Master Thesis

Evaluating Switching GARCH Volatility Forecasts During the Recent Financial Crisis

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Abstract

Forecasting volatility is a fundamental topic in in both academic and applied financial economics. Different GARCH-specifications are by far the most popular model based approach used for this purpose. This thesis evaluates the forecast accuracy of some specific GARCH-models; GARCH, EGARCH, APGARCH and MRS-GARCH. The primary purpose of the essay is to investigate whether the more flexible two-regime MRS-GARCH model outperforms the more conventional one-regime GARCH models in a very volatile time period during the recent financial crises. The evaluation period stretches from the day when Lehman Brothers went bankrupt and one year ahead. Each model is evaluated using two indexes with different characteristics; the Standard & Poor 500 and the Bombay Sensex index. The result shows that the MRS-GARCH models are superior in predictive ability on S&P500 compared to the other tested models. Conversely, the overall relative performance accuracy of the BSE is less clear-cut since none of the tested models seem to perform particularly well. Generally, the results indicate that the MRS-GARCH provides better forecasts on S&P 500 compared to the other models and that no forecast can be distinguished as entirely superior on the BSE.

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I. Introduction and background

Forecasting market volatility is of great importance in financial economics. Correct predictions of future volatility are crucial to risk managers, asset managers and other financial actors that try to minimize risk and maximize profits. The recent financial crisis emphasized the need of proper predictions in the aftermath of tightened financial regulations and common scepticism towards financial markets. Hence, understanding volatility is not only demanded by regulations but also a necessity to minimize the damage of future crises.

This essay will focus on empirical approaches originating from the Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Engle (1982). The most popular approaches used to model volatility are derived from the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model developed by Bollerslev (1986). The GARCH models are popular since they are both reasonably easy to estimate and perform diagnostic tests on. Except from user-friendliness, the popularity originates from the different models ability to capture characteristics in volatility series like nonlinearity, clustering and asymmetry (Enders, 2010).

The literature about GARCH models and their applications are incredibly comprehensive. Prominent researchers like Bollerslev (1986), Zakoian (1990), Nelson (1991), Higgins and Bera (1992), Harvey et al (1992). Ding, Granger and Engle (1993), Glosten, Jaganathan and Runkle (1993) and Klaassen (2002) have all developed noticeable specifications of the GARCH model. Consequently the large variety of GARCH-specifications can make the choice of model less straightforward. One purpose of this thesis is to elucidate the present literature by evaluating the forecast accuracy of some specific GARCH-models. The considered GARCH-models contain three of the most popular specifications and one less conventional.

To assess whether any of the more parameterized GARCH specifications increases the performance of the traditional model, the standard GARCH by Bollerslev (1986) is included in the study. Despite the GARCH models ability to capture volatility clustering and nonlinearity it has some flaws. One of those weaknesses is the ability to capture asymmetric movements in stock returns, i.e., more extensive movements to negative news than positive news. Over the years, there have been many interesting attempts to control for these asymmetric effects commonly referred to as leverage effects. This thesis evaluates two of the most influential and useful specifications built to control for asymmetric movements, namely the Exponential-GARCH (EGARCH) by Nelson (1991) and the Asymmetric Power GARCH (APGARCH) by Ding, Granger and Engle (1993). Both the EGARCH and the APGARCH models have several nice properties that makes them interesting. For instance, the output of the APGARCH model nests other well-renowned GARCH-specifications like the model of Glosten, Jaganathan and Runkle (1993) (GJR-GARCH) and the Threshold-GARCH (T-GARCH) by Zakoian (1990). Hence, by evaluating the APGARCH, the appropriateness of several other GARCH specifications are indirectly performed (He, Malmsten and Teräsvirta, 2008).

Despite the extensive variety of GARCH specifications, most of the models seem to be excessively persistent, i.e., react too slowly to movements of the market. It seems like the conditional dependency of the GARCH models helps the model to account for volatility clustering but at the same time it decreases the adaptability to shifts in stock movements (Lamoureaux and Lastrapes, 1990). Volatility series suffer from shifts that are caused by structural changes but also due to changed expectations of the market-participants. For example, the terms "Hausse" and "Baisse" refers to states with large movements of return series that causes these shifts. "Hausse" refers to rapidly increasing stock movements and "Baisse" the opposite. Both these situations are subject to periods with large variance that can be modelled as high-variance regimes. Hence, the situation where neither one of them occurs can be considered as a low-variance regime. Incorporating regimes or states in a GARCH model makes its mean-reversion state dependent. Thus, how quick the variance will get back to its long-run average will vary between the regimes. Given that there exists more than one state, a multi-state model will always be more flexible since a single-state model's parameters only represent the average mean-reversion of the states. Hence, including regimes in a GARCH framework are therefore likely to yield better estimates of the persistence and is therefore of interest (Alexander and Lazar, 2009).

Hamilton and Susmel (1994) introduced a way of modelling volatility with different states when they combined Hamiltons (1989) Markov Switching Regression with the ARCH model and introduced SWARCH. By letting volatility jump between different regimes with certain probabilities a new more flexible way of estimating volatility was born (Teräsvirta, 2006). Sprung from the SWARCH-model, a generalization soon came, i.e., the Markov Regime-Switching GARCH (MRS-GARCH) developed by Gray (1996) and Klaassen (2002). Marcucci (2005) proved that MRS-GARCH yielded a superior forecast at a short horizon on the S&P 100 index compared to the GARCH, EGARCH and GJR-GARCH. Despite the seemingly nice properties, the literature about the MRS-GARCH and its capacity is quite narrow.

This thesis aims to carry on the work of Marcucci (2005) by investigating the efficiency of the GARCH, EGARCH, APGARCH and MRS-GARCH on S&P 500 but also on the Bombay Sensex index (BSE). BSE is a more volatile index with different characteristics than S&P500. To assess the effectiveness of the different types of GARCH-models, an out of sample forecast evaluation is performed starting from the first trading day after the Bankruptcy of Lehman brothers in 2008. Since the GARCH, EGARCH and APGARCH are more acknowledged models the contribution of this thesis is foremost the evaluation of the MRS-GARCH during the financial crisis. The thesis thereby returns an answer to weather the MRS-GARCH successfully captures the characteristics of the two indexes during this erratic time-period compared to the considered single regime models.

The thesis is outlined as follows: In chapter II the evaluated models are presented. Chapter III covers the data and methodology. Chapter IV discusses the framework of forecast evaluation and loss functions used in the thesis. Chapter V presents the in-sample results and Chapter VI the results of the forecast evaluation. Finally in chapter VII the conclusions are presented.

II. GARCH-models

i) Single Regime GARCH-models

The GARCH (1,1)-model with a constant mean equation can then be written,

$$r_t = a + \eta_t h_t \tag{1}$$

with the conditional variance given by

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \tag{2}$$

and $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, which guarantees a positive conditional variance estimate. The conditional variance is useful since economic time series often violates the assumption of homoscedasticity and the variance often seems to be dependent of its recent lags. As previously discussed there are some common problems with the GARCH specification. One commonly discussed issue is that the GARCH model is too persistent, meaning that it doesn't react fast enough to changes. The persistence of a GARCH (1,1) model is calculated by summing the ARCH and GARCH parameter, α_1+β_1.

Moreover, the EGARCH was built to explain financial returns known tendency to react differently to news depending on whether they are positive or negative. Nelson (1991) specified the EGARCH with the logarithm of the conditional variance to ensure a positive measure without any constraints. The model standardises ε_{t-1} which according to Nelson

(1991) allows for a more natural interpretation of the size and persistence of shocks. Additionally the asymmetry parameter ξ captures the leverage effect. The EGARCH are defined:

$$\log(h_t^2) = \alpha_0 + \alpha_1 \left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \xi\left(\frac{\varepsilon_{t-1}}{h_{t-1}}\right) + \beta_1 \log(h_{t-1}^2) \tag{3}$$

The APGARCH's by Ding, Granger and Engle (1993) way of controlling for leverage effects resembles the EGARCH in many ways. There is no standardisation or logarithm of the conditional variance in the APGARCH but the asymmetry parameter is still given by ξ. The model also allows the power δ of the heteroskedasticity equation to be estimated from the data. The APGARCH model is specified as,

$$h_t^\delta = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \xi\varepsilon_{t-1})^\delta + \beta_1(h_{t-1})^\delta \tag{4}$$

It is noticeable that the APGARCH under certain circumstances will yield other GARCH specifications. The taxonomy of nested ARCH specifications adapted from McKenzie and Mitchell (2002) in the APGARCH model is presented in table 1.

Table 1. Taxanomy of the Asymmetric Power GARCH

Model	$\boldsymbol{\delta}$	$\boldsymbol{\alpha_1}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\xi}$
ARCH	2	free	0	0
GARCH	2	free	free	0
Leverage ARCH	2	free	0	$\lvert\xi\rvert \leq 1$
Leverage GARCH	2	free	free	$\lvert\xi\rvert \leq 1$
GJR-ARCH	2	$\alpha_1(1+\xi)^2$	0	$-4\alpha_1\xi$
GJR-GARCH	2	$\alpha_1(1+\xi)^2$	free	$-4\alpha_1\xi$
Taylor ARCH	1	free	0	0
Taylor GARCH	1	free	free	0
TARCH	1	free	0	$\lvert\xi\rvert \leq 1$
Generalized TARCH	1	free	free	$\lvert\xi\rvert \leq 1$
NARCH	free	free	0	0
Power GARCH	free	free	free	0
Asymmetric Power ARCH	free	free	0	$\lvert\xi\rvert \leq 1$
Asymmetric Power GARCH	free	free	free	$\lvert\xi\rvert \leq 1$

The one day ahead forecasts of the Single-Regime GARCH models are obtained by $\hat{h}_{t+1}^2$ which only are directly dependent on the values from the present time period. For example, the forecast of the GARCH (1,1) is calculated by

$$\hat{h}_{t+1}^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_t^2 \qquad (5)$$

All single-regime GARCH models are estimated using maximum likelihood in Eviews. Since an out of sample evaluation is performed, one estimation is made of each time period in the out-sample. The set of log likelihood functions are thereby given by

$$\ell_w = \sum_{t=-D+1+w}^{T+w} log[f(\theta_w)] \qquad (6)$$

where $w = \{0,1,\dots,n\}$ and D is the chosen number of trading days considered for the in-sample. A rolling window of log likelihood functions yielding out-sample estimates are thereby created. Hence, the appealing name of the first forecast, i.e., $\hat{h}_{t+1,1}^2$ is retrieved by choosing $T = 0$. Thus, if $w = 0$ we obtain the in-sample estimate and with $w = 1$ we obtain the first out-of sample estimate and so on

ii) Markov Regime-Switching GARCH

While previous research assumed asymmetry in the volatility series Hamilton and Susmel (1994) suggested that volatility could be considered as regime-switching. In their SWARCH-model, persistent jumps in the volatility series are defined as switches between regressions. The different regimes or states of the world are administrated by a state variable s_t that affects the probability of shifting to another regime. The evolution of the discrete state variable s_t is said to follow an i^{th} order Markov-chain meaning that s_t is assumed to be dependent solely on the i^{th} previous states.

The SWARCH model became influential but the known problems of long lag structures remained from the original ARCH-model. It is however problematic to include a state-dependent conditional variance term in a regime-switching ARCH. Generalizing the ARCH process within a regime switching context requires integration over unobserved regime paths that increases exponentially with sample size and makes estimation intractable, (Klaassen, 2002). To avoid the conditional variance to be a function of all previous states, several estimation techniques were suggested. Gray's (1996) model was the first to generalize the model into a Switching regression GARCH but his model was unable to forecast multiple periods forward. The Markov Regime Switching GARCH used in this thesis was developed by Klassen (2002) and is an improved version of Gray's (1996) that allows for multiple period forecasts.

