Modelling and Simulation of a Groundwater aquifer in Vomb

- How does artificial recharge of groundwater affects the landscape of the surface?

by

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Picture on front page: Picture from simulations in MATLAB. Belongs to the author.

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Abstract
In Sweden the groundwater cycle is by nature at steady state. When using groundwater as a freshwater resource it is common to artificially recharge the aquifer with approximately the same volume as is pumped up by the wells. The recharge to and the outtake from the aquifer effect the landscape of the saturated surface by generating higher peaks and lower valleys. In this thesis the Groundwater flow equation is studied from literature and from the study a model can be built which is simulated and compared with data given from the fields of *Vomb-verket.*

Popular Scientific Summary

Underneath almost the entire surface of the earth there is water present. This water is called groundwater and holds about 30% of all the freshwater on earth. The groundwater is naturally stored in different reservoirs that are called aquifers. From this it is easy to understand that groundwater is an important source of freshwater for humans around the world. The usage of groundwater as drinking water goes far back in history from when the first wells were mined. Today groundwater is used as drinking water and as a source to water farming lands and gardens. The heavy use has led to a bigger outtake from the aquifer than the natural input and from this many of the aquifers are drained.

Vombverket, a waterworks outside of Lund that provides the city of Malmö with drinking water, use groundwater as the source of freshwater. To make sure that draining of the aquifers does not occur the aquifers below the fields are recharged with lake water from Vombsjön. The principal of the artificial recharge is to imitate precipitation but heavily increased and only at a small area, in a pond. On the fields surrounding Vomb fifty ponds is mined. The amount of water that the aquifer is recharged with is the same amount that is pumped from the aquifer to the waterworks; this prevents the aquifer from draining.

Inside an aquifer, both naturally and artificially recharge movement occurs. This movement is caused by the look of the surface. The surface consists of peaks and valleys that arises from the slope of the bedrock and the input and output of water to and from the aquifer.

This master thesis investigates the movement of the water mass inside the aquifer and how pumping affects the groundwater surface.

Preface

This thesis is based on data from the waterworks at Vomb. I would like to thank VA Syd for providing me with the data and making this thesis possible, and the persons working at Vomb for all their help and valuable inputs regarding this project.

I would also like to thank Bernt Nilsson and Anders Holmqvist for guiding me through this thesis. I would also like to specially thank Niklas Andersson for both all the help regarding the modeling and simulation, and all valuable inputs and support.

At last I like to thank Matilda, Victor, Ferran, Boris and Fredrik for making it fun to spend everyday in a minimal room in front of a computer.

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1 Introduction

The waterworks at Vomb use groundwater as source for freshwater. The unconfined aquifer below the fields of Vomb is artificially recharged with water from the close-by lake Vombsjön. The groundwater is then pumped from the aquifer and pumped into the water-works.

When artificially recharge the aquifer peaks directly below the pond is formed, this change the looks of the surface. The opposite occur when a pump is taking out water from the aqui-fer. The outtake generates a local sink in the surface.

1.1 Aim

The aim with this thesis is to compute a model from which simulations over the surface of the groundwater can be done. From this model it can be obtained how the landscape of the sur-face is affected when the recharge is changed.

2 Theory

2.1 Groundwater

Beneath almost the entire face of the earth there is water present; this water is called groundwater. The groundwater surface marks the line between the unsaturated zone and the saturated zone, e.g. below the surface all the pores of the soil are filled with water. The use of groundwater as freshwater supply is common all over the world. Roughly one third of all of the freshwater is stored as groundwater (Rodhe, 2014).

2.1.1 Water cycle

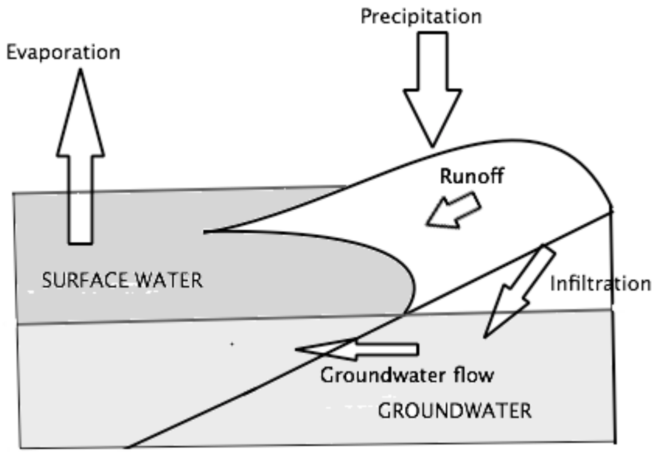


Figure 2.1. Picture describing a simplified version the hydrological cycle.

As can be seen in figure 2.1 above, the groundwater is a part of the water cycle. All the precipitation that does not runoff or evaporates immediately oozes down through the soil and eventually meets the groundwater surface. When hitting the surface the water can move in whatever direction depending on the surrounding landscape. The part of the water cycle concerning only the groundwater is called the groundwater cycle (Rodhe, 2014).

Groundwater cycle

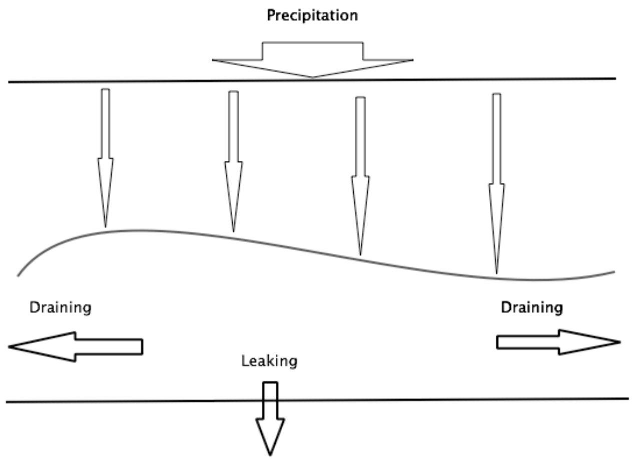


Figure 2.2. Schematic sketch showing the groundwater cycle in two dimensions.

The groundwater cycle is schematically illustrated in two dimensions in figure 2.2. In Sweden the cycle is considered to be in annual steady state, e.g. the inflow from precipitation and the outflow from draining and leaking are roughly equal (Rodhe, 2014).

As the water oozes through the ground it is naturally separated from impurities and parasites as the soil works as filter (Rodhe, 2014).

The draining occurs naturally as the groundwater diffuses towards an open surface. The groundwater follows the landscape of the surface (more about the groundwater surface in section 3.1.2) until it is ingested by a watercourse, lake or similar. At that moment the water leaves the groundwater cycle and becomes a part of the surface water (Rodhe, 2014).

The groundwater that is leaking does not leave the groundwater cycle but instead will percolate down through the soil with lower permeability. The structure of the groundwater is described in section 3.1.3 (Rodhe, 2014).

Artificial groundwater

Artificial groundwater, or artificial recharge of groundwater, can be described as an increased precipitation on a small area which leads to a local greater supply of water to the below aquifer (Committee on Ground Water Recharge, 1994). This can be used as a way to save a drained aquifer or to 'produce' drinking water. The benefit of using this method of producing drinking water from fresh water is the low-cost equipment and the environmentally friendly process.

2.1.2 Groundwater surface

The groundwater is constantly moving towards the surface water even if at a very slow pace. The pace and the direction of the groundwater flow are decided by the surging landscape of the surface and the height compered to the sea level. If the groundwater is at a height above sea level it will naturally diffuse towards lower ground following the peaks and valleys of the surface – on the other hand – if the surface is beneath sea level the water will percolate downwards (Rhode, 2014).

2.1.3 Aquifers

The groundwater available for pumping is gathered in what is called aquifers. Depending on if the aquifer stands in direct contact with the unsaturated zone or not; it is *confined* or *unconfined*.

Unconfined aquifer

Accessible water that stands in direct contact with the unsaturated zone is called an unconfined aquifer. The bottom of an aquifer, both confined and unconfined, is called confining layer. The confining layer is built up by soils that are less permeable than the layer above - and by that a basin is formed. Water leaks, or percolates, through the confining layer until reaches the next aquifer. The main supply of water to an unconfined aquifer is precipitation (Drobot, 2014).

Confined aquifer

In comparison to the unconfined aquifer where the residence time is of the order months, the residence time in a confined aquifer is of order years up to centuries and even millenniums. The main supply of water to confined aquifers is leaking from surrounding aquifers except for more shallow ones where precipitation and draining can reach the aquifer (Drobot, 2014).

2.2 Vombverket*

**All information regarding the chapter "Vombverket" derives from study visits at the facility.*

Vombverket is one of the freshwater facilities in the Malmö-Lund area and the only one using groundwater as water source. The water is pumped from shallow underlying unconfined aquifers whose main supply of water is artificial recharge of groundwater with fresh water from the near-by Vombsjön.

2.2.1 The process

The process in Vomb can be described by the simple flowchart below.

- **Pumping** –The freshwater is pumped from Vombsjön to the plant and will then be distributed over the fifty ponds, with the ability to choose which pond or ponds to fill and with what amount. The water level in the lake can be regulated by increasing or decreasing the flow to the plant, but also by regulating valves along Kävlingeån.
- **Filling** – When distributed into one of the ponds the first purification step has started. The first purification step is the biofilm that is formed in the bottom the pond.
- **Infiltration** – The second part of the purification is the infiltration or percolation of the water by letting it ooze through the approximately 25 m thick unsaturated zone.
- **Pumping 2** – The groundwater is pumped up from one of many wells distributed over the area. The pumping is regulated to match the artificial recharge of groundwater.
- **Purification** –When the water has been pumped up from the wells it moves on to the water works where the final purification is performed.

From lake-water to groundwater - purification

Before the biofilm is formed the water from Vombsjön just passes through the pond with a velocity higher than the filling velocity. With time a biofilm is formed and it will affect the flux by slowing it down. The first step in the purification is the biofilm, which is an important step since it acts as a barrier against parasites. The second step and last step before the waterworks is the unsaturated zone in which microorganisms purify the water in a series of bio reactions. After the unsaturated zone the water reaches the groundwater and will in approximately three months be pumped up and the rest of the purification continues inside the waterworks.

