Phillips Curve and Monetary Policy

Evaluation of an Expectations-Augmented Phillips Curve in Sweden 1997-2011

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Abstract

In this study, an expectations-augmented model of the Phillips curve will be provided for the Swedish market during the period 1997-2011. This will be done for four different measures of the expectation gap. These four measures were acquired by pairing up two measures of inflation (CPI and CPIF) with two measures of inflation expectations (that of the firms and that of employee/employer organisations). The method utilized for this evaluation has been the prediction error method (PEM). Moreover, in order to evaluate the long-term nature of the Phillips curve, different bootstrap methods have been used to provide information regarding the expectation gap in the long run. The overwhelming weight of the results, show that the expectation gap is zeros, and that the long-term Phillips curve is vertical.

Key words: *expectations augmented Phillips curve*, *expectation gap*, *Sweden*, *prediction error method (PEM)*, *bootstrap.*

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AIC	Akaike's Information Criterion
AR	Autoregressive
ARIMA	AR Integrated MA
ARMAX	ARMA with input X
CI	Confidence Interval
CPI	Consumer Price Index
CPIF	Consumer Price Index with a Fixed interest rate
EDF	Empirical Density Function
EEO	Employer/Employee Organisations
FPE	Final Prediction Error
FW-Tests	The Five Whiteness Tests
GARCH	Generalized AR Conditional Heteroskedasticity
LS	Least Squares
MA	Moving Average
MSE	Mean Square Error
NAIRU	Non-Accelerating Rate of Unemployment
NIER (KI in Swedish)	National Institute of Economic Research (Konjunkturinstitutet)
NRH	The Natural Rate Hypothesis
NRU	Natural Rate of Unemployment
OLS	Ordinary LS
PEM	Prediction Error Method
SCB	Statistics Sweden (Statistiska Centralbyrån in Swedish)
SARIMA	Seasonal ARIMA
SARIMAX	SARIMA with input X
VARMAX	Vector ARMAX
WLS	Weighted LS

1 Introduction

In this study, the notion of the Phillips curve and its effect on and relevance for monetary policy will be scrutinized. This will be done by using the theory regarding the short-term and the long-term Phillips curve to analyse the data on inflation, expected inflation and unemployment (short- and long-term) in Sweden during the period 1997-2011 (quarterly measurements).

The outlines of the monetary situation in Sweden is the following. The Swedish Riksbank (the Swedish central bank) has the monetary goal of holding the inflation low and constant at the value of 2 percent with the error margin of one percentage point. In recent years, however, some economists - such as Svensson (2014a,b) - have argued that the Swedish Riksbank has consistently affected an inflation rate under the 2 percent goal (i.e. a negative *inflation gap*), while others - such as Andersson and Jonung (2014) - have argued to the contrary. Furthermore, some of the discussion has focused on whether the expected rate of inflation differs from the actual rate of inflation; that is, whether the Riksbank having set the goal of 2 percent, while allegedly undershooting it, have created a gap between the inflation expectations in the market and the actual rate of inflation (referred to as the *expectation gap* hereafter).

The reason why Svensson, and the economists agreeing with him, view the inflation gap and the expectation gap as a problem, is that they claim that the gaps cause unemployment. The theory, used to motivate this argument is that of the Phillips curve. The original Philips curve, presented by Phillips (1958), asserted a negative relation between inflation and unemployment. Later studies, however, abandoned the original Phillips curve, which was deemed to be simplistic, and replaced it with two curves: the short-term and the long-term Phillips curves. It was theorized, and argued for statistically, that in the short term there is a negative relation between inflation and unemployment rate, while in the long run, there is no relation between the two measures. Hence, the long-term Phillips curve would be vertical.

However, the stagflation incidents of the seventies weakened the notion of a Phillips curve further, and gave rise to the fierce criticism of it by the monetarists, such as Friedman (1968), Lucas (1972) and Phelps (1968). It was hypothesised that there was a so called *natural rate of unemployment (NRU)*, a level of unemployment particular to any economy which is determined by "real", rather than monetary factors, such as level of competition and protectionist policies, etc. This hypothesis was referred to as the *natural rate hypothesis*. The monetarists, then, mainly argued that the negative relation is not between the inflation and the unemployment rate per se, but rather between the expectation gap and the unemployment gap (the gap between the unemployment rate and the natural rate of unemployment). They argued that as the employer and employee organisations get a better understanding of the economic climate, the expected inflation rate approaches the actual inflation rate, rendering the expectation gap close to zero. Furthermore, the employers will consequently know how many employees are needed, while the employees or employee organisations understand how much the demand for their skills is, making the unemployment rate approach the natural rate of unemployment.

Both Svensson (2014a,b) and Andersson and Jonung (2014) adhere to the natural rate hypothesis. However, apart from adhering to different models, they also disagree whether the expectation gap, in the long run, is zero or not. Svensson (2014a, pp. 11-12), argues that the expectation gap is negative, leading to a tilted long-term Phillips curve and a positive unemployment gap. To make his point, he uses CPI-inflation against the households' inflation expectations, and the firms' one year and two years ahead expectations.

Andersson and Jonung (2014, pp. 42-44), on the other hand, point out that different measures of the expectation gap give at best ambiguous results. In other words, there are both positive and negative gaps depending on which measures are used. This is illustrated using CPI- and CPIF-inflation against different measures of expected inflation (employee/employer organisations, firms and households) in different models (Neoclassical and New-Keynesian). Furthermore, they present, different measures of the unemployment during the aforementioned period, and point out that the gap, according to the different measures, can both be positive and negative. This indicates that the conclusion of a positive unemployment gap and a non-vertical (here downward sloping) long-term Phillips curve may be hasty.

As mentioned above, the debate has centred around two issues: the expectation gap (whether it is zero or not) and, subsequently, the nature of the long-term Phillips curve. Regarding the first issue, the debate has been focused on the estimated means of the different measures. This focus, however, lacks a certain perspective regarding the *significance* of the estimates. In other words, no hypothesis testing is done with respect to the estimates. In this study, bootstrap methods are used to add this perspective to the debate. Regarding the second issue, the models used, in other studies, are either exclusively calculated in accordance with New Keynesian or Neoclassical theory, using OLS. This strict adherence may result in neglecting certain aspects of the data, that is theoretically hard to motivate, but significant statistically. In order to avoid this issue, in this study, rather than projecting a rigid theory onto the model, a hybrid version of the Neoclassical model[1] is chosen, where lower and higher lags than the one-year lag is allowed. The particular lags used are motivated by the structure present in the data.[2] Furthermore, the method used will be the prediction error method (PEM). Nevertheless, as the economists above, this study will adhere to the natural rate hypothesis. Moreover, the Phillips curve utilized here is an expectations-augmented one.

In short, the aim of this study is firstly to make a more rigorous evaluation

[1]That is, only backward-looking components.

[2]Such as the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the data.

of whether there is a significant (non-zero) expectation gap during the period 1997-2011 in the Swedish market. Moreover, it will be attempted to produce significant models of the short-term expectations-augmented Phillips curve that explain the expectation gap as a function of the unemployment gap, as in accordance with the monetarist theory. These models, combined with the result from the evaluation of the expectation gap, will be used to comment on the nature of the long-term Phillips curve. Hence, the questions trying to be answered are: For the period 1997-2011 in the Swedish market,

1. Is the expectation gap zero?
2. Is the long term Phillips curve vertical?

Using the evidence provided in this study, it will be argued that one cannot reject the hypothesis that the expectation gap, in the long run, is equal to zero. It will be illustrated that this holds for three of four measures of the expectation gap employed in this study. Furthermore, it will be argued, that the weight of the evidence points to the notion that the long-term Phillips curve is vertical. This will be argued for, using the results mentioned regarding the measures of the expectation gap, together with the reasonably specified models of the short-term expectations-augmented Phillips curve.

2 Theory

2.1 The Phillips Curve

Phillips (1958) found a stable negative relation between the unemployment rate and the rate of change of money-wage rates in the UK (1861-1957). The model was based on Keynes' notions of money-wage dynamics (Phelps, 1968, p. 679). For a period, most of the research focused on the relation between the mentioned measures. However, as more research was done, the theory became more nuanced. For instance, Friedman (1968, pp. 8-9) argued that Phillips had failed to distinguish between *nominal* and *real* wages. Later on, economists, linked the money-wage change to price changes and many tried to clarify the relationship between wage change and price change (Friedman, 1976, pp. 269-71). Furthermore, during the later 60s and the 70s, rates of inflation previously experienced along with low levels of unemployment were manifested along with high levels of unemployment. The notion of *stagflation* - i.e. simultaneously high levels of inflation and unemployment - was brought into economic discourse.

As a consequence, the stability of the Phillips curve came into question by many economists, among them monetarists such as Friedman (1968 and 1976), Phelps (1967 and 1968), Lucas and Rapping (1969). They all maintained that the unanticipated changes in the nominal, and consequently real, measures of wage and/or money, along with long-term labor commitments were at fault. This would result in a lagged adjustment to the unanticipated changes and would depend on the expected changes of prices during the period the commitments correspond to (Friedman, 1976, p. 271). In other words, a discrepancy between the short-term and long-term course of events was deduced.

