LUND UNIVERSITY

School of Economics and Management

NEKP01 Master's Thesis
2015-08-19

Measuring Risk for WTI Crude Oil

An application of Parametric Expected Shortfall

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2015-08-19

Abstract

Oil is the most traded commodity in the world and is an important part in the global economy. The change in the price of oil has an effect on all sectors of the economy, and the ability to capture its risk is an important research topic. This study calculates the risk of one benchmark crude oil (West Texas Intermediate) over the period 1986-2015 by estimating the Value-at-Risk (VaR) and the Expected Shortfall (ES) on daily spot returns. More specifically, this is done by using a GARCH (1, 1) model with the normal distribution, the t-distribution, and the Generalized Error Distribution (GED). The study uses a rolling window to estimate these risk measurements creating 7125 estimates for each distribution in each tail. The normal distribution was the worst performing distribution on both ES and VaR according to the backtests. The t-distribution performed good ES estimates; however it was not as accurate when calculating VaR. The GED performed the best when calculating VaR but constantly underestimated ES. The main conclusion is that both GED and the t-distribution are needed when estimating the risk for WTI.

Keywords: Risk, Expected shortfall, VaR, Backtesting, Crude oil

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List of Abbreviations

CC	Conditional Coverage
DOF	Degree of Freedom
ES	Expected Shortfall
GED	Generalized Error Distribution
LR	Likelihood Ratio
PDF	Probability Density Function
UC	Unconditional Coverage
VaR	Value-at-Risk
WTI	West Texas Intermediate

1. Introduction

1.1 Background

Oil is the most traded commodity in the world and forty percent of the world's energy originates from crude oil. [1] There are numerous types of crude oils, and these are priced in relationship to the two main crude oil benchmarks that sets the world price of oil. Specifically, the West Texas Intermediate (WTI), which is the US benchmark, and the Brent crude, which is the European Benchmark.[2] The spot price of these benchmark are immensely important since: "The prices of these benchmarks are used by oil companies and traders to price cargoes under long-term contracts or in spot market transactions; by futures exchanges for the settlement of their financial contracts; by banks and companies for the settlement of derivative instruments such as swap contracts; and by governments for taxation purposes."[3] The price of the benchmarks has been notoriously volatile since the collapse of the OPEC pricing system, and the increased competition and deregulation since the 1980's.[4] These high fluctuations in price are driven by political events, such as the Iraqi invasion of Kuwait, when the price almost doubled in response to the invasion, but also by the business cycle that drives supply and demand imbalances.[5] The price of oil can have an important effect on government finances in oil producing countries, since the budget is balanced for a certain oil price, and any deviation from this price could lead to large deficits or surpluses.[6] The risks in these benchmarks are of great interest because of the impact the price has on both the public and private sector in the world economy.[7] The high volatility of oil prices and the increased focus on risk management in recent years are the reasons for this study. The study will therefore estimate the risk of oil prices using methods in the forefront of today's research. Next section will explore the different risk measurement tools most used to estimate the risk of an asset, which will be followed by a literature review and the outline of the objective of this study.

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[1] International Energy Agency (2014) Key World Energy STATISTICS [Online].
Available: http://www.iea.org/publications/freepublications/publication/KeyWorld2014.pdf [Accessed 12 May 2015]

[2] Edwards, Davis W. "Energy Trading and Investing." (2010) p. 142

[3] Fattouh, Bassam. An anatomy of the crude oil pricing system. Oxford, England: Oxford Institute for Energy Studies, 2011 p. 24

[4] Ibid p. 6

[5] Giot Pierre, and Sébastien Laurent. "Market risk in commodity markets: a VaR approach." Energy Economics 25.5 (2003): p. 435-457. p. 437

[6] Farzanegan, Mohammad Reza, and Gunther Markwardt. "The effects of oil price shocks on the Iranian economy." *Energy Economics* 31.1 (2009): 134-151.

[7] Kilian, Lutz. "The economic effects of energy price shocks." (2007).

1.2 Risk measurements

There has been an increased focus on risk management in the last 20 years. An expansion of the financial markets and derivative trading, along with an extensive list of companies suffering financial disaster because of improper risk management have spurred the development of better risk management practices.[8] The next section will explain the most widespread methods for calculating the risk on asset returns.

1.2.1 Value-at-Risk [9]

The most commonly used market risk measurement tool is the Value-at-Risk (VaR). This measurement was introduced by Morgan Stanley in the document Riskmetrics in 1994, and was adopted by the Basel accord to be used by regulators to calculate the capital requirements of banks.[10] The study has the point of view of two companies where one has a long position, and the other has a short position, on a portfolio of one barrel of oil. The risk is therefore the relative price changes of this barrel, i.e. the arithmetic returns according to the following formula: $R_t = 100 * \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)$. Since this risk measure is focused on the distribution of returns, VaR_α is defined as the largest return, such that the probability of observing a return less than this is equal to 1-α, equation 1.[11]

$$VaR_\alpha = \Pr(R \leq -VaR_\propto) \leq 1 - \alpha^{12} \qquad (1)$$

The VaR_α can therefore be seen as the α-quantile of the return distribution. Which position is taken dictates which tail under the return distribution is considered the risk, a short position, the right tail, while a long position the left tail of the return distribution. On a long position an asset with a correctly estimated VaR will suffer a negative return greater than –VaR, a so-called VaR violation, with a probability of 1-α over the holding period.[13] Common choices for α is 99% and 95% meaning that a VaR violation will occur on average once every 100 days for α=99%, and five times per 100 days for α=95%. The 99% VaR for an asset for which the returns are normally distributed, with variance one and mean zero is 2.326. Therefore a negative return greater than -2.326% will occur

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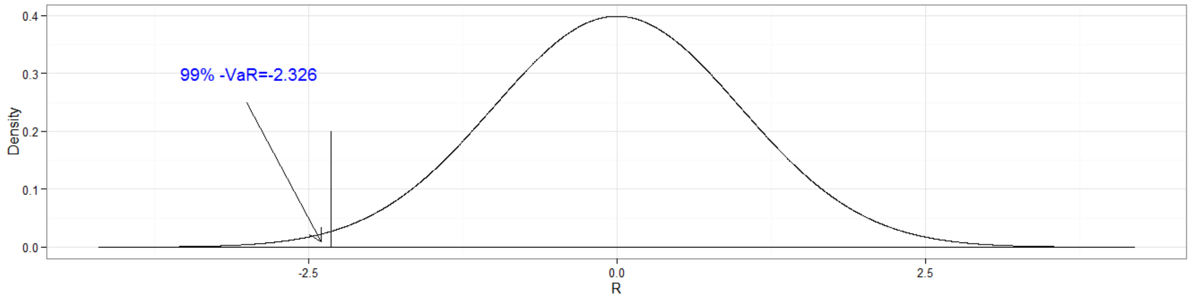
[8] Dowd, Kevin. Measuring market risk. John Wiley & Sons, 2005. p. 1-4
[9] Ibid p. 30
[10] Chen, James Ming. "Measuring Market Risk Under the Basel Accords: VaR, Stressed VaR, and Expected Shortfall." *Stressed VaR, and Expected Shortfall (March 19, 2014)* 8 (2014): 184-201.
[11] Nilsson, Birger(2014) "*Value-at risk"* lecture notes in. *NEKN83/TEK180 spring 2014. Lund University* p. 2
[12] Nilsson, Birger(2014) "*Value-at- risk"* p. 2
[13] The holding period determines how far in the future the VaR estimates are calculated for. The VaR for holding periods over one day the calculations are as follows, $\sqrt{Holdingperiod} * VaR$. For regulatory purposes it is 10 days, however in this study the holding period is one day, and will therefore be omitted in all formulas.

on average once every 100 days, meaning that the area to the left of -2.326 under the probability density function is 0.01=1-α. (Graph 1)



Graph 1: Illustration of parametric VaR, created in R.

