Estimating expected lifetime of revolving credit facilities in an IFRS 9 framework

Jonas Berglund

Faculty of Engineering, Lund University
Centre for Mathematical Sciences

January 6, 2016

Abstract

This paper sets out to estimate expected lifetime of revolving credit facilities (e.g. credit card products) and is motivated by the introduction of the International Financial Reporting Standard 9 (IFRS 9) and its requirements for loan impairments. The reporting entity is required to estimate lifetime expected credit losses for certain financial instruments. In practice, maximum contractual period for revolving credit facilities cannot be used in defining lifetime for the facility and credit risk mitigation actions need to be considered. A data set for a retail credit card portfolio was provided by a Nordic bank and for the lifetime definition derived, a model based on a conditional Markov chain was selected. Expected lifetime was estimated and an analytical expression for expected lifetime of revolving credit facilities was derived and validated.

Preface

I would like to thank the Risk Department of the bank that generously provided data and its staff to assist with this project. The topic for this thesis project was suggested by the Quantitative Advisory Services (QAS), at EY Denmark. I would like to thank Per Thåström, EY Denmark, and Magnus Wiktorsson, the Faculty of Engineering at Lund University, for their valuable contributions. I would also like to thank the QAS team for providing resources and an inspiring environment.

Contents

1 Background

The purpose with the following chapter is to provide a background and motivate the relevance of the study. First, section 1.1 motivates the development of the International Financial Reporting Standard 9 (IFRS 9, the Standard). Second, the purpose with this study is presented in section 1.2. Third, relevant previous studies are presented in section 1.3, from the industry and academia.

1.1 Objectives of the IFRS 9 standard

In response to the Great Financial Crisis, it was noticed that the current standard for loan loss provisioning, International Accounting Standard 39 (IAS 39), did not require the financial sector to provision at levels high enough for the loan losses observed. It was also concluded that provisioning occurred too late (often summarized as "too little, too late"). Hence, measures intended to resolve those concerns were included in IFRS 9.

The issues in IAS 39 were confronted as follows. In handling that provisions were deemed too low based on IAS 39 requirements, all instruments in scope of the impairment requirements of IFRS 9 require provisions for loan losses. Furthermore, an expected loss model is introduced in order to demand provisions in line with actual expectations of losses rather than incurred losses. The shift to expected losses is also central in provisioning in a timely manner. It is also argued that the requirement of recognizing (the greater amount of) loss allowances upon indication of significant increase in credit risk contribute as well (rather than based on the objective evidence of impairment in IAS 39).

1.2 Purpose of this study

In measuring lifetime expected credit losses of instruments for which a significant increase in credit risk has been identified, there is a need to define and estimate expected lifetime. Current models used for the purpose of calculating expected losses are in general based on the risk of a default occurring within the next 12 months. Thus, a method for adjusting the 12-month probability of default, to a probability of default over the entire lifetime of the instrument is relevant for practitioners.

An important difference between expected lifetime of revolving credit facilities (e.g. credit card products) and other types of loan instruments is that contractual lifetime is not applicable in practice in considering expectations of lifetime. Instead, expected credit risk mitigation actions play an important role in defining expected lifetime for the facility under IFRS 9. Furthermore, the facility includes an off-balance sheet exposure required to be considered in IFRS 9, which in IAS 39 did not demand provisioning.

Therefore, the purpose of the present study will be to provide an overview

of the standard, define expected lifetimes of revolving credit facilities for when lifetime expected credit losses are required as basis for loan loss provisions, and implement this in a model for estimating expected lifetime.

1.3   Related research

The field of credit risk management has been important for practitioners for an extensive period of time, and a wide range of models have been developed, including e.g. CreditRisk+ [4], CreditRisk and KMV [16, p. 2]. Several aspects of credit risk management are covered in the literature, often with the basis in the probability that a counterparty will default, known as probability of default (PD) or credit rating models.

Methods for modelling PD may be divided into either a top-down approach, where an entire portfolio of instruments is considered (considered in e.g. [6]), or bottom-up (as in e.g. [1]), where counterparty-specific models are developed. The literature on credit risk models considers mainly two approaches; structural and reduced-form. Structural models are based on a value process of the counterparty (the most common to use is the stock price process), where the company is considered to be in default if the process is less than some threshold, usually a proportion of the firm's debt [1, p. 1]. Reduced-form models consider the timing of defaults as unpredictable and instead utilize a default intensity, where important contributions have been made by e.g. Duffie and Singleton [5], and Hull and White [9].

When applying a structural approach in retail credit, selecting a value process corresponding to the stock price process for corporations is not straight-forward. Hence, there is instead a great number of reduced-form models for retail credit risk. Furthermore, retail portfolios are often managed on a collective basis which make the use of top-down models suitable. There are some examples in the literature of structural models for retail portfolios, e.g. in [1], where a counterparty's behavioral score is used as value process (behavioral scoring models are discussed in section 4.4.1).

Markov chains has been used for the task of modelling rating migrations in PD models, e.g. in [14]. With the increasing attention to structured finance within academia, e.g. asset-backed securities (ABS), the *competing risks framework* has been developed for rating and valuation of ABS instruments, where not only default is considered, but also uncertainty arising from other types of premature contract termination (e.g. prepayment). The competing risks framework applied to mortgage-backed securities (MBS) are for instance implemented in [6]. The framework is close to what is intended with this study since multiple causes for end of lifetime of an instrument will be considered. Markov chains have been implemented for the purpose of modelling competing risks, e.g. in [7].

There have been some papers related to impairment modelling within IFRS 9 published (or working papers). Examples include studying the concepts *lifetime* PD and transformation of what is known as through-the-cycle (TTC) PD, into what is expected to be in line with the requirements of the Standard (in the report referred to as *point-in-time* PD) [15]. The study has a different focus than the present, however, since the lifetime PD is considered for general loan instruments and not the specific requirements for revolving credit facilities.

2 Banking risks, accounting and the IAS 39 financial reporting standard

The purpose of the following chapter is to provide the context for the accounting standard in question. The chapter first presents an overview of relevant risks to financial institutions[1] in section 2.1. A presentation of accounting and financial statements follow in section 2.2, providing a background for the instruments in scope of the Standard.

2.1 Risks faced by financial institutions

Credit risk is defined as *"the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms."* [3]. Credit risk management is of great importance for the core business of financial institutions, and the essence of credit risk management is the analysis of uncertainty [4, p. 23]. Other risks facing financial institutions include interest rate, equity, liquidity and foreign exchange risks. Although risks may be seen as detrimental for the bank, risks are a vital aspect for enabling future profitability.

2.2 Accounting and financial reporting

The purpose of financial statements is to provide financial information about the issuer of the statement. Annual reports required to be prepared by the issuer include the *cash flow statement*, the *income statement* and the *balance sheet*. The international standard for financial accounting is known as International Financial Reporting Standard (IFRS), developed by the International Accounting Standards Board (IASB). The specific standard is subsequently implemented into each jurisdiction, where actual implementation may differ. For the purpose of reporting standards, the entity is an issuer of a financial statement.

2.2.1 The balance sheet

The balance sheet is the financial statement describing the financial position of the issuer in terms of its assets, liabilities and equity. For the balance sheet, the *accounting identity* holds, i.e.

$$Assets = Liabilities + Equity \tag{1}$$

The balance sheet and examples of its components are found in Fig. 1. Assets are comprised of financial assets and non-financial assets. The present study will regard financial assets.

---

[1]it should be noted, however, that entities that will be required to comply with the Standard are not restricted to financial institutions

Assets	Equity & Liabilities
Short-term Assets	Short-term Liabilities
- Cash	- Commercial Paper
- Receivables	- Line of Credit
- Treasury Bills	
- Other cash-equivalent Assets	Long-term liabilities
	- Deposits
Long-term Assets	- Debt Securities
- Loans	- Hybrid Securities
- Securities	
	Equity
Non-financial Assets	- Shareholder's Equity
- Real Estate	- Retained Earnings
- Goodwill	- Preferred Shares
- Foreign Subsidaries	

Figure 1: The Balance sheet and examples of its components

2.2.2 Financial assets

According to IFRS 9, financial assets are classified as subsequently measured at *amortized cost*, *fair value through other comprehensive income* or *fair value through profit or loss*. The classification of financial assets is based on the entity's business model for the instrument and on the characteristics of the cash flows. The following two conditions must be met for a financial asset to be measured at amortized cost, [10, §4.1.2]:

- the financial asset is held within a business model whose objective is to hold financial assets in order to collect contractual cash flows and

- the contractual terms of the financial asset give rise on specified dates to cash flows that are solely payments of principal and interest on the principal amount outstanding.

The first criterion concerns whether the financial asset is held with trading intent or not. If it is, the instrument is classified as measured at fair value through profit and loss. A financial asset is classified as subsequently measured at fair value through other comprehensive income if the following conditions are met, [10, §4.1.2A]:

- the financial asset is held within a business model whose objective is achieved by both collecting contractual cash flows and selling financial assets and

- the contractual terms of the financial asset give rise on specified dates to cash flows that are solely payments of principal and interest on the principal amount outstanding.

The classification of financial assets is illustrated in Fig. 2.
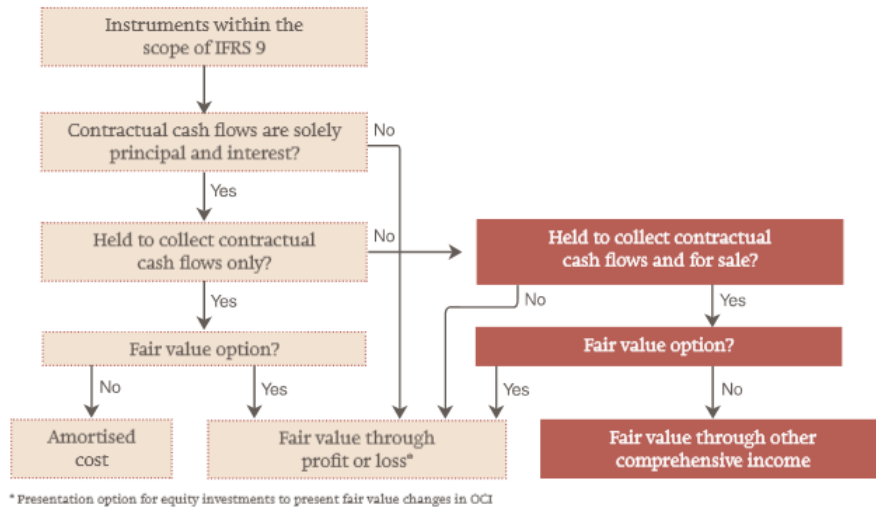
Figure 2: Classification approach for financial assets in IFRS 9 [12, p. 7]

2.2.3 Definitions of items in financial statements and related concepts

A list of items in financial statements and in general for accounting purposes relevant to the IFRS 9 impairment requirements will follow.