These insights resulted in a hypothesis that attempted to explain this discrepancy, namely the *Natural Rate Hypothesis (NRH)* (Ibid, pp. 271-274). This hypothesis postulated that there is a *natural rate of unemployment (NRU)* which depends on "real" factors rather than monetary ones. These real factors could be the effectiveness of the market in question, the fluidity and flexibility of the labor market, the level of competition, the existence of protectionist policies, etc. The hypothesis then further states that the deviance from this natural rate of unemployment is then correlated with the unanticipated inflation. Hence, the hypothesis states no stable trade-off between inflation (π) and unemployment (u), as such (i.e. $\pi \propto -u$). Instead, the relation is stated as between two different measures. These measures are two differences: on one hand, the difference between the actual rate inflation (π) and the expected rate of inflation (π^e), i.e. the expectation gap, and, on the other hand, the difference between the actual unemployment rate (u) and the natural rate of unemployment (u^*), i.e. the unemployment gap (in other words, $[\pi - \pi^e] \propto -[u - u^*]$). This new curve, is dubbed "the natural rate", "accelerationist", "expectations-adjusted" or "expectations-augmented Phillips curve". This is the type of Phillips curve adhered to in this study.

The simplest expectations-augmented Phillips-curve, in a time-series format, could be presented in the following way (Svensson, 2014b, p. 9):

$$\pi_t - \pi_t^e = -\beta(u_t - u_t^*) + e_t, \beta > 0 \tag{1}$$

where, π_t is the actual inflation rate; π_t^e the expected inflation; u_t the unemployment rate; u_t^* is the long-term or natural rate of unemployment and e_t a shock or noise process with mean zero ($E[e_t] = 0$) all at time t, or as in this study, quarter t. Furthermore, β is a positive parameter. In the more general cases, however, some lag operators also account for the general delayed response in the market. Even trend and seasonal operators may be included.[3]

Taking the expectation (i.e. the unconditional mean) in (1) will yield the generic long-term Phillips curve:

$$\pi = \pi^e - \beta(u - u^*), \tag{2}$$

where $\pi = E[\pi_t]$, $\pi^e = E[\pi_t^e]$, $u = E[u_t]$ and $u^* = E[u_t^*]$. If the inflation expectations are equal to the inflation rate in the long run (i.e. if the unconditional mean of the expectation gap is zero), then:

$$\pi = \pi^e, \tag{3}$$

and, hence according to NRH,

$$u = u^*, \tag{4}$$

that is, the unemployment rate will be equal to the natural rate of unemployment in the long run, and thus, the long-term Phillips curve would be vertical. However, if (3) does not hold, the long-term Phillips curve will be tilted, negatively sloped to be specific. This would indicate that, in the long run, there is a negative relationship between inflation and unemployment. Furthermore, in general, NRH could be evaluated through the potency of NRU in explaining and evaluating the Phillips curve, particularly in the long run (Gordon, 1997).

There are two main models of the Phillips curve:[4] the New Keynesian (where π_t^e is estimated as $\pi_{t+4|t}^e$) and the Neoclassical (where π_t^e is estimated as $\pi_{t|t-4}^e$), that is:

$$\pi_t - \pi_{t+4|t}^e = -\beta(u_t - u_t^*) + e_t \text{ (New Keynesian model)}, \tag{5}$$

$$\pi_t - \pi_{t|t-4}^e = -\beta(u_t - u_t^*) + e_t \text{ (Neoclassical model)}, \tag{6}$$

[3]See the Method section.

[4]t being quarterly measurements.

Generally, however, there are two main problems with regards to the selection of the measures of inflation expectations: the problem of simultaneity of the measures and the problem of selecting subjects whose inflation expectations are assumed to be representative of the inflation expectations of the economy as a whole.

The simultaneity problem is present in (1) and even more so in the New Keynesian model. In the expectations-augmented Phillips curve in (1) specifically, there exists a simultaneity problem with regards to the inflation expectations, π_t^e. The simultaneity problem indicates that the measure of expected inflation cannot be acquired at the exact time, t, when the actual rate of inflation is retrieved. The reason is the simple fact that the market players, after acquiring the actual rate of inflation at time t, would adjust their expectations in accordance with the inflation rate retrieved.

According to the Neoclassical model, however, the expected inflation is the expected inflation during previous periods, as opposed to the New Keynesian model, according to which it is equal to the future expectations (Andersson and Jonung, 2014, p. 42). In other words, the Neoclassical model postulates backward-looking operators while the New Keynesian model assumes forward-looking ones.[5] Due to the backward-looking nature of the Neoclassical model, it does not have any simultaneity problem. The New Keynesian model, on the other hand, worsens the problem even further. When one forms a prediction of the future inflation rate, one very well bases this forecast on the inflation rate today.

Therefore, in this study, in order to avoid the said simultaneity problem, a hybrid version of Neoclassical model, and hence, only backward-looking components have been chosen.[6] In the hybrid version of the Neoclassical model used in this study, we have allowed even for lags other than merely $t-4$ (i.e. the one year lag) to play a part in forming the inflation expectations. This has been done in order to avoid imposing a strict theory on the data, and letting the structure of the data determine which lags are significant in forming the inflation expectations. In this way, one avoids missing certain characteristics of the data, that may be theoretically complicated or difficult to explain, but are statistically significant nonetheless and, consequently, help explain the behaviour of the Phillips curve better.

The second problem regarding the selection of the measures of the inflation expectations[7] is more difficult to manage. While the choice of economic model

[5]Generally, inflation expectations (π_t^e) are assumed to be a linear combination of both backward- and forward-looking components (Debelle and Vickery, 1997, p. 3).

[6]This choice is furthermore partly motivated by the angle of the study which is focused on the *inertia* in the inflation process, and subsequently the Phillips curve.

[7]That is, the problem of selecting subjects whose inflation expectations are assumed to be representative of inflation expectations of the economy as a whole.

can be motivated by theoretical and practical issues, the choice of the market player who is the most representative of the economy as a whole, and further relevant to the study is less apparent. Indeed, there is a wide range of market players such as the firms, the employer organisations, the employee organisations, the money market players, the households etc. Nonetheless, below some motivations for the choice of the measures of the inflation rate and inflation expectations are presented.

The measures of inflation chosen in this study are the CPI-inflation and the CPIF-inflation. These measures were chosen, since they are commonly cited (e.g. Svensson, 2014a,b; Andersson and Jonung, 2014) and used by the Swedish Riksbank. Moreover, the inflation expectations are that of the firms and of the employee/employer organisations (EEO).[8] The inflation expectations of these market players has been assumed to be a good representative of the economy as a whole. The firms have an immediacy to the market in general, which makes them an interesting agent. The EEO, on the other hand, play a major part in the wage-setting dynamics, affecting the unemployment rate. Furthermore, EEO is one of the most referred-to market players, when theorizing with respect to the Phillips curve (e.g. Friedman, 1968 , pp. 10-13).

Regardless of any assumptions, modelling the expectation gap, measured as the difference between the different rates of inflation (CPI and CPIF) and the measures of the inflation expectations (the firms and EEO) is interesting. This is due the fact that, firstly, the firms and the EEO are significant market players. Moreover CPI- and CPIF-inflation is used by the Swedish Riksbank, which sets the repo-rate and has tremendous influence on the market.

2.2 Relevance for the Choice of Monetary Policy

The nature of the Phillips Curve, provided that it is stable, will obviously have consequences for the monetary policy. There are several reasons why this is the case. The main reason, however, is that unemployment and inflation are both politically significant. Hence, if the pegging of the unemployment rate for more than limited periods is possible, i.e. if the long-term Phillips curve is tilted, then a priority shift in monetary policy could be argued for. Monetarist theory, however, denies that monetary policy could peg the rate of unemployment for more than very limited periods, due to the difference between the immediate and the delayed consequences of monetary policy (Friedman, 1968, p. 5). In other words, any relationship between the inflation rate and the rate of unemployment is accordingly temporary. How long, though, this temporary period will be, is not certain (Ibid, p. 11).[9]

[8] A mean of inflation expectations the employee and the employer organisations, at each quarter, have been chosen as one category.

[9] Another reason for the significance of the theory regarding the Phillips curve, is the phenomenon of hysteresis (Gordon, 1989). Hysteresis is the phenomenon of increasing natural (long-term) rate of unemployment as a result of a period of high rate of unemployment. In other words, there is a tendency that, after a certain period where the unemployment

Accepting the natural rate hypothesis, the question of the nature of the long-term Phillips curve boils down to the nature of the expectations gap. A negative (positive) gap, would result in a higher (lower) rate of unemployment than the natural rate (Andersson and Jonung, 2014, p. 42). Hence, if the market players, disregard the inflation rate being systematically lower than the two percent goal, and peg their expectations at said goal, the unemployment rate would be higher than NRU. In such a case, the Riksbank probably should either address the over-zelous belief in its goal, in order to avoid a higher unemployment rate than NRU, or peg the inflation at the goal it promises.

