Factors driving the Euro senior funding costs for Swedish Banks

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Master's thesis
2015:E33

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Abstract

In this thesis we show that the best forecast for near future yield curves are performed with a full specification of our chosen model and for far future yield curves the forecast is better with a partial specification of the model. We conclude that the driving factors for the banks' yield curves are not as closely related to Swedbank's as amongst the other peers which show that Swedbank stand out in terms of dynamics for the yield curve. We also show why we have chosen a Nelson Siegel model with five driving factors by performing a PCA and we illustrate how we have estimated the 47 free parameters in the model with a Kalman filter and an optimization algorithm in matlab. If that was not enough we also validate our model by simulating yields and checking that our estimates correspond to the parameters used in the simulation.

Keywords: Nelson Siegel, Kalman filter, Parameter estimation, Treasury, PCA

Swedbank

Acknowledgements

We would first off like to thank our professional supervisors Kimmy and Gunilla that gave us the chance and the tools to successfully do this thesis on site at the Swedbank headquarters in Sundbyberg. Secondly we want to thank our academic supervisor Erik without whom the content of the thesis would be quite scarce. Thank you Erik for giving us valuable insights in the field of financial Statistics and for the support throughout this thesis. Apart from our supervisors we would like to thank all of our colleagues at Swedbank Treasury who have made our stay at the office very pleasant and our lunches a lot more fun. Last of all we would like to thank our families and CSN for the endless support throughout our five years in Lund and abroad without you this would not have been possible. Thank you.

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1. Introduction

1.1 Background

Since the financial crisis and the fall of Lehman Brothers credit risk and treasury management has become hot topics in the financial industry. The need of managing credit and counterparty risk has become clear and differences between how investors view different banks' bonds has changed. This stress has led to more regulations and a segmentation of what is truly risk free. Since the onset of the GIIPS/Euro-crisis Sweden has been considered a safe haven for investment but among the Swedish banks there are differences in the cost of issuing bonds to finance the operations of the banks. These differences change over time, especially in times of crisis. Many Swedish banks were helped by the Swedish Riksbank to get out of the worst liquidity crunch. In the aftermath of the crisis the cost was quite high for funding in the regular market. For this reason Swedbank wants to investigate which factors make the credit spread differ among the Swedish banks, as indications show that the credit rating itself can't explain this difference in spread.

During the Summer of 2014 a database of historical issuances of bonds was established at Swedbank (volumes, prices, spreads etc.). The result of the findings were that some banks more than others still are carrying expensive funding that was raised in the aftermath of the financial crisis. In discussions on the topic why the costs of issuing bonds differ among Swedish banks, a problem worthy exploring through a master thesis arose.

The environment for investors today is that a ”decent” yield in combination with low risk is harder and harder to find. The reason for this is not evident, but there are indications that this depends on factors such as Quantitative Easing (QE) performed by central banks and that European banks has de-leveraged a large part of their balance sheets. These factors cause the supply of bonds to decrease while demand has not decreased in the same manner leaving investors forced to accept lower and lower yields. This may imply that investors today consider the actual credit spread in form of default intensity less and a stable supply of bonds more. Investigating these dynamics will be the starting point for this thesis.

1.2 Overview - A typical Balance Sheet

In order to get a grip on where our master thesis fits in the operations of a ordinary bank we try to illustrate this with help of a typical balance sheet, in this case Nordea's. As can be seen in Figure 1.1 ”Debt securities in issue”, (red

Balance sheet[2]

EURm	31 Dec 2014
Cash and balances with central banks	31,067
Loans to central banks and credit institutions	19,175
Loans to the public	348,085
Interest-bearing securities and pledged instruments	99,261
Derivatives	105,119
Other assets	66,635
Assets held for sale	—
Total assets	**669,342**
Deposits by credit institutions	56,322
Deposits and borrowings from the public	197,254
Liabilities to policyholders	51,843
Debt securities in issue	194,274
Derivatives	97,340
Subordinated liabilities	7,942
Other liabilities	34,530
Liabilities held for sale	—
Equity	29,837
Total liabilities and equity	**669,342**

Figure 1.1: Balance Sheet, Nordea, as per 2014-12-31

box) is essentially as big as "Deposits and borrowings from the public" (yellow box) which represents accounts like yours and ours salary/savings account. We can also see that these two roughly represent "Loans to the public" (green box) with an overshoot that can be found in "Loans to central banks and other credit institutions" and in "Cash balances with central banks". Our analysis is within the "Debt securities in issue" box. This represent all funding from the market in forms of long term funding i.e. Covered bonds and senior unsecured bonds and short term funding i.e. Commercial Papers and Certificates. Thus this is the box where our analysis can give an input on the cost of this funding and the development of it. As also can be seen the funding represent roughly 1/3 of the total balance sheet which makes this valuable information in funding plans and how to structure the funding.

1.3 Relevant Literature

On the topic of timing and amount of issued debt by financial institutions, ECB has extended the two-part modelling framework presented by Cragg 1971 (ECB, 2014), which gives nice indications of correlations between amount of issued debt by European financial institutions and incentive programs such as quantitative easing. The paper presents a model where financial reputations factors (such as rating) are modelled against timing and amount of issued debt.

A specific knowledge of several interest rate models and credit risk models that can model probability of default and default intensity can be attained by reading Brigo & Mercurio (2006). The book gives an understanding of how investors determine what yield to accept for different issuers depending on the risk premium expected from the market and of course the expectations of the interest rate. It is explained how these models can be used as an outside predictor for the markets view on the issued bonds. Typically these models use CDSs and/or corporate bond yields as input. This book has a very quantitative approach, and for someone who wants to get a somewhat more qualitative understanding of credit risk, the book Duffie & Singleton (2012) is preferred.

A more general knowledge on risk-neutral valuation as well as arbitrage theory, which is the foundation of the financial mathematics, can be gained by reading Björk (2009).

Details on the Nelson Siegel modelling framework and estimation of its parameters can be attained by reading up on Christensen & Lopez (2012).

1.4 The purpose of the thesis

We aim to collect and model historic data for outstanding EUR senior unsecured debt. Our goal is to draw conclusions and better understand the driving factors for the credit spread amongst EUR senior unsecured bonds issued by Nordic banks. Evaluating these parameters and forecast future yields form the main purpose for this thesis.

1.5 Delimitations

This thesis is performed together with Swedbank's Group Treasury. We are thus naturally comparing funding costs for similar banks, i.e. SEB, SHB and Nordea. We also aim to draw conclusions from credit spreads why we need to delimit our thesis around instruments with enough depth so we can produce a full yield curve for every time instance. In our analysis of the funding market and the variation of credit spreads between the Swedish banks we have chosen to only look at the most liquid market namely the EURO senior unsecured market. Further explanation why the EURO senior market is the most liquid is described in section 3.1.3.

2. Theory

This chapter goes through the theory that form the foundation of this thesis. Building blocks such as analysis on which model we chose, the actual model and relevant theory we have used throughout this thesis make up the principal parts.

2.1 Principal Component Analysis (PCA)

We start with a PCA in order to help us understand how we should model the yield data. The PCA focus on what variables explain variation in data. By looking at the eigenvectors and knowing that the first, second and third vector correspond to the variables level, slope and curvature. This theory explains how we can determine what explanatory power each variable have and demonstrate how much variation we model by choosing a set of Level, Slope and curvature, for further details see, Shlens (2014).

2.1.1 Computing PCA using the Covariance method

Definition 2.1. *Given a $n \times m$ dataset matrix X where number of rows correspond to number of observations and number of columns to number of variables.*

$$\mu(j) = \frac{1}{n}\sum_{i=1}^{n} X(i,j) \quad i = 1,\dots,n \qquad j = 1,\dots,m \tag{2.1}$$

$$B(i,j) = X(i,j) - \mu(j) \quad \textit{take away mean} \tag{2.2}$$

$$C = \frac{1}{n-1} B^T B \qquad \textit{calculate sample covariance} \tag{2.3}$$

$$V^{-1}CV = D \qquad \textit{compute eigenvalues/eigenvectors } D/V \tag{2.4}$$

Sort D and V by descending value of diag(D) and sum diag(D)

$$g(j) = \sum_{k=1}^{j} D(k,k) \tag{2.5}$$

Choose an overall explanatory level α and the number of principal components L that satisfy $1 \le L \le m$.

$$\frac{g(L)}{g(m)} \ge \alpha \tag{2.6}$$

Adjust the number of principal components to fulfil the chosen α. Respective principal component have its own explanatory level and can be illustrated by *constructing a vector with elements*

$$explained = \left[\frac{D(1,1)}{g(m)}, \ldots, \frac{D(L,L)}{g(m)}\right] \tag{2.7}$$

2.2 Default-free and defaultable bonds

2.2.1 Zero-Coupon Bonds and the Risk Free Short Rate

Definition 2.2. *A zero-coupon bond is a contract which gives the holder a payment of a nominal amount (most often 1) at maturity time T. If the contract is bought at time t the price(value) of the bond is denoted by P(t,T) for $t \leq T$. At time $t = T$ the value of the contract is (intuitively) 1.*

Definition 2.3. *The risk-free short rate at time t, r_t, is the (arguably theoretical) interest rate at which a market participant can both borrow and lend money, if there's absence of arbitrage (Björk, 2009).*

In the continuous world the relationship between the price of a default-free bond and the short rate is:

$$P(t,T) = \mathbb{E}^{\mathbb{Q}}[e^{-\int_t^T r_s ds}|\mathcal{F}_t], \tag{2.8}$$

where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the risk-neutral measure and $\mathcal{F}_t$ is the filtration at time t.
The price is easily calculated if the short rate is deterministic. Unfortunately, in a real world application, the future short rate is not observable. However, the prices of what are considered as risk-free (non-defaultable) bonds can be observed. From that price one can calculate a risk-free rate.

2.2.2 Zero coupon curve

Using the theory in Section 2.2 on bonds with different maturities one finds that the implied short rate is not constant over time, but varies depending on maturity. This leads us to the concept of yield.

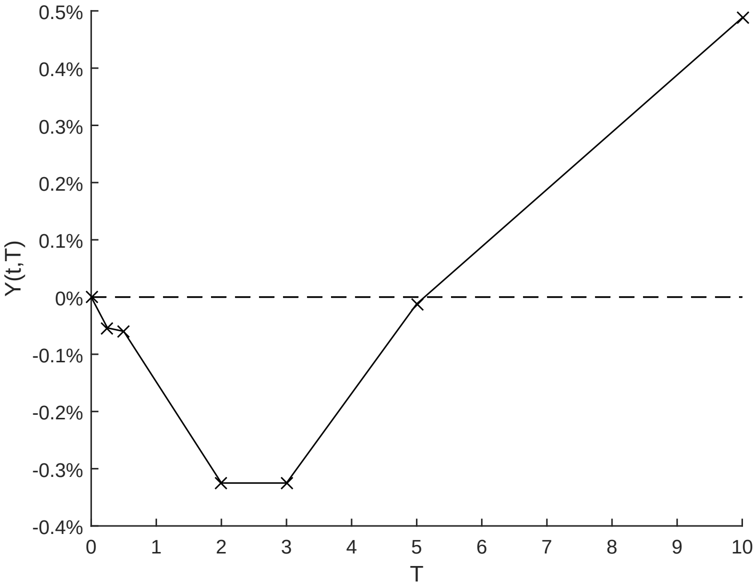
Definition 2.4. *Y(t,T) is the constant rate at time t for the maturity T at which an investment has to be made to produce an amount of one unit of currency at maturity if the initial investment was P(t,T), when reinvesting the obtained amounts once a year. This is also called the continuously-compounded spot interest rate.*

We have

$$e^{-Y(t,T)(T-t)} = P(t,T) \Rightarrow Y(t,T) = -\frac{1}{T-t}\log P(t,T). \tag{2.9}$$

Given the above equation, we can create a Zero-Coupon (or yield) curve, plotting $Y(t,T)$ for different T given at time t. This is also called the term structure of interest rates. An illustrative example can be seen in Figure 2.1.

Figure 2.1: ZCB curve for Swedish T-bills and T-bonds as per 2015-02-17



2.2.3 Defaultable Zero Coupon Bonds

Similar to a default-free zero coupon bond, a defaultable zero coupon bond pays a nominal amount at maturity time T. The difference from a risk-free bond lies in that this asset type is not considered risk-free, as there is a risk that the issuer of the bond might default before maturity, resulting in a potential loss for the investor. The price $\bar{\mathrm{P}}$(t,T) of a defaultable zero-coupon bond is calculated as:

$$\mathbb{1}_{\{\tau>t\}}\bar{\mathrm{P}}(\mathrm{t,T}) = \mathbb{E}\left[D(t,T)\mathbf{1}_{\tau>T}|\mathcal{F}_t\right]. \tag{2.10}$$

$D(t,T)$ is the discount factor over the interval [t,T], often a risk-free zero coupon bond. τ is the time of default. $\mathcal{F}_t$ is all the available information at time t. Given that the probability of default and the discount rate are often considered to be independent this can be rewritten as:

$$\mathbb{1}_{\{\tau>t\}}\bar{\mathrm{P}}(\mathrm{t,T}) = P(t,T)\mathbb{E}\left[\mathbf{1}_{\tau>T}|\mathcal{F}_t\right] \tag{2.11}$$

2.2.4 Default intensity

In the previous section(s) we've described the price of a defaultable bond as dependent on the default time τ. This is convenient for basic comprehension of how a defaultable bond is priced and behaves in continuous time. We now move into the world of intensity models, where we rather than try to estimate a time of default, we try to model the intensity of "default". The word default is within quotation marks here as in a real world application, the expected probability of default within a certain time, is not the only factor that affects the credit spread (Definition 2.5) for a bond (Brigo & Mercurio, 2006).

Definition 2.5. *A credit spread is the difference in yield between a defaultable bond and a risk-free bond of the same maturity.*

The default time τ is now considered the first increment of a jump process. In continuous time, the probability of jumping (i.e. defaulting) in the next time increment dt is

$$\mathbb{Q}\left(\tau \in [t, t+dt] | \tau > t, \mathcal{F}_t\right) = \lambda(t)dt \tag{2.12}$$

λ is what we call default intensity or hazard rate. Brigo & Mercurio (2006) show how this relation results in the expression

$$\mathbb{Q}\left(\tau > t\right) = e^{-\int_0^t \lambda(u)du} \tag{2.13}$$

If the default-free interest rate and probability of default are uncorrelated, the price of a defaultable bond maturing at time T can be written as

$$\mathbb{1}_{\{\tau > t\}} \bar{\mathrm{P}}(\mathrm{t,T}) = \mathbb{E}\left[e^{-\int_t^T \lambda(u) + r(u)du}\right] \tag{2.14}$$

or if there's a recovery ratio REC

$$\mathbb{1}_{\{\tau > t\}} \bar{\mathrm{P}}(\mathrm{t,T}) = \mathbb{E}\left[e^{-\int_t^T \lambda(u)(1-REC) + r(u)du}\right] \tag{2.15}$$

When dealing with credit spreads, one has to be careful with which conclusions can be deducted from the value of it. As mentioned earlier, other factors than the market expectation of probability of default (i.e. default intensity) affect the credit spread. In discussions with people participating in the market, there are theories of supply/demand (liquidity) and quantitative easings affecting the credit spread. With this in mind, we will further consider λ as an intensity affecting the credit spread rather than a default intensity. Due to this, Christensen & Lopez (2012) suggest rewriting the expression as

$$\mathbb{1}_{\{\tau > t\}} \bar{\mathrm{P}}(\mathrm{t,T}) = \mathbb{E}\left[e^{-\int_t^T s(u) + r(u)du}\right] \tag{2.16}$$

where $s(u)$ is the instantaneous credit spread. As we are interested in investigating the spread rather than the default probability, we can do this without any loss of generality.

2.3 Deriving zero-coupon yield from coupon yielding instruments

In order to determine a zero coupon yield curve when we have fixed coupon bond data we need to begin with calculating the yield to maturity (YTM) for each bond. In some cases Newton's method can be a necessary tool to do this YTM-calculation, i.e. when there is a large number of remaining coupons. When YTM has been calculated we want to derive corresponding zero coupon yield.

2.3.1 Yield to maturity

The YTM is the discount rate which yields the bond's present value PV and for a bond maturing in n periods with coupon c and a face value FV we have

$$PV = \frac{c}{(1+YTM)} + \frac{c}{(1+YTM)^2} + \ldots + \frac{c+FV}{(1+YTM)^n} \tag{2.17}$$

2.3.2 Newton's method

Newton's method, also called Newton-Raphson method, is one of the most well-known and powerful numerical methods for solving a root-finding problem (Atkinson, 1989). Newton's method is initiated with a first guess x_0 and generates the sequence $\{x_n\}_{n=0}^{\infty}$ by

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \qquad for \quad n \geq 1 \tag{2.18}$$

Theorem 2.6 (Convergence of Newton's Method)**.** *Let $f \in C^2[a,b]$ and consider a $x \in (a,b)$ such that $f(x) = 0$ and*
$f'(x) \neq 0$. Then there exist a $\delta > 0$ such that Newton's method generates a sequence $\{x_n\}_{n=1}^{\infty}$, defined by

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \tag{2.19}$$

converging to x for an initial approximation

$$x_0 \in [x - \delta, x + \delta] \tag{2.20}$$

Hence Newton's method converges locally but not globally.

