were the same for this system as for Jupiter and Saturn.

The most significant results extracted from the simulation runs include, and is hopefully limited to:

1. The presence of Uranus and Neptune in addition to Jupiter and Saturn is detrimental to stability. An obvious critical configuration lies between the $[750, 59]$ and $[750, 69]$ au/° configurations.

2. Uranus and Neptune are in general most and equally likely to be removed during the chaos.

3. Collisions are on the whole less common than they were in the Jupiter and Saturn system.

4. The Solar collisions orbits are probably too short-lived for significant tidal interaction, though they're longer than they were in the Jupiter and Saturn system.

	50	100	150	200	250	300	400	500	750	1000
49	1080	1034	917	758	727	719	589	552	0	0
59	1080	1078	1071	966	867	813	726	694	0	0
69	1080	1080	1080	981	863	832	763	728	535	0
79	1080	1081	1083	1029	884	850	789	672	632	0
89	1346	1286	1234	1176	945	918	823	772	654	0

Table 7: Number of removed planets for each configuration of the Solar giants system.

5. As for Jupiter and Saturn, the accumulated time that single survivors spent on orbits the periastra of which were smaller than 0.05 au is typically millions of years (enough for significant tidal interaction), and does not seem to correlate with configuration.

6. The production of hot Jupiter candidates is better when compared to Jupiter and Saturn, but still worse than single-planet systems.

3.2.1 Result #1 - less stable than Jupiter and Saturn

Figure 8 is the stability plot for this system. Comparing it to the corresponding one for Jupiter and Saturn, it is clear that the Solar giants system is more prone to instability. Somewhere between $[750, 59]$ and $[750, 69]$ au/° lies a critical configuration at which the secular oscillations overpower the Kozai effect.

In table 7, which contains the total number of removed planets in each configuration, one can see that in contrast to the Jupiter and Saturn system, there are no apparent hints of multiple critical configurations. The idea that there is a critical *curve* of configurations is not directly disproved however. If one were able to look below 49° at 500 au, one would probably find a stable configuration.

Keep in mind that the maximum possible number of planet removals is 1440 (corresponding to removal of all four planets, in all the 360 runs of a single configuration), rather than 720 as was the case for Jupiter and Saturn.

3.2.2 Result #2 - Jupiter and Saturn are most likely to survive

Table 8 shows each planet's fraction out of the total number of removals (table 7). Each cell contains four numbers, the topmost of which is the fraction of planet removals due to Jupiter's removal. The second number is Saturn's fraction, followed by Uranus and Neptune.

Table 9 shows the mean number of removed planets in each configuration.

It takes some effort to find trends amongst all the numbers, but when looking at both tables 8 and 9, it is somewhat obvious that Uranus and Neptune are removed most of the time, along with Saturn if the semi-major axis is below 250 au or so. Jupiter is almost never removed unless the configurations are to the far left and bottom. As was the case for Jupiter and Saturn, high inclinations seem

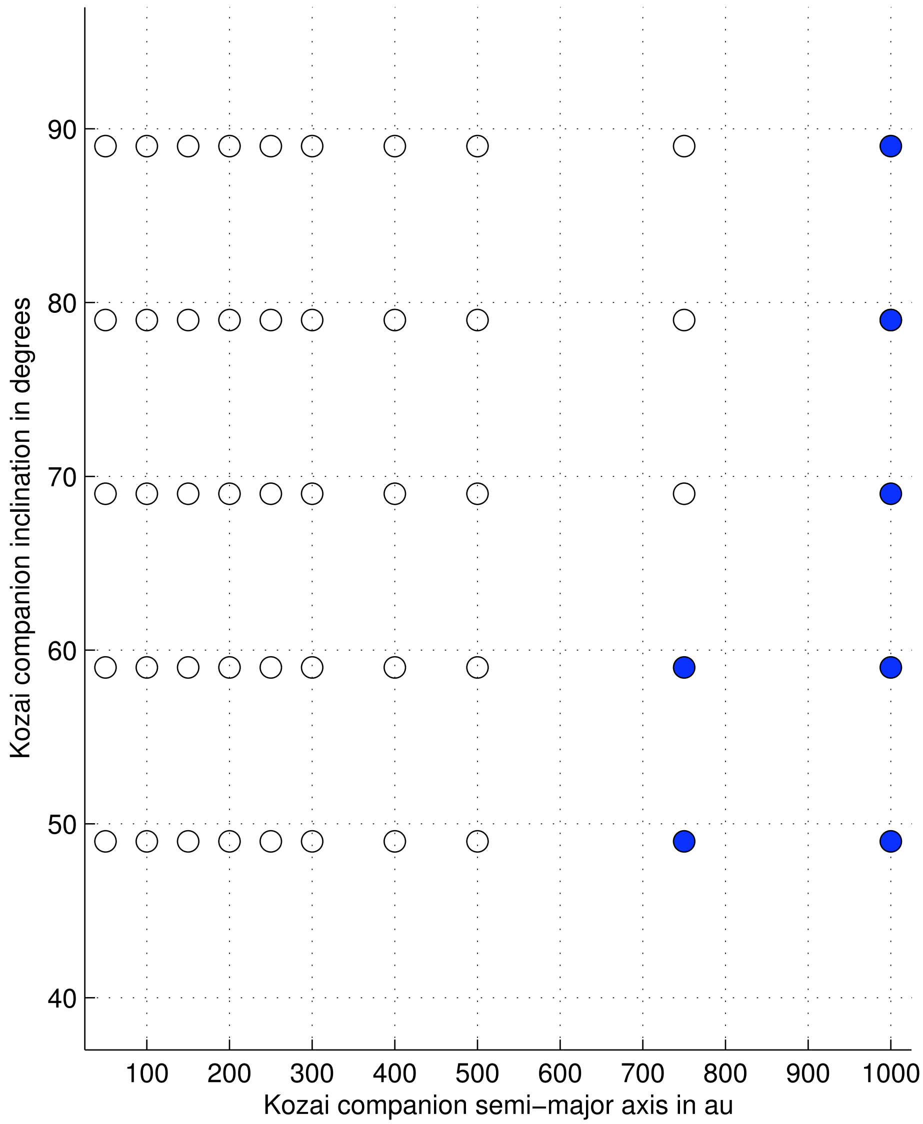

Figure 8: Stability plot for the Solar System giants with a Kozai companion. As in Jupiter and Saturn's corresponding plot, empty markers denote that at least one of the 360 runs of a particular configuration removed a planet within the integration time, while a filled marker denotes stability throughout the integration time.

	50	100	150	200	250	300	400	500	750	1000
49	0.0019 0.3315 0.3333 0.3333	0.0010 0.3472 0.3124 0.3395	0.0000 0.3762 0.3817 0.2421	0.0000 0.2876 0.4604 0.2520	0.0000 0.2105 0.4732 0.3164	0.0000 0.1530 0.4757 0.3713	0.0000 0.0832 0.5772 0.3396	0.0000 0.0562 0.5761 0.3678	-	-
59	0.0009 0.3324 0.3333 0.3333	0.0009 0.3330 0.3321 0.3340	0.0000 0.3361 0.3333 0.3305	0.0000 0.2681 0.3685 0.3634	0.0000 0.1811 0.4129 0.4060	0.0000 0.1402 0.4354 0.4244	0.0000 0.0895 0.4738 0.4366	0.0000 0.0735 0.4481 0.4784	-	-
69	0.0046 0.3287 0.3333 0.3333	0.0028 0.3306 0.3333 0.3333	0.0028 0.3306 0.3333 0.3333	0.0031 0.2640 0.3670 0.3660	0.0046 0.1645 0.4171 0.4137	0.0012 0.1382 0.4315 0.4291	0.0000 0.0773 0.4626 0.4600	0.0000 0.0659 0.4451 0.4890	0.0000 0.0654 0.3981 0.5364	-
79	0.0645 0.2726 0.3315 0.3315	0.1027 0.2313 0.3330 0.3330	0.0702 0.2650 0.3324 0.3324	0.0369 0.2663 0.3489 0.3479	0.0226 0.1652 0.4061 0.4061	0.0200 0.1341 0.4224 0.4235	0.0025 0.0887 0.4525 0.4563	0.0060 0.0804 0.3795 0.5342	0.0047 0.0665 0.3877 0.5411	-
89	0.2615 0.2036 0.2675 0.2675	0.2589 0.2379 0.2255 0.2776	0.2820 0.2164 0.2472 0.2545	0.1973 0.2245 0.2883 0.2900	0.1016 0.1471 0.3757 0.3757	0.0850 0.1351 0.3878 0.3922	0.0474 0.0863 0.4289 0.4374	0.0531 0.0829 0.4016 0.4624	0.0260 0.0443 0.4618 0.4679	-

Table 8: Inclination along the leftmost column, semi-major axis along the top row. The four numbers in each cell give, from top to bottom, Jupiter's fraction of the total amount of planet removals, followed by Saturn, Uranus, and Neptune at the bottom.

