

Two Earths in one Solar System

Dan Wallsby

Lund Observatory
Lund University



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Supervisor: Melvyn B. Davies

Lund Observatory
Box 43
SE-221 00 Lund
Sweden

Abstract

The motivation for the project is to investigate what happens if I add a second Earth in the Solar System and project it forward in time. I add an Earth mass planet (called Earth two) in the same orbit as the Earth on the other side of the Sun (i.e. six months ahead of the original Earth). I then investigate how the perturbation of the second Earth effects the Solar System in the short and the long term to see how the interactions between planets occur. These range from the relation between Earth and Earth two, the effect on all terrestrial planets and then on to the whole Solar System. I also consider on what time scales any effects occur. Already in the short term we know that there is no Earth two out there. If it were, it would be in a horseshoe orbit with respect to the Earth and therefore easy to observe. In the long term I find that the system would be unstable. After a number of simulations with small perturbations the vast majority of them end in strong interactions between Venus and either Mercury, Earth or Earth two. The outer planets are practically unaffected by the presence of the extra planet.

Populärvetenskaplig beskrivning

Hur vet vi att det inte finns en annan planet i vårt solsystem bakom solen som alltid är dold för våra ögon? Eller om det fanns en planet där, hur skulle den i så fall påverka vårt solsystem? Dessa två frågor är motivationen bakom det här arbetet, som ger både intressanta och förvånande förklaringar till hur vi skulle märka av en extra planet. I projektet används ett datorprogram som simulerar planeters omloppsbanor och deras påverkan på varandra i form av rörelsemängdsmoment, gravitation och energiutbyte. Först undersöker jag dynamiken i solsystemet idag, sedan stoppar jag in en kopia av Jorden på andra sidan av solen i samma omlopps bana och undersöker vad som händer på korta och långa tidsskalor, det vill säga allt från några tusen år upp till flera miljoner år.

Att undersöka dynamik i planetsystem är en viktig del i att förstå hur planetsystem bildas och hålls stabila. Mycket inom astronomi är fråga om tidsskalor. Ofta är frågan inte om något kan eller kommer att hända, utan hur lång tid det tar. Om till exempel tiden för att planeter skall växelverka är längre än solens livstid så kommer det aldrig att hinna inträffa.

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Chapter 1

Introduction

This thesis is about the dynamical effects of adding an Earth mass planet in the Solar System on the opposite side of the Sun (i.e. six months ahead of the original Earth). I want to examine planetary system stability due to planet-planet interactions (Davies et al. 2014). I will look at different time scales, firstly short time scales that are in the order of the orbital time scales. Secondly the longer time scales in the order of secular time scales extending up to 10^8 /years. The time scales will be explained in further detail in their respective sections. After every orbit the planets will interact and exchange angular momentum. The effect of every interaction is very small. So why do planet-planet interactions matter? Because they happen very many times in the time scale of the system. So the small interactions build up over time and determine the dynamics of the system. The planets do the work on each other by angular momentum exchange.

When looking at planetary system stability it is good to remember what the conserved quantities, variables of interest and the relations between them are. Eccentricity of an orbit is given by (Davies et al. 2014),

$$e^2 = 1 + \left(\frac{2E_b J^2}{G^2 \mu^3 (M_p + M_s)^2} \right) \quad (1.1)$$

where J is the angular momentum, M_p is the mass of the planet, G the gravitational constant, μ is the reduced mass. Given by,

$$\mu = \frac{M_p M_s}{M_p + M_s} \quad (1.2)$$

and E_b is the binding energy of an orbit, given by,

$$E_b = -\frac{GM_p M_s}{2a} \quad (1.3)$$

where a is the semi-major axis. Note that the binding energy of an orbit does not depend

on eccentricity. This is worth remembering because it means that by angular momentum exchange between planets the eccentricity can change without any energy exchange or change in the semi-major axis. The two quantities that are preserved in the system are energy and angular momentum. Therefore when interactions occur there is no loss in the system, only transfer between the planets.

To run the simulations the MERCURY code (Chambers 1999) was used to simulate an N-body system with one star. In the case of collisions the code assumes a perfect merger, so the larger planet absorbs the smaller one. This is not however the only possible outcome of collisions. They could fragment (i.e break each other up) or partially merge and leave some debris or they could perfectly merge (Leinhardt & Stewart 2012). To analyse all the data and to make all the plots from it, I made my own MATLAB code.

Chapter 2

Our Solar System

I will clarify the points of interest by looking at the Solar System the way it is today, Pluto included. Note that smaller objects like asteroids are ignored. And the mass of the moons are added to the planets they orbit and are therefore not separate bodies. This section will introduce concepts and points of interest when looking at the dynamics in a planetary system.

In the short term one such concept is the elongation angle. This is important when we want to think about observing a planet from the Earth. If the angle θ given by equation 2.1 is very small, a planet can be blocked by the Sun and be hard to observe. To calculate it we can project two vectors originating from Earth; one going to the Sun and the other to Venus. Then the angle between these vectors would be the elongation angle between Earth and Venus. Generalized, this can be done between any two planets as shown in figure 2.1. To calculate the elongation angle θ

$$\cos(\theta) = \frac{\vec{r}_{12} \cdot \vec{r}_{1s}}{|\vec{r}_{12}| |\vec{r}_{1s}|} \quad (2.1)$$

On longer time scales we want to know if the orbits of the planets change. From equation 1.1 we know that we can have angular momentum exchange without changing the semi-major axis of an orbit. So if the eccentricity changes and the semi-major axis does not we know that there is angular momentum exchange but no energy exchange. On longer time scales it is useful to plot eccentricity vs semi-major axis to see if the orbits change. For more details if the orbits do change, one can look closer at how the eccentricity changes with time. Because if one planet gains angular momentum, another planet must give it up. Then we can understand which planets are interacting.

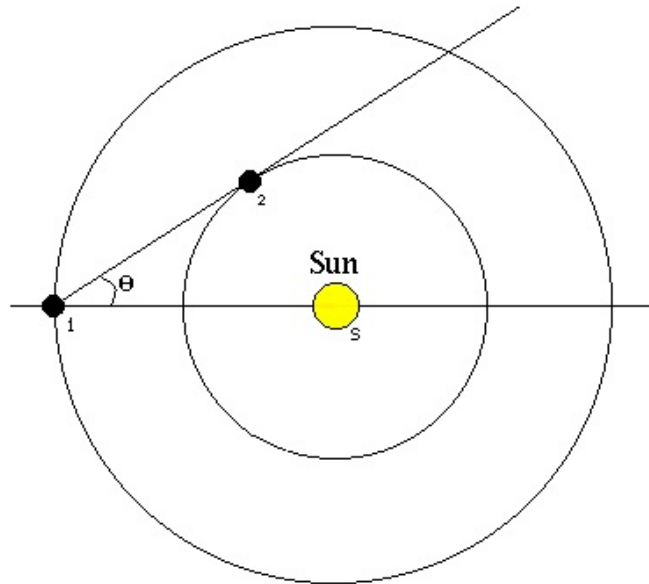


Figure 2.1: Showing elongation angle θ between an outer planet 1 and an inner planet 2 and the star s.

2.1 Short term Solar System

By looking at the elongation angle between Earth and Venus many facts can be noted, importantly that the Solar System is not flat. From figure 2.1 one would assume that the angle will be zero twice per orbit of the inner planet in respect to the outer one, at the closest and the furthest away points. If that were true we would see a transit of Venus in front of the Sun every time it is on the point closest to the Earth, but we do not see this. Why? By plotting the angle vs time it reminds us of an important detail, even if planetary systems are flat most planetary orbits do have some inclination to the system's plane, as shown in figure 2.2. This is the reason we don't have a transit of Venus every time it is close to Earth. From the Earth's point of view Venus passes over or under the Sun making it hard or impossible to see.

So by "short term" we really mean time scales of the order of orbital time scales. Ranging from around 0.5 years for the inner most planet up to 165 years for the outer planets. So looking at dynamics of a solar system evolves in the short term would mean well under the secular time scale (which is explained in the next section).

