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ASSET PRICING WITH HIGHER COMOMENTS

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Abstract

Empirical and theoretical research has for some time argued that investors also expect rewards for bearing risk related to higher moments. This thesis examines if inclusion of coskewness and cokurtosis helps to explain the variation in asset returns. These factors are added to models that also account for market risk , Fama-French factors and momentum. We use methodology of realized moments to estmate our proxies for coskewness and cokurtosis. A simulation shows that the estimator work well under certain return characterstics. We find that cokurtosis often is significant and adds explaining power, but the evidence is not always consistent. Coskewness does not prove to be an important factor in our model.

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Contents

Chapter 1

Introduction

The nature of asset returns is a crucial area in the field of finance, both from the academic and professional perspective. The significance becomes clear when one examines the vast research that has been dedicated to this particular field. A major obstacle to finding an acceptable model is the elusive link between theory and practice. The relationship between risk and return is great example of this. Intuitively, higher risk should result in greater returns, but this is at odds with empirical evidence. Researchers are instead turning towards a behavioral explanation for this risk-return paradox. (e.g. Bowman, 1980; Fiegenbaum, 1990; Shefrin, 2001).

A major breakthrough, and impetus for further research, came with the Capital Asset Pricing Model (CAPM). Due to strong underlying assumptions, this model boils down to a simple equation where the only relevant risk factor is the co-variation between asset return and the returns of the market portfolio. Diversifiable risk elements are not relevant (Markowitz, 1959; Sharpe, 1964; Lintner, 1965). Much research has been dedicated on extending and improving the CAPM. For example, size and value add explaining power in addition to the market factor (Banz, 1981; Fama and French, 1992). Another implication of the CAPM is that the stochastic discount factor is a linear function of market returns, but research suggests that square returns are also significant (Dittmar, 2002). This implies that mean and variance are not enough to describe investor preference. Omission of higher moments is possibly one of the reasons behind the failure of the CAPM (Rossi and Timmermann, 2010; Xu, 2010). Therefore, including the third and fourth moments is an interesting direction towards better models (e.g. Kraus and Litzenberger 1976; Fang and Lai, 1997; Harvey and Siddique, 2000). There is no lack of examples where researcher test higher moments such as skewness and kurtosis by including them alongside market beta; this results in higher moment CAPM-models. However, less attempts have been made to incorporate these moments in other pricing models multifactor models, such as the Fama-French (1993) and Carhart (1997). Consequently, the aim of the thesis is to investigate if higher moments can provide additional explaining power to these multifactor models. The methodology is inspired by Xu (2010), where realized higher co-moments are estimated using daily data. Realized moments are proven to work well in extracting important information in high frequency data, and have the advantage of being model-free (e.g. Andersen and Bollerslev, 1998; Andersen et al., 2003). The empirical part is however more extensive compared to Xu (2010) since several test portfolios are used. Also, the data is more recent which should be interesting because of the financial turbulence since the turn of the millennium. The empirical results indicate that higher moments are significant factors, particularly co-kurtosis.

Chapter 2

Theoretical background

2.1 Distribution of returns

The normal distribution is central and convenient to work with in econometrics. Unfortunately, research shows that it is not always realistic to assume normality. Some studies show that the distribution of stock returns are leptokurtic, and are better described by so called stable Paretian distribution. Returns also exhibit more outliers than the normal distribution would predict (Mandelbrot, 1963; Fama, 1965). Other studies also argue that the Student t-distribution is more suitable than the Paretian distribution (Blattberg and Gonedes, 1974). Hsu (1982) apply Bayesian methods to account for long tails that depart from the normal distribution. The authors state the normal distribution is inadequate because it does not account for shifts in risks (i.e. assumes fixed variance).

Non-normal (or non-symmetric) return distributions invalidate the CAPM framework because distribution is not only described by mean and variance. Rubinstein (1973) suggests that investors will care about all moments of return once the assumption of normal distribution is relaxed (in addition to investors not having quadratic utility function). Leland (1999) argues that in a market portfolio with IID returns, the mean-variance assumption will be inefficient and investors will have preference to higher moments. Zhu (1993) strongly rejects the hypothesis of multivariate normality using data of the security prices value weighted index from the period between 1926 and 1986. However he cannot find enough evidence to reject univariate normality. Richardson and Smith (2001) find significant evidence of non-normality, and their description of the empirical results is that the multivariate normality assumption is not justified. Accordingly, the higher moments (skewness and kurtosis) must be priced.

