

VALUING CREDIT DEFAULT SWAPS WITH A STRUCTURAL APPROACH

PER MÖLLER

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LUND UNIVERSITY

Faculty of Engineering
Centre for Mathematical Sciences
Mathematical Statistics

Abstract

Valuing single-name Credit Default Swaps (CDS) is a difficult task since in order to make a fair valuation, one needs to assess the credit risk of the corresponding company. Many different models exist when it comes to modelling the credit risk, this report specifically focuses on the branch of models named structural models. The aim of this thesis is to, for a number of companies, model the CDS-spreads given by the market by first modelling the credit risk of the company in the aforementioned models, and then using key metrics calculated in the models to value the corresponding CDS-contract.

A couple of different models are tested with different sets of key parameters, and the results show that a certain implementation of the Black-Cox model produces best results, which also happens to be the model with the most real-world like features. The model manages to follow both major changes in the CDS-spreads, as well as minor changes. The corresponding residuals show some stationarity-features and are slightly improved when adjusting for the current level of volatility, however, they do not appear to be white noise as corresponding SACF-plots show clear correlation for many lags. The Black-Cox model also proves to be better than a simple regression model, used as a benchmark.

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1 Introduction

1.1 Background

During the 21st century, the market for credit derivatives has grown to one of the world's biggest financial markets [1], which in turn has generated a demand for successfully being able to evaluate the credit risk of a firm, since the value of a credit-related security more or less boils down to the credit risk of the company. In short, credit risk can be summarised as the risk that a borrower will default on its debt, i.e. being unable to deliver on the predetermined cash flows of the debt.

One of the most common credit derivatives used today are credit default swaps. They essentially work as an insurance for bond holders in case the bond issuer is unable to deliver the promised cash flows on the debt. When determining the price of a credit default swap, one must have some opinion of the credit risk of the reference company. How one decides to estimate the credit risk is essential for the resulting price of the credit default swap. One way to do this is through the use of structural models, assigning dynamics to a company's assets that connects its debt and its equity.

Using structural schemes to model a firm's stock price is one of the fundamental breakthroughs in financial mathematics. This was first done by Black & Scholes which ultimately led to the elegant closed-form solutions used when valuing European call and put options [5].

The market value of a firm's assets consists of the the market value of its equity (essentially the market capitalisation) and the market value of its debt. The market capitalisation is directly observable in the market for listed companies, the market value of debt however is not as easily assessed. A firm's debt can consist of several different types of liabilities, ranging from issued bonds, bank loans, accounts payable and so on. Bonds are usually traded on the secondary market, or over the counter (OTC) which it is often referred to as. Although the bonds are not listed on any exchange, the prices they trade for are however often publicly available, making it easy to assess the market value of a company's debt that is in the form of bonds. Other types of debt however is more or less only observable four times per year, when firms state their financial results for the past quarter, which includes the current state of the balance sheet where the latest information of the current book value of debt is available. Therefore, the only part of the debt where the market value can be easily observed is debt raised through bonds. But how does one assess the market value of the remainder of the debt? Due to the relative lack of observability of the company debt, the asset value of the firm consequently becomes difficult to determine.

One of several approaches to assessing the credit risk of a company is through a model approach, i.e. assigning certain dynamics to, for example, the company's assets, and through the model, assuming the dynamics are true, estimating the credit risk of a company.

1.2 Short history of model types

There exists primarily two types of models for modelling credit risk, structural models and reduced form models. Reduced form models model the time of default τ simply as an exogenous random variable, with some default intensity λ . These models argue that default occurs at a completely random point in time [2]. Structural models however try to connect the default time with economic properties of the company. One of the first attempts at this was presented in 1974 by Merton, whom proposed the firm assets had similar dynamics as that of an individual stock presented earlier by Black & Scholes. The Merton model states that a default occurs if the company's assets has fallen below some predetermined default barrier after a certain period of time has passed [15].

There is of course both positives and negatives with the two approaches. The reduced form models generally give more flexibility, but may not be as intuitive as the structural model [12]. The structural model however is more easy to understand in the sense that it assigns a dynamic to the firms assets and from these dynamics estimates credit risk. Though it is difficult to implement these models successfully due to the fact that what the model is trying to model, the company assets, are not observable.

1.3 Objective of the thesis

This thesis will focus on trying to value credit default swaps when the market value of the firm's assets are given dynamics according to structural models, hence the reduced form models will not be further investigated. Through the models, different measures of credit risk can be found, which can be used in order to value different types of credit related instruments.

The models that will be analysed begin from the Merton model, the simplest of the models. Then, more and more complex models will be tested, all of which are in one way or another derived from the Merton model. Since the asset value of the company is not directly observable like its equity, validation of the model is not straight forward. To accommodate this issue, validation of the models themselves will not be done. Instead, the goodness of fit of a model will be determined directly by its ability to value credit default swaps (CDS), whose prices (spreads) are observable.

By starting with a simple model and adding more and more complexity to it, one will be able to assess whether a complex model is favourable to a simpler model. One can expect that there is a trade off between performance in terms of accuracy of valuing the CDS-contracts and complexity of the models.

The key question of the report can be summarised as: *Is it possible to successfully replicate market CDS-spreads through a structural model approach?*

As a test to assess whether an advanced model scheme is necessary at all, a simple regression will be implemented to price the CDS-contracts. Although an advanced structural model can outperform a simpler structural model, it is not evident that it needs to outperform a regression model.

1.4 Credit Default Swaps

1.4.1 How the contract works

Credit default swaps are a credit derivative with a bond as an underlying security. In a CDS-contract there is always a protection buyer and a protection seller. For some predetermined time T and notional amount N , the protection seller promises to insure the protection buyer against a credit event in the underlying bond. A credit event can be a number of things, but mostly it is a default, in other words, the company fails to pay its liabilities. The protection seller is compensated for taking on this extra risk by being paid a fee by the protection buyer known simply as the spread, which is fixed throughout the contract time. The spread, denoted in basis points (bps), is the total annual fee, but payments are often done quarterly. The total annual payment is the spread multiplied by the notional amount N . In case of a credit event, the payment streams to the protection seller stops and instead, the protection seller must provide the protection buyer with the difference between the notional amount N and recovered value of the bond [3].

For example, assume person A agrees to sell credit protection to person B at a notional amount of \$10MM, contract length of 5 years and a spread of 200 bps, with a bond of company C as underlying security. Nothing happens during the first year of the contract, so at this point, person B has paid a total of $\$0.02 \cdot 10\text{MM} = \0.2MM to person A. However, during the first quarter of year 2, company C files for bankruptcy, hence defaults on all its bonds. A credit event is triggered and no more payments are done from person B to person A. Instead, person A must now pay person B an amount equal to $\$(1 - R) \cdot 10\text{MM}$, where R denotes the recovery rate of the underlying bond, usually assumed to be 40% when pricing. Therefore, if person B actually would have owned the bond, which he need not do in order to enter the CDS-contract, he would have been completely reimbursed due to the CDS-contract.

A higher spread is generally associated with a higher credit risk, as the protection seller gets larger payments the higher the spread is, naturally because the premium required tends to increase with the associated risk.

During 2009 the convention of how CDS contracts worked was changed. Prior to the change, the contract worked as described in the example above, in terms of cash flows. It was very simple to enter a contract, regardless of if one was buying or selling credit protection, since there was no cost or fee associated with entering. Now however, the contracts conceptually work the same way as they provide the same sort of insurance against a credit event, but what has changed is that the CDS trade on fixed coupons and require an upfront payment from one of the parties in the contract. The upfront payment is a percentage of the total notional sum to be paid. In the United States, the fixed coupons are either 100 bps or 500 bps, depending on credit quality [7].

Given the points upfront that the CDS currently trades at, the fixed coupon associated with the specific company and the interest rate curve, the implied CDS-spread can be calculated, since the points upfront are essentially the expected present value between the CDS-spread and the fixed coupon. Once this

upfront payment is made, the contract works as before, with the exception of the annual payments from the protection buyer to the protection seller is equal to the fixed coupon.

Because of these changes, CDS:s are often quoted in terms of points upfront instead of the rolling spread, however, since everyone is so used to viewing a CDS in terms of its spread, one still commonly uses this notation, and it will be used throughout this report.

1.4.2 Valuation

When valuing a CDS-contract, it all comes down basically to finding the spread that makes the contract give the same expected return, regardless of if one would buy or sell credit protection. The contract can thus be divided into two legs, the premium leg and the protection leg [3]. The premium leg is the expected payments to the protection seller, while the protection leg is the expected payments to the protection buyer. In the setting of the CDS-contract prior to the change to points upfront in 2009, the payments to the protection seller consists of the coupon payments paid each quarter and a single payment of accrued premium in the event of a default. The payments to the protection buyer consists only of the payment that occurs in case of a default. Define the following variables:

- $D(t_i)$ = Present value of \$1 at time t_i discounted using the risk free rate of return
- $q(t_i)$ = Risk neutral probability of no default up until time t_i
- S = Spread of the credit default swap
- R = Recovery rate of the reference bond
- d_i = Time between coupon payments, 0.25 assuming quarterly payments
- N = Total number of planned payments over the length of the contract, $4 \cdot 5 = 20$ for a 5 year CDS-contract

Assuming a notional sum of \$1 and that defaults occur on average between two payment dates, using the notation above, the premium leg and protection leg can be found as

$$PV_{ProtectionLeg} = (1 - R) \sum_{i=1}^N D(t_i) [q(t_{i-1}) - q(t_i)]$$

$$PV_{PremiumLeg} = \sum_{i=1}^N D(t_i) q(t_i) S d_i + \sum_{i=1}^N D(t_i) [q(t_{i-1}) - q(t_i)] S \frac{d_i}{2}$$

The risk neutral CDS-spread is the spread that makes the value of the legs the same, thus solving for the spread S yields

$$S = \frac{(1 - R) \sum_{i=1}^N D(t_i) [q(t_{i-1}) - q(t_i)]}{\sum_{i=1}^N D(t_i) q(t_i) d_i + \sum_{i=1}^N D(t_i) [q(t_{i-1}) - q(t_i)] \frac{d_i}{2}} \quad (1)$$

Note that there exist even more detailed and advanced expressions for the CDS-spread, see for instance research provided by OpenGamma [19]. The expression obtained in equation (1) is more or less derived from those expressions, with some points simplified, and will be sufficient for this study.

2 Methodology

This section describes the outline of the analysis and the models to be tested. First, a section describing some of the models and work that has been done in the field of structural credit risk modelling is presented, followed by a section presenting more in depth the method of how the analysis will be done.

2.1 Models and earlier work

2.1.1 Merton's Model

One of the first attempts to use a structural scheme to model a firm's assets was presented by Merton in 1974 [15]. He proposed the asset value of a firm to follow similar dynamics to that of a single stock presented a few years earlier by Black & Scholes [5]. The model has become famous all over the world as it presents a very simple way of modelling the assets, providing elegant closed form solutions and relations between parameters.

Merton proposes the following dynamics of the assets of a firm:

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t, V_0 > 0 \quad (2)$$

which has the solution

$$V_t = V_0 e^{(\mu_V - \frac{\sigma_V^2}{2})t + \sigma_V W_t} \quad (3)$$

Equation (2) can be restated under the risk neutral measure \mathbb{Q} with $dW_t^* = dW_t + \frac{\mu_V - r}{\sigma_V} dt$

$$dV_t = rV_t dt + \sigma_V V_t dW_t^*, V_0 > 0 \quad (4)$$

Where V_t denotes the asset value of the firm at time t , r denotes the risk free interest rate and W_t^* is a geometric Brownian Motion. Moreover, Merton suggests that the firm debt is simply a zero coupon bond with amount K due at time T . A default is said to occur if at time T , $V_T < K$, i.e. if the asset value of the firm at time T is less than the amount of debt due, the firm defaults. In case of a default, the owners of the company debt are highest prioritised in terms of liquidation of the firm, therefore their cashflow at time T will be $\min(V_T, K)$. Equity holders on the other hand will receive what is left after the debt holders have been compensated, i.e. their cashflow at time T will be $\max(0, V_T - K)$. The cashflow of the equity holders is exactly the same as that of a European call option with strike $= K$ and underlying security V_t . Since the dynamics of equation (4) is identical to the dynamics of the stock presented by Black & Scholes, the value of equity at time t is therefore

$$\mathbb{E}_{\mathbb{Q}}[\max(0, V_T - K)|\mathcal{F}_t] = V_t N(d_1) - e^{-r(T-t)} K N(d_2), \quad (5)$$

$$\text{where } d_1 = \frac{\log(\frac{V_t}{K}) + (r + \frac{\sigma_V^2}{2})(T-t)}{\sigma_V \sqrt{T-t}}$$

$$\text{and } d_2 = d_1 - \sigma_V \sqrt{T-t}$$

$N(\cdot)$ simply denotes the standard Gaussian cumulative distribution function evaluated at point (\cdot) and $\{\mathcal{F}_t\}$ is the filtration generated by the Brownian Motion W_t^* . However, at each point in time, the value of equity is known and observable, it is simply the market capitalisation E_t of the company, calculated as the number of shares multiplied by the current price per share. Hence, the following relation with equation (5) must be true

$$E_t = V_t N(d_1) - e^{-r(T-t)} K N(d_2) \quad (6)$$

Equation (6) provides a relationship between the unobservable asset value V_t and the observable value of equity E_t . Therefore, at each point in time, it is possible to infer the current asset value of the firm given the current value of equity, but in order to do this properly, the volatility of the assets σ_V must be known. Fortunately, another relation can be found which relates the volatility of the equity to the volatility of the assets.

Assuming first that the value of equity has the same dynamics as the assets, i.e.

$$dE_t = \mu_E E_t dt + \sigma_E E_t dW_t \quad (7)$$

Since under the assumptions in the Merton model, equity is a call option on the assets of the firm and equity is assumed to follow an Ito-process, one can apply Ito's lemma on the equity process [14]. This yields

$$\begin{aligned} dE_t &= \frac{\partial E_t}{\partial V_t} dV_t + \frac{\partial E_t}{\partial t} dt + \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} dV_t^2 + \dots = \{\text{Replacing } dV_t \text{ with (2)}\} = \\ &= \frac{\partial E_t}{\partial V_t} (\mu_V V_t dt + \sigma_V V_t dW_t^V) + \frac{\partial E_t}{\partial t} dt + \\ &+ \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} (\mu_V^2 V_t^2 (dt)^2 + 2\mu_V \sigma_V V_t^2 dt dW_t^V + \sigma_V^2 V_t^2 (dW_t^V)^2) = \\ &= \{\text{as } dt \rightarrow 0, (dt)^2 \rightarrow 0 \text{ \& } dt dW_t^V \rightarrow 0 \text{ \& } (dW_t^V)^2 \rightarrow dt\} = \\ &= \frac{\partial E_t}{\partial V_t} (\mu_V V_t dt + \sigma_V V_t dW_t^V) + \frac{\partial E_t}{\partial t} dt + \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} \sigma_V^2 V_t^2 dt = \\ &= \left(\frac{\partial E_t}{\partial V_t} \mu_V V_t + \frac{\partial E_t}{\partial t} + \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} \sigma_V^2 V_t^2 \right) dt + \frac{\partial E_t}{\partial V_t} \sigma_V V_t dW_t^V \end{aligned} \quad (8)$$

Now, comparing diffusion terms in equation (7) and (8) the following relation is obtained

$$\sigma_E E_t = \frac{\partial E_t}{\partial V_t} \sigma_V V_t \quad (9)$$

Moreover, from equation (6) one can find that $\frac{\partial E_t}{\partial V_t} = N(d_1)$. Plugging this into equation (9) the final relation is found:

$$\sigma_E E_t = N(d_1) \sigma_V V_t \quad (10)$$

By combining equation (6) and (10), one has two equations and two unknowns (σ_V and V_t). By solving these two equations simultaneously, values of asset volatility and current asset value can be found [11].

Another fundamental formula can be derived in the Merton model, namely the probability of default. As mentioned before, the company is said to default if and only if $V_T < K$, the firm cannot default prior to T . Hence, the risk neutral probability of default, evaluated at time $t = 0$ is calculated the following way

$$\begin{aligned} \mathbb{Q}[V_T < K] &= \mathbb{Q}[\log(V_T) < \log(K)] = \{\text{Logarithm of equation (3)}\} = \\ \mathbb{Q}[\log(V_0) + (r - \frac{\sigma_V^2}{2})T + \sigma_V W_T^* < \log(K)] &= \{W_T^* = \sqrt{T} \cdot X, X \sim N(0, 1)\} = \\ \mathbb{Q}[\sigma_V \sqrt{T} \cdot X < \log(K) - \log(V_0) - (r - \frac{\sigma_V^2}{2})T] &= \\ \mathbb{Q}[X < -\frac{\log(\frac{V_0}{K}) + (r - \frac{\sigma_V^2}{2})T}{\sigma_V \sqrt{T}}] &= \mathbb{Q}[X < -d_2] = N(-d_2). \end{aligned} \quad (11)$$

One of the major advantages of the Merton model is the fact that it makes it possible to directly apply the theory of standard European options, resulting in simple to use formulas. However, it has also been subject to criticism. First of all, it is not very realistic that the debt structure of the firm only consists of a single zero coupon bond. Companies usually have many types of debt outstanding, ranging from zero coupon bonds, bonds with coupons, convertible bonds etc. Related to this, there is also an issue of different seniority levels of debt. Not all debt is prioritised the same in the case of a default, this becomes a big issue if one would use the Merton model to try to model CDS-spreads linked to both senior and subordinated debt since the Merton model does not make it possible to separate seniority of debt.

Another point is the fact that the model only allows for default at time T . For instance, if a company's assets become extremely small just before time T , but somehow then manages to recover and exceed the default threshold K at time T , the firm did not default according to Merton. In real-life, it is natural to assume that a company can default or file for bankruptcy at any point in time, which is not possible in the Merton model.

2.1.2 Moody's KMV

Moody's KMV is the name of a particularly successful implementation of the Merton model, now used commercially by the rating agency Moody. The KMV implementation looks initially very similar to the Merton scheme, but later diverges and makes use of new assumptions and relations. The KMV model name three key steps in determining the default probability of a firm

- Estimating the asset value and asset volatility
- Calculating the distance-to-default (DD)
- Calculating the default probability

When estimating asset value and asset volatility, the KMV model makes use of the option-pricing relation, i.e. equation (6). They chose as value of equity the current market capitalisation of the firm, and find the debt K as the book value of liabilities for the predetermined maturity time T . It should be noted that KMV puts the default point K somewhere in the range of short term liabilities and total debt, depending on the choice of T [9]. Moreover, they set equity volatility simply as the volatility of the underlying stock of the firm. Given all the variables, asset value and asset volatility are inferred through equation (6).

The next step is evaluating the so called distance-to-default. This metric is defined as $DD_t = \frac{\log(\frac{V_t}{K}) + (r - 0.5\sigma_V^2)(T-t)}{\sigma_V \sqrt{T-t}}$ and gives a rough estimate of how "far" the firm is from default at a given point in time. The smaller the value of DD_t , the closer the firm is to default. Although V_t may be close to K , the firm can have a very small asset volatility, and thus obtain a fairly large value of DD_t . This metric is defined exactly the same as d_2 used in equation (5). The distance-to-default metric is used to calculate the probability of default, and this is where the two models diverge from each other.

In the Merton model, DD_t was assumed to be Gaussian, as seen in equation (11). KMV on the other hand instead introduces a new mapping of finding the probability of default from the given DD_t . Essentially, they rely on their huge data base where they have calculated, at each point in time, the current DD_t for a very large set of firms, many, many years back in time to present. The probability of default, or expected default frequency (EDF) as they call it, is calculated by looking into the database, finding how many cases of firms having a similar DD_t defaulted within the specified maturity time T and then dividing that by the total number of firms that had the same DD_t [9].

Since the KMV model is more or less identical to the Merton model up until the point of calculating the EDF, the previous assumptions made in the Merton model hold here as well, making it valid to use the relations presented in equations (6) and (10) to estimate asset value and asset volatility.

In terms of criticism to the model, one can argue against the lack of ability to make a distinction between senior and subordinated debt just like for the Merton model. However, the KMV model is used commercially by Moody's nowadays, and apparently seems to provide them with adequate results.

2.1.3 Black-Cox Model

The Black-Cox model is the simplest of the so called first passage models [12]. This model has a very similar model setup as the Merton model, however there are two major differences between the models. The Black-Cox model makes room for default at any point in time, not only at time T , which is a major restriction of the Merton model. It also allows for a time varying default threshold $K(t)$ which is a much more realistic assumption [4]. There are many possible

choices of the default threshold, it can be chosen to be a stochastic process, a monotone increasing function with time etc. In the case of the latter, if the default threshold is defined as $K(t) = K_0 e^{kt}$, where $K(T)$ is the debt due at time T , and $k > 0$ and $K_0 < K(T)$, the probability of default can be expressed in closed form.

In this model-scheme, default occurs when $V_t < K(t)$. This event can be reformulated the following way

$$\begin{aligned} \{V_t < K(t)\} &= \{V_t < K_0 e^{kt}\} = \{\log(V_t) < \log(K_0) + kt\} = \\ &= \{\log(V_0) + (r - \frac{\sigma_V^2}{2})t + \sigma_V W_t^* < \log(K_0) + kt\} = \\ &= \{W_t^* + \frac{(r - \frac{\sigma_V^2}{2} - k)t}{\sigma_V} < \frac{1}{\sigma_V} \cdot \log(\frac{K_0}{V_0})\} = \\ &= \{X_t < d\}. \end{aligned}$$

Where $X_t = W_t^* + mt$, $m = \frac{(r - \frac{\sigma_V^2}{2} - k)}{\sigma_V}$ and $d = \frac{1}{\sigma_V} \cdot \log(\frac{K_0}{V_0})$. Hence, a default has occurred on the time frame $[0, T]$ if $\{\min_{s \leq T} X_s < d\}$. The risk neutral probability of default can therefore be shown to be [12]

$$\mathbb{Q}[\min_{s \leq T} X_s < d] = 1 - N\left[\frac{-d + mT}{\sqrt{T}}\right] + e^{2md} N\left[\frac{d + mT}{\sqrt{T}}\right]. \quad (12)$$

However, note that this relation only holds in the case when the default threshold is assumed to follow $K(t) = K_0 e^{kt}$. Most other default threshold dynamics would require Monte Carlo simulation in order to calculate the default probability.

Another possible choice of default threshold, which may be considered more realistic, is choosing the default threshold $K(t)$ as the debt due at each time t for the company. At each point in time, the debt structure of the company is observable in terms of outstanding bonds, when they are due and what cash flows they will make. The company may have a large part of its debt due in two years, and after that they have relatively little debt, which most likely will result in a probability of default much higher in the two first years than in the following two years, if calculated using the information available today.

The default threshold can also be chosen to be stochastic, for example following similar dynamics to that of the assets. An argument for using the type of threshold could be that one considers default to be an event that occurs at a random point in time, which may or may not be true. For instance, one may choose

$$dK_t = \mu_K K_t dt + \sigma_K K_t dW_t^K \quad (13)$$

as the dynamics of the default threshold. Also, one may consider some correlation between the movements of the company assets and the default threshold, i.e. $dW_t^K dW_t^V = \rho dt$. However, the choice of σ_K and ρ are not straight forward and it is unclear how one would go about estimating these parameters.

A drawback with the Black-Cox model compared to Merton's original model is the absence of simple equations that relate asset value and asset volatility to market implied data. Only in the special case where one sets $K(t)$ to be constant over time, one can once again use the relation in equation (6). The reason for this is because the value of equity in the Black-Cox setting instead has become the value of an American call option (an option that can be exercised at any point in time until T), but since no dividends of the firm are included in this model scheme, the value of the American call option becomes exactly the same as the value of the European call option, since in this specific case, one can show that one should never exercise an American call option before maturity [17]. Note though that the only difference in this specific case of the Black-Cox model and Merton's model is the fact that default can occur at any point, the default threshold is still said to be the same over time.

To further visualise the difference between the Merton and the Black-Cox model, see figure 1. The figure shows two simulations of a fictive company's assets, where the risk free rate of return has been set to 5%, the asset volatility has been set to 35% and finally the default threshold has been set to \$500M.

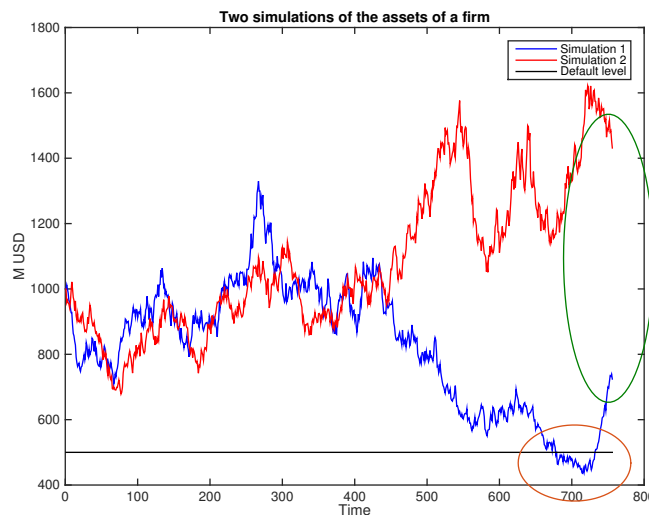


Figure 1: Simulated trajectories of a company's assets along with a default level

The green ellipse shown in the figure illustrates that both cases of the company assets in this case did not default, if they had been modelled through the Merton model. Since both trajectories end up above the default threshold at maturity, the Merton model states that they did not default.

The red ellipse however shows that the blue trajectory of the company assets did fall below the default threshold during a period, but then managed to recover and emerge from the abyss. This is where the two models diverge, the Merton model still does not consider the blue trajectory to have defaulted as that model is not path dependant and only care about the terminal value of the assets, while

the Black-Cox model would have deemed the blue trajectory as a default as soon as it dipped below the default threshold.

2.1.4 Adding Complexity

2.1.4.1 Stochastic Volatility

One possible extension to both the Merton model and the Black-Cox model is the addition of stochastic volatility. It is natural to assume that a company can be more or less stressed at times, which calls for different levels of volatility. The Merton model and the Black-Cox model in their natural state do not account for this, which may be considered as restrictive, as the choice of volatility of the assets is one of the key factors when evaluating the credit risk of the entity.

One natural choice of scheme to model stochastic volatility together with the assets is the Heston model [6], giving the following dynamics

$$\begin{aligned}dV_t &= rV_t dt + \sqrt{\nu_t} V_t dW_t^V \\d\nu_t &= \kappa(\theta - \nu_t) + \xi \sqrt{\nu_t} dW_t^\nu \\dW_t^V dW_t^\nu &= \rho dt.\end{aligned}$$

Where ν_t denotes the instantaneous asset volatility. The parameter θ can be interpreted as the long run variance, i.e. as t becomes large, one can expect ν_t to approach θ . Moreover, κ is the parameter which decides how quickly ν_t tends to θ , and finally ξ is the volatility of ν_t , i.e. how much one expects ν_t to vary.

If one was to incorporate this scheme into either the Merton model or the Black-Cox model, the same "rules of default" would still hold true, the only change would be the fact that one allows for the asset volatility to vary over time. Also, no closed-form solutions of the default probability would exist, which implies that one would have to use Monte Carlo-simulations in order to get estimates of such probabilities.

The biggest issue however if choosing to implement this scheme is the fact that one would have to estimate the parameters κ , θ , ξ and ρ somehow. Although one has a theoretical interpretation of the parameters, it is far from evident how one would go about deciding the parameters. Since the assets V_t are not observable, one cannot just take a snippet of time-series data on V_t and use that to estimate the parameters, like one could have been able to do if one was to model the equity of the firm instead. Because of this issue, stochastic volatility will not be incorporated in any of the models in this report, although it would have been an interesting feature to have in the study.

