

Avdelningen för Konstruktionsteknik  
Lunds Tekniska Högskola  
Box 118  
221 00 LUND

Division of Structural Engineering  
Faculty of Engineering, LTH  
P.O. Box 118  
S-221 00 LUND  
Sweden

## **Stability of slender columns**

- A study of initial curvature and braces influence on a column's load bearing capacity

Kevin Hailemichael  
Björn Liivo

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Supervisors:  
Anders Klasson, PhD student  
Eva Frühwald Hansson, Assistant Professor  
Division of Structural Engineering

Examiner  
Roberto Crocetti, Professor  
Division of Structural Engineering

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## Abstract

Failures due to instability phenomena will happen suddenly and can cause the whole structure to collapse. It's therefore in the engineer's best interest to have a good knowledge about these phenomena. One example of instability phenomena is column buckling, which is the main focus of this thesis.

The main cases that are being evaluated have a single brace in the middle of a timber column with hinged ends, where a part of the report revolves around a comparison of calculation methods for column buckling in Eurocode and the U.S. Building code.

The second part revolves around the influence of the initial curvature shape and magnitude on a column. During this evaluation second order is taken into account in the numerical analysis. Finally the brace stiffness and reaction force on a column is evaluated, to gain a better understanding of its overall influence on a column's load bearing capacity.

In the study it was shown that Eurocode and the U.S. Building code weren't comparable due to formation of respective building code. A comparison between the two building codes was however done, to get a better understanding of the difference between the two.

Results show that a larger initial curvature leads to a larger reduction to the overall load bearing capacity for a column. The assumed shape of the initial curvature has a large impact on the load bearing capacity. There exist a large discrepancy between the simplified shape of the initial curvature and the least beneficial, which depends on the size and shape of the initial curvature.

Findings show that an increase of a column's brace stiffness contributes to the load bearing capacity even though the stiffness might be small. The European standard however demands a minimum stiffness for a brace to be considered acceptable.

The study also shows that the brace force is dependent on the initial curvature of the column and the bracing stiffness. Yura's & Helwig's brace force expression is studied and compared with the result of a numerical analysis.

Keywords: Column buckling, bracing, building codes, stiffness, initial curvature, imperfection

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Lund, September 2014

*Kevin Hailemichael*

*Björn Liivo*

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**Appendix A - Unit converter**

# Nomenclature

## Abbreviations

<i>Eurocode 5</i>	SS-EN 1995-1
FEM	Finite Element Method

## Latin upper case letters

$A$	Area [ $\text{m}^2$ ] or [ $\text{inch}^2$ ]
$C_D$	Load duration factor
$C_M$	Wet service factor
$C_t$	Temperature factor
$C_F$	Size factor
$C_P$	Column stability factor
$C_i$	Inscised factor for sawn lumber
$E$	Young's modulus [Pa] or [psi]
$E_{0,05}$	Modulus of elasticity associated with the axis of column buckling [Pa]
$E'$	Modulus of elasticity associated with the axis of olumn buckling [psi]
$F$	Force [N]
$F_C$	Tabulated compressive stress parallel to grain [psi]
$F_{CE}$	The Euler critical buckling stress for columns [psi]
$F'_C$	The design load for the compressive stress parallel to the grain [psi]
$F_C^*$	Tabulated compressive stress parallel to grain multiplied by all adjustment factors except $C_P$ [psi]
$I$	Moment of inertia [ $\text{m}^4$ ]
$K_{CE}$	0,3 for visually graded lumber
$K_e$	Effective length factor
$L$	Length [m]
$M$	Moment [Nm]
$N$	Normal force [N]
$N_d$	The design load [N]
$P$	Force [N]
$P_{cr}$	Critical force [N]
$W$	Section modulus [ $\text{m}^3$ ]

## Latin lower case letters

$b$	Width [m] or [inch]
$h$	Height [m] or [inch]
$c$	The buckling and crushing interaction factor for columns
$f_{c,0,k}$	Compressive stress capacity parallel to the grain [Pa]
$f_{ck}$	Compressive stress capacity parallel to the grain [Pa]
$f_c$	the compressive stress parallel to the grain [psi]
$i$	Gyration ratio
$k$	Brace stiffness [N/m]

$k_c$	Reduction factor
$k_{ideal}$	Ideal brace stiffness [N/m]
$k_{req}$	The required brace stiffness to prevent side sway [N/m]
$l$	Effective length [m]

**Greek lower case letters**

$\beta$	Euler's buckling factor
$\beta_c$	Straightness requirement factor
$\sigma$	Stress [Pa]
$\lambda$	Slenderness ratio
$\lambda_{rel}$	Relative slenderness ratio

# 1 Introduction

*This chapter is an introduction to this report, where the underlying background and the objective for this report will be given. The chapter starts with an introduction to the subject, where the problem formulation of the report will be presented. Later in this chapter, the reader will get more detailed description about the objective and the method of the report.*

## 1.1 Background

When instability phenomena occurs in a structure the consequences can become dire fast (Bauchau & Craig, 2009). Failures due to instability phenomena can happen suddenly and can cause the whole structure to collapse. It's therefore in the engineer's best interest to have a good knowledge about these phenomena. Examples of instability phenomena are local buckling, column buckling and lateral torsional buckling.

Instability problems such as column buckling can be counteracted by bracing. When bracing a column for column buckling the brace reduces the effective length of the column, thus increases the load bearing capacity for the structure and it becomes more stable. During the design of a brace, the bracing is normally considered to be infinitely stiff to enforce a certain kind of failure. However an infinitely stiff bracing is never the case in reality.

However it's hard to calculate the stiffness for the bracing and the way it's calculated differs dependent on the method. It is therefore of importance to gain a better understanding of how a brace influences the column buckling phenomenon.

## 1.2 Objectives and aim

Calculation methods to design a column due to column buckling vary between the different building codes. In this report the difference between how the European and the U.S. building code calculates the design load for a column in compression are presented. The design loads is compared to each other and a numerical analysis.

The second part of this thesis is a parametric study where the effects of initial curvature on the critical load bearing capacity are evaluated. During this evaluation a numerical analysis is performed to take into account non linearity.

By bracing a column, column buckling can be prevented by increasing the columns load bearing capacity. It is however hard to evaluate what kind of stiffness properties that is required for obtaining an effective brace for a wooden column. It is also hard to anticipate the magnitude of the reaction force in the brace.

### **Aim**

- How does the design procedure and results differ between Eurocode and the U.S. building code, when it comes to timber columns in compression?

- How does the shape and magnitude of initial curvature influence the load bearing capacity of a column?
- How does the stiffness of a brace affect the load bearing capacity in the column and the resulting force in the brace? And how does the influence of several braces with different stiffness affect the load bearing capacity of a column?

### 1.3 Scope

This thesis focuses on column buckling. The main cases that are being evaluated have a single brace in the middle of the column with hinged ends. The study will revolve around timber columns.

The columns that are being evaluated for timber have a rectangular cross-section with the material properties of timber class c27.

During calculations eccentricities, residual stresses and inclination are not taken into account unless assigned a specific value. Other deciding factors such as long time deformations, moisture content for the columns are not considered. While comparing European with the U.S. building code only calculations with critical load due to column buckling are done.

The evaluation of a brace's contribution to the load bearing capacity will only be done in a linear analysis in FEM. This report evaluates the phenomena up to the second order (by using computer modeling), this means that no larger deformations are taken into account.

### 1.4 Method

A literature review of current and relevant knowledge of stability phenomena will be the base foundation of this report. The procedure of the literature review will be done analytically and systematically.

To quantify and clarify the relations mentioned in the literature review, a calculation part will be carried out in the report. These calculations will be based on the European and the U.S. building codes for structural design and computer model in Brigade.

The European building code for timber is based on SS-EN 1995-1, also called Eurocode 5. The basis of the European building code for this report comes from the book "Byggekonstruktion – Regel och formelsamling" by Isaksson and Mårtensson (2010).

The review of the U.S. building code in this report revolves around the book "Design of Wood Structures – ASD" by Breyer et al (2003). The book goes through the basis for structural design and includes practical literature. The literature includes publication and design criteria's of the National Design Specification for Wood Construction (NDS), Allowable Stress Design Manual for Engineered Wood Construction and the International Building Code.

Brigade/plus 5.1-4 is an add on program from Scanscot Technology AB to Abaqus from Dassault Systèmes, that offers a variety of extra features in design of bridges. Brigade is the preferred and used finite element program for this report, this since it has all the right properties needed to do advanced calculations but also because the authors are used to its interface. The finite element method is necessary in this case to obtain the second order and also used as a comparison to the hand calculations.

## **1.5 Outline of this thesis**

**Chapter 1** – Introduction, introduces the problem studied in this report

**Chapter 2** – Literature review, underlying theory that serves as a foundation for the report

**Chapter 3** – Standards and building codes, presents theory and methods behind the different standards and building codes

**Chapter 4** – Modeling, this part walks through how the calculations and studies are performed in this report.

**Chapter 5** – Results and analysis, presents a results and discussion part, where the calculations and findings are analyzed and discussed.

**Chapter 6** - Discussion and conclusion, a conclusion of the result and discussion is done to answer the objectives of the report.



## 2 Literature review

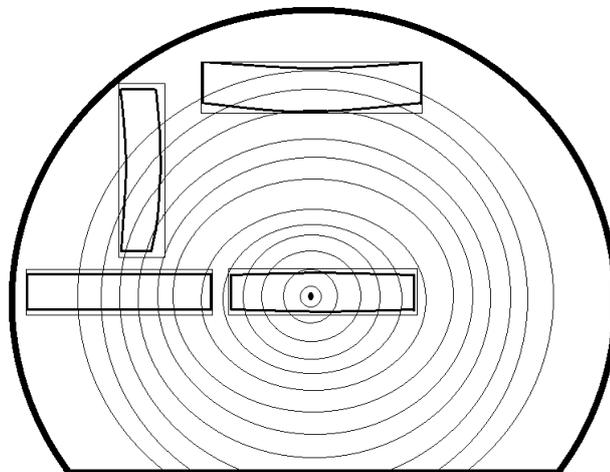
*This chapter is a literature review, where the underlying theory for the report will be presented. The chapter begins with a short review of the materials that are studied, and then to describe the theory behind stability theory and the column buckling phenomena. In connection to the column buckling part, there'll be a section that describes the influence of bracing.*

### 2.1 Material properties

In this subchapter general information about the materials wood and steel are presented from a structural point of view.

#### 2.1.1 Timber

Timber comes in all shapes and sizes, since it's organic it varies in quality. Many of the weaknesses of wood comes from its growth deviations such as knots, bark that grows inwards or tree sap (Burström, 2007). The woods anisotropic properties will cause the wood to absorb moisture differently in each direction. The moisture levels in newly cut timber are approximately 30-35% at the core (Burström, 2007). When the wood planks later dry out they will shrink and bend dependent on where they are cut out of the timber log. To illustrate this see picture 2.1 below, this initial bending can cause irregularities when used.

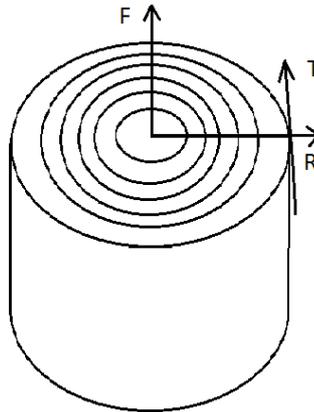


*Figure 2.1 Shapes of dried out wood planks (Burström, 2007)*

The strength properties of wood is very complex due to its anisotropic structure (Burström, 2007). Most types of timber have the largest strength capacity when it is subjected to pure tensile force along the fibers. The lowest strength capacity is obtained when timber is subjected to pure compressive force along the fibers. Since bending is something in between the strength will also be something in between tensile and compression strength. Since wood is anisotropic its strength capacity also becomes dependent on how the wooden fibers are take on the load. The tensile- and compressive strengths are at a high point when the wood takes on load parallel to the wooden fibers and noticeably weaker perpendicular to the same fibers. If the load is applied perpendicular to the wooden fibers the largest

strength is reached parallel to the annual rings of the timber, even known as the tangential direction. For further viewing of how the three main directions are defined see figure 2.2 below.

- Fiber direction (along the trunk)
- Radial direction (perpendicular towards the annual rings and the fiber direction)
- Tangential direction (perpendicular the fiber direction but parallel towards the annual rings)



*Figure 2.2 The timbers main directions F- fiber, R-radial and T-tangential*

The strength properties of wood are directly dependent on the on density, moisture content, fiber direction, temperature and dimensions (Burström, 2007).

## 2.2 Stability theory

In the world of mechanics one separates different equilibrium states from each other (Höglund, 2006). This is done by classifying certain types of scenarios into three main categories,

- Stable
- Unstable
- Neutral

To illustrating the stable case imagine a ball in a valley, if a small interference changes its position it will self-regulate and get back to its original state (Höglund, 2006). In the unstable case on the other hand, a small interference leads to an increase of force acting against the balls equilibrium state. In the neutral state the interference will only move the case further away it will not affect the equilibrium of the ball. These cases are illustrated in the figure 2.3 below.



*Figure 2.3 Stability cases, from left to right stable, unstable and neutral (Höglund, 2006)*

Preconditions for a body are the determining factor in how the body will act when it's subjected to a load (Höglund, 2006). Thus one can say that a structure is in a stable equilibrium if it will self-regulate back to its original position after exposure to a small interference. While examining stability phenomena one often presume certain restrictions for instance,

- Material properties are linear elastic
- The shape of the structural element has an ideal form
- Constitutive properties are based on presumptions of small deformations

Dependent on what kind of mechanical phenomena that occurs on the load bearing structure different failure modes can occur (Höglund, 2006). If a structure is subjected to tensile forces the material yield strength, fatigue or breaking point becomes critical for failure. However compressive forces enforce column buckling or local buckling to take precedence above other parameters concerning failure for slender columns.

### **2.3 Failure due to geometry**

Depending on the column's geometry, the allowable stress for a given material can vary and be divided into three groups (Efunda, 2014). The three groups are the short, intermediate and long column.

The materials strength limit is the dominating factor for a short column. For an intermediate respective a long column it's however the inelastic and the elastic limit that are the bounding factor for the column members (Efunda, 2014). The slenderness or the stiffness of the column becomes more and more important as a column becomes longer. The capacity of the material in a column, that is long and slender, will not be fully utilized. The column will buckle before the stresses in the column reaches the stress limit of the material.

For an illustration of the correlation between the strength limit and the slenderness ratio for the different groups, see figure 2.4 below.

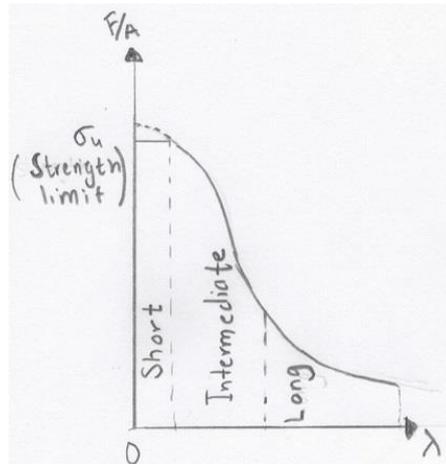


Figure 2.4 Slenderness-ratio dependent on height (efunda, 2014)

With other words, it's the material's strength properties (as the yield strength and Young's modulus) and the geometry (slenderness ratio; the cross-section and the length) that decides in which group a column resides.

### 2.3.1 Buckling

The critical load for a column, in pure buckling, is defined by Gere & Timoshenko (1961) as the axial force that is sufficient to keep the bar in slightly bent form. By assuming that the beam acts as an ideal column, the critical load for the compressed beam can be calculated. The fundamental case of column buckling, where you have the case with a bar with hinged ends, you can obtain a general equation for the critical load, see equation 2.1 below,

$$P_{cr} = \frac{\pi^2 EI}{(\beta L)^2} \quad (2.1)$$

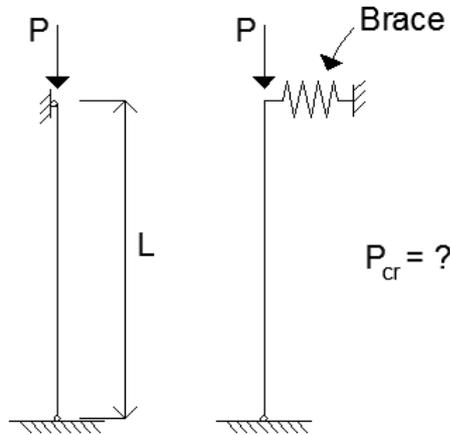
## 2.4 Bracing of columns

To prevent buckling the easiest way is to reduce the length in some form or to change the shape of the cross section. The length reduction can be done by the use of a brace (Winter, 1958). The brace can either be considered to be elastic or ideal. An elastic brace takes on the load and deflects out as well as the bracing gets displaced from its origin. The displacement is dependent on the stiffness of the brace, when it becomes stiff enough it will remain in place and be considered as ideal.