	50	100	150	200	250	300	400	500	750	1000
49	3	2.8722	2.5472	2.1056	2.0194	1.9972	1.6361	1.5333	0	0
59	3	2.9944	2.9750	2.6833	2.4083	2.2583	2.0167	1.9278	0	0
69	3	3	3	2.7250	2.3972	2.3111	2.1194	2.0222	1.4861	0
79	3.0167	3.0028	3.0083	2.8583	2.4556	2.3611	2.1917	1.8667	1.7556	0
89	3.7389	3.5722	3.4278	3.2667	2.6250	2.5500	2.2861	2.1444	1.8167	0

Table 9: The mean number of removed planets in each configuration. The most obvious trend is that the numbers decrease towards the top and right.

	50	100	150	200	250	300	400	500	750	1000
49	0.0898	0.2244	0.1810	0.1187	0.1100	0.0695	0.0662	0.0362	-	-
59	0.1324	0.2996	0.2698	0.1884	0.1592	0.0910	0.0716	0.0634	-	-
69	0.1537	0.3380	0.3491	0.2783	0.2005	0.1959	0.1219	0.0975	0.0692	-
79	0.2109	0.3839	0.4635	0.3819	0.2964	0.2471	0.1470	0.1354	0.0854	-
89	0.4376	0.6003	0.7634	0.6531	0.4931	0.3976	0.2977	0.2345	0.1177	-

Table 10: The collision frequencies for the Solar giants configurations. In general lower than for the Jupiter and Saturn system.

Orbit lifetime	Solar collisions
$< 10^4$	10207
$10^4 < 5 \cdot 10^4$	108
$5 \cdot 10^4 < 10^5$	37
$10^5 < 5 \cdot 10^5$	46
$5 \cdot 10^5 < 10^6$	15
$10^6 < 2 \cdot 10^6$	1

Table 11: On the left are the lifetimes of the pre-collisional orbits, and on the right is their frequency (total number of collisions is 10414).

to render all planets fair game. It is probably no coincidence that Jupiter and Saturn (but mostly Jupiter) experience a significant change in stability between 200 and 250 au. To the right of these configurations, Jupiter is quite safe even at the highest inclinations.

3.2.3 Result #3 - Ejections and Solar collisions

Table 10 is to the Solar giants system what table 2 is to the Jupiter and Saturn system. It contains the collision frequencies of each configuration.

Notable trends include an overall lower frequency than was seen for the Jupiter and Saturn system, especially at low inclinations, and also the skew peak between 100 and 150 au. This peak is slightly shifted towards 150 au, which actually makes it even more coincidental with the period of Jupiter and Saturn's secular oscillation (the timescales are between $1.5 \cdot 10^4$ and $4 \cdot 10^4$, rather than 10^4 and $4 \cdot 10^4$).

3.2.4 Result #4 - Pre-collisional orbits live longer, but not long enough

The pre-collisional orbits in the Solar giants system last considerably longer than they did in the Jupiter and Saturn system. It is probably not enough however. Only one pre-collisional orbit out of 10414 collisions lasted for more than a million years. Table 11 lists the lifetimes of the solar collisions and their respective frequency.

3.2.5 Result #5 - Accumulated time at eccentricity peaks

Why this is investigated is explained in the corresponding Jupiter and Saturn result. For the Solar giants, the plot looks like figure 9. What's noteworthy here is that the number of plot markers is almost five times as high as it was for Jupiter and Saturn (56 vs 256). The accumulated times are also shifted slightly up towards 10^7 years, though the maximum time (almost $2 \cdot 10^7$) is a bit lower than the maximum time in Jupiter and Saturn's corresponding plot (almost $3 \cdot 10^7$).

One might think that the number of markers is high because the Solar giants system was sensitive to Kozai cycles all the way up to 750 au, meaning a large part of all configurations featured large semi-major axes, which in turn means longer Kozai timescales that increase the probability for successful accumulation of time at the eccentricity peaks. However, almost half of the markers in figure 9 stem from the $[200, 79 \rightarrow 89]$ and $[250, 79 \rightarrow 89]$ au/° configurations, not the 500 or 750 au ones (to be fair, the 300 and 400 au configurations contribute with about 35 markers each). 27 markers stem from 500 au, and 15 markers from 750 au. The remaining markers stem fairly evenly from all other unstable 79° and 89° configurations.

The fact that the 'marker density' is peaking around 200 and 250 au is interesting. The Kozai companion's ability to remove planets and produce single survivors is evidently (from table 7) stronger at smaller semi-major axes, but at the same time the Kozai timescale increases with larger semi-major axes, increasing the likelihood of resolving the eccentricity peaks. Could it be that the peak represents the optimal compromise between the strength of the Kozai companion's influence and the Kozai timescale?

There are markers near the bottom, just above the output timestep. Also present in Jupiter and Saturn's corresponding plot, these markers could represent eccentricity peaks the duration of which is *sometimes* longer than the output timestep, but mostly not. Alternatively, they represent eccentricity peaks whose peak value fluctuates over the value corresponding to a periastron smaller than 0.05 au[21]. It is plausible that the markers between 10^4 and 10^6 represent eccentricity peaks that are successfully accumulated to varying degrees.

3.2.6 Result #6 - Hot Jupiter candidate production

With a method identical to the one used for Jupiter and Saturn, the Solar giants system's affinity for producing hot Jupiters is summarized in table 12. Comparing the numbers with those in tables 5 and 6, it would seem as though the Solar giants system is better at producing hot Jupiters than the Jupiter and Saturn system is, but still not as good as any of the theoretical single-planet systems.

[21] If it hasn't been mentioned already, Kozai cycles are not perfectly uniform in time. The cause is unclear. Perhaps it's simply a result of integration error, but it could also be a short timescale perturbation caused by occasional 'close' encounters between a planet and the Kozai companion. This would mean that it's no coincidence that the lower markers tend to the left (where the configurations with smaller semi-major axes are plotted).

