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Systemic risk measurement in the Eurozone

A multivariate GARCH estimation of CoVaR

by

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Abstract

In this essay the systemic risk contributions of financial institutions in the European Monetary Union are analyzed. For this purpose the CoVaR measure, first introduced by Adrian and Brunnermeier (2011), is applied. The definition of CoVaR is changed in the way that 1) the definition of financial distress is changed from an institution being exactly at its VaR-level to being at most at its VaR, and 2) the CoVaR measure is extended to allow for measuring the systemic risk contribution of a group of banks. For the calculations of CoVaR an underlying student t-distribution for the returns is assumed. Volatility and time-varying correlations between the institutions and the system are modeled using a GARCH-DCC approach. The systemic risk contribution is then obtained by solving numerically for ΔCoVaR. The calculations are based on daily return data of 32 banks from 10 Eurozone countries covering the period 1st May 2005 to 1st May 2015. The analysis of the results of the collective systemic risk contribution by country receives extra attention.

Keywords: systemic risk, CoVaR, Multivariate GARCH, DCC model

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Abbreviation Index

AR – Autoregressive

BIC – Bayesian information criterion

CoES – Conditional expected shortfall

CoVaR – Conditional value-at-risk

DCC – Dynamic conditional correlation

DIP – Distress insurance premium

ES – Expected shortfall

GARCH – Generalized autoregressive conditional heteroskedasticity

GJR GARCH – Glosten-Jagannathan-Runkle GARCH

MA – Moving average

MES – Marginal expected shortfall

QML – Quasi-maximum likelihood

SES – Systemic expected shortfall

SIFI – Systemically Important Financial Institution

SRISK – Systemic risk measure

USD – United States dollar

VaR – Value-at-risk

VIX – CBOE Volatility Index

1 Introduction

In today's globalized financial world the downfall of an individual financial institution can have severe consequences for both the financial system and the real economy. A prime example for such a scenario is the bankruptcy of Lehman Brothers, which struck the financial system heavily, led to several bail-outs of big financial institutions in the time after the event, and demonstrated the fragility of the whole financial system. Financial crises impose high costs for the society through the spillovers from the financial system to the real economy, which drops into a deep recession, and through the bailouts of big financial institutions with taxpayers' money. Hence, the regulation of financial institutions and the managing of such systemic risks is a desirable goal for the society as a whole. In the light of the global financial crisis, researchers and policymakers have recognized the importance of managing systemic risk. Before the events of the global financial crisis, banking regulation was solely based on idiosyncratic risk measures, as implemented in the Basel I and Basel II accords. With the acknowledgement of the importance of systemic risk, regulation authorities are moving towards a new regulation framework, such as Basel III, that incorporates macro prudential policies which focus on the mitigation of systemic risks.

A rich literature on measuring systemic risk has evolved ever since the global financial crisis and numerous attempts have been made to apply the different systemic risk measures. One of the most famous systemic risk measures is $\Delta CoVaR$, introduced by Adrian and Brunnermeier (2008). $\Delta CoVaR$ measures the systemic risk contribution of an individual financial institution in the financial system. It is defined as the difference between the VaR of the system, conditional on an institution being in financial distress and the VaR of the system, conditional on this institution being in its benchmark state. Several studies, such as Cao (2013) and Girardi and Ergün (2013), have extended the $\Delta CoVaR$ measure and introduced new ways to calculate it. Cao (2013) introduced an extension to the original $\Delta CoVaR$ which allows for conditioning on several financial institutions being in distress at the same time. The extension is then applied to a French and Chinese banking panel. Girardi and Ergün (2013) proposed a way of calculating $\Delta CoVaR$ using multivariate GARCH models. Using their new methodology they measure the systemic risk contribution of US financial firms. This essay relies on key features from both studies as well as the original study by Adrian and Brunnermeier (2011).

As the epicenter of the global financial crisis, systemic risks in the financial system of the United States have been a main focus of research so far. However, little attempts have been made to analyze systemic risk contributions in the financial system of the European Monetary Union. Furthermore, the phenomenon of several institutions being simultaneously in financial distress has received little attention. Even though the collective failure of a group of banks is not just a theoretical construction but has been observed in practice, research on this phenomenon is very limited.

The aim of this essay is to analyze individual systemic risk contributions of 32 financial institutions from 10 different European Monetary Union countries, as well as their collective systemic risk contribution. Furthermore, the essay tries to identify the countries that are home to the most systemically risky financial institutions and to analyze the collective systemic risk contribution for cases of countrywide negative shocks to the associated banks. Extra attention is paid on the analysis of the collective systemic risk contribution of a group of banks and the underlying drivers of a group's collective systemic risk contribution.

The remainder of the essay is structured in the following way. Chapter 2 provides an overview over existing research with respect to CoVaR and introduces the concept of systemic risk. The estimation and calculation methodology for $\Delta CoVaR$ is described in Chapter 3. Chapter 4 presents the data used for estimations and calculations. Empirical results for both the systemic risk contribution of individual banks and a group of banks are described in Chapter 5. The essay ends with a discussion and a summary in Chapter 6.

2 Related Literature and Theoretical Review

This chapter will consist of a review of previous research concerning the CoVaR measure as well as a theoretical discussion about systemic risk.

2.1 Previous Research on CoVaR

While systemic risk caught broad attention only after the global financial crisis, the literature on measuring and managing systemic risk is already rich and comprehensive. Several attempts have been made to categorize the different systemic risk approaches. Borri et al. (2012), for example, identify two main strands in the literature on systemic risk. The first one, referred to as *network analysis*, focuses on the interconnectedness of the entities in the financial system and thus is concerned with the joint distribution of losses. It assesses the impact of a failing network entity on the other network components' viability. Further discussions regarding *network analysis* lie beyond the scope of this essay. However, interested readers are referred to Martínez-Jaramillo et al. (2010) and Markose et al. (2010), as prime examples. The second strand, called *micro-evidence approach* measures systemic risk contribution of individual financial institutions. CoVaR, which is the measure of choice in this essay, is part of the *micro-evidence approach* strand. Thus focus in this essay is put on studies following this strand.

2.1.1 The first introduction of CoVaR

One of the most famous systemic risk measures is the so-called CoVaR measure, first introduced by Adrian and Brunnermeier in their paper *CoVaR* in 2008. It lead to a widespread application and analysis of their CoVaR measure. CoVaR is defined as the VaR of the financial system conditional on an institution i being in financial distress (Adrian & Brunnermeier, 2011). The measure can be categorized as a tail measure and thus focuses on the co-dependence in the tails of equity returns between financial institutions or an institution and the financial system (Hansen, 2013). This is also emphasized by the name of the measure, chosen by the authors, in which "Co" stands for conditional, contagion or comovement (Adrian & Brunnermeier, 2011).

The objectives of their paper are to 1) measure the contribution of a financial institution to systemic risk which is achieved through the measure ΔCoVaR, and to 2) create a forward looking indicator based on firm characteristics to predict future risk contributions of financial institutions which they call "forward ΔCoVaR". ΔCoVaR is defined as the difference between the q%-CoVaR of the system j conditional on institution i being in financial distress and the q%-CoVaR of the system j conditional on institution i being in its benchmark state,

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=Median^i}.$$

The authors define institution i's state of financial distress as institution i's equity returns being at their 1%-VaR level and its benchmark state as its equity returns being at their median level (50%-VaR level).

Adrian and Brunnermeier (2011) estimate both an unconditional version of the measure, which results in a constant CoVaR over time, and a conditional version of the measure, which varies over time. In order to estimate a time-varying conditional CoVaR measure, the authors include systemic state variables that model the changes in tail risk dependence over time. The vector of state variables contains the aggregate credit spread, the VIX as the implied equity return volatility, and the slope of the yield curve. For all of their estimations they use the so-called Quantile Regression which allows them to focus on the tails of equity returns. Their estimations are based on weekly equity return data of 1226 financial institutions, from 1986Q1 to 2010Q4, belonging to the four sectors: commercial banks, security broker-dealers (including investment banks), insurance companies and real estate companies.

The authors furthermore create the forward looking “forward ΔCoVaR” which is obtained by regressing the previously estimated ΔCoVaR on several firm characteristics such as size, market-beta, maturity mismatch, market-to-book ratio, and leverage. They find that a higher systemic risk contribution is related to a larger size, a higher leverage, and more maturity mismatch. Furthermore, “forward ΔCoVaR” is countercyclical, which means it is negatively correlated with the contemporaneous ΔCoVaR and thus captures the fact that systemic risk builds up in periods of tranquil market environments.

Another important finding by Adrian and Brunnermeier (2011) is the loose relation between conventional VaR and ΔCoVaR; that a high VaR does not automatically imply a high contribution to systemic risk. This implies that financial regulation based solely on idiosyncratic risks is not sufficient to protect against systemic risks. However, they do find a strong relation between an institution’s VaR and its systemic risk contribution ΔCoVaR in the time series dimension.

Since Adrian and Brunnermeier (2008) laid the foundation for the CoVaR measure a number of applications to different datasets and within different environments have emerged, such as in Arias, Mendoza, and Pérez-Reyna (2010), Borri et al. (2012), or Karkowska (2015). Assessments, extensions and customizations of the measure, such as in Cao (2013), Girardi and Ergün (2013), Karimalis and Nomikos (2014), Bernardi, Maruotti, and Petrella (2013), Benoit et al. (2013), and Mainik and Schaaning (2012), contributed to further developments of the CoVaR measure.

2.1.2 Multi-CoVaR and Shapley value

Cao (2013) in *Multi-CoVaR and Shapley value: A Systemic Risk Measure,* extends the basic CoVaR measure to a multivariate approach. By conditioning on more than one institution being in financial distress the Multi-CoVaR is able to measure the change in systemic risk when several institutions face financial difficulties at the same time.

The author follows a two-step procedure in order to calculate the contribution of a financial institution to systemic risk. First, the total systemic risk contribution for the case that all institutions in the system are in financial distress at the same time is calculated, which represents total systemic risk. Secondly, the systemic risk contribution of each institution is obtained by applying an allocation algorithm to the overall systemic risk. Thus, the definition of CoVaR slightly changes in the following way

$$\Delta CoVaR_{q,t}^{1,\dots,S} = CoVaR_{q,t}^{r^1 \leq VaR_q^1,\dots,r^S \leq VaR_q^S} - CoVaR_{q,t}^{-\alpha\sigma_t^1 \leq r_t^1 \leq \alpha\sigma_t^1,\dots,-\alpha\sigma_t^S \leq r_t^S \leq \alpha\sigma_t^S},$$

where $\Delta CoVaR_{q,t}^{1,\dots,S}$ is the total systemic risk contribution of all institutions $\{1,\dots,S\}$ at time t and confidence level q, $CoVaR_{q,t}^{r^1 \leq VaR_q^1,\dots,r^S \leq VaR_q^S}$ is the CoVaR of the system for all institutions being in financial distress, and $CoVaR_{q,t}^{-\alpha\sigma_t^1 \leq r_t^1 \leq \alpha\sigma_t^1,\dots,-\alpha\sigma_t^S \leq r_t^S \leq \alpha\sigma_t^S}$ is the CoVaR of the system for all institutions being in their benchmark state. Adrian and Brunnermeier (2011) define the benchmark state as institution i's returns being at their median, while Cao (2013) defines the benchmark state as the institution's returns being at an $\alpha \times \sigma_t$-event around the mean, where α is constant, and σ_t is the institution's standard deviation at time t. Furthermore, while Adrian and Brunnermeier (2011) only focus on a single institution being in financial distress, Cao (2013) calculates the total systemic risk contribution of all institutions in the system being in financial distress at the same time, and allocates the systemic risk contribution of a single institution in a second step.

In the second step Cao (2013) applies the so-called Shapley value methodology to the total systemic risk. The Shapley value was initially introduced for cooperative games where players create together one outcome for the whole group. For a single player the Shapley value is the expected marginal contribution to the outcome over the set of all permutations of players. It is a fair and efficient method to allocate the systemic risk contribution among the institutions and fulfills a set of favorable characteristics such as additivity[1].

Cao (2013) also uses a different methodology compared to Adrian and Brunnermeier (2011). He starts by assuming an underlying multivariate student t-distribution for the returns of the system and each institution. Then, GARCH modeling for estimating the time-varying volatility of the returns and the DCC approach, introduced by Engle (2002), for estimating the time-varying correlations between the returns is used to obtain the joint distribution of the returns. For the VaR of each institution however, Cao (2013) uses a Bootstrap approach which does not depend on a distributional assumption and is thus less restrictive. Given the VaR of each institution and the joint distribution of the system's and institutions' returns, Cao (2013) solves numerically for the CoVaR value of the system conditional on the adverse state and the benchmark state, and obtains ΔCoVaR.

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[1] For CoVaR, additivity means that the joint CoVaR of all institutions combined is equal to the sum of each individual CoVaR-value. Thus the following must hold: $CoVaR_{q,t}^{1,\dots,S} = \sum_{i=1}^{S} CoVaR_{q,t}^{i}$.

The estimations are based on two panels, a French and a Chinese one, which consist of the five biggest institutions of each country for which weekly returns are extracted for the time between 19th April 2002 and 29th January 2012 for the French panel, and between 27th October 2006 and 29th June 2012 for the Chinese panel. His results show that French banks are much more affected by the global and the European financial crisis than Chinese banks are, which indicates a lower global interconnectedness of the Chinese banks.

Another important finding is that while the Multi-CoVaR using Shapley values exhibits additivity, the Multi-CoVaR without Shapley values is smaller than the sum of the individual corresponding CoVaR's.

2.1.3 CoVaR estimation using multivariate GARCH models

Another paper extending the CoVaR approach is *Systemic risk measurement: Multivariate GARCH estimation of CoVaR* by Girardi and Ergün (2013). In their paper the authors change the definition given by Adrian and Brunnermeier (2011) of an institution being *exactly* at its VaR level as its distress state to an institution being *at most* at its VaR level as its distress state, which is formulated as

$$\Pr\left(R_t^i \leq CoVaR_{q,t}^{i|j} \middle| R_t^j \leq VaR_{q,t}^j\right) = q,$$

where R_t^i is the returns of the system i, R_t^j is the returns of institution j, $VaR_{q,t}^j$ is the VaR of institution j, and q is the confidence level. This new definition also takes into account more severe outcomes than the VaR level and it opens up the possibility to back test the CoVaR measure as well as improves its consistency with respect to the dependence parameter between institution j and the system i, as shown by Mainik and Schaanning (2012).

Similarly to Cao (2013), Girardi and Ergün (2013) define the benchmark case as a one-standard deviation around the mean return event. Furthermore, they also use a GARCH and DCC approach to obtain skewed-t and Gaussian joint distributions of the returns of the system and institution j, which are then used to solve numerically for CoVaR.

Their estimations and calculations are based on data of 74 US financial institutions with market value greater than 5bln USD for the time period between 26th June 2000 and 29th February 2008. The authors backtest their CoVaR results based on both the Gaussian and the skewed-t distribution using the Kupiec and the Christoffersen test. While the results based on the skewed-t distribution assumption pass all tests, the results from assuming a Gaussian distribution exceed the confidence level in the unconditional coverage test. Furthermore, they find that the CoVaR estimates based on the skewed-t distribution are all higher than the estimates based on the Gaussian distributional assumption.

The authors also investigate the relation between firm characteristics and systemic risk contribution of an institution. Contrary to Adrian and Brunnermeier (2011), they do not find a strong relation between the VaR of an institution and its systemic risk contribution $\Delta CoVaR$ in the time series dimension, which implies that monitoring only the tail risk of an institution is not enough to predict its systemic risk contribution.

2.1.4 CoVaR estimation using Copula functions

Karimalis and Nomikos (2014) in *Measuring systemic risk in the European banking sector: A Copula CoVaR approach* also extend the CoVaR approach in the sense that they propose a new methodology for its calculation. In order to obtain the joint probability distribution of the system and an institution, the authors use so-called Copula functions. Copula functions can replicate the true multivariate joint distribution function using only the univariate marginal distribution of each series and a copula which describes the dependence between the two series.

Karimalis and Nomikos (2014) calculate the joint distributions using different types of copulas from different copula families, for both the definition of CoVaR given by Adrian and Brunnermeier (2011) $\left(CoVaR_q^{j|X^i=VaR_q^i}\right)$ and for the definition given by Girardi and Ergün (2013) $\left(CoVaR_q^{j|X^i\leq VaR_q^i}\right)$. Furthermore, they extend their calculations to obtain the conditional expected shortfall (CoES) of the system.