2.3.3 Zero coupon Yield curve

In a zero coupon yield curve each maturity has it's unique discount rate and a coupon bond's price can be determined as

$$PV = \frac{c}{(1 + ZCY_1)} + \frac{c}{(1 + ZCY_2)^2} + \ldots + \frac{c + FV}{(1 + ZCY_n)^n}, \tag{2.21}$$

where c is the coupon and FV is the final value.
This means that in order to derive the full zero coupon yield curve we need a sufficiently large number of outstanding bonds to determine the ZCY_i for each maturity $i = 1, \ldots, n$.

$$\frac{c + FV}{(1 + ZCY_1)} = PV = \frac{c + FV}{(1 + YTM)} \tag{2.22}$$

In 2.22 we can solve for ZCY_1 since it is the only unknown variable. Putting ZCY_1 into the expression for a coupon bond maturing in two years we can solve for ZCY_2 and continuing solving ZCY_i we can put together a zero coupon yield curve for all maturities $i = 1, \ldots, n$.

2.4 The Nelson-Siegel Model(s)

The Nelson-Siegel model we go through here comes from the family of models usually described as affine term-structure models.

2.4.1 The treasury model

The static Nelson-Siegel model fits the yield curve with the simple functional form, Nelson & Siegel (1987),

$$y(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\phi\tau}}{\phi\tau}\right) + \beta_2\left(\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right), \tag{2.23}$$

where $y(\tau)$ is the zero-coupon yield with τ years to maturity, and $\beta_0, \beta_1, \beta_2$, and ϕ are model parameters.

This representation is often used by financial market practitioners to fit the yield curve at a point in time. Even if this static representation may appear useful for some purposes a dynamic model is needed in order to understand the development of the bond market over time. Therefore Diebold & Li (2006) reinterpret the β coefficients as time-varying factors L_t, S_t, and C_t, (which can be interpreted as Level, Slope and Curvature) so

$$y(\tau) = L_t + S_t\left(\frac{1-e^{-\phi\tau}}{\phi\tau}\right) + C_t\left(\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right), \tag{2.24}$$

Empirically, the dynamic model is highly tractable and typically fits well. Theoretically, however, the model does not demand that the dynamic change of yields blend such that arbitrage opportunities are omitted. Indeed, the results of Filipovic (1999) imply that whatever stochastic dynamics are chosen for the Dynamic Nelson-Siegel factors, it is impossible to rule out arbitrage at the bond prices implied in the resulting Nelson-Siegel yield curve. Therefore Christensen *et al.* (2011) developed the the Arbitrage-Free Nelson-Siegel Model (AFNS). In the paper, several variations of the model are discussed. We will be focusing on the non-correlated factor AFNS, as it is the most flexible and supposedly produce the best forecast among the models (Christensen *et al.*, 2011).

In the model, the short rate r_t can be expressed as the sum of L_t and S_t. Which can be shown by letting τ go to zero in equation 2.24.

$$r_t = L_t + S_t \tag{2.25}$$

Under the $\mathbb{P}$-measure, the behaviour of the state-variables (L_t S_t C_t) are modelled as

$$\begin{bmatrix} dL_t \\ dS_t \\ dC_t \end{bmatrix} = \kappa \left[\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} - \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix}\right] dt + \sigma \begin{bmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \end{bmatrix}, \tag{2.26}$$

$$\sigma = \begin{bmatrix} \sigma_{L_t} & 0 & 0 \\ 0 & \sigma_{S_t} & 0 \\ 0 & 0 & \sigma_{C_t} \end{bmatrix} \tag{2.27}$$

κ is a parameter which is allowed to take different forms depending on what you assume about the evolution of the mean, the most general specification is

$$\textit{Full } \kappa \textit{ matrix} \quad \kappa = \begin{bmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} \end{bmatrix} \tag{2.28}$$

To derive yields for a given maturity T at time t, we have

$$y_t(\tau) = L_t + S_t \frac{1}{\tau}\frac{1-e^{-\phi\tau}}{\phi} + C_t \frac{1}{\tau}\left(\frac{1-e^{-\phi\tau}}{\phi} - \tau e^{-\phi\tau}\right) + a(\tau), \tag{2.29}$$

$a(\tau)$ is the arbitrage-free yield adjustment term which has the quite long expression

$$a(\tau) = -\frac{\sigma_1^2}{6}\tau^2 - \sigma_2^2\left(\frac{1}{2\phi^2} - \frac{1-e^{-\phi\tau}}{\phi^3\tau} + \frac{1-e^{-2\phi\tau}}{4\phi^3\tau}\right) -$$
$$\sigma_3^2\left(\frac{1}{2\phi^2} + \frac{1}{\phi^2}e^{-\phi\tau} - \frac{1}{4\phi}\tau e^{-2\phi\tau} - \frac{3}{4\phi^2}e^{-2\phi\tau} - \frac{2}{\phi^3\tau}1-e^{-\phi\tau} + \frac{5}{8\phi^3\tau}1-e^{-2\phi\tau}\right) \tag{2.30}$$

$\theta_1,\ \theta_2,\ \theta_3$ are the mean level reversions of the state variables $L_t,\ S_t,\ C_t$. σ_k is the standard deviation for the state variable k with $k \in \{L_t, S_t, C_t\}$. ρ_{ij} is the correlation coefficient for the state variables i and j with $i, j \in \{L_t, S_t, C_t\}$. To the reader who wants the full derivation of these expressions we refer to reading both Christensen *et al.* (2011) and Krippner (2015). The latter uses a notation more consistent with the one we have chosen. The earlier gives an in-depth explanation of how these results are achieved by solving a system of ordinary differential equations derived from the price of a zero-coupon bond expressed in affine-form.

2.4.2 The Credit Spread Model

The credit spread model we have chosen to implement include the level, slope and curvature factors from the risk-free short rate model as well as level and slope factors for the credit spread in order to improve the explanatory decomposition Christensen & Lopez (2012).

Definition 2.7. *Continuously compounded credit spread can be expressed in terms of continuously compounded yields*

$$s_t^i = y_t^i(\tau) - y_t^T(\tau), \qquad \begin{array}{l} i \in \{\textit{Swedbank, Nordea, SHB and SEB}\}, \\ T \in \{\textit{Treasury/Central Bank}\}. \end{array} \tag{2.31}$$

The instantaneous credit spread, s_t^i is described as a function of the firm specific level and slope factor. The curvature factor is omitted since the findings of Christensen & Lopez (2012) show that adding a curvature factor to the credit spread doesn't give further explanation of the behaviour of the credit spread. This is not a general result but rather an observation made by Christensen & Lopez (2012) for modelling corporate bond yields. Firm specific level and slope are denoted as $L^S(i)$ and $S^S(i)$ and from now on the risk-free short rate level, slope and curvature are denoted as L^T, S^T and C^T. The instantaneous credit spread can be expressed as

$$s_t^i = \alpha_0^i + \alpha_{L^T}^i L_t^T + \alpha_{S^T}^i S_t^T + \alpha_L^i L_t^S(i) + \alpha_S^i S_t^S(i) \tag{2.32}$$

Under the $\mathbb{P}$-measure, the behaviour of the state-variables $(L_t^T\ S_t^T\ C_t^T\ L_t^S\ S_t^S)$

are modelled as

$$\begin{bmatrix} dL_t^T \\ dS_t^T \\ dC_t^T \\ dL_t^S(i) \\ dS_t^S(i) \end{bmatrix} = \kappa \left[\begin{bmatrix} \theta_{L^T}^{\mathbb{P},i} \\ \theta_{S^T}^{\mathbb{P},i} \\ \theta_{C^T}^{\mathbb{P},i} \\ \theta_{L^S}^{\mathbb{P},i} \\ \theta_{S^S}^{\mathbb{P},i} \end{bmatrix} - \begin{bmatrix} L_t^T \\ S_t^T \\ C_t^T \\ L_t^S(i) \\ S_t^T(i) \end{bmatrix} \right] dt + \sigma \begin{bmatrix} dW_t^{L^T,\mathbb{P}} \\ dW_t^{S^T,\mathbb{P}} \\ dW_t^{C^T,\mathbb{P}} \\ dW_t^{L^S(i),\mathbb{P}} \\ dW_t^{S^S(i),\mathbb{P}} \end{bmatrix} \tag{2.33}$$

$$\sigma = \begin{bmatrix} \sigma_{L_t^T} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{S_t^T} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{C_t^T} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{L_t^S}^i & 0 \\ 0 & 0 & 0 & 0 & \sigma_{S_t^S}^i \end{bmatrix} \tag{2.34}$$

$$\textit{Full } \kappa \textit{ matrix} \quad \kappa = \begin{bmatrix} \kappa_{11}^{\mathbb{P},i} & \kappa_{12}^{\mathbb{P},i} & \kappa_{13}^{\mathbb{P},i} & \kappa_{14}^{\mathbb{P},i} & \kappa_{15}^{\mathbb{P},i} \\ \kappa_{21}^{\mathbb{P},i} & \kappa_{22}^{\mathbb{P},i} & \kappa_{23}^{\mathbb{P},i} & \kappa_{24}^{\mathbb{P},i} & \kappa_{25}^{\mathbb{P},i} \\ \kappa_{31}^{\mathbb{P},i} & \kappa_{32}^{\mathbb{P},i} & \kappa_{33}^{\mathbb{P},i} & \kappa_{34}^{\mathbb{P},i} & \kappa_{35}^{\mathbb{P},i} \\ \kappa_{41}^{\mathbb{P},i} & \kappa_{42}^{\mathbb{P},i} & \kappa_{43}^{\mathbb{P},i} & \kappa_{44}^{\mathbb{P},i} & \kappa_{45}^{\mathbb{P},i} \\ \kappa_{51}^{\mathbb{P},i} & \kappa_{52}^{\mathbb{P},i} & \kappa_{53}^{\mathbb{P},i} & \kappa_{54}^{\mathbb{P},i} & \kappa_{55}^{\mathbb{P},i} \end{bmatrix} \tag{2.35}$$

$$\textit{Diagonal } \kappa \textit{ matrix} \quad \kappa = \begin{bmatrix} \kappa_{11}^{\mathbb{P},i} & & & & \\ & \kappa_{22}^{\mathbb{P},i} & & & \\ & & \kappa_{33}^{\mathbb{P},i} & & \\ & & & \kappa_{44}^{\mathbb{P},i} & \\ & & & & \kappa_{55}^{\mathbb{P},i} \end{bmatrix} \tag{2.36}$$

We can calculate the yield at a given time t for a time to maturity τ and company i as

$$\begin{aligned} y_t^i(\tau) = L_t^T + \frac{1-e^{-\phi\tau}}{\phi\tau} S_t^T + \left[\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right] C_t^T + a_1(\tau) \\ +\alpha_{L^T}^i L_t^T + \alpha_{S^T}^i \frac{1-e^{-\phi\tau}}{\phi\tau} S_t^T + \alpha_{S^T}^i \left[\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right] C_t^T + a_2(\tau) \\ +\alpha_0^i + \alpha_L^i L_t^S(i) + \alpha_S^i \frac{1-e^{-\phi_i\tau}}{\phi_i\tau} S_t^S(i) + a_3(\tau) \end{aligned} \tag{2.37}$$

The credit spread, s_t^i , is calculated as $s_t^i = y_t^i - y_t^T$, where y_t^T is the treasury yield and first specified in equation 2.29 and also found on the first row in eq. 2.37. This gives us

$$\begin{aligned} s_t^i(\tau) = \alpha_{L^T}^i L_t^T + \alpha_{S^T}^i \frac{1-e^{-\phi\tau}}{\phi\tau} S_t^T + \alpha_{S^T}^i \left[\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right] C_t^T + a_2(\tau) \\ +\alpha_0^i + \alpha_{L^S}^i L_t^S(i) + \alpha_{S^S}^i S_t^S(i) \frac{1-e^{-\phi_i\tau}}{\phi_i\tau} + a_3(\tau) \end{aligned} \tag{2.38}$$

where

$$a_2(\tau) = -\frac{\sigma^2_{L^T}(\alpha^i_{L^T})^2}{6}\tau^2 -$$
$$\sigma^2_{S^T}(\alpha^i_{S^T})^2\left(\frac{1}{2\phi^2} - \frac{1-e^{-\phi\tau}}{\phi^3\tau} + \frac{1-e^{-2\phi\tau}}{4\phi^3\tau}\right)$$
$$-\sigma^2_{C^T}(\alpha^i_{S^T})^2\left(\frac{1}{2\phi^2} + \frac{1}{\phi^2}e^{-\phi\tau} - \frac{1}{4\phi}\tau e^{-2\phi\tau}\right.$$
$$\left.-\frac{3}{4\phi^2}e^{-2\phi\tau} - \frac{2}{\phi^3\tau}1 - e^{-\phi\tau} + \frac{5}{8\phi^3\tau}1 - e^{-2\phi\tau}\right) \quad (2.39)$$

and

$$a_3(\tau) = -\frac{(\sigma^i_{L^S})^2(\alpha^i_{L^S})^2}{6}\tau^2 - (\sigma^i_{S^S})^2(\alpha^i_{S^S})^2\left(\frac{1}{2\phi^2} - \frac{1-e^{-\phi\tau}}{\phi^3\tau} + \frac{1-e^{-2\phi\tau}}{4\phi^3\tau}\right) \quad (2.40)$$

$a_1(\tau)$ is expressed similarly as in 2.30, $a_2(\tau)$ and $a_3(\tau)$ is expressed as in 2.39 and 2.40. As can be seen, the model takes into account that the credit spread is possibly not only affected by it's own corresponding state variables $L^S_t(i)$ and $S^S_t(i)$, but also by the level, slope and curvature of the underlying treasury curve.

2.4.3 Comparing specifications of models by Likelihood ratio test

When different cases of for example κ matrices are considered the likelihood of each specification is compared and evaluated as specified in a likelihood ratio test.

Comment 2.1. *Likelihood ratio test*
A likelihood ratio test can only be performed on nested models i.e. where a more complex model can be transformed into a simpler model with a set of constraints.

$$D = 2\Big(\ln(likelihood\ for\ alternative\ model) - \ln(likelihood\ for\ null\ model)\Big) \sim \chi^2(Degrees\ of\ freedom) \quad (2.41)$$

Degrees of freedom is calculated as $D.f. = (number\ of\ parameters\ in\ alternative\ model) - (number\ of\ parameters\ in\ null\ model)$

Now the probability of observing D in the $\chi^2(D.f.)$ *distribution can be calculated, i.e. the p-value, if this probability is lower than a chosen level of significance* α *the alternative model can be ruled out/rejected. If however p-value* $> \alpha$*, the alternative model can't be rejected.*

2.5 The Kalman filter

A Kalman filter is a method with which one can estimate hidden state variables when observing a process which is a result of the hidden variables as well as some (Gaussian) noise. A linear operator is applied to the state to generate a new state with some noise mixed in. Then another linear operator and more noise is applied and generates the true (hidden) state. We use the Kalman filter within a negative log-likelihood function which we minimize in order to estimate the state variables $(L_t^T, S_t^T, C_t^T, L_t^S, S_t^S)$.

The following section explains how the Kalman filter was applied for both the treasury model as well as for the credit model. For the reader not familiar with the Kalman filter we recommend reading the original article (Kalman, 1960).

2.5.1 Applied to treasury rates

We have time series of zero coupon yields denoted R for maturities τ. We make an estimate of the hidden state variables $x_t = (L_t^T, S_t^T, C_t^T, L_t^S, S_t^S)$ via the Kalman filter.

Definition 2.8. *Initial estimates and calculations for filter equations as in Definition 2.9*

$$x_0 = \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, \qquad P_0 = \int_0^\infty e^{-\kappa s} \sigma \sigma' e^{-\kappa' s} ds,$$

$$Q = \int_0^{dt} e^{-\kappa s} \sigma \sigma' e^{-\kappa' s} ds, \qquad F = e^{-\kappa dt},$$

$$\sigma \text{ and } \kappa \text{ are defined as in } 2.27 \text{ and } 2.28,$$

$$H = \begin{bmatrix} 1 & \frac{1}{\tau_1} \frac{1-e^{-\phi\tau_1}}{\phi} & \frac{1}{\tau_1}\left(\frac{1-e^{-\phi\tau_1}}{\phi} - \tau_1 e^{-\phi\tau_1} \right) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1}{\tau_n} \frac{1-e^{-\phi\tau_n}}{\phi} & \frac{1}{\tau_n}\left(\frac{1-e^{-\phi\tau_n}}{\phi}\phi - \tau_n e^{-\phi\tau_n} \right) \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \sigma^2_{\epsilon^T}(\tau_1) & 0 & \dots & 0 \\ 0 & \sigma^2_{\epsilon^T}(\tau_2) & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2_{\epsilon^T}(\tau_n) \end{bmatrix}$$

Q is the state noise covariance, H is the observation matrix and Ω is the observation noise matrix.