2.2 Capital Asset Pricing Model and further extensions

Markowitz (1952) presents a mathematical method for choosing optimal portfolios. Depending on preference, an investor will choose a portfolio with minimal variance for a given mean return (the efficient set). This mean-variance framework is the foundation for the CAPM (Sharpe, 1964; Lintner, 1965). Risk is measured by the market beta which represents the variation of the individual asset that cannot be diversified away. Black (1972) introduces the zero-beta CAPM, and relaxes the assumption that investors can borrow and lend at risk-free rate. The risk-free rate is replaced by a portfolio whose returns are uncorrelated with those of the market portfolio. Several articles compare CAPM with zero-beta CAPM, and find that zero-beta CAPM performs better (Sharpe and Cooper, 1972; Fama and MacBeth, 1974; Köseoğlu et al., 2013). Hansen and Richard (1987) assume that investments are optimized over multi-periods, which results into conditional CAPM (and beta).

Evidence has been presented in support of conditional CAPM, and of a time-varying beta (Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001; Ang and Chen, 2007). However, if the conditional model is misspecified, the unconditional CAPM performs better (Ghysels, 1998). Shanken (1990) uses macroeconomic variables to model market beta as a function of interest rate and volatility. Lettau and Ludvigson (2001) support the conditional CAPM by using the consumption to wealth ratio as variable, where their model can explain Fama-French portfolios well. Using industry and size portfolios, Bali (2008) finds significant market beta in a bivariate GARCH model. Ang and Chen (2007) test the conditional CAPM using a long sample of data from 1926 to 2001 and do not reject the model.

The consumption CAPM (CCAPM) is another derivative of the CAPM research which uses consumption data to capture risk premium (consumption beta) (Lucas, 1978; Breeden, 1979). Breeden, Gibbons and Litzenberger (1989) test the CCAPM and compare it to the traditional CAPM. The market price of risk they test is significantly positive, and they find comparable performance of CAPM and a model that uses a portfolio of consumption growth as a factor. Wheatley (1988) uses the CCAPM in testing the international equity market integration but reject the model. Cochrane (1996) finds that CAPM outperforms the consumption CAPM in pricing size portfolios. In Hansen and Singleton (1982,1983) reject the CCAPM that is tested on US data. In their model, the investor has time separable power utility of consumption.

2.3 Multifactor models

The CAPM relies on the assumption that market beta is the only necessary factor for explaining average returns. The Arbitrage Pricing Theory (APT) by Ross (1976) allows for multiple risk factors and does not require identication of market portfolio (Cambell et al., 1997). The assumptions underlying the APT model are less restrictive than CAPM and each investor is assumed to choose a unique set of factors to explain returns (Devinga et al., 2011). APT and CAPM have in common that testability is difficult. CAPM requires a true market portfolio, while APT requires identification of relevant factors (Shanken, 1982). This is perhaps the reason why CAPM is more popular since the theoretical foundation is more solid. Nai-Fu and Chen (1983) do not reject APT and conclude that the model performs better than CAPM using data from S&P500. In favor of the APT, several researchers conclude that single-factor models are not sufficient in explaining returns (Rosenberg and Marathe, 1977; Lee and Vinso, 1980; Langetieg, 1978). There is also research which suggests that APT only works under certain conditions (Clare and Thomas, 1994; Priestly, 1996) Another model that researchers have seen as almost equivavlent to APT is the Intertemporal CAPM (Brennan et al., 2004). The intertemporal CAPM (ICAPM) is equilibrium model that shares some of the criticized assumptions behind CAPM, for example homogenous expectations among investors. However, due to its intertemporal nature, the model should be better than the classical CAPM in explaining returns when investors having changing investment opportunity sets (Merton, 1973). Machado et al. (2013) examine the validity of the ICAPM using data from the Brazilian market between1988 and 2012. The model is valid when applied on the entire period, but performed less well during sub-periods. Rubio (1989) rejects ICAPM using the returns on gold as hedging variable and the government bond index.

