Investigation and Comparison of Cohesive Zone Models for Simulation of Crack Propagation

Sara Eliasson and Alexander Lundberg
Division of Solid Mechanics at Lund University, Faculty of Engineering

2015-01-26

1 Abstract

Looking at crack propagation modeling today there are a few different methods that enables the possibility to simulate an advancing crack. One of these methods are cohesive zones. Cohesive zones are modeled as an interface between two continuum surfaces and are described by a constitutive law that is to represent the crack propagation.

The commercial FEM-program used to implement cohesive zones, ABAQUS CAE, gives the possibility to implement linear and exponential softening in the traction-separation law. Using a double cantilever beam the traction-separation law with a linear initial loading and a linear or exponential softening is implemented.

Looking at the results, when using the same variables for the exponential and linear softening, there is no big difference in the results of the behavior of the beam. The curves for both implementations coincide very well. There are differences in the convergence of the calculations, both methods converge well but the linear softening not as well as the exponential softening.

There will be no new theory presented, only a summary of various different theories developed by other authors.

2 Introduction

The work done is intended to be an aid in choosing a suitable traction-separation formulation for a specific problem given. Why do we use cohesive zones? Why is the choice of parameters so important to make the calculation run smoothly? How do we model them in a FE-calculation?

Looking into cohesive zones there is at first a lot of information to be found in the literature. Before starting to model a cohesive zone the theoretical aspects of cohesive zones are studied. At first understanding is built on why we use cohesive zones and then also understanding on how they work. Learning more and getting a better understanding is pivotal before implementing it in the FE-program. Modeling of the cohesive zones is done in a commercial FEM-program called ABAQUS CAE. ABAQUS is a very powerful tool for FE-calculations.

Cohesive zones are for most parts implemented in fracture analyses and used to simulate crack growth. For this report the cohesive zones are used for studying crack tip conditions at a mode I crack.

3 Linear Elastic Fracture Mechanics

Early work in the field of fracture mechanics regarding stress concentrations around elliptical holes was developed by C.E. Inglis in 1913 [9]. His theory predicted that the stresses at a perfectly sharp crack tip approach infinity. In other words; that the material would have zero strength [15]. Instead of analysing the stress state in the vicinity of the crack tip, A.A. Griffith developed a theory based on energy-balance [6].

The early work by Griffith was later developed further by G.R. Irwin[10]. Irwin defined the energy release rate, G, as the rate of change in potential energy with crack area for a linear elastic material. Crack extension occurs when the energy release rate reaches a critical value, $G = G_c$, which is a measure of fracture toughness.

The balance equation under equilibrium conditions for an incremental increase in crack area, dA, can be expressed as

$$\frac{dE}{dA} = \frac{d\Pi}{dA} + \frac{dW_s}{dA} = 0 \qquad (3.1)$$

which is equivalent to

$$-\frac{d\Pi}{dA} = \frac{dW_s}{dA}$$

where E is the total energy, W_s is the work needed to create two new surfaces and Π is the potential energy in the form of strain energy and work done by external forces. For an edge crack, two new surfaces are created when a crack is formed. Thus the expression for W_s takes the form

$$\frac{dW_s}{dA} = 2\gamma_s \qquad (3.2)$$

where γ_s is the material specific surface energy.

4 Cohesive Zone Modeling of Fracture

Failure and fracture is a big part of many fields in engineering. It is therefore important to understand and to be able to do calculations on failure processes and fractures. To analyze these phenomena efficiently in arbitrary geometries, a general numerical method is needed that can describe the initiation and evolution of a crack. The method should include and be able to simulate the initial loading, the damage initiation with initial debonding, and the damage evolution until complete separation and failure has occurred. A method used for these kind of problems is modeling with cohesive zones.

Cohesive zone models are based on theory from Barenblatt and Dugdale [2, 5]. Their method for using cohesive zones to represent a crack propagation path is very similar to Griffith's theory based on a surface energy that measures the resistance against crack advance. When using cohesive elements a constitutive equation describes the behavior of the failing cohesive material elements in front of the crack tip. The constitutive relation for a cohesive interface is such that the traction across the interface will vary depending on the separation of the crack. With increasing separation, the traction will reach a maximum, start to decrease and eventually be reduced to zero.

A typical cohesive stress-displacement diagram is shown in figure 1.

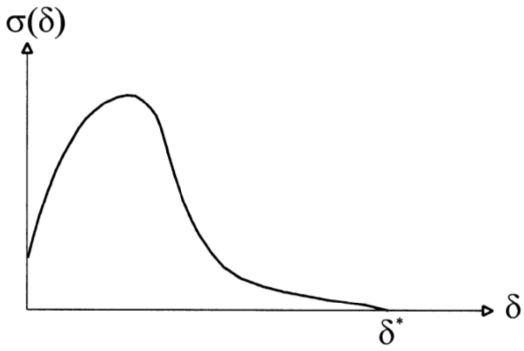

Figure 1: A typical stress-displacement diagram for a cohesive element, taken from [1].