Definition 2.9. *Filter Equations*

Prior state estimates

$$\hat{x}_{t|t-1} = (I - F)\theta + F\hat{x}_{t-1|t-1}$$
$$P_{t|t-1} = FP_{t-1|t-1}F' + Q$$

Measurement update

$$\eta = R - a - H\hat{x}_{t|t-1} \tag{2.42}$$

$$S = HP_{t|t-1}H' + \Omega$$
$$K = P_{t|t-1}H'S^{-1}$$

Posterior state Estimates

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K\eta$$
$$P_{t|t} = (I - KH)P_{t|t-1}$$

a in 2.42 is the yield adjusting term, defined as in 2.30.

If you are interested in the derivation of the prior state estimates as well as the measurement update you can find a full derivation in Christensen *et al.* (2011).

2.5.2 Applied to the credit model

When estimating the parameters for the credit model, both treasury and single name credit rates were used. This gives us the following filter equation system:

Definition 2.10. *Initial estimates and calculations for filter equations as in Definition 2.11*

$$x_0 = \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}, \qquad P_0 = \int_0^\infty e^{-\kappa s}\sigma\sigma' e^{-\kappa' s} ds,$$

$$Q = \int_0^{dt} e^{-\kappa s}\sigma\sigma' e^{-\kappa' s} ds, \qquad F = e^{-\kappa dt},$$

where σ and κ are defined as in 2.34 and 2.35,

$$H = \begin{bmatrix}
1 & \frac{1}{\tau_1}\frac{1-e^{-\phi\tau_1}}{\phi} & \frac{1}{\tau_1}\left(\frac{1-e^{-\phi\tau_1}}{\phi} - \tau_1 e^{-\phi\tau_1}\right) & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & \frac{1}{\tau_n}\frac{1-e^{-\phi\tau_n}}{\phi} & \frac{1}{\tau_n}\left(\frac{1-e^{-\phi\tau_n}}{\phi} - \tau_n e^{-\phi\tau_n}\right) & 0 & 0 \\
\alpha^i_{L^T} & \alpha^i_{S^T}\frac{1}{\tau_1}\frac{1-e^{-\phi\tau_1}}{\phi} & \alpha^i_{S^T}\frac{1}{\tau_1}\left(\frac{1-e^{-\phi\tau_1}}{\phi} - \tau_1 e^{-\phi\tau_1}\right) & \alpha^i_{L^S} & \alpha^i_{S^S}\frac{1-e^{-\phi_C\tau_1}}{\phi_C\tau_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\alpha^i_{L^T} & \alpha^i_{S^T}\frac{1}{\tau_n}\frac{1-e^{-\phi\tau_n}}{\phi} & \alpha^i_{S^T}\frac{1}{\tau_n}\left(\frac{1-e^{-\phi\tau_n}}{\phi} - \tau_n e^{-\phi\tau_n}\right) & \alpha^i_{L^S} & \alpha^i_{S^S}\frac{1-e^{-\phi_C\tau_n}}{\phi_C\tau_n}
\end{bmatrix},$$

$$\Omega = \begin{bmatrix} \sigma^2_{\epsilon^T}(\tau_1) & 0 & \dots & \dots & \dots & 0 \\ 0 & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \sigma^2_{\epsilon^T}(\tau_n) & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \sigma^2_{\epsilon^i} & \vdots & 0 \\ 0 & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \sigma^2_{\epsilon^i} \end{bmatrix}$$

where $i \in \{SHB,\ SWD,\ SEB,\ NDA\}$

Definition 2.11. *Filter Equations*

$$\hat{x}_{t|t-1} = (I - F)\theta + F\hat{x}_{t-1|t-1} \tag{2.43}$$
$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \tag{2.44}$$

Measurement update

$$\eta = [R,\ C]' - a - H\hat{x}_{t|t-1} \tag{2.45}$$

$$S = HP_{t|t-1}H' + \Omega$$
$$K = P_{t|t-1}H'S^{-1}$$

Posterior state Estimates

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K\eta$$
$$P_{t|t} = (I - KH)P_{t|t-1}$$

a as in eq. 2.45 is the yield adjusting term, defined as $a = \begin{bmatrix} a_1(\tau) \\ a_2(\tau) + a_3(\tau) \end{bmatrix}$ *where $a_1(\tau),\ a_2(\tau)$ and $a_3(\tau)$ are defined as in equation 2.30, 2.39 and 2.40.*

2.6 Maximum Likelihood Estimation

When estimating the parameters we use Maximum Likelihood Estimation (MLE) as presented in chapter 5 in Lindstrom *et al.* (2015)

$$\hat{\theta}_{MLE} = \underset{\theta \in \Theta}{argmax} L(\theta) \tag{2.46}$$

where

$$L(\theta) = p_\theta(x_0, \dots, x_n) \tag{2.47}$$
$$= \prod_{n=1}^{N} \left(p_\theta(x_n | x_{n-1}, \dots, x_0) \right) (p_\theta(x_0)) \tag{2.48}$$

The argument maximization of $L(\theta)$ is not affected by a logarithmic transformation and thus a simpler optimization can be attained as

$$\hat{\theta}_{MLE} = \underset{\theta\in\Theta}{argmax} \log p_\theta(x_0) + \sum_{n=1}^{N} \log p_\theta(x_n|x_0, \ldots, x_{n-1}) \tag{2.49}$$

The MLE parameter estimate is consistent under rather general conditions.

2.7 Standard deviation, Fisher Information and the Hessian

In order to determine the standard deviation for the parameters estimated with maximum likelihood estimation (MLE), we need following results as presented in chapter 32 in Cramér (1946)

Definition 2.12. *The probability density function for X and also the log-likelihood function for θ is denoted as $p_\theta(X)$. Let θ be a $N \times 1$ vector and the Fisher Information (FIM) $I_{i,j}(\theta)$ will take the form of an $N \times N$ matrix with elements as*

$$I_{i,j}(\theta) = \mathbb{E}_X\left[\left(\frac{\partial}{\partial\theta_i}\log p_\theta(X)\right)\left(\frac{\partial}{\partial\theta_j}\log p_\theta(X)\right)\middle|\theta\right]. \tag{2.50}$$

Under certain regularity conditions, the FIM may also be written as

$$I_{i,j}(\theta) = -\mathbb{E}_X\left[\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log p_\theta(X)\middle|\theta\right] = -\mathbb{E}_X[H_{i,j}(\theta)|\theta] \tag{2.51}$$

Where $H_{i,j}(\theta)$ denote the Hessian matrix.

Since we minimize the negative log-likelihood instead of maximizing the positive log-likelihood we get the FIM by calculating the Hessian of the negative log-likelihood straight off. Having the FIM we can calculate the standard deviations.

Definition 2.13. *Let $\hat{\theta}_{MLE}$ be the $N \times 1$ vector of MLE parameters then*

$$Var(\hat{\theta}_{MLE}) = [I_{i,j}(\hat{\theta}_{MLE})]^{-1} \tag{2.52}$$

For the asymptotic distribution of $\hat{\theta}_{MLE}$ we can write

$$\hat{\theta}_{MLE} \overset{a}{\sim} \mathcal{N}\big(\theta_0, \big[I_{i,j}(\hat{\theta}_{MLE})\big]^{-1}\big) \tag{2.53}$$

where θ_0 are the true parameters. The standard deviation is finally

$$SD(\hat{\theta}_{MLE}) = \sqrt{[I_{i,j}(\hat{\theta}_{MLE})]^{-1}} = \sqrt{[-\mathbb{E}_X[H_{i,j}(\hat{\theta}_{MLE})|\hat{\theta}_{MLE}]]^{-1}} \tag{2.54}$$

2.7.1 Numerical calculation of the Hessian matrix

Since we are dependent on an optimization algorithm to produce a Hessian for our minimization we want to verify this Hessian by numerical approximation. We use the same approximation as on page 202 in Nocedal & Wright (2006).

$$\frac{\delta^2 f}{\delta x_i \delta x_j}(x) = \frac{f(x + \epsilon e_i + \epsilon e_j) - f(x + \epsilon e_i) - f(x + \epsilon e_j) + f(x)}{\epsilon^2} + O(\epsilon) \tag{2.55}$$

where $O(\epsilon)$ is the measurement error.

2.8 Root mean squared error RMSE

Given a sample of out-of-sample forecasts h days ahead of t, with a N number of τ_i we can compute the RMSE.

$$RMSE_m(\tau_i) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}^{(\tau_i)}_{t+h|m} - y^{(\tau_i)}_{t+h}\right)^2} \tag{2.56}$$

3. Methodology

This chapter will go through data, models and the application of the theory described in the previous chapter and describe how these have been implemented in our thesis. All discussion referred to rating are from the S&P rating-scale.

3.1 Terms for Analysis

3.1.1 Segmentation of Swedish Wholesale Funding

We define the Swedish wholesale funding market as the wholesale funding issued by SHB, SWD, SEB and Nordea.

Table 3.1: Percentage of Long Swedish Wholesale Funding

Long Swedish Wholesale Funding	EUR	USD	SEK	Total
Covered Bonds	17%	5%	52%	73%
Senior Unsecured Bonds	15%	9%	2%	27%
Total per Currency	32%	14%	54%	

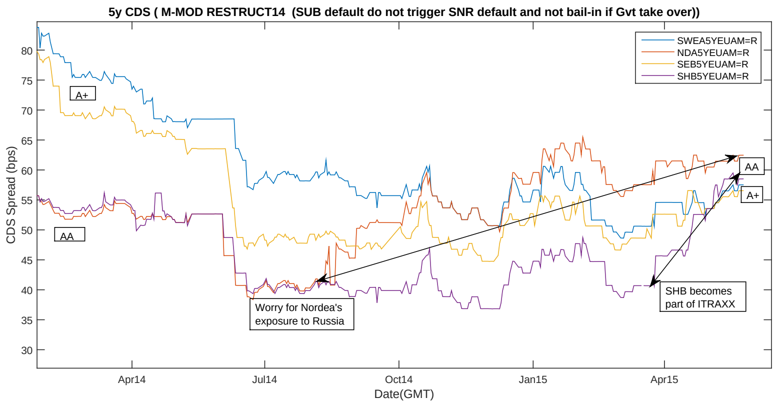
Source: Bloomberg and Issuer per Q2 2014 (CAST profile and IR reports)

As can be seen we have the highest portion in SEK covered, but since we are not interested in covered bonds, our best choice is, as can be seen, the EUR senior unsecured market, representing 15 % of the Swedish Long Wholesale Funding market.

3.1.2 Finding a suitable estimation method

The most obvious approach for modelling the credit risk of corporate bonds from single-names (i.e. not from a bulk of companies in a certain sector) is by using the implied default probability derived from credit default swaps (CDS). Brigo & Mercurio (2006) give an in-depth explanation of how this is done. For the companies we are interested in modelling there exists CDS for all of them, and they are all publicly traded. However, after discussions with our supervisor at Swedbank, we came to the conclusion that the traded CDS were far to illiquid and misleading to give an implication of the credit risk for these companies.

Figure 3.1: 5y CDS spread (bps) for Swedish Banks



As can be seen in figure 3.1 the CDS spreads May-2015 imply that SHB and Nordea who are AA-rated cost more to insure than SEB and Swedbank who are A+-rated. We can only speculate but the reason for this may lie in an increased demand for the SHB and Nordea CDSs. This demand could have been driven by SHB becoming a constituent of the latest series of the ITRAXX European Senior Financials index and possibly the market has overreacted on Nordea's exposure to Russia. Even if we can't isolate and prove the reason for this abnormality we can conclude that the CDSs are misleading in an evaluation of single name credit risk.

The natural choice was therefore modelling the credit risk from the outstanding debt that the companies have issued. All four companies are reliant on funding by issuing bonds and have issued bonds with relevant maturities for the scope. The main problem with this approach is that it is not as well discussed within the field as the CDS approach. Therefore a lot of time was spent reading relevant literature to find which model choice best fit our needs. We ended up using a modification of the Nelson-Siegel model. Further explanation of why this was a relevant model can be found in subsection 3.2.2.

3.1.3 The choice of EUR senior market

The Euro-denominated senior-debt market is the best alternative among the available markets to perform the analysis for two main reasons.

Firstly the senior unsecured market (i.e. unsecured unsubordinated debt) was chosen since the difference in credit spread across the banks better reflects the markets perception of the level of risk in investing in the banks. The banks also issue a significant amount of covered bonds, which are collaterized. These bonds have a structured security mass (cover pool). This cover pool usually consists of assets in the form mortgage loans to the private sector. The covered bonds issued from these pools all have AAA-rating for all of the banks which we are interested in looking at. Due to this fact, we decided that looking at the covered bond market was less relevant for our purpose, even though a substantial amount of all of the banks total funding is by issuance of covered bonds.

Secondly, the choice of currency was a choice among the major funding markets; EUR, USD and SEK, the markets in which the banks are most active. Initially we had the ambition to model all of these markets simultaneously, but after contemplation we realized that this would make the problem far more complex without necessarily giving a better explanation of the yield curves of the banks.

The SEK senior market was ruled out due to the issuance of senior debt in this market being relatively small. Most of the funding in SEK for the banks we are investigating is done by issuance covered bonds.

In the USD market there are documentational differences (144A vs 3(a)2 vs SecReg) and peculiarities that affect especially non-us domestic issuers and especially the banks in scope. This affects the prices of the bonds in the secondary and primary market and consequently the market's perceived risk of investing in the bond is not fully captured by a USD senior market analysis. I.e. the Swedish banks issue their debt under different programs, where 144A reaches more investors, making the bonds more liquid.

The EUR senior market however, is a market where the banks in scope issue a significant amount of bonds under the same conditions, regardless of bank. These bonds are also liquid as there are many investors in this market.

3.1.4 Restrictions in data

We operate in an environment where data is available only if banks have chosen to fund themselves via debt securities in a structured way. Before the financial crisis the majority of debt was taken up with maturity of 1-2 years. This was done mainly by issuing certificates and commercial papers for debt maturing in the coming year and as senior for debt up to two years. During the crisis in 2008 and 2009 the investors became reluctant to make new investments, making the market illiquid. This made it difficult to issue new debt at reasonable rates. Gradually, after the crisis, the banks were able to issue new debt, but mostly in the form as covered bonds but some in senior as well even if the cost was very high. It was not until 2010-2011 that the banks of interest started issuing relevant amounts of senior debt with maturities longer than 2 years. This means that EUR senior data for nodes further out on a banks yield curve are very scarce for dates before 2011. A sober variety of maturities can first be seen at 2012-2013. This market summary is the main reason for missing data but there

are other reasons too. Even if our analysis has led us to believe that the EUR senior market is the most suitable for our purposes to compare funding costs the choice of market is still up to each bank and they all have different views on when, for how long and in which currency the bank should issue bonds. These decisions are quite similar across the banks and thus have less of a effect but nonetheless it compromises the period which we can use in the analysis. For example Nordea has maturities spanning from 10 years to 3 months, SHB from 15 years to 3 months, SEB from 7 years to 3 months and Swedbank from 7 years to 3 months. These observations has led us to use data for maturities ranging from 3 months-7 years and from dates after mid 2013 up until April this year (2015).
The German treasury rates were never a restriction in the analysis. The rates very easily retrieved for the relevant maturities and times for our analysis, as Bloomberg has data for these yields for a very long time. The choice of Germany for treasury rates in EUR was due the general view that Germany is the most stable economy in the Eurozone and therefore has the lowest yields, i.e. being closest to what can be considered risk-free.

3.1.5 Estimating Zero coupon yields (ZCY) from yields to maturity (YTM)

We wanted to perform the analysis solely with zero coupon yields as Gurkaynak *et al.* (2007) did with the treasury yield curve and also Christensen & Lopez (2012) in their paper on credit risk modelling. This way we keep the absence of arbitrage in the model. Thus we face the problem that we want to estimate zero coupon yields while we have yields to maturities as input data from respective bank. This is due to the fact that the bonds we are interested in are coupon bonds. In order to estimate zero coupon yields from coupon bearing instruments we use the theory as described in section 2.3. Results from the estimation is illustrated in figure B.9.