2.1.4.2 Stochastic Interest Rates

One might also be interested in modelling using stochastic interest rates. Obviously, rates are not constant over time, so it is reasonable to model them somehow. One possible choice of model scheme for the interest rates is the Vasicek model, presented in 1977 by Oldrich Vasicek [18]. He proposes the following dynamics of the interest rate

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

where W_t is a Brownian Motion under the risk neutral probability measure. Here b can be seen as the "long term mean level" of the interest rates, a can be viewed as how quickly the trajectories of the interest rate regroups around b , a is also assumed non-negative. Finally, σ can be seen as the volatility of interest rate where a large value of σ , will yield more volatile trajectories of the interest rate.

Just as for the case when incorporating stochastic volatility, no closed form solutions for the default probability would be possible to obtain, meaning that one is stuck with Monte Carlo-simulation. Moreover, once again, one would have the issue of estimating parameters in a reasonable way. Although it feels natural to allow for varying interest rates, the complexity of the implementation rapidly increases. Hence, stochastic interest rates will also not be included in this study.

2.1.5 Regression model

As a benchmark model against the structural models, a very simple regression model will be used. It is reasonable to assume that a company during times with high CDS-spreads will tend to have a higher equity volatility than during other times, as the firm most likely is more distressed during this period. Hence, an interesting regression would be one where the equity volatility, along with a constant, is regressed against the market spreads. The following regression model will thus be implemented: $y_{CDS}^{(t)} = \alpha + \beta \cdot \sigma_E^{(t)} + \epsilon^{(t)}$, where $Var(\epsilon^{(t)}) = \sigma_\epsilon^2$ and $\mathbb{E}[\epsilon^{(t)}] = 0$ and $\epsilon^{(t)}$ is assumed Gaussian.

2.2 Method of approach

The previous section presented some of the models that will be analysed in this report. This section will present how the models will be implemented and validated.

Each model implemented will be tested on a number of different companies, coming from different industry sectors in order to get as good an assessment as possible of the models. The only requirement of each company included in the analysis is that it has outstanding bonds, and therefore tradable CDS-contracts, and is "large enough", so that there is liquidity in the market, making the CDS-spreads trustworthy.

For each model and company, metrics such as probability of default, asset volatility, asset value and so forth will be calculated at each point in time (each day) and used to value CDS-contracts according to equation (1). Only CDS-contracts with a reference bond of 5 years maturity will be valued, as these are by far the most liquid in the market, implying more trustworthy spreads [8]. Therefore, the implementation scheme of the models will yield a time series of CDS-spreads, which in turn can be compared with the market spreads. By determining how well the model-implied spreads follow the market spreads, one can validate whether or not a model is considered good.

Although the CDS-spreads returned from a model are different from the market spreads, it does not necessarily mean that the model is wrong. The market spreads can of course also be wrongly valued, but since it is very difficult to assess whether this is the case or not, in terms of validating the spreads returned from a model, it is easiest to assume the market spreads to be true, fair values of the contracts.

Validation of the models can be done both by looking at the absolute error of the model-implied CDS spreads versus the market spreads, but it can also be done by looking at the relative error of the model-implied CDS spreads versus the market spreads. Perhaps one may allow for a certain level of error, relative to the market spreads. Hence, the absolute error in the CDS spreads may vary over time, but the relative error of the CDS spreads may be fairly stable.

2.2.1 Data

In order to model the company's credit worthiness according to the structural model approach the following data is required: market capitalisation, book value of the company's debt, volatility of the company's market capitalisation, value of the risk free rate of return, asset value and volatility of the assets. And of course, in order to validate the models, spreads of the company's 5 year CDS is needed.

The asset value and asset volatility of the company are calculated through the models. The remaining data is observable in the market and can be downloaded using the Bloomberg Terminal. However, some further details regarding the data should be clarified.

Market capitalisation

The values of the market capitalisation are simply taken as the close price at each day of the underlying stock, multiplied by the number of shares outstanding.

Equity volatility

There are many options of how the volatility could be chosen. For example, one could, at each day, estimate the volatility of the log returns of the stock, trailing n days back. This would however require one to decide how many days trailing to calculate from, and would require daily calculations.

Another approach which seems more attractive is simply to take the implied volatility of vanilla American call or put options. However, one then faces the problem of deciding what level strikes to use and what maturity. This is an issue since different level strikes can yield different values of implied volatility. This phenomenon is known as skew, or smile, depending on the situation [13]. One of the reasons why this occurs is because the market sometimes is, for instance, very bullish towards a certain stock, yielding a greater demand for OTM (Out-of-The-Money) call options, thus pushing the price for those options up, and consequently the implied volatility for those options increase. One advantage with using realised volatility would be that one then gets the volatility that is really given by actual movements in the underlying stock, unlike implied volatility from options that may be subject to market anticipations on the underlying stock, which may overstate or understate the volatility.

Since it is the 5-year CDS to be valued, one may argue that the implied volatility should come from a option with a maturity of five years. This is of course a valid point, as one could perhaps expect a option with the same maturity as the CDS-contract to have a implied volatility more true, and more suitable to the analysis. However, the simple reason why this volatility is not used instead is simply because options with such a long maturity are hardly traded, hence there is little to no data accessible.

Preferably, one would want volatility from the most liquid options as one can expect those volatility levels to be most fair. Without spending too much time with this issue, a quick and empirical investigation was done to see what level options seemed to be most liquid. This resulted in 90% OTM put options with a one year maturity, i.e. put options with strike level = 90 % of current spot price, that mature in one year. Therefore, these options will be the source of the equity volatility.

Book value of company debt

The debt of a company is observable in the sense that it is updated and reported once per quarter and can be observed in the balance sheet of the company. Here, both short term debt and long term debt has been downloaded for each day that the study is done, which of course means that the debt will be the same every day until the company presents a new balance sheet.

Risk free rate of return

Just as for the equity volatility, a couple of alternatives of what interest rate to use exist. However, as will be presented shortly, the companies to be analysed are all listed in the United States (and Canada), therefore the choice of risk free interest rate as the 12 month Interbank Offered Rate in London (LIBOR) for the US dollar seems like a valid choice.

The LIBOR is the average interest rate that a bank would be charged if it was to borrow from other banks in London.

CDS-spreads

Since CDS-contracts are traded over-the-counter (OTC), they are not directly observable per se, however companies such as Markit and CMA collect and provide data for CDS-trades each day and are available in the Bloomberg Terminal. Just as for the market capitalisation, the close price each day is used.

Recovery rate

The recovery rate of the reference bond can in reality be anything between 0% and 100%. However, the market convention for quotation of CDS-contracts referring to senior unsecured bonds is a recovery rate of 40%, which will be used in this report.

Companies to be analysed

Table 1 presents the companies to be analysed and what industry sector they are active in. All of the chosen companies are active in very distinct industries and should thus provide a good variation to the analysis. Note however that not a single company from the financials sector has been included, and this is done on purpose. Whether it is a bank, an investment company or an insurance company, assessing the credit risk in those type of firms is notoriously difficult as there are so many factors affecting them that are not observable for a bystander.

Banks for instance are very exposed to regulators, which means that new rules can change the entire industry over night. Therefore, for ease of implementation, they are simply excluded from the analysis.

Table 1: Companies to be analysed and their respective industry group and sector

Name	Industry Sector	Industry Group
United States Steel Corp	Basic Materials	Iron/Steel
J.C. Penney Co	Consumer, Cyclical	Retail
Avon Products Inc	Consumer, Non-cyclical	Cosmetics
Bombardier Inc	Industrial	Misc. Manufacturer
Peabody Energy Corp	Energy	Coal
Advanced Micro Devices	Technology	Semiconductors

Data will be collected from 2011-07-01 up until 2015-02-09, each trading day. Why this period of time is chosen is due to a couple of things. Firstly, during the second half of 2011, the financial markets were being highly affected by the Euro-crisis, resulting in overall uncertainty and volatility. Moreover, during this period of time, many of the companies to be analysed have at least during one period been distressed, solely due to the individual company. Thus, in the data set, there is a period of time when macroeconomic variables affect all the companies, but also times when the companies individually are under pressure. A good model should be able to handle all situations, hence, the chosen period of time and the list of companies to be analysed should be a good combination for this thesis. All plots of time series will show this time period, unless else is specified.

Figures 2 to 4 shows the market capitalisation, equity volatility and CDS-spreads that will be used during the period to be analysed for US Steel. Figures 5 to 7 show the corresponding data for J.C. Penney, figures 8 to 10 for Avon Products, figures 11 to 13 for Bombardier, figures 14 to 16 for Peabody Energy and figures 17 to 19 for Advanced Micro Devices.

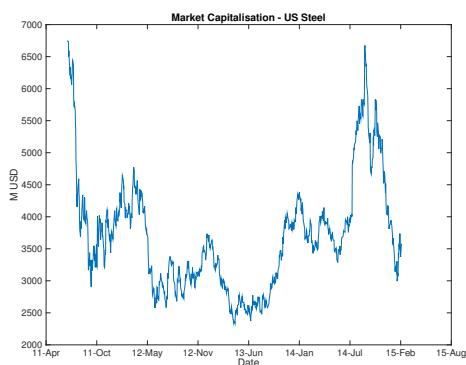


Figure 2: Observed market capitalisation of US Steel

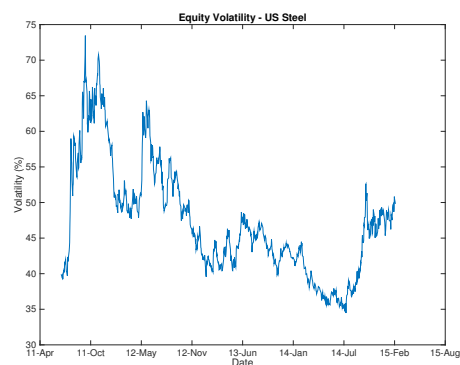


Figure 3: Implied equity volatility for US Steel

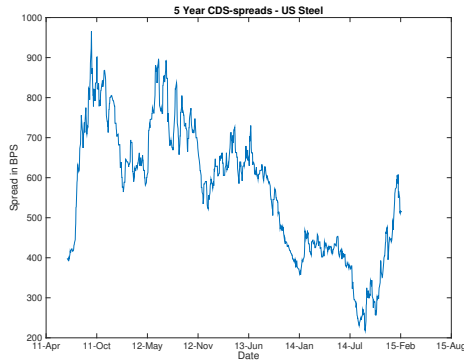


Figure 4: Observed spreads of US Steel’s 5 year CDS

As one can see for US Steel, the beginning of the period is fairly calm, with a relative high market capitalisation, however, something appears to happen short after as the volatility sky rockets in tandem with the CDS-spreads. By the end of the period, things appear to have calmed down, though there seems to be something going on again at the very end.

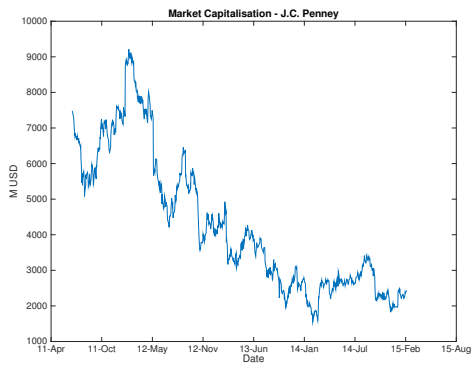


Figure 5: Observed market capitalisation of J.C. Penney

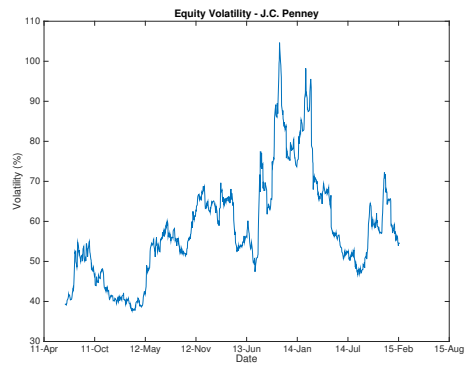


Figure 6: Implied equity volatility for J.C. Penney

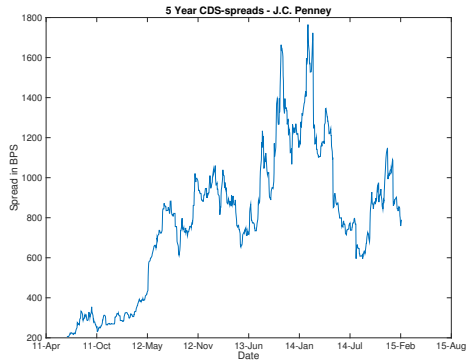


Figure 7: Observed spreads of J.C. Penney’s 5 year CDS

For J.C. Penney, the trend appears to be negative, apart from the beginning where the market capitalisation increases and the equity volatility decreases. When the market capitalisation had reached its peak however, things seem to be in a negative spiral according to all metrics.

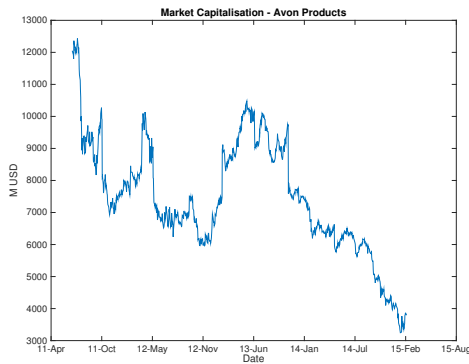


Figure 8: Observed market capitalisation of Avon Products

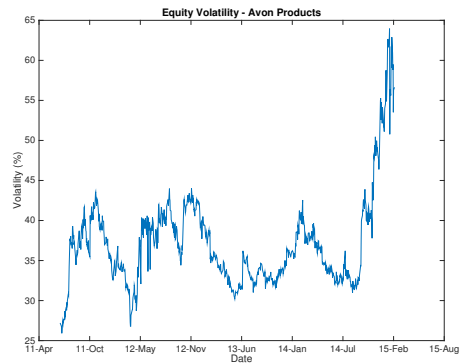


Figure 9: Implied equity volatility for Avon Products

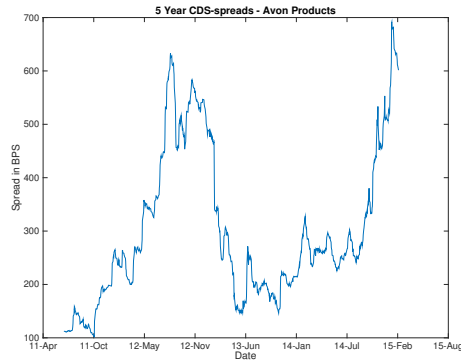


Figure 10: Observed spreads of Avon Products’s 5 year CDS

Avon Products appear to have an unusual trajectory, according to the CDS. Things are nice and clam at first, then in May 2012, there seems to be something going on as the CDS rapidly increases. One year later however, the CDS is back to previous levels, but at the very end of the period, the CDS once again increases aggressively back to prior heights.

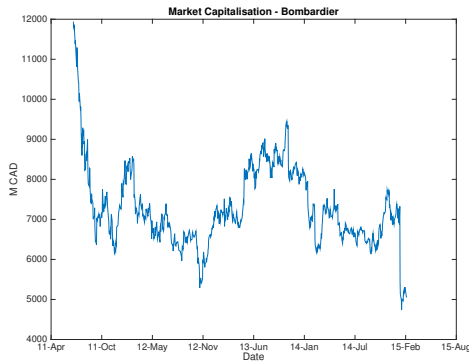


Figure 11: Observed market capitalisation of Bombardier

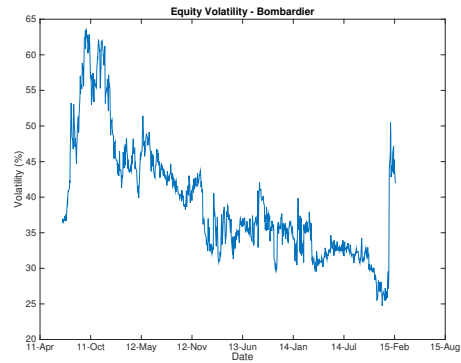


Figure 12: Implied equity volatility for Bombardier

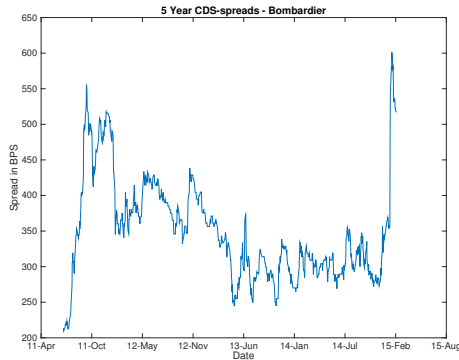


Figure 13: Observed spreads of Bombardier's 5 year CDS

Bombardier has a rapid increase in the CDS at the very beginning, which then slowly decays to lower level. Though at the absolute end of the period, the CDS appears to double in a matter of days, from 300 bps to 600 bps. This can also be seen in the volatility, which appears to have a spike during the same period of time.

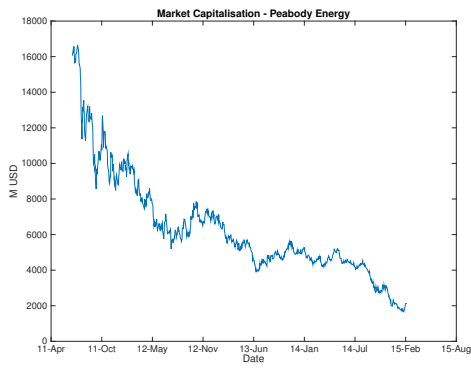


Figure 14: Observed market capitalisation of Peabody

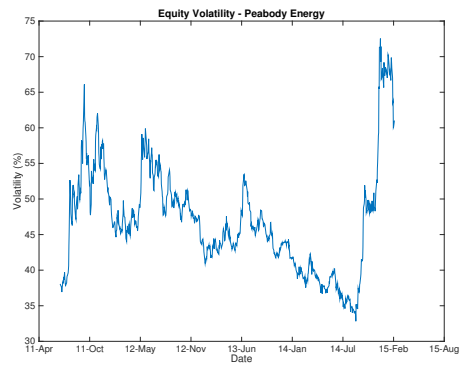


Figure 15: Implied equity volatility for Peabody

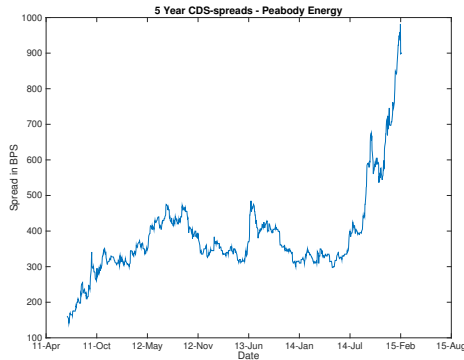


Figure 16: Observed spreads of Peabody’s 5 year CDS

Peabody Energy appears to be in an overall negative trend throughout the period. The market capitalisation steadily decreases while the CDS looks to increase during the whole period, with a couple of short period where the CDS appears to be moving sideways.

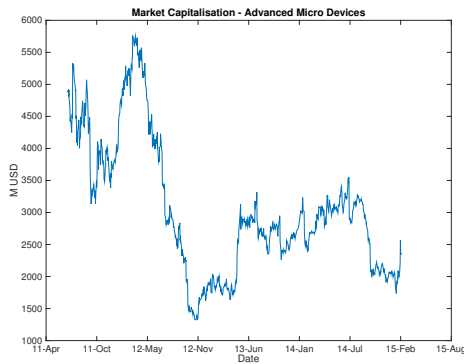


Figure 17: Observed market capitalisation of Advanced Micro Devices

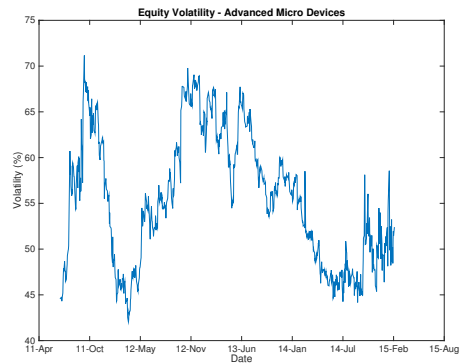


Figure 18: Implied equity volatility for Advanced Micro Devices

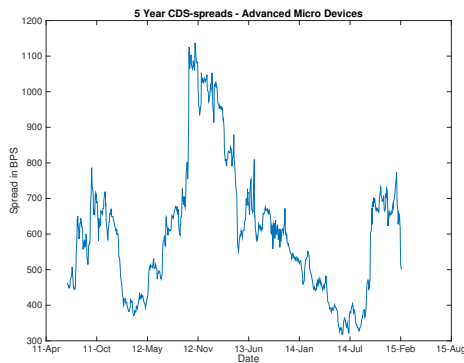


Figure 19: Observed spreads of Advanced Micro Devices’s 5 year CDS

Finally, Advanced Micro Devices seems to be very volatile in terms of all the metrics. This is especially clear in the CDS-curve as the spreads appear to vary quite aggressively throughout the period. The lows are around 300 bps while the highs are larger than 1100 bps.

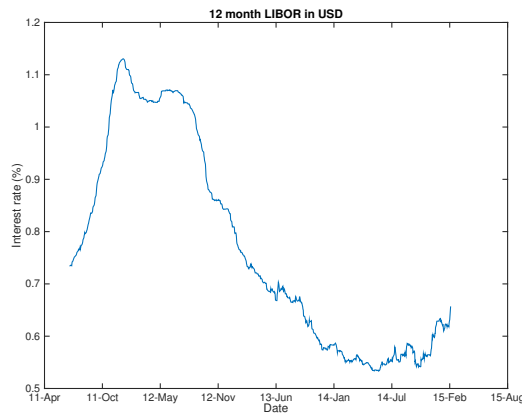


Figure 20: 12 month LIBOR rate in USD

Figure 20 shows the risk free rate of return used, i.e. 12 month LIBOR for the US dollar. During the entire period, interest rates have been very low, which will lead to discounting having a minor effect.

3 Implementation

3.1 The Merton Model

As presented, the Merton model is one of the first attempts of structural modelling of a firm's assets. The assumptions made in the model may not be very realistic, for instance assuming all the company's debt is a single zero coupon bond, but the model offers other attractive features such as elegant closed form solutions and the fact that the model is fairly simple to interpret.

Before digging into specifics regarding parameter estimation and how to choose certain parameters, a rough scheme of how the implementation will be done is presented.

3.1.1 Outline of implementation

As mentioned earlier, for each company, the reference 5 year CDS will be valued at each day during the interval of analysis. So for every day, new data has become available in terms of updated market capitalization, equity volatility, risk free rate of return and so on. These updates in data will further imply changes in variables derived from market data, namely V_t and σ_V . Essentially, if one was to plot for instance σ_V over the interval to be analysed, one will see that the parameter changes with time, most likely. This appears to be a contradiction as the Merton model states that the variance of the assets does not vary over time. However, this is actually not an issue since the model states that the variance of the assets is the same over the period of analysis, so at each point in time, the asset volatility calculated will be assumed to be true over the length of maturity T . The next day, new data has become available, yielding a new estimate of σ_V , which then will be assumed to be true over the maturity T . The same goes for the debt K , given the level of debt K today, this value is assumed true and constant over the modelling period T , but when valuing the CDS the next day, new data on the debt K has become available, resulting in a potentially new value of K .

Essentially, the following steps will be done during the implementation at each point in time

1. Update latest values of E_t , σ_E , r and K .
2. Estimate values of σ_V and V_t .
3. Use data to calculate probability of default according to equation (11).
4. Calculate 5 year CDS-spread.

After this has been done for each day during the time interval, a validation procedure is done on the resulting CDS-spreads from the modelling.

However, a couple of points need further specification, namely how one chooses T and K , how one estimates V_t and σ_V and in detail how one calculates CDS-spreads given a probability of default on some time horizon T .

3.1.2 Choosing T and K

The choice of maturity T and debt level K are not predetermined by Merton, so before one can start modelling using the Merton model, one must first make an assessment of these parameters. The debt level K is, according to the model, the debt that is due at time T , i.e. at this time, the company assets must exceed the debt level, otherwise the company is said to default. In order for the implementation to be as realistic as possible, one should choose, for some maturity T , the debt level that is due at this time according to what the company presents. Of course, a company has payments due at many points in time during, for instance, a year, so when choosing some T and a K connected to that maturity, in terms of reality, that does not mean that all debt is due exactly at time T . However, the Merton model interprets the input this way. So when choosing these parameters, one should be aware that the exact interpretation of the parameters are not the same in the model world and real life. Hence, one must find a maturity and debt level that somehow balances this distinction.

The data related to debt that is available is the data presented in the balance sheet plus the debt distribution of the company's issued bonds and loans. The balance sheet distinguishes between short-term liabilities and long-term debt. Short-term meaning debt due within one year and long-term meaning any debt with maturity of more than one year. The debt presented in the balance sheet includes all the different debt types, ranging from debt raised from bond issuance, taxes, bank loans etc. The only drawback is that the maturity of debt is not categorised more than short-term and long-term. Here, the debt distribution of bonds has an advantage as one can clearly see when in time the bonds are due, making it possible to vary the maturity time T and get the exact amount of debt due at that time. However, this detailed distinction of the debt is exclusive to bonds and certain loans, hence completely leaving out other liabilities. Moreover, since the implementation will be done on historic data, it appears that historic debt distributions of bonds and loans are not as easily accessible as the current. For simplicity, only the data related to debt provided in the balance sheet will be used, although this means it is a bit more unclear how the long term debt is distributed, one can at least easily access historic balance sheets, making this choice of debt source more attractive.

Two immediate choices of T and K are available. The first being simply choosing $T = 1$ year and K equal to short term liabilities. This is a good choice since it is known for a fact that the short term liabilities are due within one year. The other choice is choosing K equal to long term debt + short term liabilities, i.e. all the debt that the firm is said to have, and then choosing T as some fairly long maturity when one assumes the majority of the long term debt to be due within.

The long-term debt contains the bulk of the total debt of the firm usually, therefore it is reasonable to include it. However, in doing so the choice of maturity T is not obvious, since for some companies, $T = 20$ years may be true, while for others $T = 5$ years is more appropriate. A quick screening in Bloomberg for different companies indicate that $T = 10$ years is a reasonable maturity on average.

Figures 21 through 26 shows the market capitalisation, the short-term liabilities and the total debt of the companies to be analysed, related to $T = 1$ and $T = 10$ years, respectively. Looking at the graphs, one can see that the overall trends in the short-term debt and total debt usually follow each other, however for J.C. Penney and Peabody, the long term debt makes a "jump" during one point in time where the short-term debt does not appear to follow the trend. Because of differences like this, one would expect to get different end results, at least for these two companies, since the long- and short-term debt data appears to give different information at times.