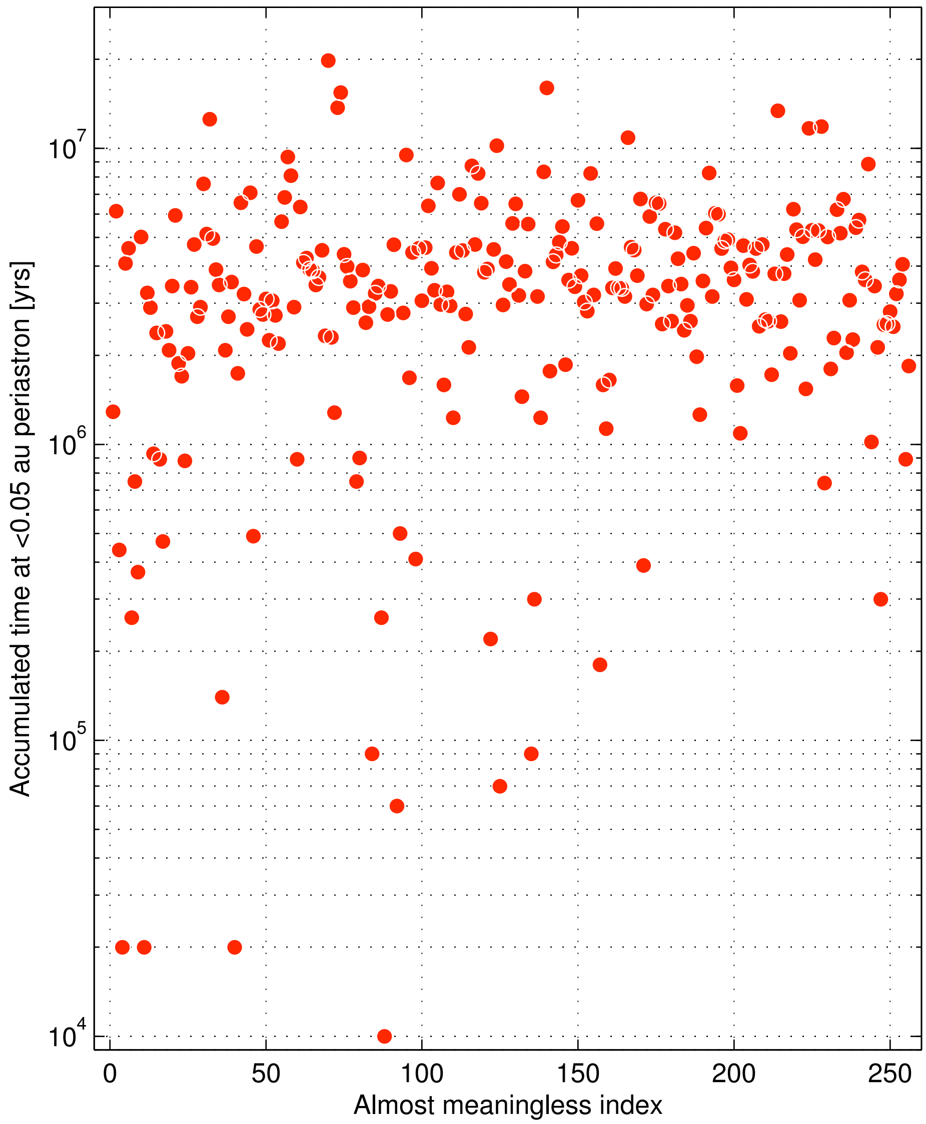

Figure 9: Each marker represents a single simulation run's successful accumulation of time spent on orbits whose periastra were smaller than 0.05 au during the Kozai cycle eccentricity peaks. The total amount of markers is 256, of which almost half come from the [200, 89] and [250, 89] au/° configurations. The rest come fairly evenly from the other 79° and 89° configurations.

	Cluster fraction	In-and-out binary fraction
Conservative	0.00019	0.0041
Optimistic	0.00023	0.0049

Table 12: In this table are the conservative and optimistic hot Jupiter production fractions in the whole cluster (middle column), and in the in-and-out binary subset (right column), that the Solar giants system produced.

The reason for why the Solar giants system is better at hot Jupiter production than the Jupiter and Saturn system is probably tied to the destabilizing impact of Uranus and Neptune. This effectively opens up the larger semi-major axis configurations, so that they can contribute with their hot Jupiter candidates.

3.3 Kepler-30

Kepler-30 is a planetary system detected with the Kepler satellite. The star's mass and radius are 0.99 Solar masses and 0.94 Solar radii respectively. Kepler-30 was chosen for investigation because

- It contains three detected and confirmed planets, as opposed to two or four planets as in the previously examined systems.
- It is a realistic system, since it evidently occurs in nature, and
- Its planetary orbits extend to roughly 0.5 au which makes it one of the bigger Kepler systems - a trait that hopefully de-biases the Kepler detection bias slightly.
- Secular oscillations occur (figure 10)

The unknown parameters of the planets in this system i.e. inclinations, eccentricities, arguments of periastron, longitudes of ascending node, and the mean anomalies, were all randomly chosen from the intervals $[0°, 5°]$, $[0, 0.05]$, and the rest with $[0°, 360°]$ respectively. The integration time is 10^7 years, which is shorter than the standard time of 10^8 years. It is shorter because Kepler-30 is very dense, with bodies on tight orbits (0.5, 0.3 and 0.2 au) which slows the integrator down significantly. Using the randomly chosen orbital parameters, Kepler-30 was found to be stable throughout the integration time when the Kozai companion was not present.

There was supposed to be a list of interesting results here, but as it turns out, the Kepler-30 system is stable even at aggressive 25 au configurations, regardless of inclination. The planets hardly seem to notice the imposing Kozai companion. In figure 11 is graphed the mutual inclination between the three planets and the companion, taken from run #118 of the $[25, 89]$ au/° configuration as an example. The mutual inclinations are more or less constant. The amplitudes of the oscillations that do occur are about 1°, 2° and 3° for the innermost

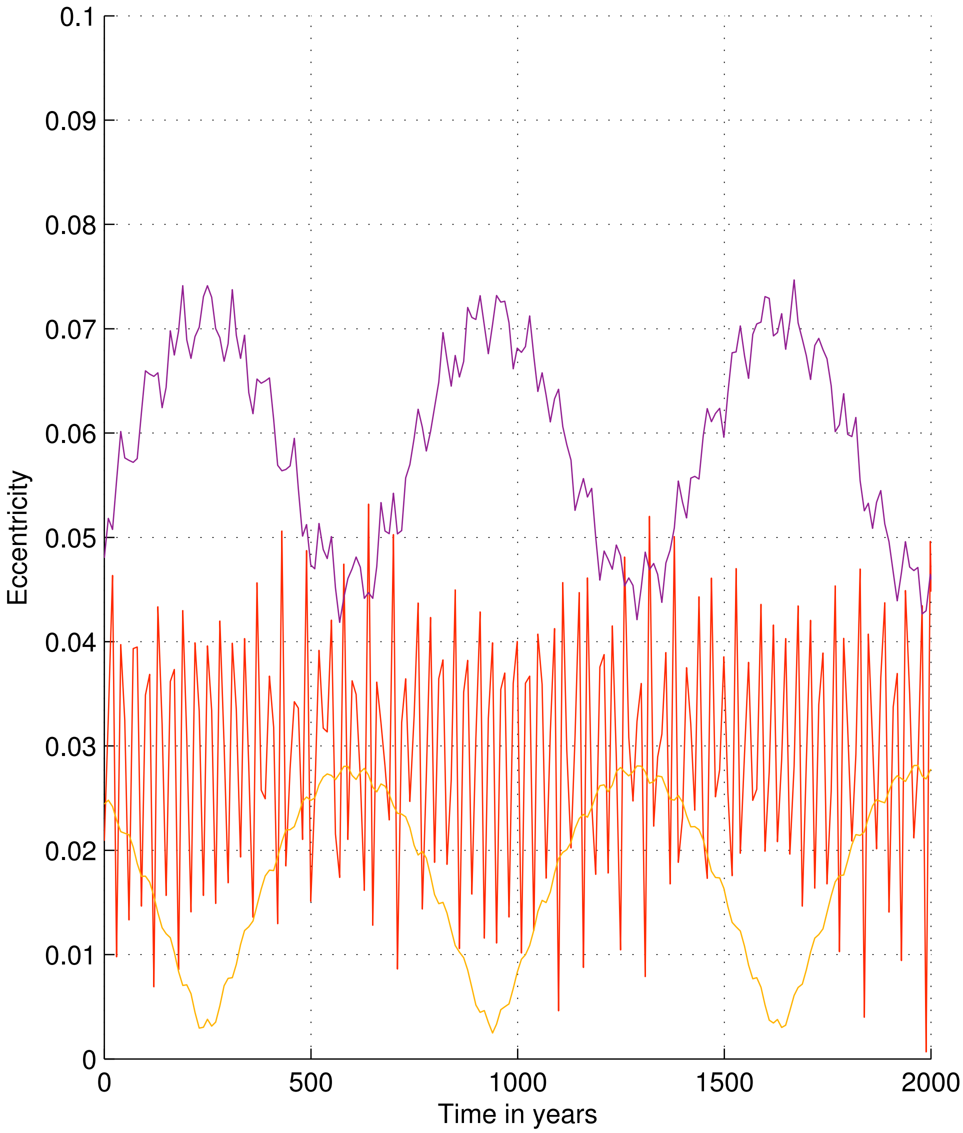

Figure 10: The secular oscillations in the Kepler-30 system, in terms of eccentricity. The innermost orbit is in red, followed by orange and purple (outermost).

(red), middle (orange), and outermost (purple) planets respectively. The secular oscillations themselves (figure 10), have an inclination amplitude around 1° (not graphed). What can be concluded from this is that the Kozai companion's presence is felt after all, but very little.

The shorter integration time of the Kepler systems might be to blame for the seemingly stable behaviour (indeed, there is a break in the oscillation trend at $\sim 7 \cdot 10^6$ years in figure 11), but integrating for another $9 \cdot 10^7$ years would exceed the thesis timeframe.

3.4 Kepler-62

Kepler-62 is a planetary system detected with the Kepler satellite. The star's mass and radius are 0.69 Solar masses and 0.6 Solar radii respectively. Kepler-62 was chosen for investigation because

- It contains five detected and confirmed planets, as opposed to two, three, or four planets as in the previously examined systems.
- It is a realistic system, since it evidently occurs in nature, and
- Its planetary orbits extend to almost 0.7 au which makes it one of the biggest Kepler systems - a trait that hopefully de-biases the Kepler detection bias slightly.
- Secular oscillations occur (graphed in figure 12. The colours give the size of the orbit. Red represents the smallest orbit (0.054 au), followed by orange, purple, blue and finally cyan, which is for the largest orbit (almost 0.7 au)).

