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Observer-Based Adaptive Control

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Unfortunately the way to use it in combination with the previous observer-controller has not been found and it has not been used in the experiments. In the Appendix A some technical details about the al-gorithm implementation are included, and in the Appendix B a paper already submitted to the 2002 Conference in Decision and Control is included, in which the adaptive output-feedback control scheme is extended for ship control. All the work has been conducted in the De-partment of Automatic Control, Lund Institute of Technology, Lund University.

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Francesco

Contents

1. Introduction

In control engineering the objective is to achieve a feasible control signal, that makes the considered plant behave in the desired way; this control signal is based on the mathematical model of the plant, on reference signals to be tracked, and on measured signals. There are several situations where we have not only uncertainties in the model, but also partial measurement of the state, and so this provides motivation in development of observer-based adaptive control schemes.

1.1 Observers

In many applications it can be very expensive, or even impossible, to install a physical sensor to measure directly certain quantities; in these cases, since the knowledge of the unmeasurable quantities is desirable anyway, a good solution is to provide an estimate by an observer. An observer is an algorithm that reconstructs the internal unmeasurable states of the system from the measurable output; in the linear case the observer theory is well investigated and the observability and detectability properties are closely connected to the existence of observers with strong convergence properties. However in the nonlinear case, the observer design problem has a systematic solution only if nonlinearities are functions of the measurable output and the input.

Observers for linear systems

Consider the linear system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1.1}$$

Under observability/detectability assumptions on the pair $[A, C]$ an observer for the system (1.1) can be constructed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}) \\ \hat{y} = C\hat{x} \end{cases} \tag{1.2}$$

where $K(y-\hat{y})$ is the linear output injection. If the gain matrix K is chosen such that $(A-KC)$ is Hurwitz, then the error dynamics are asymptotically stable, that is $\lim_{t\to\infty} \hat{x}(t) = x(t)$.

Observers for nonlinear systems

The notion of output injection can be applied also to a class of nonlinear systems in the form

$$\begin{cases} \dot{x} = Ax + f(y, u) \\ y = Cx. \end{cases} \tag{1.3}$$

As the nonlinearity f only depends on the measurable output and the known control signal, an observer for the system (1.3) can be constructed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + f(y,u) + K(y - \hat{y}) \\ \hat{y} = C\hat{x}. \end{cases} \tag{1.4}$$

Still, if the gain matrix K is chosen such that $(A - KC)$ is Hurwitz, then the linear error dynamics are asymptotically stable.
Even if the system description is not directly in the form (1.3), there could exist an invertible state transformation $\chi = S(x)$ to make the system in the form (1.3), with linear error dynamics. The convergence $\lim_{t\to\infty} \hat{\chi} = \chi$ then implies $\lim_{t\to\infty} \hat{x} = x$.

Thau [28] has considered a system in the form

$$\begin{cases} \dot{x} = Ax + f(x,u,t) + \phi(y,u,t) \\ y = Cx \end{cases} \tag{1.5}$$

For the observer

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + f(\hat{x},u,t) + \phi(y,u,t) + L(y - \hat{y}) \\ \hat{y} = C\hat{x} \end{cases} \tag{1.6}$$

conditions for asymptotic stability of the error are given, provided that the nonlinearity $f(x,u,t)$ is Lipschitz with respect to the state x.

Arcak and Kokotovic have suggested an observer design for a class of systems with monotone sector nonlinearities in the unmeasured states [1]. They have considered the system

$$\begin{cases} \dot{x} = Ax + G\psi(Hx) + \varphi(y,u) \\ y = Cx \end{cases} \tag{1.7}$$

and the observer

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + L(y - \hat{y}) + G\psi(H\hat{x} + K(y - \hat{y})) + \varphi(y,u) \\ \hat{y} = C\hat{x}. \end{cases} \tag{1.8}$$

The observer design decomposes the error dynamics into a linear system in feedback with a multivariable sector nonlinearity. Linear matrix inequalities are used to state the conditions for the existence of stable observer error dynamics with respect to the imposed observer structure.

1.2 Observer-based control

The observer-based control is a way to solve the output feedback control problem, that implies a restriction in the possibility to use all the states directly for feedback. Still for linear system the well- known separation

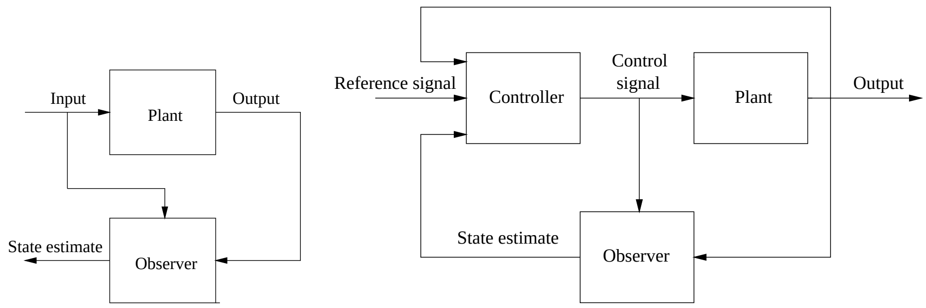

Figure 1.1 Observer scheme and observer-based control scheme

principle allows the problem to be decomposed in two sub-problems which can be solved separately: the design of a state observer, and the design of a state-feedback controller. For nonlinear system the separation principle does not apply, and the design of a state observer is generally coupled with the controller design. However, when the Input-To-State-Stability (ISS) property can be assured for disturbances entering additively to the states in a stabilizing state feedback law, estimates from a converging state observer can be used in a certainty equivalence approach. Another fundamental difference, correlated with the separation principle, in properties of linear and nonlinear systems is the effect of bounded disturbances over a finite time horizon. Consider a linear system for which there is a stabilizing state-feedback law. If the states are replaced by estimated states, then the closed loop system will still be stable if the observer errors converge to zero. This is not ensured for nonlinear systems, even if we have exponential convergence in the observer, because of the "finite escape time" phenomenon, that is illustrated in the following example [20].

EXAMPLE 1.1
Consider the system

$$\begin{cases} \dot{x} = -x + x^4 + x^2\xi \\ \dot{\xi} = -k\xi + u, \end{cases} \tag{1.9}$$

with $k > 0$. Using backstepping and defining the error variable $z = \xi + x^2$, a stabilizing state feedback-law can be found as

$$u = -cz - x^3 + k\xi - 2x\Big(- x + x^2 z\Big), \tag{1.10}$$

with $c > 0$ a design constant. The asymptotically stable closed loop system is

$$\begin{cases} \dot{x} = -x + x^2 z \\ \dot{z} = -cz - x^3. \end{cases} \tag{1.11}$$

Suppose now that the state ξ is not measured. However it can be estimated by the observer

$$\dot{\hat{\xi}} = -k\hat{\xi} + u, \tag{1.12}$$

with the estimation error $\tilde{\xi} = \xi - \hat{\xi}$ converging exponentially to zero:

$$\dot{\tilde{\xi}} = -k\tilde{\xi} \quad \Rightarrow \quad \tilde{\xi}(t) = \tilde{\xi}(0)e^{-kt}. \tag{1.13}$$

By using the estimated state $\hat{\xi}$ in the control law (1.10), we have the following closed loop system

$$\begin{cases} \dot{x} = -x + x^2 z + x^2\tilde{\xi} \\ \dot{z} = -cz - x^3 + 2x^3\tilde{\xi} \\ \dot{\tilde{\xi}} = -k\tilde{\xi}. \end{cases} \tag{1.14}$$

Consider the case of $z \equiv 0$, then we have

$$\dot{x} = -x + x^2\tilde{\xi}, \quad \tilde{\xi}(t) = \tilde{\xi}(0)e^{-kt} \tag{1.15}$$

with solution

$$x(t) = \frac{x(0)(1+k)}{[1+k-\tilde{\xi}(0)x(0)]e^t + \tilde{\xi}(0)x(0)e^{-kt}} \tag{1.16}$$

which escapes to infinity in finite time for all initial conditions $\tilde{\xi}(0)x(0) > 1+k$. □

Linear systems

Consider a linear time-invariant system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1.17}$$

which is controllable and observable. For such a system the separation principle apply for both full order and reduced order observer. The solution to the Linear Quadratic Gaussian (LQG) optimization problem is a combination of the state-feedback controller, that is the solution to the LQ problem, and the optimal Kalman filter.

Nonlinear systems

Feedback linearization with an observer For the output feedback case is not possible in general to achieve a feedback linearized system, and methods based on pseudo-linearization have been proposed [29]. The certainty equivalence approach using estimates provided with an observer in the linearizing feedback law have been studied and stability results have been proven, under Lipschitz conditions on the nonlinearities [13].

Observer backstepping In this procedure, first a nonlinear observer is designed which provides exponentially convergent estimates of unmeasured states. Then, backstepping is applied to the error between the estimated states and the desired trajectory, instead of to the error between the true states and the desired trajectories. At each step of the method, observation errors are treated as disturbances to be compensated by using nonlinear damping. Observer backstepping can be used to construct systematic design procedures applicable to nonlinear systems for which exponential observers are available.

High-gain observers and bounded control Another approach to solve the output-feedback control problem for nonlinear systems has been presented by Atassi and Khalil [5]. It consists in a high-gain observer, that robustly estimates the unmeasured states, and a globally bounded state-feedback control, usually obtained by saturating a continuous state-feedback function outside a compact region of interest, that meets the design objectives. Furthermore a separation theorem is presented, that is independent of the state-feedback design, and not only the region of attraction is shown to be recovered, but also the performance and the trajectories of the system under state-feedback.

1.3 Adaptive control

In a large number of practical problems there are significant variations in disturbances, uncertainties, or model parameters; in these cases a solution of the control problem is given by adaptive control. Intuitively, an adaptive controller is a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances. This can be made by a controller with adjustable parameters and a mechanism for adjusting the parameters. Again for linear systems with unknown parameters there are several well-known adaptive schemes that solve the control problem; instead there are restricted classes of nonlinear systems for which the design problem is solvable.
An important feature of adaptive control is its reliance on "certainty equivalence" controllers. This means that a controller is first designed as if all the plant parameters were known, and the controller parameters are calculated as functions of the plant parameters, by solving design equations. When the actual plant parameters are not known, the controller parameters are either estimated directly (direct schemes) or computed by solving the same design equations with plant parameter estimates (indirect schemes). The resulting controller is called a certainty equivalence controller. It is not at all obvious that a certainty equivalence controller will work in an adaptive feedback loop and achieve stability and tracking. Hence, it is significant that certainty equivalence controllers have been proven to be satisfactory for adaptive control of linear system.
The following example [20] show the difficulties arising in the adaptive control problem for nonlinear systems:

EXAMPLE 1.2
Consider the nonlinear system

$$\dot{x} = u + \theta x^2. \tag{1.18}$$

A certainty equivalence control law is given by

$$u = -px - \hat{\theta} x^2, \tag{1.19}$$

which, if $\hat{\theta} \equiv \theta$, result in the asymptotically stable closed-loop system $\dot{x} = -px$. Since $\dot{x}$ is not available for measurement, we filter (1.19) by $\frac{1}{s+1}$, and defining

$$x_{f1} = \frac{1}{s+1} x, \quad x_{f2} = \frac{1}{s+1} x^2, \quad u_f = \frac{1}{s+1} u, \tag{1.20}$$

we can rewrite (1.19) as

$$x - x_{f1} = u_f + \theta x_{f2} \tag{1.21}$$

and replacing θ with its estimate $\hat{\theta}$ we obtain the corresponding predicted value of x

$$\hat{x} = x_{f1} + u_f + \hat{\theta} x_{f2} \tag{1.22}$$

and the prediction error

$$e = x - \hat{x} = (\theta - \hat{\theta}) x_{f2} = \tilde{\theta} x_{f1}. \tag{1.23}$$

A possible parameter update law for $\hat{\theta}$, derived to aim at a minimum of e^2 is

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\gamma \frac{x_{f2}^2}{1 + x_{f2}^2} \tilde{\theta}. \tag{1.24}$$

It is obvious that $\tilde{\theta}$ cannot converge to zero faster than exponentially. Let us consider the most favorable case, where

$$\tilde{\theta}(t) = e^{-\gamma t} \tilde{\theta}(0). \tag{1.25}$$

Setting $p = 1$ and $\gamma = 1$ for simplicity, the nonlinear closed-loop is

$$\dot{x} = -x + \tilde{\theta} x^2 = -x + x^2 e^{-t} \tilde{\theta}(0), \tag{1.26}$$

whose explicit solution is

$$x(t) = \frac{2x(0)}{x(0)\tilde{\theta}(0)e^{-t} + [2 - x(0)\tilde{\theta}(0)]e^t} \tag{1.27}$$

which escape to infinity in finite time if $x(0)\tilde{\theta}(0) > 2$. □

This example demonstrates why a traditional estimation-based design cannot be applied to nonlinear systems, and hence, it needs faster identifiers, which can be provided by Lyapunov-based design.

1.4 Observer-based adaptive control

Consider a plant with uncertain parameters and whose state is only partially available by the measurements; for these systems a solution of the control problem is the observer-based adaptive control, that is the combination of a time-varying controller, a state observer and a parameter tuning mechanism. Although only for linear systems, thanks to the separation principle, it is ensured the possibility of decomposing the problem in two or three sub-problems, there are adaptive schemes which allow a separation of the controller, the parameter update law and the state observer also for some classes of nonlinear systems [20].

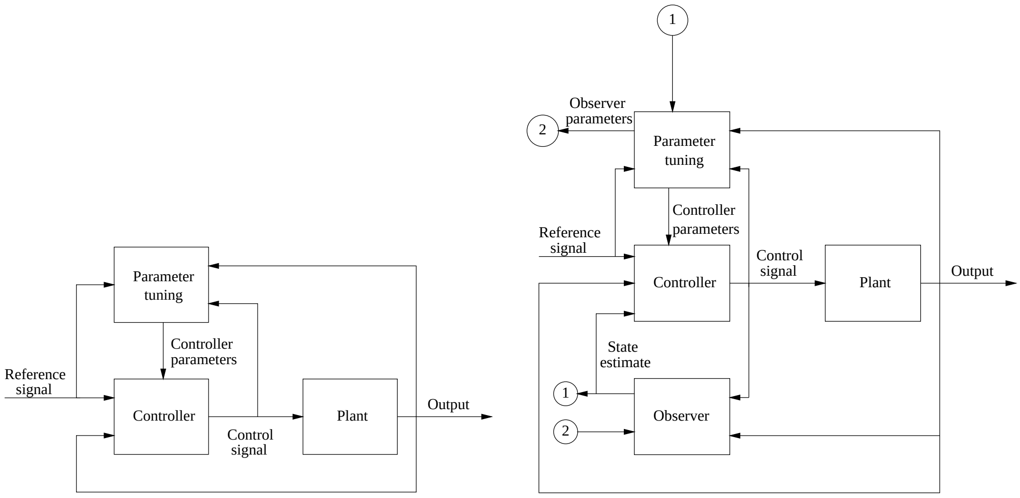

Figure 1.2 Adaptive control scheme and observer-based adaptive control scheme

Adaptive observer backstepping

Krstic et al. have proposed an observer-based adaptive control scheme for systems in output-feedback form [20], that is

$$\begin{cases} \dot{x} = Ax + \phi(y) + F^T(y,u)\theta \\ y = e_1^T x, \end{cases} \tag{1.28}$$

where θ is the unknown parameter vector, $\phi(y)$, $F^T(y,u)$ are nonlinear functions, and

$$e_1 = \begin{pmatrix} 1 \\ 0_{(n-1)\times 1} \end{pmatrix}, \quad A = \begin{pmatrix} 0_{(n-1)\times 1} & I_{(n-1)\times(n-1)} \\ 0 & 0_{1\times(n-1)} \end{pmatrix}. \tag{1.29}$$

The observer design procedure involves the K-filters as follows:
define the state estimate

$$\hat{x} = \xi + \Omega^T\theta \tag{1.30}$$

obtained with

$$\dot{\xi} = A_0\xi + ky + \phi(y) \tag{1.31}$$

$$\dot{\Omega}^T = A_0\Omega^T + F^T(y,u), \tag{1.32}$$

where the vector k is chosen so that the matrix

$$A_0 = A - ke_1^T \tag{1.33}$$

is Hurwitz. Thanks to the structure of $F^T(y,u)$, it can be written that

$$\dot{y} = \omega_0 + \omega^T\theta + \varepsilon_2 \tag{1.34}$$

where ε_2 is the second component of $\varepsilon = x - \hat{x}$, and ω_0, ω^T are the regressors, depending on Ω^T, ξ, $F^T(y,u)$ and $\phi(y)$. As only the state x_1 is measured, the backstepping is applied to the first column of Ω^T. With this procedure asymptotic tracking and boundedness of all signals are achieved.

High-gain observers and bounded control

Kahlil [19] has presented an output feedback adaptive control scheme, that combine an high-gain observer with a globally bounded control, obtained by saturating a state-feedback function outside a compact region of interest. This is done because high-gain observers exhibit a peaking phenomenon in their transient behaviour. The class of the considered systems is represented by the n-th order differential equation

$$y^{(n)} = f_0(\cdot) + \sum_{i=1}^{p} f_i(\cdot)\theta_i + \Big(g_0 + \sum_{i=1}^{p} g_i(\cdot)\theta_i\Big)u^{(m)} \tag{1.35}$$

where θ_i are unknown constant parameters, g_i are known constant parameters, u is the control input, y is the measured output, $y^{(i)}$ denotes the i-th derivative of y, and $m < n$. Furthermore the functions f_i are smooth nonlinearities depending on $y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)}$. It has been shown that the output feedback control asymptotically recovers the region of attraction achieved under state feedback.

H^∞-optimal output feedback adaptive control

Tezcan and Basar [27] have presented a systematic procedure for designing H^∞ adaptive controllers for systems in the output-feedback form (1.28). Robust identifiers are used for both unknown parameters and unmeasured states, together with backstepping and H^∞-filtering to achieve tracking of a smooth reference trajectory. It has been shown that arbitrarily small disturbance attenuation levels can be obtained at the expense of increased control effort.

2. A Reduced-Order Velocity Observer for Rigid Link Manipulators

The problem of estimating the velocity for robot manipulators is very relevant, because very few industrial robots are equipped with tachometers for velocity measurements of the links. A possible approach to this problem is to numerically differentiate the position signal or using some derivative filters; this method, known as 'dirty derivatives' is conceptually very simple and has been used extensively in applications. Although it can be adequate in some cases, its performance for very low and very high frequencies is not acceptable, and this motivates to look for an alternative method. In this chapter we propose, following [25], a reduced-order velocity observer for rigid link manipulators, analyze its stability properties and show the results of four simulations, made with Matlab/Simulink, in which the considered observer was applied to rigid link systems.

2.1 System model and properties

Model equations of a n-links robotic system can be written in matrix form as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{2.1}$$

with

q	positions and/or angular positions	$q \in \mathbf{R}^n$
$\dot{q}$	velocities and/or angular velocities	$\dot{q} \in \mathbf{R}^n$
$\ddot{q}$	accelerations and/or angular accelerations	$\ddot{q} \in \mathbf{R}^n$
$M(q)$	moment of inertia	$M \in \mathbf{R}^{n\times n}$
$C(q,\dot{q})\dot{q}$	Coriolis, centripetal and frictional forces	$C \in \mathbf{R}^{n\times n}$
$G(q)$	gravitational forces	$G \in \mathbf{R}^{n\times n}$

It is assumed that only the positions q are available for measurement. The matrices in Eq. (2.1) have the following important properties:

PROPERTY 2.1
$0 < M_m < \|M(q)\| < M_M$ where M_m, M_M are positive constants. □

PROPERTY 2.2
$C(q,\dot{q}_1)\dot{q}_2 = C(q,\dot{q}_2)\dot{q}_1$ □

PROPERTY 2.3
$\|C(q,\dot{q})\| < C_M\|\dot{q}\|$ where C_M is a positive constant. □

It is further assumed that the robot velocity is bounded by a known constant ω_{max} such that

$$\|\dot{q}(t)\| \le \omega_{max}, \qquad \forall t \in \mathbf{R}. \tag{2.2}$$

Choosing as state vector

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dot{q} \\ q \end{pmatrix} \tag{2.3}$$

we have as state representation

$$\dot{x}_1 = M^{-1}(x_2)\Big(\tau - C(x_2,x_1)x_1 - G(x_2)\Big) \tag{2.4}$$
$$\dot{x}_2 = x_1 \tag{2.5}$$

2.2 The velocity observer

We propose the following reduced-order observer structure

$$\dot{z} = M^{-1}[\tau - C(q,\hat{x}_1)\hat{x}_1 - G(q)] - K\hat{x}_1 \tag{2.6}$$
$$\hat{x}_1 = z + Kq \tag{2.7}$$

with $\hat{x}_1$ velocity estimate, $K \in \mathbf{R}^{n\times n}$ diagonal matrix and $z \in \mathbf{R}^n$ internal state of the observer.

THEOREM 2.1
Consider the observer (2.6) and (2.7) and suppose $\hat{x}_1(0) = 0$. If $\underline{\sigma} = \lambda_{min}(K) > 3C_M\omega_{max}/M_m$, then $lim_{t\to\infty}\tilde{x}_1 = 0$. □

Proof. From (2.6) and (2.7) we can eliminate the internal state z and write

$$\dot{\hat{x}}_1 = M^{-1}[\tau - C(q,\hat{x}_1)\hat{x}_1 - G(q)] + K(x_1 - \hat{x}_1). \tag{2.8}$$

Now, subtracting (2.8) from (2.4) we obtain the following dynamics for the observation error $\tilde{x}_1 = x_1 - \hat{x}_1$

$$\begin{aligned}\dot{\tilde{x}}_1 &= M^{-1}[-C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1] - K\tilde{x}_1 \\ &= M^{-1}[-2C(q,x_1)\tilde{x}_1 + C(q,\tilde{x}_1)\tilde{x}_1] - K\tilde{x}_1 \qquad (2.9)\end{aligned}$$

where we used the property 2.2. Consider now the following positive definite Lyapunov function candidate

$$V(\tilde{x}_1) = \frac{1}{2}\tilde{x}_1^T\tilde{x}_1, \tag{2.10}$$

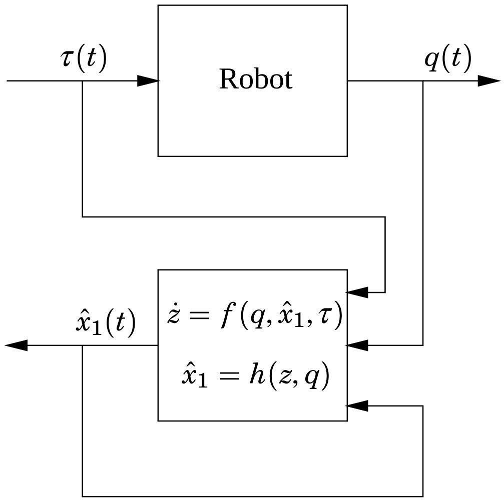

Figure 2.1 The block diagram of the observer

its time derivative along the solutions of (2.9) is

$$\begin{aligned} \dot{V} &= \tilde{x}_1^T \dot{\tilde{x}}_1 = \tilde{x}_1^T M^{-1}[-2C(q, x_1)\tilde{x}_1 + C(q, \tilde{x}_1)\tilde{x}_1] - \tilde{x}_1^T K \tilde{x}_1 \\ &\le \Big(\frac{C_M}{M_m}(2\omega_{max} + \|\tilde{x}_1\|) - \underline{\sigma}\Big)\|\tilde{x}_1\|^2 \end{aligned} \tag{2.11}$$

where we used the properties 2.1, 2.3 and assumption (2.2). Supposing that $\hat{x}_1(0) = 0$, from (2.11) we can conclude that if $\underline{\sigma} > 3C_M\omega_{max}/M_m$ then there exists a constant $\alpha > 0$ such that

$$\dot{V} < -\frac{\alpha}{2}\|\tilde{x}_1\|^2 = -\alpha V \quad \forall \tilde{x}_1 \neq 0 \tag{2.12}$$

and the desired result follows.