The advantages of VaR are that it is easily understood and intuitive, as it is probabilistic (i.e. the company will suffer a return loss greater than Y with probability X). Other attractive properties of VaR are that it is common consistent measurement over different positions as it can be applied to almost all types of portfolios, whether it is equity or a currency portfolio, as well as its ability to aggregate the risk of different sub positions.[14] These attractive properties are the reasons why VaR is so widely adopted by regulators and companies alike.[15] However, there are several disadvantages to VaR which are not to be underestimated.

VaR does not reveal anything about the size of the return loss given that a VaR violation has occurred, a so-called tail event.[16] This drawback is not negligible as two different assets with similar VaR estimate can have different risk properties, as its behaviors in the tails are not taken into consideration when estimating VaR. Also, if traders in a company are limited to how much VaR their positions can have it could incentivize them to take positions that are not beneficial to their employer. By taking positions that suffer small losses unless a tail event occurs, at which point a very large loss occurs, the trader can take a riskier position than the company wants the trader to take.[17]Another major drawback of VaR is that it is not a subadditive risk measurement. This means that this risk measure does not encourage diversification as two assets separately can produce a lower VaR estimate than a portfolio of them combined. "The failure of VaR to be subadditive is a fundamental problem because it means that VaR has no claim to be regarded as a 'proper' risk measure at all. A VaR is merely a quantile. It has uses as a

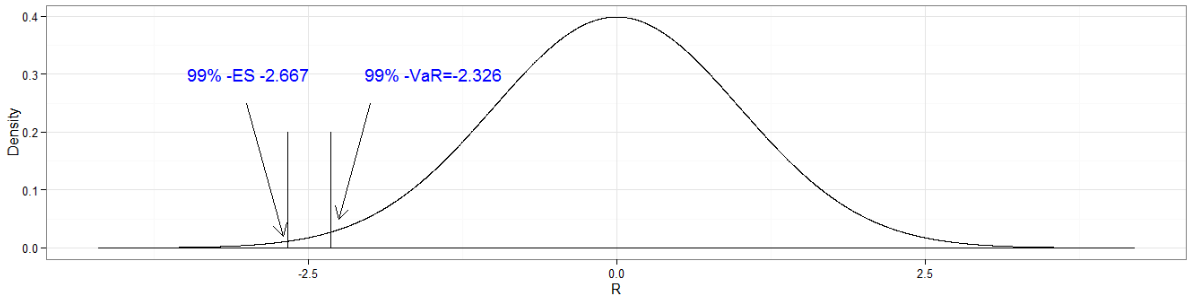
[14] Dowd 2005 p. 12
[15]Ibid p. 13
[16] Ibid p. 13
[17] Ibid p. 14

quantile, but is very unsatisfactory as a risk measurement."[18] Due to the drawbacks of VaR the measurement expected shortfall (ES) is proposed

1.2.2 Expected Shortfall [19] [20]

Expected shortfall (ES) measures the size of the return given a tail event, i.e. it is the expected return given a VaR violation. ES is therefore subordinate VaR, since the VaR computations need to be performed first in order to calculate ES. By definition, ES will always be greater than VaR. Equation 2 below provides the mathematical definition of Expected Shortfall in the continuous case, where f(R) is the probability density function of the returns.

$$-ES_{\propto,t}(R) = \frac{1}{1-\propto}\int_{-\infty}^{VaR_{\propto,t}(R)} R * f(R)dR \qquad (2)$$



Graph 2: Comparison of VaR and ES

ES measures the expected returns when a tail event occurs, and therefore reveals what is to be expected in a bad state.[21] Following the case of normally distributed returns with mean 0 and variance 1, the expected negative return is -2.667% given that the loss is greater than -2.328%. In addition to providing information on the tail behavior to the risk managers, ES is also considered to be an improved measurement compared to VaR since it is always subadditive. Two assets in a portfolio will produce equal or lower ES estimates compared to calculating ES on the assets separately, and thereby encouraging diversification in the portfolio. ES also makes it more difficult for traders to take positions that are not beneficial to the company since it is harder to "optimize" in the way explained earlier, as the tail behavior of the returns is taken into consideration. For

[18] Dowd 2005 p. 34
[19] Dowd p. 34
[20] Other names for ES are Expected Tail Loss(ETL), Conditional Value at Risk (CVaR) , and Average Value at Risk (AVaR).
[21] Dowd 2005 p. 34

all the reasons above, ES is generally considered to be a superior risk measurement compared to VaR. [22]

While ES is considered to be a better risk measurement than VaR, the financial regulations have been slow to switch from VaR to ES. One of the main reasons for this is that there is no consensus on which method is best used to backtest the ES estimates.[23] Some say ES is not even possible to backtest since is it does not have the property of elicitability,[24] this is however disputed. [25] [26] Nevertheless, the literature provides some backtesting procedures which make it possible to test the ES estimates.

1.3 Literature review

The objective of this section is to give an overview of the results of the existing literature regarding parametric estimations for both ES and VaR on WTI crude oil. This will form the basis for the objective of the study.

Chen and Hung calculated one day ahead conditional VaR on WTI spot returns using rolling window estimation on daily data from January 2002 to March 2009.[27] The GARCH (1, 1) with normal distribution performed worse than the GED and t-distribution over the out of sample period from January 2003 to March 2009. At a 99% confidence, level the GED and the t-distribution performed equally well. Fan et al. estimated in sample VaR using various GARCH with the normal distribution and GED, for daily logarithmic WTI spot returns over the period from May 1987 to August 2005. [28] GARCH (1, 1) performed better compared to any other number of lags for the normal GARCH model, but the TGARCH model performed slightly better results than the GARCH. The results indicate that negative shocks have more effect on the volatility than positive ones. They also concluded that WTI price returns have excess kurtosis, also known as fat tail properties, and that GED performed better than the normal distribution at the 99%

[22] Dowd 2005 pp. 35

[23] Acerbi, Carlo, and Balazs Szekely. "Back-testing expected shortfall." *Risk* 27.11 (2014). p. 2

[24]). Chen, James Ming. "Measuring Market Risk Under the Basel Accords: VaR, Stressed VaR, and Expected Shortfall." *Stressed VaR, and Expected Shortfall (March 19, 2014)* 8 (2014): 184-201. p. 197

[25] Acerbi and Szekely 2014 p. 2

[26] A discussion of elicitability is outside the scope of this thesis. The interested reader can refer to the sources cited.

[27] Cheng, Wan-Hsiu, and Jui-Cheng Hung. "Skewness and leptokurtosis in GARCH-typed VaR estimation of petroleum and metal asset returns." *Journal of Empirical Finance* 18.1 (2011): 160-173

[28] Fan, Ying, et al. "Estimating 'Value at Risk'of crude oil price and its spillover effect using the GED-GARCH approach." *Energy Economics* 30.6 (2008): 3156-3171

level in both tails. The normal distribution consistently underestimated the risk. On a 95% level both distributions performed well and no statistical difference was discovered.[29] This is in complete contradiction to Xiliang and Xi. They found that GARCH with GED is the best model for calculating VaR on logarithmic returns at a 95% confidence level. In addition they found that GARCH with normal distribution performed best at a 99% confidence level. This result was obtained by applying a rolling window estimation over the out sample period from October 2004 to November 2008.[30]

Hung, Lee, and Liu estimated VaR for one-day-ahead logarithmic WTI spot price returns using GARCH with the heavy-tailed distribution, normal distribution and t-distribution. [31] The study used observations from November 1996 to September 2006 of which the last 500 observations are the out of sample period. Applying a rolling window to calculate VaR, GARCH with the t-distribution and the normal distribution performed poorly at low confidence intervals while at high they performed well. The heavy tail distribution was the most accurate and most efficient measure at all confidence intervals except for $\alpha \geq 99\%$, where it was not statistically different from the normal and t-distribution.