The unknown parameters of the planets in this system i.e. inclinations, eccentricities, arguments of periastron, longitudes of ascending node, and the mean anomalies, were all randomly chosen from the intervals $[0°, 5°]$, $[0, 0.05]$, and the rest with $[0°, 360°]$ respectively. The integration time is 10^7 years, as Kepler-62 is as dense or denser than Kepler-30. Using the randomly chosen orbital parameters, Kepler-62 was found to be stable throughout the integration time when the Kozai companion was not present.

Unfortunately, Kepler-62 is so dense that the time required to finish integrating a sensible number of configurations wouldn't fit into this thesis' timeframe. The meager results obtained can still be used to conclude that the system was unstable with a Kozai companion at 25 au, to a degree shown in table 13. In addition to the familiar columns, there is an additional one that lists the relative masses of the planets[22]. The lightest and innermost planets appear to be least lucky, while the outermost planets are mostly out of harm's way. This hints at that ejections are not dominant among the removals. Indeed, out of all removals, 97.67 and 99.07 % are due to collisions in the 49° and 59° configurations respectively. This is higher than for any other system. The reason is

[22] A planet's relative mass is its mass divided by the most massive planet's mass. Kepler-62d is the most massive planet at roughly 0.01 Jupiter masses.

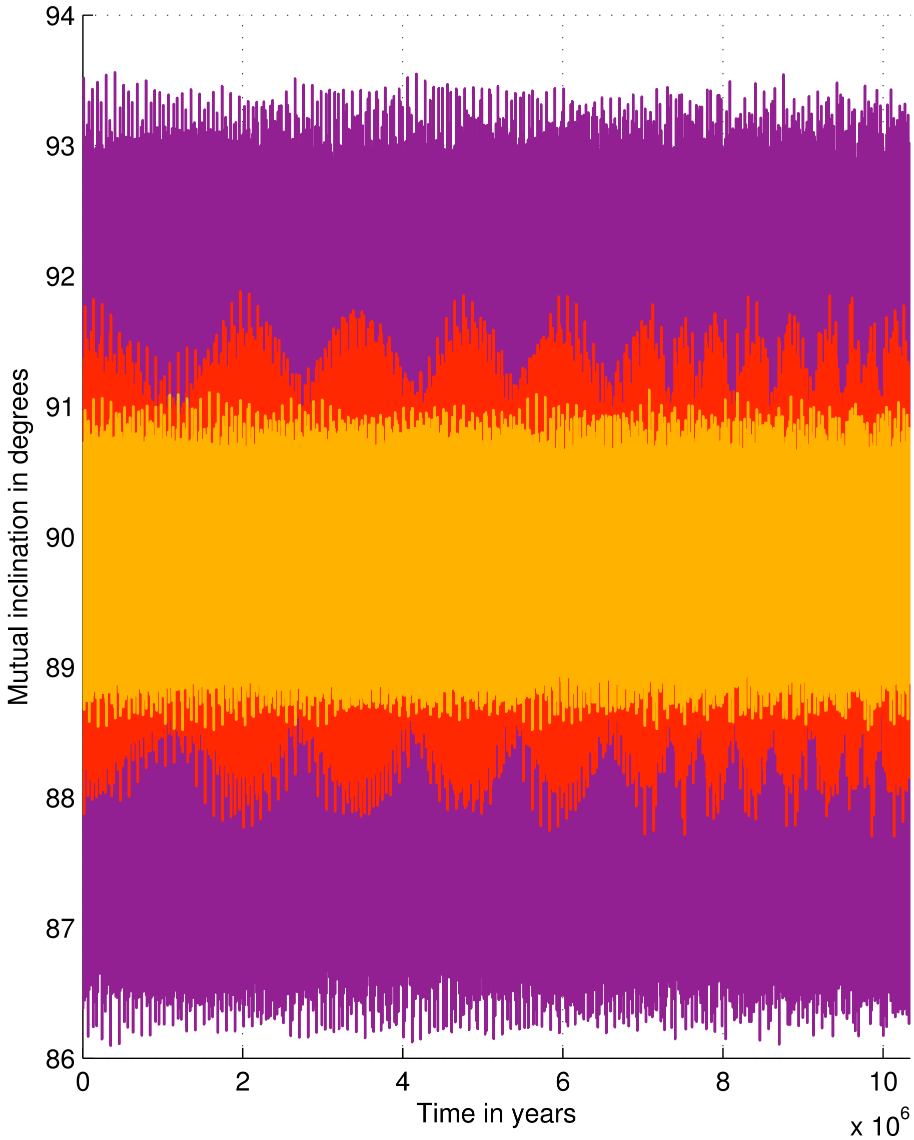

Figure 11: The evolution of mutual inclination between each planet and the Kozai companion in the $[25, 89]$ au/° configuration of the Kepler-30 system, run #118.

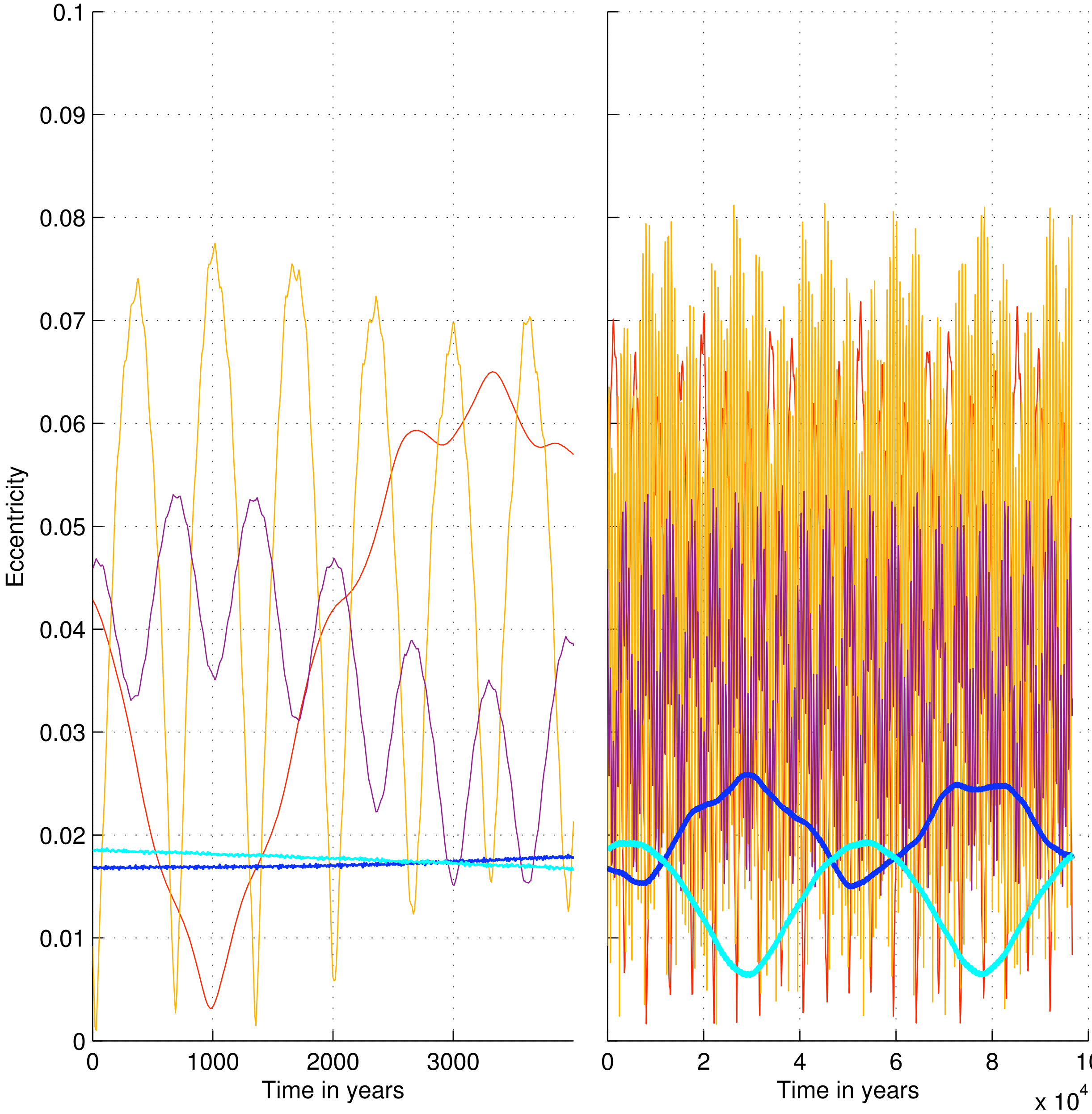

Figure 12: Secular oscillations in terms of eccentricity between the planets of the Kepler-62 system. Short timescales on the left, long on the right.

