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Black Swan Investing

An empirical study in context of efficient markets

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ABSTRACT

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Keywords: Black Swans, Efficient market hypothesis, Fat-tails, Kelly criterion, Outliers, Trading strategy, Unknown unknowns.

Purpose: The purpose of this paper is to assess the sustainability of the efficient market theorem when accounting for extreme events, which are of the essence in a Black Swan investment philosophy.

Methodology: Quantitative approach.

Empirical foundation: Historical secondary monthly and daily data of various variables during January 1, 1996 - December 31, 2012.

Conclusions: The Black Swan investment strategy applied in this paper does not generate risk adjusted returns or pure returns that significantly differ from market equilibrium returns. The empirical results hence supports the efficient market theorem but puts a question mark on market efficiency when it comes to pricing deep out of the money put options.

Limitations: The methodological approach resulted in a number of shortcomings that had implication for the overall empirical result. The validity of the results presented should be taken with some caution since the timeframe of the study was rather limited due to option data availability. Furthermore, the time restriction and operationalization process required some significant assumptions and delimitations in order to ensure hindsight research bias and data processability.

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1. INTRODUCTION

1.1 Background

> Most traders were just picking pennies in front of a steamroller, exposing themselves to the high-impact rare event yet sleeping like babies, unaware of it. (Taleb 2007:19)

What do Black Monday in 1987, the 9/11 terrorist attacks and the recent global financial crisis have in common? They are all rare events, beyond the realm of normal expectations, which resulted in a major impact on the financial markets and investor wealth. Such extreme events have become known as "Black Swans", "Fat-tailed events" or "unknown unknowns"[1] and are the Achilles heel of financial models since their occurrence are commonly underestimated using the assumption of normally distributed asset prices (Mandelbrot and Hudson 2004; Taleb 2007).

Under the assumption of normally distributed asset prices, the target of most investment philosophies is to generate small excess return on regular basis through actively held "medium risk" positions in markets with positions being exposed to possible Black Swans. Such strategies could be described as "Nickel strategies"; strategies that track market index or earn small excess returns most of the time but occasionally sustain large losses (Malliaris & Yan 2009).

A Black Swan investment philosophy on the other hand is based on the occurrence of extreme events such as the ones mentioned above through investments in tail risk derivatives.[2] Taleb (2007) describes the strategy of investing the majority of capital at hand in safe instruments such as treasury bills with the remaining part in highly diversified speculative bets benefiting from Black Swans such as deep out of the money options. A Black Swan strategy would hence generate small losses most of the time but occasionally reap large excess returns; one would invest in the belief of "time arbitrage" (i.e. in time, unpredictable events will occur that will create large exploitable price differentials). Conversely, Black Swan investments come with the obvious downside of non-performance during long stretches of financial market stability with aggregation of losses as a result.

The concept and awareness of Black Swans in finance has increased, much due to Taleb's (2007) literary book, the New York Times bestseller "The Black Swan". This, in combination with the turbulence in financial markets during the last decade has lead to a growing number

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[1] The term was first coined by the former U.S. Defence Secretary Donald Rumsfeld at a news conference in 2002. He later earned the "Foot in Mouth Award" for the puzzling comment.
[2] As an example, billion of U.S. dollars were made out of put options on airlines in the week of 9/11.

of private and exchange traded investment funds[3] adopting the Black Swan investing philosophy (Kim 2010). However, the well known "efficient market hypotheses" (henceforth referred to as EMH) state that one cannot consistently achieve returns in excess of average market returns on a risk adjusted basis by only trading on known information (Fama 1970). From this perspective, very little (if anything) has been written in academia regarding the performance of Black Swan investment strategies.

1.2 Problem discussion

Black Swans are in brief defined by rarity, extreme impact and retrospective predictability (Taleb 2007). The outcome for investors is hence an inability to predict such extreme events and correctly incorporate their impact into asset market prices. Instead of admitting to the Knightian uncertainty[4] related to Black Swans, investors try to apply financial models, based on assumed known probabilities, to calculate and predict real life financial outcome instead of admitting that one simply cannot predict the future.

What is known to market participants however, is the existence of Black Swans and that they occur more often than what the normal distribution foretells, in fact many of the flaws of the standard normal assumption are widely known amongst today's academics and practitioners.[5] There is a vast amount of literature rejecting the standard normal assumption[6] (particularly for daily returns) in favour for more fat tailed distributions in a variety of different asset markets (Aparicio and Estrada 2001; Estrada 2008; Taleb 2009). This fact was observed over 50 years ago by the pioneering work of Mandelbrot (1963) and Fama (1965) who concluded that stock markets are riskier than what is implied by the volatility of the standard normal assumption and exhibit a more fat-tailed behaviour. The standard normal assumption is however, still to this day, implicitly and explicitly, used and abused by financial market participants through the use of financial risk measurements (Mauboussin 2006; Taleb 2007; Estrada 2008; Taleb 2009).

The Knightian uncertainty of Black Swans makes accurate pricing of extreme risk discretionary. Pricing deep out of the money options is not an exact science, it is commonly known that deep out of the money options are under-priced by the widespread Black-Scholes

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[3] Along with other investment products; Most of these products are targeting institutional investors such as pension funds, endowments and high-net-worth families (Kim 2010).

[4] "Knightian uncertainty" refers to the fact that there is a distinction between quantifiable and un-quantifiable part of risk since, in Knights (1921) opinion, uncertainty is immeasurable and not subject to probabilities.

[5] For instance, the work of Mills (1927); Aparicio and Estrada (2001); Mauboussin (2006); Browne (2007); Bogle (2008); Estrada (2008); Estrada (2009) and Taleb (2009) provide insight on outlier occurrence and impact on investor wealth; returns are not generated even and steadily over time but rather irregularly as a result of outliers.

[6] However, due to the central limit theorem monthly returns are reasonably well described by the normal distribution. See the work of Aparicio and Estrada (2001) for a study of stock returns on 13 European securities markets.

model and that a premium is notable to the actual market prices to combat this issue (Hull & White 1987).[7] This premium in itself can be a problem because there is no consensus of how to price it, instead it is largely driven by supply and demand (Haug & Taleb 2011).

Relating to this discussion is the work of Samuelson (1965) and Fama (1970) which states that the financial market can be considered as efficient; in other words, *there is no way to consistently beat the market by analysing and trading on currently available information*, the market price already reflects all known information (also known as stale information). Malkiel (1973) put it more elegantly and famously stated that a chimpanzee throwing darts has the same chance of beating the market as a fund manager since none of them can predict the future. More up to date studies have since then reinforced these results, rational investors make rational decisions, this leads to that if there is a mispricing, arbitrageurs will exploit this fact and trade on that information until prices reflect the fair value of that information (Holmes 2007; Fama and French 2010). Further evidence in support of the EMH can be found in the empirical studies of Malkiel (1995) and Gruber (1996).

However, one chink in the EMH armour is the assumption of rational investors (Rubinstein 2001). Ivkovic and Weisbenner's (2009) study shows that investor rationality can be questioned when taking risk and uncertainty into consideration, investor's focus much more on total return instead of focusing on the risk adjusted return. Not taking risk and uncertainty into consideration in extreme scenarios and just focusing on returns is one reason why Black Swan investing might fare better and produce excess returns.

This would go against Nobel laureate Eugene Fama's (1970) seminal work of the EMH mentioned above ruling out any trading systems which are based solely on stale information; such systems should have no ability to generate risk adjusted returns in excess of a passive index fund. But it comes back to investor rationality though, Nobel laureate Daniel Kahneman shows in several studies that an investor does not always maximize his or hers utility, i.e. people are not as rational as the models want them to be (Kahneman & Tversky 1992). More recent research comes to the same conclusion; when it comes to risk and uncertainty investors are not always rational and do not have a risk neutral mind set, they are more risk avert and tend to value and anchor their values on what they have, not what they can obtain (Kahneman and Tversky 1992; Shleifer 2000; Shiller 2003). Individuals also tend to fear losses more than gains and be loss averse instead of risk neutral (Kahneman & Tversky 1984), they also have

[7] However, various alternative option pricing models exist which relaxes the strict assumptions of the original Black-Scholes model and counteracts its shortcomings. A discussion is provided in section 2.1.

an availability bias and tend to put too much emphasis on what is happening when calculating future probabilities (Kahneman and Tversky 1974; Barberis and Thaler 2003).

If some of the above characteristics on irrational behaviour are true then some investment strategies should be able to beat the market. Relating this argumentation to a Black Swan investing strategy one could see why the market might not be efficient when accounting for mispriced extreme risk. If there is no consensus of how to price extreme uncertainty (Knightian) and no way of knowing the risk ex-ante then pricing becomes very dubious.

By assessing the ability of a Black Swan investment strategy to generate risk adjusted return relative a passive index investment, based upon only stale information, this paper aims to assess the sustainability of the EMH when accounting for extreme risk. This is done through the inclusion of derivatives (namely deep out of the money put options) reflecting markets perception of extreme risk in their prices. Furthermore, this is justifiable since the effectiveness of arbitrage (and hence market efficiency) relies crucially on the availability of financial instrument substitutes. Exact or close substitutes are available for derivatives such as options and hence should pricing of derivatives be more efficient then for example a strategy including a stock portfolio (Shleifer 2000).

The empirical work in this paper will contribute to the general finance literature by providing insight on a rather recent investment phenomena and simultaneously assessing the efficiency of the EMH, one of the most central theorems within financial economics. Furthermore, by including put options in the applied Black Swan strategy, the paper will provide insights regarding prices of extreme risk in financial markets (by commenting the price premium of the options used in the strategy) and hence contribute to the literature of risk management. This is of relevance since an important input in risk management is the prices of instruments used for hedging[8] and hence the price of risk determines a firm's cost of reducing its exposure in financial markets (Oxelheim & Wihlborg 2008). Oxelheim and Wihlborg (2008) further stresses the importance of the price premium on risk in financial markets since it determines the inducement of market participants to bear the risk that is hedged away.

1.3 Purpose and research question

The purpose of this paper is to assess the EMH theorem when accounting for extreme events (Black Swans). The following research question is formulated in order to operationalize our purpose:

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[8] Where options represent a popular choice when financial derivatives are considered.

- Does a Black Swan investment strategy, based upon stale information, generate risk adjusted returns that differ from market equilibrium returns?

1.4 Delimitations

The main limitation of this paper is due to the availability of historical option data, the years studied are limited to 1996-2012. This is due to no access of option data before or after this period for full calendar years. Due to time constraints, considering the immense amount of data, the option contracts are limited to 30-day maturity put options. To avoid potential hindsight bias the option strategy is limited to only options on S&P 500.

1.5 Structure of the thesis

This paper comprises five chapters, including this introductory chapter that has presented the general background of the research matter, problematization, purpose as well as acknowledged research delimitations.

Chapter two will introduce the reader to the theoretical and empirical framework of the paper with the aim of creating an understanding for the research matter. The chapter will start by shortly discussing option pricing and present a model used to determine optimal bet sizes since these two concepts are used in the methodological framework of the paper. Furthermore, the concept of Black Swans, their importance in finance, a strategy based on their occurrence as well as arguments relating to the EMH and the underlying theory of behavioural finance will be presented. The chapter will end by presenting the research hypotheses and a model for operationalization.

Chapter three will outline the methodological framework used in this paper. The data collection process will be defined, motivations and explanations of variable selection criteria will be presented along with any made assumptions.

Chapter four will outline the empirical findings of the study and present a relating analytical discussion in the context of existing theoretical and empirical literature.

Chapter five ends this paper by presenting the most eminent conclusions, discussing theoretical contributions and suggestions for future research.

2. LITERATURE REVIEW

2.1 Option pricing

Since the investment strategy applied in this paper is based upon put options, we open up this chapter with a discussion on option pricing and by presenting the widespread Black-Scholes option pricing model. Furthermore, the Black-Scholes model relates to the Black Swan discussion on financial models made below and the methodology of the paper since it includes historical option prices and the using of the model itself.[9]

Options have a long history and have actively been trading and priced since at least the 17th century (Vega 1688). Entering the era of modern finance, a plethora of different option pricing models have evolved. However, since being derived by Black and Scholes in 1973[10] (Black & Scholes 1973), the Black-Scholes option pricing model is widely used in practice[11] and has lead to a boom in option trading mainly due to its simplicity and approximation abilities (MacKenzie 2006; Bodie, Kane and Marcus 2008; Hull 2012). Hence Hull (2012) concludes that the Black-Scholes has a huge influence on how traders price options in practice.

In deriving the formula, Black and Scholes (1973) made the following underlying assumptions:

> a) The short-term interest rate is known and is constant through time.
> b) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is log-normal. The variance rate of the return on the stock is constant.
> c) The stock pays no dividends or other distributions.
> d) The option is "European," that is, it can only be exercised at maturity.
> e) There are no transaction costs in buying or selling the stock or the option.
> f) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
> g) There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date. (Black & Scholes 1973:640).

Given these assumptions, the formula, which provides a theoretical approximation of the price of a European style put option can be written (using the more common notations from Hull

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[9] For fundamentals on options and other financial derivatives see the very comprehensive work of Hull (2012).
[10] The work of Robert C. Merton (1973) expanded the mathematical understanding of the model by Black and Scholes and hence coined the term "Black-Scholes option pricing model". Scholes and Merton later received the 1997 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel for the work. Due to his death in 1995, Fischer Black was mentioned as a contributor to the model by the Swedish Academy. As the reader might note, the model should actually be referred to as the "Black-Scholes-Merton option pricing model".
[11] Although often with adjustments and corrections; see the work of Bakshi, Cao and Chen (1997).

(2012)) as:[12]

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1) \qquad (1)$$

Where:

$$d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad (2)$$

$$d_2 = \frac{\ln(S_0/K)+(r-\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad (3)$$

And the payoff to the holder of a long position (i.e. has bought the option) in this put option as:

$$\max(K - S_T, 0) \qquad (4)$$

In equation (1), the variable (p) is the put option price. For all of above equations (S_0) is the underlying assets price at time zero, (K) is the strike price of the option, (r) is the continuously compounded risk-free rate, (σ) is the underlying assets volatility and (T) is the time to maturity of the option. The variables (d_1) and (d_2) are probabilities and can be interpreted as measures of *"moneyness"* in standard deviations (i.e. the relative position of the current price of the underlying asset with respect to the strike price). (In the method chapter of this paper (d_1) will be used to sort out suitable put options to use in the Black Swan strategy). The functions of N(x) are the cumulative probability distribution functions for a standardized normal distribution. Equation (4) illustrates the fact that a bought put option will only generate a positive payoff to the holder if the underlying asset price (S_T) is below the strike price (K) at maturity, otherwise the put option will expire useless. Equation (4) provide the simply basis for calculating any option generated returns by the Black Swan strategy in the method chapter.

The strict assumptions of the original Black-Scholes model presented above, has lead to severe criticism of the model and an increase during the last three decades of alternative models relaxing some of these assumptions (Bakshi, Cao & Chen 1997). One of the more significant limitations of the original model is the assumption of log-normal asset prices, the meaning of this limitation will be discusses more in section 2.3.2 below. As an example of popular alternative models recognizing the shortcomings of the original Black-Scholes model are the models of Heston (1993) and Duan (1995). In the Heston model, Heston (1993)

[12] It is beyond the scope of this paper to present any derivation of the Black-Scholes formula and we further limit the scope by only presenting the formula for put options since only put options are used in the study.

acknowledges the widespread evidence that volatility is stochastic (i.e. non-deterministic and not constant, hence following a random process) and that the distribution of asset returns has non log-normal properties. Duan (1995) present an option pricing model (called the GARCH[13] option pricing model) which counteracts some of the significant systematic biases associated with the original Black-Scholes model such as; the under pricing of deep out of the money options (which was mentioned in the problem discussion); the under-pricing of short maturity options; the under-pricing of options on low volatility securities as well as the U-shaped implied volatility curve (also know as the volatility smile) in relation to the exercise price.

The above discussion concludes that even though the Black-Scholes model is widely used in practice and has huge influence on option trading, there are associated shortcomings (namely, of relevance for this paper, under pricing of deep out of the money options). Since being derived by Black and Scholes in 1973, a lot of different option pricing models have evolved which relaxes and counteracts some of the original models shortcomings.