In the framework of MRS-GARCH it is assumed that there exists a state variable, which evolves according to a first-order Markov chain with transition probabilities defined as

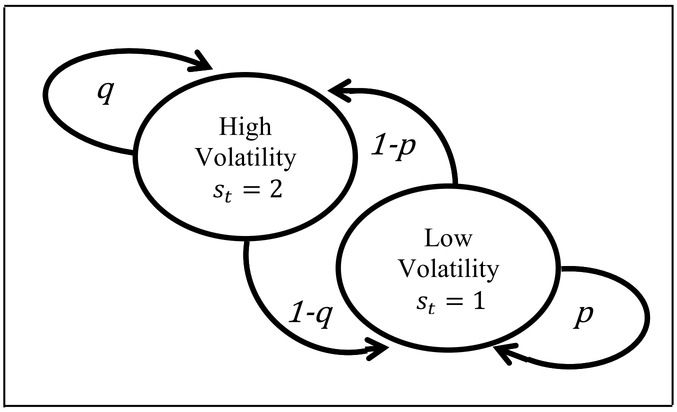
$$\Pr(s_t = j | s_{t-1} = i) = p_{i,j} \tag{7}$$

The state variable s_t gives the probability of switching from state i at time $t-1$ into state j at t, which grouped together form the transition matrix. The transition matrix in (8) is based on a two state Markov chain like the one depicted below in figure 1 which means that $i = 1$ and $j = 2$

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p & (1-q) \\ (1-p) & q \end{bmatrix} \tag{8}$$

Volatility that follows a two state Markov-chain can then be displayed as in Figure 1.

Figure 1. Two State Volatility Markov Chain



The unconditional probability or ergodic probability of $s_t = 1$ is given by $\pi_1 = \frac{(1-p)}{(2-p-q)}$

The MRS-GARCH with two regimes expressed in a generalized form is then given by

$$r_t | \varsigma_{t-1} \sim \begin{cases} f\left(\theta_t^{(1)}\right) \text{ with probability, } p_{1,t} \\ f\left(\theta_t^{(2)}\right) \text{ with probability, } \left(1 - p_{1,t}\right) \end{cases} \tag{9}$$

where the density function $f\left(\theta_t^{(i)}\right)$ displays the assumed conditional distributions of the two regimes. The ex-ante probability, $p_{1,t}$ gives the probability of being in the first regime given all available information at time $t-1$. The ex-ante probability is dependent on $\varsigma_{t-1} = \{r_{t-1}, r_{t-2}, \dots, r_{t-n}\}$, which is the information set at $t-1$ inferred by all observed variables at $t-1$, i.e., the sigma algebra. A more thorough formula of the ex-ante probability is given in equation (18) provided below. For now it is sufficient to know that the ex-ante probability encompasses

$$p_{1,t} = \Pr[s_t = 1|\varsigma_{t-1}] \tag{10}$$

Furthermore, the vector $\theta_t^{(i)}$ denotes time-varying parameters of the i^{th} regime, which can be divided into three elements. That is, the conditional mean, conditional variance and the shape parameter of the conditional distribution, i.e.

$$\theta_t^{(i)} = (\mu_t^{(i)}, h_t^{2|(i)}, \nu_t^{(i)}) \tag{11}$$

Thus, the MRS-GARCH can be divided into four elements: the conditional mean, conditional variance, regime process and the conditional distribution. The conditional mean equation modelled as an AR(0)-process is given by,

$$r_t = \mu_t^{(i)} + \varepsilon_t = \delta^{(i)} + \varepsilon_t \tag{12}$$

with the conditional mean $\mu_t^{(i)}$ defined as

$$\mu_t^{(i)} \equiv E[r_t|\varsigma_{t-1}] \tag{13}$$

and

$$\varepsilon_t = \eta_t h_t \tag{14}$$

Where η_t is a zero mean and unit variance process and the conditional variance is defined

$$h_t^{2|(i)} \equiv Var[r_t|\varsigma_{t-1}] \tag{15}$$

and the conditional variance equation in the MRS-GARCH (1,1) framework is then expressed

$$h_t^{2|(i)} = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)} h_t^{2|(i)} \tag{16}$$

The one step ahead forecast of the MRS-GARCH is then estimated as the sum of the potential expected conditional variances under each regime weighted by the ex-ante probability given in (18). Hence the one step ahead forecast is then calculated by

$$\hat{h}_{T,T+1}^{2|(i)} = \Pr(s_t = 1|\varsigma_{t-1})\left(\alpha_0^{(1)} + \alpha_1^{(1)}\varepsilon_t^2 + \beta_1^{(1)} h_t^{2|(1)}\right) + \Pr(s_t = 2|\varsigma_{t-1})\left(\alpha_0^{(2)} + \alpha_1^{(2)}\varepsilon_t^2 + \beta_1^{(2)} h_t^{2|(2)}\right) \tag{17}$$

The ex-ante probability $p_{1,t}$ i.e. the probability of being in the first regime at time t given the information at time $t-1$, with the specification from Hamilton (1989) is given by

$$\Pr[s_t = 1|\varsigma_{t-1}] = (1-q)\left[\frac{f(r_{t-1}|s_{t-1}=2)(1-p_{1,t-1})}{f(r_{t-1}|s_{t-1}=1)p_{1,t-1} + f(r_{t-1}|s_{t-1}=2)(1-p_{1,t-1})}\right] + p\left[\frac{f(r_{t-1}|s_{t-1}=1)p_{1,t-1}}{f(r_{t-1}|s_{t-1}=1)p_{1,t-1} + f(r_{t-1}|s_{t-1}=2)(1-p_{1,t-1})}\right] \tag{18}$$

Where p and q are the transition probabilities and $f(\cdot)$ is the density functions in (9). The set of log-likelihood functions is finally given by

$$\ell = \sum_{t=-D+1+w}^{T+w} log\left[p_{1,t} f(r_t|s_t = 1) + (1 - p_{1,t}) f(r_t|s_t = 2)\right] \qquad (19)$$

where $w = \{0,1, \dots, n\}$, D is the number of trading days considered for the in-sample and $f(\cdot\,|s_t = i)$ is the conditional distribution given that regime i occurs at time t, (Marcucci, 2005). A more in-depth derivation of the MRS-GARCH is found in the appendix.

III. Data and Methodology

The data used to estimate both the single regime and multi regime GARCH models in this thesis consists of the daily rate of return of the S&P 500 and BSE. The calculations of the "true volatility" used to evaluate the performance of the models are except from the closing price based on the intra-daily extreme values of the stock returns. The total sample consist of observations that stretches from September 1, 1997 to September 15, 2009 which due to different holidays yields 3072 observations from the S&P 500 and 3022 from BSE. The rate of return is specified as

$$r_t = 100(\log(p_t) - \log(p_{t-1})) \qquad (20)$$

where p_t is the closing price of the selected stock market index at time t. The time index of r_t are then divided into two subsamples, specifically an in-sample and out-sample. The total sample stretches from $t = \{-D + 1, -D + 2, \dots, n\}$, where D is the chosen number of trading days considered for the in-sample and n the total number of considered returns in the out-sample. The in-sample are defined by $t = \{-D + 1, -D + 2, \dots, 0\}$ and the out-sample with $t = \{1,2, \dots, n\}$. The in-sample covers a sample period from September 1, 1997 to September 12, 2008 and the out-sample stretches from September 15, 2008 to September 15, 2009.

The in-sample yields the estimated parameters and goodness of fit presented in table 3,4,5 and 6 while the out-sample is used to produce the forecast series presented in diagram 1 and 2. The forecast series are constructed as a one day ahead forecast corresponding to each trading day given in the out-sample. Each forecast is estimated with a rolling window of observations corresponding to the number of observations defined by the in-sample. In other words, the first forecast, $\hat{h}_{t+1,1}$ will be obtained from the in-sample i.e. $t = \{-D + 1, \dots, 0\}$, the second forecast $\hat{h}_{t+1,2}$ will be obtained with $t = \{-D + 2, \dots, 1\}$ and so on. The in-sample of the S&P 500 and BSE consists of 2819 and 2779 observations respectively. Consequently, the out-sample consists of the remaining 253 observations of the S&P 500 and 243 for BSE.

Descriptive statistics of the rate of return from the two indexes are summarized in table 2

Table 2. Sample properties of the rate of return on Standard and Poor's 500 and Bombay Sensex index

	μ	σ	Kurtosis	Skewness	Min	Max	ARCH(12)	$Q^2(12)$	Jarque-Bera
S&P 500	0.0054	1.3757	10.2555	-0.1567	-9.4695	10.9572	95.5611	2871.80	6748.595
BSE	0.0444	1.8004	7.9419	-0.1013	-11.8536	15.9456	25.1191	627.73	3079.382

The statistics are calculated based on the whole sample. The ARCH (12) and $Q^2(12)$ is Engle's ARCH test and Ljung-Box-Q-test calculated from the squared residuals of r_t regressed on a constant. As expected, both Engle's ARCH test and the Ljung-Box-Q-test indicate ARCH-effects up to the 12^{th} lag of both the indexes. The kurtosis of both BSE and S&P 500 are significantly higher than 3 which is the kurtosis of the Gaussian distribution. Thus, a fat tailed distribution is likely to suit the data better. There is also presence of negative skewness which indicates that negative returns often are more below the average than positive returns are above. Furthermore the null of normality is rejected in the Jarque-Bera test. Accordingly, the data of both indexes seems to exhibit of leverage effects and follow some leptokurtic distribution. There is however interesting differences in the sample of the considered indexes that supports the choice of additionally evaluating BSE. The leverage effects seem to be more extensive in S&P500 than in BSE which indicates a more leptokurtic distribution. The average standard deviation of BSE is also approximately 30% higher than S&P500 during the considered time span.

Both the standard GARCH and the two asymmetric GARCH models are estimated using the regular maximum likelihood and the MRS-GARCH is estimated using quasi-maximum likelihood. Either way, the logarithm of the likelihood function is maximized and both the conditional mean and variance are jointly estimated. The estimation procedure of the MRS-GARCH is conducted in MATLAB using the code from Juri Marcucci's (2005) awarded essay and the single regime GARCH models are estimated in Eviews. The optimization of the likelihood function in the MRS-GARCH is derived by Broyden, Fetcher, Goldfarb, and Shanno's (BFGS) quasi-Newton numerical optimization algorithm and the single regime models uses Marquardts optimization algorithm.

To evaluate the performance of the models a comparison is made between the forecasts of the different GARCH specifications and the "true" volatility. However, estimating the "true" daily volatility is not as straightforward as it might seem. This thesis therefore adopts two different measures of the "true volatility". The classical volatility estimator used is called the Close-Close Volatility Estimator. The Close-Close Volatility Estimator is simply calculated by

$$\sigma_{cc} = \sqrt{252\frac{1}{n}\sum_{1=1}^{n} r_i^2} \quad (21)$$

This is the fundamental historical volatility estimator which often is used due to its simplicity. Andersen and Bollerslev (1998) pointed out that the squared returns can be an inaccurate measure and that a more precise measurement can be derived using intra-daily data. One commonly used measure is the realized volatility measure developed by Koopman, Jungbacker and Hol (2004) which is calculated with intra daily stock prices sorted into 5-minutes intervals. However, there is very hard to access intra-daily data with the required frequencies. This thesis therefore uses an extreme value estimate of the returns that only need the daily high and daily low of each trading day. This technique is called the Realized Range Estimator and was originally developed by Parkinson (1980). Andersson and Bollerslev (1998) pointed out that the realized range estimator of daily volatility by Parkinson (1980) performs as well as realized volatility calculated from intra-daily data with intervals between 2-3 hours. The realized range estimator is an extreme value estimator that uses the differences between the daily high H_t and daily low L_t and is defined,

$$\sigma_{HL} = \sqrt{\frac{1}{4\ln(2)}\frac{252}{n}\sum_{t=1}^{n} ln\left(\frac{Ht}{Lt}\right)^2} \quad (22)$$

This estimate is proved by Parkinson (1980) to be approximately five times more efficient than the Close-Close Estimator. Hence, this thesis uses the Realized Range estimator to determine the forecast accuracy. The Close-Close proxy is foremost used to graphically illustrate the differences among the volatility proxies.