In this study, no discussion of whether the Riksbank keeps the promise of the two percent goal is mentioned. Instead, the study focuses on the expectation gap, trying to see whether it is zero or not. The results of the study, subsequently, are then interpreted to shed light on the consequences of the monetary policy, and the nature of the long-term Phillips curve.

rate exceeds the NRU, the natural rate is elevated to a higher level. Therefore, if certain monetary policy leads to higher unemployment levels, the economy could be susceptible to a deteriorating unemployment level. This situation is further complicated by the evidence that the hysteresis phenomenon is asymmetrical (Ibid). In other words, while accelerating inflation increases NRU, decelerating inflation and accelerating deflation does not seem to yield a lower NRU. Hence, if hysteresis is irreparable, at least through monetary means, then the actual nature of the Phillips curve, and subsequently, the relation between inflation rate and the unemployment, becomes even more crucial.

3 Data Description

The variables used in this study, as previously mentioned, are *the inflation rate*, *the expected rate of inflation*, *the unemployment rate* and *the natural rate of unemployment (NRU)*[10].

As the inflation rate two measures were chosen: CPI-inflation and CPIF-inflation. They were chosen due to their use by the Swedish Riksbank and them being commonly cited. As the expected rate of inflation two measures were chosen as well: the firms expected inflation rate and the mean of employer and employee organisations' (EEO:s) inflation expectations one year ahead.[11] The unemployment rate and NRU in this study is that of the population between the ages of 16-64 years old in the Swedish market. For NRU, the Swedish Riksbank's measure of *long-term unemployment* was chosen.

The period of 1997-2011 (quarterly measurements[12]) was chosen. The reasons for the choice of the period were threefold. Firstly, it is a recent period. Secondly, it has sparked a topical debate regarding the Swedish Riksbank's adherence to its monetary goals (Svensson 2014a,b and Andersson and Jonung 2014). Thirdly and finally, the period was deemed to be long enough to make a sound estimation with regards to the nature of the long term Phillips curve in the Swedish market, and the expectation gap as well.

The source of the actual rate of inflation (CPI and CPIF) and unemployment rate were Statistics Sweden (SCB)[13]. The long term rate of unemployment was retrieved from the Swedish Riksbank. The expected inflation rate of the firms was provided by the National Institute of Economic Research (NIER)[14]. The EEO:s expected rate of inflation was acquired from the global market information and insight group TNS Sifo Prospera.[15]

[10] Typically measured as the concept of *non-accelerating inflation rate of unemployment (NAIRU)*.

[11] The EEO:s inflation expectations for 2000Q3 was not available, and hence the mean expectations for 2000Q2 and 2000Q4 was used as an estimator.

[12] The measures of CPI-inflation, CPIF-inflation and the unemployment rate provided by the source (SCB) were given as monthly measurements. In order to acquire a quarterly measurement, the mean of each quarter was used for each variable.

[13] Statistiska Centralbyrån in Swedish.

[14] Konjunkturinstitutet (KI) in Swedish.

[15] For more details see the "Sources of the Data" section.

4 Method

4.1 Evaluation of the Long-Term Expectation Gap

The expectation gap, as mentioned before, is the difference between the actual rate of inflation (π_t) and the inflation expectations (π^e_τ). Notice that the time indexes of rate of inflation, t, and that of the inflation expectations, τ, are different. This is due to the simultaneity problem mentioned in section (2.1), according to which the exact relation between t and τ is unknown. The particular aspect of the expectation gap that is of interest in this study, is to see whether the expectation gap is zero in long run or not. Formally, the evaluation of the expectation gap in the long run, could be stated as a hypothesis test:

$$\begin{cases} H_0 : E[\pi_t] - E[\pi^e_\tau] = 0 \\ H_1 : E[\pi_t] - E[\pi^e_\tau] \neq 0 \end{cases} \tag{7}$$

One would argue that the expectation of the expectation gap, expressed as a time-series, could be evaluated directly ($E[\pi_t - \pi^e_\tau]$). However, as mentioned earlier, due to the simultaneity problem (2.1), the relation between t and τ is unknown. Hence, our approach, theoretically as well as in practice, will adhere to the form expressed in (7). In other words, the difference between the unconditional mean of inflation, (CPI or CPIF) and the unconditional mean of inflation expectations (firms or EEO) will be estimated, i.e. $E[\pi_t] - E[\pi^e_\tau]$.

The statistical tool used, in this study, to make this evaluation, is the notion of *bootstrap* (Efron and Gong, 1983). Simply put, bootstraps are tools that aid the research process by providing numerical measures to establish the error or bias of the estimator in question. They are invoked when it is reasonable to doubt whether the number of observations is large enough to guarantee a low enough variability, and hence, a high enough stability of the estimator. The estimator that is of interest for this study, is the confidence interval. Since confidence intervals and hypothesis testing correspond to each other, creating a confidence interval that includes zero, results in the inability to reject the null hypothesis in (7), and vice versa.

There are different kinds of bootstraps. Three kinds of bootstraps have been utilized in this study: *parametric*, *non-parametric* and *semi-parametric* bootstraps. In the parametric method, some distribution is assumed to correspond to the data at hand, while in the non-parametric case no such assumption is made. In the parametric case, the normal distribution was chosen, and hence, the parameters searched for are the *mean* and *standard deviation* of the data. In the non-parametric case, simulations are done from the *empirical distribution or density function (EDF)*. In practice, this amounts to simulating a data set, equal to the size of the data set being studied, by choosing between the observed values, where each observed value is given equal weight. In the semi-parametric case, some variables (here, inflation expectations of firms and EEO) are assumed to come from a certain distribution (here, normal distribution) while the other ones (here, CPI- and CPIF-inflation) are simulated from their respective EDF.

In short, in order to evaluate whether the expectation gap is zero or not, the statistical tool of bootstrap was used to estimate different kinds of confidence intervals. The confidence intervals, in turn, were used to determine whether the null hypothesis in (7) can or cannot be rejected. This was done by pairing all the measures of inflation rate (CPI and CPIF) with all the measures of inflation expectations (firms and EEO) used in this study.

4.2 Selection of Model for the Short-Term Phillips Curve

There is a wide range of possible candidates for the short-term Phillips curve. Scully (1974, p. 387) cites three main Phillips curves in the time domain: (1) *instantaneous versions* (e.g. (1)) (2) *delay versions*, such as the one-period lag (e.g. Svensson, 2014b), and (3) *distributed lag versions.* Further nuances include *linear and non-linear* models, models *with and without* speed limits (e.g. Debelle and Vickery, 1997), and *dynamic and non-dynamic* models. Furthermore, *models with additional or substitutive variables*, such as job growth, have been suggested (Guha and Visviky, 2001, pp. 457-8).

When such wide range of choices are at hand, and while there is no consensus with regards to which model type is the most satisfactory, the task of choosing a suitable model becomes difficult and, to some degree, arbitrary. Nevertheless, this study operates within the purview of Neoclassical economics. Moreover, focus lies on monetarist theory and adherence to the natural rate hypothesis (NRH). Hence, the number of possible and motivated choices will become less. However, some assumptions will be inevitable. The model used in this study will be a *linear non-dynamic distributed lag version of the expectations-augmented Phillips curve, without speed limits and without additional or substitutive variables.* This type of model will be described below.

4.2.1 The Expectations-Augmented Phillips Curve

In (1), the most simple expectations-augmented Phillips curve was introduced. However, generally there is little evidence that only one time period, t, is relevant for the estimation. Hence, we need to introduce some lag operators to the model. The general model could then be expressed by the following form:

$$\begin{cases} A(z)(\pi_t - \pi_\tau^e) = B(z)(u_t - u_t^*) + C(z)e_t \\ A(z) = 1 + a_1 z^{-1} + ... + a_p z^{-p}, \\ B(z) = b_d z^{-d} + ... + b_r z^{-r}, \\ C(z) = 1 + c_1 z^{-1} + ... + c_q z^{-q}, \end{cases} \tag{8}$$

where z^{-1} is the one lag operator, $a_i, i = 1, ..., p; b_k, k = 0, ..., r; c_k, k = 1, ..., q$ are unknown parameters, d is the lag delay of the exogenous input[16], (p, q, r) is model order and $e_t \sim N(0, \sigma)$ is the zero-mean normally distributed noise with standard deviation σ.[17] Furthermore, notice that the π_t^e in (1) has been substituted with π_τ^e, due to the simultaneity problem discussed in (2.1). This model is popularly referred to as an ARMAX-model.[18] The model orders motivated by the behaviour of the data, proved to have close to no MA component (i.e. $C(z) = 1$), and hence ARX-models were produced rather than ARMAX.