The authors base their estimations on return data of 42 European banks from 1st April 2002 to 31st December 2012. Similar to Girardi and Ergün (2013), they backtest their results and obtain similar results concerning the Gaussian and the skewed-t distribution.

Furthermore, the authors suggest ways for stress testing using their copula approach. These ways include changing the marginal distributional assumptions, changing the copula functions, or changing the dependence structure between the series.

Karimalis and Nomikos (2014) also analyze the relation between VaR and ΔCoVaR and come to the conclusion that depending on the definition of the benchmark case, the link between VaR and ΔCoVaR is strong if the benchmark case is defined according to Adrian and Brunnermeier (2011), and weak if the benchmark case is defined as in Girardi and Ergün (2013). Following the work of Adrian and Brunnermeier (2011), Karimalis and Nomikos (2014) also assess the influence of common market factors as well as firm characteristics on the systemic risk contribution of an institution and receive similar results, namely that size, leverage and equity beta are key drivers of systemic risk.

2.1.5 Systemic risk measures in comparison

Benoit et al. (2013) in *A Theoretical and Empirical Comparison of Systemic Risk Measures* analyze several systemic risk measures, including CoVaR, both theoretically and empirically. For their analysis they focus on the three systemic risk measures: MES, SRISK, and ΔCoVaR.

Through their theoretical assessment of ΔCoVaR, the authors come to the conclusion that, given the definition of ΔCoVaR by Adrian and Brunnermeier (2011), the measure is simply a linear projection of the institution's VaR and thus can be expressed as

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it}[VaR_{it}(\alpha) - VaR_{it}(0.5)],$$

where $\gamma_{it} = \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}$ is the linear projection coefficient of the market return on the firm return, ρ_{it} is the correlation between the market and the firm at time t, σ_{mt} is the volatility of the market at time t, and σ_{it} is the volatility of the firm at time t. Benoit et al. (2013) find that, due to the scaling nature of γ_{it}, ranking the institutions by their VaR and by their ΔCoVaR may lead to different results. However, they further find that for an individual institution, forecasting VaR is sufficient to predict the evolution of its systemic risk contribution ΔCoVaR.

The authors also compare the ranking of the institutions' systemic risk contribution as measured by their MES and ΔCoVaR, and conclude that the rankings may not be equivalent. However, they also find that the higher the correlation between an institution's returns and the system's returns, the more likely it is that MES and ΔCoVaR lead to the same systemic risk contribution ranking. Thus, even though the systemic risk measures have differing definitions, under certain conditions they will produce the same output. For the relation between SRISK and ΔCoVaR, the authors find that only under certain restrictive conditions, namely a high leverage of the bank and a high correlation of the bank's returns with the system's returns, the two measures will lead to equivalent results.

Besides their theoretical analysis of the three systemic risk measures, Benoit et al. (2013) also conduct an empirical comparison of the measures. They base their results on data of 94 US financial firms with market capitalization above 5 bn. USD for the period between 3rd January 2000 and 31st December 2010. The ΔCoVaR outputs that are used for the assessment are obtained through a Quantile Regression. The authors conduct their comparison by ranking the ten so-called SIFI's (systemically important financial institution) as the institutions with the highest systemic risk contribution according to the three measures. They find that the different systemic risk measures lead to different identification of SIFI's and report little overlap between the identified SIFI's. In addition, their empirical results confirm what has been concluded previously on a theoretical basis, namely that there is only a weak link between the VaR of an institution and its systemic risk contribution ΔCoVaR in the cross-sectional dimension, but a strong relation in the time-series dimension.

2.1.6 Summary and limitations of CoVaR

The rich literature on CoVaR, including applications of the measure, in-depth analysis of its properties, and numerous extensions to the measure, show that the measure indeed makes an important contribution to the understanding, measuring, and managing of systemic risk. It has been shown that regulating financial institutions according to their idiosyncratic risk measured by VaR is not sufficient, given the loose link between its VaR level and its actual contribution to systemic risk measured by ΔCoVaR. Thus ΔCoVaR can be considered a useful tool in the process of identifying and managing systemic risks.

However, there are also limitations to the CoVaR approach. While in the cross-sectional dimension there is only a weak relation between an institution's VaR and its ΔCoVaR, in the time-series dimension the evolution of the VaR of an institution resembles its ΔCoVaR evolution very closely. Thus it can be argued that ΔCoVaR does not capture the multiple facets of systemic risk and hence adds only little additional information beyond its idiosyncratic VaR measure (Benoit et al., 2013). Furthermore, systemic risk is characterized by a multitude of key features encompassed in it, of which ΔCoVaR only captures a small amount and is silent about features such as spillover effects to the real economy or the path systemic risk takes when spreading across a network.

2.2 Systemic Risk

So far, no consensus has been reached by researchers regarding the definition of systemic risk but rather a multitude of systemic risk definitions have evolved (Smaga, 2014). The European Central Bank (2009), for example, defines systemic risk as *"the risk that financial instability becomes so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially"* (p. 134). While the definition by the European Central Bank is rather vague, Bisias et al. (2013) list in their survey of systemic risk measures more precise characteristics on which other authors have focused on for their definition of systemic risk. The list includes aspects such as imbalances, spillover effects to the real economy, asset bubbles, correlated exposures of banks, feedback reactions of financial institutions, contagion, information disturbances, and negative externalities.

Smaga (2014) analyzes definitions of systemic risk in the literature with respect to the most common features associated with systemic risk. His analysis takes definitions from 55 different papers, studies and articles into consideration, dating from 1995 to 2014, and he summarizes his results as follows:

- The transmission of shocks between the interconnected institutions of the financial system, which eventually leads to possible adverse results for the real economy, is a key feature of systemic risk.
- In a large part of the literature it is highlighted that systemic risk affects the whole financial system or a majority of the financial institutions, and interferes with the operations and the purpose of the system, e.g. financial intermediation. However, the loss of confidence in the financial system that systemic risk causes is considered only by a small group of authors.
- The first definitions of systemic risk were introduced between 1995 and the time of the global financial crisis and focused mainly on the contagion effect and the wide range of affection of this occurrence. After the financial crisis the number of research on systemic risk increased dramatically and with it the number of definitions of systemic risk. Furthermore, part of the emphasis was shifted towards the disruption of the features of the financial system and the negative spillover effects this causes for the real economy.

Systemic risk can be summarized as a complex phenomenon that manifests itself through a wide range of different characteristics and affects the entire financial system which yields adverse results for both the financial system and the real economy through spill-over effects.

The multitude of definitions of systemic risk is a reflection of the undoubtable complexity of the phenomenon, and consensus on a single all-encompassing definition might never be reached. Naturally, the high number of different definitions results in a similarly high number of different systemic risk measures, and just as the various definitions emphasize on different aspects of systemic risk so are the systemic risk measures based on different aspects of the phenomenon.

The main focus of this essay lies on ΔCoVaR to measure systemic risk. However, as mentioned above there exists a long list of different systemic risk measures. A selection of systemic risk measures are briefly introduced in Appendix A of the essay.

3 Methodology

As described in Section 2, several extensions of the ΔCoVaR measure with respect to the methodology have been developed. In this essay, the methodology is based on the work of Cao (2013) and Girardi and Ergün (2013), who use a GARCH-DCC approach to model the time-varying joint distribution of the system and a single institution, and calculate the systemic risk contribution of one or more institutions. The advantage of a GARCH-DCC approach, compared to a quantile regression, which Adrian and Brunnermeier (2011) use to calculate ΔCoVaR, lies in the feature that the GARCH-DCC approach allows to take into account time-varying linkages between the system and one or more institutions without having to rely on systemic state variables (Girardi & Ergün, 2013).

3.1 VaR as a starting point

Recall that VaR is defined as the q-quantile of the return distribution and thus can be formulated in terms of returns in the following way,

$$\Pr\left(r_t \leq VaR_t^q\right) = q, \tag{1}$$

where r_t is the return at time t, and $VaR_{i,t}^q$ is the q-quantile of the returns r_t at time t. This implies that VaR can also be written as the upper boundary of an integral in the following formulation,

$$\int_{-\infty}^{VaR_t^q} pdf_t(r_t)\, dr_t = q, \tag{2}$$

where $pdf_t(r_t)$ is the probability density function of the returns at time t.

3.2 Definition of CoVaR

Recall from the previous section, that CoVaR is defined as the VaR of the financial system conditional on an institution i being at its VaR^q-level, which represents financial distress for this institution. By changing the definition of CoVaR to the VaR of the system conditional on an institution i being *at most* at its VaR^q-level, the CoVaR measure exhibits favorable characteristics with respect to dependence consistency. Mainik and Schaanning (2012) show that only with the latter definition of CoVaR, the measure is an increasing continuous function of the dependence parameter between the system and institution i. Furthermore, the definition can be extended to account for N institutions being in financial distress at the same time as introduced by Cao (2013). Thus following the definition of CoVaR by Girardi and Ergün (2013) and Cao (2013), in this essay, CoVaR for the general case of N institutions being at most at their VaR^q-level is defined as the q-quantile of the following conditional distribution,

$$\Pr\left(r_t^S \le CoVaR_{q,t}^{S|1,\dots,N} \middle| r_t^1 \le VaR_{q,t}^1, \dots, r_t^N \le VaR_{q,t}^N\right) = q, \tag{3}$$

where r_t^S is the return of the system S at time t, r_t^i is the return of institution i at time t, $CoVaR_{q,t}^{S|1,\dots,N}$ is the CoVaR measure conditional on institutions $\{1,\dots,N\}$ being in distress, and $VaR_{q,t}^i$ is the VaR of institution i at time t at confidence level q. Following Girardi and Ergün (2013), the CoVaR measure of the system conditional on N institutions being in their benchmark state $b^{1,\dots,N}$, is in this essay defined as the one standard-deviation around the mean event $\mu_t^i - \sigma_t^i \le r_t^i \le \mu_t^i + \sigma_t^i$, where μ_t^i is the mean of institution i at time t and σ_t^i is the standard deviation of institution i at time t. Given both the CoVaR measure for the distress state and the CoVaR measure for the benchmark state, the combined systemic risk contribution ΔCoVaR of N institutions can be expressed as

$$\Delta CoVaR_{q,t}^{S|1,\dots,N} = CoVaR_{q,t}^{S|1,\dots,N} - CoVaR_{q,t}^{S|b^{1,\dots,N}}, \tag{4}$$

where $CoVaR_{q,t}^{S|1,\dots,N}$ is the CoVaR measure of the system conditional on institutions $\{1,\dots,N\}$ being in financial distress, and $CoVaR_{q,t}^{S|b^{1,\dots,N}}$ is the CoVaR measure of the system conditional on institutions $\{1,\dots,N\}$ being in their benchmark state.

3.3 Calculations steps of the CoVaR measure

In order to increase the traceability of the calculations needed to obtain the ΔCoVaR measure, the calculation procedure is divided into three steps.

3.3.1 Step 1: Calculation of individual VaR

In a first step, a distributional assumption about the returns of the institutions in the system and the system itself has to be made. For the returns an underlying student t-distribution is assumed which accounts for fatter tails than a normal distribution, a well-known stylized fact of financial returns. Hence, the returns are defined as

$$r_t^i \sim t_\nu\left(\mu_t^i, \sigma_{i,t}^2\right),$$

where μ_t^i is the mean, $\sigma_{i,t}^2$ is the variance, and ν are the degrees of freedom of the student t-distribution. By assuming an underlying student t-distribution for the returns, the VaR for each institution, and the system itself, can be calculated using the following formula

$$VaR_t^q(r_t^i) = \mu_t^i + \sqrt{\frac{\nu - 2}{\nu}}\sigma_{t+1}^i t_{q,\nu}, \tag{5}$$

where $t_{q,\nu}$ is the q-quantile of the student t-distribution with ν degrees of freedom (Nilsson, 2015).

In order to obtain the inputs required to calculate the VaR for each institution and the system, a univariate GJR GARCH(1,1) model is estimated. The GJR GARCH model, first introduced by Glosten, Jagannathan and Runkle (1993), accounts for the so-called leverage effect which is commonly observed for financial data. The leverage effect usually refers to negative shocks having a bigger effect on changes in volatility than positive shocks. Here, the variance equation of the GJR GARCH model, in the formulation introduced by Ding, Granger and Engle (1993), is being used. The mean equation of the model is specified in the following way

$$r_t^i = a_0 + \sum\nolimits_{j=1}^{p} a_j r_{t-j}^i + \varepsilon_t^i + \sum\nolimits_{k=1}^{q} b_k \varepsilon_{t-k}^i, \tag{6}$$

with

$$\varepsilon_t^i = \sigma_t v_t$$

and

$$v_t \sim iid(0,1),$$

where a_0 is an intercept term, $a_j r_{t-j}^i$ is the autoregressive component, $b_k \varepsilon_{t-k}^i$ is the moving average component, and ε_t^i are the error terms.

The number of lags in the AR part, denoted by p in the sum, and the number of lags in the MA part, denoted by q in the sum, in Equation (6) are determined by the Bayesian Information Criterion (BIC).

The variance equation of the model is specified as

$$\sigma_t^\delta = \omega + \alpha_1\left(\left|\varepsilon_{t-1}^i\right| - \gamma_1 \varepsilon_{t-1}^i\right)^\delta + \beta_1 \sigma_{t-1}^\delta, \tag{7}$$

where α_1 measures the size of the impact of a shock on volatility, β_1 measures the persistence of shocks on volatility, δ indicates if the variance equation represents the conditional variance or the conditional standard deviation, and γ_1 captures the leverage effect. For $\gamma_1 > 0$, negative shocks increase the conditional variance more than positive shocks and vice versa for $\gamma_1 < 0$. For $\gamma_1 = 0$, the GJR GARCH model boils down to a standard symmetric GARCH model.

The model is estimated using the quasi-maximum likelihood estimation method, which yields estimates for the conditional mean μ_t, the conditional variance σ_t^2, and the degrees of freedom ν. These estimates for each institution and the system are then used to calculate its VaR using the VaR-formula in Equation (5).

3.3.2 Step 2: Estimation of the joint probability density function

For the estimation of the correlations between the system's returns and the returns of one or more institutions, the dynamic conditional correlation approach (DCC) by Engle (2002) is used.

In order to calculate the CoVaR measure, another distributional assumption for the joint distribution of the system's returns and the N institutions' returns has to be made. To take into account the well-known fact of fat tails of financial data, a multivariate student t-distribution is assumed for the joint distribution

$$\begin{pmatrix} r_t^S \\ r_t^1 \\ \vdots \\ r_t^N \end{pmatrix} \sim t_\nu \left(\begin{pmatrix} \mu_t^S \\ \mu_t^1 \\ \vdots \\ \mu_t^N \end{pmatrix}, \begin{pmatrix} \sigma_{S,t}^2 & \rho_{S1,t}\sigma_{S,t}\sigma_{1,t} & \cdots & \rho_{SN,t}\sigma_{S,t}\sigma_{N,t} \\ \rho_{1S,t}\sigma_{1,t}\sigma_{S,t} & \sigma_{1,t}^2 & \cdots & \rho_{1N,t}\sigma_{1,t}\sigma_{N,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{NS,t}\sigma_{N,t}\sigma_{S,t} & \rho_{N1,t}\sigma_{N,t}\sigma_{1,t} & \cdots & \sigma_{N,t}^2 \end{pmatrix} \right),$$

where $r_t = (r_t^S, r_t^1, \dots, r_t^N)'$ is the $(N+1 \times 1)$ vector of returns, $\mu_t = (\mu_t^S, \mu_t^1, \dots, \mu_t^N)'$ is the $(N+1 \times 1)$ vector of conditional means, and Σ_t is the $(N+1 \times N+1)$ conditional covariance matrix, $\sigma_{i,t}$ is the conditional standard deviation of i, and $\rho_{ij,t}$ is the conditional correlation between i and j.

The correlation coefficients defined as the $(N+1 \times N+1)$ matrix of conditional correlations can be denoted as $R_t = diag(\Sigma_t)^{-1/2}\Sigma_t diag(\Sigma_t)^{-1/2} = \rho_{ij,t}$. Following Engle (2002), the conditional correlation matrix is estimated by the following model:

$$R_t = diag(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t diag(\mathrm{Q}_t)^{-1/2}, \tag{8}$$

where

$$\mathrm{Q}_t = (1 - \delta_1 - \delta_2)\bar{Q} + \delta_1(\varepsilon_{t-1}^* \varepsilon_{t-1}^{*}{}') + \delta_2 Q_{t-1}. \tag{9}$$

$\bar{Q}$ is the unconditional covariance matrix of the standardized residuals $\varepsilon_t^* = \varepsilon_t \times diag(\mathrm{Q}_t)^{-1/2}$, and δ_1 and δ_2 are non-negative scalars that fulfill the following condition of stationarity $\delta_1 + \delta_2 < 1$.