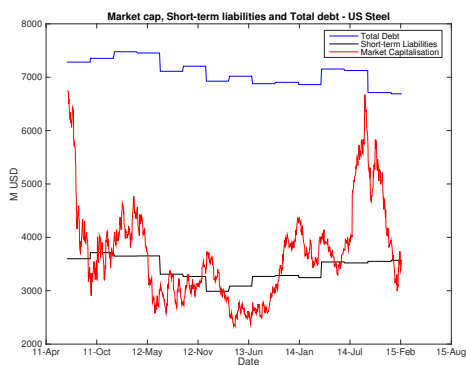


Figure 21: Market capitalisation, short-term debt and total debt of US Steel

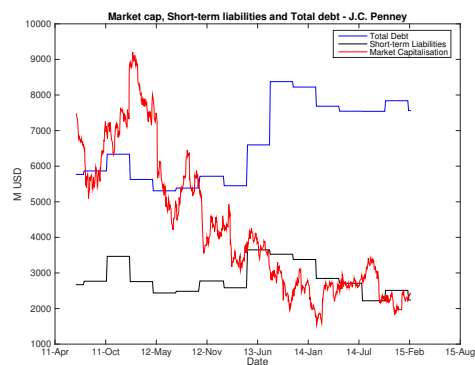


Figure 22: Market capitalisation, short-term debt and total debt of J.C. Penney

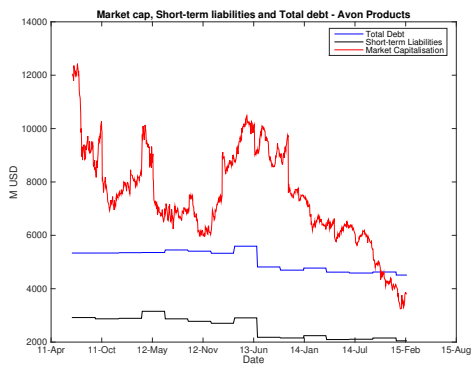


Figure 23: Market capitalisation, short-term debt and total debt of Avon Products

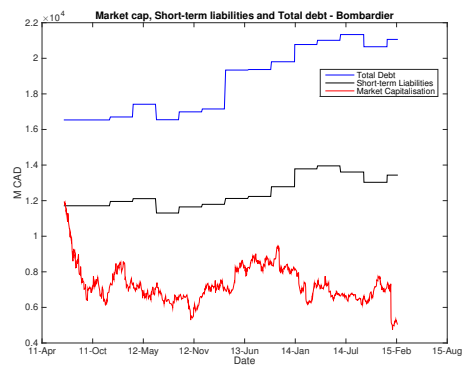


Figure 24: Market capitalisation, short-term debt and total debt of Bombardier

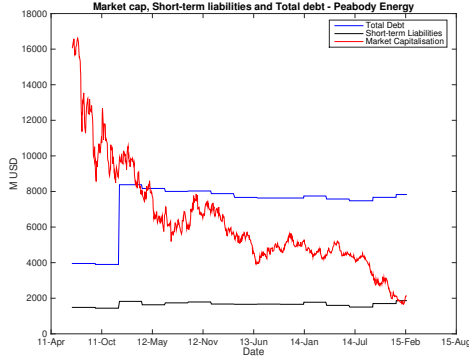


Figure 25: Market capitalisation, short-term debt and total debt of Peabody Energy

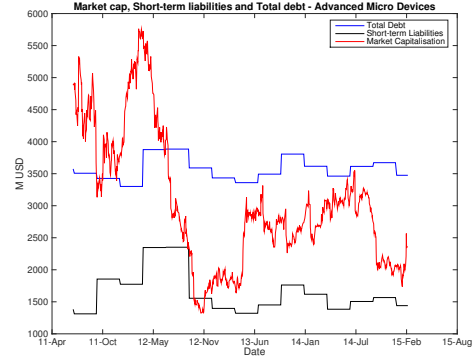


Figure 26: Market capitalisation, short-term debt and total debt of Advanced Micro Devices

3.1.3 Estimating V_t and σ_V

In the section presenting the Merton model, two vital relations are found used for estimating V_t and σ_V , namely equation (6) and (10). In order to successfully find values of asset and asset volatility, these equations must be restated so that they can be solved numerically and simultaneously. The LHS (Left Hand Side) in both equation (6) and (10) are known, so one has the possibility of subtracting the LHS to the RHS (Right Hand Side) and setting both equations equal to zero and solving for V_t and σ_V . However, empirical tests show that the order of magnitude of equation (6) is significantly larger than equation (10). A better choice is found to be simply to divide the RHS by the LHS in both equations and solving for V_t and σ_V when both equations are set equal to 1. Equationwise, this translates to

$$f_V = \frac{V_t N(d_1) - e^{r(T_t)} KN(d_2)}{E_t} - 1$$

$$f_{\sigma_V} = \frac{N(d_1) \sigma_V V_t}{\sigma_E E_t} - 1.$$

And solving for V_t and σ_V when $f_V = 0$ and $f_{\sigma_V} = 0$. In order to solve these equations simultaneously, a new function is defined, merging the two functions together, using the fact that the true value is found when both functions equal zero. Empirical tests show that σ_V is more volatile than V_t , and that f_{σ_V} is much more dominant in deciding σ_V than f_V . Hence, more weight is put on solving f_{σ_V} accurately. The following combined function has shown to achieve good results

$$F = 100 \cdot f_{\sigma_V}^2 + f_V^2. \quad (14)$$

In order to get as good results as possible, a fixed point iteration procedure is first applied on F , just to get rough estimates of V_t and σ_V , since the true values of V_t and σ_V are found when $F = 0$. Then, starting from the point found

in the fixed point iteration, a minimisation routine is run on F . Obviously, the smallest value possible for F to attain is zero, also the point where the true parameter values are found. It is in this step the factor 100 becomes important, otherwise, the minimisation routine focuses too much on f_V and returns poor values of σ_V .

The factor 100 was found through empirical tests. For a choice of factor, V_t and σ_V were estimated. The resulting parameters were then plugged into $|f_V|$ and $|f_{\sigma_V}|$ and then analysing the order of magnitude of the absolute error for the respective functions. The choice of factor of 100 yielded the smallest errors as well as the most balanced errors between the functions.

The following pseudo-code outlines the major points in finding the parameters:

1. At time t , retrieve latest values of E_t , σ_E , r and K .
2. Define f_V and f_{σ_V} anew with up to date data, letting V_t and σ_V be free variables.
3. Define F using f_{σ_V} and f_V , then apply a fixed point iteration on F in order to find estimates \hat{V}_t and $\hat{\sigma}_V$.
4. Apply minimisation routine on F , starting in the point $(\hat{V}_t, \hat{\sigma}_V)$ to obtain estimates (V_t^*, σ_V^*) .

3.1.4 Calculating the CDS-spread

Although equation (1) gives a nice formula to calculate the CDS-spread, one quickly realises that the probability of default at each point in time is required. From the modelling stage, only the probability of default at time T is returned. One needs to impose some scheme in order to transform a probability of default at some time T for an arbitrary time t . Infinitely many such transformations exist, which one to use depends on how one believes the probability of default to be distributed over time.

One approach is to assume constant conditional default probabilities, i.e. the probability of default during year one is the same as the probability of default during year 2, given that there was no default during year one, etc. This of course translates to all time-intervals. Formally, this can be represented the following way (assuming T is denoted in years), letting p_T be the default probability within T years, and p_t be the default probability within t years, where $n \cdot t = T$.

$$1 - p_T = (1 - p_t)^n \rightarrow p_t = 1 - (1 - p_T)^{1/n} \quad (15)$$

From the modelling step, p_T will be known so equation (15) provides a simple relation between p_t and p_T . Moreover, equation (1) can be simplified. Note that the factor $(q(t_{i-1}) - q(t_i))$ is the conditional probability of a default on the time interval $[t_{i-1}, t_i]$. Since it is assumed that the conditional default probabilities are constant, this factor simply becomes p_t , where t is the length of the interval $[t_{i-1}, t_i]$. In this case, t will be equal to 3 months, or 0.25 years.

As stated, many other transformations of the probability exist, but assuming constant conditional default probabilities, given the information at hand at each

point in time seems fairly reasonable as the debt level K is constant until maturity T . Since this model-scheme does not impose any further specification of how the default level is distributed until maturity, assuming constant conditional default probabilities probably is the best guess.

The discount factor $D(t_i)$ is simply defined the following way, assuming continuous discounting: $D(t_i) = e^{-r \cdot t_i}$, where r is the annual risk free rate of return.

3.2 The Black-Cox Model

3.2.1 Outline of implementation

The major difference between the Black-Cox model and the Merton model is the fact that default is allowed to occur at any point. This is an advantage in the sense that it may be considered a more real world-like feature. However, the most obvious drawback with this property is that, unless one assigns certain dynamics to the default threshold, closed-form solutions of the probability of default is no longer possible. But also, the equations used to find values of V_t and σ_V in the Merton model are no longer necessarily true.

Two different dynamics of the default threshold K will be tested. The first being when one assumes the dynamics to be $K(t) = K_0 e^{kt}$, $k \geq 0$, i.e. continuously increasing over time by a factor k , and the second being when one assumes the default threshold to be constant over time, hence making it possible to use similar relations as for the Merton model. Note however that the case with constant default threshold is just a special case of $K(t) = K_0 e^{kt}$ with $k = 0$.

A possible interpretation of the continuously increasing default threshold can be seen when one rearranges the payoff of equity holders, seen later in equation (16). The process X_t can now be viewed as an asset process continuously having some negative cash flow of a factor k , which can be thought of as a combination of dividends of the stock, interest on loans etc which seems very reasonable. This interpretation makes this choice of default threshold seem like a more valid option.

Another issue emerges with this choice of default threshold. No companies are identical to each other, some pay dividends, some do not. Hence, perhaps the factor k should be unique for each company. Though, it is a little unclear how one would go about estimating the value for each company. The dividend yield is of course possible to obtain, but the average interest rates on loans and bonds going back in time are not as easily obtainable. Moreover, it is neither obvious if the value of k should vary from day to day, like the volatility. Since there appears to be some issues with deciding the value of k , the easy route is chosen, i.e. the same values of k will be used for the different companies, and will be the same throughout the period of analysis. However, one can bear in mind the interpretation mentioned above, so that one has some idea of what is going on.

For this choice of default threshold, it will still be possible to use closed form solutions of the default probability according to equation (12), though the relations connecting asset volatility and asset value that emerge when assuming a continuously increasing default threshold are far from as short and simple as

the equivalent relations of the Merton model. However, if one was to consider using a stochastic default threshold, or another advanced dynamic for the default threshold, one is all but left with Monte Carlo simulation of the company assets and the default threshold.

3.2.2 Estimating V_t and σ_V

First of all, in the case of assuming a default threshold following the dynamics $K(t) = K_0 e^{kt}$, $k \geq 0$, the payoff to equity holders can be stated the following way

$$E_T = \begin{cases} (V_T - K_T)^+, \min_{s \leq T} (V_s - K_s) \geq 0 \\ 0, \min_{s \leq T} (V_s - K_s) \leq 0. \end{cases}$$

I.e., if no default has occurred, equity holders get what is left after the debt K_T has been paid. However, if a default occurs, equity holders get nothing.

This is exactly the payoff of a down-and-out European call-option with a time-dependant barrier (constant in the case of $k = 0$) which at maturity is equal to the strike of the option. This option is path-dependent, i.e. not only the terminal value of the underlying security is of interest in terms of the cash flows of the derivative, but also the route the underlying security takes is of importance. Knowing this, the value of equity at time t can be calculated as

$$\begin{aligned} E_t &= \mathbb{E}_{\mathbb{Q}}[E_T | \mathcal{F}_t] = e^{-r(T-t)} \mathbb{E}_{\mathbb{Q}}[(V_T - K_T)^+ \mathbb{I}_{\{\min_{s \leq T} (V_s - K_s) \geq 0\}} | \mathcal{F}_t] = \\ &e^{-r(T-t)} e^{kT} \mathbb{E}_{\mathbb{Q}}[(e^{-kT} V_T - K_0)^+ \mathbb{I}_{\{\min_{s \leq T} e^{-ks} V_s \geq K_0\}} | \mathcal{F}_t] \end{aligned}$$

Here $e^{-kt} V_t$ is set equal to X_t , where X_t is an adjusted value of assets. The process X_t can now be viewed as an asset process continuously having some negative cash flow of a factor k mentioned earlier. I.e., X_t has the following dynamics

$$dX_t = X_t(r - k)dt + X_t \sigma_V dW_t^* \quad (16)$$

Hence, the value of equity at time t can be rewritten as

$$E_t = e^{kT} e^{-r(T-t)} \mathbb{E}_{\mathbb{Q}}[(X_T - K_0)^+ \mathbb{I}_{\{\min_{s \leq T} X_s \geq K_0\}} | \mathcal{F}_t]. \quad (17)$$

The reason for rewriting the expression like this is simply because one then has removed the time-variability of the default threshold, making it possible to find closed form solutions to the expression, which can then be used in order to find values of V_t and σ_V .

In order to find the closed-form solution of the down-and-out European call option, first note the cash-flows of the down-and-in European call option denoted C_T^{DI} . The underlying security needs to dip below a certain threshold during

some point in time in order to not be worthless, as opposed to the down-and-out call option. Let C_T^{DO} denote the payoff of the down-and-out European call option and C_T denote the payoff of a vanilla European call option.

$$C_T^{DI} = \begin{cases} 0, \min_{s \leq T} (X_s - K_0) \geq 0 \\ (X_T - K_0)^+, \min_{s \leq T} (X_s - K_0) \leq 0. \end{cases}$$

$$C_T^{DO} = \begin{cases} (X_T - K_0)^+, \min_{s \leq T} (X_s - K_0) \geq 0 \\ 0, \min_{s \leq T} (X_s - K_0) \leq 0. \end{cases}$$

If one was to combine the payoffs of the down barrier options, one would then get the following payoff:

$$C_T^{DO} + C_T^{DI} = (X_T - K_0)^+ = C_T$$

This is exactly the payoff of a vanilla European call option. Hence, the combined price at any given time $t \leq T$ of the down-and-out call option and the down-and-in call option must equal the price of the vanilla call option due to identical cash-flows, otherwise there would be an arbitrage one could take advantage of. Therefore, the price of the down-and-out call option must be

$$C_t^{DO} = C_t - C_t^{DI}.$$

Where C_t^{DO} , C_t^{DI} and C_t denotes the prices of the down-and-out call option, the down-and-in call option and the vanilla call option respectively at time t .

The price of the down-and-in call option with underlying security X_t , following the dynamics of equation (16), with strike equal to K_0 and barrier equal to K_0 can be shown to be [10]

$$C_t^{DI} = X_t e^{-k(T-t)} \left(\frac{K_0}{X_t} \right)^{2e_1} N(e_2) - K_0 e^{-r(T-t)} \left(\frac{K_0}{X_t} \right)^{2e_1-2} N(e_2 - \sigma_V \sqrt{T-t}) \quad (18)$$

with

$$e_1 = \frac{r - k + \frac{\sigma_V^2}{2}}{\sigma_V^2}$$

$$e_2 = \frac{\log \left[\frac{K_0}{X_t} \right]}{\sigma_V \sqrt{T-t}} + e_1 \sigma_V \sqrt{T-t}.$$

Moreover, using the same notation, the price of the vanilla call option can be expressed as

$$C_t = X_t e^{-k(T-t)} N(f_1) - K_0 e^{-r(T-t)} N(f_2) \quad (19)$$

with

$$f_1 = \frac{\log \left[\frac{X_t}{K_0} \right] + (T-t)(r-k + \frac{\sigma_V^2}{2})}{\sigma_V \sqrt{T-t}}$$

$$f_2 = f_1 - \sigma_V \sqrt{T-t}.$$

Combining these expressions presented in equation (18) and (19), the price of the down-and-out European call option at time t can be found as

$$C_t^{DO} = C_t - C_t^{DI} =$$

$$X_t e^{-k(T-t)} N(f_1) - K_0 e^{-r(T-t)} N(f_2) - X_t e^{-k(T-t)} \left(\frac{K_0}{X_t} \right)^{2e_1} N(e_2) +$$

$$+ K_0 e^{-r(T-t)} \left(\frac{K_0}{X_t} \right)^{2e_1-2} N(e_2 - \sigma_V \sqrt{T-t})$$

Tying it all together and returning to equation (17), the value of the company's equity at time t is

$$E_t = e^{kT} \cdot C_t^{DO} \quad (20)$$

where, as previously stated, $X_t = e^{-kt} V_t$.

Another relation can be found in the same way as for the Merton model. Using the fact that equation (9) still holds true, one only needs to find $\frac{\partial E_t}{\partial V_t}$ to complete the relation.

Let $E_t = g(V_t, t)$, then by Ito's lemma, the term $\frac{\partial E_t}{\partial V_t} = \frac{\partial g}{\partial x}(V_t, t)$, where one simply substitutes X_t with $e^{-kt} V_t$. After calculating the differential (see section 9.1 in Appendix A), one can show that

$$\frac{\partial E_t}{\partial V_t} = N(f_1) + \left(\frac{K_0 e^{kt}}{V_t} \right)^{2e_1} \left[e^{-(r-k)(T-t)} (2 - 2e_1) \frac{V_t}{K_0 e^{kt}} N(e_2 - \sigma_V \sqrt{T-t}) - \right.$$

$$\left. N(e_2)(1 - 2e_1) \right]$$

By inserting this in equation (9), one has an additional relation between equity and assets, which now also includes a relation between the asset volatility and the equity volatility. This expression is far from as elegant as the equivalent expression in the Merton model, but is just as important here.

Hence, one has two expressions for the equity of the firm, and thus a relation between the assets and the asset volatility. Solving these equations like in the case for the Merton model, one gets estimates for the assets V_t at time $t = 0$ and the asset volatility σ_V . However, since the choice of k plays a crucial part in these expressions, one must first decide this value.

3.2.3 Chosing k

There is no straightforward academic methodology in choosing this value, it all depends on how aggressive one wants the growth of the default threshold to be.

Values to be used in this thesis will be $k = 0$, $k = 0.01$ and $k = 0.05$ for the same maturities as for the Merton model. Here, the value of $k = 0$ is reasonable since one then assumes a constant default threshold, which is a natural thing to test. Moreover, the choice of $k = 0.01$ is around the same level as the risk-free annual rate of return during the time period analysed in this thesis, see figure 20. One could argue that one then perhaps should chose k simply as the current LIBOR-rate, but $k = 0.01$ is chosen to keep things simple. Finally, the choice of $k = 0.05$ is chosen to have one case where the default threshold has a quite aggressive increase over time.

3.3 Regression Model

3.3.1 Outline of implementation

The parameters in the regression model will be based on the first year of data of the companies in the analysis, hence resulting in a set of parameters that will be constant throughout the validation for each company. One can argue that it would be better to use a regression model based on a rolling window of which the model parameters are based on, re-estimated each day, using the most recent information and discarding old and possibly irrelevant data. It seems natural that such a scheme will yield better results than when using a fixed time interval used to estimate the parameters on, the model is constantly fed the most relevant data. However, in doing so, the describing variable (the equity volatility in this case) becomes less important and needs not be a significant regressor to still yield adequate results, this would be especially clear if using a very short window-width.

Since the equity volatility intuitively seems to be negatively correlated with the CDS-spreads, a fix time frame of one year of which the model parameters are based should be sufficient to capture the relation, and hopefully result in an adequate, simple model.

4 Validation

There are a couple of issues that arise in the validation step. Firstly, as previously stated, the assets V_t are not observable, hence, it is impossible to directly validate the chosen model against the assets. Secondly, one does not have a distribution one can expect the resulting model implied CDS-spreads to follow, making it difficult to assess residuals like one would do in regression analysis for instance. Instead, one is forced to look into other methods of validation.

Since the object of the thesis is to model CDS-spreads through the structural models, assuming the market-spreads to be true, one would preferably want the model-implied CDS-spreads to be in the range of the true spreads, not diverging too much from the true spreads. Hence, one would preferably want the residuals to be stationary, i.e. having the same mean and volatility over time. However, since the volatility of debt or equity may not be the same over time, one may perhaps need to adjust the residuals for the current level of volatility before checking stationarity. In other words, during times when the CDS-spreads are more volatile and trading at higher absolute levels, one may expect also the absolute residual error to be larger than during calm times. Therefore, by adjusting for the current level of volatility, one can perhaps end up with stationary residuals.

Another point worth looking into is the fact that the spreads found through the structural model may show to constantly understate or overstate the market spreads. In this case, one would first need to check the average level of error the model produces and adjust for this before digging into the residual analysis.

Analysing the autocorrelation of the residuals is of interest as well. After one has adjusted for a potential constant error level, one would preferably want the residuals to be uncorrelated for all time lags $|\tau| > 0$. If one can show that the residuals in fact are uncorrelated for all lags, then one can argue that there is no further structure in the market spreads that can be modelled, i.e. all of the potential structure in spreads have been taken into account, and what remains is purely noise.

Finally, the validation will be more or less model-independent, i.e. regardless of model up for analysis, the validation methods will be the same.

As a final test, the generated model-spreads and their performance will be compared with the performance of a very simple and naïve regression of the market spreads, regressed against the equity volatility. As one can see in figures 2 through 19, presenting the data to be used in the analysis, one sees that the equity volatility appears to correlate quite strongly with movements in the CDS-spreads, further arguing in favour of the regression.

The regression will simply be based on the first year of data for each firm, and then validated on the remainder of the data. The residuals can then be analysed, and since one now also has an assumption on the distribution of the errors, one can check if the residuals follow said distribution or not.

5 Results

5.1 The Merton Model

5.1.1 $T = 1$ Year

With $T = 1$ year, the debt level is set equal to the short-term liabilities. Figure 27 through 32 shows the calculated asset values of all the companies analysed along with the market capitalisation and the debt level.

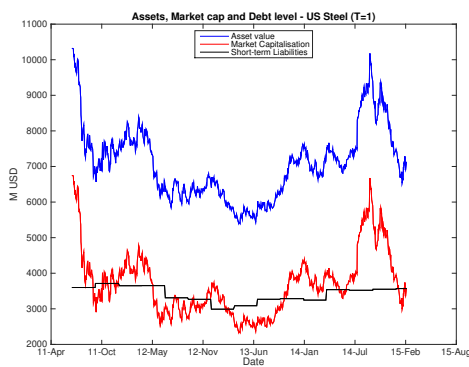


Figure 27: Asset value, market capitalisation and short-term liabilities of US Steel

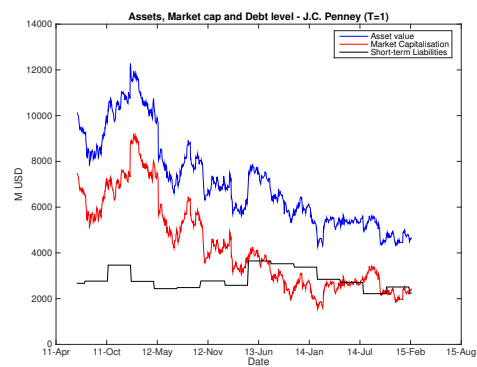


Figure 28: Asset value, market capitalisation and short-term liabilities of J.C. Penney

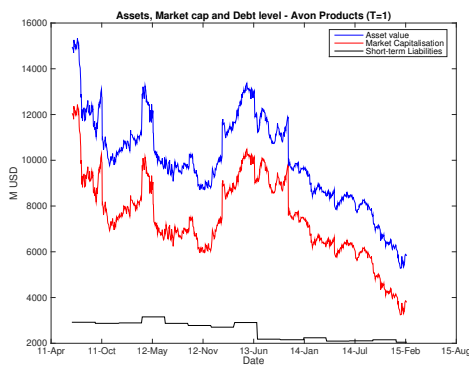


Figure 29: Asset value, market capitalisation and short-term liabilities of Avon Products

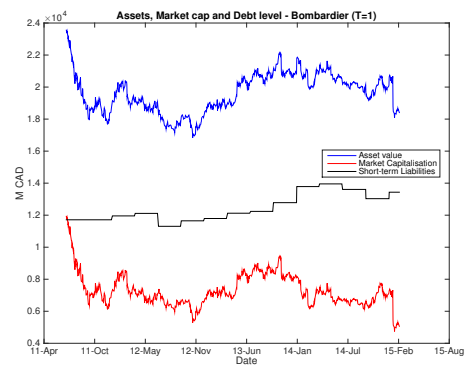


Figure 30: Asset value, market capitalisation and short-term liabilities of Bombardier

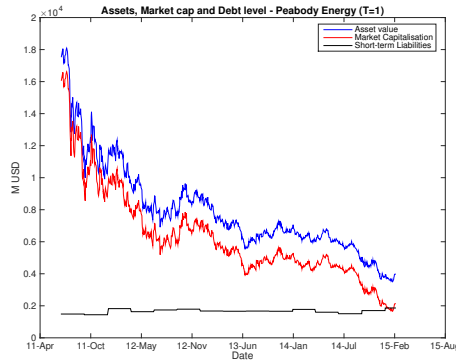


Figure 31: Asset value, market capitalisation and short-term liabilities of Peabody Energy

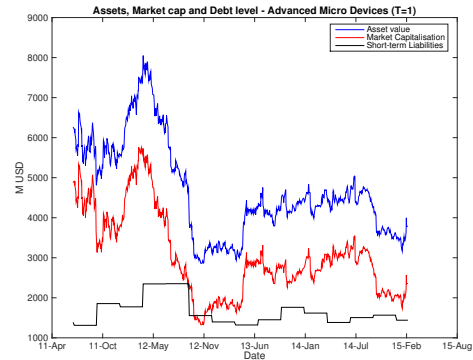


Figure 32: Asset value, market capitalisation and short-term liabilities of Advanced Micro Devices

A very rough estimate of the market value of assets when the maturity T is small is the sum of book value of debt and market capitalisation. The reason for this is simple. When the maturity is very small, there is also little room for the assets to vary, since under the lognormal assumption, nothing too crazy can happen during a short period of time. Hence, the value of the option today roughly becomes the payoff of the option, i.e. $E_t \approx V_t - K$.

Figures 33 through 38 shows the estimated values of σ_V and σ_E for the companies to be analysed. One can clearly see that the values of σ_V tend to be much smaller than the values of σ_E , implying that the volatility of the debt of the company is less volatile. Moreover, one can see that the asset volatility seems to be following the same trend as the equity volatility, although it appears to be much less volatile. This becomes evident for Bombardier, which can be seen in figure 36. The equity volatility varies up and down quite aggressively in the beginning, with a distinct spike in volatility at the very end of the time period. The asset volatility in this case however looks to be slowly decaying, with an upward turn at the very end, though far from as major as for the equity volatility.