Almli and Rege used data from July 1996 to April 2011 to estimate ES and VaR for WTI. [32] Their estimations were done by applying a rolling window to forecast the one day ahead risk measure on the last 500 observations for both a long, and short position on futures data. They concluded that the normal distribution was the worst performing distribution for the GARCH VaR estimate while the t-distribution performed better. The GED was neither best nor worst, and absolute best performing distribution was the skewed t-distribution. When estimating ES, the normal distribution produced more accurate results compared to the t-distribution and GED, which both consistently overestimated the risk.

[29] Fan et al. 2008

[30] Xiliang, Zhao, and Zhu Xi. "Estimation of Value-at-Risk for Energy Commodities via CAViaR Model." *Cutting-Edge Research Topics on Multiple Criteria Decision Making*. Springer Berlin Heidelberg, 2009. 429-437

[31] Hung, Jui-Cheng, Ming-Chih Lee, and Hung-Chun Liu. "Estimation of value-at-risk for energy commodities via fat-tailed GARCH models." *Energy Economics* 30.3 (2008): 1173-1191

[32] Almli, Eldar Nikolai, and Torstein Rege. "Risk Modelling in Energy Markets: A Value at Risk and Expected Shortfall Approach." (2011).

Aloui and Marouk estimated the one day ahead VaR and ES on WTI spot using data from January 1986 to March 2007, and concluded that WTI does not follow a normal distribution, as the series have excess kurtosis and is asymmetric to the left.[33] The result showed that a skewed t-distribution performed better compared to a symmetric t-distribution.

When reviewing previous literature some questions emerge. Which distribution produces the best estimates for conditional VaR and ES estimates? What role if any, does the backtesting procedure have when evaluating the ES estimates? These questions lead to the objective of this study.

1.4 Objective

The objective of this study is to answer the two following questions.

- **(i)** When calculating parametric 99% 1-day ahead ES on WTI returns, what distribution is superior?
- **(ii)** Is the best distribution for estimating ES also the best for estimating VaR?

Given the previous research the expectation of the results are as follows:

E(i): The fat tailed distributions will be better at estimating ES compared to the normal distribution, and the GED will be superior to the t-distribution.

E(ii): The same distribution that provides the best VaR estimate will also provide the best ES estimate.

By evaluating (i) and (ii) over the period 1986-12-15 to 2015-03-25, this study hopes to contribute to the literature with a greater understanding about the risk quantifications of WTI spot prices. This is done by applying several backtesting methods on the ES estimates, and thereby providing more robust results compared to previous literature.

1.4.2 Delimitations

This study excludes risk measurement estimation methods that depend on the extreme value theorem as well as methods depending only on historical data without any parametric assumptions, so-called non-parametric methods. This is done to focus solely

[33] Aloui, Chaker, and Samir Mabrouk. "Value-at-risk estimations of energy commodities via long-memory, asymmetry and fat-tailed GARCH models." *Energy Policy* 38.5 (2010): 2326-2339.

on conditional parametric estimations of VaR and ES. Another reason for this is that some of the backtesting procedures that will be used need some distributional assumptions which made the focus on parametric estimations natural. In order to calculate the conditional parametric volatility, a normal symmetric GARCH (1, 1) is used with symmetric distributions. The reason to only include GARCH (1, 1) is that it is a fairly simple model that has been proven to perform well when trying to capture volatility processes.[34] This study furthermore excludes asymmetric distributions. If a model is rejected in one tail but not rejected in the other, this would indicate that the distributions of returns are asymmetric. Only the confidence level of 99% will be used, both on long and short positions due to the interest in this study lie in the ability of the distributions to capture extreme events in both tails.

2 Methods

This section will present a description of the methods used. First the method used to estimate the risk measurements is presented, followed by the backtesting procedures used to evaluate these estimates. The section will conclude with a general discussion of the methodology.

2.1 Parametric estimation of the Risk measurements

Parametric estimation of a risk measurement is done by fitting a probability distribution over the data. While it can be a powerful technique since the user has information inferred from the distribution function, it is also a risky one because if an incorrect distribution is used, the estimates produced can be completely inaccurate.[35]

There are two main ways of performing distribution fitting, unconditional and conditional. The unconditional fitting does not depend on any conditional factors, and is often applied on longer holding periods. Unconditional fitting will overestimate the risk during calm periods, and more importantly, underestimate during volatile periods. In contrast fitting a distribution conditional on an assumed volatility process will account for calm and volatile periods. Conditional fitting is usually applied on shorter holding

[34] Hansen, Peter R., and Asger Lunde. "A forecast comparison of volatility models: does anything beat a GARCH (1, 1)." *Journal of applied econometrics"* 20.7 (2005): 873-889.
[35] Dowd 2005 p. 151

periods. [36] As the holding period in this study is one day, only conditional fitting will be applied to estimate the volatility.

2.1.1 GARCH[37]

The most commonly used model when calculating conditional volatility is the GARCH (1, 1) model.[38] This model is presented in equation 4 below. The GARCH accounts for volatility clustering because the conditional volatility is based on previous error terms and volatilities. If there is high variance at t, the GARCH model will predict high variance at t+1. Therefore, the variance is not constant in the sample, and the model can account for volatility clustering. The model is also symmetric, meaning that negative and positive shocks have the same effect on volatility.

$$\sigma_t^2 = \omega_t + \alpha_{1,t}\epsilon_{t-1}^2 + \beta_{1,t}\sigma_{t-1}^2 \tag{4}$$

β measures the persistence of the shocks on volatility while α measures the impact new shocks have on the volatility. This model is crucial in this study as it will be used to forecast the volatility for all the risk measurement. The volatility at t will be used to forecast the volatility for t+1 as detailed in equation 5 below.[39]

$$\hat{\sigma}_{t+1}^2 = \omega_t + \alpha_{1,t}\varepsilon_t^2 + \beta_{1,t}\sigma_t^2 \tag{5}$$

There are two parts to a univariate time series, the variance equation, which in this case is the GARCH model, and the mean equation. The mean equation in this study will be set to zero since it is assumed that in a one day ahead forecast on returns the effect of the mean equation will be negligible. In addition to the mean and variance equation, parametric assumptions need to be made about the error terms. This is due to the fact that the GARCH model is estimated by maximum likelihood, and a requirement for using this method is that distributional assumption about the errors terms have to be made.[40] In this study, as stated before, three such distributions will be used, the normal distribution, the t-distribution and the Generalized Error Distribution (GED).

[36] Dowd 2005 p. 152
[37] Reider, Rob. "Volatility forecasting I: GARCH models." *New York* (2009).
[38] Hansen and Lunde 2005
[39] Reider 2009
[40] Bollerslev, Tim. "A conditionally heteroskedastic time series model for speculative prices and rates of return." *The review of economics and statistics* (1987): 542-547.

For the GARCH estimates in this study, a rolling in-sample window of 250 observations will be used to forecast the risk measurement for one day ahead. This implies that the GARCH parameters are continuously re-estimated over the out of sample period from 1987-12-02 to 2015-03-24, leading to 7125 GARCH estimations. This is done because of the long out of sample window since it is probable that the magnitudes of the parameters in the GARCH model are not constant over the whole sample.

2.1.2 Normal Distribution[41]

The normal distribution or Gaussian distribution is an extremely commonly used distribution, which exhibits many nice properties when performing statistical tests. One of these is that the whole distribution is explained exclusively by two parameters; the mean and variance as seen in in the PDF (Probability Density Function) below.[42]. Also, the calculations for VaR are simple as can be seen in equation 6

$$\mathrm{VaR}_{\alpha}(\mathrm{R}_{t+1}) = \hat{\sigma}_{t+1} \mathrm{Z}_{\alpha,} \tag{6}$$

$$f(R) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{-1}{2}\frac{(R-\bar{R})^2}{\sigma^2}\right\}$$ [43]

The normal distribution does not account for excess kurtosis, which is often present in financial returns.[44] When using a GARCH model, even the normal distribution can to a limited extent account for excess kurtosis since the distribution is conditionally normal.[45] Nevertheless, it is a reasonable idea to include two distributions which will produce even fatter tails, since they follow a conditional distribution with excess kurtosis.