IV. Loss functions and Forecast Evaluation

The forecasting performances of the models used in this thesis are evaluated by specific statistical loss functions. The most popular way of determining the performance of volatility forecast in present literature is to measure the Mean Squared Error, i.e., MSE_i. Evidently, the model that performs the best is the one that yields the lowest value of MSE_i. However, the MSE_i is rather criticised and there is a lot of loss functions that are argued as better choices. Unfortunately it does not exist a superior loss function that alone provides sufficient information of how accurate the models are compared to each other. The criticism towards evaluating forecast performances are foremost derived from the difficulties of choosing appropriate loss functions. This thesis adopts the evaluating framework of Marcucci's (2005) thesis. Instead of focusing on a particular loss function that the researcher claim to be superior to the others a battery of loss functions are chosen. The certain statistical loss functions are:

$$MSE_1 = n^{-1}\sum_{t=1}^{n}\left(\hat{\sigma}_{t+1} - \hat{h}_{t+1|t}\right)^2 \tag{23}$$

$$MSE_2 = n^{-1}\sum_{t=1}^{n}\left(\hat{\sigma}_{t+1}^2 - \hat{h}_{t+1|t}^2\right)^2 \tag{24}$$

$$QLIKE = n^{-1}\sum_{t=1}^{n}\left(\log(\hat{h}_{t+1|t}^2) + \hat{\sigma}_{t+1}^2\hat{h}_{t+1|t}^{-2}\right) \tag{25}$$

$$R2LOG = n^{-1}\sum_{t=1}^{n}\left(\log\left(\hat{\sigma}_{t+1}^2\hat{h}_{t+1|t}^{-2}\right)\right)^2 \tag{26}$$

$$MAD_1 = n^{-1}\sum_{t=1}^{n}\left|\hat{\sigma}_{t+1} - \hat{h}_{t+1|t}\right| \tag{27}$$

$$MAD_2 = n^{-1}\sum_{t=1}^{n}\left|\hat{\sigma}_{t+1}^2 - \hat{h}_{t+1|t}^2\right| \tag{28}$$

$$HMSE = T^{-1}\sum_{t=1}^{T}\left(\hat{\sigma}_{t+1}^2\hat{h}_{t+1|t}^{-2} - 1\right)^2 \tag{29}$$

The equations given in (23) and (24) yields the mean squared error previously discussed. The metric found in (24) and (26) differ only since the logarithm of the parameters is used in the latter. They both equal the R^2 values of regressing a constant and the forecasted variance $\hat{h}_{t+1|t}^2$ on the actual volatility from the same time period $\hat{\sigma}_{t+1}^2$. As long as the forecasts are unbiased this is the Mincer-Zarnovitz (1969) regression. The metric in (26) originally suggested by Pagan and Schwert's (1990) has the ability of punishing forecast errors in low volatility periods more extensive than in high volatility periods. The $R2LOG$ therefore penalizes the models accordingly with the leverage effect. The $QLIKE$ found in (25) is similar to the $R2LOG$ in a sense since it also punishes forecasts that underestimate volatility more heavily. The loss function was originally suggested by Bollerslev, Engle and Nelson (1994) and retrieves the standardised forecasts errors centred around 1 given that $\hat{h}_{t+1|t}^2 \approx \hat{\sigma}_{t+1}^2$. The $QLIKE$ therefore returns the loss of a Guassian likelihood and are less sensitive to the largest variations among the observations and is therefore more robust (Hansen and Lunde, 2001). The loss functions given in (27) and (28), i.e., the mean absolute deviation MAD_i are argued to be more robust than the MSE_i under the presence of outliers. Finally the last equation (29) suggested by Bollerslev and Ghysel (1994) adjusts for heteroskedasticity in the MSE. The $HMSE$ is useful since MSE_i may return a defective value if there is presence of heteroskedasticity in the forecast errors, (Marcucci, 2005).

Intuitively, the loss functions provide the researcher with information about how reasonable the performances of the models are. The model which yields the smallest value of the loss functions is the one that performed the best. Nevertheless, there is a high possibility that different model returns good results on different loss functions or that some models seem to perform equally good. To be able to determine whether a forecast is significantly better than another one, Diebold-Mariano's (1995) (DM) test of superior predictive ability can be applied. Taking the difference between two loss functions yields the series d_t with average $\bar{d}$, which is equal to zero under the null of no difference between the forecasts. The DM-test statistic is calculated as follows

$$DM = \frac{\bar{d}}{\sqrt{\widehat{Var}(\bar{d})}} \sim N(0,1) \tag{30}$$

which has an asymptotically standard normal distribution with $H_0\colon E(d_t) = 0\ \forall t$ and $H_1\colon E(d_t) \neq 0\ \forall t$. However it is claimed that the DM test can be over-sized and reject the null too often. This is especially true for small sample sizes and long forecast horizons. Harvey, Leybourne and Newbold (1997) therefore introduced the Modified DM test which basically multiplies the DM statistic with

$$\sqrt{\frac{n+1-2h+\frac{h(h-1)}{n}}{n}} \geq 0 \tag{31}$$

This thesis only compares series with short forecasts horizons (one-day ahead), and the out-sample being evaluated contains 253 or 243 observations which are reasonably large. Nevertheless, both the MDM and DM statistic are calculated to get as much information as possible (Marcucci, 2005). A more thorough description about the DM-test can be found in the appendix.

V. In-sample Estimates

The estimates of the two indexes from the different single state GARCH specifications are presented in table 3 and 4 and the results from the Markov Switching GARCH in table 5 and 6. Each model is estimated from both the considered indexes with an in-sample period from September 1, 1997 to September 12, 2008. Each model is also estimated under three different distributional assumptions, namely the normal, student's t and the GED –distribution. The tables display the parameter values, significance level and standard errors and in-sample goodness of fit statistics are disregarded. This is because the purpose of the thesis foremost is to evaluate the predictive ability of the models.

i) Single Regime GARCH

- **S&P 500**

The in-sample results from S&P500 of the single regime GARCH models are given in table 3.

Table 3: Estimates of Standard GARCH models with different conditional distributions on Standard & Poor's 500 index

	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	APGARCH-N	APGARCH-t	APGARCH-GED
a	0.0377** (0.0180)	0.0463*** (0.0167)	0.0526*** (0.0165)	-0.0023 (0.0171)	0.0128 (0.0164)	0.0177 (0.0163)	-0.0016 (0.0174)	-0.0146 (0.0165)	-0.0189 (0.0165)
α_0	0.0121*** (0.0020)	0.0071** (0.0028)	0.0085*** (0.0027)	-0.0706*** (0.0102)	-0.0673*** (0.0119)	-0.0693*** (0.0123)	0.0205*** (0.0027)	0.0138*** (0.0032)	0.0159*** (0.0033)
α_1	0.0736*** (0.0067)	0.0655*** (0.0090)	0.0674*** (0.0092)	0.0926*** (0.0126)	0.0854*** (0.0151)	0.0875*** (0.0157)	0.0647*** (0.0113)	0.0582 (0.0690)	0.0608*** (0.0272)
β_1	0.9198*** (0.0075)	0.9318*** (0.0090)	0.9288*** (0.0096)	0.9807*** (0.0024)	0.9857*** (0.0026)	0.9845*** (0.0027)	0.9305*** (0.0079)	0.9373*** (0.0094)	0.9353*** (0.0097)
ξ				-0.1259*** (0.0074)	-0.1227*** (0.0104)	-0.1231*** (0.0000)	1.0000*** (0.2391)	1.0000 (1.9589)	1.0000 (0.6977)
δ							1.0331*** (0.1149)	1.1310*** (0.1674)	1.0687*** (0.1596)
ν		8.4136*** (1.0403)	1.4413*** (0.0410)		10.6390*** (1.4494)	1.5533*** (0.0438)		10.7403*** (1.4902)	1.5540*** (0.0456)
	-4121.161	-4074.867	-4079.048	-4046.660	-4015.189	-4022.453	-4048.345	-4018.009	-4024.566

Note: The standard errors are provided in the parenthesis. Significance levels (*): p<0,1;*, p<0,05**, p<0,01;***.

Almost all the estimates from the S&P 500 indicate high significance. The Exceptions are the conditional mean parameter a in the EGARCH and APGARCH as well as the asymmetry parameter ξ and ARCH-coefficent α_1 in the APGARCH model with t-distribution. The negative ξ in the EGARCH models indicates that there is presence of leverage effects which furthermore is supported by ξ in the APGARCH-N. Moreover, the power term δ in the APGARCH models is not statistically different from 1 and at the same time $|\xi| \leq 1$ under all distributions. This means that our estimates of the APGARCH model has yielded the nested Threshold GARCH (TGARCH). The persistence in the GARCH and APGARCH models are as expected high for all models under all distributions. In the EGARCH models the persistence are solely captured by β_1 which in this case also indicates high persistence under all distributions. The kurtosis of the GARCH, EGARCH and APGARCH under the t-distribution is calculated by $3(\nu-2)/(\nu-4)$ which returns 4.3594, 3.9038 and 3.8902 indicating that the returns follow a fat-tailed distribution. This is further emphasized from the models under the GED-distribution where the GED parameter ν for all models lie between 1 and 2, (Marcucci, 2005).

- **BSE**

The estimates of the Single Regime GARCH models on BSE are given in table 4.

Table 4: Estimates of Single State GARCH Models with different conditional distributions on Bombay Sensex Index

	GARCH-N	GARCH-t	GARCH-GED	EGARCH-N	EGARCH-t	EGARCH-GED	APGARCH-N	APGARCH-t	APGARCH-GED
a	0.1356*** (0.0236)	0.1349*** (0.0246)	0.1410*** (0.0242)	0.0802*** (0.0245)	0.1041*** (0.0243)	0.1066*** (0.0243)	0.0889*** (0.0256)	0.1053*** (0.0247)	0.1092*** (0.0246)
α_0	0.0987*** (0.0134)	0.0799*** (0.0187)	0.0888*** (0.0188)	-0.1379*** (0.0147)	-0.1538*** (0.0213)	-0.1475*** (0.0208)	0.1418*** (0.0172)	0.1153*** (0.0223)	0.1292*** (0.0228)
α_1	0.1368*** (0.0101)	0.1315*** (0.0162)	0.1327*** (0.0146)	0.2573*** (0.0202)	0.2596*** (0.0287)	0.2597*** (0.0281)	0.1296*** (0.0146)	0.1300*** (0.0204)	0.1302*** (0.0199)
β_1	0.8334*** (0.0110)	0.8455*** (0.0175)	0.8403*** (0.0164)	0.9240*** (0.0077)	0.9386*** (0.0165)	0.9304*** (0.0103)	0.8074*** (0.0138)	0.8207*** (0.0197)	0.8127*** (0.0192)
ξ				-0.1261*** (0.0111)	-0.1131*** (0.0165)	-0.1192*** (0.0158)	0.4585*** (0.0625)	0.3939*** (0.0829)	0.4240*** (0.0840)
δ							1.6060*** (0.1732)	1.6746*** (0.2553)	1.6320*** (0.2414)
ν		7.7335*** (0.9281)	1.4635*** (0.0419)		8.5119*** (1.1244)	1.5143*** (0.0465)		8.7103*** (1.1449)	1.5211*** (0.0458)
	-5087.288	-5030.219	-5044.814	-5055.376	-5010.965	-5023.672	-5051.217	-5007.504	-5020.359

Note: The standard errors are provided in the parenthesis. Significance levels (*): p<0,1;*, p<0,05;**, p<0,01;***.