Mechanism of the pond

The cycle is started when the water flux into the pond is switched on. From the start there is no biofilm in the bottom of the pond, which in turn causes the water to ooze down through the unsaturated zone in the ground. By time the biofilm is built-up, the film generates a resistance (in the mathematical model described as fouling on a membrane) that slows down the oozing into the ground. When a full water mirror is formed the flux is kept constant until the maximal water level is reached, then the flux is slowed down and later turned off. Since the flux from the pond into the ground is driven by the pressure from the water column and is retarded by the resistance generated from the biofilm. In the end these parameters will be of the same size and the flux be equivalent to zero. When this occurs the leftover water is pumped from the pond. The biofilm is removed by skimming and the skimming of the biofilm marks the end of the cycle and the pond is now ready to be used again.

2.3 Groundwater flow model

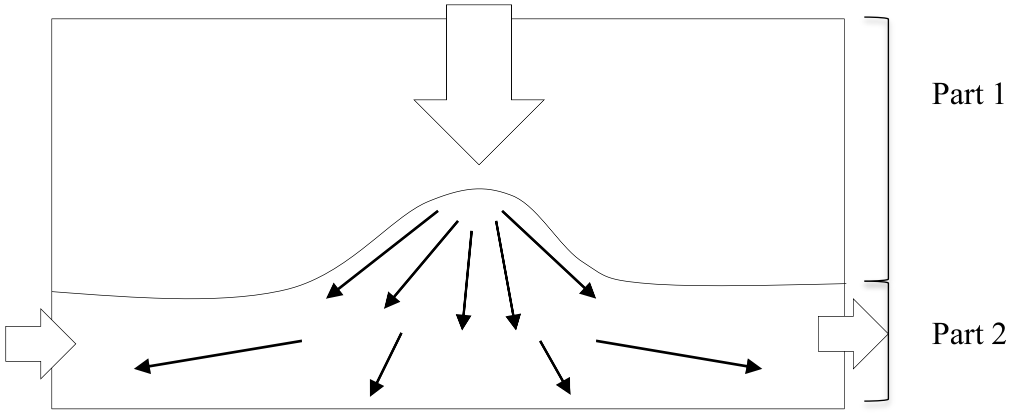


Figure 2.3. Simple scetch over the movement of the groundwater and both vertical and horizontal inflow in two dimensions.

When analysing what effect the artificial recharge to the aquifer has on the surface and movement of the groundwater a mathematic model has to be set up. The model is consistent of two parts; the first part describes the vertical in- and outflow from the aquifer, i.e. the artificial flows, and the second part describes the movement of the water mass in the aquifer.

General Mass balance (x,y,z direction)

In figure 2.3 above the aquifer is pictured as a two dimensional system, in real life the system is three-dimensional and consist of two horizontal axes, x and y respectively, and one vertical axis, z. The general mass balance over this system can be determined as equation 2.1:

$$\frac{\Delta M_{stor}}{\Delta t} = \frac{M_{in}}{\Delta t} - \frac{M_{out}}{\Delta t} - \frac{M_{gen}}{\Delta t} \leftrightarrow \rho \frac{\Delta V_{stor}}{\Delta t} = \rho \left(\frac{V_{in}}{\Delta t} - \frac{V_{out}}{\Delta t} - \frac{V_{gen}}{\Delta t} \right) \tag{2.1}$$

Since not water can be generated inside the aquifer the genereal mass balance can be seen as below:

$$\frac{\Delta M_{stor}}{\Delta t} = \frac{M_{in}}{\Delta t} - \frac{M_{out}}{\Delta t} + N \tag{2.2}$$

Where N is the vertical flows summarized.

Assumptions regarding the General Mass balance

- No water can be generated inside the aquifer.
- The area of the aquifer is constant, therefore the change of mass depends on the change in heigt.

2.3.1 Vertical in- & outflow

The artificial vertical in- and outflow to and from the aquifer derives from the artificial recharge and the wells. The system is illustrated in figure 3.4.

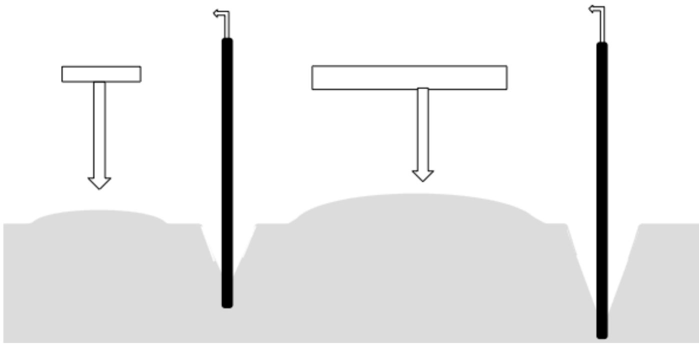


Figure 2.4. Picture illustrating the vertical in- and outflows affecting the aquifer.

Inflow – artificial precipitation

The Resistance of the Pond

The resistance of the pond increase over time from the moment a water surface is formed. The flux depends on this fouling but also on the height of the head of water. The equation describing the inflow is derived from Darcy's Law (described more closely in chapter 2.3.2) and determined to:

$$J\left[\frac{m}{s}\right] = \frac{1}{A}K(t)\frac{dh}{ds} \qquad (2.3a)$$

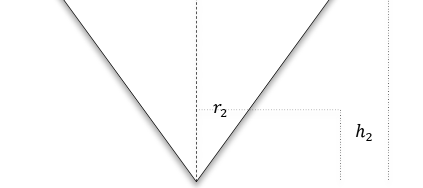
In this case K is the "resistance" of the ground and it increases by time due to the fouling. The water head is dependent on the difference of the inflow from the lake and how much water that oozes down in the ground.

The Unsaturated Zone

The flux from the pond that is oozing through the unsaturated zone are hitting the surface of the saturated zone with the same flux as above, but with a delay hence porosity of the soil.

Outflow – The Wells

The profile of the groundwater surface surrounding a well is described as a cone of depression. It can be pictured as a cone standing upside down with the well in its centre. The equation of the drawdown is described by the equation below where Q is the volumetric flow of the pump and K is the permeability coefficient of the soil. The knowledge gained from these calculations is applied when modelling the surface of the groundwater (Veslind, Peirce and Weiner ,1994).



$$Q = \frac{\pi K\left(h_1^2 - h_2^2\right)}{\ln\frac{r_1}{r_2}} \qquad (2.4)$$

Figure 2.5. Picture with properties of the cone of depression connected to the equation 2.4 to the right. In the figure h1 is the heigt of the groundwater surface (outside the cone of depression and h2 is just any height inside the cone with a corresponding radius.

Assumptions regarding the vertical recharge and discharge

- The artificial "precipitation" is much greater than the natural precipitation; hence the natural precipitation can be neglected. This assumption derives from analysing the data received from Vomb.
- The leakage as a discharge is minor to the discharge from the wells; therefore the discharge is assumed to only be from the wells.
- The horizontal recharge and discharge from the natural flow adding up to zero over the system. Therefore it is not taken into account.

2.3.2 Movement of the water mass

The importance of knowing how, how much and at what pace the groundwater is moving is general for almost any groundwater related question. For this thesis the importance lies in knowing what way and how much a local vertical inflow impacts the surrounding surface of interest. To be able to calculate this, a mathematic model for the movement has to be set-up.

Darcy's Law

According to Darcy (1865) the flow through a bed of saturated sand is proportional to the ratio between the height difference in head, Δh, and the difference between the manometers, Δs. When making further experiments Darcy came to the conclusion that the type of sand used in the trial gave an impact on the results. This impact is the hydrologic conuctivity, K_i [1/s]. From these conclusions *Darcy's Law* in one dimension derives (equation 2.5). A principal sketch over the set-up used for the experiment is showed in figure 2.6. The flow, Q, is the volumetric crossflow, hence A is thecrossflow area [m^2], in s-direction.

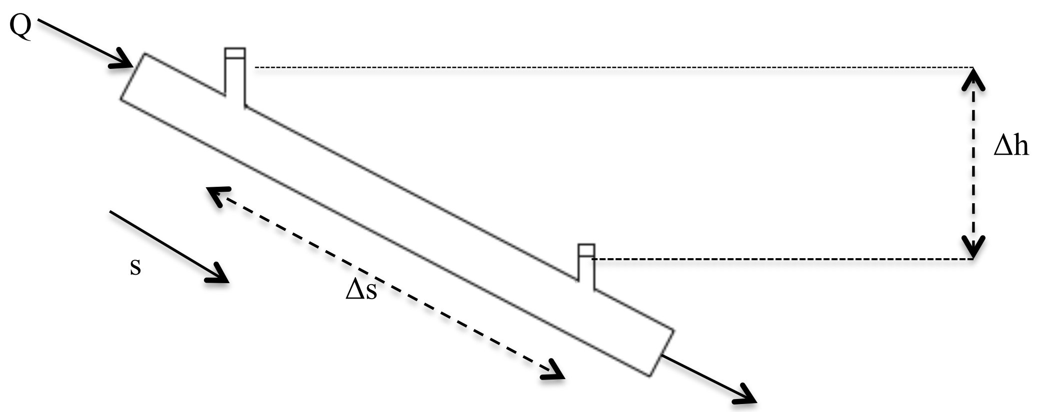


Figure 2.6. Sketch over the set-up used for Darcy's experiment. The manometers are the pipes rears over the column.

$$Q_s = -K_s \frac{\Delta h}{\Delta s} A \qquad (2.5)$$

Assumptions regarding Darcy's Law

- The negative contribution to the movement that comes from the adsorption on the surface of the soil particles is neglected since the flow from Darcy's Law is much greater.
- The hydrologic conductivity is constant in the aquifer.

Storativity

The storativity (S) of an aquifer is a measurement of its water-holding capacity. It is consistent of two parts; the specific storage ($S_s b$) and the specific yield (S_y) and is simply calculated by summarizing the storage and the specific yield (Fitts, 2012).