3.1.6 Filling missing data by fitting Nelson Siegel

Since not every bank has issued a full spectrum of bonds with yields Y and we only have a number i of available bonds, there will be missing data. Now if we want to estimate the whole yield curve we need to fit a Nelson Siegel model. One of the alternatives would be to linearly interpolate and extrapolate. The problem with this methodology is that data can at some points be scarce and the linear model does not represent the typical form of a yield curve well, which leads to inconsistent values. Hence we fit a Nelson Siegel as specified in 2.23 and visualized in B.1.4. We fit parameters by minimizing the square difference between the data points and a Nelson Siegel curve. The matlab function used performing the minimization of squared differences is `lsqnonlin()`.

3.1.7 Excluding bonds with a floating coupon

The type of bond data we take as input is commonly referred to as fixed coupon bonds. This means that the bond has a fixed coupon opposed to a floating coupon. The reason for excluding floating coupon bonds is that their YTM cannot be determined with certainty. YTM for floaters change after each fixing in the underlying rate. The investor can approximate the YTM via a forward curve or a swap curve but he can never be sure it will be the final YTM if he decides to keep the bond until maturity. Since this estimation impose more uncertainty and the fact that issuance of floating coupon bonds is less frequent than issuance in the form of fixed coupon bonds, we exclude them from this analysis.

3.1.8 Data becomes irrelevant very close to maturity

A debt investor is often interested in holding a fixed income asset in order to receive coupon or a discount at issuance. The incentives of posting correct prices for the bonds decrease the closer the bonds get to maturity. These different periods for the bond are referred to as being "on the run" and "off the run". In our case we found data far from correct just prior to maturity for some bonds which lead us to adjust these values manually as no clear pattern could be identified across the banks. This correction was done for bonds maturing within 1-5 days.

3.1.9 Weekends and Non Trading days

We have left out weekends and Non Trading days leaving us with 260 work days per year and a constant $dt = 1/260$ over all data. In French (1980) and Asai & McAleer (2007) the consequences of this is discussed. Simply put there are statistical significance in weekend and non trading day data but any straightforward way of for example estimate values as last known or as a mean would not resolve the bias and could even infer new complication. Knowing what we have introduced and the gain in computation time we feel comfortable with this choice.

3.1.10 Back-testing model and choice of window

When we control that the model can produce decent forecast we run a back-test to evaluate the performance of the model in terms of RMSE from the true model as described in section 2.8. We have chosen to forecast one week, one month, two months, three months and six months. The estimation window is set between 03-Jun-2013 until 03-Jun-2014.

3.2 Choice of model

3.2.1 PCA analysis

PCA analysis help us determine to what extent a component explains variation in the data. As can be seen in Tables 3.2 - 3.6 the degree of explanation for first, second and third component can be retrieved by studying the bottom line. The first, second and third component are the level, slope and curvature factors Christensen & Lopez (2012).

Table 3.2: Eigenvectors GER

Maturity	First	Second	Third
3M	-0.10	0.60	-0.18
6M	-0.10	0.55	-0.12
1Y	-0.12	0.46	0.09
2Y	-0.17	0.24	0.57
3Y	-0.24	-0.00	0.48
4Y	-0.33	-0.07	0.31
5Y	-0.43	-0.20	0.16
6Y	-0.50	-0.13	-0.16
7Y	-0.57	-0.07	-0.49
Explain	98.39	1.15	0.33

Table 3.3: Eigenvectors SHB

Maturity	First	Second	Third
3M	-0.07	-0.74	-0.04
6M	-0.02	-0.56	0.07
1Y	0.07	-0.32	0.31
2Y	0.22	-0.05	0.59
3Y	0.33	0.09	0.46
4Y	0.39	0.08	0.19
5Y	0.44	0.01	-0.04
6Y	0.48	-0.05	-0.26
7Y	0.51	-0.13	-0.48
Explain	98.27	1.26	0.43

Table 3.4: Eigenvectors SWD

Maturity	First	Second	Third
3M	-0.05	0.63	0.29
6M	-0.06	0.48	0.31
1Y	-0.10	0.23	0.41
2Y	-0.18	-0.15	0.38
3Y	-0.26	-0.31	0.38
4Y	-0.34	-0.30	0.27
5Y	-0.42	-0.16	0.09
6Y	-0.50	0.04	-0.17
7Y	-0.58	0.30	-0.50
Explain	96.85	2.22	0.81

Table 3.5: Eigenvectors SEB

Maturity	First	Second	Third
3M	-0.01	-0.70	-0.15
6M	-0.03	-0.46	-0.33
1Y	-0.09	-0.12	-0.55
2Y	-0.23	0.17	-0.54
3Y	-0.34	0.26	-0.33
4Y	-0.42	0.22	-0.03
5Y	-0.45	0.07	0.10
6Y	-0.47	-0.12	0.25
7Y	-0.46	-0.36	0.31
Explain	97.37	1.79	0.48

The PCA analysis has been performed individually for each bank's data. The analysis supports the framework to exclude curvature in the estimation without losing much explanatory energy.

Table 3.6: Eigenvectors NDA

Maturity	First	Second	Third
3M	-0.14	0.75	0.08
6M	-0.08	0.56	-0.09
1Y	0.03	0.29	-0.37
2Y	0.19	0.03	-0.61
3Y	0.30	-0.05	-0.50
4Y	0.38	-0.03	-0.17
5Y	0.43	0.05	0.09
6Y	0.48	0.11	0.25
7Y	0.53	0.16	0.36
Explain	91.97	7.34	0.58

We can conclude that the explanatory level are quite similar for German Government and SHB. Usually a PCA analysis would tell us that the explanatory levels for SHB are as for SWD and SEB but it seams SHB have similar dynamics as German government and thus have high explanatory energy in the Level variable. SEB and Swedbank have explanatory levels around the same as found for the banks in Christensen & Lopez (2012). Nordea stand out with lower explanatory power in the Level variable and this may be due to the fact that Nordea has more outstanding senior bonds in EUR compared to the other banks. Nordea has more outstanding senior bonds in EUR because they are an active EUR bank and thus their need in EUR is larger than for the others. With less data as in the case for SWD, SEB and SHB the Nelson Siegel filling smoothens the curve so that the level explains more than it does for the not as smooth curve from Nordea.

3.2.2 Nelson Siegel

Based on the findings in Krishnan *et al.* (2007) which is coherent with our own PCA analysis we have decided to use a five-factor credit spread model to explain the treasury and credit spread over time. The five factors are: level, slope and curvature for treasury and: level and slope for each bank. These state variables are parametrized as in a Nelson-Siegel model and we present the details in section 2.4 and subsection 2.4.2. The only alteration to the theory presented is that we fix values for the α-parameters in the actual estimation.

3.2.3 Alternative approach - Merton's model

As described in A.1 we could estimate the value of a firm's debt via the Black and Scholes pricing formula but since this approach imply so many assumptions that take away the essence of the micro structure in the credit spread we will not approach the problem via Merton's model. However it is a simplified view to get a quick estimate of what an investor could expect of the yield for a firms outstanding bonds.

3.3 Verification of model estimation by simulation

Verification that parameters in a model are correctly estimated is paramount when evaluating dynamics and driving factors. Zero coupon yields is the underlying data and the only way to evaluate the optimization is to simulate synthetic zero coupon yields and take note of which parameters we use in the simulation and see how they compare to parameters suggested by the full estimation via the kalman filter. The simulation is quite straightforward and it follows the following equations.

$$\begin{pmatrix} L_t^T & S_t^T & C_t^T & L_t^S & S_t^S \end{pmatrix} = (\mathbb{I}_5 - F)\theta + FX_{t-1} + Q\epsilon_t \tag{3.1}$$

where $\epsilon_t \sim \mathcal{N}(0,1)$

$$Y_t^T(\tau) = \begin{pmatrix} L_t^T & S_t^T & C_t^T \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\tau}\frac{1-e^{-\phi\tau}}{\phi} \\ \frac{1}{\tau}\left(\frac{1-e^{-\phi\tau}}{\phi} - \tau e^{-\phi\tau}\right) \end{pmatrix} + a(\tau) + \sigma^2_{\epsilon^T}(\tau)\eta_t, \tag{3.2}$$

where $\eta_t \sim \mathcal{N}(0,1)$

$$Y_t^S(\tau) = \begin{pmatrix} L_t^T & S_t^T & C_t^T \end{pmatrix} \begin{pmatrix} 1+\alpha_{L^T} \\ (1+\alpha_{S^T})\frac{1-e^{-\phi\tau}}{\phi\tau} \\ (1+\alpha_{S^T})\left[\frac{1-e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right] \end{pmatrix} +$$

$$\begin{pmatrix} L_t^S & S_t^S \end{pmatrix} \begin{pmatrix} \alpha_L^i \\ \alpha_S^i \frac{1-e^{-\phi_i\tau}}{\phi_i\tau} \end{pmatrix} + \alpha_0 + a_1(\tau) + a_2(\tau) + a_3(\tau) + \sigma^2_\epsilon \nu_t \tag{3.3}$$

where $\nu_t \sim \mathcal{N}(0,1)$

As can be seen these are the equations from the theory namely equations 2.37 and 2.29 with the additional noise terms. The method in Matlab to produce $\epsilon_t,\ \eta_t\ and\ \nu_t$ is `randn()`.

With the simulated data and the true parameters we can evaluate the performance of the estimation algorithm. We started off by evaluating the estimation with a generic start-vector, to save place we put the Tables in the appendix section B.1.5. As can be seen the algorithm was quite sensitive for the initial guess which we could mitigate with multiple runs and successively altering the generic start vector and thus avoid local minimums. The parameters in the kappa matrix and the fourth and fifth theta parameters were very sensitive when we ran the optimization with a generic start-vector. This lead us to believe that local minimums are possible with alterations of the kappa matrix and the theta parameters. With this knowledge we exclude any analysis of kappa and theta parameters individually. A more powerful and proven accurate way is to analyse the state variables and their correlations. We also look at forecasts of yield curves and see how they perform for a variation of forecasting periods.

3.3.1 Convergence towards the true parameters

In order to check that our optimization algorithm converges to the true parameters eventually we created a synthetic start-vector and put it into the optimization algorithm. We produced the new start-vector by taking the vector with true parameters and then we added individual noise terms $\epsilon_i \sim \mathcal{N}(0, 0.0001)$ for each parameter i in the true parameter-vector. This mimic the start-vector that we end up with, after we have altered the start-vector and then reran the optimization. We found that the model estimated parameters with a better loglikelihood score and as we expected the parameters converge to their true value. Results from this exercise are shown below.

Table 3.7: Comparison of loglikelihood scores for different start-vectors

	(Negative) loglikelihood
Generic start-vector	27 181
True parameters + ϵ_i	27 223

As can be seen in Table 3.7 we improve the likelihood and thus the fit of the yield-curve with the close to true start-vector as we expected.

Table 3.8: Estimated parameters and standard deviations

Parameter	ϕ	θ_1	θ_2	θ_3	θ_4	θ_5	ϕ_C
Upper	0.7347	0.0367	0.0075	0.0605	0.0570	0.0188	0.7367
Estimate	0.4926	0.0202	-0.0274	0.0268	0.0361	-0.0174	0.5034
Lower	0.2505	0.0036	-0.0623	-0.0070	0.0152	-0.0536	0.2702
True	0.5000	0.0200	-0.0250	0.0270	0.0350	-0.0200	0.5000

Table 3.9: Estimated noise parameters and standard deviations

Parameters	σ_1	σ_2	σ_3	σ_4	σ_5
Upper	0.0169	0.0161	0.0096	0.0545	0.0097
Estimate	0.0004	0.0004	0.0004	0.0003	0.0004
Lower	-0.0161	-0.0152	-0.0088	-0.0538	-0.0089
True	0.0004	0.0005	0.0006	0.0007	0.0008

As can be seen in Table 3.9 we have a negative lower bound for the σ-values. This is due to the fact that we numerically calculate the hessian with a quite large delta making the intervals for these σ-values too wide, the lower bound should be greater or equal to zero.

Table 3.10: Estimated noise parameters and standard deviations

Parameters	$\sigma_{\epsilon^T}(\tau_1)$	$\sigma_{\epsilon^T}(\tau_2)$	$\sigma_{\epsilon^T}(\tau_3)$	$\sigma_{\epsilon^T}(\tau_4)$	$\sigma_{\epsilon^T}(\tau_5)$
Upper	0.0012	0.0015	0.0013	0.0011	0.0010
Estimate	0.0006	0.0008	0.0007	0.0006	0.0005
Lower	0.0000	0.0001	0.0001	0.0000	0.0000
True	0.0006	0.0008	0.0007	0.0006	0.0005

Table 3.11: Estimated noise parameters and standard deviations

...	$\sigma_{\epsilon^T}(\tau_6)$	$\sigma_{\epsilon^T}(\tau_7)$	$\sigma_{\epsilon^T}(\tau_8)$	$\sigma_{\epsilon^T}(\tau_9)$	$\sigma_{\epsilon^{SimCredit}}$
...	0.0008	0.0006	0.0008	0.0009	0.0012
...	0.0004	0.0003	0.0004	0.0005	0.0010
...	0.0000	0.0000	0.0000	0.0000	0.0007
...	0.0004	0.0003	0.0004	0.0005	0.0008

Table 3.12: Estimated Kappa matrix and standard deviations

Parameters	κ_{11}	κ_{12}	κ_{13}	κ_{14}	κ_{15}	...
Upper	4.0473	1.4020	3.5179	2.2125	1.0147	...
Estimate	1.1029	0.4974	0.5856	0.4449	0.5853	...
Lower	-1.8414	-0.4073	-2.3467	-1.3228	0.1560	...
True	1.0000	0.5000	0.5000	0.5000	0.5000	...

Table 3.13: Estimated Kappa matrix and standard deviations

...	κ_{21}	κ_{22}	κ_{23}	κ_{24}	κ_{25}	...
...	4.7315	1.4089	3.4772	3.6512	0.8687	...
...	0.4738	1.1124	0.2906	0.5653	0.4880	...
...	-3.7838	0.8159	-2.8961	-2.5206	0.1072	...
...	0.5000	1.0000	0.5000	0.5000	0.5000	...

Table 3.14: Estimated Kappa matrix and standard deviations

...	κ_{31}	κ_{32}	κ_{33}	κ_{34}	κ_{35}	...
...	1.9349	1.3632	2.6046	1.2506	1.0620	...
...	0.5389	0.5318	0.9377	0.4798	0.5257	...
...	-0.8571	-0.2995	-0.7291	-0.2910	-0.0106	...
...	0.5000	0.5000	1.0000	0.5000	0.5000	...

Table 3.15: Estimated Kappa matrix and standard deviations

...	κ_{41}	κ_{42}	κ_{43}	κ_{44}	κ_{45}	...
...	1.6297	3.9049	3.1692	2.6468	2.2060	...
...	0.5300	0.6232	0.4693	1.1104	0.5977	...
...	-0.5696	-2.6586	-2.2306	-0.4260	-1.0105	...
...	0.5000	0.5000	0.5000	1.0000	0.5000	...

Table 3.16: Estimated Kappa matrix and standard deviations

...	κ_{51}	κ_{52}	κ_{53}	κ_{54}	κ_{55}
...	1.7173	1.5517	3.6637	5.2724	6.4862
...	0.5690	0.5425	0.5973	0.5375	1.1147
...	-0.5794	-0.4667	-2.4691	-4.1975	-4.2569
...	0.5000	0.5000	0.5000	0.5000	1.0000

All 47 free parameters their estimates, upper bound, lower bound and true values have been presented in the Tables above and we can conclude that our model comes very close to the true parameters even if the confidence interval is quite wide we see that the estimates are almost spot on. With this verification of the model we can conclude that the estimation algorithm converge toward the true parameters which was the intention of this exercise.

Table 3.17: True Exponential Kappa Matrix: $e^{-\kappa_{True} dt}$

0.9962	-0.0019	-0.0019	-0.0019	-0.0019
-0.0019	0.9962	-0.0019	-0.0019	-0.0019
-0.0019	-0.0019	0.9962	-0.0019	-0.0019
-0.0019	-0.0019	-0.0019	0.9962	-0.0019
-0.0019	-0.0019	-0.0019	-0.0019	0.9962

Table 3.18: Estimated Kappa Matrix: $e^{-\kappa_{Est} dt}$

0.9958	-0.0019	-0.0022	-0.0017	-0.0022
-0.0018	0.9958	-0.0011	-0.0021	-0.0018
-0.0020	-0.0020	0.9964	-0.0018	-0.0020
-0.0020	-0.0024	-0.0018	0.9958	-0.0023
-0.0022	-0.0021	-0.0023	-0.0020	0.9958

Table 3.19: Descriptive Statistics for true vs estimated State Variables

	L^T	S^T	C^T	L^S	S^S
True Mean	0.0199	-0.0254	0.0276	0.0353	-0.0197
Estimated Mean	0.0200	-0.0255	0.0276	0.0351	-0.0192
MSE	0.0000	0.0000	0.0000	0.0000	0.0000

3.4 Forecast Yield Curves

We forecast future yield curves through a combination of previously mentioned theory and methodology. The forecast periods considered are 1 week, 1 month, 2 months, 3 months and 6 months. We use estimated parameters where we evaluate both cases of the kappa matrix i.e. full and diagonal, as in definition 2.35 and 2.36. We put the estimated parameters and adjust $(T-t)/dt$ in equation 2.43 which yields the predicted state variables. The predicted state variables are then used in equations 2.37 and 2.29 for each τ to produce the full yield-curve. For convenience the forecasting scheme is summarized below.

