LUND UNIVERSITY
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Implementation of Heston-Nandi GARCH model on OMXS30

- Put and call valuation, pre and post the financial crisis of 2008

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Authors: Jakob Bengtsson Ekström, Oscar Sjögren

Supervisor: Professor Hans Byström

Lund University, Department of Economics

Abstract

This paper evaluates the performance of Heston and Nandi's closed form option pricing model (2000) on the OMXS30 (Swedish stock index), pre and post the financial crisis. The main purpose is to investigate if the more realistic assumptions of Heston and Nandi yield more accurate price estimates, than the computationally more simplistic Black-Scholes model. Both periods are evaluated in-sample and out-of-sample and the parameters of the model are generated by Maximum Likelihood Estimation. The out-of-sample analysis reveals some mixed results, but put options are in general more accurately estimated than call options, especially out-of-the money. Some periods experience large pricing errors, due to poor parameter estimates. One natural extension would thus be to perform the study by estimating the parameters by the Nonlinear Least Squares method, indeed implemented by Heston and Nandi.

Keywords: *Financial Crisis, Heston and Nandi, HN-GARCH, OMXS30, Option Pricing.*

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1. Introduction

Black and Scholes (1973) and Merton (1973) developed the famous Black-Scholes-Merton model (henceforth BS), for pricing European stock options. The importance of this model was accentuated when Scholes and Myron were the recipients of the 1997 Nobel Prize for Economics. The model has been crucial in the development of pricing and hedging options, much due to its accuracy relative to its simplicity. However, the BS-model builds on certain assumptions such as lognormal distribution of stock returns and constant volatility, which has lead people to question the robustness of the model. Stock returns have empirically been shown not to follow a lognormal distribution and volatility is seldom constant (e.g. Hull 2014). The parabolic curvature of the implied volatility curve, or the volatility smile, has lead researchers to express implied volatility as a quadratic function of moneyness and time to maturity (e.g. Rouah and Vainberg 2007). Dumas, Fleming and Whaley described this relationship when they introduced their deterministic volatility function (1998).

One of the major difficulties in pricing options, and other financial instruments, as accurate as possible is the complexity of the market volatility and how it should be estimated in an efficient and accurate way. This has been the objective of many researchers throughout the years, and has subsequently led to a number of option pricing models incorporating stochastic volatility, e.g. Hull and White (1987) and Stein and Stein (1991). Another evolutionary branch of option pricing are the binomial three volatility models, where the volatility depends solely on time and present value of the underlying asset. Rubenstein (1994) and Duprie (1994) are examples of models featuring this volatility process.

In 1993 Heston constructed a stochastic volatility option pricing model in continuous time, which featured a closed form solution just like the BS-model. In his paper, Heston discusses how the distribution of the returns of the underlying asset affects option prices, and how the distribution in turn, is affected by the correlation between the variance and the returns of the underlying asset. He specifically examines the parameters that affect the distribution's skewness and kurtosis.

Different versions of general autoregressive conditional heteroscedasticity (henceforth GARCH) are commonly used as generating processes for the stochastic volatility when pricing options (Hull 2014). Bollerslev (1986) further developed the work of Engle (1982) when he introduced the GARCH process, a generalized version of the autoregressive conditional heteroscedasticity (ARCH), and argued for its suitability as an economic modelling tool. A renowned pricing model, that incorporates a type of GARCH process (non-linear GARCH) was introduced by Duan (1995) and more have followed, e.g. Ritchken and Trevor (1998). An unfortunate trait for many of these models is, however, the absence of an easy to use option pricing formula. For the computation of option prices, one is instead often omitted to numerical simulations and approximations.

In 2000, Heston and Nandi presented the Heston-Nandi GARCH model (henceforth HN-GARCH) that offers a closed form pricing formula for European options. This model resembles the continuous Heston model from 1993 in many of its assumption, but it is easier to implement with respect to observable data. The main difference between HN-GARCH and BS is the view on which variance to use when pricing options, where the HN-GARCH assumption, of a non-lognormal distribution of returns and stochastic volatility is considered as the more realistic one (Heston and Nandi (2000)). Their study showed promising results, improving the valuation of both put and call options on the S&P500 compared to the BS-model.

Huskaj and Sharlett (2007) tested the single lag HN-GARCH model by implementing it on the OMXS30 for the time period 2005-2006. They concluded that the HN-GARCH(1,1) was significantly outperformed by the BS-model when valuing call options in this period. Harding (2013) also tested the single lag HN-GARCH by implementing it on the OMXS30 for the time period 2011-2012 and comparing it to the BS-model. He found that the HN-GARCH(1,1) gave a worse prediction of call option prices than the BS-model, for almost all maturities and moneyness. The market conditions of this time period were unstable in light of the economic crisis, and the result might have been affected by this fact.

The purpose of this paper is thus to implement the HN-GARCH(1,1) on the OMXS30, on one period prior to the crisis and on one period post crisis, in order to investigate if a change in the model's performance can be detected. The prices estimated by the HN-GARCH(1,1) model are compared, in-sample and out-of-sample, with the prices obtained by the BS-model. Both call options and put options are considered and the number of options is more extensive than in the two previous studies.

There are three limitations embedded in this paper. The first one regards the parameter estimations of the HN-GARCH(1,1) model and the choice of method. Heston and Nandi use the non-linear least squares method (NLLS) and the maximum likelihood estimation (MLE), while this paper only uses the latter and simpler method. Secondly, each day's last price is used as the observed option price, instead of the intra daily mid bid-ask spread used by Heston and Nandi. The last restriction regards the effect of dividends, which is neglected in this study. This should however not have any considerable effects on the result, since dividend payments are limited to two months each year.

The introduction of this paper is followed by section 2 in which the theoretical framework of the model is presented. Section 3 presents and discusses the data selection, while section 4 consists of in-sample and out-of-sample empirics. The last part is section 5 that concludes the paper.

2. Theoretical Framework

This section presents some of the theoretical ideas, assumptions and models fundamental to this paper and option pricing in general. Areas covered are risk neutral valuation, the Black-Scholes-Merton model, implied volatility, the GARCH process and the Heston and Nandi GARCH(p,q) model. A brief explanation of the likelihood ratio test is given and the error measures used are also presented.

2.1 Risk-Neutral Valuation

In the real world, investors will always demand excessive expected rate of returns above the risk free rate, for assets that have non-idiosyncratic risk. This is due to the fact that future prices of risky assets are uncertain. However, in a risk-neutral world investors do not demand a risk premium when exposing themselves to risk. Therefore all assets have an expected rate of return equal to the risk free rate.

An important principle in pricings of option and other derivatives is the risk-neutral valuation. It states that we can assume that all investors are risk-neutral when evaluating derivative prices and still calculate the right price in all worlds, not just the risk-neutral. This assumption can be made since risk-preferences are already incorporated into the price of the underlying asset. Thus the price of the derivative is the expected payoff at maturity, discounted at the risk-free rate. We will denote parameters in the risk-neutral world with*. For example, the price of a European call option at time t with maturity at time T, will be:

$$c = e^{-(T-t)} E^{*}\left[max\left((S_T - K), 0\right)\right] \tag{1}$$

where K is the strike price.

A very neat consequence that follows from the risk-neutral assumption is that we can price options without any regard to risk aversion. The risk aversion of buyers and sellers are incorporated in, and thus affecting, the stock price. The relationship between stock prices and options prices is however the same. Including the unknown risk aversion in option pricing would be very bothersome and the valid assumption of risk neutrality simplifies the pricing process a great deal.[1]

2.2 The Black-Scholes-Merton Model

The BS-model was introduced in 1973 and has since been the most frequently used, and most important, model for pricing options. It relies on two particular assumptions, namely that the underlying asset is to have constant variance and that the returns of that asset are, in turn, log-normally distributed. In addition, the returns of the underlying asset are assumed to follow a Geometric Brownian motion.[2]

A complete derivation of the model will not be given[3], the Black-Scholes-Merton differential equation is nonetheless given by

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{2}$$

where f is the value of the derivative, S is the value of the underlying stock and σ is the volatility (standard deviation) of the stock and *r* is the risk-free interest rate. Given certain boundary conditions, one can solve this differential equation for different derivatives.

For European call options the boundary condition is given by

$$f = max(S - K, 0), \text{ when } t = T$$

and the solution is

$$c_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{3}$$

[1] For further information about risk-neutral valuation, see any textbook on derivatives, e.g. Hull 2014.

[2] See e.g. Hull (2014) chapter 13.

[3] The interested reader is referred to any textbook on derivatives, e.g. Hull (2014) chapter 14.

where

$$d_1 = \frac{log\,(S_t/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{(T - t)}} \tag{4}$$

and

$$d_2 = \frac{log\,(S_t/K) + (r - \sigma^2/2)(T - t)}{\sigma\sqrt{(T - t)}} \tag{5}$$

c_t is the call option price at time *t*. $N(d_i)$ is the probability that a standard normal distributed variable attains a value less than d_i. S_t is the stock price at *t*, *K* is the strike price and σ is the volatility of the stock and *r* is the continuously compounded interest rate. Finally *T* is the option's maturity date.

2.3 Implied Volatility and the Volatility Smile

Implied volatility is, in contrast to the backward looking historical volatility, a forward looking measure often used as an estimate of the future volatility of an option's underlying asset. Implied volatility is obtained by finding the value that, if inserted in a pricing model would yield the actual market price, *ceteris paribus*. The most common model used to calculate implied volatility is the Black Scholes model. It is however not possible to invert equation (3) to express the volatility as a function of the other variables. Thus in order to find the implied volatility corresponding to the BS model, one has to approximate it using a numerical approach like the Newton-Raphson method (Rouah and Vainberg 2007)

Implied volatility is of great interest for option traders since it can be seen as the markets opinion or belief of future volatility. It is thus a measure of what the future volatility might be. Implied volatility can also be used to estimate probabilities, or the likelihood, for stocks to attain certain values in a specific period of time. Furthermore, due to the fact that option prices tend to vary at a greater extent than implied volatility, traders often quote the implied volatility instead of the actual option price.