The DCC approach follows a two-step procedure. For both steps the QML estimation method is used. In the first step, the volatility part of the joint student t-distribution from above is estimated. For this purpose the respective underlying GJR GARCH (1,1) model from the previous step is fitted to each institution's return series. In addition, the shape parameter given by the degrees of freedom ν is estimated for the joint student t-distribution. In the second step, the correlation part is estimated using the model described in Equation (8) and (9), with the estimates from step one serving as inputs. Thus, given the estimates from the DCC approach the conditional covariance matrix Σ_t can be specified entirely.

Given the estimates for the conditional mean vector $\mu_t = (\mu_t^S, \mu_t^1, \dots, \mu_t^N)'$, the conditional covariance matrix Σ_t, and the degrees of freedom ν of the multivariate student t-distribution, the joint probability density function of the system and N institutions can be obtained.

3.3.3 Step 3: Calculating CoVaR

After obtaining all the inputs needed to calculate CoVaR, the parameters are used in the following equations to obtain the systemic risk measure.

Recall the definition of CoVaR from above,

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|1,\ldots,N} \middle| r_t^1 \leq VaR_{q,t}^1, \ldots, r_t^N \leq VaR_{q,t}^N\right) = q. \tag{10}$$

By means of conditional probabilities, the above definition can be reformulated as

$$\frac{\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|1,\ldots,N} \cap r_t^1 \leq VaR_{q,t}^1, \ldots, r_t^N \leq VaR_{q,t}^N\right)}{\Pr\left(r_t^1 \leq VaR_{q,t}^1, \ldots, r_t^N \leq VaR_{q,t}^N\right)} = q. \tag{11}$$

Given the VaR estimates from Step 1 and the joint probability density function of the returns obtained in Step 2, the denominator $\Pr\left(r_t^1 \leq VaR_{q,t}^1, \ldots, r_t^N \leq VaR_{q,t}^N\right)$ of the above equation can easily be solved, yielding a joint probability q_d

$$\int_{-\infty}^{VaR_{q,t}^1} \ldots \int_{-\infty}^{VaR_{q,t}^N} pdf_t(r_t^1, \ldots, r_t^N)\, dr_t^1 \cdots dr_t^N = q_d. \tag{12}$$

Thus, by plugging in the result from Equation (12) and through simple rearrangements of terms, Equation (11) becomes

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|1,\ldots,N} \cap r_t^1 \leq VaR_{q,t}^1, \ldots, r_t^N \leq VaR_{q,t}^N\right) = q \times q_d. \tag{13}$$

In the same fashion as the denominator in Equation (11) has been reformulated, Equation (13) can be reformulated the following way,

$$\int_{-\infty}^{CoVaR_{q,t}^{S|1,\ldots,N}} \int_{-\infty}^{VaR_{q,t}^1} \ldots \int_{-\infty}^{VaR_{q,t}^N} pdf_t(r_t^S, r_t^1, \ldots, r_t^N) dr_t^S\, dr_t^1 \cdots dr_t^N = q \times q_d. \tag{14}$$

Since all other parameters are given, Equation (14) can be solved numerically with respect to $CoVaR_{q,t}^{S|1,\ldots,N}$ for each time period t, yielding the CoVaR measure conditional on N institutions being in financial distress.

In order to calculate the combined systemic risk contribution of the N institutions, a benchmark case of the CoVaR measure is needed. Following Girardi and Ergün (2013) and Cao (2013), the CoVaR for the benchmark state is defined as

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^{1,\ldots,N}} \middle| \mu_t^1 - \sigma_t^1 \leq r_t^1 \leq \mu_t^1 + \sigma_t^1, \ldots, \mu_t^N - \sigma_t^N \leq r_t^N \leq \mu_t^N + \sigma_t^N\right) = q, \tag{15}$$

where $\mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i$ is the one standard-deviation event around the mean.

Again, as in the distress case, Equation (15) can be reformulated by means of conditional probabilities,

$$\frac{\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^{1,\dots,N}} \cap \mu_t^1 - \sigma_t^1 \leq r_t^1 \leq \mu_t^1 + \sigma_t^1, \dots, \mu_t^N - \sigma_t^N \leq r_t^N \leq \mu_t^N + \sigma_t^N\right)}{\Pr(\mu_t^1 - \sigma_t^1 \leq r_t^1 \leq \mu_t^1 + \sigma_t^1, \dots, \mu_t^N - \sigma_t^N \leq r_t^N \leq \mu_t^N + \sigma_t^N)} = q. \tag{16}$$

For the denominator in Equation (16), the joint probability p_d in the benchmark case can be obtained by solving the following equation

$$\int_{\mu_t^1-\sigma_t^1}^{\mu_t^1+\sigma_t^1} \dots \int_{\mu_t^N-\sigma_t^N}^{\mu_t^N+\sigma_t^N} pdf_t(r_t^1, \dots, r_t^N)\, dr_t^1 \cdots dr_t^N = p_d. \tag{17}$$

Plugging in the result from Equation (17) into Equation (16) and rearranging terms yields

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^{1,\dots,N}} \cap \mu_t^1 - \sigma_t^1 \leq r_t^1 \leq \mu_t^1 + \sigma_t^1, \dots, \mu_t^N - \sigma_t^N \leq r_t^N \leq \mu_t^N + \sigma_t^N\right) = q \times p_d. \tag{18}$$

Given all other parameters, by rewriting Equation (18) in terms of a multiple integral and a probability density function, it is once again possible to solve numerically for $CoVaR_{q,t}^{S|b^{1,\dots,N}}$ using the following formula,

$$\int_{-\infty}^{CoVaR_{q,t}^{S|b^{1,\dots,N}}} \int_{\mu_t^1-\sigma_t^1}^{\mu_t^1+\sigma_t^1} \dots \int_{\mu_t^N-\sigma_t^N}^{\mu_t^N+\sigma_t^N} pdf_t(r_t^S, r_t^1, \dots, r_t^N) dr_t^S\, dr_t^1 \cdots dr_t^N = q \times p_d. \tag{19}$$

Given the CoVaR measure for the benchmark and the distress state, the combined systemic risk contribution of N financial institutions can be calculated by taking the difference of the two CoVaR measures

$$\Delta CoVaR_{q,t}^{S|1,\dots,N} = CoVaR_{q,t}^{S|1,\dots,N} - CoVaR_{q,t}^{S|b^{1,\dots,N}}. \tag{20}$$

The above described calculations and estimations apply both for the case of measuring the combined systemic risk contribution of several financial institutions, and for the case of measuring the systemic risk contribution of an individual institution. In the latter case, some of the estimations and equations can be simplified. Appendix B provides information about the estimations and calculations used for measuring the systemic risk contribution of an individual institution.

4 Data

For the estimations and calculations in this essay, stock market data of 32 European banks from 10 different Eurozone countries are used. The data consists of daily stock prices covering a time period of 10 years from 1st May 2005 to 1st May 2015 and has been obtained through Datastream.

A complete overview of all included banks can be found in Appendix C. Table 1 provides an overview of the number of banks per country included in the sample. The number of countries that participated in the European Monetary Union over the whole sample period is limited to a total of 12 countries. These are the 11 member states that founded the union in 1999 and Greece, which joined the union in 2001. Finland and Luxembourg however have not been included in the sample due to the lack of data on listed banks from the respective countries.

Table 1 *Number of banks per country*

Country	Austria	Belgium	France	Germany	Greece	Ireland	Italy	Netherlands	Portugal	Spain
No. of Banks	2	2	4	4	4	2	5	2	2	5

Percentage returns for each institution have been generated using the following formula,

$$r_t^i = \frac{P_t^i - P_{t-1}^i}{P_{t-1}^i},$$

where r_t^i is the return of institution i at time t, P_t^i is the closing price of institution i at time t, and P_{t-1}^i is the closing price of institution i at time $t-1$. Using returns instead of prices brings the advantage of a unit free measure with it.

As a representation of the financial system, a market-value-weighted return index of the 32 European banks has been created, using the following formula,

$$r_t^S = \sum_{i=1}^{32} w_t^i r_t^i,$$

where

$$w_t^i = \frac{MV_t^i}{\sum_{i=1}^{32} MV_t^i}.$$

w_t^i is the weight of institution i at time t, MV_t^i is the market value of institution i at time t, $\sum_{i=1}^{32} MV_t^i$ is the total market value of all 32 institutions at time t, and r_t^S is the market-value-weighted return index that represents the financial system at time t.

Following the hypothesis of efficient financial markets, using publicly available stock prices has the advantage of taking all available information into account, as they are reflected by the prices. While these prices are sufficient for measuring the historical and contemporaneous systemic risk contribution of an institution, their use in forecasting future systemic risk contributions of an individual bank is limited. In order to forecast the systemic risk contribution of an institution, information regarding firm specific characteristics are needed. Unfortunately, this kind of information is only available to the regulation authorities, and not to the public.

5 Empirical Results

In this chapter, the results and findings from the estimations and calculations will be presented. Given the sample size, the output will be aggregated on a country level using averages. However, the individual estimation output for all 32 financial institutions in the sample can be found in Appendix E. First, summary statistics concerning the dataset will be presented. Second, a summary of the estimation parameters of the GARCH and the DCC model, as well as an overview of the VaR of the financial institutions will be given. Third, the time-varying ΔCoVaR outputs will be presented and analyzed. Fourth, a worst-case scenario by country is presented, where CoVaR is calculated conditional on all financial institutions of a certain country being in distress at the same time.

5.1 Summary statistics

Table 2 below gives an overview of the characteristics of the return data used for estimations and calculations. Average daily returns for the period 1st May 2005 to 1st May 2015 were highest for Belgian, French and Austrian banks. The lowest average daily returns are exhibited by Greek, Irish, and Portuguese financial institutions. Average standard deviation, which can be seen as a proxy for risk, was highest for banks from Belgium, Ireland and Greece. Spanish, Dutch and Portuguese financial institutions experienced the lowest volatility, as measured by the average standard deviation. Another interesting characteristic is the average kurtosis of the return data. Banks from all 10 European countries considered in the sample show an average kurtosis more than two times larger than three. In fact, none of the returns series of the financial institutions exhibit a kurtosis lower than five, as Appendix D shows. Since the kurtosis of a normal distribution is three, these results imply that estimations of CoVaR based on the assumption of a normal distribution may be incorrect as they do not take into account the fatter tails of the actual underlying distribution. This supports the assumption of an underlying student t-distribution.

Table 2 *Summary statistics of return data aggregated at country level (01.05.2005 – 01.05.2015)*

Country	Mean (%)	St. Dev (%)	Max (%)	Min (%)	Kurtosis	Skewness
Austria	0,0244	3,1102	18,6160	-21,3831	8,9038	0,1073
Belgium	0,0870	5,9719	74,9533	-30,5980	25,2728	1,6526
France	0,0388	2,9197	27,4777	-16,0813	13,0472	0,7863
Germany	-0,0104	3,0203	31,3208	-22,2003	20,9547	0,7374
Greece	-0,0683	4,7344	29,4866	-28,3954	10,2808	0,5149
Ireland	-0,0189	5,2441	45,7898	-56,6888	19,4301	0,5736
Italy	-0,0140	2,7802	18,6175	-16,8706	8,7257	0,2082
Netherlands	0,0114	2,4234	20,8717	-18,3403	13,2168	0,4646
Portugal	-0,0156	2,6236	26,9663	-14,0615	13,8034	1,0392
Spain	0,0136	2,2008	19,7384	-10,9791	9,9640	0,7131

5.2 Estimation output GJR GARCH, DCC, and VaR

5.2.1 Estimation output

In order to estimate the individual VaR of each financial institution, an underlying student t-distribution is assumed. The underlying distribution can be defined by its mean, its variance and its degrees of freedom. By fitting a GJR GARCH model to each return series, the necessary inputs for the underlying distribution are obtained. Summary statistics of the conditional mean, the conditional variance, and the dynamic conditional correlations, all aggregated at country level, are presented in Table 3.

The estimation results in the first column of Table 3 show that the fitted models for banks from France, Belgium, Austria, and Spain yield a positive average conditional mean, with French, Belgian, and Austrian banks exhibiting the highest average conditional mean. A positive conditional mean implies that the return distributions for these banks are shifted to the right compared to a standard student t-distribution, which is centralized around zero. In contrast, the fitted GJR GARCH models for financial institutions from Portugal, Italy, Ireland, Germany, the Netherlands, and Greece, yield negative average conditional means, suggesting that their return distributions are located to the left of zero. Among the financial institutions whose return distribution is shifted to the left, Portuguese, Italian, and Irish banks' return distributions are located the furthest from zero since they exhibit the lowest average conditional means. The estimation results of the average conditional mean, as presented in the first column of Table 3, show that assuming a student t-distribution that is centralized around zero may lead to incorrect estimates of VaR and CoVaR, due to its neglection of the actual location shift of the underlying distribution.

The estimation results in the second column of Table 3 show that banks from Belgium, Ireland, and Greece experienced on average the highest conditional volatility. Whereas, Spanish, Dutch, and Italian financial institutions exhibited on average the lowest conditional volatility. Appendix E provides a comprehensive overview of the fitted coefficients. It is worth noting that the γ_1 coefficient, which captures the leverage effect in the GJR GARCH model, is positive for the return series of all institutions and in addition significant at the 5% level for all but one institution. This implies that the return series in fact exhibit leverage effects, which should be accounted for in the model.

Column three of Table 3 presents the average dynamic conditional correlations. It shows that the return series of banks from France, Spain, and Italy have the highest correlations with the return series of the system. Whereas, the return series of Greek, Portuguese, and Dutch financial institutions exhibit the lowest correlation with the return series of the financial system. The dynamic conditional correlation is needed to obtain the time varying joint distribution of the individual institutions' returns and the system's returns, which is required to calculate the CoVaR measure of the system.

Table 3 *Summary statistics of average conditional mean, average conditional variance, and average dynamic conditional correlation aggregated at country level (01.05.2005 – 01.05.2015)*

Country	Average conditional mean (%)	Average conditional variance (%)	Average dynamic conditional correlation (%)
Austria	0,0032	0,1024	64,1936
Belgium	0,0061	0,3811	61,2916
France	0,0123	0,0861	79,9829
Germany	-0,0112	0,1053	51,9774
Greece	-0,0024	0,2415	42,3725
Ireland	-0,0177	0,2938	52,8188
Italy	-0,0244	0,0816	71,8088
Netherlands	-0,0060	0,0690	51,0440
Portugal	-0,0254	0,0839	50,4063
Spain	0,0032	0,0508	77,1157

In order to obtain estimates of the individual and the joint underlying student t-distributions, the degrees of freedom as the shape parameter of the distribution are estimated using the quasi-maximum likelihood method. The average degrees of freedom by country, for the underlying individual and joint student t-distribution, as well as average respective p-values are presented in Table 4.

The first column of Table 4 represents the average estimated degrees of freedom by country for the underlying return distribution of an individual bank. The second column shows the average p-value of the respective degrees of freedom estimate. As the first two columns of Table 4 show, all average estimated degrees of freedom lie between three and seven and are highly significant. In fact, even at firm level all the estimates of the degrees of freedom lie between three and nine, as Appendix E shows, which implies fatter tails than a normal distribution. These results are in accordance with the findings from the summary statistics of the return data, which rejected the normal distribution as the underlying distribution due to the large kurtosis.

In column three, the average estimated degrees of freedom by country for the joint distribution of an individual bank and the system are presented. Column four shows the respective aggregated p-values. Again, the estimates are highly significant and lie between five and seven. As Appendix D shows, at the firm level the estimates lie between four and eight. These results support the assumption of an underlying student t-distribution for the joint return distribution of an institution and the financial system.