2.1.1 Implied volatility

Relating to the use of the Black-Scholes model in option pricing is the concept of implied volatility. The one parameter in equation (2) and (3) that cannot be directly observed is the underlying assets volatility (σ) which has to be estimated.[14] The basic idea behind implied volatility is to invert the Black-Scholes option pricing formula and insert observed market values ((p), (S_0), (K), (r) and (T)) and hence obtain the market's anticipation of future volatility in the underlying asset, this is also what is done in practice by market participants (Hull 2012). The concept of implied volatility is briefly presented here since the concept will return in the methodological framework in the use of the market volatility index (VIX) published by Chicago Board Options Exchange (henceforth referred to as CBOE).

2.2 The Kelly Criterion – Optimize your "bet" size

When devising option investment strategies such as the one applied in this paper it is not only important to determine what to buy, equally important is determining how much you want to buy. Daniel Bernoulli acknowledged this concern in 1738; he argued that when multiple options are available for an individual, one should always choose the one outcome with the highest geometric mean. John L. Kelly (1956) popularized this idea and put it in formal use in

[13] "Generalized Autoregressive Conditional Heteroskedasticity".
[14] The concept of historical volatility is also applicable here but not of interest for the study since it is the concept of implied volatility that will be used in the methodological framework.

his seminal paper. The underlying gist of Kelly's (1956) paper, which has later become known as the Kelly criterion, is the following: If one is allowed to take a large number of consecutive bets, the criterion will define how much to put in each bet in order to maximize the long term growth rate. This differs from just maximizing your utility in every bet (i.e. bet the maximum amount if you have an edge) (Kelly 1956; Thorp 2000).

The Kelly criterion can be written as:

$$F = \frac{(Expected\ \ Gain)}{Gain} = \frac{(PB-Q)}{B} = \frac{(P(B+1)-1)}{B} \quad (5)$$

In equation (5), the variable (F) is the percent of capital allocated to each bet, this is independent of total amount of capital at hand. (F) is always denominated in relative terms. (P) is the probability of profit, this is merely the odds that the profit will occur. (B) is the ratio between max profit and max loss. Max profit is determined by equation (4) on page seven ($\max(K - S_T, 0)$) and max loss is determined by the option price. (B) symbolizes how large the payout is in case of a win. Finally, (Q) is the probability of loss: $(1 - P)$. (Q) is merely the odds that profit will not occur.

There are a few things that need to be addressed when applying the Kelly criterion; one is the risk of overestimating the probability of profit. Overestimating this probability could result in what Thorp (2000) calls the "drag down effect" meaning that one inadvertently bet too much relative to the true optimal bet size resulting in a negative growth rate. This is the main reason that a lot of the proponents of the Kelly criterion are suggesting sacrificing some potential upside by applying a "fractional Kelly" (i.e. betting ½ - ¼ of what the Kelly criterion implies) (Maclean, Ziemba & Blazenko 1992). An illustrative example is provided in figure 1. The figure illustrates a simulated game (coin toss) using the Kelly criterion with a probability of success of 0,5 and an edge of 2 (you win 2 units for each win and your counterpart wins 1 unit for each win). The red dot (stationary point) in figure 1 illustrates the optimal bet size which is approximately 0,25 meaning that one should bet this fraction of any capital at hand. Note however that if one bet more than 0,5 of the capital at hand, one will enter the shaded area depicting the "drag down effect" and hence generate negative long term growth and eventually going bust.

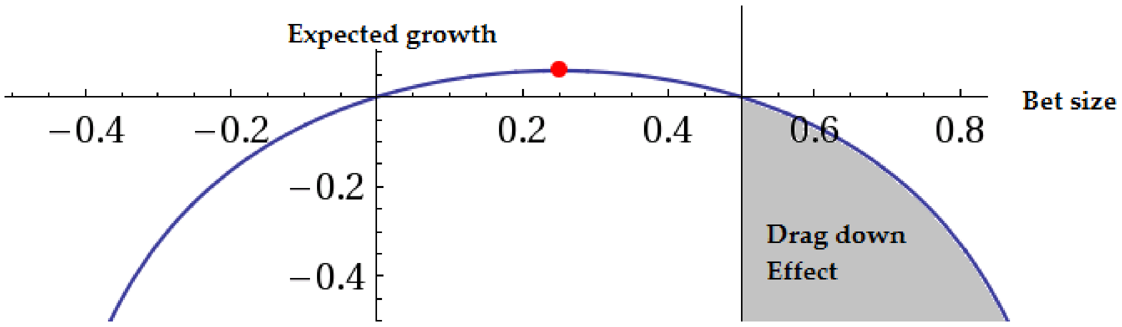

Figure 1. Coin toss game simulation with a 0,5 probability of success and an edge of 2 using the Kelly criterion.
(Calculation made with help from The Wolfram Alpha website: *www.wolframalpha.com*, 2014)

The Kelly criterion implies zero amount bets in fair games (Kelly will always imply zero amounts bets if the EMH holds up since EMH is a fair game). This approach is quite similar to the "gambler's ruin" argumentation suggesting that, in a fair game, if one have a finite amount of wealth and goes up against someone with an infinite amount of wealth (the market) one will eventually go bust (Huygens 1657; Shoesmith 1986). The conclusion is hence that games without a competitive edge (i.e. fair) is not worth betting on, in fact if the Kelly Criterion comes up negative it is beneficial to take the other side of the equation.

Given the insights above it can be concluded that at least one of the two following are needed:

- A probability of profit greater than 0,5; this factor is of less importance in this paper since the applied Black Swan strategy includes buying deep out of the money put options with a low probability of profit.
- A potential profit greater than potential loss; this is of significance in this paper since the options hold great potential upside if expiring in the money.

We end this section by presenting the underlying assumptions and characteristics of the Kelly criterion since they are of importance for the research method:

- Assumes no correlation between the corresponding bets.
- Assumes independence of action (since you have a perceived edge you want to make as many small bets as possible to reduce the potential volatility).
- Assumes no exiting positions ahead of expiration.
- Heavily dependent on accurate probabilities.

2.3 Black Swans

The expression "Black Swan" is a metaphor used to describe highly improbable events that are unpredictable and carry with them an extreme impact; in statistical terms a Black Swan represent an outlier and can have positive or negative impact (Taleb 2007).[15] The expression is not bound to economic events but rather applies to all aspects of our world such as social-, cultural-, historical-, technological events et cetera (Ibid.). The importance of the metaphor lies in the illustration of fragility in conclusions and predictions made upon past observations; attributes that are central in theorization and decision making, not at least in finance (Taleb 2009). Since Black Swan events underlie the purpose of this paper, we here establish an important reference point through the definition of a Black Swan.[16] Taleb (2007) defines a Black Swan event as an event containing three characteristics:

> First, it is an *outlier*, as it lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility. Second, it carries an extreme impact […] Third, in spite of its outlier status, human nature makes us concoct explanations for its occurrence *after* the fact, making it explainable and predictable. (Taleb 2007:xxii)

In short, a Black Swan event is rare, has an extreme impact and is predictable in retrospective.[17] As an example, consider 19 October 1987, also known as Black Monday, the Dow Jones Industrial Average declined by 22,61 percent, almost twice as much the largest previous decline of 13 percent that occurred in 1929 (Bogle 2008). Black Monday was in terms of standard deviations an event approximately 21 standard deviations above the mean and hence its probability of occurring was almost non-existent (figure 2)(Bogle 2008; Estrada 2008). In relation to the definition made by Taleb (2007), Black Monday was with its rarity, extremeness and retrospective predictability a Black Swan event that had substantial impact on investor wealth.[18]

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[15] The saying is that the expression originates from the presumption that all swans were white since all historical observations had confirmed only the existence of white swans. That belief changed when Willem de Vlamingh, a Dutch explorer discovered black swans in Western Australia in 1697. Hence it only took one observation to invalidate the beliefs made upon countless historical observations.
[16] However informally defined, as one will notice reading the methodology chapter.
[17] Note that highly expected events that do not occur are also Black Swans.
[18] From late August 1987 until the stock market closed on the occurrence of Black Monday, some 1 trillion U.S. dollars was erased from the total value of U.S. stocks (Bogle 2008).

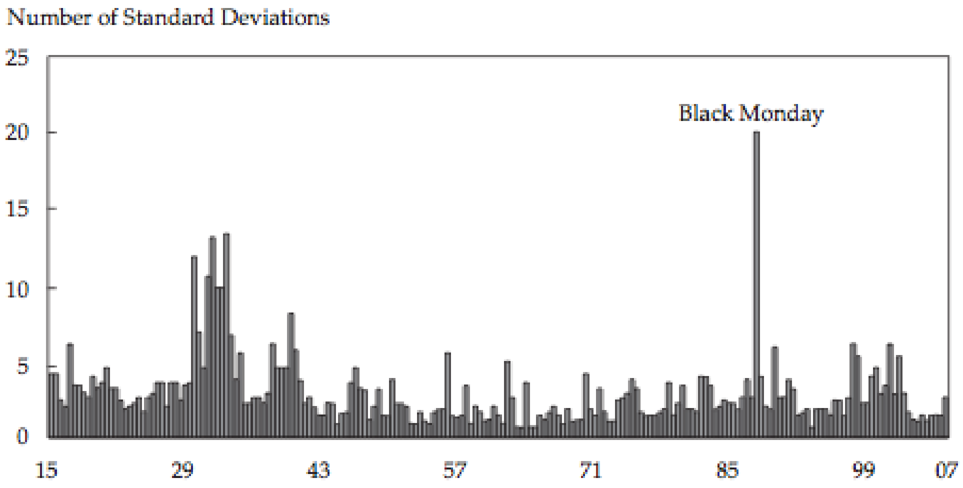


Figure 2. Daily changes (standard deviations) in the Dow Jones Industrial, 1915 - 2007.
(Bogle, *Black Monday and Black Swans*, 2008:32)

The Black Monday example, although extreme, illustrates the inability to predict Black Swan events as well as the role and impact of such events in financial markets. The combination of low predictability and major impact make Black Swan events significantly important in finance. In fact, as presented below, shocks in financial markets do not necessarily have to be of the Black Monday magnitude to have substantial impact on wealth and investment performance.

2.3.1 Empirical evidence - Impact on investment performance

The idea that Black Swan events[19] occur far more often then what is predicted by the normal distribution is not new and is known by rational market participants. For instance, Frederick C. Mills illustrates as early as 1927 how outliers affect asset prices and that a distribution over such assets may widely depart from the normal distribution and exhibit a more fat-tailed behaviour (Mills 1927, see Haug and Taleb 2011). To mention more recent studies of outlier impact on performance; Mauboussin (2006) proclaims that from January 1978 to October 2005, the S&P-500 index delivered a mean annual return of 9.6 percent, however, by excluding the worst 50 days increases the mean return to 18.4 percent, and by excluding the best 50 days lowers the mean return to just 2.2 percent (Figures not accounting for dividends). By using evidence from 15 international equity markets and defining Black Swan events as returns more than three standard deviations away from the mean,[20] Estrada (2008) concludes that such events have a massive impact on long-term portfolio performance. Estrada's (2008) study shows that on average across all 15 markets, missing the best 10 days resulted in

[19] Or by market participants more commonly known as just "outliers".
[20] Such events are hence far more "common" then events such as Black Monday in terms of standard deviations. To clarify on the "common" part; events three standard deviations from the mean are according to the normal distribution only expected to occur approximately every 1/370 or 0,27 percent of the total time.

portfolios 51 percent less valuable than a passive index investment and avoiding the worst 10 days resulted in portfolios 150 percent more valuable than a passive index investment. Using the same definition of Black Swan events as above, Estrada (2009) further studies long term portfolio performance on 16 emerging equity markets.[21] The evidence show that on average and across all 16 emerging markets missing the best 10 days resulted in portfolios 69.3 percent less valuable than a passive investment and avoiding the worst 10 days resulted in portfolios 337.1 percent more valuable than a passive investment (Estrada 2009). The same conclusion is hence made; Black Swans have a significant impact on long-term performance and occur far more often than what the normality assumption would lead investors to believe.

Further evidence on outlier occurrence and impact on investments in financial markets can be found in the work of Aparicio and Estrada (2001), Browne (2007) and Taleb (2009). However, by examining time series of daily and monthly stock returns for 13 European securities markets, Aparicio and Estrada (2001) importantly conclude that even thought the normality assumption is clearly rejected for daily returns the *central limit theorem* make the normal distribution a reasonable approximation to the empirical distribution of monthly returns.[22]

The empirical evidence presented here hence concludes that returns to investors are not generated even and steadily over time, as suggested by the normal distribution, but rather irregularly as a result of both positive and negative outliers or Black Swan events. Furthermore, the evidence concludes that negative Black Swans result in larger impact on investment performance than positive ones (not always occurring more frequently then positive ones however).

2.3.2 Financial models, the normality assumption and Knightian uncertainty

Closely relating to the empirical evidence presented above is financial models used to make projections about the future (that underlies decision making) by market participants. Irrespective of a rational market participant's investment philosophy, the fact that conventional financial models are incapable of computing and hence accounting for Black Swan events are known. For instance, there is a vast amount of literature rejecting the common used assumption of normally distributed prices for a variety of asset classes; one could mention the work of Mandelbrot (1963), Fama (1965), Aparicio and Estrada (2001), Estrada (2008) and Taleb (2009). Furthermore, Taleb (2009) argues that not just models based

[21] Markets such as Argentina, Indonesia and Sri Lanka to mention a few.
[22] The central limit theorem states that the longer the time intervals for which returns are computed, the resulting distribution of returns should conform to the standard normal distribution due to aggregation (Aparicio & Estrada 2001).

on the standard normal assumption are incapable of properly accounting for the impact and occurrence of Black Swans in practice, but rather all models based on least square methods or variance as a measure of dispersion.[23] The result is projections (and decisions) flawed buy greater estimation errors then anticipated. In the case of financial models applying the standard normal assumption, and hence thinner tails then what the market implies, the result is underestimation of Black Swan events. As a practical example that also relates to the Black Swan investment strategy applied in this paper, is the inability to price such extreme events. This is reflected in the pricing of deep out of the money options by the well-used Black-Scholes model which under-prices deep out of the money options when compared to real observable market prices (just consider the strict and fragile assumptions of the model mentioned on page six)(Hull & White 1987). However, as noted in section 2.1 a number of alternative option pricing model are available for market participants.

The underlying issue related to the mentioned estimation bias is the assumed *known* probability when there in fact is none. Probability is subject to quantifiable risk not to unquantifiable uncertainty[24] (Knight 1921; Taleb 2007). In Knights (1921) view there is a distinction between risk, which is subject to probability (as with the roll of the dice in a Casino), and uncertainty that is immeasurable[25] and hence not subject to probabilities; Black Swan events refer to the latter (Taleb 2007). Taleb (2007) states this fact regarding financial models bias to Knightian uncertainty, and call them sterilized constructions of risk that works well in theory (or in a Casino) were probabilities are known and *not* uncertain. However, when applied to practice the computable probabilities of “certain uncertain” outcomes do not tell the whole truth about the future uncertainty and hence the models estimation becomes subject of an increasing bias (Ibid.). To clarify; if an event is certain uncertain, just as the odds in a casino, then falsification of your projections is not possible (lets say, just like one single black swan can prove your theory false) since you know your odds and hence all outcomes are known (but uncertain).

2.3.3 Coping with Black Swan events - Black Swan Investing

Given the severe impact of Black Swans in finance, their unpredictability and financial models estimation bias, Taleb (2009) advocates robustness to such events when introducing the idea of the “Fourth quadrant” which represent the problem of complex decision-making under high uncertainty. For instance, Taleb (2007) suggest that one should apply a trading

[23] Even models based on ARCH/GARCH methods do not capture the impact and occurrence of Black Swan in practice (in spite of their success on in-sample data) according to Taleb (2009).
[24] Such as the uncertainty regarding Black Swan events; do remind the definition of a Black Swan made above.
[25] And hence coined “Knightian uncertainty”.

philosophy of being both hyper conservative and hyper aggressive at the same time instead of being mildly conservative or aggressive. Taleb's (2007) "barbell" strategy implies putting some 85 to 90 percent of the capital at hand in extremely safe instruments such as Treasury bills and the remaining part in extremely speculative bets, as leveraged as possible (like deep out of the money options). By doing this, the portfolio is not affected by risk management estimation bias and has a "floor" to severe adverse effects of possible Black Swan events. Although informally defined, the "barbell" strategy captures the main idea behind the Black Swan investment philosophy.