REMARK 2.1
Although we supposed for simplicity that $\hat{x}_1(0) = 0$, it is important to note that for any given bound of the velocity ω_{max}, we are free to choose the observer gain K such that we can guarantee exponential convergence for all initial values satisfying

$$0 < \|\tilde{x}_1(0)\| < \frac{M_m}{C_M}\underline{\sigma} - 2\omega_{max}. \tag{2.13}$$

As this region can be increased systematically by the gain K, we have semi-global exponential stability. □

2.3 Simulations

Free pendulum

Consider a free pendulum with mass m [Kg], length l [m], frictional constant c [mKg/s]. Then the pendulum model can be written as (2.1) with

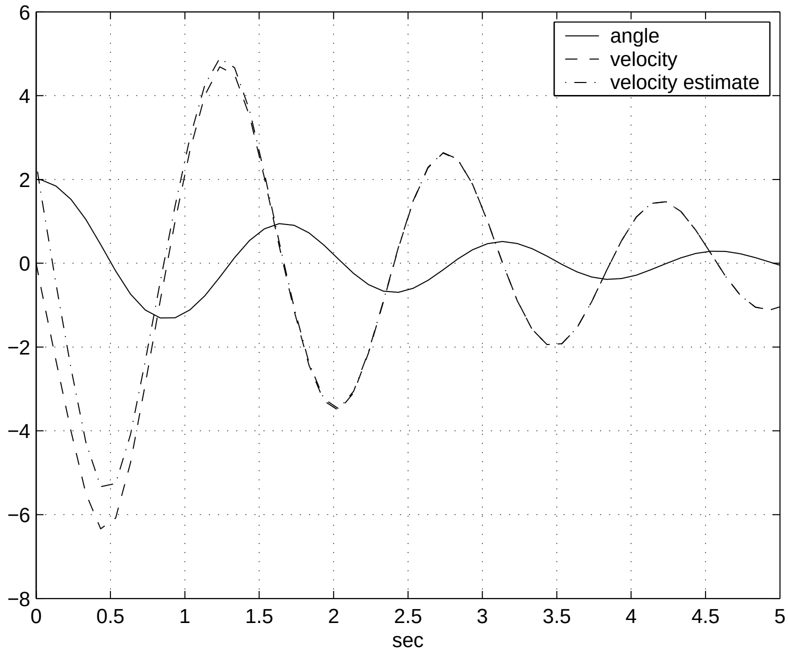

Figure 2.2 velocity estimate for a free pendulum

$M = ml^2$, $C = c$, $G = mgl\sin(q)$ and $\tau = 0$.
Simulation parameters:

$$K = 2 - (c/ml^2), \quad m = 0.5\,[\text{Kg}], \quad l = 0.5\,[\text{m}], \quad c = 0.1\,[\text{mKg/s}],$$
$$q(0) = 2\,[\text{rad}], \quad \dot{q}(0) = 0\,[\text{rad/s}], \quad \hat{x}_1(0) = 2.2\,[\text{rad/s}].$$

then , as shown in figure 2.2, the velocity estimate converges successfully to the real velocity.

Two-link manipulator

Consider a two-link manipulator with masses m_1, m_2 [Kg], lengths l_1, l_2 [m], angles q_1, q_2 [rad], frictional constants c_1, c_2 [mKg/s]; then the model equations can be written as (2.1) with

$$M(q) = \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + (m_1 + m_2) l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\ m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 \end{pmatrix}$$

$$C(q, \dot{q}) = \begin{pmatrix} c_1 - 2m_2 l_1 l_2 \sin(q_2)\dot{q}_2 & -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_1 & c_2 \end{pmatrix}$$

$$G(q) = \begin{pmatrix} m_2 l_2 g \cos(q_1 + q_2) + (m_1 + m_2) l_1 g \cos(q_1) \\ m_2 l_2 g \cos(q_1 + q_2) \end{pmatrix}$$

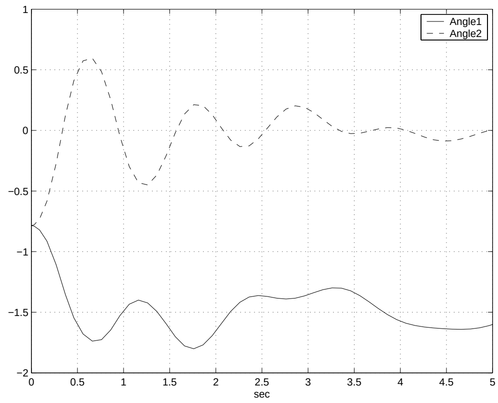

Figure 2.3 two-link manipulator: joint angle trajectories

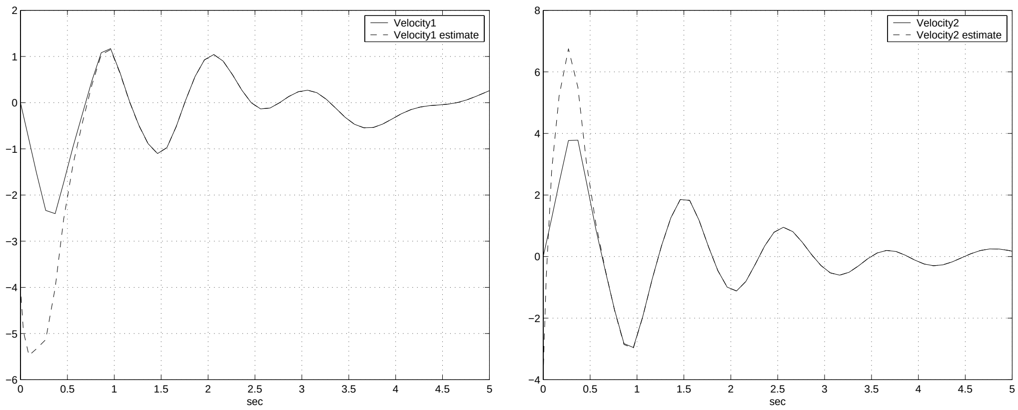

Figure 2.4 $\dot{q}_1$ and $\dot{q}_2$ estimates for the two-link manipulator

Simulation parameters:

$$
\begin{gathered}
K = 5I, \quad m_1 = 0.5\,[\text{Kg}], \quad m_2 = 0.7\,[\text{Kg}], \quad l_1 = 1\,[\text{m}], \\
l_2 = 1.5\,[\text{m}], \quad c_1 = 0.5\,[\text{mKg/s}], \quad c_2 = 0.1\,[\text{mKg/s}], \quad \tau_1 = 2\,[\text{Nm}], \\
\tau_2 = 1\,[\text{Nm}], \quad q_1(0) = q_2(0) = -\tfrac{\pi}{4}\,[\text{rad}], \quad \dot{q}_1(0) = \dot{q}_2(0) = 0\,[\text{rad/s}], \\
\hat{x}_1(0) = \begin{pmatrix} -4 & -4 \end{pmatrix}\,[\text{rad/s}].
\end{gathered}
$$

The results are shown in figures 2.3 and 2.4.

Pendulum-on-the-coach

Consider an inverted pendulum with mass m_2 [Kg], length l [m], angle q_2 [rad], frictional constant c_2 [mKg/s], connected to a sliding mass m_1 [Kg] with position q_1 [m] and frictional constant c_1 [Kg/s]. The model equations can be written as (2.1) with

$$
M(q) = \begin{pmatrix} m_1 + m_2 & m_2 l \cos(q_2) \\ m_2 l \cos(q_2) & m_2 l^2 \end{pmatrix}
$$

$$
C(q, \dot{q}) = \begin{pmatrix} c_1 & -m_2 l \sin(q_2) \dot{q}_2 \\ 0 & c_2 \end{pmatrix}
$$

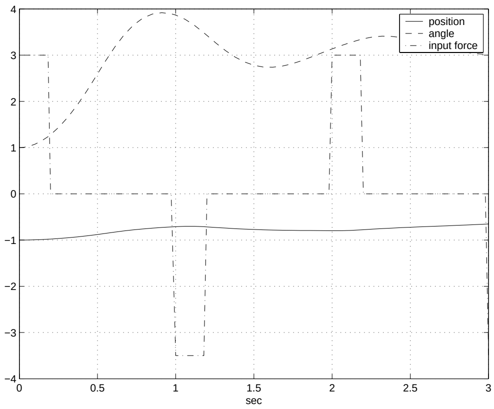

Figure 2.5 Pendulum-on-the-coach: angle and position trajectories

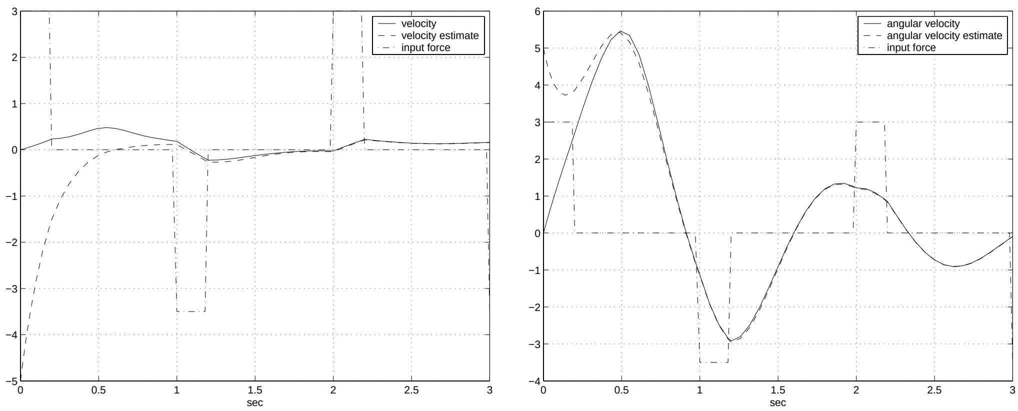

Figure 2.6 $\dot{q}_1$ and $\dot{q}_2$ estimates for the Pendulum-on-the-coach

$$G(q) = \begin{pmatrix} 0 \\ -m_2 l g \sin(q_2) \end{pmatrix}$$

Simulation parameters:

$$\begin{gathered} K = 5I, \quad m_1 = 2\,[\text{Kg}], \quad m_2 = 0.2\,[\text{Kg}], \quad l = 0.5\,[\text{m}], \\ c_1 = 0.4\,[\text{Kg/s}], \quad c_2 = 0.1\,[\text{mKg/s}], \quad \tau_1 = F(t)\,[\text{N}], \quad \tau_2 = 0\,[\text{Nm}], \\ q_1(0) = -1\,[\text{m}], \quad q_2(0) = 1\,[\text{rad}], \quad \dot{q}_1(0) = 0\,[\text{m/s}], \quad \dot{q}_2(0) = 0\,[\text{rad/s}], \\ \hat{x}_1(0) = \begin{pmatrix} -5 & 5 \end{pmatrix}[\text{m/s, rad/s}]. \end{gathered}$$

The results are shown in figures 2.5 and 2.6.

Furuta pendulum

Consider the Furuta pendulum, that consists in a center pillar with moment of inertia J [m^2Kg], rigidly connected to a horizontal arm with length l_1 [m] and homogeneously line distributed mass m_1 [Kg]. The pendulum arm with length l_2 [m] and homogeneously line distributed mass m_2 [Kg], and the balancing body with point distributed mass m_3. Furthermore there are two frictional constants c_1 and c_2 [mKg/s]. Introducing

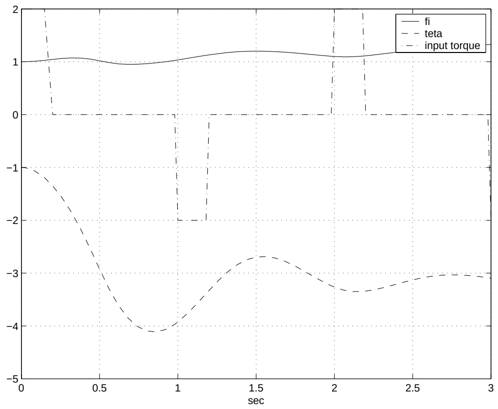

Figure 2.7 Furuta pendulum: angles trajectories

$$\alpha = J + (m_3 + \tfrac{1}{3}m_1 + m_2)l_1^2, \quad \beta = (m_3 + \tfrac{1}{3}m_2)l_2^2,$$
$$\gamma = (m_3 + \tfrac{1}{2}m_2)l_1 l_2, \qquad \delta = (m_3 + \tfrac{1}{2}m_2)g l_2,$$

the model equations can be written as (2.1) with

$$M(q) = \begin{pmatrix} \alpha + \beta\sin^2(q_2) & \gamma\cos(q_2) \\ \gamma\cos(q_2) & \beta \end{pmatrix}$$

$$C(q,\dot{q}) = \begin{pmatrix} c_1 + \beta\cos(q_2)\sin(q_2)\dot{q}_2 & \beta\cos(q_2)\sin(q_2)\dot{q}_1 - \gamma\sin(q_2)\dot{q}_2 \\ -\beta\cos(q_2)\sin(q_2)\dot{q}_1 & c_2 \end{pmatrix}$$

$$G(q) = \begin{pmatrix} 0 \\ -\delta\sin(q_2) \end{pmatrix}$$

Simulation parameters:

$$K = 5I, \quad J = 1\,[\mathrm{m^2Kg}], \quad m_1 = 0.4\,[\mathrm{Kg}], \quad m_2 = 0.1\,[\mathrm{Kg}],$$
$$m_3 = 0.5\,[\mathrm{Kg}], \quad l_1 = 0.7\,[\mathrm{m}], \quad l_2 = 0.5\,[\mathrm{m}], \quad c_1 = 0.4\,[\mathrm{mKg/s}],$$
$$c_2 = 0.2\,[\mathrm{mKg/s}], \quad \tau_1 = \tau(t)\,[\mathrm{Nm}], \quad \tau_2 = 0\,[\mathrm{Nm}],$$
$$q_1(0) = 1\,[\mathrm{rad}], \quad q_2(0) = -1\,[\mathrm{rad}], \quad \dot{q}_1(0) = \dot{q}_2(0) = 0\,[\mathrm{rad/s}],$$
$$\hat{x}_1(0) = \begin{pmatrix} -5 & -5 \end{pmatrix}\,[\mathrm{rad/s}].$$

The results are shown in figures 2.7 and 2.8.

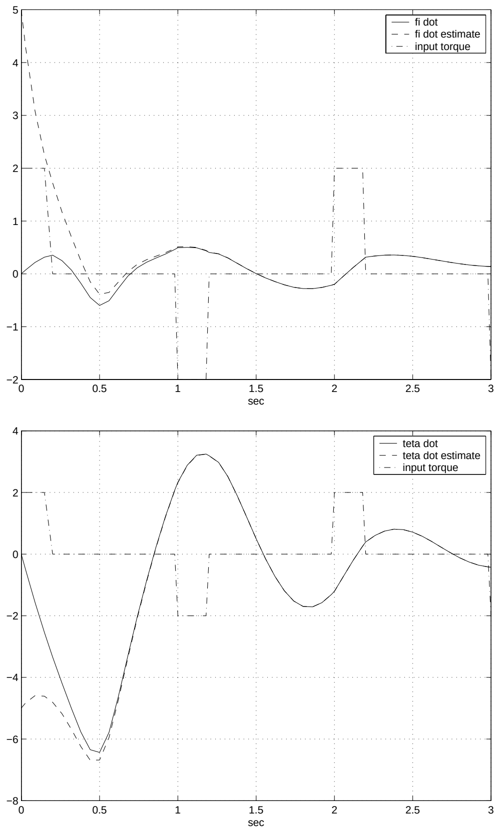

Figure 2.8 $\dot{q}_1$ and $\dot{q}_2$ estimates for the Furuta pendulum

3. On-Line Parameter Estimation for Affine Parametric Nonlinear Systems

In this chapter we shall examine a linear estimation algorithm for affine parametric nonlinear systems following the method proposed by Middleton and Goodwin in [23] and simulate it for some rigid-link manipulators with Matlab/Simulink. In particular we shall show how estimation may be performed based solely on measurement of the state variables, and how this method can be applied to estimate the inertial parameters of rigid link manipulator systems, when the link accelerations are not available for measurements.

3.1 On-line parameter estimation

Consider the following affine parametric nonlinear model

$$\begin{cases} \dot{x}_1 = f_0(x) + F^T(x,u)\theta \\ \dot{x}_2 = x_1 \end{cases} \tag{3.1}$$

with

$x_1, x_2 \in \mathbf{R}^n$		state variables
$f_0(x) \in \mathbf{R}^n$,	$F^T(x,u) \in \mathbf{R}^{n\times p}$	nonlinear functions
$\theta \in \mathbf{R}^p$		vector of unknown constant parameters

and suppose that x_1 and x_2 are known, but not $\dot{x}_1$. Let us apply the stable filter $\frac{1}{s+1}$ to the first equation of (3.1), then we have

$$\frac{s}{s+1}x_1 = \frac{1}{s+1}f_0(x) + \frac{1}{s+1}F^T(x,u)\theta \tag{3.2}$$

Denoting the filtered versions of the known quantities x_1, $f_0(x)$ and $F^T(x,u)$ by

$$x_{1,f} = \frac{1}{s+1}x_1 \tag{3.3}$$

$$f_{0,f}(x) = \frac{1}{s+1}f_0(x) \tag{3.4}$$

$$F_f^T(x,u) = \frac{1}{s+1}F^T(x,u) \tag{3.5}$$

we can rewrite (3.2) as

$$x_1 = f_{0,f}(x) + F_f^T(x,u)\theta + x_{1,f} \tag{3.6}$$

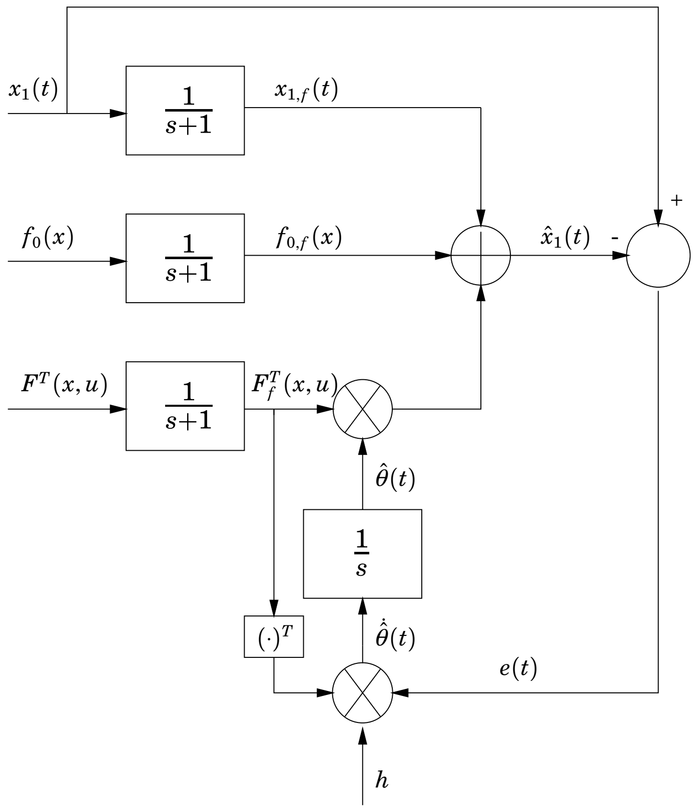

Figure 3.1 the block diagram of the estimator

If instead of the unknown θ we use its estimate $\hat{\theta}$, the corresponding predicted value of x_1 is

$$\hat{x}_1 = f_{0,f}(x) + F_f^T(x,u)\hat{\theta} + x_{1,f} \tag{3.7}$$

and the prediction error e is related to the estimation error $\tilde{\theta} = \theta - \hat{\theta}$ as follows:

$$e = x_1 - \hat{x}_1 = \tilde{x}_1 = F_f^T(x,u)\tilde{\theta} \tag{3.8}$$

We propose the following unnormalized gradient-type estimation algorithm:

$$\dot{\hat{\theta}} = hF_f(x,u)e \tag{3.9}$$

with h positive constant
The following lemma establishes some of the properties of the above estimator.

LEMMA 3.1
The estimator (3.9) applied to the system (3.8), yields the following properties:
1) $\tilde{\theta}$ is bounded.
2) If $rank\Big(F_f(x,u)F_f^T(x,u)\Big) = p$, then $\lim_{t\to\infty}\tilde{\theta} = 0$. $\square$

Proof. Consider the Lyapunov function

$$V(\tilde{\theta}) = \frac{1}{2}\tilde{\theta}^T\tilde{\theta} \tag{3.10}$$

using (3.9) and (3.8) we can show that

$$\dot{V} = -h\tilde{\theta}^T F_f(x,u)F_f^T(x,u)\tilde{\theta} \tag{3.11}$$

$\dot{V}$ is negative definite whenever $rank\Big(F_f(x,u)F_f^T(x,u)\Big) = p$, otherwise is negative semidefinite, then the desired results follow.

3.2 Simulations

In this section we use the previous algorithm to estimate the inertial parameters of rigid-link manipulators. Although these systems are not in the form (3.1), it is possible to reorder the robot equation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{3.12}$$

as

$$\tau = \varphi(\ddot{q},\dot{q},q)\theta + \varphi_0(\ddot{q},\dot{q},q) \tag{3.13}$$

and after the filtering with $\frac{1}{s+1}$ we have

$$\tau_f = \varphi_f(\dot{q},q)\theta + \varphi_{0,f}(\dot{q},q) \tag{3.14}$$

where $\varphi_f = \frac{1}{s+1}\varphi$ and $\varphi_{0,f} = \frac{1}{s+1}\varphi_0$ are not affected by $\ddot{q}$. Replacing θ with its estimate $\hat{\theta}$ we have

$$\hat{\tau}_f = \varphi_f(\dot{q},q)\hat{\theta} + \varphi_{0,f}(\dot{q},q) \tag{3.15}$$

and

$$e = \tau_f - \hat{\tau}_f = \varphi_f(\dot{q},q)\tilde{\theta} \tag{3.16}$$

so we can use the algorithm (3.9).

Pendulum

Consider a pendulum with mass m [Kg], length l [m]. Then the pendulum model can be written as (3.12) with $M = ml^2$, $G = mgl\sin(q)$.

Simulation 1 We consider the mass as unknown and piecewise constant parameter, then we can write the pendulum equation as (3.13) with $\varphi_0 = 0$ and $\varphi = l^2\ddot{q} + gl\sin(q)$.
Simulation parameters:

$$q(0) = 1\,[\text{rad}], \quad \dot{q}(0) = 0\,[\text{rad/s}], \quad \hat{m}(0) = 1\,[\text{Kg}],$$
$$l = 1.5\,[\text{m}], \quad h = 10.$$

Results in figure (3.2) show good tracking properties.