Researchers have found several firm level factors that help to explain asset returns, e.g. size (Banz, 1981) and leverage (Bhandari,1988). Fama and French (1992) challenge the CAPM model and provide evidence of its insufficiency. Fama and French (1993) present the Three-factor model where they have included size and value as factors. Jegadeesh and Titman (1993) study momentum strategies and nd that buying winners, while selling losers, can earn prots. By adding this factor, improvements are made on the Fama-French model (Carhart, 1997).

2.4 Higher moments in asset pricing models

Risk-averse investors prefer positive skewness over negative skewness (or no skewness). In general, investors have a positive preference for odd moments and negative preference for even moments. Consequently, investors require higher premiums for being exposed to variance and kurtosis (Scott and Horvath, 1980; Fang and Lai, 1997). An early attempt to include skewness was by Kraus and Litzenberger (1976) in a Three-moment CAPM. They find that skewness factor is significant and performs better than CAPM. Several other studies draw the same conclusions (Hwang and Satchell, 1999; Lim , 1989). Johansson (2005) finds that the Three-moment CAPM explains the returns better than the other models. He concludes that co-kurtosis is significant and positive, while co-skewness has a sign that differs from what theory predicts. Some studies find that coskewness premium is significant and time-varying, and that it adds explaining power to the Fama-French model (Jondeau and Rockinger, 2003; Smith, 2007). Research suggests that omission of co-skewness can mislead inference, and this means that CAPM suffers from omitted variable bias. Also, size is correlated with co-skewness which could explain why Fama-French model performs better than CAPM (Barone-Adesi et al., 2004). Liow and Chan (2005) find that co-skewness is significant and time-varying on the real-estate market. They conclude that co-kurtosis has more explaining power than co-skewness. Another study shows that skewness in stocks is not persistent. Therefore, investors who prefer skewness should combine either options or convertible bonds in their portfolios (Singleton and Wingender, 1986).

Chapter 3

Methodology and data

3.1 Methodology

Several different econometric techniques are applied in asset pricing test. According to Jagannathan et al.(2009), these techniques can be grouped into three classes: cross sectional regression methods, the maximum likelihood method, and the generalized method of moments. Maximum likelihood estimation (MLE) is an intuitive method but often fails due to strong underlying assumptions. An early application of MLE is for example in Gibbons (1982). The MLE approach not only avoids the errors-in-the-variables problem, but also gives more accurate risk premiums. However, this method requires assumptions on the distribution of stock returns. This is not a requirement when applying the generalized method of moments (GMM), see e.g. Cochrane (2001). GMM relies on the assumption of stationary return process, and allows for serial dependence and conditional heteroskedasticity. MacKinlay and Richardson (1991) compare the GMM and MLE, and recommend the former due to robustness. The authors show that MLE (and the Wald test) is biased when returns are conditionally heteroskedastic.

Furthermore, all three models are asymptotically equivalent under conditional homoskedasticity. The empirical results of this thesis are based on cross-sectional regression proposed by Fama-Macbeth (1973). This is an important and widely used method in empirical research (Bailer and Martin, 2007). Various papers discussed in this thesis (several of them seminal) apply the FM methodology in their studies (Kraus and Litzenberger, 1976; Fama and French, 1993; Carhart, 1997; Harvey and Siddique, 2000) . In the first stage of this method, a time series regression is made to calculate the sensitivities, or betas, on the factors. The estimated betas are then used in a cross-sectional regression to calculate factor premiums. The aim of this thesis is to test several factors and Fama-Macbeth is therefore advantegous. Despite the usefulness of FM, some disadvantages with this method must be mentioned. Fama and Macbeth (1973) discuss the "errors-in-variables" problem that arise because of using estimated betas from the first step. Another problem, presented by Roll (1977), is that market returns are unobservable since they represent every individual asset that is traded. In this thesis, a proxy for market are the traded stocks on NASDAQ and New York Stock Exchange. This poses a problem since our additional factors, coskewness and cokurtosis, are estimated with this proxy.

