Simulation of Coupled Systems by Aggregation

Pukashawar Pannu

A major concern when simulating coupled systems is maintaining proper coupling relations throughout model evaluation. The concept of aggregation aims to tackle this issue and also solve the problem of handling sub-system discontinuities.

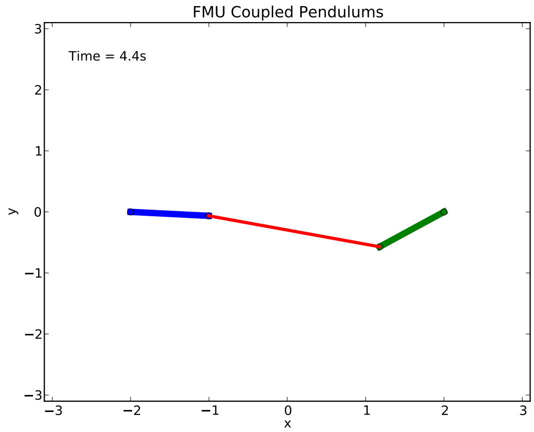
Physical systems such as cars, planes or swinging pendulums can be described as mathematical models. The modeller is required to define and setup all equations and attributes needed to describe the system. When replacing specific components, such as tyres on a car, the entire system has to be changed. A modelling paradigm of creating standalone systems that are coupled together to form large complex models is gaining momentum. The systems communicate with one another through input and output signals that define coupling relations. Imagine modelling a car. All pieces have been assembled and the model is ready for use. Now you want to replace the engine with a newer version. As previously mentioned, replacing vital parts would require much work, but with a coupling strategy specific component can be swapped out without altering other systems. Be it to switch engines or simply change from summer to winter tyres.

Problems when simulating coupled systems come down to how to properly evaluate the system dynamics. Systems considered for coupling are described as ordinary differential equations,

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u})$$
$$\bar{y} = \bar{g}(\bar{x}, \bar{u})$$

where $\bar{x}$ are states, $\bar{u}$ inputs and $\bar{y}$ outputs. The dot above $\dot{\bar{x}}$ indicates derivative with respect to time. Complex systems can be modelled by taking such systems and defining coupling relations. In order to solve coupled systems the concept of aggregation is introduced. It aggregates all system dynamics into one problem,

$$\dot{x} = f(x, u)$$
$$y = g(x, u)$$
$$u = c(y, w)$$



Simulation of separately modelled pendulums coupled with a linear spring and external excitations on pivots.

where x, u and y are aggregated states, inputs and outputs. Variable w represents time dependent external excitations that may be present in some subsystems, such as wind acting on a car. Function $c(y, w)$ defines the coupling relations. With this structure all sub-systems have been aggregated into one unified system that is responsible for correctly evaluating system dynamics.

The idea has been implemented and tested in Python. The models used were either directly written in Python or in Modelica, a language for modelling physical systems. Modelica models were compiled to FMUs with the open source JModelica.org tool. An FMU, Functional Mock-up Unit, follows an industrial standard for model exchange called Functional Mock-up Interface (FMI). It is used to export models from one tool to another to test models in different simulation environments or make use of specialized solvers. A problem class for simulating coupled systems with Assimulo solvers has been developed. Assimulo is an open source Python package with numerical solvers.

Several coupled systems with different characteristics were tested. Mainly a system with two separately modelled pendulums coupled with a linear spring and external excitations on the pivots. Simulation results show potential for the concept.