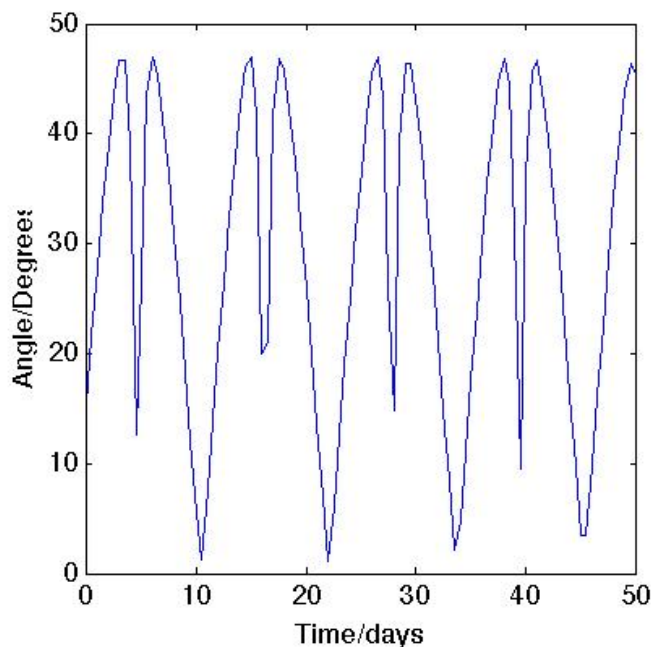


Figure 2.2: Elongation angle vs time for Earth to Venus over 50 days.

2.2 Long term Solar System

When a planetary system is observed in the long term it can no longer be viewed as a star in the middle with planets orbiting it. Because after many orbits long term effects come in to play, called secular interactions. A planet in an orbit can then be viewed as a ring of mass, since the planet will be in most places in it's orbit repeatedly. This gives interactions between rings of mass. To look at the elongation-angle to Venus like I did in figure 2.2 the short term will not give any useful information in long term. Instead the eccentricity e is a good variable to look at over time, because it will show how the planets exchange angular momentum. So if there is instability in the system it will show in the eccentricity. By integrating the Solar System over $2 \cdot 10^5$ years we find that the Jupiter-Saturn oscillation is stable figure 2.3. The orbits are not static so they change over time, but the semi-major axis or the eccentricity of the orbits do not grow with time. So the system is stable. In figure 2.3 we can see that one cycle for the angular momentum exchange between Jupiter and Saturn is just over 50000/years, so this is the order of magnitude for secular interactions.

Looking at the e for the terrestrial planets shows a chaotic system, because there are multiple planets that all interact with each other at the same time as they are getting perturbed by Jupiter. Meaning that the inclination of Jupiter's orbit is affecting the inner planets eccentricities. This effect is called the Lidov-Kozai mechanism (Kozai 1962). But one must remember that just because a system is chaotic, it do not have to be unstable. The question is what is chaotic in the system. So the system must be observed over a long time to see that the system is stable.

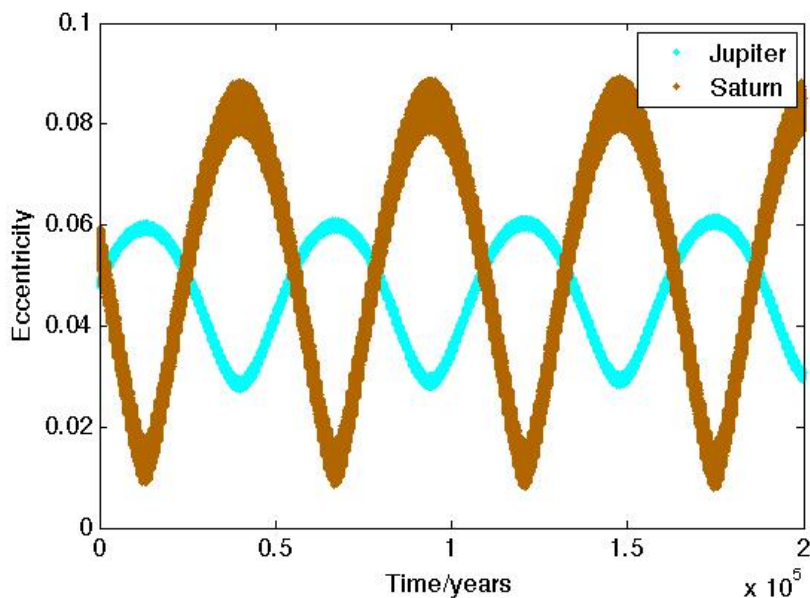


Figure 2.3: Eccentricity vs time for Jupiter and Saturn over 200000/years. Showing an oscillating angular momentum exchange.

In figure 2.4 we see that even if there are many interactions none of the terrestrial planet's eccentricities are growing with time. As a result they do not get close enough to have any close encounters, close encounters being when a planet gets within the Hill sphere of another planet. The Hill radius is given by

$$r_{\text{hill}} = a(1 - e) \left(\frac{m_p}{3M_s} \right)^{1/3} \quad (2.2)$$

The Hill radius gives the range of the potential well. So any object well within it will be bound to that planet rather than the main body. In more precise terms, the Hill sphere approximates the gravitational sphere of influence of a smaller body in the face of perturbations from the main body; moons are an example of this. The Earth's Hill radius is roughly $1.5 \cdot 10^6$ km and the moon orbits on an average distance of $0.384 \cdot 10^6$ km from the Earth, well within the Hill radius and has no risk of unbinding without any outer perturbations.

We have now seen that the Solar System is stable. We can however think about it the other way around. What is needed to destabilize a system? To get strong encounters, being collisions or ejections, the first thing that must happen is that a planet gets within the Hill radius of another planet so we get enough energy exchange. For the planets to get within the Hill radius of another planet, their orbits must come close, and since no orbits are that close today it means that for us to have any strong interactions the orbits must change. So if the semi-major axis a is plotted vs e over an even longer time we can see whether the orbits change, or if there is only angular momentum exchange.

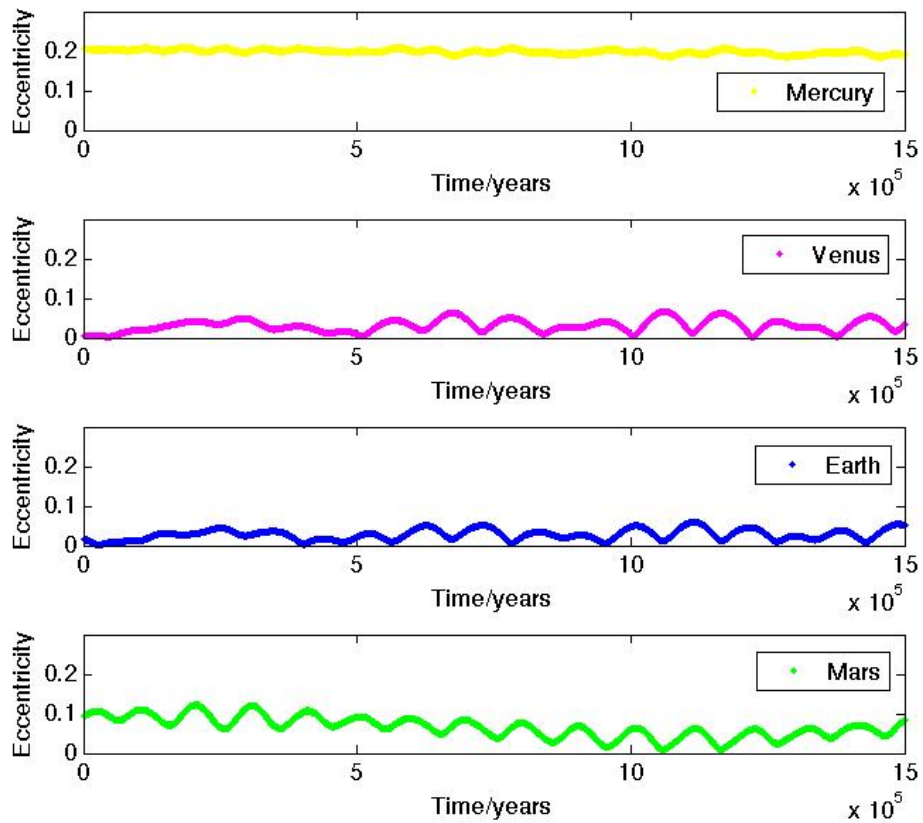


Figure 2.4: Eccentricity vs time for the terrestrial planets in the Solar System over $1.5 \cdot 10^6$ /years.