To acquire an effective bracing against buckling the bracings required strength and the rigidity of the whole structure needs to be taken into account also the fact that there will always be imperfections of shapes or loads (Winter, 1958). It is common knowledge that imperfections in placements of loads will create unwanted moments that in turn causes a deflection of the column, in the case of column buckling the critical load is not affected by this however the total deflection becomes larger.

### 2.4.1 Bracing at the end of a column element

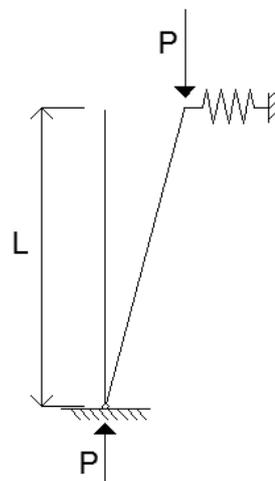
If a column is subjected to an axial load, hinged on the ends with the assumption of adequate stiffness to the restraints, the critical load is given by Euler's load (Helwig & Yura, 1996). For illustration for the case, see figure 2.5 and equation 2.2 below,



**Figure 2.5** Hinged and braced top of a column (Helwig & Yura, 1996)

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (2.2)$$

Consider now that the top end is elastic and that the bracing is inadequate, this will result in the inevitable deflection of the column but also a displacement of the bracings original position, see figure 2.6 below.



**Figure 2.6** Inadequate stiffness gives side sway (Helwig & Yura, 1996)

The manner of which the structure acts can be described by the moment equilibrium state at the hinge. The expression of the equilibrium can according to Helwig & Yura (1996) be illustrated and expressed by figure 2.7 and equation 2.3 below,

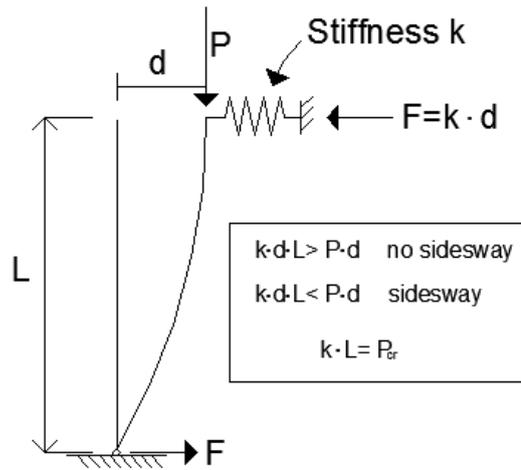


Figure 2.7 Equilibrium state (Helwig & Yura, 1996)

$$P \cdot d = F \cdot L \quad (2.3)$$

The force,  $F$ , in the spring can be expressed by the stiffness of the brace and the deformation of the column this can be seen in equation 2.4 below. By implementing 2.4 into the equation 2.3 the following expression is given, see equation 2.5 below,

$$F = k \cdot d \quad (2.4)$$

$$P \cdot d = k \cdot d \cdot L \quad (2.5)$$

This can be further derived to equation 2.6 below,

$$P = k \cdot L \quad (2.6)$$

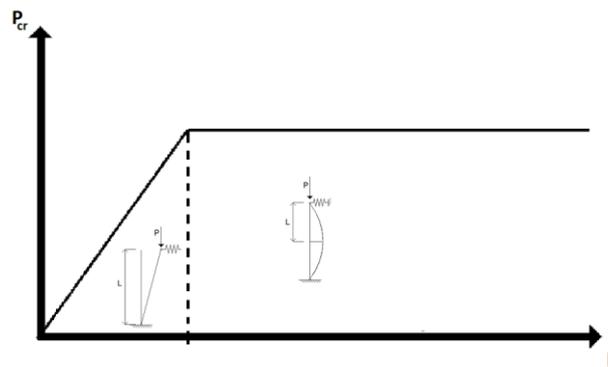
When this equilibrium state between the stiffness of the brace and the applied load is reached see equation 2.3, the column will not sway out sideways. The equilibrium state between the support reaction and acting force together with the two buckling cases is illustrated in figure 2.7 above.

The first buckling load prescribed in figure 2.7 is described as a linear relation, which is dependent on the stiffness of the spring (sidesway). When the second buckling load is reached it triggers the spring to act more like a hinge (no sidesway), by doing so the load can be expressed with Euler's critical load as in equation 2.6 above. The ideal stiffness for the bracing, can according to Winter (1958), be derived from equation 2.7 to equation 2.8 below,

$$k_{ideal} = \frac{P_{cr}}{L} \quad (2.7)$$

$$k_{ideal} = \frac{\pi^2 EI}{l^3} \quad (2.8)$$

The correlation between the rigidity of the bracing and the critical buckling load is illustrated in the figure 2.8 below. The critical buckling load will increase as the stiffness of the bracing increases. When the critical buckling hit the second buckling mode, an increased stiffness won't improve the load capacity as the buckling mode will govern the load capacity (Galambos & Surovek, 2008).



**Figure 2.8** Relation between the stiffness and critical load when bracing at the top (Helwig & Yura, 1996)

It is therefore shown that it isn't necessary to strive for larger brace stiffness, the fully braced buckling mode can be acquired through sufficient bracing (Winter, 1958). The only requirement for the bracing is to have sufficient strength and rigidity to withhold the effects from the critical load.

#### 2.4.2 Bracing at the middle of a column element

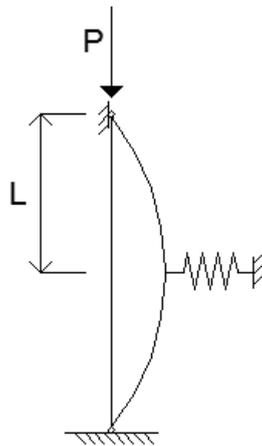
The usual way to stabilize a column with two hinged supports at each end, is to connect it to a bracing, this can be done for example in the middle (Winter, 1958). The value of the necessary stiffness of that bracing can be determined by previously defined correlations in the section 2.4.1 above. On that basis one could even calculate more than one bracing between two supports.

Winter (1958) states when a column has a bracing in the middle with an adequate stiffness, the buckling mode will follow the appearance of two half sine waves. Helwig & Yura (1996) illustrates it like figure 2.9 below.



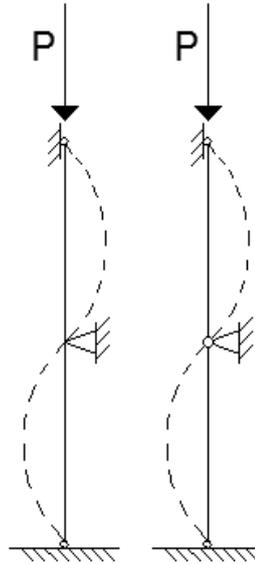
**Figure 2.9** Column with sinus shaped curvature (Helwig & Yura, 1996)

If however the support were to not to have an adequate stiffness the result would be an displacements of the bracings origin and the column will get a buckling mode similar one described in figure 2.10 below.



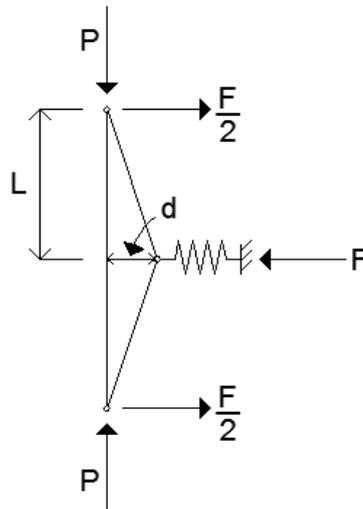
**Figure 2.10** Inadequate stiffness (Helwig & Yura, 1996)

When an ideal column with hinged ends is improved by an unyielding support at a miniscule portion of the middle and tested, it buckles into two half sine-waves (Winter, 1958). If this ideal column's unyielding support got replaced by a real or fictitious hinge at the middle of the column the result becomes the same where the column still buckle out in two half sine-waves. Since both cases express the same shape a fictitious hinge can in this case be introduced at the middle with insignificant error. To illustrate this see picture 2.11 below.



**Figure 2.11** Illustrating column that is subjected to column buckling with a minuscule unyielding support versus a hinge in the middle.(Winter, 1958)

The moment equilibrium of the column seen in figure 2.12, can therefore be described by introducing a fictitious hinge in the middle of the column to describe the equilibrium correlations (Winter, 1958). In this case the equilibrium can be expressed and illustrated as in the equation 2.9 and figure 2.12 below,



**Figure 2.12** Moment equilibrium (Helwig & Yura, 1996)

$$P \cdot d = \frac{k \cdot d}{2} L \quad (2.9)$$

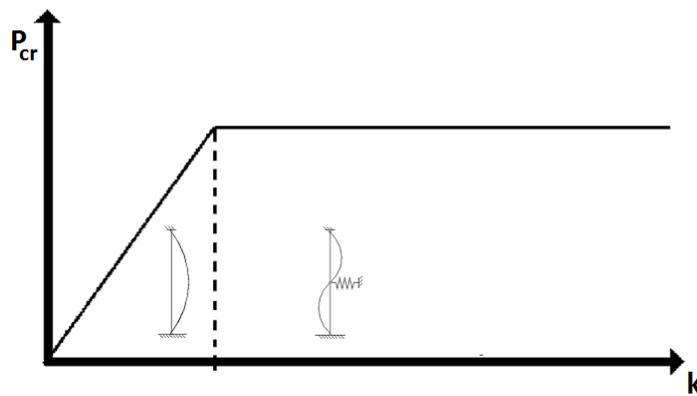
The formulation can later be simplified as in equation 2.10,

$$k = 2 \cdot \frac{P_{cr}}{L} \quad (2.10)$$

Based on this correlation one can later evolve it to the expression in equation 2.11 for the necessary stiffness for the bracing,  $k_{ideal}$ , in the same manner as in previous sections,

$$k_{ideal} = 2 \cdot \frac{\pi^2 EI}{L^3} \quad (2.11)$$

The figure 2.13 below illustrates the relationship between the critical buckling load and the bracing's stiffness. As in section 2.4.1 the stiffness of the bracing has a great influence of the critical buckling load, in the first buckling mode. An increased stiffness will increase the load capacity. But when the column reach the second buckling load, the increase of stiffness becomes less relevant and the buckling mode will govern the capacity of the column according to Helwig & Yura, (1996).



**Figure 2.13** Relation between stiffness and critical load with a bracing in the middle. (Helwig & Yura, 1996)

#### 2.4.2.1 Imperfections

If the imperfections in the column are taken into account, the formulation of the required stiffness will change slightly from the earlier expression. Winter (1958) uses an equilibrium equation about the hinge to describe the correlations of the bracing, which is illustrated in fig 2.14 and stated in equation 2.12 below,

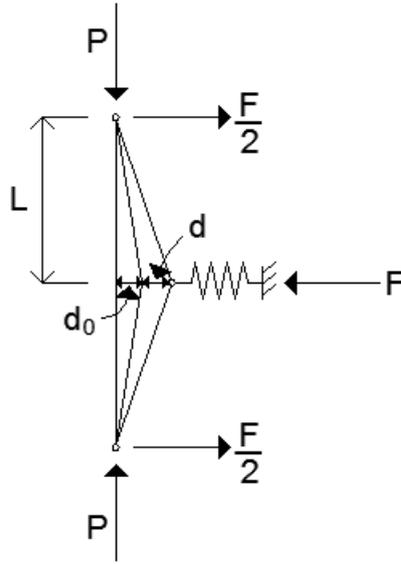


Figure 2.14 Equilibrium with initial curvature (Winter, 1958)

$$M = \frac{FL}{2} - P_e(d_0 + d) = 0 \quad (2.12)$$

By implementing the equation 2.4 in the formulation 2.12 above, the required stiffness of a full braced column is acquired. The formulation is presented in the equation 2.13 below,

$$k_{req} = \frac{2P_e}{L} [(d_0/d) + 1] \quad (2.13)$$

According to Winter (1958) this demonstrates that a bracing's rigidity for an imperfect column exceeds the need of an ideal column, to produce full bracing. This means that a larger imperfection requires a stronger and more rigid bracing.

#### 2.4.2.2 Brace force

According to Yura & Helwig (1996) the initial curvature has a big influence on the brace force. The brace force can be expressed as equation 2.4. The formulation can according to Yura & Helwig, due to initial curvature, be expressed as equation 2.14. This formulation can be further derived and expressed as equation 2.15 below,

$$F_{br} = \frac{2P}{L} (d_0 + d) \quad (2.14)$$

$$F_{br} = \frac{2P}{L} \frac{d_0}{1 - \frac{k_{ideal}}{k}} \quad (2.15)$$



### 3 Standards and building codes

*This chapter is a review of the methods and design standards that are in use in the report. The chapter starts with an introduction that describes the differences between an ideal- and a real member. The reader will also get a presentation of the design building codes in Europe and U.S. for timber columns in compression. A section about the Finite Element Method (FEM) will later be presented, to describe the method in general.*

#### 3.1 Theoretical members opposed to real members

The difference between theoretical and the practical member is the existence of different unspecified factors that together with the deviation from the linear elastic behavior acts on the member (Runesson et al., 1992). During design procedures these conditions need to be taken into account in general, for instance:

- Inhomogeneity in material properties
- Initial stresses in the form of residual stresses
- Initial curvature
- Eccentricity of axial load point of application

Real members are never perfectly straight nor are there load applied without eccentricities (Trahair & Bradford, 1994). To simplify the problem these imperfections can be equal to an addition in initial curvature since the behavior is similar.

Initial curvature is a form of geometrical imperfection where a straight beam or column has a natural curvature to its shape often caused by residual stresses (Trahair & Bradford, 1994). The maximum allowed stresses and design rules that takes initial curvature into account are based on semi-empirical studies. Residual stresses are those stresses that act internally on a structural member in an unloaded state. By definition this means that those stresses are in equilibrium since they are there without external forces (Höglund, 2006). However residual stresses are not being taken into account in this report.

The shape of how the initial curvature acts in a column is very irregular and different in each case. When the column becomes subjected to a load the buckling effect is added with the initial curvature and thereby speeds up the process of failure due to column buckling. The model assumption is that the shape for column buckling also is the shape for the initial curvature of the column. In this case the total deflection becomes the contribution from the column buckling and the contribution from the initial curvature.

When adding all deviations on a perfect member the critical load is reduced dependent on how much and of what the actual member is exposed to (Trahair & Bradford, 1994). To illustrate an example of a real member compared with a purely theoretical member see figure 3.1 below,

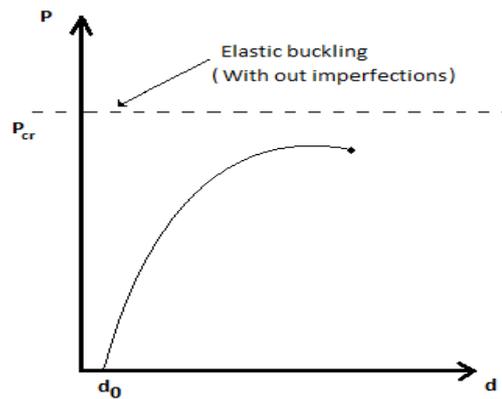


Figure 3.1 example of a real members behavior compared to the theoretical value in the dotted line (Trahair & Bradford, 1994)

### 3.2 Design according to Eurocode 5

In a previous section 2.1  $P_{cr}$  is described as the critical axial force that is needed for a column to buckle. This value of  $P_{cr}$  is a pure theoretical value, in a perfect case where a column is subjected to an axial force without any eccentricity or any other imperfections. It is simply impossible to recreate it in the real world since a lot of different factors can interfere for instance (Crocetti and Mårtensson, 2011).