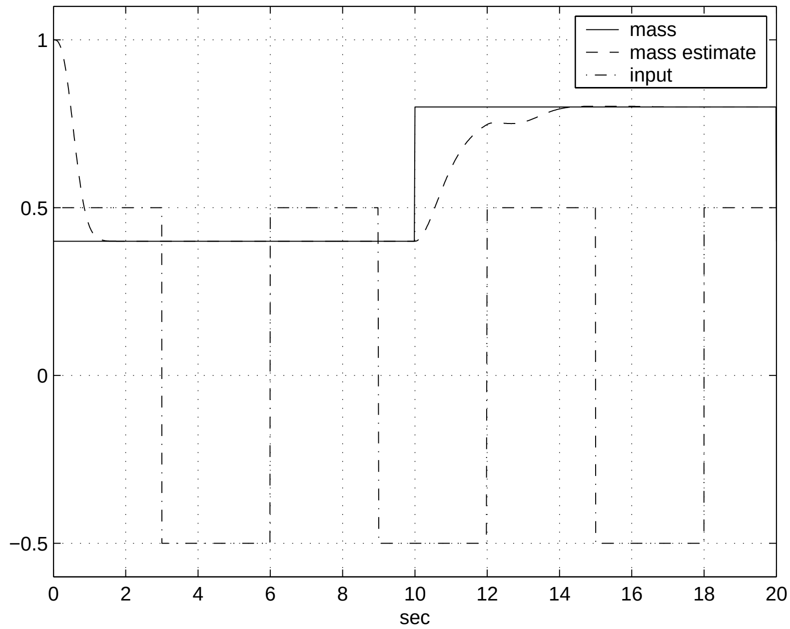

Figure 3.2 estimation of the mass for the pendulum

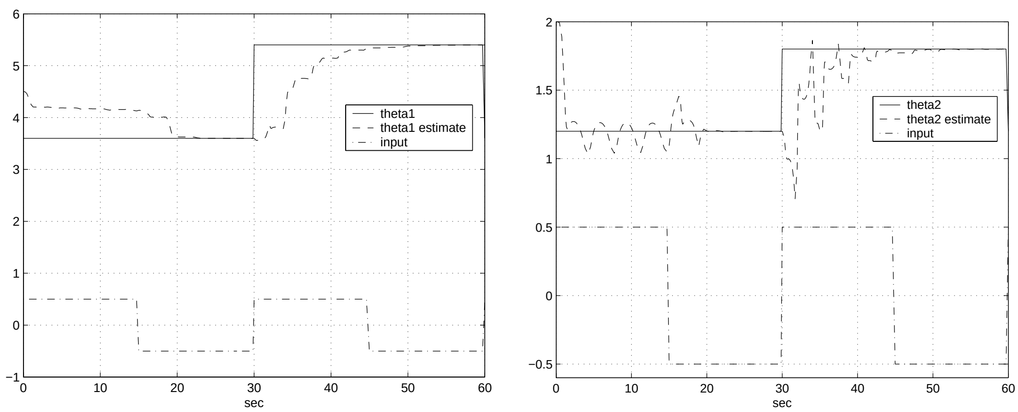

Figure 3.3 estimation of ml^2 and ml for the pendulum

Simulation 2 Now we consider unknown both the mass and the length of the pendulum and, choosing $\theta = \begin{pmatrix} ml^2 \\ ml \end{pmatrix}$, we can write the pendulum equation as (3.13) with $\varphi_0 = 0$ and $\varphi = \begin{pmatrix} \ddot{q} & g\sin(q) \end{pmatrix}$.
Simulation parameters:

$$q(0) = 0\,[\text{rad}], \quad \dot{q}(0) = 0\,[\text{rad/s}], \quad \hat{\theta}_1(0) = 4.5\,[\text{m}^2\text{Kg}],$$
$$\hat{\theta}_2(0) = 2\,[\text{mKg}], \quad h = 100.$$

Results in figure (3.3) show that the estimates converge again to the true values, although this property is not guaranteed anymore, because in this case $rank\Big(F_f(x,u)F_f^T(x,u)\Big) < p$.

4. A Reduced-Order Adaptive Velocity Observer for Robot Manipulators

Adaptive observers can be used together with adaptive controllers to build observer-based adaptive control schemes; in [9] a combination of an adaptive control law with a sliding state observer is proposed, resulting in an asymptotically stable closed-loop system. However, a drawback of this method is that the switching could still excite unmodeled high frequency dynamics. In order to avoid this problem, in this chapter we want to extend the reduced-order velocity observer proposed in chapter 2 for adaptation, following the method shown by Erlic and Lu in [12]; the result is that the signal produced by the observer is smooth and the excitation of unmodeled high frequency dynamics is less likely.

4.1 System model and properties

Let us remind the robot equation (2.1)

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{4.1}$$

It is assumed that only the positions q are available for measurement and that some parameters of the matrices in (4.1) are unknown but constant. The matrices in (4.1), in addition to 2.1, 2.2 and 2.3, have the following two properties:

PROPERTY 4.1
$\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric. □

PROPERTY 4.2
$M(q)\psi + C(q,\xi)\xi + G(q) = \varphi_0(q,\xi,\psi) + \varphi(q,\xi,\psi)\theta$
where $\xi, \psi \in \mathbf{R}^n$ and $\theta \in \mathbf{R}^p$ is the unknown parameter vector. □

Furthermore the bounded velocity assumption (2.2) still holds. As in chapter 2, we define the state vector as

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dot{q} \\ q \end{pmatrix} \tag{4.2}$$

then, from Eq. (4.1) we can write

$$\dot{x}_1 = M^{-1}(x_2)\Big(\tau - C(x_2,x_1)x_1 - G(x_2)\Big) \tag{4.3}$$

$$\dot{x}_2 = x_1 \tag{4.4}$$

4.2 The reduced-order adaptive observer

Consider a reduced-order adaptive observer for estimating the angular velocity and the unknown parameter vector, when the angle is measurable. The observer equation is given by

$$\begin{aligned} \dot{\hat{x}}_1 &= \psi(q, \hat{x}_1, \tau, \hat{\theta}) + K\tilde{x}_1 && (4.5) \\ \psi(q, \hat{x}_1, \tau, \hat{\theta}) &= \hat{M}(q)^{-1}\Big(\tau - \hat{C}(q, \hat{x}_1)\hat{x}_1 - \hat{G}(q)\Big) && (4.6) \end{aligned}$$

where $\hat{x}_1$ is the velocity estimate, $\tilde{x}_1 = x_1 - \hat{x}_1$ is the observation error, $K > 0$ is a diagonal gain matrix. The estimated parameters used in (4.5) and (4.6) are obtained with the following adaptation law:

$$\dot{\hat{\theta}} = -\Gamma\varphi^T(q, \hat{x}_1, \psi)\tilde{x}_1 \qquad (4.7)$$

where $\varphi^T(q, \hat{x}_1, \psi)$ is the regressor determined by property 4.2 and $\Gamma > 0$ is a diagonal gain matrix.

4.3 Stability of the observer

The following theorem establish the stability properties for the above observer:

THEOREM 4.1
Consider the observer (4.5) and (4.6) with the adaptation law (4.7). Define

$$\underline{\sigma} = \lambda_{min}(KM(q) + M(q)K)/2, \qquad (4.8)$$

the initial estimation error as

$$e(0) = \begin{pmatrix} \tilde{x}_1^T(0) & \tilde{\theta}^T(0) \end{pmatrix}^T \qquad (4.9)$$

and

$$p_l = \lambda_{min}(P), \quad p_u = \lambda_{max}(P) \qquad (4.10)$$

with $P = \operatorname{diag}\left\{ \begin{matrix} M(q) & \Gamma^{-1} \end{matrix} \right\}$. If

$$\underline{\sigma} > C_M\omega_{max} + \beta \qquad (4.11)$$

where C_M is given in property 2.3, $\beta > 0$ is a fixed constant, and the initial estimation error $e(0)$ belongs to the ball B_e, defined by

$$B_e = \left\{ e(0) \in \mathbf{R}^{n+p} : \|e(0)\| < \sqrt{\frac{p_l}{p_u}}\left(\frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}\right) \right\} \qquad (4.12)$$

then

$$\lim_{t\to\infty} \tilde{x}_1 = 0 \qquad (4.13)$$

□

Proof. Consider the Lyapunov function

$$V(e(t)) = \frac{1}{2}e^T(t)Pe(t), \tag{4.14}$$

where

$$e(t) = \begin{pmatrix} \tilde{x}_1^T(t) & \tilde{\theta}^T(t) \end{pmatrix} \tag{4.15}$$

it follows that

$$\frac{1}{2}p_l\|e(t)\|^2 \leq V(e(t)) \leq \frac{1}{2}p_u\|e(t)\|^2. \tag{4.16}$$

The time derivative of $V(e(t))$ along trajectories of $\tilde{x}_1$ and $\tilde{\theta}$ is

$$\dot{V} = \tilde{x}_1^T M(q)\dot{\tilde{x}}_1 + \frac{1}{2}\tilde{x}_1^T \dot{M}(q)\tilde{x}_1 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}. \tag{4.17}$$

Subtracting (4.5) from (4.3), we have

$$M(q)\dot{\tilde{x}}_1 = -C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1 - M(q)K\tilde{x}_1 - \Theta \tag{4.18}$$

where

$$\Theta = \Big(M(q)\psi + C(q,\hat{x}_1)\hat{x}_1 + G(q)\Big) - \Big(\hat{M}(q)\psi + \hat{C}(q,\hat{x}_1)\hat{x}_1 + \hat{G}(q)\Big) \tag{4.19}$$

By applying property 4.2, we have

$$\begin{aligned} \Theta &= \varphi_0(q,\hat{x}_1,\psi) + \varphi(q,\hat{x}_1,\psi)\theta - \varphi_0(q,\hat{x}_1,\psi) - \varphi(q,\hat{x}_1,\psi)\hat{\theta} \\ &= \varphi(q,\hat{x}_1,\psi)\tilde{\theta} \end{aligned} \tag{4.20}$$

and (4.18) becomes

$$M(q)\dot{\tilde{x}}_1 = -C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1 - M(q)K\tilde{x}_1 - \varphi(q,\hat{x}_1,\psi)\tilde{\theta}. \tag{4.21}$$

Now, substituting (4.21) into (4.17) and using property 2.2, we have

$$\begin{aligned} \dot{V} &= -\tilde{x}_1^T\Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1)\Big)\tilde{x}_1 + \tilde{x}_1^T\Big(\frac{1}{2}\dot{M}(q) - C(q,x_1)\Big)\tilde{x}_1 \\ &+ \tilde{\theta}^T\Big(-\Gamma^{-1}\dot{\hat{\theta}} - \varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1\Big) \end{aligned} \tag{4.22}$$

which in conjunction with (4.7), properties 2.1, 2.3 and 4.1, and assumption (2.2), gives

$$\dot{V} \leq -(\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2. \tag{4.23}$$

Hence $\dot{V} \leq -\beta\|\tilde{x}_1\|^2$ if

$$\underline{\sigma} > C_M\omega_{max} + C_M\|\tilde{x}_1\| + \beta. \tag{4.24}$$

Since $\|e\| > \|\tilde{x}_1\|$, (4.24) holds if

$$\|e\| < \frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}. \tag{4.25}$$

From (4.23), (4.25) and (4.16), if follows that if

$$\|e(0)\| < \sqrt{\frac{p_l}{p_u}}\Big(\frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}\Big) \tag{4.26}$$

then

$$\dot{V} \leq -\beta\|\tilde{x}_1(t)\|^2 \quad \forall t \geq 0 \tag{4.27}$$

and the desired result follows.

The main assumption made in Theorem 4.1 is that joint velocities are bounded and the bound is known. Although this assumption could appear to be quite restrictive, the result of Theorem 4.1 can be interpreted as a quantitative relation of the region of attraction to the magnitude of joint velocity. Furthermore, this assumption can be eliminated when the proposed observer is combined with an adaptive controller in a feedback loop for robot motion control. Finally, Theorem 4.1 also quantifies the relation of the observer gain K to the region of attraction, indicating how the gain matrix K is selected to make the observer work for a given $e(0)$.

REMARK 4.1
It is important to note that the observer (4.5), (4.6), (4.7) is not implementable in case the velocity measurements are not available. This is because (4.5) and (4.7) involve the use of $\tilde{x}_1 = x - \hat{x}_1$. □

4.4 An implementable approximation of the observer

Integrating (4.5) and (4.7) over the time interval $[t_0, t]$ and using the estimated initial conditions $\hat{x}_1(t_0)$ and $\hat{\theta}(t_0)$ we obtain

$$\hat{x}_1(t) = f(t) + \int_{t_0}^{t} [\psi(q, \hat{x}_1, \tau, \hat{\theta}) - K\hat{x}_1]\mathrm{d}t \tag{4.28}$$

where

$$f(t) = \hat{x}_1(t_0) + K[q(t) - q(t_0)] \tag{4.29}$$

is known, and

$$\hat{\theta}(t) = \hat{\theta}(t_0) - \Gamma \int_{t_0}^{t} \varphi^T(q, \hat{x}_1, \psi)\tilde{x}_1\mathrm{d}t. \tag{4.30}$$

Using $\tilde{x}_1 = x - \hat{x}_1$ with $x = dq/dt$, (4.30) becomes

$$\hat{\theta}(t) = \hat{\theta}(t_0) - \Gamma \int_{q(t_0)}^{q(t)} \varphi^T(q, \hat{x}_1, \psi)\mathrm{d}q + \Gamma \int_{t_0}^{t} \varphi^T(q, \hat{x}_1, \psi)\hat{x}_1\mathrm{d}t. \tag{4.31}$$

The system of integral (4.28) and (4.31), together with initial conditions $\hat{x}_1(t_0)$ and $\hat{\theta}(t_0)$, provides an equivalent version of the adaptive observer

(4.5) and (4.7), but is also not implementable since the evaluation of $\hat{\theta}(t)$ in (4.31) involves the differentiation of the joint position $q(t)$.
From (4.28) and (4.31), it follows that for an arbitrary fixed time interval $\Delta > 0$, we have

$$\hat{x}_1(t) = \hat{x}_1(t-\Delta) + K[q(t) - q(t-\Delta)] + \int_{t-\Delta}^{t} [\psi(q, \hat{x}_1, \tau, \hat{\theta}) - K\hat{x}_1]\mathrm{d}t \quad (4.32)$$

and

$$\hat{\theta}(t) = \hat{\theta}(t-\Delta) - \Gamma \int_{q(t-\Delta)}^{q(t)} \varphi^T(q, \hat{x}_1, \psi)\mathrm{d}q + \Gamma \int_{t-\Delta}^{t} \varphi^T(q, \hat{x}_1, \psi)\hat{x}_1 \mathrm{d}t. \quad (4.33)$$

Assuming that $\psi(q, \hat{x}_1, \tau, \hat{\theta})$, $\varphi(q, \hat{x}_1, \psi)$ and $q(t)$ are continuous time functions and that Δ is sufficiently small, (4.32) and (4.33) suggest a discrete implementation of the proposed observer as follows

$$\hat{x}_1(i) = (I - \Delta K)\hat{x}_1(i-1) + \Delta\psi(i-1) + K[q(i) - q(i-1)] \quad (4.34)$$
$$\hat{\theta}(i) = \hat{\theta}(i-1) + \Gamma\varphi^T(i-1)[\Delta\hat{x}_1(i-1) - q(i) + q(i-1)]. \quad (4.35)$$

REMARK 4.2
Although (4.34) and (4.35) are only an approximation of the proposed observer (4.28) and (4.31), anyway they are implementable and approach to (4.28) and (4.31) as Δ approaches to zero. Therefore (4.34) and (4.35) stand for a good representative of the observer if the sampling interval Δ is sufficiently small. □

4.5 Passivity

Consider the observation error system

$$\begin{aligned} M(q)\dot{\tilde{x}}_1 &= \Big(-2C(q, x_1) + C(q, \tilde{x}_1) - M(q)K \Big)\tilde{x}_1 \\ &- \varphi(q, \hat{x}_1, \psi)\tilde{\theta} \quad (4.36) \\ \dot{\tilde{\theta}} &= \Gamma\varphi^T(q, \hat{x}_1, \psi)\tilde{x}_1 \quad (4.37) \end{aligned}$$

obtained from (4.1), (4.5) and by using property 2.2. This system has an important passivity property. We can write

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{1}{2}\tilde{x}_1^T M(q)\tilde{x}_1\Big) &= \tilde{x}_1^T\Big(-2C(q, x_1) + C(q, \tilde{x}_1) - M(q)K \Big)\tilde{x}_1 \\ &+ \frac{1}{2}\tilde{x}_1^T \dot{M}(q)\tilde{x}_1 - \tilde{x}_1^T\varphi(q, \hat{x}_1, \psi)\tilde{\theta} \quad (4.38) \end{aligned}$$

and, following the proof or Theorem 4.1, we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{1}{2}\tilde{x}_1^T M(q)\tilde{x}_1\Big) &\leq -(\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2 \\ &- \tilde{x}_1^T\varphi(q, \hat{x}_1, \psi)\tilde{\theta}. \quad (4.39) \end{aligned}$$

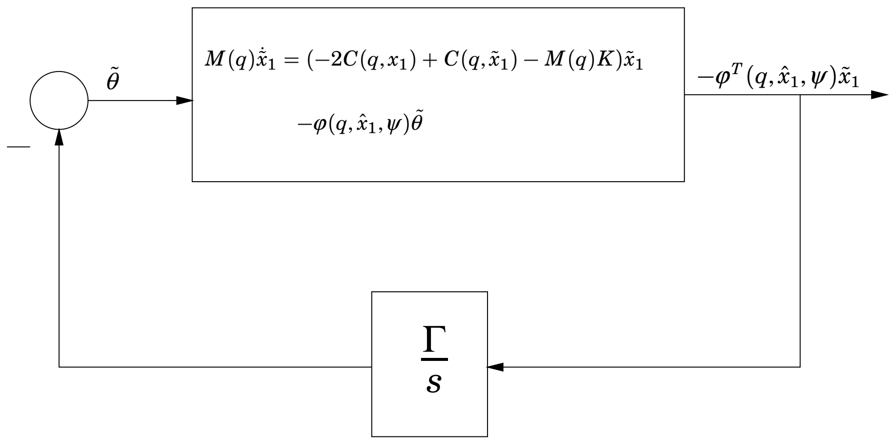

Figure 4.1 The resulting negative feedback connection for the observation error system

By integrating (4.39) over $[0, t]$, we get

$$\int_0^t (-\varphi(s)^T \tilde{x}_1(s))^T \tilde{\theta}(s)\mathrm{d}s \geq \frac{1}{2}\tilde{x}_1^T(t)M(t)\tilde{x}(t) - \frac{1}{2}\tilde{x}_1^T(0)M(0)\tilde{x}(0) + \int_0^t (\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1(s)\|)\|\tilde{x}_1(s)\|^2\mathrm{d}s. \quad (4.40)$$

If the condition

$$\underline{\sigma} > C_M\omega_{max} + C_M\|\tilde{x}_1\| \quad (4.41)$$

is fulfilled, (4.40) implies that the system

$$\begin{aligned} M(q)\dot{\tilde{x}}_1 &= \Big(-2C(q,x_1) + C(q,\tilde{x}_1) - M(q)K\Big)\tilde{x}_1 \\ &- \varphi(q,\hat{x}_1,\psi)\tilde{\theta} \end{aligned} \quad (4.42)$$

is strictly passive with $\tilde{\theta}$ as its input, $-\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1$ as its output, $V(\tilde{x}_1) = \frac{1}{2}\tilde{x}_1^T M(q)\tilde{x}_1$ as the storage function, and

$$\xi(\tilde{x}_1) = (\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2 \quad (4.43)$$

as the dissipation rate. Furthermore, the integrator system

$$-\tilde{\theta} = -\frac{\Gamma}{s}\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1 \quad (4.44)$$

is passive from $-\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1$ to $-\tilde{\theta}$. Hence, the observation error system represents a negative feedback connection of the strictly passive system (4.42) with the passive system (4.44), as shown in figure 4.1.

4.6 The problem of the ill-conditioning of $\hat{M}(q)$

As $\dot{\hat{x}}_1$ contains the term $\hat{M}(q)$, a problem in the continuity of the velocity estimation could arise if the adaptation of $\hat{\theta}$ causes an ill-conditioned $\hat{M}(q)$.

One way to avoid this is to restrict the estimated parameters to lie in a fixed compact region about the true parameters. If the parameter estimate leaves this fixed region, the estimation algorithm should be designed to reset $\hat{\theta}$ to the boundary of the region. In order to do this, according to [11], we modify the update law for the parameter vector θ with the reset conditions

$$\begin{cases} \hat{\theta}_i(t^+) = l_i, & \text{if} \quad \hat{\theta}_i(t) \leq l_i - \mu \\ \hat{\theta}_i(t^+) = h_i, & \text{if} \quad \hat{\theta}_i(t) \geq +h_i + \mu \end{cases} \tag{4.45}$$

where we know that the actual value θ_i lies between l_i and h_i, the lower and the upper bound, respectively, and $\mu > 0$ is chosen such that $\hat{M}(q)$ remains positive definite. Now we can write the value of the Lyapunov function before and after the reset of θ_i to its lower bound, for instance, at time t_j as

$$V(t_j) = \tilde{x}_1^T M(q)\tilde{x}_1 + \sum_{\substack{k=1 \\ k\neq i}}^{p} \frac{1}{\gamma_k}\tilde{\theta}_k^2 + \frac{1}{\gamma_i}(\theta_i - l_i + \mu)^2 \tag{4.46}$$

$$V(t_j^+) = \tilde{x}_1^T M(q)\tilde{x}_1 + \sum_{\substack{k=1 \\ k\neq i}}^{p} \frac{1}{\gamma_k}\tilde{\theta}_k^2 + \frac{1}{\gamma_i}(\theta_i - l_i)^2, \tag{4.47}$$

where $\Gamma = \text{diag}[\gamma_1, \ldots, \gamma_p]$. Therefore the change in V due to the resetting of $\hat{\theta}_i$ at time t_j is

$$-\varepsilon_j = V(t_j^+) - V(t_j) = -(2(\theta_i - l_i) - \mu)\frac{\mu}{\gamma_i} \tag{4.48}$$

where $\varepsilon_j > 0$. With this addition of parameter resetting, (4.23) becomes

$$\dot{V} \leq -(\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2 - \sum_{j}^{r} \delta(t - t_j)\varepsilon_j, \tag{4.49}$$

where r resets take place and $\delta(\cdot)$ is the unit impulse function. Hence, the parameter resetting maintains the nonpositiveness of $\dot{V}$, and the system goes on being semiglobally asymptotically stable with respect to the velocity error and stable with respect to the unknown parameters.

4.7 Robustness to bounded disturbances

Consider the robot equation

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \tag{4.50}$$

and suppose that the angle q only is available for measurement.

Force disturbances and modeling error

Let us first consider the case of a disturbance vector ν added to the equation (4.50) as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \nu, \tag{4.51}$$

where $\nu(t)$ is completely unknown but is upper bounded by

$$\|\nu(t)\| \leq \nu_{max}. \tag{4.52}$$

The vector ν can be interpreted as an external force disturbance or a modeling error, or both of them. Consider the adaptive observer (4.5), (4.6) and (4.7); choosing as Lyapunov function

$$V(\tilde{x}_1,\tilde{\theta}) = \tilde{x}_1^T M(q)\tilde{x}_1 + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}, \tag{4.53}$$

and following the proof of theorem (4.1), we obtain

$$\dot{V} \leq -(\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2 + \tilde{x}_1^T\nu. \tag{4.54}$$

that is less than zero when

$$(\underline{\sigma} - C_M\omega_{max})^2 > 4\nu_{max} \tag{4.55}$$

and

$$\begin{aligned}\frac{\underline{\sigma} - C_M\omega_{max} - \sqrt{(\underline{\sigma} - C_M\omega_{max})^2 - 4\nu_{max}}}{2} &< \|\tilde{x}_1\| \\ &< \frac{\underline{\sigma} - C_M\omega_{max} + \sqrt{(\underline{\sigma} - C_M\omega_{max})^2 - 4\nu_{max}}}{2}\end{aligned} \tag{4.56}$$

This result means that the disturbance ν can actually destabilize the system, because if the condition (4.55) is not fulfilled, boundedness of $\tilde{x}_1$ and $\tilde{\theta}$ is not guaranteed. On the other hand, if $\underline{\sigma}$ is chosen such that (4.55) is fulfilled, and the initial estimation error is sufficiently small, we have an upper bound for $\tilde{x}_1$, that is

$$\|\tilde{x}_1\| \leq \frac{\underline{\sigma} - C_M\omega_{max} - \sqrt{(\underline{\sigma} - C_M\omega_{max})^2 - 4\nu_{max}}}{2} \tag{4.57}$$

and this upper bound can be made arbitrarily small increasing $\underline{\sigma}$, as

$$\lim_{\underline{\sigma}\to+\infty} \frac{\underline{\sigma} - C_M\omega_{max} - \sqrt{(\underline{\sigma} - C_M\omega_{max})^2 - 4\nu_{max}}}{2} = 0. \tag{4.58}$$

Note that the above analysis does not guarantee that $\tilde{\theta}$ remains bounded. However, with the resetting rules seen in the previous section, we are assured that the estimates will remain bounded at all times.

