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Added Turbulence and Optimal Power Distribution in Large Off-shore Wind Farms

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Abstract
Wind power is subject to intensive research in the development of using more renewable energy. With a growing number of countries aiming at reducing carbon dioxide emissions, increasing investments are being made in wind turbines and wind farms. An intriguing option is to build large wind farms at sea.
There are many challenges with building large offshore wind farms. This Master's thesis addresses the challenge of the wind turbines interacting with each other within the farm. Observations and measurement data indicate that wind turbines standing in rows behind other turbines experience a lower mean wind speed and a higher level of turbulence. With the wind turbines controlled individually, this means that upwind turbines will produce more power than downwind turbines, and downwind turbines will be subject to a higher degree of fatigue than the upwind turbines, as a result of the higher level of turbulence.
The starting point of the thesis is the proposition that controlling the wind turbines in the farm as a team, taking the interaction with the other turbines into account, forms an optimization problem of finding power references to each turbine, such that total power output of the farm is maximized and the total fatigue is minimized. The ambition is to present a way of formulating the optimization problem and finding the optimal power references, in order to investigate if this proposition is plausible. A model for the added turbulence induced by turbines upwind from other turbines is presented, along with a model for the wind deficit. These models are then used to formulate the optimization problem, which is solved using a gradient descent algorithm.
Our results indicate that there is a potential benefit in controlling wind turbines in a farm with respect to the other turbines. By including the wind deficit and added wake turbulence in determining the power references, we have found that the total power output could be increased while at the same time reducing the total amount of turbulence experienced by the turbines.

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Preface

The growing environmental conscience of our time demands ever greater efforts in developing renewable energy that is reliable, efficient and preferably cheap.

As our final step in the academic journey as students of engineering, the process of writing this Master's thesis has let us work with people in the forefront of the interesting and challenging research of wind energy, and we are grateful for have been given the opportunity.

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Naturally, we send our heartiest gratitude to our families and friends who have put up with our endless elaborations about wind and turbines; an interest they do not always share with us but have done their best to appreciate. Particularly the lunch meetings with Morten gave us considerable support at turbulent times.

Lund, October 2010

Thomas Alexander Clevenhult and Fredrik Himmelman

Abstract

Wind power is subject to intensive research in the development of using more renewable energy. With a growing number of countries aiming at reducing carbon dioxide emissions, increasing investments are being made in wind turbines and wind farms. An intriguing option is to build large wind farms at sea.

There are many challenges with building large offshore wind farms. This Master's thesis addresses the challenge of the wind turbines interacting with each other within the farm. Observations and measurement data indicate that wind turbines standing in rows behind other turbines experience a lower mean wind speed and a higher level of turbulence. With the wind turbines controlled individually, this means that upwind turbines will produce more power than downwind turbines, and downwind turbines will be subject to a higher degree of fatigue than the upwind turbines, as a result of the higher level of turbulence.

The starting point of the thesis is the proposition that controlling the wind turbines in the farm as a team, taking the interaction with the other turbines into account, forms an optimization problem of finding power references to each turbine, such that total power output of the farm is maximized and the total fatigue is minimized. The ambition is to present a way of formulating the optimization problem and finding the optimal power references, in order to investigate if this proposition is plausible. A model for the added turbulence induced by turbines upwind from other turbines is presented, along with a model for the wind deficit. These models are then used to formulate the optimization problem, which is solved using a gradient descent algorithm.

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1. Introduction

To set the scene, we will first introduce wind energy in general terms and figures, and give a motivation to the thesis. We will then move on to defining the purpose and objectives, as well as the approach we have chosen for reaching them. Last in this chapter there is a reading guidance for those who want a short but detailed go through guide to the thesis.

1.1 Background and motivation

The growing environmental awareness of our time has changed the course of investments done by energy stakeholders. With global warming at stake, the demand is ever larger for ways of producing renewable energy with low emissions of carbon dioxide. The European Union has set as target to reach 20% of the total energy consumption in the member countries to come from renewable power by the year 2020 (European Commission, 2010). Wind power is a renewable energy alternative in focus of attention. According to Pullen, Hays and Knolle (2009), it is the most developed, clean and affordable of the renewable energy technologies, which explains why it is the technology primarily chosen by countries to reach the 20% target.

As the number of wind turbines increases, it is becoming common to put them in large groups. So called wind farms, acting like wind power plants, are being built both on land and at sea. Building large offshore wind farms is interesting for many reasons, most of them concerning the limited supply of land and the vast areas of unexploited water offshore with untapped wind energy potential. It is still a young area of research, and issues related to construction, connectivity to the power grid and the lack of experience make it an expensive business. Nevertheless, several countries have proclaimed large offshore wind farms to be the best option. As an example, Germany has an objective of a 25 000 MW offshore wind power capacity by 2030, compared to the country's total wind power capacity of 26 000 MW today, of which 400 MW is offshore (Global Wind Energy Counsil, 2010). In other words, a substantial expansion of the present installed base of wind farms, offshore in particular, is expected. (Pullen, Hays and Knolle, 2009)

Wind turbines extracting energy from the wind influence the wind flow behind them. The wake phenomena can be said to be characterized by two things; a decrease in mean wind speed (*wind deficit*), and an increase in turbulence (*added wake turbulence*). In wind farms, the turbines are placed in the vicinity of each other and thus, an upwind turbine will influence the wind coming into downwind turbines. In other words, a wind turbine in the wake of another turbine will experience a lower mean wind speed and a higher level of turbulence. As a result of the wind deficit, the power loss in a large offshore wind farm is calculated to be of the order 10–20%, seen as an average over a year and all wind directions (Barthelmie, Frandsen *et al.*, 2007). Fatigue loads are strongly connected to the turbulence as fluctuations in wind speed, since they affect the turbine with fluctuating forces (Danish Wind Industry Association, 2003).

Today, turbines in a wind farm are controlled similar to individual turbines, i.e. they do not account for the interaction with other turbines in the farm, and upwind turbines extract power without regards to the impact they have on downwind turbines (Svensson, 2010). Together with this egoistic feature, the two parts of the wake phenomena pose an interesting question: By letting the wind turbines in a large offshore wind farm act as a team and give them power references that take the wind conditions of the other turbines into account; can the total power output of the farm be increased? And can the total level of turbulence and thus structural loads on the turbines be decreased, without lowering the total power output of the wind farm?

In the current European research project Aeolus is worked on ways of modeling wind flow within large scale offshore farms, in order to incorporate them in real time predictions and distributed control methods that account for the interaction between turbines. Participating in the project are both the Department of Automatic Control at Lund University, Faculty of Engineering, and Vestas Wind Systems A/S. As part of the development of the new control paradigms, there is a need of good operating points for turbines in a farm, i.e. an optimal power distribution. They should be based on a flow model that captures the wake phenomena, and an adequate objective function relating to the cost of fatigue loads. (Aeolus, 2010)

1.2 Purpose and objectives

The overall purpose of the thesis is to investigate the potential increase in power output and decrease in turbulence level by distributing the stationary power references of turbines in a large offshore wind farm with respect to *all* turbines in the farm, instead of letting each turbine act as an egoistic individual aiming to extract as much power as possible from the wind.

The thesis was divided into two parts, each with one main objective[1]:

1. “Find an intuitively quantitative model of the added wake turbulence in large offshore wind farms. Since the model will be used for distributed control, the model should strive to have a distributed information propagation structure."

2. “Find the stationary power distribution between the turbines that optimizes power and fatigue in the farm according to an objective function. In this part, the wake turbulence model from part 1 should be used together with a wake deficit model, and applied on a large farm."

[1]Taken from the thesis project plan.

The model of part 1 is based on criteria formulated from research of literature and studies on observed behavior of the wake turbulence and the structure of turbulence within large offshore wind farms. The purpose of the model is to catch the wake phenomena in an intuitive manner, disregarding largely the complexity of the physical relationships and dynamics of the wind flow behind a turbine, in order to use it in the optimization process of part 2.

Since the overall aim is to investigate the potential winnings of distributing the power references of the turbines with respect to the turbulence model, the results of part 2 is strongly connected to the accuracy of the model. This is brought up more in detail in the final discussion in Section 5.2.

Approach

To clarify the starting point and frame of the thesis, some aspects are important to emphasize:

As mentioned in the previous section, the process of modeling the added wake turbulence is based on *intuitive criteria*, and not physical relationships and equations. The model is set out to capture the observed behavior in a way that suits the main purpose of finding the optimal power distribution that maximizes power output and minimizes fatigue. Thus, the model gives only a very simplified picture of the real wake phenomena and should not be compared to advanced CFD models[2] or such, models too complex to use in the optimization in Chapter 4 or the distributed control algorithms being developed in Aeolus, because of the large amount of computing power they demand. It is also a *horizontal* model, in the sense that the turbines are represented as points with no spatial extension vertically.

As will be further described in Chapter 2, fatigue is connected to turbulence in an almost linear manner. The standing point taken for the thesis is that a higher level of turbulence implies a higher degree of fatigue, and no further decomposition of the different kinds of fatigue (blades, tower, generator, etc.) is made. This also means that the dynamics of the turbine is disregarded in the optimization.

Lastly, wind speed, turbulence and also turbine states are treated as *stationary*, as described further in Section 2.2.

[2]Computational fluid dynamics; highly complex computer simulations for fluid dynamics, using numerical methods and computer power demanding algorithms.

1.3 Reading guide

Chapter 2 summarizes the literature search on which the models, presented in Chapter 3, as well as the optimization, found in Chapter 4, are based. The theory describes various components of wind technology in both a general and a detailed manner, depending on relevance for the progress of the thesis, i.e. finding the optimal power references. The conclusions drawn from the selected theory are summarized in the end of Chapter 2. Two models regarding the wake phenomenon will be elaborated on, before they are used in the optimization algorithm. In Chapter 5, the results of the optimization are analyzed, and the final conslusions are presented. Lastly, the educational requirements and the learning outcomes are reflected on in the appendices.

2. Theoretic frame of reference

Our intention with this chapter is to make a summary of the learning outcomes relevant to the process of modeling added wake turbulence and finding the optimal stationary power distribution, which we will present in the coming chapters 3 and 4. Keeping it as light as possible, the theory includes turbine technology, wind and turbulence, turbine wakes and structural loading.

2.1 Wind turbines

A modern wind turbine is assigned one main task, to extract energy from wind as cost efficient as possible. For this purpose, the wind energy industry continuously strives for more sufficient technologies and better turbine solutions in order to achieve more energy at a lower cost. The evolution of wind turbines has the last twenty years increased the power capacity by a factor larger than 100, and is in the moment of writing more than 5 MW. Moreover, the cost efficiency has been increased by a factor larger than 5 during the same period. (Morthurst *et al.*, 2009)

Figure 2.1 The recently released Vestas 3 MW turbine V112. (Picture taken by Clevenhult, 2010)

Modern wind turbines in general

Figure 2.1 shows the most common turbine design called HAWT (Horizontal Axis Wind Turbine), which consists of a tower with a house mounted on its top (*nacelle*), to which three blades are symmetrically attached to a nose cone (*hub*). Furthermore, the hub is connected to a generator through a drive train located inside the hub. In order to face the incoming wind perpendicular to the rotor area, the nacelle is equipped with a slow speed motor that together with a yaw controller ensures an upwind position for the rotor.

Power control

All turbines have a wind speed interval in which they can operate, limited by the maximum wind for which they can produce power without risking damages (*cut off speed*), and the minimum wind for which they can produce power at all (*cut in speed*). Controlling the power output is necessary when it is possible for the turbine to extract more energy from the wind than is demanded by the operator, or when the wind is stronger than what the turbine needs to reach its maximum capacity.

For modern wind turbines, there are three common types of power control systems; pitch control, stall control or a mix of both.

On a pitched controlled turbine, a controller obtains the current power output several times per second and in order to achieve demanded power, it uses electric or hydraulic motors, located in the hub, to change the attack angle of the blades (*pitch*).

Stall control can be divided into two methods; passive and active stall control. A passive stall controlled turbine has its blades fixed to the hub, and uses aerodynamically designed blade features to reduce the lifting force on the blades. The certain blade features prevents the moment of the blades from becoming too high by creating turbulence on the backside of the blades, which counteracts the lifting effect (*stall*) when the wind increases.

Active stall control is a combination of pitch control and passive stall control. In order to achieve sufficient torque at low wind speeds, the turbine is pitching to increase the angle of attack of the blades. The activity is the opposite for higher wind speeds than rated. This method gives more accurate control than pure passive stall control. (Danish Wind Industry Association, 2003)

Energy extraction

The concept of extracting energy from the wind with rotating blades is basic and has been used since medieval times, but a decomposition of a modern wind turbine reveals a complexity that requires many high technological parts. The amount of kinetic energy that is extracted by the rotor is determined by the aerodynamic coupling between the blades and the incoming wind flow, and the design of the blades plays an important part.

Two turbine specific coefficients are used for mapping the aerodynamic coupling; the power coefficient C_P and the thrust coefficient C_T. The power coefficient C_P relates to the power extracted from the wind and is defined as:

$$C_P = \frac{P}{\frac{1}{2}\rho v^3 A} \tag{2.1}$$

where ρ is the air density, v the speed of incoming wind, P the extracted power and A the rotor disc area. In the same way, the extracted momentum is described by the thrust coefficient C_T as:

$$C_T = \frac{F_T}{\frac{1}{2}\rho v^2 A} \tag{2.2}$$

where F_T is the thrust force on the rotor. Both C_P and C_T are wind turbine specific, and properties such as blade geometry, rotational speed of the rotor and the applied control strategy for the wind turbine have influence on the coefficients. (Burton *et al.*, 2001)

NREL 5 MW

The NREL 5 MW is a research HAWT model developed by the American National Renewable Energy Laboratory (NREL). The model is fully open and was provided for use in the thesis project by Aeolus[1], and has been an important means for the authors for understanding the different parts considered when designing control systems, how they interact, and how they are connected to wind speed and power output. The model has also been used for including turbine properties when validating the proposed wake models of Chapter 3. A selection of properties for the NREL 5 MW is given in Table 2.1.

Capacity	5 MW
Rotor	3 blades, upwind
Controller	Variable speed and pitch
Drive train	High speed, multiple stage gearbox
Rotor radius	63 m
Hub height	90 m
Cut in, rated, cut out wind speed	3 m/s, 11.4 m/s, 25 m/s
Rated tip speed	80 m/s

Table 2.1 A selection of properties for the research turbine NREL 5 MW. (Jonkman, 2009)

The model

The model is implemented in Simulink and provides the turbine's full dynamics; from power reference and wind speed, to various parameters including power output P_{out}, thrust coefficient C_T, power coefficient C_P, tip speed ratio λ and pitch angle β. High level schematics for the model can be seen in Figure 2.2. The aerodynamics block includes a model of the aerodynamic coupling for C_P and C_T as functions of λ and β. Tip speed ratio is given from the relation:

$$\lambda = \frac{\omega_r v}{R} \tag{2.3}$$

[1]The model is available on: http://www.ict-aeolus.eu/SimWindFarm/index.html (2010-10-23)

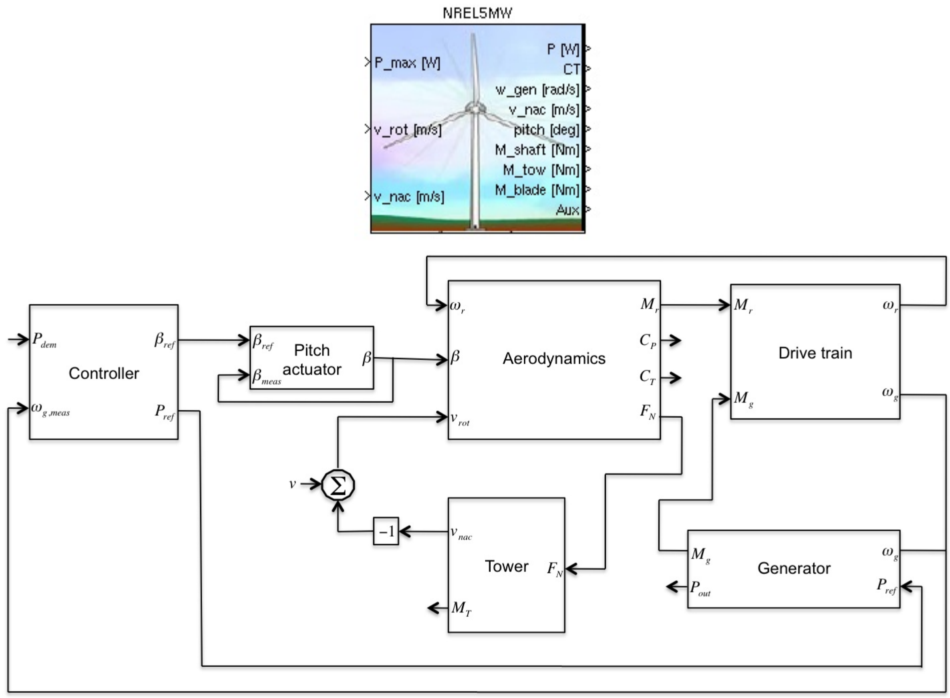


Figure 2.2 Top: The Simulink block for using the NREL 5 MW model for a power reference (P_{ref}) at a wind speed (v_{rot}) (Jonkman, 2009). (NREL, 2010; provided by Aeolus)
Bottom: High level schematics of the NREL 5 MW model.

where w_r is the rotational speed of generator axis. The coefficients are obtained in discrete look up tables. An interpolation of the tables can be seen in Figure 2.3.

The assigned controller is of hybrid model, switching between two main modes depending on which region the turbine is operating in. When the power reference is set to be less than the available power, a gain scheduled PI controller tracks the signal by pitching the blades. In the opposite case, when the power reference exceeds the available power, the signal is set according to a nonlinear function for determining the optimal torque. As the NREL 5 MW does not measure the wind speed, the controller gets feedback from the speed of the generator axis. (Spudić *et al.*, 2010)

2.2 Wind and turbulence in general

Wind and turbulence are complex phenomena in nature, and understanding both of them is important for wind energy development because of their continuous interaction with turbines.

Wind is related to movement of particles (gases) in the air on a large scale. It occurs as a result of varying pressure in the atmosphere and the sun plays the largest role in wind creation.