- Strength/stiffness - A materials modulus of elasticity and compressive strength varies.
- Geometry of the member - Creation of members has a certain precision therefore it might have small variation in cross section, initial curvature and length.
- Support conditions - Countered by effective buckling length
- Material imperfection - Grown materials vary in quality and imperfections. Density, moisture and effect of compression are all properties that vary dependent on type of material, where it's created and where it's stored.
- Geometry imperfections - The human factor is always present during erection of buildings and can cause imperfections such as eccentricities and inclination. This together with a materials initial curvature needs to be taken into account.

To counter this theoretical value Eurocode 5 introduce a reduction factor  $k_c$ ,  $k_c$  depends on previous mention factors but also the slenderness ratio  $\lambda$ .

#### 3.2.1 Column Subjected to Compression

In Eurocode 5 the general expression is presented for a column subjected to compression. These expressions have been redone for a more user friendly guideline in “Bygghkonstruktion: Regel- och formelsamling” by Isaksson and Mårtensson and are presented below.

The load capacity for this column is calculated in the following manner see equation 3.1 below,

$$N_d = f_{co,k} \cdot A \cdot k_c \quad (3.1)$$

Where	$f_{co,k}$	=	compressive stress capacity parallel to the grain
	$A$	=	Area of the cross-section
	$k_c$	=	Reduction factor

$k_c$  is a reduction factor that is taking into account the risk of plane buckling. When determining the reduction factor the column's slenderness is of grave importance. The slenderness factor is prescribed as equation 3.2 below,

$$\lambda = \frac{\beta \cdot L}{i} \quad (3.2)$$

Where	$\beta$	=	Euler's Buckling factor
	$L$	=	Length of the column
	$i$	=	$\sqrt{\frac{I}{A}}$

Thereafter the relative slenderness ratio can be expressed as equation 3.3,

$$\lambda_{rel} = \frac{\lambda}{\pi} \sqrt{\frac{f_{co,k}}{E_{0,05}}} \quad (3.3)$$

Where	$f_{co,k}$	=	Compressive stress capacity parallel to the grain
	$E_{0,05}$	=	modulus of elasticity associated with the axis of column buckling

Finally the reduction factor will be decided according to equation 3.4,

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}}, \quad \text{for} \quad \lambda_{rel} > 0,3 \quad (3.4)$$

Where	$k$	=	$0,5(1 + \beta_c(\lambda_{rel} - 0,3) + \lambda_{rel}^2)$
	$\beta_c$	=	Straightness requirement factor

**Comments:**

The reduction factor  $k_c$  has been determined by large quantity testing (Crocetti and Mårtensson, 2011). The columns that were tested was picked at random had different deviations in material properties, geometry and initial curvature. Property values between different columns were taken in to account for correlation. By studying the tested columns using second order one could calculate the ultimate load.

**3.2.2 Live loads in Eurocode**

Design guidance on how to design for imposed loads for structural designs, are also given by Eurocode. The characteristic values are divided in different categories, which indicate on its occupancy.

The live load that is of an interest in this report is,

School, classroom:  $2,5 \text{ kN/m}^2$

**3.2.3 Single members in compression**

For each single member in compression, the required minimum resistance of the lateral support is given by  $C$  in Eurocode 5 (2004). The minimum stiffness of the brace is given by equation 3.5 below.

$$C = k_s \frac{N_d}{a} \quad (3.5)$$

Where	$k_s$	=	Is a modification factor varied from 1 to 4, where 4 is recommended.
	$N_d$	=	Is the mean design compressive force in the element
	$a$	=	Is the bay length between the support and spring.

$N_d$  is given by equation 3.1 above,

**3.3 Design according to the U.S. standard**

This section will go through the U.S. structural design approach for axially loaded column.

**3.3.1 Axially loaded column**

The general expression for a column subjected to an axially load is presented in “Design of Wood Structures – ASD” (2003). The formula 3.6 below, is a control of the capacity of an axially loaded wood column.

$$f_c = \frac{P}{A} \leq F'_c \quad (3.6)$$

Where	$f_c$	=	the compressive stress parallel to the grain
	P	=	the members axial compressive force
	A	=	the area of the cross-section
	$F'_c$	=	the design load for the compressive stress parallel to the grain

The design load is taking in to account several different factors in addition to the column stability. Example of the different factors is the temperature, the moisture content in wood and the load duration. The different factors are manifested as adjustment factors.

The formula for the allowable stress in a column is presented in equation 3.7 below.

$$F'_c = F_c(C_D)(C_M)(C_t)(C_P)(C_i) \quad (3.7)$$

Where	$F'_c$	=	allowable load for the compressive stress parallel to the grain
	$F_c$	=	tabulated compressive stress parallel to grain
	$C_D$	=	load duration factor
	$C_M$	=	wet service factor
	$C_t$	=	temperature factor
	$C_F$	=	size factor
	$C_P$	=	column stability factor
	$C_i$	=	incised factor for sawn lumber

More detailed information about the various adjustment factors can be found in the book “Design of Wood Structures – ASD” (2003).

As specified above, the adjustment factor,  $C_P$ , considers the columns stability. The  $C_P$  factors is defined as specified as the equation 3.8 below,

$$C_P = \frac{1+F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1+F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.8)$$

Where	$F_{cE}$	=	the Euler critical buckling stress for columns
	$F_c^*$	=	tabulated compressive stress parallel to grain multiplied by all adjustment factors except $C_P$
	c	=	the buckling and crushing interaction factor for columns

The c factor varies dependent on what kind of column it is. For more detailed values look below,

c	=	0,8 for sawn lumber
	=	0.85 for round timber
	=	0,9 structural composite or glulam lumber columns

U.S standard describes the failure of a column with two failure modes, the buckling and the crushing mode. The second failure takes in to account the crushing of the wood fibers, e.g. the material capacity before plasticity. It's given by the formula 3.9 below,

$$F_c^* = F_c(C_D)(C_M)(C_t)(C_i) \quad (3.9)$$

$F_c^*$  includes the tabulated compressive stress parallel to grain,  $F_c$ , and all the adjustment factors except the column stability factor,  $C_p$ .

The first buckling mode is given by Euler critical buckling stress. To use the Euler stress in allowable stress design (ASD), a factor of safety is used and divided with the Euler expression. The Euler critical buckling stress for columns is therefore expressed in NDS as equation 3.10 below,

$$F_{cE} = \frac{K_{cE}E'}{(l_e/d)^2} \quad (3.10)$$

Where	$E'$	=	modulus of elasticity associated with the axis of column buckling
	$K_{cE}$	=	0,3 for visually graded lumber
		=	0,384 for MEL
		=	0,418 for products with less variability such as MSR lumber and glulam

The safety factor and  $\pi^2$  are included in the  $K_{cE}E'$  term.

The formula for the general slenderness ratio is expressed as equation 3.11 below.

$$\frac{l_e}{r} \quad (3.11)$$

Equation 3.11 can be derived from the equation 3.12 seen below,

$$\left(\frac{l_e}{d}\right)_{max} = \left(\frac{K_e l}{d}\right)_{max} \quad (3.12)$$

Where	$l_e$	=	effective unbraced length of a Column
	$r$	=	least radius of gyration of column cross section
	$d$	=	least cross-sectional dimension of column
	$l$	=	unbraced length
	$K_e$	=	effective length factor

### 3.3.2 Live loads in U.S. Building code

Design guidance and actions on how to design for imposed loads for structural designs, are also given by the U.S. building code. However the regulation in U.S. differs between the states.

In this study the live loads were taken for New York City, which was given by the International Council (Iccsafe, 2014).

School, classroom: 40 *psf*

### 3.4 Finite Element Method and modeling

In mechanics physical phenomena are often described and modeled by differential equations (Ottosen & Petersson, 1992). The studied problems are often too complex to be solved by the classical analytical methods. The finite element method solves differential equations in an approximate manner with a numerical approach. Since the differential equations describe a physical problem, one can assume it acts over a certain region. This region might be one-,two- or three-dimensional in nature. This approximation of the region is often done by a polynomial. What the finite element method does is divide the region into smaller pieces called finite elements, the approximation is then done in each element instead of done over the whole region at once.

The whole cluster of finite elements are often referred to as finite element mesh or mesh (Ottosen & Petersson, 1992). After the approximation of the physical phenomena is made over the each single element in the mesh, the reacting behavior will be determined in each element. When all elements behavior has been determined they can be patched together according to the systematic formation of the mesh, this gives the entire region. This in turn provides an approximate solution for the entire body's behavior. The element adopts the general approximation to see how it changes over the element. The approximation becomes an interpolation over the element, where one assumes to know values at certain points in the element. At the boundary of each element one often find these points even known as nodal points.

The behavior between the nodal points varies on approximation it may linear, quadratic, cubic and so on (Ottosen & Petersson, 1992). The finite element method is matrix based

because it enables one to use thousands of unknown variables in a compact fashion. The number of elements is crucial for the accuracy, more nodes means more accurate approximation which in turn means that the solution will converge towards the actual case. When one uses FE programs in practice the user still need to have the understanding of underlying theories otherwise the result might be irrelevant.

### 3.4.1 Mathematical modeling

This section is a review of the different approaches in mathematical modeling within FEM, which is implemented to interpret the mechanical behavior.

#### 3.4.1.1 *First order theory*

In the first order theory, the applied load is directly proportional to the deformations and the section forces, when a structure is of a linear elastic material (Runesson et al, 1992). This is true as long as one assumes that the deformations are small and neglect able during equilibrium equation. The original geometry of the structure is thus analyzed as a whole in the first order theory. Each load can therefore be treated separately and later on be added to the result since its linear, this is also referred as the superposition principle

#### 3.4.1.2 *Second order theory*

When some structures is subjected to utility loads significant deformations can occur, it's therefore important that one takes the size of the deformations into account alongside the other geometry of the structure (Runesson et al, 1992).

In the second-order theory the structural deformations are still assumed to be small (Runesson et al, 1992). But compared to the first-order theory the equilibrium equations in the second-order theory consider the deformations. The superposition principle does therefore not apply anymore and the relationship between the deformation and the load generally isn't linear, even for linear elastic material.

#### 3.4.1.3 *Third order theory*

By increasing realism further in the third order, deformations are no longer small and no simplifications of geometry can be made, since the real geometry is the ground pillar to the equilibrium equation (Runesson et al, 1992). Superposition principle is no longer possible, neither is the possibility for simplifications for example, one can't use the reduced expression for curvature everything needs to be taken into account.

### **Comments:**

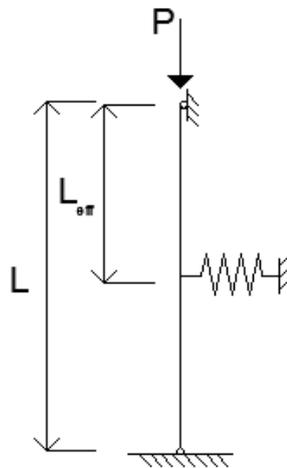
To gain a better understanding of the linear elastic and inelastic behavior of the material the FEM calculations will consider up to the second order.

## 4 Modeling

*In this chapter methods and models of the different cases will be presented. The calculations and modelling that are done in this report are based on the methods prescribed in this chapter. The aim of this chapter is to describe the approach we take on solving certain problems.*

### 4.1 Case study

The cases that are studied have hinged ends with a bracing at the mid-span and a length of 2 m. The preconditions are illustrated in the figure 4.1 below. The effective length of the column are 1/2 times L since the Euler case and support conditions describing the problem will not interfere or create any reduction to the effective length.



**Figure 4.1** Reference case

I

#### 4.1.1 Material properties

The columns that are analyzed vary in size of the different cross-sections. The differences are presented in the table 4.1 below. The material-properties are the same for the various columns.

Case	A	B	C	D	E
Height [m]	0,145	0,145	0,145	0,145	0,145
Width [m]	0,022	0,028	0,034	0,045	0,07
Length [m]	2	2	2	2	2
Timber class	C27	C27	C27	C27	C27
fck [MPa]	22	22	22	22	22
E [MPa]	11 500	11 500	11 500	11 500	11 500

**Table 4.1** The different cross-section and material properties

The cases studied are all targeted for column buckling at the weak side, this causes the slenderness to be dependent by the width of the cross-section rather than the height in a rectangular cross-section. The height is therefore kept constant for all cases. Other geometrical information can be found in the following table 4.1 above.

## **4.2 Finite Element Method (FEM) model**

This section introduces the method of how the FEM calculations are made. During FEM calculations all cases are based on the reference case seen in 4.1.

Initial curvature is also pre-determined in all cases to 2 mm,  $L/500$ , unless it is otherwise specified. The cross sections that are studied in second order analysis and have a rectangular shape. The bracings are always placed on the weak direction since it's the side that will give in first. Other presumptions during FEM are that each case is restricted in the strong direction and rotations are presumed to be zero to prevent rigid body motion in those cases it might occur.

### **4.2.1 Geometry**

The cases that are being studied are modeled in FEM into wire models. The reason behind the use of the wire type is because of its accuracy and it gives a good approximation of the case.

To create the model start of by draw a line of equivalent length to the case studied, then apply the cross-section and material properties of the object. Since it's a wire element the user needs to specify an orientation of the line as well.

### **4.2.2 Load case**

When the geometry is done the load is applied in the form of a concentrated force on the wire object.

During the linear perturbation buckling the force from the load is put to 1 so the resulting eigenvalue will give you the load in Newton. The force is applied in the top of the wire object.

In the second order the columns get a small initial curvature of  $L/500$  so the simulation reassembles a more realistic case. The loads in these cases are dependent on cross sections and therefore different for all cases, to evaluate the load the linear perturbation buckling calculation are made where the load is given in the shape of the eigenvalue.

### **4.2.3 Boundary conditions**

When it comes to limitations the program needs to know a reference in space where the object is restricted. This is first done in assembly where the object is placed in a coordinate system. Later one adds boundary conditions where the object gets given restriction properties of how it should act in different directions in this space. Together it forces the object in place when adding the applied load. The boundaries in these cases are given in three points the two hinges at top and bottom together with the spring brace system in the middle.

The boundary at the top is only restricted in x and z axis where the bottom is restricted in the y axis as well as the others. The hinge boundary is placed in the middle of the top and bottom part of the objects, this is done to best resemble a hinge.

When it comes to the spring on the other hand it is created by a specific type of boundary condition called “spring to ground”, this causes one connection point to get a spring restriction in one direction. The stiffness of the spring needs to be specified to know what kind of resistance it causes to the system. During the second order the spring is considered to be indispensable, and can therefore withstand forces acting on it. During other calculations where the spring stiffness is evaluated it varies.

Since the cases are done in a three dimensions the nodes takes into account six different values at each nodal point, displacements and rotations in x,y and z. Rotations are restricted to prevent rigid body motion.

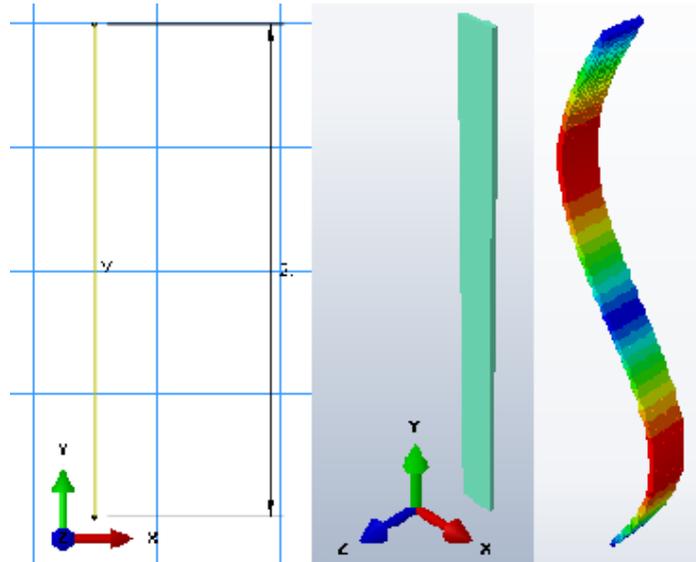
#### 4.2.4 Mesh

The key in a finite element method is the mesh, the matrix based appearance of the original shape. Together with boundary conditions, load cases and the mesh FEM is capable to calculate objects in a three dimensional space. The mesh size is dependent on the size of the analyzed case, larger dimensions increases the mesh. More detailed mesh increases the accuracy however and this cannot be over emphasized more detailed mesh takes more computer power. In a perfect world one would have used infinitesimally small elements in a mesh, but since the computer advancements of this day and age doesn't allow it simplifications needs to be made.