Measurement disturbances

If a bounded disturbance vector w is added to the measurement, we have

$$M(q+w)\ddot{q} + C(q+w,\dot{q})\dot{q} + G(q+w) = \tau \tag{4.59}$$

where the the matrices M, C, G are nonlinear in w. If w is sufficiently small, we can assume M, C, G linear in w and the left side of (4.59) can be rewritten as

$$\begin{aligned}&M(q+w)\ddot{q} + C(q+w,\dot{q})\dot{q} + G(q+w)\\ &\cong M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + h(q,\dot{q},\ddot{q})w.\end{aligned} \tag{4.60}$$

EXAMPLE 4.1
Consider a two-link manipulator that is described by (4.50) with

$$M(q) = \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 c_2 + (m_1+m_2) l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \tag{4.61}$$

$$C(q,\dot q) = \begin{pmatrix} -2m_2 l_1 l_2 s_2 \dot q_2 & -m_2 l_1 l_2 s_2 \dot q_2 \\ m_2 l_1 l_2 s_2 \dot q_1 & 0 \end{pmatrix} \tag{4.62}$$

$$G(q) = \begin{pmatrix} m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{pmatrix} \tag{4.63}$$

with the short notation $c_2 = \cos(q_2)$, $c_{12} = \cos(q_1+q_2)$, etc.
Using the formulas

$$\begin{aligned} \cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) && (4.64) \\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) && (4.65) \end{aligned}$$

and the assumption that w is sufficiently small, we can write

$$\cos(w) \cong 1, \quad \sin(w) \cong w \tag{4.66}$$

and

$$\begin{aligned} \cos(q_i+w_i) &\cong \cos(q_i) - w_i \sin(q_i) \quad (i=1,2) && (4.67) \\ \sin(q_i+w_i) &\cong \sin(q_i) + w_i \cos(q_i) \quad (i=1,2) && (4.68) \\ \cos(q_1+q_2+w_1+w_2) &\cong \cos(q_1+q_2) - (w_1+w_2)\sin(q_1+q_2) && (4.69) \end{aligned}$$

Hence, the system can be rewritten as (4.60) with

$$h(q,\dot q,\ddot q) = \begin{pmatrix} \begin{matrix} -m_2 l_2 g s_{12} \\ -(m_1+m_2) l_1 g s_1 \end{matrix} & \begin{matrix} -m_2 l_1 l_2 (2 s_2 \ddot q_1 + s_2 \ddot q_2 \\ +2 c_2 \dot q_1 \dot q_2 + c_2 \dot q_2^2) - m_2 l_2 g s_{12} \end{matrix} \\ -m_2 l_2 g s_{12} & -m_2 l_1 l_2 (s_2 \ddot q_1 - c_2 \dot q_2^2) - m_2 l_2 g s_{12} \end{pmatrix}$$

□

LEMMA 4.1
Consider the system

$$M(q)\ddot q + C(q,\dot q)\dot q + G(q) + h(q,\dot q,\ddot q) w = \tau \tag{4.70}$$

with the adaptive observer ($x_1 = \dot q$)

$$\begin{aligned} \dot{\hat x}_1 &= \psi(q,\hat x_1,\tau,\hat\theta) + K\tilde x_1 && (4.71) \\ \psi(q,\hat x_1,\tau,\hat\theta) &= \hat M(q)^{-1}\Big(\tau - \hat C(q,\hat x_1)\hat x_1 - \hat G(q)\Big) && (4.72) \\ \dot{\hat\theta} &= -\Gamma\varphi^T(q,\hat x_1,\psi)\tilde x_1. && (4.73) \end{aligned}$$

Using the definitions in theorem (4.1), if

$$\underline{\sigma} > C_M \omega_{max} + C_M \|\tilde{x}_1\|, \tag{4.74}$$

then a strictly passive mapping exists from w to $-h^T(q,\dot{q},\ddot{q})\tilde{x}_1$ with

$$V(\tilde{x}_1,\tilde{\theta}) = \frac{1}{2}\Big(\tilde{x}_1^T M(q)\tilde{x}_1 + \tilde{\theta}^T \Gamma^{-1}\tilde{\theta}\Big) \tag{4.75}$$

as the storage function and

$$\xi(\tilde{x}_1) = (\underline{\sigma} - C_M \omega_{max} - C_M \|\tilde{x}_1\|)\|\tilde{x}_1\|^2 \tag{4.76}$$

as the dissipation rate. □

Proof. Following the same method of theorem (4.1) we can write

$$\dot{V} \leq -(\underline{\sigma} - C_M \omega_{max} - C_M \|\tilde{x}_1\|)\|\tilde{x}_1\|^2 - (h^T(q,\dot{q},\ddot{q})\tilde{x}_1)^T w \tag{4.77}$$

and by integrating (4.77) over $[0,t]$, and using the definition of strictly passive mapping, the desired result follows.

4.8 Simulation Results

In this section we will show the results of a simulation, made with Matlab/Simulink, in which the above adaptive observer is applied to a pendulum. Consider a free pendulum with mass m [Kg] unknown and length l [m]; then the pendulum model can be written as (4.1) with $M = ml^2$, $C = 0$, $G = mgl\sin(q)$ and $\tau = \tau(t)$. For the implementation of the observer we have $\theta = m$,

$$\psi(q,\hat{x}_1,\tau,\hat{\theta}) = \frac{\tau}{\hat{\theta}l^2} - \frac{g}{l}sin(q),$$

and

$$\varphi(q,\hat{x}_1,\psi) = \frac{\tau}{\hat{\theta}}.$$

Simulation parameters:

$$K = 20, \quad m = 1\,[\text{Kg}], \quad l = 1.5\,[\text{m}], \quad \Gamma = 10, \quad \hat{\theta}(0) = 0.1\,[\text{Kg}],$$
$$q(0) = 0\,[\text{rad}], \quad x_1(0) = \dot{q}(0) = -1\,[\text{rad/s}],$$
$$\hat{x}_1(0) = 0\,[\text{rad/s}], \quad \Delta = 10\,[\text{ms}].$$

As shown in figure 4.2, the velocity estimate converges successfully to the real velocity.

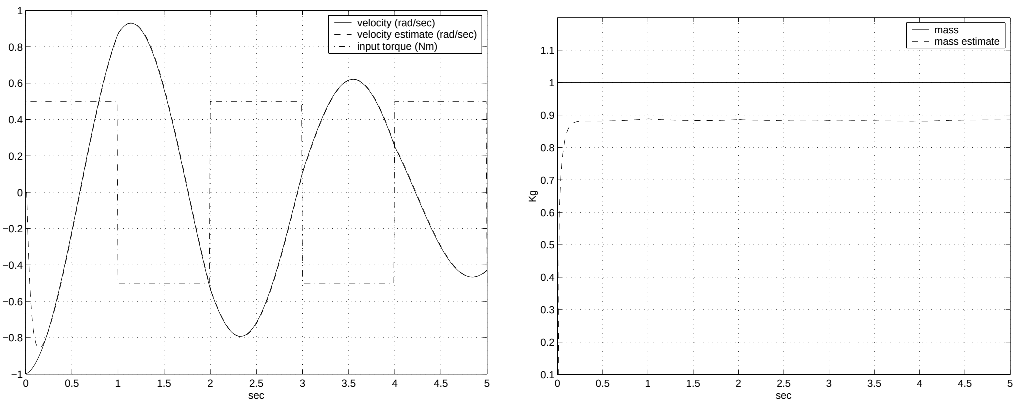

Figure 4.2 Velocity and parameter estimates for the pendulum

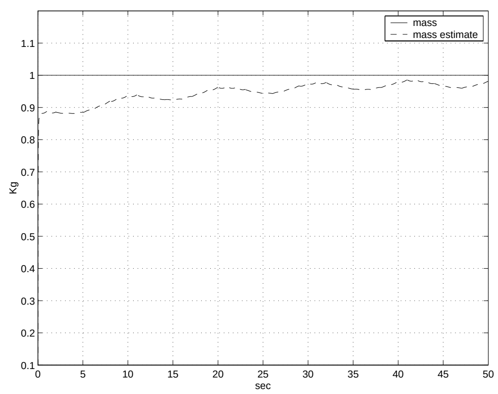

Figure 4.3 Long-term parameter estimate for the pendulum

5. Output Feedback Adaptive Control of Robot Manipulators Using Observer Backstepping

In this chapter we shall show how the problem of observer-based adaptive control for robot manipulators can be solved using an observer backstepping method. This procedure allows the adaptive observer to be designed independently from a state-feedback controller, that uses damping terms to compensate the presence of the estimation error in the tracking error dynamics. As the adaptive observer design problem has been solved in the previous chapter, in this one we shall use those results in combination with a state-feedback controller. The idea of backstepping is to design a controller recursively by considering some of the state variables as "virtual controls" and designing for them intermediate control laws. This approach is more flexible than feedback linearization design and do not force the designed system to appear linear. It can avoid cancellations of useful nonlinearities and often introduces additional nonlinear terms to improve transient performance [20].

5.1 Observer Backstepping

Consider the robot equation (2.1), and suppose that the estimates of the unmeasured velocity x_1 and the unknown parameters θ are given by the adaptive observer (4.5), (4.6) and (4.7). Define a smooth reference trajectory q_d satisfying

$$\ddot{q}_d, \dot{q}_d, q_d \in \mathcal{L}_\infty. \tag{5.1}$$

and the first error variable $z_1 = q - q_d$. We have

$$\dot{z}_1 = x_1 - \dot{q}_d. \tag{5.2}$$

The main idea of backstepping is to choose one of the state variables as virtual control. It turns out that

$$\xi_1 = \hat{x}_1 = z_2 + \alpha_1 \tag{5.3}$$

is an excellent choice for the virtual control. ξ_1 is defined as the sum of the next error variable z_2, and α_1 which can be interpreted as a stabilizing function. Hence

$$\dot{z}_1 = z_2 + \alpha_1 + \tilde{x}_1 - \dot{q}_d. \tag{5.4}$$

We choose the following stabilizing function

$$\alpha_1 = -C_1 z_1 - D_1 z_1 + \dot{q}_d \tag{5.5}$$

where $C_1 \in \mathbf{R}^{n\times n}$ is a strictly positive constant feedback design matrix, usually diagonal, and $D_1 \in \mathbf{R}^{n\times n}$ is a positive diagonal damping matrix defined as

$$D_1 = \mathrm{diag}[d_1, \ldots, d_n] \tag{5.6}$$

where $d_i > 0\,(i = 1, \ldots, n)$. The damping term $-D_1 z_1$ has been added because $\tilde{x}_1$ in (5.2) can be treated as a disturbance term to be compensated. Then we can write

$$\dot{z}_1 = -\Big(C_1 + D_1\Big)z_1 + z_2 + \tilde{x}_1. \tag{5.7}$$

The next step is to specify the desired dynamics of z_2; from (5.3), we have

$$\begin{aligned} \dot{z}_2 &= \dot{\xi}_1 - \dot{\alpha}_1 \\ &= \dot{\hat{x}}_1 + \Big(C_1 + D_1\Big)\dot{z}_1 - \ddot{q}_d \\ &= -\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)(z_2 + \tilde{x}_1) \\ &- \ddot{q}_d + \hat{M}(q)^{-1}\Big(\tau - \hat{C}(q, \hat{x}_1)\hat{x}_1 - \hat{G}(q)\Big) + K\tilde{x}_1. \end{aligned} \tag{5.8}$$

Now we choose the control law as follows

$$\begin{aligned} \tau &= -\hat{M}(q)\Big[-\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)z_2 - \ddot{q}_d + C_2 z_2 + D_2 z_2 + z_1\Big] \\ &+ \hat{C}(q, \hat{x}_1)\hat{x}_1 + \hat{G}(q), \end{aligned} \tag{5.9}$$

where $C_2 \in \mathbf{R}^{n\times n}$ is a strictly positive constant feedback design matrix, usually diagonal. Substituting (5.9) into (5.8), we have

$$\dot{z}_2 = -C_2 z_2 - D_2 z_2 - z_1 + \Omega\tilde{x}_1 \tag{5.10}$$

where

$$\Omega = \Big(C_1 + D_1\Big) + K. \tag{5.11}$$

The damping matrix $D_2 \in \mathbf{R}^{n\times n}$ is defined in terms of the rows of Ω as

$$D_2 = \mathrm{diag}[d_{n+1}\omega_1^T\omega_1, \ldots, d_{2n}\omega_n^T\omega_n] \tag{5.12}$$

where $\Omega^T = [\omega_1, \ldots, \omega_n]$ and $d_i > 0 \quad (i = n+1, \ldots, 2n)$.

5.2 Stability Analysis of the Closed-Loop System

From (5.7), (5.10) and (4.21), we can write the error dynamics as

$$\dot{z} = -(C_z + D_z + E)z + W\tilde{x}_1 \tag{5.13}$$

$$\begin{aligned} M(q)\dot{\tilde{x}}_1 &= -C(q, x_1)x_1 + C(q, \hat{x}_1)\hat{x}_1 - M(q)K\tilde{x}_1 \\ &- \varphi(q, \hat{x}_1, \psi)\tilde{\theta} \end{aligned} \tag{5.14}$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad C_z = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \quad D_z = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \tag{5.15}$$

$$E = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad W = \begin{bmatrix} I \\ \Omega \end{bmatrix}. \tag{5.16}$$

Consider the following Lyapunov function candidate

$$V(z, \tilde{x}_1, \tilde{\theta}) = \frac{1}{2}\Big(z^T z + \tilde{x}_1^T M(q)\tilde{x}_1 + \tilde{\theta}^T \Gamma^{-1}\tilde{\theta}\Big) \tag{5.17}$$

its time derivative along the solutions of (5.13) and (5.14) is

$$\begin{aligned} \dot{V} &= -z^T C_z z - z^T D_z z + z^T W \tilde{x}_1 - \tilde{x}_1^T \Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1)\Big)\tilde{x}_1 \\ &+ \tilde{x}_1^T \Big(\frac{1}{2}\dot{M}(q) - C(q,x_1)\Big)\tilde{x}_1 - \tilde{\theta}^T \Big(\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1 + \Gamma^{-1}\dot{\hat{\theta}}\Big) \end{aligned} \tag{5.18}$$

where we have used the fact that $z^T E z = 0$, and property 2.2. Now, using (4.7), property 4.1 and adding the zero term

$$\frac{1}{4}\Big(\tilde{x}_1^T P \tilde{x}_1 - \tilde{x}_1^T P \tilde{x}_1\Big) = 0 \tag{5.19}$$

(5.18) becomes

$$\begin{aligned} \dot{V} &= -z^T C_z z - z^T D_z z + z^T W \tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P \tilde{x}_1 \\ &- \tilde{x}_1^T \Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1) - \frac{1}{4}P\Big)\tilde{x}_1. \end{aligned} \tag{5.20}$$

Defining the matrix P as

$$P = pI \tag{5.21}$$

where

$$p = \sum_{i=1}^{6} \frac{1}{d_i} \tag{5.22}$$

we have

$$-z^T D_z z + z^T W \tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P \tilde{x}_1 \leq 0. \tag{5.23}$$

Actually, consider the left side of (5.23), which can be expanded as

$$\begin{aligned} &- z^T D_z z + z^T W \tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P \tilde{x}_1 + \\ &= -z_1^T D_1 z_1 - z_2^T D_2 z_2 + z_1^T \tilde{x}_1 + z_2^T \Omega \tilde{x}_1 - \frac{p}{4}\tilde{x}_1^T \tilde{x}_1. \end{aligned} \tag{5.24}$$

Using definitions (5.6), (5.11), and (5.12) together with $z_1 = [\bar{z}_1, \bar{z}_2, \bar{z}_3]^T$ and $z_2 = [\bar{z}_4, \bar{z}_5, \bar{z}_6]^T$, (5.24) can be rewritten as follows

$$\begin{aligned} & - z^T D_z z + z^T W \tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P \tilde{x}_1 \\ &= -\sum_{i=1}^{3} \Big[d_i \Big(\bar{z}_i - \frac{1}{2d_i}\tilde{x}_1 \Big)^T \Big(\bar{z}_i - \frac{1}{2d_i}\tilde{x}_1 \Big) \\ &+ d_{i+3} \Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{x}_1 \Big)^T \Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{x}_1 \Big) \Big] \leq 0 \end{aligned} \quad (5.25)$$

because all the quadratic terms in (5.25) are less than or equal to zero. Hence we can write

$$\dot{V} \leq -z^T C_z z - \tilde{x}_1^T \Big(M(q)K + C(q, x_1) - C(q, \tilde{x}_1) - \frac{1}{4}P \Big) \tilde{x}_1 \quad (5.26)$$

and using properties 2.1, 2.3, and assumption (2.2), we have

$$\dot{V} \leq -z^T C_z z - (\underline{\sigma} - C_M \omega_{max} - C_M \|\tilde{x}_1\| - \frac{1}{4}p)\|\tilde{x}_1\|^2, \quad (5.27)$$

where $\underline{\sigma} = \lambda_{min}(KM(q) + M(q)K)/2$.
Hence $\dot{V} \leq 0$ if

$$\underline{\sigma} > C_M \omega_{max} + C_M \|\tilde{x}_1\| + \frac{1}{4}p. \quad (5.28)$$

As the region of attraction can be arbitrarily increased by the gain K, we have semi-global exponential stability.

REMARK 5.1
Using again (4.34) and (4.35) for the implementation of the adaptive observer, the implementation of controller (5.9) involves simply the calculation of $\tau(t)$ at time instant $t = i\Delta$. □

5.3 A Simulated Example

We consider the two-link example from chapter 2, with masses m_1, m_2 [Kg], lengths l_1, l_2 [m], angles q_1, q_2 [rad], and torques τ_1, τ_2 [Nm]. The end-effector load m_2 is assumed to be unknown but constant. The equations are

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \qquad \theta = m_2 \quad (5.29)$$

$$M(q) = \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 c_2 + (m_1 + m_2) l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \quad (5.30)$$

$$C(q, \dot{q}) = \begin{pmatrix} -2m_2 l_1 l_2 s_2 \dot{q}_2 & -m_2 l_1 l_2 s_2 \dot{q}_2 \\ m_2 l_1 l_2 s_2 \dot{q}_1 & 0 \end{pmatrix} \quad (5.31)$$

Control law:
$\tau = -\hat{M}(q)\Big[-\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big) z_2 - \ddot{q}_d + C_2 z_2 + D_2 z_2 + z_1\Big]$
$+\hat{C}(q, \hat{x}_1)\hat{x}_1 + \hat{G}(q)$
$z_1 = q - q_d$
$z_2 = \hat{x}_1 - \alpha_1$
Stabilizing function:
$\alpha_1 = -C_1 z_1 - D_1 z_1 + \dot{q}_d$
Observer:
$\dot{\hat{x}}_1 = \psi(q, \hat{x}_1, \tau, \hat{\theta}) + K\tilde{x}_1$
$\psi(q, \hat{x}_1, \tau, \hat{\theta}) = \hat{M}(q)^{-1}\Big(\tau - \hat{C}(q, \hat{x}_1)\hat{x}_1 - \hat{G}(q)\Big)$
$\dot{\hat{\theta}} = -\Gamma\varphi^T(q, \hat{x}_1, \psi)\tilde{x}_1$
Damping:
$\Omega = \Big(C_1 + D_1\Big) + K$
$\Omega^T = [\omega_1, \dots, \omega_n]$
$D_1 = \mathrm{diag}[d_1, \dots, d_n]$
$D_2 = \mathrm{diag}[d_{n+1}\omega_1^T\omega_1, \dots, d_{2n}\omega_n^T\omega_n]$
Design matrices and constants:
K positive definite
$\underline{\sigma} = \lambda_{min}(KM(q) + M(q)K)/2$
$\underline{\sigma} > C_M\omega_{max} + C_M\Vert\tilde{x}_1\Vert + \frac{1}{4}p$
C_1 strictly positive
C_2 strictly positive
$d_i > 0\,(i = 1, \dots, 2n)$

Table 5.1 Observer backstepping: summary

$$G(q) = \begin{pmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{pmatrix} \tag{5.32}$$

with the short notation $c_2 = \cos(q_2)$, $c_{12} = \cos(q_1 + q_2)$, etc. The model parameters are $m_1 = 1\,[\mathrm{Kg}]$, $m_2 = 1.5\,[\mathrm{Kg}]$, $l_1 = 1\,[\mathrm{m}]$, $l_2 = 1\,[\mathrm{m}]$. Furthermore the regressor associated with the unknown parameter m_2 is

$$\varphi(q, \dot{q}, \ddot{q}) = \begin{pmatrix} (l_2^2 + 2l_1 l_2 c_2 + l_1^2)\ddot{q}_1 + (l_2^2 + l_1 l_2 c_2)\ddot{q}_2 \\ -(2l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 + l_1 l_2 s_2 \dot{q}_2^2) + (l_2 g c_{12} + l_1 g c_1) \\ \\ (l_2^2 + l_1 l_2 c_2)\ddot{q}_1 + l_2^2 \ddot{q}_2 + l_1 l_2 s_2 \dot{q}_1^2 + l_2 g c_{12} \end{pmatrix} \tag{5.33}$$

The velocity estimate provided by the reduced-order adaptive observer at the ith time instant is calculated with

$$\hat{x}_1(i) = (I - \Delta K)\hat{x}_1(i-1) + \Delta\psi(i-1) + K[q(i) - q(i-1)] \tag{5.34}$$

$$\hat{\theta}(i) = \hat{\theta}(i-1) + \Gamma\varphi^T(i-1)[\Delta\hat{x}_1(i-1) - q(i) + q(i-1)]. \tag{5.35}$$