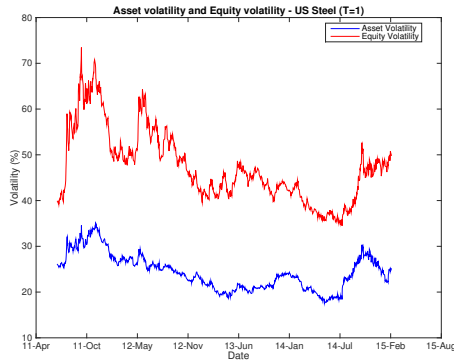


Figure 33: Asset volatility and equity volatility of US Steel

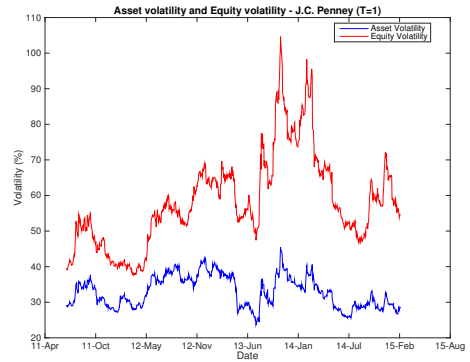


Figure 34: Asset volatility and equity volatility of J.C. Penney

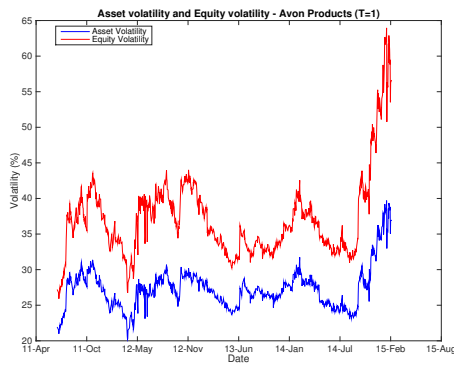


Figure 35: Asset volatility and equity volatility of Avon Products

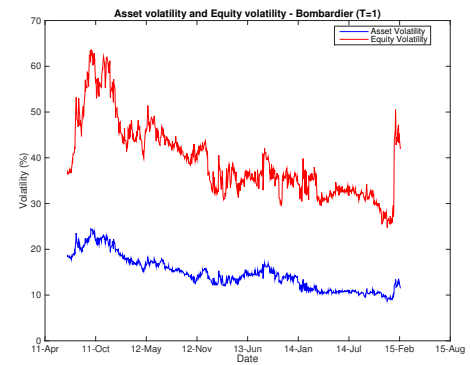


Figure 36: Asset volatility and equity volatility of Bombardier

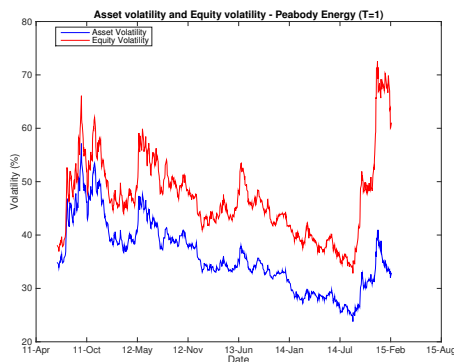


Figure 37: Asset volatility and equity volatility of Peabody

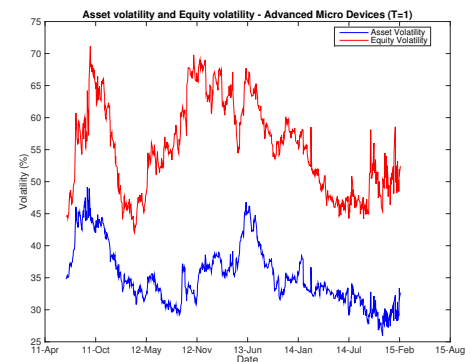


Figure 38: Asset volatility and equity volatility of AMD

The presented values of assets V_t and asset volatility σ_V are then used to calculate the Distance-to-Default, which then in turn is used to calculate the one year default probability. With the obtained default probabilities, the CDS-spreads can be inferred. Figure 39 through 44 shows for each company the calculated CDS-spreads versus the market spreads, the difference between the spreads, the ratio between the spreads and finally the market spreads versus the model spreads adjusted for the average error over the period.

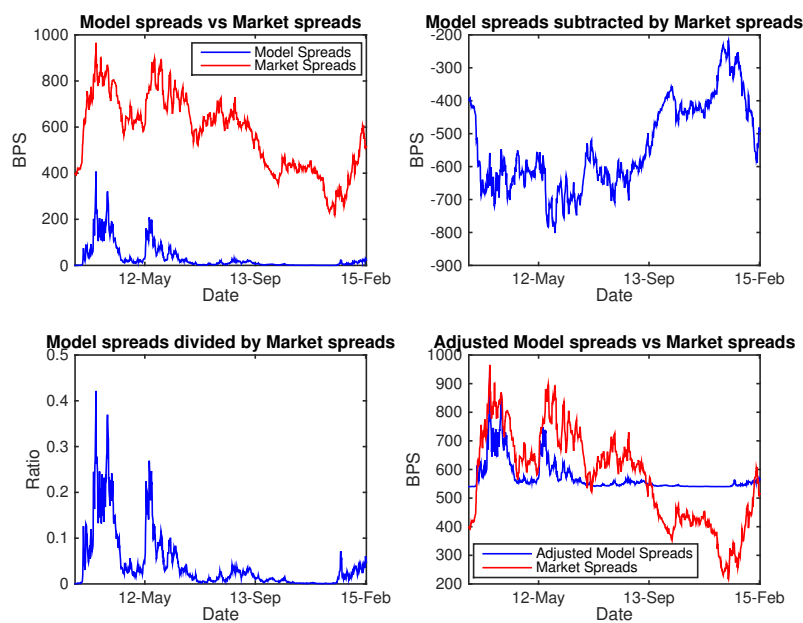


Figure 39: Results and CDS-spreads for US Steel

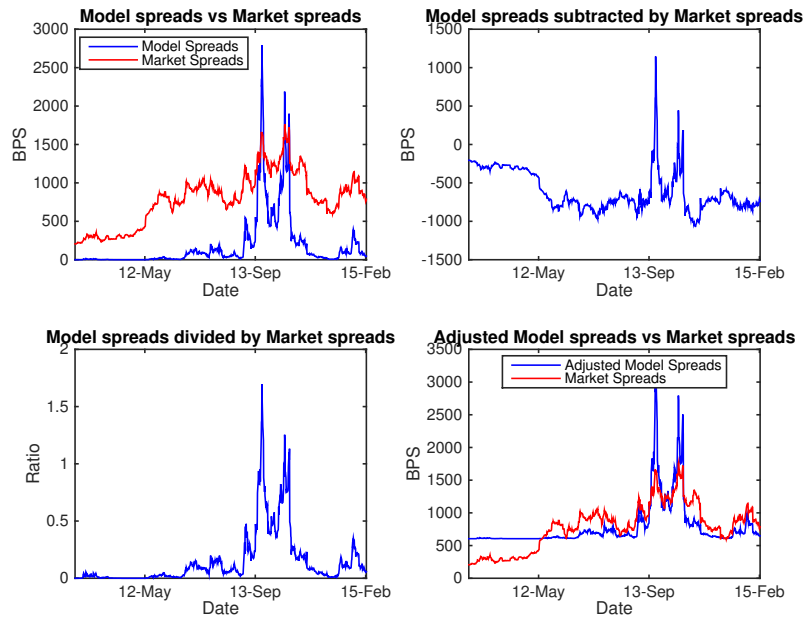


Figure 40: Results and CDS-spreads for J.C. Penney

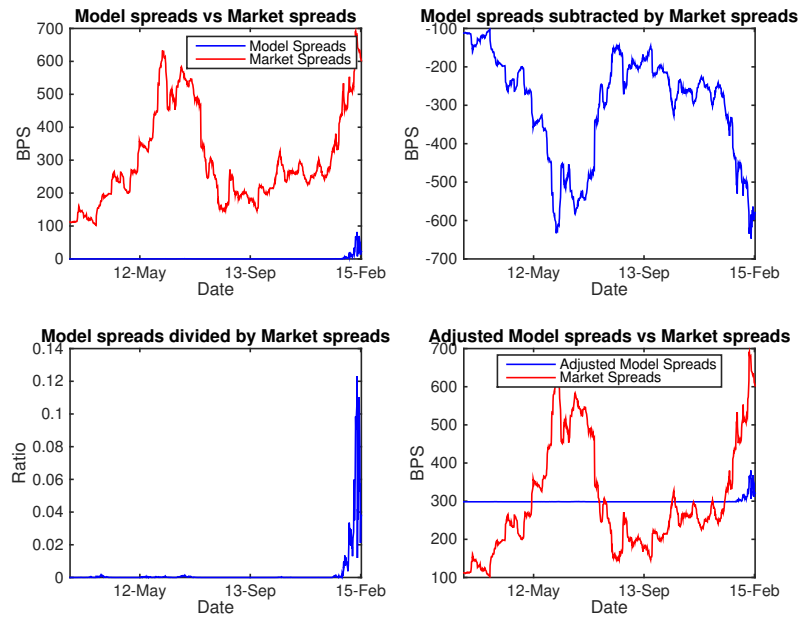


Figure 41: Results and CDS-spreads for Avon Products

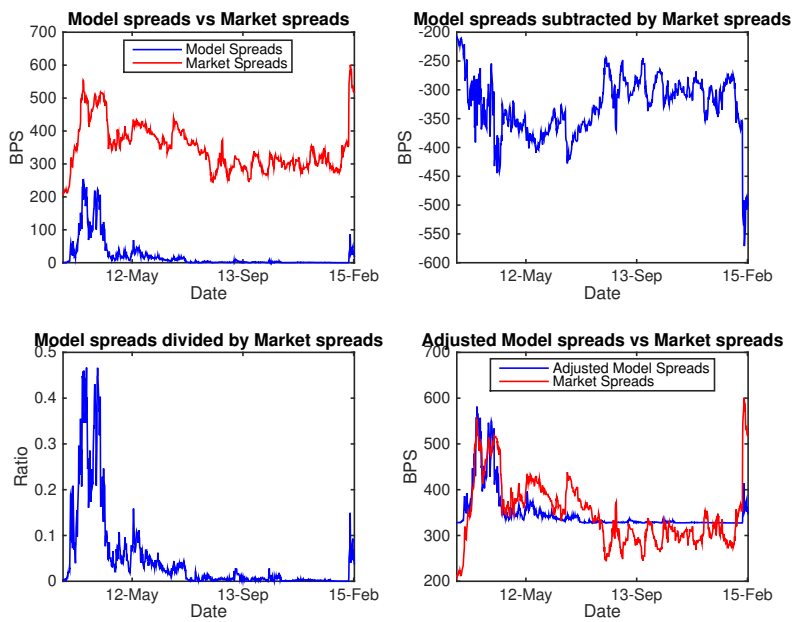


Figure 42: Results and CDS-spreads for Bombardier

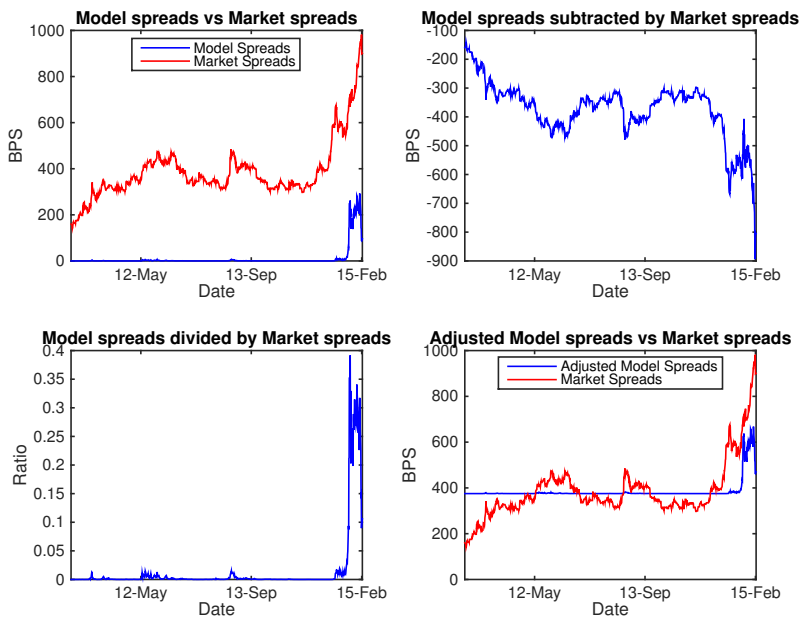


Figure 43: Results and CDS-spreads for Peabody Energy

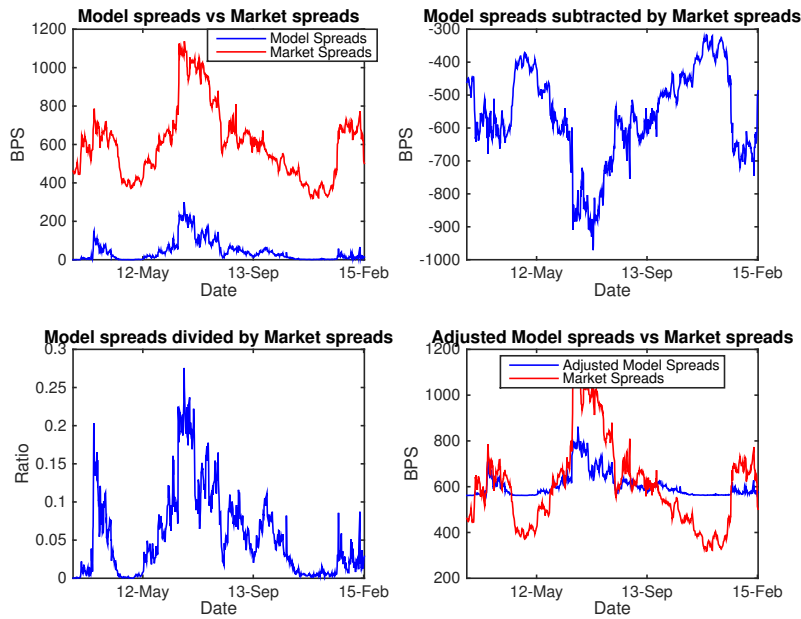


Figure 44: Results and CDS-spreads for Advanced Micro Devices

Obviously, it goes without saying that the resulting model-spreads are very poor. The absolute level of the model spreads are far below the market spreads. Looking at the residuals, it is clear that they are far from stationary. One could possibly argue that the residuals seen in figure 42 for Bombardier are somewhat stationary, but the reason for this seems to be because the market spreads themselves seem to be stationary in combination with the fact that the model spreads are very close to zero. Moreover, looking at the adjusted model spreads, one sees that the model spreads does a poor job in following movements of the market spreads for all companies, only major movements in the market spreads do the model spreads seem to follow.

Table 2 shows the average residual error and the standard deviation of the residuals for each company. The metrics are calculated based on the residuals shown in the top right chart of figures 39 through 44. Here Bombardier appears to have the smallest standard deviation, which is due to the relative stationarity of the residuals.

Table 2: Average residual error and standard deviation of residuals for $T = 1$ year

Name	Average Error (BPS)	SD of Residuals (BPS)
United States Steel Corp	-539.94	133.13
J.C. Penney Co	-605.07	270.37
Avon Products Inc	-298.40	137.87
Bombardier Inc	-327.37	51.11
Peabody Energy Corp	-375.21	98.32
Advanced Micro Devices	-561.90	139.09

The bottom left chart in all figures show the ratio between model spreads and market spreads. The time series for this metric does not look stationary for any company. The biggest problem is the fact that the model spreads seem to understate the CDS-spreads so much that they sometimes appear to be around zero.

It is clear that the choice of maturity as one year and the default level as the short-term liabilities generate very poor results when modelling CDS-spreads. Further results can be presented, but since the overall performance of the model spreads are so poor, there really is no point in doing so.

5.1.2 $T = 10$ Years

In this case with $T = 10$ years, the debt level is set equal to the short-term liabilities plus the long-term debt. One can perhaps expect that the results from this choice of maturity and default level should be better than for $T = 1$ year since more data specific to the company is taken into account.

Figure 45 through 50 shows the calculated asset values of the companies along with the market capitalisation and the debt level. These figures can be compared with figures 27 through 32 for the maturity $T = 1$ year. One can observe that the overall trend of the asset values for $T = 10$ years are about the same as for $T = 1$ year, but the absolute level of the asset values seem to have had an overall increase. This is connected to the fact that the debt level is increased as well.

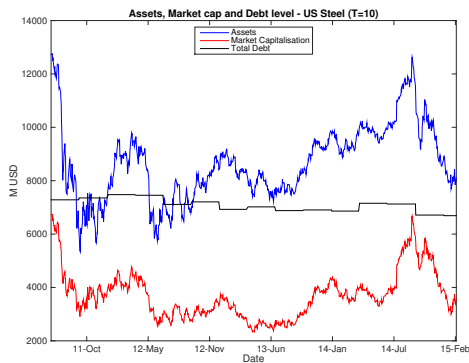


Figure 45: Asset value, market capitalisation and short-term liabilities of US Steel

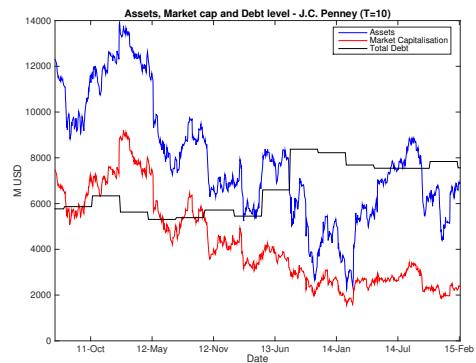


Figure 46: Asset value, market capitalisation and short-term liabilities of J.C. Penney

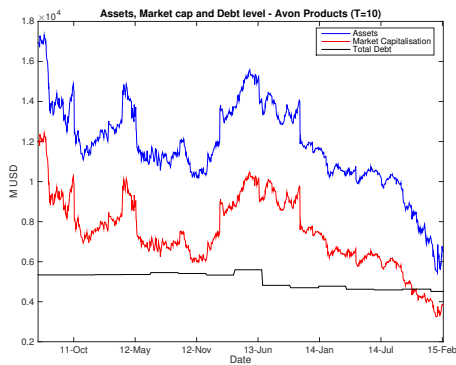


Figure 47: Asset value, market capitalisation and short-term liabilities of Avon Products

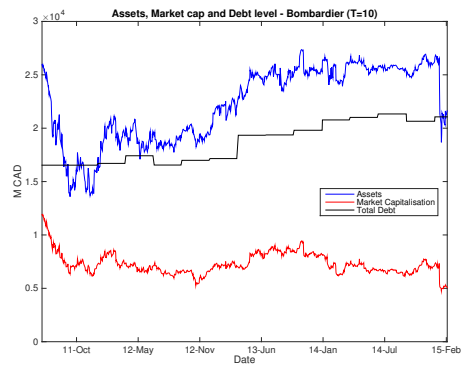


Figure 48: Asset value, market capitalisation and short-term liabilities of Bombardier

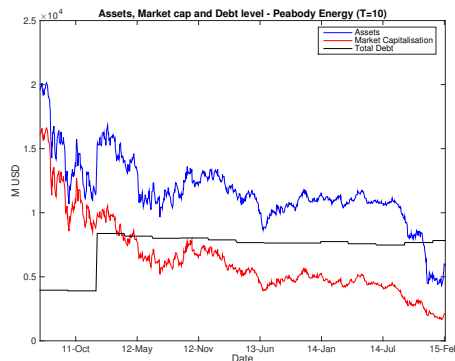


Figure 49: Asset value, market capitalisation and short-term liabilities of Peabody Energy

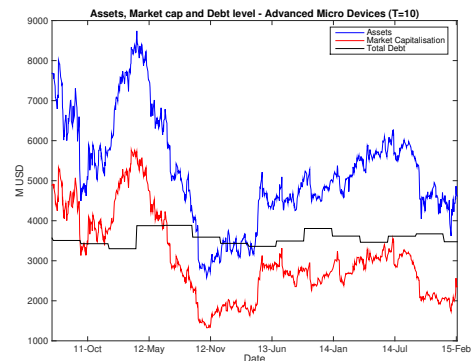


Figure 50: Asset value, market capitalisation and short-term liabilities of Advanced Micro Devices

Moreover, figure 51 through 56 shows the calculated asset volatility and the equity volatility. The charts can be compared with figures 33 through 38 respectively, which are the equivalent graphs but for $T = 1$ year. Here one can see that the asset volatility has had an overall increase, although there are a couple of period for certain companies where the asset volatility has decreased compared with for $T = 1$ year. The asset volatility also seems to follow the movements of the equity volatility better also.

Because of this increase in asset volatility, one can expect higher default probabilities, since a higher volatility leads to larger movements in the assets, which in turn yields higher probabilities of the assets falling below the default threshold. Consequently, one can expect an overall increase in CDS-spreads, which, considering the results for $T = 1$ year, would be an improvement.

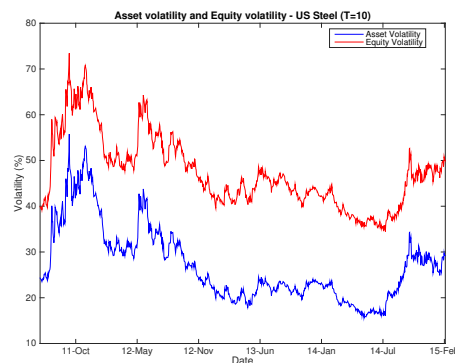


Figure 51: Asset volatility and equity volatility of US Steel

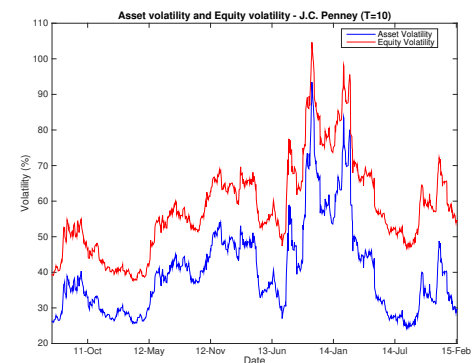


Figure 52: Asset volatility and equity volatility of J.C. Penney

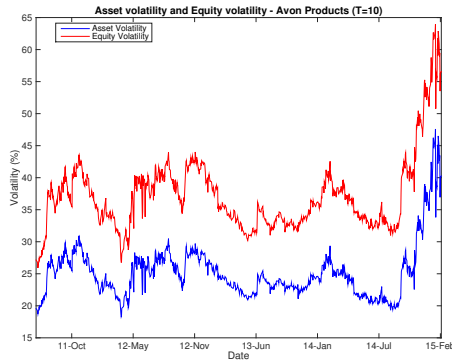


Figure 53: Asset volatility and equity volatility of Avon Products

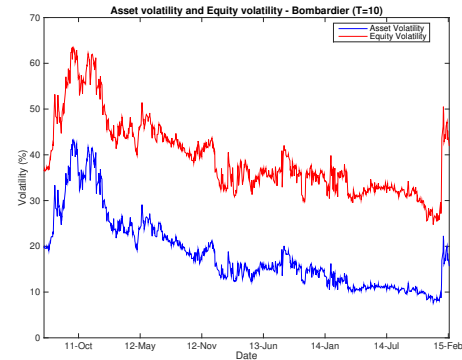


Figure 54: Asset volatility and equity volatility of Bombardier

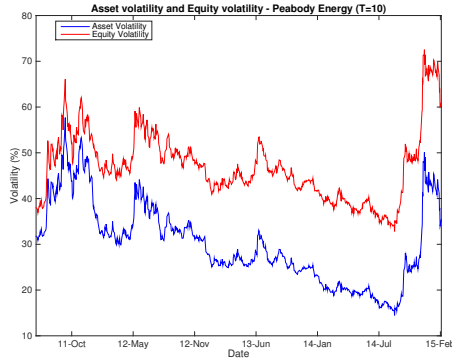


Figure 55: Asset volatility and equity volatility of Peabody

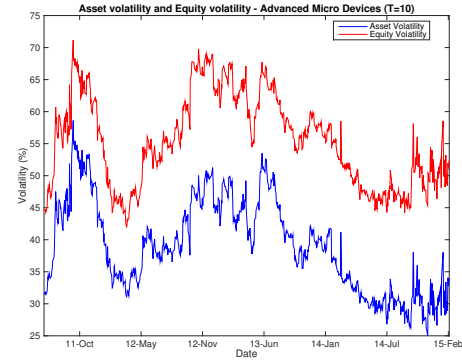


Figure 56: Asset volatility and equity volatility of AMD

Figures 57 through 62 shows the resulting CDS-spreads of the model along with the market spreads, the difference between the spreads, the ratio between the spreads and the market spreads versus the model spreads adjusted for the average error over the period. As suspected, the spreads have now become larger, although a little too large in many cases. Now however, one can see that the model-spreads appear to follow the trend of the market-spreads quite nicely in several cases.

Table 3 shows the average residual error and the standard deviation of the residuals for the analysed companies. Only in the case of Avon Products is the average residual error negative, it also happens to be the company where the standard deviation is the smallest. Overall, the model does an acceptable job in following the major trends in the market-spreads, although it fails to keep up with all the movements in the market. The major trends are obviously followed quite well, though the relative error is not perfectly stationary as seen in the charts, but is still a great improvement over the equivalent charts for $T = 1$

year.

Table 3: Average residual error and standard deviation of residuals for $T = 10$ years

Name	Average Error (BPS)	SD of Residuals (BPS)
United States Steel Corp	68.20	173.99
J.C. Penney Co	256.30	415.22
Avon Products Inc	-92.86	111.87
Bombardier Inc	121.30	201.68
Peabody Energy Corp	134.33	202.71
Advanced Micro Devices	158.11	148.31

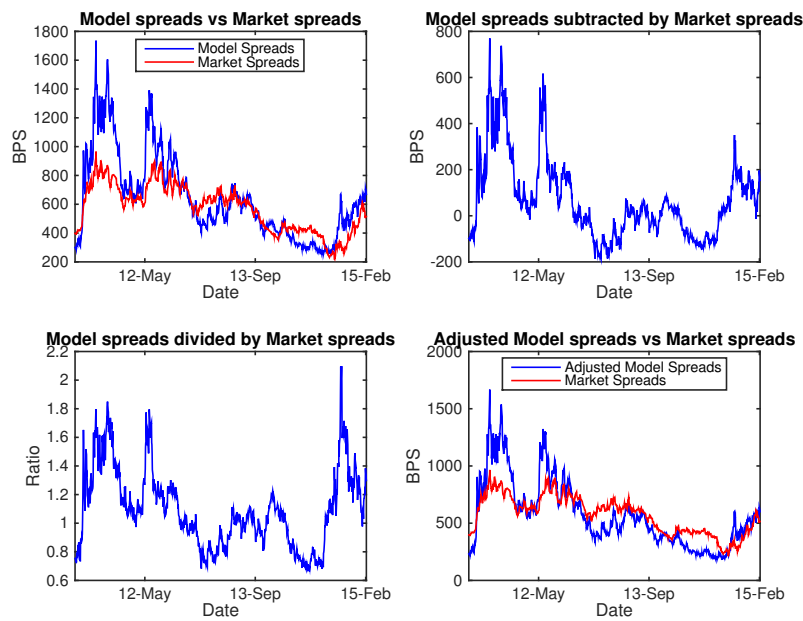


Figure 57: Results and CDS-spreads for US Steel

For US Steel, the model overstates the CDS-spreads during many time periods, but the overall market trend appears to be somewhat captured by the model. However, neither the residuals nor the model spreads divided by the market spreads appear to be particularly stationary.

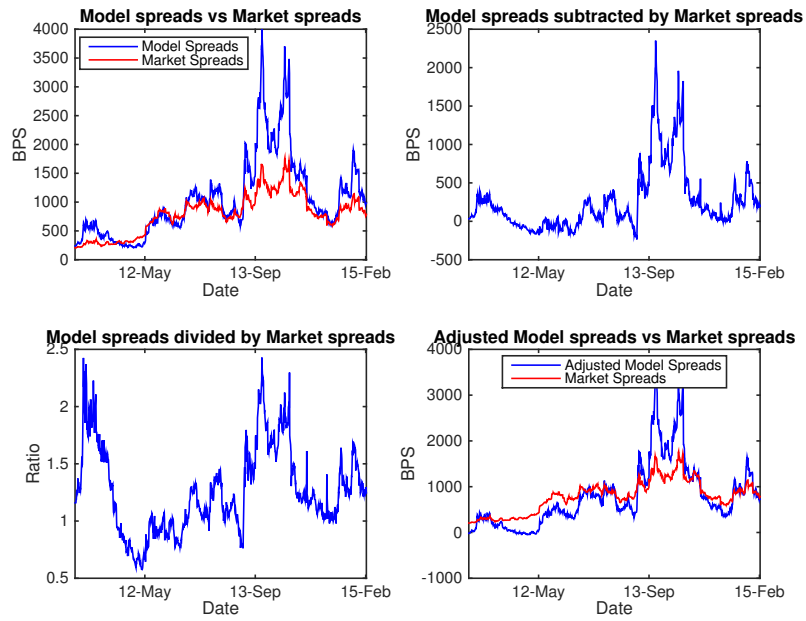


Figure 58: Results and CDS-spreads for J.C. Penney

As for US Steel, the model overstates the CDS-spreads on several occasions for J.C. Penney, though the trends seems to be followed in an acceptable way. Obviously, the residuals are far from stationary.