If the Black Scholes implied volatility for a specific maturity is plotted with respect to strike price, one obtains the implied volatility curve, commonly referred to as volatility smile. The volatility smile is important since it illuminates the problems related to the assumption of lognormal returns and constant variance of the underlying asset. If this assumption were true, the volatility curve would simply be a horizontal line. However, the volatility curve of equity options has been observed to be downward sloping, hence establishing a negative relationship between strike price and volatility. This volatility smirk is consistent with the more realistic assumption of a skewed distribution of returns. Thus highlighting the systematic biases incorporated in Black and Scholes model and its assumptions.

The implied volatility for put and call options, with same time to maturity and strike price, are due to the put call parity the same. This implies that the volatility smile is the same for European put and call options. The type of option is consequently redundant information when studying the volatility smile. It is moreover possible to deduce the probability distribution/density function from the volatility smile.[4]

2.4 GARCH

The general autoregressive conditional heteroscedasticity (GARCH) is a commonly used process when estimating current and future volatility of an asset. It exist numerous GARCH processes with different number of lags, but the most commonly used is the GARCH(1,1). In this process the variance is estimated by assigning appropriate weights in the formula

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

where V_L is the long run average variance rate and u_{n-1} is the most recent percent change of the asset. The weights are given by γ, α and β respectively, and according to unity, the following condition must be satisfied

$$\gamma + \alpha + \beta = 1$$

and the process is said to be stable if

$$\alpha + \beta < 1$$

[4] For more information about implied volatility and the volatility smile see e.g. Hull (2014) chapter 19.

A common abbreviation is to put

$$\gamma V_L = \omega$$

yielding the following expression

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

By estimating the variance in this way, one is assuming that the variance of the asset is affected by the most recent asset return and variance, as well as by the long run average variance.[5]

2.5 HN-GARCH

The closed form option pricing formula presented by Heston and Nandi in 2000 builds on two assumptions that differ from the, somewhat limiting, assumptions made in the BS-model. The first of the two assumptions, is that the underlying asset follows a GARCH process over a time period Δ,

$$log(S(t)) = log(S(t-\Delta)) + r + \lambda h(t) + \sqrt{h(t)}z(t) \tag{6}$$

$$h(t) = \omega + \sum_{i=1}^{p} \beta_i h(t-i\Delta) + \sum_{i=1}^{q} \alpha_i \left(z(t-i\Delta) - \gamma_i \sqrt{h(t-i\Delta)}\right)^2 \tag{7}$$

Equation (6) is the mean model where *r* is the continuously compounded interest rate for a time period of length Δ, $z(t)$ is a standard normal disturbance term and λ is a parameter. $h(t)$ is the log return's conditional variance from time $t-\Delta$ to time *t*, and is generated by equation (7) and information known at time $t-\Delta$.

[5] For more information about GARCH processes see e.g. Hull (2014) or the original paper by Bollerslev (1986)

Spot returns are, due to the appearance of conditional variance in the mean model, dependent on the level of risk, where λ is a constant that has a negligible effect on the option price. $\lambda h(t)$ is regarded as the return premium. Consequently, if the variance is equal to zero, the expected value of the mean model is the risk free interest rate, which eliminates the possibility of arbitrage.

This paper is focusing on the single lag version of the model, which limits to Heston's (1993) continuous time pricing model, when Δ approaches zero (Heston and Nandi (2000)). The conditional variance in the HN-GARCH(1,1) is given by

$$h(t) = \omega + \beta_1 h(t-\Delta) + \alpha_1 \left(z(t-i\Delta) - \gamma_1 \sqrt{h(t-\Delta)}\right)^2 \tag{8}$$

By solving for $z(t)$ in equation (6) and inserting this into equation (8), one can via some algebraic manipulation arrive at the following expression for the conditional variance at time $t+\Delta$.

$$h(t+\Delta) = \omega + \beta_1 h(t) + \alpha_1 \frac{\left(\log\big(S(t)\big) - \log\big(S(t-\Delta)\big) - r - \lambda h(t) - \gamma_1 h(t)\right)^2}{h(t)} \tag{9}$$

For the process to be stationary we require that

$$\beta_1 + \alpha_1 \gamma_1^2 < 1 \tag{10}$$

the kurtosis of the log-distribution is controlled by α_1 which also can be interpreted as the standard deviation of the standard deviation. γ_1, on the other hand, determines the skewness of the log-return distribution and signals how the conditional variance responds to shocks. It is obvious from equation (8) that a negative shock has a greater effect on variance, assuming positive γ_1. In the special case where β_1 and α_1 are approaching zero, the conditional variance becomes the same as in the BS-model.

Also, the covariance between the spot returns and the variance process is given by

$$cov_{t-\Delta}\left[h(t-\Delta), log\big(S(t)\big)\right] = -2\alpha_1 \gamma_1 h(t) \tag{11}$$

A negative correlation between spot returns and variance will appear when α_1 and γ_1 are positive. This relationship gives a somewhat intuitive explanation to the empirical evidence of a negatively skewed log-return distribution[6]. This phenomenon is often referred to as the leverage affect, introduced by Black (1976).

The second assumption of Heston and Nandi is necessary in order to price options and it transforms equations (6), (7) and (9) to risk neutral form. To achieve this, Heston and Nandi changes the parameters λ to λ^* in the mean model and γ_1 to γ_1^* in the volatility process. They also define $z^*(t)$ as a random variable that follows a normal distribution under the risk neutral probability.

$$\lambda^* = -\frac{1}{2}$$

$$\gamma_1^* = \gamma_1 + \lambda + \frac{1}{2}$$

$$z^*(t) = z(t) + \left(\frac{1}{2} + \lambda\right)\sqrt{h(t)}$$

Heston and Nandi justify these changes in their "*proposition 1*" where they prove that inserting these parameters in the mean model, results in that the one period return from investing in the underlying asset is equal to the risk free rate. They thus conclude that the risk neutral versions of equation (6) and (7) are the following:

$$log\big(S(t)\big) = log\big(S(t-\Delta)\big) + r - \frac{1}{2}h(t) + \sqrt{h(t)}z^*(t) \tag{12}$$

$$h(t) = \omega + \sum_{i=1}^{p} \beta_i h(t - i\Delta) + \sum_{i=2}^{q} \alpha_i \left(z(t - i\Delta) - \gamma_i\sqrt{h(t - i\Delta)}\right)^2 \tag{13}$$
$$+ \alpha_1 \left(z^*(t-\Delta) - \gamma_1^*\sqrt{h(t-\Delta)}\right)^2$$

[6] e.g. see Rubenstein (1985) and Heston and Nandi (2000).

The next step in the process of producing option values, is to solve for the generating function of the GARCH process in (6) and (7). Heston and Nandi (2000) denote the conditional generating function of the underlying asset by $f(\phi)$, which is also the moment generating function of the logarithm of *S(T)*.

$$f(\phi) = E\left[S(T)^{\phi}\right]$$

$f^{*}(\phi)$ is furthermore used to denote the generating function in the risk neutral case. Heston and Nandi (2000) formalize this in their "*proposition 2*" where they state that the generating function has the following log-linear form

$$f(\phi) = S(t)^{\phi}\exp\left(A(t;T,\phi) + \sum_{i=1}^{p} B_i(t;T,\phi)h(t+2\Delta-i\Delta) + \sum_{i=1}^{q-1} C_i(t;T,\phi)(z(t+\Delta-i\Delta)-\gamma_i\sqrt{h(t+\Delta-i\Delta)})^2\right) \tag{14}$$

where, in the single lag case[7], i.e. when $p = q = 1$,

$$A(t;T,\phi) = A(t+\Delta;T,\phi) + \phi r + B_1(t+\Delta;T,\phi)\omega - \frac{1}{2}\log\left(1-2\alpha_1 B_1(t+\Delta;T,\phi)\right) \tag{15}$$

$$B_1(t;T,\phi) = \phi(\lambda+\gamma_1) - \frac{1}{2}\gamma_1^2 + \beta_1 B_1(t+\Delta;T,\phi) + \frac{\frac{1}{2}(\phi-\gamma_1)^2}{1-2\alpha_1 B_1(t+\Delta;T,\phi)} \tag{16}$$

[7] The general case is presented and deduced in the appendix of the paper of Heston and Nandi (2000).

The coefficients can, furthermore, be calculated recursively from the terminal conditions:

$$A(T; T, \phi) = 0$$

$$B_1(T; T, \phi) = 0$$

We have that the characteristic function of the logarithm of the spot price is given by $f(i\phi)$. This is due to the fact that the generating function of the spot price is the moment generating function of the logarithm of the spot price. Thus in order to use the characteristic function, ϕ is substituted by $i\phi$ in equations (15) and (16). This leads to the third proposition of Heston and Nandi's paper (2000), where they state that if $f(i\phi)$ is the characteristic function, the expected payoff of a call option then is,

$$E_t[Max(S(T) - K, 0)] = f(1)\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi} f(i\phi + 1)}{i\phi f(1)}\right] d\phi\right)$$
$$-K\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi} f(i\phi)}{i\phi}\right] d\phi\right)$$

where $Re[\,]$ is the real part of a complex number.