In order to solve the simultaneity problem, a relationship between t and τ is needed to be either postulated or found. As mentioned in section (2.1), inflation expectations is assumed to be linearly composed of both backward- and forward looking components (Debelle and Vickery, 1997, p. 3):

$$\begin{cases} \pi_\tau^e = A'(z)\pi_\tau^e + B'^{-1}(z)\pi_\tau^e + C'(z)\eta_\tau \\ A'(z) = a_s z^{-s} + ... + a_{\tilde{p}} z^{-\tilde{p}}, \\ B'(z) = b_l z^{-l} + ... + b_{\tilde{q}} z^{-\tilde{q}}, \\ C'(z) = 1 + c_1 z^{-1} + ... + c_{\tilde{r}} z^{-\tilde{r}} \end{cases} \tag{9}$$

where s and l are respectively the past and future delays, $(\tilde{p}, \tilde{q}, \tilde{r})$ is the model order, and η_τ is a noise process with zero mean ($E[\eta_\tau] = 0$). In such a case, then π_τ^e is taken to be represented by:

$$\hat{\pi}_t^e = A'(z)\pi_t^e + B'^{-1}(z)\pi_t^e \tag{10}$$

As mentioned in section (2.1), this study is conducted within the purview of a hybrid version the neoclassical model. Hence, only backward-looking components have been used, i.e.

$$\hat{\pi}_t^e = A'(z)\pi_t^e \tag{11}$$

This is equivalent with finding an AR-model for the inflation expectations.[19]

Therefore, the model in (8) was ultimately estimated as the following hybrid ARX-model:

$$\begin{cases} A(z)(\pi_t - \hat{\pi}_t^e) = B(z)(u_t - u_t^*) + e_t \\ \hat{\pi}_t^e = A'(z)\pi_t^e \end{cases} \tag{12}$$

where $A(z)$ and $B(z)$ are as described in (8) and $A'(z)$ is given by (9). The precise nature of these polynomial lag operators, is of course determined by the behaviour present in the data. In this study two different measures of

[16]If $d = 0$, the input's influence is not delayed.

[17]One should be aware of the fact that ergodicity, and hence stationarity, is assumed in this system. Furthermore, it should be noticed by the reader that linearity in parameters, and an absence of any non-linear transform has been assumed as well.

[18]One could generalize the ARMAX-model to a SARIMAX-model. However, any SARIMAX-model, ultimately could be reduced to some ARMAX-model. Hence, for the sake of simplicity, such generalization was deemed unnecessary, and therefore, not explicated.

[19]One could also find ARMA-models for the inflation expectations. This was done as well. However, the AR-models were much more significant, and subsequently were adopted and preferred.

the inflation rate (CPI and CPIF), are paired with two different measures of the inflation expectations (firms and EEO), which amounts to four different measures of the expectation gap ($\pi_t - \hat{\pi}_t^e$). The particular estimations retrieved by this author, are presented in the "Results and Analysis" section.[20]

4.2.2 Model Order Identification

Model order identification is a difficult task which is partially skill-based. However, there are statistical tools and methods aiding the process. The main tools of selecting the model order in the linear case are the *autocorrelation function (ACF)* and the *partial autocorrelation function (PACF)* (Jakobsson, 2013). Furthermore, certain tables and algorithms are used to identify the so called transfer function for the ARMAX model. The details of these methods will not be discussed here however, due to the wide range of mathematical and statistical background needed to explain them properly.[21]

4.2.3 Estimation: the Prediction Error Method (PEM)

Estimation, in this study, was done through the prediction error method (PEM).[22] In PEM, instead of minimizing the sum of the squared residuals between observed and fitted values, the sum of the difference between the squared residuals between observed values and first step predictions (i.e. the first step prediction errors) given the model are minimized (Jakobsson, pp. 155-8). The minimization is done according to some minimization criteria i.e. some norm.

In this study, the L^2-norm was chosen as the minimization criteria. This choice was primarily due to its wide use and familiarity. It is namely the same norm used in OLS. Secondly, the use of other norms such as the L^1-norm, the supremum norm (uniform norm or infinity norm) or weighted norms (for instance WLS) either could not be motivated, as there was no theoretical ground to discriminate between different input variables, or the motivations were deemed to be weak or *ad hoc*.

The PEM and the usual LS estimates (i.e. the ones using the fitted values) will coincide if the regressor (the independent variable) is uncorrelated with the measured noise values (the residuals). However, the PEM estimates generally have lower variance than the ones calculated through LS. this was the main reason for choosing PEM over the usual LS-methods of estimation.

[20]The difference between the π_t^e measures and the corresponding $\hat{\pi}_t^e$ estimated in this study, was also calculated. The difference was not significantly different from zero (95% significance).

[21]For more information see references of the time-series analysis field, such as Jakobsson (2013).

[22]The program used in order to conduct the calculations was the technical and numerical environment MATLAB.

4.2.4 Criteria for the Selection of a Specific Model

In order to make the comparison of the differently specified models meaningful, one should specify some motivated criteria, according to which the models are selected. Otherwise, one is susceptible to comparing categorically different models. The criteria used for the selection of the models, in this study, were the following:

1. No correlation between the input (the regressor) and the residuals;
2. Whiteness of the final residual of the model and all mediate residuals in the steps leading to the final estimation of the model. The whiteness was evaluated by the satisfaction of *all* the following five whiteness tests: the *Ljung-Box-Pierce test*, the *Monti test*, the *McLeod-Li test*, the *sign change test* and the *cumulative periodogram test*.[23] All the tests were conducted at 5% rejection significance;[24]
3. Significance of *at least* one standard deviation ($\pm\sigma$) of *all* the coefficients involved (about 70% significance)[25];
4. *Once all three previous criteria are met*, the model with the lowest possible *final prediction error (FPE)* among the models motivated by statistical analysis has been chosen.[26]

The criteria above were used to identify significant models, with motivated orders, for the four different measures of expectation gap utilized in this study. After finding these models, the comparison between them was done using different measures of fit, but mainly R^2 (coefficient of determination), adjusted R^2 and mean square error (MSE).

4.3 Methodological Framework: a Summary

Using the methods mentioned, the general methodology boils down to the following steps:

First, it will be evaluated whether the expectation gap is non-zero in the long run. This is done primarily through employing *bootstraps* via Monte-Carlo simulation of data.

Secondly, attempts will be made to construct expectations-augmented Phillips curve in the short run. This will be done through pairing different measures of inflation (CPI and CPIF) with different measures of inflation expectations (firms and EEO).

[23]These five tests will hereafter be referred to as the *Five Whiteness Tests* (FW-Tests).

[24]See Jakobsson (2013, pp. 176-180) for details of the FW-tests.

[25]Some of the coefficients have, of course, better significance as mentioned in the *Results and Analysis* section.

[26]FPE is an information criterion which is used for model order selection: $FPE(l) = \hat{\sigma}^2_{e,l}\frac{1+l/N}{1-l/N}$, where l is the model order, $\hat{\sigma}^2_{e,l}$ is the variance of the error residuals of the model with order l and N is the number of observations (Jakobsson, 2013, pp. 172-173.). It can be be compared to Akaike's Information Criterion (AIC): $AIC(l) = N\ln(\hat{\sigma}^2_{e,l}) + 2l$.

Thirdly, if a "good" (significant) short-term expectations-augmented Phillips curve model is constructed, the existence of such model is taken as consistent with the absence of a long run inflation-unemployment trade-off, both in theory and in practice.[27]

The reader is urged to make note of the line of argument above, due its frequent use hereafter.

[27] Similar line of reasoning has been evoked by Lucas and Rapping (1969, p.349).

5 Results and Analysis

5.1 Analysis of the Expectation Gap

The aim here, is to evaluate whether the expectation gap is zero in the long run or not. This is done by the use of bootstraps. In section (4.1), it was mentioned that the different kinds of expectation gaps were studied by three types of bootstraps: parametric, non-parametric and semi-parametric. Moreover, it was mentioned that in the parametric case, a normal distribution was assumed for all the variables involved (inflation rate and inflation expectations). Furthermore, in the semi-parametric case there was a discrepancy in which variables were assumed to be normally distributed. Namely, while inflation expectations (firms and EEO) were assumed to be normal, no such such assumption was made for the inflation rate (CPI and CPIF). In other words, in the semi-parametric case, different distributions are used for the different types of measures. These assumptions will be argued for below, before the actual findings of the bootstraps are presented and analysed.

So, there is a need to evaluate the nature of the distributions of the different measures. In order to conduct said evaluation, we first look at the EDFs of the different types of inflation rate and inflation expectations in figure (1). One notices that CPI-inflation has larger spread (standard deviation) while the other measures are more compact. Moreover, the firms seem to under- and overestimate CPI-inflation equally large, but very largely at that. This could be due to the fact that the backward- and forward-looking price setting agents are biased upwards (Russell, 2011, p. 416). Another interesting observation is the fact that the biggest difference in mean, is between CPI-inflation and the inflation expectations of EEO. Furthermore, the EDFs of both CPI-inflation and CPIF-inflation are partly *bulked*, which could be a sign of innovation in the random process. It could be due to the financial crisis of 2008. Hence, a problem arises. If we are to assume normality of the data, one should evaluate whether such assumption is justified or not. In other words, the mentioned bulky nature of the EDFs of the inflation measures, may make the assumption that the data is normally distributed, problematic.