The estimates of the single-regime GARCH models of the BSE differs some from S&P 500. The conditional mean parameter a are here highly significant together with all other parameters. Moreover, the persistence is much lower for especially the EGARCH and APGARCH. The presences of leverage effects are once again found since the asymmetric parameter ξ are negative and significantly different from zero in the EGARCH models.

Moreover, the estimates of the APGARCH model under all tested distributional assumptions do not yield any of its nested models.

The kurtosis of the models with t-distribution are 4.6070, 4.3298 and 4.2738 which indicates that a fat-tailed distribution is suitable. Under the GED distribution the parameter ν once again is between 1 and 2 indicating that the returns follow fat tailed distribution here as well.

i) Markov-Regime Switching GARCH

- **S&P 500**

The in-sample estimates with corresponding standard errors of the MRS-GARCH on S&P500 are gathered in table 5. The low variance regime is given by $i = 1$ and the high variance regime by $i = 2$

Most of the parameters derived from S&P 500 are significant. The conditional mean parameter $a^{(i)}$ is highly significant under all conducted distributions for both regimes. The constant in the conditional variance equation $\alpha_0^{(i)}$ is significant for the low variance regime under each distribution. The ARCH parameter $\alpha_1^{(i)}$ is highly significant for the high variance regime but insignificant for the low variance regime under all distributional assumptions. Furthermore, the $\beta_1^{(i)}$ parameter of the conditional variance is significant for both the regimes under all tested distributions. To find evidence of whether there exists a high and low variance regime the unconditional volatility must be calculated for all models and regimes. The unconditional variance is calculated by

$$\frac{\alpha_0^{(1)}}{1-\alpha_1^{(1)}-\beta_1^{(1)}} \tag{32}$$

The unconditional variance of the normal, t- and GED-distribution for the first regime is 0.2766, 0.2761 and 0.2652 respectively and 6.2458, 2.9440 and 6.3353 for the high variance regime. Hence there is a significant difference between the two regimes under all distributions. There is also interesting to look at the values of the constants $a^{(1)}$ and $a^{(2)}$ which is significant and negative for the high variance regime. This indicates that the returns of the high variance regime are negative and could demonstrate crisis periods.

Table 5: Estimates of MRS-GARCH models with different conditional distributions on S&P500 index

	MRS-GARCH-N	MRSGARCH-t	MRSGARCH-GED
$a^{(1)}$	0.0359** (0.0173)	0,0397** (0,0170)	0,0424** (0,0169)
$a^{(2)}$	-2.3627*** (0.2192)	-2,3056*** (0,1723)	-2,3432*** (0,1844)
$\alpha_0^{(1)}$	0.0236*** (0.0032)	0,0231*** (0,0043)	0,0217*** (0,0047)
$\alpha_0^{(2)}$	0.1184 (0.1632)	0,0719 (0,1622)	0,1451 (0,1818)
$\alpha_1^{(1)}$	0.0128 (0.0092)	0,0110 (0,0117)	0,0113 (0,0123)
$\alpha_1^{(2)}$	0.0756*** (0.0000)	0,1023*** (0,0000)	0,0734*** (0,0000)
$\beta_1^{(1)}$	0.9020*** (0.0093)	0,9054*** (0,0126)	0,9067*** (0,0134)
$\beta_1^{(2)}$	0.9055*** (0.0000)	0,8733*** (0,0000)	0,9037*** (0,0000)
p	0.9775*** (0.0020)	0,9778*** (0,0026)	0,9791*** (0,0027)
q	0.1872 (0.1231)	0,1700 (0,1411)	0,1863 (0,1427)
ν		16,2025*** (4,7168)	1,6478*** (0,0715)
$Log(L)$	-4062.29	-4058,06	-4052,90
π_1	0.0269	0,0261	0,0250
π_2	0.9731	0,9739	0,9750

Note: The standard errors are provided in the parenthesis. Significance levels (*): p<0,1;*, p<0,05;**, p<0,01;***.

Despite some fluctuations in the unconditional variance between the distributions the persistence seems to be very alike. The persistence of the MRS-GARCH estimated under the normal, t- and GED-distribution for the low variance regime is 0.9148, 0.9164 and 0.9181 and for the high variance regime 0.9811, 0.9756 and 0.9771. Consequently, the high variance regime has under all distributions a persistence close to unity comparable to the single regime GARCH models and a lower persistence in the low variance regime. This demonstrates one of the mayor advantages of the MRS-GARCH. By allowing for a second regime, the persistence becomes flexible since it can vary between the regimes. The shape parameter of both the t and GED-distributions indicates that the returns follow a fat-tailed distribution.

- **BSE**

The in-sample estimates with corresponding standard errors of the MRS-GARCH on BSE are gathered in table 6. The low variance regime is given by $i = 1$ and the high variance regime by $i = 2$

Analogous with the estimates from the S&P500 the constant from the mean equation $a^{(i)}$ is highly significant for both regimes under all distributions. It is also evident that the high variance regime reflects crisis periods with lower returns compared to the low variance regimes. The MRS-GARCH under normal distributions has a negative constant for both regimes which indicates low returns in both cases but to a greater extent for the high variance regime. The intercept of the conditional variance equation $\alpha_0^{(i)}$ is highly significant for both regimes under the normal and GED-distribution but insignificant for the t-distribution. The ARCH parameter $\alpha_1^{(i)}$ is highly significant for the high variance regime under all distributions but only significant for the low variance regime under the t-distribution. The GARCH parameter $\beta_1^{(i)}$ is significant for both the regimes under all distribution. Moreover, the persistence of the high variance regime under the normal distribution are extremely close to unity and indicates non-stationary. Consequently the unconditional variance becomes unreasonably high for the high variance regime. Nevertheless, under the t- and GED-distribution the unconditional volatility for the low variance regime is 0.4468 and 0.4513 and the high variance regime yields 12.4912 and 46.0896 respectively. In addition, the persistence is close to unity in the high variance regime and the for low variance regime 0,8902 and 0.8478 respectively. Even though the true volatility is higher in BSE compared to S&P500 the unconditional volatility seem to be overestimated by the MRS-GARCH. Nonetheless, this essay focus on the forecast accuracy and since new parameters will be estimated for each forecast none of the models can be ruled out yet. The shape parameter is significant for both the t- and GED-distribution and indicates that a fat tailed distribution is suitable for the data.

Table 6: Estimates of MRS-GARCH models with different conditional distributions on BSE

	MRS-GARCH-N	MRSGARCH-t	MRSGARCH-GED
$a^{(1)}$	-0.2106***	0,3032***	0,2368***
	(0.0267)	(0,0470)	(0,0271)
$a^{(2)}$	-0.5408***	-0,8814***	-0,6519***
	(0.1117)	(0,2832)	(0,1480)
$\alpha_0^{(1)}$	0.0725***	0,0541	0,0687***
	(0.0224)	(0,0320)	(0,0235)
$\alpha_0^{(2)}$	0.9077***	0,1460	0,6789***
	(0.1385)	(0,1899)	(0,1663)
$\alpha_1^{(1)}$	0.0057	0,0690***	0,0290
	(0.0212)	(0,0219)	(0,0240)
$\alpha_1^{(2)}$	0.0134***	0,2700***	0,1004***
	(0.0000)	(0,0946)	(0,0000)
$\beta_1^{(1)}$	0.8321***	0,8212***	0,8189***
	(0.0214)	(0,0250)	(0,0218)
$\beta_1^{(2)}$	0.9866***	0,7183***	0,8848***
	(0.0000)	(0,1032)	(0,0000)
p	0.9180***	0,9223***	0,9300***
	(0.0094)	(0,0297)	(0,0103)
q	0.6972***	0,7051***	0,7470***
	(0.0290)	(0,0981)	(0,0373)
ν		8,1764***	1,6794***
		(0,8238)	(0,0677)
$Log(L)$	-5018.74	-5004,42	-5015,05
π_1	0.2131	0,2085	0,2166
π_2	0.7869	0,7915	0,7834

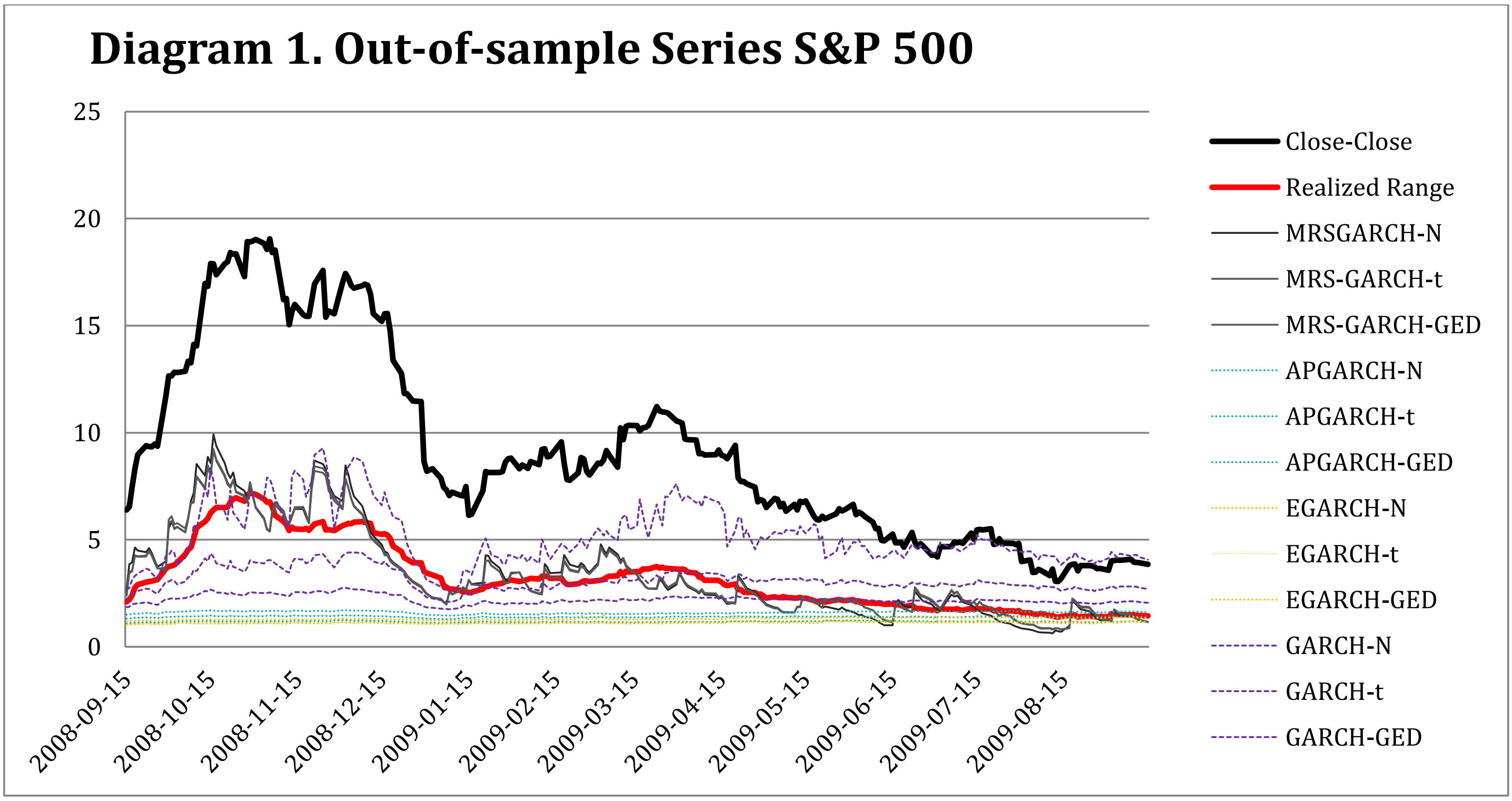
Note: The standard errors are provided in the parenthesis. Significance levels (*): p<0,1;*, p<0,05;**, p<0,01;***.