The specific storage is the basic measurement of the storativity in saturated soils. Practically the specific storativity is the quantity of water that leaves a specific volume of soil when the pore water is exposed to a change of head. For specifying the affected volume the constant b is used where b is the thickness of the aquifer (Fitts, 2012).

When the soil is saturated with water only a certain amount of water can drain. This is due to the porosity of the soil.

With this in mind the mass balance can be rewritten as equation 2.6 (Delleur, 2010):

$$S_y \frac{\Delta M_{stor}}{\Delta t} = \frac{M_{in}}{\Delta t} - \frac{M_{out}}{\Delta t} + N \leftrightarrow S_y A \frac{\Delta h}{\Delta t} = \frac{V_{in}}{\Delta t} - \frac{V_{out}}{\Delta t} + \frac{N}{A} \tag{2.6}$$

Assumptions regarding the Storativity

- For an unconfined aquifer the specific yield is much greater than the specific storage and therefore $S = S_y$.
- The soil around Vomb is assumed to be of the type fine – medium fine sand. According to Johnson (1967) the specific yield of this type would vary from 0.1-0.28 and 0.15-0.32 respectively.

Darcy's Law in two dimensions

According to the *Dupuit-Frochheimer assumption* the flow in an unconfined aquifer can be assumed to be only horizontal. This leads to the conclusion that $\frac{\partial h}{\partial z} = 0$ (Dupuit, 1863). As a result of this Darcy's Law needs to be applied in only two dimensions, even though the model is three-dimensional:

$$Q_x = -K_x \frac{\partial h}{\partial x} A \tag{2.6 a}$$

$$Q_y = -K_y \frac{\partial h}{\partial y} A \tag{2.6 b}$$

The soil in the aquifer is assumed to be isotropic and from this assumption it can be said that the hydrologic conductivity does not differ in x,y or z direction, this leads to the conclusion that $K_x = K_y = K_z$ (Muyinda, Kakuba and Mango, 2014).

Laplace's equation combined with Darcy's Law in two dimensions

For solving the system at steady state, $\frac{\partial h}{\partial t} = 0$, *Laplace's equation* for two-dimensional Cartesian coordinates is used. The Laplace operator, ∇^2, is the sum of the second derivatives of the function (Weissten 1, 2014). The equation in two dimensions is therefore stated as below:

$$\nabla^2 h = -KA\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = 0 \tag{2.7}$$

This equation 2.7 describes the diffusion of the water mass in the aquifer in two dimensions, which is the difference between the in and outflow in the general mass balance.

Groundwater flow equation

The equation for the groundwater flow derives from equations 2.1, 2.2 and 2.5-2.7 together with several of the assumptions stated above. The equation is written as following:

$$S_y \frac{\partial h}{\partial t} = K\nabla^2 h + N \leftrightarrow \frac{\partial h}{\partial t} = \frac{K\nabla^2 h}{S_y} + \frac{N}{S_y} \tag{2.8}$$

3 Mathematics

3.1 Finite Difference Method

For solving the Groundwater flow equation, which is a Laplacian equation, on the discrete grid that is formed in MATLAB the discrete Laplace operator is used. This operator translates the equation so that it is possible to solve the grid. The discrete operator is approximated from the Laplacian equation by using the five-point stencil method (Weissten 2, 2014).

3.1.1 Five-Point Stencil Method with Dirichlet Boundary conditions

The five-point stencil method approximates the function in every point by using four surrounding points; see figure 3.1 and equation 3.1 (Wang and Andersen, 1982).

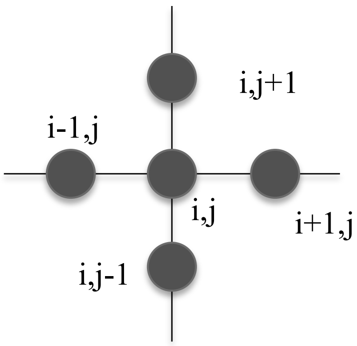


Figure 3.1. Picture graphically illustrating the method of five-point stencil discretization.

$$\nabla^2 h_{i,j} = \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right)_{i,j} \approx \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{h^2} + \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{h^2} \tag{3.1}$$

When applies this on the groundwater flow equation, and on a uniform grid the approximation is as equation 3.3 below:

$$\nabla^2 h \approx \frac{h(x-H,y) + f(\, x+H,y) + f(x,y-H) + f(x,y+H) - 4f(x,y)}{h^2} \tag{3.2}$$

In this equation H is the height of the uniform grid. For a non-uniform grid the equation will turn out a bit more complex, the equation can be found in Appendices II.

3.2 Boundary conditions

For solving a Laplace equation definition the boundary conditions is required.

Dirichlet boundary condition

The Dirichlet boundary condition is also referred to as *fixed boundary condition*. By using this boundary condition the height at the borders of the model are fixed for the simulations.

4 Computational Setup

The simulations of the model are performed in MATLAB 2014b. Both native solvers and files are used for simulation, as well as files computed by hand.

The numeric work of the thesis is divided into three chronological parts. First a theory-based model is computed and simulated. The result of this simulation is evaluated and some adjustments are made before further use. The second part begins with evaluating the data given from Vomb. The data is simulated and certain outputs of this simulation make valuable information that forms a base for the next step. The third and final step is to simulate the model with authentic parameters to understand the appearance of the saturated surface.

4.1 Physical Properties and common Assumptions

The physical data and glossary that are used is stated in Appendices I.

4.2 Part I - Simulation the based on Theory

The first part was performed were the Modelling based on only theory. The Model is build from theory and the equations are stated in previous chapters.

The extended model described further down in this chapter is based on theory, but with a grid that is based on an actual map.

4.2.1 Simple model

In the simple model no external files for computing the Laplace operator is used. The operator is computed by constructing the Laplace matrix by hand.

The grid

A fixed grid was used here. The calculations was performed directly in matrix-form and later plotted as a surface. The matrix is a 25 by 25 matrix were fictional vertical flows computed as shown in figure X. Every square that is filled makes a number in the matrix; the number is based on how much of the square that is filled. The unfilled squares are marked as a zero in the matrix. By using this method the vertical flow is switched on or off at the right place.

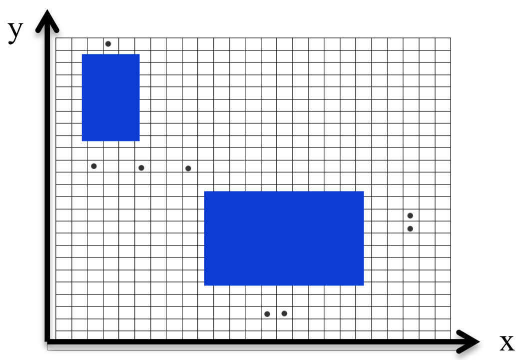


Figure 4.1. Illustration of the fictional area studied. The blue fields are the pounds and the black dots the wells.

The model is constructed in a matrix form, and the plot is shown as axes running from zero and up. Therefore it is important to flip the columns of matrix in x direction, by doing this the model and the input agrees.

Boundary conditions
The boundary conditions are in this first model set to a Dirichlet type and fixed at zero. Zero represents the start level of the surface. By using uniform Dirichlet the surface is approximated to be even – which is known from the Theory part to not be the case.

The in- and outflow
For this first simulation an on-going constant inflow is simulated. In the extended model further studies is made.

4.2.2 The Extended Model
In the extended model both the way of approximating the discrete Laplace operator, and the model of the inflow is changed. For approximations of the Laplace operator the five-point stencil method is still used, only this time it is performed by using a different m-file. The model of the inflow is made more advanced and true to reality with an changing inflow over time.

The grid
For making this model both a uniform grid and a non-uniform grid was tested. When making the grid big the runtime in MATLAB got really long and when making even bigger MATLAB ran out of memory. For making simulations of large grids possible a non-uniform grid was constructed. The grid was heavy were an input or output existed, and light were there was no flow. By doing this, the number of grid-points was decreased and the simulation was made possible.

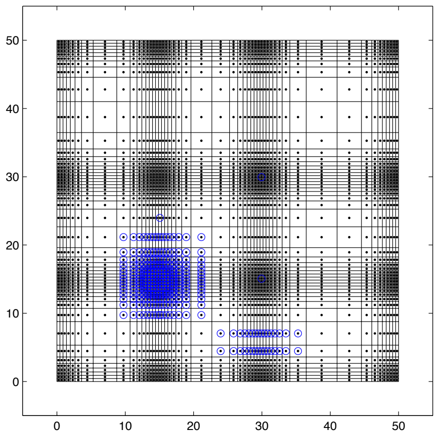
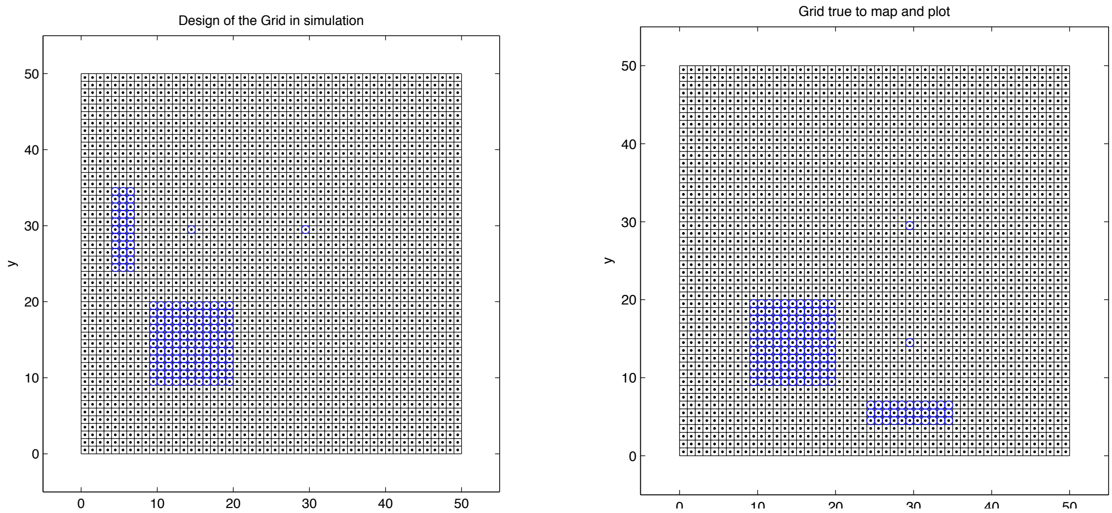


Figure 4.2. Example of a non-uniform grid. The grid get denser in a Gaussia way.