In order to conduct an evaluation of the distributions of the data, one can compare the EDF with the distributions fitted to the data. In figure (2), we can see the EDF of each measure, together with the, to each data set, fitted normal density function and fitted t-location-scale density function. First of all, one can see that the fitted t-location-scale density functions do not offer significant improvements over the normal ones, why they were discarded. Furthermore, one notices that while the measures of inflation expectations are fairly normally distributed, the measures of inflation rates are much less so. The most discrepancy is observed for CPI-inflation. Hence, inflation expectations of the firms and EEO could be assumed to be normally distributed, while the CPI- and CPIF-inflation not so. This motivates the use of asymmetric semi-parametric

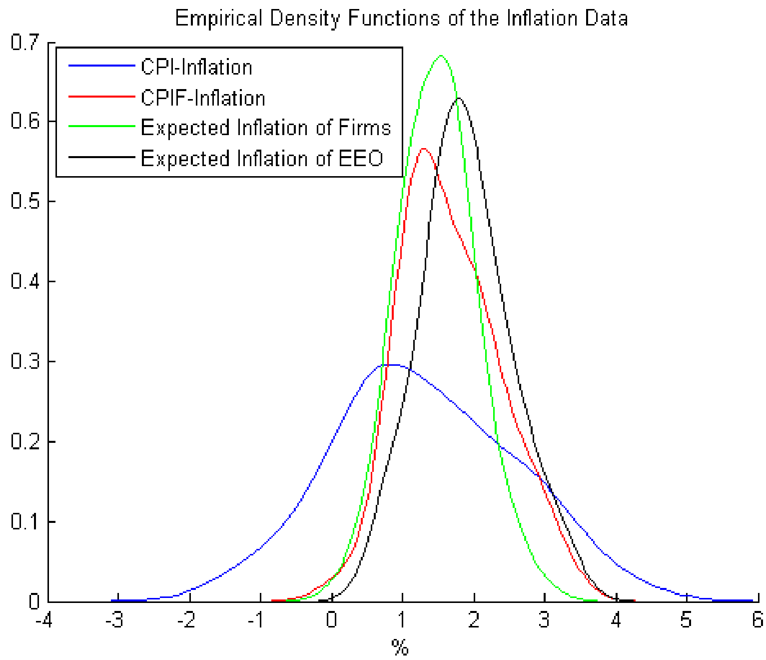

Figure 1: Empirical density functions of different inflation data.

bootstraps, where inflation expectations are assumed to be normally distributed, while no such assumption is made for the measures of the inflation rate. These bootstraps are completed of course by both completely parametric and completely non-parametric bootstraps. Let's then move on to the results provided by the confidence intervals calculated by means of the bootstraps mentioned.

In table (1), we see the evaluation of the confidence intervals via the use of the different kinds of bootstraps.[28] Notice that the confidence intervals are constructed for both 5% and 1% rejection significance. One notices that the results of the confidence intervals for each measure of the expectation gap individually and for all bootstrap methods as a whole, agree with each other. Three measures of expectation gap are deemed not to be significantly different from zero. These include expectation gaps measured via the difference of CPI- and CPIF-inflation and the inflation expectations of firms, and the expectation gaps measured via the difference of CPIF-inflation and the inflation expectations of EEO. This is due to the fact that the confidence intervals include zero. Hence, the null hypothesis in (7) for these measures cannot be rejected. In other words, for these measures, the expectation gap is not significantly different from zero in the long run.

Nevertheless, the expectation gap measured via the difference of CPI-inflation

[28]All the bootstraps in table (1) are results of one million Monte-Carlo simulations.

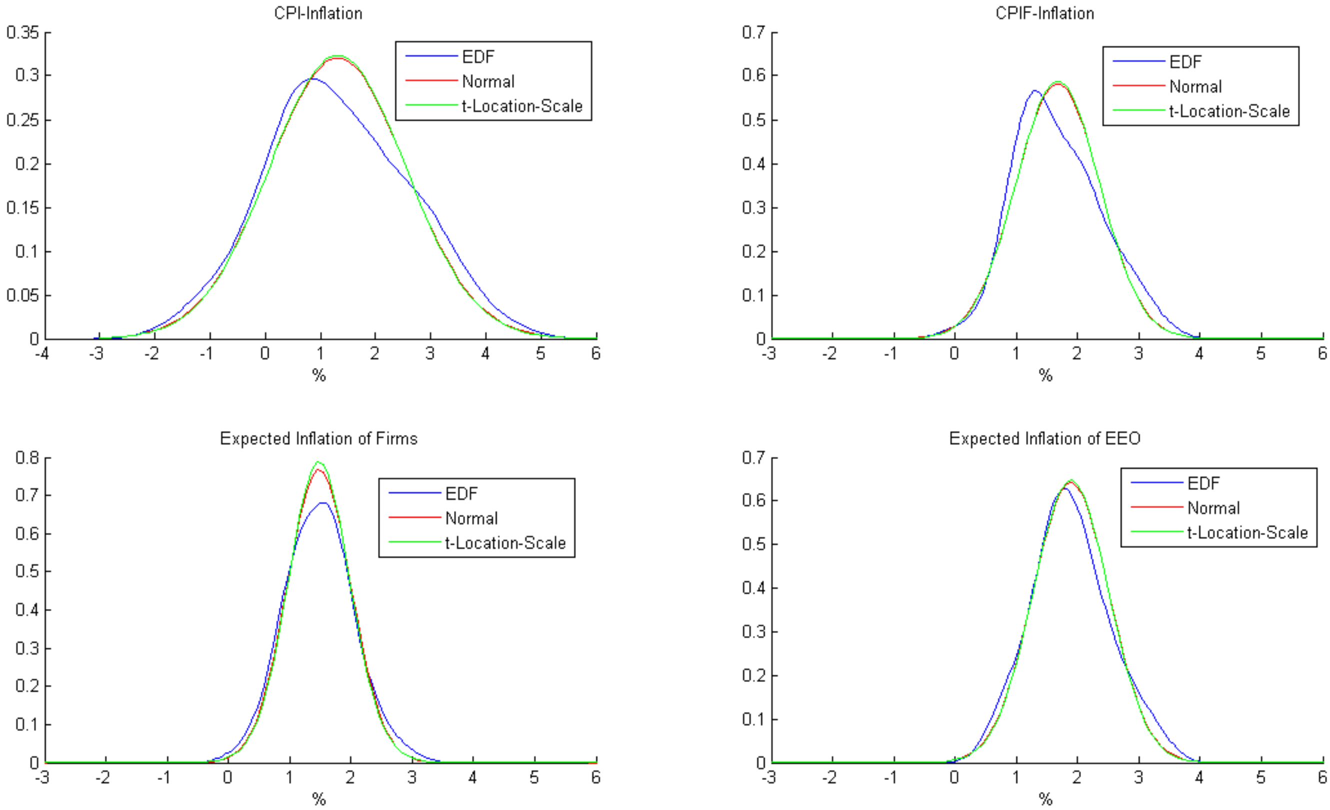

Figure 2: Different inflation data and fitted densities. Each plot includes the EDF of the data, a fitted normal density function and a fitted t-location-scale density function.

and the inflation expectations of EEO is significantly different from zero, and negative at that. This follows from the fact that the confidence intervals for this case, do not include zero and is exclusively negative. Hence, the null hypothesis in (7) for this measure cannot be rejected.

We summarize the results found. In the long run, three measures of expectation gap are deemed zero, while one measure not. Simply put, the results state that firms, in general, forecast the actual inflation rate (CPI and CPIF) well, or at least underestimate and overestimate them equally often and equally much. The EEO, however, underestimate the CPI-inflation, while forecasting CPIF-inflation well. Only the EEO underestimating the CPI-inflation, provides a possibility for a non-vertical Phillips curve. This result, however, does not necessarily pose a great problem. There needs to be a significant model of the short-term expectations-augmented Phillips curve. Otherwise, a correlation is stated between two measures (i.e. the expectation gap and the unemployment

	CPI-Firms	CPIF-Firms	CPI-EEO	CPIF-EEO
Non-par.				
95% CI	[-0.5067,0.1711]	[-0.0228,0.4078]	[-0.9249,-0.2253]	[-0.4471,0.0165]
99% CI	[-0.6139,0.2756]	[-0.0911,0.4745]	[-1.0349,-0.1171]	[-0.5214,0.0882]
Par.				
95% CI	[-0.5082,0.1751]	[-0.0240,0.4109]	[-0.9278,-0.2238]	[-0.4482,0.0195]
99% CI	[-0.6164,0.2823]	[-0.0920,0.4791]	[-1.0383,-0.1132]	[-0.5219,0.0927]
Semi-par.				
95% CI	[-0.5072,0.1715]	[-0.0245,0.4080]	[-0.9254,-0.2261]	[-0.4481,0.0167]
99% CI	[-0.6148,0.2756]	[-0.0934,0.4746]	[-1.0362,-0.1176]	[-0.5220,0.0893]

Table 1: Bootstrap confidence intervals for the mean difference between different measures of inflation and expected inflation. The bootstraps, are non-parametric, parametric (normally fitted), and semi-parametric (normally fitted for expected inflation of firms and all players, and non-parametric for CPI- and CPIF-inflation).

gap) that do not have any such significant relation. In other words, if the non-zero expectation gap, measured as the difference between CPI-inflation and the inflation expectations of the EEO, does not have any significant relation with the unemployment gap, then it is not prudent to make any deduction with respect to the long-term Phillips curve based on such a model. In the following sections, the question will be answered, whether such models are reasonably specified.