The price of a European call option is given by the expected value of the payoff calculated under the risk neutral assumption ($E_t^*[Max(S(T) - K, 0)]$), discounted to present value. The value of a European call option at time *t* is thus

$$c = e^{-r(T-t)}\, E_t^*[Max(S(T) - K, 0)] = \frac{1}{2}S(t) + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi} f^*(i\phi + 1)}{i\phi f(1)}\right]$$
$$-Ke^{-r(T-t)}\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\phi} f^*(i\phi)}{i\phi}\right] d\phi\right)$$

The price of put options can simply be obtained by the put call parity.[8]

[8] For a more detailed explanation about the prepositions and assumptions in the model, see Heston and Nandi (2000). The appendix offers the functions used to estimate the prices in the programming language R.

2.6 Likelihood-Ratio Test

When the fit of two models is to be examined, a commonly used test is the likelihood ratio test. The likelihood ratio states how much more likely it is to obtain an observed set of data, when using one model compared to using another model. One can then use this ratio to calculate a p-value, thus testing if the difference between the two models is significant. To simplify the computations, one can equally use the logarithm of this ratio without any loss of generality.

The log-likelihood ratio test is, for computational reasons, suitable in order to test the significance of the skewness parameter γ_1. The log-likelihood ratio test statistic is computed as follows:

$$\mathrm{LR} = -2\log\frac{\mathrm{L_R}}{\mathrm{L_{UR}}} = -2(\log\mathrm{L_R} - \log\mathrm{L_{UR}}) = 2\log\mathrm{L_{UR}} - 2\log\mathrm{L_R}$$

Where $\log\mathrm{L_R}$ is the log-likelihood for the restricted model, when γ_1 is equal to zero, which also is the null hypothesis. $\log\mathrm{L_{UR}}$ is the log-likelihood of our unrestricted model and LR is the test statistic, which is approximately follows a Chi-squared distribution. The degrees of freedom of the test is equal to one since we have that degrees of freedom is equal to the number of free parameters in the unrestricted model subtracted by the number of free parameters in the restricted model. If the null hypothesis would be rejected, it would imply that the returns are not symmetrically distributed about the mean, in other words not normally distributed.[9]

[9] See Berger and Casella (2002) or other statistical textbook.

2.7 Error Measures

In order to analyse the performance of the different pricing models, three error measures are used. The root mean square error (RMSE) is a measurement used to examine the difference between values predicted and generated by a model, and the actually observed values. More precisely, it is the root of the average squared error between the model and the actual price.

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{N}}\sum\left(\mathrm{p_{mod}} - \mathrm{p_{last\ price}}\right)^2}$$

Where $\mathrm{p_{last\ price}}$ is the observed market last price, $\mathrm{p_{mod}}$ is the price generated by respectively model and *N* is the number of observations. Since the error is squared before it is averaged, larger errors are given higher importance in this measurement.

The mean error (ME) measures the mean valuation error of a pricing model. It is simply the average of the difference between the pricing model and the actual market price. This measure is used as to indicate whether a model is categorically mispricing above or below the correct option price.

$$\mathrm{ME} = \frac{1}{\mathrm{N}}\sum \mathrm{p_{mod}} - \mathrm{p_{last\ price}}$$

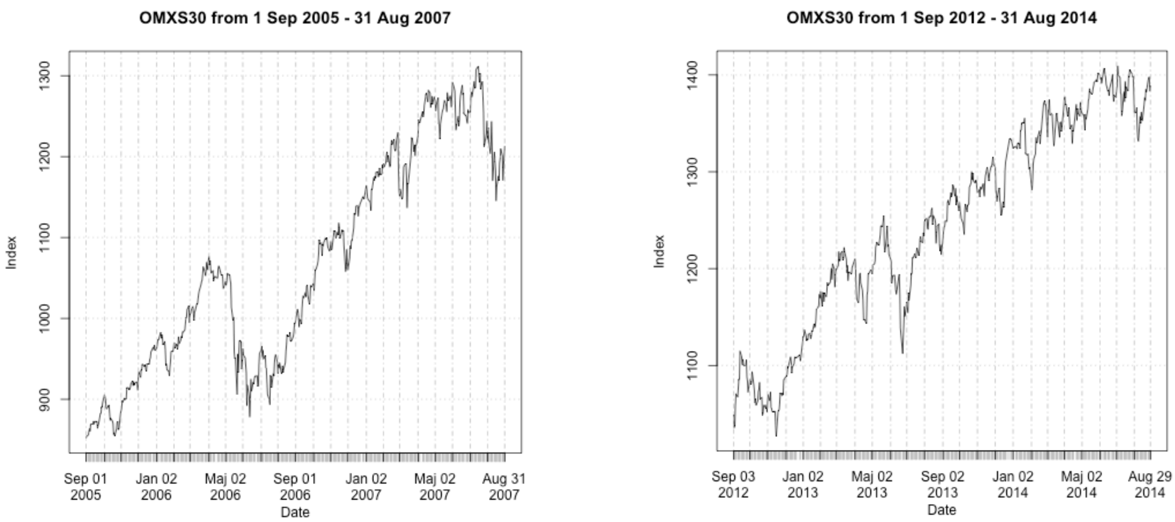
The third measure is the percentage error (PE), which illustrates how large the pricing error is relative to the actual price.

$$\mathrm{PE_{mod}} = \frac{100 * \mathrm{RMSE}}{\mathrm{meanP_{last\ price}}}$$

3 Data

In this study, daily returns and options on OMXS30 have been used from two different time periods. The first period in question is between 2005-09-01 and 2007-08-31, while the second period is between 2012-09-01 – 2014-08-31. The selection of time periods, is motivated by proximity to the financial crisis and data availability. Historical daily returns and the 3-month STIBOR rate, functioning as the risk-free rate, have been obtained from *Bloomberg Terminal*. Option prices were provided by the exchange (NasdaqOMX). For both the daily returns and the option prices, closing daily prices are used. The influence of dividends has been neglected, due to the fact that most companies on the OMXS30 pay dividend one time each year during a period in April-May. Making this restriction should in general not influence our estimations. However, it should be kept in mind when evaluating the results.

Figure 1 – OMXS30 index prices



Plots of the daily OMXS30 price for the time periods 2005-09-01 – 2007-08-31 and 2012-09-01 – 2014-08-31

In addition to removing zero-price options, two criteria are put on the option data. We choose only to include options that have maturity between 7 and 120 days. The second restriction involves removing options with an absolute moneyness[10] less than or equal to 0.1.[11] After cleansing the data, the number of option prices is reduced from 745,214 to 48,971, where 19,811 of the observations belong to the first period and 29,160 to the second period. The

[10] Defined as |K/S - 1|.

[11] For further details see Dumas, Fleming and Whaley (1998).

average price for the options during the first period is 19.82 SEK and 17.61 SEK during the second period. For the first period the in-sample valuation consists of 5,524 puts and 4,892 calls, whereas the second period consists of 4,775 puts and 4,640 calls. Corresponding numbers for the second period are 5,946 for puts and 5,839 for calls in-sample and the out-of-sample numbers are 9,275 for puts and 8,100 for calls. These numbers are presented in table 1 and the different average prices in table 2, e.g. the average price for puts in-sample is 13.35 SEK.

Table 1 – Number of observations in option price data

Year		Puts	Calls	Total
2005-2007				
	In-sample	5,524	4,892	10,416
	Out-of-sample	4,755	4,640	9,395
	Total	**10,279**	**9,532**	**19,811**
2012-2014				
	In-sample	5,946	5,839	11,785
	Out-of-sample	9,275	8,100	17,375
	Total	**15,221**	**13,939**	**29,160**

Table 1 contains the options remaining after the data cleaning, categorised by type of option and time period. Period 1 ranges from 2005-09-01 to 2006-08-31 in-sample and from 2006-09-01 to 2007-08-31 out of sample. Period 2 ranges from 2012-09-01 to 2013-08-31 in-sample and from 2013-09-01 to 2014-08-31 out-of-sample.

Table 2 – Average observed option prices in data set (in SEK)

Year		Puts	Calls	Total
2005-2007				
	In-sample	13.35	18.03	15.55
	Out-of-sample	21.93	27.26	24.56
	Total	**17.32**	**22.52**	**19.82**
2012-2014				
	In-sample	16.62	22.56	19.56
	Out-of-sample	14.46	18.37	16.28
	Total	**15.31**	**20.12**	**17.61**

Table 2 contains the average price for each option category. Period 1 ranges from 2005-09-01 to 2006-08-31 in-sample and from 2006-09-01 to 2007-08-31 out-of-sample. Period 2 ranges from 2012-09-01 to 2013-08-31 in-sample and from 2013-09-01 to 2014-08-31 out-of-sample.

4 Results

This section begins with a demonstration of the parameter estimates for both two-year periods, as well as a justification of the HN-GARCH(1,1) model. Moreover, the performance of the models is reviewed in-sample as well as out-of-sample.

4.1 – Estimations

We implement the single lag version of the GARCH model in accordance with Heston and Nandi's paper (2000). Daily index log-returns are used to model conditional standard deviation (volatility). We thus set $\Delta = 1$. For the HN-GARCH(1,1) process, Maximum Likelihood Estimation (MLE) is used to estimate the parameters from the historical daily log-returns. To test the significance of the skewness parameter γ_1, two models are used to estimate the parameters. One is the symmetric model, where $\gamma_1 = 0$, and the second is the asymmetric model where γ_1 is a variable and not constant. In table 3 the results of the likelihood tests are presented, as well as the estimated parameters for both periods.

Table 3 - Maximum likelihood estimations of parameters and likelihood ratio test

Year		α_1	β_1	γ_1	ω	λ	θ	$\beta_1+\alpha_1\gamma^2$	LL	LR-Test
2005-2007	HN-GARCH	9.075e-06	0.8463	91.80	9.563e-95	7.239	17.21%	0.92282	2862.8	
	HN-GARCH γ_1=0	1.250e-05	0.8949	0	1.535e-28	11.18	17.31%	0.8949	2849.2	<0.00001
2012-2014	HN-GARCH	1.008e-06	0.3990	665.8	9.127e-06	7.347	12.88%	0.84603	2940.9	
	HN-GARCH γ_1=0	2.936e-06	0.8539	0	6.675e-06	11.04	12.87%	0.85395	2930.5	<0.00001

Maximum likelihood estimates of the HN-GARCH(1,1) model on daily log-returns on OMXS30, for the two periods 2005-09-01 to 2007-08-30 and 2012-09-01 to 2014-08-30. Two versions of the model are presented, one under the constraint γ_1=0 and one unrestricted when γ_1 can attain any number.