VI. Forecast evaluation

Good in-sample fit does not necessary entail accurate forecasts. Practitioners are often more interested in a better forecast accuracy than a good in-sample fit. Diagram 1 and 2 depicts the out-of-sample forecast series of both indexes and all tested models. The volatility given by the Close-Close and Realized Range- estimator are given by the black and red line (the thicker lines). Table 7 and 8 demonstrates the loss functions of both indexes calculated by the Realized Range estimator. The loss functions derived from the Close-Close proxy can be found in the appendix. In the last subsection of the chapter, the results of the Diebold Mariano tests are presented.

- **S&P 500**

It is quite obvious that the MRS-GARCH models produce a more accurate forecast series than the other models. The Realized Range volatility are clearly higher than all single-regime models except the GARCH with t-distribution. There is also a noticeable difference between the two volatility proxies. Both MRS-GARCH and the single regime models underestimates the volatility of the Close-Close estimator. However the Close-Close proxy is like previously mentioned known to produce unreliable estimates and focus should be put on the Realized Range volatility.



A noticeable difference between the forecasts and Realized Range proxy was expected during the beginning of the considered time period when the highest volatility is found. However, three models can be distinguished graphically to predict the Realized Range volatility with what seem to be a much higher accuracy than the others. Those three models are the MRS-

GARCH under all three distributional assumptions (depicted with solid lines). The GARCH under the t-distribution also seem to produce relatively accurate forecasts during the most critical time period but overvalues the volatility somewhere after New Year 's Eve 2009. The EGARCH, APGARCH and GARCH under the normal and GED distribution underestimate the volatility in the beginning but produces better forecasts in the end of the considered time period. All in all, the MRSGARCH seems at least graphically to outperform the other models forecasting accuracy. Nevertheless, to be able to draw any conclusions more information is needed.

Table 7 presents the calculated loss functions with the Realized Range proxy

Table 7. Out-of-sample Loss functions of S&P 500

	MRS-GARCH			APGARCH			EGARCH			GARCH		
	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED
MSE1	0.0597	0.0455	**0.0447**	0.4048	0.4929	0.6154	0.5543	0.6374	0.6629	0.2085	0.3621	0.1081
MSE2	0.9609	0.7346	**0.7226**	5.1060	5.8339	6.7220	6.3092	6.8823	7.0454	3.0936	5.2512	1.5057
QLIKE	2.1059	2.0884	**2.0881**	2.4794	2.6378	2.9046	2.7692	2.9640	3.0283	2.2176	2.2320	2.1316
R2LOG	0.0890	0.0610	**0.0602**	0.5630	0.7408	1.0246	0.8738	1.0758	1.1426	0.2439	0.4438	0.1463
MAD1	0.2021	0.1760	**0.1738**	0.4975	0.5652	0.6633	0.6051	0.6727	0.6935	0.3587	0.5276	0.2807
MAD2	0.7424	0.6521	**0.6436**	1.6686	1.8344	2.0571	1.9271	2.0773	2.1218	1.2824	2.0358	1.0026
HMSE	0.1252	0.0682	**0.0680**	1.8881	2.8825	4.7617	3.8625	5.3180	5.8047	0.5226	0.2011	0.1343

The volatility proxy is calculated with the Realized Range estimator

By investigating table 7 it is evident that the loss functions calculated with Realized Range yields the lowest value when the forecast series of the MRS-GARCH with GED distribution is used. The second best model is the MRS-GARCH with t-distribution and the third best is the MRS-GARCH under normal distribution. The results are not surprising, Marcucci (2005) also found that the MRS-GARCH under leptokurtic distributions outperformed the single regime GARCH models. However, to be able to tell with statistical significance whether the MRS-GARCH models actually outperform the others the DM test must be performed.

- **BSE**

Examining diagram 2 is not as clear as the previous diagram. All the MRS-GARCH once again depicted with a solid line seem to produce too high forecasts. All the forecasts seem to underestimate the Close-Close volatility except the MRS-GARCH models that both overestimates and underestimates the volatility. None of the models seem to produce relatively accurate forecast series of neither the Realized Range nor Close-Close volatility. It is really hard to determine which model that succeeds the most by only examining the series graphically. It seems like the Single Regime models fails to pick up any differences at all and merely produces a non-fluctuating constant. Consequently, taking the mean of the Realized Range seems to yield a constant that would retrieve results comparable to the best models on BSE. Hence, none of the model appears to produce particularly good predictions.

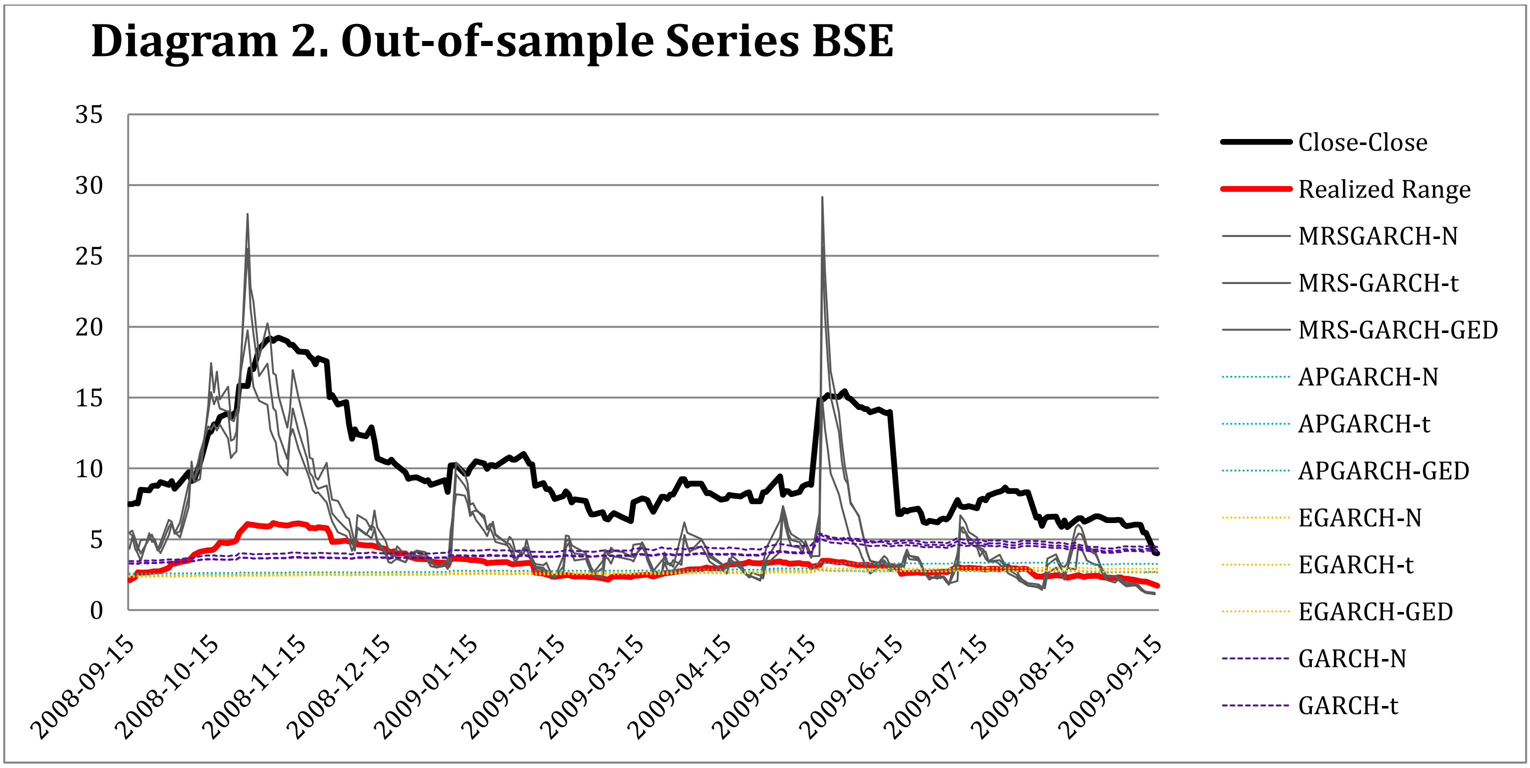


Table 8 contains the loss functions of derived from the Realized Range volatility.

Table 8. Out-of-sample Loss functions of BSE

	MRS-GARCH			APGARCH			EGARCH			GARCH		
	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED
MSE1	0.3833	0.6017	0.5834	0.1193	**0.1094**	0.1136	0.1233	0.1148	0.1193	0.1402	0.1684	0.1663
MSE2	10.5337	21.9014	20.3841	1.7449	**1.6147**	1.6865	1.7913	1.6876	1.7445	2.0253	2.4863	2.4147
QLIKE	2.2673	2.2829	2.2855	2.2417	2.2292	2.2455	2.2455	2.2369	2.2417	**2.2280**	2.2387	2.2377
R2LOG	0.2722	0.3319	0.3394	0.1356	**0.1237**	0.1270	0.1412	0.1301	0.1357	0.1610	0.1889	0.1893
MAD1	0.4582	0.5154	0.5118	0.2409	0.2456	**0.2329**	0.2459	0.2388	0.2413	0.3371	0.3635	0.3735
MAD2	2.1150	2.6279	2.5549	0.8786	0.9013	**0.8559**	0.8946	0.8728	0.8800	1.2794	1.3942	1.4315
HMSE	0.1469	0.1515	0.1542	0.2832	0.2189	0.2619	0.2975	0.2606	0.2828	**0.1225**	0.1354	0.1270

The volatility proxy is calculated with the Realized Range estimator

The results of the loss functions are like expected from the diagram not as straightforward as the result from S&P 500. The MSE_1, MSE_2 and $R2LOG$ are lowest when they are calculated with the forecast from APGARCH under t-distribution. The $QLIKE$ and $HMSE$ are lowest calculated from the GARCH under the normal distribution. Finally the best model based on MAD_1 and MAD_2 is the APGARCH with GED innovations. Nevertheless, there are not any large differences between the values of the loss functions from the single regime GARCH models and they seem to have a somewhat similar accuracy.

i) Diebold Mariano Test

Since the DM test only are able to compare the models pairwise and the seven loss functions are calculated from each model of both S&P 500 and BSE, 4032 test statistics are calculated. To uphold brevity most of the tables are decided to not be presented. The demonstrated results of the DM-test are entirely from the series of loss functions calculated with realized range. The DM-test aims to determine superior predictive ability and the Realized Range estimator is as discussed proved to be approximately five times more efficient than the Close-Close proxy. Moreover, to make the thesis as coherent as possible the two benchmark models chosen to be presented are the most efficient MRS-GARCH models based on the loss functions. Additionally, the best performing specification of each model are to be found in the appendix.