	49°	59°	Relative mass
Kepler-62 b	328	315	0.583
Kepler-62 c	358	350	0.314
Kepler-62 d	210	205	1
Kepler-62 e	73	154	0.763
Kepler-62 f	63	55	0.715

Table 13: Each planet's total number of removals in the $[25, 49]$ and $[25, 59]$ au/° configurations, plus a column with relative masses.

probably that the binding energies of the planets are very high, owing to their tight orbits. It probably helps that the planets have roughly the same binding energy (because of similar masses and high density i.e. similar orbit sizes), which makes it difficult for any one planet to eject another.

3.5 Kozurn

Kozurn is a fictional planetary system in which Jupiter and Saturn appear as in reality, but with a few changes. The system has supposedly been through a close encounter with another star, which left Saturn's orbit highly inclined. This will be an interesting test of the less contrived TCKR process. As before, each configuration is run 360 times, with different initial mean anomalies for Saturn.

Table 14 lists the removals and ejections of Saturn for each initial inclination. In short, the majority of simulation runs ejected Saturn, while the rest sent Saturn into the Sun. An exception to this would be the 49° configuration, in which both planets mostly remain after the integration time (10^8 years). For reference, Jupiter and Saturn (as they appear in reality) constitute a stable system on their own.

No simulation ever removed Jupiter. The pre-collisional orbits of Saturn were, except three with lifetimes shorter than $2 \cdot 10^4$ years, always shorter than 10^4 years. There were many runs of the 49° configuration in which both Jupiter and Saturn survived. One might think that this would be favourable for some serious TCKR, but Jupiter's periastron never reached below 2 au in those runs. Out of all the simulation runs, a few had Jupiter's periastron dipping down to 0.3 au or so, but this was rare and either way 0.3 au is much too large to have any significant tidal impact.

Bearing these results in mind, this system is a failure when it comes to producing hot Jupiters. There *was* something going on between the planets however, sometimes resembling a Kozai cycle - figures 13 and 14 show the eccentricities and semi-major axes of Jupiter (red) and Saturn (orange) up until the ejection of Saturn. These figures were taken from run #160 of the 89° configuration. The eccentricity peaks are not quite sinusoidal, which sets them apart from the regular secular oscillations. It is also obvious how the period is a function of Saturn's orbit size, by comparing the two graphs.

	49°	59°	69°	79°	89°
Removals	18	302	360	358	360
Ejections	18	298	325	308	268

Table 14: The removals and ejections of Saturn in the Kozurn system. Jupiter was never removed, and never reached interesting periastra.

One final note - it might appear odd that Saturn is eventually ejected by Jupiter when Saturn is on a 70 au orbit, but since there are only two bodies present, the orbits remain closed at every close encounter. This means that the scattered planets will sooner or later return to the point of the scattering event. So even though Saturn's orbit is 70 au big, it is simultaneously eccentric enough that its periastron is on the order of Jupiter's semi-major axis. The bodies may thus interact fiercely whenever Saturn passes through its periastron.

3.6 Koztune

Koztune is a Solar giants version of Kozurn. In this case, Neptune is the outermost planet and thus the supposedly inclined one due to a stellar close encounter. The most interesting difference between Koztune and Kozurn is that Koztune houses four planets, rather than two.

The unfortunately uninteresting result was that regardless of Neptune's initial inclination, instabilities never occured (within 10^8 years) and no planet ever reached orbital eccentricities above 0.1, meaning there is no hope for producing hot Jupiter candidates in a system like Koztune. If Neptune was more massive in relation to its inferior planets, perhaps the result would be vastly different? See next subsection!

Figure 15 shows the evolution of inclination and mutual inclination (in the 89° configuration) with respect to Neptune for the planets in Koztune (Neptune's mutual inclination with respect to itself is omitted). There is at least one mildly interesting feature in these plots - the mutual inclination is kept roughly constant throughout the integration. There must be some interaction that synchronizes the changes in each planet's ascending node.

3.7 Koziter

Koziter is just like Koztune, but with Neptune's mass enhanced to that of Jupiter's. This will hopefully amplify the Kozai effect enough to make it manifest itself at all (it did not quite manage to do so in Koztune). It should be noted that while all the other tested systems were fully stable without an inclined companion[23], Koziter was unstable 38 times out of 360. Of these runs, two removed Saturn and Uranus, and 36 removed Uranus. No interesting periastron distances were ever reached.

[23] That is to say, the systems with a stellar companion were stable without the companion, and the systems with a planetary companion were stable if the 'companion' was uninclined.

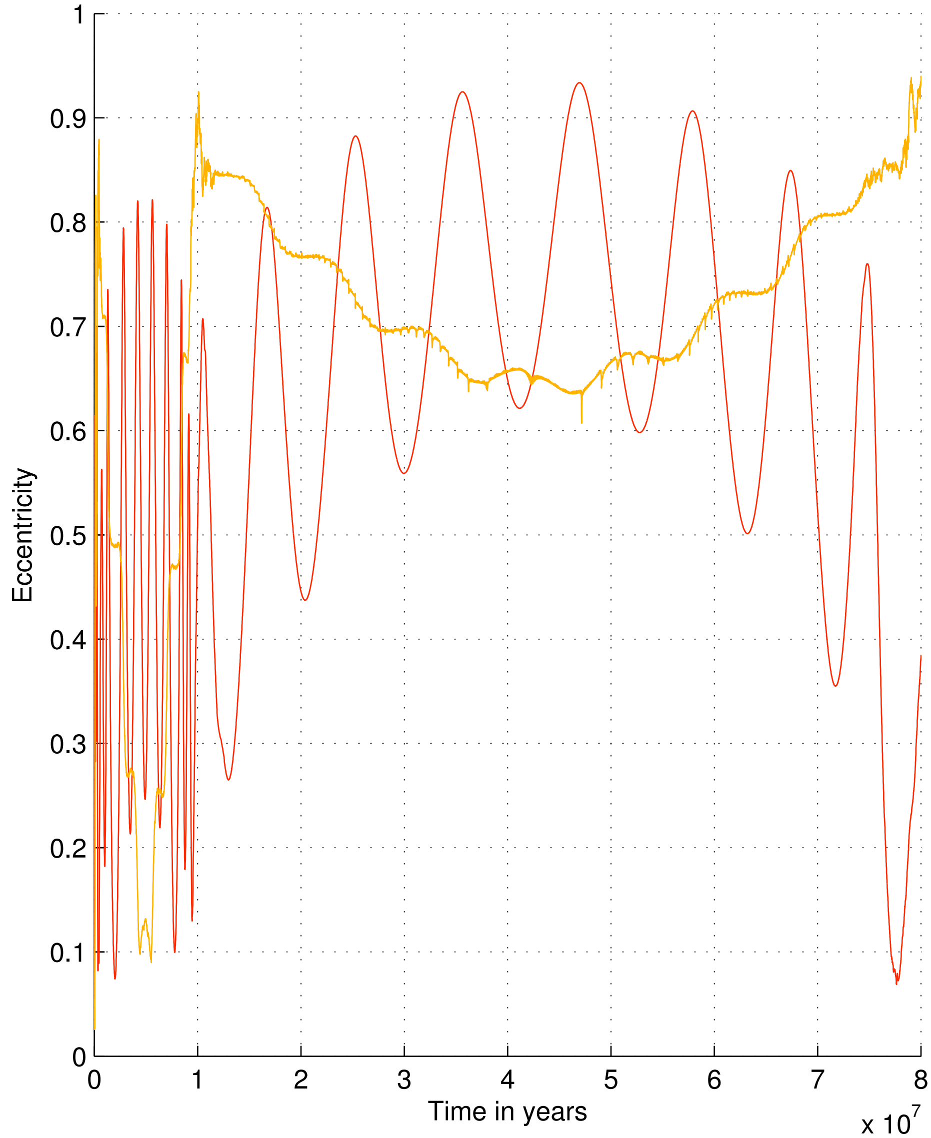

Figure 13: Abnormal Kozai cycles or abnormal secular oscillations? Saturn ejects soon after $8 \cdot 10^7$ years.