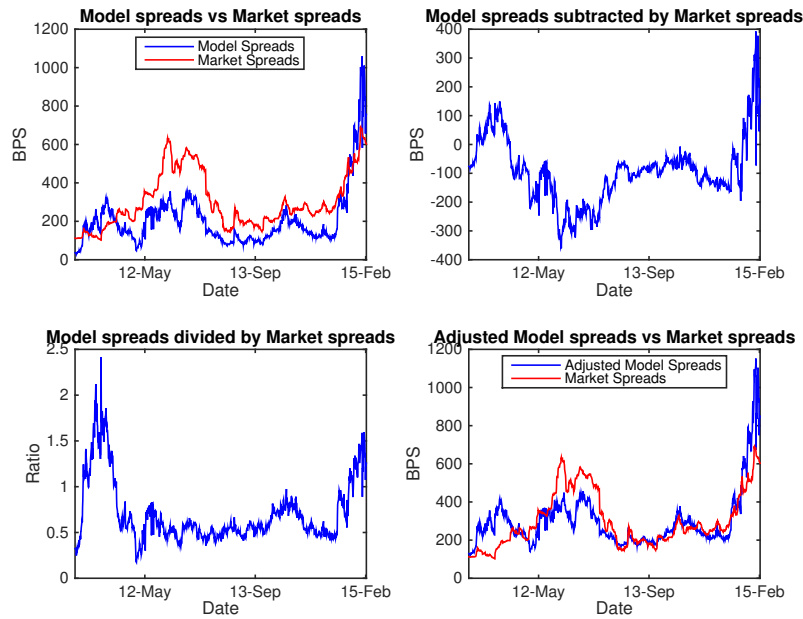


Figure 59: Results and CDS-spreads for Avon Products

The results for Avon Products are fairly good, at least when having adjusted for the average residual error. The overall trend is there, although the model cannot quite capture the rise in the CDS-spreads in the middle of the period. The model also appears to overstate the market at the very end of the period. The residuals are not stationary in this case either though, however, one can perhaps argue that the models spreads relative the market spreads, from May 2012 and forward show slight hints of stationarity.

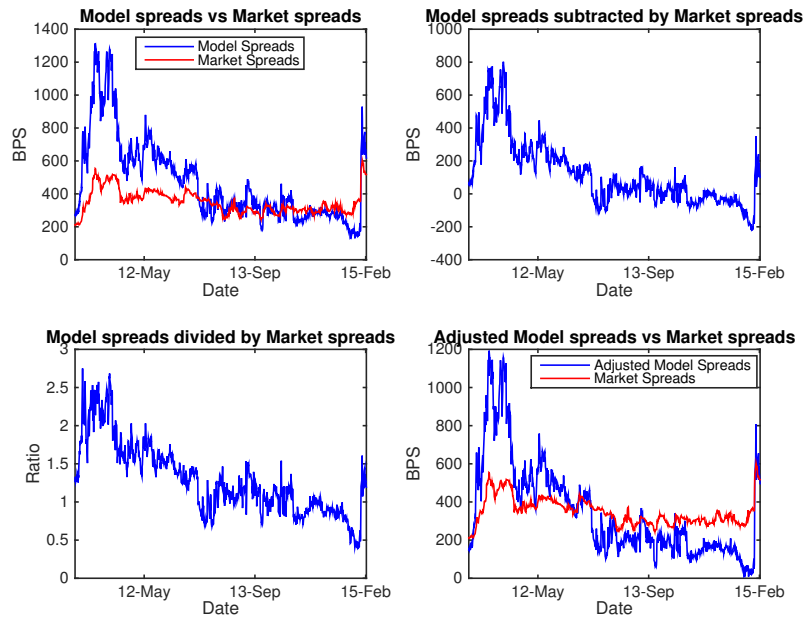


Figure 60: Results and CDS-spreads for Bombardier

The model-spreads in the case of Bombardier appears to be severely overstated almost all the time. Also, the trends seem to be massively amplified. In this case, it is also obvious that the residuals are not stationary as there seems to be a negative trend in the residuals. The same goes for the model spreads relative the market spreads.

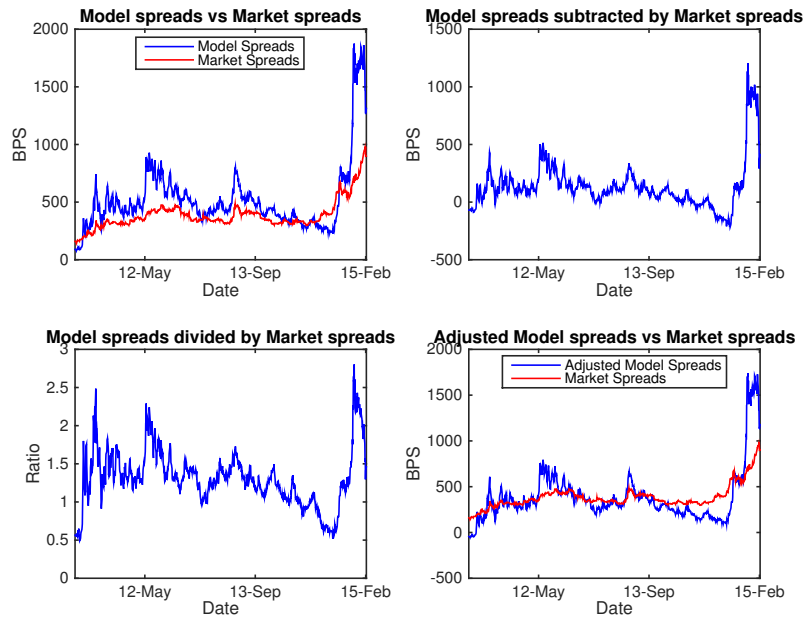


Figure 61: Results and CDS-spreads for Peabody Energy

Like for Bombardier, in the case of Peabody Energy, the model seems to mostly overstate the CDS-spreads as well as amplify the trends in the market. However, in this case, apart from the very end, the residuals look fairly stable, looking somewhat stationary.

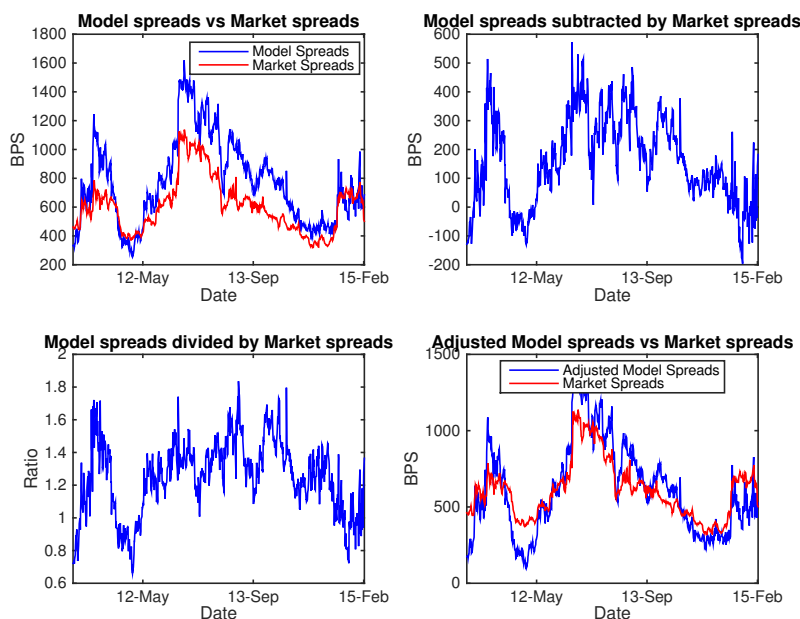


Figure 62: Results and CDS-spreads for Advanced Micro Devices

The results for Advanced Micro Devices are quite nice. When adjusting for average error level, the model spreads and the market spreads seem to move together nicely. Also, the residuals look somewhat promising here, apart from a dip in the residuals just before May 2012. Also, the model spreads relative the market spreads seem to vary between 1.2 and 1.4 during most of the period.

In summary, the results from the Merton model when one is very strict about how one chooses the maturity and debt level are rather unsatisfactory. At least for $T = 10$ years, the model manages to capture the major trends in the market-spreads, but does a poor job in following the market for minor changes. The results are even more poor for $T = 1$ year, here only very extreme changes in the market spreads are somewhat followed by the model. Due to the results for the Merton model being very poor, no further tests on stationarity and uncorrelated residuals is presented, the figures with the model-spreads versus the market-spreads speak for themselves.

5.2 The Black-Cox Model

5.2.1 $T = 1$ Year

Just as for the Merton model, with the maturity chosen as $T = 1$ year, the debt level is chosen as the short-term liabilities. Moreover, the default-threshold is assumed to continuously increase over the period according to $K(t) = K_0 e^{kt}$, i.e. at maturity, the default-threshold is equal to the short-term liabilities. Results will be presented for $k = [0, 0.01, 0.05]$.

The results for this specific maturity and choices of k are very disappointing, and are thus instead presented in the appendix. Section 9.2.1 through 9.2.3 in Appendix B shows the resulting model-spreads, residual error and ratio between the market and model-spreads for all companies analysed.

The results are very similar to those of the Merton model for the same maturity. The only case where one achieves somewhat acceptable results is for Advanced Micro Devices in the case of $k = 0.05$. Albeit, looking at the residuals, they are not perfectly stationary as there is a big jump in the model spreads somewhere around the winter of 2012-2013. However, if one overlooks this period, and then looks at the bottom right chart, where the market-spreads are plotted against the model-spreads adjusted for the average error over the period, one is left with something that actually looks reasonably stationary and actually follows the market-spreads quite nicely. However, the overall results for this choice of parameters for the other companies are far too poor, hence, no further analysis will be done for these results.

Since the Merton model showed increased performance when the maturity and debt level increased, one can hope for similar results for the Black-Cox model.

5.2.2 T = 10 Years

As for the maturity $T = 1$ year, results for $k = [0, 0.01, 0.05]$ will be presented. The major difference between these two maturities for the Black-Cox model is that the longer maturity will make the impact of the continuously increasing default-threshold more greater. During a period of one year, the threshold will only vary a factor e^{-k} from the maximum value. However, during a period of ten years, the factor will be e^{-10k} , which will be much greater of an impact.

Tables 4 through 6 shows the average error of the residuals and the corresponding estimated standard deviation of the residuals. One can very clearly see how the value of k affects the size of the resulting model spreads. For a value of k equal to zero, the average residual error is negative for all companies, implying that the model spreads are too low. However, for a value of $k = 0.05$, the opposite occurs. The average residual error instead becomes positive, meaning that the model spreads are larger than the market spreads. However, the choice of $k = 0.01$ seems to yield average residual errors close to zero, as half of the companies have positive residual errors and half have negative residual error.

Moreover, looking at the estimated standard deviations of the residuals, it is clear that the choice of $k = 0.01$ yields the best results as the standard deviation of the residuals is smallest for all the companies in this case, except for US Steel, which has the smallest standard error when $k = 0.05$.

Table 4: Average residual error and standard deviation of residuals for Black-Cox model, $T = 10$ years, $k=0$

Name	Average Error (BPS)	SD of Residuals (BPS)
United States Steel Corp	-279.69	170.66
J.C. Penney Co	-257.73	226.82
Avon Products Inc	-104.61	101.50
Bombardier Inc	-231.96	107.92
Peabody Energy Corp	-49.68	184.33
Advanced Micro Devices	-48.77	133.42

Table 5: Average residual error and standard deviation of residuals for Black-Cox model, $T = 10$ years, $k=0.01$

Name	Average Error (BPS)	SD of Residuals (BPS)
United States Steel Corp	-30.10	93.73
J.C. Penney Co	-59.73	126.71
Avon Products Inc	-54.85	101.41
Bombardier Inc	96.74	74.83
Peabody Energy Corp	108.72	94.14
Advanced Micro Devices	58.98	104.74

Table 6: Average residual error and standard deviation of residuals for Black-Cox model, $T = 10$ years, $k=0.05$

Name	Average Error (BPS)	SD of Residuals (BPS)
United States Steel Corp	537.80	67.13
J.C. Penney Co	428.81	175.97
Avon Products Inc	219.00	105.91
Bombardier Inc	800.84	75.92
Peabody Energy Corp	529.17	163.77
Advanced Micro Devices	428.83	111.72

Table 2 and 3 are the corresponding tables for the Merton model for the maturity $T = 1$ year and $T = 10$ years respectively. Comparing table 5 with these tables, it is evident that the Black-Cox model, with a maturity of 10 years and $k = 0.01$ is better than the Merton model in terms of average residual error and standard deviation of the residuals.

The resulting charts for $k = 0$ and $k = 0.05$ are presented in Appendix B, section 9.2.4 and 9.2.5. Since the results were deemed most satisfactory in the case of $k = 0.01$, these results will be presented and further analysed.

Figures 63 through 68 shows the resulting model-spreads, residual error and ratio between the market and model-spreads for all companies analysed for $k = 0.01$.

By looking at the figures, one can now see that the model spreads very nicely follow the market spreads in a way the Merton model was nowhere close to do.

There are surprisingly few cases where the model spreads and market spreads diverge significantly, the most apparent ones are at the very beginning of J.C. Penney, in the middle period for Avon Products and at the end for Peabody Energy. Other than in those cases, the model fits the market quite nicely. Note also that the average residual error is very small for all the companies, which can be seen in table 5, meaning that the model spreads adjusted for average error seen in the bottom right charts diverge little from the original model spreads.

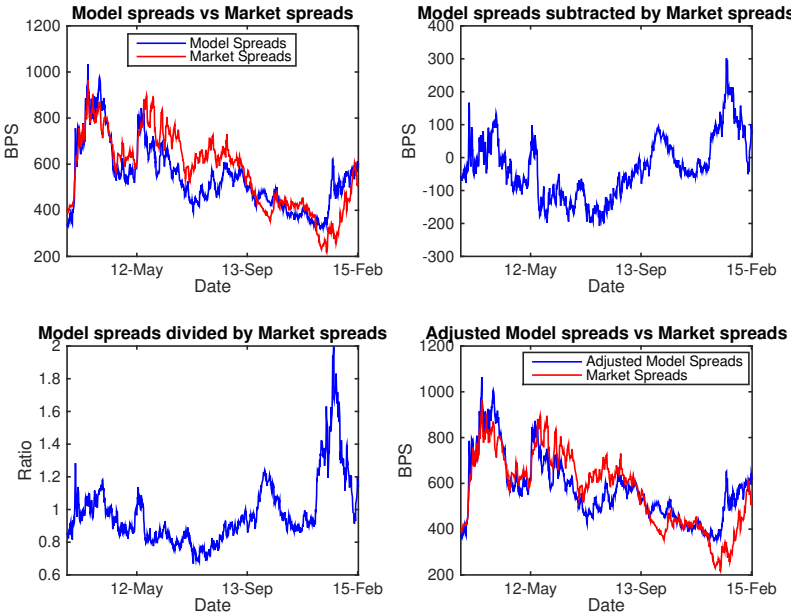


Figure 63: Results and CDS-spreads for US Steel

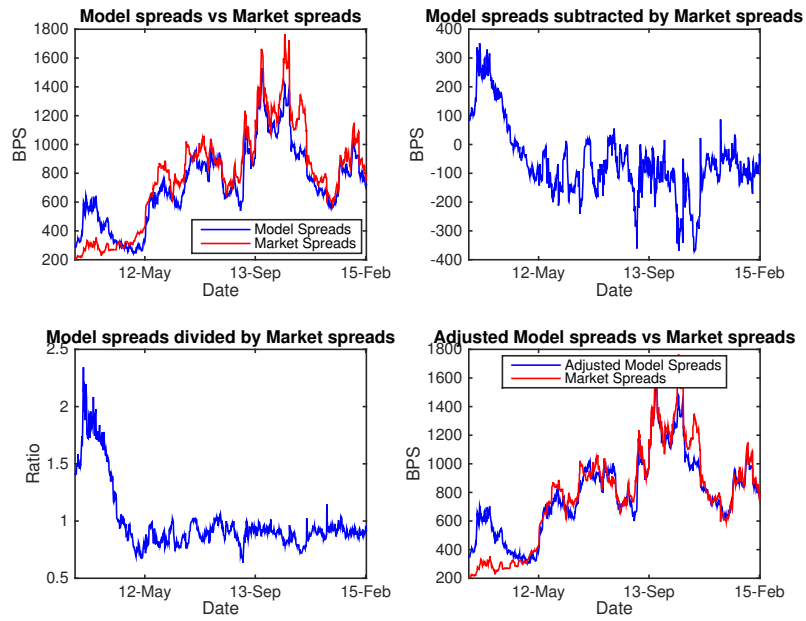


Figure 64: Results and CDS-spreads for J.C. Penney

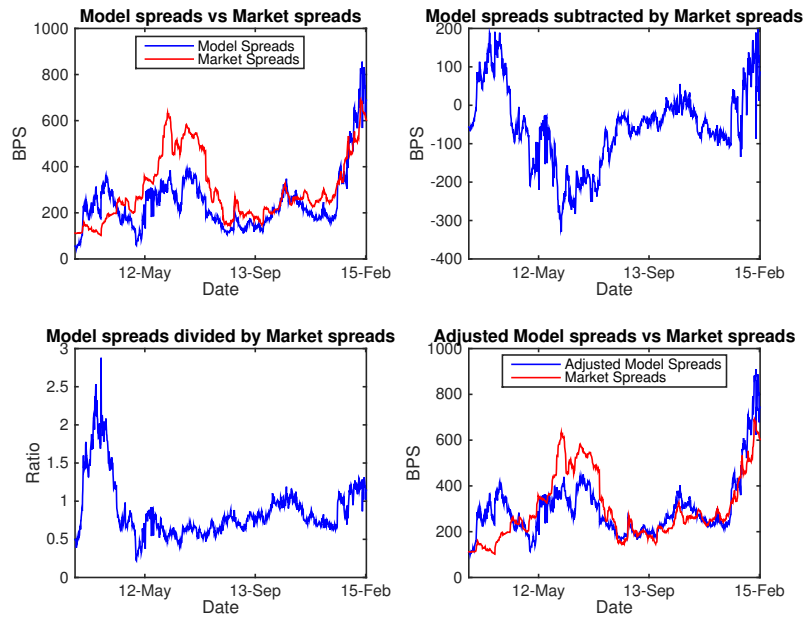


Figure 65: Results and CDS-spreads for Avon Products

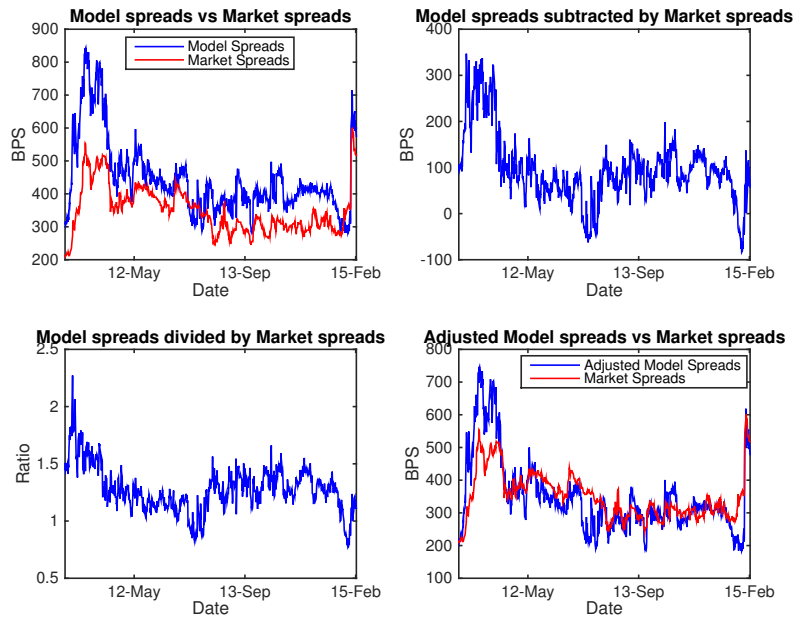


Figure 66: Results and CDS-spreads for Bombardier

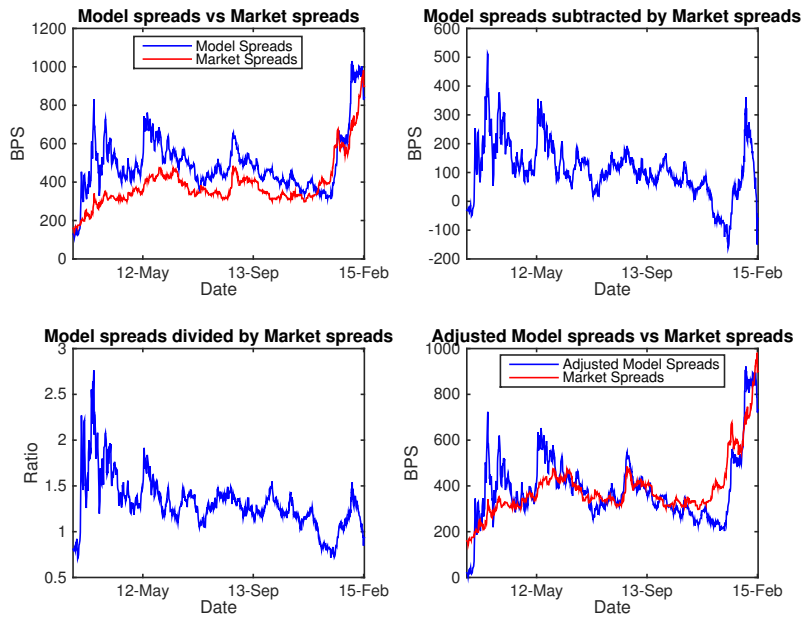


Figure 67: Results and CDS-spreads for Peabody Energy

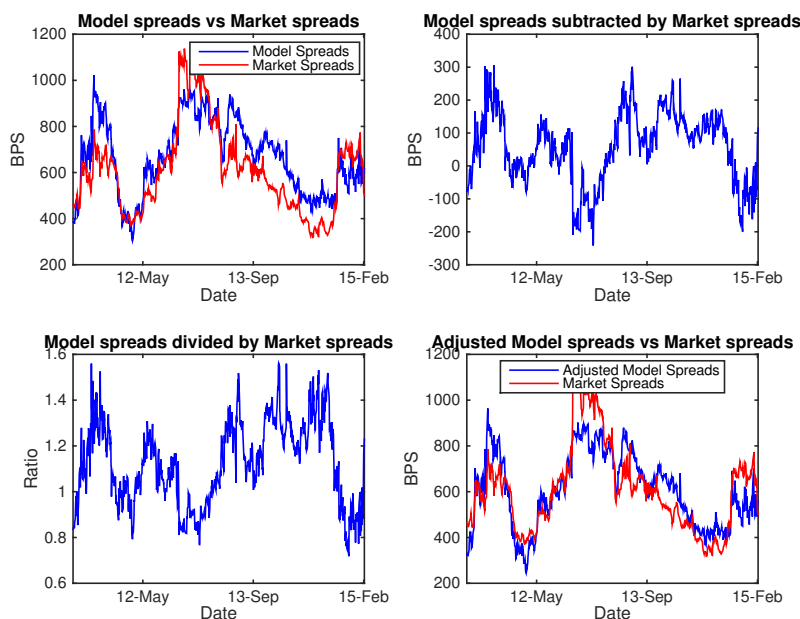


Figure 68: Results and CDS-spreads for Advanced Micro Devices

Looking at the residuals for the companies, there appears to be a lot more stationarity than for the Merton model. Especially Peabody Energy and Advanced Micro Devices show fairly nice properties. Also, Bombardier looks acceptable, if one ignores the very beginning of the period. The same can be said about J.C. Penney.

Moreover, looking at the ratio between the model spreads and the market spreads, one can see that many of the companies show a fairly stationary time series. As before, Advanced Micro Devices seems to perform best, followed by Peabody Energy and Bombardier. Avon Products and J.C. Penney also look promising, if one, like earlier, ignores the very beginning of the period. US Steel also has an overall acceptable performance, but there appears to be a "spike" in the model spreads at the very end of the period which ruins the results a bit.

Since the residuals are not perfectly stationary, a further study is made by simply taking the mean adjusted residuals and dividing them by the calculated asset volatility at each point. The reason why this analysis may be relevant is because one can potentially expect that higher residuals may be associated with a higher volatility. If one can manage to obtain more stationary residuals when adjusting for volatility, then one has managed to capture a very large part of what determines the CDS-spreads.

Figure 69 shows the mean adjusted residuals divided by asset volatility for all companies. Since the volatility is in units of percentage, the output shown in the charts can be interpreted as number of basis points per percentage of asset volatility.

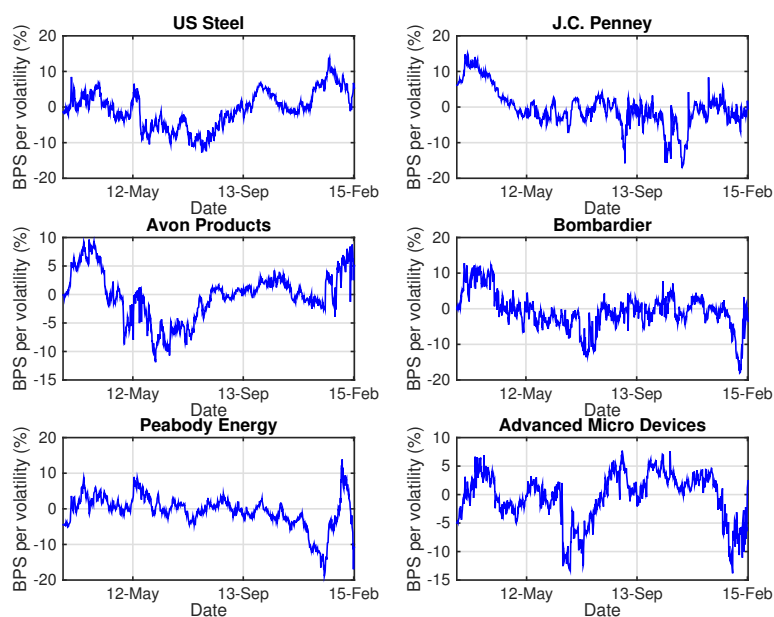


Figure 69: Mean adjusted residuals divided by corresponding asset volatility

These adjusted residuals do not look perfectly stationary either, there are however some differences between the original residuals shown in figures 63 through 68 and the ones presented here. For instance, some of the "spikes" in the residuals for US Steel have been somewhat attenuated. However, there appears to still be some sort of drift in the adjusted residuals, starting from the beginning of the period to the middle of the period. The most improvement has probably been in Peabody Energy. If one looks at the original residuals in this particular case, one sees that these were very far from stationary as they showed different levels of volatility throughout the period. Now, however, those changes in volatility has been almost completely smothered, only the very end of the period shows non-stationary features, but the rest of the data looks very nice.

Bombardier has also seen some improvement. In the original residuals, one sees in the very beginning of the period that the residuals are significantly larger than during other times. In the adjusted residuals, the same property is still present, but it has been significantly lowered, while the rest of the period, apart from the very end, looks quite nice in terms of stationarity.

Figure 70 shows auto-correlation plots of the volatility adjusted residuals for the companies for up to 100 lags. Although one could suspect that the adjusted residuals would not prove to be an uncorrelated sequence, the charts confirm this. It can be worth to note that only Peabody Energy appears to have correlations that stabilise at some point in time, from about lag 60-65. However, the overall results from the auto-correlation charts are quite disappointing.