Everything is now in place to start calculating, when done the program gives the displacements with the correlating force applied or other stresses dependent on what the user specifies in the history output. At this point the data is converted over to excel to restructure it and manage it further into tables and figures.

#### 4.2.5 Linear perturbation buckle analysis

The linear perturbation buckle analysis is preformed to gain the critical load for the studied column case. This type of analysis will later be used as a reference case for how the initial curvature shape will occur in the second order analysis.



*Figure 4.2 From start to finish with the beginning of a line to its deformed state after a linear perturbation buckle analysis.*

#### 4.2.6 Second order analysis

During the second order analysis the computer gives an increment of the load applied and the resulting deformation. When the stresses become too big failure occurs, calculations after failure are of no interest therefore calculations stop there.

### 4.3 Comparison between the building codes

As seen in previous chapter's different building codes address the same problem differently. Therefore a comparison is going to be made between Eurocode and the U.S. Building code. This will be done by firstly doing the full calculations of each standard separately and then compare it against each other.

#### Design problem

Later a study is done, where both building codes are taking on the same design problem. A column is subjected to three different influence areas when designing a class room for a school. The different influence areas of interest are  $3 \text{ m}^2$ ,  $4 \text{ m}^2$  and  $5 \text{ m}^2$ . The live load that is in use for respective building code can be found in section 3.2.2 and 3.3.2.

#### 4.3.1 According to Eurocode 5

The method to calculate the design load for timber members in compression according to Eurocode 5 is presented in section 3.2. The design load, formulated in equation 3.1, is defined by parameters as the timber's allowable stress level, the reduction factor and the area of the cross-sections.

By using the method, described in subsection 3.2.1, the design load for respective cross-sections, presented in table 4.1, are calculated.

In the results the design load is used in order to compare Eurocode with the U.S. standard.  $K_{mod}$  is given a value of 0.9 while calculating the design load.

#### 4.3.2 According to the U.S. standard

In section 3.3.1 the method to calculate the design load, according to the U.S. standard, for timber members was presented. The design load is given in equation 3.6 where it is defined that the subjected axial compressive stress in the member has to be lower than the capacity of the column, which is defined by the columns allowable compressive stress.

#### Material properties

The material properties for timber are given in NDS Supplement 4A. As the essence of this rapport is to compare the different methods and standards, the material properties of timber class 27 has been implemented and converted to the U.S. customary units. The properties used in this section are presented in table 4.2 below.

	$F_c$	$E$	$C_F$
U.S. Customary units	3191 psi	1 670 000 psi	1
SI-units	22 MPa	11 500 MPa	1

*Table 4.2 Timber properties converted from SI to U.S customary units*

For further information regarding unit conversion between U.S. customary units and SI-units can be found in Appendix A.

#### Correction factors

The U.S. standard is using correction factors for calculating the allowable stress capacity for the column. The correction factors are dependent on different factors, which are defined in chapter 3.3.1. In this report the correction factors have been decided to have the values presented in the table 4.3 below.

	Values of correction factors
$C_M$ – Wet service factor	1
$C_t$ – Temperature factor	1
$C_F$ – Size factor	1
$C_i$ – Incised factor for sawn lumber	1
$C_D$ – Load duration factor	1,25

*Table 4.3 Correction factors*

### **The correction factor considering the slenderness of the column**

The calculations start with deciding the column's capacity by calculating the slenderness of the column, see equation 3.12. The Euler critical buckling stress and the limiting compressive stress in column at zero slenderness ratio was presented in equation 3.10 respective equation 3.9. By calculating the values of these two, the correction factor that is dependent on the slenderness can be estimated.

### **The allowable force**

The allowable compressive stress and the allowable force in the column can now be calculated. It is calculated by using equation 3.6. To able to compare the result between the U.S. standard and Eurocode, the results have also been converted to SI-units in the results.

## **4.4 Initial curvature**

In this section the effects of initial curvature on the load bearing capacity of a column, are going to be evaluated. As effects of a varying initial curvature, but also how the size and shape of the initial curvature influence the load bearing capacity.

### **4.4.1 Critical force dependency to initial curvature**

In this part the effects of a varying initial curvature are going to be evaluated, therefore one cross-section case A is kept and the initial curvature for this case is changed between  $L/500$ ,  $L/400$ ,  $L/300$ ,  $L/250$  and  $L/150$ .

### **4.4.2 Shape and magnitude of Initial curvature in a column**

In previous experiments initial curvature has been set to be ideal for each case where the shape between two nodal points has been set to half a sinus curve. However initial curvature rarely acts as in the ideal case, therefore the shape and appearance of the initial curvature needs to be readjusted to a more plausible shape. In this part the shape becomes a coalition between different buckling modes dependent on how many buckling modes a column will have. As an example if a column has a brace in the midpoint the imperfection becomes dependent on two buckling modes.

The resulting shape is calculated by adding the shape of respective imperfection curve. Some of the shapes and the formulations of respective imperfection are presented figure 4.3 to 4.5 below.

### First buckling shape

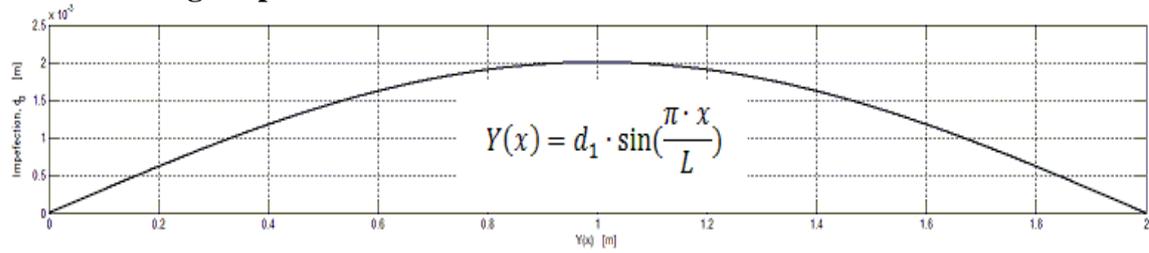


Figure 4.3 First buckling shape

### Second buckling shape

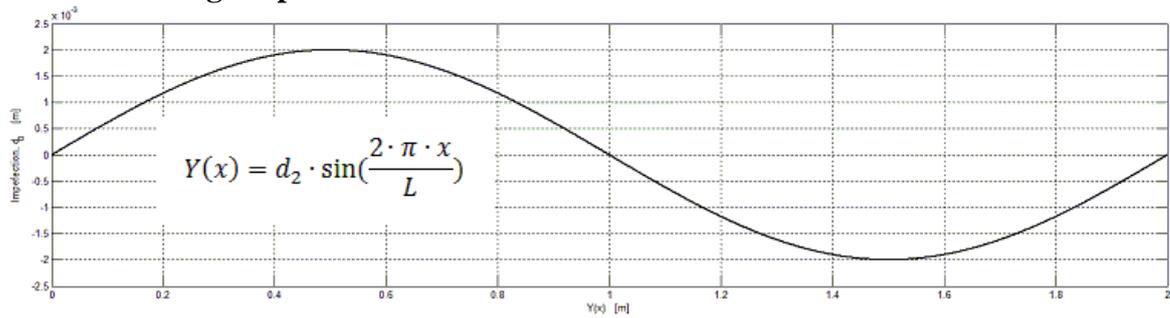


Figure 4.4 Second buckling shape

### Third buckling shape

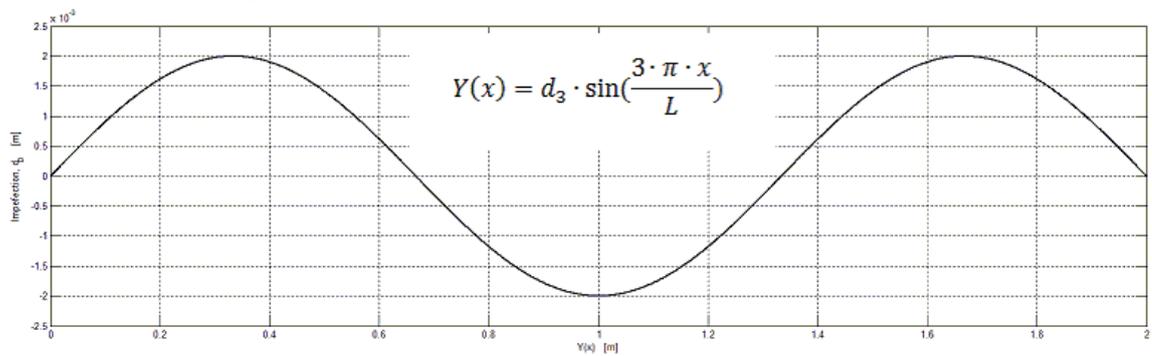


Figure 4.5 Third buckling shape

Where,

$d_i$             =            Amplitude of respective buckling shape  
 $L$                 =            Total length of the column

## 4.5 Bracing of a column

By using a brace the load bearing capacity of a column can increase. However, how much the load bearing capacity increases solely depends on how stiff the brace is.

### 4.5.1 Ideal stiffness according to Yura

In equation 2.11 the expression of calculating the ideal stiffness for a bracing, for an ideal column with a bracing at the midpoint, was given. The formulation is dependent on the critical load given by Euler's theory.

Yura's & Helwig's equation for reaction force in a brace known as equation 2.15 is evaluated. This is done by varying the critical vertical column force and stiffness of the brace. Since the critical force of a column is dependent on the stiffness of its bracing the critical force used in this evaluation is dependent on the stiffness used. The stiffness is varying from 0 to 50000 N/m with a step of 1000 in between. The critical load is calculated by the known stiffness values input into equation 2.10 in previous chapter. This is only valid until it reaches the column's maximum capacity. Only case A is used during this evaluation.

### 4.5.2 Bracing stiffness in Eurocode

In section 3.2.2 the method for calculating the minimum spring stiffness is defined. To be able to calculate the stiffness using equation 3.5 the critical bending stress needs to be calculated first. The critical bending stress together with the material strength gives the relative slenderness factor that in turn will give the reduction factor  $k_{crit}$ . The reduction factor is needed to calculate the critical moment but also to calculate the mean compressive force in the member. When the mean compressive force is done everything needed to complete equation 3.5 are acquired and can thereby be calculated.

### 4.5.3 Stiffness of a brace

The critical force a column with brace can obtain is however dependent on how stiff the bracing is. To see how it affects a column case A's bracing are varied and simulated to gain the critical load for each stiffness. During this simulation no initial curvature was applied or any other forms of disruptions.

### 4.5.4 Influence of several braces

The influence of having several braces is studied in this section to get a better understanding of how a column is affected to it compared to a single brace. The study will have a focus on two and three braces, where the stiffness of the braces will vary.

#### **Brace stiffness variation on a column with two braces**

For the two brace scenario the lower brace is varied while the upper is kept constant. The stiffness value of the brace that is kept constant is 30 kN/m, 75 kN/m, 150 kN/m and 200 kN/m.

#### **Brace stiffness variation on a column with three braces**

In the study of a column with three braces there exist two separate scenarios. The first scenario is where the middle brace is kept constant and the outer braces are varying. In the second scenario the outer braces are kept constant, and the middle is varying. In both cases the constant braces have a stiffness of 100kN/m, 200kN/m 400kN/m and 500kN/m.

#### 4.5.5 Brace force study

To evaluate the study of bracings further, a study of the reaction force in the brace is done. This is done to verify Yura's equation of a brace force seen in equation 2.15. This is compared in FEM by varying the vertical column force and the stiffness of the brace. This is done by varying the critical vertical column force and stiffness of the brace. Since the critical force of a column is dependent on the stiffness of its bracing, the force applied is always set so the critical force for each stiffness case used during this evaluation.

The critical load is calculated by linear buckling analysis which uses the stiffness prescribed. This is only valid until it reaches the columns maximum capacity. Only case A is used during this evaluation with an initial curvature of  $d_0 = 2\text{mm}$ .

#### 4.5.6 Influence of initial curvature

Lastly the initial curvature is changed to see its effect of how the reaction force in a brace is in correlation to the initial curvature, and how it behaves for a known bracing stiffness. The known brace stiffness that is evaluated is 50000 N/m and 40000 N/m. The load that acts on the column is restricted by the columns load bearing capacity. The results is therefore restricted to  $p/p_{cr} = 1$  throughout the evaluation.



## 5 Results and Discussion

*This chapter presents the results and discussion, where the underlying methods have been used to gain the data and results presented in this chapter. In the result part the authors observe and interpret of how the data acts. Some minor clarifications of what has been done are presented so the reader might gain a better understanding of how the problems is solved for a more detailed version see previous chapter. Finally an analysis of the each subchapter is made of the findings.*

### 5.1 Comparison between different building codes

In this subchapter, the design loads are presented for the cases in section 4.1. A comparison is also done between Eurocode and the U.S. building code, regarding a column's design load.

#### 5.1.1 Eurocode and the U.S. Building code

Eurocode and the U.S. building code are used to calculate the load bearing capacity for cases A to E. The result is presented in the table 5.1 below.

Case	A	B	C	D	E
Eurocode, design value [kN]	9,3	17,6	30,2	61,4	136,3
U.S. building code [kN]	5,3	10,8	19,1	43,2	145,4

*Table 5.1 Design load for respective case A to E.*

By observing the table 5.1 above it becomes clear that there exist a big difference between the two design codes. Eurocode design value is larger than the U.S. building code in each case with the exception of case E.

The large differences in the result are due to the formation of each standard. The building codes are based on different methods and thereby cannot be compared directly. The American building code is based on allowable stresses whereas Eurocode uses partial coefficient. Individual components in each respective standard can thereby not be compared directly.

#### Design study comparison between Eurocode and the U.S. Building code

Eurocode and the U.S. building code are used to design a column, based on the problem formulation in section 4.3. The calculated design load is presented in the table 5.2 below.

	Design load (3m <sup>2</sup> ) [N]	Design load (4m <sup>2</sup> ) [N]	Design load (5m <sup>2</sup> ) [N]
Eurocode	7500	10000	12500
U.S building code	5745	7660	9576

*Table 5.2 Calculated design load for 3 m<sup>2</sup> live load (class room) for respective standard*

By observing the results, it can be seen that there exist a difference in respective standards approach towards live load. The design load for Eurocode is also in this case larger than the U.S. building code.

Respective cross-section is later decided by using the calculated design loads in table 5.2, for a predetermined width of 45 mm. The result is presented in table 5.3 below.

Influence area	Design standard	Cross-section [mm]	Column's load capacity (EU) [N]	Column's load capacity (US) [N]
3 m <sup>2</sup>	Eurocode	0,045x0,095	9 095	5 142
	U.S building code	0,045x0,120	11 488	6 495
4 m <sup>2</sup>	Eurocode	0,045x0,120	11 488	6 495
	U.S building code	0,045x0,145	13 881	7 848
5 m <sup>2</sup>	Eurocode	0,045x0,145	13 881	7 848
	U.S building code	0,045x0,195	18 668	10 555

*Table 5.3 Cross-section are designed based on the design load presented in table 5.2 for respective standard, for a predetermined width of 45 mm.*

By observing table 5.3 above it becomes clear that the European building code allows for a smaller cross section, whereas the U.S. standard has a harder restriction. Since the standards are based on different approaches the result becomes very different when adding the design load into the equation. The differences between the two are not as large as previous result has pointed towards, whereas the difference is given by one standardized size.

### 5.1.2 Analysis of results

The result indicates a larger difference between the European building code and the U.S. counterpart. The authors didn't at first anticipate the big difference between the two standards. As previously mention the differences depend on the fact that both standards are not based on similar methods. It becomes therefore harder to compare the two in a smaller perspective and an overall approach needs to be made.

## 5.2 Initial curvatures effect on the load bearing capacity

In this subchapter the initial curvature's effects on the load bearing capacity are presented. A numerical analysis is performed to determine the correlation between initial curvature and load bearing capacity.