5.2 Empirical Models

In this section, the empirical models found will be presented. Firstly, the simultaneity-treated measures of inflation expectations ($\hat{\pi}_t^e$ in (11)) will be presented, both for the expected inflation of the firms and EEO. Thereafter, the models of the short-term expectations-augmented Phillips curve will be illustrated. Here, different measures of the expectation gap will be modelled. These distinct measures have been attained by the pairing of the different measures of simultaneity-treated inflation expectations ($\hat{\pi}_t^e$ of the firms and EEO) with the different measures of inflation (CPI and CPIF). Afterwards, some comments are made with regards to the nature of the long-term Phillips curve.

5.2.1 Inflation Expectations

As mentioned in sections (2.1) and (4.2.1), we need to treat the original inflation expectations data (π_τ^e) for the simultaneity problem. The model, that would perform said treatment, was presented in (11). This model has the backward-looking components corresponding to hybrid version of the Neoclassical model. Since two measures of inflation expectations (the firms and EEO) were used, two of such models was produced. The details, and exact nature, of these models is presented in table (2).[29]

[29]Observe that all the measures of fit, in this study, have *Prediction focus*, which means that the measures of fit are based on the one step predictor, as opposed to the fitted values.

One can observe that both simultaneity-treated measures of the inflation expectations are expressed as the prior one and two lag observations of the respective data series. Both models provide good measures of fit. For instance, the adjusted R^2 of the models are between 30-36%. Furthermore, both models have high significance of the first lag observation (π^e_{t-1}). The second lag observation (π^e_{t-2}) of the two models, however, differ a bit in significance. Their significances are still relatively high. Moreover, all the criteria mentioned in section (4.2.4), for the selection of a model, are satisfied.

These models have been utilized to provide different measures of the expectation gap, denoted by:

$$\Delta\pi^e_t = \pi_t - \hat{\pi}^e_t, \tag{13}$$

where the measures of inflation (π_t) are either CPI- or CPIF-inflation. These measures of expectation gap, in their turn, are modelled as an expectations-augmented Phillips curve against the unemployment gap, denoted by:

$$\Delta u^*_t = u_t - u^*_t, \tag{14}$$

where u_t is the actual unemployment rate, and u^*_t is the natural rate of unemployment.

	Firms	EEO
π^e_{t-1}	-0.9250 (0.1306) [0.0000]	-0.9289 (0.1311) [0.0000]
π^e_{t-2}	0.2611 (0.1302) [0.0496]	0.2043 (0.1312) [0.1248]
Fit		
FPE	0.1343	0.1644
MSE	0.1213	0.1484
R^2	33.00%	38.00%
Adj. R^2	30.65%	35.82%

Table 2: Models of different measures of inflation expectations. In the parentheses the standard deviation and in the brackets the p-values of the coefficients are mentioned. All the measures of fit have prediction focus.

5.2.2 The Short Run

The simultaneity-treated measures of inflation expectations modelled in the previous section, which are presented in table (2), have subsequently been used to retrieve models of the expectations-augmented short-term Phillips curve, as mentioned in (12). One should also be aware that this should be done by pairing the different measures of inflation (π_t), i.e. CPI- and CPIF-inflation, with the distinct measures of inflation expectations ($\hat{\pi}^e_t$), i.e. Firms' and EEO:s, in order to attain four different measures of the expectation gap, i.e. $(\Delta\pi_t^e)$ defined in (13). As mentioned before, these measures of expectation gap, in their turn, are modelled against the unemployment gap, i.e. (Δu_t^*) described in (14).

The different models of the short-term expectations-adjusted Phillips curve are presented in table (3).[30] Note these models all satisfy the criteria mentioned in section (4.2.4). The said criteria are the reasons why these particular models, with their specific orders, were chosen. Furthermore, note that the pairing of the different measures of the inflation rate and the inflation expectations has resulted in four models of the short-term Phillips curve, denoted as CPI-Firms, CPIF-Firms, CPI-EEO and CPIF-EEO. For the last measure two models were provided: CPIF-EEO (I) and CPIF-EEO (II).

One can observe that of the four measures of expectation gap, two were found to be reasonably specified, namely CPI-Firms and CPIF-EEO (I and II). The CPI-Firms correlation with the unemployment gap was, however, more significant than that of CPIF-EEO. The p-value of the unemployment gap in CPI-Firms is equal to 0.0620. Nevertheless, increasing the model order will further improve the significance of the correlation between the gaps of expectation of CPIF-EEO and unemployment. This can be observed if one compares the p-values of the unemployment gap in the two models of the expectation gap CPIF-EEO (I) and (II) in table (3). In (I) the p-value is equal to 0.1059, while in (II) it is 0.0990. This improvement, though, comes at the cost of deteriorated fit, e.g. lower adjusted R^2 (from 19% to 12%). All in all, in these two versions of the expectation gap (CPI-Firms and CPIF-EEO) the significance of the correlation with the unemployment gap is of the order of 90%. However, the CPI-Firms-measure of the gap produces a short-term Phillips curve with better fit, e.g. an adjusted R^2 of almost 36%.

The other two measures of expectation gap (CPIF-Firms and CPI-EEO), however, do not produce that much significant correlation with the unemployment gap. The p-values of the unemployment gap there is between 0.15 and 0.25. In other words, the significance is lower than 85%; and lowest at 75%.[31] The measures of fit for them are not that good either.

5.2.3 The Long Run

As the reader recalls, in section (5.1), we found that three measures of the expectation gap seem to be zero in the long run. These measures included CPI-Firms, CPIF-Firms and CPIF-EEO. This was achieved by a quite high significance, namely 99%. For these measures, the expectation (unconditional mean) would be equal to zero, i.e. $E[\pi_t - \hat{\pi}_t^e] = 0$. Hence, taking the uncondi-

[30]For complete models, including all lags up to lag 9 of the expectation gap, see tha table in the Appendix.

[31]These numbers may seem still high. However, this is achieved by design. Furthermore, a significance of about 70% was guaranteed since one of the criteria mentioned in section (4.2.4) - that all the models in this study should satisfy - was that very level of significance (criteria 3). With a rejection level of 10%, these models would have been rejected.

	CPI-Firms	CPIF-Firms	CPI-EEO	CPIF-EEO(I)	CPIF-EEO(II)
$\Delta\pi^e_{t-1}$	-0.9324 (0.1354) [0.0000]	-0.6165 (0.1148) [0.0000]	-0.9920 (0.1610) [0.0000]	-0.7765 (0.1368) [0.0000]	-0.7463 (0.1434) [0.0000]
$\Delta\pi^e_{t-2}$	0.2404 (0.1410) [0.0937]		0.3205 (0.2019) [0.1183]	0.2607 (0.1481) [0.0839]	
$\Delta\pi^e_{t-3}$			-0.4185 (0.2002) [0.0414]	-0.2809 (0.1476) [0.0624]	
$\Delta\pi^e_{t-4}$		0.1589 (0.118) [0.1837]	0.6996 (0.2030) [0.0011]	0.6575 (0.1482) [0.0000]	0.6539 (0.1772) [0.0006]
$\Delta\pi^e_{t-5}$			-0.4202 (0.1564) [0.0096]	-0.4058 (0.1378) [0.0047]	-0.4985 (0.1883) [0.0108]
$\Delta\pi^e_{t-8}$					0.2964 (0.1793) [0.1045]
$\Delta\pi^e_{t-9}$					-0.2282 (0.1742) [0.1962]
Δu^*_{t-1}	-0.2311 (0.1214) [0.0620]	-0.1065 (0.09085) [0.2462]	-0.1834 (0.1376) [0.1884]		
Δu^*_{t-3}			-0.2010 (0.1386) [0.1530]	-0.1098 (0.0668) [0.1059]	-0.1015 (0.06038) [0.0990]
Fit					
FPE	0.4547	0.3547	0.7598	0.3209	0.4009
MSE	0.3898	0.2803	0.4542	0.2193	0.2220
R^2	39.11%	17.55%	28.63%	27.37%	26.92%
Adj. R^2	35.85%	9.92%	17.43%	19.14%	12.01%

Table 3: Models of different measures of expectation gap as a function of unemployment gap. In the parentheses the standard deviation and in the brackets the p-values of the coefficients are mentioned. All the measures of fit have prediction focus.