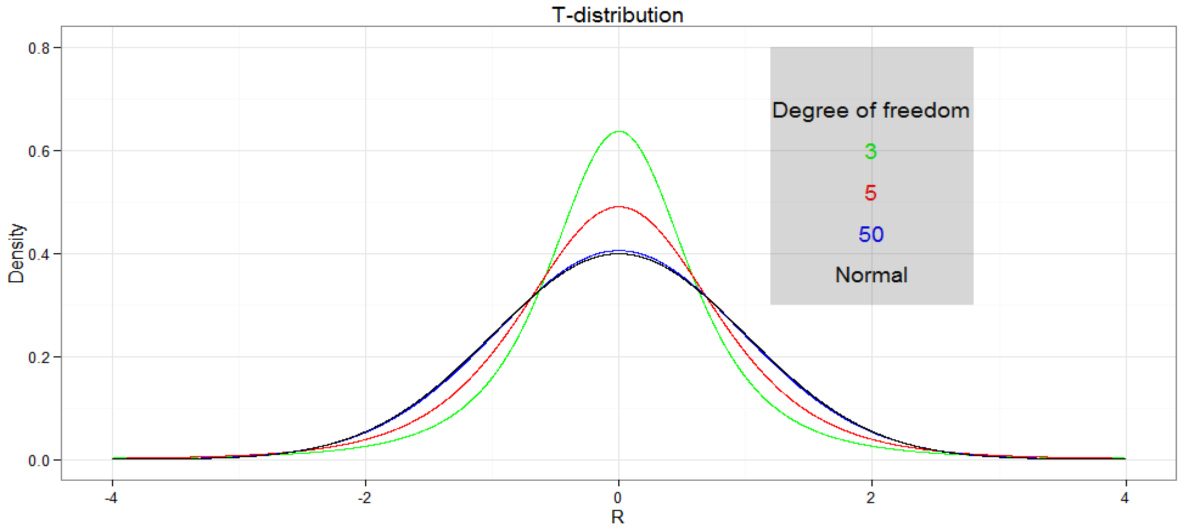
2.1.3 The t-distribution [46]

The t-distribution usually referred to as student's t-distribution, is a commonly used distribution when performing calculations on financial data.[47] This distribution is closely linked to the normal distribution, but can account for fatter tails as seen in graph 3. The shape of the distribution is dependent on three parameters, the mean, variance and the

[41] Verbeek, Marno. A guide to modern econometrics. John Wiley & Sons, 2008. p 457.
[42] Ibid p.457
[43] Ibid p.457
[44] Cont, Rama. "Empirical properties of asset returns: stylized facts and statistical issues." (2001): 223-236.
[45] Dowd 2005 p 132
[46] Hamilton, James Douglas. *Time series analysis.* Vol. 2. Princeton: Princeton university press, 1994
[47] Dowd 2005 p. 159

Degrees of Freedom (DoF). Specifically, a lower value of DoF means that the distribution has fatter tails, while a higher DoF means that the distribution will approach the normal distribution. The normal distribution is therefore a special case of the t-distribution when DOF, or v as defined in the probability density function below, approaches infinity.[48]

The calculations for VaR is a little different compared to the normal distribution as seen in equation 7.



Graph 3: Probability Density Function of the t-distribution at different Degrees of Freedom.

$$\mathrm{VaR}_{\alpha}(\mathrm{R}_{t+1}) = \sqrt{\frac{v-2}{v}}\hat{\sigma}_{t+1}\mathrm{T}_{\alpha,v} \quad (7)$$ [49]

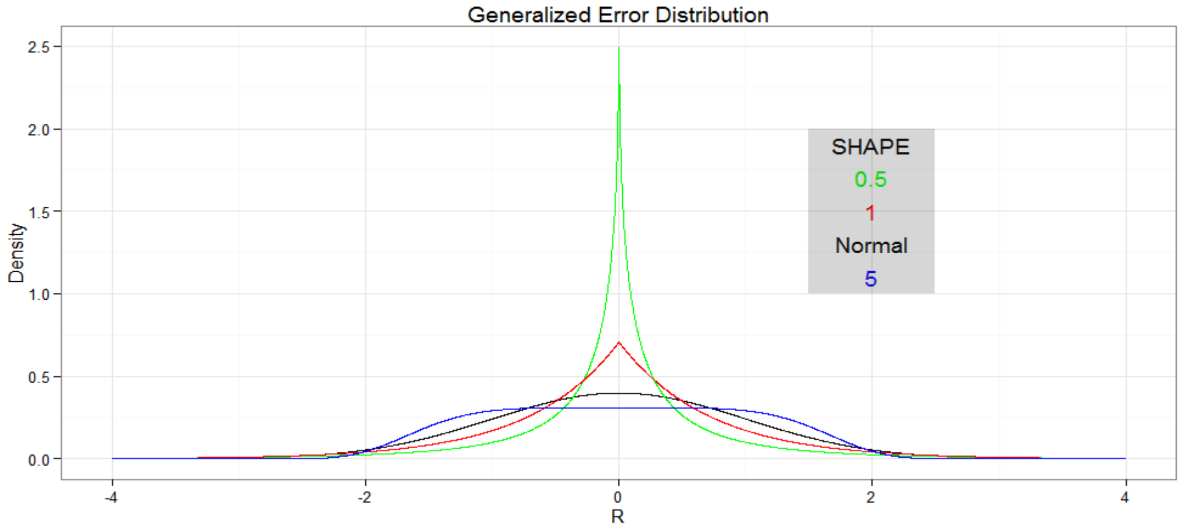
$$f_v(R) = \frac{\Gamma[v+1/2]}{\sigma\sqrt{(v-2)\pi}\Gamma(v/2)}\left[1+\frac{1}{v-2}\left(\frac{R-\bar{R}}{\sigma}\right)^2\right]^{-(v+1)/2} \quad for\ v>2, \qquad R \in (-\infty,\infty)$$ [50]

The critical value $T_{\alpha,v}$ is dependent on both the critical value α and on the DoF parameter. Thereby the critical value is not fixed as in the case for the normal distribution.

48 Hamilton 1994
49 Dowd 2005 p 159
50 Hamilton 1994

2.1.4 GED[51]

The Generalized Error Distribution, also known as Generalized Normal Distribution, or exponential power distribution, similar to the t-distribution, can account for excess kurtosis. The normal distribution is also a special case of the GED when the shape parameter v=2. The shape of the curve at different values of the shape parameter can be seen in graph 4.



Graph 4: Probability Density Function of the GED at different values of the shape parameter values.

$$f_v(R) = \frac{v^{*}\exp(\frac{1}{2}\ [R/\lambda]^{v})}{\lambda^{*}2^{[v+1/v]}\Gamma(1/v)} \quad for\ (0 \leq v \leq \infty), R\ \in (-\infty, \infty)$$ [52]

$$\lambda = \left[\frac{2^{(-1/k)}\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2}$$

As seen in the graph 4 above this is the most complex of the three distributions used. It is also the most uncommon. However, as mentioned earlier this distribution has produced good results in previous studies. Now that the estimation techniques are established for the risk measurements, the techniques of backtesting the estimates are presented.

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[51] Vasudeva, R., and J. Vasantha Kumari. "On general error distributions." 2013

[52] Ibid

2.2 Backtesting the risk measurements

In this section a summation of the methods used to backtest the VaR and ES estimates are presented, which is followed by a brief discussion about their properties.

2.2.1 Backtesting VaR[53]

The Christoffersen test is the most widely adopted method for backtesting VaR.[54] The test is divided into two parts, the unconditional part, which examines if there is correct number of VaR violations over the estimation period, and the conditional part which tests if the violations are randomly distributed in the sample.