2.3.4 Criticism

Leaving any criticism regarding Taleb's (2007) plain literary ideas and philosophical arguments on Black Swans aside, the Black Swan investment philosophy comes with the obvious downside of non-performance during long stretches of financial market stability with aggregation of losses as a result (Malliaris & Yan 2009). The time arbitrage belief is of course dependent on just that, namely time. Since Black Swans are unpredictable and market timing empirically not possible (see Estrada 2008; 2009) the time duration of the strategy becomes vital. There is no consensus on how to determine an appropriate timeframe in which the strategy is applied. This fact leads to potential scenarios where, if the strategy is applied in a managed fund, the manager faces the impending risk of getting fired before returns arrive; large scale withdrawal of funds making the strategy undesirable; and even if both manager and investors understand the nature of the strategy and are dedicate to it, the aggregation of losses over time might lead to insolvency and termination (Malliaris & Yan 2009).[26]

2.3.5 Conclusions on Black Swans

Before turning to the second part of this chapter, the significance of the discussion and findings made above needs to be stated. In the above section the following is presented; a definition of Black Swan events, empirical evidence of their importance in finance along with how financial models fail to account for their occurrence and a suggested strategy of coping with them along with some obvious criticism.

The discussion and findings simply form the foundation that underpins the Black Swan investment philosophy applied in this paper. Consider, if Black Swans occur far more often then what is predicted due to the erroneous application of probability in financial models, carry with them severe irregular impact on investment performance and are mispriced, then

[26] Consider the saying ”Markets can remain irrational longer than you can remain solvent.” supposedly coined by John Maynard Keynes.

perhaps, most investors truly are picking pennies in front of a steamroller. However, an investment strategy based upon the Black Swan philosophy is crucially dependent on the timeframe during which the strategy is applied with severe aggregation of losses as a potential outcome. Of further, and more practical importance for the purpose of this paper, is that the findings presented in above sections represent stale information known to rational market participants and provide the basis for answering the research question.

2.4 The Efficient Market Hypothesis

No one of intelligence resents the inevitable. (Clarke 1953)

Since being formulated in the late 1960's, the EMH has been a dominating theorem within financial economics, and has provided a central theoretical foundation for how financial markets act and behave when pricing assets (Berk & DeMarzo 2011). What makes the EMH so alluring is its simplicity; it merges two basic insights that together form the core of the hypothesis. The first insight is that competition imposes a relationship between returns and costs. If asset returns are in excess, then investors enter into trade on those assets until returns are aligned with costs. The second insight is that price variations are a function of information, i.e. whenever new information is revealed prices adjust directly to encompass this new information. The conclusion formulated by the EMH is thus: Competition amongst market participants' align asset returns with their costs, there is hence no excess return to be made from stale information (Ball 2009).

The man who formulated the hypothesis, Eugene Fama (1970), concluded that prices and returns should always reflect all available information. If this were to be true then the implication of this simple tidbit has far reaching consequences when it is applied to the world of investing and trading strategies. The implication is that stale information is not of value for the investor, the only information affecting prices is new information and since new information is unknown until it becomes known the market follows a random walk and is impossible to predict (Fama 1970). Fama further stated:

> The efficient market hypothesis rules out the possibility of trading systems based only on currently available information that have expected profits or returns in excess of equilibrium expected profit or return (Fama 1970).

Jensen (1978) has a slightly different view then Fama (1970) and states that prices on assets reflect information *to the point* where the marginal costs outweigh the marginal benefits. There is however, a paradox when reflecting on the two statements above in regards to investor rationality. If investors are aware of the EMH effect and are rational, then the same investors will not study nor analyze the market because they believe other investor will do that work for them. This can lead to the market becoming inefficient, the opposite of what the EMH states, this is called the Grossman-Stiglitz paradox (Grossman & Stiglitz 1980).

If this paradox is disregarded and it is assumed the EMH holds, which a lot of empirical studies have shown (Malkiel 1995; Gruber 1996; Fama and French 2010), the hypothesis itself is based on three underlying assumptions. First, investors are assumed to be rational, i.e. they are supposed to know the correct price and try to maximize their utility. Secondly, not all investors need to be rational in order for the EMH to hold, the ones that act irrational cancel each other out since they do not act in cohesion, (i.e. in the end their trades are just noise on the market). Third, even if irrational investors would act in cohesion, arbitrageurs would act on the asset mispricing generated and hence omit the irrational investors actions and bring back market efficiency (Shleifer 2000). However, this arbitrage ability is dependent on selling or buying exact or close offsetting substitutes for the initial assets. An exact substitute for a given asset is another asset (or a portfolio of assets) with *identical* cash flows in all states of the world[27] that can be used to take the offsetting market position (Ibid.). A close substitute for a given asset is another asset with *similar* cash flows (or a portfolio of *similar* cash flows) in all states of the world (Ibid.).

Another construction of the EMH is that the cost of information is to be regarded as free and that public information is the only information available, in other words there is no difference in the level of knowledge between different market participants; there is no hidden exploitable information gap (Berk & DeMarzo 2011). There are different types of known information and levels of market efficiency however. Fama (1970) formed three different layers of efficiency; weak, semi-strong and strong.

- *Weak market efficiency*: Historical information has been taken into account for the market pricing of an asset. There is no way of making excess returns by just studying history, this results in technical analysis being useless.
- *Semi-strong market efficiency*: Historical information has been taken into account as well as available public information. There is no way of making excess returns by just studying history and trading on stale information, this results in fundamental analysis being useless.
- *Strong market efficiency*: All information is reflected in the assets price. The information here encompasses all public and non-publicly known information (insider

[27] And hence has the same risk characteristics as the initial asset.

information). Hence there is no excess return to be made even when trading as an insider concluding that nothing on the market can create excess return.[28]

The EMH is however not uncontested; it has its critics as well. Most of this criticism stems from the problem of rationality in the behavior of market participants. The maximization of the investor's utility does not always seem to be the prime reason for investing, this assumption of utility maximization is heavily questioned by Kahneman and Tversky (1979; 1984; 1992).

2.5 Behavioral finance

> To be against rationalization is not the same as to be opposed to reasoning. (Hitchens 2008)

Behavioral finance is a field in academia that since its conception in the late 1970s has been in stark contrast to the EMH (Kahneman & Tversky 1979). The main argument for behavioral finance proponents against the EMH is the assumption of the rationality amongst the market participants. There have been studies since the early 1980s that goes against this assumption of rationality and that show how investors, when faced with decisions, do not always act rational (Shiller 1981; Arrow 1982; Summers 1986). Rather than acting on logic, individuals tend to "misbehave" and act on different psychological criteria's that goes against a utility maximization. This behavior can create a mispricing and have the effect that the market is not as effective as the EMH states (Shleifer 2000).

Overall there are three big themes in behavioral economics; framing, heuristics and market inefficiencies (Shrefin 2002).

- *Framing*: This is the concept of how individuals perceive reality; different answers are formulated for the same scenario depending on how the questions are framed. This goes against the rationality of an individual (Kahneman & Tversky 1981).
- *Heuristics* (decisions): Individuals, when faced with decisions, often base those decisions on pre-conceived notions. Potential reasons could be availability bias (how easy an idea can be imagined), anchoring (individuals tend to anchor their value against an already stated value) or loss aversion (individuals tend to put more emphasis on losses than gains even if they have the same monetary impact) (Kahneman and Tversky 1979; 1981; 1991; Shrefin 2002).

[28] Unsurprisingly does the strong form of market efficiency remain an appendage of the EMH since it does not perform very well empirically (See Jeng, Metrick and Zeckhauser 2003).

- *Market inefficiencies*: This is a combination of mispricing and non-rationality from the individuals regarding decisions.

Even if the behavioral finance insights on irrationality holds true, the market can still be rational according to EMH. This goes back to what is presented in section 2.4 with regards to irrational investors not acting in cohesion, or even if they do, arbitrageurs take opposite position to cancel out their actions (Shleifer 2000). But this statement does not paint the whole picture however. There could be other factors at hand that lead investors to believe they act rational, but in fact they exhibit bounded rationality and hence try to make the best of a situation with the limited resources they have available (Kahneman 2003).[29] This can lead to sub-optimal decisions that are not utility maximizing although the investor that acts is rational.

2.5.1 Irrational Behavior

2.5.1.1 Anchoring

When individuals form estimates, Kahneman and Tversky (1974) argue that they anchor their estimate on an already known initial value, i.e. if you would think of a number and then be asked of the price of something you're answer would be influenced by the number you thought about before the question. However, with regards to option pricing, one can assume that professional managers will have knowledge of anchoring. The logical conclusion will then be that anchoring effects will have no effect since the individual estimating the price will know about the phenomena and take proper measures to mitigate the effect. This is not always the case though. Strack and Mussweiler (1997) show just how hard it is to escape anchoring even if the participants know of the phenomenon. When participants know that the number they have as an anchor is wrong it still serves as an anchor for their answers, it seems to be inescapable. This discussion is of significance for this paper since anchoring can have a potential effect on option prices used in the applied Black Swan strategy. The anchoring value obtained is the one supplied by the Black-Scholes formula and since it prices deep out of the money option with too low of a value (see Hull and White 1987) this could potentially have the effect of under pricing deep out of the money options.

2.5.1.2 Availability bias

When estimating the probability of an occurrence individuals access memories of prior events or prior information. Memories or information that is fresh and salient is given a much larger

[29] Bounded rationality: In-perfect information, in-perfect cognitive ability and insufficient time to make a decision.

weight then information that is old and obscure when making a decision (Kahneman and Tversky 1974; Barberis and Thaler 2003). This can be one reason why an option strategy might work, since Black Swans are "unknown unknowns" they lie beyond the realm of normal expectations and are hence given much less weight than what is justified. Vice versa, when such an extreme event has just occurred investors might put too much emphasis on it and hence overestimate the possibility of it happening again (Ibid.).

2.5.1.3 Herd behavior

This is the behavior of a manager following the perceived herd consensus, instead of utilizing his own analysis of investments. This irrationality of not doing what you deem best to maximize utility (if you are an investment manager) and instead do what everyone else is doing is coming from a place of rationality though (Malliaris & Yan 2009). Investors that hire such a manager get a participant that makes irrational decisions from their point of view, but from the manager's point of view, decision making becomes perfectly rational. This sounds odd at first, decisions being rational and irrational at the same time. But if the manager makes decisions that goes against the herd, and is wrong, then he might face the risk of getting fired. However, if he makes decisions in coherence with the herd and those decisions turns out to be wrong, then not much would change for the manager because he would "share-the-blame" with the other participants (Scharfstein & Stein 1990). This could be one of the reasons why the market might not be efficient in the case of Black Swan strategies, because if no one is willing to take the other side and bet against the market consensus (even if they believe that consensus to be wrong) then prices could stop being efficient. This could have the effect for deep out of the money options to become underpriced because no one is trading and correcting this mispricing because of a rational fear for their own utility.

2.5.1.4 Isolation effect

When evaluating decisions with different alternatives individuals tend to have problems with assessing the entire scope of information for these alternatives. The total information is disregarded and instead only the information of what separates the alternatives from each other is regarded and evaluated (Kahneman & Tversky 1979). In the case of option pricing this can have the effect that the whole risk is not evaluated, instead just what separates the alternatives is considered. This with regards to pricing extreme risk can lead to options being bought that have a slim to none chance of ever ending up in the money.

2.5.1.5 Loss aversion

People have been shown to strongly prefer avoiding losses even if the outcome dictates that avoiding losses is not optimal (Kahneman & Tversky 1984). This goes against the first assumption in the EMH of individuals wanting to maximize their own utility. This is one behavioral aspect that speaks against a Black-Swan investment strategy, the willingness to avoid big losses and hence hedge potential downfall even if the cost is not fair. This could have the effect of pushing up option costs above what is market efficient.

2.6 Conclusions on EMH and Behavioral Finance

In short, the EMH rules out any trading system being able to generate any risk adjusted return in excess of a passive investment trading on solely stale information (Fama 1970). This theoretical statement underlies the problem discussion as well as the purpose of this paper and to some extent the formulation of the Black Swan strategy (which is based upon stale information). Of further importance is the statement that market efficiency relies somewhat on the availability of financial instrument substitutes. Since exact or close substitutes are available for derivatives such as options, the pricing should be more efficient (then for example a strategy including a stock portfolio) and hence should the EMH hold stronger and rule out any trading strategy based on derivatives (Shleifer 2000). However, the EMH rests on the assumption that the market as whole acts rational. The literature presented five behavioral reasons why this might not be the case; anchoring, availability, herd behavior, isolation effect and loss aversion.

2.7 Hypotheses and the "Model"

This chapter has presented the theoretical foundation underlying the research issue of this paper. In short, given the theoretical insights that the EMH rules out any trading system being able to generate any risk adjusted return in excess of a passive index investment, and that the empirical evidence regarding the significance of unanticipated extreme events in finance (Black Swans) as well as the flaws of well used financial models is known to rational markets participants, the two following hypotheses are formed in order to fully operationalize the purpose of this paper:

> *H_1: Returns generated by a Black Swan investment strategy do not differ from S&P 500 generated returns.*

> *H_2: Returns generated by a Black Swan investment strategy do not differ from S&P 500 generated returns on a risk adjusted basis.*

In order to test the set hypotheses a model is formed and illustrated in figure 3. A discussion of the model in purpose of creating an overview will follow below.[30]

Due to mainly data availability the model will be applied to the U.S. market during January 1, 1996 - December 31, 2012. Representing the benchmark, on which the Black Swan investment strategy will be evaluated, is the S&P 500. Hence, 100 million U.S. dollars will be passively invested in the S&P 500 during the period resulting in market equilibrium generated return at the end of the period.

The framework of the applied strategy is based upon the "barbell" strategy defined in section 2.3.3 and an equal amount of U.S. dollars will be invested during the same period. Of this capital, 85 percent will be invested in the benchmark itself and the remaining 15 percent will be used to buy rolling 30-day maturity European style deep out of the money put options (defined as having a strike price three standard deviations from the price of the underlying) on the benchmark. In order to determine the optimal fraction of capital (of the original 15 percent part) that will be used to buy options each year the Kelly criterion will be used. Furthermore, 50 percent of any available cash holdings in the strategy will be invested in rolling 1-month Treasury bills during the period (since the Kelly criterion only will suggest that a fraction of the original 15 percent part will be used for put options). Representing stale information in the model is the empirical evidence presented in earlier sections concluding, in short, that returns to investors are not generated even and steadily over time, but rather irregularly as a result of unanticipated extreme events (Black Swans) which are furthermore underestimated by many common used financial models.

Since the main purpose of the paper is to assess the EMH theorem when accounting for extreme events, the 85 percent capital part will not be invested in extreme safe instruments such as Treasure bills as the "barbell" strategy suggest but simply invested in the benchmark to allow for a more smooth comparison and evaluation of strategy performance.

Finally, at the end of the period, strategy generated returns will be compared to market equilibrium generated returns both on a pure return basis and a risk adjusted basis to determine performance and sustainability of the EMH.

[30] More detailed motivations and explanations are provided in the next chapter.

