Control of an Oscillatory System

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Abstract

Systems with structural resonances are difficult to control if fast responses are required. Rapid changes in the control signal may excite the resonant modes. In digital control with zero-order-hold sampling there will be very rapid changes in the control signal. A way to deal with the problem is to use predictive first-order-hold sampling instead. The control signal is then linear between the sampling instants. This thesis investigates trade-offs in control of a typical oscillatory system. PI, PID and state feedback controllers are examined analytically or by simulations with a model of the considered system. A comparison is also made between zero-order-hold and predictive first-order-hold sampling.

Key words

Classification system and/or index terms (if any)

Supplementary bibliographical information

Contents

Chapter 1

Introduction

Structural resonances are common in mechanical systems. Control of such systems is difficult if fast responses are required, because rapid changes in the control signal may excite the resonant modes. With digital control the problems become even worse since there will be very rapid changes in the control signal at the sampling instants. The magnitudes of these changes can of course be reduced by having a higher sampling rate but that is not always possible or preferable. Since the problem is the transitions in the control signal it would be natural to gradually increase the control signal. This can be achieved with a first-order-hold circuit, which generates a signal that is linear between the sampling instants. This thesis investigates trade-offs in control of a typical oscillatory system. The system consists of two masses connected by a spring. It has three modes represented by one pole at the origin, one pole on the negative real axis and two oscillatory poles. The performance that can be achieved with controllers of different complexity is investigated. First a PI controller is explored. With such a controller the main limitation on response speed is given by the real non-zero process pole. With a PID controller this pole can be cancelled. The limits will then instead be given by the oscillatory poles of the process. Finally a state feedback controller, which makes use of more of the knowledge of the process dynamics, is shown to give even better results. A comparison is also made between zero-order-hold and predictive first-order-hold sampling, showing on slightly better behaviour for the latter. The controllers are examined analytically and by simulations.

The investigated oscillatory system is introduced in chapter 2. A mathematical model of the system is derived. This is used for control design and simulations throughout the thesis. The chapter also contains a short description of predictive first-order-hold sampling. Chapter 3 covers PI and PID control. State feedback control is examined in chapter 4. Chapter 5 contains some conclusions and suggestions for future work on this topic.

Chapter 2

Process model

The plant model used for the simulations throughout this thesis is a model of the Rectilinear Control System Model 210 from ECP. It is an educational system and real life applications are for example robot arms with harmonic drives or the arms of disk drives and cd players. Even seemingly stiff structures can have resonances because of large accelerations. The problems with mechanical resonances are in general very common in mechatronic systems. This chapter gives a short description of the plant and the simulation model. Sampling of the model is also described and performed for digital control purposes. Details about the plant can be found in [3].
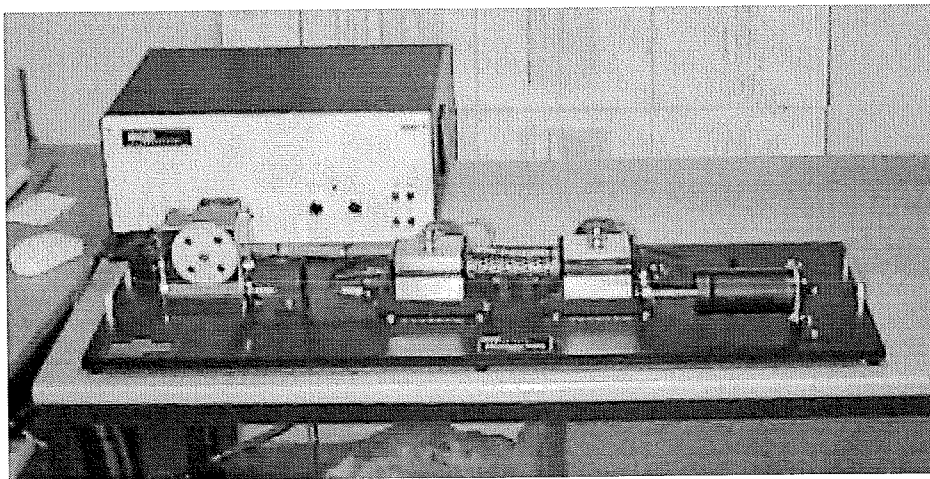
2.1 Description of the plant