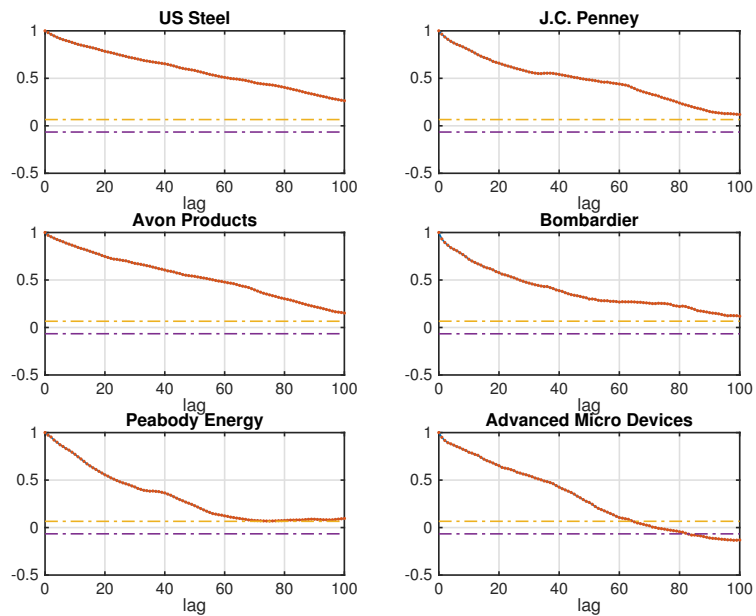


Figure 70: SACF plot of mean adjusted residuals divided by corresponding asset volatility

5.3 Regression Model

Table 7 shows the estimated parameters of the regressions for all companies as well as the standard error of the residuals. A star (*) next to the parameter means that it is significant on a 95% level. The resulting parameters confirm the suspicion that a higher equity volatility would be associated with higher CDS-spreads. This can be seen as all the slopes are positive, i.e. the parameters that are multiplied by the current level of volatility. For instance, for US Steel, the parameter value of the slope of about 15 means that for each percentage of equity volatility, 15 bps are added to the CDS, which is offset by about -136 bps to begin with.

In almost all cases, the parameters are significant, only Avon Products and Peabody Energy "fail" in this sense, so the regression variable appear to make sense.

Table 7: Estimated parameters for equity volatility regressed against market CDS-spreads

Name	Constant	Slope	SE of Residuals
United States Steel Corp	-135.87*	14.95*	118.32
J.C. Penney Co	-195.07*	12.11*	95.15
Avon Products Inc	150.91*	1.66	96.06
Bombardier Inc	-63.53*	9.38*	50.76
Peabody Energy Corp	3.77	5.98*	85.67
Advanced Micro Devices	-184.08*	13.25*	164.33

Figures 71 through 76 shows the resulting regression spreads against the market spreads along with the residuals, a normal probability plot of the residuals and plot of the residuals versus the corresponding volatility. The assumption is that the residuals should be Gaussian which the probability plot will assess, also, the residuals should be uncorrelated with the input data, in this case the volatility, which the bottom right charts in the figures will analyse.

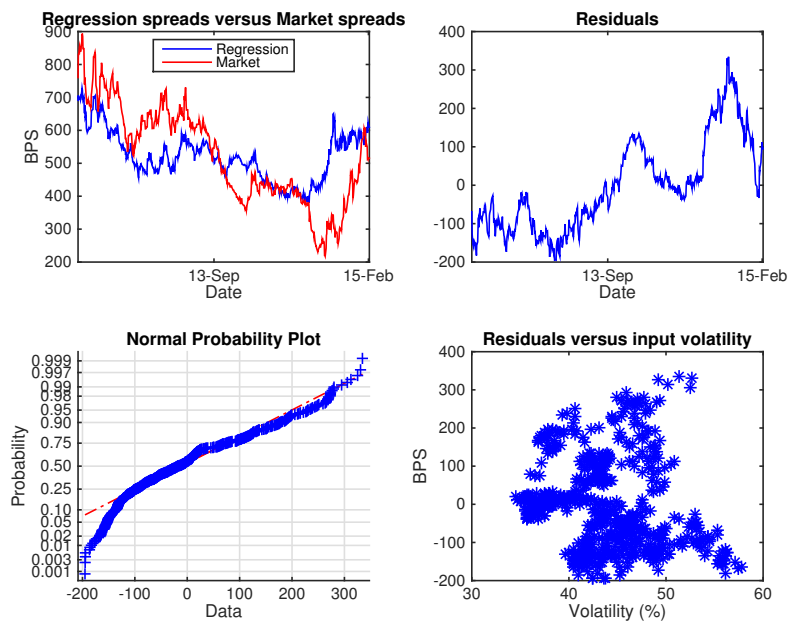


Figure 71: Regression results for US Steel

Starting with US Steel, one clearly sees that the assumptions of Gaussian errors are violated as the data points in the tail diverge from the rest of the data in the probability plot. Also, the regression does a rather poor job in modelling the spreads as seen in the top left chart. However, the residuals seems to be uncorrelated with the input, looking at the bottom right chart, as there seems to be no clear structure in the scatter plot.

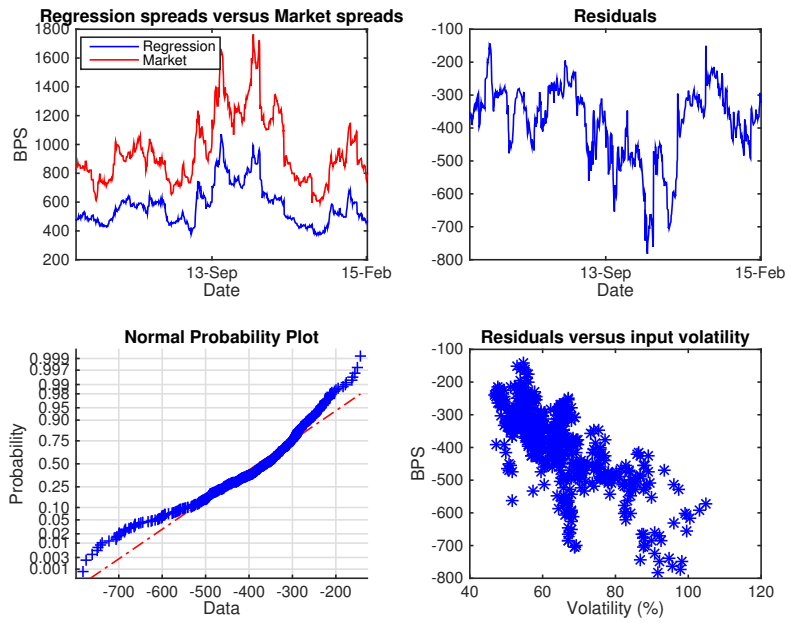


Figure 72: Regression results for J.C. Penney

J.C. Penney on the other hand looks better in the sense that the residuals look fairly stationary, there is no drift over time, although there appears to be some increase in volatility in the middle of the period. The regression also does a fairly good job at following major changes and shifts in market spreads, but as for US Steel, the normality assumptions on the residuals do not hold true. There also appears to be some negative correlation between residuals and input volatility.

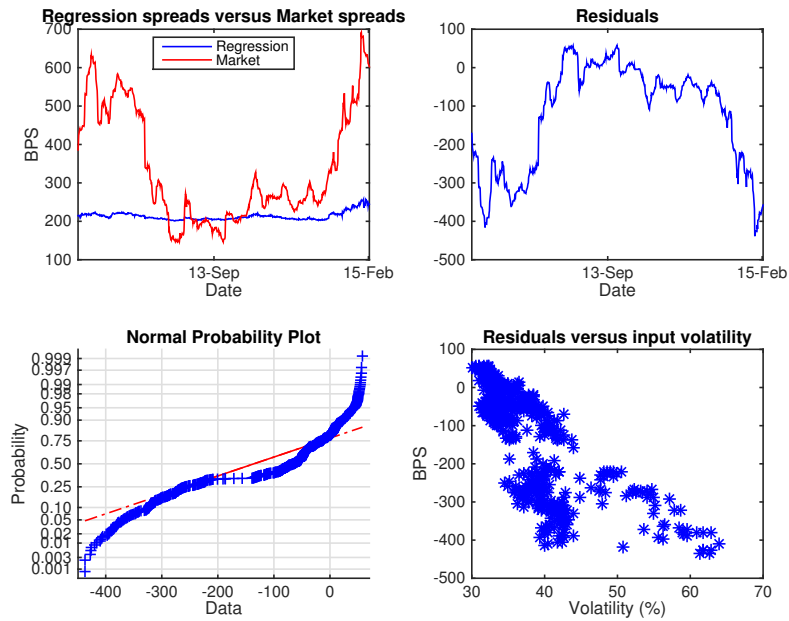


Figure 73: Regression results for Avon Products

Moving on to Avon Products, it looks as if the regression-spreads can not follow the market spreads at all. This is partly explained by the fact that the slope-variable for Avon Products showed to be non-significant, meaning that in this case, the equity volatility does not provide further information that helps determine the CDS-spreads. This can perhaps be seen even more clear in figure 9 and 10, where the original data of the equity volatility and CDS-spreads of Avon Products are shown. There does not seem to be a clear correlation between the two variables in this case, which might be the reason why the parameter resulted in being insignificant. However, like for J.C. Penney, there seems to be some negative correlation between residuals and input volatility. And finally the normal probability plot looks nothing like a normal distribution at all.

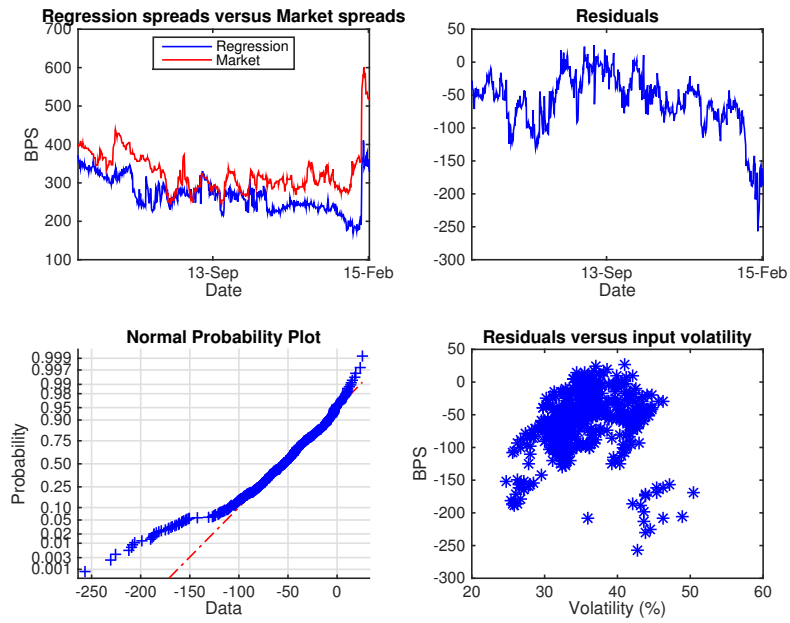


Figure 74: Regression results for Bombardier

The regression delivers acceptable results for Bombardier. The regression-spreads look almost like a constant straight line over the period, but manages to at least follow some movements in the market. The residuals look fairly stationary, apart from at the very end of the period.

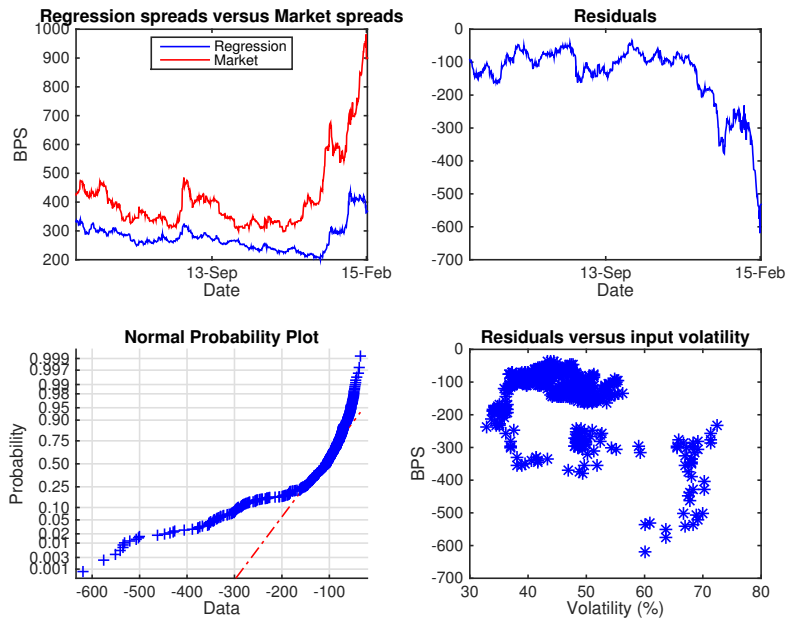


Figure 75: Regression results for Peabody Energy

Peabody Energy on the other hand shows quite disappointing results as the regression spreads continuously seem to understate the CDS-spreads. This is probably due to the fact that the period used for estimating the parameters in the regression, the volatility was fairly high, but the CDS-spreads low. After the modelling period, the volatility decays, while the CDS increases. This mismatch between volatility and CDS is most likely the cause for the look of the figure above.

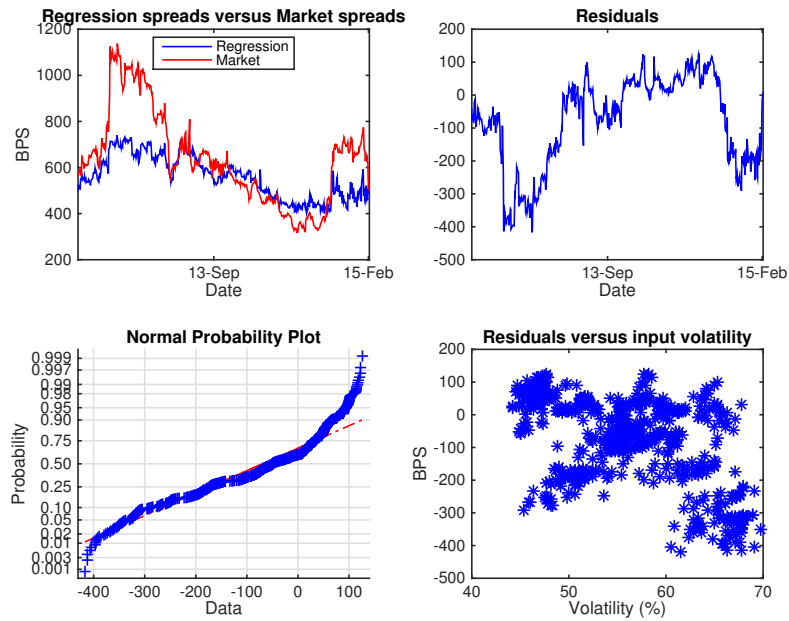


Figure 76: Regression results for Advanced Micro Devices

Finally, Advanced Micro Devices present fairly good results, the regression returns spreads in the correct order of magnitude, and follows the major trends fairly well. However, it appears to be poor when it comes to large movements in the market spreads, for instance when the increase rapidly over a short period of time. One can also see that there appears to be a negative correlation between the residuals and the corresponding volatility, implying that this simple regression is probably too simplistic to model the CDS-spreads in this case.

6 Discussion

The results-section provided mixed results. Some models perform very poorly, while especially the Black-Cox model proved to be surprisingly good at modelling the CDS-spreads for a certain set of parameters. In this section, the results are further analysed and potential points of improvement are discussed.

The Merton model showed rather poor results, regardless of maturity chosen, although the results improved when a longer maturity was chosen. What made the results poor was the fact that the model seemed unable to catch movements in the CDS-spreads other than the absolute largest ones. However, in some cases, the exact opposite occurred, i.e. the model significantly overstated movements in the CDS-spreads. This resulted in rather unsatisfactory residuals, showing little to no tendencies of stationarity. Although, for the case when the maturity was set as $T = 1$ year, residuals for some companies such as Bombardier seemed to be fairly stationary, but the only reason for this was due to the model-spreads being close to zero while the market-spreads looked fairly stationary during those times. As the model spreads in general followed the market-spreads very poorly, no further assessment of the residuals was made as for the Merton model.

There are a couple of changes with the current implementation of the model that one could do to potentially gain more favourable results. First is how the maturity of debt and the debt level is chosen. In this implementation, the two have been chosen to work well together theoretically, but in terms of performance, this may not be the best choice of combination. Looking at the results from the Merton model, it is clear that a maturity of one year is far too short, while a maturity of ten years may be too long. One could perhaps through empirical testing arrive at a better combination of the two variables, however, one has then prioritised practical performance over theoretical correctness. However, the latter can in a sense be considered to be violated from the very beginning as the Merton model is far from realistic.

The problem one faces is that if one decides to decrease the maturity T , how should one then adjust the default level? In the KMV model for instance, as mentioned in the presentation of the models, the default point is put somewhere between the short-term debt and total debt, with some appropriate maturity. However, it is not further described how they go about choosing the default point, other than its range. One potential choice is choosing $T = 5$ years and $K = \text{Short term debt} + 0.5 \cdot \text{Long term debt}$, just to get the default point somewhere in the middle with a maturity that should represent this. This combination of parameters was tested on the companies analysed in this report, but yielded no better results than the case with a ten year maturity. One could potentially arrive at better results if one fine tuned the maturity and default point even more, this was however not further investigated here.

The second point addresses more others findings of the Merton model. The creators of the KMV model use the basis of the Merton model, but diverges when it comes to calculation of the probability of default. Instead they have taken another route, obviously being dissatisfied with the results of a Gaussian assumption on the distance-to-default metric. Obviously, the data-base used is proprietary to Moody's so testing using that is not a possibility, but perhaps

one could try to change the distribution that maps the distance-to-default to the default probability. In doing so, one moves away from the classic Merton approach, and it becomes an entirely new model. A distribution with wider tails would probably be preferable as too much of the probability mass is centred around the mean value in the Gaussian distribution, making the probability of an extreme event highly improbable.

The Black-Cox model however, with the more world-like feature of allowing for default at any point, as well as enabling a dynamic default threshold, proved to be superior to the Merton model when modelling the CDS-spreads. Especially in the case when choosing a maturity of ten years and compound factor of the debt as $k = 0.01$, the results were satisfactory. The results were not perfect for all companies, but what was important was the fact that the model spreads here actually managed to follow the trends and movements of the market-spreads. However, the residuals did not prove to be perfectly stationary other than for a couple of companies during certain time periods. By correcting the residuals for current level of asset volatility, the residuals were slightly improved in terms of stationarity, however not perfect.

The volatility-adjusted residuals did not prove to be white-noise however, as the auto-correlation plots showed clear positive correlation for many lags before eventually decaying and approaching zero.

Implementation wise, the Black-Cox model enables a couple of features that were not capitalised on in this report. Specifically, one has the option of choosing the default-threshold in any way possible. In this particular implementation, the default threshold was said to increase with a continuous compound rate of k . What one could do is in a very delicate way assess how the debt of a firm is distributed, including dividends and coupons of bonds etc, over time. The debt structure would probably not be perfectly smooth, but rather have more of a discrete look, moving up and down. One could then with this debt-structure calculate exactly the default probability during a specific year and use this information to value the CDS-contract in a more exact way, instead of like in this implementation calculate only the probability of default by maturity, and then assume constant conditional default probabilities year-by-year. It is not evident that this would yield superior results, but would be an interesting study. However, the task of trying to find historic debt structures would be tedious. Moreover, the relations used in this report to link the assets and the equity would no longer hold true. Hence, how one would determine the assets V_t at each point in time is not straight forward.

In order to get the model-spreads to the right order of magnitude, the model-spreads are corrected for the average residual error over the entire period. This can impose a problem if one decides to use, for instance, the Black-Cox implementation for practical purposes, can one be sure that the average error level is the same all the time? An interesting study to test this hypothesis would be to take part of the data and calculate the average error, then use that average error to correct future market spreads to see if the resulting modified model spreads seem to map well with the market spreads. If it appears that the average error is not constant, one can accommodate this issue by perhaps correcting the model-spreads using, for instance, one year trailing average error.

As a comparison to the structural models, a very simple regression model was done on the market-spreads, simply taking the equity volatility as input. The first year of data for each company was used to build the regression. The results from this regression were quite poor, but overall could be considered better than the Merton model and the Black-Cox model with a one year maturity. Judging from the results, the regression model also seemed to be quite unreliable in the sense that in some cases, the model works fine, while in other cases, nothing seems to work, which can be seen in the cases of Avon Products and Peabody Energy. Although the regression manages to follow the market-spreads fairly well on a couple of occasions, one runs the risk of the model not working at all, which is not a feature one would want.

Obviously, the reason for assessing whether a regression produces any results close to that of the structural models is because implementing a regression is much, much, simpler. Consequently, if one manages to achieve comparable results between a structural model and a regressive model, what is the point in making things more advanced than necessary?

A possible expansion of the regression model could be to incorporate variables generated from one of the structural models, for instance the Merton model seeing as this is the easiest of the structural models to implement. Although the Merton model proved to not perform great, certain metrics generated within the model can still be of interest, for instance the distance-to-default. Remember that the KMV model uses this metric, so apparently that metric contains some information that may be of interest for a regression.

Something worth discussing that is of relevance for all the models is how the data used is chosen. Specifically two points are worth pointing out. The first point is the volatility. As argued, the implied volatility of a 90% OTM put option with a one year maturity is used, as these options seemed to be the most liquid ones when doing a quick analysis. However, one can question the use of implied volatility as the demand for certain options vary over time, and as the demand increases drastically of a certain option, one can expect the price of that option to increase, and thus also the implied volatility, without any major movements in the underlying stock needing to have occurred. One possibility to overcome this issue could be to perhaps use a volatility measure that is simply the average of several implied volatilities for different maturities and strike levels.

The second issue is the sampling rate of the data. This report has used daily close data for all variables. But perhaps, using data sampled, say, once per week would prove better. The advantage of using a lower sampling frequency would be that a longer data range could be used without the computations taking too long time. Plus, one would incorporate periods which perhaps showed different trends and features that would challenge the models further. It would have been interesting to use the last fifteen years to model on, as one then would have included the 2007-2008 financial crisis, which created extreme distress for many companies in a very short period of time. For instance, the 5-year CDS of Advanced Micro Devices traded at above 5000 bps during a period in 2008, which is an extremely high level.

Something that would make the results in the report more significant would be to include more companies to analyse. In this report, six names were used, but

all of the included companies operated in very diverging sectors, thus no further companies were included. However, using more companies would of course only be positive in terms of the significance of the results.

Moreover, one can perhaps question what companies were chosen. As it turns out, all of the companies used in this report are included in the 24th series of Markit's High Yield CDS-Index. One would perhaps want to use also Investment Grade companies as these firms have a higher credit rating, thus their respective 5-year CDS will generally trade at lower levels.

Generally, one can perhaps expect that Investment Grade companies are more "stable", making the prices of their respective bonds and CDS:s less volatile compared with High Yield companies. Since this study was done on companies, that for the moment, are High Yield, perhaps testing the best performing model on an Investment Grade name would not yield any satisfactory results, due to the smaller volatility, which may not be a feature that the Black-Cox model can capture effectively. Perhaps the Merton model would prove better in this case.

Moreover, one can also argue that other external variables affect the CDS spreads for certain Investment Grade companies more than relative increases in equity volatility or decreases in market capitalisation. For instance, Investment Grade energy companies may not see their respective CDS-spreads vary too much unless something drastic happens to the price of oil or gas. If this is in fact the case, that macro-economic variables more affect the CDS-spreads than anything else, one would potentially need sector-specific models to successfully model the CDS-spreads.

Another point that would be interesting to test, and potentially incorporate in the models, would be the impact on the CDS-spread after changes in credit rating of the respective companies. A lowered credit rating could potentially yield an increase in the CDS-spreads, and vice-versa when the credit rating is improved. However, changes in credit rating does not happen too often, and many times, the market is already aware of underlying reasons for a change in credit rating before it happens, making the impact minimal. In some cases however, the impact can be severe if the change in credit rating comes as a surprise.

7 Conclusion

The objective of this thesis was to try to replicate CDS-spreads by modelling companies assets through structural models, then calculating key metrics such as probability of default to value the corresponding 5-year CDS-contract. In a sense, this was possible, depending on how much errors one allows the model-implied CDS-spreads to have versus the market-spreads.

The Merton model is an overall disappointment and delivers very poor results when used to model the assets for all maturities tested. The reason for this may be because the normal distribution that maps the distance-to-default metric to a default probability is a poor choice.

Better results are achieved with the Black-Cox model, especially for the maturity of ten years. The Black-Cox model with the longer maturity manages to actually follow the market-spreads quite well, although the residuals did not prove to be perfectly stationary. The fact that the model enables default at any time, which is a more world-like feature, may be an underlying reason for the improved performance over the Merton model.

The Black-Cox model also performed better than a naïve regression model which only used equity volatility as input, which means that there is an incentive to use more advanced models to calculate the CDS-spreads, as a simple regression did not seem to perform better.

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9 Appendix

9.1 Appendix A - Ito Calculation

This section provides the calculation of the differentiation of a down-and-out European call option, used to obtain one further relation between equity and assets.