The unconditional part of the Christoffersen test checks if VaR violations, or days that the VaR estimate is lower than the actual loss, follows a Bernoulli distribution with the null hypothesis expressed in equation 8.[55]

$$H_0: V_{t+1} \sim i.i.d\ Bernoulli(1-\alpha) \tag{8}$$

The α is the chosen confidence level, and in this study α =0.99. This means that the expectation under the null hypothesis is that there is a violation on any given day with a probability of 0.01. To check if the observed number of actual violations in the sample is equal to the expected number of violations, the Christoffersen test uses the log likelihood ratio test (LR). In order to perform this test, both the unconstrained and a constrained value of the likelihood function are needed. The constrained part is defined as $\mathrm{L}(1-\alpha) = (\alpha)^{\mathrm{t}_0}(1-\alpha)^{\mathrm{t}_1}$, where *(1-α)* is the probability of a violation under the null, in our case 0.01. t_0 is number of observed non-violations and t_1 is number of observed violations in the sample. The unconstrained model is $\mathrm{L}(\widehat{\pi}) = (1-\widehat{\pi})^{\mathrm{t}_0}\widehat{\pi}^{\mathrm{t}1}$ where t_0 and t_1 is the same as in the constrained model, but $\widehat{\pi}$ it is the actual probability of a violation as observed in the series, $\widehat{\pi} = \mathrm{t}_1/(\mathrm{t}_0 + \mathrm{t}_1)$. These two are used in the likelihood ratio test equation 9.[56]

$$LR_{uc} = -2\,ln[L(1-\alpha)/L(\hat{\pi})] \sim \chi^2(1) \tag{9}$$

[53] Christoffersen, Peter. "Backtesting." *Encyclopedia of Quantitative Finance* (2009).
[54] Ibid
[55] Ibid
[56] Ibid

$$H_0: (1-\alpha)=\hat{\pi}$$

$$H_1: (1-\alpha) \neq \hat{\pi}$$

If H_0 is rejected, the VaR estimates have produced an incorrect number of violations in the sample. Therefore the chosen model to estimate VaR does not accurately capture the risk of the asset. [57]

The conditional component of the Christoffersen test checks if the probability of a violation at t+1 is p, conditional on what is known at t. The test examines if the violations are independently distributed over the sample period. Violations that are clustered indicate that there is an increased likelihood of a violation occurring in the next period. A model that fails the conditional coverage Christoffersen test is not ideal since the model does not accurately capture volatility clustering effects.[58]

The conditional part of the Christoffersen test, similar to the unconditional part, uses a likelihood ratio test. To create the unrestricted part, a transition matrix as described in equation 10 is needed. This matrix is used to calculate probability of a transition from one state to another. $\hat{\pi}_{01}$ is the probability of observing a non-violation followed by a violation in the sample. This means that transition probabilities based on the VaR violations given in the sample must be calculated. These estimates are then used as described in equation 11 which is the unconstrained part of the LR test.[59]

$$\prod_1 = \begin{bmatrix} 1-\hat{\pi}_{01} & \hat{\pi}_{01} \\ 1-\hat{\pi}_{11} & \hat{\pi}_{11} \end{bmatrix} \tag{10}$$

$$L(\prod_1) = (1-\hat{\pi}_{01})^{t_{00}}\hat{\pi}_{01}{}^{t_{01}}(1-\hat{\pi}_{11})^{t_{10}}\,\hat{\pi}_{11}{}^{t_{11}} \tag{11}$$

The observed transitions in the sample are t, meaning that t_{01} is number of observations that have a non-violation followed by a violation in the sample. In order to solve for the actual probabilities in the transition matrix, the first derivatives with respect to $\hat{\pi}_{01}$ and

[57] Christoffersen 2009
[58] Ibid
[59] Ibid

$\hat{\pi}_{11}$ in equation 11 are taken to produce the following formula used in the transition matrix.[60]

$$\hat{\pi}_{01} = \frac{t_{01}}{t_{00} + t_{01}}, \quad \hat{\pi}_{11} = \frac{t_{11}}{t_{10} + t_{11}}$$

From this it is now possible to calculate the unconstrained part of the test. The constrained model in the conditional part is the same as the unconstrained model in the unconditional test.[61]

$$L(\hat{\pi}) = (1 - \hat{\pi})^{t_0} \hat{\pi}^{t_1} \tag{12}$$

Equation 11 and equation 12 are then combined to create the LR test, equation 13.[62]

$$LR_{indep} = -2\,ln\left[L(\hat{\pi})/L\left(\prod\nolimits_1\right)\right] \sim \chi^2(1) \tag{13}$$

$$H_0\!: \hat{\pi}_{01} = \hat{\pi}_{11}$$

$$H_1\!: \hat{\pi}_{01} \neq \hat{\pi}_{11}$$

The null hypothesis means that the information at t does not provide any information about the probability of violation at t+1. Equation 9 and equation 13 are then combined to the Christoffersen combined test equation 14 to test the overall validity of the model.

$$LRcc = LRuc + LR_{IND} \;\sim\; \chi^2(2) \tag{14}$$

The Christoffersen test has two main parts *LRuc*, and *LRcc*, and if any of the null hypotheses are rejected in either of the tests, the model used to calculate these VaR estimates is not correct. While other backtesting methods for VaR exist, the Christoffersen test has proven to perform well and therefore other tests will be omitted.[63] With a significance level of 99%, and an out of sample period of 7125 observations in this study, a correctly estimated VaR model will produce 71 violations.

[60] Christoffersen 2009
[61] Ibid
[62] Ibid
[63] Dowd 2005 p. 329

To test the validity of the models, a critical value when performing the backtest is needed. When deciding the critical values, there is a tradeoff between type I and type II errors. Type I error is when the null hypothesis is rejected even though it should not be, and type II is when we fail to reject the null hypothesis even though it should be rejected. A critical value of 10% is selected since it has proven to strike a good balance between the errors. [64] With this confidence level we fail to reject the null if the $LRuc$ is below 2.706 and below 4.605 for LRcc.

2.2.2 Backtesting Expected Shortfall

2.2.2.1 McNeil and Frey[65]

One of the first to propose a backtesting method for ES was McNeil and Frey. The test ignores all values of the return series that not violate VaR, and measures the difference between the size of the VaR violation and the calculated expected shortfall, divided by the forecasted variance.

$$r_t = \frac{R_t - ES_{\alpha,t}}{\sigma_t} \Big| R_t < -VaR_{\alpha,t} \tag{15}$$

They argued that the resulting modified series r_t should under the null be i.i.d with zero mean and unit variance. To empirically test the null hypothesis, the non-parametric bootstrap method is used on the n observations in the modified return series, against the alternative hypothesis "Mean of excess violations of VaR is greater than zero."[66] The test will therefore only reveal if the ES estimates are consistently underestimating the risk. The bootstrap methodology used follows from Efron and Tibshirani.[67]

To create a bootstrap test, first, the statistic below is created using the results from the n observations obtained from equation 15, according to the steps below.

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

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[64] Christoffersen 2009
[65] McNeil, Alexander J., and Rüdiger Frey. "Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach." *Journal of empirical finance* 7.3 (2000): 271-300
[66] Ibid
[67] Efron, Bradley, and Robert J. Tibshirani. *An introduction to the bootstrap*. CRC press, 1994. p. 224

$$\bar{\sigma} = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2$$

$$T = t(r) = \frac{\bar{r}}{\bar{\sigma}/\sqrt{n}}$$

The n observations from equation 15 are used to created M new samples with size n. This is done by sampling with replacement, n observations from the series M times, thereby creating M new samples of size n. In order to sample under the null hypothesis of mean zero, these replacements are shifted according to the equation below ensuring that the mean is on average zero.[68]

$$\tilde{r}_j = r_j - \bar{r}, \text{ j=1,2,…,N}$$

Now these modified bootstrapped returns follow the same procedure as above to create M number of T values.[69]

$$\bar{\tilde{r}}_j = \frac{1}{n}\sum_{i=1}^{n}\tilde{r}_j$$

$$\bar{\tilde{\sigma}}_j = \frac{1}{n-1}\sum_{i=1}^{n}(\tilde{r}_j - \bar{\tilde{r}}_j)^2$$

$$\tilde{T}_i = t(\tilde{r}_i) = \frac{\bar{\tilde{r}}_i}{\bar{\tilde{\sigma}}/\sqrt{n}} \quad \text{i=1,2,…,M}$$ [70]

These values are then compared to the observed T to create a P-value for the Null hypothesis.[71]

$$\frac{1+\sum_{i=1}^{M} 1_{\{\tilde{T}_i > T\}}}{1+M} = \text{p-value}$$

[68] Efron and Tibshiran 1994 p. 329
[69] Ibid
[70] In this study M=10000
[71] Efron and Tibshiran 1994

This calculates the p-value for the McNeil test, and it reveals if the ES estimates consistently underestimate the risk or not.[72] The test however says nothing about the number of violations, except the number n obtained from equation 15. This test cannot be adopted on its own since it does not formally test the number of VaR violations. It must therefore be used in conjunction with other tests, for instance the Christoffersen test.