### 5.2.1 Effect of initial curvature

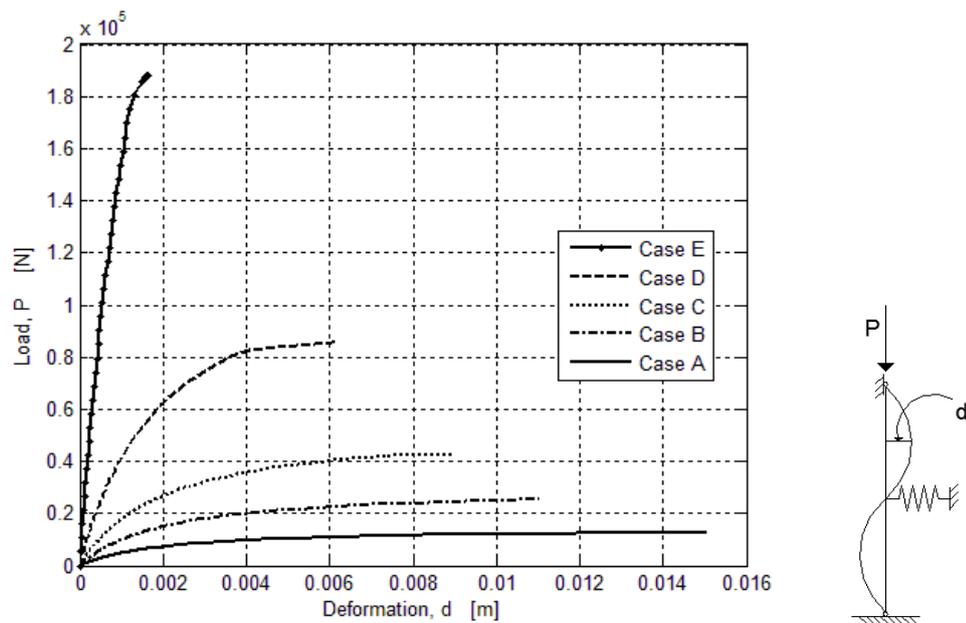
The columns load bearing capacities for case A- E are presented below in table 5.4, by using Euler and FEM.

Case	A	B	C	D	E
Euler load [kN]	14,6	30,1	53,9	125,0	470,4
FEM Linear buckling [kN]	14,6	30,0	53,7	124,3	464,3
FEM Second order (L/500) [kN]	13,5	25,2	42,4	85,7	187,7

*Table 5.4 Buckling load for respective cases A to E,*

By observing the results in table 5.4, one can see that the Euler load and the FEM linear analysis are similar. They should theoretically be the same since they both use the same method. The reason behind why Euler's load and FEM linear analysis differs could be because of Euler's load is meant for slender columns.

The force- and deformation correlation for FEM secondary analysis is presented in the figure 5.1 below.



**Figure 5.1** Force - Deformation diagram for case A to E, with an initial curvature of  $d_0 = 2$  mm

Comparing the Euler buckling load and the buckling load based on FEM second order analysis in table 5.4, a large difference becomes present. Second order indicates a smaller capacity since it takes imperfections and smaller deformations into account, which can be seen in table 5.4. The column's load bearing capacity is therefore smaller than an ideal column. Figure 5.1 illustrates the nonlinear effects of the deformation in the second order analysis.

### 5.2.2 Critical force dependency due to initial curvature

Table 5.5 and figure 5.2 below presents the resulting load bearing capacity and deformation of a column due to a varying initial curvature. The different initial curvature cases are set from the smallest  $L/500$  ( $d_{01}$ ) to the largest  $L/150$  ( $d_{05}$ ), and are based on Case A. To evaluate each imperfection with resulting deformation a bit closer figure 5.3 below will display how it behave during two known forces.

Initial curvature case	L/500	L/400	L/300	L/250	L/150
Initial curvature $d_0$ [mm]	2	2,5	3,3	4	6,7
Force [kN]	5,32	4,60	3,87	3,38	2,29
Deformation $d_1$ [mm]	1,86	1,78	1,80	1,77	1,75

**Table 5.5** Initial curvature cases

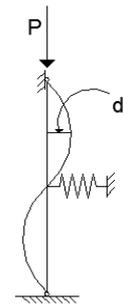
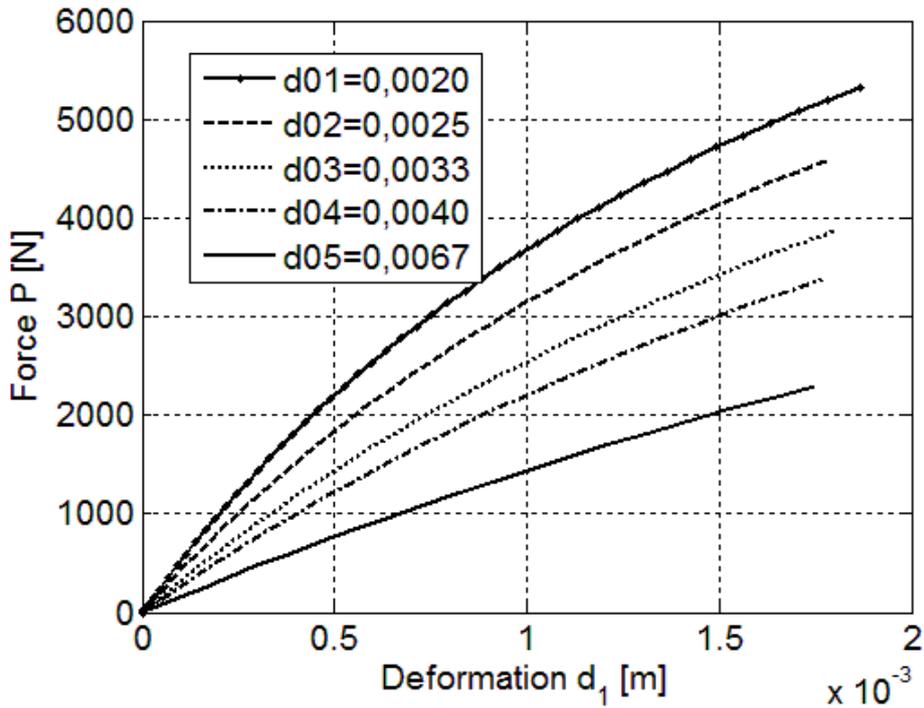


Figure 5.2 Initial curvature variation, case A

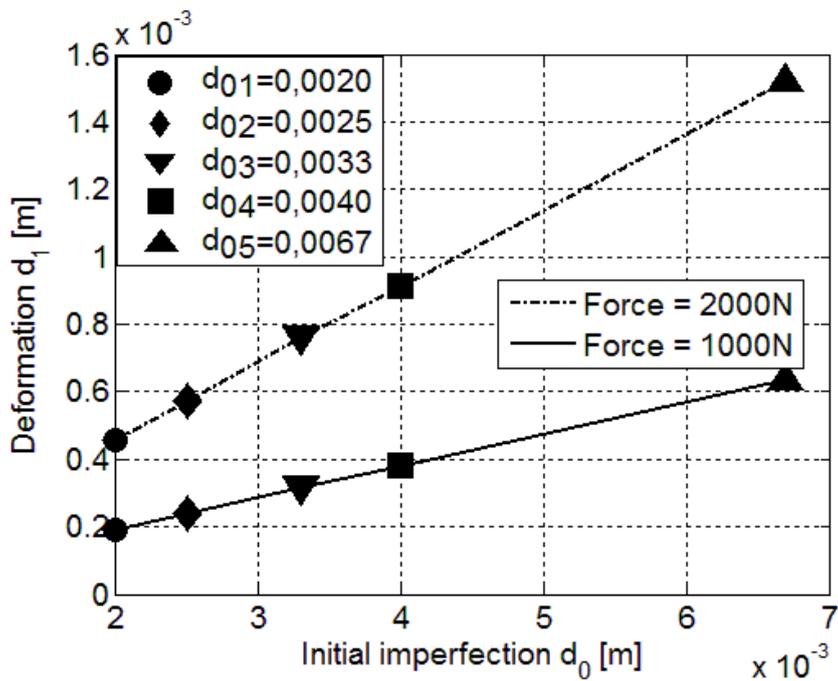


Figure 5.3 Deformation due to imperfection, case A

By studying figure 5.2, it can be seen how a larger initial curvature leads to a smaller load bearing capacity for the column. The load bearing capacity for various initial curvatures are also presented in the table 5.5 above. The deformation slightly increases as the initial curvature gets larger, except in initial curvature case  $L/400$  where it gets smaller.

In figure 5.3 one can see that larger initial curvature leads to an increased rate of deformation. The deformation rate is with other words dependent on the initial curvature of the column.

### 5.2.3 Shape and magnitude of initial curvature

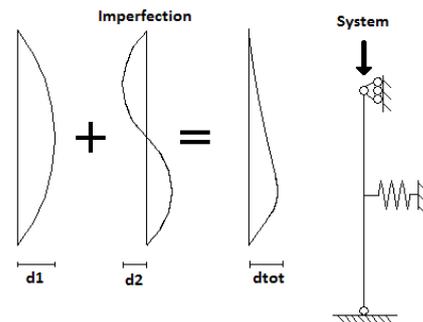
As mentioned in section 4.4.2 the shape and the form of the initial curvature in a column can vary.

#### Imperfection shape for one brace

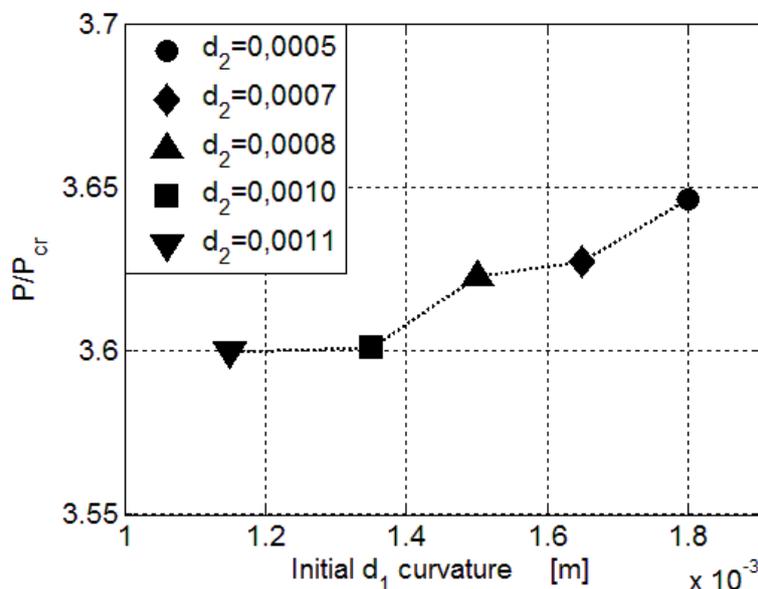
By using the mathematical models shown in section 4.4.2, the influence of the different forms are tested. The resulting buckling load due to the varied forms of the initial curvature is shown in table 5.6 below.

$D_{tot}$	$D_1$	$D_2$	Buckling load
0,002	0,0018	0,0005	13308
0,002	0,00165	0,0007	13238
0,002	0,0015	0,0008	13222
0,002	0,00135	0,001	13142
0,002	0,00115	0,0011	13137

**Table 5.6** Buckling load due to varied buckling shape. For a column with a single brace. For case A.



**Figure 5.4** Illustrates the influence of buckling shapes.



**Figure 5.5** Illustrates the buckling load capacity of the five different scenarios seen in table 5.6. For a column with a single brace due to varied shape of the initial curvature. For case A.

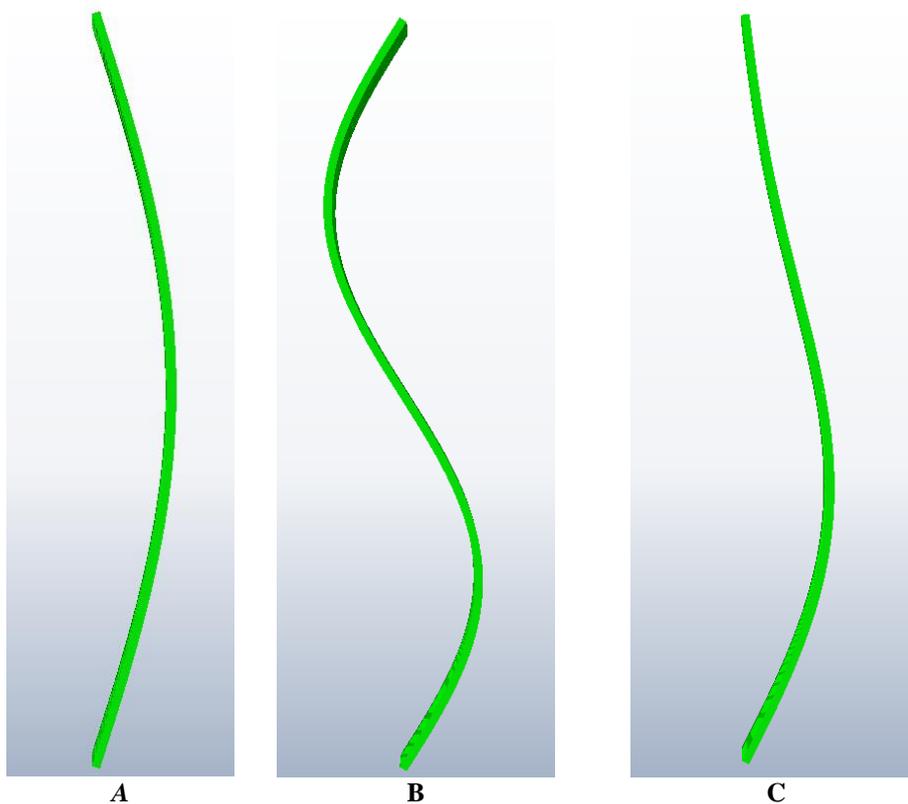
Results in table 5.6 shows that the resulting buckling load capacity is increasing as the imperfection of the first buckling shape  $d_1$  is more dominating and as the second buckling shape gets smaller. This can be interpreted that a more dominating role for the second buckling shape  $d_2$  is less beneficial for a column's load capacity. The difference in load

buckling capacity between a more dominating  $d_1$  and  $d_2$  is though very small in this example.

The shape and form of the initial curvature has with other words an influence on the load bearing capacity. When observing a single column without a brace buckle, the column will slowly indicate a shape of half a sine wave. If a brace is added to the same set up the shape will be determined by a correlation between the old shape (half a sine wave) and the second buckling shape, which has a shape of a whole sine wave.

The new shape can be adjusted by allowing different contributions from each part. However the worst scenario in this case would be when the second shape of a full sine wave gets its largest allowed amplitude.

In the figure 5.6 below, an exaggerated buckling shape  $d_1$  (5.6 A) and  $d_2$  (5.6 B) is generated to illustrate the correlation of combining the buckling shapes in FEM. The resulting initial curvature in the column is shown in figure 5.6 C below. The figures confirms figure 5.4 (above) and how the resulting initial curvature looks like.



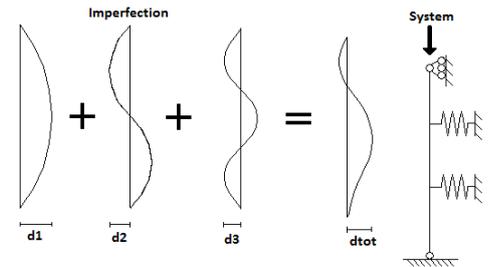
**Figure 5.6** Illustrates an exaggration of the buckling shape in FEM. Whereas A is dependent on  $d_1$  and B is dependent on  $d_2$ . C describes the correlation of A and B.

### **Imperfection shape for two braces**

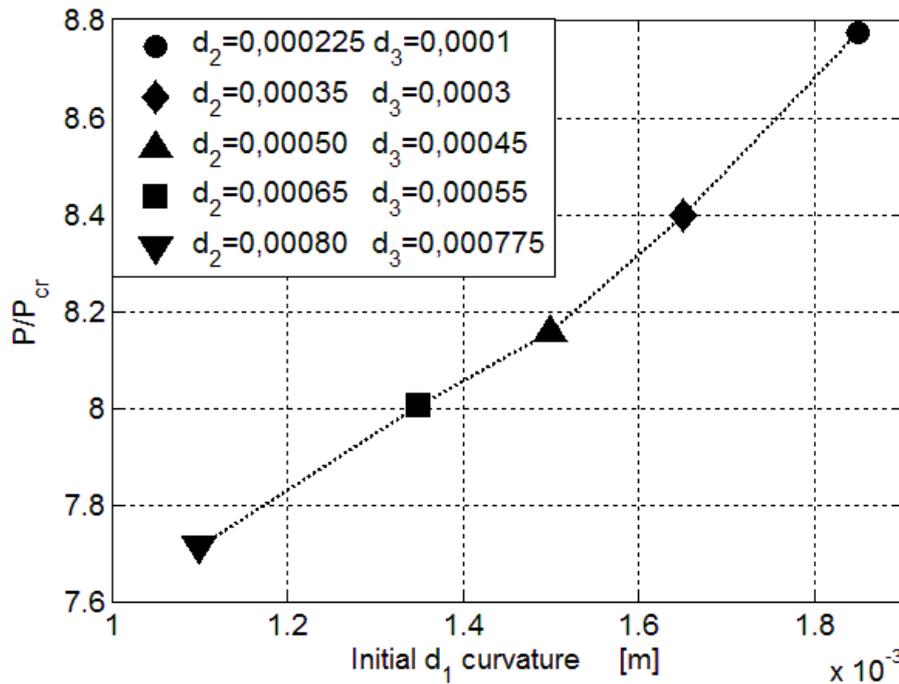
In a similar fashion when adding a second brace to the system the influence of the different forms is tested. In difference to the previous example, an additional imperfection contributor is added, to calculate the resulting initial shape of the column. In table 5.7 below the resulting buckling load due to a varied initial imperfection is shown.