Define the following variables

$$\begin{aligned} E_t &= e^{kT}(C_t - C_t^{DI}) \\ f_1 &= C_t = X_t e^{-k(T-t)} N(d_1) - K_0 e^{-r(T-t)} N(d_2) \\ f_2 &= C_t^{DI} = X_t e^{-k(T-t)} \left(\frac{K_0}{X_t}\right)^{2e_1} N(e_2) - \\ &K_0 e^{-r(T-t)} \left(\frac{K_0}{X_t}\right)^{2e_1-2} N(e_2 - \sigma_V \sqrt{T-t}). \end{aligned}$$

Where $X_t = e^{-kt} V_t$. Inserting this yields

$$\begin{aligned} f_1 &= C_t = V_t e^{-kT} N(d_1) - K_0 e^{-r(T-t)} N(d_2) \\ f_2 &= C_t^{DI} = V_t e^{-kT} \left(\frac{K_0 e^{kt}}{V_t}\right)^{2e_1} N(e_2) - \\ &K_0 e^{-r(T-t)} \left(\frac{K_0 e^{kt}}{V_t}\right)^{2e_1-2} N(e_2 - \sigma_V \sqrt{T-t}). \end{aligned}$$

Then, set $f(x, t) = E_t$, substituting V_t with x , hence $f(x, t) = e^{kT}(f_1(x, t) - f_2(x, t))$

Using this, one can calculate $\frac{\partial f(x, t)}{\partial x} = e^{kT} \left(\frac{\partial f_1(x, t)}{\partial x} - \frac{\partial f_2(x, t)}{\partial x} \right)$. The calculation can thus be split into two calculations, one for f_1 and one for f_2 . This results in

$$\begin{aligned} \frac{\partial f_1(V_t, t)}{\partial x} &= e^{-kT} N(d_1) \\ \frac{\partial f_2(V_t, t)}{\partial x} &= e^{-kT} \left(\frac{K_0 e^{kt}}{V_t}\right)^{2e_1} N(e_2) + V_t e^{-kT} (K_0 e^{kt})^{2e_1} N(e_2) (-2e_1) \left(\frac{1}{V_t}\right)^{2e_1+1} - \\ &K_0 e^{-r(T-t)} (K_0 e^{kt})^{2e_1-2} N(e_2 - \sigma_V \sqrt{T-t}) (-2e_1 + 2) \left(\frac{1}{V_t}\right)^{2e_1-1} = \\ &e^{-kT} N(e_2) \left(\frac{K_0 e^{kt}}{V_t}\right)^{2e_1} (1 - 2e_1) - \\ &e^{-r(T-t)} N(e_2 - \sigma_V \sqrt{T-t}) \left(\frac{K_0 e^{kt}}{V_t}\right)^{2e_1-2} \left(\frac{K_0}{V_t}\right) (2 - 2e_1) \end{aligned}$$

Combining these two results, one then obtains the following relation

$$\begin{aligned} \frac{\partial E_t}{\partial V_t} &= \frac{\partial f(V_t, t)}{\partial x} = e^{kT} \left(\frac{\partial f_1(V_t, t)}{\partial x} - \frac{\partial f_2(V_t, t)}{\partial x} \right) = \\ &N(d_1) + e^{kT} e^{-r(T-t)} N(e_2 - \sigma_V \sqrt{T-t}) \left(\frac{K_0 e^{kt}}{V_t} \right)^{2e_1-2} \left(\frac{K_0}{V_t} \right) (2 - 2e_1) - \\ &N(e_2) \left(\frac{K_0 e^{kt}}{V_t} \right)^{2e_1} (1 - 2e_1) = \\ &N(d_1) + \left(\frac{K_0 e^{kt}}{V_t} \right)^{2e_1} \left[e^{-(r-k)(T-t)} N(e_2 - \sigma_V \sqrt{T-t}) \left(\frac{V_t}{K_0 e^{kt}} \right) (2 - 2e_1) - \right. \\ &\quad \left. N(e_2)(1 - 2e_1) \right] \end{aligned}$$