The cross-sectional regressions in the second step is performed with 4 different specifications or models:

Model 1: CAPM:

$$\hat{\mu}_i = \lambda_0 + \hat{\beta}_i \lambda_1 + \varepsilon_i$$

Model 2: Fama-French

$$\hat{\mu}_i = \lambda_0 + \hat{\beta}_i \lambda_1 + \widehat{HML}_i \lambda_2 + \widehat{SMB}_i \lambda_3 + \varepsilon_i$$

Model 3: Fama-French + momentum

$$\hat{\mu}_i = \lambda_0 + \hat{\beta}_i\lambda_1 + \widehat{HML}_i\lambda_2 + \widehat{SMB}_i\lambda_3 + \widehat{MOM}_i\lambda_4 + \varepsilon_i$$

Model 4,5,6: Fama-French + momentum, coskewness, cokurtosis

$$\hat{\mu}_i = \lambda_0 + \hat{\beta}_i\lambda_1 + \widehat{HML}_i\lambda_2 + \widehat{SMB}_i\lambda_3 + \widehat{MOM}_i\lambda_4 + \widehat{CoS}_i\lambda_5 + \widehat{CoK}_i\lambda_6 + \varepsilon_i$$

In the classic CAPM, the only factor tested is the beta between (excess) asset and market returns. The three-factor model by Fama-French (1993) includes size and value in addition to market risk. Momentum is added to model 3. These models will be assessed and compared, both individually and when coskewness and cokurtosis factors are included. Models 4 and 5 include coskewness and cokurtosis, respectively. Finally model 6 includes all the factors. In order to model coskewness and cokurtosis premiums, we have chosen to follow Harvey and Siddique (2000). First, stocks are ordered on the basis of coskewness and cokurtosis. The 20 percent of the stocks with the lowest coskewness are placed in portfolio S, while portfolio K contains the 20 percent of the stocks with the highest cokurtosis. Harvey and Siddique weighted with 30 percent instead of 20 percent, but there is some indication (see tables in section 3.2) that there could be abnormal returns in the 3rd decile which could distort the results. Excess returns are then used in the regressions. The next issue that arises is to estimate coskewness and cokurtosis. The literature on realized moments is promising (see e.g. Andersen Amaya (2010)) but requires that large amount of data (tick by ticks prices). As discussed earlier, Xu (2010) proposes a simpler way where daily prices are used to calculate monthly comoments. Besides simplicity, this method has the advantage of being model-free as opposed to Harvey and Siddique

(1999,2000). The estimators for coskewness (CoS) and cokurtosis (CoK) are:

$$\widehat{CoS}_{i,t} = \frac{\frac{1}{J_t}\sum_{j=1}^{J_t}\left(r_{i,j}-\mu_{i,t}\right)\left(r_{m,j}-\mu_{m,t}\right)^2}{\sqrt{\frac{1}{J_t}\sum_{j=1}^{J_t}\left(r_{i,j}-\mu_{i,t}\right)^2}\left(\frac{1}{J_t}\sum_{j=1}^{J_t} r_{i,j}-\mu_{i,t}\right)^2} \tag{3.1}$$

$$\widehat{CoK}_{i,t} = \frac{\frac{1}{J_t}\sum_{j=1}^{J_t}\left(r_{i,j}-\mu_{i,t}\right)\left(r_{m,j}-\mu_{m,t}\right)^3}{\sqrt{\frac{1}{J_t}\sum_{j=1}^{J_t}\left(r_{i,j}-\mu_{i,t}\right)^2}\left(\left(\frac{1}{J_t}\sum_{j=1}^{J_t} r_{i,j}-\mu_{i,t}\right)^2\right)^{\frac{3}{2}}} \tag{3.2}$$

where J_t is the number of trading days in month t, $r_{i,j}$ and $r_{i,j}$ are returns of asset i and market on j:th trading day of month t.
In order to see how robust of coskewness estimator is, a simulation study is made using a model by Liu et al.(2013). We generate two processes that follow

$$S_{t,j} = \sigma_j W_{t,j} + \sum_{k=1}^{Nt} \xi_{k,j}, j = I, M \tag{3.3}$$

where $W_{t,j}$ are correlated standard Brownian motions. Jumps are added by using ξ which are exponentially distributed with constant mean η. N_t is a Poisson process with intensity λ. The I and M denote the processes of the asset and market, respectively.
The true coskewness according to Liu et al.(2013) is then:

$$CoS = \frac{\sum_{s\leq T} \delta_s^I \left(\delta_s^M\right)^2}{\sum_{s\leq T} \left(\delta_s^M\right)^3}. \tag{3.4}$$

δ_s^j denotes the jump of j at time s, i.e. $\delta_s^j = S_s^j - S_{s^-}^j$. All jumps in the generated processes are known which enables the estimation of true higher comoments. The estimator from Xu(2010) is then evaluated by comparing them with true value. In later section, the result of this simulation is presented.