Figure 2.5 show us that the Solar System is stable. To illustrate the difference, here figure 2.6 is the same plot but with Jupiter's mass tripled. Such a big mass change will likely yield an unstable system, as the change in semi-major axis shows. After only $0.5 \cdot 10^6$ /years Pluto is ejected and near the end of the simulation after $87 \cdot 10^6$ /years Uranus is also ejected. It is interesting to note that if the mass of Jupiter were to triple, there would be no or only very minor changes to the terrestrial planets on the first 10^8 /years.

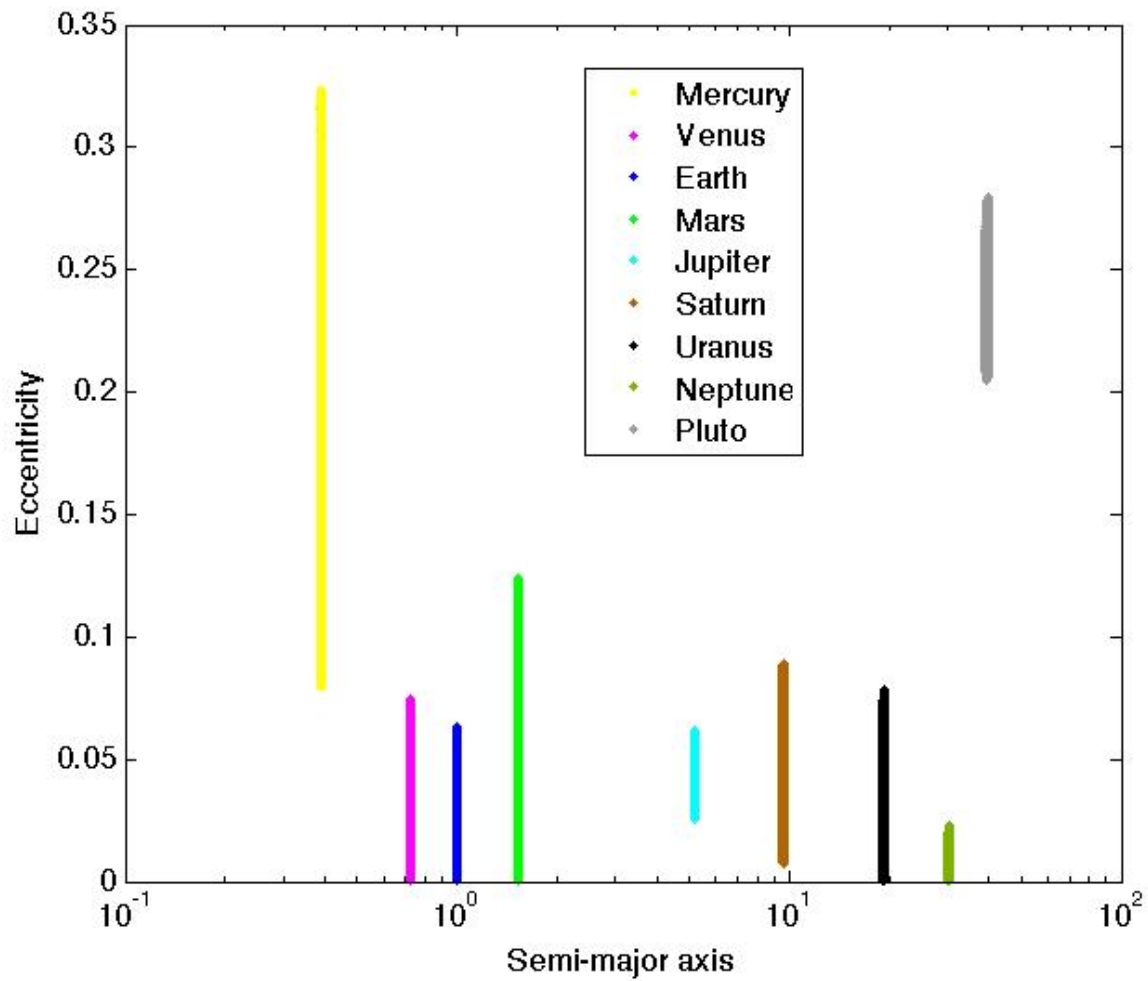


Figure 2.5: Eccentricity vs log of semi-major axis in AU for the Solar System over 10^8 /years. The stability in the semi-major axis indicate that the system is stable.

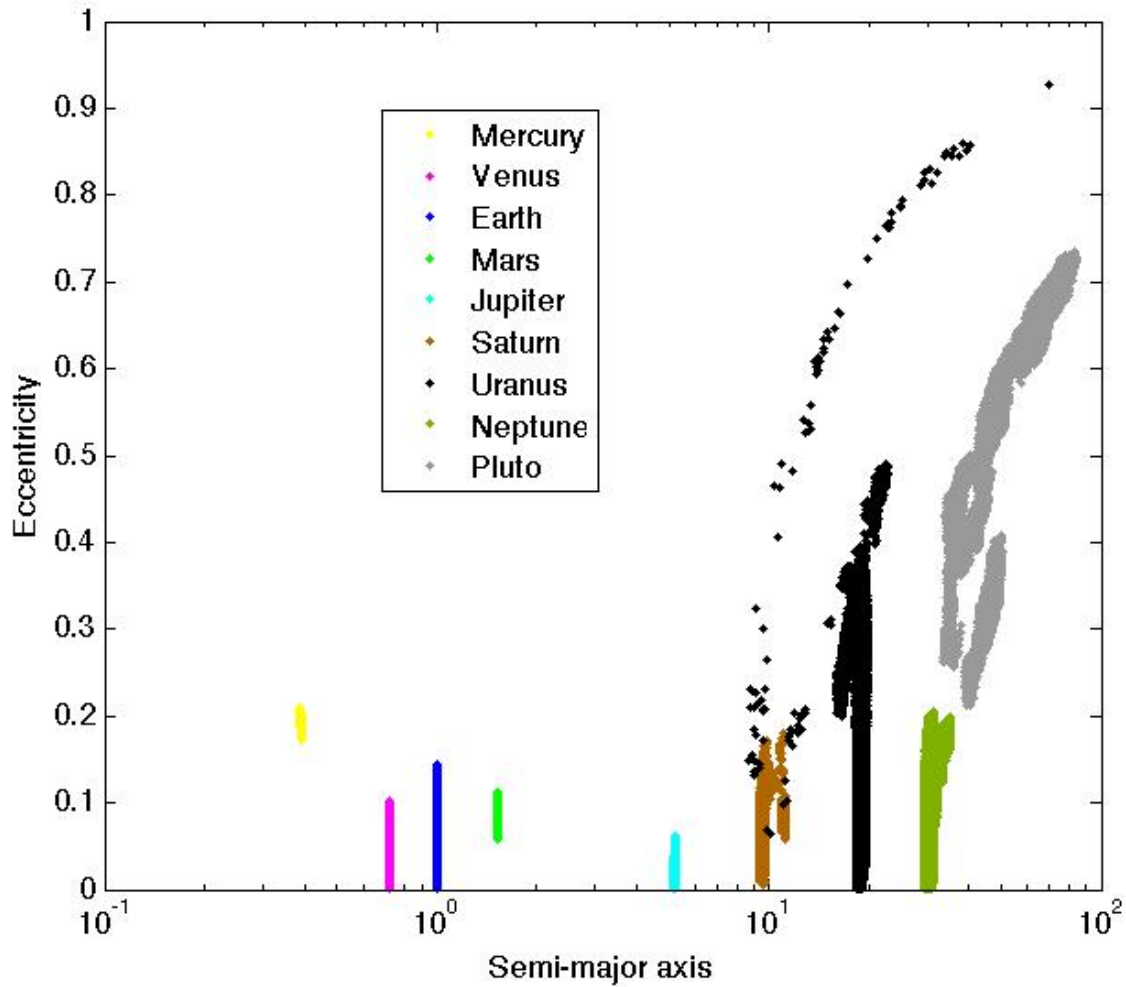


Figure 2.6: Eccentricity vs log of semi-major axis in AU over 10^8 /years with Jupiter's mass tripled. The change in Jupiter's mass is making the outer planets unstable and their semi-major axis change making their orbits overlap and strong scattering occurs.