Figure 2.1: Rectilinear Control System Model 210

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The process is basically two masses, connected with a spring, moving along a rail. A dashpot with adjustable damping can also be attached to any of the masses. There is a picture of the process in Figure 2.1. The first mass can be accelerated by a motor and the position of each mass can be measured by a sensor moving along with it. The movement of the masses is limited at each end of the rail by a bumper. However, there will also be friction between the masses and the rail which can not be neglected. This can be modelled as a damping of the masses with a different damping coefficient for each mass. A model of the process with friction taken into account but the bumpers neglected is found in Figure 2.2. The dashpot is considered to be disconnected.
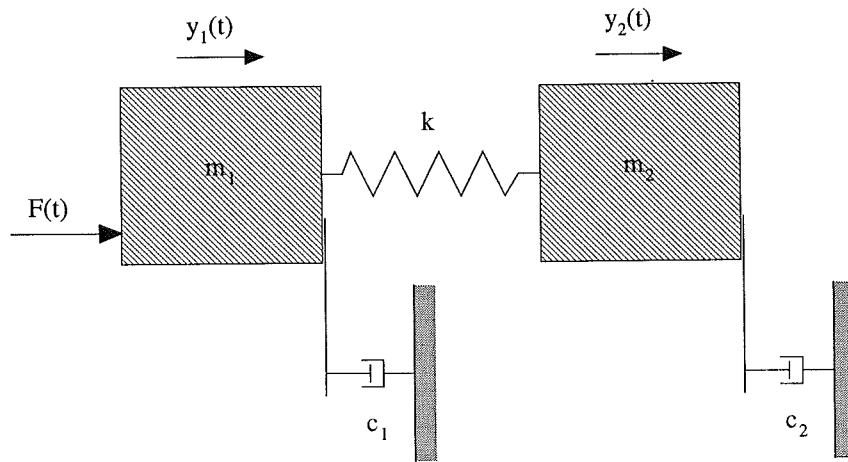

Figure 2.2: Model of the plant

2.2 Simulation model

Newtons Second Law of Motion gives

$$\begin{cases} m_1\ddot{y}_1 + c_1\dot{y}_1 + k(y_1 - y_2) &= F(t) \\ m_2\ddot{y}_2 + c_2\dot{y}_2 + k(y_2 - y_1) &= 0 \end{cases}$$

By introducing the vector x as

$$x = \begin{bmatrix} y_1 \\ \sqrt{\frac{m_1}{k}}\dot{y}_1 \\ y_2 \\ \sqrt{\frac{m_2}{k}}\dot{y}_2 \end{bmatrix}$$

the equations can be rewritten on the form

$$
\begin{cases}
\dot{x} & = & Ax + BF(t) \\
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} & = & Cx
\end{cases}
$$

with

$$
A = \begin{bmatrix}
0 & \sqrt{\frac{k}{m_1}} & 0 & 0 \\
-\sqrt{\frac{k}{m_1}} & -\frac{c_1}{m_1} & \sqrt{\frac{k}{m_1}} & 0 \\
0 & 0 & 0 & \sqrt{\frac{k}{m_2}} \\
\sqrt{\frac{k}{m_2}} & 0 & -\sqrt{\frac{k}{m_2}} & -\frac{c_2}{m_2}
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 \\
\sqrt{\frac{1}{m_1}} \\
0 \\
0
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

The x vector could have simply been chosen as the positions and the velocities of the two masses but the velocities have been weighted to avoid numerical problems due to a badly conditioned matrix A. (See [4].)

Model parameters

The parameters of the plant have been estimated to

$$
\begin{aligned}
k &= 175\,N/m \\
m_1 &= 2.79\,kg \\
m_2 &= 2.54\,kg \\
c_1 &= 3.12\,Ns/m \\
c_2 &= 3.73\,Ns/m
\end{aligned}
$$

With a system represented by

$$
\begin{aligned}
\dot{x} &= Ax + BF(t) \\
y &= Cx
\end{aligned}
$$

the matrices will be given the values

$$
A = \begin{bmatrix}
0 & 7.9198 & 0 & 0 \\
-7.9198 & -1.1183 & 7.9198 & 0 \\
0 & 0 & 0 & 8.2939 \\
8.2939 & 0 & -8.2939 & -1.4662
\end{bmatrix}
$$

$$B = \begin{bmatrix} 0 \\ 0.5987 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Transfer function

For the position of the second mass, which is going to be the primary focus of the control, the transfer function from the control signal is given by

$$G_p(s) = C_2(sI - A)^{-1}B + D \approx \frac{326}{s(s + 1.28)(s^2 + 1.3s + 131)}$$

where C_2 denotes the second row in C. This can be represented by

$$G_p(s) = \frac{K_p}{s(s + a)(s^2 + 2\zeta_p\omega_p s + \omega_p^2)}$$

where $K_p \approx 326$, $a \approx 1.28$, $\omega_p \approx 11.5$ and $\zeta_p \approx 0.0567$.