Turbulence goes under the field of fluid dynamics and is often mentioned when dealing with different kind of flows of gases and liquids. Characteristic for a turbu-

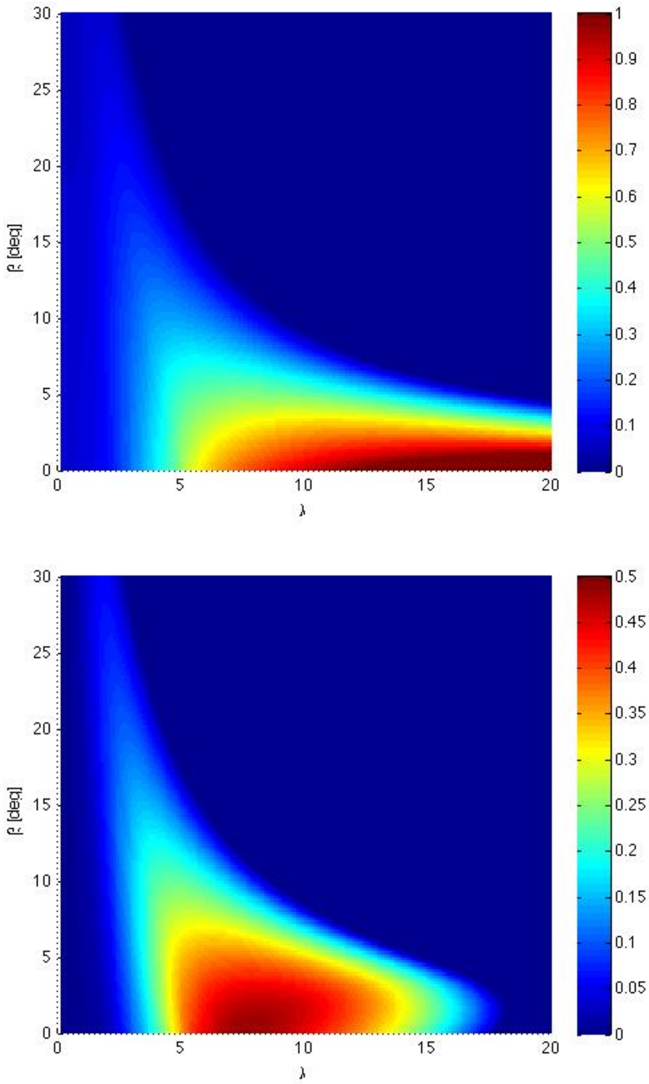


Figure 2.3 Interpolated curves of the lookup tables for thrust coefficient C_T and power coefficient C_P, as functions of tip speed ratio λ and pitch angle β, for the NREL 5 MW. Top: $C_T(\lambda, \beta)$. Bottom: $C_P(\lambda, \beta)$.

lent flow is that it has a rapid variation of pressure and velocity in space and time. If a point view is adopted when studying a turbulent flow, then velocity, direction and magnitude at a specific point in the fluid will undergo continuous changes with time. The complex process is difficult to represent simply in terms of deterministic equations, which is why turbulence generally is described from its stochastic properties instead. (Burton *et al.*, 2001)

Turbulence in ambient wind

Wind related turbulence will for the purpose of the thesis be regarded as fluctuations in wind speed, which is a typical way of describing it. The fluctuations are caused by friction between the air and the topography, disturbing the air flow and changing its movement. Also, thermal differences in the air, related to vertical movement as a result of variations in temperature and air density, is a source of the wind fluctuations. *Ambient* turbulence refers to the free flow wind turbulence, i.e. what a turbine without upwind neighboring turbines experiences. (Burton *et al.*, 2001)

An established expression for wind fluctuations is *turbulence intensity*, which also commonly is referred to as turbulence level. Turbulence intensity is defined as:

$$I = \frac{\sigma_0}{\bar{v}_0} \tag{2.4}$$

where σ_0 is the standard deviation of wind speed variations on a 10 minute basis and $\bar{v}_0$ the mean wind speed based on data for the same time period. The variations can be considered to be normally distributed around the mean wind speed $\bar{v}_0$, with the standard deviation σ_0. Measurements have shown σ_0 to be approximately constant with height, which implies that turbulence intensity decreases with height as a result of an increasing wind speed according to:

$$\bar{v}_0(z) \propto \ln(z/z_0) \tag{2.5}$$

where z is height from the ground and z_0 is the surface roughness length. (ibid)

The considerably smaller roughness length for water[2] makes turbulence intensity levels generally small offshore. Turbulence conditions are also more stable offshore, resulting in the turbulence being more preserved over larger distances than on land. (Réthoré, 2009)

2.3 Turbine wakes

The presence of a wind turbine in a wind flow changes the properties of the flow in different manners and leaves a wake behind it downwind; characterized by a reduced mean wind speed and an increased level of turbulence. The nature of the wake is in its turn dependent on properties of both the turbine and the wind flow itself.

Turbine induced turbulence and wind speed deficit

The reduction of wind speed, wind deficit, is related to the wind energy converted by the turbine into mechanical work. As given by (2.2), the thrust coefficient C_T is directly related to the extracted momentum from the wind and thus, the thrust coefficient is directly related to the wind deficit.

Wake properties

When a turbine extracts energy from the wind, the result is a reduced wind speed immediately after the turbine. More specifically, the approaching air already slows down as a result of the mere presence of the rotor, before it reaches or performs any

[2]Typical values for the surface roughness are 0.7 for cities and forests, 0.1 for landscapes, and 0.001 for water. (Burton *et al.*, 2001)

work on the blades. Slowing down without losing any of its kinetic energy, the flow makes the static pressure of the air increase and the air flow starts to expand.

When the air passes the rotor, the static pressure instantly drops below the atmospheric level, and the wake is created. Downstream, the pressure recovers gradually until it reaches the pressure level surrounding the wake. The wind speed will continue to decrease downstream, until the pressure has fully recovered. An illustration of the wake is presented in Figure 2.4. (Burton *et al.*, 2001)

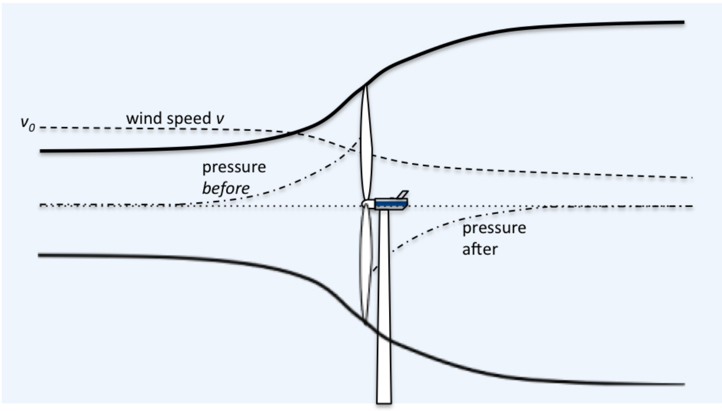


Figure 2.4 Illustration of a wake (seen from above). The pressure drops immediately behind the turbine, causing the air flow to expand. The result is a lower wind speed, decreasing downwind until the pressure is restored to surrounding level. Further downwind the wind speed recovers.

As the region of reduced wind speed spreads out downstream, *shear generated* turbulence will emerge in the intersection between the free flow and the wake because of the difference in wind velocity. A transfer of energy takes place from the surrounding flow into the wake, and the wake mixes with the ambient wind. The mixed flow moves from the edge of the wake inwards the centre, as well as outwards the ambient flow. As a result, wind deficit is reduced and the wake becomes wider and shallower until the wind speed has recovered, far downstream. (ibid)

According to Burton *et al.* (2001), the shear generated turbulence, created at the edges of the two flows where the mixing is large, will dominate the turbulence level surplus far downstream.

Also Crespo, Hernandez and Frandsen (1999) referred to the downstream mixing of the wake and the ambient flow as the dominating part of the turbulence creation. They concluded that the most important turbulence production takes place in the shear layer where the difference in wind speed between the two air flows is large.

There is also noteworthy turbulence inside the wake, as the wind speed deficit caused by the rotor is not uniform; in particular in the area of the wake closest to the turbine. Furthermore, close to the turbine, in addition to the shear generated turbulence, there is turbulence induced by the activity of the rotor, the tower, the nacelle and the blades. This mechanically induced turbulence is of high frequency character and decays relatively quickly. (Gómez-Elvira *et al.*, 2005)

Wake regions

The wake is divided into two regions; *near* and *far*. The near wake is typically defined as 2–5 rotor diameters. More generally, where the pressure has recovered and the wind speed no longer decreases, marks the end of the near wake region. Further downstream is the far wake region, where the wake is completely developed. Here, the pressure is homogenous and the wind speed is recovering. (Crespo, Hernandez and Frandsen, 1999)

In terms of turbulence, the near wake can be described as the region behind the turbine dominated by the turbulence induced by the blades. The far wake is then the region where the shear turbulence dominates, and is gradually absorbed by the ambient turbulence. (Réthoré, 2009)

The spectral properties of wind fluctuations in wakes are important for mapping the loading effects on turbines exposed to them, as will be described in Section2.5. Studies have shown that the spectral properties of the far wake region turbulence are similar to those of ambient wind. This means that the shear induced turbulence can be said to have approximately the same frequency content as the ambient turbulence. In the near wake, the mechanically induced turbulence, as described previously in this section, will still be apparent and it will affect the spectral similarity negatively because of its higher frequencies. (Höjstrup, 2009)

Wind farm conditions

In large wind farms, the wind deficit and added turbulence that a turbine inside the farm experiences, will be a result of the accumulation of the wake effects from upwind turbines. The actual wind speed and level of turbulence at a turbine both depend on the structure of the farm and the properties of its surroundings, e.g. spatial positioning of the turbines, wind direction and surface roughness. Generally, however, a turbine within the farm will be subject to a lower wind speed and higher turbulence intensity. (Burton *et al.*, 2001)

The farm itself acts as a surface roughness and therefore affects the ambient turbulence, even if all turbines are shut off. With the added wake turbulence from active turbines the level of turbulence within the farm is higher than the ambient, regardless the structure of the farm or the states of the turbines. Furthermore, the turbulence intensity is observed to reach a maximum value after the first couple of rows into the farm, independent of how many upwind rows there are. (Frandsen, 2007)

The properties of a farm will naturally influence the magnitude of the wake effects. Because of the wind deficit, the farm will be subject to a power deficit and a lowered efficiency, referred to as *wake loss* or power loss. Setting up a farm normally includes analysis of dominating wind directions and such, in order to design the farm to extract as much power as possible. Commonly, a rule of thumb is used for turbine spacing (roughly a distance of five rotor diameters in the dominant wind direction), but little emphasis is presently given to the increased turbulence levels when projecting wind farms. Focus is rather on power output because of the direct relation it has to economical estimates. (Svensson, 2010)

Figure 2.5 Wakes at Horns Rev wind farm. (Aeolus, 2010)

However, when the wind has a direction that negatively affects the wind conditions within the farm, the wake effects can be surprisingly large. Measurements at the large offshore farm Horns Rev in Denmark showed that under certain “worst case" wind conditions (and directions), the power loss was as large as 55–60%, with an additional large increase in turbulence level (Berthelmie, Rathmann *et al.*, 2007). Other investigations have revealed an increased turbine loading in farms of 15% on average (Thomsen and Sørensen, 1999).

For offshore farms, it is likely that the wake effects are larger than for onshore farms because of the longer time it takes for the wake to mix with the surrounding air, and also as a result of the larger farm size. Shortcomings in classic wind farm wake models have been observed, and they are said to be connected to the interaction within these large clusters of turbines in offshore conditions. (Gardner *et al.*, 2009)

2.4 Existing wake models

Modeling wind and turbulence can be done in many ways, ranging from advanced CFD models to linear approximations describing the overall phenomena. Below is presented a selected number of existing models that are relevant for the modeling in Chapter 3.

Wind deficit models

As will be further described in Chapter 3, the wind speed deficit caused by the turbine wakes is the starting point for modeling the added turbulence, and the choice of wind deficit model therefore plays a critical role in capturing the total wake phenomenon.

I

An analytical wind speed deficit model, intended for a single turbine, was presented by Frandsen *et al.* (2006), and is based on an approximation of the momentum equation derived by Betz and Lanchester in the early nineteen hundreds. Normalized with ambient wind speed, the model is:

$$\frac{v(x)}{v_0} = 1 - \frac{1}{2}\frac{C_T}{((\phi^{k_s/2} + \alpha x/R)^{1/k_s})^2} \tag{2.6}$$

where $v(x)$ is the wind speed at a distance x downwind from the turbine, ϕ is a parameter that is dependent on C_T, R is the rotor radius, and α and k_s are constants related to the shape of the expanding wake. (Frandsen *et al.*, 2006)

II

As a part of Aeolus, a wake deficit model better suited for distributed control was proposed by Madjidian and Rantzer (2010). The distributed model is based on intuitive assumptions related to the properties of wakes:

- The mean wind speed at a turbine should not be able to exceed ambient wind speed, or be negative.
- The wind speed in a single wake should go towards ambient wind speed downwind from the turbine.
- The wind speed should reach an equilibrium after a few turbines, assuming all turbines demand the same amount of power.
- A turbine deep inside the farm should be able to experience a higher wind speed than its upwind neighbor.

For the purpose of satisfying these requirements, the following wind deficit model for a row of turbines, facing the wind perpendicularly, was proposed:

$$v_{n+1} = (1 - k_n C_{T1} - k_{n-1} C_{T2} - \ldots - k_1 C_{Tn}) v_0 \tag{2.7}$$

where v_i is the mean wind speed at turbine[3] i and k_i are positive constants. On the current form, k_i is depending on the turbine spacing, and to which extent the activity of a turbine is coupled to downwind turbines through C_T. By introducing $k_i = k^i$ the model can be rewritten on an iterative form in line with the desired distributed property:

$$v_{n+1} = (1 - kC_{Tn})v_0 - k(v_0 - v_n) \tag{2.8}$$

In order to fulfill the requirements, it is given that $0 < k < 1$. With this model structure, the wind speed at a turbine is expressed only by the ambient wind speed and the activity of its nearest upwind neighbor. (Madjidian and Rantzer, 2010)

[3] The index represents the i:th position downwind in the row, i.e. turbine i has $i-1$ turbines standing in front of it in upwind direction.

Added wake turbulence models

There are many models that aim to describe the turbulence behind a wind turbine. Some are empirical, based on advanced wind tunnel studies or full scale simulations, others are theoretically developed from kinematic calculations. A common way to describe turbulence behind a turbine is as the sum of ambient turbulence intensity and turbine induced turbulence intensity:

$$I_{\text{wake}}^2 = I_0^2 + I_{\text{add}}^2 \tag{2.9}$$

where I_{wake} is the total turbulence intensity behind the turbine, I_0 is the ambient turbulence intensity, and I_{add} is the turbulence intensity added by the turbine. (Burton *et al.*, 2001)

I

Quarton and Ainslie (1989) presented an empirically developed expression for the turbulence intensity on a distance x downstream from the turbine, based on measurements from wind tunnel studies and full scale observations of turbines in free flow:

$$I_{\text{add}} = 4.8 C_T^{0.7} I_0^{0.68} (x/x_n)^{-0.57} \tag{2.10}$$

where x_n is the length of the near wake region, which depends on the rotor radius and the thrust coefficient. The model is meant to describe turbulence in the near wake region, as well as the far wake region.

A modification of (2.10), intended only for the far wake region, was presented by Crespo and Hernandez (1996), and was based on large scale computer simulations:

$$I_{\text{add}} = 0.73 a_t^{0.8325} I_0^{0.0325} (x/D_{\text{rotor}})^{-0.32} \tag{2.11}$$

where a_t is related to the thrust coefficient according to $C_T = 4a_t(1-a_t)$, and D_{rotor} is the diameter of the rotor. (Crespo, Hernandez, and Frandsen, 1999)

II

Frandsen (2007) concluded that the models of (2.10) and (2.11) differ foremost because they are intended for different regions of the wake, and that it is noteworthy that more or less any model including thrust coefficient and downwind distance as parameters, can be fitted to data by choosing suitable model constants. Furthermore, a weakness with the models above is that the turbulence intensity goes towards unlimited values when applied for multiple wakes, i.e. for a long row of turbines.

With the approach that a simple and more general model captures the added wake turbulence with approximately the same accuracy as the previous empirical models, Frandsen (2007) presented a conceptual model compiled to cover both the near wake region and the far wake region:

$$I_{\text{add}} = \frac{1}{C_1 + C_2 \frac{x/D_{\text{rotor}}}{\sqrt{C_T}}} \tag{2.12}$$

where C_1 and C_2 are constants for adjustment of the model to fit some desired conditions.

A concluding remark
Although numerous studies have been done regarding turbulence in the fare wake, the knowledge is limited both numerically and experimentally. Many of the models show on satisfying agreement with experimental data, which the assumptions and choice of parameters are based on. Nevertheless, the general validity for many models is still far from satisfying and often not conducted with independent data. (Vermeer, Sørensen and Crespo, 2003)

Burton *et al.* (2001) expressed the absence of satisfactory, validated models for wind farm turbulence:

> "*However, no consensus has yet emerged on a sufficiently well validated formula for turbulence intensity within a wind farm for use in wind turbine design calculations."* (Burton *et al.*, 2001)

2.5 Wind turbine loading

A wind turbine is subject to continuous loading. Gravity, wind fluctuations and the rotor movement expose the turbine components to stress in various ways. The connection between turbulence and fatigue is of special interest for the purpose of the thesis and needs to be clarified.

Loads in general

There are generally two ways of describing material loading: *extreme loads*, referring to extreme one time stress that will induce failure to the material, and *fatigue loads*, referring to the cyclic stress the material experiences throughout its lifetime. The focus here will be on fatigue, because of its relation to continuous loading, repeated over a long period of time.

Deterministic approach
A common way of calculating fatigue loads is with the *SN curve*. Conceptually, the SN curve can be described as the relationship between the maximum amount of cycles N a material can be exposed to a certain amount of stress s, and the stress resistant properties of the material. These properties are denoted k_{W} (the Wöhler coefficient) and K, and the relationship is then given by:

$$s^{k_{\mathrm{W}}}N = K \tag{2.13}$$

This applies for harmonic cycles with constant amplitude, s being the amplitude times two, or *stress range*. Thus, the SN curve describes how many times a material can be exposed to a certain stress range with homogenous cycles before it fails, depending on the material properties. It is clear that a larger s will make the material fail after a smaller amount of cycles.

When dealing with more than one stress range, it is more convenient to look at *damage*. By denoting D as the damage inflicted by one cycle, (2.13) can be rewritten as:

$$D_i = \frac{1}{N_i} = \frac{1}{K} s_i^{k_W} \tag{2.14}$$

Summarizing D over N cycles yields 1, i.e. the material fails when the sum of D reaches[4] 1. If M cycles, each with a stress range of s_i, are exposed to the material, the total damage inflicted can be expressed by:

$$D = \sum_{i=1}^{M} \frac{1}{K} s_i^{k_W} \tag{2.15}$$

Since the stress cycles most often are irregular and not harmonic with a single, fix amplitude, the SN curve must be expanded with algorithms for decomposing the true stress fluctuations into a set of cycles with stress range s_i, resulting in the same amount of total stress on the material; the *equivalent load*. Such an algorithm is Rainflow counting, which is done by iterating over every local maximum in the stress history, and assigning each maximum a local minimum. Each local extremes pair defines a specific stress range, and the total damage can then be determined with (2.15).

Stochastic approach

For stochastically distributed stress amplitudes, bundling the stress ranges into subsets and assigning a probability p_s to each subset, the expected damage per time unit can expressed as (making the number of subsets go to infinity):

$$E(d_t) = \frac{v_c}{K} \int_0^{\infty} s^{k_W} p_s(s) ds \tag{2.16}$$

where v_c is the number of cycles per time unit and $p_s(s)$ is the probability density function. The expression directly gives that the expected damage is larger if the stress ranges are larger.