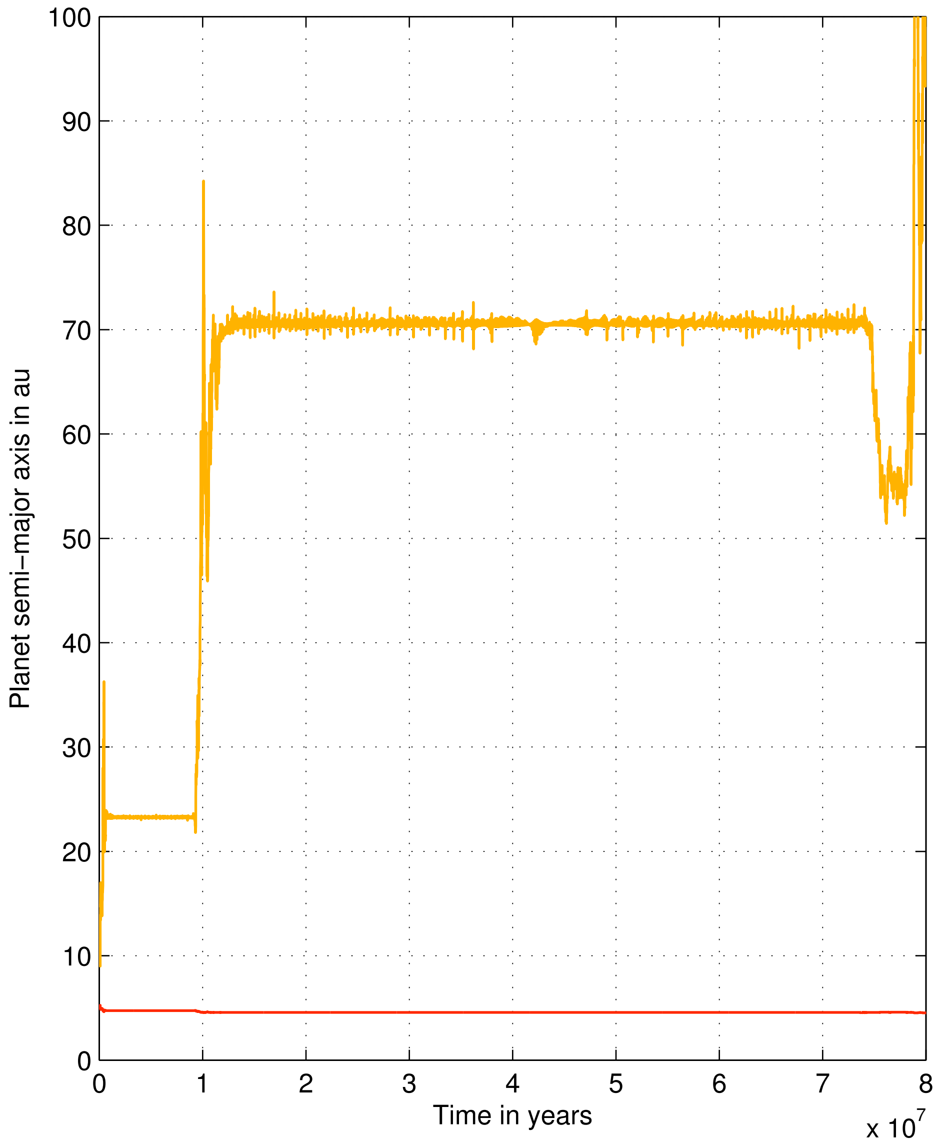

Figure 14: Saturn's chaotic voyage out of the system. Its ejection occurs after roughly $8 \cdot 10^7$years.

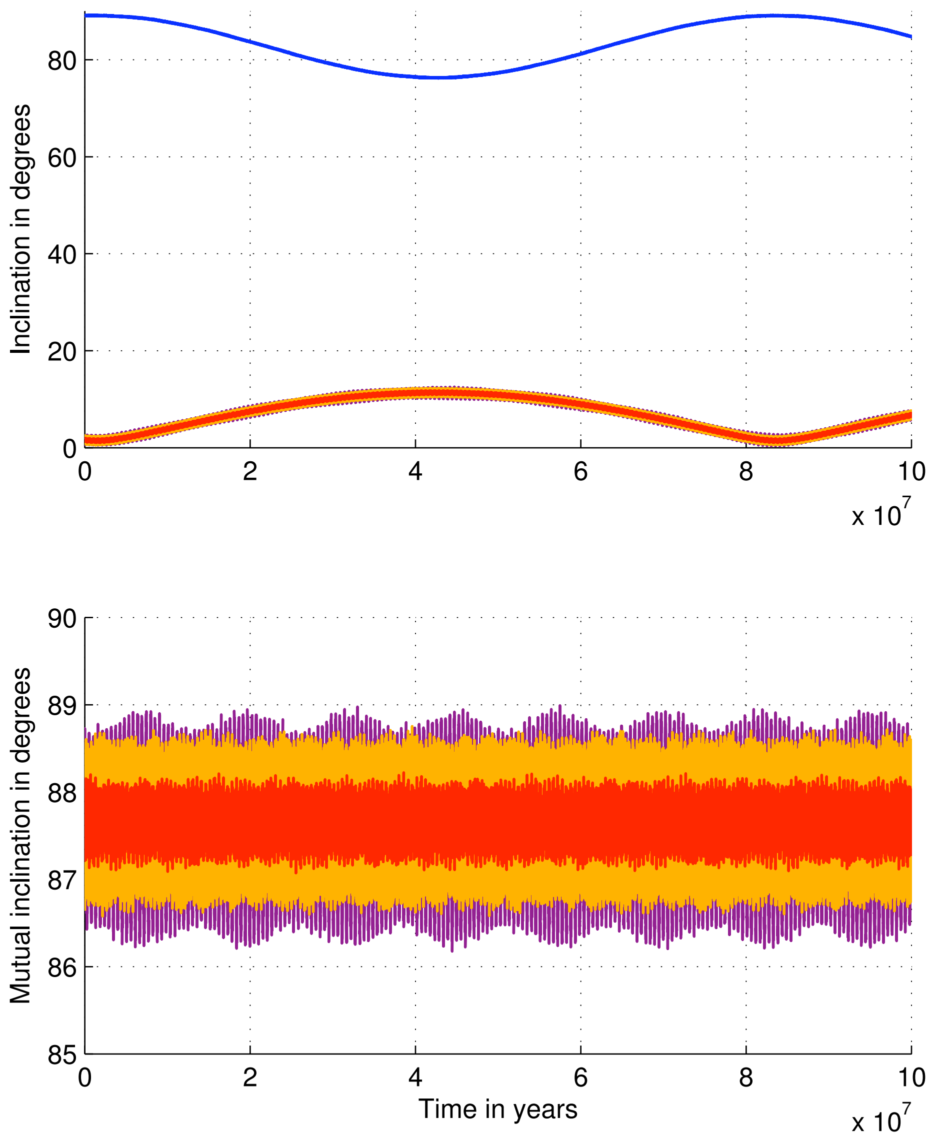

Figure 15: The inclinations (top) and mutual inclinations (bottom) of the planets in Koztune. The colours represent Neptune (blue), Uranus (purple), Saturn (orange), and Jupiter (red). The orbits appear to rotate in a synchronized fashion.

	49°	59°	69°	79°	89°
Jupiter	1	0	0	4	5
Saturn	69	81	81	100	97
Uranus	360	358	359	360	360
Neptune	0	2	0	2	7

Table 15: The number of planet removals in the five tested configurations of the Koziter system.

A heavier inclined Neptune renders the system completely unstable. Table 15 shows each configuration's number of removed planets (note that a configuration in this context is defined by initial inclination of Neptune alone). The best chance at producing a hot Jupiter is when all but one of the inferior planets are removed. By the looks of it, this occurs in about a quarter of the runs when both Uranus and Saturn are removed, leaving Jupiter to hopefully enter a Kozai cycle. Before looking into those runs however, it's worth explaining why all the other outcomes are uninteresting.

First off, all the runs that removed Neptune either removed three planets out of four, voiding any TCKR, or they removed Neptune and Uranus, leaving Jupiter on a tight orbit, and Saturn on a huge orbit of several hundred au. This is not an interesting situation, keeping in mind the fate of Kozurn.