- *Credit-impaired financial asset:* A financial instrument is credit impaired when one or more events that have a detrimental impact on the estimated future cash flows of that financial asset have occurred. [10, A371-A375]

- *Credit loss:* The difference between all contractual cash flows that are due to an entity in accordance with the contract and all the cash flows that the entity expects to receive. [10, A371-A375]

- *Derecognition:* The removal of a previously recognized financial asset or financial liability from an entity's statement of financial position. [10, A371-A375]

- *Initial recognition:* An entity shall recognize a financial asset or a financial liability in its statement of financial position when, and only when the entity becomes party to the contractual provisions of the instrument. [10, 3.1.1]

- *Loan commitment:* A contract for the commitment of an entity to extend credit to a counterparty, usually up to some limit, known as the credit

limit. This is an off-balance sheet exposure, which will be converted to an on-balance sheet exposure when the commitment is drawn down.

- *Loss allowance:* The allowance for expected credit losses on financial assets [...], the accumulated impairment amount for financial assets [...] and the provision for expected credit losses on loan commitments and financial guarantee contracts. [10, A371-A375]

Other related concepts include

- *Minimum to Pay (MTP):* The minimum amount that a counterparty to a credit card facility needs to pay in order to not breach the contractual terms.

- *Past due, Days Past Due (DPD):* A financial asset is past due when a counterparty has failed to make a payment when that payment was contractually due. Days Past Due (DPD) is the number of days past the contractual due-date of the payment. This contractual breach will also be referred to as Delinquency.

2.3 International Accounting Standard 39 (IAS 39)

IFRS 9 will replace IAS 39, currently defining the requirements for loan loss provisioning. Hence, it is important to consider relevant requirements of IAS 39 for the purpose of understanding gaps for implementing an impairment model under IFRS 9. Under IAS 39, entities are required to determine provisions based on *objective evidence of impairment*, resulting from a *loss event*, if the loss event has an impact on future cash flows [13, §59]. This is known as *incurred losses*, either incurred and at the reporting date reported losses and Incurred But Not Reported (IBNR) losses.

Such objective evidence is the result of for example the following loss events, [13, §59]:

- significant financial difficulty of the issuer or obligor;

- a breach of contract, such as a default or delinquency in interest or principal payments;

- observable data indicating that there is a measurable decrease in the estimated future cash flows from a group of financial assets since the initial recognition of those assets, although the decrease cannot yet be identified with the individual financial assets in the group.

The focus on objective evidence of impairment and the principle of basing provisioning on losses that are already incurred at the reporting date is an important difference from the IFRS 9 standard. Incurred losses would include IBNR losses, as considered in [13, §AG90]. It is also mentioned in the example, [13, §AG90], that this must not include expectations of future loss events. In the event of

limited available historical data, *"an entity uses its experienced judgement to estimate the amount of any impairment loss. Similarly, an entity uses its experienced judgement to adjust observable data for a group of financial assets to reflect current circumstances.*

This is often in practice interpreted as allowing for *expert judgement* included in determining provisions for IBNR losses. The correspondence to an *expert judgement layer*, or collective overlay, in IFRS 9, will be discussed in section 4.5.

3 IFRS 9 impairment requirements

Impairment accounting is under IFRS 9 applicable for all financial instruments classified as amortized cost and fair value through other comprehensive income (section 2.2). The impairment requirements in IFRS 9 will be covered as follows. First, differences from IAS 39 are discussed in section 3.1. Second, expected credit losses (ECL) and possible components of ECL are outlined in section 3.2, providing a context for expected lifetime. Third, timing of when lifetime ECL is recognized and derecognized for financial assets on an individual and a collective basis is studied respectively in sections 3.3 and 3.4. Furthermore, the exception for instruments with *low credit risk* is introduced in section 3.5, followed by a presentation on which information is required to be used by the entity (section 3.6). The sections provide background to the IFRS 9 Stage model introduced in the next chapter.

3.1 Expected credit losses in IFRS 9

In line with the neutrality principle of the standard, principles for calculating expected credit losses (ECL) are introduced in [10, §B5.5.42].

> The purpose of estimating expected credit losses is neither to estimate a worst-case scenario nor to estimate the best-case scenario. Instead, an estimate of expected credit losses shall always reflect the possibility that a credit loss occurs and the possibility that no credit loss occurs even if the most likely outcome is no credit loss.

The unbiased estimation of expected credit losses means that it does not, for instance, contain a conservative bias. Guidance on how to implement estimation of ECL is further given in [10, §B5.5.18]:

> When measuring expected credit losses, an entity need not necessarily identify every possible scenario. However, it shall consider the risk or probability that a credit loss occurs by reflecting the possibility that a credit loss occurs and the possibility that no credit loss occurs, even if the possibility of a credit loss occurring is very low.

3.2 Components of expected credit losses

The requirements introduced in the previous section for how to estimate ECL are discussed in the context of the components probability of default (PD), exposure at default (EAD) and loss given default (LGD, the proportion of the exposure expected to be lost, given that default has occurred).

It is required by the Standard that an entity considers *time-value of money*, requiring that future cash-flows are discounted to the reporting date (not the expected default or some other date) [10, §B5.5.44]. Furthermore, *expected prepayment* is required to be included [10, §B5.5.51]. The components are described

in the following sections to provide a context for expected lifetime.

3.2.1   Probability of default (PD)

The difference between lifetime and 12-month (assigned for instruments at initial recognition) ECL specifically relates to the PD component. The Standard stresses that the difference between measuring loss allowance for an instrument or group of instruments before or after a significant increase in credit risk (since initial recognition) has been identified, relates to estimating the probability that a default will occur within 12 months or lifetime, rather than cash shortfalls predicted to occur within the next 12 months. For instruments that an entity is required to recognize loss allowance at an amount equal to lifetime expected credit losses, the entity must estimate the risk of a default occuring on the instrument during its expected lifetime. [10, §B5.5.43]. 12-month ECL is described in the Standard as follows [10, §B5.5.43]:

> 12-month expected credit losses are a portion of the lifetime expected credit losses and represent the lifetime cash shortfalls that will result if a default occurs in the 12 months after the reporting date (or a shorter period if the expected life of a financial instrument is less than 12 months), weighted by the probability of that default occurring.

3.2.2   Loss given default (LGD)

The Standard requires the *time value of money* to be incorporated in the calculation of ECL. This may be incorporated in the estimation of LGD, since the Standard stresses the necessity in discounting cash shortfalls (the difference between contractual and expected cash-flows) to the reporting date. The Standard specifies that the discount factor to be used is the effective interest rate, determined at initial recognition (with some exceptions) [10, §B5.5.44].

3.2.3   Exposure at default (EAD)

EAD is the exposure that an instrument or a group of instruments constitute at the time of default. It is mentioned that for *loan commitments*, exposure resulting from expectations of draw-downs is to be included in the estimation over the lifetime of the loan commitment for lifetime ECL (and expected draw-down for the next 12 months, for 12-month ECL). However, it is likely that credit risk management actions will limit the exposure (discussed in section 6), e.g. for revolving credit facilities (section 5) the loan commitment component may be removed, limiting this exposure.

3.3 Timing of recognizing lifetime expected credit losses

At initial recognition of a financial instrument, a loss allowance is required to be recognized at a level equal to 12-month ECL for the instrument. The timing of recognizing lifetime ECL, is in the Standard considered as follows (in [10, §B5.5.7]): *"The assessment of whether lifetime ECL should be recognized is based on significant increase in the likelihood or risk of a default occurring since initial recognition."*

Furthermore, the Standard highlights the need for separation of definition of credit-impairment and *significant increase in credit risk since initial recognition* for an instrument, *"[an] entity cannot align the timing of significant increases in credit risk and the recognition of lifetime expected credit losses to when a financial asset is regarded as credit-impaired or an entity's internal definition of default."* [10, §B5.5.21] Thus, a model implementing the two definitions must separate the timing of when they occur for an instrument, where the criterion for significant increase in credit risk needs to be such that a significant increase in general will be observable before an instrument becomes credit-impaired [10, §B5.5.7].

3.3.1 Determining whether credit risk has increased significantly since initial recognition

The determination of whether an increase has occurred for an instrument is based on whether the risk of default (PD) has increased since initial recognition and not based on a change in the amount of expected credit losses. Determining whether an increase has occurred should be based on PD over the entire lifetime of the instrument (at each reporting date) [10, 5.5.9]. Considering the components of expected credit losses in section 3.2, this corresponds to solely monitoring changes in PD over the expected lifetime of the instrument, for the purpose of determining whether a significant increase since initial recognition has occurred.

For instruments where the default pattern is not concentrated to a specific point in time, changes in PD over the next 12 months may be a reasonable approximation of changes in PD over the lifetime of the instrument, [10, §B5.5.13]. This might be a useful simplification for implementation, since financial institutions are likely to have implemented internal credit rating models based on 12-month PD.

3.3.2 Individual assessment of significant increase in credit risk

Guidance on what to include in the assessment of increase in credit risk is found in [10, §B5.5.17], e.g.:

- significant changes in external market indicators of credit risk

- an actual or expected change in the financial instrument's external credit rating

- significant increases in credit risk on other financial instruments of the same borrower

- an actual or expected internal credit rating downgrade for the borrower or decrease in behavioral scoring used to assess credit risk internally.

3.3.3 Collective assessment of increase in credit risk

Depending on what information is used on individual basis, there may be a need to determine if credit risk has increased on a collective basis [10, §B5.5.1]. If there is an indication of significant increase in credit risk on a collective basis (on a portfolio level) not yet observed on individual level, loss allowance needs to be based on a collective recognition of significant increase in credit risk as well, in order to meet the requirements of the Standard. [10, §B5.5.1]

For the purpose of the collective determination, the Standard demands the aggregation of financial instruments based on *shared credit risk characteristics* [10, §B5.5.5]. Shared credit risk characteristics included in the Standard, [10, §B5.5.5], are for instance:

- instrument type,

- credit risk ratings,

- date of initial recognition,

- remaining term to maturity,

- industry,

- geographical location of the borrower

If an entity is not able to aggregate instruments for which the credit risk has increased significantly since initial recognition (based on shared credit risk characteristics) the entity is required to recognize lifetime expected credit losses on a portion of the instruments [10, §B5.5.6]. Furthermore, the aggregation of instruments in portfolios may change over time, highlighted in Example 2 in [11, IE38]. Furthermore, for a credit card portfolio, examples of indication of increase in credit risk on a collective basis, is the expected number of customers that will exceed their credit limit, or pay the Minimum to Pay amount.

3.4 Derecognition of lifetime expected credit losses

If an instrument no longer meets the criteria for recognition of lifetime ECL, 12-month expected credit losses will be used as the basis for recognition of loss allowance. The counterparty to the instrument must show evidence of

financial stability. Evidence that the criteria for lifetime ECL are no longer met would include a history of up-to-date and timely payments. Furthermore, it is mentioned that one payment on time would not suffice for considering the instrument to have demonstrated financial stability [10, §B5.5.27].

3.5 Financial instruments that have low credit risk at reporting date

There is an exception to the need for recognizing lifetime expected credit losses based on an increase in credit risk. If the instrument is considered to have a low credit risk at reporting date, the entity is not required to recognize lifetime expected credit losses for the instrument, [10, 5.5.10]. What is considered *low credit risk* may be an external rating of *investment grade* [10, §B5.5.23]. However, not only instruments with external ratings are applicable for the low credit risk definition. Internal credit risk ratings, or other methodologies, may be used for the purpose of determining low credit risk, if they are consistent with a global understanding of low credit risk [10, §B5.5.23]. The focus of this study is on retail portfolios and it cannot be assumed that instruments within these portfolios have an external rating.