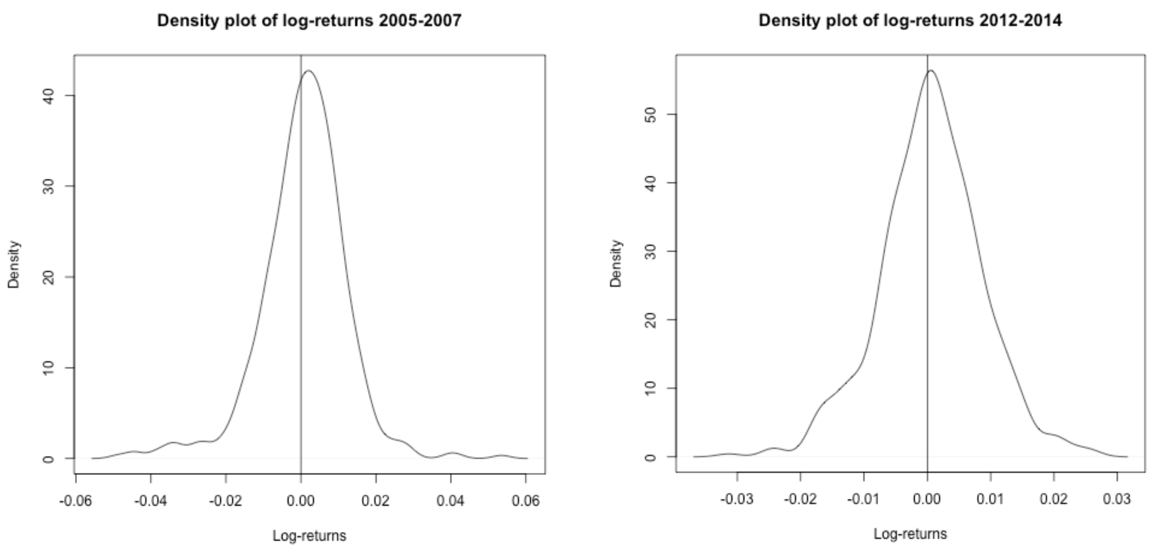
$$log(S(t)) = log(S(t-\Delta)) + r + \lambda h(t) + \sqrt{h(t)}z(t)$$

$$h(t) = \omega + \beta_1 h(t-\Delta) + \alpha_1 \left(z(t-i\Delta) - \gamma_1\sqrt{h(t-\Delta)}\right)^2$$

The log-likelihood function is $\sum_{t=1}^{T} -0.5\left(log\left(h(t)\right)\right) + z(t)^2$ where T is the number of days in the sample. θ is defined as $\sqrt{252(\omega+\alpha_1)/(1-\beta-\alpha_1\gamma_1^2)^2}$ and is the annualized (252 trading days) expected value of the conditional standard deviation (volatility), inferred by the parameter estimates .

It is obvious from the likelihood test that the γ_1 parameter is positive and significantly larger than zero for both periods, demonstrated with p-values less than 0.001%. Since we also obtain positive α_1:s, the covariance equation (11) tells us that the correlation between the index returns and the volatility is, in fact, negative. We can thus conduct, that the asymmetric model is in agreement with the leverage effect presented by Black (1976). Figure 2 displays the density plots of the log-returns for the two periods. By visually inspecting the two curves, one can anticipate the result of the log-likelihood test, noticing that the curves are not normal but instead negatively skewed – apparent from the thicker left tails and right-shifted median. Black-Scholes' assumption of log-normally distributed returns can therefore highly be questioned.

Figure 2 – Density plots of log-returns



Density plots of the log-returns for the period 2005-09-01 – 2007-08-31 as well as for the period 2012-09-01 – 2014-08-31.

From Table 3, we also see that the degree of mean reversion, $\beta_1+\alpha_1\gamma^2$, for the asymmetric model is 0.92282 for the first period and 0.84603 for the second. The volatility of the volatility, given by α_1, is 9.075e-06 and 1.008e-06 respectively. θ, which represents the annualized long-run mean volatility, decreases from 17.21% for the first period to 12.88% for the second period.

Figure 3 demonstrates the conditional variance for the symmetric model and the asymmetric model for the two periods. By inspection of the graphs in the figure, the asymmetric model produces more persistent periods of volatility before suddenly jumping. Thus better resembling the autocorrelation of the volatility movement found in markets for exampled studied by Booth and Koutmos (1998). Adding the gamma parameter to the model therefore gives a more realistic volatility process and should, in theory, add quality to the behaviour of the process.

Figure 3 - Symmetric and asymmetric GARCH

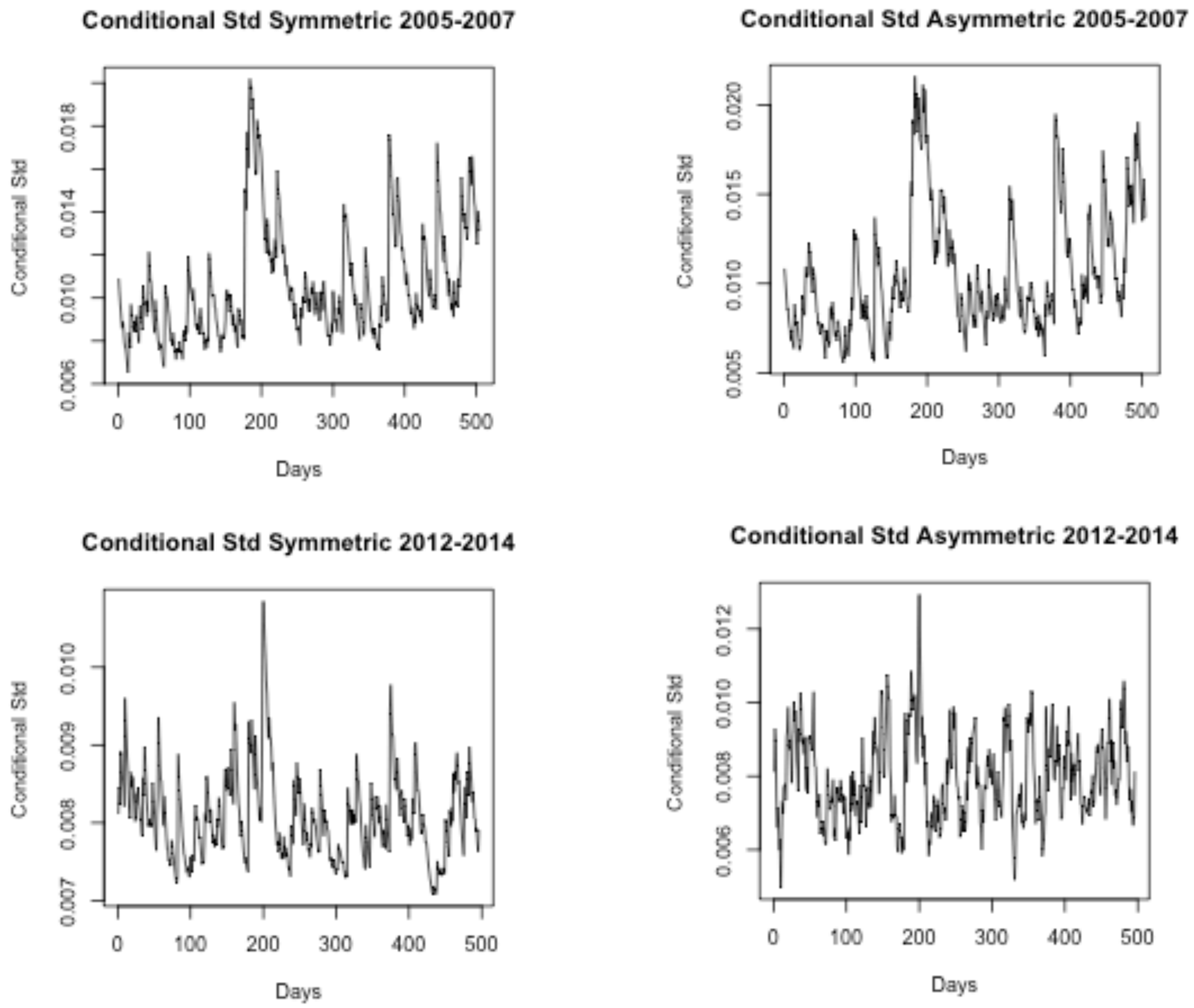


Figure 3 illustrates the GARCH process for each period. The pictures to the left is the symmetric GARCH process in which γ_1 is set equal to zero, while the pictures to the right is the asymmetric GARCH process in which no restriction is out on γ_1. Period 1 ranges from 2005-09-01 to 2007-08-31 and period 2 from 2012-09-01 to 2014-08-31.

4.2 Model Comparison – In-Sample

The in-sample comparison between the BS-model and the HN-GARCH(1,1) model is conducted by estimating, and fixing, the parameters of the HN-GARCH(1,1) model on the first twelve months period of each data set. In other words, on observable market data from the 1st of September to the 31th of August, for each time period respectively (2005 to 2006 and 2012 to 2013). Average option price for the two periods are 15.5500 SEK and 19.5620 SEK. The in-sample analysis is implemented to optimize the HN-GARCH(1,1) model's performance on a given set of historical data. Therefore it does not reveal anything about the forecasting qualities of tomorrow's option prices.