Where fc is future time of forecast and t is latest date in sample.

Predict state variables

$$\hat{x}_{fc|t} = (I - F)\theta + F\hat{x}_{t|t}$$

Calculate Treasury's forecasted yield curve

$$y_{fc}(\tau) = L_{fc} + S_{fc}\frac{1}{\tau}\frac{1 - e^{-\phi\tau}}{\phi} + C_{fc}\frac{1}{\tau}\left(\frac{1 - e^{-\phi\tau}}{\phi} - \tau e^{-\phi\tau}\right) + a(\tau),$$

Calculate Bank's forecasted yield curve

$$\begin{aligned} y^i_{fc}(\tau) = {} & L^T_{fc} + \frac{1 - e^{-\phi\tau}}{\phi\tau}S^T_{fc} + \left[\frac{1 - e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right]C^T_{fc} + a_1(\tau) \\ & + \alpha^i_{L^T}L^T_{fc} + \alpha^i_{S^T}\frac{1 - e^{-\phi\tau}}{\phi\tau}S^T_{fc} + \alpha^i_{S^T}\left[\frac{1 - e^{-\phi\tau}}{\phi\tau} - e^{-\phi\tau}\right]C^T_{fc} + a_2(\tau) \\ & + \alpha^i_0 + \alpha^i_L L^S_t(i) + \alpha^i_S\frac{1 - e^{-\phi_i\tau}}{\phi_i\tau}S^S_t(i) + a_3(\tau) \end{aligned} \tag{3.4}$$

3.5 Macro data

The macro data we chose to look at was European central bank balance sheet, ECB BS, German Financial Institutions Balance Sheets, GER FI BS and Euro zone GDP, EUR GDP. We gathered the data for the available dates which covered our time period of interest namely the sample period of 2013-Jun to 2014-Jun. Since the macro data was available less frequent than our bond data we performed a linear interpolation for data in between dates. This was necessary since we later want to look at correlations between the state variables estimated by the model and each macro data time series. The interpolated and original time series are visualized in the appendix B.1.2.

4. Results and Analysis

4.1 Estimation Results

This sections gives the results from estimation of the chosen time series by the mentioned models in previous sections. The purpose is to give a better understanding of the parameters and how they interact, essentially giving a better knowledge of how treasury and credit yields change in time.

4.1.1 The Treasury Model

Firstly estimation was performed solely on the treasury curve to give initial estimates of parameters that were used in the full estimation with of both the treasury as well as the corporate bond curve. This way, our first estimation works as a benchmark against all the later performed estimations. The estimated κ and θ for the German treasury zero-coupon yields be found in Table 4.1. The maximized log-likelihood was 18225.82 and ϕ was estimated to 0.2331. We chose to estimate a full κ matrix to give full flexibility.

Table 4.1: κ and θ for the Treasury Model

	κ		θ
8.9567	-5.5072	0.3103	0.0323
4.4632	18.4197	-0.0330	-0.0317
2.8691	20.2816	1.9785	-0.0379

4.1.2 The Credit Spread Model

The estimation was performed for all the banks individually, with the German government bond yield curve as the underlying treasury curve. For each bank we estimated two cases, the full κ-matrix as well as the diagonal κ-matrix, as stated in equation 2.35 and 2.36. In Table 4.2 the maximized log-likelihood for all banks and both κ-matrices are shown. In the rightmost column the likelihood-ratio between the models is shown, which clearly rejects the diagonal κ-matrix for all banks, as the χ^2-quantile with for a difference of 20 degrees of freedom is:

$$\chi^2_{0.005}(20) \cong 40$$

Table 4.2: Max LLH and LR for the Credit Spread model

Banks	Full κ	Diagonal κ	LR
Nordea	32403.45	32134.70	537.48
Svenska Handelsbanken	35039.52	34083.66	1911.72
SEB	32960.75	32288.72	1344.07
Swedbank	31727.47	31526.11	402.71

The estimated full κ-matrices can be seen in Tables 4.3, 4.4, 4.5 and 4.6.

Table 4.3: κ_{SHB}^{Full}

14.4589	12.8798	0.5445	-7.1709	-4.2528
-0.0876	-0.9591	1.1527	4.7835	4.2115
4.9877	0.4516	3.3675	13.5102	11.5590
2.2581	1.5126	0.2219	0.5324	0.1210
5.6390	5.0393	0.5710	1.0252	2.0454

Table 4.4: κ_{NDA}^{Full}

28.8761	0.0904	6.9796	8.0051	-0.0639
0.0756	-0.0061	2.8003	5.7115	7.8296
1.7287	0.5655	5.3472	7.7111	13.2335
7.0871	-1.6546	1.0613	8.9710	0.0024
-0.0395	0.6204	1.5440	0.0627	3.6839

Table 4.5: κ_{SEB}^{Full}

19.1726	16.5129	0.6589	-3.6594	-2.1898
0.0065	-0.0936	1.6574	5.1186	5.3201
4.5659	2.0613	3.9435	18.8496	16.8504
2.2852	1.3025	0.0722	2.9098	-0.0000
1.2963	1.2908	0.6121	0.3140	3.7969

Table 4.6: κ_{SWD}^{Full}

20.8799	0.8133	7.7767	-3.5199	0.0045
0.2684	-0.0009	3.3156	6.6465	6.8532
1.7120	-0.0388	5.5246	11.7964	10.8298
4.9768	0.0161	1.4560	2.2227	-0.0046
-0.1429	0.0990	1.1580	0.1818	2.0699

There are some similarities which can be observed for the mean reversion matrices. For example $\kappa_{1,1}$ is the most dominant factor for the mean reversion rate in all estimations, however we do not perform any deeper analysis for individual kappa values.

4.1.3 Correlations between state variables

Below you will find the estimated state parameters for the period (shown in Graph 4.1, 4.2, 4.3 and 4.4)as well as a number of correlation matrices, describing correlations for state variables both for the banks individually as well as for the Level and Slope for the credit spread between the banks. This is all from the estimation with the full κ-matrix, as this fits the data for the estimation period best.

Table 4.7: NDA state variable correlation

	L_t^T	S_t^T	C_t^T	L_t^S	S_t^S
L_t^T	1	0.1159	-0.7055	0.2434	0.2645
S_t^T		1	-0.6981	0.0738	0.5550
C_t^T			1	0.0553	-0.7224
L_t^S				1	-0.6847
S_t^S					1

These 4 Tables (Tables 4.7, 4.8, 4.9 and 4.10) show the correlation coefficients for the four banks. There are some shared properties throughout which should be noted. L_t^T and L_t^S shows positive correlation in all cases, implying that a shock in the level component of the underlying treasury curve is likely to increase the credit spread. Same thing goes for S_t^T and S_t^S, where the correlation is even stronger. As the slope component has a bigger impact for the shorter maturities, while the level component affects the whole yield curve in the same way, there seems to be evidence of the credit spread being more affected by shocks in the

Table 4.8: SHB state variable correlation

	L_t^T	S_t^T	C_t^T	L_t^S	S_t^S
L_t^T	1	0.4854	-0.7278	0.3635	0.1964
S_t^T		1	-0.8078	0.3856	0.5121
C_t^T			1	-0.0519	-0.6949
L_t^S				1	-0.5777
S_t^S					1

Table 4.9: SEB state variable correlation

	L_t^T	S_t^T	C_t^T	L_t^S	S_t^S
L_t^T	1	0.1691	-0.7014	0.3789	0.2547
S_t^T		1	-0.7262	0.1132	0.6907
C_t^T			1	-0.0512	-0.7663
L_t^S				1	-0.5141
S_t^S					1

lower end of the yield curve than for longer maturities. This is coherent with the general view that bonds with shorter time to maturity are more volatile than those with longer time to maturities in terms of percentage returns. Another conclusion which can be drawn is that pairwise behaviour between the state variables L_t^T and S_t^T is significantly different from L_t^S and S_t^S. The latter shows strong negative correlation while the earlier shows weak positive correlation. While a shock to the treasury level L_t^T might not affect the treasury slope S_t^T significantly, it seems that a shock to the credit spread level L_t^S will impact the credit spread slope S_t^S negatively, resulting in shorter maturities less affected than longer maturities. This implies that an event causing the credit spread level to increase (such as a rating reduction, default event or even an under-performing quarterly report) has a bigger impact for longer maturities. Our interpretation is that the market perceives that the risk for investing in bonds issued by the bank with a shock to the level component of the credit spread level is bigger for longer maturities than shorter, as the uncertainty increases with time. The curvature component C_t^T has strong negative correlation with all other components except for L_t^S. A shock to any of the three components L_t^T, S_t^T, S_t^S decreases the curvature, resulting in a flattening or convexity of the underlying treasury curve. It is especially interesting to notice the strong negative correlation between C_t^T and S_t^S. This could imply that a change in the curvature component of the treasury curve is counteracted to some extent by the slope component of the credit spread. It seems as if local curvature of the German government yields doesn't fully carry on to the yields of the bonds issued by the banks.

Table 4.10: SWD state variable correlation

	L_t^T	S_t^T	C_t^T	L_t^S	S_t^S
L_t^T	1	0.2216	-0.7370	0.6122	-0.0379
S_t^T		1	-0.7581	-0.1936	0.8341
C_t^T			1	-0.1228	-0.5461
L_t^S				1	-0.6485
S_t^S					1

Table 4.11: Intrabank state L_t^S and S_t^S correlation

	$L_t^{S,SHB}$	$L_t^{S,NDA}$	$L_t^{S,SEB}$	$L_t^{S,SWD}$	$S_t^{S,SHB}$	$S_t^{S,NDA}$	$S_t^{S,SEB}$	$S_t^{S,SWD}$
$L_t^{S,SHB}$	1	0.9341	0.9343	0.6740	-0.5777	-0.4686	-0.3534	-0.2411
$L_t^{S,NDA}$		1	0.9466	0.6603	-0.6913	-0.6846	-0.5753	-0.3401
$L_t^{S,SEB}$			1	0.6852	-0.6081	-0.5650	-0.5141	-0.3284
$L_t^{S,SWD}$				1	-0.5765	-0.4741	-0.4681	-0.8060
$S_t^{S,SHB}$					1	0.9411	0.9107	0.6840
$S_t^{S,NDA}$						1	0.9660	0.6322
$S_t^{S,SEB}$							1	0.7039
$S_t^{S,SWD}$								1

Table 4.11 shows the correlation coefficients for the level and slope components of the credit spreads for all the banks. There is a strong correlation between all of the level components, even though the level component of the credit spread for Swedbank stands out as the one with the weakest correlation to the other ones. The same behaviour can be seen from looking at the correlations between the slope components. This is possibly misleading, as Swedbank was the last of the banks to issue major senior benchmark bonds denominated in EUR, resulting in fewer bonds to estimate the full yield curve from. All the individual level components are negatively correlated with all the individual slope components, which coincides with the analysis performed earlier in this section.

Figures 4.1, 4.2, 4.3 and 4.4 show the estimated state variables for the estimation period. They coincide pretty well, with Swedbank as the odd one out, having significantly higher credit spread level component, but at the same time a smaller slope component, indicating Swedbank higher credit spreads for Swedbank for longer maturities.

Figure 4.1: NDA estimated state variables for the period

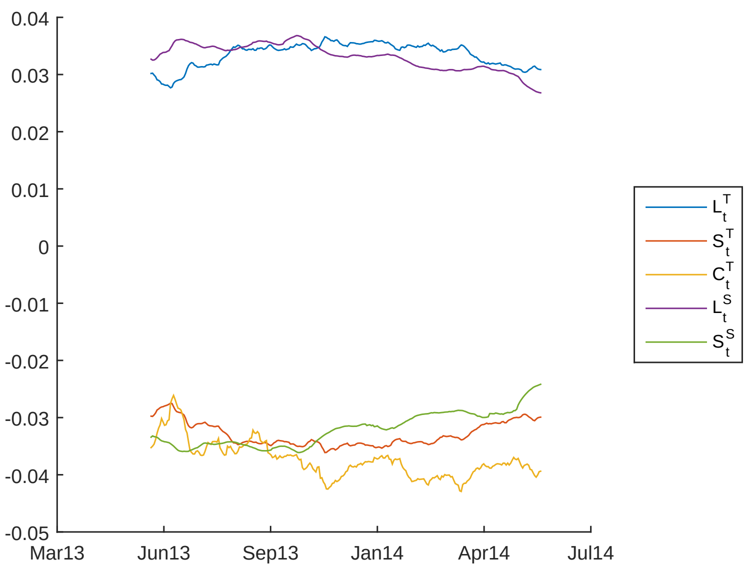


Figure 4.2: SHB estimated state variables for the period

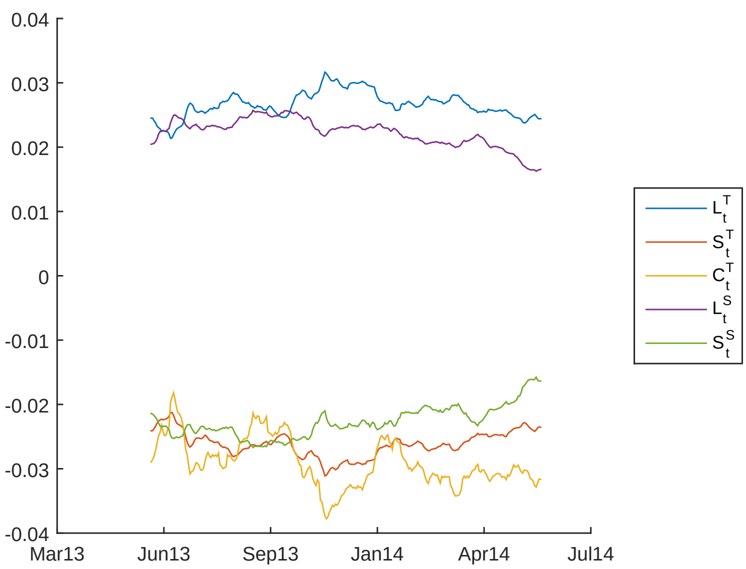


Figure 4.3: SEB estimated state variables for the period

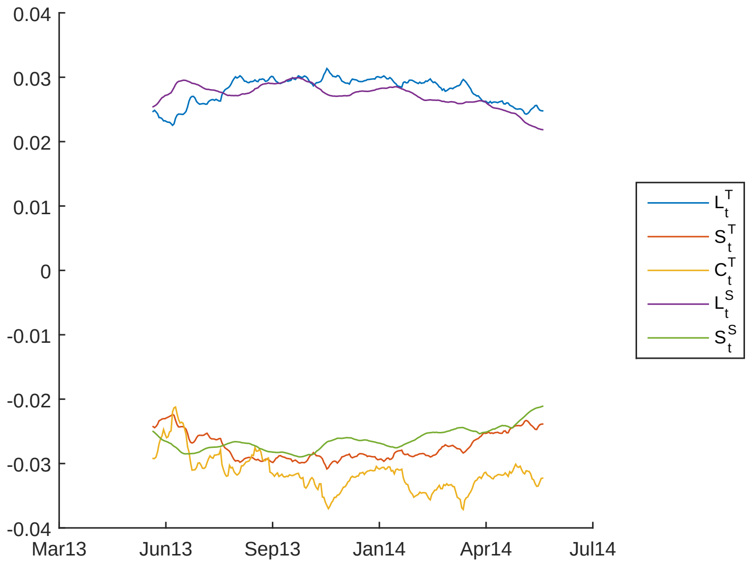
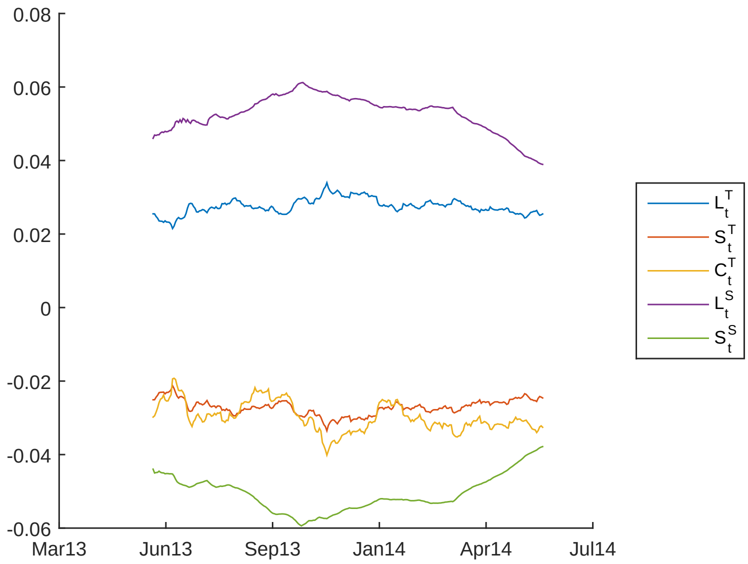


Figure 4.4: SWD estimated state variables for the period



4.1.4 Out of Sample Fit

To estimate the performance of the model in a new environment, out of sample fit was performed for the estimated parameters for all banks for a period of 6 months after the estimation period. The results can be seen in Table 4.12.