2.2.2.2 Embrechts et al.[73]

Embrechts et al. proposed two methods that evaluate the ES estimate. Equations 16, and 17 which can be combined to equation 18. Equation 16 henceforth referred to as the V_1 test compares the actual observed loss given that there is a VaR violation to the estimated expected shortfall. The V_1 test can therefore be viewed as the average deviation of the return from the ES estimate given that VaR is violated. This implies that a correctly estimated risk model will produce a V_1 value close to zero. This would indicate that on average the ES estimations are close to actual returns in case of a tail event. [74]

$$V_1 = \frac{\sum_{t=1}^{T}\left(R_t-(-ES_{\alpha,t})\right)1_{\{R_t<-VaR_{\alpha,t}\}}}{\sum_{t=1}^{T} 1_{\{R_t<-VaR_{\alpha,t}\}}} \tag{16}$$

This test can be seen as a diagnostic tool, more so than a formal statistical test since the test does not have a null hypothesis. The test however can give valuable insights to the characteristics of the VaR violations. If the number of VaR violations in the sample is incorrect however, the V_1 test could the take the mean of a sample size that is far different from the optimal size of $(T)(1-\alpha)$ observations. Because of this subordination of the VaR estimates, equation 17 was proposed, henceforth referred as the V_2 test.[75]

$$V_2 = \frac{\sum_{t=1}^{T}\left(R_t-(-ES_{\alpha,t})\right)1_{\{D_t<D^p\}}}{\sum_{t=1}^{T} 1_{\{D_t<D^p\}}} \tag{17}$$

$$D_t = R_t - \left(-ES_{\alpha,t}\right)$$

[72] McNeil and Frey 2000
[73] Embrechts, Paul, Roger Kaufmann, and Pierre Patie. "Strategic long-term financial risks: Single risk factors." *Computational Optimization and Applications* 32.1-2 (2005): 61-90.
[74] Ibid
[75] Ibid

V_2, unlike V_1, depends on the empirical p-quantile of D_t instead of the VaR estimations to decide which observations are included in the test. D_p is the value of the (T)(1-α) lowest values from series D_t The empirical quantile thereby guarantees that $D_t<D^p$ will occur (T)(1-α) times in the sample. This ensures that the correct number of observations will be used in the test. Similarly to V_1, V_2 is close to zero when ES is correctly estimated.[76]

These two measurements can be combined to create equation 18.

$$V_3 = \frac{|V_1^{ES}|+|V_2^{ES}|}{2} \tag{18}$$

2.2.2.3 Acerbi and Szekely Z_1: Testing ES after VaR[77]

The foundation to the Z_1 test is the expectation equation 19.

$$E_{H_0}\left[\frac{R_t}{ES_{\alpha,t}}+1 \middle| (R_t + VaR_{\propto,t}) < 0\right] = 0 \tag{19}$$

Equation 19 states that a correctly estimated ES will on average be equal the size of the negative return when VaR is violated. This test is subordinated to the VaR measurement because it dictates which observations are included in the test. In order for the Z_1 test to produce accurate results, the VaR measurement has to be accurate as well. The expectation equation 19 forms the basis for the Z1 test, equation 20.[78]

$$Z_1(\vec{R}) = \frac{\sum_{t=1}^{T}\frac{R_t I_t}{ES_{\alpha,t}}}{N_T} + 1 \tag{20}$$

$$I_t = (R_t + VaR_{\propto,t}) < 0$$

$$N_T = \sum_{t=1}^{T} I_t > 0$$

A correctly estimated ES estimate will produce a Z_1 value close to zero. The expectations under the null and alternative hypothesis are as follows: [79]

$$E_{H_0} = [Z_1|N_t > 0] = 0$$

$$E_{H_1} = [Z_1|N_t > 0] < 0$$

[76] Embrechts et al 2005
[77] Acerbi, Carlo, and Balazs Szekely. *"Back-testing expected shortfall." Risk* 27.11 (2014)
[78] Ibid
[79] Ibid

Under H_1, the VaR estimates are still correct since the test is subordinated VaR. If the ES estimates are incorrect, this has no effect on the VaR estimates. The test is completely unaffected by the accuracy of the VaR estimates in the sense that it is only the average of violations that matters. The test is similar to the McNeil and Frey test in that regard.[80] However, the tests are not identical since the simulations for estimating the significance of the test are different as explained in **2.2.2.5**.

2.2.2.4 Acerbi and Szekely Z_2: Testing ES directly[81]

The Z_2 test simultaneously checks for both the frequency of tail events and accuracy of the ES estimates. The foundation of the Z_2 test originates from the unconditional expectation equation 21.

$$ES_{\alpha,t} = -E\left[\frac{R_t I_t}{\alpha}\right] \tag{21}$$

This leads to the Z_2 test statistics equation 22:

$$Z_2(\vec{R}) = \sum_{t=1}^{T} \frac{R_t I_t}{T\alpha ES_{\alpha,t}} + 1 \tag{22}$$

The Z_2 test simultaneously checks if the numbers of VaR violations are correct and that the ES estimates are accurate. The null and alternative hypotheses for the Z2 test are as follows:[82]

$$H_0: P_t^{|\alpha|} = F_t^{|\alpha|}, \forall\, t$$

$$H_1: \begin{array}{c} ES_{\alpha,t}^{F} \geq ES_{\alpha,t}\, for\ all\ t\ and > for\ some\ t \\ VaR_{\alpha,t}^{F} \geq VaR_{\alpha,t}\ \forall t \end{array}$$

Where $F_t^{|\alpha|}$ is the actual observed distribution while $P_t^{|\alpha|}$is the predicted distribution. The null hypothesis will be rejected if ES, VaR, or both underestimate the risk. Since the test has a one sided alternative hypothesis, an overestimation of the risk will not lead to a rejection of the null hypothesis.[83] Basically, the difference between the Z_1 and Z_2 tests

[80] Acerbi, and Szekely 2014
[81] Ibid
[82] Ibid
[83] Ibid

is the way it sums the number of violations. However, if the number of VaR violations is correct, the Z_1 and Z_2 will produce the same output. The relationship between Z_1 and Z_2 is shown in equation 23.[84]

$$Z_2 = 1 - (1 - Z_1)\frac{N_T}{T\alpha} \tag{23}$$

2.2.2.5 Acerbi and Szekely: Testing the significance of Z1 and Z2.[85]

In order to test both Z_1 and Z_2, the predictive distribution P_t in each observation is saved. The calculation can be decomposed into several steps. Step 1: A simulation of the P_t under the null is done for all observations M times. Step 2: These M series are then used to compute the Z_1 and Z_2 scores for each series. Step 3: These M Z scores are then compared to the Z score of the observed series to create the p-value.