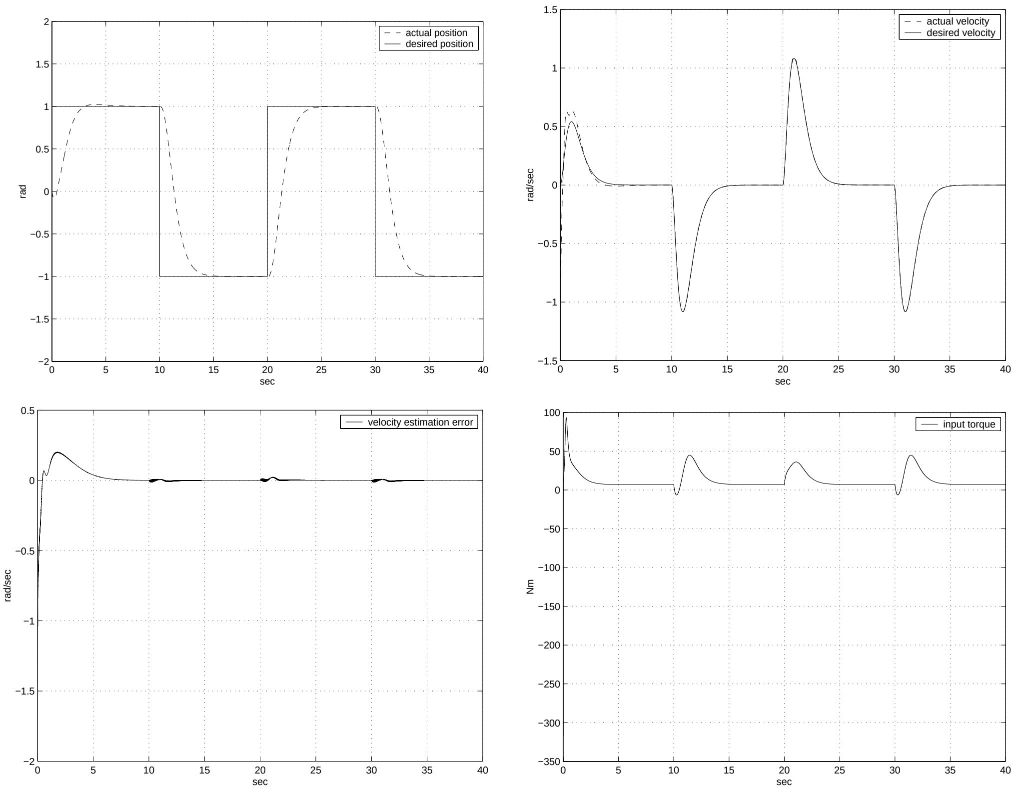

Figure 5.1 Simulation results for the first link

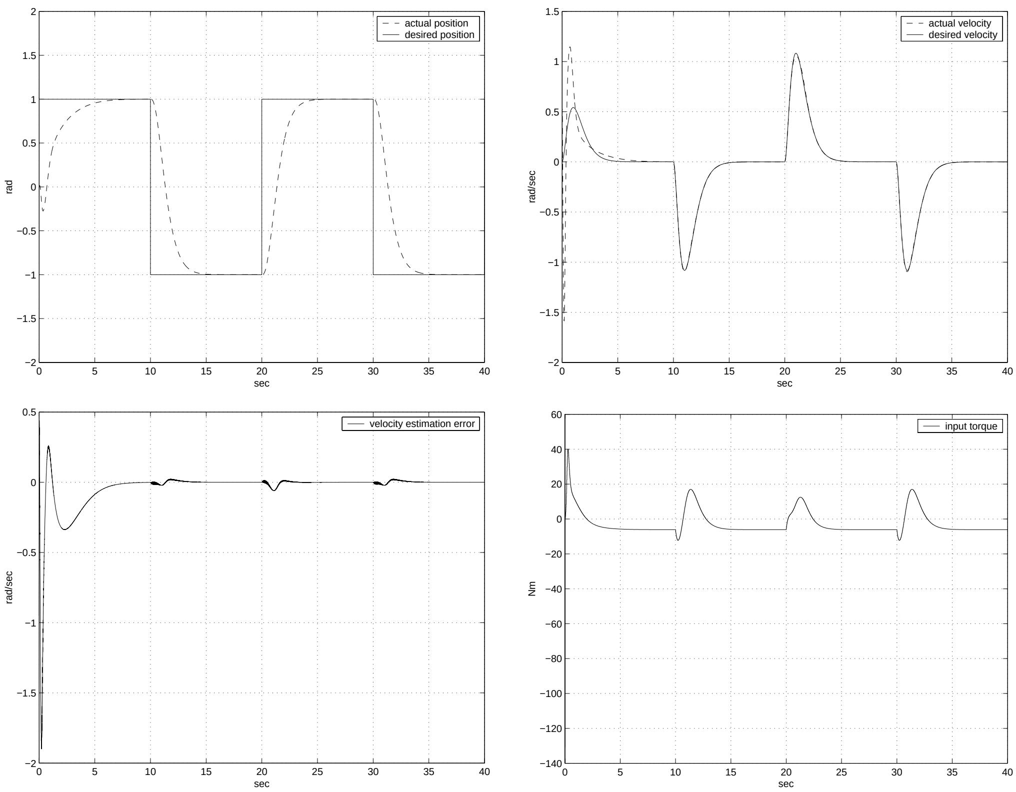

Figure 5.2 Simulation results for the second link

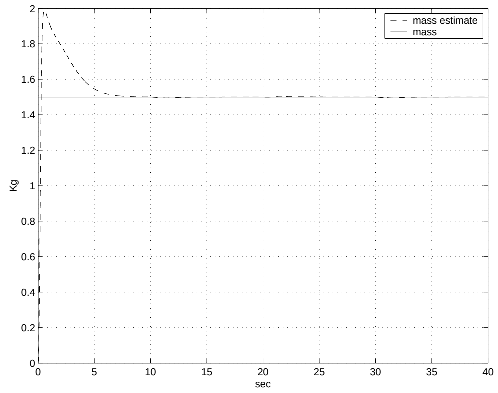

Figure 5.3 Parameter estimate

where

$$\begin{aligned}\psi(i-1) &= \hat{M}(q(i-1))^{-1}\Big(\tau(i-1) - \hat{C}(q(i-1), \hat{x}_1(i-1))\hat{x}_1(i-1) \\ &- \hat{G}(q(i-1)\Big)\end{aligned} \tag{5.36}$$

$$\hat{M}(q) = \begin{pmatrix} \hat{\theta} l_2^2 + 2\hat{\theta} l_1 l_2 c_2 + (m_1 + \hat{\theta}) l_1^2 & \hat{\theta} l_2^2 + \hat{\theta} l_1 l_2 c_2 \\ \hat{\theta} l_2^2 + \hat{\theta} l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \tag{5.37}$$

$$\hat{C}(q, \hat{x}_1) = \begin{pmatrix} -2\hat{\theta} l_1 l_2 s_2 \hat{x}_{12} & -\hat{\theta} l_1 l_2 s_2 \hat{x}_{12} \\ \hat{\theta} l_1 l_2 s_2 \hat{x}_{11} & 0 \end{pmatrix} \tag{5.38}$$

$$\hat{G}(q) = \begin{pmatrix} \hat{\theta} l_2 g c_{12} + (m_1 + \hat{\theta}) l_1 g c_1 \\ \hat{\theta} l_2 g c_{12} \end{pmatrix} \tag{5.39}$$

$$x_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{5.40}$$

$$\varphi(q, \hat{x}_1, \psi) = \begin{pmatrix} (l_2^2 + 2l_1 l_2 c_2 + l_1^2)\psi_1 + (l_2^2 + l_1 l_2 c_2)\psi_2 \\ -(2l_1 l_2 s_2 \hat{x}_{11}\hat{x}_{12} + l_1 l_2 s_2 \hat{x}_{12}^2) + (l_2 g c_{12} + l_1 g c_1) \\ \\ (l_2^2 + l_1 l_2 c_2)\psi_1 + l_2^2 \psi_2 + l_1 l_2 s_2 \hat{x}_{11}^2 + l_2 g c_{12} \end{pmatrix} \tag{5.41}$$

The reference signals $\ddot{q}_d, \dot{q}_d, q_d$ are obtained with a third-order filter with poles in $-a$, that is

$$F(s) = \frac{a^3}{(s+a)^3}. \tag{5.42}$$

Furthermore the observer-controller parameters and the initial conditions are

$$
\begin{gathered}
K = 5I, \quad \Delta = 0.01\,[\mathrm{s}], \quad \Gamma = 0.1, \quad a = 2, \quad d_i = 1\,(i = 1, \ldots, 4) \\
C_1 = 2I, \quad C_2 = 2I, \quad q(0) = \{0, 0\}\,[\mathrm{rad}], \quad \dot{q}(0) = \{0, 0\}\,[\mathrm{rad/s}] \\
\hat{x}_1(0) = \{1, 1\}\,[\mathrm{rad/s}], \quad \hat{\theta}(0) = 0.7\,[\mathrm{Kg}].
\end{gathered}
$$

Results in figures 5.1, 5.2 and 5.3 show a good behaviour of the proposed adaptive observer-controller, even if the input torques have high peaks in the very first seconds, due to the initial velocity estimation error.

6. Experiment: Furuta Pendulum

In this chapter we will show the results of some experiments made by applying the algorithms presented in the previous chapters to the Furuta pendulum. In the first three experiments the objective was to make the pendulum angle track a desired reference signal, and at the same time the arm velocity remained bounded, but without any constraints for the arm angle. In the fourth experiment we stabilized the pendulum in the upright position. The angular velocities were supposed unknown, even if analog velocity signals obtained by differentiation were available for the pendulum. In the first experiment we used the velocity observer shown in the chapter 2 together with the backstepping controller of the previous chapter. The resulting scheme was an observer-based control without adaptation. The second, the third and the fourth experiments were made by using the adaptive velocity observer of chapter 4 with the same controller, to estimate the Coulomb friction parameter in the second and fourth experiments, and both the Coulomb friction parameter and an inertial parameter in the third one. The position measurements were affected by a quantization noise of 0.001 rad for the θ-angle and 0.01 rad for the ϕ-angle. The reference signals θ_d, $\dot{\theta}_d$, $\ddot{\theta}_d$ were obtained by using the filter (5.42). Furthermore, we used the Simulink Real-Time Workshop as interface to the pendulum, with the Euler solver and a fixed step size $F_s = 7.5$ [ms]. As the algorithms implemented in Simulink was too slow, we decided to use the Dymola/Modelica environment for the implementation of the observer-controllers, and imported them in Simulink as shown in Appendix A, with a big improvement in the algorithm velocity.

6.1 The Furuta pendulum

As shown in chapter 2, we can write the equations of motions for the Furuta pendulum as

$$\begin{aligned}(\alpha + \beta\sin^2\theta)\ddot{\phi} + \gamma\cos\theta\ddot{\theta} + 2\beta\cos\theta\sin\theta\dot{\phi}\dot{\theta} - \gamma\sin\theta\dot{\theta}^2 &= \tau \qquad (6.1)\\ \gamma\cos\theta\ddot{\phi} + \beta\ddot{\theta} - \beta\cos\theta\sin\theta\dot{\phi}^2 - \delta\sin\theta &= 0 \qquad (6.2)\end{aligned}$$

Equations (6.1) and (6.2) can be written in matrix form as as (2.1) with

$$q = \begin{pmatrix} \phi \\ \theta \end{pmatrix}, \quad M(q) = \begin{pmatrix} \alpha + \beta\sin^2\theta & \gamma\cos\theta \\ \gamma\cos\theta & \beta \end{pmatrix}$$

$$C(q,\dot{q}) = \begin{pmatrix} \beta\cos\theta\sin\theta\dot{\theta} & \beta\cos\theta\sin\theta\dot{\phi} - \gamma\sin\theta\dot{\theta} \\ -\beta\cos\theta\sin\theta\dot{\phi} & 0 \end{pmatrix}$$

$$G(q) = \begin{pmatrix} 0 \\ -\delta\sin\theta \end{pmatrix}.$$

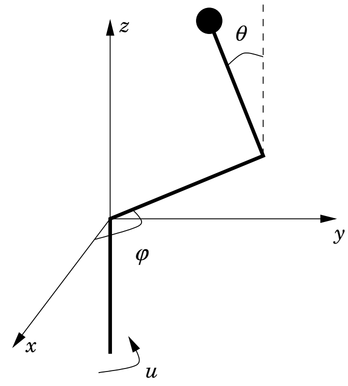

Figure 6.1 A schematic picture of the Furuta pendulum

Furthermore the external torques τ can be divided into a driving torque and dissipation terms as

$$\tau = \tau_u - \tau_F. \tag{6.3}$$

For all the experiments we used the simple Coulomb friction model, that is

$$\tau_F = \tau_C \mathrm{sgn}\dot{\phi} \tag{6.4}$$

The numeric values for the parameters, taken from [4], are

$$\alpha = 0.00354, \quad \beta = 0.00384$$
$$\gamma = 0.00258, \quad \delta = 0.103.$$

6.2 Experiment I

In first Experiment we used the continuous-time velocity observer of chapter 2 and the observer backstepping controller of the previous chapter, to build a continuous-time output-feedback control scheme without any adaptation law. We considered

$$\begin{aligned} z_1 &= \theta - \theta_d && (6.5)\\ z_2 &= \hat{x}_{12} - \dot{\theta}_d + c_1 z_1 + d_1 z_1 && (6.6)\end{aligned}$$

as tracking errors, with c_1, d_1 positive constants and $x_1 = \dot{q} = (x_{11},\, x_{12})^T$. By applying the observer backstepping procedure shown in chapter 5 to these variables we had

$$\begin{aligned} \dot{z}_1 &= z_2 - c_1 z_1 - d_1 z_1 + \tilde{x}_{12} && (6.7)\\ \dot{x}_2 &= \dot{\hat{x}}_{12} + (c_1 + d_1)\dot{z}_1 - \ddot{\theta}_d \\ &= [0,\, 1]\Big(M^{-1}(q)\Big([1,\, 0]^T\tau - C(q,\hat{x}_1)\hat{x}_1 - G(q)\Big) + K\tilde{x}_1\Big) \\ &+ (c_1 + d_1)\big(z_2 - c_1 z_1 - d_1 z_1 + \tilde{x}_1\big) - \ddot{\theta}_d && (6.8)\end{aligned}$$

and the resulting control law

$$\begin{aligned}\tau_u &= \tau_C \mathrm{sgn}\hat{x}_{11} + \frac{1}{M_{21}^{-1}(q)}\Big(-[0,\,1]\Big(M^{-1}(q)\big(-C(q,\hat{x}_1)\hat{x}_1 - G(q)\big)\Big) \\ &- (c_1+d_1)\big(z_2 - c_1 z_1 - d_1 z_1\big) + \ddot{\theta}_d - (c_2+d_2)z_2 - z_1\Big), \qquad (6.9)\end{aligned}$$

where c_2, d_2 are positive constants and $\tau_C = 0.028$ [Nm].

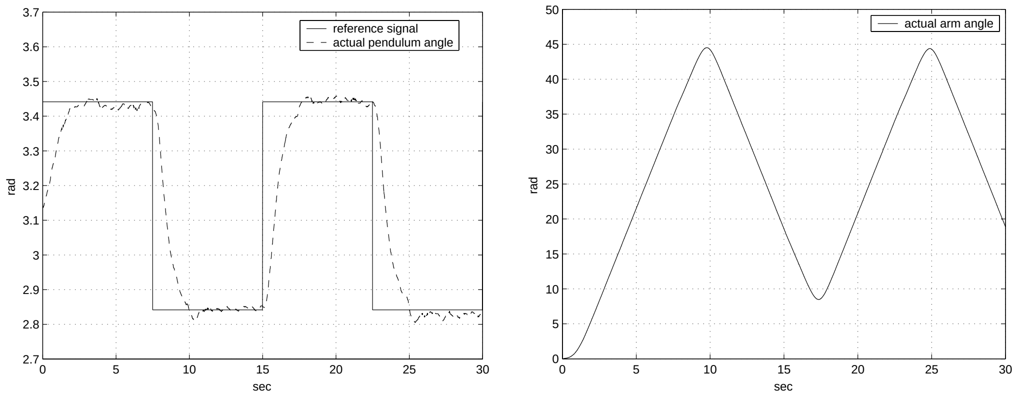

Figure 6.2 Experiment I: reference signal and actual angles

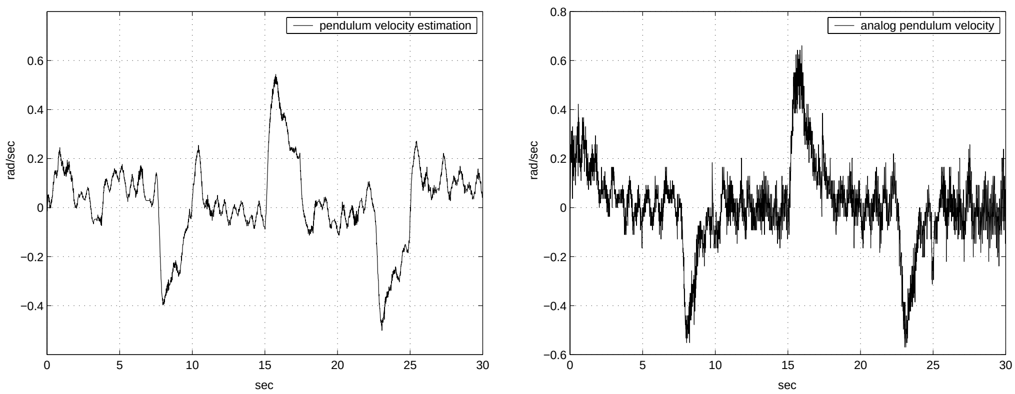

Figure 6.3 Experiment I: estimated and analog pendulum velocity

The observer-controller parameter and initial values we used are

$$\begin{gathered}K = 5I, \quad d_1 = 1, \quad d_2 = 2, \quad c_1 = 10, \\ c_2 = 10, \quad \hat{x}_1(0) = \{0, 0\} \text{ [rad/sec]}, \quad a = 3.\end{gathered}$$

Results are shown in figures 6.2, 6.3, 6.4 and 6.5.

6.3 Experiment II

In the second experiment we used the adaptive observer of chapter 4 together with the backstepping controller of the previous one, and in this

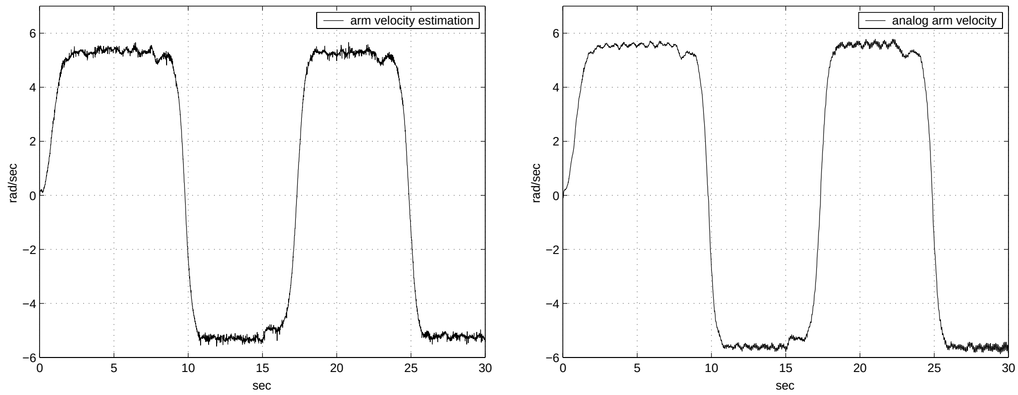

Figure 6.4 Experiment I: estimated and analog arm velocity

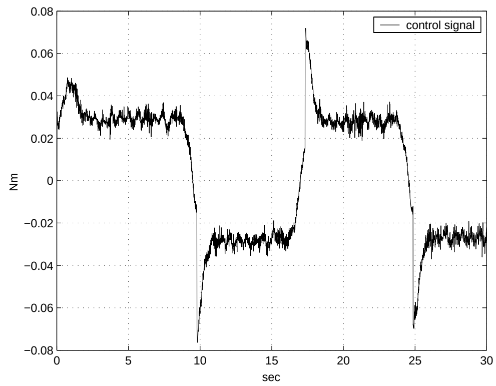

Figure 6.5 Experiment I: control signal

case we have a discrete time adaptive observer-controller. The parameter we decided to estimate with the adaptation law is τ_C, as it is slowly time-varying. Choosing again (6.5) and (6.6) as tracking errors we had as resulting control law

$$\begin{aligned}\tau_u &= \hat{\tau}_C \mathrm{sgn}\hat{x}_{11} + \frac{1}{M_{21}^{-1}(q)}\Big(- [0,\,1]\Big(M^{-1}(q)\big(- C(q,\hat{x}_1)\hat{x}_1 - G(q)\big)\Big) \\ &- (c_1+d_1)\big(z_2 - c_1 z_1 - d_1 z_1\big) + \ddot{\theta}_d - (c_2+d_2)z_2 - z_1\Big), \qquad (6.10)\end{aligned}$$

Furthermore we implemented the resetting rules shown in section 4.6 with τ_{Cl} and τ_{Ch} lower and upper bound of τ_C, respectively.

The observer-controller parameters and initial values we used are

$$\begin{gathered} K = 5I, \quad d_1 = 1, \quad d_2 = 1, \quad c_1 = 10, \\ c_2 = 10, \quad \hat{x}_1(0) = \{0,0\}\,[\text{rad/sec}], \quad a = 3, \\ \Gamma = 0.01, \quad \Delta = 0.0075\,[\text{sec}], \quad \hat{\tau}_C(0) = 0.001\,[\text{Nm}], \\ \tau_{Cl} = 0.001, \quad \tau_{Ch} = 0.04, \quad \mu = 0.0001. \end{gathered}$$

Results in figures 6.6, 6.7, 6.8 and 6.9 show a good behaviour of the discrete-time approximated algorithm, and the adaptation law seems to work very well.

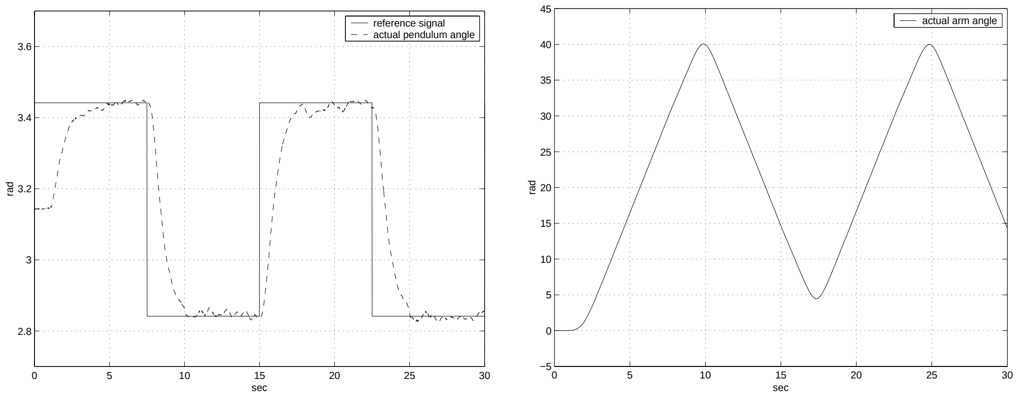

Figure 6.6 Experiment II: reference signal and actual angles

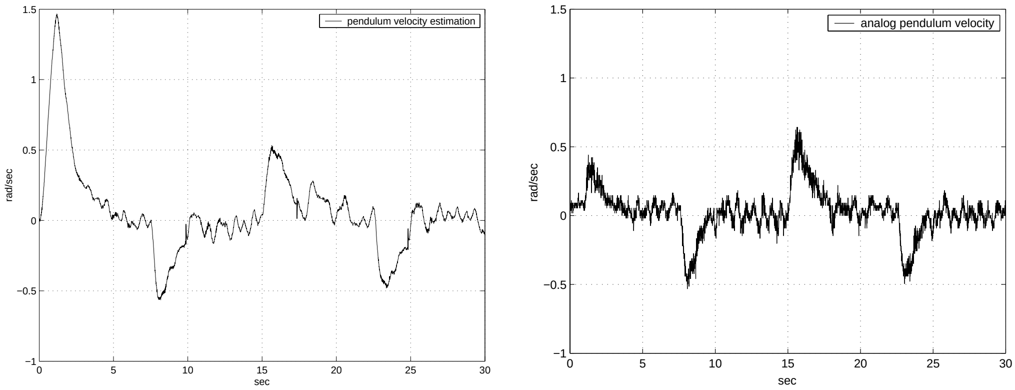

Figure 6.7 Experiment II: estimated and analog pendulum velocity

6.4 Experiment III

In the third experiment we used the same control scheme of the previous one, but with both the parameters τ_C, and α to estimate. Choosing again (6.5) and (6.6) as tracking errors we had as resulting control law

$$\begin{aligned}\tau_u \quad &= \quad \hat{\tau}_C \mathrm{sgn} \hat{x}_{11} + \frac{1}{\hat{M}_{21}^{-1}(q)} \Big(-[0,\, 1] \Big(\hat{M}^{-1}(q) \big(-C(q, \hat{x}_1)\hat{x}_1 - G(q) \big) \Big) \\ &\quad - \quad (c_1 + d_1)\big(z_2 - c_1 z_1 - d_1 z_1\big) + \ddot{\theta}_d - (c_2 + d_2) z_2 - z_1 \Big), \qquad (6.11)\end{aligned}$$

As in the previous experiment, resetting rules for parameters were implemented with τ_{Cl} and τ_{Ch} lower and upper bound of τ_C, α_l and α_h lower and upper bound of α.