Table 4 *Summary statistics of individual and joint degrees of freedom aggregated at country level (01.05.2015 – 01.05.2015)*

Country	DF	p-value	Joint DF	p-value
Austria	5,3316	0,0000	6,0903	0,0000
Belgium	5,2312	0,0000	5,6232	0,0000
France	6,6257	0,0000	5,3573	0,0000
Germany	4,4344	0,0000	5,0639	0,0000
Greece	5,1571	0,0000	6,7329	0,0000
Ireland	4,3687	0,0000	5,8443	0,0000
Italy	6,0159	0,0000	5,9731	0,0000
Netherlands	5,3096	0,0000	5,3006	0,0000
Portugal	3,7577	0,0000	5,0520	0,0000
Spain	5,6103	0,0000	5,7055	0,0000

5.2.2 Time-varying conditional variance and dynamic conditional correlations

Since Table 3 does not allow to assess the evolution of the conditional variance over time, Figure 1 depicts its development from 1st May 2005 to 1st May 2015. For clarity reasons, the average of the conditional variance of all institutions in the system is taken. The graph shows a tranquil period in the beginning followed by a period of very high volatility from mid-2008 to mid-2009, represented by the high peaks during this time. The volatility decreases again thereafter but exhibits three more peaks: one in mid-2010, one in late-2011, and one in mid-2013. While the first and largest peaks can be explained by the global financial crisis, the three peaks thereafter represent the distressing events caused by the European debt crisis. Since the global financial crisis and the European debt crisis have been major system-wide shocks, it can be expected that these events also affected the ΔCoVaR measure.

Figure 1 *Average conditional variance of all sample institutions (01.05.2005 – 01.05.2015)*

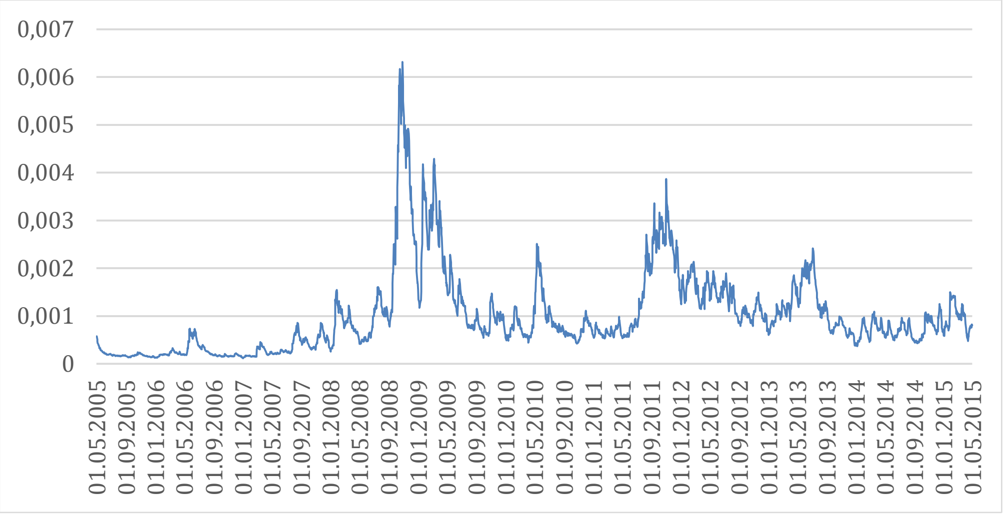
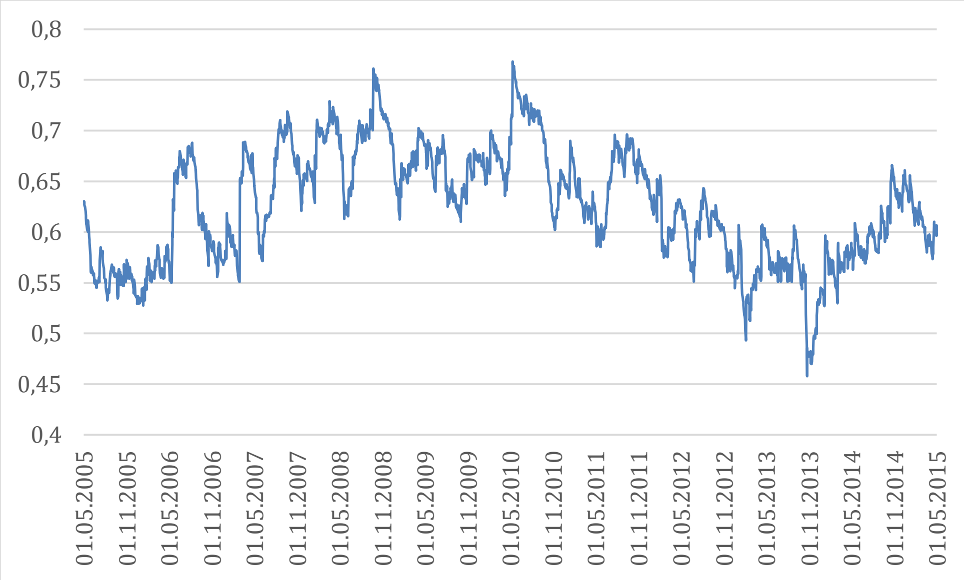


Figure 2 depicts the evolution of the dynamic conditional correlations from 1st May 2005 to 1st May 2015. Again, for clarity reasons the average of the dynamic conditional correlations of all banks in the system is taken. While the average dynamic conditional correlation is also influenced by the events of the global financial crisis and the European debt crisis, the impact is less pronounced than for the average conditional variance. Interestingly, while the average conditional variance exhibits peaks over the course of the year 2013, caused by the events of the European debt crisis, the average dynamic conditional correlations in contrast show decreasing correlations between the financial institutions and the system. This is somewhat surprising, as the correlations between a financial institution and the system are expected to increase in times of a crisis (Ang & Chen, 2002).

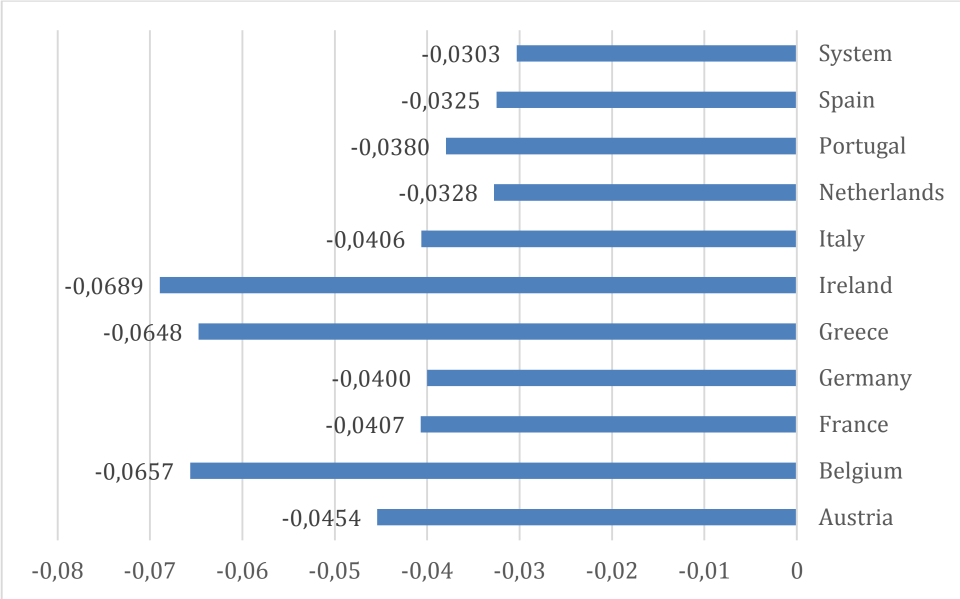
Figure 2 *Average conditional correlation of all sample institutions (01.05.2005 – 01.05.2015)*



5.2.3 Individual VaR

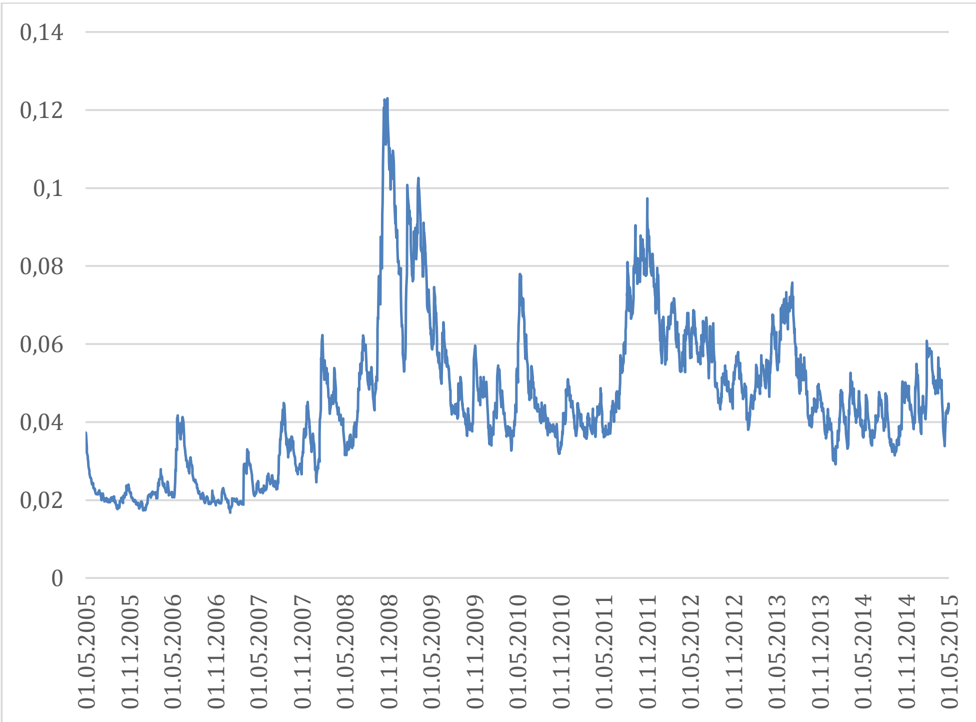
Using the estimation outputs that have been described above, daily 5% VaR-levels for each institution and the system itself have been calculated. Figure 3 depicts the average daily 5% VaR for each country. The figure shows that on average, Irish, Belgian, and Greek banks exhibited the highest individual VaR, whereas financial institutions from Spain, the Netherlands, and Portugal showed the lowest average VaR. Furthermore, Figure 3 shows that the unconditional VaR of the financial system is lower than the average VaR of each country. This can be explained by the diversification effect of the system, which consists of all sample institutions. The strong link between individual VaR and individual volatility becomes clear when comparing Table 2 to Figure 3, as the countries with the highest (lowest) average conditional variance are among the countries with the highest (lowest) average individual VaR-levels.

Figure 3 *Average daily 5% VaR aggregated at country level (01.05.2005 – 01.05.2015)*



To analyze the time-series behavior of the VaR, the evolution of the average 5% daily VaR of all sample institutions is depicted in Figure 4. Similarly to Figure 1, the average VaR first exhibits relatively low levels until mid-2008, where it begins to increase dramatically. This period of very high VaR-levels lasts until mid-2009 and can be explained by the global financial crisis. The average VaR decreases to lower levels thereafter, but like the average conditional variance shows three more peaks in mid-2010, in late-2011, and in mid-2013. These three peaks can be explained by the on-going European debt crisis. The resemblance of Figure 1 and Figure 4 is another indicator for the strong relation between individual volatility and individual VaR.

Figure 4 *Average 5% daily VaR of all sample institutions (01.05.2005 – 01.05.2015)*



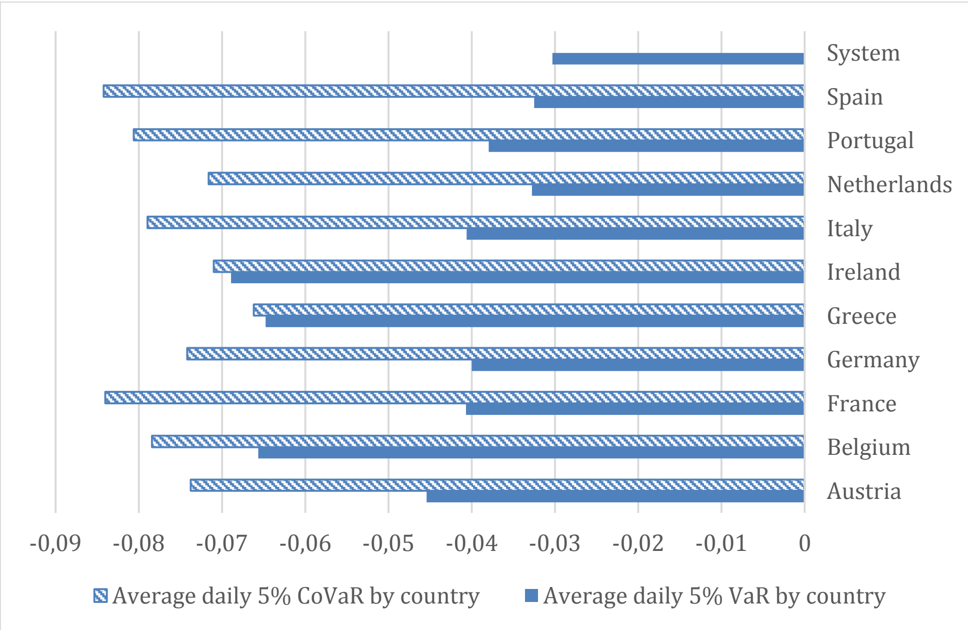
5.3 CoVaR and ΔCoVaR

5.3.1 CoVaR

After analyzing the estimation outputs and the idiosyncratic VaR of the financial institutions, the focus is now put on the empirical results regarding CoVaR and ΔCoVaR. Figure 5 below depicts the average daily 5% VaR and CoVaR by country. Recall, that the CoVaR is the VaR of the financial system conditional on an institution being in financial distress, i.e. its returns being at their VaR level.

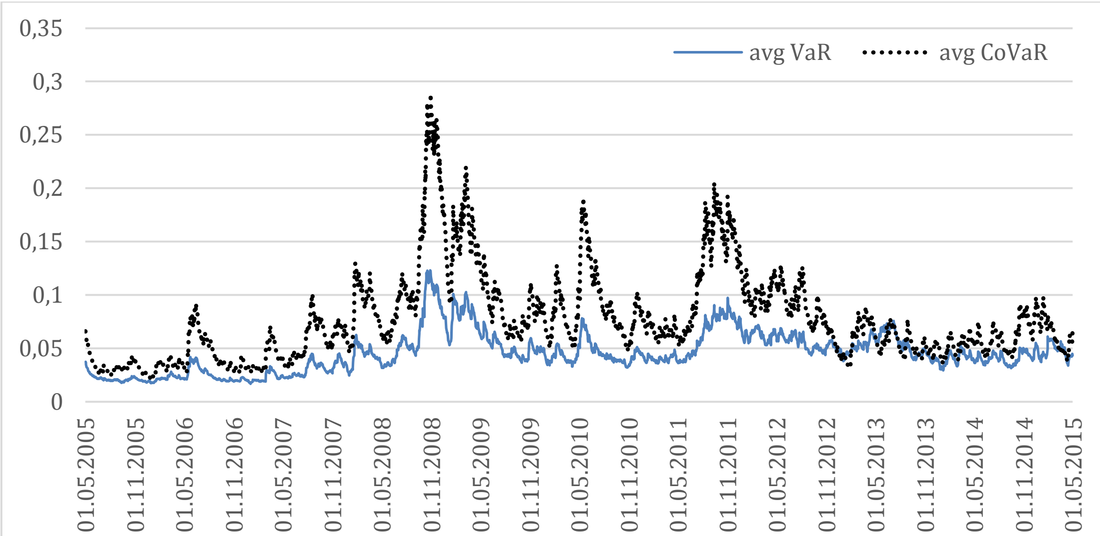
Figure 5 shows that Spanish, French, and Portuguese banks cause the highest average CoVaR levels of the system when being in financial distress, and that financial institutions from Greece, Ireland, and the Netherlands cause the lowest average CoVaR levels of the system when being in financial distress. A possible explanation of this observation is that the Greek, Irish, and Dutch banks are less correlated with the system than financial institutions from Spain, France, and Portugal. Table 3 partly supports this explanation, since the banks from Greece, Ireland and the Netherlands are among the banks which exhibit the lowest average dynamic conditional correlation with the system, and Spanish and French banks are among the banks which exhibit the highest average dynamic conditional correlation with the system. However, while Portuguese financial institutions cause high average CoVaR levels for the system, their average dynamic conditional correlation with the system are among the lowest. Thus, systemic risk seems to be determined by more factors than just correlation with the system. Figure 5 also shows that all countries' average CoVaR is larger than their average VaR. Furthermore, the relation between VaR and CoVaR appears to be rather weak, as low VaR-levels are paired with both high and low CoVaR-levels. Lastly, the average unconditional VaR of the system is, unsurprisingly, smaller than all average VaR's of the system conditional on an individual institution being in distress, as denoted by CoVaR.