1. $Simulate\ independent\ R_t^i \sim P_t . \forall t\ , \forall i = 1, \dots M$[86]
2. $Compute\ Z^i = Z(\vec{R}^i)$
3. $Estimate\ p = \sum_{i=1}^{M} \left(Z^i < Z(\vec{R})\right)/M$

This procedure requires the recording of the predictive distribution P_t, but also a simulation of returns under the null. Therefore, this method is somewhat more data intensive compared to the McNeil test. [87]

2.2.2.6 Berkowitz[88]

Risk managers are mostly interested in what happens in the tail, and a normal likelihood ratio test will asymptotically detect any departure from the null hypothesis in the first two moments over the whole distribution. Therefore, the test proposed by Berkowitz uses a censored likelihood ratio test which will only detect deviation of the two first moments in the tail. The shape of the observed tail is compared to the shape of the forecasted tail. One of the key components in this method is the Rosenblatt

[84] Acerbi, and Szekely 2014
[85] Ibid
[86] In this study K=2000
[87] Acerbi, and Szekely 2014
[88] Berkowitz, Jeremy. "Testing density forecasts, with applications to risk management." *Journal of Business & Economic Statistics* 19.4 (2001): 465-474.

transformation equation 24, where R_t is the ex post realized returns, $\hat{f}(R)$is the ex-ante forecasted return density.[89]

$$x_t = \int_{-\infty}^{R_t} \hat{f}(R)dR \qquad (24)$$

Rosenblatt proved that if the forecasted return density is correct the x_t series is i.i.d U(0,1).[90] If equation 24 is i.i.d U(0,1), then:

$$Z_t = \Phi^{-1}\left[\int_{-\infty}^{R_t} \hat{f}(R)dR\right] \sim i.i.d\ N(0,1) \qquad (25)$$

Where Φ^{-1} is the inverse Gaussian distribution. When a series follows N (0,1), the VaR estimate at α=99% is 2.326 as mentioned in 1.2.1. Therefore, a left tail VaR violation occurs when $Z_t < -2.326$. While it is possible to do a likelihood ratio test on equation 25, this would, as mentioned earlier, detect any deviation from N(0,1) over the whole distribution and since the interest lies in the tail behavior, this is not ideal. Therefore, the Berkowitz test censors the data where all observations that fail to violate VaR will be truncated according to equation 26. This truncating is done in order to treat the series as a continuous variable.[91]

$$Z_t^* = \begin{cases} -VaR_{\alpha,t} \ if \ \ Z_t \geq -VaR_{\alpha,t} \\ Z_t \ if \ \ Z_t < -VaR_{\alpha,t} \end{cases} \qquad (26)$$

Equation 26 is then used to create the log likelihood function equation 27 which is used for the joint estimation of the mean deviation from the ES estimate and variance in the tail.[92]

$$L(\mu,\sigma|Z^*) = \sum_{Z_t^*<-VaR} ln\frac{1}{\sigma}\varphi\left(\frac{Z_t^*-\mu}{\sigma}\right) + \sum_{Z_t^*=-VaR} ln\left(1-\Phi\left(\frac{-VaR-\mu}{\sigma}\right)\right) \qquad (27)$$

The estimation of the parameters in equation 27 is done by maximizing likelihood. By taking the derivative with respect to the mean and variance, and setting both to 0, it is possible to solve for the parameters that maximizes the function. Equation 27 with values of μ and σ obtained from maximizing the function is the unrestricted part of the censored likelihood ratio test. This is then compared to the restricted part of the test by

[89] Berkowitz 2001
[90] Ibid
[91] Ibid
[92] Ibid

setting μ=0 and σ=1 in order to calculate the likelihood ratio test according to equation 28[93]

$$LR_{tail} = -2\big(L(0,1) - L(\hat{\mu}, \hat{\sigma}^2)\big) \sim \chi^2(2) \qquad (28)$$

This two sided LR test will detect any deviation from the null hypothesis in the first two moments in the tail. The test is two sided so any overestimation or underestimation of the risk will lead to a rejection of H_0. If the variance is significantly different from 1, this will also be detected. Essentially the test detects if the ex-ante forecasted return density used to calculate ES accurately captures the tail behavior of the asset.[94] The critical value for a Chi square distribution at 10% confidence level and two degrees of freedom is 4.605. This means that the null hypothesis is rejected if the value of equation 28 exceeds this.

2.2.3 General discussion of the Backtesting methods.

All the tests presented for backtesting expected shortfall are fairly similar as they calculate either the variance, the mean or both regarding the first two moments in the tail. They differ in the way models punish deviation from the "optimal" value of the moments condition. Another key difference between the models is the way the procedures derive the null hypothesis, and define the rejection region. These slight variations in the backtesting methods are the reason why numerous procedures are included, as the optimal model according to one backtesting method not necessarily has to be the optimal model for all.

By definition ES is dependent on VaR, and while the V_2 test does not explicitly have VaR in the formula, the method is indirectly affected by the VaR estimates because of ES subordination to VaR. This is the reason for including the Christoffersen test, since if a

[93] Berkowitz 2001
[94] Ibid

model produces good ES estimates but poor VaR estimates, the overall validity of the ES estimates are in question[95].

One of the main problems with the ES backtesting methods presented is the reliance on large sample properties for convergence as they are only asymptotically correct.[96].This problem is especially prevalent when the VaR and ES estimates have large critical values since the numbers of violations are so small. This means that the test require large datasets to produce accurate backtests. Given the large dataset used in this study this problem will not be as important. However operators of these methods should be aware of this limitation.

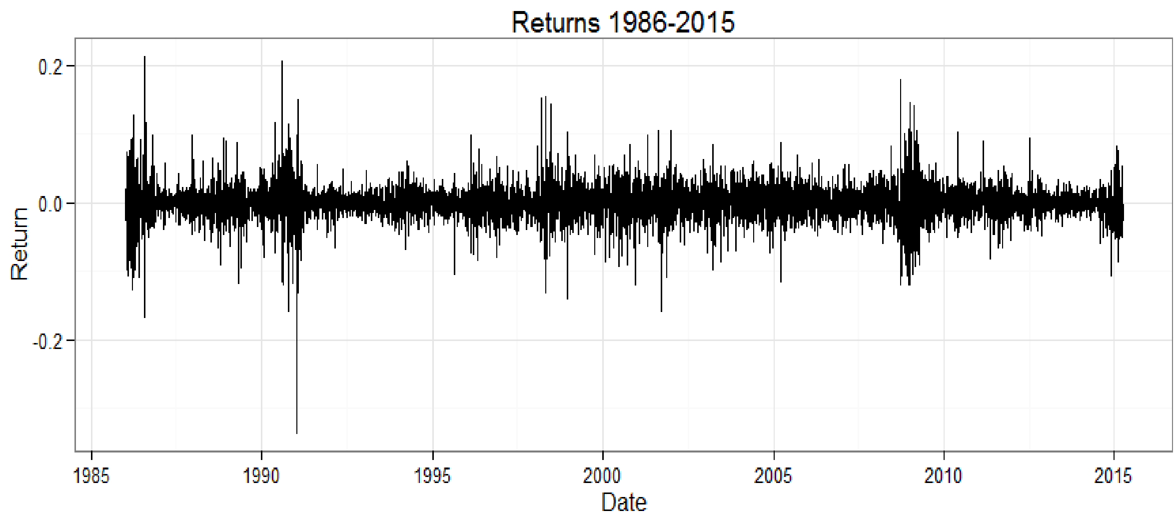
The Embrechts test does not formally have a null hypothesis. However, it is possible to create either a bootstrapped sample following McNeil and Frey, or a parametric simulation based on the Acerbi and Szekely test. Even if there is no feasible restriction limiting this, it will not be implemented in this study as both the Acerbi and McNeil tests formally test the null hypothesis of mean zero.