Placement of the grid points



Figur 4.3. The figures illustrate how the placements of the vertical in- ant outflow are changed due to the calculation technique.

The placement of the grids where the in- and outflow takes placed is different on the map to when calculating. This is due to the fact that the five-point stencil method calculates with matrixes and the area is designed as a Cartesian coordinate system.

Boundary conditions

The boundary conditions are kept the same as in the previous model.

fvmLaplace

For approximating the discrete Laplace operator a hand-computed script is made. This script allows other boundary conditions than Dirichlet to be used in the model. My supervisor *Niklas Andersson* made the script; more information regarding the *fvmLaplace*-script can be seen in the Appendices.

The inflow

To be able to run the script with real data a model for the inflow that takes into account that the flow is not constant is made. This model is using time dependent if statements for switching the flow into the pond on and off, without making the vertical inflow to aquifer stop.

4.3 Part II - Evaluation of the Data from Vomb

4.3.1 The Data

The received data from Vomb is evaluated manly by looking at plots and compare the plots with the map to find connection. This first study led to a demarcation of investigated area hence to the scant data provided.

The data is sorted and the ones from close-by observation pipes are plotted together. This data provides with a first glimpse of the landscape of the surface. As mentioned above the amount of data provided ensued in a decrease of study area. Since very little information regarding the outflow from the aquifer was provided the focus shifted from calibrating the model to real data – to instead trying to gain a better understanding of the looks and behaviour of the groundwater surface.

4.3.2 Interpolation and extrapolation of the Data

By using the MATLAB native *Curve Fitting Toolbox* the scattered data is interpolated and extrapolated to a plane. A curve fitting in three dimensions is made both one to each of the ten data sets, and one to the mean value of the ten data sets. From the last fitted plane the Dirichlet boundary condition vectors are formed. The planes from the ten different data sets is used to se if another boundary condition could be applied on the model.

Calculating the new Dirichlet boundary condition

From appendix III it is known that the boundary condition do not follow the system of the x and y axes, respectively, but the system of a matrix. By compare the two systems with each other it is learnt that the matrix-based boundary conditions can be translated as:

$$BL_{matrix} \leftrightarrow BD_{xyplot} \quad (4.1\text{ a})$$
$$BR_{matrix} \leftrightarrow BU_{xyplot} \quad (4.1\text{ b})$$
$$BD_{matrix} \leftrightarrow BR_{xyplot} reversed \quad (4.1\text{ c})$$
$$BU_{matrix} \leftrightarrow BL_{xyplot} reversed \quad (4.1\text{ d})$$

4.4 Part III - Simulation based on the Data

When making simulations based on the provided data some adjustments of the previous model has to be done. Mainly the inflow model has to be redone due to the non-existing biofilm in the pond and the on-going flow.

4.4.1 The Grid

The grid used for this simulation is adjusted to the shape of the invested area. A uniform 55 by 30 grid is used and executed together with the fvmLaplace-file. In the grid the grid points influenced by the inflow was estimated to a square of 4 grid points, the wells is still determined to one grid point.

4.4.2 Boundary Conditions

The simulation is performed with the boundary conditions obtained from the data analysis.

5 Results

5.1 Part I

5.1.1 Simple Model

As can be seen in figure 5.1 and 5.2 below the simulation of the simple model turned out as expected. The Inflow generates heads on the surface as wells as the wells generates depression cones. The height of the heads and valleys are not in reasonable numbers. This is due to the wrongly guessed constants (stated in table 4.1)

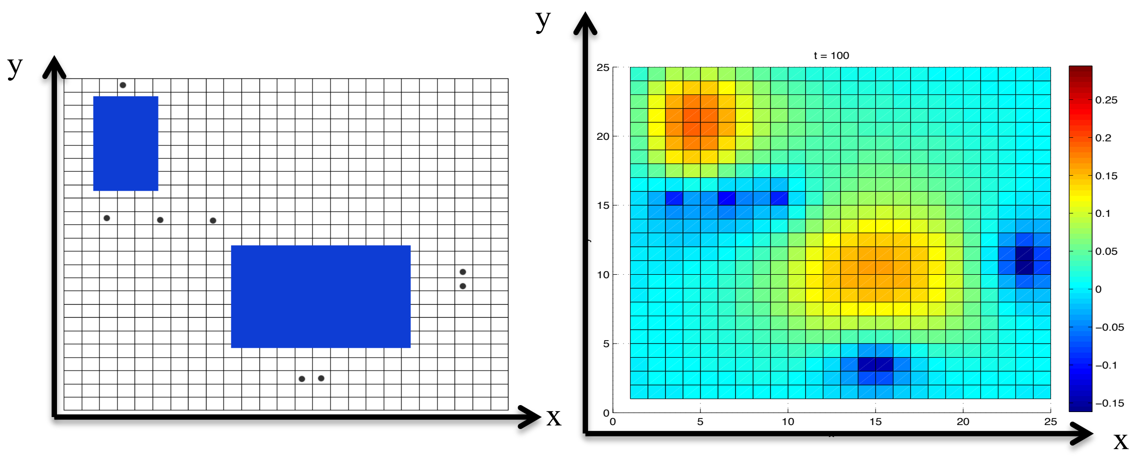


Figure 5.1. The placement of the vertical flows on the grid compared to the simulation. The ponds are pictured as the blue fields and the wells as black dots.

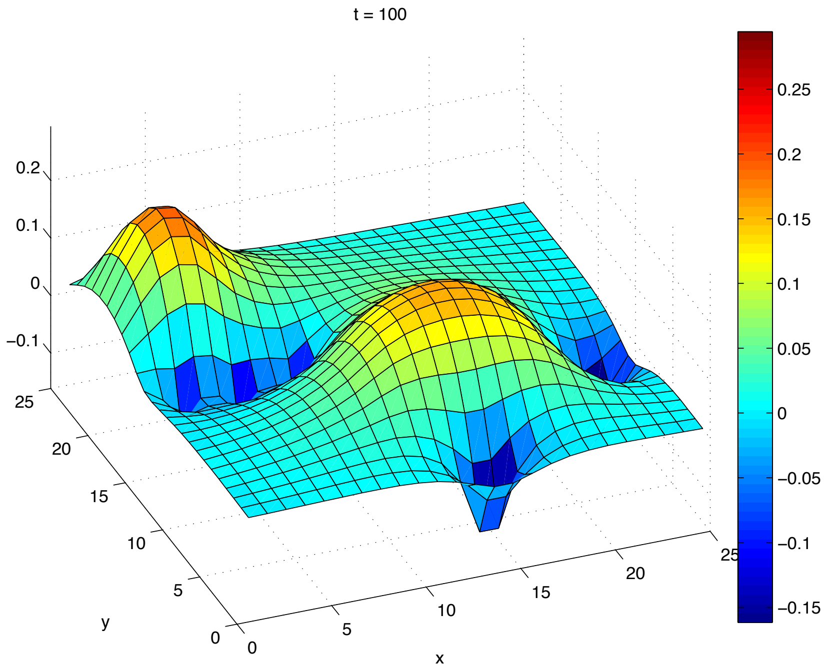


Figure 5.2. Result from the simple simulation presented in a three dimensional plot where the x- and y-axes represent the placement on the grid and the z-axis represent the water head in meters.

5.1.2 Extended Model

Non-constant inflow

The model for the changing inflow behaves as expected. When the inflow stops the velocity of the flux through the ground is decreasing.

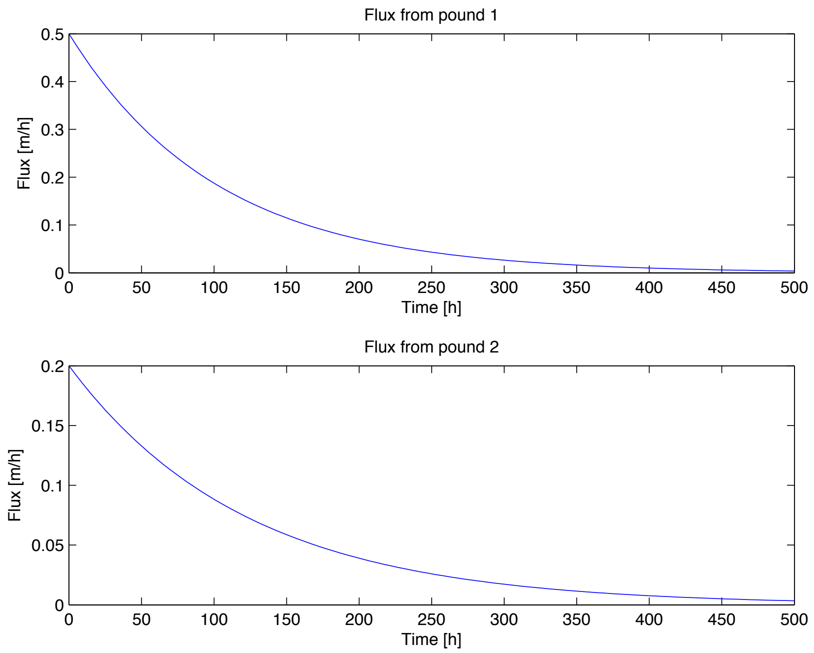


Figure 5.3. In this figure the decrease of flow velocity is presented. The decrease is due to the stop of inflow to the pound at time 200 h.