When a cracked structure is exposed to some external loads the crack surfaces are subjected to forces which restrain the surfaces from separating. These forces can be seen as cohesive forces. The cohesive stress is a function of the relative displacement between the crack surfaces, $\sigma = \sigma(\delta)$. The external loads will increase δ until it reaches δ^*, see figure 1. When δ^* is reached the bond between the crack faces breaks and new free surfaces are created.

When two new free surfaces are created, the atoms can be considered to be pulled apart. They are slowly moving out of range from their neighbours. For the process where new free surfaces are created, the cohesive stresses perform some amount of work. The work is written as

$$W = \int_0^{\delta^*} \sigma(\delta) \mathrm{d}\delta \qquad (4.1)$$

This relation is equal to the J-integral. To propagate a crack through the distance Δa, a surface energy is needed which corresponds to

$$\Delta U_s = \int_0^{\Delta a} \int_0^{\delta^*} \sigma(\delta) \mathrm{d}\delta \mathrm{d}x = \Delta a \int_0^{\delta^*} \sigma(\delta) \mathrm{d}\delta \qquad (4.2)$$

The area under the traction-separation curve is by definition twice the surface energy. Recalling equation (3.2), γ_s is the surface energy for one new free surface created and for a crack we have two new free surfaces. This gives us

$$\int_0^{\delta^*} \sigma(\delta) \mathrm{d}\delta = 2\gamma_s + \gamma_p \qquad (4.3)$$

If the cohesive zone is negligible in size compared to the characteristic lengths of the structure around the crack it can be concluded that Griffiths theory and the theory of atomic cohesive forces are identical, cf. equation (3.2).

The information above on cohesive zone models is collected from several authors, see references [1, 5, 12, 13, 14, 16, 17]

4.1 Traction-Separation Laws

A basic bilinear traction-separation law, frequently used in calculations, can be seen in figure 2, where the softening after damage initiation is linear. Another model frequently used is the one seen in figure 3, where the softening after damage initiation is described by an exponential function. Crack propagation can be simulated using different parameters that control the advance of the crack front for cohesive zone models. It can be based on either the local energy release or on the separation of the crack surfaces which corresponds to the displacement of the cohesive elements [3].

The bilinear model is uniquely defined by the set of parameters that describes the top point, (δ_n^0,t_n^0), and the end point, (δ_n^t,0), of the triangle. For both the bilinear and the exponential model the maximum traction sustainable by the cohesive element,

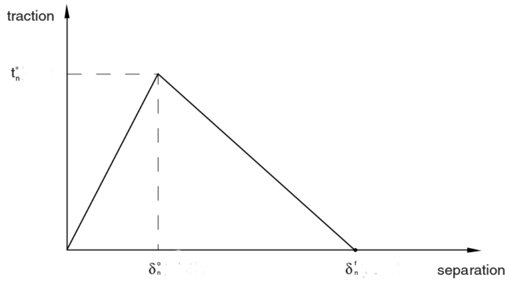

Figure 2: Illustration of a simple bilinear traction-separation law, taken from [2].

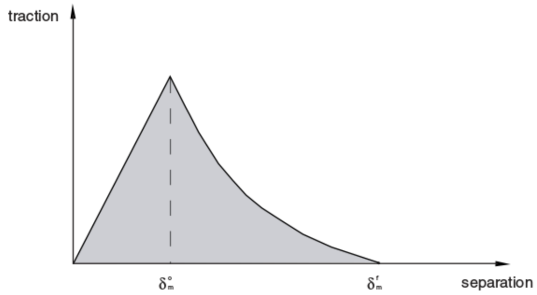

Figure 3: Illustration of exponential damage evolution in a traction-separation law, taken from [3].

t_n, is required. This threshold value is important since it governs when initiation of damage occurs. The penalty stiffness, K_0, is also an important parameter in ensuring realisitic pre-crack conditions. K_0 is the slope of the first part of the curve seen in figure 2 and 3. If the penalty stiffness is insufficient, large displacements will occur in the interface, which alters the behavior of the structure.

When choosing to prescribe failure displacement, the critical energy release rate can be calculated as the area under the curve, and vice versa.

$$\Gamma_0 = \int_0^{\delta_0} T(\delta) \mathrm{d}\delta \qquad (4.4)$$

5 Cohesive Zone Modeling in Abaqus

Different cohesive zone models can be defined in ABAQUS CAE, which is the pre- and post processor of ABAQUS. In the present study, the cohesive zone model is implemented in a double cantilever beam (DCB). A DCB is a good model to be able to investigate the basic behavior of the cohesive elements and find a traction-separation law that is suitable for predicting crack growth. DCB structures in combination with cohesive zones have been employed in, for example, [1] and [4]. For the analysis a plane stress condition is implemented for conditions of small-scale yielding where LEFM applies. Plane stress is used since we are looking at a DCB which is small in the thickness direction compared to the in-plane dimensions.