Table 4.12: Out of sample LLH and RMSE

	κ^{NDA}_{Full}	κ^{SHB}_{Full}	κ^{SEB}_{Full}	κ^{SWD}_{Full}	κ^{NDA}_{Diag}	κ^{SHB}_{Diag}	κ^{SEB}_{Diag}	κ^{SWD}_{Diag}
LLH	-13740	-21779	-21272	-25644	-26123	-22836	-24942	-25570
RMSE	1.472E-03	3.279E-04	5.385E-04	4.225E-04	4.863E-04	3.364E-03	3.389E-03	3.361E-03

4.2 Forecast results

As described in 3.4 we can forecast future yield curves. Since we have chosen the estimation window as June 2013 to June 2014 we can also verify their accuracy. We evaluate their performance by looking at the real sample data and calculate the RMSE as defined in section 2.8.

Table 4.13: RMSE (%) Full Kappa Matrix

Bank	1 Week	1 Month	2 Months	3 Months	6 Months
SHB	0.0116	0.0775	0.1597	0.2746	0.4756
SWD	0.1053	0.2885	0.4257	0.5687	0.7951
SEB	0.0756	0.1151	0.2469	0.3705	0.5547
NDA	0.3011	0.5737	0.6633	0.7387	0.8074
GER	0.1751	0.1801	0.1752	0.1879	0.2028

Table 4.14: RMSE (%) Diagonal Kappa Matrix

Bank	1 Week	1 Month	2 Months	3 Months	6 Months
SHB	0.2092	0.1963	0.1993	0.2428	0.3949
SWD	0.2149	0.1646	0.1444	0.1695	0.3523
SEB	0.2267	0.1751	0.1732	0.2119	0.3511
NDA	0.1001	0.1258	0.2207	0.3269	0.4496
GER	0.2168	0.2058	0.2039	0.2036	0.2089

Studying the RMSE we can conclude that in general the diagonal kappa matrix outperform the full kappa matrix. As one could expect based on the findings from Christensen & Lopez (2012) the full kappa matrix performs better then the diagonal for forecast periods close in time i.e. 1 week and 1 month. This is natural since the full kappa matrix capture most of today's dynamics and it is quite probable that these are still valid in the 1 week and 1 month forecast. For SHB and German Government however we get better forecast for both 1 and 2 months with the full kappa matrix and if we only look at German Government we have a better performance over all periods.

This result suggest that SHB and German government change less than the other peers and thus their dynamics stays the same for a longer period of time giving the full kappa matrix an edge toward the diagonal kappa matrix. In order to illustrate the forecast performance better we visualize the best-performing forecasts (GER and SHB with Full Kappa matrix) in the figures below.

Figure 4.5: 1 week forecast for GER - Full Kappa Matrix

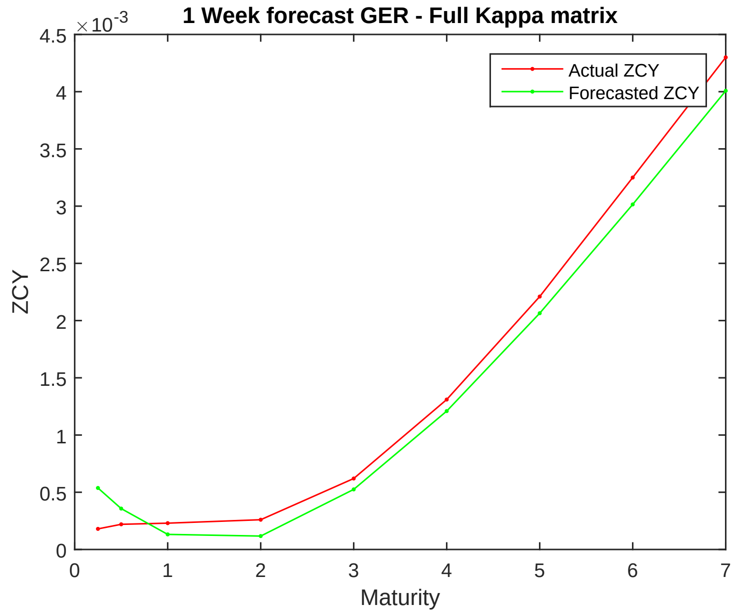


Figure 4.6: 1 month forecast for GER - Full Kappa Matrix

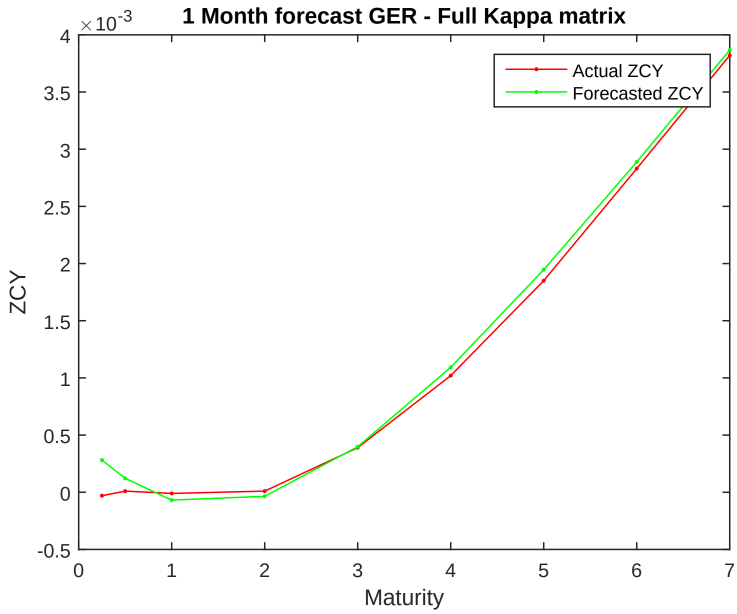


Figure 4.7: 3 months forecast for GER - Full Kappa Matrix

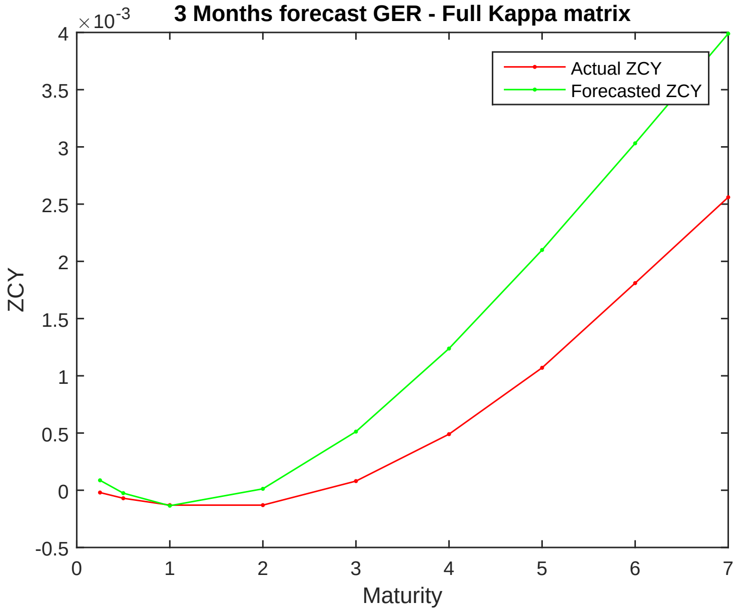


Figure 4.8: 6 months forecast for GER - Full Kappa Matrix

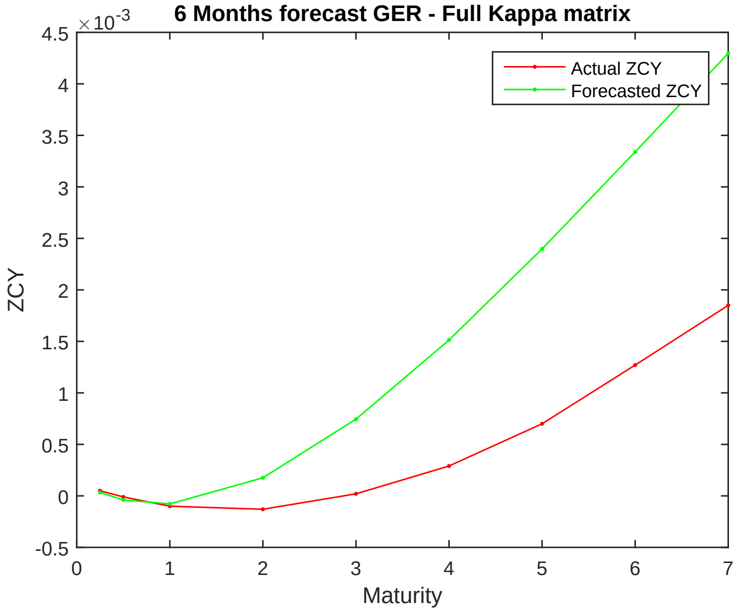


Figure 4.9: 1 week forecast for SHB - Full Kappa Matrix

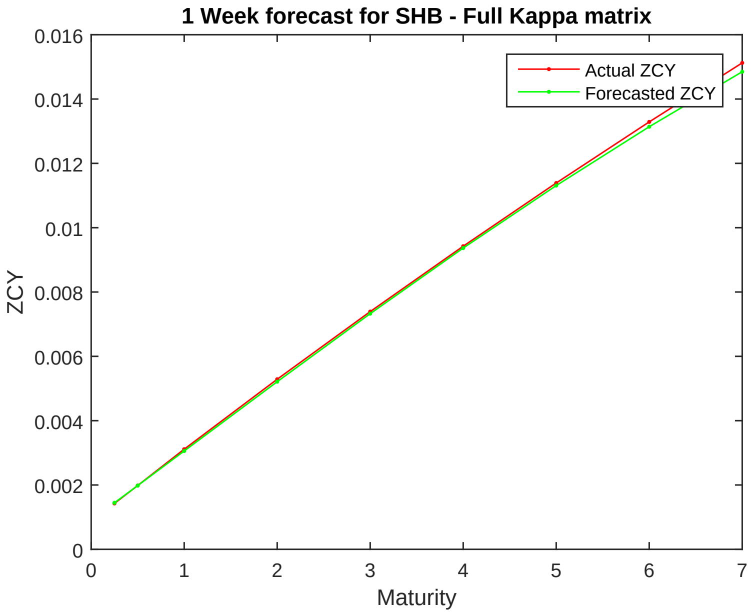


Figure 4.10: 1 month forecast for SHB - Full Kappa Matrix

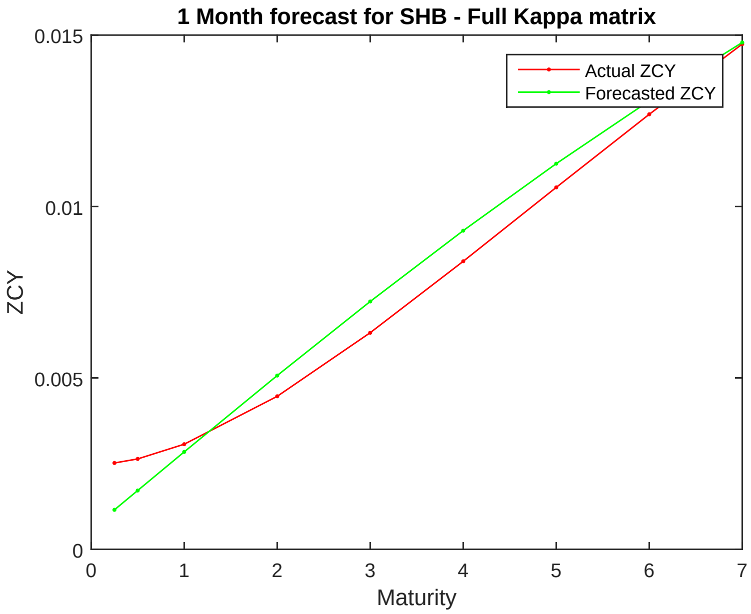


Figure 4.11: 2 months forecast for SHB - Full Kappa Matrix

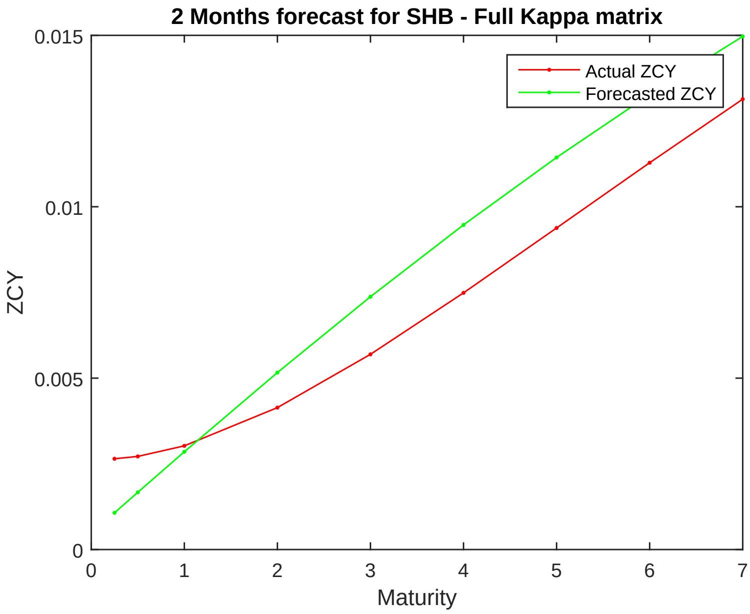
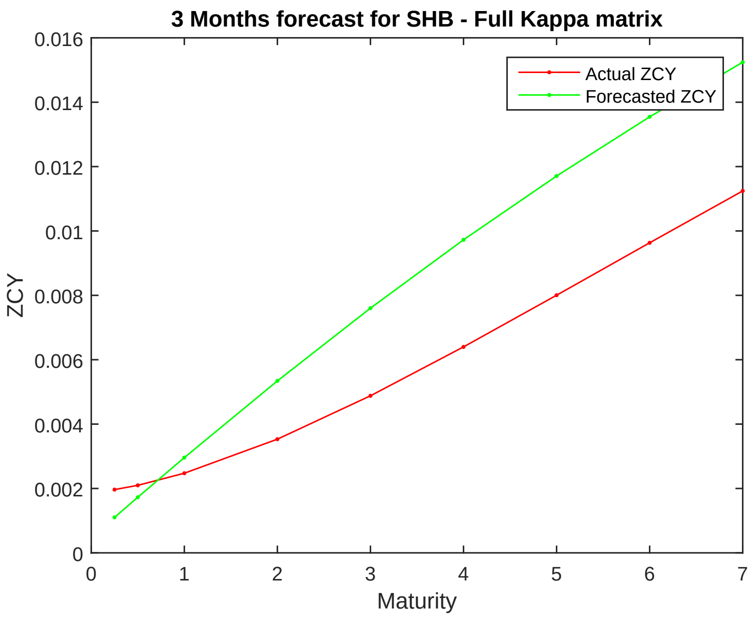


Figure 4.12: 3 months forecast for SHB - Full Kappa Matrix



As can be seen in the graphs for SHB the forecast is above the actual yield. This is the case for the other banks as well. The probable reason for this outcome is that the model predicts a similar dynamics as during the estimated period and the actual dynamics at this time shift the yield curve further down compared to what the model does. This can be mitigated by elaborating with the number of parameters estimated. Either way forecasts for shorter periods such as 1 week and 1 month are rather spot on so for this reason the model works quite well.

4.3 Macro factor correlation with state variables

We denote European Central Bank Balance Sheet as ECB BS, German Financial Institutions Balance Sheet as GER FI BS and Euro zone Gross Domestic Product as EUR GDP. Correlations are calculated on the sample period 03-Jun-2013 until 03-Jun-2014. Visualization of macro data can be found under appendix B.1.2. Comments of the correlations follow each Table.