Groundwater surface

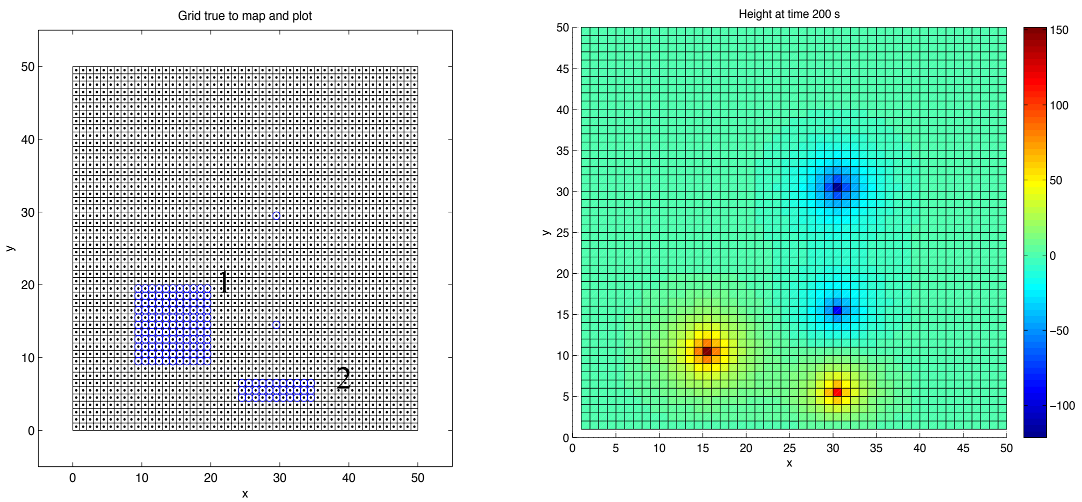


Figure 5.4. The picture to the left illustrates where on the grid the inlet of lake-water, ponds and wells are placed. The picture to the right is the result of the simulation.

The result of the simulations of extended model turns, just like from the simple model, out as expected. And similar to the simple model the numbers of the heads and valleys are not realistic.

The change of the landscape over time

In the figure 5.5 below the landscape is shown at different times of the simulation. The result is like expect, if though the heads are flatted out a bit rapid.

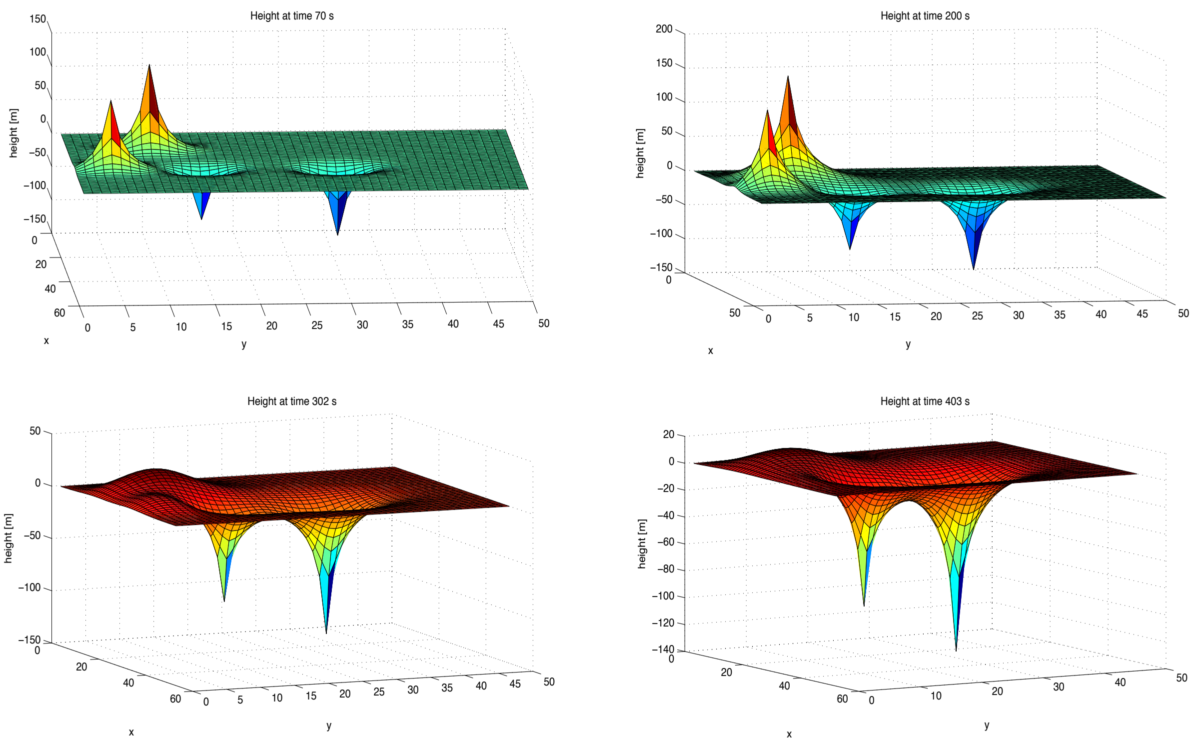


Figure 5.5. Results from the same simulation at different times. These plots show how the heads and depression cones change over time. The influence stops after 200s.

5.2 Part II

Raw data analysis

The scattered data is plotted and from the information a target-area is chosen. This area was chosen based on the amount of observation pipes. The raw data plots are presented in the Appendices.

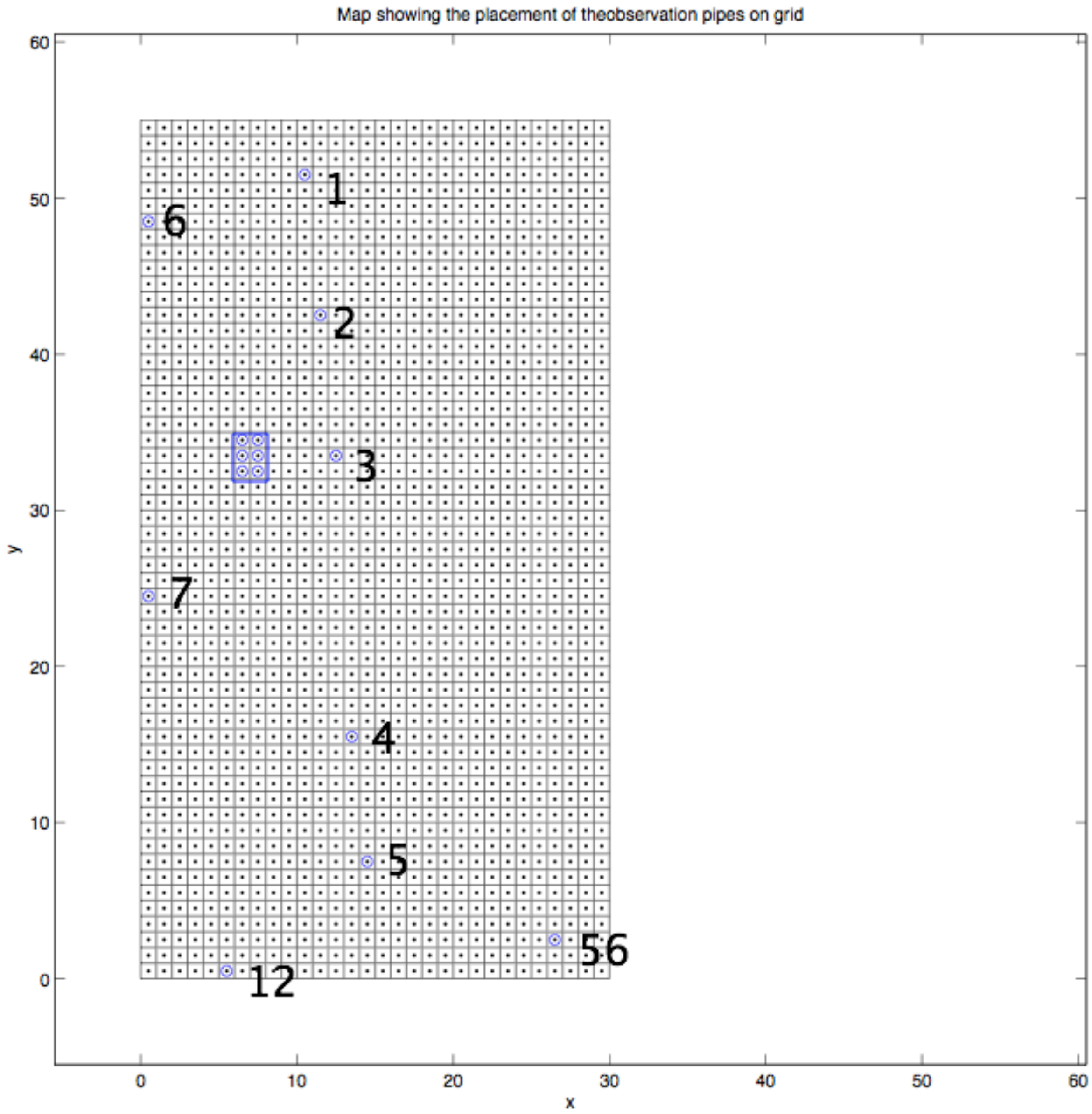


Figure 5.6. Grid showing where the datapoints are placed, This placement is based on the real placement.