6 Discussion and Conclusions

The mesh convergence study for the cohesive zone shows that no significant difference in the results are obtained when using different mesh densities. The results for the different counts of cohesive elements coincide. This shows that the number of cohesive elements will not significantly affect the results. However, the general experience from working with cohesive elements during this project is that choosing too few elements can result in instability and convergence difficulties. Using a very fine mesh in the cohesive zone can, on the other hand, lead to very long computation times. The mesh density must therefore be chosen carefully with these two points in mind.

Using an exponential softening model the calculations was perceived to converge faster. The linear softening has a good convergence as well but not as good. The behavior of the two different models does not show differences in the resulting behavior of the simulated beam. The crack extends in the same way, which is controlled by the fracture energy, and initiates at the same time, due to the choice of penalty stiffness and maximum traction. Most inequalities are seen after damage initiation which is when the damage evolution behaves according to either of the two different models. The results, however, follow the same slope and converge to about the same value.

7 Future Work

Writing this report gave us a perspective of how useful cohesive zone models can be. Even though it is not yet big in the area of crack propagation in steel it has huge potential. Most applications today are for brittle materials and very ductile materials, like polymers. The theory on the subject is endless and there are still much to look into even further. There are a lot of traction-separation models for the cohesive zone proposed by different authors and still more to come. The work on cohesive zones is a long way from being finished. For more thourough information about the cohesive zones look at the report that the article is based on [11].

A proposal on further work with crack advance is to consider Hallberg's article from 2007 [8]. In this article, a constitutive model for martensite transformation in austenitic stainless steel is derived. In a subsequent article, also by Hallberg, from 2011 [7], results are shown from a stationary crack where the martensite transformation at the crack tip is included. Using this constitutive model with a cohesive zone model on an advancing crack would make it possible to investigate how the martensite transformation influences the crack propagation.

8 References

[1] G. Alfano, M. A. Crisfield, *Finite Element Interface Models for Delaminaion Analysis of Laminated Composites: Mechanical and Computational Issues*. 2001

[2] G.I. Barenblatt, *The Mathematical Theory of Equilibrium Cracks in Brittle Fracture.* Advances in Applied Mechanics vol. 7, pp 55-129, 1962

[3] J. Chen M. Crisfield, *Predicting Progressive Delamination of Composite Material Specimens via Interface Elements.* Mechanics of Composite Materials and Structures 6:301-317, 1999

[4] M.A. Crisfield, G.A.O. Davies, H.B. Hellweg, Y. Mi, *Progressive Delamination Using Interface Elements.* Journal of Composite Materials, 1998

[5] D.S. Dugdale, *Yielding of Steel Sheets Containing Slits*, Engineering Department, Univeristy College ov Swansea, 1959

[6] A.A.Griffith, *The Phenomena of Rupture and Flow in Solids.* 1920

[7] H. Hallberg, L. Bank-Sills, M. Ristinmaa, *Crack Tip Transformation Zones in Austenitic Stainless Steel.* Division of Solid Mechanics, Lund University, 2011

[8] H. Hallberg, P. Håkansson, M. Ristinmaa, *A Consitutive Model for the Formation of Martensite in Austenitic Steels under Large Strain Plasticity.* Division of Solid Mechanics, Lund University, 2007

[9] C.E Inglis, *Stresses In a Plate Due to the Presence of Cracks and Sharp Corners.* Cambridge, 1913

[10] G.R Irwin, *Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate.* Journal of Applied Mechanics 24, 361–364, 1957

[11] A. Lundberg, S. Eliasson, *Investigation and Comparison of Cohesive Zone Models for Simulation of Crack Propagation*, Division of Solid Mechanics, Lund University, 2015

[12] N. Moës, T. Belytschko, *Extended Finite Element Method for Cohesive Crack Growth.* Department of Mechanical Engineering, Northwestern University, USA, 2001

[13] A. Needleman, *An Analysis of Decohesion Along an Imperfect Interface* - 1988. International Journal of Fracture vol.42 pp.21-40, 1990

[14] J.R. Rice, *A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks.* Journal of Applied Mechanics Vol. 35, pp. 379-386, 1968

[15] D. Roylance, *Introduction to Fracture Mechanics.* 2001

[16] I. Scheider, *Cohesive Model for Crack Propagation Analyses of Structures with Elastic-Plastic Material Behavior.* GKSS Research Center Geesthacht, 2001

[17] A. Yavari, *Generalization of Barenblatt's Cohesive Fracture Theory for Fractal Cracks.* California Institute of Technology, 2001

9 References - Images

[1] Image from figure 3b in
A. Yavari, *Generalization of Barenblatt's Cohesive Fracture Theory for Fractal Cracks.* California Institute of Technology, 2001

[2] Image from figure 31.5.6-1 in
Abaqus Analysis, User's Manual: Volume IV. Dassault Systèmes, 2011

[3] Image from figure 31.5.6-5 in
Abaqus Analysis, User's Manual: Volume IV. Dassault Systèmes, 2011