Table 4 presents each period's maximum likelihood estimates of the parameters for the in-sample HN-GARCH(1,1) model. In line with the results presented in the previous section, positive $\alpha_1$and γ_1, result in a negative correlation between the index and the variance. The distribution of the log-returns is thus negatively skewed. This holds for both periods (see table 4). A striking observation is however, the large difference in γ_1 between the two periods. We furthermore note the long run annualized volatility, θ, drops from 15,91% in the first period to 13.83% in the second period.

Table 4 – In-sample comparison for all options

Year		α_1	β_1	γ_1	ω	λ	θ	$\beta_1+\alpha_1\gamma^2$	RMSE	Average	n
2005-2006										15.5500	10,416
	BS								5.2052		
	HN-GARCH	1.100e-05	0.7957	92.83	1.403e-120	7.685	15.91%	0.89055	7.3666		
2012-2013										19.5620	11,785
	BS								10.7785		
	HN-GARCH	6.426e-07	3.484e-11	1149	1.090e-05	6.040	13.83%	0.84803	10.0582		

BS is the Black-Scholes model estimated using unconditional volatility (historical volatility) updated each day. The parameters of the HN-GARCH(1,1)-model are obtained by maximum likelihood estimation. The parameters are constant during the entire in-sample valuation. The table considers both call and put options. RMSE is the root mean square error, "Average" is the average price of the options (call and puts) in each set and "n" is the number of observations for each respective period. The first period ranges from 2005-09-01 to 2006-08-31 and the second period from 2012-09-01 to 2013-08-31.

The HN-GARCH(1,1) parameters are fixed for the in-sample valuation and average RMSE is 7.3666 SEK for the first period and 10.0582 SEK for the second period. This can be compared with the corresponding errors of the BS-model in-sample valuation of 5.2052 SEK for the first period and 10.7785 SEK for the second period. In the first period the HN-GARCH(1,1) is outperformed by the BS-model by 2.1614 SEK, while the error measures in the second period are more similar, with a slight advantage of 0.7203 SEK to the HN-GARCH(1,1).

The BS-model's option prices, are computed with the unconditional volatility (historical volatility), estimated each day - making it a somewhat more flexible estimation compared to the HN-GARCH(1,1) model. It is not unlikely that BS-model's outperformance of the HN-GARCH(1,1) is due to this flexibility. A more just comparison could thus be to implement an updated version of the HN-GARCH model with parameters updated every five-day (trading days) period. This extension is however not introduced in this section, but in the out-of-sample comparison, which is the main focus of this paper.

4.3 Model Comparison – Out-of-Sample

Out-of-sample evaluation is a method often used to test how well a specific model might perform in the future. The HN-GARCH(1,1) model is thus implemented using the parameters obtained from the in-sample estimations for period 1 and period 2 respectively. Option prices from the non-updated HN-GARCH(1,1) model are calculated with the in-sample parameters and the conditional variance, $h(t+1)$, generated by equation (9) and movements of the underlying asset. The initial value of the variance process $h(0)$ is obtained from the in-sample parameter estimations. The BS-model is tested using the one-year unconditional volatility (historical volatility), re-estimated every day, similar to the in-sample procedure.

By inspecting tables 3 and 4, we observe differences in the parameters of the distribution, implying that the distribution of the log-returns of the underlying asset varies over time. In order to capture this change, an updated version of the HN-GARCH(1,1) model, with parameters updated every week (five trading days), is introduced. The variance process is otherwise similar to the non-updated version, with the conditional variance depending on index levels and the particular parameter estimations. In contrast to the non-updated model that generates option prices with constant parameters and distribution over the entire year, the updated version is more flexible since its re-estimated parameters allows for changes in the distribution of the log-returns.

Table 5 gives a brief summary of the models performance out-of-sample for all options. The RMSE values, for the first period, are 7.4264 SEK, 7.9259 SEK and 7.3859 SEK for the BS model, the non-updated HN-GARCH(1,1) model and the updated HN-GARCH(1,1) respectively. Corresponding values for the second period are 6.9406 SEK, 8.4924 SEK and 8.2970 SEK. This tells us that the updated HN-GARCH(1,1), on average, values options better than the non-updated HN-GARCH(1,1). It additionally performs better than the BS-model in the first period, this is however not true for the second period. From the mean error values, one can furthermore see that the BS-model, on average, undervalues options while the both HN-GARCH(1,1) models overvalues. This holds for both periods and is consistent with the results of Heston and Nandi (2000) on S&P500 options.[12]

Table 5 – Out-of-sample comparison all options

Year		RMSE	ME	Averarage option price	n
2006-2007				24.564	9,395
	BS	7.4264	-1.2653		
	HN-GARCH (non-updated)	7.9259	1.4345		
	HN-GARCH (updated)	7.3859	1.5546		
2013-2014				16.285	17,375
	BS	6.4906	-0.7174		
	HN-GARCH (non-updated)	8.4924	4.5041		
	HN-GARCH (updated)	8.2970	4.1444		

RMSE stands for root mean square error, ME for mean error and "n" for the number of observations. Average option price is the average price of all puts and calls. The first out-of-sample period ranges from 2006-09-01 to 2007-08-31 and the second out-of-sample period from 2013-09-01 to 2014-08-31.

To further analyse the updated HN-GARCH(1,1) model, we examine the average parameter estimations for the two periods, presented in table 6. In accordance with Heston and Nandi, who state that, *"...option values are more sensitive to α_1 (that measures the volatility of volatility), and γ_1 (that controls the skewness of index returns) than they are to the other parameters. This stability is important for the GARCH model to fit the data reasonably*

[12] Heston and Nandi (2002), page 607, table 5a.

well…" one can suspect that the updated model's prices are imperfect due to the instability of α_1 and γ_1. The high standard deviations are partly due to some periods of substantial drops in the estimations of the α_1 and γ_1 parameters. These periods happen to coincide with periods characterised by large fluctuations and absence of any clear market trend. The poor parameter estimates, makes it possible to question the appropriateness of the maximum likelihood estimation.

Table 6 – Average out-of-sample updated parameter values

Year		α_1	β_1	γ_1	ω	λ
2006-2007						
	Mean	1.065e-05	6.156e-01	161.1	1.149e-05	6.894
	Standard deviation	6.236e-06	3.141e-01	208.9	2.860e-05	2.460
2013-2014						
	Mean	8.576e-07	5.396e-02	745.5	1.942e-05	8.650
	Standard deviation	7.490e-07	4.785e-02	429.3	1.851e-05	2.510

Average parameter values and standard deviations for the updated HN-GARCH(1,1) process. The first out-of-sample period ranges from 2006-09-01 to 2007-08-31 and the second out-of-sample period from 2013-09-01 to 2014-08-31.

Tables 7A and 7B (8A and 8B) present a more detailed view of the three models' out-of-sample performance during the first period (second period). Both call and put options are evaluated over different maturities and moneyness. Table 7A regards call option valuation for the first period. We can see that the BS-model, overall, undervalues call options out-of-the money and overvalues in the money, as one would expect by the volatility smile and Jackwerth and Rubenstein (1996).[13] This is, however not the case for the both HN-GARCH(1,1) models, which categorically overprices for all maturities and types of moneyness. Additionally, the non-updated model performs better than the updated version in terms of RMSE on all options expect far in-the money options with maturity between 40 and 70 days. The put performance is in turn presented in table 7B, and the BS-model prices are, in general, below the actual price for out-of-the money puts and above the actual price for in-the money puts. Both HN-GARCH(1,1) models, on the other hand, undervalues all options (except short term options with moneyness 0.99-0.01 for the updated version), with a slight advantage to the non-updated version, in terms of RMSE across all maturities and moneyness. The performances of the models are also illustrated in figures 4 and 5, in which

[13] Jackwerth and Rubenstein (1996) page 1612.

the percentage error is plotted against moneyness, for each group of maturity. For put options, the HN-GARCH(1,1) models perform better than the BS-model for all maturities (except for far in-the money options with maturity equal or greater to 40), while the result for call options is more ambiguous, with the relative success of the BS-model increasing with maturity.

Table 7A – Out-of-sample comparison, calls period 1

		Time to maturity								
		<40			**40-70**			**>70**		
Model	Moneyness	RMSE	% Error	ME	RMSE	% Error	ME	RMSE	% Error	ME
BS										
	<0.95	2.2529	123.19%	0.5529	5.3959	80.64%	1.2551	8.0439	72.84%	3.3166
	[0.95-0.99)	4.6206	57.84%	1.1142	8.3704	46.94%	1.0390	10.5861	42.09%	2.1726
	[0.99-1.01)	6.7129	30.71%	-0.4162	9.8746	29.27%	-0.6993	9.3372	22.88%	1.2538
	[1.01-1.05]	8.1543	18.44%	-2.1882	10.6576	20.39%	-2.1872	13.6290	20.84%	-2.2165
	>1.05	9.9376	12.00%	-0.7059	15.4456	16.17%	-5.8658	12.0626	11.31%	-3.7844
HN-GARCH (non-updated)										
	<0.95	1.5715	85.93%	0.6451	5.0009	74.74%	3.3298	9.6588	87.46%	7.9322
	[0.95-0.99)	3.8013	47.58%	2.2930	8.3537	46.85%	5.8087	13.1354	52.23%	10.6385
	[0.99-1.01)	5.5149	25.23%	3.0532	10.4975	31.11%	6.9874	14.5409	35.64%	12.4863
	[1.01-1.05)	7.0997	16.05%	2.8624	11.2548	21.54%	7.0213	16.3753	25.04%	11.9158
	>1.05	10.3819	12.54%	4.7526	14.4262	15.10%	5.4894	16.7119	15.68%	11.7456
HN-GARCH (updated)										
	<0.95	1.8829	102.96%	0.6214	5.2654	78.69%	3.1654	9.9374	89.99%	7.9503
	[0.95-0.99)	4.6674	58.42%	2.6709	8.8505	49.64%	5.8918	13.5954	54.06%	10.8771
	[0.99-1.01)	6.6166	30.27%	3.5090	10.7468	31.85%	6.9956	15.0318	36.84%	12.7909
	[1.01-1.05)	7.8082	17.66%	3.2561	11.4414	21.89%	7.0570	16.9893	25.98%	12.3514
	>1.05	10.7215	12.95%	4.9021	14.3325	15.01%	5.5527	17.1533	16.10%	12.3414

Out-of-sample valuation errors, sorted after maturity and moneyness, which is defined as $|S/K|$ for call options. RMSE and ME are valued in SEK, while % Error is given by $\mathrm{PE_{mod}} = \frac{100 * \mathrm{RMSE}}{\mathrm{meanP_{last\ price}}}$. The out-of-sample period ranges from 2006-09-01 to 2007-08-31.