Figure 5 *Average daily 5% VaR and CoVaR by country (01.05.2005 – 01.05.2015)*



In order to compare the evolution of VaR and CoVaR over time, the average of the respective measure of all sample institutions is depicted in Figure 6. While Figure 5 indicates that there is a loose cross-sectional relation between VaR and CoVaR, Figure 6 suggests a close link between VaR and CoVaR in the time-series dimension. It is interesting to note that for almost every point in time, the average systemic risk caused by the financial institutions is larger than their average idiosyncratic risk. Especially the peaks of the average systemic risk, caused by the global financial crisis and the European debt crisis, are far more pronounced than the peaks of the average idiosyncratic risk.

Figure 6 *Absolute value of average VaR and CoVaR of all sample institutions (01.05.2005 – 01.05.2015)*



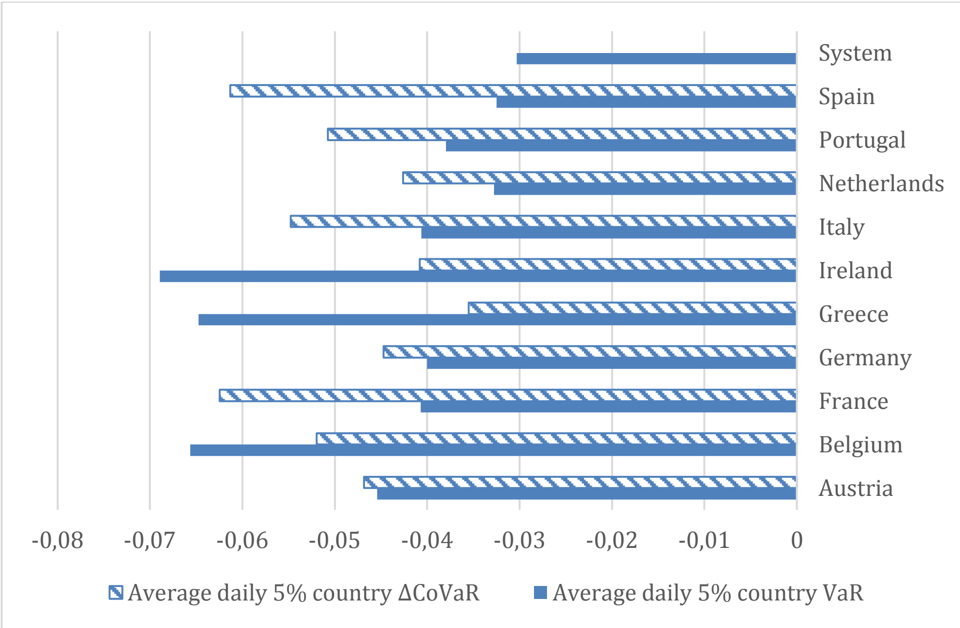
5.3.2 ΔCoVaR

By taking the difference between the distress-state CoVaR of an institution and the benchmark-state CoVaR of that institution, the systemic risk contribution of the institution can be calculated. Figure 7 provides a comparison of average 5% daily VaR and ΔCoVaR by country.

It can be seen that banks from France, Spain, and Italy exhibited the highest systemic risk contribution, while Greek, Irish, and Dutch financial institutions showed the lowest systemic risk contribution. As Appendix C shows, France, Spain, and Italy are represented in the sample by some of the largest banks in Europe such as BNP Paribas, Banco Santander, and UniCredit. Given their size and scale of international activity, it is little surprising that these countries show the highest average systemic risk contribution. Finding Greek banks among the least systemically risky institutions on the other hand is somewhat surprising, given that their home country faces a deep and long lasting economic crisis. One explanation is that financial institutions from Greece are typical commercial banks with focus on the local market but limited international activity, leading to a lower correlation with the financial system (Karimalis & Nomikos, 2014). Interestingly, French, Spanish, and Italian banks are among the banks with the lowest VaR-levels, while financial institutions from Greece and Ireland exhibit some of the highest VaR-levels. However, financial institutions from France, Spain, and Italy exhibit the highest systemic risk contributions, while Greek and Irish banks show the lowest

ΔCoVaR-levels. As in the case of CoVaR, there seems to be only a weak cross-sectional relation between average daily 5% VaR and average daily 5% ΔCoVaR. This finding is in line with the results from previous research, which showed that there is only a weak cross-sectional link between VaR and ΔCoVaR.

Figure 7 *Average daily 5% VaR and ΔCoVaR by country (01.05.2005 – 01.05.2015)*

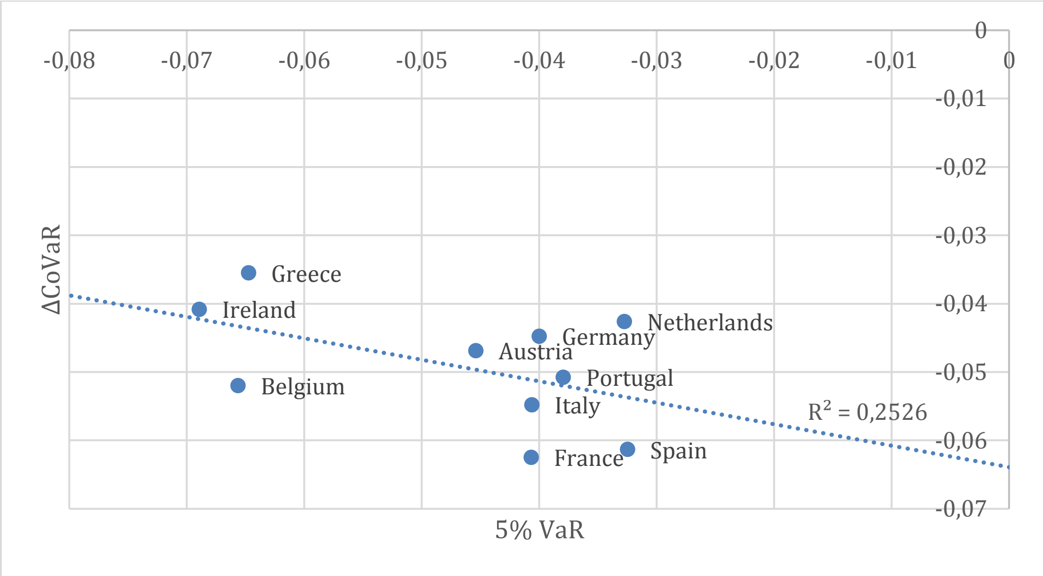


To further investigate the cross-sectional relation between VaR and ΔCoVaR, a scatterplot and a ranking comparison of the two measures are presented below.

For the ranking comparison, the financial institutions in the sample are ranked in descending order and according to their absolute value of VaR and ΔCoVaR for each day, from 1st May 2005 to 1st May 2015. Then the overlap among the 10 highest ranked banks in each ranking is analyzed. The analysis shows that only in 1.48% of the cases, the rank of an institution according to its VaR is the same as its rank according to its ΔCoVaR. Furthermore, in only 13.76% of the cases, an institution which ΔCoVaR is among the 10 highest for a certain day is also found in the top 10 of the highest VaR-levels for that day. Density plots of the matching ranking pairs can be found in Appendix F. The results from the ranking analysis further confirms the finding of a loose link between VaR and ΔCoVaR in the cross-sectional dimension.

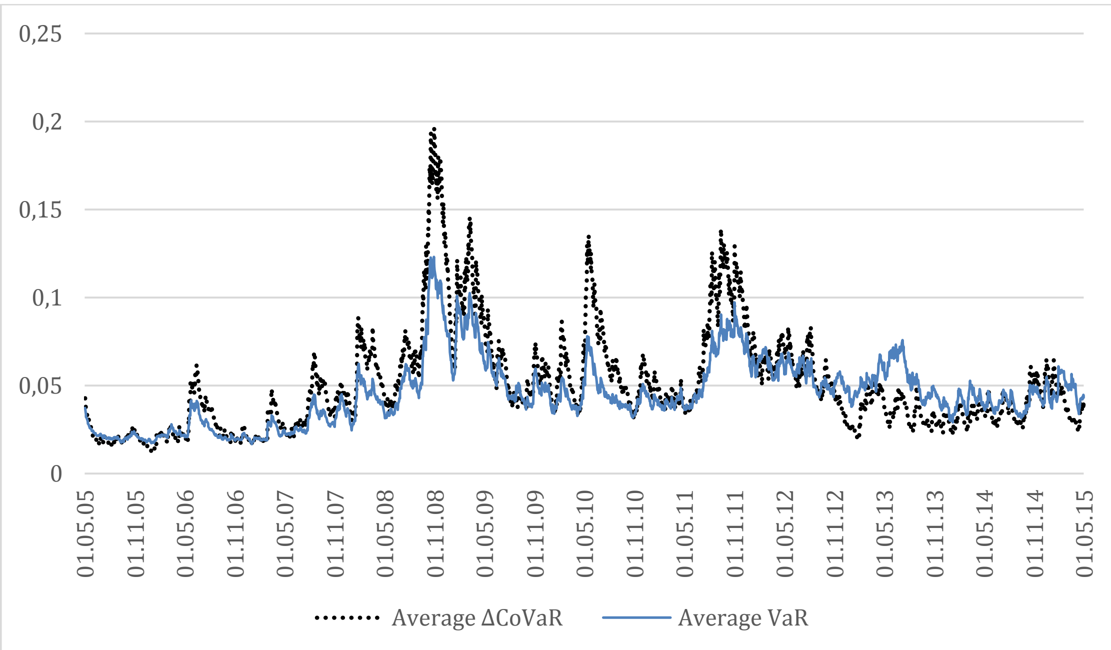
In addition, the scatterplot of the average daily 5% VaR and the average daily 5% ΔCoVaR in Figure 8 shows only a weak relation between the two measures. A visual observation of the plot and a low value for the goodness of fit ($R^2 = 0{,}2526$) confirm the prior findings.

Figure 8 *Scatterplot of average daily 5% VaR and ΔCoVaR by country (01.05.2005 – 01.05.2015)*



In the time-series dimension, however, there seems to be a strong relation between VaR and ΔCoVaR which is depicted in Figure 9. It depicts the average ΔCoVaR of all banks included in the sample and the average VaR of all banks in the sample over the period from 1st May 2005 to 1st May 2015. As the chart shows, the average ΔCoVaR is higher than the average VaR at almost every point in the sample period. Though, from 1st November 2012, the VaR values are mostly higher than the ΔCoVaR values. Furthermore, as in the case of CoVaR, the peaks caused by the global financial crisis and the European debt crisis are more pronounced for ΔCoVaR than for VaR. This indicates that financial crises increase systemic risk even more than idiosyncratic risk. The close relation between ΔCoVaR and VaR in the time-series dimension is in line with previous research on the two measures, as discussed in Chapter 2.

Figure 9 *Average ΔCoVaR and VaR of 32 European banks (01.05.2005 – 01.05.2015)*



5.4 Worst case scenario analysis

While the previous results were all based on a single bank being in financial distress, the worst case scenario analysis presents results for several institutions being in financial distress at the same time.

The worst case scenario is defined as all banks from a certain country being in financial distress at the same time. Country specific factors such as country specific banking rules, market structure, and customs and traditions might lead the banks of a certain country to develop similar business models. Hence, the banks might accumulate exposures in the same asset classes which leaves all of them vulnerable to shocks in those assets. Examples for such events where numerous financial institutions of a certain country were facing financial difficulties at the same time are the Spanish financial crisis in 2007, caused by a housing bubble, and the Cypriot financial crisis in the years 2012 and 2013. Table 5 below gives an overview of the systemic risk contribution $\Delta CoVaR_{q,t}^{S|1,\ldots,N}$ of all N banks of a certain country being in financial distress at the same time.

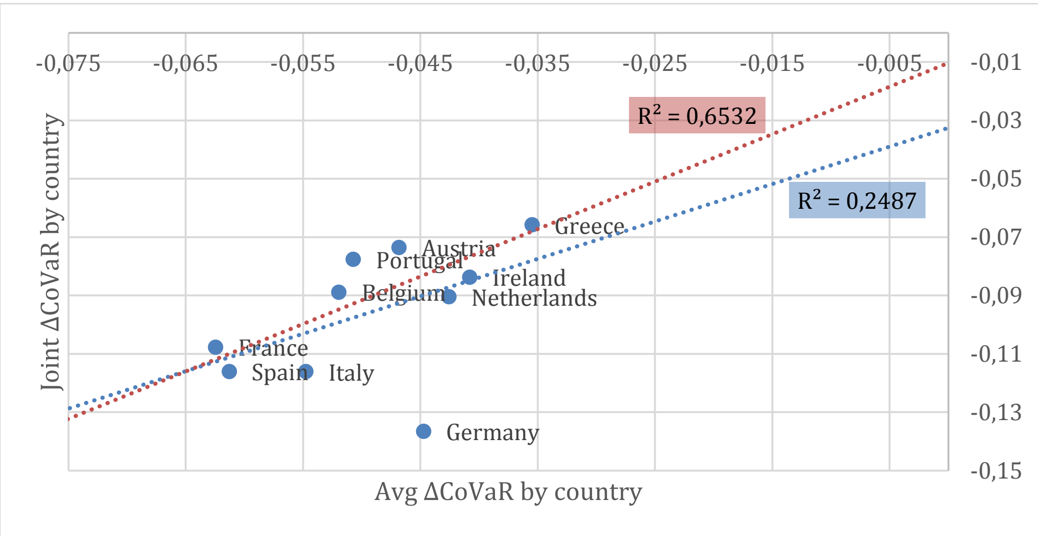
Table 5 shows that while the average joint systemic risk contribution of banks from Germany, Italy, and Spain was the highest, Greek, Austrian, and Portuguese financial institutions exhibited the lowest average combined systemic risk contribution. The results show that the average combined systemic risk contributions are higher than the average individual systemic risk contributions. This is hardly surprising since several institutions in distress are expected to have a higher systemic risk contribution than a single institution being in financial distress. Column 4 of Table 5 shows that the sum of the individual average ΔCoVaR is larger than their combined ΔCoVaR for banks from Austria, Belgium, France, Germany, Greece, Italy, Portugal and Spain. Irish and Dutch financial institutions exhibit a larger collective ΔCoVaR than sum of the individual average ΔCoVaR. This confirms the findings of Cao (2013), that ΔCoVaR does not exhibit additivity. A very interesting result from Table 5 is that the average combined systemic risk contribution of German banks is the highest and that the average combined systemic risk contribution of Greek banks is the lowest. This is somewhat surprising since banks from Spain, France, and Italy showed higher average individual ΔCoVaR values as illustrated in Figure 8. In addition, Spain and Italy are represented in the sample by five banks each, while Germany only contributes with four banks to the sample, yet their combined ΔCoVaR is lower than that of German banks. Greek banks on the other hand, exhibited both the lowest average individual systemic risk contribution as shown in Figure 7, and the lowest average combined ΔCoVaR. Greece is represented in the sample by four banks and thus is expected to have a greater influence on the financial system than countries which are represented in the sample by only two banks. Yet, the four Greek banks have a lower joint systemic risk contribution than, for example, the two banks from Belgium. This is somewhat surprising as Greece contributes a larger share to the financial system of the sample, yet exhibits a lower joint systemic risk contribution than countries with a lower share of the system.

Table 5 *Average combined ΔCoVaR by country (01.05.2005 – 01.05.2015)*

Country	Mean	Max	Min	Sum of average individual *ΔCoVaR*
Austria	-0,0736	-0,2792	-0,0200	-0,0937
Belgium	-0,0888	-0,3382	-0,0249	-0,1039
France	-0,1077	-0,3810	-0,0316	-0,2499
Germany	-0,1366	-0,4758	-0,0396	-0,1789
Greece	-0,0658	-0,2821	-0,0164	-0,1420
Ireland	-0,0837	-0,3152	-0,0232	-0,0816
Italy	-0,1161	-0,4426	-0,0392	-0,2739
Netherlands	-0,0905	-0,3441	-0,0236	-0,0852
Portugal	-0,0776	-0,2937	-0,0190	-0,1015
Spain	-0,1160	-0,4131	-0,0375	-0,3065

Figure 10 presents a scatterplot of the average individual ΔCoVaR and the average joint ΔCoVaR by country. In general there seems to be only a weak link between the average individual ΔCoVaR of a country and its average joint ΔCoVaR. This result is supported by the low goodness of fit value ($R^2 = 0{,}2487$). However, if Germany is treated as an outlier, the goodness of fit value ($R^2 = 0{,}6532$) increases significantly.