3.2 Data Description

This study uses both daily and monthly data on assets and market returns from 1994 to 2014. The main source is Datastream where daily common stocks prices are gathered from Nasdaq and NYSE. The number of stocks are over 6000, and this number varies with years. Data on market returns, and the test portfolios used in regression, are taken from the home page of Kenneth French.

In both tables, the two factors, coskewness and cokurtosis, are inversely proportional. The interpretation is not clear since the returns do not seem to have a pattern in the first table. From a theoretical perspective, the data in table 3.1 would make more sense if the returns would also be negatively related to coskewness. The first table does however not show any general pattern. Coskewness does not seem to be correlated with either returns or size. This gives some hints that coskewness probably is not a significant factor in our asset pricing model. The first decile has absolutely the lowest average returns, while deciles 2-4 show some extremly high returns. For example, the value-weighted portfolio in the second decile has an average monthly return of 1,43%. This corresponds to an annual return of almost 19 %. The equal-weighted portfolios have even more extreme cases. A possible explanation for these returns high market capitalization during the late 90's and early 20's. This could explain why value-weigthed and equal-weighted returns differ so much. Many tech companies, often young and start-up companies, were highly overvalued which is a possible explanantion for distorted return distributions. If stocks with low prices, for example up to 5 dollards, are excluded, as e.g. Amaya et al (2010) or Heaney et al. (2012) did, the figures in the tables change insignficantly. Table 3.2 shows the corresponding sorting based on cokurtosis. Cokurtosis seems to have clearer relationship to both returns and size compared to coskewness. There is general pattern where the value-weighted returns are related to cokurtosis. The value-weighted returns, in some deciles, deviate from this pattern, but the relationship is clearly positive. The equal-weighted returns do not exhibit this pattern. Cokurtosis is also related to size; the smallest firms have the highest cokurtosis and vice versa. This is in accordance with theory that positive cokurtosis is priced with a premium, but also that smaller firms have higher average returns. Cokurtosis is therefore expected to be a significant variable in the model.

Table 3.1: Assets are sorted based on their coskewness

Decile	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Coskew	-0,51	-0,28	-0,17	-0,09	-0,01	0,06	0,14	0,23	0,34	0,56
Cokurt	0,14	0,06	0,02	0,00	-0,02	-0,04	-0,05	-0,07	-0,08	-0,10
Excess ret (EW)	0,68%	2,60%	6,89%	2,43%	1,83%	1,69%	0,97%	0,70%	1,32%	2,03%
Excess ret (VW)	0,42%	1,43%	1,18%	1,37%	1,10%	0,96%	0,83%	0,63%	0,66%	1,26%
Size (logs)	8,20	8,32	8,35	8,42	8,46	8,47	8,48	8,50	8,46	8,40

Table 3.2: Assets are sorted based on their cokurtosis

Decile	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Cokurt	-1,45	-0,84	-0,55	-0,32	-0,12	0,07	0,27	0,50	0,81	1,46
Coskew	0,08	0,05	0,04	0,03	0,02	0,01	0,01	0,01	0,01	0,00
Excess ret (EW)	1,24%	1,33%	1,38%	3,31%	1,83%	2,60%	4,36%	1,70%	1,55%	1,83%
Excess ret (VW)	0,55%	0,83%	0,74%	0,90%	0,91%	1,08%	0,96%	1,22%	1,14%	1,35%
Size (logs)	8,55	8,54	8,54	8,49	8,45	8,42	8,35	8,32	8,27	8,07

Chapter 4

Empirical findings

4.1 Simulation results

Table 4.1 on the next page shows the results when the estimator by Xu (2010) is used to calculate coskewness in the model from Liu et al. (2013). For each combination of λ and η, 5000 months are simulated. The result is presented as the average deviation of the estimator from the true value. When the jumps are intense, e.g. large value of η and λ, the deviation increases drastically; the bias occurs in both directions. The conclusion is that this estimator works less when jumps intensify because of the noise generated.On the other hand, under certain conditions, such as λ and η being around values 1-2, the estimator works fairly well. A likely cause of this is that the amount of noise increase when higher moments are estimated.