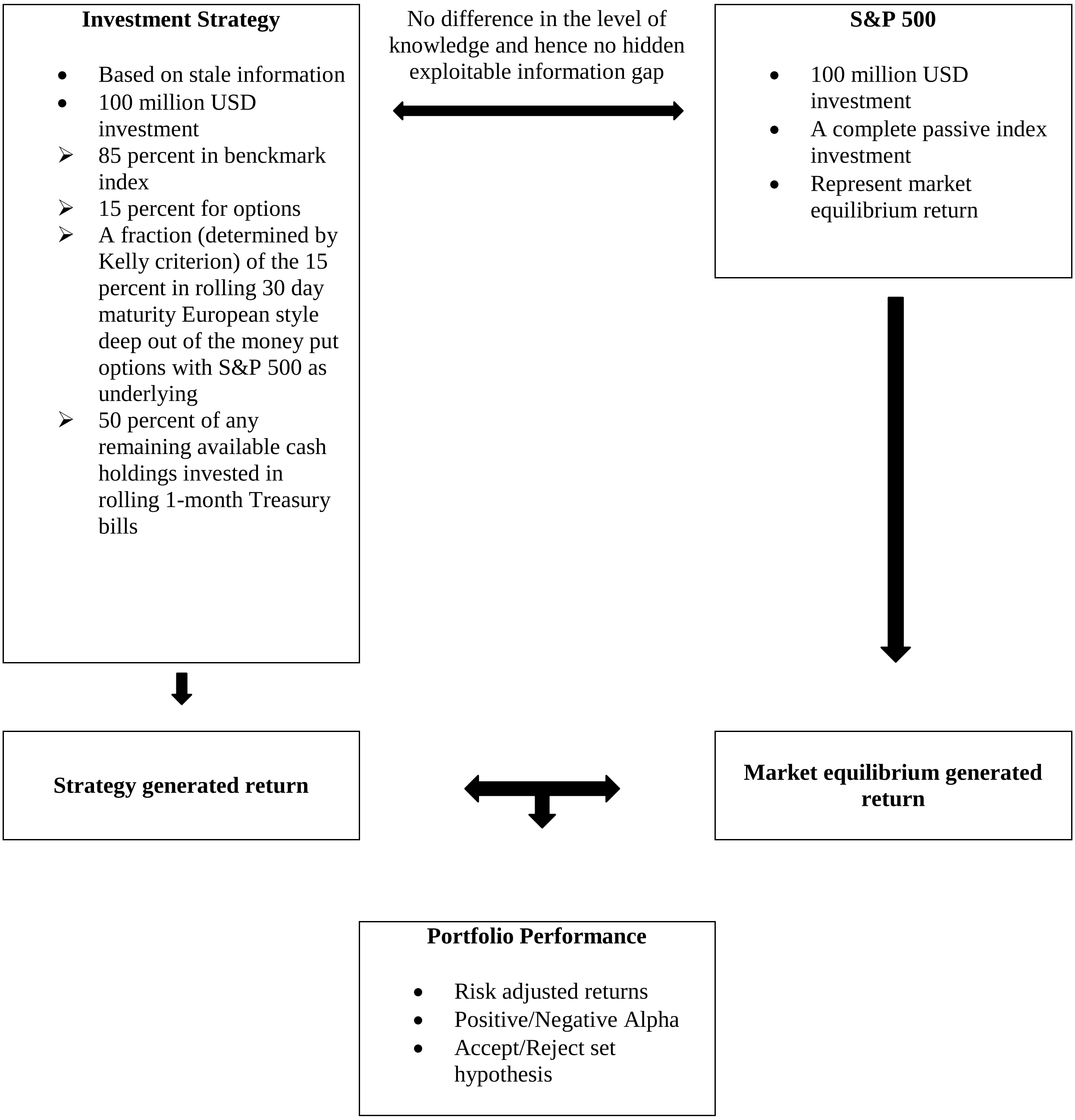


Figure 3. Model of operationalization.

3. METHODOLOGY

3.1 Data collection

3.1.1 Secondary data

Since the study is based upon empirical time-series data, secondary data provides the foundation for any data collection in this paper. Moreover, secondary data underlies the problem discussion and research question of this paper, and further use of secondary data is utilized in the literature review mainly in the form of scientific articles. Given the research nature of this paper, the main advantage of using secondary data is the input of in depth knowledge of the subject at stake which leads to a more comprehensive study then what would be possible by collecting and processing any other type of data. However, there are some well-known disadvantages of using secondary data.[31] Potential problems are the inability to control data quality and that secondary data could potentially be based upon tertiary sources and hence potentially passing multiple stages from the primary source with the result of being biased (Bryman 2008; Saunders, Lewis and Thornhill 2009). To overcome potential bias, secondary data is collected from the most reliable sources as possible; that being the work of merited and cited authors, notable academic journals and extensive, well-used databases. Furthermore, to ensure data reliability a number of cross-references are performed from alternative data sources.

3.1.2 Data sources

To implement the quantitative methods of this paper data are retrieved from the following sources:

- Thomsom Reuters - Datastream: *(cross-referencing collected data) S&P 500 (S&PCOMP), VIX index (CBOEVIX) & T-Bills (FRTBW3M; FRTBS4W)*
- Yahoo! Finance - *S&P 500 Historical Daily data (1996 - 2012)*
- Federal Reserve Bank of St.Louis - *U.S. T-Bills Historical Daily Data on 3-Month rate*
- Kenneth R. French Data Library - *U.S. T-Bills Historical Daily Data on 1-Month rate*
- Robert J. Shiller Online Data - *S&P 500 Historical Monthly Data (1900 - 2012)*
- CBOE - *VIX Historical Price Data*
- OptionMetrics - *Historical option data*

[31] Bryman (2008), Saunders, Lewis and Thornhill (2009) provide comprehensive insights on the usage of secondary data as well as general benefits and disadvantages.

In terms of literature covered in the paper, the Lund University's LUBsearch engine is used as the main source for finding relevant journal articles and books. Additionally, Google Scholar is used as a complement to LUBsearch.[32]

3.1.3 Data set and processing

The papers complete data set consists of time series data for historical 30-day maturity European style put options and S&P 500 quotes which form the two main components in the model depicted in figure 3. The put option data ranges from January 1, 1996 until December 31, 2012, which also constitutes the time period of the study. The reason is that there are no available option data, with S&P 500 as underlying, prior to 1996 in the data source and hence does this time period form the main delimitation of the study.[33] S&P 500 quotations are both collected on daily (for the time period 1996 - 2012) and monthly basis (for the time period 1900 - 2012). The daily S&P 500 data is used to calculate the strategy's (portfolio) beta (used for risk adjusting returns) and to determine any positive put option payoffs on the expiration day according to the simple payoff formula described on page seven (equation 4). The monthly S&P 500 data is used to approximate the odds of any put options expiring in the money. These odds are then plugged in to the Kelly criterion along with the put option price in order to determine an appropriate fraction of capital or "bet size" to use for option trading each year. In order to estimate a price for the put options, the variables "highest closing bid" and "lowest closing ask" is obtained for each option. Since no definite option price exists it has to be estimated and hence an average of the two variables is used.[34]

The data will be processed in yearly intervals with calculated returns and risk adjusted returns for each year and for the 17-year period as a whole. The components of the Black Swan strategy will be broken down into segments so their individual performance can be evaluated separately and some of the more interesting scenarios will be presented more in detail. Current subchapters will provide more in detailed explanations and motivations for the components used processing the data as well as selection criteria for the variables.

3.2 Selection of options and the VIX

Historical option data is rather extensive in unprocessed form considering that there exist both put and call options (for the same underlying) with different maturities and different strike

[32] LUBsearch engine is a common access point to all of Lund University's literary resources providing, through its various settings, a good overview of accessible material.
[33] The rather suboptimal time duration of the study is acknowledged and commented in section 3.6 below.
[34] Choosing the highest closing bid would be lenient towards the Black Swan strategy, this could positively skew the results. Choosing the lowest closing ask would be strict towards the Black Swan strategy, this could negatively skew the results; hence an average is used.

prices.[35] Due to a number of reasons, 30-day maturity put options are chosen to provide the basis for the Black Swan strategy and sufficient trading volume is always assumed to hold. It is acknowledged that the chosen maturity somewhat contradicts the strategy in relation to the central limit theorem effect noted in section 2.3.1 by the work of Aparicio and Estrada (2001) (i.e. the option will have a lower probability of expiring in the money when bought over 30 days then if bought on a daily basis). However, due to the time constraints of completing this study and the extensive amount of data that a daily historical option data set would generate over the period (that also would need to be processed) the Black Swan strategy is limited to 30-day maturity options. Further delimitations in selection of options are due to the fact that only put options are considered. The premise for this is the following:

First, there is a need to reduce the amount of data within the existing data set to make the data more surmountable and processable. Second, when sorting out suitable options for the strategy according to the definition and selection criteria below, an estimation of the underlying (S&P 500) volatility is needed. This volatility is provided by the VIX, which is calculated on a 30-day basis (with S&P 500 as underlying) matching the maturity of the options. Third, given the need to limit the data at hand, put options are also chosen over call options on the rational basis that the empirical evidence in section 2.3.1 (stale information) concludes that negative Black Swans result in larger impact on investment performance than positive ones, on average. This is further verified by the empirical S&P 500 data (1900 - 2012) in this paper, concluding that almost four times more negative Black Swan events were observed compared to positive Black Swan events (15/4).

In earlier sections it is stated that put options are chosen on the premise of having a strike price three standard deviations from the price of the underlying, the basis for this statement needs to be addressed. In section 2.3 a Black Swan event is defined as being rare, extreme in its impact and predictable in retrospective. Although being illustrative, this definition is not operational. Estrada (2008) recognises this inability to formally define a Black Swan and simply defines a Black Swan as an event equal to, or exceeding three standard deviations from the mean. Using this assumption means that an event of this magnitude is “only” expected to occur every 1/370 or 0,27 percent of the total time (using the standard normal assumption, using the actual distribution from historical data shows that it happens 1,4 percent of the time) hence capturing some of the initial definition. The authors will not make any further attempt to formally define the Black Swan concept in this study and hence

[35] e.g. one year of historical data for S&P 500 (for both puts and calls over all maturities) contains approximately +700 thousand options depending on the year.

Estrada's (2008) definition will be applied to sort out suitable put options for the strategy. However, this implies that options should be chosen as close as possible to the three standard deviation definition. If this is strictly implemented, the data set would end up with approximately half of the options outside of the defined Black Swan event horizon (below three standard deviations). Henceforth it is necessary to put up a more flexible criterion in the option selection process, the following criterion is formed:

- When presented with two options alternatives, the one that is the closest to and outside of three standard deviations is chosen.
- However, if there is a large discrepancy between the value outside and inside three standard deviations (e.g. 3,4 versus 2,96 standard deviations), the value closest at hand is chosen.
    - At four occasions the selection process deviates from the above stated criterion, the reason for this is that there is simply no feasible alternative at hand.[36] The market timing dilemma (not being in the market as a Black Swan event occur, see section 2.3.1) hence took precedence over the problem of buying suboptimal options. At three other occasions there are no suitable alternatives (or even semi-suitable alternatives) and hence no options are bought.[37]

Given the selection criteria above, the Black-Scholes (1973) formula for (d_1) (explained in section 2.1) is used to sort out suitable options:

$$d_1 = \frac{\ln(S_{S\&P500}/K) + (r_{3TB} + \sigma_{VIX}^2/2)T}{\sigma_{VIX}\sqrt{T}} \qquad (6)$$

For the above equation ($S_{S\&P500}$) is the S&P 500 closing quote at the time of purchase, (K) is the strike price of the put option, (r_{3TB}) is the daily 3-month Treasury bill rate, (σ_{VIX}) is the implied VIX volatility of the S&P 500 and (T) is the time to maturity of the option (always 30 days).

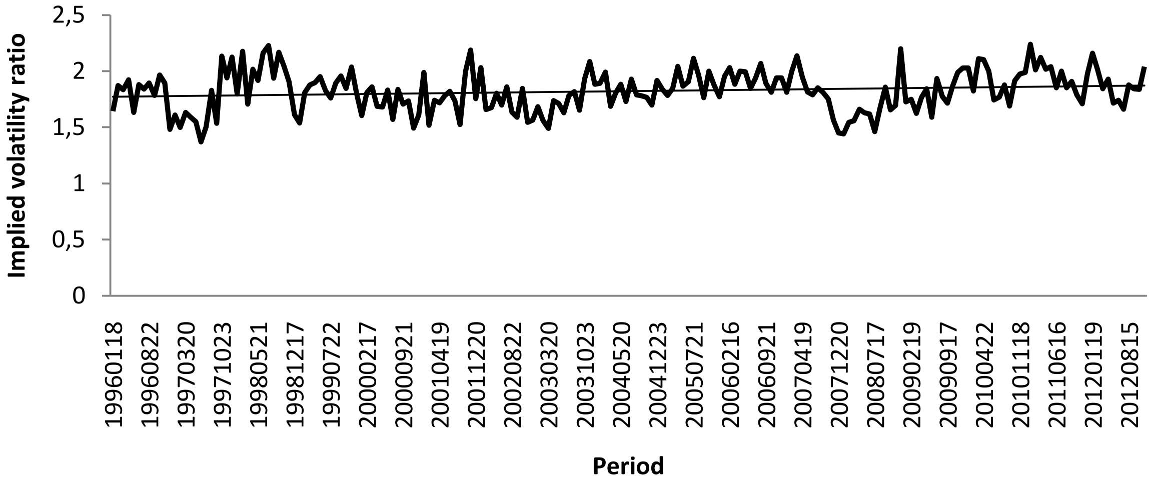
3.2.1 The Chicago Board Options Exchange Market Volatility Index - VIX

The use of the VIX index in equation (6) needs further explanation. The VIX is based on S&P 500 index option prices and measures the market's expectations of the near term volatility (30-day volatility) expressed as a weighted blend of implied volatilities available to the

[36] This is done two times in 1996 (2,81SD and 2,93SD), one time in 1998 (2,93SD) and one time in 2000 (2,89SD).
[37] The months in question are Nov-2001, Jul-2002 and Nov-2007.

market "today" (i.e. in case of this study, at the time of respective option purchase). In order to sort out suitable options by using the formula for (d_1) above, the volatility of the S&P 500 needs to be estimated. The use of the VIX is advantageous since a volatility of the S&P 500 does not need to be estimated in retrospective but simply given as unbiased market estimation, at the time of the option purchase.

However, the way that VIX is estimated is altered during time period of the study resulting in the existence of two different versions of the VIX (the VXO and VIX). The VXO version is not optimal for this study since it is based on the S&P 100 and does *not* capture the difference in implied volatility between deep out of the money and at the money options. The newer version of the VIX acknowledges that there is a difference in implied volatility between deep out of the money and at the money options and hence takes this difference into consideration. Figure 4 illustrates that there is a difference in implied volatility between deep out of the money and at the money options during the time period of this study, which is captured by the use of VIX. As is depicted, implied volatility for the deep out of the money put options sorted out in this paper is a much higher over the 17-year period.



The implied volatility ratio is calculated as the ratio between the "moneyness" ($d_1 \approx 3$) of the sorted out put options and the "moneyness" ($d_1 \approx 0$) of at the money put options with the same underlying (S&P 500) and maturity (30 days).

Figure 4. Implied volatility ratio for the sorted out deep out of the money put options, 1996 - 2012.

The VIX has the S&P 500 as underlying making it more optimal for this study; it also incorporates a broader scope of strike prices capturing a wider volatility skew. These two reasons make the VIX (compared to the old VXO version) superior for the study and thus it is chosen as the volatility measurement.

3.3 Optimizing put option trading

In order to determine the optimal fraction of capital that will be used to buy options each year probability and profitability's needs to be estimated (see section 2.2 on the Kelly criterion) according to the formula:

$$F = \frac{(PB-Q)}{B} \qquad (7)$$

In equation (7), the variable (F) is the fraction of capital (of the total 15 percent of capital that is allocated to option trading in the strategy each year) that will be used to buy put options each year. (F) is always denominated in relative terms and is independent of the total amount of capital allocated to option trading. (P) is the probability that the put option will expire in the money. (B) is the ratio between max profit and max loss. Max profit is determined by the put option payoff equation $(\max(K - S_T, 0))$ and max loss is determined by the put option price (the average of "highest closing bid" and "lowest closing ask"). (B) symbolizes how large the payout is in case of a win. Finally, (Q) is the probability that the put option will expire out of the money, or simply $(1 - P)$.

When determining the probability (P), it is known that the standard normal assumption underestimates probabilities in common asset (such as the S&P 500; which is the underlying of the put options). As stated in section 2.3.2 there is an inherent estimation bias when applying the standard normal assumption to market probabilities, the assumption of having a *known* probability is flawed. Hence, another estimation method is applied. This estimation is based upon historical observation, this is not optimal when capturing Black Swans since they are de facto impossible to predict with historical data according to Taleb's (2007) definition. However, this is the best estimation procedure at hand and since a Black Swan event is defined as a three standard deviations event it is arguably a plausible way of determining the probability (P). The actual probability is calculated as a ratio of how many monthly observations are outside of the negative three standard deviation margin, this is then divided with the total sample size.