3.6 Reasonable and supportable information

For the purposes of the Standard, "[...] *reasonable and supportable information is that which is reasonably available at the reporting date without undue cost or effort, including information about past events, current conditions and forecasts of future economic conditions*" [10, §B5.5.49].

The implementation guidelines further specifies the information that is considered to be reasonable and supportable, [10, §B5.5.51],

> The information used shall include factors that are specific to the borrower, general economic conditions and an assessment of both the current as well as the forecast direction of conditions at the reporting date. An entity may use various sources of data, that may be both internal (entity-specific) and external. Possible data sources include internal historical credit loss experience, internal ratings, credit loss experience of other entities and external ratings, reports and statistics.

The role of historical information as an anchor is stressed in [10, §B5.5.52]. An approach for determining if a significant increase in credit risk (since initial recognition) has occurred and the estimation of ECL will likely depart from historical information and be supplemented by information on current and future conditions.

4 IFRS 9 Stage model

The previous sections have covered the specific requirements of the Standard. It is convenient to summarize the impairment requirements of the Standard as stages and transitions between stages, [12, p. 16]. This chapter initially covers the three stages and how an instrument is transitioned between the stages (sections 4.1, 4.2 and 4.3). After the general stage model is presented, examples of credit risk models on a counter-party (individual) basis are introduced. It is likely that an accompanying *collective overlay* is needed in order to fulfill the requirements of the Standard (section 4.5). The delinquency model (section 4.4.2) is the type of credit risk model that will be the basis for estimating expected lifetimes.

An instrument is at initial recognition included in Stage 1 (if not regarded as credit-impaired). At the reporting date, if there is indication of a significant increase in credit risk since initial recognition (section 3.3.1), the instrument is transitioned to Stage 2, and subsequently, the requirements for Stage 2 will be applicable. If the instrument meets the criteria for *credit-impairment*, section 3.3, the instrument is transitioned to Stage 3.
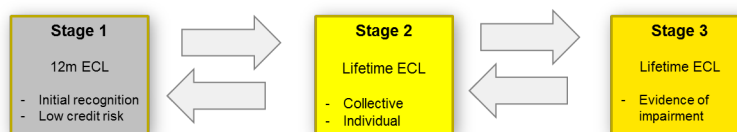
Figure 3: IFRS 9 stages

4.1 Instruments in Stage 1

At initial recognition, the instrument is included in Stage 1. The entity is for instruments in Stage 1 required to assign loss allowances based on 12-month ECL. It is discussed in section 3.2, that the difference between estimating 12-month or lifetime ECL relates specifically to estimating 12-month or lifetime PD, since the 12-month ECL is measured as cash shortfalls over the entire expected lifetime of the instrument, scaled by the 12-month PD.

4.2 Instruments in Stage 2

Upon indication of significant increase in credit risk since initial recognition for an instrument or a group of instruments, loss allowance is required to be recognized at an amount equal to lifetime ECL. The transition criterion is therefore the criterion for significant increase in credit risk and what differs from Stage 1 is that loss allowance is based on lifetime ECL rather than 12 month ECL. Transition of loans from Stage 2 to Stage 3 is based on whether the instrument

meets the entity's criterion for credit-impairment. Transition from Stage 2 to Stage 1 is based on the criterion for derecognition of lifetime ECL (financial stability, as mentioned in section 3.4).

4.3   Instruments in Stage 3

Transition to Stage 3 is based on whether an instrument is considered credit-impaired, as discussed in section 3.3. Loss allowance for instruments in Stage 3 are equal to lifetime ECL, as in Stage 2. What differs, is how to calculate interest revenue, which is not in scope for this study. Transition from Stage 1 to Stage 3 is unlikely, considering the demand of not aligning the criterion for significant increase in credit risk with the definition of credit-impairment (section 3.3.1).

4.4   An individual assessment basis

4.4.1   Stages in a behavioral scoring model

An entity is required to utilize all *reasonable and supportable information available without undue cost or effort at reporting date* (section 3.6) in order to determine if a significant increase in credit risk since initial recognition has occurred. It is specified in the implementation guidelines that, in order to assess whether an increase has occurred, a relevant factor to consider is the "behavioral scoring used to assess credit risk internally" [10, §5.5.17]. This is often in practice interpreted as utilizing observed payment behavior of the counterparty to the instrument. It is likely that if an entity utilizes a behavioral scoring model, this will be considered *reasonable and supportable information available without undue cost or effort* (section 3.6).

An entity is required to consider forward-looking information. Behavioral scores cannot be considered forward-looking information, merely forward-looking to a greater degree than delinquency information. Thus, methods to account for forward-looking information must be developed both in order to assess increases in credit risk, and in estimating expected credit losses. A discussion on implementation of a collective overlay to the individual assessment is found in section 4.5.

Stages and stage transitions will be discussed for an example behavioral scoring model. The model consists of a total of ten internal credit ratings. Eight ratings used for non-impaired (performing) instruments. A definition of low credit risk, as discussed in section 3.5, of 0.5% has been selected and the criterion for significant increase in credit risk since initial recognition is selected as an exceedence of twice the 12 month probability of default at initial recognition, *Significant increase* $\geq 2PD_{initial}$. The resulting partitioning of the portfolio is depicted in Fig. 4. It should be noted that it is assumed that for the instruments in the portfolio, no previous transitions has occurred since initial recognition.

15

Otherwise, there would not be a straight-forward mapping between the credit rating at reporting date (indicated vertically) and the partitioning of the portfolio. In practice, it would be necessary to compare the current credit rating with the credit rating at initial recognition, rather than the one at reporting date.
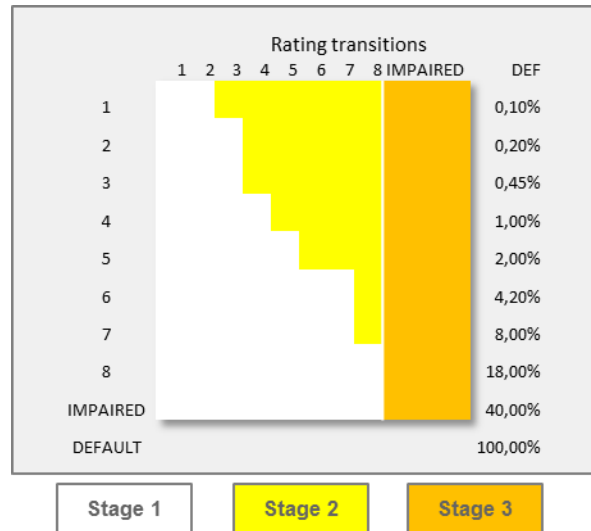
Figure 4: Incorporating criteria for stage transitions and low credit risk in an internal credit rating model (credit scoring model)

4.4.2 Reducing the behavioral model to delinquency information

It is recognized in the implementation guidelines of the Standard, [10, §B5.5.3], that for certain portfolios,

> an entity may not be able to identify significant changes in credit risk for individual financial instruments before the financial instrument becomes past due. This may be the case for financial instruments such as retail loans for which there is little or no updated credit risk information that is routinely obtained and monitored on an individual instrument until a customer breaches the contractual terms.

For the retail portfolios that are in-scope for the present study, it is likely that no other information on individual instrument level is available without undue cost or effort (section 3.6).

The delinquency-based (*Days Past Due (DPD)*) ratings, will be referred to as *risk-states*. The delinquency-based model therefore maps customers into risk-states. The risk-states used in the date set are found in section 7 and the implementation of risk-states in a model for expected lifetimes is found in section 9.
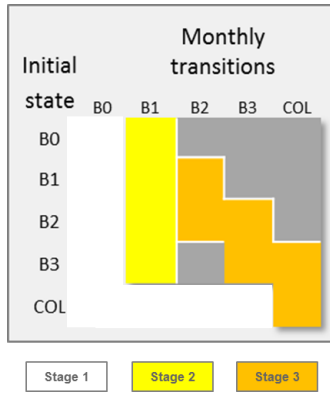
Figure 5: Possible interpretation of transitions for monthly transitions in delinquency-based model

It is recognized that implementation will be similar for different delinquency models. However, as stated in section 3.3.1, the timing of increase in credit risk and credit-impairment must not be aligned. Thus, at least one risk-state below a performing risk-state (non-delinquent) must be present before instruments are regarded as credit-impaired.

Certain aspects are not observable in risk-state transitios for revolving credit facilities (see section 5 for characteristics of revolving credit facilities). Due to the structure of the facility, the account may be drawn-down at multiple points in time, and if the counterparty repays the first amount later than when it is contractually due and at a later point become past due on another draw-down, the instrument will have a delinquent risk-state for several months, although the definition of risk-state transitions may be that an instrument is transitioned every, e.g. 30 days of DPD. This is illustrated in Fig. 6.

4.5 A collective assessment basis

The *delinquency-based* credit risk model introduced in section 4.4.2 is for the retail portfolios intended in this study utilizing all information that is *reasonable and supportable* on an individual basis, section 3.6. It is stated in [10, §B5.5.3] that *"[if] changes in the credit risk for individual instruments are not captured before they become past due, a loss allowance based only on credit information at an individual financial instrument level would not faithfully represent the changes in credit risk since initial recognition"*. [10, §B5.5.3]

The delinquency-based model, as described in section 4.4.2, is not likely to be sufficient on its own in order to detect *significant increase in credit risk*, as defined in section 3.3.1. Thus, the purpose of a *collective overlay*, is to com-
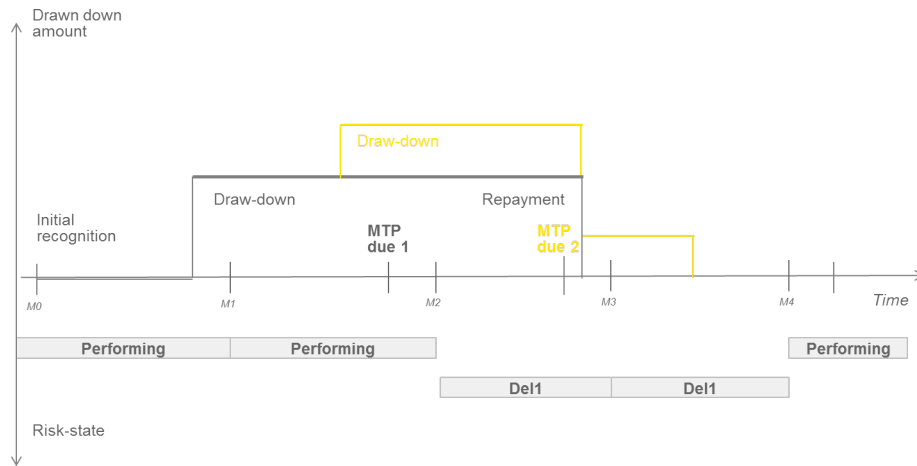
Figure 6: Risk-state deviate from transition principle due to multiple draw-downs

plement the individual approach to incorporate forward-looking information for the purpose of determining significant increases in credit risk and incorporating forward-looking information, discussed in section 3.6.