Chapter 3

Evolution on orbital time scales

3.1 Adding the second Earth

Now I add Earth two in the Solar System in the same orbit as the Earth but shifted six months, i.e. directly behind the Sun as seen from the Earth. How do we know that there is not another planet on the other side of the Sun, always hidden by it? The answer to that question and the bigger question, would it affect the system's stability? In the simulations I measured the elongation-angle from the Earth to Earth two over a short term of few thousand years, so still well under the secular time scale of Jupiter-Saturn of about 50000/years that could be seen in figure 2.3. In the orbital time scale a planet must still be viewed as a planet and not a ring of mass. So if the elongation angle grows noticeably the planet would be visible from Earth. If that were the case we would know it was there. This also matters because if the orbits are not static and the planets would change their positions in the orbit all the time they will also change there position in relation to all the other planets in the system, further fueling the already chaotic system, and possibly affecting it's stability.

3.2 Horseshoe orbit

Earth and Earth two form a horseshoe orbit in respect to each other. To show this I plot the elongation angle as a function of time over 1600 years in figure 3.1. This shows that the angle grows significantly to nearly 83 degrees, which would put Earth two in another direction than the Sun, making it visible. But how Earth two moves is not obvious from seeing how the elongation angle oscillates. To examine Earth two's movement I plot the position of Earth two in the rest frame of the Earth, see figure 3.2. This interesting movement is called a horseshoe orbit, because of its appearance. To explain it we must recall how the orbital velocity depends on the distance from the star, assuming zero eccentricity.

$$v_{\text{orb}}^2 = \frac{GM_s}{R_{\text{orb}}} \quad (3.1)$$

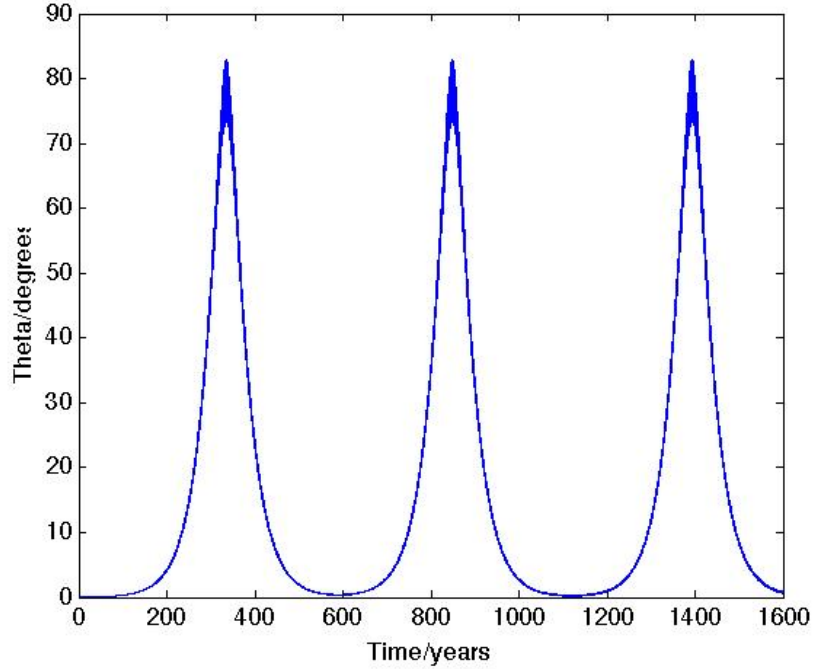


Figure 3.1: Elongation-angle between Earth and Earth two over 1600/years. The big change in the angle indicate that Earth two would be visible from the Earth.

Simply put, the closer to the star you are the faster you will orbit. Even if Earth and Earth two starts on exact opposite sides of the Sun they therefore will orbit with the same speed. One has to keep in mind that the Sun also moves in the system, it orbits the common barycentre it has with Jupiter. After only a partial orbit Jupiter and the other planets will perturb the Sun, that in turn perturb the Earth-Earth two system and one of the two planets will be closer to the Sun than the other one; giving it a higher velocity. So one of the planets (let's say Earth two) orbits faster and therefore catches up with the Earth on the orbit. But when it gets closer it is affected by the Earth's gravity and gets pulled to a wider orbit. At the same time, the same thing happens to the Earth in the opposite direction. How close the planets have to get for the gravitational force to be noticeable can be quantified by comparing to the gravitational force from Jupiter. The (attracting) gravitational force between two bodies is given by,

$$F_{\text{grav}} = \frac{GM_1M_2}{r^2} \quad (3.2)$$

so for masses in Earth mass and distance in AU we get $F_{\text{Jupiter}} = GM_{\text{Earth}}100M_{\text{Earth}}/5^2$, since Jupiter is roughly 100 times more massive then the Earth and on average 5AU away. Between Earth and Earth two we get $F_{\text{Earth}} = GM_{\text{Earth}}M_{\text{Earth}}/r^2$. Putting the forces equal

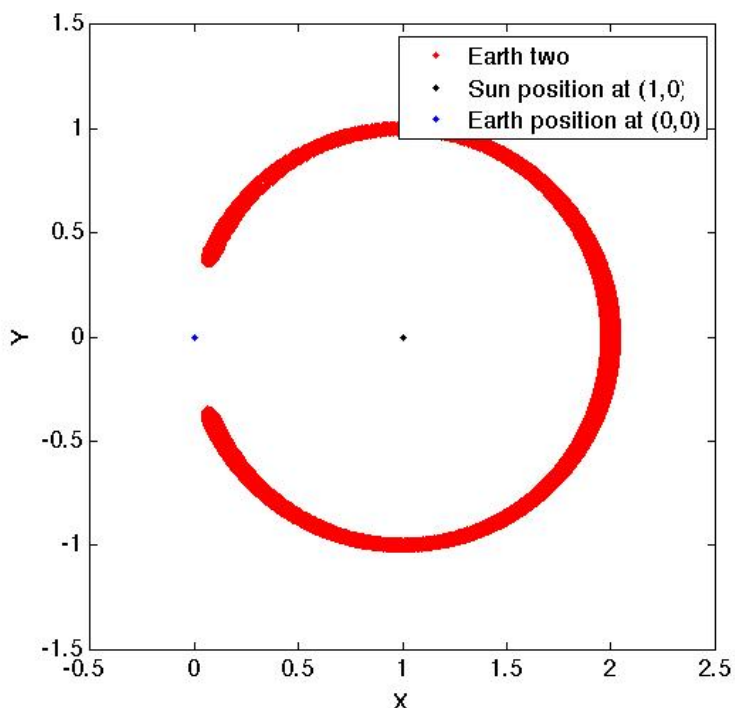


Figure 3.2: The position of Earth two in the rest frame of the Earth over 1200 years. Distances are in AU.

and solving for r gives $r = 0.5AU$. So when Earth and Earth two is roughly $0.5AU$ away from each other the gravitational force becomes noticeable. The gravitational effect happen rather quickly (before the planets get in close encounter range) Earth is on a closer orbit than Earth two and therefore assumes a higher velocity and moves away from Earth two. Seen from the Earth's rest frame it will look like Earth two catches up and interacts with the Earth and then falls behind again until it "catches up" from the other direction by falling behind long enough. The process repeats itself and gives the horseshoe orbit seen in figure 3.2. From figure 3.1 we can see the first peak in the angle after approximately 300/years and the closest point Earth two gets is roughly $0.3/AU$ away from the Earth. For reference this is close to the same distance between the Earth and Venus when they are as close to each other as possible. This could be seen with the naked eye, but we would not be able to resolve the disk of the Earth two's surface.

$0.3/AU$ is about $30r_{\text{hill,earth}}$ and the gravitational effect falls of as $1/r^2$. So the time scale for the gravitational effect to be significant will be of the order of $30^2/12=75$ years. The number 12 comes from the fact that the orbital time scale for the moon within the Earth's Hill radii is about 28 days, close to a month, so $1/12$ year. The time scale around 75 years is where we see the turn of the horseshoe orbit, being the time from when they start to affect each other until they are far enough away not to feel much gravitational pull, so over $0.5AU$ away.