Determining profitability (B) is harder since profitability is a ratio of max profit and max loss. The latter is easy to assess, this is just the price that is paid for the put option. The former is much harder to assess, since only one of the two variables, the strike price ((K) in equation (6)), is in fact known. The other variable, the future spot price ($(S_{S\&P500})$ in equation (6)), is not known and needs to be estimated. There are some possible solutions for this:

- *The strict max profit approach.*
  S&P 500 goes totally bust; equal to a future spot price of zero.
  - This solution is not chosen because it is rather drastic and unrealistic. For this to be true then every company on NYSE and NASDAQ must go bankrupt during one month.[38]
- *Backtracking the max profit value through the EMH assumption.*
  - The authors formed the idea, that if one knows the probability (P) (which is assumed) as well as the price of the put option (which it is), then it would be possible to calculate the max profit assuming that the EMH holds true (because of the perceived fair game assumption), this would simply be done by dividing the price of the put option with the probability.[39] However, this solution was not feasible due to lack of robustness; in some cases the max profit exceeds the strict max profit approach (which is impossible).
- *Imagine the unimaginable, putting a percentage on a Black Swan.*
  Future spot price of the S&P 500 is equal to today's spot price times 0,5.
  - An assumption is hence made on how much the S&P 500 can reasonable fall during a month in a worst case scenario. Empirically, the most the S&P 500 has fallen during a month is 26,6 percent (for the period 1900 - 2012). However, trying to put a number on a Black Swan event is contradicting to the Black Swan philosophy itself. Nevertheless, due to lack of further reasonable alternatives it is assumed that the absolute max profit event is an S&P 500 fall of 50 percent during a month; hopefully this will encompass all possible Black Swan events.

At five different occasions the Kelly Criterion present a negative fraction of capital, this implies that it is optimal to sell instead of buy put options.[40] Since the main goal of this paper is to invest on the occurrence of Black Swan events, and since the opportunity for market timing is next to nil an arbitrary fraction of capital of 0,5 percent will be invested.

Furthermore, to address some of the underlying assumptions of the Kelly criterion (presented on page ten) there will be only one type of put option bought each month (to avoid infringing the no correlation assumption), all put options will also be held until expiration (to avoid

[38] Even if such a scenario was to occur due to some extreme Black Swan event, it is highly questionable if any profit were to pay out at all.
[39] This can be illustrated with another coin toss example; if the probability (0.5) and the cost (1) is known then in a fair game the max profit must be (1 / 0.5) = 2 for this to hold true.
[40] The months in question are: Oct. 2008; Nov. 2008; Jan. 2009; May 2010; Aug. 2011.

infringing the no exit position assumption). However, there is a danger that the value of the put options might correlate with the other instruments in the strategy; this could be a potential problem. In order to contradict this potential problem and raise the awareness of it, a separate beta will be calculated for the option trading part of the strategy to assess if it correlates with the rest of the strategy. Finally, a full Kelly approach will be implemented.[41]

3.4 Selection of the benchmark index

The concept of the market portfolio plays a significant role in finance since it represents the optimal portfolio (in theory, containing a weighted sum of every available asset in the market in exact proportion to its market value) in which investors need to invest depending on risk attitude. Since the market portfolio is a theoretical concept and not observable, proxies for its existence are commonly used (Roll 1977; Standard and Poor 2001).

The S&P 500 composite stock market index is chosen to act as proxy for the market portfolio due to a number of reasons. The fact that unlike other major indices (such as the Fortune 500) the S&P 500 is market-value-weighted, meaning that each firm's influence on index performance is directly proportional to its market value (just as in the theoretical portfolio). This characteristic has made the index a standard proxy both in practice and academia for the evaluation of investment theories (Standard & Poor 2001). Of more importance for this paper is the fact that if the put options were to be bought on anything less general then the benchmark (such as specific sectors or equities), a more severe research bias would be present since the paper is based upon historical time series data and the authors are aware of historical outcomes.[42] Another motive for using S&P 500 as benchmark is that when equation (6) is applied to sort out suitable options according to the criteria in section 3.2, the VIX which is calculated on the S&P 500 is used.

However, it is noted that Roll (1977) states that proxies for the market portfolio, such as the S&P 500, cannot represent the entire market and hence any conclusion made upon such a proxy will be inaccurate. Nevertheless, given that the same author also notes that the market portfolio is unobservable and given the beneficial characteristics of the S&P 500 listed above we will not further address any shortcomings.

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[41] This decision is taken before hand to prevent tinkering with the optimal Kelly fraction.

[42] The research outcome would be rather worthless if we were to buy put options on, lets say, airplane equities during the event of 9/11 or on Lehman Brothers during September 2008.

3.5 Risk adjusting returns

In order to allow for a reliable comparison between the Black Swan strategy and the benchmark, any generated returns must be adjusted in relation to the level of absorbed risk. The importance of risk adjusted returns in context of the EMH is stressed by Shleifer (2000). He concludes that measuring the performance of a particular investment strategy is both difficult and controversial; whenever money making opportunities (resulting from trading on stale information) are presented, critics are quick to suggest an alternative model of risk reducing any excess returns to a fair compensation for risk taking. There is a plethora of different risk adjustment measures in academia[43] with the Treynor ratio (Treynor 1965), the Sharpe Ratio (Sharp 1966) and Jensen's alpha (Jensen 1968) being the most common and well used by researchers and practitioners (Modigliani & Modigliani 1997).

Due to its straightforward (absolute) interpretation compared to the Treynor and Sharp ratio as well as the non-need to rank multiple portfolios in the paper, Jensen's alpha is applied to risk adjust returns.[44] Jensen's approach is based on the capital asset pricing model (CAPM) which provides the theoretically appropriate required rate of return of an particular asset (based on the market portfolio described above), given the assets non-diversifiable risk (Sharp 1964; Lintner 1965; Mossin 1966).[45] Leaving the extensive litterateur regarding the CAPM's benefits and shortcomings out of this paper, we note that Shleifer (2000) states that the CAPM is a widely accepted model in context of the EMH and its empirical challengers in academia. The formula which is applied to realized returns can be written as:

$$a = R_S - [R_{1TB} + \beta_S(R_{S\&P500} - R_{1TB})] \qquad (8)$$

In equation (8), (R_S) is the logged monthly total return of the Black Swan strategy (put options, Treasury bills and S&P 500). (R_{1TB}) is the 1-month Treasury bill rate. (β_S) is the strategy's (portfolio) beta on yearly basis.[46] ($R_{S\&P500}$) is the logged monthly return of S&P 500 representing the passive investment. The conclusion is that Jensen's alpha (a) directly reflects the strategy's ability to generate return above market equilibrium return. Hence, a

[43] Cogneau and Hübner (2009a; 2009b) provide a comprehensive study of 107 performance measures by presenting classification based on objectives, properties and degree of generalization as well as discussing main strengths and weaknesses.
[44] Since the Treynor- and Sharp ratio are relative risk measures (see Treynor (1965) and Sharp (1966)) they are mostly used to rank fund performance, in which they are superior to Jensen's alpha, however, the output are not as easy interpreted as the Jensen's alpha (Jensen 1968; Cogneau and Hübner 2009a).
[45] It is beyond the scope of this paper to further explain the properties and implications of the CAPM.
[46] Where $(\beta_S) = \frac{Cov(R_S, R_{S\&P500})}{Var\,(R_{S\&P500})}$; The yearly betas are derived using daily data and each years beta contains approximately 250 observations. The risk of the portfolio was assumed to be constant over each year after a sample of yearly betas was compared to monthly beta values that differed only marginally.

positive alpha indicates returns above what is required by the market given the risk of the strategy, and a negative alpha naturally indicates the opposite (Jensen 1968).

To test for statistical significance of any pure returns and risk adjusted returns during the period, a regular two sample t-test will be performed.

3.6 Reliability and Validity

The essence of scientific methodology is the credibility of any research findings; hence the chapter is ended by critically reflecting over the applied research approach in terms of reliability and validity (Bryman 2008; Saunders et al. 2009).

In context of this paper, the concept of reliability refers to the extent to which the data and analysis procedures will yield consistent findings (Saunders et al. 2009). The secondary data which lay the foundation for this study is obtained from merited, well-used sources (see section 3.1.2) and has furthermore formed the basis for various of former peer reviewed papers. To further ensure quantitative data reliability a range of observations within all variables are cross-referenced using Datastream. The procedure of processing the data is conducted using well-merited, academically accepted models and formulas. MS Excel provides the basis for any computations, presented tables or figures made by the authors in this paper. Any used proxies or assumptions in regards data processing are clearly disclosed to the reader and some commonly used in academia.

The concept of validity enlightens the issue of how well the measurements in the methodological framework capture what they are expected to capture with respect to the research objective (i.e. whether the research findings are really about what they appear to be about) (Bryman 2008; Saunders et al. 2009).[47]

In context of ensuring validity, the premise of the papers methodological approach is to limit any potential research bias. Since the purpose of this paper is dependent on the concept of stale information and sensitive to any interference from the researcher (due to knowledge about historical outcomes), put options in the strategy are always held to maturity and only bought with the benchmark as underlying to eliminate any biased speculation in order to generate returns.

[47] Note that even though the concepts of reliability and validity can be separated in a pure analytical sense they are still closely related by the fact that validity infers reliability. This entails that a measurement cannot be valid less it is reliable (Bryman 2008).

It is debatable that the operationalization of the informal definition of Black Swan events in section 3.2 is inadequate, not so much in the sense of not capturing an extreme event but rather that the event not can be proven to be strictly unanticipated by the market (see section 2.3 for Taleb's (2007) definitions). The point is well taken; the Black Swan events in this paper are solely dependent on the methodological definition of these events. Since the main focus when operationalizing the concept is the outlier effect (defined as three standard deviations) and since there is no completely valid way of quantifying Black Swans the paper will apply the definition used in the work of Estrada (2008) and the mentioned shortcoming will merely be noted.

Another important aspect that needs to be addressed in context of validity is the length of the time period used in this paper. Due to lack of option data (with S&P 500 as underlying) prior to 1996, the study is limited to the time period January 1, 1996 - December 31, 2012. Since the Black Swan investment philosophy is based upon the belief of time arbitrage it would unquestionably be more optimal to conduct the study over a longer time span. Hence, the outcome of the study would potentially differ in term of the strategy's generated alpha. Yet another issue with potential to change this outcome is the decision to use 30-day maturity put options in the strategy due to the reasons described in section 3.2. Certainly would a strategy based on shorter term maturity options (or the ability to exercise the options at will) have the potential to generate a different alpha. The strategy's sensitivity to such influences is acknowledged and the meaning of such sensitivity to alpha is illustrated in section 4.3.1.2.

4. EMPIRICAL FINDINGS AND ANALYSIS

4.1 Findings (I) - Returns

The total data sample consists of 204 monthly observations of returns for the passive index investment (S&P 500) and the Black Swan strategy (Portfolio). The empirical findings for the period January 1, 1996 - December 31, 2012 (17-years) are presented in table 1.

Table 1. Performance of the passive investment and portfolio on a pure return basis, 1996 - 2012.

Period	S&P 500	Portfolio	SD	t-statistic	p-value	CI
1996-2012	0,82886	0,595349953	0,03943	-0,267	0,790	[0,58994; 0,60076]

Relative performance is based on logged monthly returns; actual relative performance is approximately 129 percent (S&P 500) and 81 percent (Portfolio) for the period. (SD) portfolio standard deviation. Corresponding test statistic (n=204), p-value and confidence interval at a set significance level of 5 percent $(\alpha = 0{,}05)$.

As presented, the passive index investment has generated a return of approximately 83 percent compared to the Black Swan strategy's return of approximately 60 percent. The test of any difference in strategy generated returns compared to market equilibrium returns is clearly insignificant at the set significance level indicating market efficiency during the studied period on a pure return basis. **Hence the first hypothesis of this paper cannot be rejected:**

H_1: Returns generated by a Black Swan investment strategy do not differ from S&P 500 generated returns.

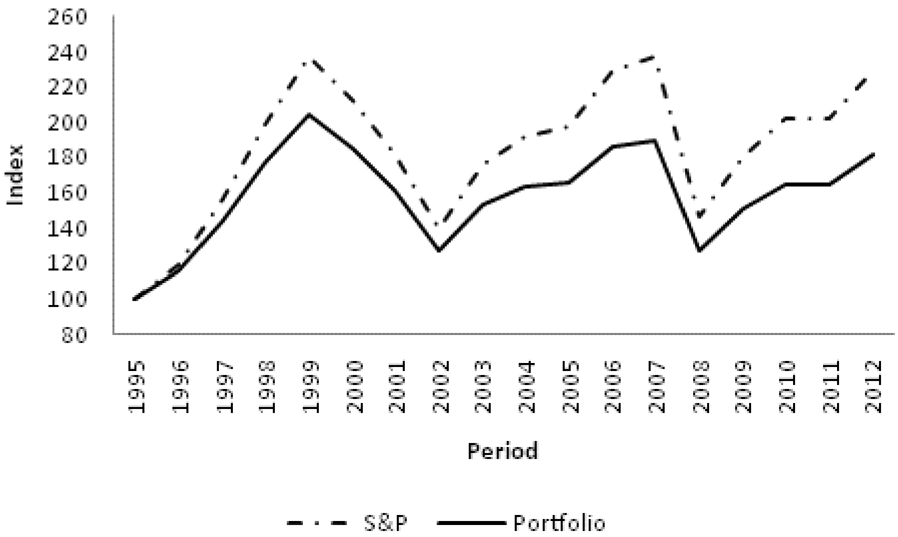
The result is unsurprising considering the performance progress of the strategy during the studied period. Table 2 presents returns for the passive index investment and strategy on a yearly basis. An illustration of the performance is further given in figure 5.

Table 2. Yearly performance on a pure return basis, 1996 - 2012.

Period	S&P 500	Portfolio	SD	t-statistic	p-value	CI
1996	0,17675	0,14353	0,02639	-0,236	0,815	[0,12860; 0,15846]
1997	0,27009	0,22520	0,03847	-0,218	0,830	[0,20343; 0,24696]
1998	0,23640	0,20003	0,05435	-0,125	0,902	[0,16928; 0,23078]
1999	0,17836	0,14698	0,03225	-0,183	0,856	[0,12874; 0,16523]

2000	-0,10691	-0,10260	0,04171	0,019	0,985	[-0,12620; -0,07900]
2001	-0,15600	-0,13941	0,05023	0,061	0,952	[-0,16783; -0,11099]
2002	-0,26613	-0,23408	0,05019	0,118	0,907	[-0,26248; -0,20569]
2003	0,23413	0,19138	0,02731	-0,293	0,772	[0,17593; 0,20683]
2004	0,08612	0,06148	0,01783	-0,259	0,798	[0,05140; 0,07157]
2005	0,02957	0,01355	0,01921	-0,157	0,877	[0,00269; 0,02442]
2006	0,14607	0,11489	0,01439	-0,408	0,687	[0,10675; 0,12303]
2007	0,03469	0,01828	0,02234	-0,132	0,896	[0,00564; 0,03091]
2008	-0,48590	-0,40234	0,05290	0,289	0,775	[-0,43227; -0,37240]
2009	0,21070	0,17279	0,05406	-0,129	0,898	[0,14221; 0,20338]
2010	0,11515	0,08705	0,04818	-0,109	0,914	[0,05979; 0,11432]
2011	-0,00003	0,00399	0,03701	0,020	0,984	[-0,01695; 0,02493]
2012	0,12580	0,09462	0,02662	-0,221	0,827	[0,07956; 0,10968]

Relative yearly performance is based on logged monthly returns. (SD) portfolio standard deviation. Corresponding test statistics, p-values and confidence intervals at a set significance level of 5 percent ($a = 0,05$).



1996-01-01 = 100. Monthly data.

Figure 5. Performance of the passive investment relative the portfolio, 1996 - 2012.

Not at a single year did the strategy significantly generate returns different to market equilibrium returns which advocating for the EMH. Given that that strategy contains 85 percent holding in the benchmark itself it is rather self-evident by table 1 and figure 5 that the option portfolio of the strategy lacked positive performance. Appendix A provides an overview of monthly performance and returns.

4.2 Findings (II) - Risk adjusted returns

The concept of risk adjusted returns was stressed in the previous chapter. Table 3 present the empirical findings for the period January 1, 1996 - December 31, 2012.

Table 3. Jensen's alpha for the Black Swan investment strategy, 1996 - 2012.

Period	Jensen's alpha	SD	t-statistic	p-value	CI
1996-2012	-0,12978	0,00596	-1,525	0,128	[-0,13059; -0,12896]

Jensen's alpha is derived using logged monthly returns. (SD) standard deviation in alpha. Corresponding test statistic (n=204), p-value and confidence interval at a set significance level of 5 percent ($a = 0{,}05$).