The MRS-GARCH with GED innovations has the lowest loss functions out of all models on the S&P 500. Hence, rejecting the null hypothesis of no difference in the forecast series implies that the MRS-GARCH with GED innovations is superior to the model it's being compared with. The opposite is almost true in table 10 where the MRS-GARCH with normal distribution is presented. Even though the MRS-GARCH with normal innovations yields the lowest loss functions compared to the Regime switching GARCH models, it has higher values on all functions except $HMSE$, compared to the single regime models. Consequently, except for the $HMSE$, when the MRS-GARCH with normal distribution is competing against a Single Regime model, rejecting the null implies that it has been outperformed.

- **S&P 500**

Table 9 presents the p-values from the DM-tests where the MRS-GARCH with GED distribution serves as the benchmark model.

Table 9: Diebold-Mariano test Benchmark: S&P 500 - MRS-GARCH(GED)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0170	0.0654	0.1167	0.0908	0.0216	0.0146	0.1637
	p_{MDDM}	0.0182	0.0676	0.1194	0.0933	0.0230	0.0157	0.1666
MRS-GARCH(T)	p_{DM}	0.6002	0.7484	0.7178	0.6253	0.3741	0.4393	0.9368
	p_{MDDM}	0.6021	0.7496	0.7192	0.6271	0.3768	0.4419	0.9371
APGARCH(N)	p_{DM}	0.0050	0.0162	0.0027	0.0015	0.0009	0.0024	0.0086
	p_{MDDM}	0.0056	0.0174	0.0031	0.0017	0.0010	0.0027	0.0094
APGARCH(T)	p_{DM}	0.0025	0.0109	0.0012	0.0004	0.0001	0.0006	0.0063
	p_{MDDM}	0.0028	0.0118	0.0014	0.0005	0.0002	0.0008	0.0069
APGARCH(GED)	p_{DM}	0.0008	0.0063	0.0003	0.0000	0.0000	0.0001	0.0037
	p_{MDDM}	0.0009	0.0069	0.0004	0.0001	0.0000	0.0001	0.0042
EGARCH(N)	p_{DM}	0.0019	0.0092	0.0011	0.0003	0.0000	0.0003	0.0069
	p_{MDDM}	0.0022	0.0100	0.0013	0.0004	0.0001	0.0004	0.0076
EGARCH(T)	p_{DM}	0.0008	0.0063	0.0004	0.0001	0.0000	0.0001	0.0047
	p_{MDDM}	0.0010	0.0069	0.0005	0.0001	0.0000	0.0001	0.0052
EGARCH(GED)	p_{DM}	0.0006	0.0055	0.0003	0.0000	0.0000	0.0000	0.0039
	p_{MDDM}	0.0008	0.0061	0.0003	0.0000	0.0000	0.0000	0.0044
GARCH(N)	p_{DM}	0.0249	0.0499	0.0184	0.0141	0.0074	0.0154	0.0295
	p_{MDDM}	0.0264	0.0519	0.0196	0.0152	0.0081	0.0166	0.0311
GARCH(T)	p_{DM}	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0006
	p_{MDDM}	0.0000	0.0000	0.0002	0.0002	0.0000	0.0000	0.0007
GARCH(GED)	p_{DM}	0.0317	0.1478	0.0203	0.0123	0.0313	0.0639	0.1249
	p_{MDDM}	0.0333	0.1506	0.0216	0.0133	0.0330	0.0661	0.1277

Loss functions calculated with the Realized Range volatility

There is evident that most of the p-values in the table are rejected with high statistical significance. The only two models where it is hard to distinguish between the forecast accuracy is the other MRS-GARCH specifications and the GARCH under GED-distribution. The MRS-GARCH under all distributions yields loss functions with similar values compared to the other models. It is therefore expected that the different series of MRS-GARCH under GED and t-distribution can't be distinguished for any loss function. Nevertheless, at least a 90% significance level is fulfilled on all loss functions except the $QLIKE$ and $HMSE$ when The MRS-GARCH under GED and normal distribution are compared. The result of the MRS-GARCH with GED stressed against the GARCH under GED is fairly similar. A significance level higher than 90% is achieved for all loss functions except the MSE_2 and $HMSE$. Consequently, the MRS-GARCH with GED innovation is without a doubt superior to all models except the other MRS-GARCH and the GARCH with GED innovations. Nonetheless, the MRS-GARCH under GED is superior to both the MRSGARCH with normal innovations and the GARCH with GED innovations on five out of seven of the considered loss functions. This could be interpreted as a weakly superior predictive ability of the MRS-GARCH under GED. The only model with inseparable forecast accuracy to the MRS-GARCH with GED innovations is the MRS-GARCH with t-distribution.

- **BSE**

The MRS-GARCH with normal distribution is presented in table 10

Table 10: Diebold-Mariano test Benchmark: BSE - MRS-GARCH(N)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(T)	p_{DM}	0.1477	0.1269	0.3729	0.2990	0.2796	0.1645	0.6782
	p_{MDDM}	0.1507	0.1298	0.3758	0.3020	0.2827	0.1675	0.6798
MRS-GARCH(GED)	p_{DM}	0.2097	0.2047	0.3251	0.2768	0.2340	0.1806	0.9453
	p_{MDDM}	0.2128	0.2078	0.3281	0.2799	0.2371	0.1836	0.9456
APGARCH(N)	p_{DM}	0.0169	0.0537	0.2023	0.0219	0.0083	0.0072	0.9481
	p_{MDDM}	0.0181	0.0559	0.2054	0.0233	0.0091	0.0079	0.9484
APGARCH(T)	p_{DM}	0.0144	0.0513	0.3334	0.0270	0.0021	0.0025	0.7434
	p_{MDDM}	0.0156	0.0535	0.3364	0.0287	0.0025	0.0029	0.7447
APGARCH(GED)	p_{DM}	0.0174	0.0535	0.4657	0.0464	0.0040	0.0035	0.6382
	p_{MDDM}	0.0186	0.0556	0.4683	0.0484	0.0045	0.0040	0.6400
EGARCH(N)	p_{DM}	0.0160	0.0526	0.3648	0.0335	0.0040	0.0038	0.7301
	p_{MDDM}	0.0172	0.0548	0.3677	0.0352	0.0045	0.0043	0.7314
EGARCH(T)	p_{DM}	0.0168	0.0531	0.4659	0.0453	0.0038	0.0034	0.6389
	p_{MDDM}	0.0181	0.0553	0.4684	0.0473	0.0043	0.0039	0.6407
EGARCH(GED)	p_{DM}	0.0183	0.0539	0.5415	0.0586	0.0049	0.0038	0.5895
	p_{MDDM}	0.0196	0.0561	0.5437	0.0609	0.0055	0.0044	0.5915
GARCH(N)	p_{DM}	0.1275	0.1128	0.2468	0.2742	0.3286	0.1889	0.3706
	p_{MDDM}	0.1304	0.1156	0.2499	0.2773	0.3316	0.1919	0.3734
GARCH(T)	p_{DM}	0.1317	0.1149	0.2401	0.2770	0.3918	0.2258	0.3210
	p_{MDDM}	0.1346	0.1177	0.2432	0.2800	0.3946	0.2289	0.3240
GARCH(GED)	p_{DM}	0.0766	0.0908	0.1094	0.1236	0.1948	0.1177	0.3002
	p_{MDDM}	0.0791	0.0935	0.1122	0.1264	0.1979	0.1205	0.3032

Loss functions calculated with the Realized Range volatility

The result from BSE is not as easy to interpret as the result of S&P 500. Except for $HMSE$, the MRS-GARCH models has under all distribution higher loss function values than the single regime models. It is evident that the $HMSE$ lacks significance independently of which model it's evaluated against. All the APGARCH and EGARCH models outperforms the MRS-GARCH with normal distribution on all loss functions except $QLIKE$. However, the forecasts of the APGARCH and EGARCH merely yields a constant almost entirely without fluctuations compared to the volatility proxy. A table of the standard deviation from the forecast series compared to the Realized Range volatility can be found in the appendix. Hence, neither the APGARCH nor EGARCH returns appealing forecasts. Taking the average of the Realized Range volatility would therefore probably also be superior in forecast accuracy to the MRS-GARCH with normal innovations. Furthermore, when the MRS-GARCH with normal distribution is evaluated against the GARCH under normal and t-distribution the null can't be rejected for any loss function. Nevertheless, when it is evaluated towards the GARCH under GED the null can be rejected under 90% significance for the MSE_1 and MSE_2.

VII. Conclusion

The aim of this thesis was to evaluate the accuracy of volatility forecast from a set of single regime GARCH models and the MRS-GARCH during the financial crisis of 2008. The forecast horizon was one day ahead and the models were estimated from both the S&P 500 and BSE. All models were estimated under three distributional assumptions, i.e., normal-, t- and GED-distribution.

The purpose was foremost to evaluate the predictive accuracy of the MRS-GARCH and additionally whether the MRS-GARCH assessed on the BSE would be successful. This evaluation was stressed by comparing a series of forecast towards a proxy for the true volatility. The Realized Range estimator was used as the true volatility, calculated using intra daily extreme values.

The predictive accuracy was measured by calculating loss functions from an out-sample period starting from the collapse of Lehman brothers in 2008 until one year ahead. Furthermore the DM-test was applied to be able to determine whether the forecasting performance differed between the models. A statistical significant difference of the performance of two models means that one models has a superior predictive ability. The DM-tests is a pairwise test where one model is appointed as benchmark. The benchmark models predictive ability is then tested towards all other models and loss functions. Hence, the conclusions of the forecasting performances are only relative to the other models included in the thesis.

The results were very different depending on which index that was evaluated. The predictive accuracy was generally far better on S&P 500 than the BSE. Both the Single and Regime Switching models had a hard time forecasting volatility on the BSE. The MRS-GARCH models was superior in predictive accuracy to the single regime models on the S&P 500 except for the GARCH with GED innovations. However, the MRS-GARCH was at least weakly superior since the null were rejected on five out of seven loss functions. The superior predictive accuracy is conversely hard to determine on the BSE. It seems like no model performed particularly well and that the characteristics of BSE was either hard to pick up or wasn't symptomatic of the out-sample period. This is also evident when the DM-test is performed and a difference in performance accuracy is much harder to find. In fact no model was entirely superior in predictive ability.

It should be noted that there are many GARCH specifications that aren't evaluated in this thesis. Nonetheless, the MRS-GARCH demonstrated an ability to produce relatively accurate short-term forecasts of S&P 500's volatility. If the aim of a researcher is to find accurate short-term forecasts, the MRS-GARCH proved itself useful. Furthermore, none of the models forecasting performance are particularly successful on the BSE. There is consequently interesting to further investigate other kind of GARCH models on this index. An especially interesting feature left for further research is to let the density function of the MRS-GARCH shift between the different regimes.