5.2.1 Interpolation and extrapolation

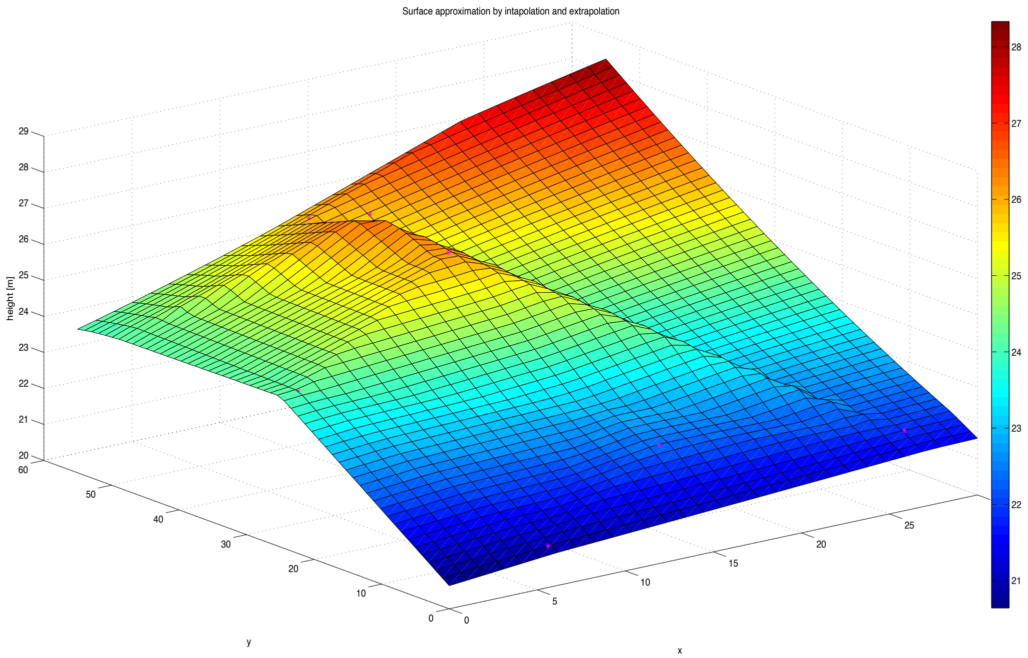


Figure 5.7. Plot showing the data points (in pink) together with the surface derived from interpolation and extrapolation.

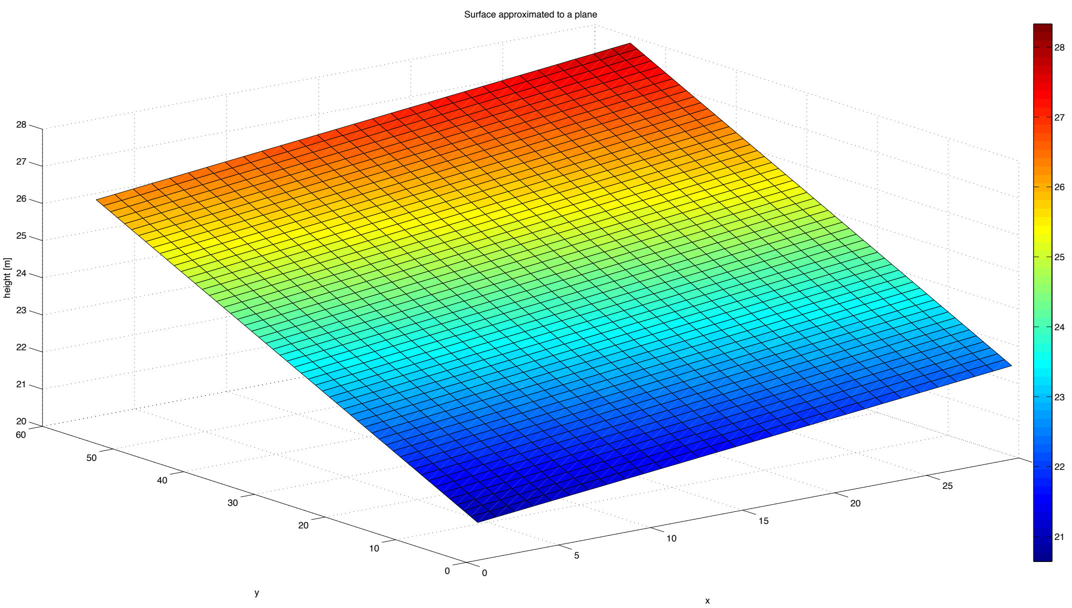


Figure 5.8. Plot showing the surface approximated as a plane. The plane is constructed from the 9 data points by linear regression.

Study of annual cycle

In figure 5.9-11 simulations of the data for each month of 2014 is showed.

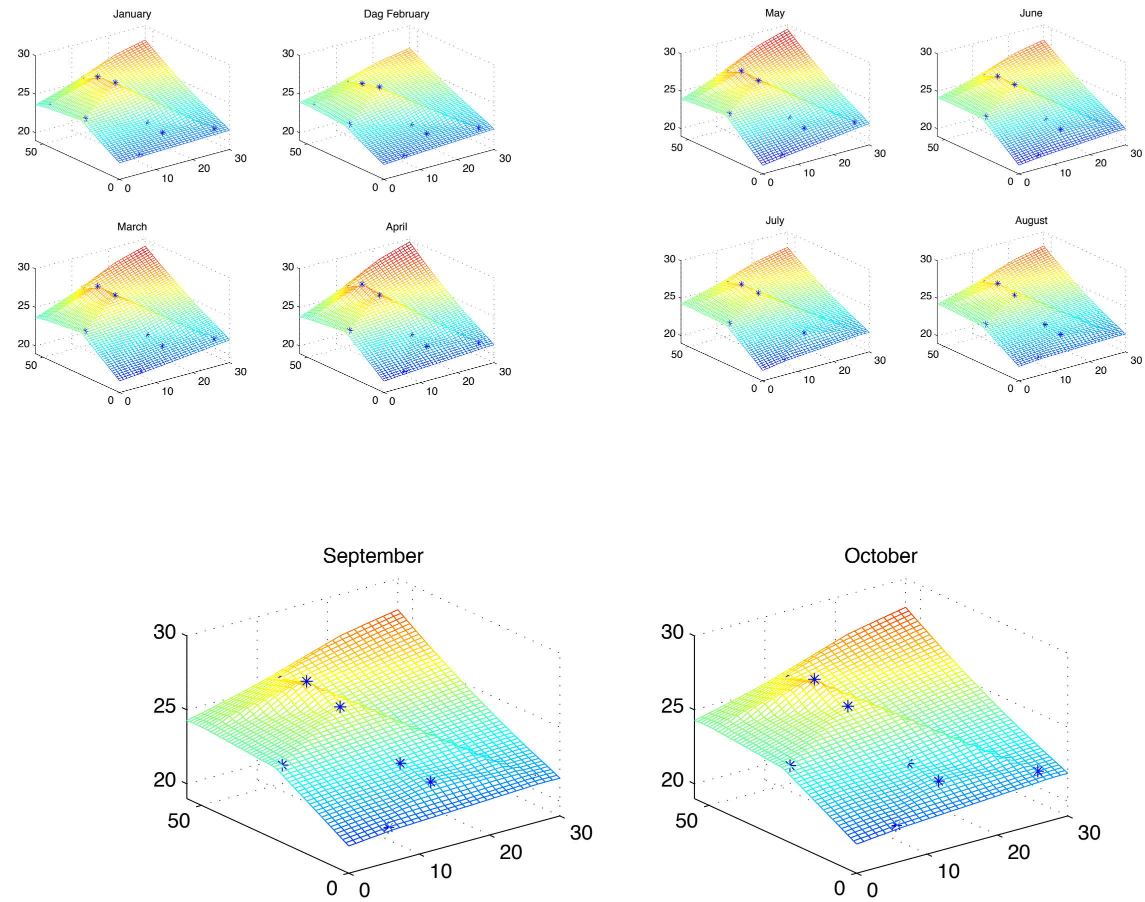


Figure 5.9. A surf plot for each month in the investigated interval is showed.

5.3 Part III

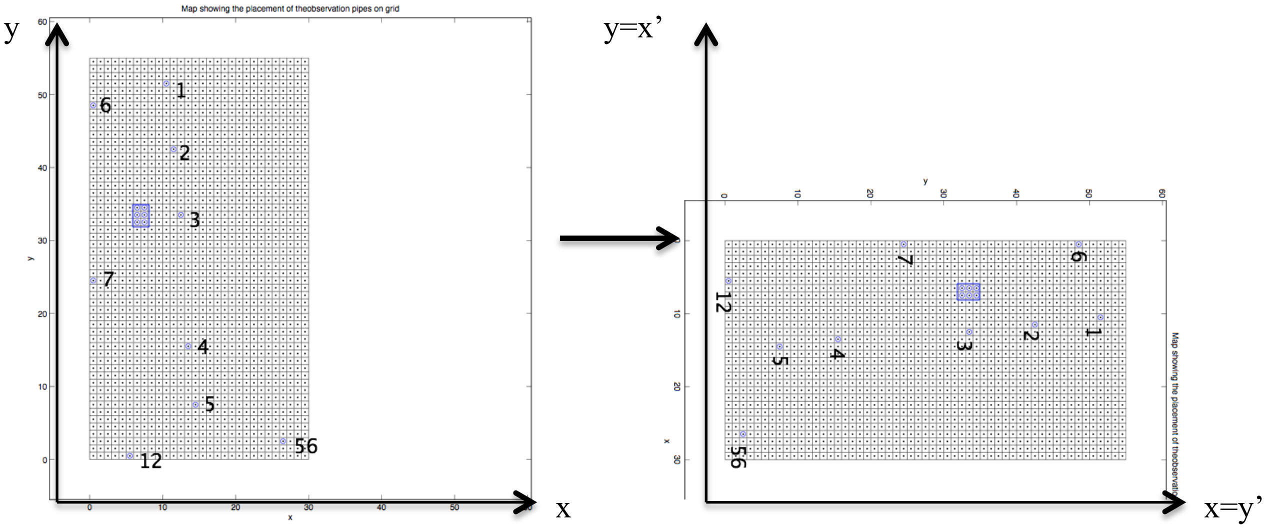
5.3.1 Simulation based on the data

Boundary conditions

The new boundary condition vectors are calculated from the plane showed in picture. The equation of the plane is approximated to

$$z = 20.92 + 0.05013x + 0.09805y \tag{5.1}$$

The four Dirichlet boundary condition vectors are calculated by using equation 5.1 together with the left, right, upper and lower vectors. The plot is remodelled so that this give the boundary conditions valid for the xy-plot, from the expressions stated in equation 4.1 a-d the boundary conditions valid for the matrix. The proportions of the investigated area for curve fitting are designed so that the boundary conditions easily could be used for further studies.