tional mean on the short-term Phillips curve, i.e. on both sides of the equation in the first line of (12), the following vertical long-term Phillips curve will be yielded:

$$E[A(z)(\pi_t - \hat{\pi}_t^e)] = E[B(z)(u_t - u_t^*) + e_t] \tag{15}$$

$$\Rightarrow (1 + \sum_{k=1}^{p} a_k) E[(\pi_t - \hat{\pi}_t^e)] = (\sum_{k=d}^{r} b_k) E[(u_t - u_t^*)] + E[e_t] \tag{16}$$

$$\Rightarrow 0 = (\sum_{k=d}^{r} b_k) E[(u - u^*)] + 0 \tag{17}$$

$$\Rightarrow u = u^* \tag{18}$$

where, $u = E[u_t]$, $u^* = E[u_t^*]$ and $b_k, k = d, ..., r$ and $a_k, k = 1, ..., p$ are the (estimated) parameters of the lag polynomials $A(z)$ and $B(z)$ respectively. Notice that $E[e_t] = 0$ since it is the noise process.[32]

One measure of the expectation gap, on the other hand, was found to be significantly negative, namely CPI-EEO. For this measure, the left hand side of (12) will not be unequivocally zero. Hence, according to this measure of the expectation gap, the long-term Phillips curve will be negatively sloped:

$$E[A(z)(\pi_t - \hat{\pi}_t^e)] = E[B(z)(u_t - u_t^*) + e_t] \tag{19}$$

$$\Rightarrow \pi = \hat{\pi}^e + \frac{\sum_{k=d}^{r} b_k}{1 + \sum_{k=1}^{p} a_k}(u_t - u_t^*) \tag{20}$$

where u, u^*, b_k and a_k as above, $\pi = E[\pi_t]$ and $\hat{\pi}^e = E[\hat{\pi}_t^e]$.

Using the line of argument above, the long-term Phillips curves, resulting from the short-term models in the previous section, are mentioned in table (4). However, as the results in the previous section have indicated, only two measures of the expectation gap produced significant models of the short-term expectations-augmented Phillips curves (CPI-Firms and CPIF-EEO in table (3)). Hence, we can judge the nature of the long-term Phillips curve solely with respect to these models, since it is only theoretically and practically prudent to base judgement on short-term curves that manifest a significant relation between the expectation gap and the unemployment gap. Otherwise, one would risk to draw conclusions based on weak foundations.

This means that of the five long-term Phillips curves mentioned above, only three are reasonably specified. We hence discard the two other ones: one vertical curve expressed by the expectation gap measure of CPIF-Firms, and one negatively sloped curve expressed by the expectation gap measure of CPI-EEO. For the other models of the expectation gap (CPI-Firms, CPIF-EEO (I) and (II)), the case is different. As argued by Lucas and Rapping (1969, p. 349), when the expectation gap is calculated as zero in the long run, the existence of a significant short-term expectations-augmented Phillips curve should be taken as

[32]This line of thought is more elaborately explained in section (4).

	Long-Term Expectations-Augmented Phillips Curve
CPI-Firms	$u = u^*$
CPIF-Firms	$u = u^*$
CPI-EEO	$\pi = \pi^e - 0.6790(u - u^*)$ where, $\pi^e = 1.8879, u^* = 7.0808$, hence: $\pi = 6.6960 - 0.6790u$
CPIF-EEO (I)	$u = u^*$
CPIF-EEO (II)	$u = u^*$

Table 4: The long-term models resulted from the short-term models in table (3). These models where calculated by taking the unconditional mean on the models found in table (3).

consistent with the absence of a trade-off between inflation and unemployment in the long run, both in theory and in practice.[33] In other words, for these measures of the expectation gap, the long-term Phillips curve is vertical.

All in all, the conclusion is that the reasonably specified models of the short-term Phillips curves, together with the results if the hypothesis testing regarding the measures of expectation gap they are based on, result in the evidence that in Sweden 1997-2011, the long-term Phillips curve is vertical. The reasonably specified models were those incorporating CPI-Firms and CPIF-EEO as the measures of the expectation gap. Moreover, the hypothesis testing with respect to these measures, resulted in not being able to reject the null hypothesis, i.e. the expectation gap being equal to zero. Hence, the evidence support the claim that the market has adapted its inflation expectations to the topical developments in the market, and has not pegged its expectation at the two percent goal. Consequently, the evidence points to the notion that the Swedish Riksbank has not affected any excess unemployment.

[33]This line of argument was mentioned in section (4.3).

6 Conclusions

Despite the fact that the results are a bit ambivalent, most of the weight of the evidence support the three following statements:

Firstly, the expectation gap is zero in the long run. That was the case for three of four measures of the expectation gap (CPI-Firms, CPIF-Firms and CPIF-EEO). Only in one case (CPI-EEO) the expectation gap was negative.

Secondly, the short-term expectations-augmented Phillips curve has explanatory power with regards to the relationship between the expectation gap and the unemployment gap. Two of four measures of the expectation gap (CPI-Firms and CPIF-EEO) stated 90% of significance for the correlation between the two said gaps. The other two stated lower significance though.

Thirdly, due to the two previous pieces of evidence, the long-term Phillips curve is vertical, rather than being negatively sloped. This is concluded by pairing, on one hand, the evidence of the expectation gaps, CPI-Firms and CPIF-EEO, being zero, with, on the other hand, the evidence of these measures of expectation gap providing reasonably specified models of the short-term expectations-augmented Phillips curve. The other long-term curves are discarded, due to their short-term counterparts not supporting a significant relation between the respective measures of expectation gap (CPI-EEO and CPIF-Firms) on one hand, and the unemployment gap on the other.

Note that all these statements, and hence most the weight of the evidence gathered, agree with NRH and monetarist theory, operating within the hybrid version of the Neoclassical model, containing merely backward-looking components.[34] Furthermore, notice that the results provide evidence for the notion that the Swedish Riksbank has not affected the unemployment rate.[35]

[34]Before moving on, however, this author would like to remind the reader of a crucial point. All the results, with respect to the expectations-augmented Phillips curve, can only be stretched so far as to indicate *correlation* between the gaps of expectation and unemployment, and *not causality*. In order to deduce causality, one needs to set up much finer conditions which has not been done here, and their elaboration lies outside the scope of a bachelor thesis.

[35]Observe, however, that whether the Riksbank, during the period studied, has undershot its two percent goal regarding inflation is another question, and has not been treated here.

7 Future Studies

The straightforward conclusion seem to be that the Swedish Riksbank has not affected the unemployment rate. Indeed, if the long-term Phillips curve is vertical, then the Riksbank, utilizing the interest rate and hence affecting the inflation rate, will not have influenced the unemployment rate. There are, however, certain measures one could take to make the conclusions stronger yet.

For one thing, the EDF of the inflation expectations of the firms and the CPI-inflation, despite having the same mean, have very different spread and variance (as witnessed by figure (1)). Whether this discrepancy in the magnitude of the inflation expected affects the unemployment rate is indeed theoretically ambiguous. However, a rush to the judgement that said discrepancy in the spreads, is irrelevant may be too much of wishful thinking and indeed hasty.

Another factor that needs attention is the wide range of theoretical and formal assumptions made. For instance, Russell (2011, p. 417) points out that "...nearly all of the empirical work on the 'modern' Phillips curve fails to adequately account for the shifts in the mean rates of inflation." In other words, stationarity (or as in most cases, ergodicity) is assumed when not motivated. Furthermore, there is evidence suggesting that the expectation horizon affect the slope of the Phillips curve (Lee and Nelson, 2007, p. 176). This issue has not been tackled here. Moreover, the instability of the statistical Phillips curves, and the number of unknown variables shifting the curve, is great enough to diminish the ability to base any policy decision on it (Lucas and Rapping, 1969, p. 349). However, as Lucas (1976, p. 41) points, this last point goes both ways. Just as basing monetary policies on unstable models would be a great folly, so would evaluating them.

Hence, there are several other interesting ways by which one could pursue the topic of the Phillips curve further. For instance, there seems to be some correlation between the raw expected inflation data (π_t^e) and the unemployment gap (Δu_t^*). This motivates using multivariate models such as VARMAX. A related topic is further pursuing the question whether non-linear relations provide better description of the relation between unemployment gap and expectations gap (e.g. Debelle and Vickery, 1997; Hasanov, Arac and Telatar, 2010). Using GARCH models have proved promising (e.g. Russell and Chowdhury, 2013). The issue of breaks and shifts in the Phillips curve, for instance in cases of crises such as financial ones, is very relevant as well (Ibid).

One could also study the nature of the so-called *curvature* of the Phillips curve (i.e. determine whether the shape is linear, concave or convex), and incorporate that information into the model selection process (Hasanov, Arac and Telatar, 2010). Another interesting perspective would be using models that incorporate speed limits (Debelle and Vickery, 1993). Furthermore, one could alter the model describing the formation of expectations, for instance use both forward- and backward-looking components (Toshihisa, 1972, p. 269). Even if these models, borrow from both the Neoclassical and New Keynesian models, and hence may be theoretically sketchy, they seem to show much promise. A last suggestion would be using dynamic modelling, for instance using Kalman filters (Debelle and Vickery, 1997). All the suggestions mentioned should also be analysed having the historical perspective of the modelling approaches in mind (Qin, 2010).