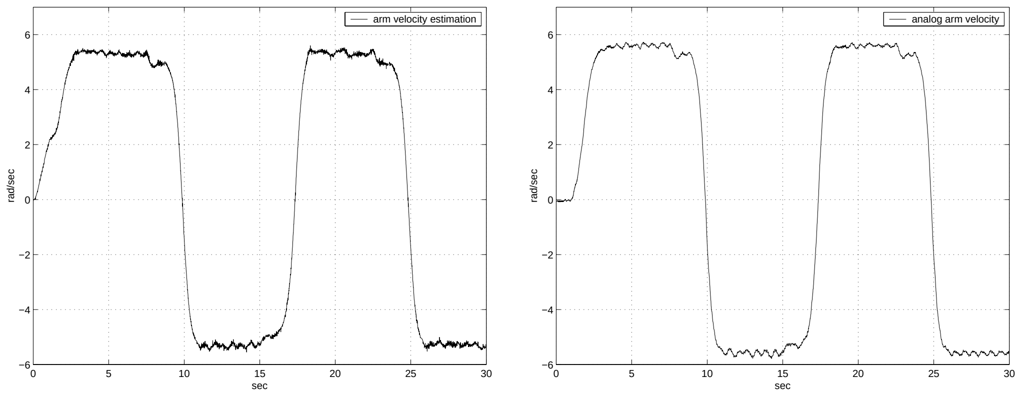

Figure 6.8 Experiment II: estimated and analog arm velocity

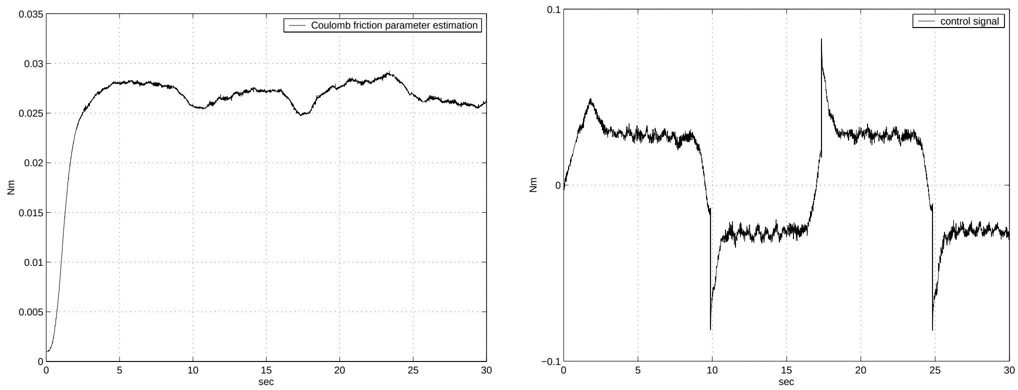

Figure 6.9 Experiment II: parameter estimation and control signal

The observer-controller parameters and initial values we used are

$$
\begin{gathered}
K = 5I, \quad d_1 = 1, \quad d_2 = 1, \quad c_1 = 10, \\
c_2 = 10, \quad \hat{x}_1(0) = \{0, 0\}\,[\text{rad/sec}], \quad a = 3, \\
\Gamma = \text{diag}\{0.01, 0.001\}, \quad \Delta = 0.0075\,[\text{sec}], \\
\hat{\tau}_C(0) = 0.001\,[\text{Nm}], \quad \hat{\alpha}(0) = 0.0001[\text{Kg m}^2], \\
\tau_{Cl} = 0.001, \quad \tau_{Ch} = 0.04, \quad \alpha_l = 0.0001, \\
\alpha_h = 0.005, \quad \mu = 0.00001.
\end{gathered}
$$

Results in figures 6.10, 6.11, 6.12, 6.13 and 6.14 show that after the transient, due to the parameter initial estimation errors, the discrete-time adaptive observer controller has a good performance.

6.5 Experiment IV

In this experiment we used the same adaptive observer-controller we used in the second one, but the objective was to stabilize the pendulum in the upright position. The observer-controller parameters and initial values we

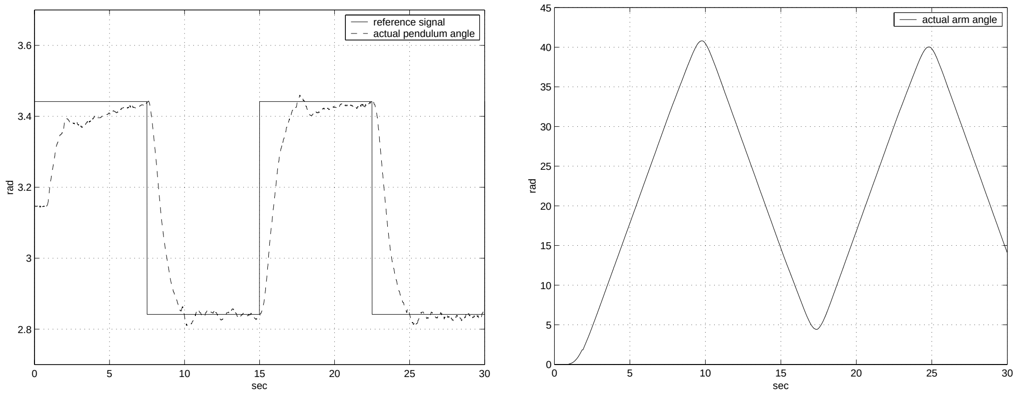

Figure 6.10 Experiment III: reference signal and actual angles

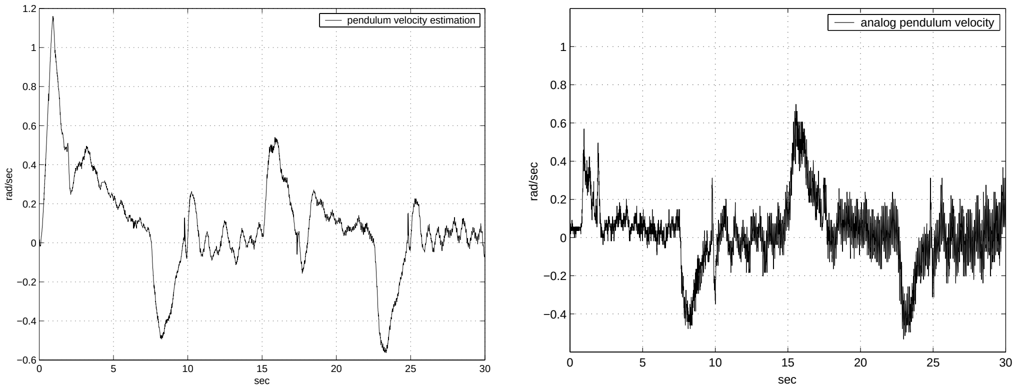

Figure 6.11 Experiment III: estimated and analog pendulum velocity

used are

$$
\begin{gathered}
K = 5I, \quad d_1 = 1, \quad d_2 = 1, \quad c_1 = 10, \\
c_2 = 10, \quad \hat{x}_1(0) = \{0, 0\}\,[\text{rad/sec}], \quad a = 3, \\
\Gamma = 0.003, \quad \Delta = 0.0075\,[\text{sec}], \quad \hat{\tau}_C(0) = 0.001\,[\text{Nm}], \\
\tau_{Cl} = 0.001, \quad \tau_{Ch} = 0.04, \quad \mu = 0.0001.
\end{gathered}
$$

Results in figures 6.15, 6.16, 6.17 and 6.18 show that although there is not any control in the arm angle and velocity, these signal remain quite small and the performance is good.

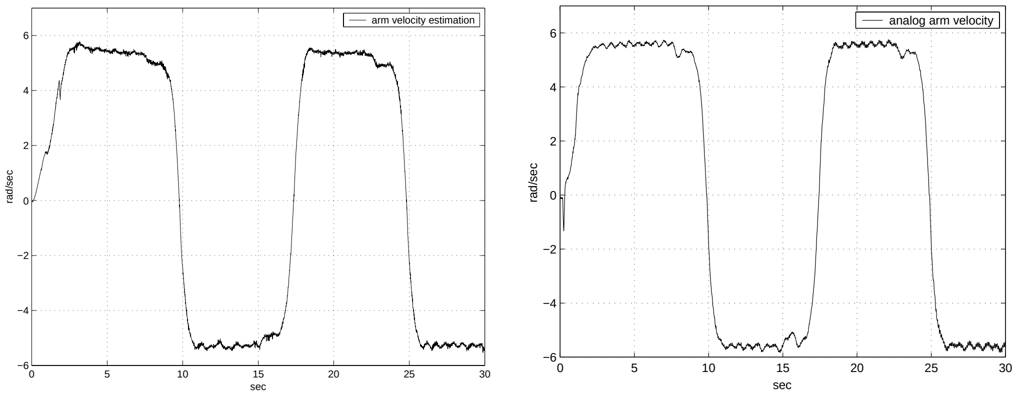

Figure 6.12 Experiment III: estimated and analog arm velocity

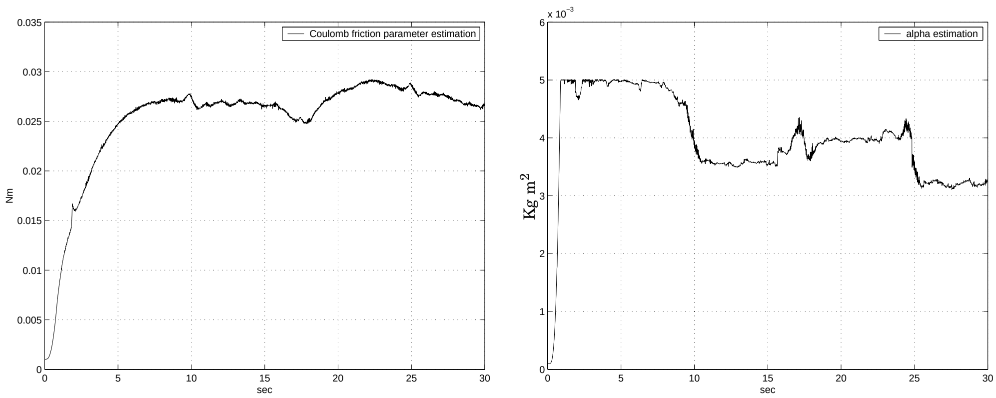

Figure 6.13 Experiment III: parameter estimations

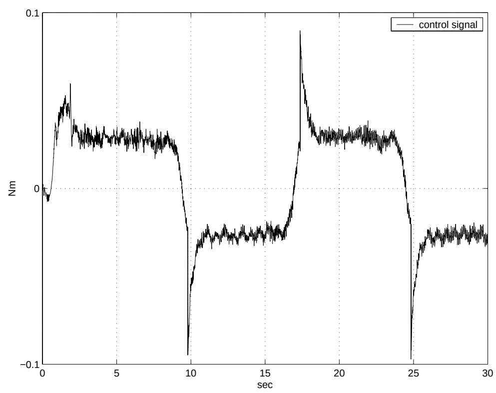

Figure 6.14 Experiment III: control signal

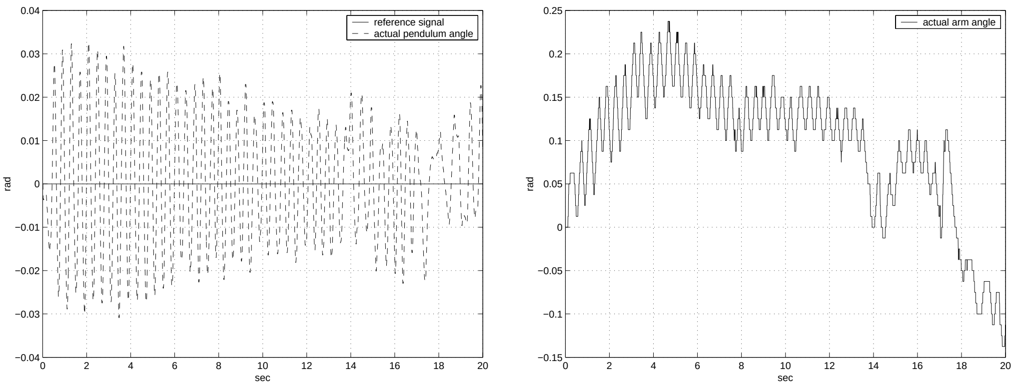

Figure 6.15 Experiment IV: reference signal and actual angles

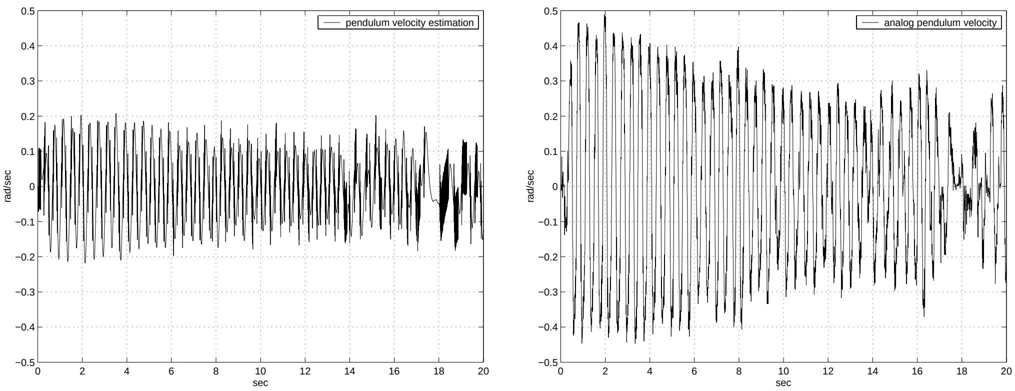

Figure 6.16 Experiment IV: estimated and analog pendulum velocity

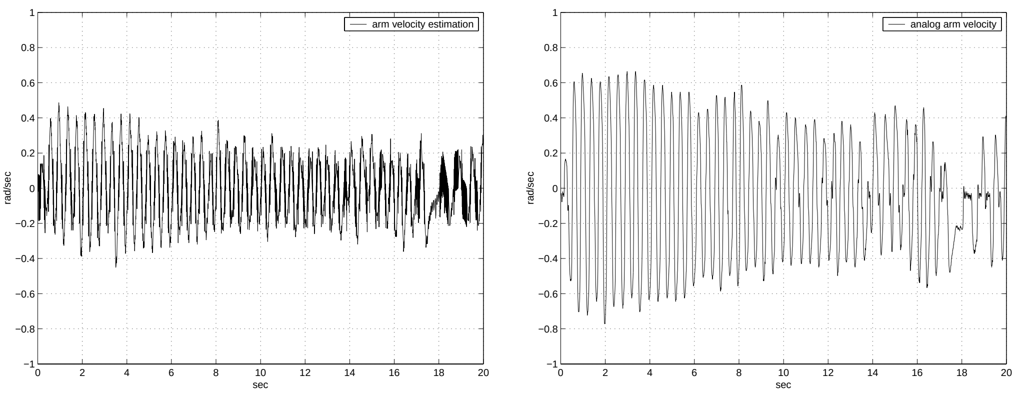

Figure 6.17 Experiment IV: estimated and analog arm velocity

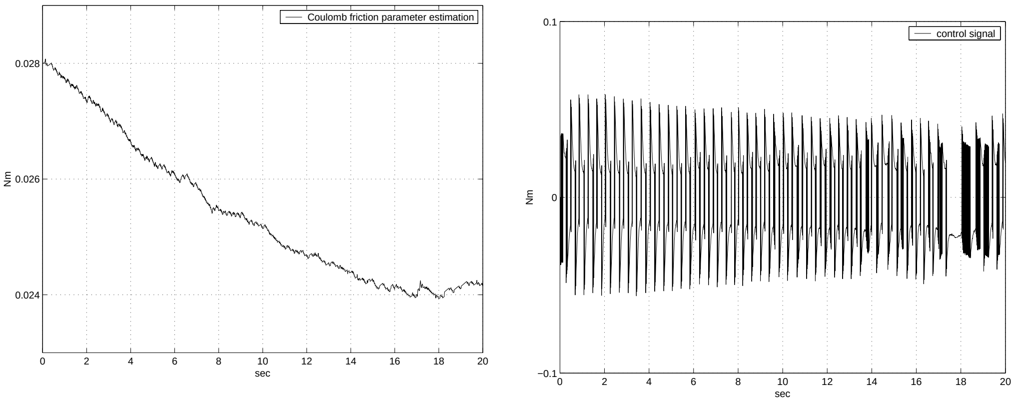

Figure 6.18 Experiment IV: parameter estimation and control signal

Figure 6.19 Furuta pendulum, Department of Automatic Control, Lund

7. Concluding Remarks

7.1 Discussion

Many output feedback control schemes for robot manipulators were presented in the last decade, based on different approaches. Nicosia and Tomei used Lyapunov design [24], Berghuis and Nijmeijer the passivity concept [7], while Lim et al. suggested an observed integrator backstepping procedure [21]. In all those schemes, an observer which reconstructs the unmeasured velocity was used in combination with a controller. As the observer exploits the physical structure of the robot, the velocity estimation is more accurate than the one obtained by using position differentiation algorithms, and consequently the controller performance is higher. The adaptive observer-controller proposed in this thesis is an extension of those results, covering also parameter uncertainties and smooth time-varying parameters, thanks to the adaptation law. Furthermore, it allows us to eliminate the need of tachometers, that are required by adaptive controllers [17] and introduce some noise anyway. With sensor noise, controller gains are not allowed to be high, so it results in larger tracking errors, and velocity filtering can be only partially a solution because of the introduced time delay that can not be accepted in high performance tracking. A passivity-based approach for designing observer-based adaptive robot control was shown by Berghuis in [6], by using a bounded adaptation law, but achieving only stability for the tracking error dynamics. Instead, as pointed out above, the control scheme presented in this paper achieves semiglobal asymptotic stability both for the observer estimation error and the tracking errors.

7.2 Conclusion

An output feedback adaptive control scheme for robot manipulators has been presented, that allows the separate design of the adaptive observer from the state-feedback controller. By applying Lyapunov stability theory, for the closed-loop system semiglobal asymptotic stability has been proven, with respect to position and velocity tracking errors and velocity estimation error. Using this approach, the behaviour of the closed-loop system seems to be good even for small observer gains, that means low sensitivity to noise and smooth control signals. Even if convergence to zero is not guaranteed for the parameter estimation error, simulations show that in absence of noise the closed loop system has this property too. Four experiments have been made on the Furuta pendulum to check the performance of the presented algorithms on a real plant. Results show a good behaviour even in presence of position measurement noise and the adaptation law seems to work very well.

7.3 Future Work

Many further works can be made from the results presented here. First, as the closed loop system seems to be asymptotically stable with respect to the parameter estimation error also, a proof of this property could be found. It may depend on some kind of persistent excitation condition. Furthermore it could be possible to include an on-line parameter identification method in the control scheme, as the one proposed in chapter 3, but using position measurement only. An interesting extension of the output-feedback adaptive control scheme proposed in this thesis could be made for flexible joint robots, or in general for other classes of Euler-Lagrange equations. In particular the adaptive observer of chapter 4 seems to be widely applicable, and an extension for ships has already been made and is shown in Appendix B.

Appendix

A. The Dymola-Simulink interface on Unix and Linux platforms

As pointed out in the previous chapter, some problems arose in the Simulink implementation of the adaptive algorithms, and in particular we could not reduce the sample time below 20 ms in the real process. By using Dymola/Modelica instead of Matlab/Simulink for the algorithm implementation, we were able to reach a sample time of 7.5 ms, with a considerable improvement of the discrete-time observer-controller performance. However, there were some problems in importing the Dymola models into the Simulink environment on Unix and Linux platforms, because a direct interface exists for Windows only. In this appendix we will show how we solved this problem.

A.1 Dymola-Simulink Windows interface

By including

- dymola\mfiles
- dymola\mfiles\traj

in the Matlab path, we can find a Dymola block in Simulink's library browser as Dymola Block/DymolaBlock, that represents the Modelica model. This block is a shield around a S-function MEX block, or in other words, the interface to the C-code generated by Dymola for the Modelica model.

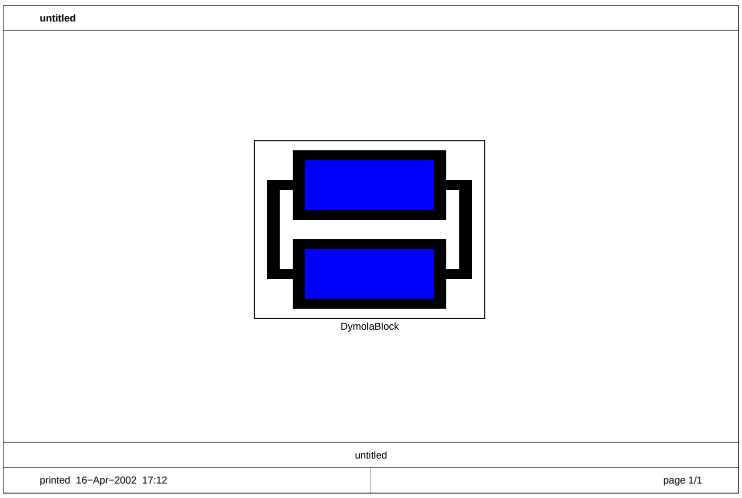

Figure A.1 Dymola block before compiling

Once we got the block in our Simulink model, a double click on it opens a dialog window where it is possible to change parameters and initial values.

By clicking on the “Compile model” button is also possible to compile the model, and as result we have a dll file executable by the S-function in the Dymola block on the Windows version of Simulink.

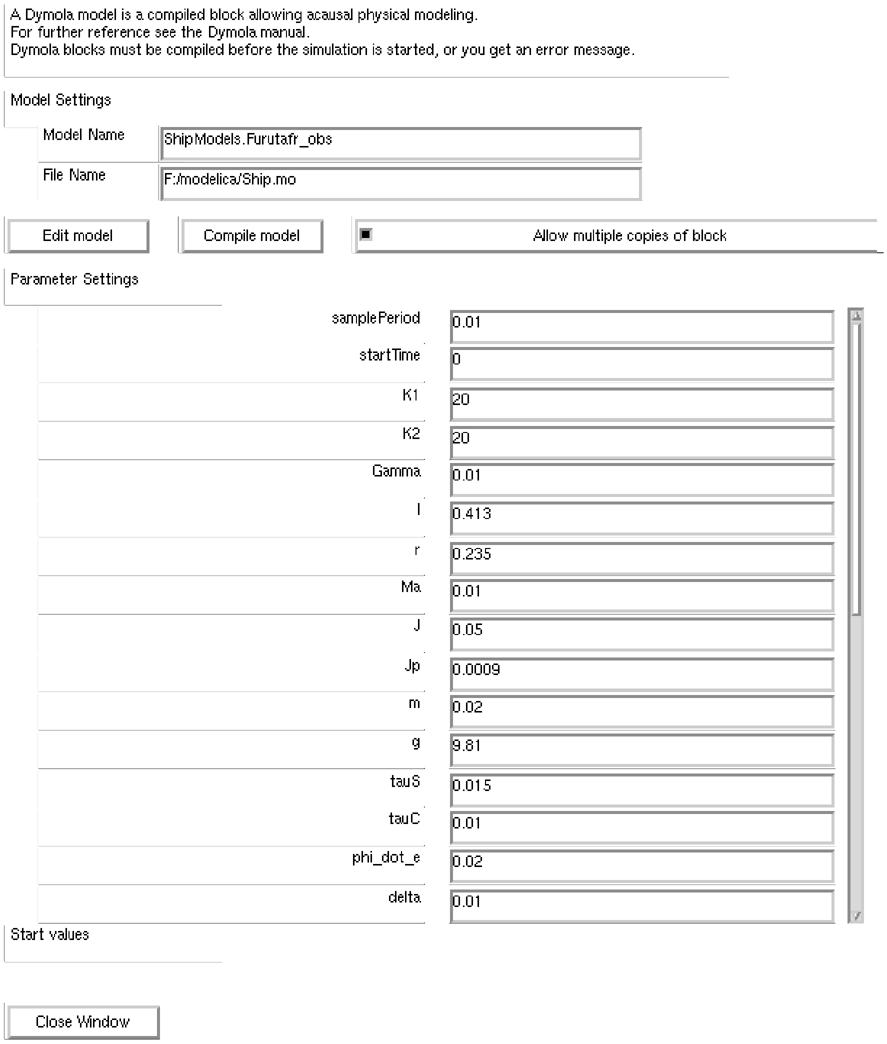

Figure A.2 Dialog window for a Dymola model in Simulink

A.2 How to import Modelica models into Simulink on Unix or Linux platforms

Unfortunately we have not a direct interfaces like the previous one on Linux or Unix, hence we must compile the models in another way. First we have to compile the model on Windows and take the temporary C-source file of our model from dymola\tmp and copy it somewhere else. Then we can use in Matlab the “dymmex” function in dymola\mfiles as dymmex(’filename.c’) to compile the model.
At this point we have a mexsol file in Unix or a mexglx file in Linux, which are executable by an S-function in Simulink. Hence we can take an S-function from the Simulink library and write the filename in the right

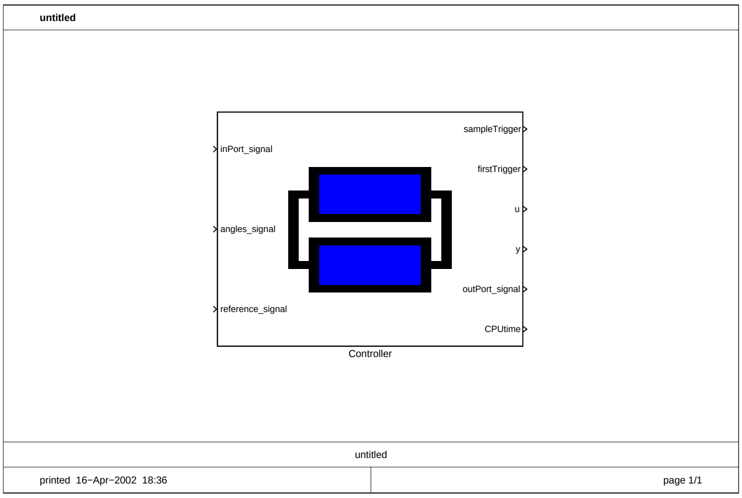

Figure A.3 Dymola block after compiling

field of the dialog box.
To pass the parameters and the initial values to the S-function, we have to know the exact order of them, and we can see it in the dialog window of the Dymola block on the Windows version of Simulink or we can use the "loaddsin" function in dymola\mfiles to load the dsin.txt file, which is in the same directory of our Modelica model and contains all the parameters and the initial values. It is important to note that we can pass to the S-function the initial values of the continuous-time variables only. For the discrete-time variables we must treat the initial values as parameters if we want to change them, and this has to be specified in the original Modelica model.
Now the S-function is ready to be used, but the last trick is that it has one input port and one output port only, even if our Modelica model is multi-input multi-output, so we have to know which are the input and output signals and in which order they are. We can see it in the Dymola block on the Windows Simulink after compiling the model, and after that use multiplexer/demultiplexer to select the desired input/output. Finally, if we decide to pass directly (without using the loaddsin function) the parameters and the initial values to the S-function, we have to remember that they must be written as column vectors.