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IX. Appendix

i) Markov Regime Switching GARCH

The conditional variance equation in the MRS-GARCH (1,1) framework is expressed as

$$h_t^{2|(i)} = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)} h_t^{2|(i)} \quad \text{(A1)}$$

The conditional variance of the rate of returns is dependent on the whole regime path $\tilde{s}_t = \{s_t, s_{t-1}, \ldots, s_{t-n}\}$ which is unobserved by the econometrician. The conditional variance is therefore given by

$$h_t^{2|(i)} = Var[\varepsilon_t|\tilde{s}_t, \varsigma_{t-1}] \quad \text{(A2)}$$

This make the estimation procedure problematic since the possible regime paths increase exponentially with time. Many prominent researchers has dealt with this issue in numerous ways but this thesis focus on the estimation procedure by (Klaassen, 2002). The conditional variance equation in (A1) for the MRS-GARCH(1,1) is then written as

$$h_t^{2|(i)}[\varepsilon_t|\tilde{s}_t, \varsigma_{t-1}] = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)} E[Var\{\varepsilon_{t-1}|\tilde{s}_{t-1}, \varsigma_{t-2}\}|s_t, \varsigma_{t-1}] \quad \text{(A3)}$$

where $s_t = \{1,2\}$ and the expectation E is across the state $\tilde{s}_{t-1}$ and conditional on the information set ς_{t-1} and current regime s_t. The constraints are the same as for the traditional GARCH model: $\alpha_0^{(i)} > 0$ and $\alpha_1^{(i)}, \beta_1^{(i)} \geq 0$ to ensure positivity of the conditional variance. With this setting it is only necessary to integrate out one single regime, s_{t-1} since (A3) can be expressed as

$$h_t^{2|(i)}[\varepsilon_t|s_t, \varsigma_{t-1}] = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)} E\left[h_{t-1}^{2|(i)}|s_t, \varsigma_{t-1}\right] \quad \text{(A4)}$$

which is independent of $\tilde{s}_t$.This is true since $Var[\varepsilon_t|\tilde{s}_t, \varsigma_{t-1}]$ by construction only depends on the present variance regime which means that $Var[\varepsilon_t|\tilde{s}_t, \varsigma_{t-1}] = Var[\varepsilon_t|s_t, \varsigma_{t-1}]$. Hence by using the law of iterated expectations, i.e., taking the conditional expectation of the lagged conditional variance on the current regime, Klaassen (2002) get rid of the path dependence problem. Consequently the conditional variance is given by

$$h_t^{2|(i)} = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)} E\left[h_{t-1}^{2|(i)}\middle|s_t, \varsigma_{t-1}\right] \quad \text{(A5)}$$

where the expected conditional variance is calculated as

$$E\left[h_{t-1}^{2|(i)}\middle|s_t, \varsigma_{t-1}\right] = \tilde{p}_{ii,t-1}\left[\left(\mu_{t-1}^{(i)}\right)^2 + h_{t-1}^{2|(i)}\right] + \tilde{p}_{ji,t-1}\left[\left(\mu_{t-1}^{(j)}\right)^2 + h_{t-1}^{2|(j)}\right] - \left[\tilde{p}_{ii,t-1}\mu_{t-1}^{(i)} + \tilde{p}_{ji,t-1}\mu_{t-1}^{(j)}\right]^2 \quad \text{(A6)}$$

with the probabilities given by

$$\tilde{p}_{ji,t} = \Pr(s_t = j|s_{t+1} = i, \varsigma_{t-1}) = p_{ji}\frac{\Pr(s_t=j|\varsigma_{t-1})}{\Pr(s_{t+1}=i|\varsigma_{t-1})} = p_{ji}\frac{p_{j,t}}{p_{i,t+1}} \quad \text{(A7)}$$

where $i, j = 1,2$.

ii) DM-test

Assume that we have two competing forecast series, one from the MRS-GARCH, $\{\hat{h}_{i,t}^{MRSGARCH}\}_t^n$ and one from the standard GARCH $\{\hat{h}_{j,t}^{GARCH}\}_t^n$, with corresponding forecast errors $\{e_{i,t}^{MRSGARCH}\}_t^n$ and $\{e_{j,t}^{GARCH}\}_t^n$. By taking the difference between two loss function $g(e_{i,t}^{MRSGARCH})$ and $g(e_{j,t}^{GARCH})$ we define

$$d_t \equiv \left(g\left(e_{i,t}^{MRSGARCH}\right) - g(e_{j,t}^{GARCH})\right) \quad \text{(A8)}$$

Under fairly weak conditions explicitly that $\{d_t\}_i^n$ is covariance-stationary and has short memory Diebold Mariano proved that the mean of the loss differential series, $\bar{d}$ is $\sqrt{n}(\bar{d} - \mu) \rightarrow N\left(0, Var(\bar{d})\right)$, where $Var(\bar{d})$ is the asymptotic or long-run variance of the sample mean loss differential series. Assuming that the conditions above holds the difficulties attached with calculating the DM statistic is that the econometrician do not observe the long run variance and therefore must estimate it with

$$\widehat{Var}(\bar{d}) = n^{-1}\left(\hat{\gamma}_0 + \sum_{k=1}^{q} \omega_k \hat{\gamma}_k\right) \quad \text{(A9)}$$

where $= \left\lfloor 4\left(\frac{n}{100}\right)^{2/9} \right\rfloor$, $\omega_k = 1 - \left(\frac{k}{q+1}\right)$ is the lag window and $\hat{\gamma}_i$ is the i^{th} order autocovariance estimated by

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}\left(d_t - \bar{d}\right)\left(d_{t-k} - \bar{d}\right) \quad \text{(A10)}$$

for $k = 1, \ldots, q$. Finally the DM test statistic is retrieved by calculating

$$DM = \frac{\bar{d}}{\sqrt{\widehat{Var}(\bar{d})}} \sim N(0,1) \quad \text{(A11)}$$

iii) Tables

Table A1. Out-of-sample Loss functions of S&P 500

	MRS-GARCH			APGARCH			EGARCH			GARCH		
	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED
MSE1	1.4590	1.4455	1.4456	3.2892	3.5756	3.9315	3.7425	3.9827	4.0551	2.5437	**0.7676**	1.7525
MSE2	39.9120	40.6670	40.6075	76.8817	80.0642	83.6154	81.8714	84.1584	84.7997	66.8292	**28.6072**	52.9460
QLIKE	4.0104	3.9142	3.9186	6.1204	6.8145	7.8810	7.3134	8.0823	8.3336	4.7978	**3.3417**	3.9413
R2LOG	1.1878	1.1005	1.1060	2.8463	3.3040	3.9431	3.5804	4.0312	4.1777	1.8691	**0.3739**	1.0770
MAD1	1.1642	1.1539	1.1557	1.6610	1.7449	1.8433	1.7851	1.8527	1.8735	1.4405	**0.6976**	1.1634
MAD2	5.6924	5.7051	5.7082	7.5080	7.7140	7.9373	7.8073	7.9576	8.0021	6.8962	**4.0292**	5.9789
HMSE	4.6999	3.9030	3.9224	28.8309	39.8277	59.3873	49.7532	64.6519	69.6028	11.8624	**0.9585**	4.3904

The volatility proxy is calculated with the Close-Close estimator

Table A2. Out-of-sample Loss functions of BSE

	MRS-GARCH			APGARCH			EGARCH			GARCH		
	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED	Normal	Student's t	GED
MSE1	0.9609	**0.8882**	0.9148	2.2113	2.4643	2.4723	2.3749	2.4602	2.4803	1.4683	1.3461	1.4961
MSE2	26.1525	**23.3308**	24.8487	60.6587	65.3370	64.9262	63.3962	64.7508	65.0688	46.3168	43.7379	46.9985
QLIKE	3.7184	**3.6883**	3.6934	4.4787	4.7314	4.7528	4.6481	4.7407	4.7662	3.8534	3.7651	3.8659
R2LOG	0.6725	**0.6449**	0.6494	1.4684	1.7094	1.7363	1.6324	1.7226	1.7440	0.8216	0.7258	0.8402
MAD1	0.8947	**0.8720**	0.8765	1.3669	1.4703	1.4689	1.4301	1.4633	1.4692	1.0579	1.0134	1.0797
MAD2	4.6021	**4.4401**	4.5064	6.8421	7.2338	7.1869	7.0582	7.1686	7.1871	5.6777	5.5049	5.7735
HMSE	2.2243	**1.9873**	2.0244	7.7815	9.6494	10.0232	9.1937	9.9555	10.2115	3.1491	2.5675	3.1935

The volatility proxy is calculated with the Close-Close estimator

Table A3: Diebold-Mariano test Benchmark: APGARCH (N)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0070	0.0193	0.0055	0.0038	0.0028	0.0053	0.0123
	p_{MDDM}	0.0077	0.0206	0.0061	0.0043	0.0032	0.0059	0.0133
MRS-GARCH(T)	p_{DM}	0.0033	0.0127	0.0022	0.0010	0.0004	0.0014	0.0081
	p_{MDDM}	0.0038	0.0137	0.0025	0.0012	0.0005	0.0017	0.0088
MRS-GARCH(GED)	p_{DM}	0.0010	0.0071	0.0005	0.0001	0.0000	0.0001	0.0044
	p_{MDDM}	0.0012	0.0078	0.0006	0.0001	0.0000	0.0002	0.0049
APGARCH(T)	p_{DM}	0.0000	0.0002	0.0001	0.0000	0.0000	0.0000	0.0033
	p_{MDDM}	0.0000	0.0002	0.0001	0.0000	0.0000	0.0000	0.0037
APGARCH(GED)	p_{DM}	0.0127	0.0000	0.0000	0.0000	0.0000	0.0000	0.0019
	p_{MDDM}	0.0138	0.0000	0.0000	0.0000	0.0000	0.0000	0.0022
EGARCH(N)	p_{DM}	0.0000	0.0003	0.0002	0.0000	0.0000	0.0000	0.0056
	p_{MDDM}	0.0001	0.0004	0.0003	0.0000	0.0000	0.0000	0.0062
EGARCH(T)	p_{DM}	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0032
	p_{MDDM}	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0037
EGARCH(GED)	p_{DM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0026
	p_{MDDM}	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0030
GARCH(N)	p_{DM}	0.0008	0.0023	0.0008	0.0003	0.0012	0.0013	0.0052
	p_{MDDM}	0.0010	0.0027	0.0010	0.0004	0.0014	0.0015	0.0057
GARCH(T)	p_{DM}	0.8168	0.9533	0.1112	0.6201	0.8537	0.5178	0.0181
	p_{MDDM}	0.8177	0.9535	0.1139	0.6220	0.8545	0.5200	0.0193
GARCH(GED)	p_{DM}	0.0127	0.0176	0.0053	0.0100	0.0386	0.0440	0.0082
	p_{MDDM}	0.0138	0.0188	0.0059	0.0109	0.0404	0.0459	0.0089

Loss functions calculated with the Realized Range volatility

Table A4: Diebold-Mariano test Benchmark: S&P 500-EGARCH (N)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0025	0.0105	0.0017	0.0006	0.0002	0.0007	0.0082
	p_{MDDM}	0.0029	0.0115	0.0020	0.0008	0.0002	0.0009	0.0090
MRS-GARCH(T)	p_{DM}	0.0011	0.0071	0.0006	0.0001	0.0000	0.0001	0.0054
	p_{MDDM}	0.0013	0.0078	0.0008	0.0002	0.0000	0.0002	0.0060
MRS-GARCH(GED)	p_{DM}	0.0008	0.0062	0.0004	0.0001	0.0000	0.0001	0.0045
	p_{MDDM}	0.0010	0.0069	0.0005	0.0001	0.0000	0.0001	0.0051
APGARCH(N)	p_{DM}	0.0000	0.0003	0.0002	0.0000	0.0000	0.0000	0.0056
	p_{MDDM}	0.0001	0.0004	0.0003	0.0000	0.0000	0.0000	0.0062
APGARCH(T)	p_{DM}	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0032
	p_{MDDM}	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0037
APGARCH(GED)	p_{DM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0026
	p_{MDDM}	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0030
EGARCH(T)	p_{DM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0013
	p_{MDDM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0016
EGARCH(GED)	p_{DM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010
	p_{MDDM}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0012
GARCH(N)	p_{DM}	0.0003	0.0012	0.0004	0.0000	0.0000	0.0000	0.0054
	p_{MDDM}	0.0004	0.0014	0.0005	0.0001	0.0000	0.0000	0.0060
GARCH(T)	p_{DM}	0.0004	0.0004	0.0004	0.0004	0.0000	0.0000	0.0004
	p_{MDDM}	0.0005	0.0005	0.0005	0.0001	0.0000	0.0000	0.0005
GARCH(GED)	p_{DM}	0.0041	0.0094	0.0017	0.0015	0.0045	0.0088	0.0067
	p_{MDDM}	0.0046	0.0102	0.0019	0.0017	0.0051	0.0096	0.0074