$D_{tot}$	$D_1$	$D_2$	$D_3$	Buckling load
0,002	0,00185	0,000225	0,0001	32028
0,002	0,00165	0,00035	0,0003	30660
0,002	0,0015	0,0005	0,00045	29774
0,002	0,00135	0,00065	0,00055	29222
0,002	0,0011	0,0008	0,000775	28163

**Table 5.7** Buckling load due to varied buckling shape. For a column with two braces. For case A.



**Figure 5.7** Illustrates the influence of buckling shapes



**Figure 5.8** Illustrates the buckling load capacity of the five different scenarios seen in table 5.7. For a column with a single brace due to varied shape of the initial curvature. For case A.

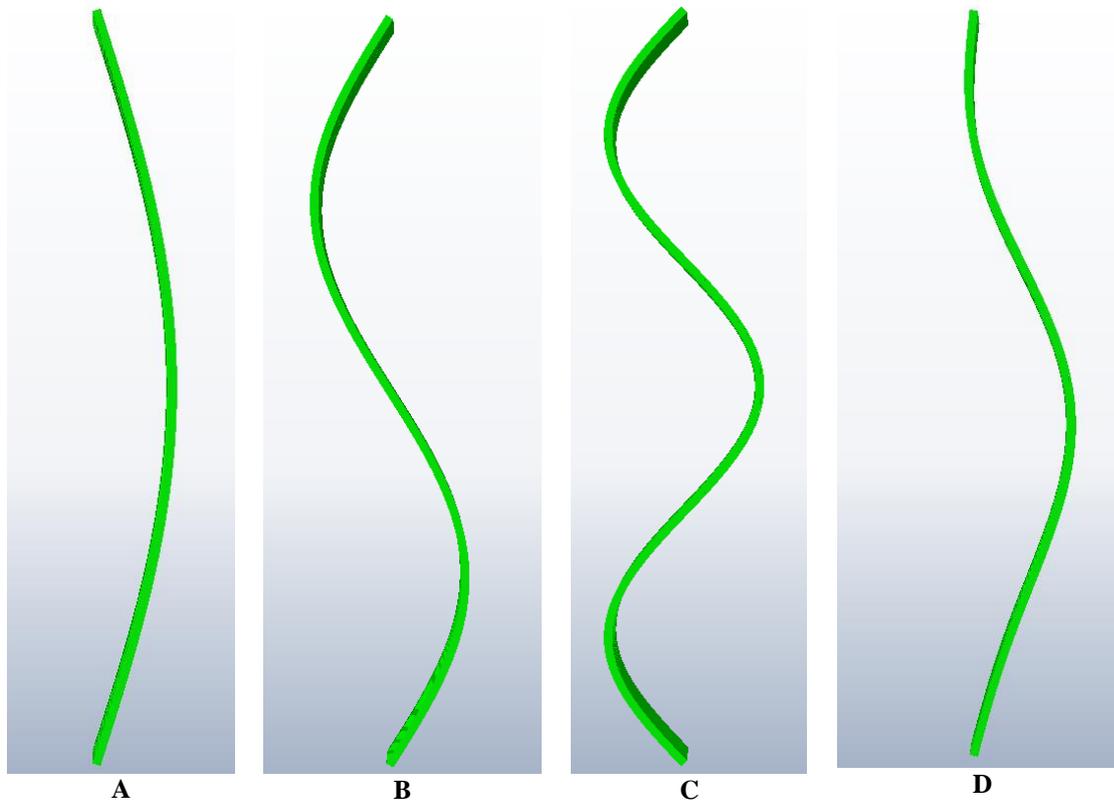
Upon viewing table 5.7 it becomes clear that as soon as  $d_1$  gets reduced and  $d_2$  and  $d_3$  gets larger,  $d_2$  and  $d_3$  will govern the outcome of the buckling load. The bearing load capacity of the column gets smaller as  $d_1$  is reduced. By studying the figure 5.8 above a distinctive difference in buckling load capacity can be seen between a more governing  $d_1$  and a smaller  $d_1$ , in difference to the previous section which had a small variation seen in figure 5.5. This can be interpreted that the addition of an initial shape has a big influence on the resulting bearing load capacity of a column.

Similar to the case with one brace a column with two braces will gain an extra shape to its shape contributors as its predecessor. In this case this would mean that the total value would be a mixture of a half a sine wave, whole sine wave and the 1,5 sine wave.

The worst contributions from this scenario would be when the third and second contribution parts gain as large amplitude as possible.

In the figures 5.9 below, an exaggerated buckling shape  $d_1$  (5.9 A),  $d_2$  (5.9 B) and  $d_3$  (5.9 C) is generated in a similar manner as in the previous example to illustrate the correlation

of combining the buckling shapes in FEM. The resulting initial curvature in the column is shown in figure 5.9 D below. The figure confirms figure 5.7(above) and how the resulting initial curvature looks like.



**Figure 5.9** Illustrates an exaggration of the buckling shape in FEM. Whereas A is dependent on  $d_1$ , B is dependent on  $d_2$  and C is dependent on  $d_3$ . Together A,B and C becomes D.

#### 5.2.4 Analysis of results

The result in table 5.4 between Euler and FEM were anticipated by the authors. As written in the results, the load bearing capacity in FEM linear buckling should equal the Euler buckling load, whereas the difference between Euler and FEM second order analysis is considerably larger. The resulting buckling load in FEM can be displayed as a ratio of Euler's buckling load to illustrate the difference better. The ratio between FEM and Euler is shown in the table 5.8 below.

Case	A	B	C	D	E
FEM Linear buckling [kN]	1,00	1,00	1,00	0,99	0,99
FEM Second order (L/500) [kN]	0,88	0,84	0,79	0,69	0,40

**Table 5.8** Ratios between FEM and Eulers buckling load.

The study on critical force dependency due to initial curvature, seen in table 5.5, shows that the initial curvature has a large impact on the load bearing capacity of the column. This was

anticipated by the authors, as a column with a larger initial curvature will have a decreased load bearing capacity compared to an ideal column.

It was also shown that the deformation and the deformation-rate of the column are dependent on the magnitude of the initial curvature, which can be seen in figure 5.3. The total deformation of the column and the deformation -rate gets larger as the initial curvature gets larger.

The most disadvantageous situation for the column with one brace is when the second buckling shape governs the total shape of the imperfection in the column. As the second shape, which has the form of a full sine wave, gets its largest allowed amplitude the column will get its largest reduction on the load bearing capacity as shown in figure 5.5. The second situation with two braces has a similar counterpart whereas the second and third shape governs the magnitude of the reduction on the load bearing capacity as shown in figure 5.8. The authors find the results both interesting and logical.

The findings indicates that if the other shape models of the initial curvature are taken into account, the total load bearing capacity of the column will decrease further. This can be studied in 5.6 and 5.7.

### 5.3 Stiffness requirement of a brace

The required stiffness of a brace for a column in compression, with a brace at the midspan, is presented in the table 5.9 below for respective method and building code.

Case	A	B	C	D	E
Yura's ideal stiffness [kN/m]	29,2	57,6	103,1	239,1	940,8
FEM- Linear analysis [kN/m]	29,2	60,1	107,5	248,6	928,6
Eurocode's minimum $k_s=1$ [kN/m]	13,3	25,4	43,6	88,7	196,9
Eurocode's maximum (recommended) $k_s=4$ [kN/m]	53,2	101,8	174,4	355	787,8

*Tabell 5.9* Required stiffness for the brace according to the following

#### 5.3.1 Yura's and Helwig's model

First of Yura's ideal stiffness is calculated for each case where neither imperfection nor any other disturbance affects the case. By reviewing the results in table 5.9 one finds yet again the linear buckling analysis in FEM to reassembles the theoretical values. Theoretically the results from linear analysis should reassemble those of Yura's and Helwig's theories, and are therefore a good check to see if the theory is accurate. As mentioned in the section 5.2.1, the deviation in the results is probably caused by the shape of the cross-section where Euler's load is best suited for analyzing slender columns.

#### Bracing stiffness

By modeling the ideal case of the column with a single brace at the midspan in FEM, with no imperfections, it can be shown that Yura's & Helwig's theories from chapter 2.4.2 are confirmed.

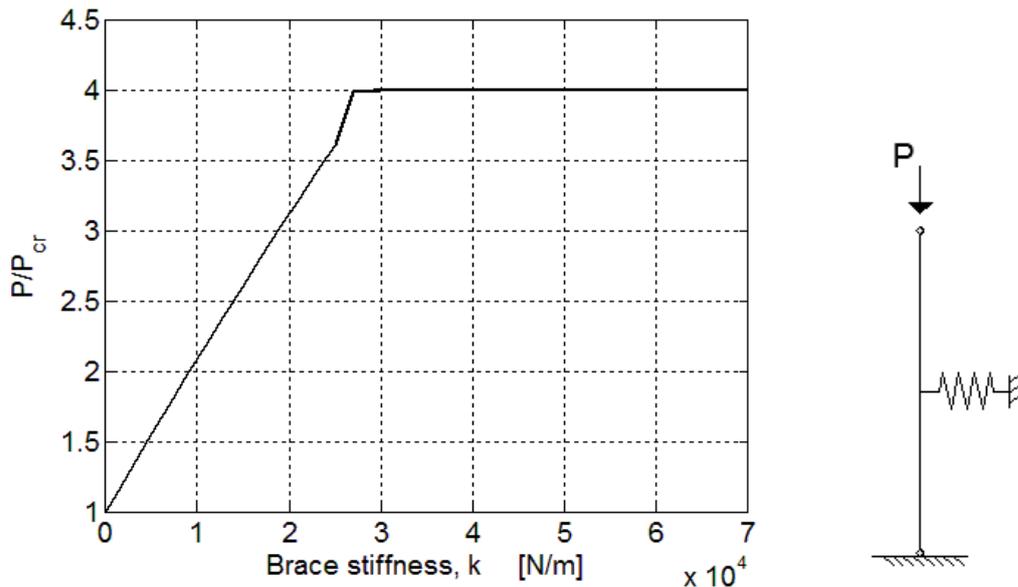
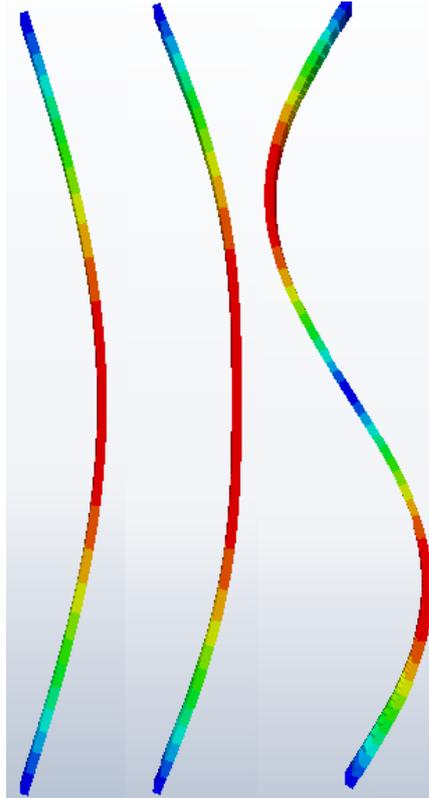


Figure 5.10 Critical force due to stiffness of the bracing, for case A

As described in chapter 2.4.2 the spring stiffness takes on critical load in a linear manner until the ideal stiffness threshold  $k_{ideal}$  is reached, afterwards the load bearing capacity cannot be increased, this can also be viewed in figure 5.10. At this point the maximum capacity of the column is obtained. Afterwards an increase of stiffness becomes unnecessary since it does not contribute to an increase in the load bearing capacity of the column.

As one can see in figures below, the column will at first buckle as a half sinus curve as the load on the column gets larger. When it later hits the threshold value  $k_{ideal}$ , the buckling shape will become a full sine curvature. This is true for an ideal case where no imperfections or disturbances act on the tested column. To illustrate the buckling modes see figure 5.11 below,



**Figure 5.11** From left to right the stiffness of the brace is 0 to 29200 N/m. For case A.

What cannot be overemphasized is that a brace only reinforces in one axis. During the design procedure the engineer needs to be aware of the systems overall capacity and of what kind of force each brace will counteract.

### 5.3.2 Eurocode's stiffness requirement

In contrast to the mathematical expression of the ideal stiffness, defined by Yura, the formulation in Eurocode isn't as specific where some correction factors can be seen more as guidelines. The expression for minimum required stiffness for timber in Eurocode is dependent on several factors, where one of these is the factor  $k_s$ . The correction factor  $k_s$  is dependent on implementation and varies from 1 to 4, where 4 is recommended to be used.

To gain a better perspective of the effects of  $k_s$ , the maximum 4 and minimum 1 are tested in the table above. When  $k_s = 4$  Eurocode's maximum stiffness requirement becomes larger than Yura's for slimmer designs.

### 5.3.3 Influence of several braces

As seen in previous sections the usage of a brace can increase the load bearing capacity of a column. By using additional braces the load bearing capacity can be increased furthermore, this can be seen in the figure 5.12 below.