[95] Dowd 2005 p. 36

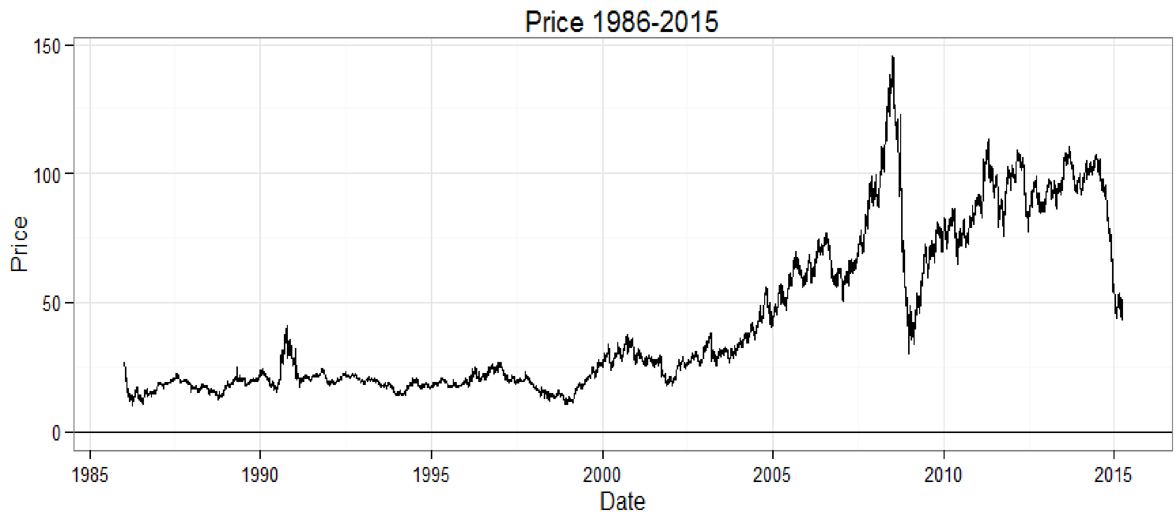
[96] Wong, Woon K. "Backtesting trading risk of commercial banks using expected shortfall." *Journal of Banking & Finance* 32.7 (2008): 1404-1415. p. 2

3 Data

The daily WTI spot price from 1986-12-15 to 2015-03-25 used in this study was retrieved from Thomson Reuters DataStream at 2015-04-08. This long period of analysis implies that many interesting events are included in the sample. For example, these events include: the collapse of the Soviet Union, both Iraq wars, the financial crisis, and the recent shale boom. The daily returns can be seen in graph 5 while graph 6 shows the price evolution. This shows that there are periods of high volatility, which is the reason why conditional risk estimates are preferred as mentioned in 2.1.



Graph 5: Daily returns



Graph 6: The evolution of prices

Descriptive statistics					
mean	max	min	Sd	kurtosis	skewness
0.04	21.0	-33.00	2.46	10.7	-0.21

Table 1: Descriptive statistics

These periods of high volatility observed in graph 5 can also be seen in the descriptive statistics table 1 above, as the largest one day price drop was 33%, while the biggest gain was 22%. The unconditional distribution of the returns over the whole period shows signs of excess kurtosis. This since the value of the kurtosis parameter is clearly above 3, which is the value it would have if the series had normal tails. The mean is close to zero as expected, and the series is slightly negatively skewed, or skewed to the left. However, the skewedness is not considered great since the absolute value is below 0.5.

4 Results

	Christoffersen						
	Actual violations	UC	P. UC	CC	P. CC		
GED Long	98	9.07	0.00	10.62	0.00		
GED Short	83	1.855	0.17	1.85	0.39		10% level
T Long	101	11.1	0.00	12.45	0.00	UC Critical	2.7
T short	90	4.6	0.03	4.61	0.1	CC Critical	4.6
Norm Long	134	44.3	0.00	44.1	0.00	Expected violations	71.25
Norm Short	114	21.9	0.00	22.5	0.00		

Table 2: VaR backtesting results

	McNeil	Embrechts			Acerbi				Berkowitz
	Boot P	V1	V2	V3	Z1	Z1 P	Z2	Z2 P	LR
GED Long	0.19	-0.35	-1.37	0.86	-0.07	0	-2.47	0.00	104
GED Short	0.108	0.47	1.09	0.78	-0.10	0	2.28	0.02	51
T Long	0.6	-0.06	-0.71	0.39	-0.01	0.42	-2.43	0.00	50
T short	0.48	0.01	0.76	0.38	-0.02	0.3	2.29	0.02	27
Norm Long	0	-0.82	-1.84	1.33	-0.15	0	-3.15	0.00	754
Norm Short	0	1.01	1.82	1.41	-0.20	0	2.92	0.00	529

Table 3: ES backtesting results

The Christoffersen backtest shows that both fat tailed distributions performed better than the normal distribution. At a p-value of zero in both tails for the normal distribution, it can uncontroversially be said that the normal distribution is not the right distribution for calculating the VaR over the chosen period as it is rejected by the Christoffersen backtest for both positions. The normal distribution also performs poor when calculating ES as it is rejected in the McNeil, Acerbi and Berkowitz tests. It also performs worst of all distributions for the V_1 test as it underestimate the risk on average by 0.82 percentage points for the long position while for the short by 1.01 percentage points. This poor performance of the normal distribution is to be expected since the series clearly exhibits excess kurtosis. The results are also consistent with previous literature which found that the normal distribution was not optimal for calculating the different risk measurements for WTI.

GED performed well when estimating VaR for the short position. With 83 violations, it fails to reject the null hypothesis for both the UC and CC part. When calculating the VaR on GED long however, it rejects the null hypothesis and thereby further indicating that the distribution is slightly skewed to the left. This was also indicated in the descriptive statistics which showed a slight negative skewedness. For the McNeil backtest, the long position performed better than the short position, which was also the case for the V_1, and Z_1 tests. This could be due to the fact that these tests are subordinate VaR and that the long position has more VaR violations, thereby extreme outliers have less effect on the results of these tests. Furthermore, the fact that on the V_2 test, which is not subordinate VaR in the same sense, the short position performed better would strengthen this perceptive. GED, though it outperformed the normal distribution, does not produce satisfactory results when backtested since all null hypotheses are rejected, except for VaR on the short position.

Similarly to the GED, the t-distribution produced better VaR estimates on short position compared to long. However, since the value obtained from the LRuc is 4.6, the null hypothesis is rejected at a 10% confidence level. For the McNeil test the null hypothesis is not rejected, indicating that the mean of excess violations is close to zero in both tails. This is furthermore indicated by the V_1 test which estimated that the t-distribution only underestimate the expected loss by less than 0.07 percentage points in both tails. The value of the Z_1 tests also indicates that the values in both tails that are close to zero. The Z_2 test further strengthens the argument that the mean is close to zero, but that there are incorrect number of violation in the sample as the Z_2 test reject the null hypothesis while the Z_1 does not. Both tails fail the Berkowitz test which is an indication that the variance in the modified series using the Rosenblatt transformation is not one. This would indicate that there are outliers that have an extreme effect on the volatility of the returns. This is not surprising since the oil market is sometimes affected by sudden jumps due to political events as explained in the introduction. The fat tailed distributions performed better than the normal distribution. However none of the distributions performed perfectly on both VaR and ES. Therefore, it would be advisable for a company to use both GED and t-distribution. It is however, extremely important to keep in mind the limitations the models have and their inability to foresee the sudden shocks in the price level.

5 Further research

This study can be extended in a number of ways. Firstly by using asymmetric distributions and asymmetric GARCH models. This is a natural extension since the results and descriptive statistics both showed some signs of negative skewedness. Another natural extension would be the use of different holding periods as well as different confidence levels on the risk measurements. This would provide more robust results as it would offer additional understanding of the behavior of the returns.

6 Conclusion

When performing conditional GARCH estimations of VaR and ES on arithmetic returns for West Texas Intermediate, the fat tailed distributions performed better than the normal distribution. The t-distribution performed better than GED when calculating ES which was not expected according to **E (i).** However when performing the VaR estimates, the t-distribution did not produce good estimates as both tails fail the unconditional coverage part of the Christoffersen test. The GED on the other hand produced better VaR estimates, especially in the right tail. Therefore, the best estimator for VaR is not the best for ES which is the opposite of was expected by **E (ii).** It would be advisable for a company interested estimating the risk of WTI to use both GED and t-distribution.

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