4.2 Fama-Macbeth regressions

Tables 4.2-4.6 below are the result of the Fama-French regression using different test portfolios. With this variety of test portfolios, the aim is to truly

Table 4.1: Simulation results: $\sigma_I = 0.2$, $\sigma_M = 0.4$, $\rho = 0.4$

		λ :					
		0.5	0.8	1	1.5	3	10
η :	0.5	-59%	-47%	-43%	-40%	-39%	-58%
	0.8	-28%	-17%	-15%	-17%	-29%	-59%
	1	-44%	-7%	-5%	-14%	-25%	-56%
	1.5	17%	17%	11%	0%	-24%	-55%
	3	50%	34%	21%	8%	-21%	-55%
	10	66%	41%	29%	6%	-18%	-56%

test the power of the model. Many empirical tests use the size-B/M portfolio but some researchers criticize this choice since it too often gives favorable results (Lewellen et al., 2010). By choosing different sets of portfolios, the empirical results should be more credible, and suffer less from biases (Lo and MacKinlay, 1990). The argument is that many models unjustifiably result in high R^2 or low errors.

The test portfolio consist of (1) 30 industry portfolios, (2) 19 portfolios sorted on E/P, (3) operating profitability and (4) size and momentum. The last table combines all of these portfolio as a final power test of the models. For each portfolio, the models discussed earlier are tested. The intention with these portfolio, which are not really related to each other, is to test the models under a variety of return distributions.

The characteristics of a well-specified model is that the intercept is as small as possible. The intercept is ideally not significantly different from 0 because this signals that the model captures most of the information.

4.2.1 30 industry

In general, this test portfolio is a real challenge for the models since the return characteristics vary extensively. The results suggest that all the models are inadequate in explaining returns, and none of them work particularly well in explaining the returns of the 30 industry portfolios. The intercepts are significant, except for models 3 and 4 when the significance level is set to 1%. None of the factors are significant except cokurtosis. Compare model 5 and 6; the adj R^2 increases when coskewness is removed from model 6.

Table 4.2: Test assets are 30 Industry portfolios. Asterisks denote statistical significance at the 1% (***), 5% (**), or 10% (*) level, respectively. t-statistic is shown in brackets.

Model	**Intercept**	**Market**	**HML**	**SMB**	**MOM**	**Coskew**	**Cokurt**	**adj R**
1	0,75***	-0,03						0,0%
	[6,85]	[-0,21]						
2	0,76***	0,00	-0,08	-0,07				0,0%
	[4,54]	[0,01]	[-1,59]	[-0,36]				
3	0,62**	0,16	-0,04	-0,11	0,53			1,0%
	[2,40]	[0,60]	[-0,47]	[-0,47]	[1,73]			
4	0,69**	0,10	-0,04	-0,12	0,51	0,34		0,0%
	[2,64]	[0,33]	[-0,41]	[-0,54]	[1,57]	[0,79]		
5	0,61**	0,16	-0,07	-0,24	0,13		0,80	14%
	[2,62]	[0,67]	[-0,97]	[-0,95]	[0,54]		[2,17]	
6	0,64**	0,13	-0,06	-0,24	0,13	0,53	0,81	11%
	[2,56]	[0,52]	[-0,93]	[-0,96]	[0,55]	[1,53]	[2,16]	

4.2.2 Earnings/Price

The results from test portfolio on E/P-ratio are good in a sense that several models are valid. CAPM is rejected because of the significant intercept. Beta is significant, but has a negative sign which is against theory. Model fit increases greatly by adding factors size and book-to-market; from 18,2% to 60% . Intercept is not sigificantly different from 0, and of the 3 variables included in model 2, only book-to-market factor is significant. When included in model 3, momentum is not significant. Model 6 is not rejected and the two significant variables are book-to-market and cokurtosis.