If the stress cycles are seen as a zero mean stochastic process $x(t)$ with variance σ_X^2, the spectral moments[5] λ_m is defined as:

$$\lambda_m \equiv \frac{1}{\pi} \int_0^{\infty} \omega^m S_X(\omega) d\omega \tag{2.17}$$

where $S_X(\omega)$ is spectral density defined as the Fourier transform of the autocovariance function R_X of the stress cycles according to:

$$\sigma_X^2 = R_X = \frac{1}{2\pi} \int_{\infty}^{\infty} S_X(\omega) d\omega \tag{2.18}$$

[4]This is not the entire truth, since the lifetime of the material also is influenced by the mean stress of the cycles. However, it is approximately true and under specific conditions, the mean stress factor can be accounted for by calibrating the material property constants k_W and K. (Frandsen, 2007)

[5]This holds for narrow band processes. Ambient turbulence is not a narrow banded process but the responses of the turbine components can be viewed as such (Frandsen, 2007).

If the frequency content is held constant, a higher variance will give an increased energy at for every frequency, resulting in a higher expected damage rate. Furthermore, the first and second derivative of $x(t)$ corresponds to the second and fourth spectral moment respectively. Choosing the first four spectral moments for describing the stress history, and combining it with an approximation of the expected rate of damage in (2.16), including the material properties k_W and K; the expected fatigue can be estimated. (Hammerum, 2006)

Turbine loads

A wind turbine is exposed to continuous loading. Sources of the loading include gravity, aerodynamics and activity of the turbine components generated by the control system. Fatigue is both a result of deterministic loading, such as rotor rotation, as well as the probabilistic feature of fluctuations in wind speed, i.e. wind turbulence is transferred to stochastic loading in the material of the turbine components. (Burton *et al.*, 2001)

The components of the turbine (blades, tower and nacelle, etc.) respond differently to the turbine activity and the wind conditions. The tower oscillates with a period of order of seconds, depending on its structure, height and the weight of the nacelle. The flapwise bending of the blades through the rotation also inflict loads on the tower every time they pass by (so called tower shadowing), as well as on the nacelle because of the uneven force of the rotor. The torque of the generator and gear box is another source of loading. (Danish Wind Industry Association, 2003)

The variance in the components will be connected to the fatigue of the turbine, according to (2.16) and (2.18). One of the sources for the variances in the turbine is turbulence in the wind. Building on the supposition that the frequency content in the far wake region is approximately the same as in the ambient turbulence, as stated in Section 2.3, the total variance in wind speed will be the critical component of the turbulence affecting a turbine in wake conditions. Figure 2.6 illustrates the conceptual view of turbulence influenced fatigue.

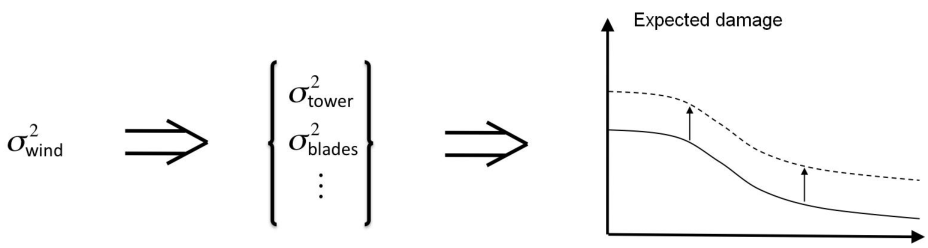


Figure 2.6 An increase in variance in wind speed (turbulence) is transferred to more variances in component materials in the turbine, resulting in an increase in expected damage — the fatigue rate becomes higher.

Fatigue and turbulence

Although it is intuitively straight forward to believe that turbulence will affect the fatigue of a wind turbine, it is less straight forward to say how it will affect the turbine and its different parts, and how great the impact will be. As mentioned previously, the dynamical responses of the turbine components are dependent on many parameters, turbulence as a variance in wind speed being one of them, and in addition, the responses are integrated with each other in a complex way.

In (2.17), the variance of the respective part of a turbine is included, which affects the expected damage derived from the use of the spectral moment and a linear model of the turbine. The results derived by Frandsen (2007) points out the variance in horizontal wind speed as the driving factor for the dynamic response of the components, or in other words; turbulence[6], under both ambient and wake conditions, is the major contributor to fatigue loading, i.e. the estimated equivalent loads in the sense of fatigue for the component, through its impact on the stress variance. (Frandsen, 2007)

From the point of view that turbulence is the most important part for the fatigue loads on a turbine, mapping the effects of wakes on added turbulence becomes vital for the control of large wind farms.

2.6 Summary

Summarizing the conclusions based on the theory presented in this chapter, some key points for the modeling in Chapter 3 and the optimization in Chapter 4 are important to emphasize:

The turbine specific, aerodynamic coefficients for power, C_P, and thrust, C_T, were described in (2.2) and (2.1) respectively, and determine the downwind wake conditions and power output for a turbine operating at a certain wind and power reference P_{ref}. Thus, at a constant wind speed, *lowering the power reference for a turbine will lower its power output and change the properties of the wake*[7]. Based on the assumption that the added wake turbulence is induced foremost by the shear phenomenon, and the turbine spacing is larger than the near wake region, it would mean that the turbulence a downwind turbine experiences also is lowered, since the wind deficit will be smaller. The same reasoning follows for an increase in power reference, assumed there is more power to extract from the wind; if the turbine extracts more power, the turbulence level of its wake is higher.

Although stated before the model (2.12) was presented, the quotation in Section 2.4 still holds (Vestas, 2010), and it is likely that *any model resembling the same behavior could be used to model added wake turbulence*, with similar results.

For large wind farms, the formation and spacing of turbines will affect the propagation and inference of wakes. Additionally, wind direction and properties of the

[6]Frandsen (2007) used the ambient standard deviation σ_u, but for the sake of simplicity it is here referred to as “turbulence".

[7]Assuming that the turbine is extracting the reference power, i.e. $P_{\text{out}} = P_{\text{ref}}$.

terrain will have a great impact on the wake effects; the latter indicating a more distinct difference between ambient and within farm wind conditions for large offshore wind farms, as a result of the low surface roughness. *Worst case scenarios for the wind direction represents the maximum negative effects of the wind deficit and added wake turbulence*.

In short, everything in the turbine is exposed to variations that inflict fatigue. It is therefore important to account for the cost to expected damage ratio for the different components of operating turbines. Mapping the transition of the variance in wind speed to stress variances in specific turbine components is complex. However, more importantly, the results will, for the case of large wind farms, depend on the added wake turbulence model chosen to evaluate the inflicted fatigue. Acknowledging that *turbulence, seen as the standard deviation of wind speed, is a large contributor to turbine fatigue*, will give a suitable starting point for addressing the problem of describing the wake related fatigue in a large wind farm.

Furthermore, an interesting prospect is given by the nonlinear relation between power output and wind deficit: If the resulting sum of power output from two turbines standing in the direction of the wind can be increased by letting the upwind turbine extract a little less power, it would mean that *total power output for a large wind farm can be increased at the same time as the total fatigue (experienced turbulence) is decreased by finding optimal power references for the turbines*.

3. Added wake turbulence model

We were first introduced to this thesis project because of the need of more advantageous stationary operating points for use in the Aeolus Project. However, in order to find well working points, we needed a model for describing the added wake turbulence phenomenon. Many of the present models do not work well when they are put in a cumulative context, like the propagation through a large wind farm. We will in this chapter present our proposed model for the added wake turbulence, and in addition, we will suggest a modification of the wind deficit model it was set out to complement.

3.1 Model prerequisites

The intuitively quantitative model aims to describe the turbulence in a large farm based on former reports on modeling and observed behavior of the wake phenomena. The model, which will be presented in the coming sections, is constructed from a set of *requirements* it needs to fulfill, and some *assumptions* have been made to define the frame for the model.

Assumptions

Below listed assumptions are important for setting the model in a context. They do not take part in the model but set the frame for which it is intended. Thus, they also acknowledge the limitations of the model, and define the approach taken to describe the turbulence phenomena and its connection to fatigue.

1. The fluctuations in wind speed within the farm have the same bell shaped distribution as ambient wind, i.e. normally distributed around 0.
2. Turbulence is the standard deviation of wind speed fluctuations and its square (variance) is additive.
3. The wind direction is perpendicular to the farm and does not change, and the turbines are homogeneously spaced with fix distances between each other. (See Figure 3.1).
4. The distance between the turbines in the farm is longer than the near wake region of each turbine.

In addition to these assumptions, the model is one dimensional and treats turbines as points, i.e. it does not account for the rotor area and the vertical differences of wind speed.

The assumptions will be further clarified in the following sections, as the model is elaborated on.

Requirements

The requirements have been chosen so to suit the purpose of the model, and selected from the collected theory about turbulence and fatigue. They make the basis for the intuition for the model and will also be used for the evaluation in the end of the chapter.

1. For a single turbine, turbulence should be the highest immediately after the turbine, and decrease with distance until it has returned to ambient level.

2. The turbulence in a farm should not be able to be lower than the ambient level.

3. In a wind farm, the turbulence level reaches a limit after a distance downwind from the first row, i.e. it should not diverge and become infinitely large for a “long" farm.

4. A turbine deep within a wind farm should be able to experience a lower, as well as a higher, turbulence than its upwind neighbor.

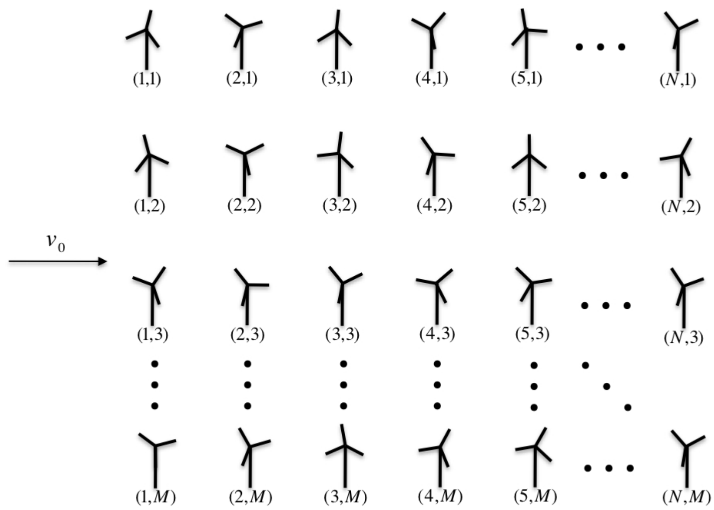


Figure 3.1 Assumed farm structure. Turbine spacing is fixed, as is the wind direction.

3.2 Modeling

The added wake turbulence model composes the conclusions in Section 2.6, and properties based on intuition. It aims to fulfill the requirements in the previous section, and to capture the wake turbulence behavior in a suitable manner. Starting with ambient turbulence and proceeding with turbine induced turbulence in large farm structures, the model will be presented in a stepwise manner.

Wind speed in general and ambient turbulence

As described in the Section 2.2, ambient wind can be seen as composed by the ambient mean wind $\overline{v}_0$ and the ambient turbulence ω_0. This is a very general view, and turbulence refers to the fluctuations in wind speed, normally distributed around 0 with variance σ_0^2, seen over 10 minutes:

$$v_0 = \overline{v}_0 + \omega_0, \;\; \omega_0 \in \mathrm{N}(0, \sigma_0) \tag{3.1}$$

The variance σ_0^2 of the ambient turbulence is the square product of the ambient turbulence intensity I_0 and ambient mean wind speed v_0.

$$\sigma_0^2 = I_0^2 \overline{v}_0^2 \tag{3.2}$$

Ambient farm turbulence

Acknowledging a higher general turbulence level within a wind farm than the ambient turbulence level, turbulence inside a farm can be said to have two components; the ambient turbulence and the added turbulence from the turbines. Not accounting for the turbulence the turbines add by their very presence, i.e. the turbines as a "roughness" (see Section 2.2), simplifies the picture slightly and the added turbulence then depends solely on the wakes of the turbines. Put differently, the turbulence at a turbine inside the farm depends on the ambient turbulence and the contribution of turbulence from upwind turbine wakes. This intuitive conclusion, along with the view of turbulence having an additive property, forms the basis for the proposed model.

The wake mixes with the ambient wind within the farm and "ambient farm turbulence" here refers to the sum of the variance of wind speed in the ambient wind and the contributions of the added variance from the wakes of upwind turbines, deep within the farm. There the sum of the upwind contributions has reached an equilibrium, or steady state. Since the ambient farm turbulence has the same properties as the ambient turbulence, as concluded in Section 2.6, it is assumed that the added wake turbulence is also normally distributed around 0.

A single turbine

Figure 3.2 illustrates a single turbine, experiencing an ambient wind $v_o = \overline{v}_0 + \omega_0$. The wind speed behind the turbine is $v = \overline{v} + \omega$. Based on the assumptions above, turbulence ω is normally distributed with variance:

$$\sigma^2 = \sigma_0^2 + \sigma_{\mathrm{add}}^2 \tag{3.3}$$

It also applies that $\sigma^2 > \sigma_0^2$ for the turbulence and $\overline{v} < \overline{v}_0$ for the mean wind speed. The added turbulence, i.e. σ_{add}^2, decreases with distance and eventually disappears. After some distance the turbulence level will have returned to ambient.
Since the model is thought to describe the turbulence a downwind turbine will experience, it is the area direct *in front of* a turbine which is focused on. Therefore, the characteristics of the turbulence in the near wake, immediately *behind* a turbine, is disregarded. This is because in the near wake there are more things affecting the

turbulence apart from the shear factor, as described in Section 2.3. Thus, σ^2_{add} refers to the shear turbulence in the far wake.

Figure 3.2 The basic model concept: Behind the turbine, the mean wind speed is $\overline{v} < \overline{v}_0$ and the turbulence is $\omega \in \mathrm{N}(0,\sigma)$, $\sigma^2 > \sigma_0^2$.

Wind deficit, thrust coefficient and added wake turbulence

Assuming that the majority of the turbulence a turbine wake adds depends on the difference in wind speed inside and outside the wake due to the shear phenomenon, the magnitude of the wind deficit can be said to determine how much turbulence is created in the wake. Conceptually, this can be seen as turbulence created by the wake is a function of either the wind deficit itself, or the state of the turbine creating the deficit, which is a more suitable representation for the objectives of this modeling.

The wind deficit in a wake is proportionate to the thrust coefficient C_T, and the coefficient has been chosen to represent the state of the turbine that determines the added wake turbulence, based on the relation between wind deficit and shear described in Section 2.3. This is also in line with the models for wind deficit and turbulence presented Section 2.4. Built on this reasoning and (3.2), the added variance in the wake can informally be represented as:

$$\sigma^2_{\text{add}} = f_{\sigma^2_{\text{add}}}(C_T, v) \tag{3.4}$$

where $f_{\sigma^2_{\text{add}}}$ is an arbitrary function. When elaborating on $f_{\sigma^2_{\text{add}}}$, it has to be accounted for that the thrust coefficient is also stochastic, since it depends on the wind speed at the turbine with a given power reference. Adjusting for the ten minutes time span, it is in its place for a rewriting according to:

$$\sigma^2_{\text{add}} = f_{\sigma^2_{\text{add}}}(\overline{C_T}, \overline{v}) \tag{3.5}$$

The added wake turbulence is a then represented as a function of the mean thrust coefficient and the mean wind speed. This suits the objectives with the model well, since it is for the stationary operating points of the turbines in a farm it is intended.

If the variance of wind speed fluctuations is approximated as proportional to the thrust coefficient, $f_{\sigma^2_{\text{add}}}$ can be represented as:

$$f_{\sigma^2_{\text{add}}}(\overline{C_T}, \overline{v}) = b\overline{C_T}\overline{v}^2 \tag{3.6}$$

where $b > 0$ and can be chosen to fit the model, depending on the desired calibration[1]. The expression above can be seen as the shear generated turbulence that a turbine gives rise to downwind, if there is no recovery of the wake, i.e. at any distance

[1]Parameters such turbine properties can be seen as implicitly included in the model by choosing b.

behind the turbine. Choosing e.g. $b = 0.08$ for $\overline{v} = 10$ m/s, $\overline{C_T} = 0.7$ and an ambient turbulence intensity $I = 0.1$, would give an added turbulence intensity of $\frac{\sigma_{\text{add}}}{\overline{v}} = 0.24$.

However, since the wake is recovering, the model has to include a component that accounts for the decrease of the added turbulence with downwind distance behind the turbine.

Consecutive turbines

Turbines standing behind other turbines will experience higher turbulence, due to the added wake turbulence. First, two consecutive turbines are considered. Following the reasoning above, $v_1 = v_0$ and the wind speed at turbine 2 is given by:

$$v_2 = \overline{v}_2 + \omega_2 \tag{3.7}$$

where ω_2 is composed by ambient turbulence and the contribution from turbine 1 and normally distributed with variance σ_2^2.

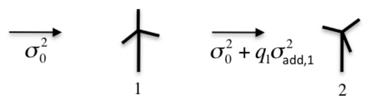


Figure 3.3 The added variance in the wake of a turbine affects the downwind neighboring turbine. q is the decay of the added turbulence between two turbines and represents the recovery of the wake.

However, this contribution is dependent on the distance between the turbines, because of the decreasing property of the added wake turbulence. In order to capture this decrease, a constant $0 < q_1 < 1$ is introduced[2]. The variance of ω_2 at turbine 2 then becomes:

$$\sigma_2^2 = \sigma_0^2 + q_1\sigma_{\text{add},1}^2 \tag{3.8}$$

More than two turbines

Expanding the row to an arbitrary number of turbines, turbine i will contribute to an added variance on turbines $i+1$ and $i+2$ by $q_1\sigma_{\text{add},i}$ and $q_2\sigma_{\text{add},i}$, respectively. By denoting $q_i = c^i$ for all $0 < q_i < 1$, the decreasing property is maintained. The variance at turbine $n+1$ in the row can then be written as:

$$\sigma_{n+1}^2 = \sigma_0^2 + c^n\sigma_{\text{add},1}^2 + c^{n-1}\sigma_{\text{add},2}^2 + \ldots + c^2\sigma_{\text{add},n-1}^2 + c\sigma_{\text{add},n}^2 \tag{3.9}$$

The turbine experiences turbulence with a variance composed by the ambient turbulence and the contributions from all upwind turbines, as illustrated in Figure 3.2.

Besides fulfilling the assumptions made earlier in an intuitive manner, the model can be rewritten as:

$$\sigma_{n+1}^2 = (1-c)\sigma_0^2 + c(\sigma_n^2 + \sigma_{\text{add},n}^2) \tag{3.10}$$

[2]Parameters such as the distance between two turbines can be seen as implicitly included in the model by choosing q_1.

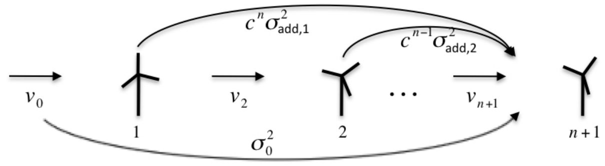


Figure 3.4 A turbine on position $n+1$ in a row of consecutive turbines (in the direction of the wind) is affected by the total added variance from its n upwind neighbors.