As is shown, the alpha of the Black Swan strategy is approximately -13 percent, implying returns below market equilibrium given the risk of the strategy for the stretch of the studied period. Moreover, following the findings for pure returns, the test of any difference in strategy generated risk adjusted returns compared to market equilibrium returns is insignificant at the set significance level; this result further supports the evidence for market efficiency. **Hence the second hypothesis of this paper cannot be rejected:**

H_2: Returns generated by a Black Swan investment strategy do not differ from S&P 500 generated returns on a risk adjusted basis.

Table 4 presents alpha for the strategy on a yearly basis. At no point in time did the strategy generate a statistically positive alpha but during three years did the strategy significantly underperform the market (shaded rows). Appendix A provides an overview of monthly alpha's as well as the strategy's beta representing the risk adjustment factor.

Table 4. Risk adjusted returns on a yearly basis, 1996 - 2012.

Period	Jensen's alpha	SD	t-statistic	p-value	CI
1996	-0,00824	0,00192	-1,239	0,228	[-0,00933; -0,00716]
1997	0,00055	0,00313	0,050	0,960	[-0,00123; 0,00232]
1998	0,00733	0,00521	0,406	0,688	[0,00438; 0,01027]
1999	0,00568	0,00522	0,314	0,756	[0,00273; 0,00863]
2000	-0,03708	0,00511	-2,095	0,048	[-0,03998; -0,03419]
2001	-0,03009	0,00499	-1,741	0,096	[-0,03291; -0,02727]
2002	-0,03303	0,00387	-2,466	0,022	[-0,03522; -0,03085]
2003	0,01750	0,00403	1,254	0,223	[0,01522; 0,01978]
2004	-0,00234	0,00321	-0,211	0,835	[-0,00416; -0,00052]
2005	-0,01595	0,00485	-0,950	0,353	[-0,01870; -0,01321]
2006	0,00204	0,00330	0,178	0,860	[0,00017; 0,00390]
2007	-0,02051	0,00580	-1,020	0,319	[-0,02379; -0,01723]
2008	-0,06434	0,00806	-2,304	0,031	[-0,06890; -0,05978]
2009	0,02162	0,00776	0,804	0,430	[0,01723; 0,02601]
2010	0,00791	0,00961	0,238	0,814	[0,00247; 0,01335]
2011	0,00385	0,00962	0,116	0,909	[-0,00159; 0,00929]
2012	0,01534	0,00727	0,609	0,549	[0,01122; 0,01945]

Yearly alpha is based on logged monthly returns. (SD) standard deviation in alpha. Corresponding test statistics, p-values and confidence intervals at a set significance level of 5 percent ($a = 0{,}05$).

The empirical results in above sections are uniform. However, they raise a number of practical and theoretical issues with regards to the performance of the strategy's option portfolio and the theoretical framework of this paper.

4.3 Analytical discussion

The set hypotheses of this paper could not be rejected implying efficient markets during the period in support of the EMH and in coherence with the previous work of Malkiel (1995), Gruber (1996), Holmes (2007), Fama and French (2010). However, during the years of 2000, 2002 and 2008 (presented in table 5) with significant negative alpha, the market is indicated to be inefficient in the sense that options is a zero-sum game which always has a counterpart benefiting of ones losses. With analytical regards to the risk adjusting method applied, this may not be the case at all however since Jensen's alpha in hindsight showed to be rather weak in accounting for proper risk taking during large market downturns. Consider Jensen's alpha below:

$$a = R_S - [R_{1TB} + \beta_S(R_{S\&P500} - R_{1TB})]$$

When ($R_{S\&P500}$) is negative, the lower the portfolio beta (β_S) is, the more negative impact it has on Jensen's alpha since the portfolio beta negates a part of the large market downturn. For instance, at the most statistical significant year of 2002 Jensen's alpha is -0,021925. However, if the portfolio beta would double, than the result would be an Alpha of 0,184217. So instead of having a risk adjusted negative return of over 0,02 by doubling the risk *and* having the same return as before the alpha would indicate a positive difference of over 0,20;[48] there is no logic behind doubling the risk, keeping the same return and generate a positive effect out of it. (i.e. the negative effect of having a low beta and positive effect of having a high beta (if the market return is negative) is erroneous (see table 5 below)). As is presented, the market return is largely negative during 2000, 2002 and 2008; this could potentially explain why the market appears to be inefficient during these years. Table 5 and figure 6 depicts that the significance coincides with the years of large market downturns.

Table 5. Jensen's alpha for statistically significant years.

Period	S&P Return	Portfolio Return	Jensen's alpha	Beta	t-statistic	p-value
2000	-10,69%	-10,26%	-0,03708	0,748	-2,095	0,048
2002	-26,61%	-23,41%	-0,03303	0,769	-2,466	0,022
2008	-48,59%	-40,23%	-0,06434	0,705	-2,304	0,031

The table depicts the erroneous effect of having a low beta during large market downturns. Yearly alpha is based on logged monthly returns. Corresponding test statistics and p-values at a set significance level of 5 percent ($a = 0{,}05$).

[48] i.e. (0,184217-(-0,021925)).

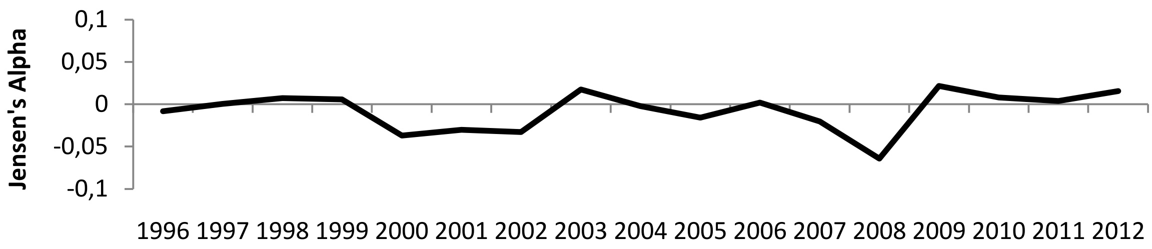


Figure 6. Jensen’s alpha, 1996 - 2012.

4.3.1 Option portfolio analysis

Analyzing the performance of the portfolio it becomes obvious that the essential option component of the Black Swan strategy has underperformed. As is illustrated in figure 7, 2011 is the only year where the option component of the portfolio actually generated a profit. This year has however only a minute impact on the overall result for the period. The accumulated losses of the option component in the strategy is approximately 30 million U.S. dollars for the period, indicating overpriced options and adhering to the risks of Black Swan investing as discussed by Malliaris and Yan (2009).

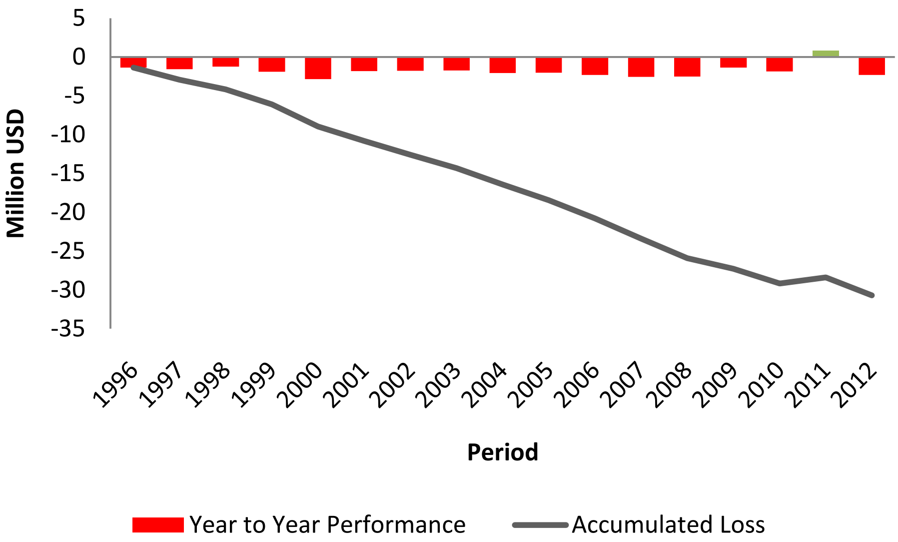
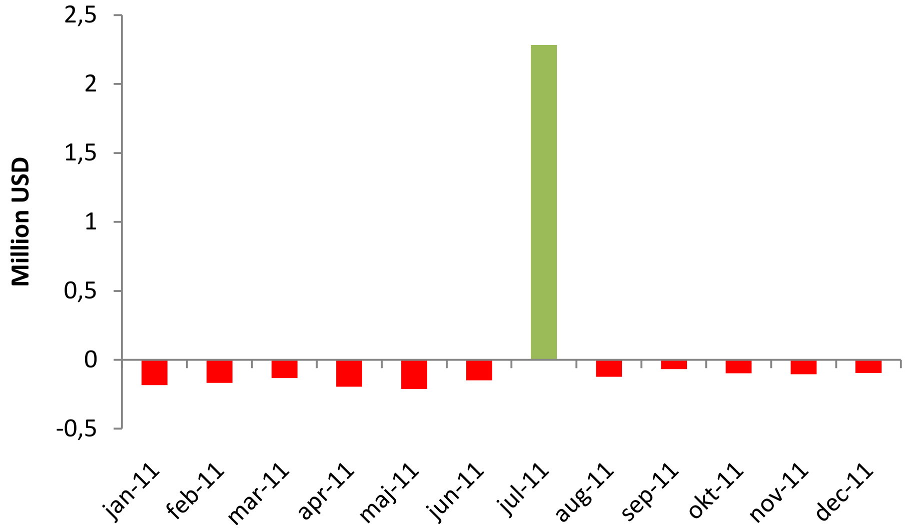


Figure 7. Performance of option portfolio, 1996 - 2012.

In 2011, only one set of option contracts expired in the money for the option portfolio, which generated that year’s profit. The contracts were bought in July 2011 and captured the effect of the European sovereign debt crisis and U.S. debt-ceiling crisis. Figure 8 illustrates the performance of the option portfolio during 2011 and captures the essence of a Black Swan investment philosophy as described by Taleb (2007), Malliaris and Yan (2009).



The year 2011 ended with a surplus of 0,762 million U.S. dollars.

Figure 8. Performance of option portfolio during 2011.

In addition, figure 9 illustrates how the U.S. debt-ceiling crisis (resolved at the beginning of August 2011) and the fear of contagion in the European sovereign debt crisis (banned short selling in parts of Europe on August 8 and August 11) pushed the S&P 500 below the strike price of the options.

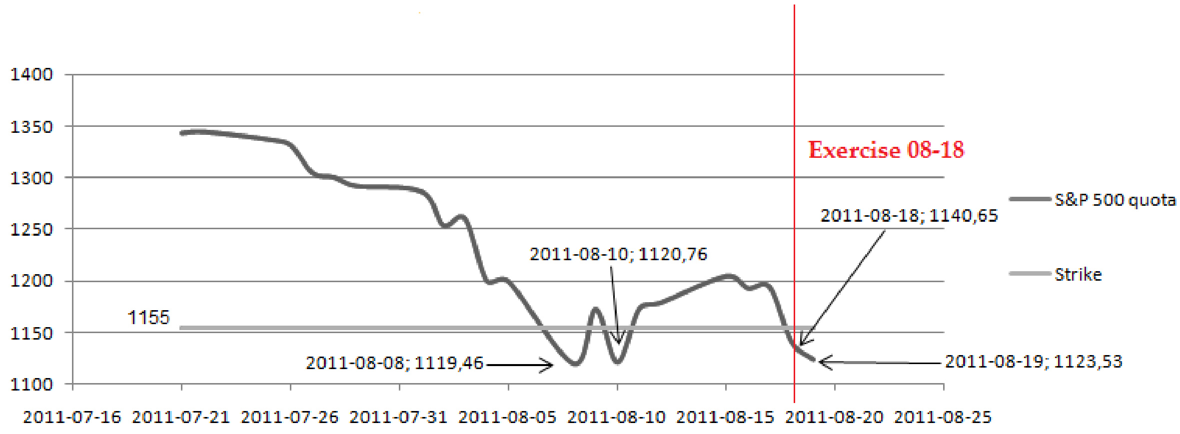
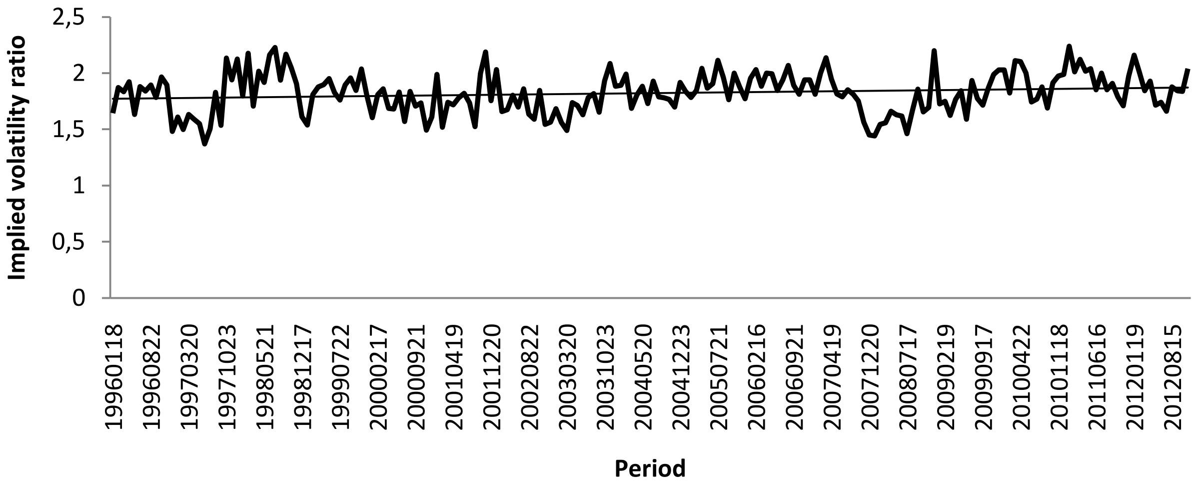


Figure 9. European sovereign debt crisis and U.S. debt-ceiling crisis, 2011.

As mentioned, the option contracts of July 2011 were the only contracts that ended up in the money at exercise, all other contracts failed to make a profit. The potential reason for this can be two-fold. Either could the sorted out put options be overpriced in general, or the applied time frame of the strategy be too short to capture the essence of the time arbitrage belief, or a combination of both. An analytical discussion of these possibilities will follow.

4.3.1.1 The sorted out put options are overpriced

This somewhat contradicts the work of Kahneman and Tversky (1974), Scharfstein and Stein (1990), Strack and Mussweiler (1997) with regards to the anchoring effect and herd behavior when pricing options. If these effects were to be present then one would expect the option prices to be lower. So one potential conclusion is that any possible anchoring- and herd behavior effects clearly is not enough to affect the option prices sufficient for the Black Swan strategy too generate positive returns. However, something reflecting the anchoring effect seems to be present though. Analyzing the premium (reflected in the implied volatility in figure 10) that is paid for the sorted out put options it seems to be approximately constant over the studied time period, so a potential effect can be present, but the options could still be overprized. The illustration in figure 10 can potentially also be related to the isolation effect discussed by Kahneman and Tversky (1979) which will be considered later on in this analysis.



The implied volatility ratio is calculated as the ratio between the "moneyness" ($d_1 \approx 3$) of the sorted out put options and the "moneyness" ($d_1 \approx 0$) of at the money put options with the same underlying (S&P 500) and maturity (30 days). Potential anchoring effect; the implied volatility ratio seems to be constant over time.

Figure 10. Implied volatility ratio for the sorted out deep out of the money put options, 1996 - 2012.

4.3.1.2 Insufficient timeframe

Buying 30-day maturity put options led to 201 observations (total months of option contracts being bought). Since a Black Swan investment strategy is built upon the concept of time arbitrage it postulates a large enough data set accumulated over a large time period to generate accurate results. This speaks for the use of daily option data which would have been superior to monthly data, not only for the added amount of observations, more importantly it would have not put the central limit theorem into effect which was acknowledged by Aparicio and Estrada (2001).