Similar to the approach discussed for estimating provisions in IAS 39 for IBNR losses (section 2.3, where expert judgement may be incorporated in assessing IBNR losses), a similar approach for adjusting the historical credit loss information where qualitative information is incorporated, may be developed motivated by [10, §B5.5.18], "In some cases the qualitative and non-statistical quantitative information available may be sufficient to determine that a financial instrument has met the criterion for the recognition of a loss allowance at an amount equal to lifetime expected credit losses. This may for example be in the form of an adjustment parameter based on where in the credit cycle the economy is at reporting date. Furthermore, criteria on a collective basis need to be considered, found in section 3.3.3, e.g. whether the number of facilities extending their credit beyond the credit limit (i.e. overdraft) has increased or is expected to increase.

5 Revolving credit facilities

The financial instruments in scope for this study are instruments known as *revolving credit facilities*, where an important example is retail credit card products. The facility consists of two components. The loan component (*the drawn amount*), and a loan commitment component (*the undrawn commitment*). Several characteristics of the contract and risk management practice associated with portfolios of revolving credit facilities make revolving credit different from other loan instruments, in terms of lifetime definitions and modelling of PD, EAD and LGD. In the present study, only characteristics relevant to lifetime definitions are considered.

The contractual terms typically include a *credit limit*, for which equality holds between

$$Credit\ limit = Drawn\ amount + Undrawn\ commitment \tag{2}$$

and a Minimum-to-Pay (MTP). The following characteristics of revolving credit facilities are especially relevant, [10, §B5.5.39]:

- the financial instruments do not have a fixed term or repayment structure and usually have a short contractual cancellation period (for example, one day);

- the contractual ability to cancel the contract is not enforced in the normal day-to-day management of the financial instrument and the contract may only be cancelled when the entity becomes aware of an increase in credit risk at the facility level; and

- the financial instruments are managed on a collective basis.

This provides background for the next chapter, where appropriate definitions are considered for the expected lifetime of the instrument.

The loan commitment component is in scope of the Standard, motivated by the fact that the concept of *expected* losses (to be compared with *incurred* losses, for which this is not true) is, from a credit risk management perspective, ”[...] *as relevant to off-balance sheet exposures as it is to on-balance sheet exposures.*” [11, §BC5.259]. Consequently, loan commitments demand provisioning in line with other financial instruments in scope for the impairment requirements of the Standard.

6 Expected lifetime

The *expected lifetime* of revolving credit facilities is the period over which to estimate lifetime ECL for the facility and thus the central concept of this paper. The chapter initially provides a background in the Standard for how lifetime is defined for the type of instruments constituting the components of the facility (sections 6.1 and 6.2). This leads into section 6.3, where specific considerations relevant for when the components are managed together on facility level are considered, including *credit risk (management) mitigation actions*. A definition is derived in section 6.5.

6.1 Period over which to estimate lifetime ECL for (installment) loan instruments

The Standard specifies that for loans, the *behavioral lifetime* shall be the basis for expected lifetime, which is the contractual lifetime adjusted by expected prepayment. The contractual lifetime of a loan is the period from initial recognition to contractual maturity. For some instruments, the contractual terms allow for prepayment, i.e. the principal and further interest payments due are repaid at an earlier date than the contractual maturity, thus shortening the period of time that the entity is exposed to credit risk arising from the instrument,

$$Behavioral\ lifetime = Contractual\ lifetime - Expected\ prepayment \qquad (3)$$

6.2 Period over which to estimate lifetime ECL for loan commitments

For loan commitments *"[...] the period over which the entity is exposed to credit risk*[2] *is the maximum contractual period over which an entity has a present contractual obligation to extend credit."* [10, §B5.5.38]. The use of maximum contractual period for loan commitments as the basis is further specified in [11, §BC5.260], where the IASB *"[...] noted that most loan commitments will expire at a specified date, and if an entity decides to renew or extend its commitment to extend credit, it will be a new instrument for which the entity has the opportunity to revise the terms and conditions."*

6.3 Definition of expected lifetime for revolving credit facilities

A lifetime definition is more complicated to design for the facility than for its components, due to the characteristics of the contract and expected management practice presented in the last chapter (chapter 5). Defining the period over which to estimate expected credit losses based on the entity's *contractual*

---

[2]which has been mentioned earlier, is the basis for defining the period over which to estimate ECL

ability to cancel the undrawn commitment and demand repayment with little or no notice would not necessitate the entity to recognize expected credit losses on the undrawn commitment. This is a significant difference from IAS 39 in how the definition must be designed. However, since this would not reflect actual expectations of loss (i.e. *expected* lifetime), there is a need to consider how the facility is actually expected to be managed and which events that would constitute the derecognition of the instrument (in how it is defined in the Standard) [11, §BC5.256, §BC5.259]. Thus, it does not seem to be relevant to consider maximum contractual period when defining expected lifetime for the facility.

Instead, an entity is required to consider the entire period that it is expected to be exposed to credit risk, for which not *credit risk management actions* would mitigate expected losses [10, §B5.5.40] (*credit risk mitigation actions*), such as the reduction or removal of the undrawn commitment component.

6.3.1 Modifications and recognition of a 'new' financial instrument

A modification of the contractual cash flows *"[...] can lead to the derecognition of the existing financial asset in accordance with this Standard. [...] the modified asset is considered a "new" financial asset[...]"* [10, §B5.5.25] It is mentioned, however, that *"[m]odifications frequently do not result in the derecognition of a financial instrument."*[11, §BC5.227]

If the undrawn commitment component of the facility is expected to be removed following some event (e.g. if the counterparty of the instrument is several days past due), it no longer constitutes an exposure to credit risk in accordance with the standard [11, §BC5.243]. If the previously removed undrawn commitment component of the facility is reinstated following a reassessment of the credit risk, a "new" instrument may be recognized, and the instrument previously considered would be derecognized.

The argumentation is summarized by the following example:

- A facility becomes delinquent, which is observed by credit risk management monitoring systems. However, no credit risk mitigation action is expected to be taken.

- Credit risk of the facility continues to increase and following some event (e.g. the instrument transitions to a higher risk-state, for a delinquency-based model), credit risk management is expected to remove the undrawn commitment component and demand repayment of the drawn amount.

One of the following two alternative evolutions will follow:

- The counterparty has repaid the drawn amount at latest on the maximum contractual period (usually defined indirectly by a monthly MTP proportion of the drawn amount). The lifetime of the facility is then ended when the drawn amount is repaid.

- The counterparty repays the drawn amount partially. Credit risk management reassesses the credit risk of the facility and reinstates the undrawn commitment component. This is considered a "new" instrument and hence the instrument in this example is derecognized, implying the end of lifetime for the facility.

A similar situation may arise if the undrawn commitment component is expected to be removed for inactive instruments (inactive may be defined as no drawn amount for a certain period of time). If the process of reinstating the undrawn commitment component involves a (significant enough) reassessment of credit risk for the facility, a possible interpretation of the Standard is that a new initial recognition is then defined for this date and thus end of lifetime for the instrument in question.

Taking credit risk management actions into account and incorporating how the facility is managed, demands information of when the undrawn commitment is removed for a specific facility. It is likely that this information available *without undue cost or effort* to the entity, although it is not directly observable variables in the data set.

6.4 End of lifetime event

This paper introduces the concept *End of lifetime event*, to bring together definitions used in the credit risk model (chapter 7) with additional definitions new to the Standard and specific for revolving credit facilities, related to expected credit risk mitigation actions. The basis for defining the End of lifetime event is that follwoing the event, there is no longer an exposure to credit risk for the instrument (as the Standard defines it). Assumptions are made on expected credit mitigation action in different situations described for the events.

6.4.1 Deactivation of contract based on inactivity

Following a period of six months of inactivity (defined for the purposes of this study as no drawn amount) credit risk management is assumed to deactivate the instrument and an activation process would involve the opportunity for reassessment of credit risk, in order for the counterparty to use the instrument again. This would enable the entity to reassess the credit risk associated with the facility; hence, motivating a new initial recognition and in consequence the End of lifetime event for the current instrument.

6.4.2 Charge-off

The instrument is charged off. It is of importance to know which credit risk management actions are taken at charge-off since it for some actions is not motivated to recognize charge-off as a termination to exposure to credit risk, and consequently an *End of lifetime event*. There are at least three possibilities,

under the assumption that the entity sells the claim (the amount still to be collected) to a Debt Collection Agency (DCA).

- The entity sells the claim and receives a single payment for the trade. In this case, the credit risk is transferred to the collection agency and the entity no longer has a present exposure to credit risk.

- The entity sells the claim and will be compensated for this in the future. In this case, the entity has an exposure to credit risk, arising from the possibility of default for the collection agency, a *Counterparty Credit Risk*. This would in turn simply shift the source of the exposure to credit risk, not terminate it.

- The entity receives a cash flow from the instrument, through the collection agency. In this case, exposure to credit risk from the instrument is still present.

It is assumed that the first action is taken, motivating that charge-off is an *End of lifetime event*.

6.4.3 Contract is terminated for other reason than breach of terms

The counterparty terminates the contract associated with the instrument, thus the instrument is no longer an exposure to credit risk for the entity.

6.4.4 Credit risk management actions taken upon evidence of credit-impairment at facility level

When the facility is considered credit-impaired, this is assumed to lead to the removal of the undrawn commitment and demanded repayment of the drawn amount. The facility determined credit-impaired is not in itself what defines the End of lifetime event (in line with last section). It is instead the end of the *behavioral lifetime* (section 6.1) of the loan component of the facility, which is approximated as the maximum contractual period.

6.5 Expected lifetime of instruments in stage 2

In line with the stages in section 4, the definition of start of lifetime, i.e. when to estimate lifetime ECL (rather than 12-month ECL), is defined by the significant increase in credit risk since initial recognition, e.g. in [10, §B5.5.40] where it is emphasized that information and experience about *"the length of time for related defaults to occur on similar financial instruments following a significant increase in credit risk"* shall be used when determining the period over which the entity is exposed to credit risk, for the purpose of estimating lifetime ECL. Expected lifetime is then measured as the period until the first End of lifetime event occurs.

6.6 Lifetime for Stage 3

The requirements in the standard differentiates between instruments of increase in credit risk since initial recognition and instruments that are credit-impaired (defined in chapter 9). The start of lifetime for Stage 3 instruments is defined as the entity's internal definition for credit-impairment. The definition of end of lifetime is chosen as the End of lifetime event in previous section. Since the End of lifetime event associated with credit-impairment coincides with the assumption on removal of the undrawn amount, Stage 3 instruments will with the presented assumptions in fact have a maximum contractual lifetime. A motivation vital for considering revolving credit facilities (no maximum contractual lifetime in practice) is thereby no longer present, hence Stage 2 instruments will be considered solely from here on.