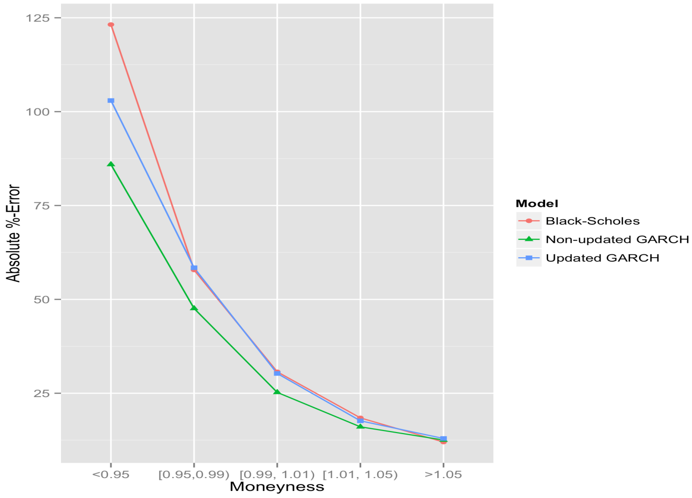
Table 7B – Out-of-sample comparison, puts period 1

		Time to maturity								
		<40			40-70			>70		
Model	Moneyness	RMSE	% Error	ME	RMSE	% Error	ME	RMSE	% Error	ME
BS										
	<0.95	3.7375	91.16%	-2.5083	7.5961	65.26%	-5.5323	8.5574	45.58%	-6.6634
	[0.95-0.99)	4.8451	44.21%	-2.1238	8.4016	37.77%	-3.9920	8.8226	28.66%	-4.2964
	[0.99-1.01)	5.3711	24.96%	-0.1080	8.6997	25.68%	-1.8268	9.5402	22.72%	-2.2319
	[1.01-1.05]	5.7649	14.24%	0.7880	9.3294	18.25%	0.4931	11.0631	18.31%	-1.6505
	>1.05	9.0870	11.89%	2.8145	11.1035	12.91%	1.4384	10.7941	11.39%	2.4937
HN-GARCH (non-updated)										
	<0.95	2.2033	53.74 %	-0.9007	4.7645	40.93%	-2.5272	6.2462	33.27%	-4.1368
	[0.95-0.99)	3.1966	29.05%	-0.5917	5.9862	26.91%	-2.0295	6.9380	22.51%	-2.9782
	[0.99-1.01)	4.4050	20.46%	-0.1908	7.0567	20.83%	-1.8620	7.2705	17.31%	-2.2807
	[1.01-1.05]	5.2332	12.93%	-0.8673	8.2045	16.05%	-2.1734	9.1022	15.07%	-2.8645
	>1.05	8.4897	11.10%	0.7041	11.0945	12.90%	-3.2251	11.2818	11.90%	-3.8159
HN-GARCH (updated)										
	<0.95	2.4747	60.36%	-0.7589	5.2690	45.26%	-2.4947	6.4592	34.41%	-4.0134
	[0.95-0.99)	3.7612	34.18%	-0.2173	6.8515	30.80%	-1.9972	7.6683	24.91%	-2.9926
	[0.99-1.01)	4.8928	22.73%	0.2451	8.0891	23.88%	-2.0292	8.2742	19.70%	-2.5941
	[1.01-1.05]	5.4922	13.57%	-0.8218	9.1106	17.82%	-2.3977	10.7397	17.78%	-4.0089
	>1.05	8.6056	11.26%	-0.2296	12.0640	14.03%	-4.3701	11.7341	12.38%	-4.9281

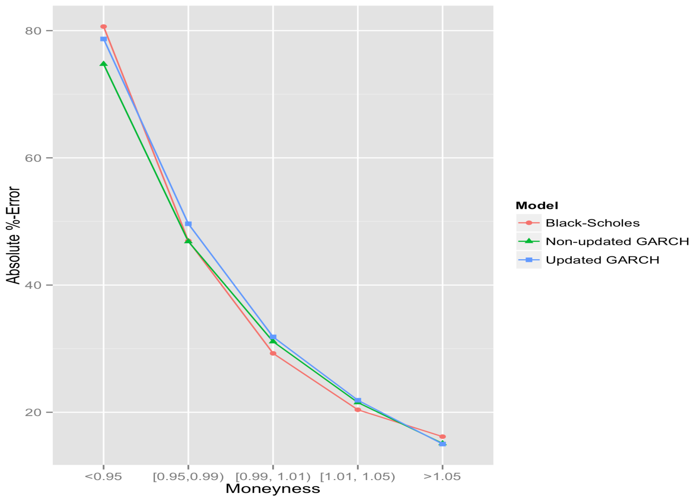
Out-of-sample valuation errors, sorted after maturity and moneyness, which is defined as $|K/S|$ for put options. RMSE and ME are valued in SEK, while % Error is given by $\mathrm{PE}_{\mathrm{mod}} = \frac{100*\mathrm{RMSE}}{\mathrm{meanP}_{\text{last price}}}$. The out-of-sample period ranges from 2006-09-01 to 2007-08-31.

Figure 4 – Out-of-sample percentage pricing errors period 1, calls

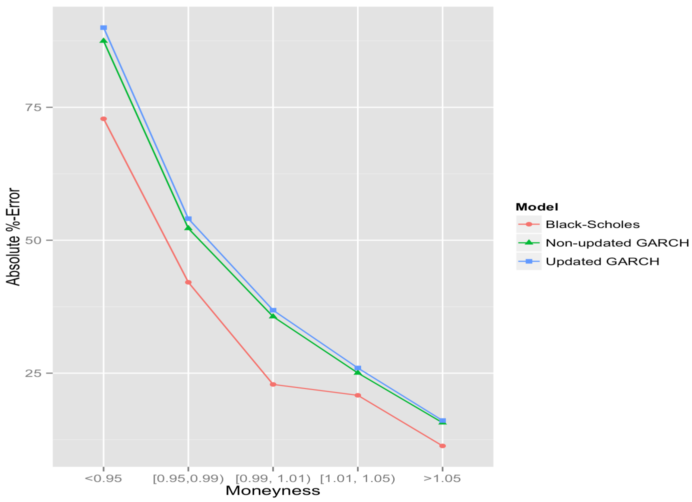
4A – Maturity less than 40 days



4B – Maturity between 40 and 70 days



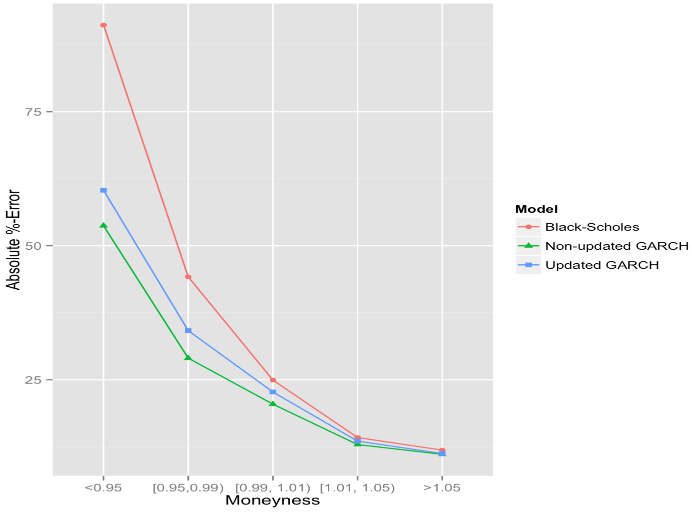
4C – Maturity larger than 70 days



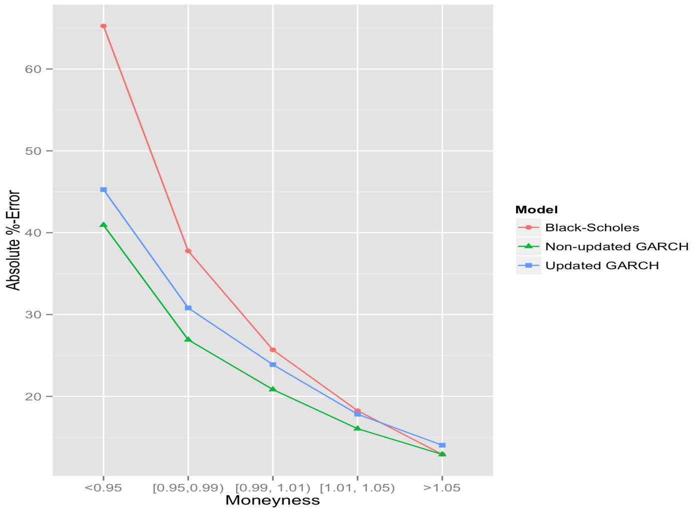
The figure above contains three plots illustrating the models'pricing performance, as the percentage error (defined by $\mathbf{PE_{mod}} = \frac{\mathbf{100*RMSE}}{\mathbf{meanP_{last\ price}}}$) with respect to maturity, of call options out-of-sample. The out-of-sample period renges from 2006-09-01 tot 2007-08-31. Figures 4A, 4B and 4C represent options that have less than 40 days, between 40 and 70 days and more than 70 days to maturity respectively.