Figure 10 *Scatterplot of average individual ΔCoVaR and average joint ΔCoVaR by country (01.05.2005 – 01.05.2015)*



To analyze the drivers of the joint systemic risk contribution of a country, the combined ΔCoVaR by country is plotted against the average cumulative VaR by country. This is depicted in Figure 11. There seems to be a moderate link between the combined ΔCoVaR of a country and the sum of the country's banks' VaR. This is supported by a goodness of fit value of $R^2 = 0{,}4727$. Interestingly the relation between the combined ΔCoVaR of a country and the sum of the county's banks' VaR seems stronger than the relation between the average individual ΔCoVaR by country and the average individual VaR by country, as depicted in Figure 8. The finding of a stronger relation between the combined ΔCoVaR and the cumulative VaR should not be overrated, as this might be simply due to the fact that a higher number of institutions in a group results in a higher cumulative VaR of that group, while a higher number of banks also implies a higher share of the system and thus a higher combined systemic risk contribution.

Figure 11 *Scatterplot of average cumulative VaR and average comined ΔCoVaR by country (01.05.2005 – 01.05.2015)*

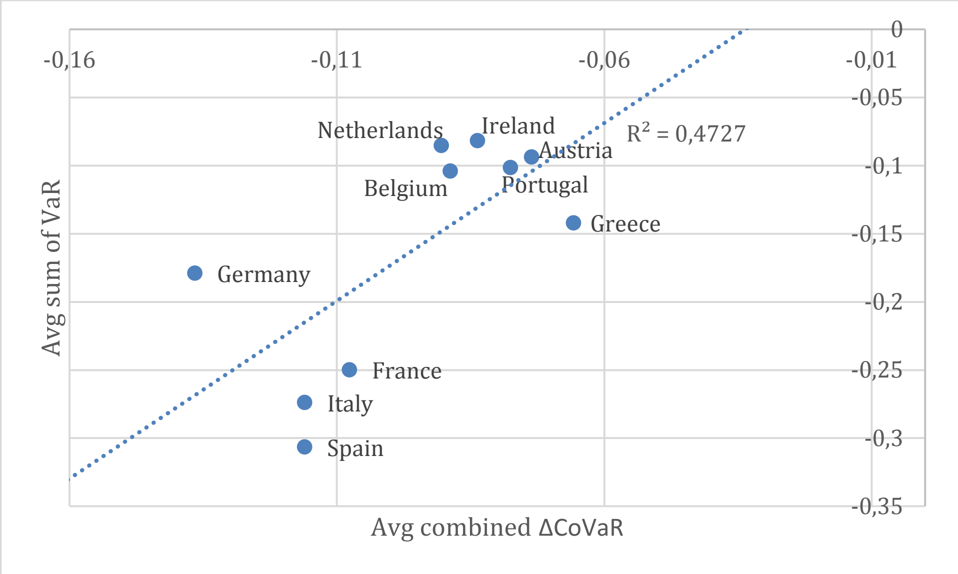


Figure 12 depicts a scatterplot of the average correlation between the institutions of a certain country and the combined ΔCoVaR of that country. The plot shows that there is only a very weak relation between the correlation between the banks of a certain country and their combined ΔCoVaR. This is supported by the very low goodness of fit ($R^2 = 0{,}011$). This result implies that the correlations between the members of a group do not influence the groups combined systemic risk contribution. This is somewhat surprising since the correlation between the banks determine their likeliness of being simultaneously in financial distress. Hence, a group of banks that is less likely to be collectively in financial distress does not necessarily have a higher combined systemic risk contribution. One explanation for the weak link can be found in Section 3.3. The correlations between the banks of a group are used to obtain their joint probability density function. This joint probability function is then used to calculate the joint probability of the banks of that group being simultaneously in distress, as Equation (12) shows. However, the same probability density function is also used to calculate the joint probability of the institutions being in their benchmark state, as Equation (17) shows. Thus the correlations between the banks of a group influence both the CoVaR when the institutions are in their benchmark *and* in their distress state. Hence, the effect of the correlations between the members of a group on the group's combined systemic risk contribution is ambiguous.

Figure 12 *Scatterplot of average correlation between institutions and average combined $\Delta CoVaR$ by country (01.05.2005 – 01.05.2015)*

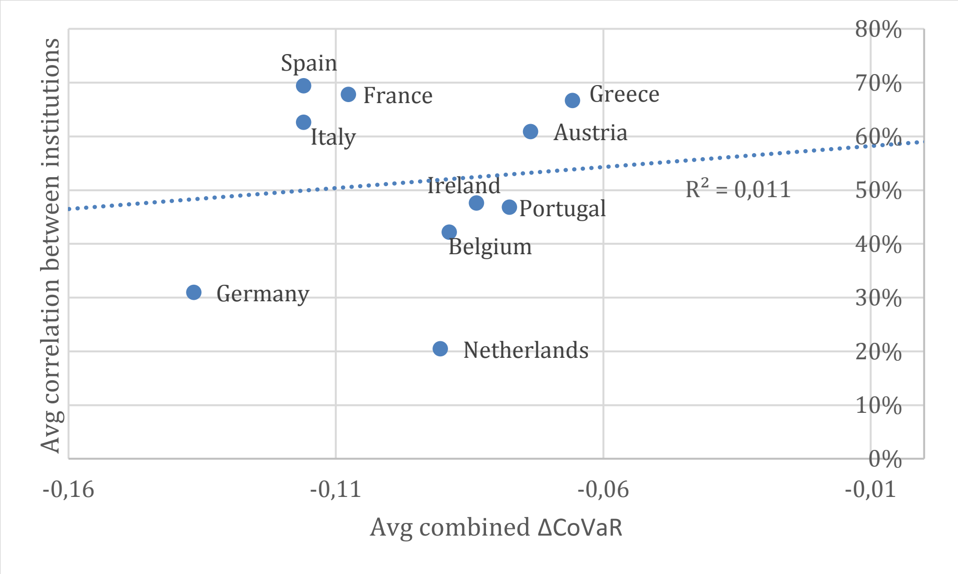
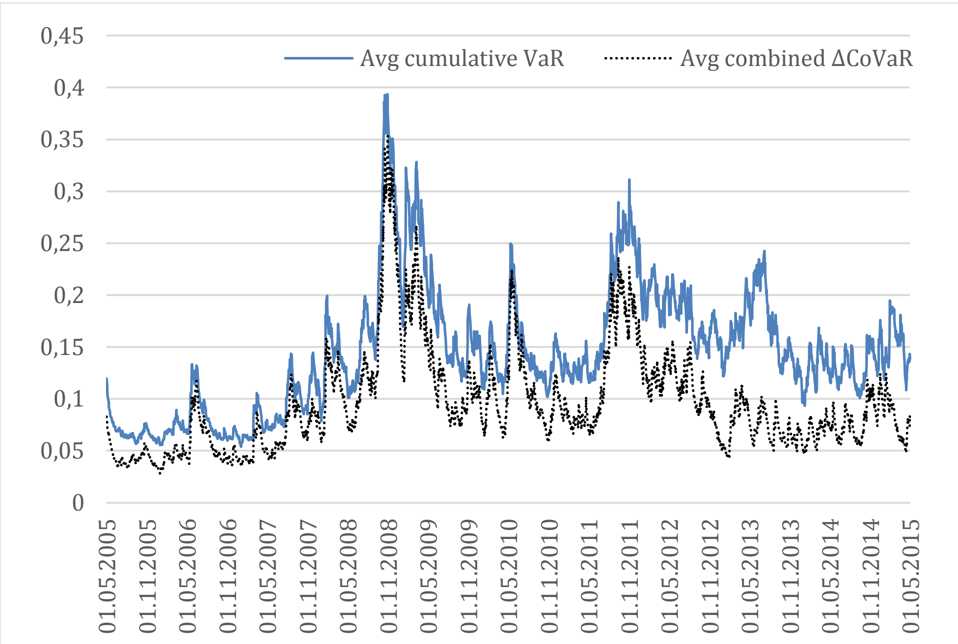


Figure 13 presents the evolution of the average cumulative VaR and the average combined ΔCoVaR of all countries. As in the cases of individual ΔCoVaR there seems to be a strong link between cumulative VaR and the combined ΔCoVaR in the time-series dimension. Both series exhibit peaks, which can be associated with the global financial crisis and the European debt crisis. Interestingly, the two series follow each other very closely until early-2012 but seem to diverge thereafter. Furthermore, the average cumulative VaR exhibits another peak in mid-2013 while the average combined ΔCoVaR does not. Hence, there seems to be a distress event that increased the average cumulative VaR of all countries but did not affect their combined systemic risk contribution, as the combined ΔCoVaR series does not show a peak.

Figure 13 *Average cumulative VaR and average combined $\Delta CoVaR$ of all countries (01.05.2005 – 01.05.2015)*



6 Summary and Conclusion

The aim of this essay was to measure systemic risk contributions within the financial system of the European Monetary Union and to identify the countries that are home to the banks that contribute the highest risks to the system. Systemic risk contributions have been measured by applying the CoVaR measure introduced by Adrian and Brunnermeier (2011). Multivariate GARCH models with underlying student t-distributions and DCC models, to estimate the correlations between institutions and the system, were used to obtain the joint return distribution of the financial system and the financial institutions. Given the joint probability density function of the system and the banks, the CoVaR measure was calculated. This approach also allows to calculate the joint systemic risk contribution of a group of financial institutions, which enables to measure the collective systemic risk contribution of all banks of a country. The dataset used for estimations and calculations consists of daily stock prices of 32 financial institutions from 10 Eurozone countries covering a period of 10 years from 1st May 2005 to 1st May 2015.

The empirical findings show that financial institutions from France, Spain, and Italy exhibited on average the highest systemic risk contribution, while Greek, Irish, and Dutch banks on average contributed the least to systemic risk. One possible explanation for the lower systemic risk contribution of Greek, Irish, and Dutch banks is that these banks are typical commercial banks with a focus on the local market and limited international activity, which leads to a lower correlation with the financial system (Karimalis & Nomikos, 2014). France, Spain, and Italy on the other hand contribute to the sample data with some of the largest European financial institutions. Given their size and scale of international activity, it is little surprising that these countries exhibit the highest average systemic risk contributions.

The empirical results further show that the unconditional VaR of the financial system is lower than its VaR conditional on an institution being in financial distress, as measured by CoVaR. This result confirms the intuition that a bank in financial difficulties increases the overall risk of the financial system.

A finding that has also been reported by previous studies is that ΔCoVaR and VaR differ significantly in the cross-sectional dimension. This result has important implications as regulation based on idiosyncratic risk of financial institutions would not be sufficient to protect from systemic risk. The fatal outcomes of basing banking regulations on idiosyncratic risks have been visualized by the global financial crisis.

Besides the systemic risk contribution of individual institutions, the collective systemic risk contribution of groups of banks has been assessed. The empirical results show that the joint systemic risk contribution of banks from Germany, Italy, and Spain was the highest, while Greek, Austrian, and Portuguese financial institutions had the lowest collective systemic risk contribution. Interestingly, there seems to be only a weak link between the individual systemic risk contribution by country and the collective systemic risk contribution by country. This implies that countries which are home of the individually most systemically risky banks do not necessarily exhibit the highest combined systemic risk contribution.

The relation between a country's combined systemic risk contribution and the correlations between the institutions of that country is found to be very weak. This implies that the joint probability of being in financial distress, which is determined by the correlation between the banks, does not have an impact on the combined systemic risk contribution. As shown, this can be explained by the fact that the joint probability density function, which is based on the inner group correlations, is used for calculating both the benchmark and the distress state CoVaR. Since ΔCoVaR is the difference between the benchmark and the distress state CoVaR, the influence of the inner group correlations on the collective ΔCoVaR of the group is ambiguous.

Furthermore, the relation between the average cumulative VaR and the combined $\Delta CoVaR$ by country has been assessed. The results show that the link between the cumulative VaR and the combined ΔCoVaR of a group of banks is stronger than the link between the VaR and the ΔCoVaR of an individual institution. However, as described above the relation might be simply due to the obvious fact that bigger groups of banks have both a larger cumulative VaR and a larger share of the financial system, which leads to a larger effect on systemic risk.

The findings show that in practice regulation should not be based on idiosyncratic risks. Furthermore, given the weak link between a group's average ΔCoVaR of the individual institutions and the group's combined ΔCoVaR, regulation should in addition be group-based and a group of banks should be regulated according to their collective systemic risk contribution.

Certainly, CoVaR allows for further investigations and research. The ability to forecast systemic risk contribution as measured by CoVaR is an important feature needed to regulate financial institutions. It would be interesting to further investigate the firm specific drivers of systemic risk and construct a regulation framework based on these characteristics. This however requires bank specific data that is only available to regulation authorities. Furthermore, the collective systemic risk contribution of a group of institutions has received little attention so far. It would be interesting to further analyze what drivers within a group of banks affect the systemic risk contribution of the whole group. In addition, a regulation rule that takes into account both the individual as well as collective systemic risk contribution of a group of banks could prove to be very favorable for protecting the financial system from systemic risks.

This essay contributes to the growing research on systemic risk and systemic risk contribution by measuring both systemic risk contributions of individual institutions and joint systemic risk contributions of a group of banks in the financial system of the European Monetary Union. It puts an extra focus on the collective systemic risk contribution of a group of banks, as this phenomenon has received little attention in the literature so far.

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Appendix A

1. Value-at-risk and expected shortfall

Even though Value-at-Risk (VaR) and expected shortfall (ES) are not capable of measuring systemic risk and are therefore not classified as systemic risk measures, the two measures are nevertheless briefly described. The reasons are that 1) until the outbreak of the global financial crisis, regulation of financial institutions and the financial system was mainly based on VaR, and 2) a number of systemic risk measures, such as CoVaR, are based on those two idiosyncratic risk measures.

As explained by Nilsson (2015), VaR is defined as the smallest loss ℓ such that the probability of receiving a future loss L that is bigger than ℓ is less than or equal to $1-\alpha$. In mathematical terms it is defined as

$$VaR_\alpha(L) = min\{\ell \colon \Pr(L > \ell) \leq 1-\alpha\}.$$

In case of a continuous loss distribution the following definition can be used,

$$\Pr\big(L > VaR_\alpha(L)\big) = 1-\alpha.$$

In other words, VaR is the α-quantile of the loss distribution. (Nilsson, 2015)

Nilsson (2015) further provided a definition for expected shortfall. It is defined as the average of VaR for confidence levels greater than or equal to α. Hence it also takes into account losses that are larger than the VaR and thus gives an estimate of the magnitude of the loss if a tail event occurs. Mathematically it is defined as

$$ES_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_x(L)\,dx.$$

In case of a continuous loss distribution,

$$ES_\alpha(L) = E[L \colon L > VaR_\alpha(L)].$$

(Nilsson, 2015)

The global financial crisis has painfully shown that banking regulation based on idiosyncratic risk measures is not sufficient to protect from systemic risk. The new Basel III accords are the first to include macro prudential regulation approaches to take systemic risk into account. (Bank for International Settlements, 2015)

2. CoVaR

CoVaR is one of the most important systemic risk measures and thus should not be missing in this selection of systemic risk measures. Recall from above, that it is defined as the VaR of the financial system conditional on an institution being in financial distress. Furthermore, the systemic risk contribution ΔCoVaR of an institution is defined as the difference between the distress state CoVaR and the benchmark state CoVaR. In contrast to VaR and ES, CoVaR is an actual systemic risk measure and thus suitable for regulation purposes. (Adrian & Brunnermeier, 2011)

3. CoES

Adrian and Brunnermeier (2011) also shortly elaborate on further "corisk measures" that can be derived from CoVaR, such as CoES which stands for co-expected shortfall. Similarly to the relation between VaR and ES, CoES is defined as the average of CoVaR for confidence levels smaller than or equal to β. In mathematical terms it is defined as follows:

$$CoES_{\beta}^{i,VaR} = \frac{1}{\beta}\int_0^{\beta} CoVaR_y^{system|X^i=VaR_q^i}\, dy.$$

And in the benchmark case

$$CoES_{\beta}^{i,bench} = \frac{1}{\beta}\int_0^{\beta} CoVaR_y^{system|X^i=b^i}\, dy.$$

Hence the contribution of institution i to systemic risk, as measured by CoES, is defined as

$$\Delta CoES = CoES_{\beta}^{i,VaR} - CoES_{\beta}^{i,bench}.$$

4. Systemic risk beta

Besides the before mentioned measures CoVaR and CoES there is a large number of other systemic risk measures, as the discussion on the definition of systemic risk already implies. These risk measures may focus on other key characteristics of systemic risk, differ in the data required or are simply extensions or advancements of existing systemic risk measures.