7 Data set and credit risk model

The data set that has been used was provided by a Nordic bank and consists of 20 months of observations of on-balance sheet exposures (for revolving credit facilities, corresponding to the drawn amount) and risk-states (for the credit risk model described in the next section) for retail loan instruments in one of the countries that the bank operates. The portfolio was further segmented in order to achieve a homogeneous group of instruments, in terms of *shared credit risk characteristics*, discussed in section 3.3.3, into product groups and products. The credit card product group was studied. Within the credit card product group, one product was selected. The balance is a continuous variable ranging from negative values (corresponding to a customer repaying more than needed) to positive. Negative values are considered as zero (0). Additional variables are included, since they indirectly are available in the data set and considered *reasonable and supportable information*. These are maximum risk-state observed from earlier time points and the number of months that an instrument has not had a drawn amount.

7.1 The credit risk model

In line with the type of model described in section 4.4.2, the observed risk-states indicate if the contract is active and if so, how late the corresponding instrument is with payments. If the contract is not active, it is either written off from the books (risk-state **Written-off**) or **Closed**. The bank monitors each instrument at discrete time points (for this study a monthly frequency is assumed). If the counterparty to the instrument is not late with payments or less than a defined *grace period* (for which the counterparty is late with a payment although it is not yet considered delinquent), the instrument is assigned risk-state **Performing**.

If the counterparty is past due, although less than a defined limit (e.g. 30 DPD), the instrument is assigned risk-state **Del1**. If DPD is greater than this limit but less than the definition for being charged off (from here on, Charge-off will be used instead of default since they are expected to be defined sufficiently similarly), **Del2** is assigned as risk-state. If the counterparty is later, it is assigned the risk-state **Charge-off**, which is used interchangeably with default in this study (which is a simplification). The assigning of risk-states to instruments is outlined in Fig. 7.

Allowed transitions between risk-states from an observation time to the next are found in Fig. 8. The credit risk model presented will be the basis for the expected lifetime model implemented in section 9.

7.2 Maximum historical risk-state

Maximum historical risk-state is considered reasonable and supportable information since it is believed that segmenting instruments based on maximum
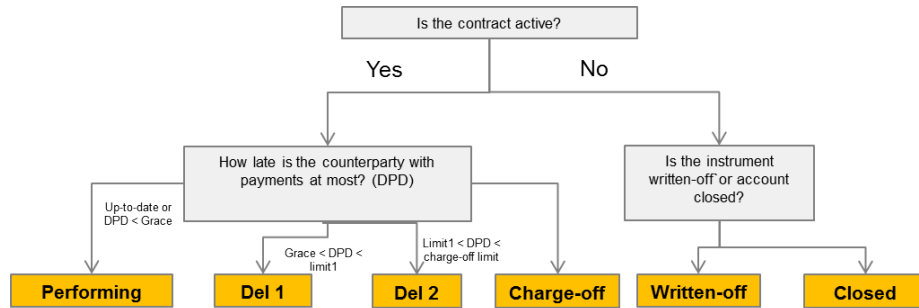
Figure 7: Visualization of how risk-states are assigned to instruments. Grace corresponds to the grace-period and limit1 to the defined limit between the delinquent risk-states
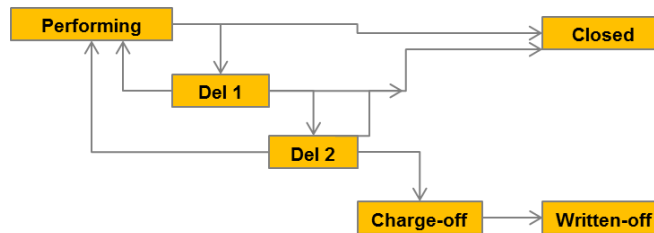
Figure 8: Dynamics of the credit risk model - allowed transitions between risk-states

historical risk-state will lead to significantly different expected lifetimes. This should be considered when estimating lifetime for the portfolio [10, §B5.5.5].

7.3 Number of consecutive months of inactivity

It is expected that instruments with no drawn amount will differ significantly from other instruments, in terms of behavior related to expected lifetime and credit risk. Furthermore, it is expected that this behavior will differ depending on the amount of time of inactivity. This will be utilized in the implementation of a definition of expected lifetime (chapter 9) for Model $M_{3'}$.

8 Markov processes

The purpose with the following chapter is to provide theoretical motivation for the choice of methodology, the models selected as candidates and the measure derived for estimation of expected lifetime. The chapter is arranged as follows. Concepts with general relevance to Markov processes are discussed in a first-order Markov chain context (section 8.1). Then absorption time is defined and a method for measuring absorption time with modifications relevant for lifetime of revolving credit facilities is derived (section 8.2). Extensions to the first-order Markov chain is first incorporated in a dependence on covariates for the process (section 8.3); and second, in a higher-order model (section 8.4). The different extensions will correspond to the different candidate models developed in preceding chapters.

8.1 Theory for discrete first-order Markov processes

8.1.1 Stochastic processes and the Markov property

A Discrete Markov chain is a stochastic process,

$$\{X_k(t)\}_{t>0} \tag{4}$$

defined on the probability space $(\Omega, \mathcal{F}, P)$ (defined in [17]) and the discrete state-space $X_t \in S = \{s_0, s_1, ..., s_n\} \quad \forall t = 1, 2, ...,$ for instrument k (k will be omitted from here on, since instruments are expected to be segmented into groups based on *shared credit risk characteristics*, as described in section 3.3.3, and in consequence is expected to be homogeneous within this group), with the following property, known as the *Markov property*,

$$P\left(X_{t+1}|X_t, X_{t-1}, ..., X_0\right) = P\left(X_{t+1}|X_t\right) \tag{5}$$

The one-step transition probability matrix, t.p.m., is defined accordingly as,

$$P = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n,1} & p_{n,2} & \cdots & p_{n,n} \end{pmatrix} \tag{6}$$

where each element, $p_{j,k}$ is the probability

$$p_{j,k} = P\left(X_{t+1} = k|X_t = j\right) \tag{7}$$

8.1.2 Absorbing and transient states in the Markov chain

An absorbing state is defined as not having positive transition probabilities to other states, or equivalently $p_{i,i} = 1$ for absorbing state i. For a transient state, i, it holds that $\sum_{j \in m} p_{i,j} < 1$, for transient state $i \in m$, where m is the set of transient states.

8.1.3 Absorbing Markov chain

A Markov Chain is defined in [17] to be absorbing when it satisfies the following two conditions,

- *The chain has at least one absorbing state; and*

- *It is possible to transition from each transient state to some absorbing state (perhaps in multiple steps)*

In *canonical form*, the transition probability matrix (6) is reorganized into transient and absorbing states,

$$P = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix} \tag{8}$$

where Q is the $n_{trans} \times n_{trans}$ matrix, representing transitions within the r transient states, R is the $n_{trans} \times n_{abs}$ matrix representing transitions from transient to absorbing states, 0 is the $n_{abs} \times n_{trans}$ matrix containing all zeros, and I is the $n_{abs} \times n_{abs}$ identity matrix. With the n_{trans} states, $X = 0, ..., X = r - 1$, corresponding to transient states and the n_{abs} states, $X = r, X = r + 1, ..., X = n$, corresponding to absorbing states.

8.2 Absorption time

Absorption time is defined in [8] as,

$$T_i = \min_{t \geq 0} \left(X_t \geq r \mid X_0 = i \right) \tag{9}$$

where $X \geq r$ implies an absorbing state.

The probability mass function of T, $p_T(t)$ for multiple absorbing states,

$$p_T(t) = Q^{t-1} R \, \mathbf{1}_{n_{abs} \times 1} \tag{10}$$

where row i of $p_T(t)$ corresponds to the ith transient state as initial state, $X_0 = i$.

For the defined submatrices in (8), and the Chapman-Kolmogorov equation [8] the t-step t.p.m., becomes

$$P^t = \begin{pmatrix} Q^t & (I + Q + Q^2 + ... + Q^{t-1})R \\ 0 & I \end{pmatrix} \tag{11}$$

8.2.1 The fundamental matrix

Vital to the study of absorption behavior, is the *fundamental matrix*, N, derived from the limiting distribution, (as $t \to \infty$).

$$P^\infty = \begin{pmatrix} 0 & NR \\ 0 & I \end{pmatrix} \tag{12}$$

where $N = I + Q + Q^2 + ... = (I - Q)^{-1}$.

The expected time spent in a transient state j, conditional on that the process is initiated in transient state i ($X_0 = i$) is given by $N_{\{i,j\}}$, and naturally

$$E(T) = N\mathbf{1}_{1 \times n_{trans}} \tag{13}$$

is the *expected absorption time*, conditional on initial state, $X_0 = i$. The expected absorption time will in later chapters be used for lifetime of an instrument in Stage 2.

8.2.2 Absorption probability

The absorption probability matrix, B, is the probability that a process is absorbed by some absorbing state, j, given initial state i, $B_{\{i,j\}}$. The absorption probability is computed as,

$$B = NR \tag{14}$$

8.2.3 Expected absorption time for a deterministic delay in absorption to one absorbing state

Consider an absorbing Markov chain, where a delay, t_d, is added to the absorption time of one of the absorbing states, s_d. The p.m.f. of the modified absorption time, T', is then

$$p_{T'}(t) = Q^{t-1} R e^c_{s_d} + I(t > t_d) Q^{t-1-t_d} R e_{s_d} \tag{15}$$

where $I(\cdot)$ is the indicator function ($I(\cdot)$ is 1 if the condition in the parenthesis is true and 0 otherwise), e_{s_i} and $e^c_{s_i}$ are

$$e_{s_d} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix} \tag{16}$$

and

$$e^c_{s_d} = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ 1 \\ \vdots \end{pmatrix} \qquad (17)$$

where the row of the 1 and 0 entry in the respective arrays correspond to the row of s_d in the t.p.m. (6) and the lengths are equal to n_{abs}.

Expectation of the modified absorption time, $E(T')$ is derived in the appendix, and the resulting formula is given below.

$$E(T') = E(T) + B_{*,s_d} \cdot t_d \qquad (18)$$

where $B_{*,k}$ is the kth column of the absorption probability matrix, (14).

8.3 First-order Markov chain dependent on covariates

It is assumed here that some transitions are dependent on a covariate, C, where C may take non-numerical values, in the set $S_C = \{C_1, C_2, C_3\}$. C may also be macro-economic variables, such as the rate of unemployment in a country or region. Transition probabilities with a dependence on C, would then be expressed as

$$p_{i,j}(c) = P(X_t = j | X_{t-1} = i, C = c), \quad i, j \in S', c \in S_C \qquad (19)$$

where $S' \subseteq S$ is the set of states that have a dependence on C.

8.4 Higher-order discrete Markov processes

8.4.1 Definition and characteristics

The Markov chains outlined in previous sections depend only on the past states, $X_{t-1}, ..., X_0$, through the current state, X_t, in determining the distribution for future state, X_{t+1}, known as the Markov property defined in (5). If the $t+1$ distribution depends on the past as well, the process is a higher-order Markov process, and the number of states that the $t+1$ distribution depends on decides what order the Markov process is of. For an lth order process, the lth lag is present in the distribution.

$$P(X_{t+1} | X_t, X_{t-1}, ...X_{t-l}, ..., X_0) = P(X_{t+1} | X_t, X_{t-1}, ..., X_{t-l}) \qquad (20)$$

8.4.2 Transforming a higher-order Markov process

Since the higher-order Markov process is not dependent only on present state, the t.p.m. defined in (6), is not valid. There is a need to transform the process in order to ensure that it fulfills the Markov property, (section 5), and employ the methodology outlined in section 8.2. This will be performed through extending the state-space and deriving at a two-dimensional process.