The turbulence level at a turbine depends then solely on that of the upwind neighbor, and the ambient turbulence level. This distributed feature makes it very similar to the deficit model, which was one of the objectives. Restructuring and substituting $\sigma^2_{n,add}$ according to (3.6) gives:

$$\sigma^2_{n+1} = \sigma_0^2 + \underbrace{cb\overline{C_T}_n\bar{v}_n^2}_{\text{upwind neighbor}} + \underbrace{c(\sigma_n^2 - \sigma_0^2)}_{\text{other upwind turbines}} \tag{3.11}$$

The model then describes the turbulence as composed by the three parts ambient turbulence, directly induced turbulence from the upwind neighbor, and the added turbulence from other upwind turbines. As a final step, the constants are renamed according to $cb = c_1$ and $c = c_2$.

$$\sigma^2_{n+1} = \sigma_0^2 + c_1\overline{C_T}_n\bar{v}_n^2 + c_2(\sigma_n^2 - \sigma_0^2) \tag{3.12}$$

The wind speed at the turbine is:

$$v_{n+1} = \bar{v}_{n+1} + \omega_{n+1},\ \ \omega_{n+1} \in \mathrm{N}(0, \sigma_{n+1}) \tag{3.13}$$

Multiple rows

For wind turbines in a farm, the added turbulence will not only come from those standing in the same row but also from upwind turbines in parallel rows. The shear phenomenon is directly dependent on the difference in wind speed of the wake and its surrounding wind. To include this, the model of (3.12) is expanded to include the turbulence level at the two upwind turbines of the two neighboring rows. For turbine with farm index $n+1, m$, the turbulence will then become:

$$\sigma^2_{n+1,m} = \sigma_0^2 + c_1\overline{C_T}_{n,m}\bar{v}_{n,m}^2 + c_2(\sigma_{n,m}^2 - \sigma_0^2) + c_3(\sigma^2_{n,m-1} + \sigma^2_{n,m+1} - 2\sigma_0^2) \tag{3.14}$$

The addition is meant to capture the influence of the turbines in the parallel rows. Figure 3.2 illustrates the multiple rows situation. It also illustrates which sources of information that are needed to describe the turbulence at turbine $n+1, m$. First, the mean wind speed and thrust coefficient of turbine n are needed, for determining the direct contribution of added variance. Second, the turbulence at the same turbine is needed, representing the addition from the rest of the turbines in the same row in upwind direction. Third, the turbulence at turbines $n, m+1$ and $n, m-1$ are needed, giving the addition from the rest of the upwind turbines.

The model has now the structure to describe the turbulence propagation in a farm, in the sense that the variance in wind speed that a turbine experiences depends on the states of all upwind turbines. A turbine will experience a higher turbulence than the ambient. It will be dependent on the upwind turbines in the same row, and also the effect all upwind turbines in the farm have on the “ambient farm conditions".

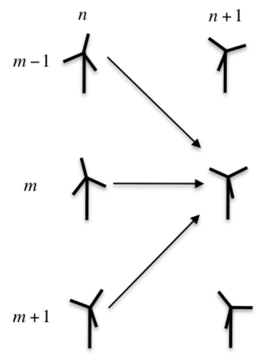


Figure 3.5 A turbine is affected by its upwind neighbors also in parallel rows. To determine the turbulence at turbine $n+1, m$, information about the turbulence at the three closest upwind neighbors is needed, together with the wind speed and thrust coefficient of the closest neighbor n, m.

3.3 Assessment

In this section the model will be evaluated according to firstly the criteria and secondly the turbulence model from Frandsen (2007). The latter was not originally presented in the same distributed form. However, in lack of other ways of assessing the proposed model, this is the chosen benchmark for assessing the model for a single row of turbines. For the farm, it is only the criteria and intuitive behavior that have been taken into account.

Model configuration

Choosing the constants c_1, c_2 and c_3 is critical for the model behavior. The approach chosen here is to calibrate c_1, determining how much the direct influence from the closest upwind neighbor in the same row is. c_2 and c_3 will then determine the equilibrium in criteria 3 and are given suitable values.

Assigning the turbulence level at turbine 2 and the same value as for the model by Frandsen (2007), gives the reference point for c_1. Then c_2 is adjusted so the equilibrium value is 25%, . For the farm, c_3 is chosen to give an equilibrium of 30%, which is seen as an reasonable contribution from the other rows[3].

[3] c_1 and c_2 were chosen according to Frandsen's model, since it was originally configured to fit measurement data. c_3 was given a “modest" value, but is chosen arbitrary by the authors. See Section 5.1 for further elaboration on the model calibration.

The resulting values are:

$$\begin{aligned} c_1 &= 0.0352 \\ c_2 &= 0.4200 \\ c_3 &= 0.1000 \end{aligned}$$

Since the constants do not represent a direct relation, it might be the case that having different constants for different turbines, operating modes, etc. would improve the accuracy of the model. The model assessment will be focused on fulfilling the general requirements in a way suited for the optimization.

Plots

The following plots are chosen to compare the proposed model to the requirements. The model by Frandsen (2007) is included as a comparison in the plots for a single turbine or a single row of consecutive turbines, even though, as mentioned earlier, it was not used for this propagation structure in its original form.

Single turbine

According to the first requirement, the turbulence should return to ambient level after a distance downwind from the turbine. Figure 3.6 shows the turbulence after a single turbine (distance as counted in the assumed, fix turbine spacing). The ambient conditions of 10 m/s wind speed and 10% turbulence intensity. These ambient conditions will be the same for all comparisons in this section and if nothing else is mentioned, the turbines try to extract as much effect as possible (i.e. $P_{\text{ref}} = P_{\text{max}}$).

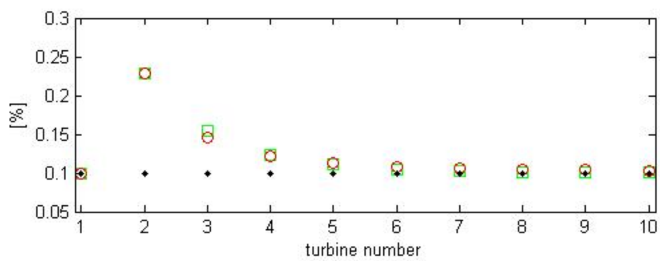


Figure 3.6 Turbulence intensity decreases downwind from a single turbine, until it has returned to ambient level. The proposed model is marked as circles and the comparison model as stars.

Consecutive turbines

Figure 3.7 shows the turbulence intensities for a single row of 10 consecutive turbines. The calibration is set to make the equilibrium to 25%.

That the model also fulfills requirement 4 is illustrated in the bottom plot in the figure. A turbine is set to extract less power, which gives it a lower thrust coefficient; the others are the same as previously. This is an important feature for the model, since the optimization will depend on the dependency between induced turbulences and the power references given to the turbines in the farm.

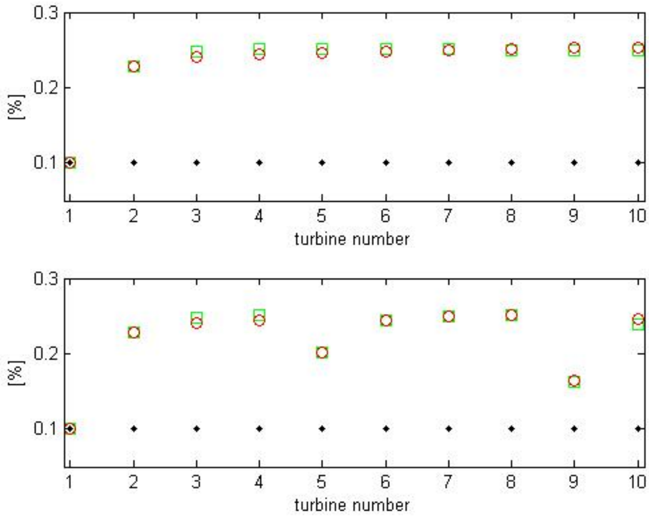


Figure 3.7 The turbulence intensity for a single row of 10 consecutive turbines steadies at 25%. The proposed model is marked as squares and the comparison model as circles. Top: $P_{\text{ref}} = P_{\max}$ for each turbine. Bottom: Turbine 4 has a lower thrust coefficient (due to a lower P_{ref}).

Multiple rows
For multiple rows, the comparison model is left out and is simply showed to fulfill the requirements also for a large farm. Figure 3.8 illustrates a farm of 10x10 turbines. The calibration is set to make the turbulence intensity deep within the farm steady at approximately 30%. Observe that the turbulence is lower for the turbines in the side rows at the edges of the farm (i.e. for $m = 1$ and $m = 10$), which goes nicely with the intuition of the wake interference deeper in the farm in comparison with the ambient conditions surrounding it.

3.4 Modification of the wind deficit model

As mentioned in Section 3.2, the added wake turbulence will be determined by the magnitude of the wind deficit. Thus, the choice of wind deficit model will impact the turbulence experienced by the turbines in the farm. It is important that the wind deficit model suits our objectives of capturing the phenomena of wake propagation throughout the farm, especially when the turbines are given different power references during the optimization presented in the next chapter.

Initially, the wind deficit model (2.7) was used for obtaining the turbulences at the turbines (i.e. for calculating v_n). However, the model showed to be sensitive to changes in power references in a way that did not suit the objectives for the farm; a lower thrust coefficient for a turbine deep in the farm had too large an impact on the downwind wind deficit and turbulence level. That led to a modification in the wind deficit

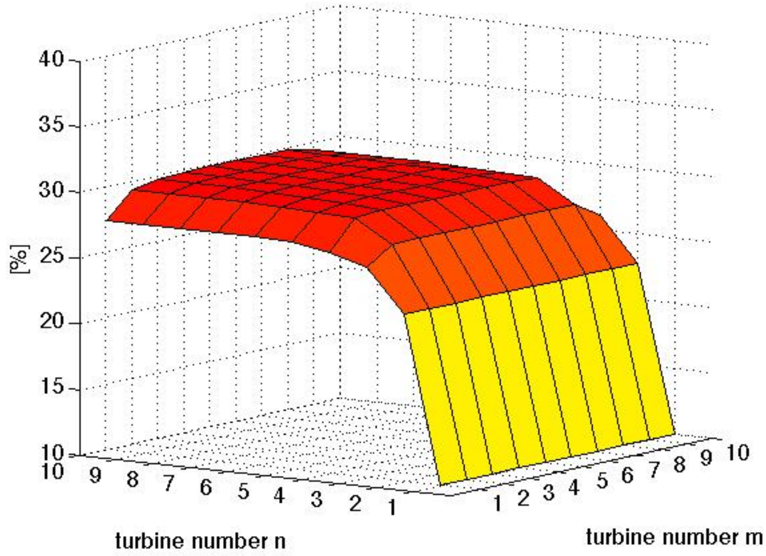


Figure 3.8 Turbulence intensities for a farm of 10x10 turbines, derived with (3.14). The turbulence intensity steadies at 30 % for the turbines furthest downwind. Note: The turbulence intensities relate to the turbulence just in front of the turbines and do not account for its behavior between them.

model, which will be presented in this section. The modification makes the wind deficit very similar to the turbulence model. It is also similar to the initial model, with the difference that the deficit is dependent on the wind surrounding the wake of the specific turbine, and not the ambient wind.

It should also be emphasized that for the previous figures in this chapter (as well as the results presented in Chapter 4) were used the modified wind deficit model.

The model

Following the same reasoning as for the added wake turbulence, the modification is done on the wind deficit that each turbine contributes with. Thus, just as the added wake turbulence was seen as a function of the incoming mean wind speed (surrounding the wake) and the mean thrust coefficient of the turbine (see (3.4)), the wind deficit with which a turbine affects the downwind flow can be seen as a function of the same variables, but with the difference that the wind deficit is proportionate to the incoming wind and not its square:

$$\overline{v}_{\text{def}} = f_{v_{\text{def}}}(\overline{C_T}, \overline{v}) = a\overline{C_T}\overline{v} \tag{3.15}$$

where a is a constant (corresponding to b in (3.4)) and v_{def} is the wind deficit. Following the same reasoning as in Section 3.2, the wind speed at turbine $n+1$ in a single row of consecutive turbines can be written as:

$$\overline{v}_{n+1} = \overline{v}_0 - \underbrace{k_1\overline{C_T}_n\overline{v}_n}_{\text{upwind neighbor}} - \underbrace{k_2(\overline{v}_0 - \overline{v}_n)}_{\text{other upwind turbines}} \tag{3.16}$$

The positive constants k_1 and k_2 corresponds to the constants c_1 and c_2 of the turbulence model (a being absorbed by k_1 through $k_1 = ak_2$). This structure is the same as for the turbulence model. The wind coming in to turbine $n+1$ is lower than ambient, with the total deficit composed by the wake effect of closest neighbor, and the wake effects from remaining upwind turbines.

Choosing k_1 and k_2 to agree with the configuration of the initial deficit model (2.7), a comparison is presented in Figure 3.9.

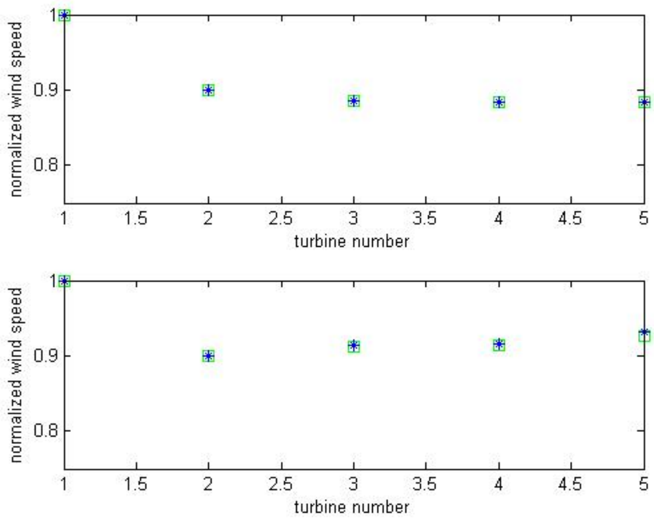


Figure 3.9 Comparison between the deficit model (2.7) (stars) and the modified model (3.16) (squares) with $k_1 = 0.143$ and $k_2 = 0.24$. The two models show similar behavior. Top: $C_T = [0.7\ 0.7\ 0.7\ 0.7\ 0.7]$. Bottom: $C_T = [0.7\ 0.5\ 0.5\ 0.4\ 0.7]$.

The models are identical for this setup. Expanding the model to the case of multiple rows finalizes the modification and gives the wind speeds for turbines in a farm according to:

$$\bar{v}_{n+1,m} = \bar{v}_0 - k_1 \overline{C_T}_{n,m} \bar{v}_{n,m} - k_2(\bar{v}_0 - \bar{v}_{n,m}) - k_3(2v_0 - \bar{v}_{n,m-1} - \bar{v}_{n,m+1}) \tag{3.17}$$

Since Figure 3.9 shows only the case of five consecutive turbines in a row with explicitly assigned thrust coefficients, comparisons need to be done for thrust coefficients determined by the power references and the wind speeds at the turbines, as well as for farm conditions. Furthermore, based on the conclusions of Chapter 2, the wind speed deficit, which the models are calibrated for, can be considered larger for a worst case wake effect scenario in a single row of turbines[4].

Configuration

Calibrating the modified wind deficit, the constants k_1, k_2 and k_3 can be chosen in the same way as for the turbulence model. Adopting a higher wind deficit, with the

[4]Referring to e.g. Barthelmie, Frandsen (2007).

same ambient wind speed of 10 m/s, k_1 is assigned a value so that the wind speed at a neighboring turbine downwind is 8.5 m/s (i.e. a wind deficit of 15%). For a single row of 10 turbines, k_2 is given a value making the "steady state" wind speed 8.15 m/s, fitting it with (2.7), and respectively for the farm, k_3 is adjusted to yield a farm stationarity of 8 m/s. The resulting values are[5]:

$$\begin{aligned} k_1 &= 0.1916 \\ k_2 &= 0.3400 \\ k_3 &= 0.1020 \end{aligned}$$

In Figure 3.10, illustrating a single row of ten turbines according to the new configuration, the slight difference between the models can be seen, i.e. the purpose of the modification. The wind propagation in 10x10 farm derived with (3.17) is shown in Figure 3.11.

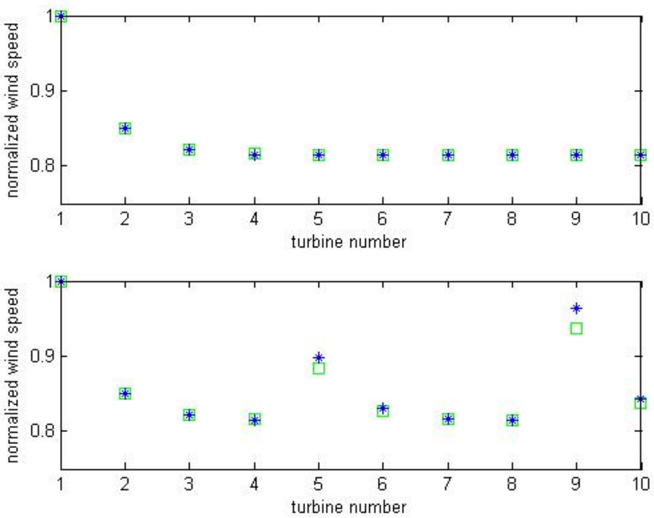


Figure 3.10 Wind speeds at turbines in a row of consecutive turbines. The modified wind deficit model is marked as square and the initial as stars. Top: $P_{\text{ref}} = P_{\max}$ for all turbines. Bottom: $P_{\text{ref}} = 0.5P_{\max}$ for turbine 4, and turbine 8 is shut off (i.e. $P_{\text{ref}} = 0$ and thus, $C_T = 0$). The proposed model returns slower to ambient level.

3.5 Concluding remarks

It is important to emphasize the implicit meaning of the model constants of (3.17) and (3.14). Taking neither turbine properties nor farm specific features into account, the models are focused on capturing the overall behavior of the turbine wakes in an arbitrary large offshore farm setting, related to wind deficit and added turbulence.

Choosing the overall behavior according to the collected literature and research presented in Chapter 2 is a "natural" approach of configuring the wake models. As an

[5] In addition, the thrust coefficients for the turbines are calculated from the wind speed at each turbine, along with maximum power references (i.e. $P_{\text{ref}} = P_{\max}$).

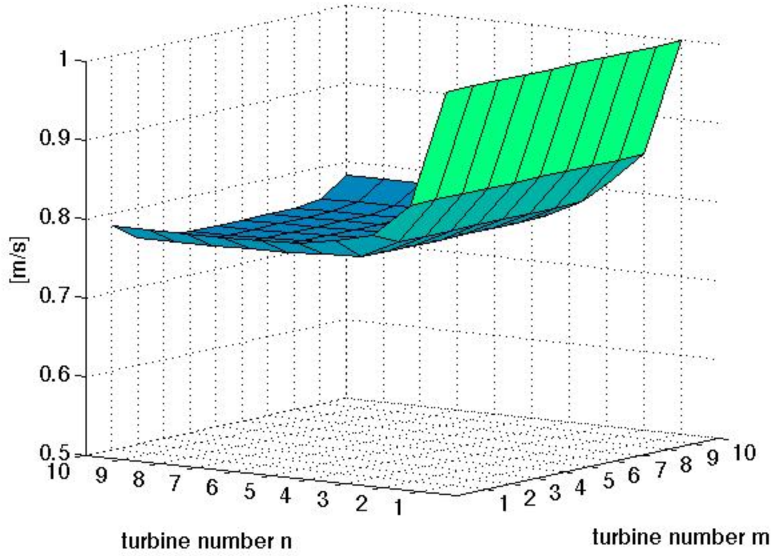


Figure 3.11 Normalized wind speeds at the turbines in a 10x10 farm, derived with (3.17). The lower wind speed steadies at 75 % of ambient for turbines furthest downwind. Note: The wind speeds relate to the wind just in front of the turbines and do not account for its behavior between them.

interesting mean of comparison is provided by the worst case scenario power loss at Horns Rev of roughly 50% (as cited in Section 2.3) for the large offshore farm under certain wind directions and velocities. Figure 3.12 shows the power deficit for the same 10x10 farm used above. The total power loss for the model calibration is 48% (compared to the an equal amount of turbines standing in free flow).