One of the many systemic risk measures is the systemic risk beta proposed by Hautsch, Schaumburg and Schienle (2015). It is defined as the marginal effect of institution i's VaR on the VaR of the financial system. In mathematical terms it is defined as

$$\frac{\partial VaR_{p,t}^{s}\left(V_t^{(i)}, VaR_{q,t}^{i}\right)}{\partial VaR_{q,t}^{i}} = \beta_{p,q}^{s|i},$$

where $V_t^{(i)}$ are firm specific control variables. In an inverse analogy to asset pricing, the systemic risk beta can be interpreted as the sensitivity of the system's VaR to changes in institution i's VaR. The authors further define the realized systemic risk beta as

$$\bar{\beta}_{p,q}^{s|i} \coloneqq \beta_{p,q}^{s|i} VaR_t^i.$$

The realized systemic risk contribution visualizes the absolute effect of an increase in bank i's VaR on the VaR of the financial system. In order to estimate the before defined measures Hautsch, Schaumburg and Schienle (2015) use a two-step procedure. In a first step they select significant factors that influence the VaR of a single institution i. In a second step they regress the VaR of the system onto these significant factors and the VaR of firm i in order to obtain an estimate of the marginal systemic risk contribution $\beta_{p,q}^{s|i}$. The significant background factors consist of firm characteristics, macroeconomic state variables and the VaR values of other institutions in the system.

5. Marginal expected shortfall

Another systemic risk measure is the marginal expected shortfall (MES) first introduced by Acharya et al. (2010). It is defined as the marginal contribution of firm i to systemic risk, measured in terms of expected shortfall of the financial system. The conditional expected shortfall of the financial system can be defined as

$$ES_{mt}(C) = \mathbb{E}_{t-1}(r_{mt}|r_{mt} < C) = \sum_{i=1}^{N} w_{it}\mathbb{E}_{t-1}(r_{it}|r_{mt} < C),$$

where C is a given threshold. Typical thresholds are $VaR_{0.95}$, $VaR_{0.975}$ or $VaR_{0.99}$. Consequently, MES is defined as

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = \mathbb{E}_{t-1}(r_{it}|r_{mt} < C).$$

Hence MES measures the increase in systemic risk caused by a marginal increase in the weight of institution i. Thus, the higher the MES of an institution the higher its contribution to overall risk in the financial system. (Acharya et al., 2010)

6. Systemic expected shortfall

Acharya et al. (2010) also introduce an extension to MES, the so-called systemic expected shortfall (SES). The SES measures the contribution of a financial institution to systemic risk and is defined as the propensity of a firm i to be undercapitalized when the entire financial system is undercapitalized. The authors show that the SES measure consists of three components: i) excess ex ante leverage, ii) the MES based on pre-crisis data, and iii) an adjustment term. In mathematical terms it is defined as

$$SES_{it} = (k\, L_{it} - 1 + \theta\, MES_{it} + \Delta_i)\, W_{it},$$

where L_{it} is the leverage (A_{it}/W_{it}), A_{it} are the total assets, W_{it} are the market value of the equity and k, θ and Δ_i are constants. The formulation supports the fact that the SES for institution i increases with a higher leverage and a higher MES.

7. Distress Insurance Premium

Another measure that allows to assess the contribution of a financial institution to systemic risk is the distress insurance premium (DIP) introduced by Huang, Zhou and Zhu (2011). The DIP is a systemic risk indicator and is measured by the insurance price against systemic financial distress. It is defined as the insurance premium that shields against the distressed losses of the system portfolio, which consists of the total liabilities of all institutions in the system. Thus the total loss of the system portfolio is defined as $L = \sum_{i=1}^{N} L_i$, where L_i is the loss of institution i's liability. The DIP is then given by the risk-neutral expectation of the loss exceeding a certain threshold. In mathematical terms it is defined as

$$DIP = E^Q[L|L \geq L_{min}],$$

where L_{min} is the minimum loss threshold, similar to a deductible. In order to obtain the marginal contribution of a single bank to systemic risk, the partial derivative of the DIP is taken with respect to institution i,

$$\frac{\partial\, DIP}{\partial\, L_i} \equiv E^Q[L_i|L \geq L_{min}].$$

8. Further measures

There are several attempts in the literature to taxonomize the multitude of different systemic risk measurements. Hansen (2013) for example divides systemic risk measures in four groups depending on the approach of measurement of systemic risk they are following. The four groups are tail measures, contingent claims analysis, network models and dynamic, stochastic macroeconomic models. The first group of tail measures focus on co-dependence in the tails of equity returns of financial firms. Adrian and Brunnermeier's CoVaR measure as well as the MES and SES systemic risk measures can be classified to this group. Contingent claims analysis, the second group of measuring approaches, is based on option pricing theory. By assuming an underlying stochastic process for the value of the assets of an institution, equity can be expressed as a call option on the firm's assets and debt as a put option. In order to measure systemic risks, the contingent claims analysis is extended by aggregating balance sheet data for entire sectors of the economy. Third, network models focus on the interconnectedness of financial firms in the system. The fourth group of measurement approaches, dynamic, stochastic macroeconomic models, focus on the spillover effects from distressed financial markets to the macro economy (Hansen, 2013).

There are many ways to assess systemic risk, and depending on different criteria such as availability of data, the supervisory scope or temporal category, one measurement approach might be preferred over another. An extensive overview of systemic risk measures, their key characteristics and their specific outputs, can be found in Bisias et al. (2013).

Appendix B

Appendix B presents the calculation of ΔCoVaR for an individual institution. The steps presented in Section 3.3 remain the same. However, some simplifications can be used.

The first step, as described in Section 3.3.1, remains the exact same and the individual VaR of each institution is obtained.

Secondly, the joint student t-distribution is obtained. Since the ΔCoVaR of an individual institution is calculated, the joint student t-distribution boils down to a bivariate distribution,

$$\begin{pmatrix} r_t^S \\ r_t^i \end{pmatrix} \sim t_\nu \left(\begin{pmatrix} \mu_t^S \\ \mu_t^i \end{pmatrix}, \begin{pmatrix} \sigma_{S,t}^2 & \rho_{Si,t}\sigma_{S,t}\sigma_{i,t} \\ \rho_{iS,t}\sigma_{i,t}\sigma_{S,t} & \sigma_{i,t}^2 \end{pmatrix} \right),$$

where $r_t = \left(r_t^S, r_t^i\right)'$ is the (2×1) vector of returns, $\mu_t = \left(\mu_t^S, \mu_t^i\right)'$ is the (2×1) vector of conditional means, and Σ_t is the (2×2) conditional covariance matrix, $\sigma_{i,t}$ is the conditional standard deviation of i, and $\rho_{ij,t}$ is the conditional correlation between i and j.

Hence, in a similar fashion the DCC model boils down to a bivariate one. The (2×2) correlation matrix is estimated as described in Equation (8) and Equation (9).

Given all the inputs needed, the joint bivariate student t-distribution is obtained.

Third, $CoVaR$ for the distress and the benchmark state is calculated. The distress sate $CoVaR$ is defined as

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|i} \middle| r_t^i \leq VaR_{q,t}^i\right) = q.$$

By means of conditional probabilities the equation can be reformulated as,

$$\frac{\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|i} \cap r_t^i \leq VaR_{q,t}^i\right)}{\Pr\left(r_t^i \leq VaR_{q,t}^i\right)} = q.$$

Given the definition of VaR, the denominator of the equation above can be written as,

$$\Pr\left(r_t^i \leq VaR_{q,t}^i\right) = q.$$

Plugging in this result into the formulation of $CoVaR$ in terms of joint probabilities gives:

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|i} \cap r_t^i \leq VaR_{q,t}^i\right) = q^2.$$

Thus $CoVaR_{q,t}^{S|i}$ is obtained by solving numerically for the upper boundary of the double integral,

$$\int_{-\infty}^{CoVaR_{q,t}^{S|i}} \int_{-\infty}^{VaR_{q,t}^{i}} pdf_t(r_t^S, r_t^i) dr_t^S \, dr_t^i = q^2.$$

In a similar fashion, the benchmark state $CoVaR$ can be calculated. The benchmark $CoVaR$ is defined as:

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^i} \middle| \mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i\right) = q.$$

In terms of conditional probabilities it can be written as:

$$\frac{\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^i} \cap \mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i\right)}{\Pr(\mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i)} = q.$$

Given the probability of the benchmark event $\Pr(\mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i) = p_t$, the above equation becomes:

$$\Pr\left(r_t^S \leq CoVaR_{q,t}^{S|b^i} \cap \mu_t^i - \sigma_t^i \leq r_t^i \leq \mu_t^i + \sigma_t^i\right) = q \times p_t.$$

The benchmark state $CoVaR$ is obtained by solving numerically for the upper boundary of the double integral,

$$\int_{-\infty}^{CoVaR_{q,t}^{S|b^i}} \int_{\mu_t^i - \sigma_t^i}^{\mu_t^i + \sigma_t^i} pdf_t(r_t^S, r_t^i) dr_t^S \, dr_t^i = q \times p_t.$$

The systemic risk contribution of institution i is then given by the difference between the distress state and the benchmark state $CoVaR$.

$$\Delta CoVaR_{q,t}^{S|i} = CoVaR_{q,t}^{S|i} - CoVaR_{q,t}^{S|b^i}.$$

Appendix C

Appendix C presents the banks included in the sample and their respective home country.

Institution	Country
Erste Group Bank AG	Austria
Raiffeisen Bank International AG	Austria
KBC Group NV	Belgium
Dexia N.V./S.A.	Belgium
BNP Paribas S.A.	France
Crédit Agricole S.A.	France
Société Générale S.A.	France
Natixis S.A.	France
Commerzbank AG	Germany
Deutsche Bank AG	Germany
IKB Deutsche Industriebank AG	Germany
Deutsche Postbank AG	Germany
Alpha Bank S.A.	Greece
National Bank of Greece S.A.	Greece
Piraeus Bank S.A.	Greece
Eurobank Ergasias S.A.	Greece
Bank of Ireland Ltd.	Ireland
Allied Irish Banks p.l.c.	Ireland
Banca Monte dei Paschi di Siena S.p.A.	Italy
Banco Popolare Soc.Coop.	Italy
Intesa Sanpaolo S.p.A.	Italy
Unione di Banche Italiane S.c.p.A.	Italy
UniCredit S.p.A.	Italy
ING Groep N.V.	Netherlands
Van Lanschot N.V.	Netherlands
Banco Comercial Português S.A.	Portugal
Banco Português de Investimento S.A.	Portugal
Bankinter S.A.	Spain
Banco Bilbao Vizcaya Argentaria S.A.	Spain
Banco Popular Español S.A.	Spain
Banco de Sabadell S.A.	Spain
Banco Santander S.A.	Spain

Appendix D

Appendix D presents the summary statistics at the bank level.

Institution	Mean	St. Dev	Max	Min	Kurtosis	Skewness
ERSTE GROUP BANK	0,0003	0,0302	0,1854	-0,1810	8,9137	0,0918
RAIFFEISEN BANK INTL.	0,0002	0,0320	0,1869	-0,2466	8,8938	0,1229
KBC GROUP	0,0006	0,0363	0,4991	-0,2492	24,9516	1,0206
COMMERZBANK	-0,0003	0,0311	0,2148	-0,2461	10,8260	0,3254
DEUTSCHE BANK	0,0001	0,0256	0,2499	-0,1653	15,0648	0,8229
BANKINTER	0,0004	0,0234	0,1450	-0,0807	6,4013	0,6673
BBV.ARGENTARIA	0,0002	0,0217	0,2203	-0,1278	10,8887	0,6260
BANCO POPULAR ESPANOL	-0,0003	0,0237	0,2068	-0,1232	8,7727	0,6970
BANCO DE SABADELL	0,0001	0,0192	0,1827	-0,0764	10,8172	1,0018
BANCO SANTANDER	0,0003	0,0220	0,2322	-0,1409	12,9400	0,5732
BNP PARIBAS	0,0004	0,0264	0,2090	-0,1724	12,5836	0,7567
CREDIT AGRICOLE	0,0003	0,0286	0,2632	-0,1337	10,3606	0,6525
SOCIETE GENERALE	0,0003	0,0294	0,2389	-0,1623	10,3122	0,4525
NATIXIS	0,0006	0,0324	0,3880	-0,1748	18,9325	1,2836
ALPHA BANK	-0,0001	0,0447	0,2941	-0,2703	9,7972	0,7082
NATIONAL BK.OF GREECE	-0,0007	0,0444	0,2915	-0,2678	9,5887	0,4500
BANK OF IRELAND	0,0001	0,0509	0,4810	-0,5476	22,3568	0,8686
BANCA MONTE DEI PASCHI	-0,0007	0,0294	0,2134	-0,2150	11,8589	0,3151
BANCO POPOLARE	-0,0002	0,0293	0,1894	-0,1636	7,6276	0,3176
INTESA SANPAOLO	0,0003	0,0265	0,1968	-0,1686	9,4941	0,0855
UNIONE DI BANCHE ITALIAN	0,0000	0,0242	0,1219	-0,1236	5,6795	0,1386
UNICREDIT	-0,0001	0,0296	0,2093	-0,1727	8,9686	0,1843
ING GROEP	0,0005	0,0325	0,2924	-0,2748	18,1213	0,7544
BANCO COMR.PORTUGUES	-0,0004	0,0276	0,2690	-0,1507	11,7338	0,7985
ALLIED IRISH BANKS	-0,0005	0,0540	0,4348	-0,5862	16,5035	0,2786
BANK OF PIRAEUS	-0,0007	0,0472	0,2990	-0,2955	10,8908	0,5074
EUROBANK ERGASIAS	-0,0013	0,0531	0,2948	-0,3023	10,8462	0,3940
BANCO BPI	0,0001	0,0249	0,2704	-0,1305	15,8730	1,2799
DEXIA	0,0011	0,0831	1,0000	-0,3627	25,5940	2,2845
IKB DEUTSCHE INDSTRBK.	-0,0004	0,0419	0,6367	-0,2387	36,2487	2,8086
DEUTSCHE POSTBANK	0,0003	0,0222	0,1515	-0,2379	21,6795	-1,0072
VAN LANSCHOT	-0,0002	0,0160	0,1250	-0,0920	8,3124	0,1748
INDEX_SYSTEM	0,0004	0,0214	0,1943	-0,1053	10,4001	0,5655

Appendix E

Appendix E presents the estimation output at the bank level. The Appendix is divided into 4 tables. Table 1 of Appendix E shows the estimation output of the mean equation as presented in Equation (6).

Institution	a_0	a_1	b_1	b_2
ERSTE GROUP BANK	0,0003	-	-	-
RAIFFEISEN BANK INTL.	-0,0003	-	-	-
KBC GROUP	0,0005**	-	0,0223***	-
COMMERZBANK	0,0000	-	-	-
DEUTSCHE BANK	0,0002	-	-	-
BANKINTER	-0,0001	-	-	-
BBV.ARGENTARIA	0,0000	-	0,0637***	-
BANCO POPULAR ESPANOL	-0,0001	-	0,0661***	-
BANCO DE SABADELL	0,0000	0,0825***	-	-
BANCO SANTANDER	0,0003	-	-	-
BNP PARIBAS	0,0000	-	-	-
CREDIT AGRICOLE	0,0001	-	-	-
SOCIETE GENERALE	0,0000	-	0,0589***	-
NATIXIS	0,0003	-	-	-
ALPHA BANK	0,0003	-	-	-
NATIONAL BK.OF GREECE	-0,0001	-	-	-
BANK OF IRELAND	0,0001	-	-	-
BANCA MONTE DEI PASCHI	-0,0007**	-	0,0456**	-
BANCO POPOLARE	-0,0004	-	0,0302	-
INTESA SANPAOLO	-0,0001	-	-	-
UNIONE DI BANCHE ITALIAN	-0,0002	-	-	-
UNICREDIT	0,0002	-	-	-
ING GROEP	0,0004	-	-	-
BANCO COMR.PORTUGUES	-0,0005*	-	0,0318	-
ALLIED IRISH BANKS	-0,0004	-	-0,0116	-
BANK OF PIRAEUS	0,0004	-	-	-
EUROBANK ERGASIAS	-0,0006	-	0,0578**	-
BANCO BPI	0,0000	-	-	-
DEXIA	-0,0002	-0,9572***	0,8332***	-0,1239***
IKB DEUTSCHE INDSTRBK.	-0,0008***	-	-0,2486***	-
DEUTSCHE POSTBANK	0,0003***	-	-	-
VAN LANSCHOT	-0,0005**	-0,1207***	-	-
INDEX_SYSTEM	0,0005**	-	-	-

Significance levels: 1%: ***; 5%: **; 10%: *

Table 2 of Appendix E shows the estimation output of the variance equation as presented in Equation (7).