The Markov process, $\{Z_t\}$, is defined as

$$\{Z_t\}_{t \geq 1} = \{X_{t-1}, X_t\}_{t \geq 1} \tag{21}$$

Hence, the state space of $Z_t \quad \forall t \geq 1$ will be pairs of states for $\{X_t\}_{t \geq 0}$, $S_Z = \{(s_1, s_1), (s_1, s_2), ..., (s_n, s_n)\}$. The fact that X_{t-1} is present in both Z_t and Z_{t-1} will produce 0's for most entries in the t.p.m.,

$$P\left(Z_t = (l, m) | Z_{t-1} = (j, k)\right) = 0 \quad \forall j \neq k \tag{22}$$

9 Implementation of candidate expected lifetime models

The purpose of this chapter is to implement the definitions of expected lifetime in models based on Markov chains, utilizing the credit risk model provided (section 7). In section 9.1, it is defined how relevant concepts from the stage model relate to the credit risk model. In section 9.2, the definition for lifetime is included in a Markov chain based on the credit risk model. Section 9.3 covers the candidate models to be validated in the following chapters.

9.1 Risk-states and IFRS 9 Stages

The Standard specifies that lifetime expected credit losses shall be recognized for instruments that satisfy the criteria for significant increase in credit risk since initial recognition, which in section 4 was defined as Stage 2, and for credit-impaired instruments, Stage 3. Thus, there is a need for including in the implementation for expected lifetime estimation, criteria for when an instrument is transitioned to either of the stages and when an *End of lifetime event* occurs.

It is motivated in section 4.4.2, that the only available information *without undue cost or effort* on an individual basis and indicative of a significant increase in credit risk is past-due information. It is clarified in the Standard [10, §B5.5.4] that if only past-due information is available on individual level, this does not suffice for estimating lifetime expected credit losses. This was further discussed in section 4.5 and it is assumed that such a *collective overlay* is already present. Thus, transition to stage 2 is defined as the transition to Del1 (the first observation of an increase in credit risk for individual facilities). Since expected lifetime for stage 2 instruments only relates to the stage model in defining the beginning of lifetime, there is no need to further specify the relationships.

9.2 Expected lifetime model based on Markov chain

There are three areas in which the definition of expected lifetime and the Markov chain need to be tied together:

- beginning of lifetime (for stage 2 instruments) corresponds to initial state in Markov chain

- instruments initially in stage 2 transitioning between risk-states (before an End of lifetime event occurs) corresponds to state dynamics for transient states (i.e. how the process evolves in time, governed by the t.p.m.); and,

- End of lifetime events corresponding to absorbing states in the chain.

The same name for the states in the Markov chain as in the risk-states in the credit risk model are used. There will be a need to extend the state-space for

the Markov chain (beyond the risk-states from the credit risk model), to include all End of lifetime events. The following will signify the Markov chain (along with the areas above):

- beginning of lifetime is defined as Del1 in the Markov chain, $X_0 = \text{Del1}$,

- Performing, Inactive (no drawn amount) and Del1 are transient states; and,

- Del2, Charge-off, Closed and Deactivated are absorbing states.

The Inactive, Deactivated and Del2 states need clarification. The state Inactive corresponds to that there is no drawn amount of the facility at the observation date. Deactivation is defined to occur after six consecutive months of inactivity.

The transition probabilities of the t.p.m., (6), were previously assumed constant. However, including covariates for some or all of the transition probabilities in (6) is motivated for the following reasons:

- segmentation into homogeneous portfolios of instruments (groups of shared credit risk characteristics) may demand that instruments which differ significantly in terms of expected lifetime based on this variable are separated,

- the Markov property is assured for the process conditional on the covariate.

- in order to account for future economic conditions, the covariate may be dependent on some exogenous (macroeconomic) variable (e.g. unemployment)

The present study will consider segmenting into portfolios with shared characteristics in terms of expected lifetime, motivated by the Standard and in establishing the Markov property for the least complex model (in terms of model parameters) sufficient for the modelling purpose. It is recognized that implementing a model conditional on macroeconomic variables, would be conceptually similar to the one selected. The covariate that will be implemented in the model is the maximum historical risk-state, described in section 7.

9.3   Candidate models

Motivated by the previous sections in the present chapter, candidate models are developed with increasing complexity, initially solely based on the credit risk model, eventually implementing the model for the lifetime definition. Previous models are used for validation purposes.

9.3.1   First-order Markov process, Model M_0

M_0 is the first-order Markov chain corresponding to the credit risk model in section 7 with no alterations. Hence, absorbing and transient states differ from

the definitions above. If no a priori information (other than absorbing and transient states) is given, $(n_{trans} + n_{abs} - 1) \, n_{trans}$ parameters need to be estimated. Since the Markov chain is based on the credit risk model provided, some a priori information may be included in the model, to reduce the parameter-space. The allowed transitions are in (9.3.4) as parameters and not allowed transitions as 0. Since the absorbing states in the Markov chain has been defined, only the parameters in Q and R, of the t.p.m. in canonical form (8), need to be estimated.

$$P_{M_0} = \begin{pmatrix} p_{P,P} & p_{P,D_1} & 0 & p_{P,CO} & p_{P,Cl} \\ p_{D_1,P} & p_{D_1,D_1} & p_{D_1,D_2} & p_{D_1,CO} & p_{D_1,Cl} \\ p_{D_2,P} & 0 & p_{D_2,D_2} & p_{D_2,CO} & p_{D,Cl} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

where P is state Performing, D_1 is Del1, D_2 is Del2, CO is Charge-Off and Cl is Closed.

9.3.2 First-order Markov process with dependence on covariate, Model M_1 and $M_{1'}$

M_2 is the first-order Markov process with *Maximum historical risk-state* as a covariate. The number of parameters in this model would no a priori knowledge of the structure is $n_c \, (n_{trans} + n_{abs} - 1) \, n_{trans}$. It is noted that C may be stochastic, which leads to a conditional model. This is described as,

$$p_{i,j} \, (c) = P \, (X_{t+1} = j | X_t = i, C = c) \quad \forall i, j \in S, c \in S_C \tag{24}$$

and the t.p.m. is with the same allowed probabilities as for M_0,

$$P_{M_1}(c) = \begin{pmatrix} p_{P,P}(c) & p_{P,D1}(c) & \cdots & p_{P,Cl}(c) \\ p_{D_1,P}(c) & p_{D_1,D1}(c) & \cdots & p_{D1,Cl}(c) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{25}$$

where $S_C = \{P, D_1, D_2\}$ is the domain of C. Naturally, current risk-state X_t (rows in (25)), cannot be of a higher risk-state (i.e. corresponding to the instrument being more delinquent) than maximum historical risk-state c (since c would then be updated to X_t).

Furthermore, not all transitions are expected to be dependent on C. Since the purpose is to reduce the parameter-space (while maintaining an accurate model), an approach of validating the model based on dependence only for one row at a time will be performed. The first such model, $M_{1'}$, is assumed to depend on C for transitions from the Performing state (26).

$$P_{M_1}(c) = \begin{pmatrix} p_{P,P}(c) & p_{P,D1}(c) & \cdots & p_{P,Cl}(c) \\ p_{D_1,P} & p_{D_1,D1} & \cdots & p_{D1,Cl} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{26}$$

9.3.3 Second-order Markov process, Model M_2

M_2 is the second-order Markov process, with n_2 parameters,

$$n_2 = n_{trans}^2 \left(n_{trans} + n_{abs} - 1 \right) \tag{27}$$

Based on the theory for second-order processes, the model is transformed into a Markov chain (described for $\{Z_t\}_{t \geq 1}$ in section 8). This model is also reduced, restricting the estimation of transition probabilities to include only transition probabilities associated with allowed transitions.

9.3.4 Implementing expected lifetime, Model $M_3, M_{3'}$ and $M_{3''}$

Model M_3 is the implementation of the states in section 9.2 with the addition that the first row depends on the covariate C, as in $M_{1'}$. Hence, the t.p.m. of M_3 is,

$$P_{M_3}(c) = \begin{pmatrix} p_{P,P}(c) & p_{P,D_1}(c) & p_{P,In}(c) & 0 & p_{P,CO}(c) & p_{P,Cl}(c) & 0 \\ p_{D_1,P} & p_{D_1,D_1} & p_{D_1,In} & p_{D_1,D_2} & p_{D_1,CO} & p_{D_1,Cl} & 0 \\ p_{In,P} & 0 & 0 & p_{In,In} & 0 & 0 & p_{In,De} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where In is the state Inactive, De is the state Deactivated. All other notations are unchanged.

When including the state Inactive, it is expected that the transition probability will be dependent on the past for a longer history than for other states, since the deactivation criterion is based on the consecutive number of months the process occupies the Inactive state anterior to present state. Hence, in Model $M_{3'}$ the Inactive state is separated into five Inactive states, depicted in Fig. 9.
Furthermore, absorption time for M_3 must take into account that the *End of lifetime event* for removal of the undrawn commitment (absorption to $Del2$ in the Markov chain) is delayed, i.e. the absorptipon to the $Del2$ state in the Markov chain does not coincide with the end of the lifetime for the instrument. In section 8.2.3, expected absorption time, where a delayed absorption time for one of the absorbing states was introduced. The modified absorption time for the Markov chain, T', accurately incorporates the delayed *End of lifetime event*
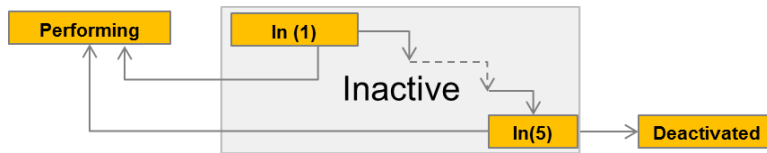
Figure 9: Inactive state is depedent on time in state for $M_{3'}$

for the drawn amount. From section 8.2.3, state s_d is incorporated in Model M_3 as Del2, and t_d is the *residual lifetime* of the drawn amount.

$M_{3''}$ is the second-order process of the extended state-space in M_3, without the dependence on C; hence, M_3 corresponds to $M_{3''}$ in the same way as $M_{1'}$ corresponds to M_2 (and will also be used for validation in the same way).