However, a decomposition of the general behavior of the wake phenomenon, referring to the initial conditions of wind speed and turbulence level, can be done by distributing the weights of the constants; i.e. how the components of the models are weighted according to each other. Thus, k_1 and c_1 will determine how much direct influence a turbine will have on its downwind neighbor, k_2 and c_2 how much influence it will have on rows further downwind, and k_3 and c_3 on the rows parallel to the downwind direction. There is no similarly “natural" approach of making the trade off between the components. Large offshore wind farms have not been subject to measurements long enough to have a full understanding of the downwind propagation of wakes, or the effects it has on the turbines.

The choice of model constants will be further elaborated on in Section 4.4, when investigating the consequence it has for finding the optimal stationary power distribution.

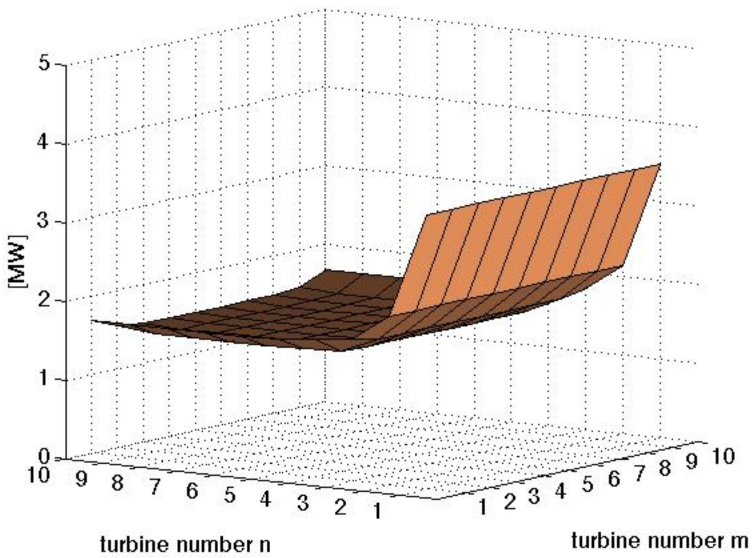


Figure 3.12 The power deficit for each turbine, with the same model calibration as in Figure 3.8 and Figure 3.11. The total power loss is 48%.

4. Optimal static power references

This is the Optimization Chapter, in which we are going to derive and calculate the optimal static power references for a large offshore wind farm, according to the models presented in the previous chapter. Since it is also the main purpose of the thesis, the results presented here are the most important for our final analysis in Chapter 5. First, we will describe the optimization in general terms of objective functions, algorithm and implementation. Thereafter, a summary of the solutions for the optimal power references we found is presented on table form, for various initial conditions.

4.1 Optimization prerequisites

Because the turbines in a wind farm are interacting through the downstream wakes, it is relevant to investigate the opportunity to find a more beneficial way of distributing static power references individually for each turbine, instead of demanding maximum output from every turbine. First, a procedure for reaching better farm performance is suggested according to different objectives as outlined in Section 2.6. Second, the procedure is done for certain initial conditions of wind speed and turbulence, etc.

The frame for the optimization is set according to:

- Farm structure and wind direction are as presented in Figure 3.1, representing the “worst case scenario" for the wake conditions. Furthermore, the turbine states and wind conditions are interconnected using (3.14) and (3.17).
- The algorithm chosen for the optimization is the *gradient descent method*, with the aim of finding an arbitrary set of power references for the turbines which is better than the initial (“better" referring to the optimization objective in question).
- Turbulence a turbine in the farm experiences is expressed as the standard deviation of wind speed, because of its relation to fatigue (see Section 2.6).
- With wind speed v, and power coefficients C_P and C_T refers in the following to the mean values (as described in Section 3.2).

4.2 Objective functions

Three relevant optimization objectives for a large wind farm with respect to distributed power references were addressed:

1. *Power maximization:*
   Maximize the total farm power output

2. *Power-Fatigue trade off:*
   Maximize the total farm output and minimize total farm fatigue based on a arbitrary trade off cost.

3. *Power plant optimization*
   Minimize the total farm fatigue for a given total farm power output.

Power maximization

Without considering fatigue, it is a reasonable first approach to investigate whether it is possible to extract more power from the farm with individually distributed power references to the turbines. The value of comparison is that of maximum power output ($P_{\text{ref}} = P_{\max}$) demanded for each turbines, which is the present approach used in wind farms (Svensson, 2010). The objective function is the sum of power outputs of each turbine:

$$\begin{aligned} \max_{\mathbf{u}} \quad & \Gamma = \sum_{i=1}^{N \cdot M} P_{\text{out}}(u_i, v_i) \\ s.t. \quad & 0.1 \leq u_i \leq 5 \end{aligned} \tag{4.1}$$

where P_{out} is the turbine power output, $N \cdot M$ is the total number of turbines in the farm, and $\mathbf{u}$ is the power references for the turbines according to[1]:

$$\mathbf{u} = \begin{pmatrix} P_{\text{ref}_{1,1}} & P_{\text{ref}_{1,2}} & \cdots & P_{\text{ref}_{1,M}} & P_{\text{ref}_{2,1}} & \cdots & P_{\text{ref}_{N,M-1}} & P_{\text{ref}_{N,M}} \end{pmatrix} \tag{4.2}$$

and $\mathbf{v}$ holds the wind speeds at respective turbine.

Power-Fatigue trade off

If fatigue is considered in relation to power output, a trade off is introduced. The trade off is determined by the “cost" assigned to turbulence in terms of power output, and highly affects the outcome of the optimization. Therefore, it is important to choose an adequate cost based on economic estimations of the turbulence in relation to fatigue (maintenance, turbine lifetime, etc.). This estimation is considered out of scope for the thesis and the cost was chosen more or less arbitrary[2], in order indicate the potential of the optimization: $\delta = 0.1$ MWs/m.

The cost function is the sum of power outputs minus the sum of the cost weighted

[1]Note that the indexation with n and m refers a turbines position in the farm according to Figure 3.1, and i refers to the corresponding position in the vectors used for the optimization according to (4.2).

[2]A short reasoning show the effect of the assigned value of δ: If $T = 25$ the estimated lifetime of a turbine in years, and $P = 1.5$ is the estimated lifetime averaged power output in MW, then $T \cdot \Delta P = \Delta T \cdot P$. Lowering the average power output according to $\Delta P = -0.1$, yields $\Delta T = -1.33$, i.e. the change in lifetime which corresponds to the same loss of total produced power. With $\delta = 0.1$ MWs/m, an increase of σ with 1 (seen as a lifetime average) inflict the same “cost" as a reduced P with 0.1 MW, or a shorter lifetime of 1.33 years. Whether or not this is a reasonable value will not be discussed further in the scope of the thesis; but it shows that δ is not assigned a value yielding a shorter life time in the order of 10 years or similar.

turbulence:

$$\begin{aligned} \max_{\mathbf{u}} \quad & \Gamma = \sum_{i=1}^{N \cdot M} P_{\text{out}}(u_i, v_i) - \delta \sum_{i=1}^{N \cdot M} \sigma_i \\ s.t. \quad & 0.1 \leq u_i \leq 5 \end{aligned} \tag{4.3}$$

where δ is the assigned cost of turbulence σ_i for respective turbine.

Power plant optimization

The last objective approaches the view of the farm as a power plant, and proposes the interesting possibility for a farm operator to produce cheaper power. Currently, farms are designed to produce as much power as possible at any given wind. However, although rarely used in practice, the ability to deliver a fix total power output is a requirement for modern farms. It might be of interest for future use of large wind farms as power plants to produce this fix power demand to the lowest possible cost in the sense of fatigue. (Svensson, 2010)

A way of expressing the optimization problem for this objective is to restrict the farm output by adding a condition to the cost function. The cost function is the sum of turbulence experienced by the turbines:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \Gamma = \sum_{i=1}^{N \cdot M} \sigma_i \\ s.t. \quad & \gamma = \sum_{i=1}^{N \cdot M} P_{\text{out}}(u_i, v_i) = P_{\text{plant}} \\ & 0.1 \leq u_i \leq 5 \end{aligned} \tag{4.4}$$

where P_{plant} is the fixed power demand for the farm.

4.3 Implementation

In order to find solutions to the different objective functions in Section 4.2, the embedded MatLab function `fmincon` was used. `fmincon` is included in the MatLab Optimization toolbox, and can be used for solving standard and large-scale optimization problems for multivariable nonlinear functions with equality, inequality and nonlinear constraints. The solver does not garantuee that a global extreme point is found for the solution.

There are other algorithms to use for solving the optimization problems but it is considered out of scope for the thesis, as is further analysis of convergence or local and global minima. The approach taken is simply that if a beneficial point is found, it indicates the potential of finding new power distributions.

4.4 Results

As discussed in Section 3.5, choosing the constants k_i and c_i will make the wake effects more or less obvious, and more or less dependent on the different model components. Additionally, the wind deficit model will directly influence the added wake turbulence, since the latter depends on the wind speeds at the turbines. Therefore, the three objectives have been solved for two different model calibrations. A "worst case scenario" behavior of the wake effects forms the general approach, but the models have been tuned differently. The farm structure is according to that used in Chapter 3, i.e. a 10x10 farm is considered.

To begin with, the model configurations in Section 3.2 and Section 3.4 have been applied to the cost functions for the optimization. This setup will be referred to as *original* in the following presentation of the results. Thereafter, the results from the same optimizations but with an altered model configuration, with higher values for the model constants, will be presented. The wake effects have been given a more significant impact on the power loss and experienced turbulence for the farm. The second setup will be referred to as the *boosted* configuration, and the model constants were given following values[3]:

$$\begin{aligned} k_1 &= 0.20 & \qquad c_1 &= 0.05 \\ k_2 &= 0.60 & \qquad c_2 &= 0.50 \\ k_3 &= 0.05 & \qquad c_3 &= 0.10 \end{aligned}$$

The initial total power loss, due to the wake wind deficit, gives an indication of how large the impact of the wind model configuration in question has on the farm for the different wind speeds, and it will be presented for the initial conditions of the optimizations as a point of reference.

Four ambient wind speeds were chosen for the optimizations: 8 m/s, 10 m/s, 12 m/s and 14 m/s. Ambient turbulence intensity was set to 10%.

Solutions were found for each of the objectives, indicating beneficial power distributions than the initial. The magnitude of the improvements vary with both ambient wind and also which of two the model configurations was used.

Power maximization

Ambient wind speeds, initial farm outputs ($P_{\text{out}}^{\text{farm}}$), initial power losses, initial total experienced turbulences (σ^{farm}) and the resulting power output and experienced turbulence differences ($\Delta P_{\text{out}}^{\text{farm}}$ and $\Delta\sigma^{\text{farm}}$) can be seen in Table 4.1 and Table 4.2 for the original and the boosted model configurations respectively. Note that the total turbulence experienced by the turbines in the farm is lowered for the found solutions, although it was not accounted for in the cost function (4.1). The optimizations found improved power distributions for the wind speeds 10 m/s, 12 m/s and 14 m/s, but not for 8 m/s. For the boosted configuration, the improvements were higher than for the

[3]The implication of the boosted model constants can be seen e.g. by comparing the left columns in Figure 4.3 and Figure 4.4. For the boosted configuration, the wind speed goes down to 0.65% of ambient, and turbulence intensity reaches 40%.

original.

Figure 4.1 (original model configuration) and Figure 4.2 (boosted model configuration) show power references, wind speeds, and power outputs for the turbines in the farm before and after the optimization, for 12 m/s.

v_0	*initial* $P_{\text{out}}^{\text{farm}}$	*initial power loss*	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	89 MW	50%	168	+-0%	+-0%
10 m/s	181 MW	48%	207	+0.33%	-1.08%
12 m/s	317 MW	37%	242	+1.69%	-3.43%
14 m/s	484 MW	3%	259	+0.18%	-0.60%

Table 4.1 Optimization results for the Power maximization objective, cost function (4.1) with the original model configuration. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, improving the farm power output, as well as total experienced turbulence.

v_0	*initial* $P_{\text{out}}^{\text{farm}}$	*initial power loss*	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	69 MW	61%	196	+-0%	+-0%
10 m/s	145 MW	58%	242	+2.10%	-4.60%
12 m/s	260 MW	48%	281	+3.23%	-5.62%
14 m/s	410 MW	18%	307	+1.51%	-2.91%

Table 4.2 Optimization results for the Power maximization objective, cost function (4.1) with the boosted model configuration. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, improving the farm power output, as well as total experienced turbulence.

Power-Fatigue trade off

The assigned cost for δ was set to 0.1 MWs/m, as described in Section 4.2.

Table 4.3 and Table 4.4 show the initial conditions and results for the optimizations, in the same manner as for the Power maximization objective. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, resulting in a higher farm power output and a lower total amount of experienced turbulence; but not for 8 m/s.

Figure 4.3 (original model configuration) and Figure 4.4 (boosted model configuration) show power references, wind speeds, turbulence intensity and power outputs for the turbines in the farm before and after the optimization, for 12 m/s.

v_0	*initial* $P_{\text{out}}^{\text{farm}}$	*initial power loss*	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	89 MW	50%	168	+-0%	+-0%
10 m/s	181 MW	48%	207	+0.30%	-1.65%
12 m/s	317 MW	37%	242	+1.68%	-3.82%
14 m/s	484 MW	3%	259	+0.18%	-0.60%

Table 4.3 Optimization results for the Power-Fatigue trade off objective, cost function (4.3), with the original model configurations. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, lowering the total experienced turbulence in the farm, and also increasing the farm power output.

v_0	*initial* $P_{\text{out}}^{\text{farm}}$	*initial power loss*	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	69 MW	61%	196	+-0%	+-0%
10 m/s	145 MW	58%	242	+1.71%	-5.22%
12 m/s	260 MW	48%	281	+3.26%	-5.70%
14 m/s	410 MW	18%	307	+1.49%	-3.01%

Table 4.4 Optimization results for the Power-Fatigue trade off objective, cost function (4.3), with the boosted model configurations. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, lowering the total experienced turbulence in the farm, and also increasing the farm power output.

Power plant optimization

The approach for the Power plant optimization was to investigate if the same total power output for the farm can be obtained, with a lower cost in terms of experienced turbulence. Thus, the initial $P_{\text{out}}^{\text{farm}}$ from Table 4.1 and Table 4.2 was chosen for P_{plant}, giving the upper limit for the power output. Table 4.5 and Table 4.6 show the initial conditions, with ambient wind, demanded farm output, total experienced turbulence; and the resulting differences in turbulence for the optimizations, for the two model configurations respectively. Beneficial power reference distributions were found for all four wind speeds, and for both the original and the boosted model configurations; indicating that the fixed demanded farm output was achieved with a lower experienced turbulence.

Figure 4.5 (original model configuration) and Figure 4.6 (boosted model configuration) show power references, wind speeds and turbulence intensity for the turbines in the farm before and after the optimization, for 12 m/s.

Setting the fixed power demand for the farm lower than the initial values in Table 4.5 and Table 4.6, e.g. 80%, should be expected to yield similar result, given that the initial power references are equally distributed. However, for the purpose of the Power plant optimization it is seen as sufficient to show the “100%" case.

v_0	P_{plant}	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	89 MW	168	–	+-0%
10 m/s	181 MW	205	–	-2.52%
12 m/s	317 MW	227	–	-6.56%
14 m/s	388 MW	216	–	-0.20%

Table 4.5 Results for the Power plant optimization objective, cost function (4.4), with the original model configurations, and fixed farm output according to Table 4.1. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, lowering the total experienced turbulence in the farm.

v_0	P_{plant}	*initial* σ^{farm}	$\Delta P_{\text{out}}^{\text{farm}}$	$\Delta\sigma^{\text{farm}}$
8 m/s	69 MW	196	–	%
10 m/s	145 MW	242	–	-6.14%
12 m/s	260 MW	281	–	-11.58%
14 m/s	328 MW	255	–	-3.56%

Table 4.6 Results for the Power plant optimization objective, cost function (4.4), with the boosted model configurations, and fixed farm output according to Table 4.2. Beneficial power distributions were found for 10 m/s, 12 m/s and 14 m/s, lowering the total experienced turbulence in the farm.

4.5 Illustrative examples

In Figures 4.1–4.6 are given illustrating examples from the optimizations, showing power references, wind speeds, power outputs and turbulence intensities for before and after the optimizations, for both the original model configuration and the boosted. The left column in the figures shows the initial conditions and the right column shows the conditions with the optimal power references. The wind speed chosen for the examples is 12 m/s.

With initial distribution of power references, $P_{\text{ref}} = P_{\text{max}}$ for every turbine. However, the turbines will not be able to extract P_{max} at 12 m/s. Therefore, the “effective" initial power references are shown for the sake of comparison with the optimal, and refer to the power output for every turbine before the optimization.

As mentioned in Section 4.1, the approach taken is that the fatigue, subject to the optimization, refers to the standard deviation of wind speed fluctuations (see Section 2.6). However, it is hard to make an intuitive conclusion from the values of σ^{farm}, even though the difference $\Delta\sigma^{\text{farm}}$ indicates the direction in which the results point.

In order to illustrate the turbulence at each turbine, before and after the optimization, the fluctuations are instead represented in the figures as turbulence intensity (calculated according to (2.4), with the wind speed at the turbines given by (3.17)).