Table 4.15: Correlations German Government State Variables

	L^T	S^T	C^T	ECB BS	GER FI BS	EU GDP
L^T	1.000	-0.986	-0.642	-0.271	-0.317	0.093
S^T		1.000	0.538	0.157	0.211	0.044
C^T			1.000	0.517	0.402	-0.512
ECB BS				1.000	0.947	-0.948
GER FI BS					1.000	-0.890
EU GDP						1.000

Treasury Level move against ECB BS and it seams that the effects of quantitative easing show in this correlation Table. We expect lower rates with larger ECB BS due to QEs. Treasury Level move against GER FI BS which is unexpected. This may be since a senior bond is not comparable with a government bond. It make sense that the level is not so dependent of the supply of bonds in the senior market or even move against it as this Table suggest.

Table 4.16: Correlations SHB State Variables

	L^{SHB}	S^{SHB}	ECB BS	GER FI BS	EU GDP
L^{SHB}	1.000	-0.983	0.582	0.428	-0.754
S^{SHB}		1.000	-0.604	-0.448	0.758
ECB BS			1.000	0.947	-0.948
GER FI BS				1.000	-0.890
EU GDP					1.000

SHB level moves with ECB BS this is counterintuitive but our hypothesis is that this is a lag-effect. Rates continue to move down from previous QEs even if ECB BS has come down during the last year the effect from before was stronger. SHB Level also move with GER FI BS which supports the statement that the level move down with less supply of bonds on the EUR senior market. These conclusions can not proofed in any wider sense but the hypothesis is at least supported.

Table 4.17: Correlations Swedbank State Variables

	L^{SWD}	S^{SWD}	ECB BS	GER FI BS	EU GDP
L^{SWD}	1.000	-0.819	0.148	0.039	-0.377
S^{SWD}		1.000	0.047	0.131	0.166
ECB BS			1.000	0.947	-0.948
GER FI BS				1.000	-0.890
EU GDP					1.000

SWD show same direction as SHB but but with somewhat weaker correlations.

Table 4.18: Correlations SEB State Variables

	L^{SEB}	S^{SEB}	ECB BS	GER FI BS	EU GDP
L^{SEB}	1.000	-0.979	0.517	0.391	-0.701
S^{SEB}		1.000	-0.584	-0.467	0.759
ECB BS			1.000	0.947	-0.948
GER FI BS				1.000	-0.890
EU GDP					1.000

SEB's Table has same directions as SHB's and similar magnitude of correlations.

Table 4.19: Correlations Nordea State Variables

	L^{NDA}	S^{NDA}	ECB BS	GER FI BS	EU GDP
L^{NDA}	1.000	-0.707	0.723	0.613	-0.859
S^{NDA}		1.000	-0.586	-0.489	0.713
ECB BS			1.000	0.947	-0.948
GER FI BS				1.000	-0.890
EU GDP					1.000

NDA's Table also has same directions as SHB's but with somewhat stronger correlations.

5. Conclusion and suggestions for further research

5.1 Conclusion

In this thesis we have researched the possibilities to model the behaviour of bonds issued by Swedish banks. After evaluating different possible models as in section 3.2.3, we implemented the arbitrage free Nelson-Siegel model to identify the dynamics of the yield curve and forecasting performance for EUR senior unsecured bonds issued by Swedish banks. We show that in-sample-fit not necessarily indicates good out-of-sample fit and forecasting performance for periods longer into the future. However, the specifications giving better in-sample-fit give significantly better forecasting performance not too far in the future. We can show some support for our hypothesises around correlations with macro-data and between treasury and the banks. We have also concluded that there are possibilities to do a lot of minor alterations and modifications to the model, which is discussed in the section 5.2. All in all we can say that we have fulfilled the purpose outlined in section 1.4.

5.2 Further Research

To take this thesis further we recommend to further investigate improvements of the model by altering some of the specifications of it and check for better in sample fit as well as improving forecasting performance and out of sample fit. The most straightforward ways of doing this are to alter the kappa matrix and/or try a different sampling frequency. Since we chose a daily frequency we think the risk of capturing non important dynamics might be higher than if we would for example have tried with a less frequent sampling. The choice of daily sampling was based on the fact that data was scarce and we were under the impression we needed more data points than what a weekly or monthly sampling would have given us. However from today going forward more useful data is becoming available which make a less frequent sampling approach more viable. Another suggestion and a natural step in this field would be to confirm the results we have gotten from the model based on the EUR senior market and look at USD senior market. Even if there are different jurisdiction between the banks it would be interesting to perform the analysis nonetheless. The model we used in this thesis is a so called mean-reversion model, where the state variables revert to a mean level. As there was a downwards trend in the rates for the estimation period (see subsection B.1.1), further research could be conducted to investigate other types of models (we believe that a trending model would be a suitable choice). Last but not least we see that an implementation, integration and automation of the model would be the next step to take this model from being a prototype to be a powerful quantitative analysis tool in the daily work on the funding desk.

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Appendices

A. Appendix A

A.1 Merton's model

Merton (1974) uses the results of Black & Scholes (1973) option pricing model in order to value corporate liabilities. This approach is heavily simplified and assumptions of capital structure, interest rate and default intensity are made. Capital structure of the firm at time t is assumed to consist of equity of value E_t and a zero-coupon bond with a value of $z(t,T)$, face value of D and a maturity T. The firm't asset value is denoted as V_t. At maturity if $V_t > D$ the firm does not default and vice versa. Other assumptions Merton (1974) does is that there are no transaction cost, bankruptcy cost, taxes or problems with indivisibilities of assets. Remaining assumptions are that there is; continuous time trading, unrestricted borrowing ad lending at a constant interest rate r, no restrictions on the short selling of the assets, the value of the firm is invariant under changes in its capital structure and that the firm's asset value follows a diffusion process. The diffusion process under the risk neutral measure is specified as

$$dV_t = rV_t dt + \sigma_V V_t dW_t \tag{A.1}$$

where σ_V denote the assets volatility and W_t is a Brownian motion. Equity-holders and bond-holders can expect following pay-off depending how V_T has evolved.

$$E_T = \max\{V_T - D, 0\}, \tag{A.2}$$

$$z(T,T) = V_T - E_T \tag{A.3}$$

Applying Black Scholes pricing formula, the value of Equity is hence

$$\mathbb{E}_t(V_t, \sigma_V, T-t) = e^{-r(T-t)}[e^{r(T-t)}V_t\Phi(d_1) - D\Phi(d_2)] \tag{A.4}$$

$$d_1 = \frac{\ln(\frac{e^{r(T-t)V_t}}{D}) + \frac{1}{2}\sigma_V^2(T-t)}{\sigma_V\sqrt{T-t}} \tag{A.5}$$

$$d_2 = d_1 - \sigma_V\sqrt{T-t} \tag{A.6}$$

Probability of default would hence be

$$\mathbb{Q}[V_T < D] = \Phi(-d_2) \tag{A.7}$$

And finally the value of debt is $z(t,T) = V_t - E_t$.

A.2 Monte Carlo methods - Sample Paths

In order to sample a path for a given SDE we use Monte Carlo methods. For example if we consider the evolution of the stock price in the standard Black-Scholes model given as

$$dS_t = \mu S_t dt + \sigma dW_t. \tag{A.8}$$

In order to sample this path from time 0 to T we make a partition and we end up with $\delta t = \frac{T}{M}$ where M is the number of simulations. The stock price will then evolve as

$$S(k, \delta t) = S(0)e^{\sum_{i=1}^{k}[(\mu - \frac{\sigma^2}{2})\delta t + \sigma \epsilon_i \sqrt{\delta t}]} \tag{A.9}$$

for each k between 1 and M and each ϵ_i is a draw from a standard normal distribution, $\epsilon_i \sim \mathcal{N}(0, 1)$.

A.3 Discretization with Euler and Maruyama

In order to approximate a numerical solution to a continous SDE we will use a discretization scheme named after Leonard Euler and Gisiro Maruyama (Kloeden & Platen, 1992). Consider the SDE

$$dX_t = a(X_t)dt + b(X_t)dW_t \tag{A.10}$$

With initial condition $X_0 = x_0$ and again W_t is a wiener process. Discretization then follows as

$$0 = \tau_0 < \tau_1 < \ldots < \tau_N = T \quad and \quad \Delta t = \tfrac{T}{N}; \text{ set } Y_0 = x_0$$

$$Y_{n+1} = Y_n + a(Y_n)\Delta t + b(Y_n)\Delta W_n, \tag{A.11}$$

$$\text{where} \quad \Delta W_n = W_{\tau_{n+1}} - W_{\tau_n}$$

B. Appendix B

B.1 Data

B.1.1 Visualization of yield data

Figure B.1: German Government Bonds zero coupon yield Euro

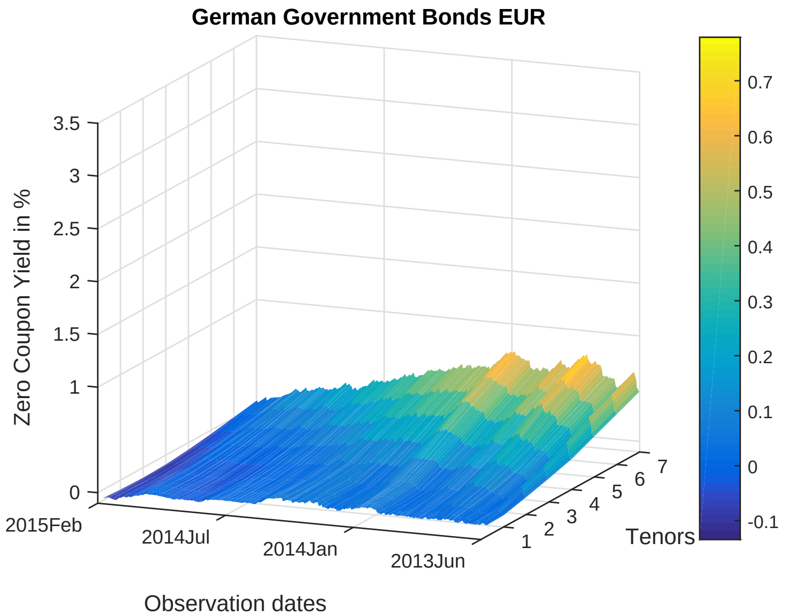


Figure B.2: Nordea senior unsecured zero coupon yield Euro

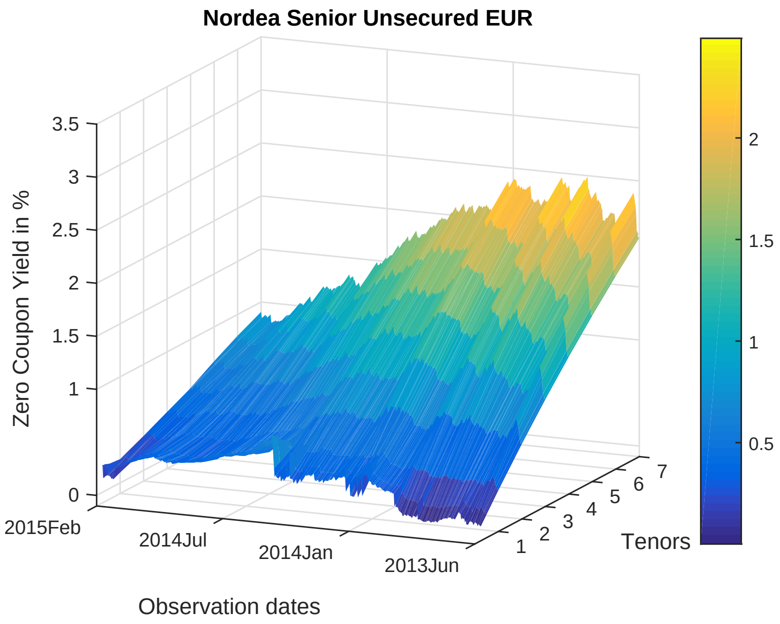


Figure B.3: SHB senior unsecured zero coupon yield Euro

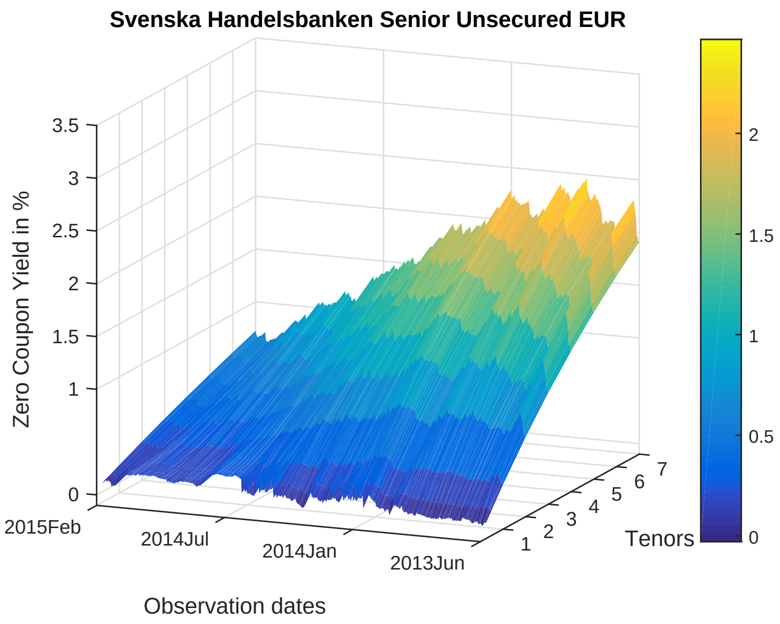


Figure B.4: SEB senior unsecured zero coupon yield Euro

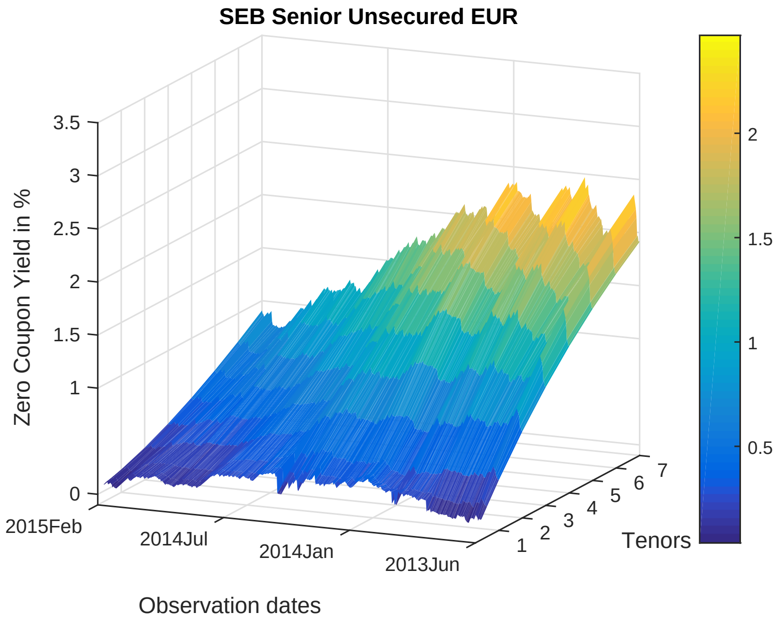
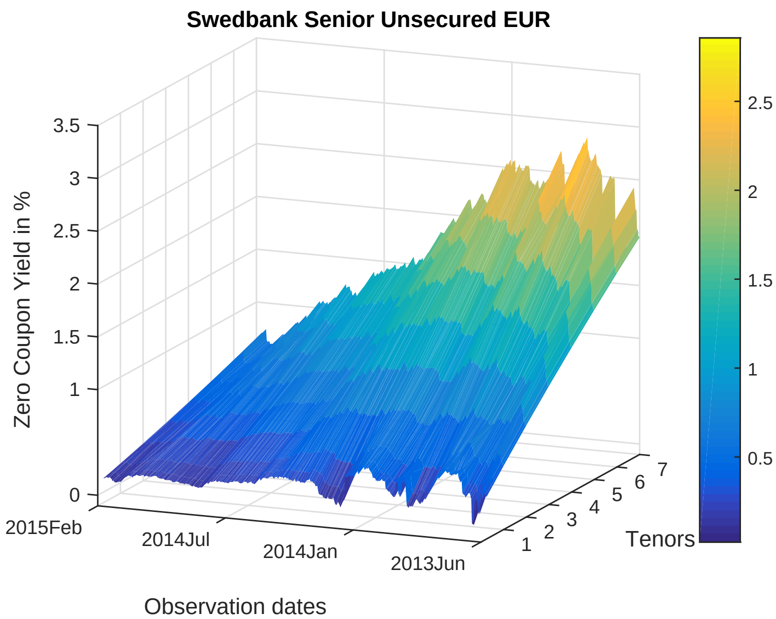