An observant thinker might also ask why the orbit is all filled in, why is it not a thin line? Our theory for this is that the eccentricity is playing its part. To investigate if that is the case I made a system with only Earth and Earth two and again plotted the movement of Earth two in the Earth's rest frame. In figure 3.3 we see that without any perturbation

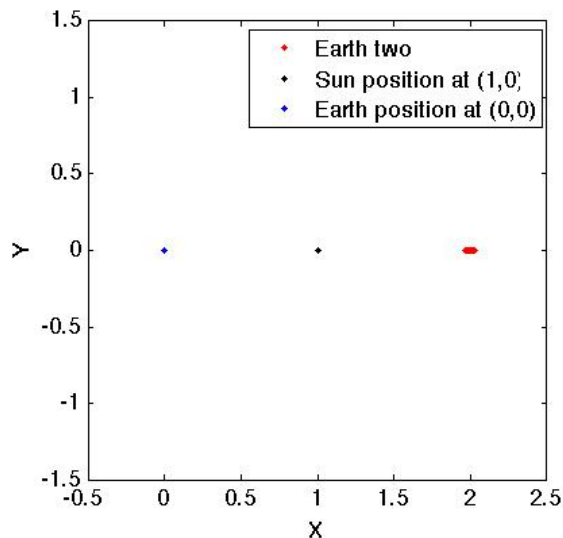


Figure 3.3: The position of Earth two in the rest frame of the Earth over 1200 years with only those two planets in the system. Distances are in AU.

in the system the planets will remain hidden from each other's line of sight. But we do see a small change in Earth two's position, and of course the Earth is undergoing a similar motion. So what we see is the angular momentum exchange between the planets affecting their eccentricities. If this wobble was in the shape of a horseshoe it would look like what we saw in figure 3.2. In order to confirm that a perturbation in the position will result in the horseshoe orbit, I again used only the two planets but moved Earth two 10^{-4} AU closer to the Sun and plotted it. As expected we get the horseshoe orbit figure 3.4, confirming the previously stated reason for the horseshoe orbit.

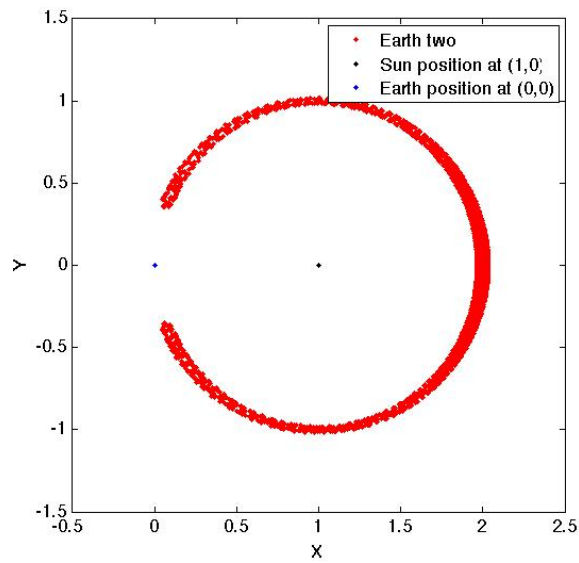


Figure 3.4: The position of Earth two in the rest frame of the Earth over 1200 years with only those two planets in the system. Earth two has been perturbed 10^{-4} /AU. Distances are in AU.

Chapter 4

Evolution on secular time scales

4.1 Stability issues

With the same initial placement of Earth two in the Solar System I now examine how the long-term stability is affected by this perturbation. First we want to know if the system remains stable or not. But we also want to determine the evolution of the horseshoe orbit over long time-scales, because this can tell us if or when the Earth-Earth two system gets perturbed. This in turn can help us understand why other interactions occur. If the system becomes unstable, we want to look closer into the time scale in which they go unstable. If it goes unstable in a time scale that is shorter than the time scale for the secular interactions between the outer planets (mainly Jupiter), it would tell us that it is the interactions between the terrestrial planets that makes the system go unstable and not the effect from Jupiter. Lastly of course we want to know what happens when the system goes unstable.

4.2 Results

The first simulation we look closer into have the same initial conditions as the long-term run with the Solar System, with the addition of Earth two. First in order to see if the orbits change I plot eccentricity e vs the semi-major axis a . Here in figure 4.1 the system appears unstable. We see orbits changing among the terrestrial planets and overlapping each other, which is a strong indication that there were close encounters. We do not see any strong interaction with the outer planets so we take a closer look at the terrestrial planets by plotting e vs time. In figure 4.2 we see that Venus has some strong early interactions so we take a closer look up to the time when Venus is lost. Here in figure 4.3 we see Earth, Earth two and Venus beginning to get unstable after a few million years Earth gets hit by Venus. After this collision the system stabilizes as seen in figure 4.2 and no more strong interactions occur.

We remember that Earth and Earth two are in a horseshoe orbit with respect to each other. So how does the excitation of Venus affect the relation between Earth and Earth two over a longer time? To investigate this I plot Earth two in the Earth's rest frame again

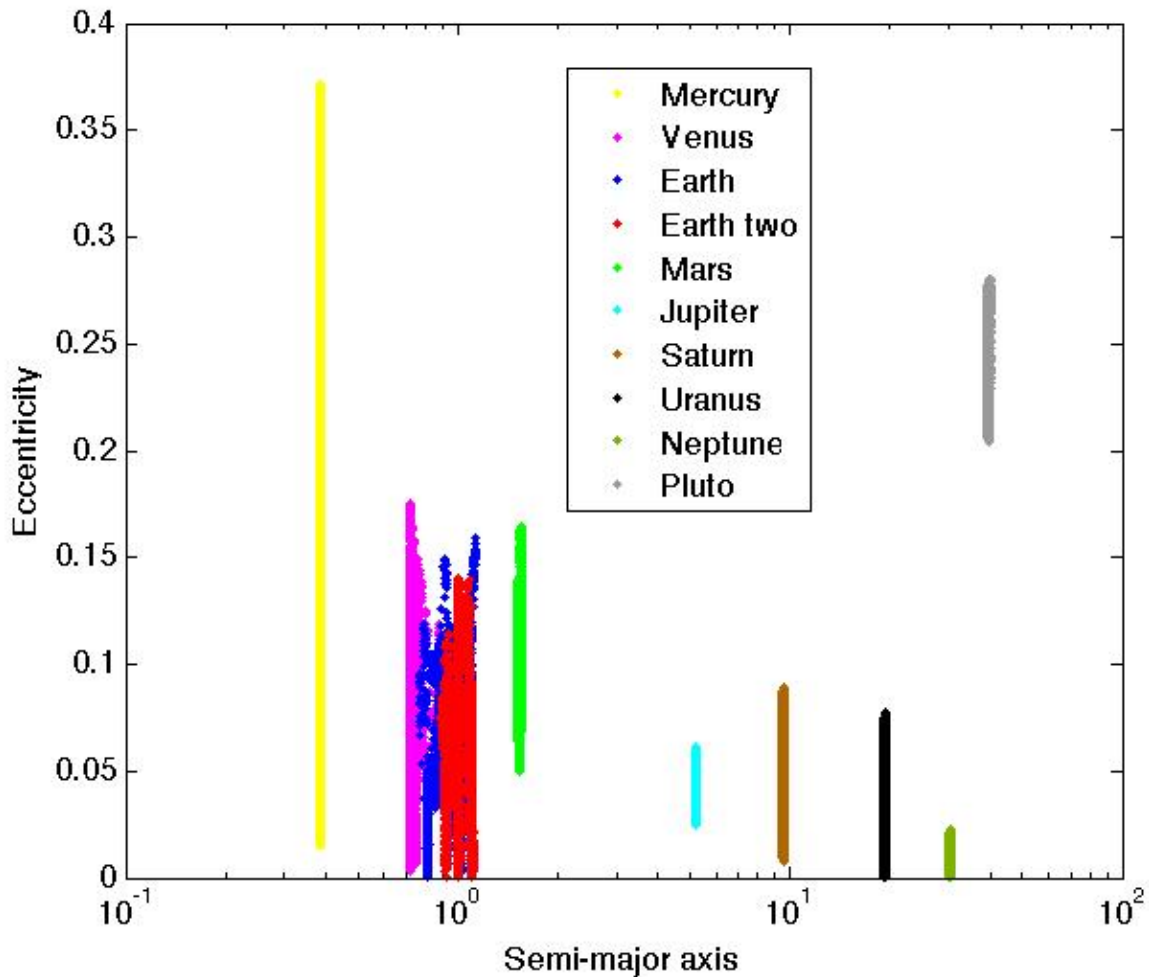


Figure 4.1: Eccentricity vs the log of semi-major axis in AU over 10^8 /years for the Solar System with the addition of Earth two in the system. The change in the semi-major axis seen in the terrestrial planets indicate that the system is unstable.

in time intervals to see if/how it changes when the orbit of Venus changes. In figure 4.4 it is clear that the perturbation in the Earth and Earth two system is growing and changing the orbits in respect to each other at the same rate as Venus is getting excited, so it is the Earth and Earth two that together is making Venus more eccentric until it finally collides with the Earth. Judging from the orbits of Earth and Earth two at the time of collision it appears equally likely for ether planet to be hit.