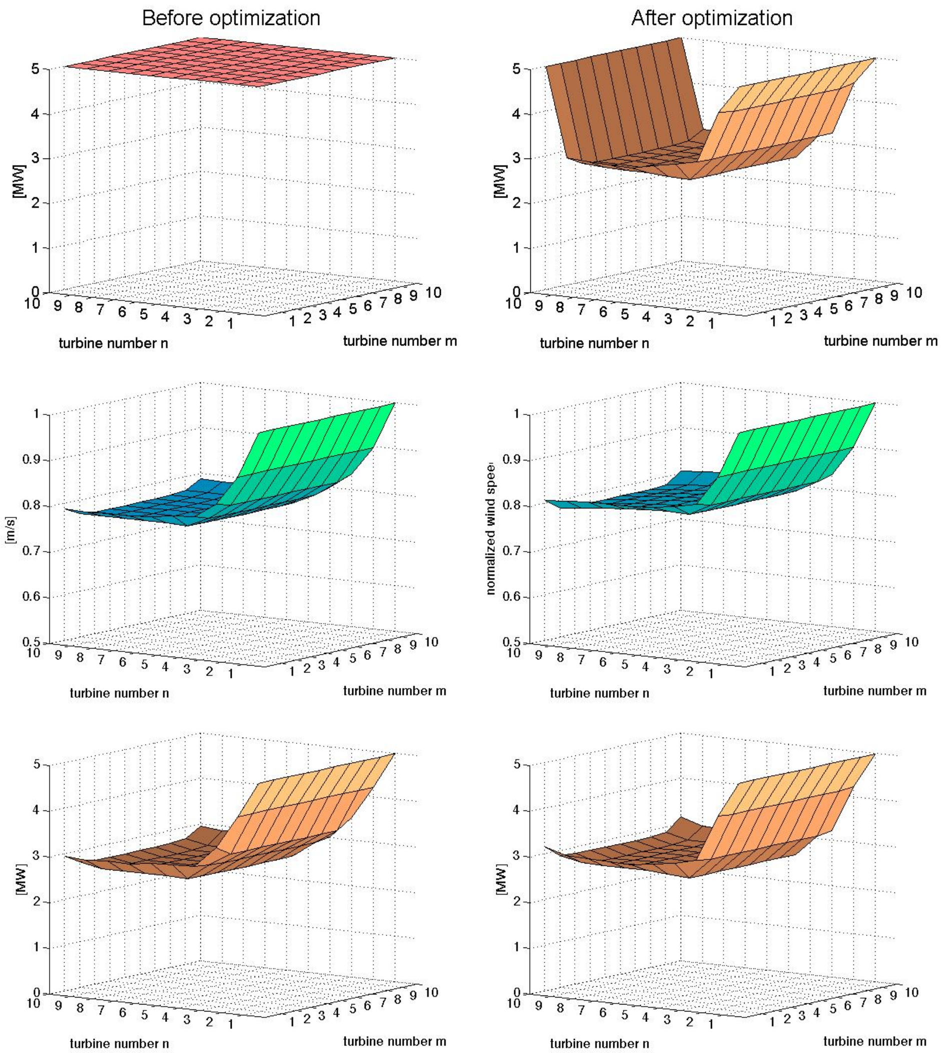


Figure 4.1 Power references, wind speeds, and power distribution for a 10x10 farm, before and after optimization, for *Power maximization, original model configuration.* Ambient wind speed is 12 m/s. Top: Power references. Middle: Wind speed. Bottom: Power distribution.

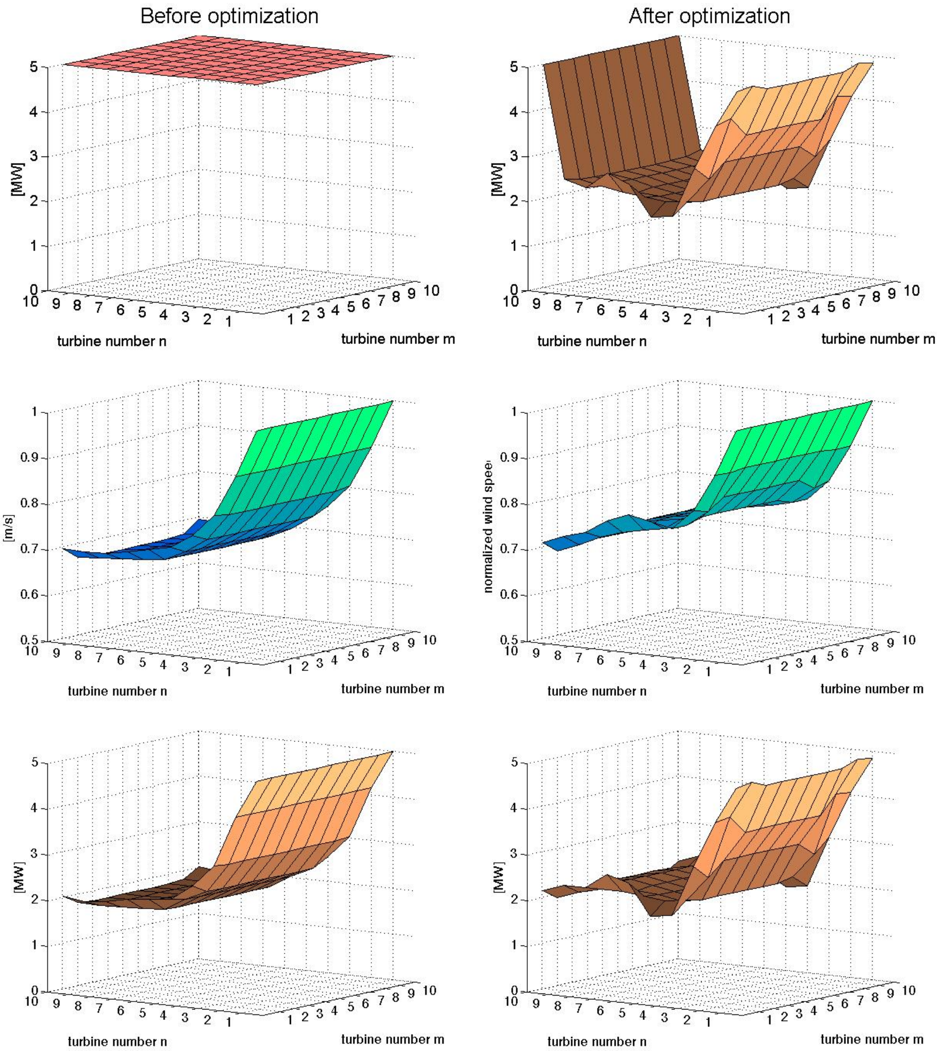


Figure 4.2 Power references, wind speeds, and power distribution for a 10x10 farm, before and after optimization, for *Power maximization, boosted model configuration.* Ambient wind speed is 12 m/s. Top: Power references. Middle: Wind speed. Bottom: Power distribution.

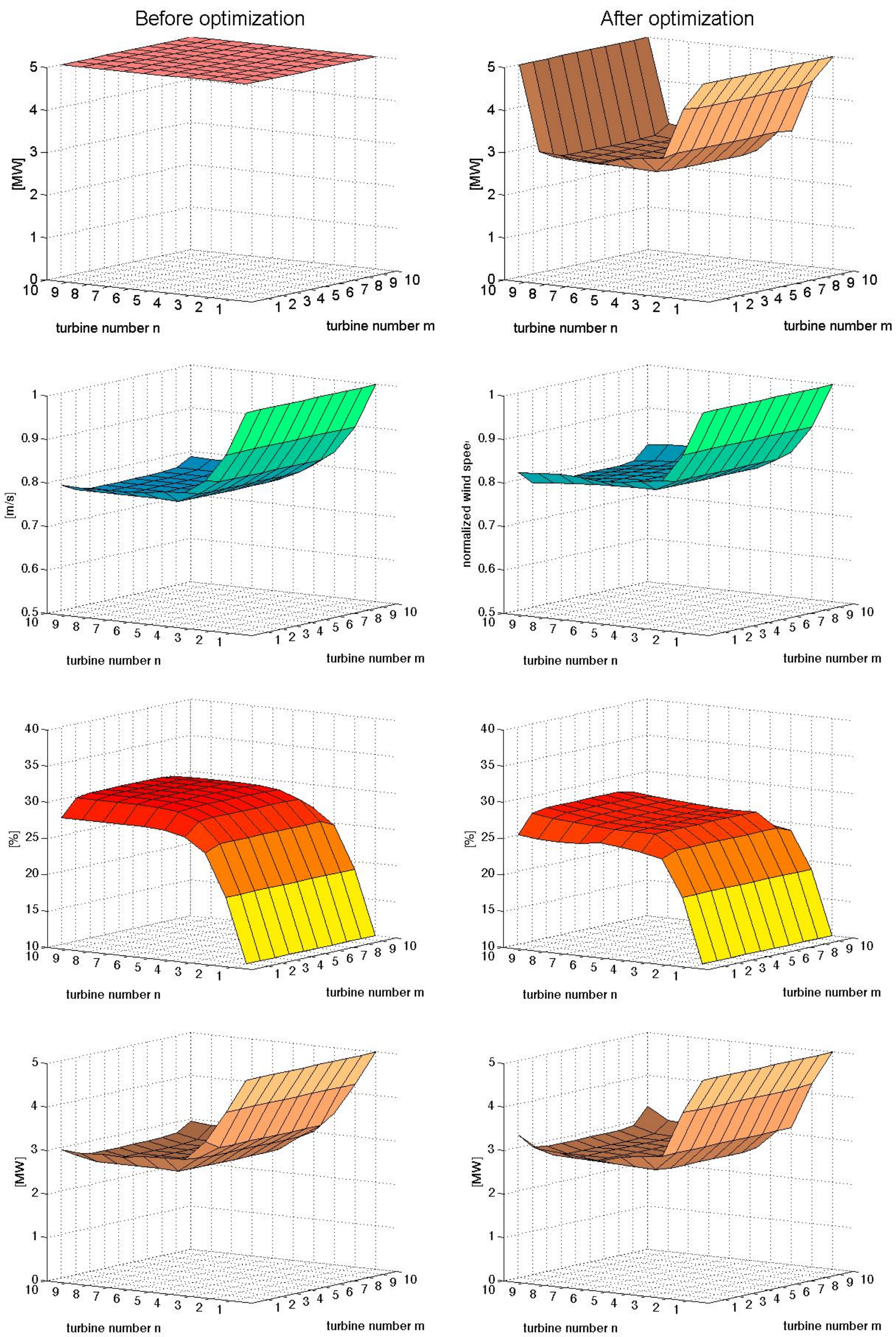


Figure 4.3 Power references, wind speeds, turbulence intensity, and power distribution for a 10x10 farm, before and after optimization, for *Power-Fatigue trade off, original model configuration.* Ambient wind speed is 12 m/s and ambient turbulence intensity is 0.1%. Top: Power references. Second from top: Wind speed. Second from bottom: Turbulence intensity. Bottom: Power distribution.

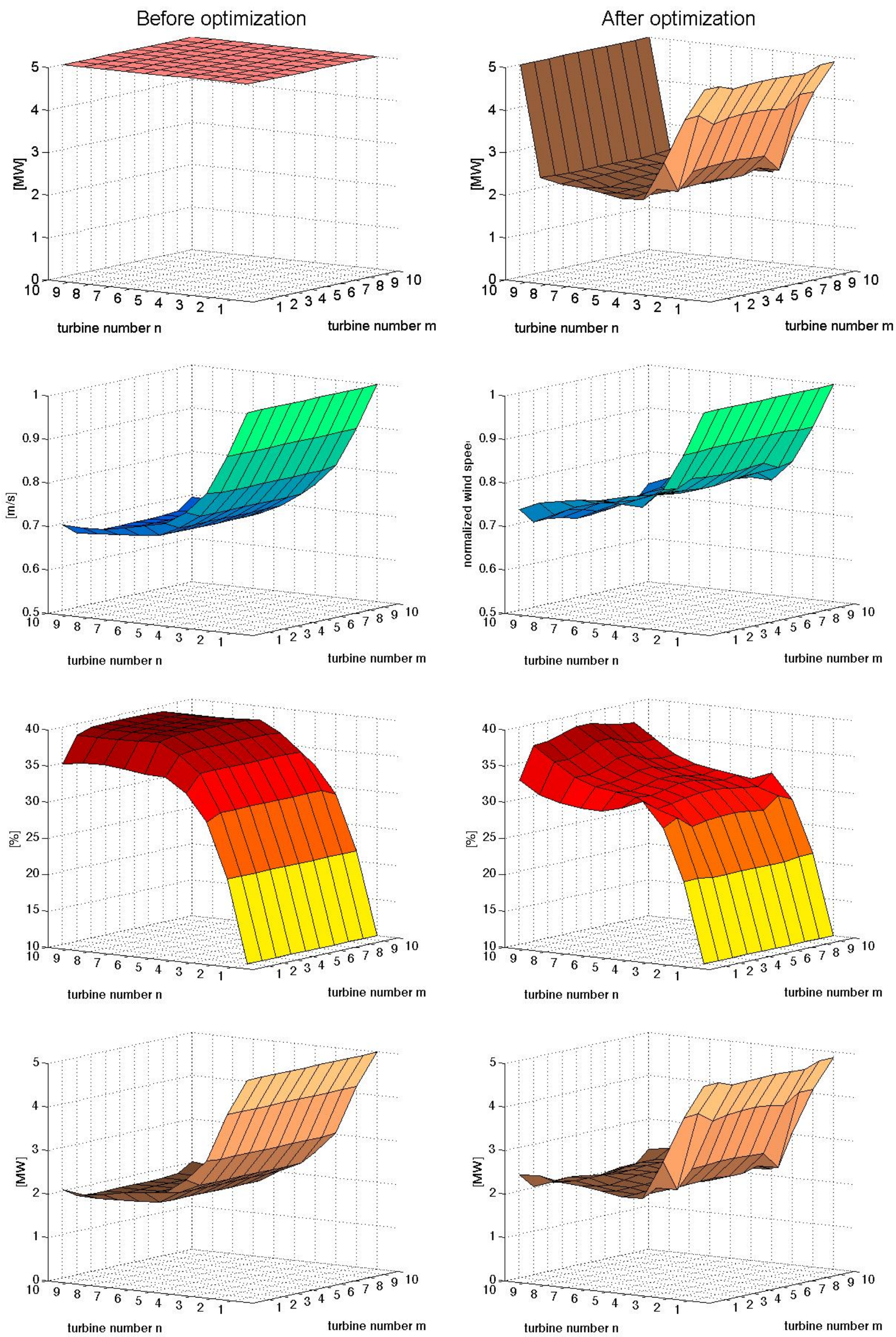


Figure 4.4 Power references, wind speeds, turbulence intensity, and power distribution for a 10x10 farm, before and after optimization, for *Power-Fatigue trade off, boosted model configuration.* Ambient wind speed is 12 m/s and ambient turbulence intensity is 0.1%. Top: Power references. Second from top: Wind speed. Second from bottom: Turbulence intensity. Bottom: Power distribution.

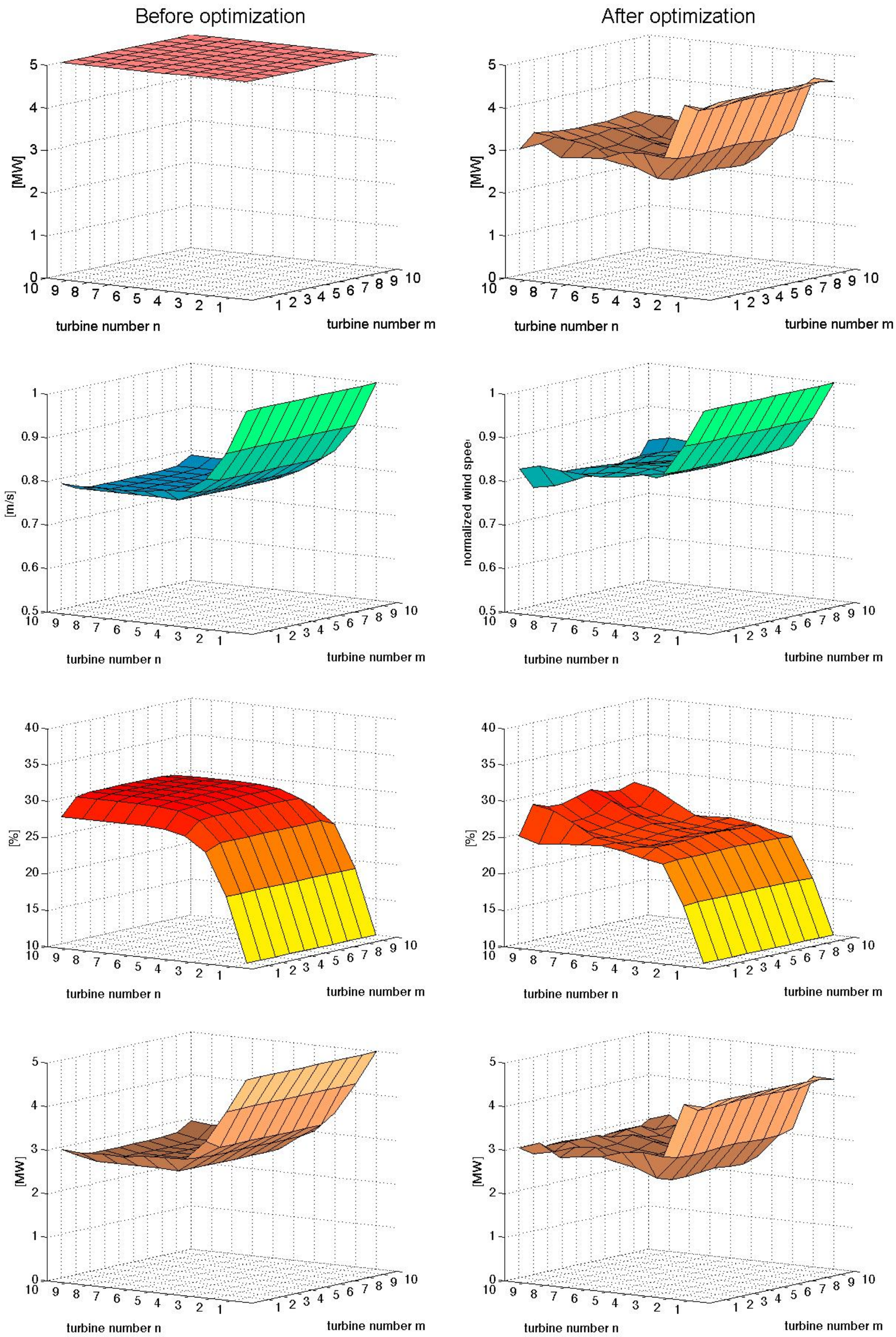


Figure 4.5 Power references, wind speeds, turbulence intensity, and power distribution for a 10x10 farm, before and after optimization, for *Power plant optimization (100%), original model configuration.* Ambient wind speed is 12 m/s and ambient turbulence intensity is 0.1%. Top: Power references. Second from top: Wind speed. Second from bottom: Turbulence intensity. Bottom: Power distribution.