10 Estimation and validation

The following chapter will provide the statistical methodology used in order to validate and subsequently select the most suitable model for estimating expected lifetime. In order to make inference about the data set and eventually arrive at estimates for expected lifetimes, the maximum likelihood estimators (MLEs) for the models (section 10.1) and the *likelihood function* will be key for the development of statistical tests. First a generalized likelihood ratio test applicable for nested models (i.e. where one model is a restriction of the other, section 10.2) and subsequently, information theory-based tests are introduced (section 10.3). The hierarchy of the models are introduced in Fig. 10 The chapter sets out the methods used to achieve the results presented in the next chapter.
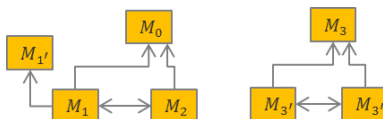

Figure 10: Hierarchy of the models. The two-sided arrows indicate that tests for non-nested models will be required. Arrows point to the nested model

10.1 Maximum likelihood estimators for discrete Markov processes

10.1.1 First-order process

The observations contained in the data set are from N realizations of the same Markov process, since it is assumed that the instruments in each portfolio are homogeneous. In consequence, all observed transitions are used for inference to the transition probabilities of the underlying Markov process. The *Maximum likelihood estimator (MLE)* of the parameters in the first-order process is defined in [17, p. 19] as,

$$L(P; \mathbf{x}) = \prod_{k,t} p_{x_t, x_{t+1}} \tag{28}$$

where the dependence of the estimator on the data sample, $\mathbf{x}$, is explicitly denoted (this has been omitted for the other estimators). The sample, $\mathbf{x}$, is the sequence of observations, $x_1, x_2, ..., x_N$, aggregated in some suitable way, considering that the sample consists of several i.i.d. chains.[3]

For the log-likelihood, a summation of transitions between the different states is performed and aggregated into

---

[3]how the sequence is aggregated in practice is of little relevance, since the data is of a different format, and instead the transitions are observed without first having explicit sequences for each chain

$$\ln L(P; \mathbf{x}) = \sum_i \sum_j N_{i,j} \ln p_{i,j} \tag{29}$$

The derivation is found in [17] to be

$$\hat{p}_{i,j} = \frac{N_{i,j}}{\sum_j N_{i,j}} \tag{30}$$

where $N_{i,j}$ is the number of observed transitions in the portfolio, from state i to state j during the time period.

10.1.2 Second-order process

The MLE for the t.p.m. of M_2, is derived in a similar fashion to the first-order process in section 10.1.1. Following the definition of the second-order process, 8.4.2, it is noted that the states of $\{Z_t\}_{t \geq 1}$ are pairs of states, corresponding to the original process, $\{X_t\}_{t \geq 0}$. The log-likelihood function for the parameters in M_2 is,

$$\ln L(P; \mathbf{x}) = \sum_{ijk} N_{ij,jk} \ln p_{ij,jk} \tag{31}$$

and the maximum likelihood estimators, derived in [17], are,

$$\hat{p}_{ij,jk} = \frac{N_{ij,jk}}{\sum_k N_{ij,jk}} \tag{32}$$

10.2 Generalized likelihood ratio tests for nested models

A Generalized Likelihood Ratio (GLR) test will be employed in order to test for the order of the process. These models are nested in nature, i.e. a lower-order model is a restriction of the higher-order model. The test statistic is,

$$2 \left[\ln L_2(P) - \ln L_0(P)\right] \sim \chi^2(f) \tag{33}$$

where L_0, L_1 represent the likelihood functions of the null model and the alternative model respectively. The null-hypothesis, i.e. that the null model is sufficient (additional parameters in the alternative model is not providing a significantly better fit to the observations), is rejected at some significance level, α, if

$$2 \left[\ln L_2(P) - \ln L_0(P)\right] \geq \chi^2_\alpha(f) \tag{34}$$

where $\chi^2_\alpha(f)$ is the inverse of the chi-squared distribution with f degrees of freedom, evaluated at α.

10.2.1 Validating the Markov property

The Markov property defined in section 8, is tested by

$$2\left[\sum_{ijk} N_{ij,jk} \ln \hat{p}_{ij,jk} - \sum_{ij} N_{i,j} \ln \hat{p}_{i,j}\right] \sim \chi^2(f) \qquad (35)$$

where f is the degrees of freedom, given by the difference in the dimensions of parameters of the models to be tested, or presently, the number of parameters. [17, p. 21]

10.2.2 Finding the true order of a Markov process

Similar to the test in section 10.2.1, the true order of the model is found through testing the l-order model against the $(l+1)$-order model. If the l-order model cannot be rejected, l is regarded the true order of the model.

10.3 Information theory-based tests

The tests derived in previous sections all are for models that are nested, i.e. one of the models is a restriction of the other. When testing M_1 against M_2, this will not be the case, and a different test is needed. This will be done through tests regarding *Information criterion*. The test to be employed will be based on Bayes' Information Criterion (BIC).

Bayes' information criterion is (for P in accordance with (28))

$$BIC(P) = 2 \sup_P \ln L(P) - K \ln(N) \qquad (36)$$

where $\sup_P \ln L(P)$ is the maximized likelihood function, given K observations, is the number of parameters in the model and N is the number of observations. The better model is the one with the higher BIC[4].

Akaike's Information Criterion is defined as,

$$AIC(P) = 2 \sup_P \ln L(P) - K \qquad (37)$$

with the same definitions of variables as in (36).

There are several advantages to using tests based on information theory over GLR; one being that the models tested do not need to be of a nested structure (i.e. where one is a restricted form of the other). It is mentioned in [17, p. 21] that tests utilizing AIC will overestimate the true order, which is why BIC has been used.

---

[4]it should be noted here that for discrete processes, the BIC will be negative and a higher value is a value closer to 0 and not of greater magnitude

11   Results

This section divided into two parts. First, model validation is performed in section 11.1, involving all candidate models. This first part of the chapter is organized from a least complex model and additional complexity is considered when the tests from last chapter deem it necessary. Finally, the most suitable model is selected. It should be noted that the credit risk model was intentionally considered without the extensions for lifetime, since the additional states possess very different statistical properties, and also introduce additional parameters, not motivated in credit risk practice. Instead, in the second part of the chapter, expected lifetime for the selected model is assessed (section 11.2) and sensitivity to the most relevant additional parameter is analyzed (residual lifetime of the drawn amount). This also serves as an empirical validation of the theoretical results derived for the expectation of the modified lifetime, in (18).

11.1   Model validation

11.1.1   Validating M_0 - is a first-order model sufficient?

The Markov property has been assessed in the two ways described in section 10.2.1. Results from the tests are found in Fig. 11 and Fig. 12 respectively. The tests are in agreement and the Markov property for the first-order model, M_0 is rejected.

Measure	Value	Result
Test-statistic	40204.47	
$\chi^2_{0.05}(18)$	28.86	rejected
$\chi^2_{0.01}(18)$	34.81	rejected
$\chi^2_{0.001}(18)$	42.31	rejected

Figure 11: Generalized Likelihood Ratio test with M_0 as null-hypothesis against M_2, at different segnificance levels

Measure	Value
BIC, M_0	-698716.99
BIC, M_2	-658778.50

Figure 12: Bayes' Information Criterion for M_0 and M_2

11.1.2 Is a second-order model necessary? Comparison of M_1 and M_2

The second-order model, M_2, is compared to the first-order model dependent on a covariate, M_1. BIC for the two models are found in Fig. 13 and the M_1 has a higher BIC. This implies that the Markov property can be established for a first-order model if a dependence on the maximum riskstate is included.

Measure	Value
BIC, M_1	-563437.69
BIC, M_2	-658778.51

Figure 13: Bayes' Information Criterion for M_1 and M_2

11.1.3 Reducing the parameter space of M_1

It was hypothesized that only transitions from the first risk-state (Performing) will have a large dependence on the covariate, since for other states, it is likely that the maximum risk-state will coincide with the present state. In Fig. 14, it is shown that a reduction of the parameters dependent on the covariate preserves the Markov property. It is further shown, that the BIC is only slightly lower for the reduced model compared to the unreduced model.

Measure	Value
BIC, M_1	-563437.69
$BIC, M_{1'}$	-565914.92
BIC, M_2	-658778.51

Figure 14: Bayes' Information Criterion for M_1, $M_{1'}$ and M_2

11.1.4 Assessing the extensions for lifetime modelling - validation of M_3, $M_{3'}$ and $M_{3''}$

In order to choose a suitable model for lifetime estimation, the state-space is extended as described in section 9.3.4. In order to assess validity of the Markov property for the extended model, a second-order model with otherwise same setup as M_3 is used. M_3 is the model with only one state representing facilities with no drawn amount. M_3 is compared to the second-order model, M_3'', in order to assess if M_3 has the Markov property. This is rejected, based on the BIC,

as depicted in Fig. 15. Thus, M_3 is extended with states for each observation time that the facility does not have a balance, thus instead of including one state for all instruments of no balance, five states are included. The results are depicted in Fig. 15, in the comparison for M_3' with the other models. It cannot be rejected that M_3' has the Markov property, since the BIC is lower than for the second-order model. Hence, M_3' is selected as the final model for lifetimes.

Model				BIC
BIC, M_3	-293313	19	746390	-586883
$BIC, M_{3'}$	-186321	23	746390	-372953
$BIC, M_{3''}$	-257669,1	53	735633	-516054

Figure 15: Bayes' Information Criterion for M_3, M_3' and M_3''

11.2 Estimating expected lifetime

11.2.1 The lifetime distribution - p.m.f. of Lifetime, $p_{T_i}(t)$

Lifetime is estimated with Model M_3'. From the data set, the parameters in the t.p.m. of M_3' are estimated and lifetime is based on the calculations described in section 8. The estimated t.p.m. is found in Fig. 16.

	Performing	Del1	In(1)	In(2)	In(3)	In(4)	In(5)	Del2	Chargeoff	Closed	Deactivated
Performing	89,59%	7,98%	2,28%	0%	0%	0%	0%	0,04%	0,10%	0%
Del1	49,17%	22,21%	2,60%	0%	0%	0%	0%	19,05%	6,95%	0,03%	0%
In(1)	11,95%	0%	0%	88,05%	0%	0%	0%	0%	0%	0%	0%
In(2)	6,69%	0%	0%	0%	93,31%	0%	0%	0%	0%	0%	0%
In(3)	4,42%	0%	0%	0%	0%	95,58%	0%	0%	0%	0%	0%
In(4)	3,14%	0%	0%	0%	0%	0%	96,86%	0%	0%	0%	0%
In(5)	0,58%	0%	0%	0%	0%	0%	0%	0%	0%	0%	99,42%
Del2	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%
Chargeoff	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%
Closed	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%
Deactivated	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%

Figure 16: Estimated t.p.m. for $M_{3'}$ for $C =$ Del1 and $In(\cdot)$ representing Inactive states

The p.m.f. and cumulative distribution of Lifetime, T_i with initial state i being Del1 (with Performing as a comparison) are shown in Fig. 17 and 18 respectively. The peak in the p.m.f. corresponds to the instruments transitioning from $Del1$ to $Del2$ at the first month, where the *Residual life* of the drawn amount has been accounted for. In this analysis, a deterministic value of 24 months has been used.