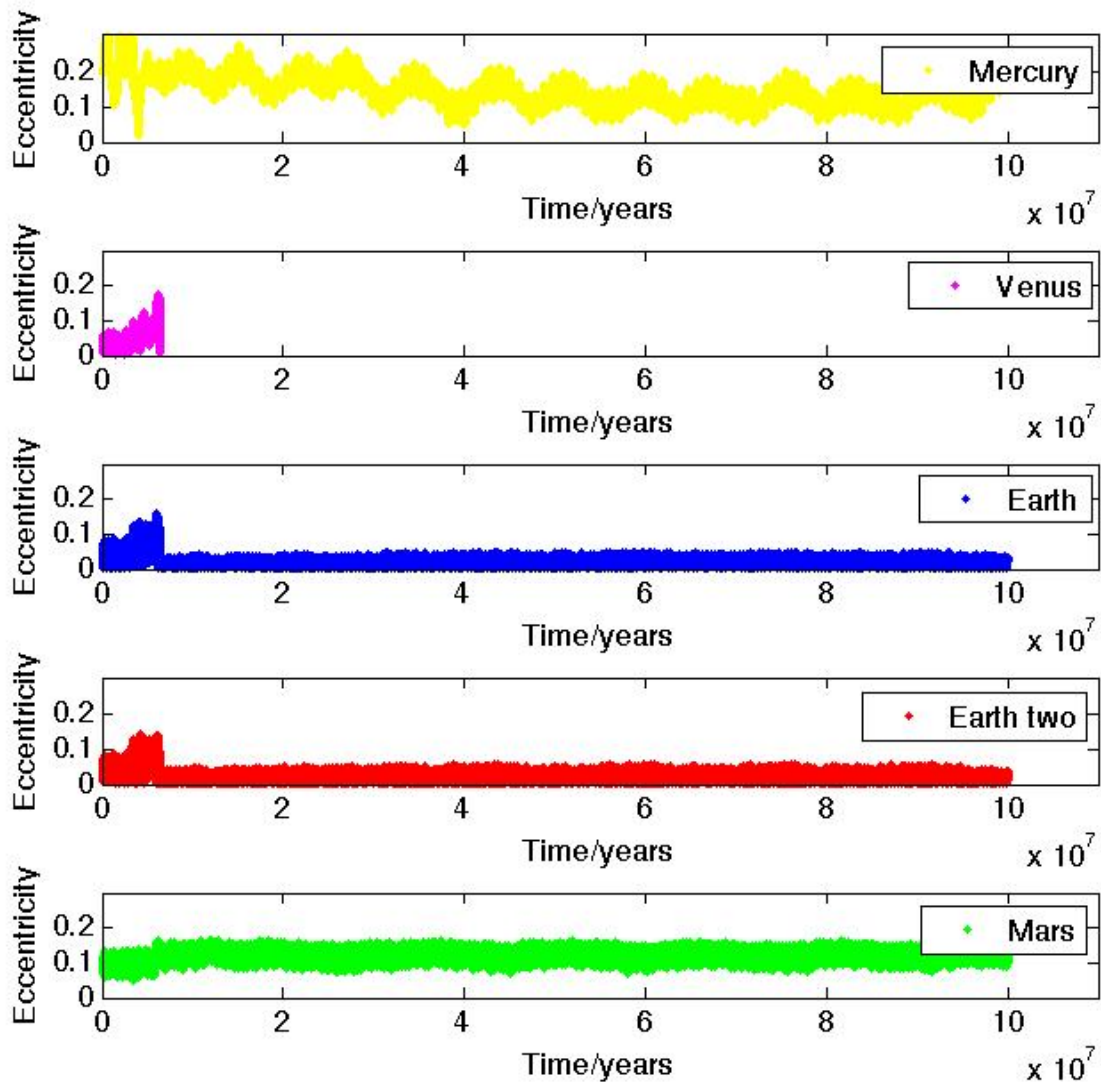


Figure 4.2: Eccentricity vs time for the terrestrial planets over 10^8 /years. Showing a unstable system with many interactions in the early part. Then Venus collides with the Earth and is lost from the system. After the collision the systems appears more stable.

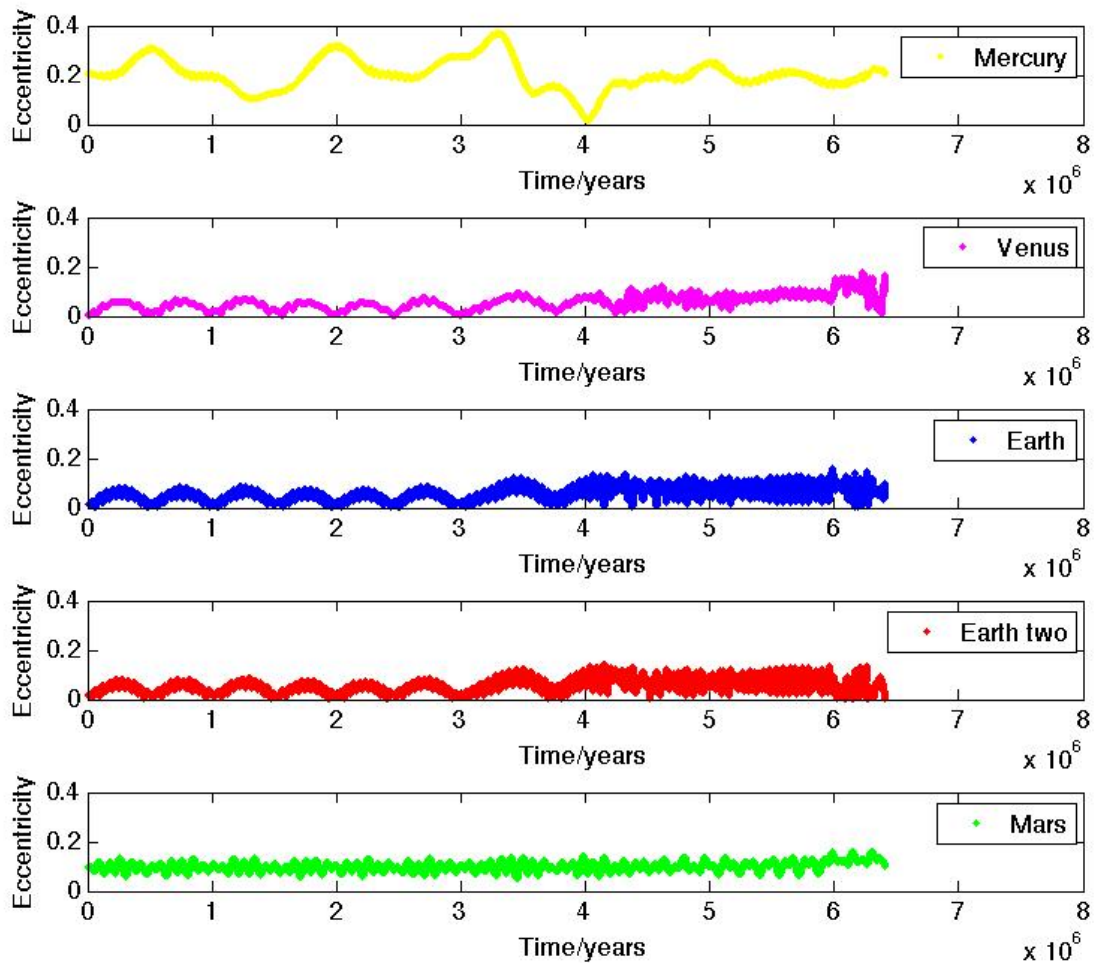


Figure 4.3: Eccentricity vs time for the terrestrial planets over $8 \cdot 10^6$ /years, up to the point when Venus collides with the Earth. Showing that the system stays stable for $3 \cdot 10^6$ /years, then the instability in the system is noticeable and the orbits begin to change. Leading to Earth, Earth two and Venus orbits crossing, resulting in Venus colliding with the Earth.