If the decision to hold options until expiration had not been taken then the central limit theorem effect would not have mattered as much. The “buy and hold” limitation was decided on sound logic. First, if options could have been sold whenever, then the strategy would have been open for criticism of obvious hindsight bias. Second, for the Kelly criterion to be valid it requires that the investment is held until expiration (Kelly 1956; Thorp 2000). However, both of these requirements would have been satisfied if the strategy were instead based on daily data. This subsequent self-criticism can be illustrated more clearly in figure 11 (options held during the Global financial crisis, Sept. and Oct. 2008). The figure illustrates exactly how the “buy and hold” limitation hampered the ability to make a profit.

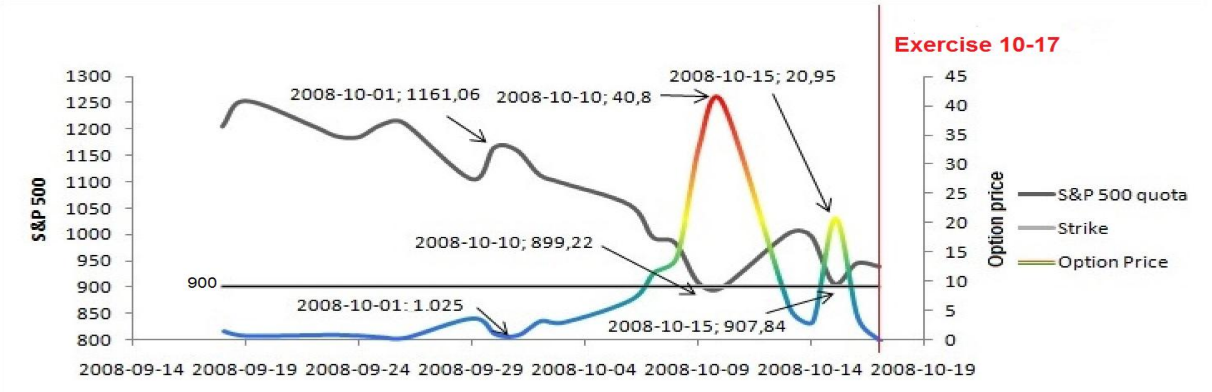


Figure 11. Put options held during the global financial crisis, 2008.

As illustrated, potential profit could have been made if the options would have been sold for either 41 or 21 U.S. dollars per option, since the options are prohibited from being sold before expiration they end up worthless. Hence, the central limit theorem effect ends up cancelling out some of the potential Black Swan returns in the portfolio.

4.3.1.3 Loss aversion, availability bias and the isolation effect

The following month of the Global financial crisis (November 2008) shows how loss aversion, availability bias and the isolation effect discussed by Kahneman and Tversky (1974; 1979; 1984), Barberis and Thaler (2003) could potentially have affected option prices (see figure 12).

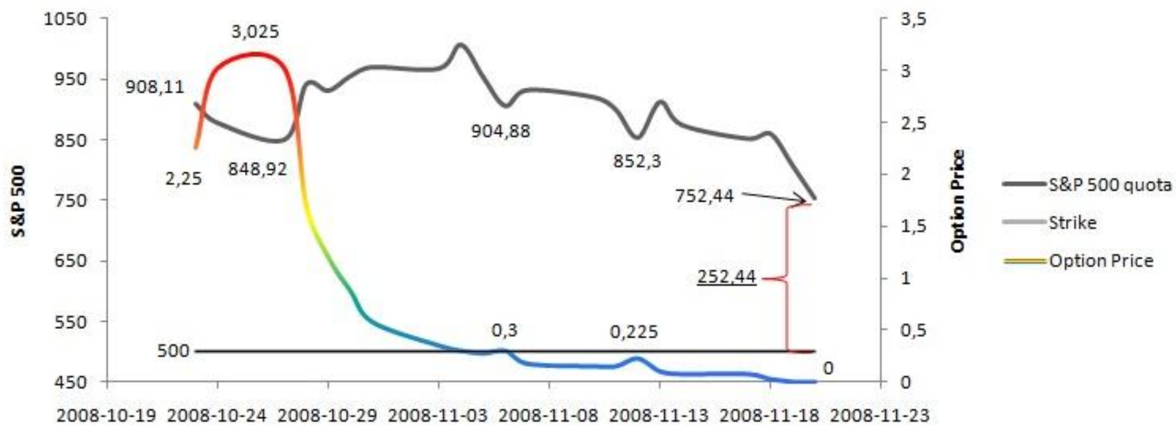


Figure 12. Put options held during October and November 2008.

Loss aversion could potentially be seen in the willingness to try and avoid catastrophic losses by buying put options that have a very slim chance of ever end up in the money.[49] The option contracts bought during these months have an implied volatility of 113 percent. For the options to end up in the money the S&P 500 needs to fall by 45 percent. This is extremely improbable, the worst month ever in S&P 500 during the time period 1900 - 2012 is a decline of 26,6 percent. So a fall by 45 percent somewhat goes against investor rationality during these months. As is illustrated in figure 12, even with a large fall of nearly 18 percent (908 to 752) the index were never even close of approaching the strike price, in fact the S&P 500 would have needed to fall another 252 for it to end up at the money. One potential explanation for this overprice could be the Grossman-Stiglitz paradox (1980): This implies that the market is assumed to be efficient when it in turn is not, rendering the price inefficient.

November 2008 is one of five months were the Kelly criterion actually presented a negative number in the sample (hence implying to short (sell) the put options at hand) and there was furthermore no need to assume sufficient trading volume during this month. The volume for the contracts was 10724 with a median price of 225 U.S. dollars per contract, all in all, everything considered, this raises the question of why these put options were bought when the probability of them ever generating profit was next to nil. One potential answer to this question can be the isolation effect; instead of looking at the option contract at hand the investors only looked at the difference between the option contracts. So instead of analyzing the contract in its entirety they just compare the differences to the at the money alternative (this difference is illustrated in figure 10). Another potential explanation for the trading volume can be found in the availability bias. The two month period prior to November 2008

[49] It could also be seen as speculation but that seems rather unlikely considering the outrageous implied volatility of the options.

show strong negative development in the S&P 500, this fact can have influenced investors to make irrational decisions when trading in options.

4.3.2 Conclusions

Concluding from above made analysis is that there are a lot of reasons why option pricing might not be efficient when it comes to Black Swan events. As written in section 3.3, the max profit scenario is based on an assumption of how much the most severe Black Swan event can potentially affect the S&P 500. If the assumption of a 50 percent decline is an overstatement, than the option strategy would look very much different. If instead the assumption was made that the worst historical scenario was to be the foundation of the max profit scenario (i.e. -26,6 percent in November 1929, the start of the great depression) then the Kelly criterion would advice against investing in over 60 months instead of 5 months. This is a major difference and a weakness since it exactly refers to the concept of Knightian uncertainty (Knight 1921).

Another example of why the assumption of a 50 percent decline could be too severe is the following: if a question would be asked to the public of what the most severe, rare and extreme event of the period at hand (1996 - 2012) were, the answer would surely be the 9/11 terrorist attacks. Figure 13 presents the S&P 500 index and the strike price for the option contracts held during 9/11. As is illustrated, even an extreme event such as 9/11 leads to no profit for the option strategy.[50]

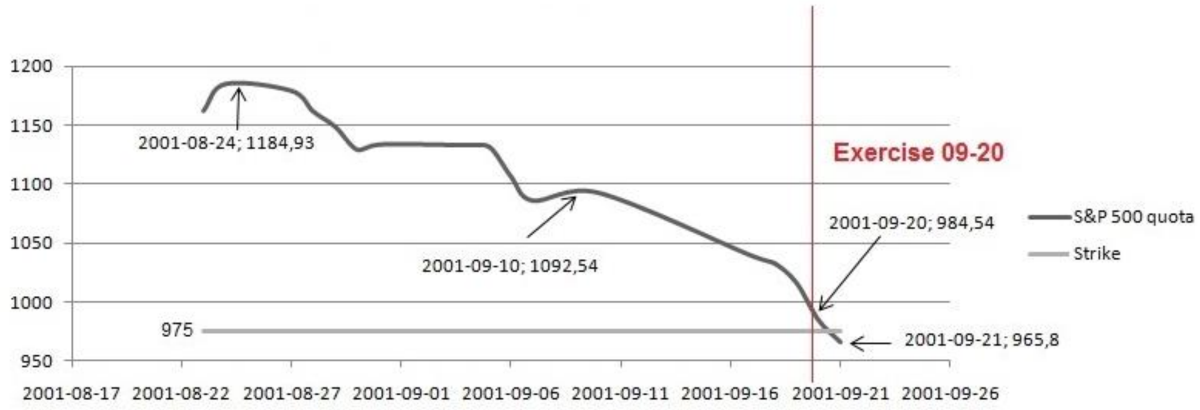


Figure 13. Put options held during the 9/11 Terrorist attacks, 2001.

Still, with all of this said, it is hard to draw a conclusion if the market is efficient or inefficient when it comes to extreme risk. Figure 7 suggests that the market might be inefficient when it

[50] Comparing figure 13 with figure 9 shows that both these option contracts would have made a lot more money if the exercise date were merely one day into the future.

comes to pricing Black Swan events (it suggests that there is money to be made by shorting (selling) deep out of the put money options), however this is precisely what Taleb (2007) calls picking pennies in front of a stream roller. Also with a suboptimal timeframe it could be that there is simply not enough time to properly evaluate the Black Swan investment strategy.

5. CONCLUDING DISCUSSION

5.1 Conclusions

The research purpose of this paper was to assess if the efficient market hypothesis is valid when accounting for extreme events (Black Swans). This was done by assessing the ability of a Black Swan investment strategy to generate risk adjusted returns relative a passive index investment, based upon only stale information. The strategy accounted for extreme events through the inclusion of long positions in deep out of the money put options reflecting the markets perception of extreme risk.

The empirical results show that the investment strategy proposed in this paper does not generate risk adjusted returns that differ from market equilibrium returns, which indicates that the efficient market hypothesis holds. This was also true on a pure return basis.

However, with regards to the option component of the Black Swan strategy, the findings imply that the strategy has the characteristics of not being efficient, at least in the allotted time period of the study. Selling the sorted out put options would have been highly profitable during every year except 2011. Some of the put options were bought with such a high implied volatility that there was next to no chance of them ever ending up in the money. The Kelly criterion used in the method of this paper suggested against buying these contracts even when it's parameters was based on a max profit assumption of the S&P 500 falling by *50 percent in a month* (almost double the largest historic monthly fall ever). This strongly insinuates that the market is not efficient when it comes to option pricing and fat-tailed events.

Worth mentioning is that the methodological approach resulted in a number of shortcomings that had implications for the overall empirical result. The validity of the results presented should be taken with some caution since the timeframe of the study was rather limited due to option data availability. Furthermore, the time restriction and operationalization process required some significant assumptions and delimitations in order to ensure hindsight research bias and data processability.

All in all, this paper supports the efficient market hypothesis but puts a question mark on the efficiency of the market when it comes to option pricing. Selling option contracts relates to the "Nickel strategy"- philosophy described in section 1.1 and can be seen as *picking pennies in front of the steamroller*. However, some of the contracts have the same risk as if the steamroller is down at the end of the street; even if you fall, there is plenty of time to brush off and get up, hence the probability of one being crushed is rather low. So the question remains: *What about that steamroller?*

5.2 Theoretical Contributions

Disregarding any methodological implications, this study has contributed to the general finance literature by exploring and assessing the performance of a rather recent investment phenomenon. The empirical findings and indications in this paper furthermore complement the existing literature by cohering to work on option pricing and outlier occurrence. The indication of possible overpricing on the S&P 500 deep out of the money put options during the period corresponds well with the findings of Hull and White (1987). Furthermore, given the indication of overpricing and that only one set of option contracts generated return during the studied period (2011) suggest that the marginal cost of hedging catastrophic events (using these index options) far outweigh the marginal benefits from a macro economic risk management perspective for the studied period. The importance of such an indication is stressed by Oxelheim and Wihlborg (2008). Additionally, the fact that a total of 19 outlier events[51] (15 negative and 4 positive) were acknowledged for S&P 500 (1900 - 2012) cohere to the literature mentioned in section 2.3.1.

5.3 Suggestions for future research

Given the empirical findings and research implications in this study the following subject for future research is suggested:

- Extend and intensify the current research topic of this paper by including shorter time maturity options in the Black Swan strategy. Using weekly or even daily option data would counteract the central limit theorem effect and hence enhance the impartial foundation for the Black Swan strategy. Furthermore, more frequent data would increase the number of observation in the data set which would improve the statistical accuracy of any empirical results.

[51] Defined as an event exceeding or equal to three standard deviations on a monthly basis (see section 3.2).

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Appendix A. Monthly figures

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
1996-01-01	100	100	-	-	-	-	-
1996-01-31	102,46	102,01	0,02433	0,01995	-0,00041	0,802	0,0043
1996-02-29	103,17	102,56	0,00691	0,00536	-0,00095	0,802	0,0039
1996-03-29	103,99	103,21	0,00789	0,00630	-0,00080	0,802	0,0039
1996-04-30	105,39	104,32	0,01334	0,01073	-0,00088	0,802	0,0046
1996-05-30	108,21	106,65	0,02644	0,02209	0,00006	0,802	0,0042
1996-06-28	108,04	106,38	-0,00159	-0,00260	-0,00212	0,802	0,0040
1996-07-31	103,10	102,13	-0,04683	-0,04073	-0,00408	0,802	0,0045
1996-08-30	105,04	103,90	0,01864	0,01715	0,00139	0,802	0,0041
1996-09-30	110,73	108,39	0,05279	0,04236	-0,00083	0,802	0,0044
1996-10-31	113,62	110,74	0,02577	0,02141	-0,00008	0,802	0,0042
1996-11-29	121,96	117,70	0,07081	0,06096	0,00338	0,802	0,0041
1996-12-31	119,33	115,43	-0,02174	-0,01944	-0,00292	0,802	0,0046
1997-01-30	126,33	121,05	0,05698	0,04752	0,00144	0,792	0,0045
1997-02-28	127,40	121,87	0,00844	0,00674	-0,00076	0,792	0,0039
1997-03-31	121,97	117,32	-0,04355	-0,03804	-0,00443	0,792	0,0043
1997-04-30	129,10	122,93	0,05676	0,04671	0,00085	0,792	0,0043
1997-05-30	136,66	129,05	0,05693	0,04859	0,00247	0,792	0,0049
1997-06-30	142,60	133,87	0,04254	0,03668	0,00221	0,792	0,0037
1997-07-31	153,74	142,91	0,07524	0,06533	0,00482	0,792	0,0043
1997-08-29	144,91	135,63	-0,05918	-0,05227	-0,00623	0,792	0,0041
1997-09-30	152,61	141,74	0,05179	0,04405	0,00211	0,792	0,0044
1997-10-31	147,35	138,10	-0,03509	-0,02604	0,00088	0,792	0,0042
1997-11-28	153,92	142,70	0,04362	0,03282	-0,00256	0,792	0,0039
1997-12-31	156,34	144,59	0,01561	0,01311	-0,00025	0,792	0,0048
1998-01-30	157,92	145,85	0,01010	0,00868	-0,00008	0,769	0,0043
1998-02-27	169,05	154,52	0,06808	0,05775	0,00452	0,769	0,0039
1998-03-31	177,49	161,15	0,04874	0,04201	0,00364	0,769	0,0039
1998-04-30	179,10	162,17	0,00904	0,00631	-0,00163	0,769	0,0043
1998-05-29	175,73	159,49	-0,01901	-0,01663	-0,00294	0,769	0,0040
1998-06-30	182,66	164,69	0,03868	0,03207	0,00139	0,769	0,0041
1998-07-31	180,54	163,03	-0,01168	-0,01016	-0,00210	0,769	0,0040