Table 4.3: Test assets are 19 portfolios sorted on the ratio Earnings/Price. Asterisks denote statistical significance at the 1% (***), 5% (**), or 10% (*) level, respectively. t-statistic is shown in brackets.

Model	**Intercept**	**Market**	**HML**	**SMB**	**MOM**	**Coskew**	**Cokurt**	**adj R**
1	1,11***	-0,40						18,2%
	[12,63]	[-4,76]						
2	0,22	0,43	0,44	-0,22				68,0%
	[0,35]	[0,71]	[7,26]	[-0,46]				
3	0,19	0,49	0,41	-0,11	1,23			71,0%
	[0,33]	[0,92]	[7,26]	[-0,22]	[0,66]			
4	0,13	0,54	0,41	-0,1	1,36	-0,02		70,7%
	[0,22]	[0,96]	[7,11]	[-0,13]	[0,76]	[-0,04]		
5	0,23	0,44	0,4	-0,1	1,12		0,48	68,4%
	[0,37]	[0,76]	[6,91]	[-0,2]	[0,68]		[0,95]	
6	0,77	-0,10	0,40	0,14	-0,33	-0,81	1,33	75,0%
	[1,18]	[-0,17]	[7,10]	[0,26]	[-0,23]	[-1,04]	[3,0]	

4.2.3 Operating profitability

The models are better at explaning the returns of portfolios that are sorted on operating profitability. CAPM and Fama-French model are rejected even though model fit is high. All three factors are insignificant at 5%. When momentum is included, the intercept is insignificant, and only momentum is priced with p-value of 2,5%. When coskewness and cokurtosis are included as variables, the model improves. At 5%, cokurtosis is marginally significant, while coskewness is not priced. Momentum is still significant together with size. The size premium has the right sign and is -0,52% on average per month.

Table 4.4: Test assets are 25 portfolios sorted on size and momentum. Asterisks denote statistical significance at the 1% (***), 5% (**), or 10% (*) level, respectively. t-statistic is shown in brackets.

Model	Intercept	Market	HML	SMB	MOM	Coskew	Cokurt	adj R
1	1,36***	-0,73						61,0%
	[14,25]	[-9,47]						
2	1,27***	-0,63	-0,21	-0,34				71,0%
	[4,01]	[-2,03]	[-1,21]	[-2,07]				
3	0,74	-0,08	0,14	-0,35	1,41			76,0%
	[1,68]	[-0,19]	[0,52]	[-2,09]	[2,53]			
4	0,83	-0,17	0,09	-0,35	1,39	0,12		74,9%
	[1,58]	[-0,33]	[0,3]	[-1,88]	[2,12]	[0,23]		
5	0,62	0,04	0,28	-0,44	1,48		0,63	79,6%
	[1,65]	[0,11]	[1,19]	[-3,06]	[2,15]		[1,74]	
6	0,30	0,35	0,51	-0,52	1,58	0,21	1,17	80,1%
	[0,61]	[0,72]	[1,62]	[-3,21]	[2,64]	[0,61]	[2,29]	

4.2.4 Size and momentum

The results from size and momentum test portfolios are interesting. Coskewness is close to being significant at 5%, while cokurtosis is not significant. The intercept in model 4 is also marginally significant at this level. As expected, size and momentum are priced with on monthly averages of 0,46% and 1,48%, respectively. Momentum is strongly signifcant, while size is close to being rejected at 5% because the p-value is close to 4%. The goodness of fit is high as expected because of the factor structure in size variable.

Table 4.5: Test assets are 25 portfolios sorted on size and momentum. Asterisks denote statistical significance at the 1% (***), 5% (**), or 10% (*) level, respectively. t-statistic is shown in brackets.