8 Sources of the Data

Measure	**Source**
CPI-Inflation	Statistics Sweden (SCB)
CPIF-Inflation	Statistics Sweden (SCB)
Inflation Expectations of the Firms	National Institute of Economic Research (NIER) (Konjunkturinstitutet (KI) in Swedish)
Inflation Expectations of the EEO (one year ahead)	TNS Sifo Prospera (The reports on the inflation expectations 1997-2011)
Unemployment Rate (16-64 years old)	Statistics Sweden (SCB)
Long-Term (Natural) Rate of Unemployment (16-64 years old)	The Swedish Riksbank (Penningpolitisk Rapport October 2010)

9 References

Andersson, Fredrik N. G.; Jonung, Lars (2014), *Riksbanken och inflationen 1995-2012 - missar Svensson målet?* Ekonomisk Debatt, Number 3, pages 36-48.

Debelle, Guy; Vickery, James (1997), *Is the Phillips Curve a Curve? Some Evidence and Implications for Australia.* Reserve Bank of Australia, Research Discussion Papers, Number RDP9706.

Efron, Bradley; Gong, Gail (1983, February), *A Leisurely Look at the Bootstrap, the Jackknife, and Cross-Validation.* The American Statistician, Volume 37, Number 1, pages 36-48.

Friedman, Milton (1968, March), *The Role of Monetary Policy.* American Economic Review. Volume 58, Number 1, pages 1-17.

Friedman, Milton (1976, December), *Inflation and Unemployment.* Nobel Memorial Lecture, The University of Chicago, Illinois, USA.

Gordon, Robert J. (1989, May), *Hysteresis in History: Was There Ever a Phillips Curve?* American Economic Review. Volume 79, Number 2, pages 220-225.

Gordon, Robert J. (1997) *The Time-Varying NAIRU and Its Implications for Economic Policy.* Journal of Economic Perspectives, Volume 11, Number 1, pages 11-32.

Guha, Debashis; Visviki, Dimitra (2001, July-September), *What determines inflation in the US, job growth or unemployment?* International Journal of Forecasting, Volume 17, Issue 3, pages 447-458.

Hasanov, Mubariz; Arac, Aysen; Telatar, Funda (2010, September), *Nonlinearity and structural stability in the Phillips curve: Evidence from Turkey.* Economic Modelling, Volume 27, Issue 5, pages 1103-1115.

Jakobsson, Andreas (2013), *An Introduction to Time Series Modeling.* Lund, Studentlitteratur AB.

Lee, Jaejoon; Nelson, Charles R. (2007, January/February), *Expectation Horizon and the Phillips Curve: The Solution to an Empirical Puzzle.* Journal of Applied Econometrics, Volume 22, Number 1, pages 161-178.

Lucas, Robert E. JR. (1967, August), *Adjustment Costs and the Theory of Supply.* The Journal of Political Economy. Volume 75, Number 4, Part 1, pages 321-334.

Lucas, Robert E. JR. (1976, January), *Econometric policy evaluation: A critique.* Carnegie-Rochester Conference Series on Public Policy, Volume 1, Pages 19-46.

Lucas, Robert E. JR.; Rapping, Leonard A. (1969, June), *Price Expectations and the Phillips Curve..* American Economic Review. Volume 59, Number 3, pages 342-350.

Lucas, Robert E. JR.; Rapping Leonard A. (1969, September/October), *Real Wages, Employment and Inflation.* The Journal of Political Economy. Volume 77, Number 5, pages 721-754.

Lucas, Robert E. JR. (1972, April), *Expectations and the Neutrality of Money.* Journal of Economic Theory, Volume 4, Number 2, pages 103-124.

Phelps, Edmund S. (1967, August), *Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time.* Economica, New Series, Volume 34, Number 135, pages 254-281.

Phelps, Edmund S. (1968), *Money-Wage Dynamics and Labor-Market Equilibrium.* Journal of Political Economy, Volume 76, Number 4, Part 2: Issues in Monetary Research, 1967 (Jul. - Aug., 1968), pages 678-711.

Phillips, A. W. (1958, November), *The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957.* Economica, New Series, Volume 25, Number 100, pages 283-299.

Qin, Duo (2010), *Modelling of the Inflation-Unemployment Tradeoff from the Perspective of the History of Econometrics.* Working Papers 661, Queen Mary, University of London, School of Economics and Finance. Retrieved July 20, 2014 from `http://www.econ.qmul.ac.uk/papers/doc/wp661.pdf`.

Russell, Bill (2011, September), *Non-stationary inflation and panel estimates of United States short and long-run Phillips curves.* Journal of Macroeconomics, Volume 33, Issue 3, pages 406-419.

Russell, Bill; Chowdhury, Rosen Azad (2013, March), *Estimating United States Phillips curves with expectations consistent with the statistical process of inflation.* Journal of Macroeconomics, Volume 35, pages 24-38.

Scully, Gerald W. (1974, August), *Static vs. Dynamic Phillips Curves.* The Review of Economics and Statistics, Volume 56, Number 3, pages 387-390.

Svensson, Lars E. O. (2014a), *De senaste årens penningpolitik - 'leaning against the wind'.* Ekonomisk Debatt. Number 3, pages 6-24.

Svensson, Lars E. O. (2014b), *The Possible Unemployment Cost of Average Inflation below a Credible Target.* Forthcoming in American Economic Journal: Retrieved June 10, 2014 from http://larseosvensson.se/files/papers/Phillips.pdf.

Toyoda, Toshihisa (1972, August), *Price Expectations and the Short-Run and Long-Run Phillips Curves in Japan, 1956-1968.* The Review of Economics and Statistics, Volume 54, Number 3, pages 267-274.

10 Appendix: Full Model Order Table

In this appendix, a full version of table 3 including all the the lags of the expectation gap, up to lag 9, is included. Note that for these models, the criteria mentioned in section (4.2.4) is not necessarily satisfied.

	CPI-Firms	CPIF-Firms	CPI-EEO	CPIF-EEO
$\Delta\pi^e_{t-1}$	-0.8939 (0.1782) [0.0000]	-0.5785 (0.1662) [0.0010]	-0.7701 (0.2763) [0.0076]	-0.7019 (0.1706) [0.0001]
$\Delta\pi^e_{t-2}$	0.0970 (0.2317) [0.6772]	-0.1195 (0.1840) [0.5189]	0.6701 (0.3467) [0.0592]	0.0004 (0.2077) [0.9986]
$\Delta\pi^e_{t-3}$	0.01385 (0.2299) [0.9522]	-0.1817 (0.1814) [0.3212]	-0.7740 (0.3928) [0.0546]	-0.2048 (0.2043) [0.3211]
$\Delta\pi^e_{t-4}$	0.2555 (0.2276) [0.2670]	0.3585 (0.1822) [0.0546]	0.2376 (0.4252) [0.5789]	0.7054 (0.1977) [0.0008]
$\Delta\pi^e_{t-5}$	-0.0581 (0.2286) [0.8003]	0.0367 (0.1985) [0.8541]	-0.2037 (0.4404) [0.6458]	-0.2794 (0.2340) [0.2381]
$\Delta\pi^e_{t-6}$	-0.0609 (0.2201) [0.7830]	-0.1121 (0.1807) [0.5377]	0.1799 (0.4237) [0.6731]	-0.2425 (0.1967) [2234]
$\Delta\pi^e_{t-7}$	-0.0265 (0.2197) [0.9045]	-0.2432 (0.1806) [0.1841]	-0.0355 (0.3778) [0.9256]	-0.0993 (0.1984) [0.6191]
$\Delta\pi^e_{t-8}$	0.2043 (0.2243) [0.3667]	0.1307 (0.1837) [0.4801]	-0.2693 (0.3246) [0.4109]	0.3132 (0.2047) [0.1323]
$\Delta\pi^e_{t-9}$	-0.0803 (0.1779) [0.6536]	0.1150 (0.1634) [0.4848]	0.1111 (0.2275) [0.6275]	-0.0868 (0.1963) [0.6602]
Δu^*_{t-1}	-0.1552 (0.1087) [0.1595]	-0.0888 (0.0952) [0.3553]	-1.062 (0.4590) [0.0250]	
Δu^*_{t-3}			0.6925 (0.4484) [0.1290]	-0.1018 (0.0607) [0.0997]
Fit				
FPE	0.7283	0.5029	2.286	0.4121
R^2	42.20%	23.31%	-2.28%	30.86%
Adj. R^2	30.41%	7.66%	-28.39%	16.75%

Table 5: Models of different measures of expectation gap as a function of unemployment gap. The models here are complete versions of the models in table (3), meaning all the lags, up to lag 9, of the expectation gap are included. In the parentheses the standard deviation and in the brackets the p-values of the coefficients are mentioned. All the measures of fit have prediction focus.