Institution	ω	α_1	β_1	γ_1	δ
ERSTE GROUP BANK	0,0001	0,0536***	0,9546***	0,5179***	1,0492**
RAIFFEISEN BANK INTL.	0,0002	0,0988***	0,9115***	0,2989***	1,3223***
KBC GROUP	0,0002	0,1013	0,9167***	0,4799**	1,0579***
COMMERZBANK	0,0002	0,1146**	0,8984***	0,2960***	1,2890***
DEUTSCHE BANK	0,0000	0,0678***	0,9353***	0,4328***	1,4374***
BANKINTER	0,0001	0,1049***	0,9089***	0,3865***	1,3830***
BBV.ARGENTARIA	0,0001	0,0629***	0,9373***	0,7939***	1,2482***
BANCO POPULAR ESPANOL	0,0000	0,0753***	0,9405***	0,3434**	1,2482***
BANCO DE SABADELL	0,0000	0,0525***	0,9518***	0,2057**	1,8034***
BANCO SANTANDER	0,0002	0,0799***	0,9235***	0,8033***	1,0796***
BNP PARIBAS	0,0001	0,0531*	0,9416***	0,9887***	1,2758***
CREDIT AGRICOLE	0,0001	0,0639***	0,9413***	0,5051***	1,3217***
SOCIETE GENERALE	0,0000	0,0758***	0,9302***	0,5097***	1,3812***
NATIXIS	0,0003*	0,1103***	0,9094***	0,4691***	1,0431***
ALPHA BANK	0,0000	0,0865***	0,9261***	0,1956***	1,5345***
NATIONAL BK.OF GREECE	0,0001	0,1253***	0,8925***	0,2659***	1,5096***
BANK OF IRELAND	0,0001	0,0812**	0,9380***	0,3333**	1,1576***
BANCA MONTE DEI PASCHI	0,0001	0,1141***	0,9066***	0,2272***	1,3666***
BANCO POPOLARE	0,0001	0,0566***	0,9539***	0,5586***	1,0381***
INTESA SANPAOLO	0,0001	0,0606***	0,9484***	0,6539***	1,0425***
UNIONE DI BANCHE ITALIAN	0,0001	0,0603***	0,9517***	0,4718***	1,0718***
UNICREDIT	0,0001	0,0762***	0,9297***	0,5572***	1,3066***
ING GROEP	0,0001	0,0861***	0,9246***	0,5975***	1,1236***
BANCO COMR.PORTUGUES	0,0002	0,1669***	0,8744***	0,2841***	1,1841***
ALLIED IRISH BANKS	0,0001	0,1436***	0,8905***	0,1542***	1,3354***
BANK OF PIRAEUS	0,0001	0,1228***	0,9030***	0,2524***	1,4343***
EUROBANK ERGASIAS	0,0000	0,1373***	0,8849***	0,1489**	1,5757***
BANCO BPI	0,0001	0,1944***	0,8605***	0,2266***	1,1406***
DEXIA	0,0001	0,1725***	0,8684***	0,2709***	1,3108***
IKB DEUTSCHE INDSTRBK.	0,0016	0,1935***	0,8483***	0,1993**	0,8830***
DEUTSCHE POSTBANK	0,0004	0,2602***	0,8122***	0,1002*	1,0036***
VAN LANSCHOT	0,0013	0,2355***	0,7073***	0,1812**	1,1669***
INDEX_SYSTEM	0,0000	0,0745***	0,9322***	0,5213***	1,2913***

Significance levels: 1%: ***; 5%: **; 10%: *

Table 3 of Appendix E shows the degrees of freedom of the student t-distribution of an individual institution and the degrees of freedom of the joint student t-distribution of an institution and the system.

Institution	ν	joint-ν
ERSTE GROUP BANK	6,2563***	6,5640***
RAIFFEISEN BANK INTL.	4,4068***	5,6165***
KBC GROUP	6,2559***	5,8134***
COMMERZBANK	4,5883***	5,1260***
DEUTSCHE BANK	6,7258***	5,8420***
BANKINTER	5,8059***	5,9662***
BBV.ARGENTARIA	6,8817***	6,5838***
BANCO POPULAR ESPANOL	4,9180***	5,4498***
BANCO DE SABADELL	3,4491***	4,4085***
BANCO SANTANDER	6,9966***	6,1190***
BNP PARIBAS	8,3186***	5,8942***
CREDIT AGRICOLE	7,0494***	5,4790***
SOCIETE GENERALE	7,2275***	5,5523***
NATIXIS	3,9074***	4,5039***
ALPHA BANK	6,2264***	7,6508***
NATIONAL BK.OF GREECE	5,5344***	7,0707***
BANK OF IRELAND	4,4855***	5,9685***
BANCA MONTE DEI PASCHI	4,1802***	4,8428***
BANCO POPOLARE	5,1296***	5,5057***
INTESA SANPAOLO	6,7464***	6,3367***
UNIONE DI BANCHE ITALIAN	6,1250***	6,4003***
UNICREDIT	7,8983***	6,7801***
ING GROEP	7,6158***	5,7993***
BANCO COMR.PORTUGUES	3,7260***	5,0891***
ALLIED IRISH BANKS	4,2520***	5,7200***
BANK OF PIRAEUS	4,0775***	5,7376***
EUROBANK ERGASIAS	4,7903***	6,4724***
BANCO BPI	3,7895***	5,0148***
DEXIA	4,2066***	5,4329***
IKB DEUTSCHE INDSTRBK.	3,0487***	4,5981***
DEUTSCHE POSTBANK	3,3749***	4,6895***
VAN LANSCHOT	3,0035***	4,8019***
INDEX_SYSTEM	8,4314***	-

Significance levels: 1%: ***; 5%: **; 10%: *

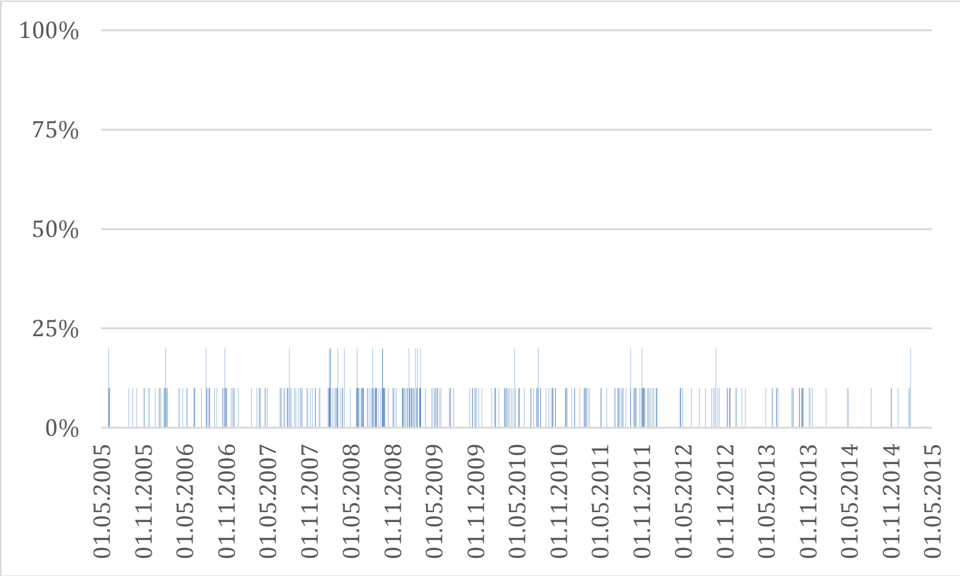
Table 4 of Appendix E shows the estimation output of the DCC model as presented in Equation (9).

Institution	δ_1	δ_2
ERSTE GROUP BANK	0,0300***	0,9513***
RAIFFEISEN BANK INTL.	0,0268***	0,9481***
KBC GROUP	0,0467***	0,9120***
COMMERZBANK	0,0222***	0,9693***
DEUTSCHE BANK	0,0061**	0,9918***
BANKINTER	0,0475***	0,9212***
BBV.ARGENTARIA	0,0339***	0,9431***
BANCO POPULAR ESPANOL	0,0449***	0,9410***
BANCO DE SABADELL	0,0444*	0,9241***
BANCO SANTANDER	0,0462***	0,9185***
BNP PARIBAS	0,0665***	0,8774***
CREDIT AGRICOLE	0,0414***	0,9162***
SOCIETE GENERALE	0,0509***	0,8630***
NATIXIS	0,0343***	0,9605***
ALPHA BANK	0,0073***	0,9911***
NATIONAL BK.OF GREECE	0,0109**	0,9862***
BANK OF IRELAND	0,0255***	0,9572***
BANCA MONTE DEI PASCHI	0,0383***	0,9541***
BANCO POPOLARE	0,0249***	0,9662***
INTESA SANPAOLO	0,0267***	0,9641***
UNIONE DI BANCHE ITALIAN	0,0234***	0,9708***
UNICREDIT	0,0465***	0,9152***
ING GROEP	0,0444***	0,9166***
BANCO COMR.PORTUGUES	0,0313***	0,9549***
ALLIED IRISH BANKS	0,0246***	0,9720***
BANK OF PIRAEUS	0,0062***	0,9932***
EUROBANK ERGASIAS	0,0097***	0,9877***
BANCO BPI	0,0159*	0,9809***
DEXIA	0,0287***	0,9706***
IKB DEUTSCHE INDSTRBK.	0,0142***	0,9856***
DEUTSCHE POSTBANK	0,0212***	0,9788***
VAN LANSCHOT	0,0202	0,8001***
INDEX_SYSTEM	-	-

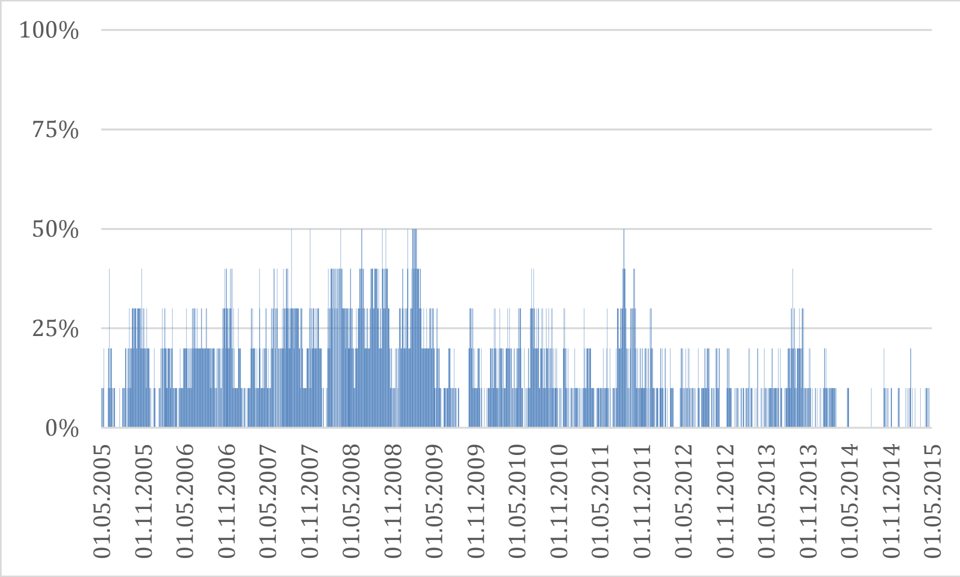
Significance levels: 1%: ***; 5%: **; 10%: *

Appendix F

Appendix F presents the density plots of the matches of the VaR-ΔCoVaR top 10 ranking. The first figure shows the percentage of exact rank matches per day. An exact rank match means that the position of an institution in the VaR ranking is the exact same position in the ΔCoVaR ranking.



The second figure shows the percentage of the top 10 overlaps per day. This means that an institution simultaneously exhibits a ΔCoVaR-value that is among the 10 highest ΔCoVaR-values *and* a VaR-value that is among the 10 highest VaR-values.



Appendix G

Appendix G presents the summary statistics of VaR and ∆CoVaR at the bank level.

	VaR			$\Delta CoVaR$		
Institution	Mean	Max	Min	Mean	Max	Min
ERSTE GROUP BANK	-0,0428	-0,1493	-0,0178	-0,0464	-0,1846	-0,0092
RAIFFEISEN BANK INTL.	-0,0480	-0,2254	-0,0209	-0,0473	-0,1900	-0,0107
KBC GROUP	-0,0464	-0,2595	-0,0111	-0,0546	-0,2079	-0,0147
COMMERZBANK	-0,0432	-0,1676	-0,0151	-0,0614	-0,2283	-0,0159
DEUTSCHE BANK	-0,0345	-0,1447	-0,0119	-0,0573	-0,2096	-0,0162
BANKINTER	-0,0359	-0,0876	-0,0131	-0,0526	-0,1803	-0,0128
BBV.ARGENTARIA	-0,0313	-0,1121	-0,0109	-0,0559	-0,2058	-0,0152
BANCO POPULAR ESPANOL	-0,0349	-0,0918	-0,0092	-0,0638	-0,2483	-0,0172
BANCO DE SABADELL	-0,0295	-0,0715	-0,0133	-0,0728	-0,2617	-0,0190
BANCO SANTANDER	-0,0310	-0,1257	-0,0098	-0,0614	-0,2237	-0,0173
BNP PARIBAS	-0,0367	-0,1357	-0,0139	-0,0615	-0,2148	-0,0161
CREDIT AGRICOLE	-0,0413	-0,1337	-0,0162	-0,0685	-0,2491	-0,0181
SOCIETE GENERALE	-0,0420	-0,1551	-0,0132	-0,0605	-0,2144	-0,0168
NATIXIS	-0,0428	-0,1522	-0,0114	-0,0594	-0,2201	-0,0074
ALPHA BANK	-0,0627	-0,1940	-0,0174	-0,0304	-0,1299	-0,0076
NATIONAL BK.OF GREECE	-0,0623	-0,2449	-0,0185	-0,0376	-0,1675	-0,0096
BANK OF IRELAND	-0,0649	-0,2919	-0,0148	-0,0416	-0,1763	-0,0101
BANCA MONTE DEI PASCHI	-0,0427	-0,2015	-0,0136	-0,0565	-0,2252	-0,0093
BANCO POPOLARE	-0,0438	-0,1232	-0,0141	-0,0511	-0,1871	-0,0130
INTESA SANPAOLO	-0,0379	-0,1213	-0,0137	-0,0585	-0,2177	-0,0131
UNIONE DI BANCHE ITALIAN	-0,0366	-0,0878	-0,0106	-0,0536	-0,2037	-0,0091
UNICREDIT	-0,0421	-0,1598	-0,0106	-0,0541	-0,1967	-0,0138
ING GROEP	-0,0414	-0,2530	-0,0118	-0,0588	-0,2191	-0,0160
BANCO COMR.PORTUGUES	-0,0401	-0,1288	-0,0105	-0,0508	-0,2213	-0,0120
ALLIED IRISH BANKS	-0,0730	-0,4015	-0,0130	-0,0400	-0,1727	-0,0110
BANK OF PIRAEUS	-0,0637	-0,2698	-0,0128	-0,0368	-0,1660	-0,0103
EUROBANK ERGASIAS	-0,0704	-0,3489	-0,0162	-0,0372	-0,1738	-0,0099
BANCO BPI	-0,0358	-0,1227	-0,0059	-0,0506	-0,2064	-0,0091
DEXIA	-0,0849	-0,6358	-0,0099	-0,0493	-0,2224	-0,0041
IKB DEUTSCHE INDSTRBK.	-0,0535	-0,3630	-0,0158	-0,0252	-0,1304	-0,0039
DEUTSCHE POSTBANK	-0,0289	-0,2330	-0,0043	-0,0350	-0,1525	-0,0014
VAN LANSCHOT	-0,0241	-0,0738	-0,0146	-0,0263	-0,1051	-0,0073
INDEX_SYSTEM	-0,0303	-0,1106	-0,0083	-	-	-