9.2 Appendix B - Black-Cox Results

9.2.1 T = 1 Year, k = 0

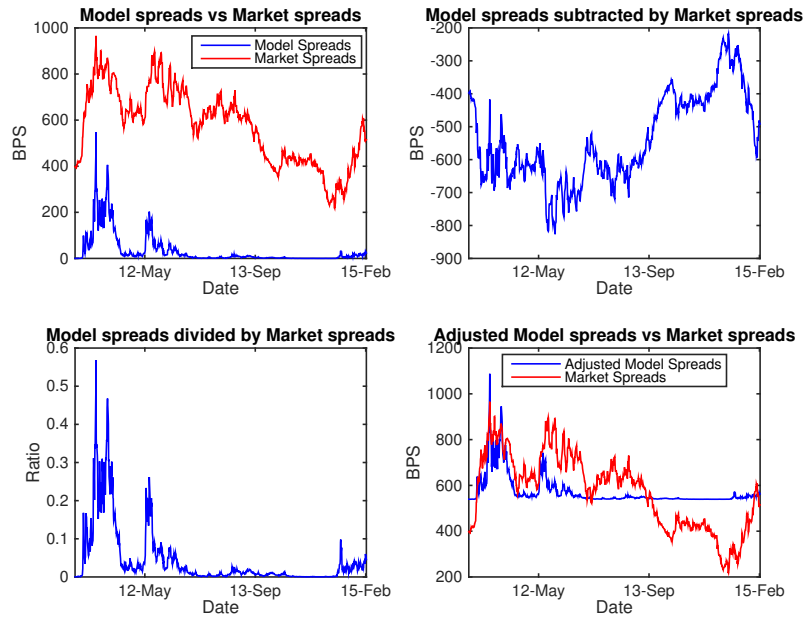


Figure 77: Results and CDS-spreads for US Steel

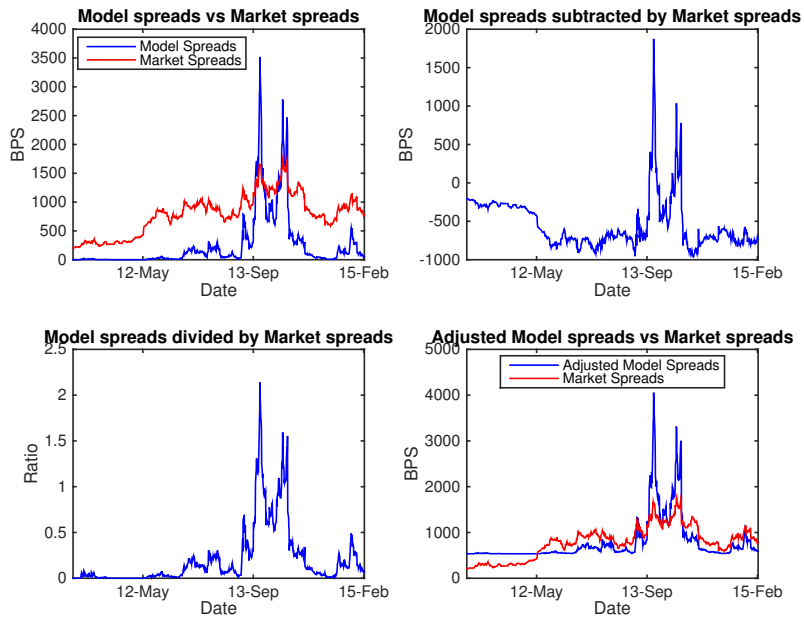


Figure 78: Results and CDS-spreads for J.C. Penney

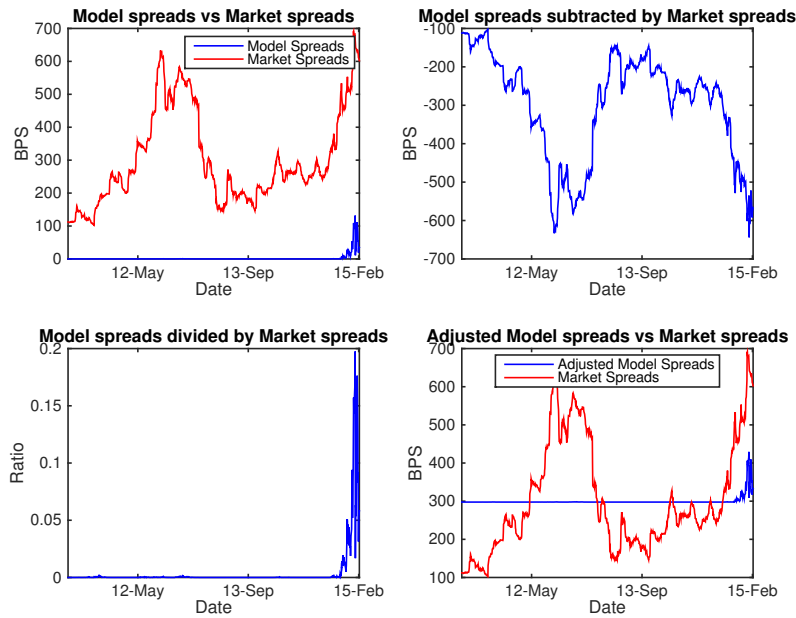


Figure 79: Results and CDS-spreads for Avon Products

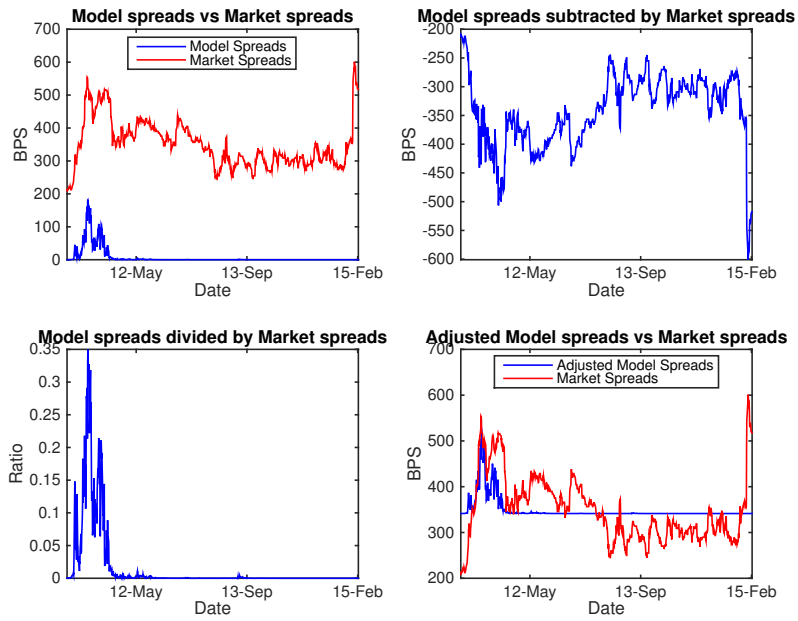


Figure 80: Results and CDS-spreads for Bombardier

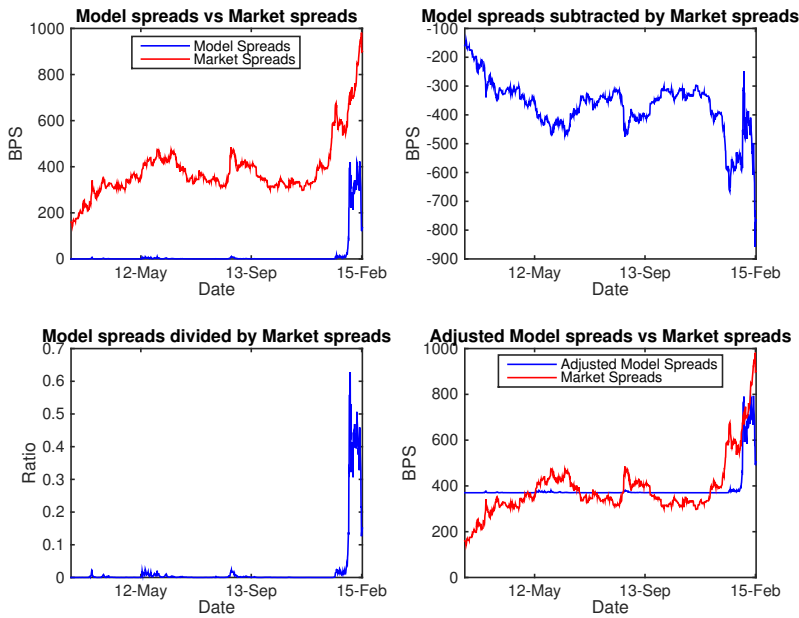


Figure 81: Results and CDS-spreads for Peabody Energy

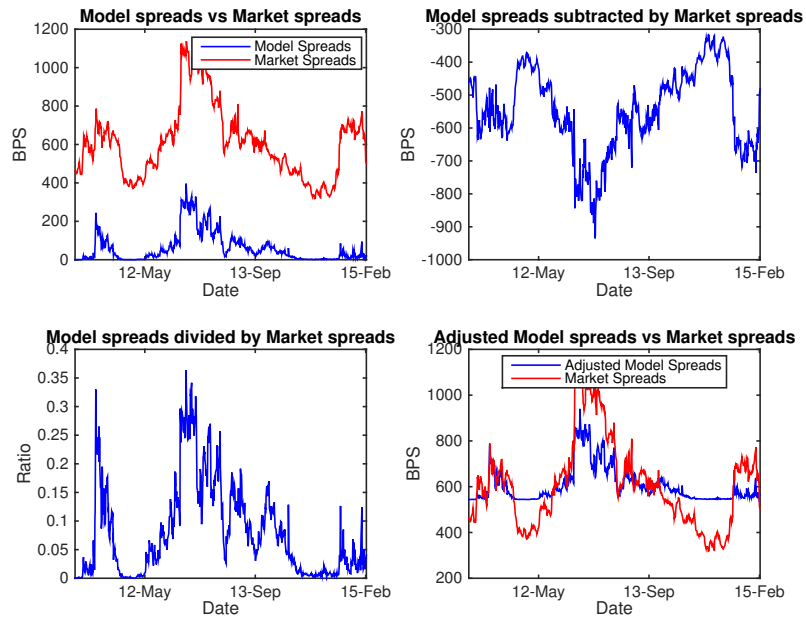


Figure 82: Results and CDS-spreads for Advanced Micro Devices

9.2.2 $T = 1 \text{ Year}, k = 0.01$

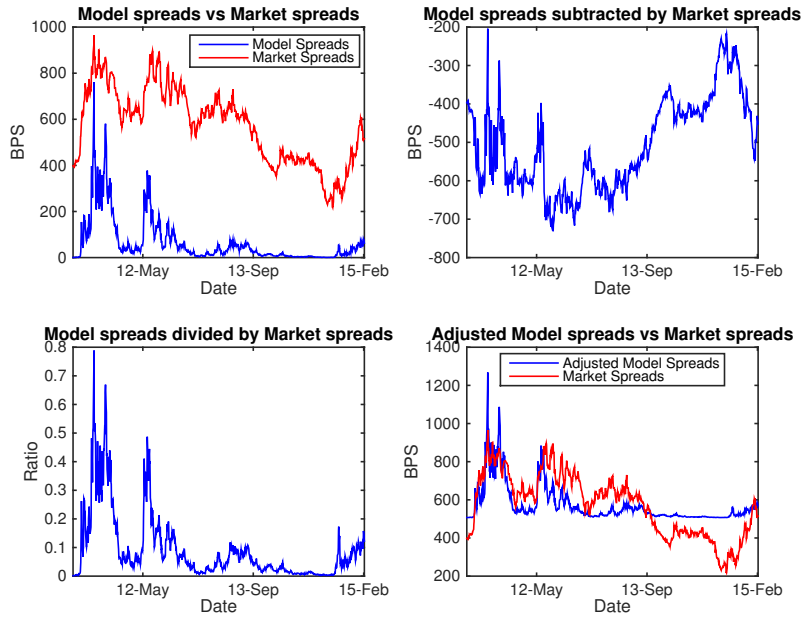


Figure 83: Results and CDS-spreads for US Steel

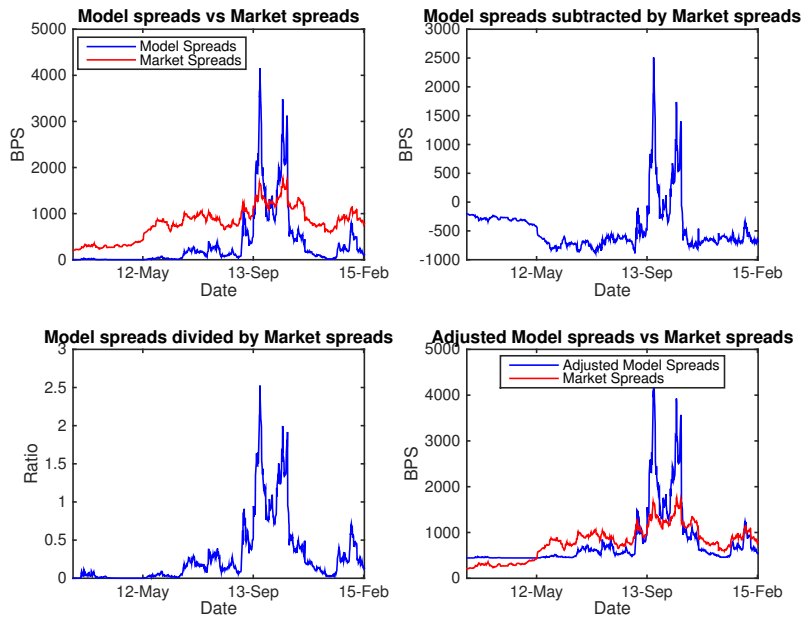


Figure 84: Results and CDS-spreads for J.C. Penney

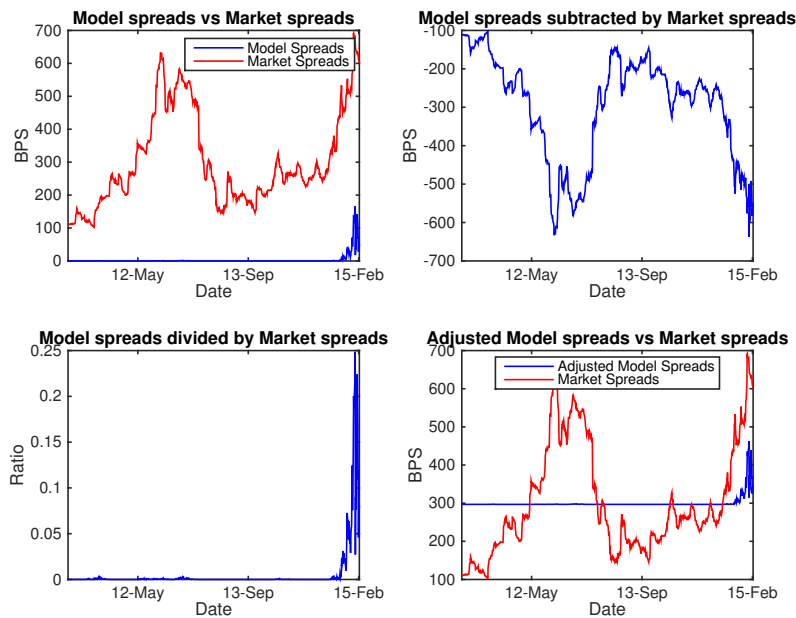


Figure 85: Results and CDS-spreads for Avon Products

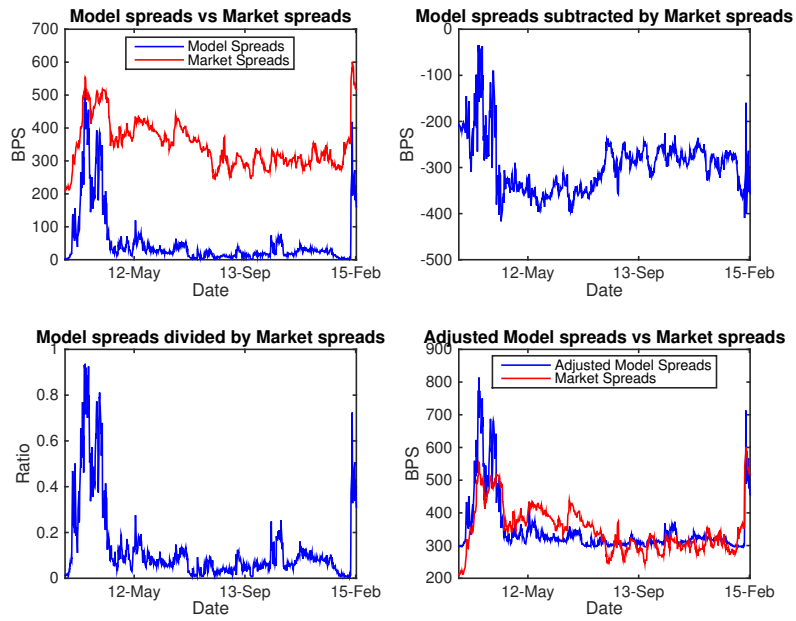


Figure 86: Results and CDS-spreads for Bombardier

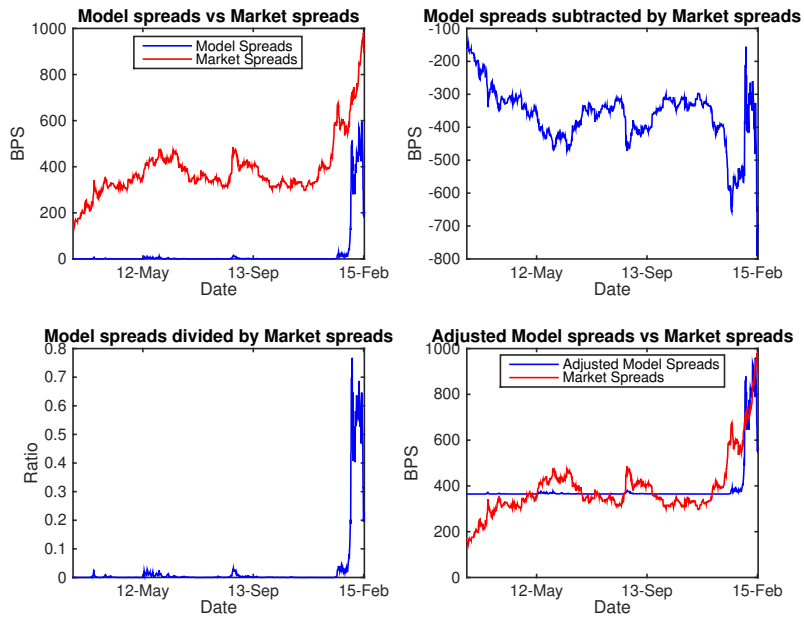


Figure 87: Results and CDS-spreads for Peabody Energy

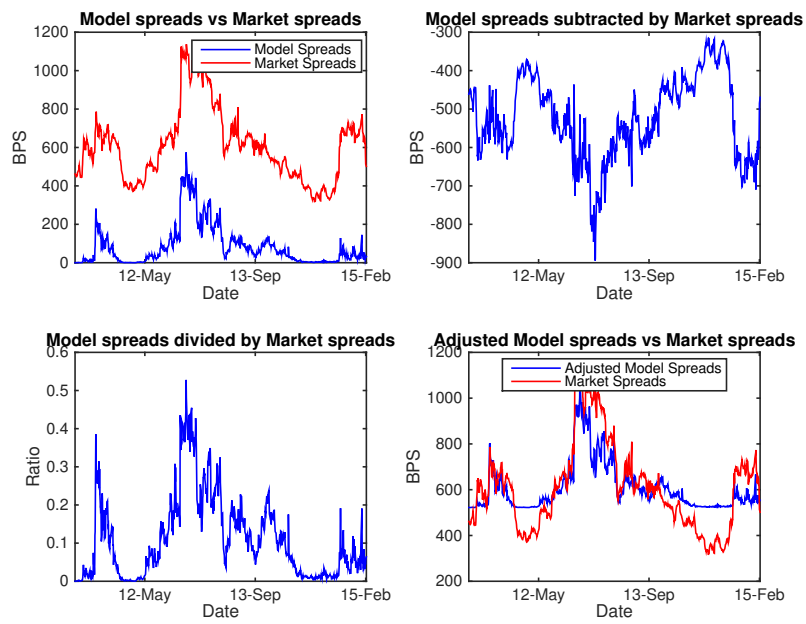


Figure 88: Results and CDS-spreads for Advanced Micro Devices

9.2.3 $T = 1 \text{ Year}$, $k = 0.05$

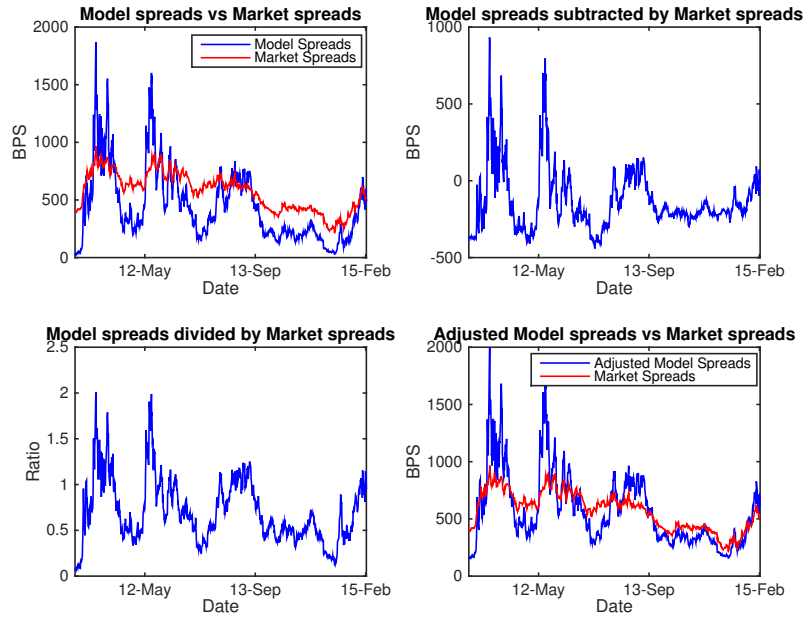


Figure 89: Results and CDS-spreads for US Steel

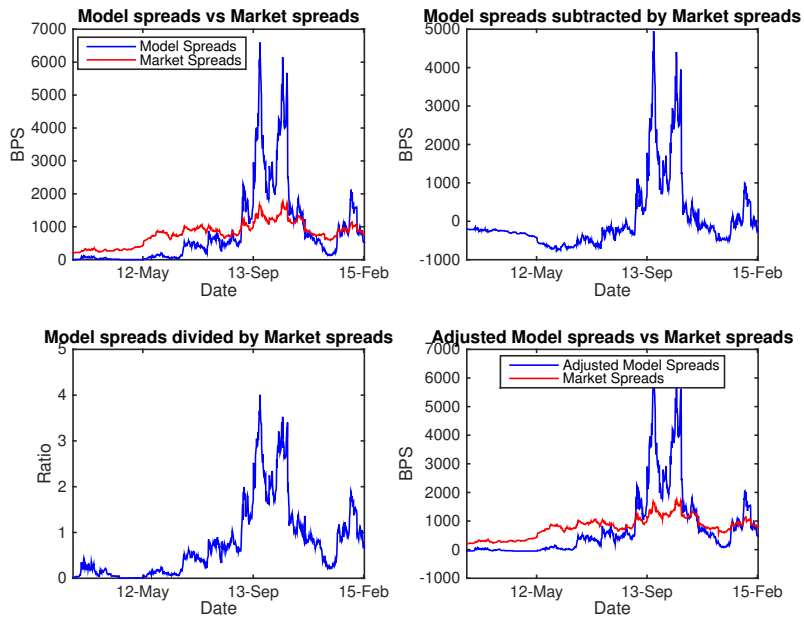


Figure 90: Results and CDS-spreads for J.C. Penney

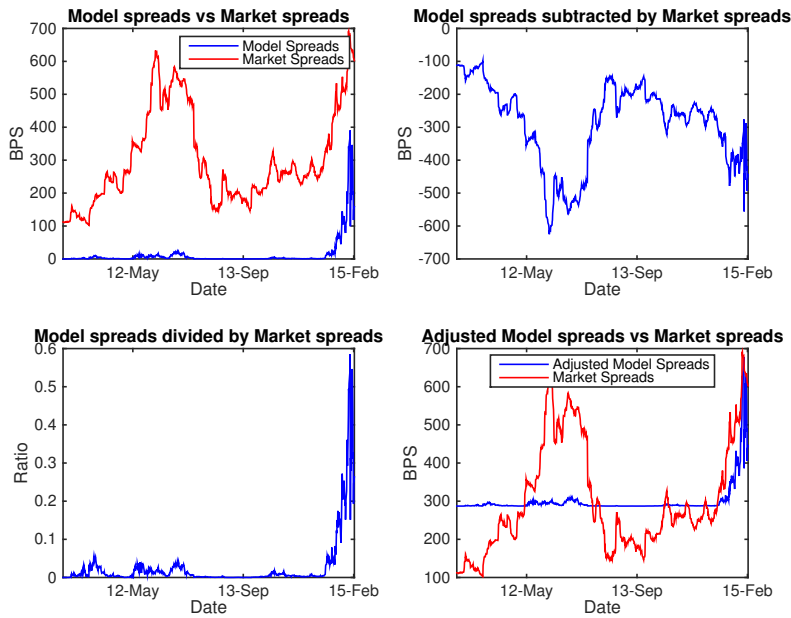


Figure 91: Results and CDS-spreads for Avon Products

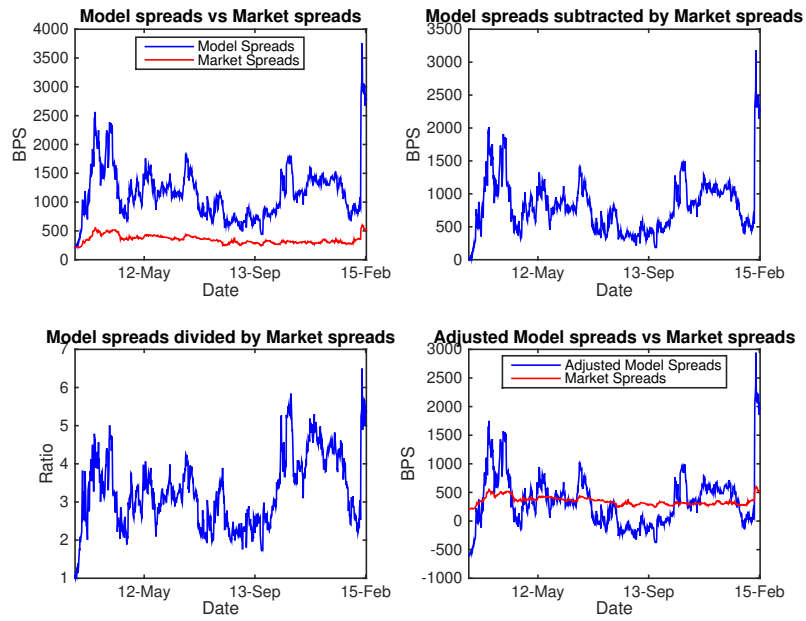


Figure 92: Results and CDS-spreads for Bombardier

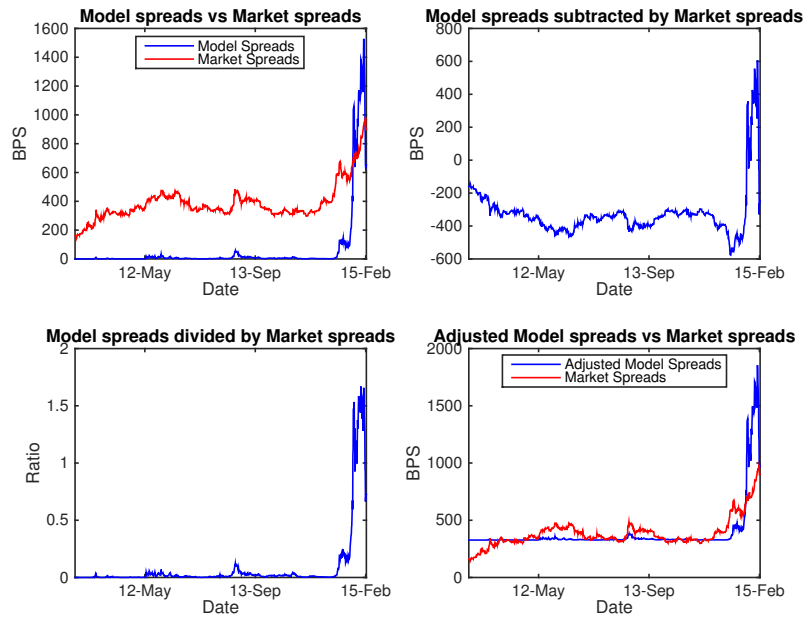


Figure 93: Results and CDS-spreads for Peabody Energy

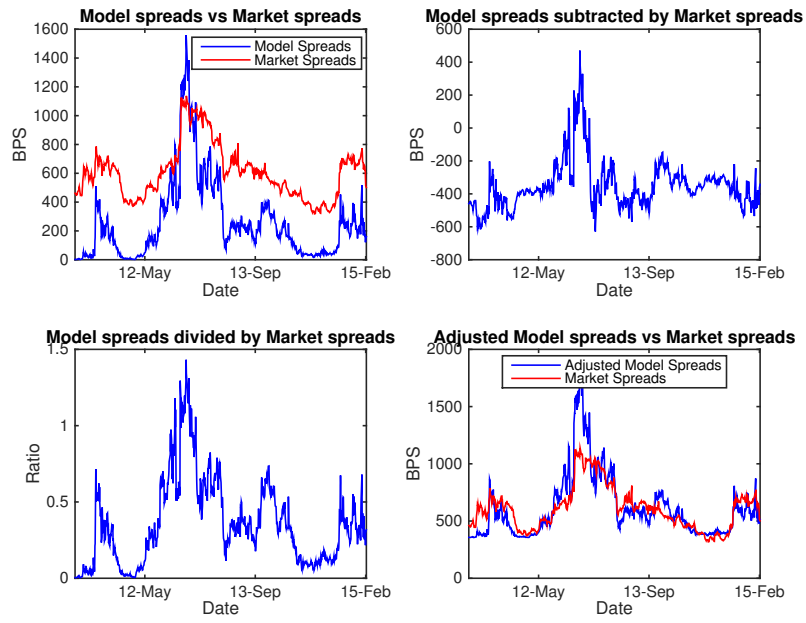


Figure 94: Results and CDS-spreads for Advanced Micro Devices

9.2.4 $T = 10$ Years, $k = 0$

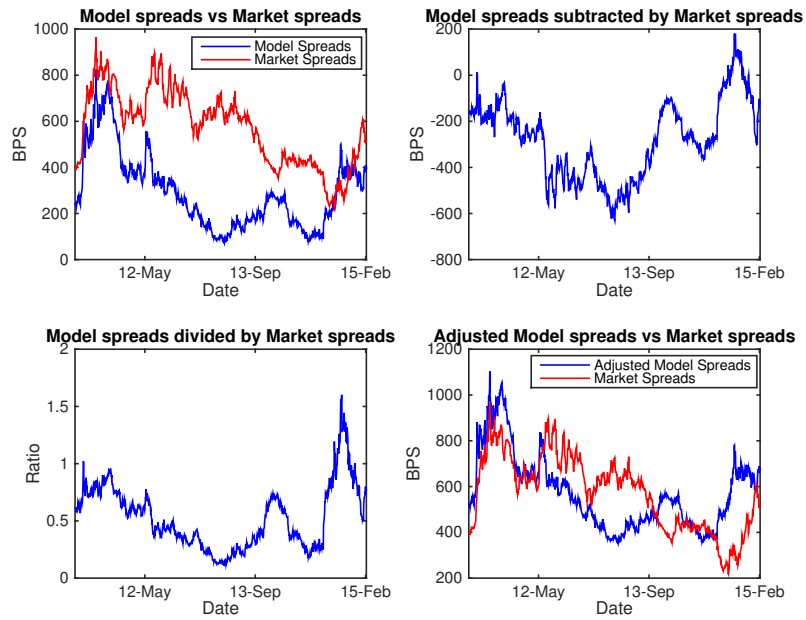


Figure 95: Results and CDS-spreads for US Steel

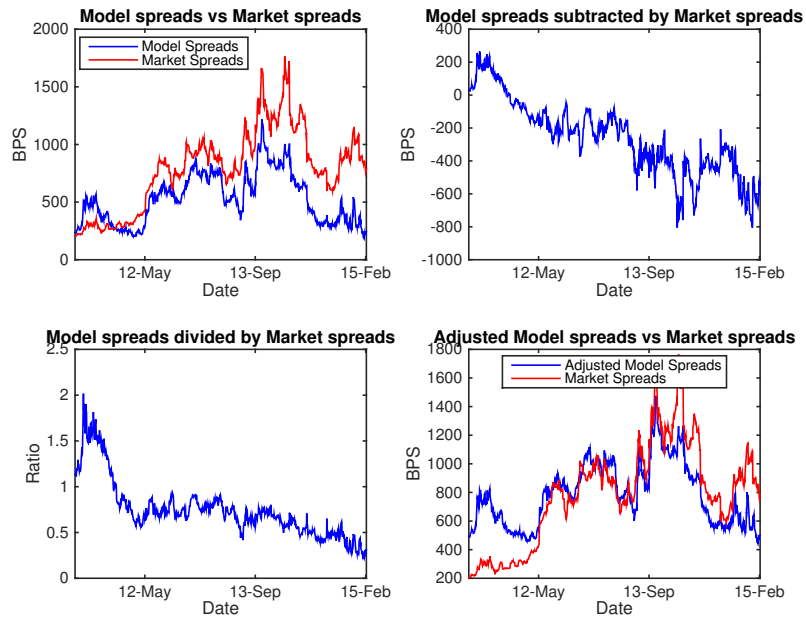


Figure 96: Results and CDS-spreads for J.C. Penney

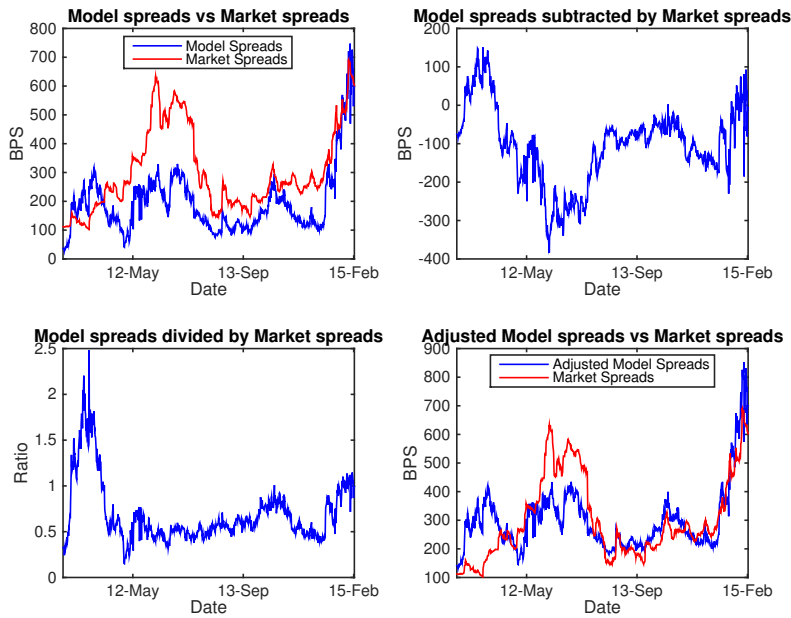


Figure 97: Results and CDS-spreads for Avon Products

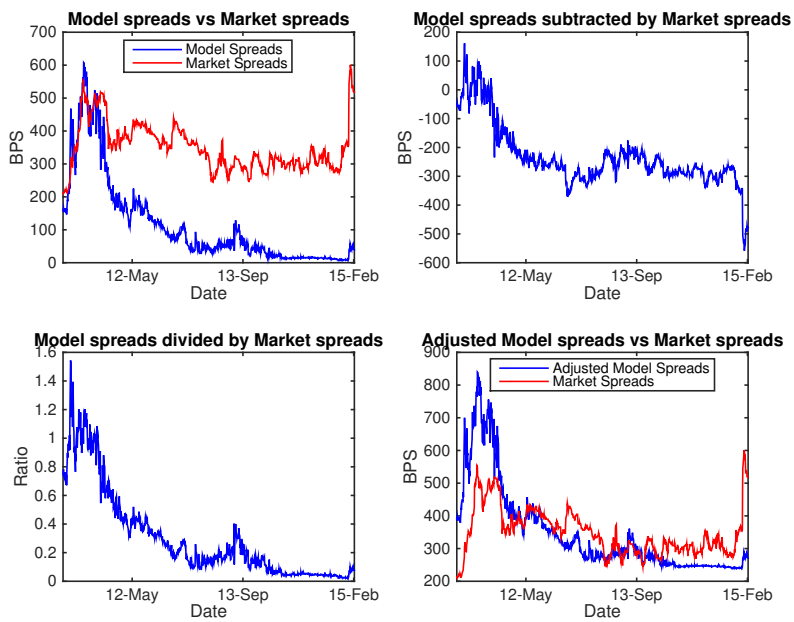


Figure 98: Results and CDS-spreads for Bombardier

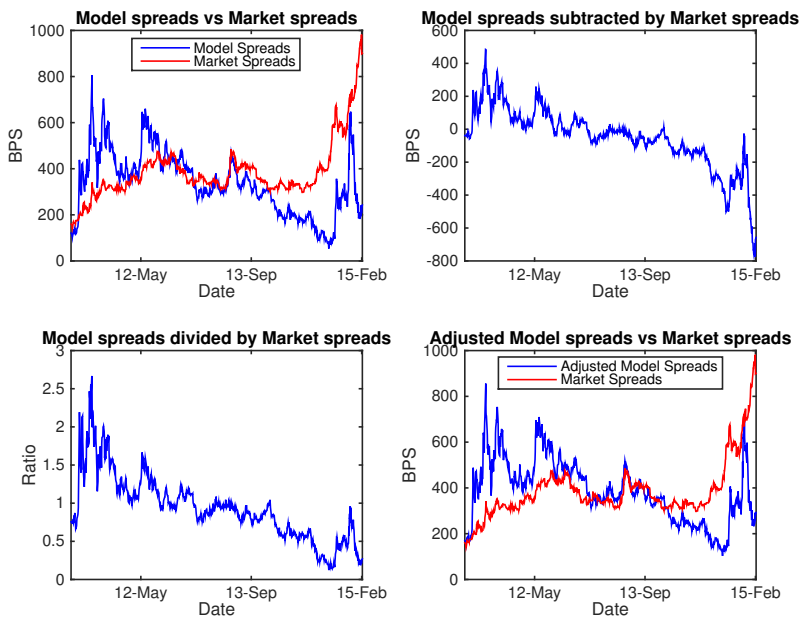


Figure 99: Results and CDS-spreads for Peabody Energy

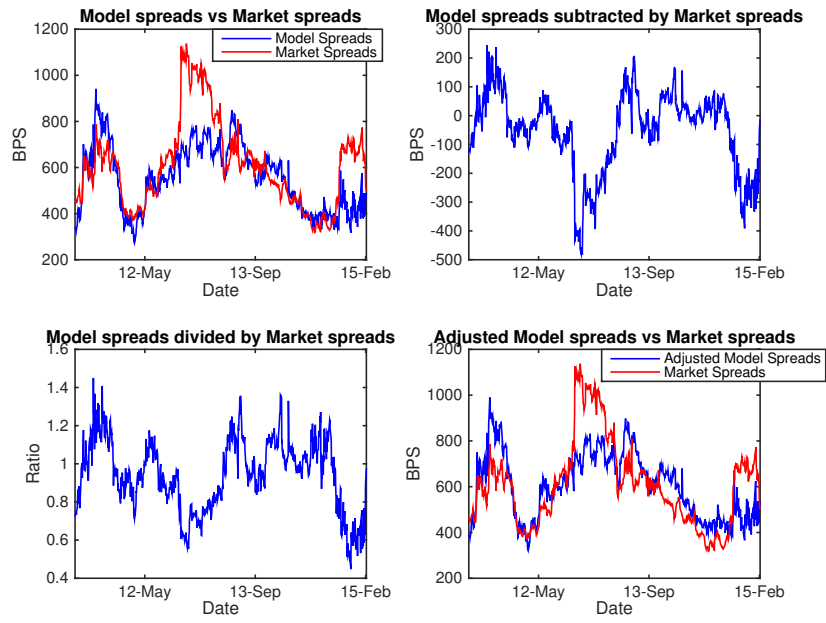


Figure 100: Results and CDS-spreads for Advanced Micro Devices

9.2.5 $T = 10$ Years, $k = 0.05$

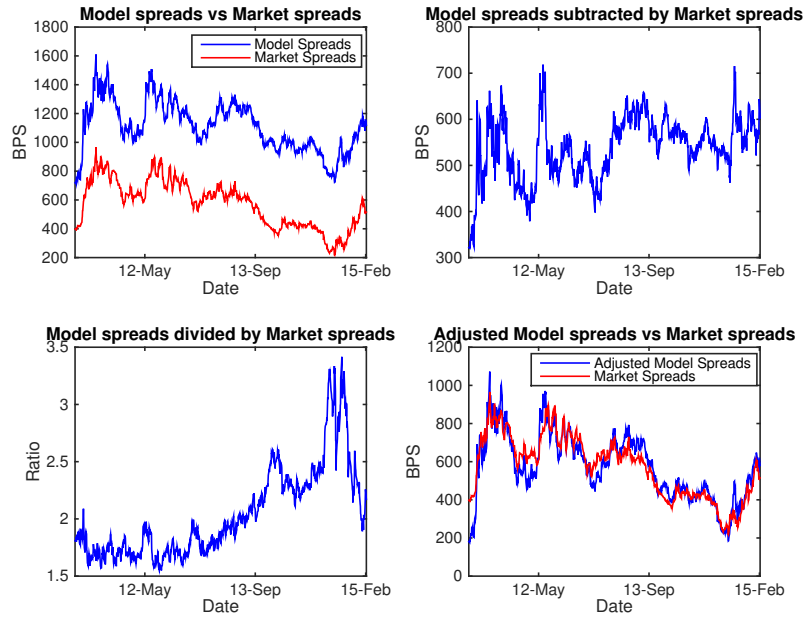


Figure 101: Results and CDS-spreads for US Steel

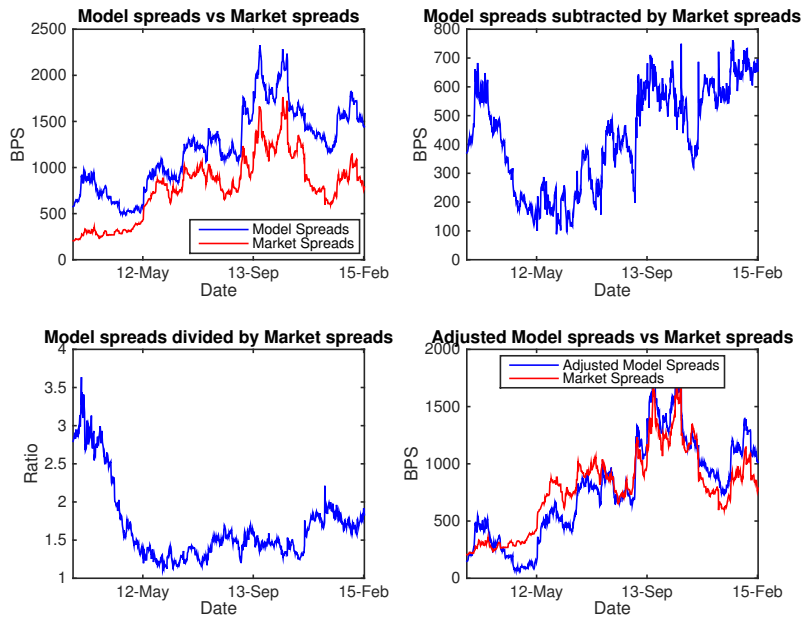


Figure 102: Results and CDS-spreads for J.C. Penney

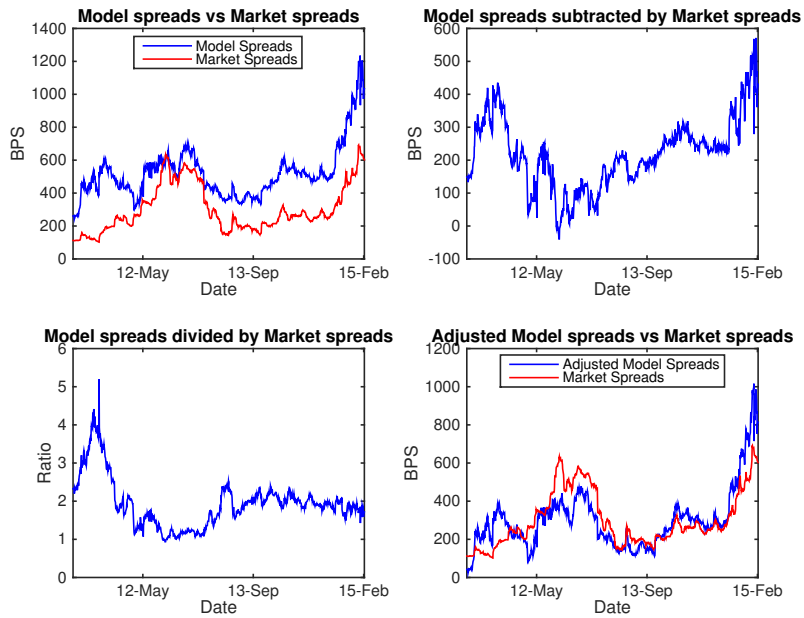


Figure 103: Results and CDS-spreads for Avon Products

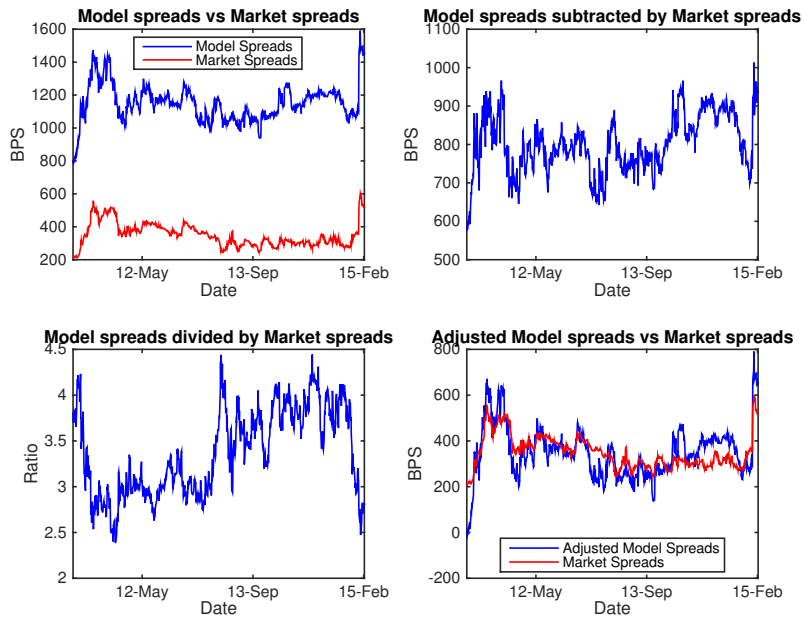


Figure 104: Results and CDS-spreads for Bombardier

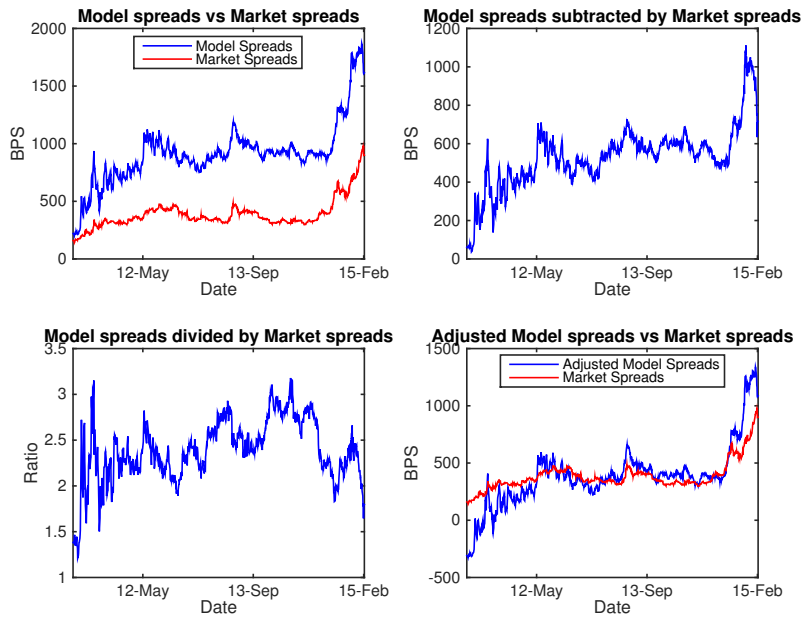


Figure 105: Results and CDS-spreads for Peabody Energy

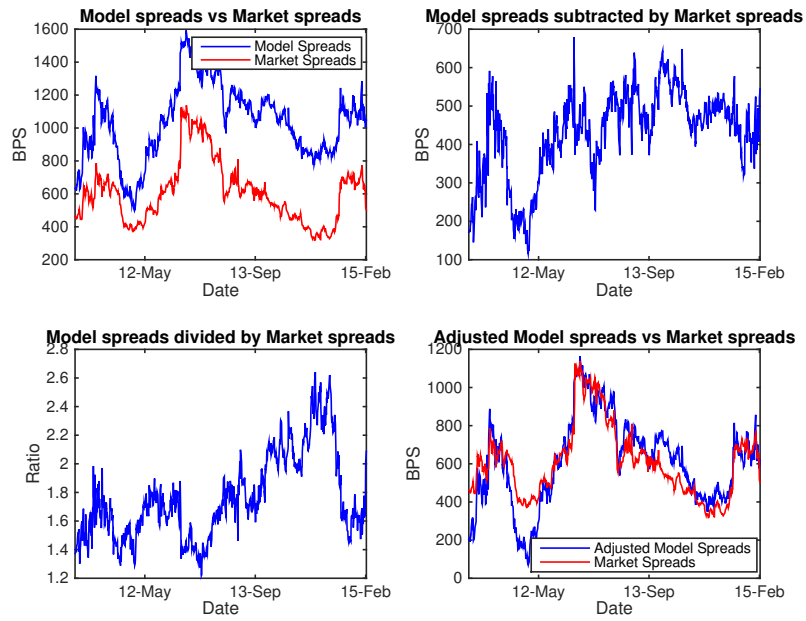


Figure 106: Results and CDS-spreads for Advanced Micro Devices

Valuing Credit Default Swaps with a Structural Approach

Per Möller, Lund University, Faculty of Engineering

During the 21st century, the market for credit related financial instruments has grown to one of the worlds largest. The thesis addresses whether it is possible to successfully determine the value of some of these through mathematical formulas and equations.

The financial instrument that is investigated in this thesis is the so called credit default swap, often called in short a CDS. The contract is also what one would call a derivative, as its value is derived from an underlying financial security, namely a bond. A bond is basically an IOU which promises to pay back the borrowed amount in the future, plus some interest. What the CDS does is that it acts as insurance against that a company, that has issued bonds, files for bankruptcy, and thus cannot pay back the money it borrowed through the bonds. The idea is that if you own a bond, and also decide to buy insurance on the bond through a CDS, you will be guaranteed to get the money back on the bond, regardless of if the company defaults on its debt or not. Of course, buying insurance on the bond through a CDS costs money. That cost is quoted in terms of a so called spread, which is the number of basis points (bps), where 100 basis points is one percentage, it would cost to buy protection per year. The amount actually paid each year is the spread multiplied by the total insured amount, which usually ranges from \$1MM to \$10MM. The CDS is only triggered if the company becomes bankrupt, or anything related occurs to the company. To determine what fair price (or fair spread) of a CDS is, one must have some idea of how likely it is that the company defaults.

In this thesis, structural models have been used to

determine the probability of default. Structural models is a family of models which assigns certain dynamics to the assets of the company and imposes certain rules by which the company is said to default. Through the models and the rules, one can calculate the probability of default. The default probability is then plugged into the valuation formula of the CDS, which returns a fair value of the spread, according to the models. Mainly two different models in this thesis have been analysed; the Merton model and the Black-Cox model. The Merton model is very simple and states that the company defaults if the assets of the company is below some default barrier after a certain amount of time. The Black-Cox model however also enables the possibility of default at any time, as soon as the assets of the company dips below the default barrier, the company defaults. The Black-Cox model also makes it possible to use default barriers which are time dependent.

At each day and for each model, the current probability of default is calculated and used to value the CDS. The resulting CDS-spreads are then compared with the spreads that are seen in the market to see whether or not the model manages to replicate the market spreads.

The Black-Cox model proved to be the best model of the two. The spreads calculated from the model appeared to follow the spreads given by the market quite well, both in terms of major trends, but also minor trends. Since the results were quite satisfying, and the model used actually was fairly simple, one may want to further analyse these types of models and perhaps add more features that make the models more realistic, in an attempt to even better model the market spreads.

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Centre for Mathematical Sciences

Lund University

Box 118, SE-221 00 Lund, Sweden

<http://www.maths.lth.se/>