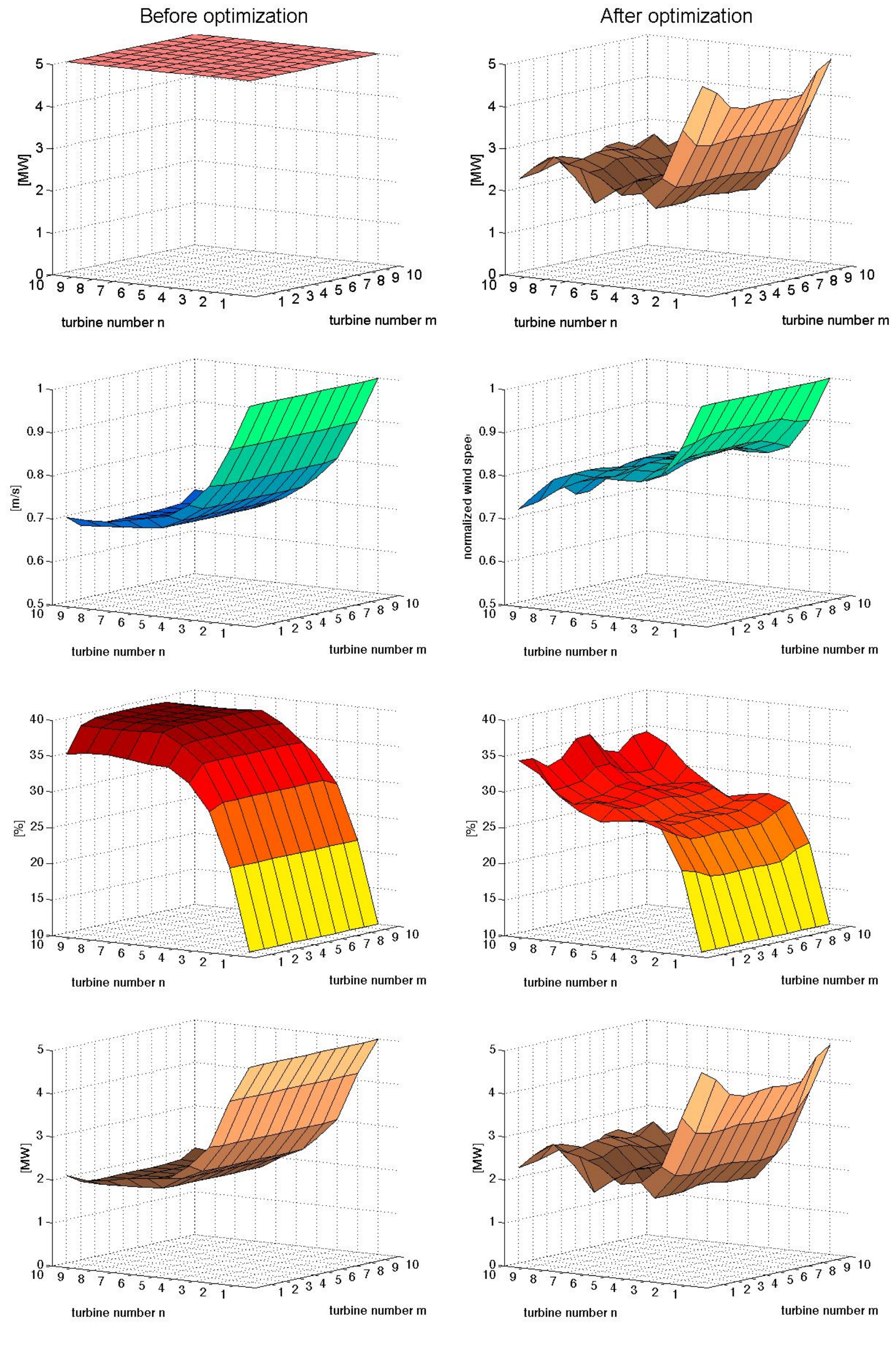


Figure 4.6 Power references, wind speeds, turbulence intensity, and power distribution for a 10x10 farm, before and after optimization, for *Power plant optimization (100%), boosted model configuration.* Ambient wind speed is 12 m/s and ambient turbulence intensity is 0.1%. Top: Power references. Second from top: Wind speed. Second from bottom: Turbulence intensity. Bottom: Power distribution.

5. Analysis and conclusions

In this final chapter finalizes the main theme of the thesis. Here we will present the analyze the results from the Optimization chapter, and present the conclusions we have come to regarding both the models, which we presented in the Modeling chapter, and the distribution of stationary optimal power references that we found for the solved optimization problems. Finally, we give some recommendations on further work regarding the subject of our thesis — large scale plant control.

5.1 Analysis

Not surprisingly, the two model configurations gave varying results for the optimization. When the wake effects are given a larger impact on the wind and turbulence situation of the downwind turbines, the effect of adjusting the power references will also be of greater impact. The relationship between wind and power cubic, and consequently, a small adjustment of wind will impact the power output with a power 3. For 8 m/s, better power reference distribution for the optimization objectives Power maximization and Cost minimization could not be found. This indicates that for the proposed models, the wake effects in lowered power output for an upwind turbine could not be regained with a surplus for downwind turbines. However, for higher wind speeds, ranging between 10 and 14 m/s, beneficial distributions were found. The most potential was found for 12 m/s, indicating an area of wind speed giving rise to the most trade off of extracting less power from the wind for the turbines standing in the front rows, in favor for the downwind turbines. This is the result of 12 m/s being the closest wind speed to rated wind speed for the NREL 5 MW, i.e. the possibility to affect the turbine states with changed power references is the largest.

The solutions for Power maximization and Cost minimization objectives resulted an increase in total farm output and at the same time a lower total amount of experienced turbulence, even though turbulence was not accounted for in the cost function of the former. This is in line with the interconnection between the wind deficit and turbulence accounted for in the proposed turbulence model, since the wind speed coming in to a turbine, together with the surrounding wind, determines the added wake turbulence.

The C_T and C_P table oppose a difficulty, since the interpolations performed on them for obtaining the coefficients and their derivatives with respect to P_{ref} do not give exact values of the coefficients. The result is that the values for each iteration is slightly off, even though adjusted as described in Appendix 4.3, and the resulting differences should foremost be seen as indications, rather than exact figures.

5.2 Conclusions

There are surely better ways of giving turbines in large offshore wind farms a better distribution of power references. There *is* a wind deficit caused by the wakes of turbines within a farm, and it results in a lower power output. The turbulence level within the farm *is* higher, and higher fluctuations are connected to higher fatigue in the turbines. The wind deficit affects the turbulence level, and vice versa. There is yet a need to acknowledge conceptual models to be used for real time calculations of wind flow within large offshore wind farms.

The models presented in this thesis are based on former presented models, and motivated by their similar features, if adjusted to fit a certain behaviour. The starting point was to develop simple models for use in the optimization, and for that purpose they were adequate. Whether or not they capture the real wake phenomena or not is harder to say, because of the varying reports from measurements and models that were found in the literature search. Two model configurations were presented, from which the first was based on models presented by Frandsen *et al.* (2006) and Madjidian and Rantzer (2010) (wind deficit), and Frandsen (2007) (turbulence); and the second from worst case scenario measurements from the large offshore wind farm Horns Rev presented by Barthelmie, Rathmann *et al.* (2009) and conversations with experienced people in the field (wind deficit, turbulence).

In the models, the turbine thrust coefficient is the variable given to describe the downwind wake. If the model configuration makes the turbine leave a larger wake for the same thrust coefficient, then there will be more to be gained by lowering the thrust coefficient through the power reference.

It is a rough approximation that the relationship between turbulence, as standard deviations in wind speed, and turbine fatigue is directly linear. Nevertheless, the approach taken for the thesis, that a lower turbulence level should yield in a lower total fatigue for the farm, should not be completely out of hand, since there is a strong connection between the wind fluctuations and turbine loading. Therefore, the results point out that there is a lot to be done with controlling large offshore wind farms as a team, and not a group of egoistic individuals; beginning with finding the optimal power references for different wind speeds and power demands. What is more, if the relationship given by the models resembles the real wake effects under bad conditions, the power output could be increased in addition.

5.3 Further work

- Validation of the results from the optimization is key to assess the final conclusions of Section 5.2. Applying the models to real turbines would of course require a large buffer of security, but comparison with measurement data at hand is a more appropriate first step.
- In order to better map the effects of the turbulence on different components in the turbines, dynamics need to be considered. In addition, the models need to be expanded to account for more than points addressed to the turbine hubs.

- Giving turbine loading fatigue a cost in terms of power output is not straight forward, and more analysis is needed for the full coupling between turbulence and fatigue to be mapped.

- A superior optimization algorithm would provide more accurate results. Global and local maxima, convergence and error analysis would also make the method more reliable. In addition, there is an uncertainty in the lookup tables for C_T and C_P. Applied to a reliable source for the cofficients combined with a thorough optimization analysis might impact the outcome.

- The models point to another issue, related to the large offshore farms being built in the vicinity of each other. Downwind *farms* are also affected by the wind deficit from other farms, implying that there need to be more investigations for large offshore wind farm planning (Svensson, 2010).

References and bibliography

Aeolus, (2010). *http://www.ict-aeolus.eu/index.html*, as of 2010-10-10.

Barthelmie, R. J., Frandsen, S. *et al.*, (2007). 'Modelling and measurements of power losses and turbulence intensity in wind turbine wakes at Middelgrunden offshore wind farm.' *Wind Energ.*, **10**, 517–528.

Barthelmie, R. J., Rathmann, O. *et al.*, (2007). 'Modelling and measurements of wakes in large wind farms.' *J. of Physics: Conference Series*, **75**, 012049. IOP Publishing Ltd, UK.

Boyd, S. and Vandenberghe, L., (2009). *Convex optimization*. Cambridge University Press, USA.

Burton *et al.*, (2001). *Wind energy handbook*. John Wiley & Sons, Ltd., England.

Crespo, A. and Hernandez, J., (1996). 'Turbulence characteristics in wind turbine wakes.' *J. of Wind Eng. and Ind. Aerodyn.*, **61**, 71–85.

Crespo, A., Hernandez, J. and Frandsen, S., (1999). 'Survey of modelling methods for wind turbine wakes and wind farms.' *Wind Energ.*, **2**, 1–24.

Danish Wind Industry Association, (2003). 'Wind turbine design: Basic load considerations.' Retrieved from: *http://guidedtour.windpower.org/en/tour/design/index.htm#anchor989446*, 2010-10-10.

European Commission, (2010). 'The EU climate and energy package.' Retrieved from: *http://ec.europa.eu/environment/climat/climate_action.htm*, 2010-10-10.

Frandsen, S., (2007). *Turbulence and turbulence generated structural loading in wind turbine clusters.* Ph.D. thesis, Risø National Laboratory for Sustainable Energy, Denmark.

Frandsen, S. *et al.*, (2006). 'Analytical modelling of wind speed deficit in large offshore wind farms.' *Wind Energ.*, **9**, 39–53.

Gardner, P. *et al.*, (2009). 'Technology.' *Report from Wind Energy - The Facts.* European Wind Energy Association consortium.

Global Wind Energy Counsil, (2010). 'Germany.' Retrieved from: *http://www.gwec.net/index.php?id=129*, 2010-10-29.

Gómez-Elvira, R. *et al.*, (2005). 'Anisotropy of turbulence in wind turbine wakes.' *J. of Wind Eng. and Ind. Aerodyn.*, **93**, 797–814.

Hammerum, K., (2006). *A fatigue approach to wind turbine control.* M.Sc. thesis, Technical University of Denmark.

Höjstrup, J., (1999). 'Spectral coherence in wind turbine wakes.' *J. of Wind Eng. and Ind. Aerodyn.*, **80**, 137–146.

Jonkman, J. *et al.*, (2009). 'Definition of a 5-MW reference wind turbine for offshore system development.' *Technical report NREL/TP-500-38060*. National Renewable Energy Laboratory, USA.

Madjidian, D. and Rantzer, A., (2010). 'A stationary turbine interaction model for control of wind farms.' *Article submitted to 18th IFAC World Congress, 2011*. Department of Automatic Control, LTH, Lund University, Sweden.

Morthorst, P. E. *et al.*, (2009). 'The economics of wind power.' *Report from Wind Energy - The Facts.* European Wind Energy Association consortium.

Pullen, A., Hays, K. and Knolle, G., (2009). 'Industry and markets.' *Report from Wind Energy - The Facts.* European Wind Energy Association consortium.

Réthoré, P.-E., (2009). *Wind turbine wake in atmospheric turbulence.* Ph.D. thesis, Aalborg University, Denmark; Risø National Laboratory for Sustainable Energy, Denmark.

Svensson, J., (2010). Interview with Ass. Prof. Ph.D. Jörgen Svensson, IEA, Lund University, Sweden.

Thomsen, K. and Sørensen, P., (1999). 'Fatigue loads for wind turbines operating in wakes.' *J. of Wind Eng. and Ind. Aerodyn.*, **80**, 121–136.

Vermeer, L. J., Sørensen, J. N. and Crespo, A., (2003). 'Wind turbine wake aerodynamics.' *Progress in Aerospace Sci.*, **39**, 467–510.

Vestas, (2010). Conversations with staff at Technology R&D, Aarhus, Denmark.

A. Optimization by hand

As a part of the thesis, the optimization was done by hand. This appendix describes the process of differentiating the cost functions, and how the implementation was done. The results differ a little from those obtained with MatLab (see Section 4.3) and are left out. However, the implementation done by hand gave us valueable insight in the interdependencies between the turbines, and also provided an opportunity to learn more about optimization in general. The appendix can be left aside for readers interested in the results, and is merely a documentation of our learning outcomes.

A.1 Differentiation of the cost functions

For this optimization, the gradient descent method was used. Independent which cost function is referred to, the process for the method is the same with the exception that the Power plant optimization objective includes a requirement on total power output (restraining the reference signals).

The gradient of the cost function Γ is calculated with respect to $\mathbf{u}$ according to:

$$\nabla\Gamma = \begin{pmatrix} \frac{\partial\Gamma}{\partial u_{1,1}} & \frac{\partial\Gamma}{\partial u_{1,2}} & \cdots & \frac{\partial\Gamma}{\partial u_{1,M}} & \frac{\partial\Gamma}{\partial u_{2,1}} & \cdots & \frac{\partial\Gamma}{\partial u_{N,M-1}} & \frac{\partial\Gamma}{\partial u_{N,M}} \end{pmatrix} \tag{A.1}$$

Once the gradient is determined a step has been taken in its direction, $\mathbf{u}$ will be updated to represent a new set of power references. The new set is the sum of the previous set, and the normalized gradient times the step length:

$$\mathbf{u}_{\text{updated}} = \mathbf{u}_{\text{previous}} + \frac{\nabla\Gamma}{\|\nabla\Gamma\|} \cdot step \tag{A.2}$$

By iterating the procedure a more beneficial set of power references is identified according to the chosen objective function (as long as the gradient finds a direction to make “the descent" in).

For the Power plant optimization objective, the procedure is extended to include the constraint on the reference signals with respect to P_{plant}, through *gradient projection*. The resulting gradient to use for the step according to (A.2) becomes with the projection:

$$\nabla\Gamma_{\text{proj}} = \nabla\Gamma - \frac{\nabla\Gamma(\nabla\gamma)^T}{\nabla\gamma(\nabla\gamma)^T}\nabla\Gamma \tag{A.3}$$

where $\nabla\gamma$ is:

$$\nabla\gamma = \begin{pmatrix} \frac{\partial\gamma}{\partial u_{1,1}} & \frac{\partial\gamma}{\partial u_{1,2}} & \cdots & \frac{\partial\gamma}{\partial u_{1,M}} & \frac{\partial\gamma}{\partial u_{2,1}} & \cdots & \frac{\partial\gamma}{\partial u_{N,M-1}} & \frac{\partial\gamma}{\partial u_{N,M}} \end{pmatrix} \tag{A.4}$$

The interdependencies of the turbines are illustrated in Figure A.1 and the specific differentiations are described in Appendix A.1.

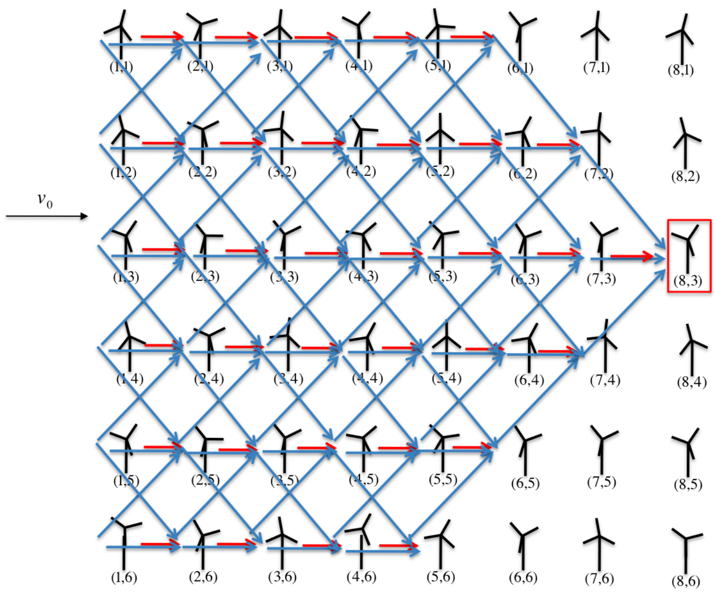


Figure A.1 The interdependencies of the turbines in a farm and their wakes, with turbine at position $8,6$ is marked as example. Red line represent the direct influence from the closest upwind neighbor on a turbine. The blue lines represent the indirect influence from other upwind turbines, through their impact on the wake properties surrounding the downwind turbines.

A.2 Algorithm

The gradient descent method is simple and straightforward. By differentiating the objective function, a local descent can be found. The strategy is to iterate many small steps in the direction of the gradient in order to identify a minimum. An issue with the method used in its simplest form is the imminent risk to end up in a local minimum instead of a more favorable global minimum. Another disadvantage with the method is that it is time consuming. Since the step length will affect the accuracy of the solution, the tradeoff between step length and number of iterations will make the solution slightly off its exact value. (Boyd and Vandenberghe, 2009)

Gradient descent algorithm in practice

According to chosen objective function, the practical implementation involves deriving the partial derivatives for the gradients and iterating the procedure of calculating them for each set of $\mathbf{u}$ and $\mathbf{v}$, for a chosen number of steps taken from the initial conditions of $P_{\text{ref}} = P_{\max}$ for all turbines[1].

As mentioned in Section A.2, the chosen step length will affect the accuracy of the solution and the algorithm's ability to find a solution. Since time is a critical factor for testing various initial conditions in wind and turbulence, and since the focus of the thesis is on investigating the potential with addressing the optimization problem, rather than analyzing the solutions by means of optimization theory; step length and number of iterations have been chosen based on intuition. The step length was chosen

[1]Note that the turbines standing in the furthest downwind will always have the power reference $P_{\text{ref}} = P_{\max}$, because their wake will not affect any other turbines.

to 0.01 MW for lower wind speeds (8 m/s and 10 m/s) and 0.1 MW for higher wind speeds (12 m/s and 14 m/s). See Appendix **??** for a selection of code implementing the algorithm.

The lookup tables for C_T and C_P are not given upper or lower limits for the possible power output calculations of the turbines static states, which had to be taken into account when updating the new power references for each iteration of the algorithm. Before each step was taken, checks were made to ensure that the new power references for the next iteration would not be larger than $P_{\text{max}} = 5$ MW (i.e. the maximum power output), or smaller than $P_{\text{min}} = 0.1$ MW (i.e. the lower limit of the lookup tables).

For the Power Plant Optimization, the initial power reference distribution was chosen as P_{plant} divided equally over the turbines. However, before the algorithm was initiated, checks were made to ensure that P_{plant} was possible to achieve with the wind deficit model configuration in use, and to ensure that P_{plant} really was achieved with the equal distribution, if possible. If the latter was not fulfilled, the equal distribution was increased until the initial power output yielded P_{plant}, and then the algorithm was started. Since the solution from the algorithm not is exact, there will be a small difference in power output.

A.3 C_P and C_T lookup tables modification

From the cost functions follow that C_P and C_T need to be expressed as functions of P_{ref} and v. However, in the lookup tables for NREL 5 MW, the coefficients are expressed as λ and β. Before differentiating the cost functions, new lookup tables were made. In addition, the new lookup tables were also modified in order to get “smoother" functions. See Appendix A.3 for a further description of the procedure, along with interpolated plots of the original and modified functions.