Figure B.5: Swedbank senior unsecured zero coupon yield Euro



B.1.2 Visualization of Macro data

Figure B.6: ECB BS in Billion EUR

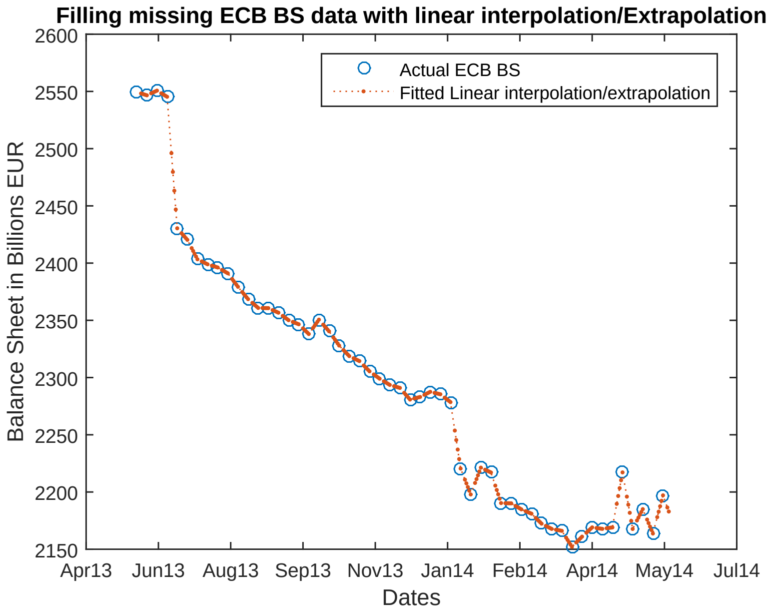


Figure B.7: EU GDP in Billion EUR

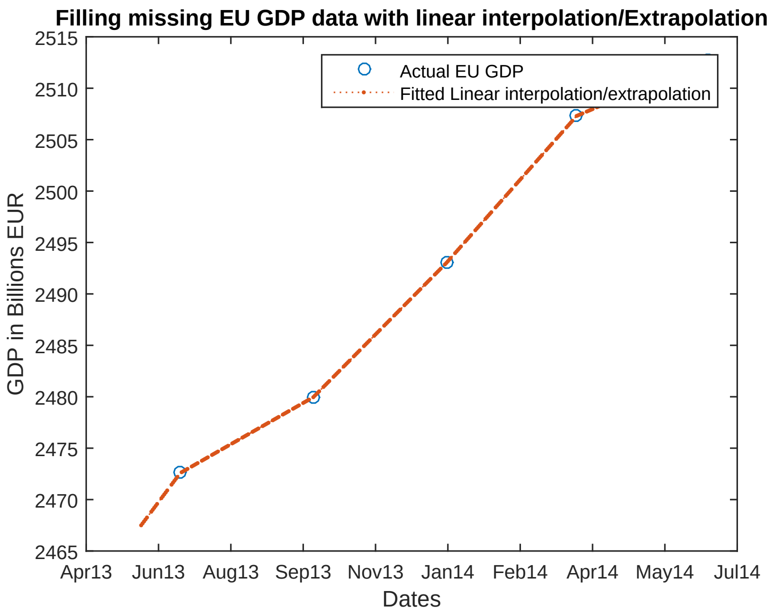
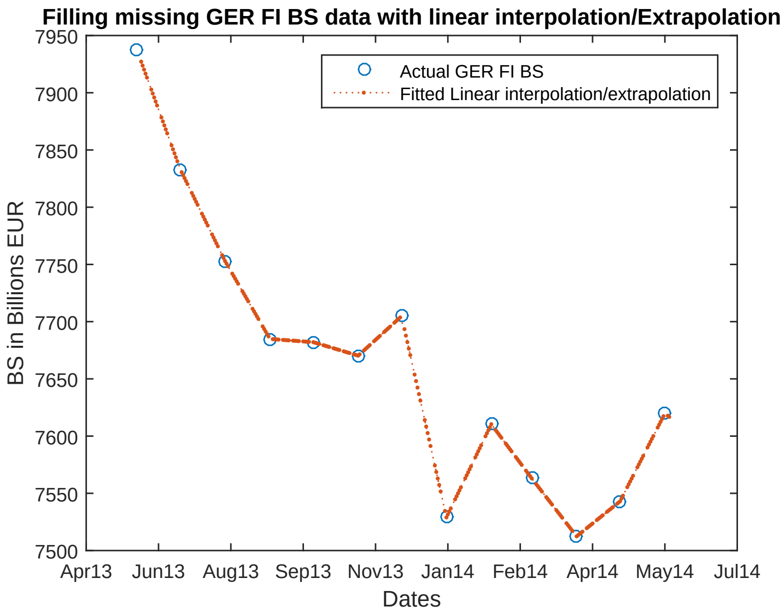


Figure B.8: German Financial Institutions BS in Billion EUR



B.1.3 Descriptive Statistics

Table B.1: Descriptive Statistics GER

Maturity	Mean	Std	Kurtosis	Skewness
3M	0.00	0.05	3.05	-0.73
6M	0.00	0.05	3.02	-0.75
1Y	0.01	0.05	2.71	-0.69
2Y	0.02	0.07	2.35	-0.39
3Y	0.07	0.10	2.03	-0.23
4Y	0.14	0.13	1.86	-0.17
5Y	0.23	0.17	1.81	-0.16
6Y	0.32	0.20	1.81	-0.27
7Y	0.41	0.23	1.83	-0.35

Table B.2: Descriptive Statistics NDA

Maturity	Mean	Std	Kurtosis	Skewness
3M	0.28	0.16	2.39	0.05
6M	0.31	0.12	2.29	0.08
1Y	0.38	0.10	2.37	-0.37
2Y	0.56	0.19	1.54	-0.20
3Y	0.78	0.28	1.45	-0.12
4Y	1.01	0.36	1.47	-0.14
5Y	1.25	0.43	1.50	-0.21
6Y	1.48	0.49	1.55	-0.28
7Y	1.69	0.55	1.60	-0.36

Table B.3: Descriptive Statistics SEB

Maturity	Mean	Std	Kurtosis	Skewness
3M	0.20	0.09	1.61	-0.01
6M	0.23	0.09	2.04	-0.24
1Y	0.32	0.12	1.92	-0.53
2Y	0.52	0.22	1.52	-0.25
3Y	0.75	0.32	1.42	-0.16
4Y	0.97	0.41	1.43	-0.15
5Y	1.20	0.47	1.47	-0.17
6Y	1.41	0.51	1.54	-0.22
7Y	1.62	0.53	1.63	-0.31

Table B.4: Descriptive Statistics SHB

Maturity	Mean	Std	Kurtosis	Skewness
3M	0.12	0.08	2.34	0.19
6M	0.18	0.07	3.77	-0.64
1Y	0.30	0.11	2.44	-0.55
2Y	0.53	0.22	1.50	-0.19
3Y	0.75	0.32	1.44	-0.15
4Y	0.97	0.41	1.46	-0.16
5Y	1.17	0.48	1.51	-0.19
6Y	1.37	0.53	1.55	-0.23
7Y	1.54	0.58	1.61	-0.27

Table B.5: Descriptive Statistics SWD

Maturity	Mean	Std	Kurtosis	Skewness
3M	0.24	0.11	2.91	0.82
6M	0.28	0.11	2.35	0.55
1Y	0.37	0.14	1.89	0.08
2Y	0.56	0.21	1.48	-0.19
3Y	0.77	0.31	1.45	-0.17
4Y	0.99	0.41	1.47	-0.15
5Y	1.21	0.50	1.50	-0.14
6Y	1.43	0.60	1.56	-0.12
7Y	1.65	0.69	1.63	-0.10

B.1.4 Visualization of filling missing data with NS-curve fitting

Figure B.9: SHB filling missing data with NS-curve fitting 17-Jun-2013

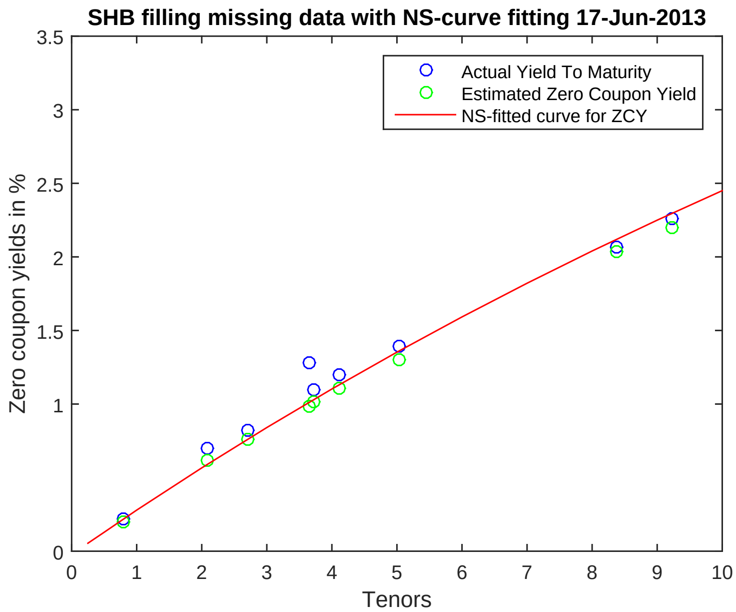


Figure B.10: SWD filling missing data with NS-curve fitting 17-Jun-2013

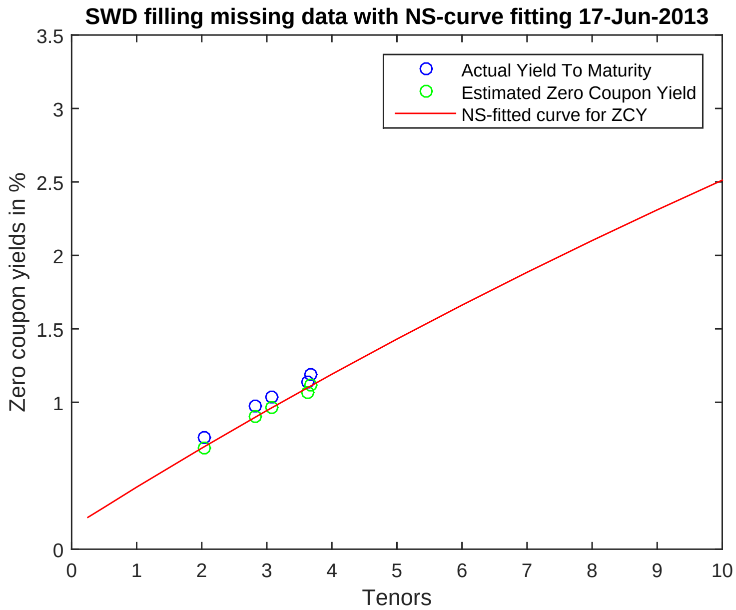


Figure B.11: SEB filling missing data with NS-curve fitting 17-Jun-2013

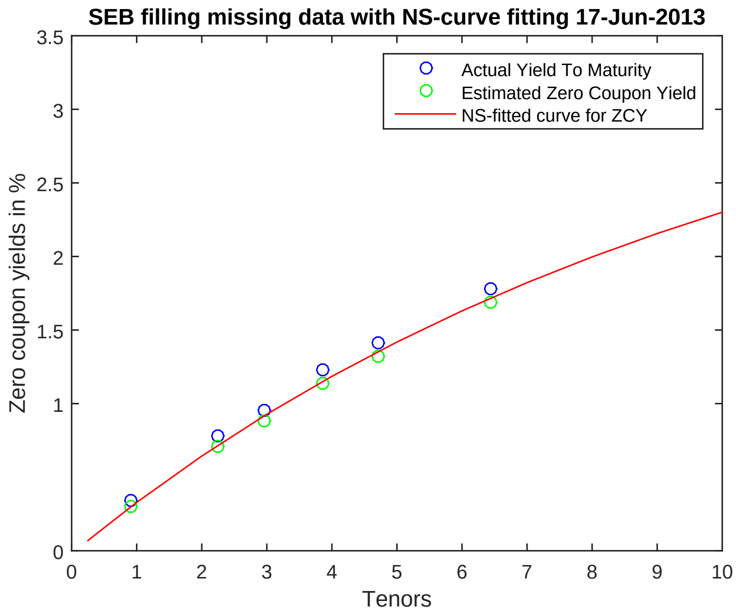
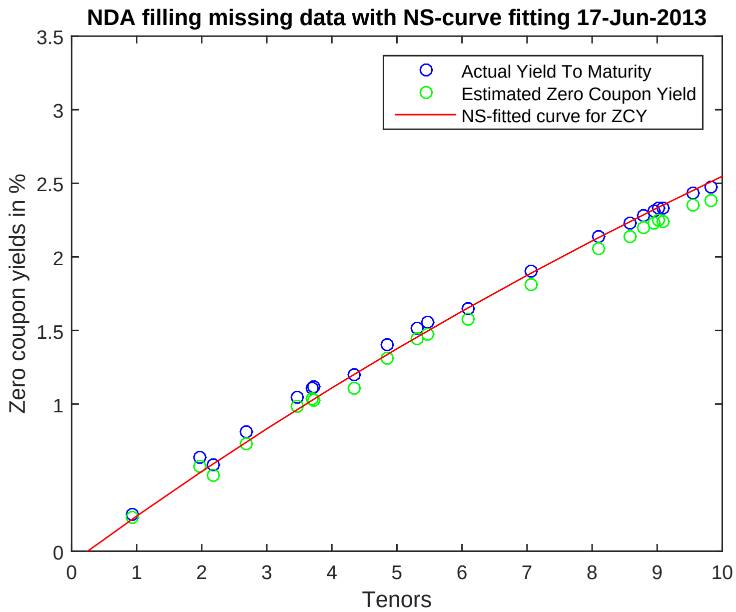


Figure B.12: NDA filling missing data with NS-curve fitting 17-Jun-2013



B.1.5 Estimation data for generic start vector

Figure B.13: Evaluation of State Variables - Simulated vs. Estimated

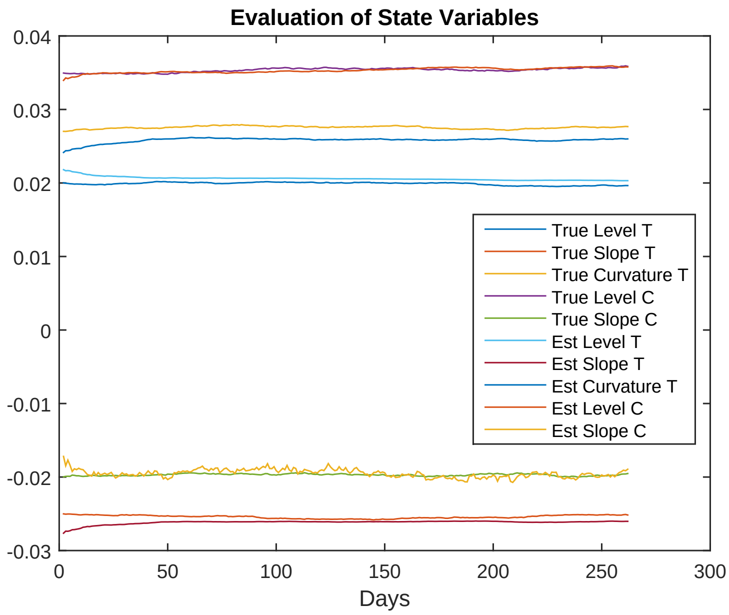


Figure B.14: Descriptive Statistics for true vs estimated State Variables

	L^T	S^T	C^T	L^S	S^S
True Mean	0.019905	-0.025412	0.027553	0.035349	-0.019700
Estimated Mean	0.020606	-0.026164	0.025827	0.035309	-0.019497
MSE	0.000001	0.000001	0.000003	0.000000	0.000000

Table B.6: Estimated parameters and standard deviations

Parameter	ϕ	θ_1	θ_2	θ_3	θ_4	θ_5	ϕ_C
Upper	0.509157	0.069589	-0.021437	0.032079	0.002043	0.226371	0.501897
Estimate	0.509063	0.040117	-0.050857	0.000628	0.000619	0.219265	0.501689
Lower	0.508969	0.010645	-0.080278	-0.030824	-0.000805	0.212158	0.501481
True	0.500000	0.020000	-0.025000	0.027000	0.035000	-0.020000	0.500000

Table B.7: True one day mean rev. matrix: $e^{-\kappa_{True}dt}$

0.996198	-0.001896	-0.001896	-0.001896	-0.001896
-0.001896	0.996198	-0.001896	-0.001896	-0.001896
-0.001896	-0.001896	0.996198	-0.001896	-0.001896
-0.001896	-0.001896	-0.001896	0.996198	-0.001896
-0.001896	-0.001896	-0.001896	-0.001896	0.996198

Table B.8: Estimated one day mean rev. matrix: $e^{-\kappa_{Est}dt}$

0.975880	0.008394	-0.013705	-0.008896	0.000108
-0.002700	0.989299	-0.002211	0.007972	0.000030
-0.005026	0.024849	0.969866	-0.000250	-0.000249
-0.074396	0.006945	-0.052389	0.984784	-0.000966
0.021079	-0.001880	-0.008110	-0.042071	0.990939

Master's Theses in Mathematical Sciences 2015:E33
ISSN 1404-6342

LUTFMS-3285-2015

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