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
1998-08-31	154,22	142,74	-0,15759	-0,13286	-0,01272	0,769	0,0043
1998-09-30	163,84	149,87	0,06053	0,04868	0,00109	0,769	0,0046
1998-10-30	177,00	160,14	0,07723	0,06630	0,00619	0,769	0,0032
1998-11-30	187,46	168,28	0,05744	0,04961	0,00474	0,769	0,0031
1998-12-31	198,03	176,61	0,05484	0,04827	0,00524	0,769	0,0038
1999-01-29	206,15	182,63	0,04019	0,03352	0,00359	0,720	0,0035
1999-02-26	199,50	177,49	-0,03282	-0,02854	-0,00589	0,720	0,0035
1999-03-31	207,24	183,27	0,03806	0,03205	0,00343	0,720	0,0043
1999-04-30	215,10	189,13	0,03724	0,03150	0,00364	0,720	0,0037
1999-05-28	209,73	184,91	-0,02529	-0,02255	-0,00529	0,720	0,0034
1999-06-30	221,14	193,26	0,05301	0,04414	0,00485	0,720	0,0040
1999-07-30	214,06	187,84	-0,03257	-0,02842	-0,00603	0,720	0,0038
1999-08-31	212,72	186,65	-0,00627	-0,00638	-0,00295	0,720	0,0039
1999-09-30	206,65	181,99	-0,02897	-0,02526	-0,00549	0,720	0,0039
1999-10-29	219,57	191,68	0,06066	0,05186	0,00708	0,720	0,0039
1999-11-30	223,75	194,82	0,01888	0,01623	0,00163	0,720	0,0036
1999-12-31	236,70	204,57	0,05623	0,04883	0,00710	0,720	0,0044
2000-01-31	224,65	195,40	-0,05224	-0,04586	-0,00782	0,748	0,0041
2000-02-29	220,13	191,85	-0,02031	-0,01833	-0,00422	0,748	0,0043
2000-03-31	241,42	207,39	0,09232	0,07787	0,00764	0,748	0,0047
2000-04-28	233,99	201,71	-0,03128	-0,02777	-0,00554	0,748	0,0046
2000-05-30	229,16	197,87	-0,02086	-0,01920	-0,00486	0,748	0,0050
2000-06-30	234,34	201,57	0,02235	0,01850	0,00077	0,748	0,0040
2000-07-31	230,51	198,82	-0,01648	-0,01371	-0,00260	0,748	0,0048
2000-08-31	244,50	208,83	0,05893	0,04913	0,00380	0,748	0,0050
2000-09-29	231,42	199,02	-0,05497	-0,04814	-0,00832	0,748	0,0051
2000-10-31	230,28	197,94	-0,00496	-0,00544	-0,00314	0,748	0,0056
2000-11-30	211,84	184,33	-0,08346	-0,07125	-0,01012	0,748	0,0051
2000-12-29	212,70	184,62	0,00405	0,00159	-0,00270	0,748	0,0050
2001-01-31	220,07	189,93	0,03405	0,02837	0,00123	0,759	0,0054
2001-02-28	199,76	175,54	-0,09683	-0,07880	-0,00624	0,759	0,0038
2001-03-30	186,93	165,35	-0,06636	-0,05983	-0,01048	0,759	0,0042
2001-04-30	201,29	175,86	0,07401	0,06162	0,00451	0,759	0,0039
2001-05-31	202,31	176,45	0,00508	0,00337	-0,00126	0,759	0,0032
2001-06-29	197,25	172,53	-0,02535	-0,02247	-0,00391	0,759	0,0028

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
2001-07-31	195,13	170,74	-0,01080	-0,01042	-0,00295	0,759	0,0030
2001-08-31	182,62	161,47	-0,06626	-0,05583	-0,00630	0,759	0,0031
2001-09-28	164,99	148,18	-0,10150	-0,08588	-0,00953	0,759	0,0028
2001-10-31	167,98	150,35	0,01794	0,01456	0,00042	0,759	0,0022
2001-11-30	180,61	159,67	0,07248	0,06016	0,00474	0,759	0,0017
2001-12-31	181,98	160,60	0,00755	0,00576	-0,00033	0,759	0,0015
2002-01-31	179,14	158,32	-0,01570	-0,01431	-0,00255	0,769	0,0014
2002-02-28	175,42	155,39	-0,02098	-0,01868	-0,00283	0,769	0,0013
2002-03-28	181,87	160,09	0,03608	0,02981	0,00175	0,769	0,0013
2002-04-30	170,70	151,63	-0,06338	-0,05431	-0,00588	0,769	0,0015
2002-05-31	169,15	150,23	-0,00912	-0,00923	-0,00253	0,769	0,0014
2002-06-28	156,89	140,76	-0,07521	-0,06513	-0,00756	0,769	0,0013
2002-07-31	144,50	131,40	-0,08230	-0,06878	-0,00579	0,769	0,0015
2002-08-30	145,20	132,02	0,00487	0,00468	0,00061	0,769	0,0014
2002-09-30	129,23	119,81	-0,11656	-0,09704	-0,00767	0,769	0,0014
2002-10-31	140,40	128,09	0,08291	0,06678	0,00266	0,769	0,0014
2002-11-29	148,41	134,05	0,05550	0,04553	0,00255	0,769	0,0012
2002-12-31	139,46	127,08	-0,06223	-0,05342	-0,00579	0,769	0,0011
2003-01-31	135,63	124,26	-0,02780	-0,02240	-0,00235	0,731	0,0010
2003-02-28	133,33	122,05	-0,01715	-0,01796	-0,00567	0,731	0,0009
2003-03-31	134,44	122,79	0,00832	0,00602	-0,00034	0,731	0,0010
2003-04-30	145,34	131,04	0,07793	0,06503	0,00780	0,731	0,0010
2003-05-30	152,73	136,62	0,04965	0,04174	0,00521	0,731	0,0009
2003-06-30	154,46	137,81	0,01126	0,00865	0,00015	0,731	0,0010
2003-07-31	156,97	139,57	0,01609	0,01271	0,00076	0,731	0,0007
2003-08-29	159,77	141,61	0,01772	0,01449	0,00135	0,731	0,0007
2003-09-30	157,87	140,15	-0,01202	-0,01037	-0,00180	0,731	0,0008
2003-10-31	166,54	146,51	0,05350	0,04440	0,00510	0,731	0,0007
2003-11-28	167,73	147,37	0,00710	0,00585	0,00047	0,731	0,0007
2003-12-31	176,24	153,88	0,04952	0,04323	0,00682	0,731	0,0008
2004-01-30	179,29	156,03	0,01713	0,01384	0,00164	0,700	0,0007
2004-02-27	181,48	157,39	0,01214	0,00872	0,00005	0,700	0,0006
2004-03-31	178,51	155,00	-0,01649	-0,01532	-0,00404	0,700	0,0009
2004-04-30	175,51	152,77	-0,01693	-0,01453	-0,00292	0,700	0,0008

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
2004-05-28	177,63	154,00	0,01201	0,00802	-0,00057	0,700	0,0006
2004-06-30	180,83	156,14	0,01783	0,01386	0,00113	0,700	0,0008
2004-07-30	174,63	151,40	-0,03489	-0,03085	-0,00672	0,700	0,0010
2004-08-31	175,03	151,60	0,00228	0,00131	-0,00062	0,700	0,0011
2004-09-30	176,67	152,64	0,00932	0,00681	-0,00004	0,700	0,0011
2004-10-29	179,14	154,34	0,01392	0,01113	0,00105	0,700	0,0011
2004-11-30	186,06	159,22	0,03787	0,03109	0,00413	0,700	0,0015
2004-12-31	192,10	163,64	0,03194	0,02741	0,00457	0,700	0,0016
2005-01-31	187,24	159,87	-0,02562	-0,02329	-0,00750	0,639	0,0016
2005-02-28	190,78	162,35	0,01873	0,01534	0,00279	0,639	0,0016
2005-03-31	187,13	159,50	-0,01930	-0,01770	-0,00613	0,639	0,0021
2005-04-29	183,37	156,60	-0,02031	-0,01832	-0,00610	0,639	0,0021
2005-05-31	188,86	160,30	0,02951	0,02331	0,00359	0,639	0,0024
2005-06-30	188,83	160,12	-0,00014	-0,00108	-0,00182	0,639	0,0023
2005-07-29	195,62	164,95	0,03534	0,02973	0,00629	0,639	0,0024
2005-08-31	193,43	163,21	-0,01129	-0,01065	-0,00452	0,639	0,0030
2005-09-30	194,77	163,99	0,00692	0,00480	-0,00067	0,639	0,0029
2005-10-31	191,32	161,37	-0,01790	-0,01612	-0,00566	0,639	0,0027
2005-11-30	198,05	166,06	0,03458	0,02866	0,00544	0,639	0,0031
2005-12-30	197,86	165,87	-0,00095	-0,00112	-0,00167	0,639	0,0032
2006-01-31	202,90	169,23	0,02515	0,02005	0,00217	0,664	0,0035
2006-02-28	202,99	169,16	0,00045	-0,00044	-0,00188	0,664	0,0034
2006-03-31	205,24	170,55	0,01103	0,00816	-0,00042	0,664	0,0037
2006-04-28	207,74	172,20	0,01208	0,00965	0,00041	0,664	0,0036
2006-05-31	201,32	167,44	-0,03140	-0,02803	-0,00861	0,664	0,0043
2006-06-30	201,33	167,31	0,00009	-0,00075	-0,00215	0,664	0,0040
2006-07-31	206,11	170,57	0,02346	0,01925	0,00233	0,664	0,0040
2006-08-31	210,50	173,56	0,02105	0,01737	0,00198	0,664	0,0042
2006-09-29	215,67	177,06	0,02427	0,02000	0,00250	0,664	0,0041
2006-10-31	222,46	181,67	0,03102	0,02572	0,00373	0,664	0,0041
2006-11-30	226,13	184,24	0,01633	0,01401	0,00174	0,664	0,0042
2006-12-29	228,98	186,07	0,01254	0,00990	0,00023	0,664	0,0040
2007-01-31	232,20	187,95	0,01396	0,01006	-0,00035	0,628	0,0044
2007-02-28	227,13	185,45	-0,02209	-0,01340	-0,00095	0,628	0,0038
2007-03-30	229,39	185,72	0,00993	0,00145	-0,00638	0,628	0,0043

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
2007-05-31	247,11	197,47	0,03203	0,02602	0,00439	0,628	0,0041
2007-06-29	242,71	194,42	-0,01798	-0,01556	-0,00576	0,628	0,0040
2007-07-31	234,95	190,25	-0,03250	-0,02169	-0,00277	0,628	0,0040
2007-08-31	237,97	190,65	0,01278	0,00208	-0,00751	0,628	0,0042
2007-09-28	246,49	196,33	0,03517	0,02940	0,00612	0,628	0,0032
2007-10-31	250,14	198,66	0,01471	0,01176	0,00134	0,628	0,0032
2007-11-30	239,12	191,06	-0,04504	-0,03901	-0,01200	0,628	0,0034
2007-12-31	237,06	189,50	-0,00867	-0,00816	-0,00373	0,628	0,0027
2008-01-31	222,56	181,86	-0,06311	-0,04116	0,00273	0,705	0,0021
2008-02-29	214,82	176,41	-0,03538	-0,03043	-0,00586	0,705	0,0013
2008-03-31	213,54	175,15	-0,00598	-0,00715	-0,00343	0,705	0,0017
2008-04-30	223,70	181,95	0,04645	0,03807	0,00479	0,705	0,0018
2008-05-30	226,09	183,40	0,01062	0,00795	-0,00006	0,705	0,0018
2008-06-30	206,65	169,94	-0,08988	-0,07624	-0,01336	0,705	0,0017
2008-07-31	204,61	168,05	-0,00991	-0,01117	-0,00462	0,705	0,0015
2008-08-29	207,11	169,63	0,01212	0,00934	0,00042	0,705	0,0013
2008-09-30	188,30	156,47	-0,09518	-0,08078	-0,01410	0,705	0,0015
2008-10-31	156,40	134,24	-0,18564	-0,15325	-0,02258	0,705	0,0008
2008-11-28	144,69	125,99	-0,07780	-0,06342	-0,00864	0,705	0,0003
2008-12-31	145,83	126,73	0,00779	0,00587	0,00038	0,705	0,0000
2009-01-30	133,34	117,44	-0,08955	-0,07611	-0,01197	0,716	0,0000
2009-02-27	118,68	106,56	-0,11646	-0,09724	-0,01386	0,716	0,0001
2009-03-31	128,81	113,97	0,08195	0,06726	0,00850	0,716	0,0002
2009-04-30	140,91	122,83	0,08977	0,07480	0,01047	0,716	0,0001
2009-05-29	148,39	128,27	0,05172	0,04338	0,00634	0,716	0,0000
2009-06-30	148,42	128,12	0,00020	-0,00117	-0,00133	0,716	0,0001
2009-07-31	159,42	136,19	0,07152	0,06108	0,00982	0,716	0,0001
2009-08-31	164,77	139,95	0,03301	0,02725	0,00358	0,716	0,0001
2009-09-30	170,66	144,20	0,03510	0,02988	0,00471	0,716	0,0001
2009-10-30	167,29	142,53	-0,01996	-0,01161	0,00268	0,716	0,0000
2009-11-30	176,89	148,48	0,05578	0,04088	0,00093	0,716	0,0000
2009-12-31	180,03	150,63	0,01761	0,01439	0,00175	0,716	0,0001
2010-01-29	173,37	145,83	-0,03768	-0,03243	-0,00664	0,685	0,0000
2010-02-26	178,32	149,13	0,02811	0,02241	0,00317	0,685	0,0000

Date	S&P 500	Portfolio	S&P 500 Return	Portfolio Return	Jensen's Alpha	Beta	Interest
2010-04-30	191,59	158,47	0,01465	0,01295	0,00289	0,685	0,0001
2010-05-28	174,98	146,22	-0,09068	-0,08048	-0,01844	0,685	0,0001
2010-06-30	165,55	139,60	-0,05539	-0,04628	-0,00840	0,685	0,0001
2010-07-30	176,94	147,38	0,06652	0,05423	0,00866	0,685	0,0001
2010-08-31	168,54	141,22	-0,04861	-0,04270	-0,00945	0,685	0,0001
2010-09-30	183,30	151,54	0,08393	0,07049	0,01300	0,685	0,0001
2010-10-29	190,05	156,21	0,03619	0,03041	0,00560	0,685	0,0001
2010-11-30	189,62	155,75	-0,00229	-0,00300	-0,00146	0,685	0,0001
2010-12-31	202,00	164,34	0,06326	0,05368	0,01035	0,685	0,0001
2011-01-31	206,57	167,35	0,02239	0,01817	0,00452	0,608	0,0001
2011-02-28	213,18	171,70	0,03146	0,02568	0,00652	0,608	0,0001
2011-03-31	212,95	171,33	-0,00105	-0,00217	-0,00158	0,608	0,0001
2011-04-29	219,02	175,42	0,02810	0,02359	0,00651	0,608	0,0000
2011-05-31	216,06	173,13	-0,01359	-0,01314	-0,00488	0,608	0,0000
2011-06-30	212,12	170,21	-0,01843	-0,01702	-0,00582	0,608	0,0000
2011-07-29	207,56	167,63	-0,02171	-0,01529	-0,00209	0,608	0,0000
2011-08-31	195,78	161,06	-0,05847	-0,03996	-0,00445	0,608	0,0001
2011-09-30	181,73	151,29	-0,07447	-0,06256	-0,01729	0,608	0,0000
2011-10-31	201,30	164,73	0,10231	0,08510	0,02290	0,608	0,0000
2011-11-30	200,28	163,91	-0,00507	-0,00499	-0,00190	0,608	0,0000
2011-12-30	201,99	164,99	0,00850	0,00657	0,00141	0,608	0,0000
2012-01-31	210,80	170,86	0,04266	0,03492	0,00811	0,628	0,0000
2012-02-29	219,35	176,61	0,03979	0,03311	0,00810	0,628	0,0000
2012-03-30	226,23	181,26	0,03085	0,02602	0,00664	0,628	0,0000
2012-04-30	224,53	179,81	-0,00753	-0,00807	-0,00334	0,628	0,0000
2012-05-31	210,46	169,85	-0,06470	-0,05694	-0,01632	0,628	0,0001
2012-06-29	218,79	175,43	0,03879	0,03231	0,00793	0,628	0,0000
2012-07-31	221,54	177,22	0,01252	0,01016	0,00230	0,628	0,0000
2012-08-30	224,78	179,23	0,01451	0,01127	0,00211	0,628	0,0001
2012-09-28	231,40	183,57	0,02901	0,02393	0,00566	0,628	0,0001
2012-10-31	226,82	180,12	-0,01999	-0,01900	-0,00647	0,628	0,0001
2012-11-30	227,46	180,35	0,00284	0,00131	-0,00052	0,628	0,0001
2012-12-31	229,07	181,37	0,00704	0,00560	0,00114	0,628	0,0001