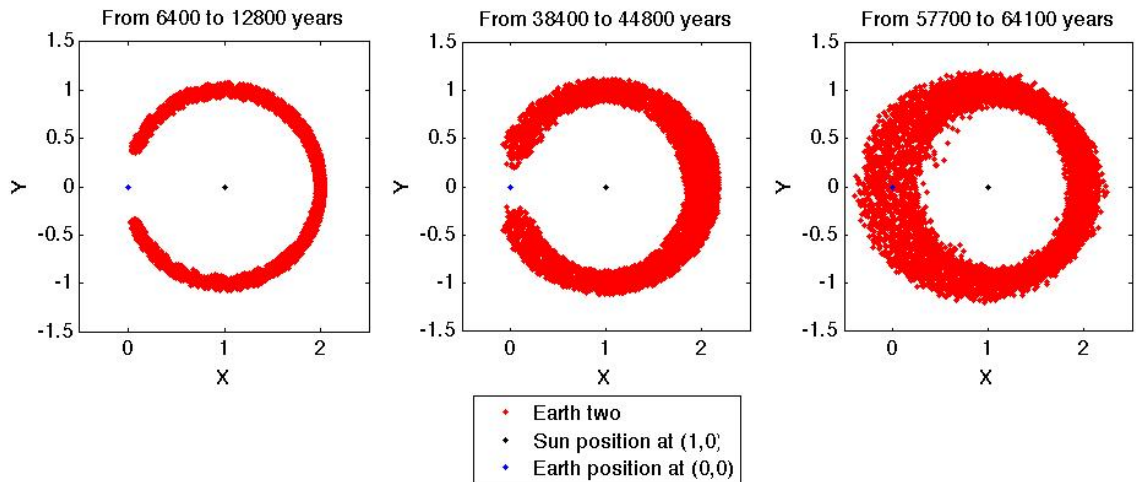


Figure 4.4: The position of Earth two in the Earth's rest frame in three time intervals before Venus collides with the Earth. Initially Earth two follows a horseshoe orbit (left) which breaks up (middle, left) as the system becomes more perturbed. This leads to the point where Earth and Earth two are practically on independent orbits. Distances are in AU.

4.3 Perturbed system

In order to obtain some statistics on the outcome for the long-term simulations I set up multiple versions of the Solar System with Earth two in place. All with different small perturbations in Earth two's initial placement. It would be an entire project to look in detail on all the simulations so I set them up to stop the simulation when the first close encounter occurs, so when a planet get within the Hill sphere of another planet. This will give us statistics on the time scale for the system to go unstable, and give a hint as to what will happen. We can assume (but not be sure) that it is likely that the two planets that have the first close encounter are likely to also have the first strong interaction. Note that by definition a close encounter will always happen before a strong interaction, being collision or ejection. Then I plot the ending reason for the simulations vs time.

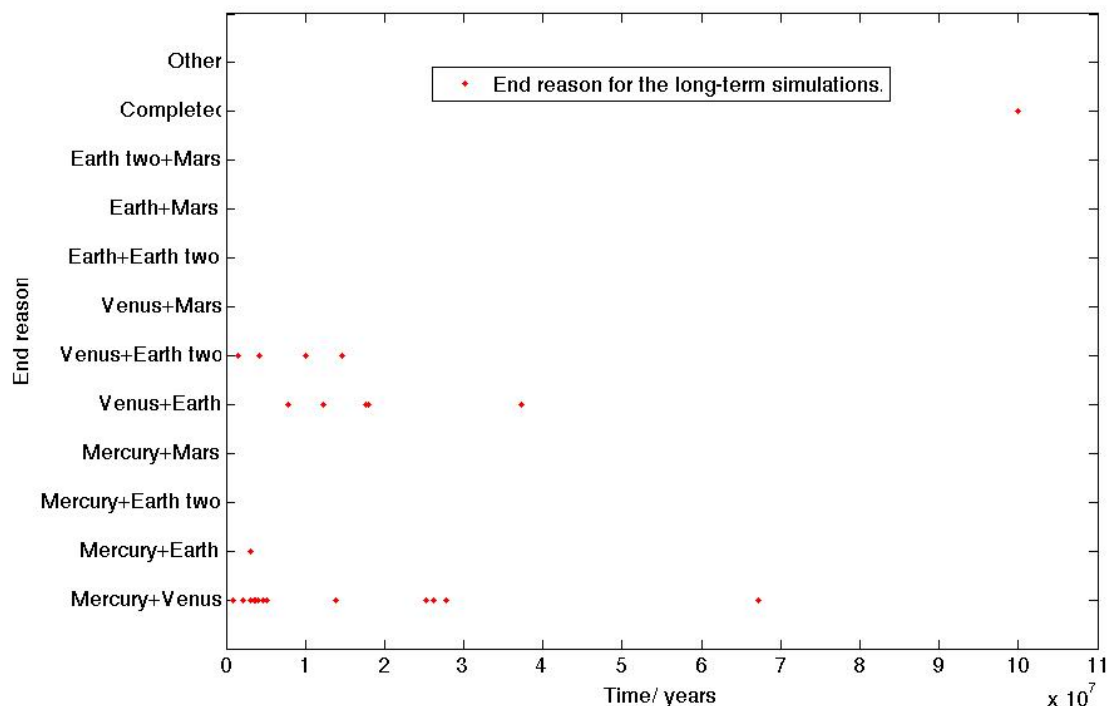


Figure 4.5: Marking the end reasons for 27 long-term simulations. All simulations in this plot end when two planets have a close encounter or when the simulation is completed after 10^8 /years. On the vertical axis is which two planets have the close encounter. Note that there were two simulations that completed.

For the purpose of investigating the end outcomes after a close encounter I took a sample of five simulations (from figure 4.5) and ran them again with the same initial conditions, but without stopping them on the first close encounter. With the intent to examine the end outcome and if possible, the time scale for the first strong interaction. I found that in

four of the five cases they led to collisions. Out of those four, two of them later resulted in a planet getting ejected from the system. In one case Mercury got ejected and in the other Mars got ejected. This is a bit unexpected because of the Safronov number Θ (Goldreich et al. 2004) given by,

$$\Theta^2 = \left(\frac{M_p}{M_s}\right) \left(\frac{R_p}{a_p}\right)^{-1} \Rightarrow \Theta^2 = \frac{1}{2} \frac{v_{\text{esc}}^2}{v_{\text{orb}}^2} \quad (4.1)$$

where the fraction between the escape velocity given by equation 3.1 and the orbital velocity given by equation 4.2 assuming low eccentricity.

$$v_{\text{orb}}^2 = \frac{2GM_p}{r_p} \quad (4.2)$$

The Safronov number will be higher the more massive the planet is, and the higher number the more likely strong scattering is, leading to ejections. In the case of smaller planets like the terrestrial planets in our Solar System the number will be low and make collisions more likely. It is all about how close the planets have to be to affect each other enough for strong scattering to be possible. For example a small planet might have to pass another small planet on a distance shorter than their radius to make strong scattering possible, and that will not happen because they physically hit each other first.

The sample was too small to determine any clear time scale between the first close encounter and the first strong interaction. But it seems to be significantly shorter than time scale from the simulations start to the first close encounter.

4.4 Earth mass doubled

If one thinks of the system as rings of mass, is it then the same as if we double the mass of the Earth or place another identical planet in the orbit? Both give the same two Earth mass ring. So a naive guess based on secular interactions would be that these systems would have the same outcome. I did a long-term simulation without the extra planet and doubled the mass of the Earth and then I investigated the stability in the system. As before I plot eccentricity e vs the semi-major axis a over 10^8 /years. In figure 4.6 we see what looks like a very stable system. Compared to figure 2.5 showing the unperturbed Solar System we see that Venus and the double mass Earth orbits are even less eccentric. Mars's orbit has also changed, the range of eccentricities have become smaller but the maximum eccentricity is close to the same. So the system appears relatively stable.