Model	**Intercept**	**Market**	**HML**	**SMB**	**MOM**	**Coskew**	**Cokurt**	**adj R**
1	1,36***	-0,49						19,8%
	[7,29]	[-3,25]						
2	1,55***	-0,75	-0,17	0,28				68,6%
	[10,48]	[-6,85]	[-1,60]	[3,42]				
3	0,68	0,03	0,29	0,23	0,47			72,5%
	[1,19]	[0,06]	[0,87]	[2,25]	[5,78]			
4	1,3**	-0,57	0,23	0,34	0,45	1,5		74,4%
	[2,22]	[-1,03]	[0,98]	[3,69]	[8,12]	[2,06]		
5	0,73	-0,03	0,39	0,26	0,48		0,89	73,5%
	[1,42]	[-0,05]	[1,32]	[2,45]	[6,49]		[1,08]	
6	1,26**	-0,53	0,27	0,34	0,46	1,48	0,75	76,3%
	[2,15]	[-0,95]	[1,21]	[3,61]	[7,89]	[2,03]	[1,19]	

4.2.5 All portfolios

As expected, the model fit from the regressions in the last table is lower. The adjusted R^2 is greatly increased when adding additional factors to the CAPM model. The coskewness and cokurtosis do not provide any explaining power, and they are not significant. All models are rejected which indicates that these factors are not enough to explain all the variation. Size and momentum are significant in all models where they are included. Book-to-market is always insignificant, while beta shows mixed results.

Table 4.6: Test assets are all (four of the above) portfolios combined. Asterisks denote statistical significance at the 1% (***), 5% (**), or 10% (*) level, respectively. t-statistic is shown in brackets.

Model	**Intercept**	**Market**	**HML**	**SMB**	**MOM**	**Coskew**	**Cokurt**	**adj R**
1	0,93***	-0,20						4,5%
	[10,46]	[-2,14]						
2	1,17***	-0,46	0,02	0,24				20,4%
	[9,86]	[-3.83]	[0,27]	[2,95]				
3	0,89***	-0,18	0,11	0,21	0,46			30,0%
	[6,18]	[-1,21]	[1,07]	[2,64]	[6,11]			
4	0,94***	-0,23	0,1	0,22	0,46	-0,01		29,6%
	[5,55]	[-1,38]	[1,0]	[2,66]	[6,2]	[-0,04]		
5	0,93***	-0,22	0,1	0,22	0,45		0,19	31,1%
	[7,35]	[-1,69]	[1,0]	[2,71]	[6,84]		[0,83]	
6	0,95***	-0,24	0,10	0,22	0,46	0,02	0,19	30,4%
	[5,98]	[-1,52]	[0,97]	[2,67]	[6,88]	[0,10]	[0,83]	

Chapter 5

Summary and conclusions

Aside from being one of the most famous results in the field of finance, the Capital Asset Pricing Model (CAPM) has also been well-established among practioners. Since its introduction, empirical research has been conducted to test its validity, and the results have not been entirely favorable. The suggestions on where the model fails is also not in a complete accord. In line with Roll (1977), there is no precise way of testing the model until all the inputs are correct. Nevertheless, several suggestions have been presented on how to improve the model. Models such as Fama and French (1993) and Carhart (1997) have added some explaining power but are not theoretically justified. The purpose of this thesis has been to test extensions of the CAPM. More specifically, those models where investors also care about the third and fourth moment of return distribution, i.e. skewness and kurtosis. As with CAPM, only the comoments have been used because of diversification effects. The simulation results indicate that the estimators work well under certain return characteristics (i.e. intensity of jumps). No investigation has however been done on the behaviour of actual returns since it was considered out of scope.

Several papers have reported results where these preferences matter in asset pricing. Some researchers, such as Harvey and Siddique (1999; 2000), work with models, while our research has been conducted using model-free estimators for coskewness and cokurtosis. The empirical results supports the hypothesis that cokurtosis is important in explaining the returns, and there is little evidence of coskewness being important. Including higher moments seems to improve the models, at least marginally. However, judging from this empirical study, and earlier research, the results are far from assertive because of inconsistency. Potential reasons for this could be many. Firstly, the model is linear in factors, and this assumption could be a severe limitation. Second, even if the true model is linear in reality, the determinants used as input seem to be inadequate and perhaps unobservable. This results in endogeneity problems and other errors. Finally, the use of high frequency data, such as minute by minute prices, could add information and also improve the model. On the other hand, that would also require more computational power and new sources of data..

To find a truly acceptable model is elusive and challenging. It seems that researchers will be engaged for a long time in their effort to find a model that delivers consistent results.

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