Second, all the runs that removed Jupiter removed three planets in total, except for runs #98 and #130 of the 89° and 79° configurations respectively. In both of these, Saturn is left on an inferior orbit to that of Neptune's, after Jupiter and Uranus are gone. What follows in both is what appears to be a mixture between a Kozai cycle and regular secular oscillations. Saturn's eccentricity never reaches above 0.8, which unfortunately renders these runs uninteresting.

A simple argument for why the runs in which only Uranus is removed are uninteresting, is that any significant Kozai cycle entered by either Saturn or Jupiter would cause them to cross orbits and scatter. If neither planet has been removed, odds are that nothing worth investigating occured[24].

With all that said, let's look into the runs that left Jupiter and Neptune. Figure 16 shows the evolution of eccentricity for Jupiter and Neptune a short time after Uranus and Saturn have been removed. Figure 17 shows the corresponding evolution of inclination. Jupiter's plotted inclination (red) is actually its mutual inclination with respect to Neptune, while Neptune's inclination (blue) is with respect to a static reference plane. It does not quite resemble a 'pure' Kozai cycle as we know it (e.g. figure 2), but in this example, run #332 of the 89° configuration, Jupiter reached a periastron of 0.03 au, which is promising. Promising enough to warrant a look into accumulated time at periastra smaller than 0.05 au. Figure 18 plots the results. This plot is completely analogous to figures 7 and 9. There are 10 markers. The eight rightmost of them stem from the 89° configuration, while the two remaining stem from the 69° (left)

[24] However unlikely, it is possible that Saturn is scattered to a large orbit far from both Jupiter and Neptune. This should have Jupiter enter a Kozai cycle.

	Cluster fraction	Close encounter fraction
Conservative	0.001031	0.002098
Optimistic	0.001183	0.002407

Table 16: In this table are the conservative and optimistic hot Jupiter production fractions that the Koziter system produced.

and 79° (right) configurations. 10 is a modest number when compared to those of the Jupiter and Saturn and Solar giants systems (56 and 256 respectively)[25]. The values of the accumulated times are also relatively short, not counting the leftmost marker (run #286 of the 69° configuration).

The number of Jupiters that ever reached periastra below 0.05 au was 11, which is one more than the number of runs that accumulated time at such orbits. The conservative and optimistic approaches to estimating the produced number of hot Jupiter candidates will thus be much the same. Table 16 lists those results. The Cluster fraction, as before (tables 5 and 12), is the hot Jupiter frequency in the entire cluster, while the 'Close encounter' fraction is the frequency among the subset of stars that underwent close encounters.

The frequencies are quite high in relation to those obtained for the Jupiter and Saturn and Solar giants systems. The cause is partly that the close encounter frequency is more than a magnitude higher than the in-and-out binary exchanges, but also because of the optimistic assumption that every single close encounter causes the outermost planet to become inclined. A more realistic assumption could probably bring the production frequencies down a magnitude or so.

[25] Though to be fair, 56 isn't particularly impressive compared to 10 when considering that 56 a sum over 35 configurations, while 10 is a sum over five.

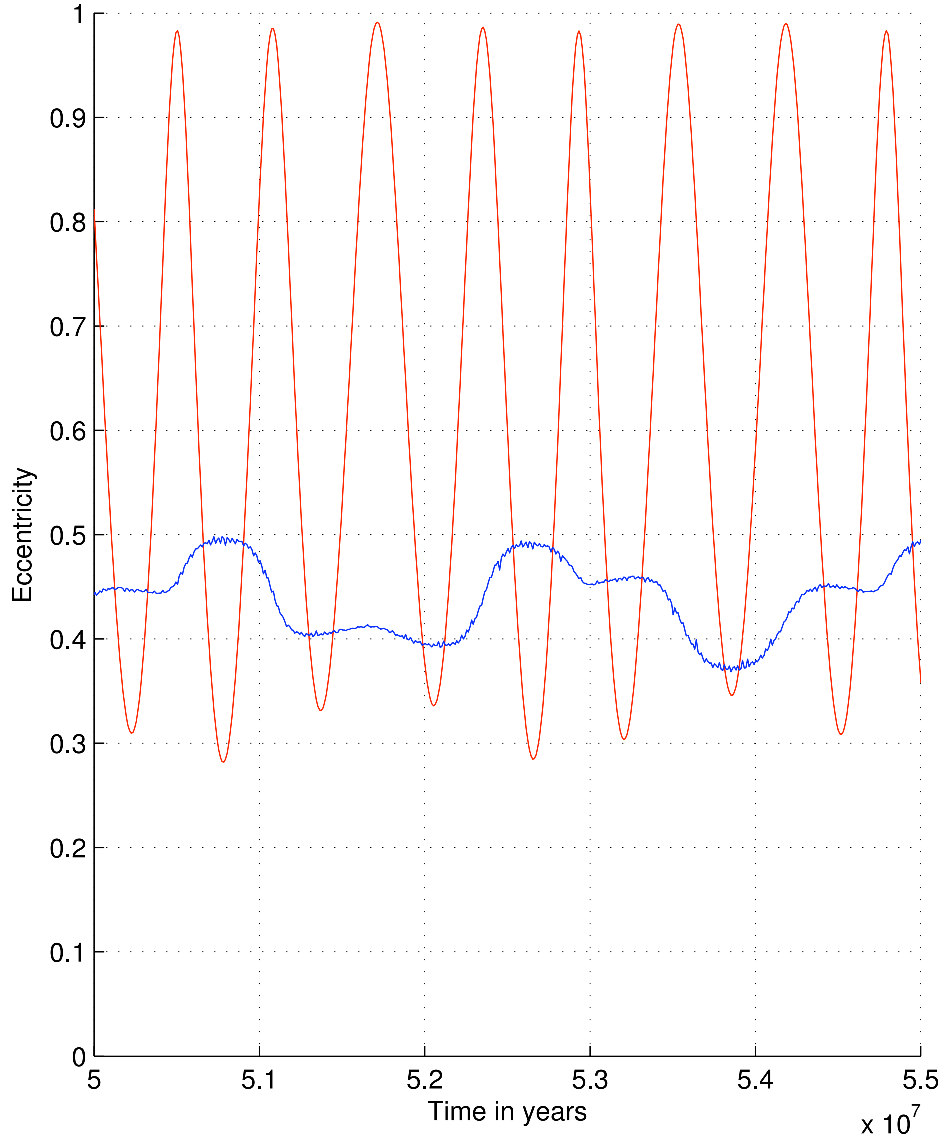

Figure 16: An excerpt from run #332 of the Koziter system's 89° configuration. Plotted are the eccentricities of Neptune (blue) and Jupiter (red).