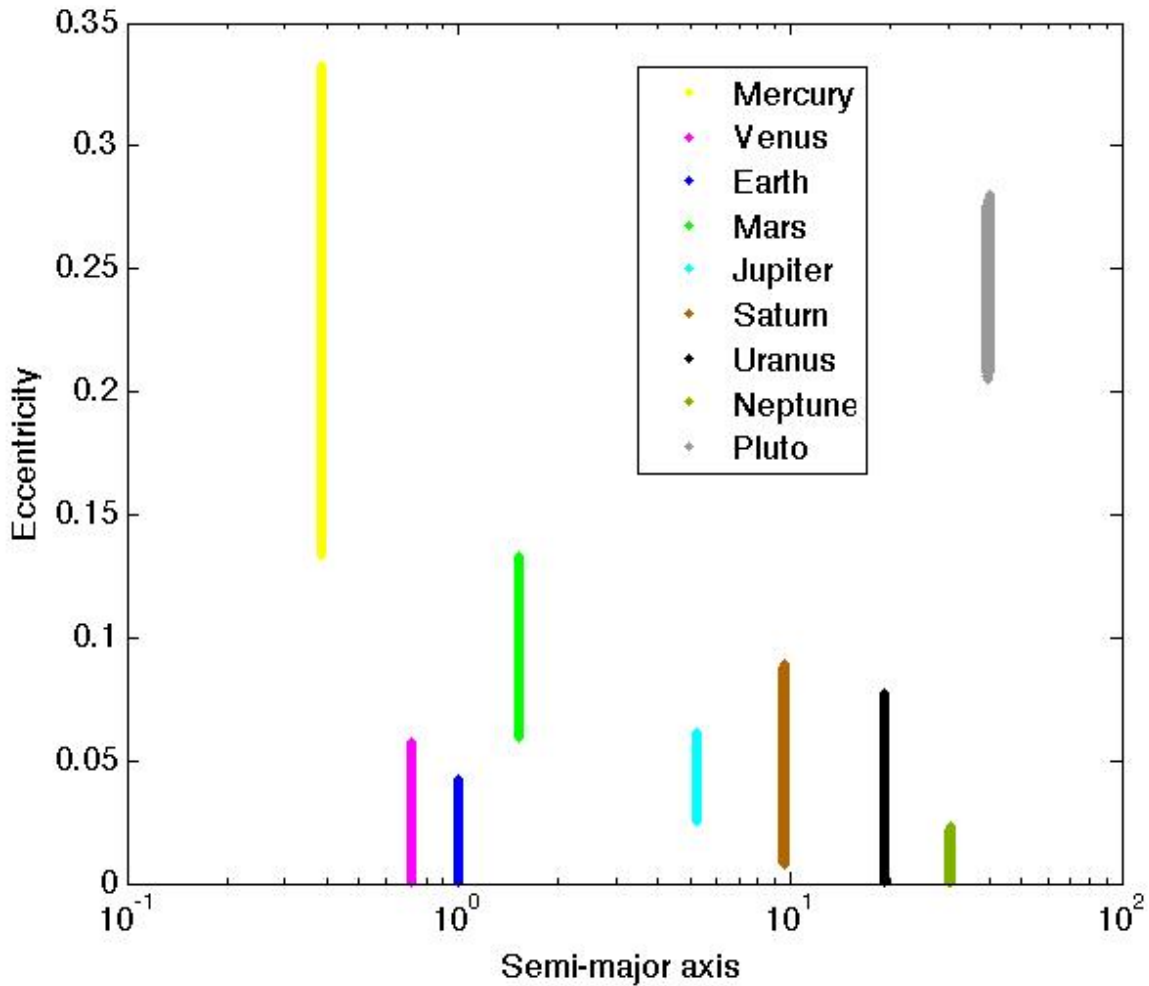


Figure 4.6: Eccentricity vs Semi-major axis in AU over 10^8 /years with Earth's mass doubled. Showing a very small change in the eccentricity range for the terrestrial planets. The system seems as stable as the Solar System.

Chapter 5

Conclusions

5.1 Summary

The Solar System is stable and there is no Earth two behind the Sun. But we can conclude that if there were a Earth two there, over a short time the system would remain stable and Earth and Earth two would make a horseshoe orbit with respect to each other. The addition of Earth two makes the orbit of Venus more eccentric over time scales comparable to those of secular interactions between Jupiter and Saturn; Until (in most cases) Venus gets too eccentric and has a strong interaction with a nearby planet, making the system unstable. We also saw that small perturbations affect the system noticeably, changing both the end result and the time scale in which close encounters occur. What could be found to be a bit unexpected is that there was not a single case where Earth and Earth two had a close encounter.

There are no close encounters between any of the outer planets. This is to be expected since the addition of another earth mass planet is a small change in the total mass of the system.

If instead of adding the second Earth, the mass of the original Earth is doubled. The Solar System as a whole would be rather unaffected; Telling us that more planets are more likely to make systems unstable than fewer more massive ones. This is likely because there will be many more interactions taking place, greatly reducing the time scales for orbits to change and strong interactions to occur.

Also if Jupiter's mass tripled it would destabilize the outer planets but only make the orbit of Jupiter less eccentric, and with that lessen the Kozai effect. With a reduced Kozai effect on the inner planets, their orbits become less eccentric. So since the orbit of Jupiter is not significantly changed, neither will the orbits of the inner planets.

Finally I made a time scale plot for an overview on the different events and interactions mentioned in this project, in figure 5.1. The color indicates intervals for the corresponding interaction or event.

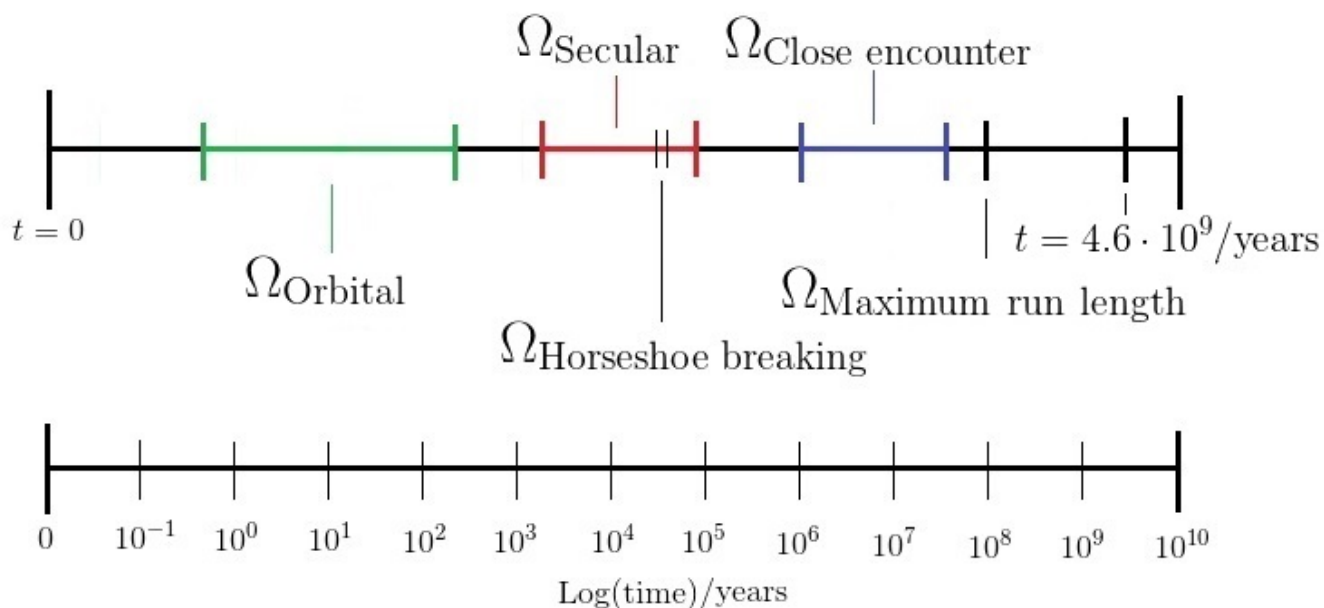


Figure 5.1: Showing an overview of the time scales in log scale. Where Ω_i is the time scale for the event i . The color showing intervals for the corresponding time scales. This shows that close encounters that leads to system instability would take place early in the lifetime of a system, telling us that a system with Earth two in it is highly unlikely to exist.

5.2 Further details

With results come more questions: what would be the next step if this project were to be continued? The first thing I would do is to gather more statistics on the outcomes for the long simulations. I would also re-run all the previous simulations without stopping them on a close encounter, and get statistics on the time scale between the first close encounter and the first strong interaction thereafter.

The strong interaction that dominates is collision, and in this project all collisions have been assumed to result in perfect mergers. This is far from the only likely outcome from collisions between terrestrial planets. With enough statistics to determine the most likely collision to happen, we want to know the most likely outcome of that collision, and use this information when the simulations proceed beyond that collision. This can greatly affect later events, like the two occasions I found planets to be ejected in the later parts of the simulations.

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