Left Boundary (xy – axes) → Upper Boundary (x'y' – axes)
Right Boundary (xy – axes) → Lower Boundary (x'y' – axes)
Upper Boundary(xy – axes) → Right Boundary (x'y' – axes)
Lower Boundary (xy – axes) → Left Boundary (x'y' – axes)

Simulation with new Boundary conditions and initial values

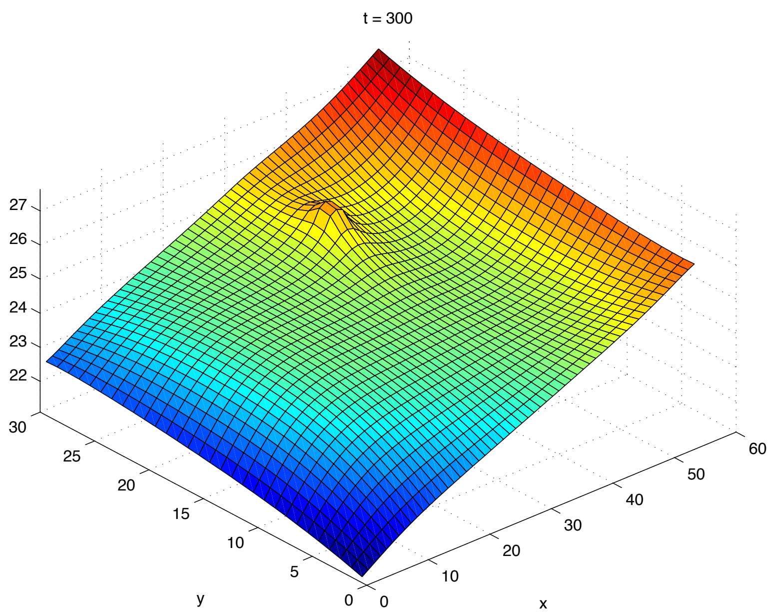


Figure. 5.11. This figure shows the plot with initial values and boundary conditions obtained from the simulations performed in part II.

6 Discussion

6.1 Dirichlet Boundary Condition

The use of fixed boundary condition at each margin of the area does not allow for any inflow to, or outflow from the studied area. The approximation that the in- and outflow are of the same size may not be realistic.

6.2 Part I

Both the simple model and the extended model behave as expected. But the result of the simulation is not realistic. For the simple model the values of both the draw down from the cone of depression and the head from the artificial recharge are very low. The opposite applies for the extended model where the values are unrealistically high.

No further calibration is done due to the fact that when these models were computed when the goal for this report was to calibrate the constants to real data. The model is flexible and useful for eventual further studies in the subject.

6.3 Part II

From evaluation of the data a small area of the fields was chosen for further studies. The data was used for making more realistic simulations possible. The scattered data was as can be seen in picture 5.7 and 5.8 interpolated and extrapolated with two different methods. The first one gives a realistic indication of how the looks of the landscape of the saturated surface. Due to the small amount of data points the surface is not reliable enough to use as boundary conditions or initial values for further simulations, therefore a second method is used. By making a plane of the nine data points, more safe values are obtained. By safer it is meant that if one of the data points are incorrect the peak or valley derived from this is smaller.

A study to see if the surface moved in a certain cycle over the year was done. One surf plot for each moth was executed. The expectations were to see that the mean height of the surface would change by a certain pattern. This pattern would be used as mixed boundary condition.

6.4 Part III

The aim from start was to make a model over the impact that the artificial recharge and drain of groundwater have on the saturated surface. A model is computed that behaves as the reality is doing based on theory. None of the parameters could be calibrated to real data due to the scant data sets and lack of necessary information.

In figure 5.11 a simulation based on the extended model with the initial values and Dirichlet boundary conditions obtained in studies from the previous part was used. Hence this simulation is the most realistic model that could be obtained with the given data.

6.5 Future work

The model used for these simulations is a good foundation to have for further work in the field. For the future work there is some parts that could be approved.

Improvements of the model

The boundary conditions could be further investigated. The use of fixed boundary conditions is not realistic since the surface edges will vary as well. For future work it would be recommended to consider mixed boundary conditions to take the horizontal flow in consideration.

The model for the recharge could be changed to better fit the reality. For those ponds where a water mirror not is obtained a model which takes an resistance in the ground in consideration could be derived.

If given a more frequently measured data denser measured over the investigated area the constants used in the mathematical model could be calibrated to fit the real area and this obtains a useful tool for investigation of the surface of the groundwater.

A model that describes the suspected annual variations of the surface could be developed by further investigating the data given from Vomb. This annual variation can than be used as a boundary condition as well as being apart of the existing model.

These simulations do not take temperature or concentrations of trace element in consideration. If this type of data is given a model which include these parameters can be developed.

Future applications of the model

When the improvement stated above is made the model could have a lot of applications. The main one is to help the waterworks at Vomb to improve their way of running the field. If seeing how the surface is affected of the recharge and outtake the pumps may be controlled differently and this could run the works more efficiently. The model could also be helpful when choosing which ponds to fill.

7 Conclusions

- Artificial recharge of groundwater in form of artificial precipitation affects the surface of the groundwater by increasing heads. When the recharge stops the surface slowly return to its basic mode.
- Outtake from wells affects the surface informs of cones of depressions. These cones primarily have an affect on the local surroundings. The change of surface is temporary and as soon as the outtake stops the level will return to its normal.
- To make an accurate model it is fundamental to have a basic knowledge regarding the original landscape of the surface. This information is used for approximation of the initial values in z directions on the grid and to form the boundary conditions.
- It is suspected that the level of the surface varies on an annual basis, even though it could not be proved in this thesis.
- The model used for these simulations are accurate and realistic. The use of Dirichlet boundary conditions did not allowed for any water transferring horizontally in and out from the studied area, which it does.

8 Reference List

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9 Appendices

I. Glossary
II. Equation regarding non-uniform grid
III. Boundary Conditions fvmLaplace
IV. Results - Boundary Conditions
V. Raw data plots

I – Glossary

			Part I	Part II	Part III
Q	Volumetric Flow	m^3/s			
q	Specific discharge	m/s			
J	Flux	m/s			
S_y	Specific Yield	-	0.21	0.21	0.21
ρ	Density	kg/m^3	998	998	998
K	Hydrolgic Condyctivity	s^1	0.0036	0.0180	0.0180
A	Area	m^2		4 grids	4 grids

II – Approximation for non-uniform grid

Divergence, with $\Delta x_{i+1/2} = x_{i+1} - x_i$ and $x_{i+1/2} = (x_{i+1} - x_i)/2$

$$\overline{\overline{\nabla \cdot u}}_{i+1/2,j+1/2} = \frac{1}{A_{i+1/2,j+1/2}} \int_{\partial\Omega_{i+1/2,j+1/2}} d\nabla u \cdot \hat{n}$$

$$= \frac{\overline{f}_{i+1,j+1/2}\Delta y_{j+1/2} - \overline{f}_{i,j+1/2}\Delta y_{j+1/2} + \overline{f}_{i+1/2,j+1}\Delta x_{i+1/2} - \overline{f}_{i+1/2,j}\Delta x_{i+1/2}}{\Delta x_{i+1/2}\Delta y_{j+1/2}}$$

$$= \frac{\frac{\overline{\overline{f}}_{i+3/2,j+1/2}+\overline{\overline{f}}_{i+1/2,j+1/2}}{2}\Delta y_{j+1/2} - \frac{\overline{\overline{f}}_{i+1/2,j+1/2}+\overline{\overline{f}}_{i-1/2,j+1/2}}{2}\Delta y_{j+1/2}}{\Delta x_{i+1/2}\Delta x_{i+1/2}} +$$
$$\frac{\frac{\overline{\overline{f}}_{i+1/2,j+3/2}+\overline{\overline{f}}_{i+1/2,j+1/2}}{2}\Delta x_{i+1/2} - \frac{\overline{\overline{f}}_{i+1/2,j+1/2}+\overline{\overline{f}}_{i+1/2,j-1/2}}{2}\Delta x_{i+1/2}}{\Delta x_{i+1/2}\Delta x_{i+1/2}}$$
$$= \frac{\overline{\overline{f}}_{i+3/2,j+1/2} - \overline{\overline{f}}_{i-1/2,j+1/2}}{2\Delta x_{i+1/2}} + \frac{\overline{\overline{f}}_{i+1/2,j+3/2} - \overline{\overline{f}}_{i+1/2,j-1/2}}{2\Delta y_{j+1/2}} \tag{1}$$

Laplacian

$$\overline{\overline{\nabla \cdot d\nabla u}}_{i+1/2,j+1/2} = \frac{1}{A_{i+1/2,j+1/2}} \int_{\partial\Omega_{i+1/2,j+1/2}} d\nabla u \cdot \hat{n}$$

$$= \frac{\overline{d}_{i+1,j+1/2}\frac{\overline{\overline{u}}_{i+3/2,j+1/2}-\overline{\overline{u}}_{i+1/2,j+1/2}}{x_{i+3/2}-x_{i+1/2}}\Delta y_{j+1/2} - \overline{d}_{i,j+1/2}\frac{\overline{\overline{u}}_{i+1/2,j+1/2}-\overline{\overline{u}}_{i-1/2,j+1/2}}{x_{i+1/2}-x_{i-1/2}}\Delta y_{j+1/2}}{\Delta x_{i+1/2}\Delta y_{j+1/2}} +$$
$$\frac{\overline{d}_{i+1/2,j+1}\frac{\overline{\overline{u}}_{i+1/2,j+3/2}-\overline{\overline{u}}_{i+1/2,j+1/2}}{y_{j+3/2}-y_{j+1/2}}\Delta x_{i+1/2} - \overline{d}_{i+1/2,j}\frac{\overline{\overline{u}}_{i+1/2,j+1/2}-\overline{\overline{u}}_{i+1/2,j-1/2}}{y_{j+1/2}-y_{j-1/2}}\Delta x_{i+1/2}}{\Delta x_{i+1/2}\Delta y_{j+1/2}} \tag{2}$$

konstant d

$$d\frac{\frac{\overline{\overline{u}}_{i+3/2,j+1/2}-\overline{\overline{u}}_{i+1/2,j+1/2}}{x_{i+3/2}-x_{i+1/2}}\Delta y_{j+1/2} - \frac{\overline{\overline{u}}_{i+1/2,j+1/2}-\overline{\overline{u}}_{i-1/2,j+1/2}}{x_{i+1/2}-x_{i-1/2}}\Delta y_{j+1/2}}{\Delta x_{i+1/2}\Delta y_{j+1/2}} +$$
$$d\frac{\frac{\overline{\overline{u}}_{i+1/2,j+3/2}-\overline{\overline{u}}_{i+1/2,j+1/2}}{y_{j+3/2}-y_{j+1/2}}\Delta x_{i+1/2} - \frac{\overline{\overline{u}}_{i+1/2,j+1/2}-\overline{\overline{u}}_{i+1/2,j-1/2}}{y_{j+1/2}-y_{j-1/2}}\Delta x_{i+1/2}}{\Delta x_{i+1/2}\Delta y_{j+1/2}} \tag{3}$$