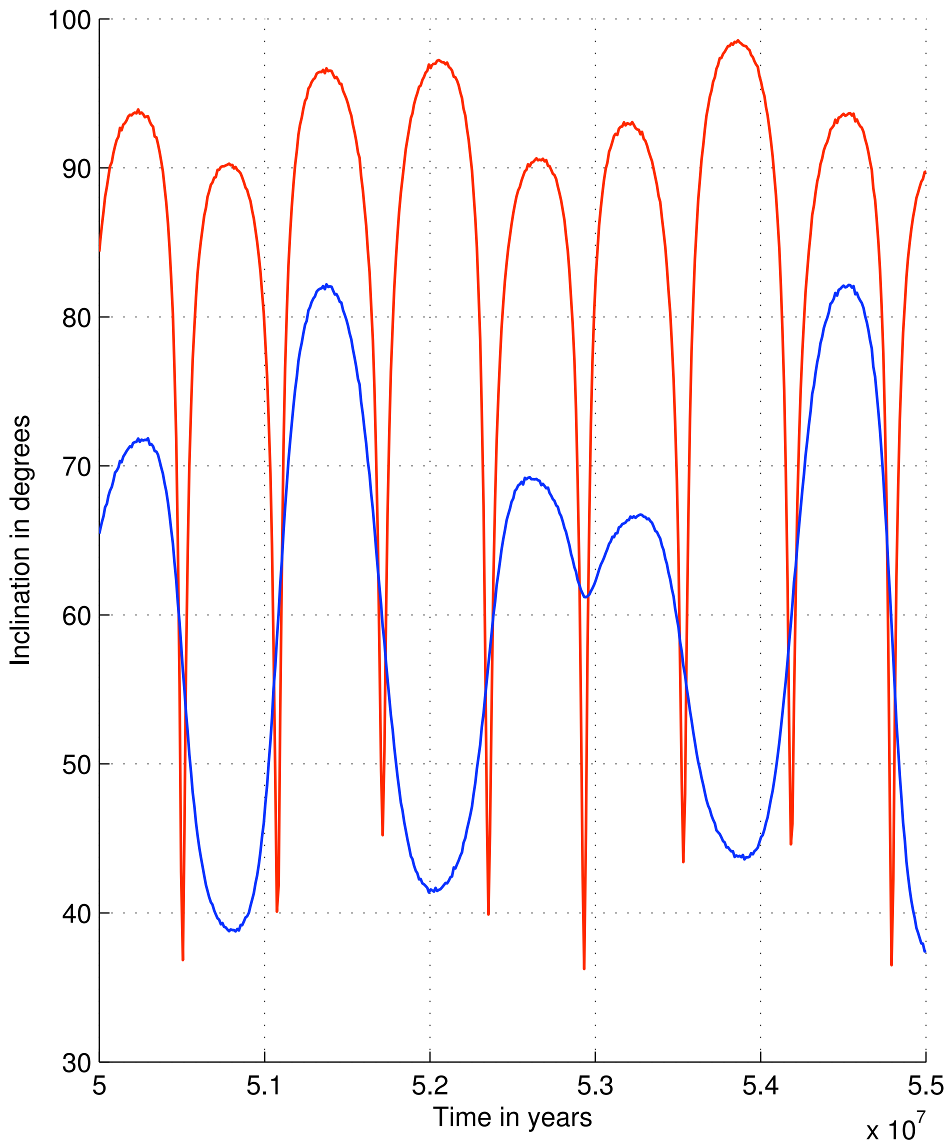

Figure 17: An excerpt from run #332 of the Koziter system's 89° configuration. Plotted are the inclination of Neptune (blue), and mutual inclination of Jupiter with respect to Neptune (red).

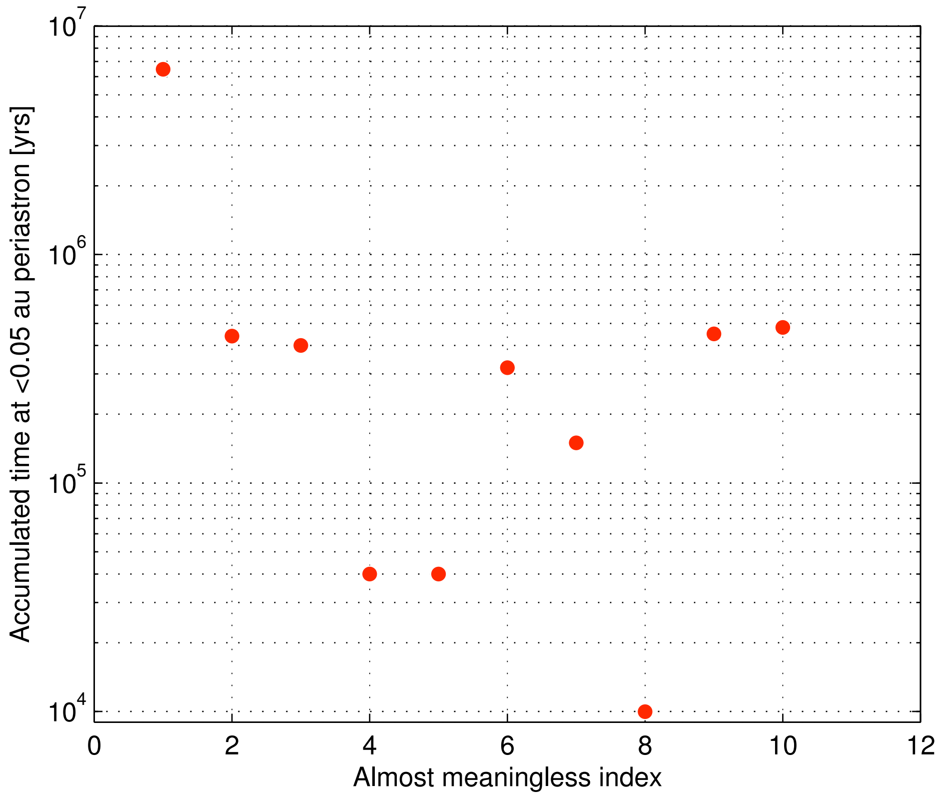

Figure 18: Each marker represents a single simulation run's successful accumulation of time that the Koziter system's Jupiter spent on orbits whose periastra were smaller than 0.05 au.

4 Conclusions

This thesis has been an investigation into how some planetary systems may react to stellar and/or planetary Kozai companions. The answer is in retrospect strongly dependent on which system is being investigated. In order to draw the most interesting conclusions, one would have to investigate many more systems than was done in this thesis. This requires vast amounts of computer power.

Nevertheless, some (perhaps precarious) conclusions can still be drawn:

- Secular oscillations do appear to play an important role when under the influence of a Kozai companion. This was most evident in table 10, where there is a Solar collision frequency peak when the Kozai timescale coincides with the period of Jupiter and Saturn's secular oscillation. It is also evident when, past some critical configuration ($\sim$200, $\sim$1000, and < 25 au for the Jupiter and Saturn, Solar giants, and Kepler-30 systems respectively[26]), the Kozai companion is unable to induce instability. Denser planetary systems appear to be less affected by a Kozai companion, since Jupiter and Saturn was less affected than the Solar giants system, and since Kepler-30 was stable even with a Kozai companion at 25 au, a smaller orbit than Neptune's.

- Multiplanetary systems seem, on the whole, less prone to produce hot Jupiters by means of TCKR than single-planetary systems are. However, at medium-high inclinations, the chaos in multiplanetary systems, as opposed to non-chaotic single-planet systems, can actually increase the likelihood for very eccentric ($e \gtrsim 0.99$) orbits i.e. the production of hot Jupiter candidates, from zero to finite. The reason is that while there is a particular initial inclination that leads to a planet periastron < 0.05 au (eq. 1), the chaos in multiplanetary systems can leave a single remaining planet at a higher mutual inclination than it was initially. This effectively allows for a greater span in initial inclination that is able to, in the end, produce a hot Jupiter candidate.

- The hot Jupiter production frequencies calculated may not be realistic, but they should give an order-of-magnitude idea of a real value. If one chooses to trust the estimates, it must be either be that TCKR is not the only way of making hot Jupiters, *or* the multiplanetary systems investigated in this thesis are atypically unaffected by Kozai companions. Simply because the (upper limit) estimates in this thesis are at least one magnitude lower (~ 0.001) than the estimated hot Jupiter frequency from observations (~ 0.01). Since there were no actual tidal effects incorporated into the N-body integrator, it is a bit precarious to say too much about how the results in this thesis fit into the spin-orbit misalignment distribution in figure 1. But if TCKR produces misalignments between $\sim 39°$ and $\sim 141°$, then that hints at (even though the misalignments in figure 1 are projected,

[26]While there is no practical evidence, surely there must be a semi-major axis at which even the Kepler-30 system breaks up into chaos.

i.e. lower limits) that there is some other or multiple production channel(s) that produce(s) low misalignment hot Jupiters.

- Planetary Kozai companions are inefficient producers of hot Jupiters if their masses are roughly on par with the other planets's masses. This can be concluded from the facts that Saturn and Neptune in the Kozurn and Koztune systems respectively did not manage to raise the eccentricity of the inferior orbits significantly. Only when Neptune was given a jovian mass did some hot Jupiter candidates emerge in the end. However if for some reason the outermost planet is significantly more massive than its inferior planets (which, it could be argued, is expected, see section 1.2), it could very well be that planetary Kozai companions are the main producers of hot Jupiters (since close encounters are relatively common in stellar clusters).

In short, this thesis does not present results convincing enough to suspect that the Kozai effect is the only viable production channel for hot Jupiters. It does however present evidence for that secular oscillations interact with the Kozai effect.

To improve the validity of the results and conclusions in this thesis, the following measures may be taken:

- Simulating many more planetary systems, so as to reduce biases introduced by the choice of investigated systems.
- Incorporating tidal forces into the N-body integrator. This would relieve us of the assumptions about tidal circularization (i.e. conservative/optimistic hot Jupiter *candidates*), and yield more realistic results.
- Simulating the entire stellar cluster, so as to avoid assumptions about the effects of close encounters on planetary systems, and the properties of in-and-out binaries.
- Expanding the configuration space with parameters such as eccentricity and mass, for stellar and planetary Kozai companions alike.
- Increasing the configuration space resolution so as to, if it exists, find the critical configuration curve.

These measures are listed in order of believed importance, with the top measure deemed most important. So if resources are limited, the author suggests starting at the top, and work downwards as far as possible.

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