Figure 5 – Out-of-sample percentage pricing errors period 1, puts

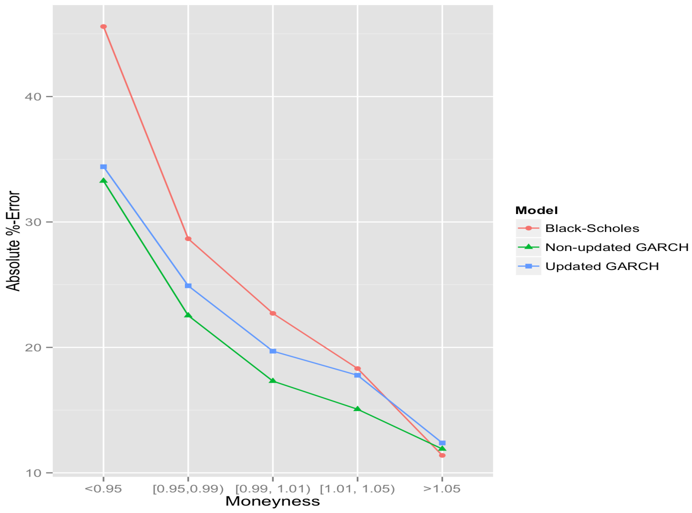
5A – Maturity less than 40 days



5B – Maturity between 40 and 70 days



5C – Maturity larger than 70 days



The figure above contains three plots illustrating the models'pricing performance, as the percentage error (defined by $\mathbf{PE_{mod}} = \frac{\mathbf{100*RMSE}}{\mathbf{meanP_{last\ price}}}$) with respect to maturity, of put options out-of-sample. The out-of-sample period renges from 2006-09-01 tot 2007-08-31. Figures 4A, 4B and 4C represent options that have less than 40 days, between 40 and 70 days and more than 70 days to maturity respectively.

Tables 8A and 8B present the corresponding error measures for the second period. Both of the HN-GARCH(1,1) models are outperformed quite substantially by the BS-model, when it comes to pricing call options. We note that the HN-GARCH(1,1) models, analogously to the result of the first period, continues to overvalue call options across all maturities and moneyness, and that the updated version performs, somewhat, better than the non-updated model in terms of RMSE . Unlike the first period however, the BS-model also overvalues calls (except for far in-the-money options with time to maturity longer than 70 days) in the second period. In figure 6, the percentage errors are plotted and one can conclude that the HN-GARCH models are greatly outperformed by the BS-model.

Table 8B shows a more mixed result than table 8A. The average BS-model put option price is below the observable price for all maturities and moneyness, except for short maturity options with moneyness equal or greater than 1.01 and far in-the money options with maturity between 40 and 70 days. Options with a maturity less than 0.95 are undervalued by both the non-updated and updated versions of the HN-GARCH(1,1) model across all maturities, while all other types of moneyness result in an overvaluation of put options. For short-term maturity options, the updated version performs better than the non-updated model, the same holds for long-term options, with the exception of options with moneyness less than 0.95. When the maturity is between 40 and 70 days, the interrelationship between the two models varies. We can furthermore see that the valuation of out-of-the money puts are, similar to the first period, improved considerably by the HN-GARCH(1,1) models compared to the BS-model. The BS-model improves close to at-the money options and produces smaller RMSE for in-the money puts. This course of event is illustrated further in Figure 7. A striking similarity between the two out-of-sample valuation periods, is thus that the HN-GARCH(1,1) models perform better when it comes to valuing put options, than when it comes to valuing call options.

Table 8A – Out-of-sample comparison period 2, calls

		Time to maturity								
		<40			40-70			>70		
Model	Moneyness	RMSE	% Error	ME	RMSE	% Error	ME	RMSE	% Error	ME
BS										
	<0.95	0.6376	116.26%	0.3397	1.5046	74.55%	1.0748	2.4047	38.57%	1.1532
	[0.95-0.99)	2.6269	81.80%	1.5596	4.6013	50.61%	2.6354	5.9979	38.07%	1.9800
	[0.99-1.01)	4.5284	37.08%	2.1269	7.4110	31.32%	2.6142	9.2372	29.13%	2.2660
	[1.01-1.05]	9.8171	28.80%	3.2805	13.9187	33.76%	5.5034	13.1685	26.56%	3.9237
	>1.05	8.4008	9.08%	1.4614	12.5906	13.46%	3.0517	12.2002	12.85%	-0.9651
HN-GARCH (non-updated)										
	<0.95	1.9057	347.51%	1.4113	4.9789	246.68%	4.5593	9.0965	145.91%	8.5361
	[0.95-0.99)	5.6868	177.08%	4.6759	10.7194	117.91%	9.9403	14.1215	89.62%	12.9794
	[0.99-1.01)	8.2890	67.88%	7.0053	14.0326	59.32%	12.3876	17.8543	56.30%	15.6631
	[1.01-1.05]	12.8466	37.68%	8.5338	19.9589	48.42%	15.6446	21.6271	43.61%	17.7484
	>1.05	9.5760	10.35%	4.4651	16.0752	17.18%	10.5115	16.7256	17.61%	11.4096
HN-GARCH (updated)										
	<0.95	1.6749	305.42%	1.2013	4.7110	233.41%	4.2741	8.8663	142.22%	8.2957
	[0.95-0.99)	5.3019	165.09%	4.2102	10.4003	114.40%	9.5785	13.9091	88.28%	12.7240
	[0.99-1.01)	7.8324	64.14%	6.4039	13.7477	58.11%	12.0208	17.6867	55.77%	15.4398
	[1.01-1.05]	12.5972	36.95%	8.1892	19.7225	47.84%	15.3178	21.4828	43.32%	17.5787
	>1.05	9.5733	10.34%	4.3027	16.0112	17.12%	10.3852	16.6682	17.55%	11.3107

Out-of-sample valuation errors, sorted after maturity and moneyness, which is defined as |S/K| for call options. RMSE and ME are valued in SEK, while % Error is given by $\mathrm{PE_{mod}} = \frac{100*\mathrm{RMSE}}{\mathrm{meanP_{last\ price}}}$. The out-of-sample period ranges from 2013-09-01 to 2014-08-31.

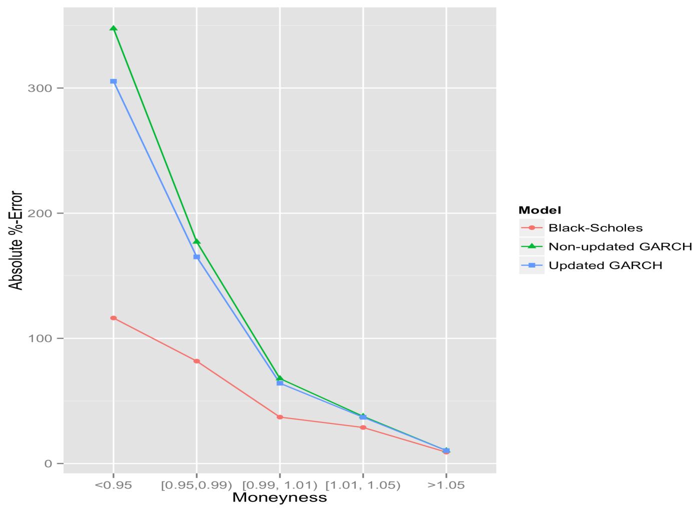
Table 8B – Out-of-sample comparison period 2, puts

		Time to maturity								
		<40			40-70			>70		
Model	Moneyness	RMSE	% Error	ME	RMSE	% Error	ME	RMSE	% Error	ME
BS										
	<0.95	2.7184	115.25%	-2.0060	5.8162	80.19%	-5.1101	9.2946	68.84%	-8.3094
	[0.95-0.99)	4.0238	64.15%	-2.4420	7.6352	45.78%	-5.6821	10.6727	40.34%	-8.5340
	[0.99-1.01)	5.2452	34.36%	-0.9734	7.6459	26.02%	-3.3202	7.7986	19.94%	-4.7350
	[1.01-1.05]	9.7936	28.01%	1.3331	9.7725	21.83%	-0.0274	9.3182	17.99%	-2.7531
	>1.05	18.6691	25.60%	6.2528	15.7186	20.04%	3.3540	5.0621	5.79%	-0.3218
HN-GARCH (non-updated)										
	<0.95	1.6881	71.56%	-0.5233	2.8245	38.94%	-0.5669	4.2887	31.76%	-1.3086
	[0.95-0.99)	3.5928	57.28%	1.2605	5.7778	34.64%	1.7380	7.0276	26.56%	1.0936
	[0.99-1.01)	6.2699	41.07%	3.2498	8.3974	28.58%	4.4145	8.2426	21.07%	4.8963
	[1.01-1.05]	10.6641	30.50%	4.0416	11.7248	26.20%	5.9974	10.9980	21.23%	5.5732
	>1.05	18.6095	25.52%	6.0726	15.9174	20.29%	4.7966	7.1786	8.21%	3.2202
HN-GARCH (updated)										
	<0.95	1.6257	68.92%	-0.6482	2.8660	39.51%	-0.7474	4.3390	32.14%	-1.4543
	[0.95-0.99)	3.4604	55.17%	0.8504	5.6732	34.01%	1.4132	6.9575	26.30%	0.8475
	[0.99-1.01)	5.9715	39.12%	2.6543	8.1940	27.89%	3.7631	8.1035	20.72%	4.6267
	[1.01-1.05]	10.4123	29.78%	3.4294	11.5380	25.78%	5.4714	10.8051	20.86%	5.1931
	>1.05	18.5629	25.46%	5.8267	15.9418	20.32%	4.3887	6.8636	7.85%	2.8803