The thrust coefficient C_T is commonly expressed in terms of tip speed ratio λ and pitch angle β, both depending on P_{ref} and v. As $C_T(\lambda, \beta)$ is included in the wind speed deficit model as well as the turbulence model, it needs to be mapped to P_{ref} and v — i.e. as $C_T(P_{\text{ref}}, v)$ — for the optimization described further on in this section. The same concerns C_P, which links wind speed with power output (see Section 2.1 for more details on the two coefficient functions).

Mapping C_T and C_P to P_{ref} and v is complex and involves the turbine dynamis. A Matlab script provided by Aeolus (Spudić *et al.*, 2010) calculates turbine states (i.e. static parameters) for the NREL 5MW for a given combination of power reference and wind speed, including C_T and C_P. The coefficients are obtained from a look up table for different combinations of λ and β. The first step for the modification was to transform these tables into look up tables for different combinations of P_{ref} and v.

Large irregularities can be identified for both functions on the edge of their drop offs. This behaviour is related to a specific operating mode for NREL 5 MW, which occurs for a certain set of combinations of P_{ref} and v. Since this set contains few combinations but result in extreme values, both functions (i.e. look up tables) have been smoothen out. The modified functions will be a better basis for the optimization, since for real conditions the wind speed includes variations and as this mode occurs within a very limited interval and is not desirable to track. Figure A.2 shows the interpolated $C_T(P_{\mathrm{ref}}, v)$ and $C_P(P_{\mathrm{ref}}, v)$ look-up tables of NREL 5MW. The modified functions are shown in Figure A.3.

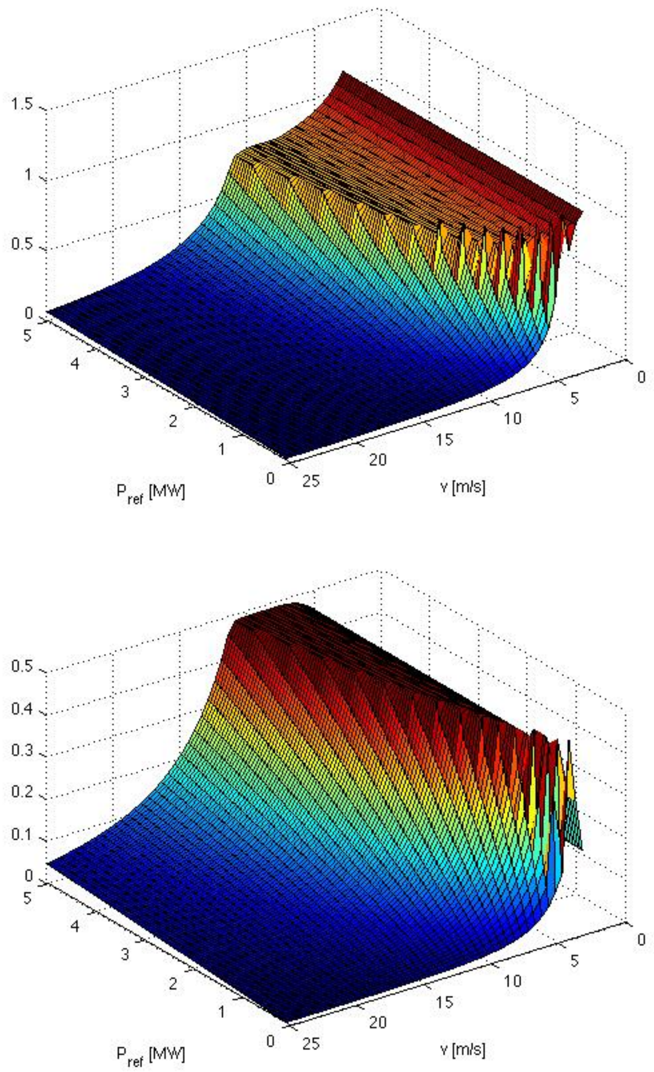
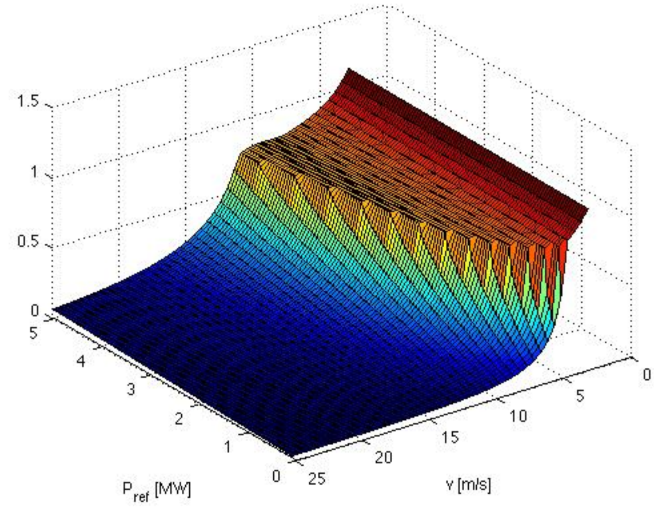


Figure A.2 Interpolation of the derived lookup tables for $C_T(P_{\mathrm{ref}}, v)$ and $C_P(P_{\mathrm{ref}}, v)$ for NREL 5 MW, before modification (see Figure 2.3 for the original tables).



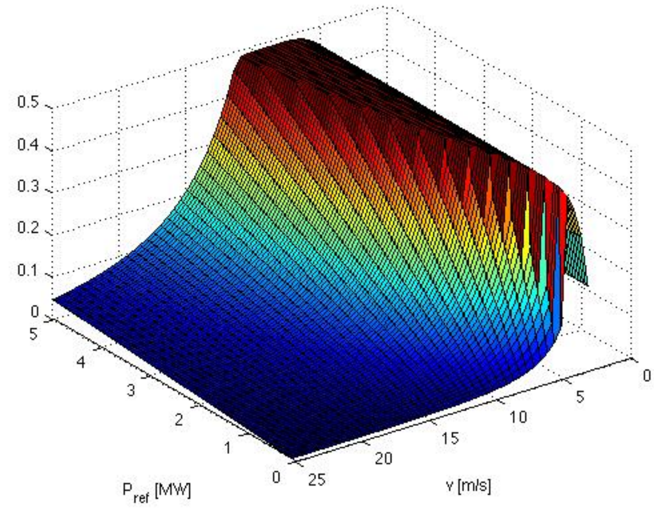


Figure A.3 Interpolation for the modified lookup tables for $C_T(P_{\text{ref}}, v)$ and $C_P(P_{\text{ref}}, v)$ for NREL 5 MW. Mode 5 has been taken out of the tables, resulting in a smoother curve.

B. Educational requirements and project plan

A compulsory part of Master's thesis projects done in pairs is to describe how the work has been divided between the thesis workers. This appendix gives a description of how the responsibilities have been divided between us and also contains the project plan and objectives for the project.

B.1 Responsibilities

The work has been been continuously done by both thesis workers, but certain areas of responsibilities have been divided in order to meet the educational requirements:

Project plan	—	Fredrik
Modeling	—	Fredrik & Thomas
Optimization in MatLab	—	Thomas
Report content and formatting	—	Fredrik & Thomas
Report text	—	Fredrik
Report figures	—	Thomas

Not in the report:

Contact person	—	Thomas
Presentation design	—	Fredrik
Scientific article	—	Fredrik
Accounting	—	Thomas
Optimization by hand	—	Fredrik & Thomas
Script implementation	—	Fredrik
Dynamic configuration	—	Thomas
NREL linearization	—	Fredrik & Thomas

Project plan

Master Thesis

Added wake turbulence and optimal power distribution in large off-shore wind farms.

2010-02-05
Lund University
Fredrik Himmelman and Thomas Alexander Clevenhult

LUNDS UNIVERSITET
Lunds Tekniska Högskola

in cooperation with
Vestas

1 Introduction

The purpose of this document is to create a blueprint of how we have planned to reach the assigned objectives for this master thesis project. We have identified and detailed a number of activities on different levels of the project and from these activities generated a draft time schedule based on time consumption and deadlines we think are proportionate for each activity. Not said this 'blueprint' is final, quite the opposite: We will continuously reevaluate it in order to make the best of the assigned time for the project.

2 Overall objectives

The execution part of this thesis project is divided into two phases, each with one overall objective:

1. Find a qualitative model of the added wake turbulence in wind farms. Since the model will be used for distributed control, the model should (if possible) have a distributed information propagation structure.
2. Find the stationary power reference distribution between the turbines that optimizes power and fatigue in the farm according to a given objective function. In this part the wake turbulence model from part 1 will be used together with existing turbine and wake deficit models to model the whole farm.

3 Organization

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Examinator:

Anders Rantzer (Lund University)	rantzer@control.lth.se

4 Results

- A qualitative model of the added wake turbulence in wind farms.
- The stationary power reference distribution that optimizes power and fatigue in a wind farm, given an objective function.
- Master thesis report.

5 Activity decomposition

By decomposing this thesis project into a number of simpler components bound over short time periods, we hope to easier maintain control throughout the whole process. First, the whole project has been divided into five main parts, or overall activities, as shown in Figure 1. Each of those has then been divided into more specific activities presented further on in this document.

When specifying the partial goals (milestones) we have adopted the SMART[1] goals technique to support us staying focused on the tasks leading to our main objectives. We believe it will work both as motivation as well as to help us deliver results.

The activities are summarized in tables in Appendix 1.

5.1 Project initiation

The initiation of the project is divided into two parts:

- To define and formulate the thesis objectives in a Thesis Project objectives document.
- To identify and specify all necessary activities and schedule them. The activity decomposition and time schedule will be summarized and described in this document, the Thesis Project plan.

After the initiation, the project will be defined, decomposed into activities and scheduled according to the time scope of the project. This will be the done during the first week of the project.

5.2 Phase 1

During the first phase literature will be studied and assessed in order to develop a suitable model for added wake turbulence. An iterative method will be used:

- Search for and assess relevant literature.
- Select and compile material.
- Develop a model prototype, built on the previous iteration or not.
- Evaluate the model prototype.

The purpose of the iterative method is to make the literature search narrower and the literature compound more refined for each iteration. This will make it possible to evaluate the modeling frequently on a sort of prototype basis and we can start simple and step by step gather deeper knowledge and direct the progress towards a model that fulfils our requirements[2]. The iterations have initially been set to twelve days. Searching and assessing literature for the first iteration start during the first week parallel to the project planning.

At the end of Phase 1 we would like to suggest a one-week stay at Technology R&D, Vestas Wind Systems A/S in Aarhus. The purpose of the visit would be to present the progress of the project up to that point and to get feedback from appointed staff with experience in the subject.

[1] SMART stands here for Specific, Measurable, Adjustable, Relevant and Time-bound. There are a few variations on the method as well as disagreements of who first developed it.
See for example: http://www.rapidbi.com/created/WriteSMARTobjectives.html#HistoryandoriginsoftheSMARTobjectivesacronym
[2] The requirements for the evaluation process will be specified in later reports.

5.3 Phase 2

The second phase of the thesis project is directly dependent on the first. The model from Phase 1 will be merged with a model for wake deficit in wind farms developed in the Aeolus Project. A cost function from the same project will also be given. The combined models and the cost function will form the optimization problem to be solved in this phase. Depending on the characteristics of the problem, the project plan will be reevaluated accordingly and activity table and time schedule will be specified then.

5.4 Project finalization

Other mandatory parts included in a master thesis, such as an oral presentation, a popular scientific article and a summary will be produced during this part. At this point most research work (executional tasks of Phase 2) should be completed or about to be completed and focus will turn gradually on finishing the report. There is a short over-lapping of Phase 2 and the Finalization part where we intend to work simultaneously with both.

6 Opposition

Critically reviewing another master thesis project is a mandatory part in the criteria of Lund University and includes opposing at the final presentation of the reviewed master thesis. The opposition will be handled parallel to the other activities during part of Phase 1.

7 Follow-up

Follow-up of our progress will be done continuously throughout the whole project.

During the initiation and creation of the Objectives and Project plan documents respectively, possible issues with definition and planning will be assessed in order to deal with them in an early stage of the project. This will ensure setting off Phase 1 in the right direction and will positive for the whole process.

In the end of each iteration of Phase 1 our model prototypes will be evaluated by us and our supervisor at Lund University. Methodically working towards an improved model will be gainful for the end product of Phase 1 and will affect the outcome of the project. Also, a visit to the Technology and Research department at Vestas Wind Systems A/S in the end of Phase 1 would give us an opportunity to get expert feedback and useful critique on our progressed model from our supervisors and other personnel in the department.

Although Phase 2 will be planned more in detail later in the project, follow-up and assessment of the progress will be the same: Frequent meetings with our supervisor at the university will be held.

Finally, the report will be reviewed by one or possibly two students also writing a master thesis as well as by our supervisors and examinator during the Finalization part of the project.

Appendix 1. Activity tables

Overall project activity		Duration (weeks)	Depends on activity	Milestone
OA1	Project initiation	1	-	Project plan
OA2	Phase 1	7,5	OA1	Added wake model
OA3	Phase 2	9	OA2	Reference distribution
OA4	Project finalization	3	OA1, OA2	Thesis report
OA5	Opposition	2	-	Opposition report

Table 1. The overall activities for this thesis project.

Project initiation activity		Duration (days)	Depends on activity	Milestone
PIA1	Objectives definition	2	-	Objectives document
PIA2	Project planning	5	PIA1	Project plan

Table 2. The activities of initiating the project.

Phase 1 iteration activity		Duration (days)	Depends on activity	Milestone
P1A1	Literature search	2	OA1	Literature list
P1A2	Literature compilation	3	P1A1	Literature compound
P1A3	Model prototyping	6	P1A2	Model prototype
P1A4	Prototype evaluation	1	P1A3	Evaluated model
P1A5	Proj. plan reevaluation	1	P1A4	Reevaluated proj. plan
P1A6	Suggested Vestas visit	5	-	-

Table 3. The activities of Phase 1. Iterative activities are P1A1-P1A5.

Phase 2 activity		Duration (days)	Depends on activity	Milestone
P2A1	Proj. plan reevaluation	1	-	Reevaluated proj. plan
P2A2	Phase formulation	?	OA2	Optimization problem
P2A3				
P2A4				
…				

Table 4. The activities of Phase 2. This table will be specified later in the project.

Project finalization activity		Duration (days)	Depends on activity	Milestone
PFA1	Report completion	14	-	Master thesis report
PFA2	Article writing	5	PFA1	Scientific article
PFA3	Presentation	10	PFA1	Examination

Table 5. The activities of finalizing the project.

Appendix 2. Gantt charts of the time scheduling

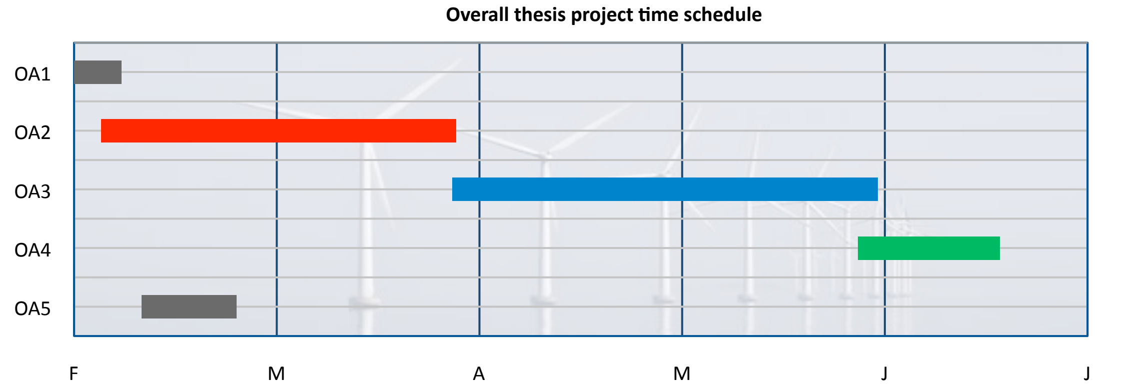

Figure 1. Overall activity time schedule.

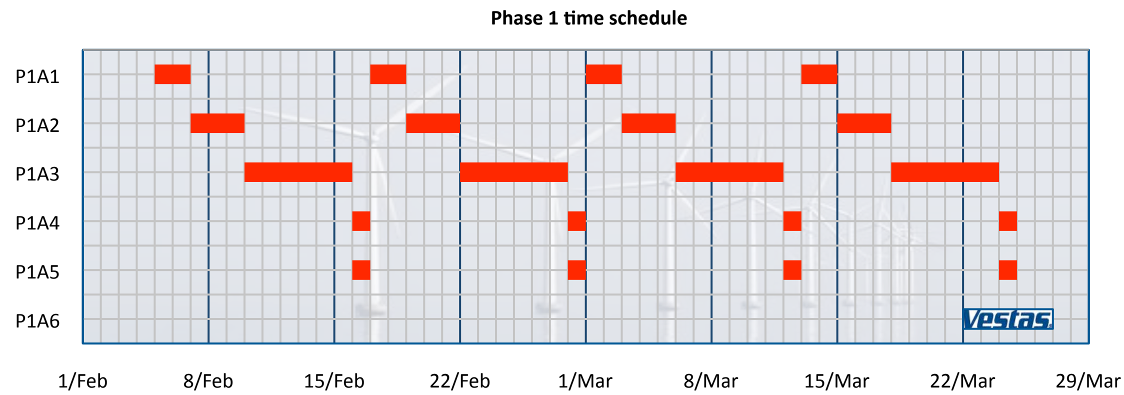

Figure 2. Phase 1 activity time schedule. Red bars show the iterative activities.

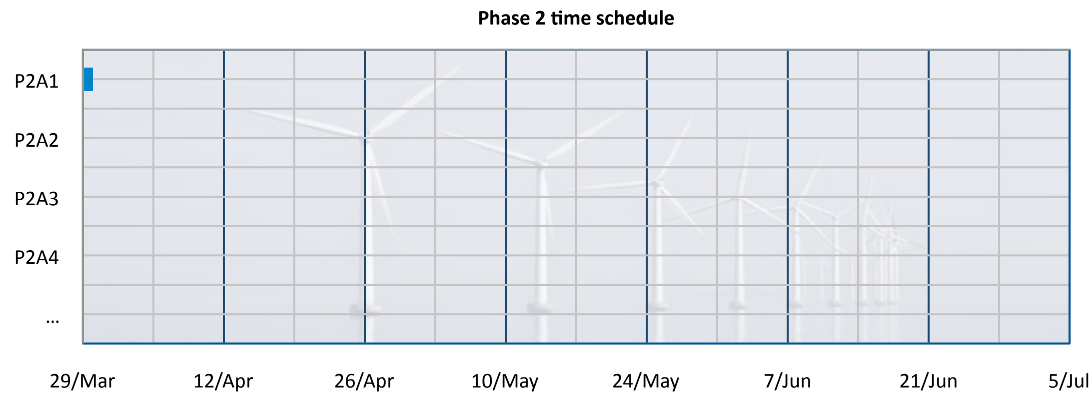

Figure 3. Phase 2 activity time schedule. This time schedule will be specified later in the project.