B. An Adaptive Observer for Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping

Fossen and Grøvlen proposed an observer-based backstepping method that allows the decomposition of nonlinear output feedback control into an observer and a state feedback control [14]. However the observer design does not cover unstable ship dynamics, and an extension for these cases have been proposed by Robertsson and Johansson in [25], under a detectability condition. The adaptive observer proposed in this chapter is a modified version of the reduced-order one proposed by Erlic and Lu in [12] for manipulator control, and does not require any condition for its application, except a bound for the unknown parameters. However, in this case a full-order observer is required, in order to have a good filtering of x and y, which are measured by DGPS, with a noise in the range of 1-3 [m]. The yaw angle ψ is assumed to be measured by using a gyro compass, which is quite accurate (the noise will be less than 0.1 [deg]). Furthermore, the control law is the one shown in [14], where the unknown parameters are replaced by estimates obtained with the proposed adaptive observer.

B.1 Ship Model and Properties

The earth-fixed positions (x, y) and yaw angle ψ of the vessel is expressed in vector form as $\eta = [x, y, \psi]^T$, and the body-fixed velocities are represented by the vector $\nu = [u, v, r]^T$. The elements in η and ν describe the surge, sway, and yaw modes, respectively. Using the problem formulation from [14], we have the following system model:

$$\begin{aligned} \dot{\eta} &= J(\eta)\nu && \text{(B.1)} \\ M\dot{\nu} + D\nu + K\eta &= \tau, && \text{(B.2)} \end{aligned}$$

where

$$J(\eta) = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \qquad \text{(B.3)}$$

$$D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} > 0, \; M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} > 0. \qquad \text{(B.4)}$$

$J(\eta)$ is the rotation matrix in yaw, M is the inertia matrix, K represents the mooring forces and τ is the control vector of forces from the thruster

system. For details about the model, see [14], and the references therein. We suppose that some parameters of the matrices M, D, K are unknown but constant, and that only position (η) measurements are available.
From the structure of the model, we can write the left side of (B.2) as

$$M\dot{\nu} + D\nu + K\eta = \varphi_0(\dot{\nu}, \nu, \eta) + \varphi(\dot{\nu}, \nu, \eta)\theta \tag{B.5}$$

where $\theta \in \mathbf{R}^p$ is the unknown parameter vector, and we suppose to know a bound for M, D and K, that is

$$\begin{aligned} 0 &< M_{min} < \|M\| < M_{max} && \text{(B.6)} \\ 0 &< D_{min} < \|D\| < D_{max} && \text{(B.7)} \\ 0 &< \|K\| < K_{max}. && \text{(B.8)} \end{aligned}$$

Furthermore, it is important to note that $J^{-1}(\eta) = J^T(\eta)$, and $\|J(\eta)\| = 1$.

REMARK B.1
As in [25] and [14], eq. (B.2) can be rewritten as

$$\dot{\nu} = A_1\eta + A_2\nu + B\tau \tag{B.9}$$

where $A_1 = -M^{-1}K$, $\quad A_2 = -M^{-1}D$, $\quad B = M^{-1}$, but, from a viewpoint of parameter identification, eq. (B.2) is a better description of the system. If, for instance, only the inertia matrix M is unknown, using (B.2) we shall have to estimate only the matrix M, but using (B.9) we shall have to estimate A_1, A_2 and B, because all these matrices contain M. □

B.2 Observer Design and Stability Analysis

We propose the following adaptive observer for the system (B.1) and (B.2):

$$\begin{aligned} \dot{\hat{\eta}} &= J(\eta)\hat{\nu} + K_1(\eta - \hat{\eta}) && \text{(B.10)} \\ \dot{\hat{\nu}} &= \hat{M}^{-1}\left(\tau - \hat{D}\hat{\nu} - \hat{K}\hat{\eta}\right) + K_2(\nu - \hat{\nu}) && \text{(B.11)} \\ \dot{\hat{\theta}} &= -\Gamma\varphi^T(\xi, \hat{\nu}, \hat{\eta})[\nu - \hat{\nu}] && \text{(B.12)} \end{aligned}$$

where $\hat{\eta}, \hat{\nu}, \hat{\theta}$ are the position, velocity, and parameter estimates, respectively, $\xi(\hat{\eta}, \hat{\nu}, \hat{\theta}, \tau) = \hat{M}^{-1}\left(\tau - \hat{D}\hat{\nu} - \hat{K}\hat{\eta}\right)$, and $K_1 > 0$, $\quad K_2 > 0$ are constant gain matrices.
Subtracting (B.10) from (B.1), and (B.11) from (B.2), we have the observation error dynamics

$$\begin{aligned} \dot{\tilde{\eta}} &= J(\eta)\tilde{\nu} - K_1\tilde{\eta} && \text{(B.13)} \\ M\dot{\tilde{\nu}} &= -\tilde{M}\left(\dot{\hat{\nu}} - K_2\tilde{\nu}\right) - \tilde{D}\hat{\nu} - \tilde{K}\hat{\eta} - D\tilde{\nu} - K\tilde{\eta} - MK_2\tilde{\nu} && \text{(B.14)} \end{aligned}$$

where $\tilde{\eta} = \eta - \hat{\eta}$ and $\tilde{\nu} = \nu - \hat{\nu}$ are the position and velocity estimation errors, respectively, and $\tilde{M} = M - \hat{M}$, $\quad \tilde{D} = D - \hat{D}$, $\quad \tilde{K} = K - \hat{K}$. Let us define the parameter estimation error $\tilde{\theta} = \theta - \hat{\theta}$ and consider the following Lyapunov function candidate:

$$V(\tilde{\eta}, \tilde{\nu}, \tilde{\theta}) = \frac{1}{2}\Big(\tilde{\eta}^T \tilde{\eta} + \tilde{\nu}^T M \tilde{\nu} + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}\Big), \tag{B.15}$$

its time derivative along the solutions of (B.13) and (B.14) is

$$\begin{aligned} \dot{V} &= \tilde{\eta}^T \dot{\tilde{\eta}} + \tilde{\nu}^T M \dot{\tilde{\nu}} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &= -\tilde{\eta}^T K_1 \tilde{\eta} - \tilde{\nu}^T \Big(D + M K_2\Big)\tilde{\nu} + \tilde{\eta}^T \Big(J(\eta) - K\Big)\tilde{\nu} \\ &- \tilde{\nu}^T \Big(\tilde{M}\xi + \tilde{D}\hat{\nu} + \tilde{K}\hat{\eta}\Big) + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}}. \end{aligned} \tag{B.16}$$

Using the property (B.5) and noting that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$ for constant parameters, (B.16) becomes

$$\begin{aligned} \dot{V} &= -\tilde{\eta}^T K_1 \tilde{\eta} - \tilde{\nu}^T \Big(M K_2 + D\Big)\tilde{\nu} + \tilde{\eta}^T \Big(J(\eta) - K\Big)\tilde{\nu} \\ &- \tilde{\theta}^T \Big(\varphi^T(\xi, \hat{\nu}, \hat{\eta})\tilde{\nu} + \Gamma^{-1}\dot{\hat{\theta}}\Big), \end{aligned} \tag{B.17}$$

and furthermore, using Eq. (B.12) and assumptions (B.6), (B.7) and (B.8), we have

$$\dot{V} \leq -\underline{\sigma}_1 \|\tilde{\eta}\|^2 - \Big(\underline{\sigma}_2 + D_{min}\Big)\|\tilde{\nu}\|^2 + \Big(1 + K_{max}\Big)\|\tilde{\eta}\|\|\tilde{\nu}\|, \tag{B.18}$$

where $\underline{\sigma}_1 = \lambda_{min}(K_1)$ and $\underline{\sigma}_2 = \lambda_{min}(K_2^T M^T + M K_2)/2$. Rewriting (B.18) as

$$\dot{V} \leq -[\|\tilde{\eta}\|, \|\tilde{\nu}\|] Q(\underline{\sigma}_1, \underline{\sigma}_2) [\|\tilde{\eta}\|, \|\tilde{\nu}\|]^T, \tag{B.19}$$

it can be verified readily that Q is positive definite if

$$\underline{\sigma}_1 > \frac{(1 + K_{max})^2}{4(\underline{\sigma}_2 + D_{min})}, \quad \underline{\sigma}_2 > 0 \tag{B.20}$$

and in this case we have global asymptotic stability with respect to the ship positions and velocities, and global stability with respect to the unknown parameters.

REMARK B.2
The observer (B.10), (B.11) and (B.12) is not directly implementable because of the presence of the unknown signal ν into the equations (B.11) and (B.12). However a discrete-time approximation of the above observer can be implemented as shown below. □

B.3 The Discrete-Time Approximation of the Adaptive Observer

Integrating (B.10), (B.11) and (B.12), we have

$$\hat{\eta}(t) = \hat{\eta}(t_0) + \int_{t_0}^{t} [J(\eta)\hat{\nu} + K_1(\eta - \hat{\eta})]\mathrm{d}t \quad \text{(B.21)}$$

$$\hat{\nu}(t) = \hat{\nu}(t_0) + \int_{t_0}^{t} [\xi(\hat{\eta}, \hat{\nu}, \hat{\theta}, \tau) - K_2\hat{\nu}]\mathrm{d}t + \int_{\eta(t_0)}^{\eta(t)} K_2 J^T(\eta)\mathrm{d}\eta \quad \text{(B.22)}$$

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t_0) + \Gamma \int_{t_0}^{t} \varphi^T(\xi, \hat{\nu}, \hat{\eta})\hat{\nu}\mathrm{d}t \\ &- \Gamma \int_{\eta(t_0)}^{\eta(t)} \varphi^T(\xi, \hat{\nu}, \hat{\eta}) J^T(\eta)\mathrm{d}\eta, \end{aligned} \quad \text{(B.23)}$$

and replacing t_0 with $t - \Delta$, $\Delta > 0$, we can write

$$\hat{\eta}(t) = \hat{\eta}(t-\Delta) + \int_{t-\Delta}^{t} [J(\eta)\hat{\nu} + K_1(\eta - \hat{\eta})]\mathrm{d}t \quad \text{(B.24)}$$

$$\begin{aligned} \hat{\nu}(t) &= \hat{\nu}(t-\Delta) + \int_{t-\Delta}^{t} [\xi(\hat{\eta}, \hat{\nu}, \hat{\theta}, \tau) - K_2\hat{\nu}]\mathrm{d}t \\ &+ \int_{\eta(t-\Delta)}^{\eta(t)} K_2 J^T(\eta)\mathrm{d}\eta \end{aligned} \quad \text{(B.25)}$$

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t-\Delta) + \Gamma \int_{t-\Delta}^{t} \varphi^T(\xi, \hat{\nu}, \hat{\eta})\hat{\nu}\mathrm{d}t \\ &- \Gamma \int_{\eta(t-\Delta)}^{\eta(t)} \varphi^T(\xi, \hat{\nu}, \hat{\eta}) J^T(\eta)\mathrm{d}\eta. \end{aligned} \quad \text{(B.26)}$$

Assuming that Δ is sufficiently small, (B.24), (B.25) and (B.26) suggest a discrete implementation of the proposed observer as follows

$$\hat{\eta}(i) = \hat{\eta}(i-1) + \Delta\Big(J(i-1)\hat{\nu}(i-1) + K_1\tilde{\eta}(i-1)\Big) \quad \text{(B.27)}$$

$$\begin{aligned} \hat{\nu}(i) &= \Big(I - \Delta K_2\Big)\hat{\nu}(i-1) + \Delta\xi(i-1) \\ &+ K_2 J^T(i-1)\Big(\eta(i) - \eta(i-1)\Big) \end{aligned} \quad \text{(B.28)}$$

$$\begin{aligned} \hat{\theta}(i) &= \hat{\theta}(i-1) \\ &+ \Gamma\varphi^T(i-1)\Big[\Delta\hat{\nu}(i-1) - J^T(i-1)\Big(\eta(i) - \eta(i-1)\Big)\Big]. \end{aligned} \quad \text{(B.29)}$$

REMARK B.3
Obviously (B.27), (B.28) and (B.29) are only an approximation of the proposed observer (B.10), (B.11) and (B.12). However, they are implementable and stand for a good representation of the observer if the sampling interval Δ is sufficiently small. □

B.4 Observer Backstepping

Referring to [14], we define a smooth reference trajectory $\eta_d = [x_d, y_d, \psi_d]^T$

satisfying

$$\ddot{\eta}_d, \dot{\eta}_d, \eta_d \in \mathcal{L}_\infty. \tag{B.30}$$

Since the measurement of η is affected by sensor noise and the observer guarantees that $\hat{\eta} \to \eta$, the tracking error $\eta - \eta_d$ is replaced by $\hat{\eta} - \eta_d$, and is used for observer backstepping. Defining $z_1 = \hat{\eta} - \eta_d$ we have

$$\dot{z}_1 = J(\eta)\hat{\nu} + K_1\tilde{\eta} - \dot{\eta}_d. \tag{B.31}$$

The main idea of backstepping is to choose one of the state variables as virtual control. It turns out that

$$\xi_1 = J(\eta)\hat{\nu} = z_2 + \alpha_1 \tag{B.32}$$

is an excellent choice for the virtual control. ξ_1 is defined as the sum of the next error variable z_2, and α_1 which can be interpreted as a stabilizing function. Hence

$$\dot{z}_1 = z_2 + \alpha_1 + K_1\tilde{\eta} - \dot{\eta}_d. \tag{B.33}$$

We choose the following stabilizing function

$$\alpha_1 = -C_1 z_1 - D_1 z_1 + \dot{\eta}_d \tag{B.34}$$

where C_1 is a constant strictly positive feedback design matrix, usually diagonal, and D_1 is a positive diagonal damping matrix defined as

$$D_1 = \begin{bmatrix} d_1 k_1^T k_1 & 0 & 0 \\ 0 & d_2 k_2^T k_2 & 0 \\ 0 & 0 & d_3 k_3^T k_3 \end{bmatrix} \tag{B.35}$$

where $d_i > 0\,(i = 1 \ldots 3)$, and $k_i\,(i = 1 \ldots 3)$ are the column vectors of $K_1^T = [k_1, k_2, k_3]$. The damping term $-D_1 z_1$ has been added because $K_1\tilde{\eta}$ in (B.31) can be treated as a disturbance term to be compensated. Then we can write

$$\dot{z}_1 = -\Big(C_1 + D_1\Big) z_1 + z_2 + K_1\tilde{\eta}. \tag{B.36}$$

The next step is to specify the desired dynamics of z_2; from (B.32), we have

$$\begin{aligned} \dot{z}_2 &= \dot{\xi}_1 - \dot{\alpha}_1 = J(\eta)\dot{\hat{\nu}} + \dot{J}(\eta)\hat{\nu} + \Big(C_1 + D_1\Big)\dot{z}_1 - \ddot{\eta}_d \\ &= -\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)(z_2 + K_1\tilde{\eta}) - \ddot{\eta}_d \\ &+ \dot{J}(\eta)\hat{\nu} + J(\eta)\Big(- \hat{M}^{-1}\hat{K}\hat{\eta} - \hat{M}^{-1}\hat{D}\hat{\nu} + \hat{M}^{-1}\tau + K_2\tilde{\nu}\Big). \end{aligned} \tag{B.37}$$

Defining

$$\rho = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}, \quad S(\rho) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{B.38}$$

and $\tilde{\rho} = \rho - \hat{\rho}$, we can write

$$\dot{J}(\eta) = J(\eta)S(\rho) = J(\eta)S(\tilde{\rho}) + J(\eta)S(\hat{\rho}) \tag{B.39}$$

and

$$\begin{aligned} \dot{J}(\eta)\hat{\nu} &= J(\eta)S(\tilde{\rho})\hat{\nu} + J(\eta)S(\hat{\rho})\hat{\nu} \\ &= J(\eta)T(\hat{\nu})\tilde{\nu} + J(\eta)S(\hat{\rho})\hat{\nu} \end{aligned} \tag{B.40}$$

where

$$T(\hat{\nu}) = \begin{bmatrix} 0 & 0 & -\hat{v} \\ 0 & 0 & \hat{u} \\ 0 & 0 & 0 \end{bmatrix}. \tag{B.41}$$

Substituting (B.40) into (B.37), yields

$$\begin{aligned} \dot{z}_2 &= -\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)(z_2 + K_1\tilde{\eta}) \\ &- \ddot{\eta}_d + J(\eta)T(\hat{\nu})\tilde{\nu} + +J(\eta)S(\hat{\rho})\hat{\nu} \\ &+ J(\eta)\Big(- \hat{M}^{-1}\hat{K}\hat{\eta} - \hat{M}^{-1}\hat{D}\hat{\nu} + \hat{M}^{-1}\tau + K_2\tilde{\nu}\Big). \end{aligned} \tag{B.42}$$

Now we choose the control law as follows

$$\begin{aligned} \tau &= -\hat{M}J^T(\eta)\Big[- \Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)z_2 - \ddot{\eta}_d + C_2 z_2 + D_2 z_2 + z_1\Big] \\ &- \hat{M}S(\hat{\rho})\hat{\nu} + \hat{K}\hat{\eta} + \hat{D}\hat{\nu}, \end{aligned} \tag{B.43}$$

where C_2 is a constant strictly positive feedback design matrix, usually diagonal. Substituting (B.43) into (B.42), we have

$$\dot{z}_2 = -C_2 z_2 - D_2 z_2 - z_1 + \Omega_1\tilde{\eta} + \Omega_2\tilde{\nu} \tag{B.44}$$

where

$$\Omega_1 = \Big(C_1 + D_1\Big)K_1 \tag{B.45}$$

$$\Omega_2 = J(\eta)\Big(T(\hat{\nu}) + K_2\Big) \tag{B.46}$$

The damping matrix D_2 is defined in terms of Ω_1 and Ω_2 as

$$\begin{aligned} D_2 &= \mathrm{diag}[d_4(\omega_1^T\omega_1 + \omega_4^T\omega_4), \quad d_5(\omega_2^T\omega_2 + \omega_5^T\omega_5), \\ &\quad d_6(\omega_3^T\omega_3 + \omega_6^T\omega_6)] \end{aligned} \tag{B.47}$$

where $\Omega_1^T = [\omega_1, \omega_2, \omega_3]$, $\Omega_2^T = [\omega_4, \omega_5, \omega_6]$ and $d_i > 0\,(i = 4\ldots 6)$.

B.5 Stability Analysis of the Closed-Loop System

We can write the error dynamics as

$$
\begin{aligned}
\dot{z} &= -(C_z + D_z + E)z + W_1\tilde{\eta} + W_2\tilde{\nu} && \text{(B.48)}\\
\dot{\tilde{\eta}} &= J(\eta)\tilde{\nu} - K_1\tilde{\eta} && \text{(B.49)}\\
M\dot{\tilde{\nu}} &= -\tilde{M}\left(\dot{\hat{\nu}} - K_2\tilde{\nu}\right) - \tilde{D}\hat{\nu} - \tilde{K}\hat{\eta} - D\tilde{\nu} - K\tilde{\eta} - MK_2\tilde{\nu} && \text{(B.50)}
\end{aligned}
$$

where

$$
\begin{aligned}
z &= \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad C_z = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \quad D_z = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}, && \text{(B.51)}\\
E &= \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad W_1 = \begin{bmatrix} K_1 \\ \Omega_1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0 \\ \Omega_2 \end{bmatrix}. && \text{(B.52)}
\end{aligned}
$$

Consider the following Lyapunov function candidate

$$
V(z,\tilde{\eta},\tilde{\nu},\tilde{\theta}) = \frac{1}{2}\left(z^T z + \tilde{\eta}^T\tilde{\eta} + \tilde{\nu}^T M\tilde{\nu} + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\right) \tag{B.53}
$$

its time derivative along the solutions of (B.48), (B.49) and (B.50) is

$$
\begin{aligned}
\dot{V} &= -z^T C_z z - z^T D_z z + z^T W_1\tilde{\eta} + z^T W_2\tilde{\nu}\\
&- \tilde{\eta}^T K_1\tilde{\eta} - \tilde{\nu}^T\left(MK_2 + D\right)\tilde{\nu} + \tilde{\eta}^T\left(J(\eta) - K\right)\tilde{\nu}\\
&- \tilde{\theta}^T\left(\varphi^T(\xi,\hat{\nu},\hat{\eta})\tilde{\nu} + \Gamma^{-1}\dot{\hat{\theta}}\right) && \text{(B.54)}
\end{aligned}
$$

where we have used the fact that $z^T Ez = 0$. Now, using (B.12), and adding the zero terms

$$
\begin{aligned}
\frac{1}{4}\left(\tilde{\eta}^T G_1\tilde{\eta} - \tilde{\eta}^T G_1\tilde{\eta}\right) &= 0 && \text{(B.55)}\\
\frac{1}{4}\left(\tilde{\nu}^T G_2\tilde{\nu} - \tilde{\nu}^T G_2\tilde{\nu}\right) &= 0 && \text{(B.56)}
\end{aligned}
$$

(B.54) becomes

$$
\begin{aligned}
\dot{V} &= -z^T C_z z - z^T D_z z + z^T W_1\tilde{\eta} + z^T W_2\tilde{\nu}\\
&- \frac{1}{4}\left(\tilde{\eta}^T G_1\tilde{\eta} + \tilde{\nu}^T G_2\tilde{\nu}\right) - \tilde{\eta}^T\left(K_1 - \frac{1}{4}G_1\right)\tilde{\eta}\\
&- \tilde{\nu}^T\left(MK_2 + D - \frac{1}{4}G_2\right)\tilde{\nu} + \tilde{\eta}^T\left(J(\eta) - K\right)\tilde{\nu}. && \text{(B.57)}
\end{aligned}
$$

Defining the matrices G_1 and G_2 as

$$
G_1 = g_1 I, \quad G_2 = g_2 I \tag{B.58}
$$

where

$$
g_1 = \sum_{i=1}^{6}\frac{1}{d_i}, \quad g_2 = \sum_{i=1}^{3}\frac{1}{d_{i+3}} \tag{B.59}
$$

we have, as shown below,

$$-\quad z^T D_z z + z^T W_1 \tilde{\eta} + z^T W_2 \tilde{\nu} - \frac{1}{4}\Big(\tilde{\eta}^T G_1 \tilde{\eta} + \tilde{\nu}^T G_2 \tilde{\nu}\Big) \le 0. \quad \text{(B.60)}$$