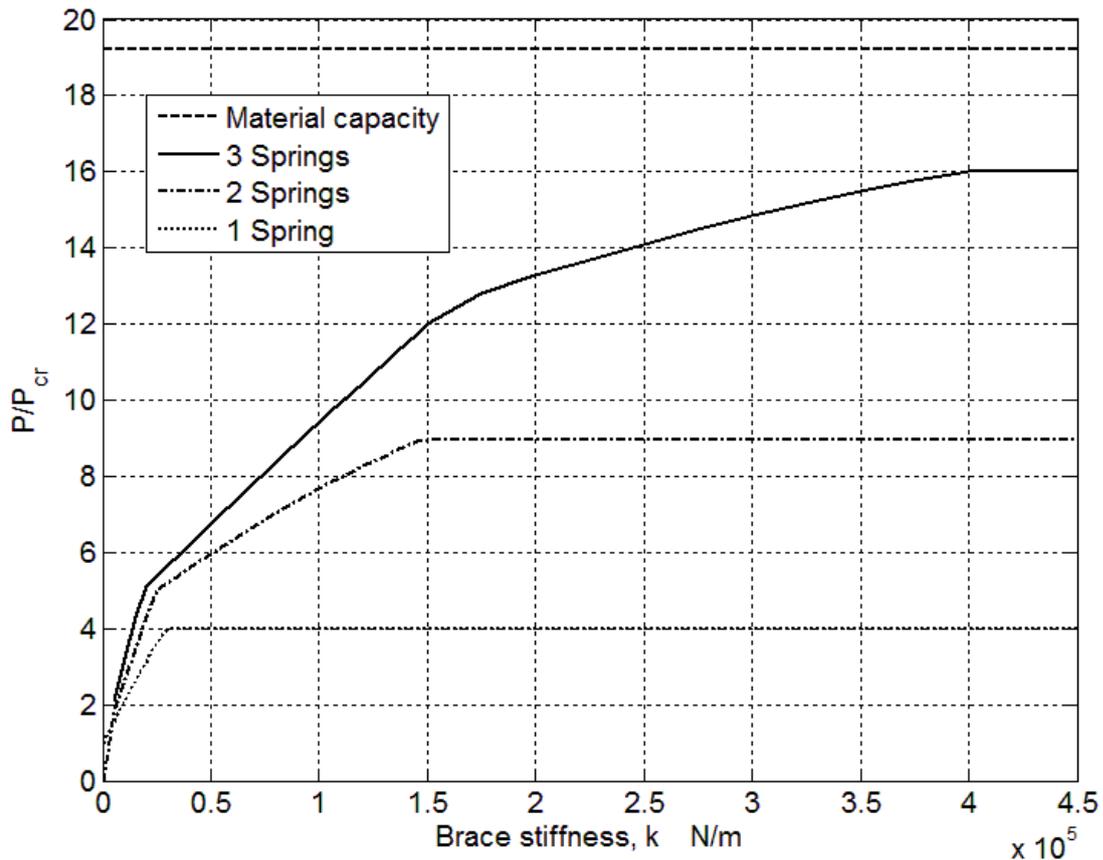
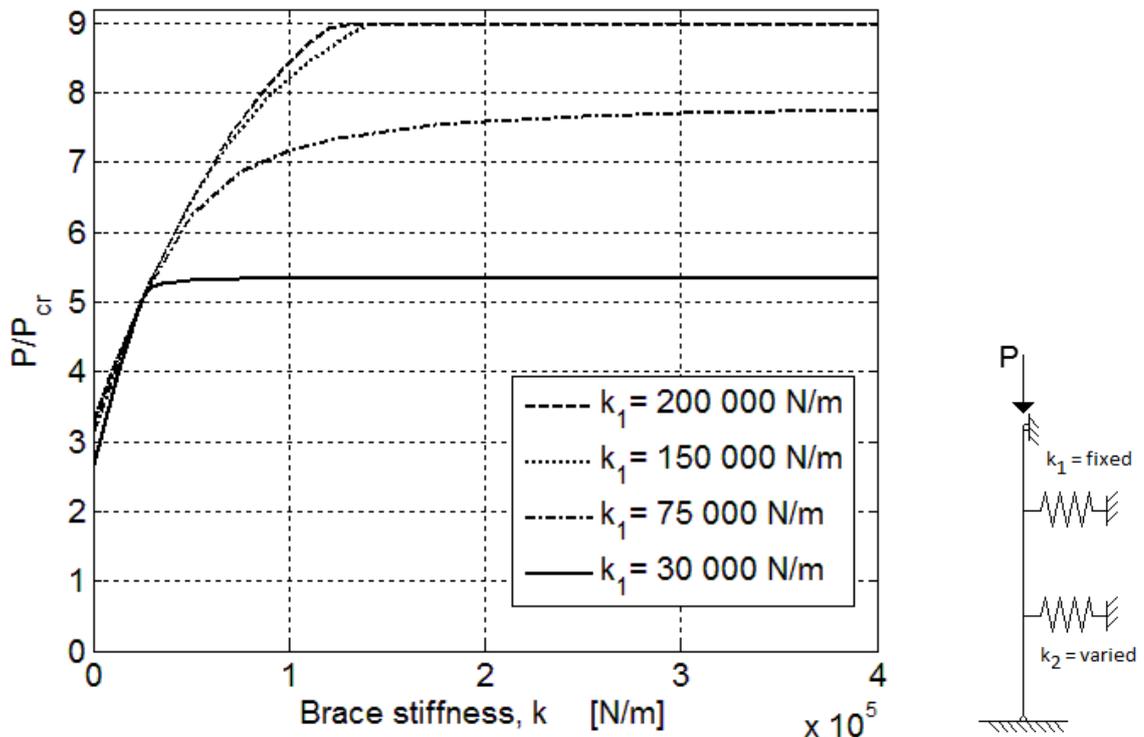


Figure 5.12 Illustrating the effect of having several braces on a column with a cross section of 22x145 mm<sup>2</sup>.

By studying the figure above it is shown that additional braces will contribute to a larger load bearing capacity for column. For every brace that is added to the column the load bearing capacity will be increased, as shown in the figure 5.12 above. However dependent on the total amount of braces the size of the increased capacity will vary, it is not profitable to add unlimited braces since after a while the materials capacity limit is reached and thereby cancels out the beneficial increase from the brace. This means that the material properties will govern the column buckling failure instead of the geometrical properties.

### Influence of two braces

In the previous section the braces of the column was presumed to have the same stiffness. In reality the stiffness of the braces doesn't have to be the same, it can vary. The influence of a varying stiffness between two braces is shown in the figure 5.13 below. Where one of the braces stiffness is constant and the second brace stiffness is varied between 0 to 400 000 N/m.



**Figure 5.13** Illustrating the influence of a varying stiffness between two braces. One of the braces stiffness is constant and the second brace stiffness is varied.

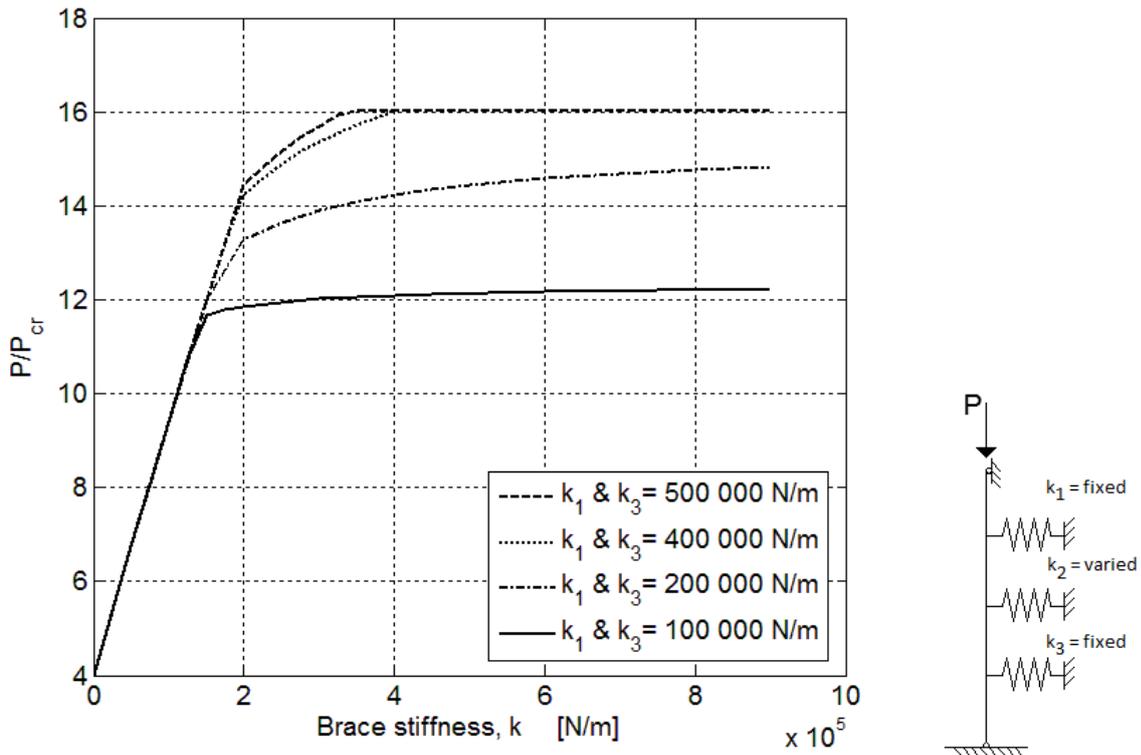
The study shows that the resulting load bearing capacity of a column has a strong relation to its brace stiffness. If  $k_1$  has a larger stiffness, the load bearing capacity of the column will have a larger total increase rate in load bearing capacity than one with a brace with a lower stiffness. The maximum load bearing capacity of the column will be reached earlier. To be able to reach the maximum buckling load, the minimum requirement is either that both braces at least have a value of  $k_{ideal}$ , which is 150000 N/m, or one value over and the other is close under.

### **Influence of three braces**

In a similar manner as in the previous section, the braces stiffness is varied to study its influence on the load bearing capacity. This section is divided into two parts, where the braces stiffness's are varied in different ways/manners to get a better grasp of its influence.

### Test variation 1

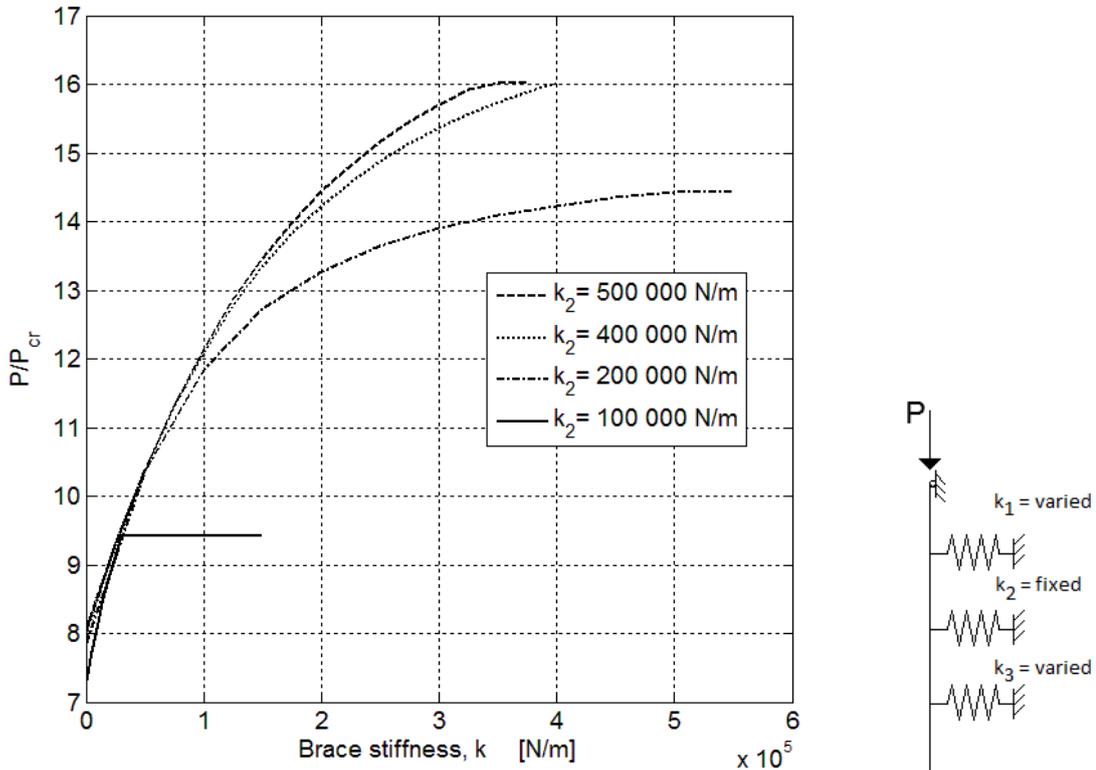
In this case, the stiffness's of the outer braces  $k_1$  and  $k_3$  are kept constant while the stiffness of the brace in the middle  $k_2$  is varying. The result is shown in the figure 5.14 below, where the influence of the brace stiffness is plotted against the resulting buckling load.



**Figure 5.14** In the figure it can be seen that the increase stiffness for  $k_2$  in the beginning results in the same load bearing capacity for the column. However as the brace stiffness gets larger the increase rate starts to differ. The brace stiffness combo with the lowest stiffness will stagnate quicker than those of a larger magnitude.

### Test variation 2

In this case, the stiffness's of the outer braces  $k_1$  and  $k_3$  are varying while the stiffness of the brace in the middle  $k_2$  is constant. The result is shown in the figure 5.15 below, where the influence of the brace stiffness is plotted against the resulting buckling load.



*Figure 5.15 Similar to figure 5.14, this figure illustrates  $k_1$  and  $k_3$  stiffness increase in relation to the load bearing capacity of the column.*

In the figure 5.15 above it can be seen that the lowest buckling load for respective correlation have a larger value than the previous case, due to set up of the braces. The increasing rate of the column's load bearing capacity is also isn't as fast as the previous case. The curves for respective object are stagnating in a higher pace. The  $k_2$  brace with the lowest stiffness value has a really low load bearing capacity, which can be due to that the brace stiffness isn't enough to make the column change buckling mode. It becomes present that the stiffness of the middle brace at least needs to be able to withstand a buckling mode shift.

#### 5.3.4 Analysis of results

The result of the correlation between the load bearing capacity of the column and the brace stiffness was anticipated by the authors. As the stiffness of the brace gets more robust, the column will be able to take on more force. But as the study shows there is a limit on how much the load bearing capacity can be increased.

In comparison to Yura's and Helwig's model, the brace stiffness in Eurocode isn't as absolute. Instead it's varying due to the chose one makes as their  $k_s$  factor. When  $k_s = 4$ , Eurocode's minimum stiffness requirement becomes larger than Yura's for slimmer designs. The authors think it's reasonable since Yura uses an ideal case and Eurocode does not. The interesting part though is when the cross-section causes the overall slenderness to become smaller, Yura's ideal case gets a larger stiffness then Eurocode. It can be explained as when the cross section causes the overall slenderness to become smaller, the columns failure behavior changes with it. The column will get a higher tendency to have a failure caused by the material capacity being reached, than the risk of failure due to column buckling.

As shown in figure 5.12 the use of several braces will lead to an increase of load bearing capacity for the column. This was something the authors anticipated, since the effective length of the column will become shorter.

In the study of multiple braces, the placements of the braces were always placed in the ideal position to acquire the most profitable  $L_{eff}$ . In this type of scenario, the findings shows that a column with three braces will have a better strategic placement than a column with two braces if only one brace has a larger stiff, due to its positioning of the middle brace.

The results show that if a brace in the middle has a larger stiffness than the outer braces (in the three brace case), the load bearing capacity can be increased further if it were the other way around. However, in the case with two braces it doesn't exist a more beneficial situation which is as distinguishable as in the case with three braces. Since the two braces has the same influence on the column.

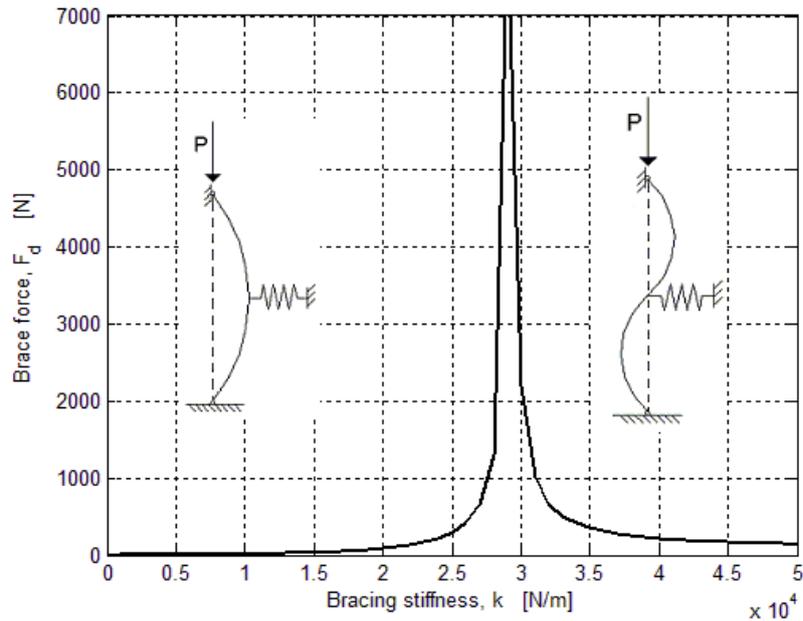
## 5.4 Brace force

In this chapter a study is preformed of how the reaction forces are affected by the correlation between the acting load on the column and the stiffness of the brace. The results are presented as a comparison between Yura's & Helwig's mathematical expression found in equation 2.15 and a numerical analysis.