Loss functions calculated with the Realized Range volatility

Table A5: Diebold-Mariano test Benchmark: S&P 500-GARCH (GED)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0249	0.0499	0.0184	0.0141	0.0074	0.0154	0.0295
	p_{MDDM}	0.0264	0.0519	0.0196	0.0152	0.0081	0.0166	0.0311
MRS-GARCH(T)	p_{DM}	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0006
	p_{MDDM}	0.0000	0.0000	0.0002	0.0002	0.0000	0.0000	0.0007
MRS-GARCH(GED)	p_{DM}	0.0317	0.1478	0.0203	0.0123	0.0313	0.0639	0.1249
	p_{MDDM}	0.0333	0.1506	0.0216	0.0133	0.0330	0.0661	0.1277
APGARCH(N)	p_{DM}	0.0001	0.0005	0.0001	0.0000	0.0000	0.0000	0.0027
	p_{MDDM}	0.0001	0.0007	0.0001	0.0000	0.0000	0.0000	0.0031
APGARCH(T)	p_{DM}	0.2624	0.6051	0.0075	0.0687	0.4171	0.9686	0.0054
	p_{MDDM}	0.2653	0.6069	0.0082	0.0710	0.4197	0.9687	0.0060
APGARCH(GED)	p_{DM}	0.0017	0.0061	0.0005	0.0003	0.0005	0.0022	0.0036
	p_{MDDM}	0.0020	0.0068	0.0006	0.0003	0.0007	0.0025	0.0040
EGARCH(N)	p_{DM}	0.0001	0.0005	0.0001	0.0000	0.0000	0.0000	0.0046
	p_{MDDM}	0.0001	0.0006	0.0001	0.0000	0.0000	0.0000	0.0035
EGARCH(T)	p_{DM}	0.2035	0.0005	0.0049	0.0407	0.3266	0.8805	0.0054
	p_{MDDM}	0.2065	0.5419	0.0055	0.0426	0.3295	0.8811	0.0060
EGARCH(GED)	p_{DM}	0.0013	0.0053	0.0004	0.0002	0.0000	0.0013	0.0038
	p_{MDDM}	0.0015	0.0059	0.0005	0.0002	0.0000	0.0015	0.0043
GARCH(T)	p_{DM}	0.2348	0.2541	0.8555	0.2013	0.0005	0.0025	0.1606
	p_{MDDM}	0.2378	0.2571	0.8562	0.2043	0.0006	0.0028	0.1635
GARCH(GED)	p_{DM}	0.0006	0.0016	0.0031	0.1895	0.2495	0.0022	0.2324
	p_{MDDM}	0.0008	0.0019	0.0035	0.1924	0.2524	0.0025	0.2353

Loss functions calculated with the Realized Range volatility

Table A6: Diebold-Mariano test Benchmark: BSE-APGARCH (T)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0496	0.0847	0.0912	0.0343	0.0123	0.0202	0.9873
	p_{MDDM}	0.0517	0.0873	0.0939	0.0361	0.0134	0.0215	0.9873
MRS-GARCH(T)	p_{DM}	0.0460	0.0832	0.1440	0.0295	0.0032	0.0114	0.7670
	p_{MDDM}	0.0481	0.0858	0.1470	0.0312	0.0037	0.0124	0.7682
MRS-GARCH(GED)	p_{DM}	0.0491	0.0847	0.2225	0.0395	0.0048	0.0133	0.6515
	p_{MDDM}	0.0512	0.0872	0.2256	0.0414	0.0054	0.0144	0.6533
APGARCH(N)	p_{DM}	0.5183	0.4538	0.2562	0.5841	0.6724	0.6209	0.0776
	p_{MDDM}	0.5207	0.4565	0.2593	0.5862	0.6741	0.6228	0.0802
APARCH(GED)	p_{DM}	0.0053	0.0066	0.0080	0.0045	0.0185	0.0197	0.0233
	p_{MDDM}	0.0059	0.0074	0.0088	0.0051	0.0198	0.0211	0.0248
EGARCH(N)	p_{DM}	0.3114	0.3590	0.3532	0.2733	0.4357	0.4381	0.5694
	p_{MDDM}	0.3145	0.3619	0.3562	0.2764	0.4384	0.4408	0.5715
EGARCH(T)	p_{DM}	0.0093	0.0123	0.0152	0.0073	0.0020	0.0022	0.0402
	p_{MDDM}	0.0102	0.0134	0.0164	0.0080	0.0024	0.0026	0.0421
EGARCH(GED)	p_{DM}	0.0070	0.0095	0.0118	0.0052	0.0018	0.0018	0.0345
	p_{MDDM}	0.0077	0.0104	0.0129	0.0058	0.0021	0.0022	0.0363
GARCH(N)	p_{DM}	0.4685	0.4390	0.9364	0.4977	0.1648	0.1148	0.2680
	p_{MDDM}	0.4711	0.4417	0.9367	0.5002	0.1679	0.1177	0.2712
GARCH(T)	p_{DM}	0.4971	0.4968	0.9324	0.5016	0.1416	0.0964	0.2817
	p_{MDDM}	0.4996	0.4993	0.9327	0.5040	0.1446	0.0991	0.2848
GARCH(GED)	p_{DM}	0.7332	0.7432	0.7303	0.7250	0.2323	0.1707	0.2246
	p_{MDDM}	0.7346	0.7446	0.7317	0.7264	0.2355	0.1738	0.2278

Loss functions calculated with the Realized Range volatility

Table A7: Diebold-Mariano test Benchmark: BSE-EGARCH (T)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.0482	0.0842	0.1645	0.0355	0.0059	0.0143	0.7509
	p_{MDDM}	0.0503	0.0868	0.1676	0.0373	0.0066	0.0155	0.7522
MRS-GARCH(T)	p_{DM}	0.0488	0.0845	0.2217	0.0390	0.0048	0.0133	0.6523
	p_{MDDM}	0.0509	0.0871	0.2248	0.0409	0.0054	0.0144	0.6540
MRS-GARCH(GED)	p_{DM}	0.0502	0.0850	0.2737	0.0451	0.0056	0.0141	0.5991
	p_{MDDM}	0.0523	0.0876	0.2767	0.0471	0.0063	0.0152	0.6011
APGARCH(N)	p_{DM}	0.3114	0.3590	0.3532	0.2733	0.4357	0.4381	0.5694
	p_{MDDM}	0.3145	0.3619	0.3562	0.2764	0.4384	0.4408	0.5715
APGARCH(T)	p_{DM}	0.0093	0.0123	0.0152	0.0073	0.0020	0.0022	0.0402
	p_{MDDM}	0.0102	0.0134	0.0164	0.0080	0.0024	0.0026	0.0421
APGARCH(T)	p_{DM}	0.0070	0.0095	0.0118	0.0052	0.0018	0.0018	0.0345
	p_{MDDM}	0.0077	0.0104	0.0129	0.0058	0.0021	0.0022	0.0363
EGARCH(N)	p_{DM}	0.1474	0.1123	0.0727	0.1930	0.7176	0.7519	0.0339
	p_{MDDM}	0.1504	0.1151	0.0752	0.1960	0.7191	0.7532	0.0357
EGARCH(GED)	p_{DM}	0.0047	0.0068	0.0085	0.0034	0.0030	0.0030	0.0284
	p_{MDDM}	0.0053	0.0076	0.0094	0.0038	0.0034	0.0035	0.0300
GARCH(N)	p_{DM}	0.4967	0.4592	0.8758	0.5346	0.1890	0.1289	0.2412
	p_{MDDM}	0.4991	0.4618	0.8764	0.5369	0.1920	0.1318	0.2443
GARCH(T)	p_{DM}	0.5321	0.5242	0.8680	0.5450	0.1648	0.1102	0.2533
	p_{MDDM}	0.5344	0.5265	0.8687	0.5473	0.1678	0.1130	0.2564
GARCH(GED)	p_{DM}	0.7746	0.7772	0.6729	0.7743	0.2706	0.1967	0.2026
	p_{MDDM}	0.7757	0.7783	0.6745	0.7755	0.2737	0.1998	0.2057

Loss functions calculated with the Realized Range volatility

Table A8: Diebold-Mariano test Benchmark: GARCH (N)

		MSE_1	MSE_2	$QLIKE$	$R2LOG$	MAD_1	MAD_2	$HMSE$
MRS-GARCH(N)	p_{DM}	0.1275	0.1128	0.2468	0.2742	0.3286	0.1889	0.3706
	p_{MDDM}	0.1304	0.1156	0.2499	0.2773	0.3316	0.1919	0.3734
MRS-GARCH(T)	p_{DM}	0.1317	0.1149	0.2401	0.2770	0.3918	0.2258	0.3210
	p_{MDDM}	0.1346	0.1177	0.2432	0.2800	0.3946	0.2289	0.3240
MRS-GARCH(GED)	p_{DM}	0.0766	0.0908	0.1094	0.1236	0.1948	0.1177	0.3002
	p_{MDDM}	0.0791	0.0935	0.1122	0.1264	0.1979	0.1205	0.3032
APGARCH(N)	p_{DM}	0.2999	0.2876	0.8224	0.3096	0.1101	0.0747	0.3639
	p_{MDDM}	0.3029	0.2906	0.8233	0.3126	0.1129	0.0771	0.3668
APGARCH(T)	p_{DM}	0.3423	0.3564	0.8567	0.3321	0.1033	0.0698	0.3719
	p_{MDDM}	0.3453	0.3593	0.8574	0.3350	0.1061	0.0722	0.3668
APGARCH(T)	p_{DM}	0.5470	0.5700	0.9216	0.5263	0.1774	0.1278	0.2958
	p_{MDDM}	0.5492	0.5721	0.9221	0.5286	0.1804	0.1306	0.2988
EGARCH(N)	p_{DM}	0.4271	0.3997	0.9729	0.4550	0.1557	0.1064	0.2786
	p_{MDDM}	0.4298	0.4024	0.9730	0.4576	0.1587	0.1091	0.2817
EGARCH(T)	p_{DM}	0,4967	0,4592	0,8758	0,5346	0,1890	0,1289	0,4967
	p_{MDDM}	0,4991	0,4618	0,8764	0,5369	0,1920	0,1318	0,4991
EGARCH(GED)	p_{DM}	0.6944	0.7047	0.7546	0.6868	0.2282	0.1653	0.2325
	p_{MDDM}	0.6959	0.7062	0.7558	0.6884	0.2312	0.1684	0.2356
GARCH(T)	p_{DM}	0.8978	0.7621	0.8901	0.9694	0.0123	0.0201	0.5300
	p_{MDDM}	0.8983	0.7633	0.8906	0.9695	0.0133	0.0215	0.5323
GARCH(GED)	p_{DM}	0.0102	0.0089	0.0113	0.0002	0.5983	0.0168	0.0088
	p_{MDDM}	0.0111	0.0098	0.0122	0.0002	0.6003	0.0180	0.0097

Loss functions calculated with the Realized Range volatility

Table A9. Standard deviation of Forecast series.

	APGARCH(N)	APGARCH(T)	APGARCH(GED)	EGARCH(N)	EGARCH(T)	EGARCH(GED)	RR-proxy
Std.dev	0.1189	0.2863	0.1198	0.1432	0.1892	0.1301	1.0488