Actually consider the left side of (B.60), which can be expanded as

$$\begin{aligned}
- &\quad z^T D_z z + z^T W_1 \tilde{\eta} + z^T W_2 \tilde{\nu} - \frac{1}{4}\Big(\tilde{\eta}^T G_1 \tilde{\eta} + \tilde{\nu}^T G_2 \tilde{\nu}\Big) \\
= &\quad -z_1^T D_1 z_1 - z_2^T D_2 z_2 + z_1^T K_1 \tilde{\eta} + z_2^T \Omega_1 \tilde{\eta} + z_2^T \Omega_2 \tilde{\nu} - \frac{g_1}{4}\tilde{\eta}^T \tilde{\eta} \\
- &\quad \frac{g_2}{4}\tilde{\nu}^T \tilde{\nu} \qquad \text{(B.61)}
\end{aligned}$$

Using definitions (B.35), (B.45), (B.46) and (B.47) together with $z_1 = [\bar{z}_1, \bar{z}_2, \bar{z}_3]^T$ and $z_2 = [\bar{z}_4, \bar{z}_5, \bar{z}_6]^T$, (B.61) can be rewritten as follows

$$\begin{aligned}
- &\quad z^T D_z z + z^T W_1 \tilde{\eta} + z^T W_2 \tilde{\nu} - \frac{1}{4}\Big(\tilde{\eta}^T G_1 \tilde{\eta} + \tilde{\nu}^T G_2 \tilde{\nu}\Big) \\
= &\quad -\sum_{i=1}^{3} \Bigg[d_i \Big(\bar{z}_i k_i - \frac{1}{2d_i}\tilde{\eta}\Big)^T \Big(\bar{z}_i k_i - \frac{1}{2d_i}\tilde{\eta}\Big) \\
+ &\quad d_{i+3}\Big(\bar{z}_{i+3}\omega_i - \frac{1}{2d_{i+3}}\tilde{\eta}\Big)^T \Big(\bar{z}_{i+3}\omega_i - \frac{1}{2d_{i+3}}\tilde{\eta}\Big) \\
+ &\quad d_{i+3}\Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{\nu}\Big)^T \Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{\nu}\Big)\Bigg] \le 0 \qquad \text{(B.62)}
\end{aligned}$$

because all the quadratic terms in (B.62) are less than or equal to zero. Hence we can write

$$\begin{aligned}
\dot{V} &\le -z^T C_z z - \tilde{\eta}^T \Big(K_1 - \frac{1}{4}G_1\Big)\tilde{\eta} - \tilde{\nu}^T \Big(MK_2 + D - \frac{1}{4}G_2\Big)\tilde{\nu} \\
&+ \quad \tilde{\eta}^T \Big(J(\eta) - K\Big)\tilde{\nu} \qquad \text{(B.63)}
\end{aligned}$$

and using assumptions (B.6), (B.7), (B.8) we have

$$\begin{aligned}
\dot{V} &\le -z^T C_z z - \Big(\underline{\sigma}_1 - \frac{1}{4}g_1\Big)\|\tilde{\eta}\|^2 - \Big(\underline{\sigma}_2 + D_{min} - \frac{1}{4}g_2\Big)\|\tilde{\nu}\|^2 \\
&+ \quad \Big(1 + K_{max}\Big)\|\tilde{\eta}\|\|\tilde{\nu}\| \\
&= -z^T C_z z - [\|\tilde{\eta}\|, \|\tilde{\nu}\|]\bar{Q}(\underline{\sigma}_1, \underline{\sigma}_2)[\|\tilde{\eta}\|, \|\tilde{\nu}\|]^T. \qquad \text{(B.64)}
\end{aligned}$$

It can be verified that $\bar{Q}$ is positive definite if

$$\underline{\sigma}_1 > \frac{1}{4}g_1 + \frac{(1 + K_{max})^2}{4(\underline{\sigma}_2 + D_{min} - \frac{1}{4}g_2)} \qquad \text{(B.65)}$$

$$\underline{\sigma}_2 > \max\Big[0, \quad \Big(\frac{1}{4}g_2 - D_{min}\Big)\Big] \qquad \text{(B.66)}$$

and in this case we have global asymptotic stability with respect to the ship positions and velocities, and global stability with respect to the unknown parameters.

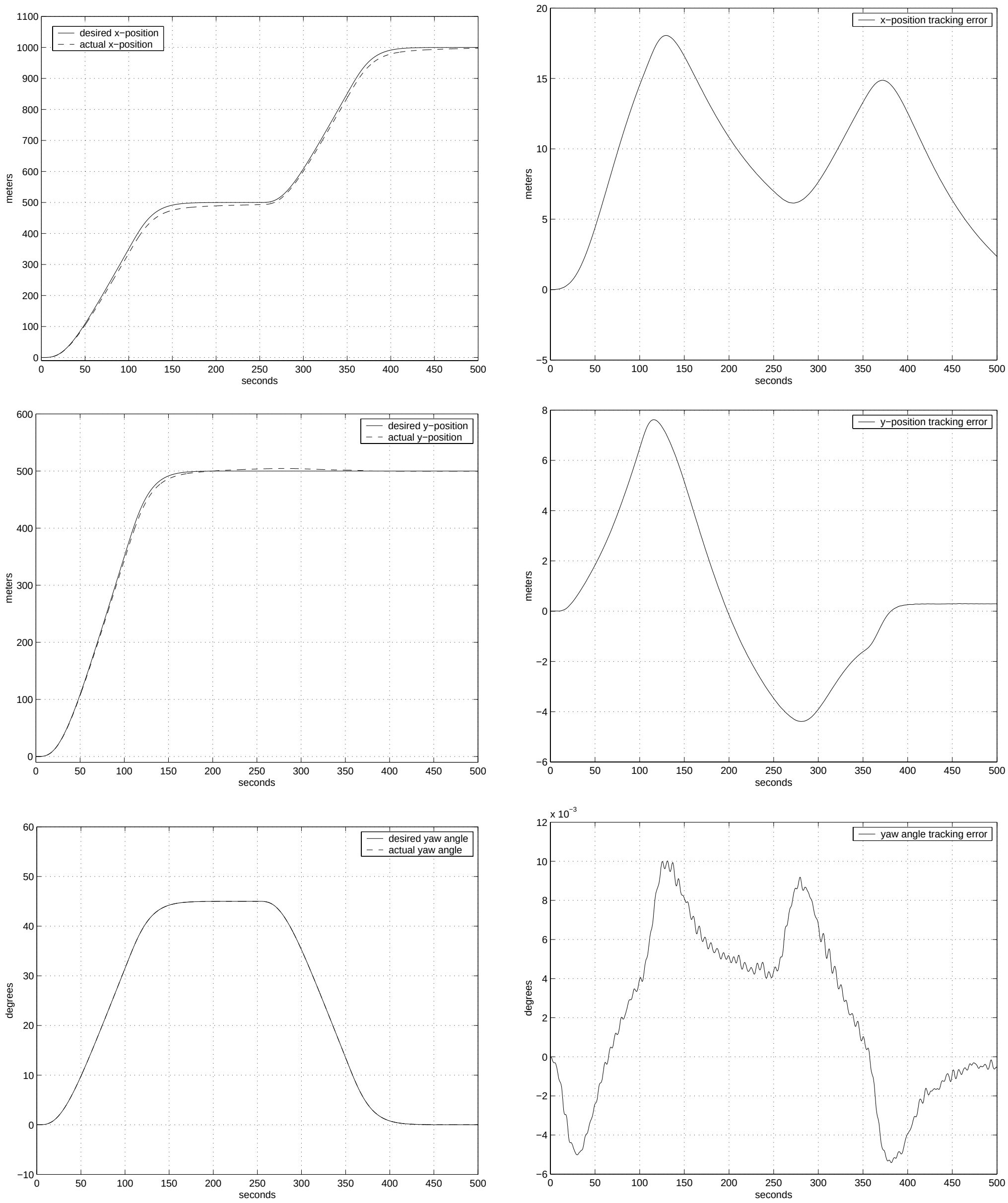

Figure B.1 DP of a supply vessel: tracking of a time-varying reference trajectory and tracking errors

REMARK B.4
Using (B.27), (B.28) and (B.29) for the implementation of the adaptive observer, the implementation of controller (B.43) involves simply the calculation of $\tau(t)$ at time instant $t = i\Delta$. $\square$

B.6 Simulation Results

To show the performance of the proposed adaptive observer-controller, we consider the case of dynamic positioning of an offshore supply vessel (see

Fig. 8 in [15]), which is described by the following matrices [15]

$$
\begin{aligned}
M &= \begin{bmatrix} 5.3122\cdot 10^6 & 0 & 0 \\ 0 & 8.2831\cdot 10^6 & 0 \\ 0 & 0 & 3.7454\cdot 10^9 \end{bmatrix} \\
D &= \begin{bmatrix} 5.0242\cdot 10^4 & 0 & 0 \\ 0 & 2.7229\cdot 10^5 & -4.3993\cdot 10^6 \\ 0 & -4.3993\cdot 10^6 & 4.1894\cdot 10^8 \end{bmatrix} \\
K &= 0.
\end{aligned}
$$

We suppose that the inertial parameter m_{11} is unknown, that is $\theta = m_{11}$. The observer-controller parameters are chosen according to

$$
\begin{gathered}
K_1 = 10^{-3}I, \quad K_2 = 10^{-3}I, \quad \Gamma = 10^3, \\
C_1 = 0.1I, \quad C_2 = 0.1I, \quad d_i = 0.1\,(i = 1, \ldots, 6).
\end{gathered}
$$

Reference trajectories are generated by using a third-order filter with poles in -0.1, that is

$$
F(s) = \frac{0.1^3}{(s+0.1)^3} \tag{B.67}
$$

Furthermore the sampling time Δ is 0.1 [s], and white noise is added to the measurements in order to illustrate the filtering properties of the observer. Results in Figs. B.1 ... B.4 show a good performance of the proposed adaptive observer-controller.

B.7 Conclusion

In this chapter an adaptive observer has been proposed and combined with an adaptive controller for dynamically positioned ship control. Global asymptotic stability of both the observer and the control law and global stability of the parameter update law have been proven by applying Lyapunov stability theory. In order to have a good filtering of noisy position measurements, a full-order observer has been used. Although only an approximate implementation of the proposed adaptive observer-controller is possible, this solution overcomes the difficulties in designing adaptive observers for nonlinear systems in which the unknown parameters and the unmeasured states are coupled. Therefore, the approximated implementation of this control scheme approaches the real one as the sampling interval approaches zero. The proposed adaptive observer does not require any conditions for its application, except a bound for the unknown parameters. In particular it is an extension of the one proposed in [14], as it covers unstable ship dynamics, parameter uncertainties and smooth time-varying parameters. Furthermore simulation results show good filtering and tracking properties also in presence of highly noise contaminated measurements.

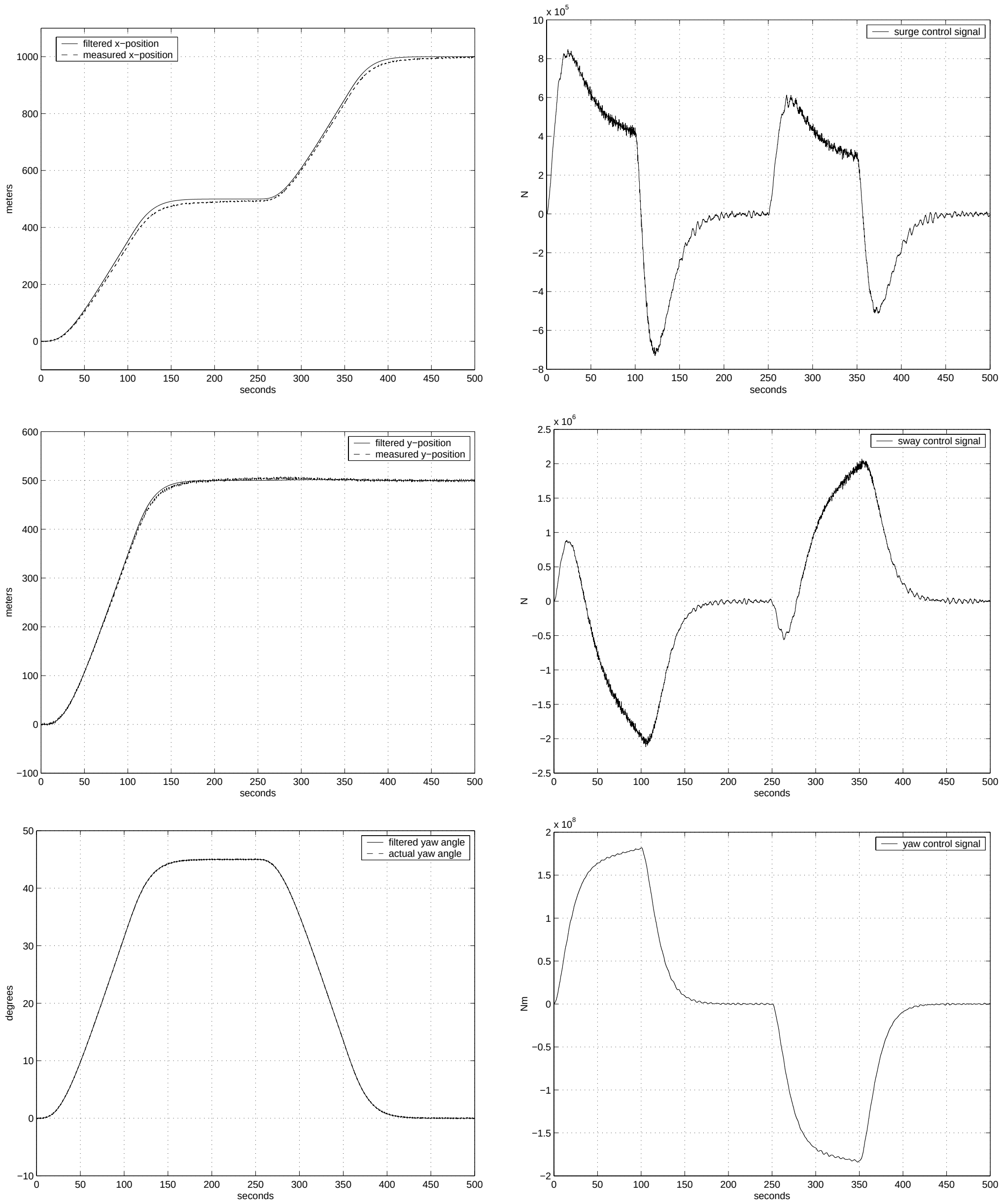

Figure B.2 DP of a supply vessel: measured and filtered positions, control signals

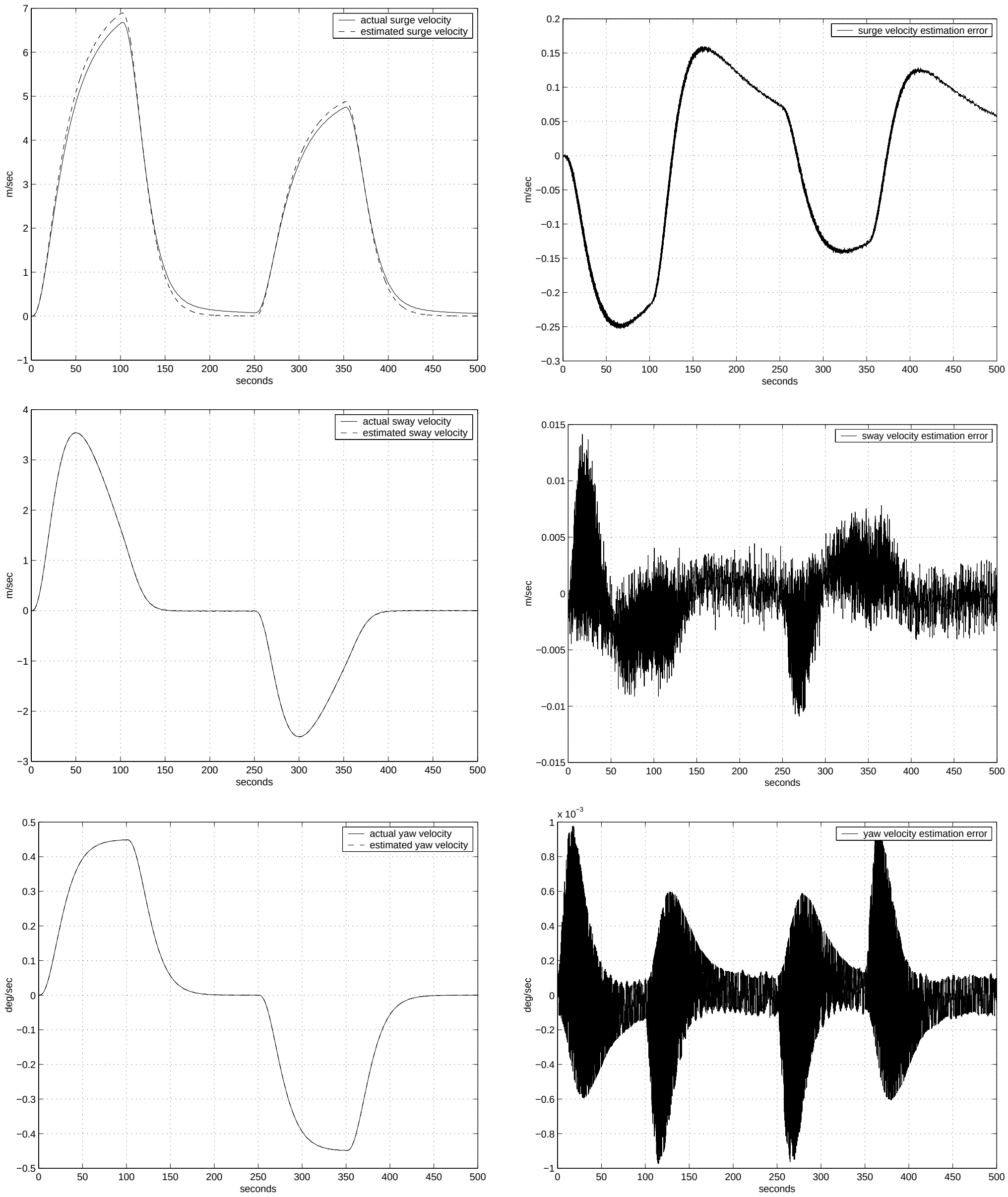

Figure B.3 DP of a supply vessel: actual and estimated velocities, velocity estimation errors

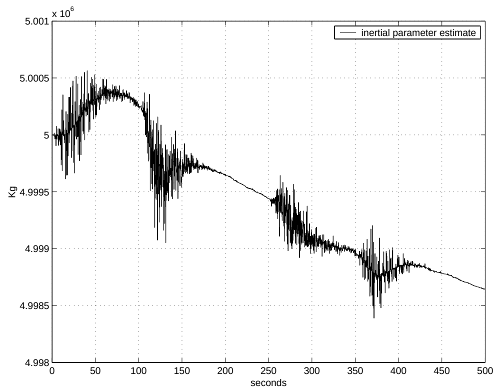

Figure B.4 DP of a supply vessel: inertial parameter estimate

C. Bibliography

[1] M. Arcak, P. Kokotovic: Nonlinear Observers: A Circle Criterion Design (Proc. of 38th Conf. on Decision and Control, December 1999)

[2] M. Arcak, P. Kokotovic: Observer-Based Control of Systems with Slope-Restricted Nonlinearities (IEEE Transaction on Automatic Control, Vol. 46, No. 7, July 2001)

[3] K.J. Åström, B. Wittenmark: Adaptive Control (Addison-Wesley 1995)

[4] K.J. Åstrom, J. Åkesson: Dynamics of the Furuta Pendulum (2002)

[5] A.N. Atassi, H.K. Khalil: A Separation Principle for the Stabilization of a Class of Nonlinear Systems (IEEE Transaction on Automatic Control, Vol. 44, No. 9, September 1999)

[6] H. Berghuis: Model-Based Robot Control: From Theory to Practice (PhD thesis, University of Twente, The Netherlands, 1993)

[7] H. Berghuis, H.Nijmeijer: A Passivity Approach to Controller-Observer Design for Robots (IEEE Transactions on Robotics and Automation, Vol. 9, No. 6 December 1993)

[8] C. Canudas De Wit, N. Fixot: Robot Control via Robust Estimated State Feedback (IEEE Transaction on Automatic Control, Vol. 36, No. 12, December 1991)

[9] C. Canudas De Wit, N. Fixot: Adaptive Control of Robot Manipulators via Velocity Estimated Feedback (IEEE Transaction on Automatic Control, Vol. 37, No. 8, August 1992)

[10] Y.M. Cho, R. Rajamani: A Systematic Approach to Adaptive Observer Synthesis for Nonlinear Systems (IEEE Transactions on Robotics and Automation, Vol. 42, No. 4, April 1997)

[11] J.J. Craig: Adaptive Control of Mechanical Manipulators (Addison-Wesley, 1988)

[12] M. Erlic, W.-S. Lu: A Reduced-Order Adaptive Velocity Observer for Manipulator Control (IEEE Transactions on Robotics and Automation, Vol. 11, No. 2, April 1995)

[13] M. Etchechoury, J. Solsona, C. Muravchik: On the Stability of Nonlinear Plants that Include an Observer for Their Feedback Linearization (International Journal of Systems Science, 1996)

[14] T.I. Fossen, Å. Grøvlen. Nonlinear Output Feedback Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping (IEEE Transactions on Control Systems Technology , Vol. 6, No. 1, January 1998)

[15] T.I. Fossen, J.P. Strand. Passive Nonlinear Observer Design for Ships Using Lyapunov Methods: Full-Scale Experiments with a Supply Vessel (May 1998)

[16] M. Jankovic: Adaptive Nonlinear Output Feedback Tracking with a Partial High-Gain Observer and Backstepping (IEEE Transaction on Automatic Control, Vol. 42, No. 1, January 1997)

[17] R. Johansson: Adaptive Control of Robot Manipulator Motion (IEEE Transactions on Robotics and Automation, Vol. 6, No. 4, August 1990)

[18] H.K. Khalil: Nonlinear Systems (Prentice Hall, 1996)

[19] H.K. Khalil: Adaptive Output Feedback Control of Nonlinear Systems Represented by Input-Output Models (IEEE Transaction on Automatic Control, Vol. 41, No. 2, February 1996)

[20] M. Krstic, I. Kanellakopoulos, P. Kokotovic: Nonlinear and Adaptive Control Design (Wiley, 1995)

[21] S.Y. Lim, D.M. Dawson, K. Anderson: Re-Examining the Nicosia-Tomei Robot Observer-Controller from a Backstepping Perspective (IEEE Transactions on Robotics and Automation, Vol. 4, No. 3, May 1996)

[22] R. Marino, G.L. Santosuosso, P. Tomei: Robust Adaptive Observers for Nonlinear Systems with Bounded Disturbances (IEEE Transaction on Automatic Control, Vol. 46, No. 6, June 2001)

[23] R.H. Middleton, G.C. Goodwin: Adaptive Computed Torque Control for Rigid Link Manipulators (Proc. 25th IEEE Conf. Decision Control, 1986)

[24] S. Nicosia, P. Tomei: Robot Control by Using Only Joint Position Measurements (IEEE Transaction on Automatic Control, Vol. 35, No. 9, September 1990)

[25] A. Robertsson: On Observer-Based Control of Nonlinear Systems (1999)

[26] M.W. Spong, M. Vidyasagar: Robot Dynamics and Control (Wiley, 1989)

[27] I.E. Tezcan, T. Basar: Disturbance Attenuating Adaptive Controllers for Parametric Strict Feedback Nonlinear Systems with Output Measurements (Proceedings of the American Control Conference, June 1997)

[28] F. Thau: Observing the State of Nonlinear Dynamic Systems (International Journal of Control, 1973)

[29] J. Wang, W.J. Rugh: On the Pseudo-Linearization Problem for Nonlinear Systems (System and Control Letters, 1989)

[30] E. Zergeroglu, D.M. Dawson, M.S. de Queiroz, M. Krstic: On Global Output Feedback Tracking Control of Robot Manipulators (Proc. of 39th Conf. on Decision and Control, December 2000)