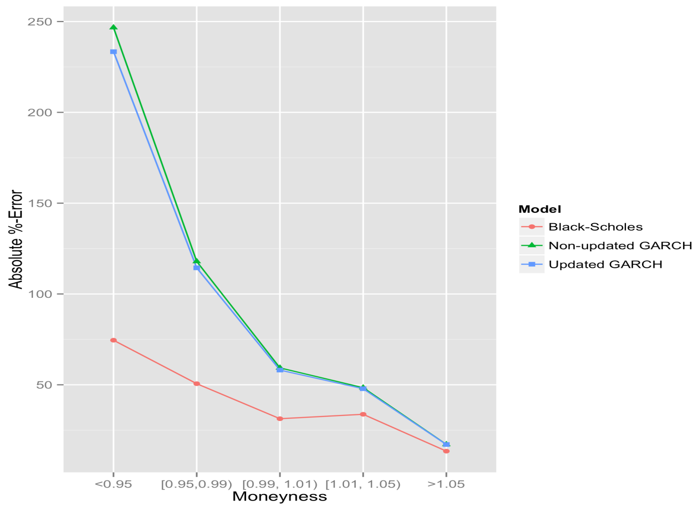
Out-of-sample valuation errors, sorted after maturity and moneyness, which is defined as |K/S| for put options. RMSE and ME are valued in SEK, while % Error is given by $PE_{mod} = \frac{100*RMSE}{meanP_{last\ price}}$. The out-of-sample period ranges from 2006-09-01 to 2007-08-31.

Figure 6 – Out-of-sample percentage pricing errors period 2, calls

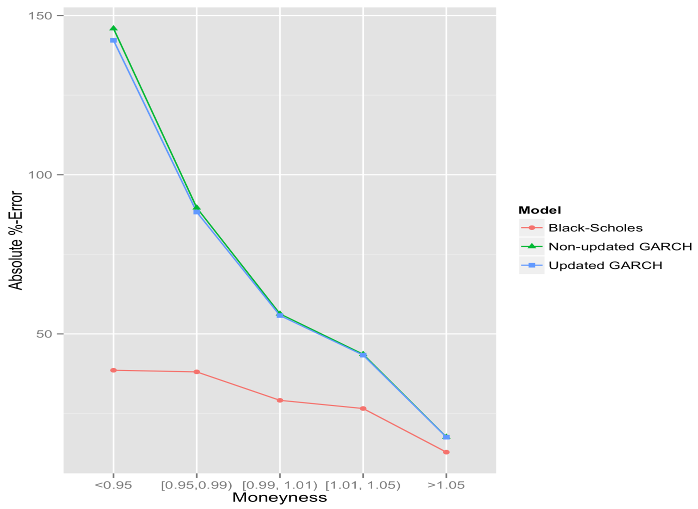
6A – Maturity less than 40



6B – Maturity between 40 and 70 days



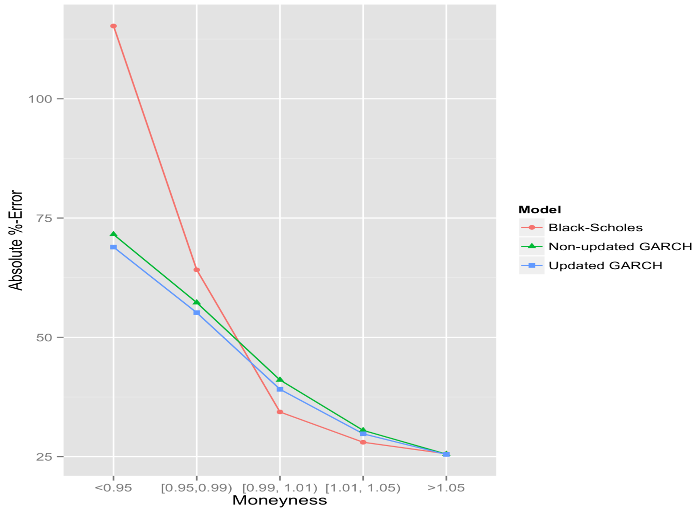
6C – Maturity larger than 70 days



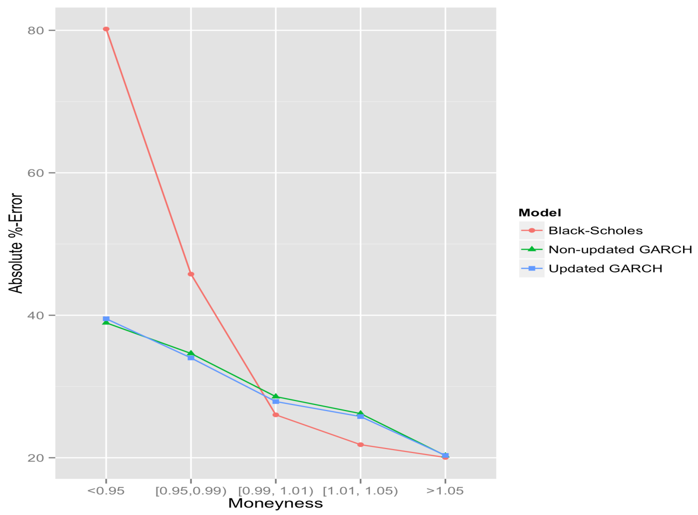
The figure above contains three plots illustrating the models'pricing performance, as the percentage error (defined by $\mathbf{PE_{mod}} = \frac{\mathbf{100*RMSE}}{\mathbf{meanP_{last\ price}}}$) with respect to maturity, of call options out-of-sample. The out-of-sample period renges from 2013-09-01 tot 2014-08-31. Figures 4A, 4B and 4C represent options that have less than 40 days, between 40 and 70 days and more than 70 days to maturity respectively.

Figure 7 – Out-of-sample percentage pricing errors period 2, puts

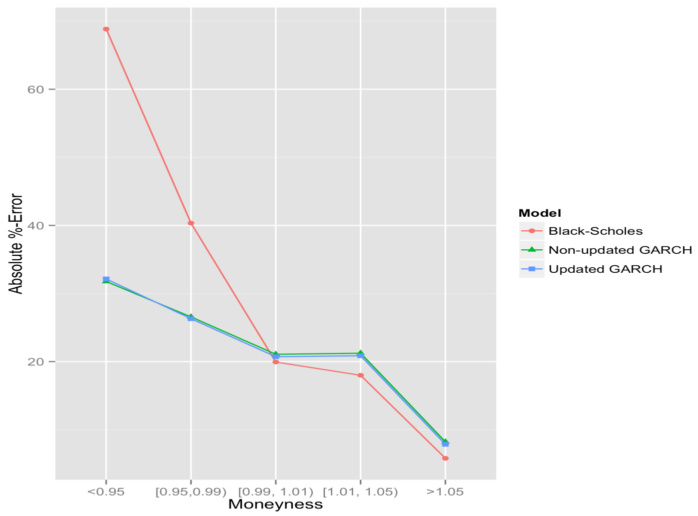
7A – Maturity less than 40 days



7B – Maturity between 40 and 70 days



7C – Maturity larger than 70 days



The figure above contains three plots illustrating the models'pricing performance, as the percentage error (defined by $\mathbf{PE_{mod}} = \frac{\mathbf{100 * RMSE}}{\mathbf{meanP_{last\ price}}}$) with respect to maturity, of put options out-of-sample. The out-of-sample period renges from 2013-09-01 tot 2014-08-31. Figures 4A, 4B and 4C represent options that have less than 40 days, between 40 and 70 days and more than 70 days to maturity respectively.

5. Conclusion

This paper evaluates the performance of Heston and Nandi's closed form option pricing model on the OMXS30 (Swedish stock index) options for two different time periods. The periods were chosen with respect to the financial crisis, in order to investigate if any changes, in terms of accuracy, could be identified. Heston and Nandi (2000) assume that the variance of the returns of underlying asset follows a GARCH(*p,q)* process. The HN-GARCH(1,1) model, that is tested in this thesis, limits to Heston's (1993) continuous time pricing model, when Δ approaches zero.

Unlike Heston and Nandi (2000), who estimate their parameters by using the nonlinear least square method and intra-daily option prices, the parameters are estimated from daily index returns using the maximum likelihood method. This approach leads to unstable out-of-sample parameter estimates for the updated HN-GARCH(1,1), which most likely affect the model's performance. However, the pricing errors of the updated and non-updated HN-GARCH(1,1) models follow each other closely. When it comes to valuing out-of-the money put options, the HN-GARCH(1,1) model performs significantly better than the famous BS-model, successfully incorporating the volatility smile and the skewness of the distribution of the underlying asset. On the other hand, the HN-GARCH(1,1) model's ability to price call options is, often, far from satisfactory. The inconclusive performance of the HN-GARCH(1,1) model, is likely due to the weak parameter estimates. Instead of using historical index returns and the maximum likelihood method, a different approach would be to use the nonlinear least square method and option prices to regress the parameters. The latter technique is as mentioned implemented by Heston and Nandi (2000), who successfully manages to improve the pricing of both put and call options compared to the BS-model. An extension of this thesis, could thus be to implement the model using the nonlinear least squares method and investigate how this affects the performance of the updated HN-GARCH(1,1) model. Another possible extension would be to conduct future studies on other markets and indices, to examine how the model's general pricing ability has, if at all, changed due to the financial crisis. Although it involves more work and computations, an implementation of the model with multiple lags would, despite the uncertainty of significant improvements, be an interesting addition to the subject. The model could finally also be tested on other types of options.

6. References

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