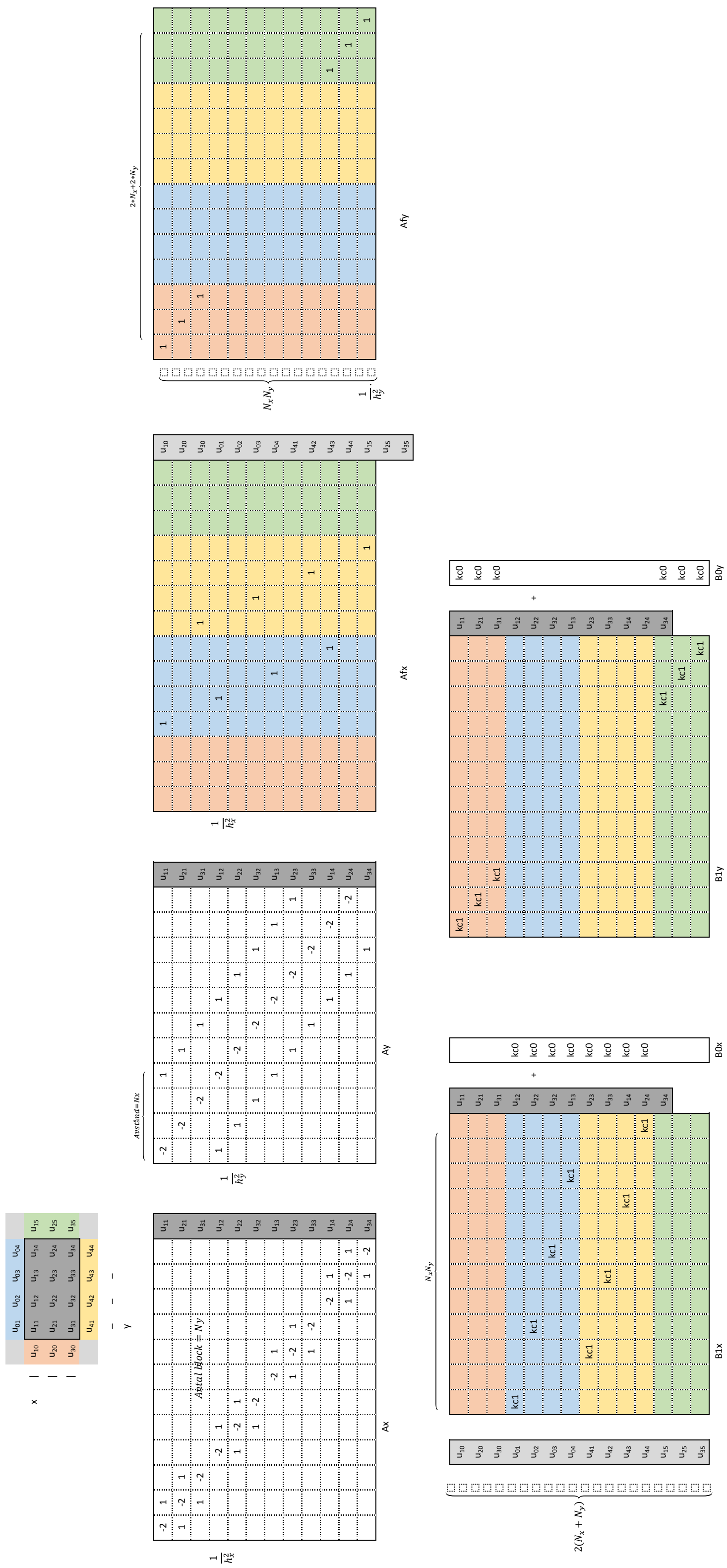
$$d\frac{\overline{\overline{u}}_{i+3/2,j+1/2} - \overline{\overline{u}}_{i+1/2,j+1/2}}{(x_{i+3/2} - x_{i+1/2})\Delta x_{i+1/2}} - \frac{\overline{\overline{u}}_{i+1/2,j+1/2} - \overline{\overline{u}}_{i-1/2,j+1/2}}{(x_{i+1/2} - x_{i-1/2})\Delta x_{i+1/2}} +$$
$$d\frac{\overline{\overline{u}}_{i+1/2,j+3/2} - \overline{\overline{u}}_{i+1/2,j+1/2}}{(y_{j+3/2} - y_{j+1/2})\Delta y_{j+1/2}} - \frac{\overline{\overline{u}}_{i+1/2,j+1/2} - \overline{\overline{u}}_{i+1/2,j-1/2}}{(y_{j+1/2} - y_{j-1/2})\Delta y_{j+1/2}} \tag{4}$$

$$d\frac{1}{(x_{i+1/2}-x_{i-1/2})\Delta x_{i+1/2}}\overline{\overline{u}}_{i-1/2,j+1/2}+d\frac{1}{(x_{i+3/2}-x_{i+1/2})\Delta x_{i+1/2}}\overline{\overline{u}}_{i+3/2,j+1/2}-$$
$$\frac{1}{(x_{i+1/2}-x_{i-1/2})\Delta x_{i+1/2}}+\frac{1}{(y_{i+3/2}-y_{i+1/2})\Delta x_{i+1/2}}+\frac{1}{(y_{j+1/2}-y_{j-1/2})\Delta y_{i+1/2}}+\frac{1}{(x_{i+3/2}-x_{i+1/2})\Delta y_{i+1/2}}\Bigg)\overline{\overline{u}}_{i+1/2,j+1/2}+$$
$$d\frac{1}{(y_{i+1/2}-y_{i-1/2})\Delta y_{i+1/2}}\overline{\overline{u}}_{i+1/2,j-1/2}+d\frac{1}{(y_{i+3/2}-y_{i+1/2})\Delta y_{i+1/2}}\overline{\overline{u}}_{i+1/2,j+3/2} \tag{5}$$

Om ekvidistant $h_x = x_{i+1/2} - x_{i-1/2}$ och ekvidistant $h_y = y_{j+1/2} - y_{j-1/2}$

$$\begin{aligned} &d\frac{\overline{\overline{u}}_{i+3/2,j+1/2}-2\overline{\overline{u}}_{i+1/2,j+1/2}+\overline{\overline{u}}_{i-1/2,j+1/2}}{h_x^2}+\\ &d\frac{\overline{\overline{u}}_{i+1/2,j+3/2}-2\overline{\overline{u}}_{i+1/2,j+1/2}+\overline{\overline{u}}_{i+1/2,j-1/2}}{h_y^2} \end{aligned} \tag{6}$$

III - Boundary Conditions fvmLaplace



IV - Results - Boundary Conditions in x-y

Boundary Left - BL	Boundary Right - BR	Upper Boundary - BU	Lower Boundary - BD
20.9200	22.4239	20.9200	26.3128
21.0199	22.5238	20.9719	26.3646
21.1197	22.6236	21.0237	26.4165
21.2196	22.7235	21.0756	26.4683
21.3195	22.8234	21.1274	26.5202
21.4193	22.9232	21.1793	26.5720
21.5192	23.0231	21.2312	26.6239
21.6191	23.1230	21.2830	26.6758
21.7189	23.2228	21.3349	26.7276
21.8188	23.3227	21.3867	26.7795
21.9187	23.4226	21.4386	26.8313
22.0185	23.5224	21.4904	26.8832
22.1184	23.6223	21.5423	26.9351
22.2183	23.7222	21.5942	26.9869
22.3181	23.8220	21.6460	27.0388
22.4180	23.9219	21.6979	27.0906
22.5179	24.0218	21.7497	27.1425
22.6177	24.1216	21.8016	27.1943
22.7176	24.2215	21.8535	27.2462
22.8174	24.3213	21.9053	27.2981
22.9173	24.4212	21.9572	27.3499
23.0172	24.5211	22.0090	27.4018
23.1170	24.6209	22.0609	27.4536
23.2169	24.7208	22.1127	27.5055
23.3168	24.8207	22.1646	27.5574
23.4166	24.9205	22.2165	27.6092
23.5165	25.0204	22.2683	27.6611
23.6164	25.1203	22.3202	27.7129
23.7162	25.2201	22.3720	27.7648
23.8161	25.3200	22.4239	27.8167
23.9160	25.4199		
24.0158	25.5197		
24.1157	25.6196		
24.2156	25.7195		
24.3154	25.8193		
24.4153	25.9192		
24.5152	26.0191		
24.6150	26.1189		
24.7149	26.2188		
24.8148	26.3187		

24.9146	26.4185		
25.0145	26.5184		
25.1144	26.6183		
25.2142	26.7181		
25.3141	26.8180		
25.4140	26.9179		
25.5138	27.0177		
25.6137	27.1176		
25.7136	27.2175		
25.8134	27.3173		
25.9133	27.4172		
26.0132	27.5171		
26.1130	27.6169		
26.2129	27.7168		

V – Raw Data Plots