### 5.4.1 Yura's & Helwig's model

The reaction force in the brace due to initial curvature defined by Yura & Helwig is illustrated in figure 5.16 below,

This is done be varying the critical column force on the column and the stiffness of the brace where the force relation is kept to  $p/p_{cr} = 1$ . Only case A is used during this evaluation. The initial curvature was set to  $d_0 = 2$  mm

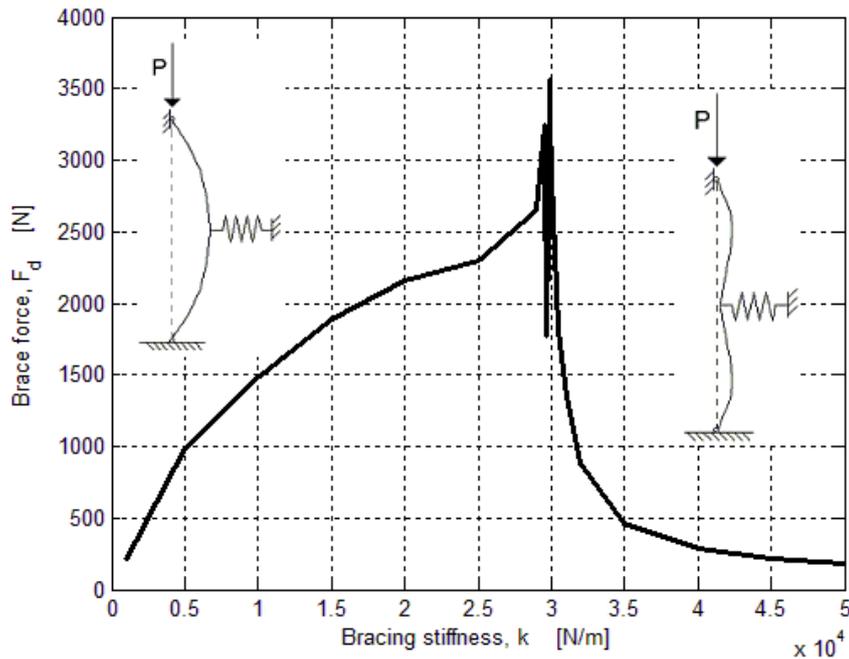


**Figure 5.16** Brace force - Bracing stiffness, by means of varying stiffness and load acting on the column. When the stiffness hits the threshold value the brace force goes towards infinity. But this is not shown in the figure due to limitations of the software.

In the figure 5.16 it can be seen that the brace force gets larger as the stiffness of the brace and the force acting at the column gets larger. This correlation is applied until the point where the stiffness of the bracing has reached the ideal stiffness. When the ideal stiffness for the column is reached, the brace force goes to the infinite. This is the point where the column goes from the first buckling mode to the second buckling mode. Thereafter the reaction force in the spring will decrease, as the stiffness of the bracing is increased mean while the load acting at the column is constant.

#### 5.4.2 Brace force due to imperfections

To study how the brace is dependent on different variables and verify Yura's and Helwig's equation 2.15 a numerical analysis is made. In this model the stiffness of the brace and the force that acts on the column are changed and analyzed for each step. Where the force has a relation of  $p/p_{cr} = 1$ . For more information about the procedure see section 4.5.5. During this procedure the initial imperfection was set to  $d_0 = 2$  mm. Results is displayed in figure 5.17 below

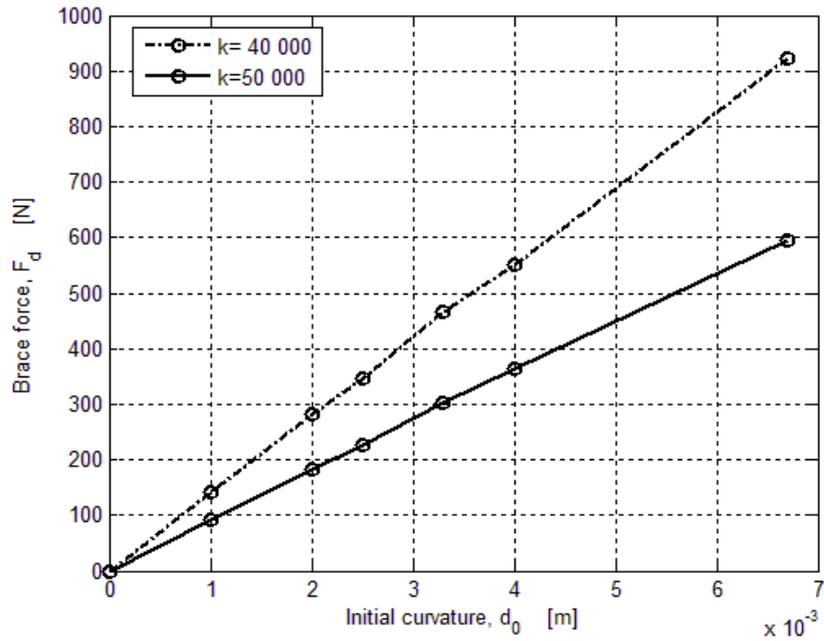


**Figure 5.17** Brace force - Bracing stiffness, by means of varying stiffness and load acting on the column

Out of figure 5.17 it can be seen that the brace force is increasing as the stiffness of the bracing and the load acting at the column gets larger. The reaction force is increasing until it reaches the region surrounding the required stiffness. In the surrounding region of the required stiffness the reaction force becomes unstable. It both shows high and low reaction forces in the brace. When the stiffness is later increased and passes the region, the resulting reaction force in the brace decreases.

#### 5.4.3 Influence of initial curvature for the brace force

In the next study the influence of the initial curvature for the brace force is studied. The result is presented in figure 5.18 below. In the model every variable, except from the initial curvature is constant.



*Figure 5.18 Brace force - Initial curvature*

The result of the initial curvature evaluation shows a linear behavior between the brace force due to increased initial curvature. In figure 5.18 it is shown that a larger initial curvature leads to a higher brace force. It also shows that higher brace stiffness affects the brace force to be smaller.

#### 5.4.4 Analysis of results

The numerical analysis seen in figure 5.17 partially confirms the mathematical modeling seen in figure 5.16. To clarify the differences further the two figures are plotted close together in figure 5.19 below,

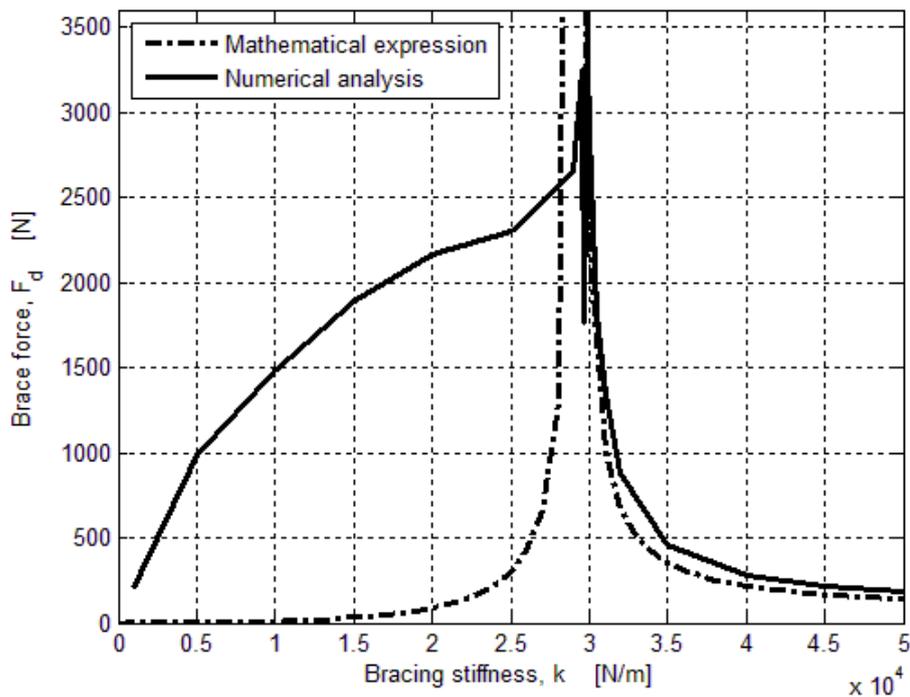


Figure 5.19 Comparison between Yura's & Helwig's model and the results from the numerical analysis

By comparing the two methods it can be seen in figure 5.19 that the behavior in the respective models is similar, after and close to the required stiffness region. However it exist a difference in how the brace force is increasing at stiffness values beneath the ideal stiffness. The brace force is larger and increases in a higher pace than the mathematical model, until the stiffness of the brace reaches the ideal stiffness. This is the point where the force goes to the infinity in the mathematical model, in contrast to the numerical analysis model where the reaction force gets unstable. After the required stiffness region, the numerical analysis shows as mentioned before similarities to the behavior of the Yura's & Helwig's model where the reacting force decreases rapidly. The authors believe that this indicates that Yura's & Helwig's mathematical formulation is only valid for stiffness values above  $k_{required}$ , due to the shown behavior in figure 5.19 above.

In the study of the initial curvature in correlation to the brace force it can be seen that the relation is linear and dependent on stiffness. It also shows that a larger imperfection leads to a larger reaction force in the brace.

## 6 Discussion and Conclusions

*In this chapter the authors present the conclusions of the findings from the result and analysis part in the previous chapter in this report.*

### **Comparison between Eurocode and the U.S. Building code**

In the start the main objective in this comparison was to study and clarify the difference between Eurocode and the U.S. building code. But as the process went on, it was made clear for the authors that the both standards couldn't be compared due to the design of respective standard.

The main difference between the two is that Eurocode uses the Ultimate Limite State design (ULS) approach and that the U.S. building code is in use of allowable stresses. The result of the two standards is therefore not comparable, as one can see in table 5.1, the load bearing capacity of the two standards differs a lot. This information was unfortunately revealed late for the authors.

If one were to analyze the result, one would think that the U.S. building code is more restrictive for slender columns than Eurocode, and that Eurocode is more restrictive as the column gets more robust.

Since time was running short a compromise had to be made, whereas the two building standards were both faced with the same design problem. A comparison between the two revealed that the U.S building code seem to be more restrictive than Eurocode, since Eurocode always have the smaller cross-section of the two.

### **Influence of initial curvature**

In the study it is shown that an initial curvature in a column will affect the overall load bearing capacity. Initial curvature has a negative effect on the column, since it decreases its total load bearing capacity. The reduction is dependent on the how large the magnitude of the initial curvature is. It is shown in the result that a larger initial curvature will lead to larger reduction of the load bearing capacity of the column, which was expected by the authors.

It's not only the load bearing capacity of the column that is affected by the initial curvature. The total deformation and deformation rate is also affected by it. The findings show that the total deformation and the deformation rate are increased, as the magnitude of the initial curvature in the column gets larger.

The results were based on a simplified shape model of the initial curvature, whereas the first buckling mode is the only one to affect the initial curvature. In reality the initial curvature is a coalition of several different buckling modes, therefore the shape and appearance of the initial curvature needs to be readjusted to a more plausible shape.

The findings indicates that if a correlation of the buckling modes is taken into account the total load bearing capacity of the column will decrease compared to the simplified imperfection model. The magnitude of the additional reduction on the load bearing capacity of the column is however dependent on the resulting shape of the imperfection.

The study indicates that when higher tiers of the buckling modes govern the shape of the initial curvature, the overall load bearing capacity will decrease further. The load bearing capacity can with other words vary, dependent on the model one assume. The findings indicate that the most disadvantageous situation for a column occurs when the higher buckling shapes acquire its largest allowed amplitude.

### **Influence of brace stiffness**

It is shown in the study that the stiffness of a brace has a contribution to the total load bearing capacity of the column. The contribution is in an ideal column case proportional to the increase in stiffness. Even though the increase is small, the increase is always present as long as the stiffness is above zero. However, as the results shows there is a limit on how much the load bearing capacity can be increased. As the stiffness of the brace reaches the threshold value  $k_{ideal}$  a new buckling shape occurs, which will govern the load bearing capacity. The load bearing capacity cannot thereafter be increased further.

By using additional braces the results shows that the column's load bearing capacity can be increased further. Dependent on the total amount of braces that are in use, the increase of load bearing capacity will vary. However there exists a limit/ where it isn't profitable to add more braces. When this limit is reached the material capacity of the column will govern the failure.

The study was continued with comparing the influence on a varying brace stiffness of multiple braces, As mentioned before, the study indicates that an increase of brace stiffness will result in a larger load bearing capacity. However as the stiffness of respective brace is varied, the findings shows that there may exist a more favorable correlation between the braces, if one is restricted.

But something the authors found was interesting was how a varying stiffness of respective brace influenced on the total load bearing capacity for the column. It is revealed in the result that even though a more robust brace lead to an increase in load bearing capacity for the column, there are good things to keep in mind when designing the braces. It appears to exist strategically places where one brace is more suitable than two/three less strategically placed braces, when one has insufficient brace stiffness. A rule of thumb should be to minimize the effective length of the column as much as possible. As in the case with three braces, it is for example smarter to put a stiffer brace in the midpoint of the column with two supporting braces at the sides.

The design guidance for a column's brace stiffness in Eurocode isn't as absolute, as in Yura's and Helwig's model. As shown in the results, the brace stiffness can vary due the  $k_s$  factor one can choose in Eurocode. The study has shown that a smaller stiffness contributes to a larger load bearing capacity. Eurocode does not calculate stiffness contribution per say, it checks if the stiffness of the brace is rigid enough to be considered as a lateral support or not. The authors think Eurocode should therefore be reevaluated in order to take into account contributions from braces with a smaller stiffness.

### **Reaction force in a brace**

This study shows that the brace force increases proportionally to the columns imperfection, due to its dependence on the columns overall deformation. This correlation does only coincide as the brace stiffness goes towards the required stiffness of the column, to make it

change its buckling shape. As the brace stiffness either become lesser or larger than the required stiffness value, the brace force will decrease.

In the findings it is also shown that the numerical analysis has a large similarity to Yura's and Helwig's mathematical model. However Yura's and Helwig's model is questioned, due to its large difference in the area where the brace stiffness values is lower than  $K_{req}$ . The authors believes that the mathematical model of Yura and Helwig is only applicable for describing correlations for a brace force that has brace stiffness larger than  $K_{req}$ .



## 7 References

### Books

Bauchau, O. A., Craig, J. I. (2009), Structural analysis: With Applications to Aerospace Structures. Dordrecht : Springer Netherlands

Breyer, Donald E. (2003). *Design of wood structures ASD*. 5. ed. New York: McGraw-Hill

Burström, Per Gunnar (2007). *Byggnadsmaterial: uppbyggnad, tillverkning och egenskaper*. 2. uppl. Lund: Studentlitteratur

Galambos, Theodore V. & Surovek, Andrea (2008). *Structural stability of steel: concepts and applications for structural engineers*. Hoboken, N.J.: John Wiley & Sons

Helwig, Todd A. & Yura, Joseph A. (1996), *Bracing for stability*, University of Houston and University of Texas at Austin

Isaksson, Tord & Mårtensson, Annika (2010). *Bygghkonstruktion: regel- och formelsamling* : based on Eurokod. 2. ed. Lund: Studentlitteratur

Runesson, Kenneth, Samuelsson, Alf & Wiberg, Nils-Erik (1992).*Byggnadsmekanik - knäckning*. Lund: Studentlitteratur

Stålbyggnadskommité (1973). *Knäckning, vippning, buckling: kommentarer till Stålbyggnadsnorm 70*. Stockholm

Timoshenko, Stephen & Gere, James M. (1961). *Theory of elastic stability*. 2., rev. ed. New York: McGraw-Hill

Trahair, Nicholas S. & Bradford, M. A. (1994). *The behaviour and design of steel structures*. Rev. 2. ed. London: Spon

Winter, G. (1958). Lateral bracing of columns and beams , Proc. ASCE, 84, pp. 561–1561 (ST2)

Yura, J. (1996). Winter's bracing approach revisited. *Engineering Structures* Volume: 18 Issue: 10 (1996-10-01) p. 821 - 825.

### Elektronic available literature:

efunda. 2014, Columns: Introduction. [Electronic] Article

Engineering Fundamentals. Available:

<[http://www.efunda.com/formulae/solid\\_mechanics/columns/intro.cfm](http://www.efunda.com/formulae/solid_mechanics/columns/intro.cfm)> (2014-04-07)

ICC. 2014, Chapter 16: Structural Design. [Electronic] Part of book

International Code Council. Available:

<[http://www2.iccsafe.org/states/newyorkcity/Building/PDFs/Chapter%2016\\_Structural%20Design.pdf](http://www2.iccsafe.org/states/newyorkcity/Building/PDFs/Chapter%2016_Structural%20Design.pdf)>



## **Appendix A – Unit Converter**

*This Appendix is a reference document to convert U.S. customary units into SI- units.*

### **Loads**

$$1 \text{ lb} = 4,448 \text{ N}$$

$$1 \text{ k} = 4,448 \text{ kN}$$

### **Modulus of elasticity and stresses**

$$1 \text{ psi} = 6,895 \text{ kPA}$$