The estimates of the expected lifetime for Stage 2 instruments with and without a residual lifetime for the drawn amount, are found in Fig. 19.
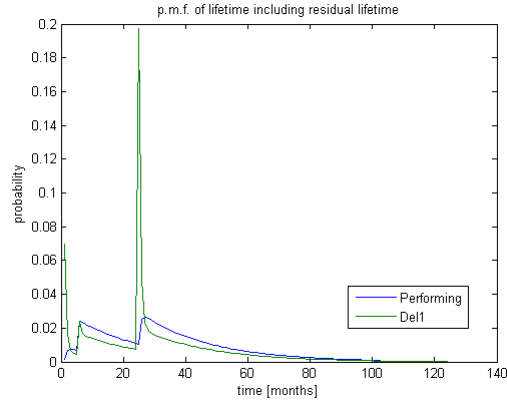
Figure 17: Probability mass function for Lifetime for M_3', where the p.m.f. for *Performing* as initial state is included

11.3   Sensitivity to the residual lifetime parameter

Since the residual lifetime of the drawn amount is not observable in the data set, discussed in section 9.3.4, this has been selected without validation in the data. Sensitivity to the *residual lifetime* for the drawn amount is depicted in Fig. 20. It is shown that the dependency is linear and $\frac{\Delta T}{\Delta t_{res}} = 0.502$. Indeed, this is in line with the expected results, considering the analytical expression of the modified expected lifetime, presented in section 8.2.3.
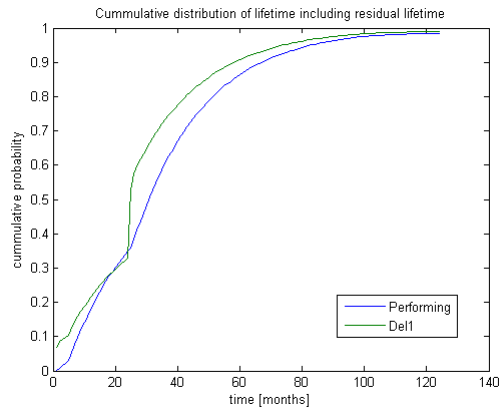
Figure 18: Cumulative distributions for Lifetime for M_3', where Performing as initial state is included

Residual lifetime [months]	Expected lifetime [months]
0	16.7
12	22.7
24	28.8

Figure 19: Expected lifetime for Stage 2 instruments with and without a residual lifetime for the drawn amount
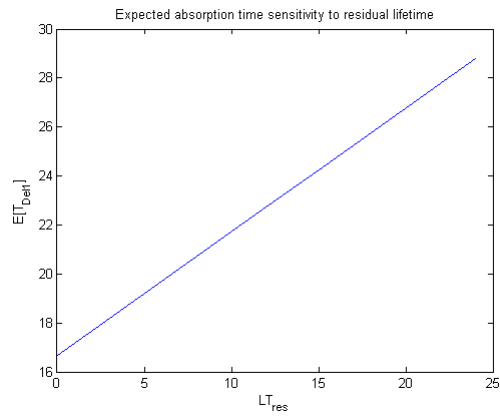

Figure 20: Expected lifetime for initial state *Del1* for different choices of residual lifetime of the drawn amount

12 Discussion

The discussion chapter is organized on the basis of the results presented. An initial discussion of the expected lifetime definition, the credit risk model and the Standard is found in section 12.1. The results conducted from validating the models and inference on the data set follows in section 12.2. The estimated expected lifetime is discussed in section 12.3. Conclusions and suggestions for further research is suggested in sections 12.4 and 12.5 respectively.

12.1 Definition for expected lifetime for revolving credit facilities

A definition has been designed for expected lifetime of revolving credit facilities based on interpretation of the IFRS 9 financial instruments standard and accompanying documents. Since the Standard was published recently, a significant proportion of this paper has been dedicated to presenting the impairment requirements of the Standard. The definition of lifetime is non-trivial for revolving credit facilities and attention to the requirements of the Standard on this topic is motivated since the requirements differ both in relation to other loan instruments and in relation to IAS 39.

Central to the definition is the implications of incorporating expectations on future events in the definition. Both in how expected credit risk management actions will affect the expected end to exposure to credit risk for the facility (thus the end of expected lifetime) and how the undrawn commitment component will affect the definition. The credit risk model provided was extended to account for these aspects. Furthermore, the concept of End of lifetime events was introduced in order to bring together the credit risk model and the events relevant specifically to the definition of expected lifetime of revolving credit facilities.

12.2 Model validation and selection

Implementation of the expected lifetime definition in a Markov chain has been proposed. Through validation based on seven candidate models, a Markov chain of first order dependent on a covariate was selected. Central to the model selection was the credit risk model provided and the observations within this credit risk model (i.e. transitions between risk-states) were utilized in order to extend the information within the model (by introducing maximum historical risk-state and number of months with no drawn amount). These variables aided in the model selection, as it was shown that their inclusion allowed for the use of a first-order Markov chain rather than a higher-order (which may have necessitated the use of an altogether different modelling approach, rather than the less complex Markov chain subsequently selected).

Furthermore, it was shown that including a dependence on the covariate only

for the transitions from the Performing state (i.e. the first row in the t.p.m.) was sufficient, in order to establish the Markov property and in consequence select a first-order Markov chain. In terms of accuracy, it is noted that this is convenient, since instruments in the portfolio assigned to the Performing risk-state (and hence observed transitions from the Performing state) constitutes a large portion of the portfolio.

The model with only one Inactive state was rejected, as it did not possess the Markov property. It was shown, however, that using the introduced variable indicating months of no drawn amount and including the number of months of inactivity in the Markov chain as separate Inactive states $(In(1), ..., In(5))$ allowed for the use of the first-order Markov chain, $M_{3'}$.

12.3 Estimation of expected lifetime

A model and methodology for estimating expected lifetime of revolving credit facilities has been developed. An analytical method (based on the estimated t.p.m. and absorption probability) for a modified absorption time was presented and validated against a method based on the p.m.f. of the absorption time. The two methods were in agreement. Reliance on residual lifetime for the drawn amount, was shown to be linear, with the slope corresponding to the absorption probability.

12.4 Conclusion

The purpose of this study was threefold. First, the impairment requirements in the IFRS 9 standard has been interpreted. Second, a definition for expected lifetime for revolving credit facilities (e.g. credit card products) has been proposed, for the instruments for which lifetime expected credit losses are applicable. For this purpose, the concept End of lifetime event was introduced and possible definitions were considered and a suggested definition was provided. Third, a methodology for estimating expected lifetime has been proposed on the basis of the provided credit risk model, implemented in a Markov chain. For the data set provided, a relatively simple model was found suitable through validation against several higher-order models.

12.5 Further research

It has been concluded in previous sections that incorporating forward-looking information, specifically forecasts of future economic conditions, is required if expected future conditions deviate from historical events that the observations are based on. Due to a short observation period in the data set and an alternative focus more specific to revolving credit facilities, this has not been implemented. Furthermore, the present study considered several definitions of expected lifetime, some found to lead to significantly different values. As the value becomes large (relative to the observation period), it may be relevant to

consider alternative modelling approaches. A suggestion if this is expected to be true, is to consider survival analysis in order to handle censoring and truncation.

References

[1] de Andrade, F. Thomas, L., (2004), *Structural models in consumer credit*, University of Edinburgh, Edinburgh

[2] Basel Committee on Banking Supervision (2000), *Principles for the Management of Credit Risk*, Bank for International Settlement (BIS), Basel

[3] Basel Committee on Banking Supervision (2010), *Basel III: A global regulatory framework for more resilient banks and banking systems*, Bank for International Settlements (BIS), Basel

[4] Credit Suisse First Boston, (1997), *CreditRisk+, A credit risk management framework*, Credit Suisse First Boston International

[5] Duffie, D. Singleton, K., (2003), *Credit Risk: Pricing, Measurement, and Management*, Princeton University Press, Princeton

[6] Fermanian, J., (2011), *A top-down approach for Asset-Backed securities: a consistent way of managing prepayment, default and interest rate risks*, The Journal of Real Estate Finance and Economics, Volume 46, Issue 3

[7] Gaffney, E., Kelly, R. McCann, F., (2014), *A transitions-based framework for estimating expected credit losses*, Central Bank of Ireland (Research technical paper), Dublin

[8] Grinstead, C. Snell, J., (2009), Introduction to Probability, American Mathematical Society, Providence

[9] Hull, J. White, A., (2001), *The General Hull–White Model and Supercalibration*, Financial Analysts Journal, Volume 57, Issue 6

[10] International Financial Reporting Standards 9 (IFRS 9), (2014), *Financial Instruments - The Standard and Implementation Guidelines*, International Accounting Standards Board, London

[11] International Financial Reporting Standards 9 (IFRS 9), (2014), *Financial Instruments - Basis for conclusions*, International Accounting Standards Board (IASB), London

[12] International Financial Reporting Standards 9 (IFRS 9), (2014), *Project Summary*, International Accounting Standards Board (IASB), London

[13] International Accounting Standard 39 (IAS 39) (as of 18 February 2011), (2011), *Financial Instruments: Recognition and Measurement*, International Accounting Standards Board (IASB), London

[14] Koopman, S., Lucas, A. Monteiro, A. (2008), *The multi-state latent factor intensity model for credit rating transitions*, Journal of Econometrics, 142

[15] Perederiy, V. (2015), *Endogenous Derivation and Forecast of Lifetime PDs*, Working paper, Social Sciences Research Network

[16] Perly, R. Nayda, W., (2003), *Economic and Regulatory Capital Allocation for Revolving Retail Exposures*, Finance and Economics Discussion Series, Federal Reserve (U.S.)

[17] Rajarshi, M., *Statistical Inference for Discrete Time Stochastic Processes* (2013), Springer India, New Dehli

13 Appendix

13.1 Derivation of expectation of the modified absorption time, $E\left(T'\right)$

The p.m.f. of the modified absorption time, T', was given in (18) as,

$$p_{T'}(t) = Q^{t-1}Re_{s_d^C} + I\left(t > t_d\right)Q^{t-1-t_d}Re_{s_d} \tag{38}$$

with the same notations as in (18). The expectation operator for a discrete random variable is given as,

$$E\left(X\right) = \sum_{k=-\infty}^{\infty} kP(X = k) \tag{39}$$

which for (38) becomes,

$$E\left(T'\right) = \sum_{t=1}^{\infty} tQ^{t-1}Re_{s_d^C} + \sum_{t=1}^{\infty} tI\left(t > t_d\right)Q^{t-1-t_d}Re_{s_d} \tag{40}$$

After rewriting the second summation and changing its summation variable to $\tau = t - t_d$, the following expression is obtained.

$$E\left(T'\right) = \sum_{t=1}^{\infty} tQ^{t-1}Re_{s_d^C} + \sum_{\tau=1}^{\infty} (\tau + t_d)Q^{\tau-1}Re_{s_d} \tag{41}$$

After rewriting the summations, the following equivalent expression is obtained,

$$E\left(T'\right) = \sum_{t=1}^{\infty} tQ^{t-1}R \cdot (e_{s_d} + e_{s_d^C}) + t_d \sum_{\tau=1}^{\infty} Q^{\tau-1}Re_{s_d} = E\left(T\right) + t_d \cdot B_{*,s_d} \quad q.e.d.$$

where again B_{*,s_d} is the column corresponding to absorbing state s_d of the absorption probability matrix, B, (14). It is noted that the ith row of $E\left(T'\right)$ corresponds to initial state i.