Impulse response of the process

The impulse response of the process is shown in Figure 2.3 As can be seen any impulse from the servo motor or the environment will cause the masses to oscillate at frequency ω_p. This will be a problem if a fast controller is to be designed for the plant and it will be even worse if a control signal that changes step wise is used, for example a signal obtained from sampling with a zero-order-hold circuit.

2.3 Zero-order-hold sampling

Zero-order-hold sampling of the system will yield a discrete time state-space representation on the form

$$\begin{cases} x_{k+1} &= \Phi x_k + \Gamma u_k \\ y_k &= C x_k \end{cases}$$

where the matrices Φ and Γ can be calculated from

$$\Phi = e^{Ah}$$
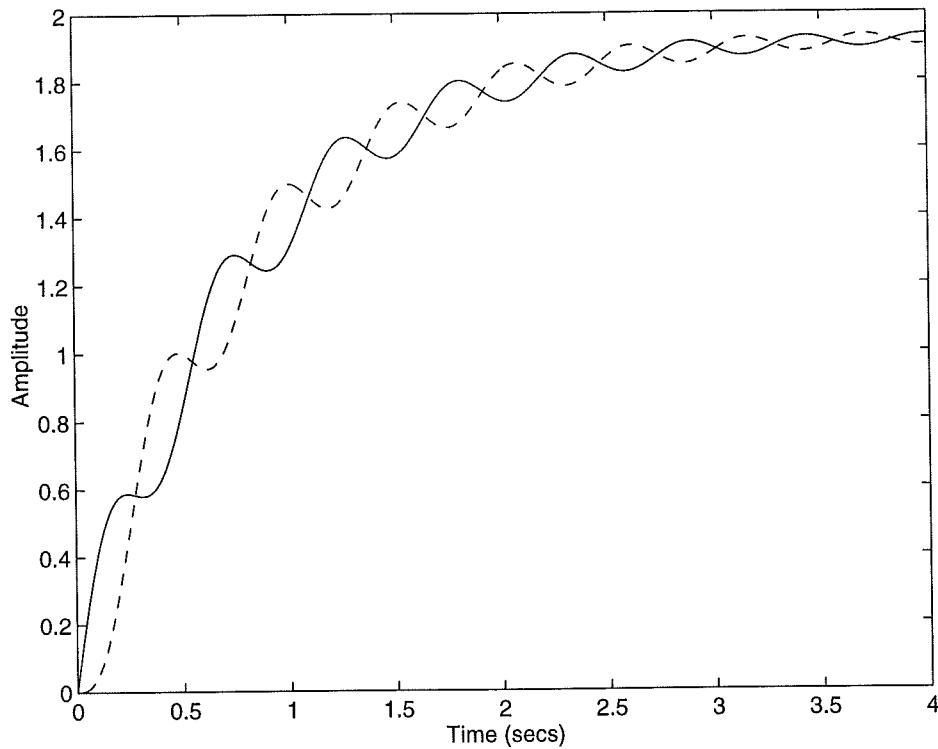
$$\Gamma = \int_0^h e^{As} ds B$$

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Figure 2.3: Impulse response of the plant. Full line is the first mass; dashed line is the second.

The sampling interval should at least be short enough to give the controller a chance to discover the oscillations of the process at the natural frequency ω_p. To get 20 samples per period of the natural oscillations h will have to be equal to

$$h = \frac{\pi}{10\omega_p}$$

With $\omega_p = 11.5$ this gives an h equal to about 27.4 ms. The sampled system matrices will then be

$$\Phi = \begin{bmatrix} 0.9769 & 0.2120 & 0.0231 & 0.0017 \\ -0.2120 & 0.9469 & 0.2120 & 0.0239 \\ 0.0253 & 0.0018 & 0.9747 & 0.2208 \\ 0.2191 & 0.0239 & -0.2191 & 0.9357 \end{bmatrix}$$

$$\Gamma = 10^{-2} \begin{bmatrix} 0.1755 \\ 1.6029 \\ 0.0008 \\ 0.0132 \end{bmatrix}$$

Zero-order-hold sampling and the choice of a sampling interval are described in [1].

2.4 Predictive first-order-hold sampling

A control signal created with zero-order-hold sampling contains a lot of high frequencies. When controlling a system with poorly damped oscillatory poles this limits controller performance, especially for fast controllers. This section presents some general formulas for predictive first-order-hold sampling, which, by changing the control signal linearly between the sampling instants, injects much less high frequencies into the process. The theory presented here was taken from [1] and [2].

The system to be sampled is assumed to be described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

and the sampling interval h is assumed to be constant. With a first-order-hold circuit the input signal u(t) is linear between the sampling instants. Integration of the system over one sampling period then gives

$$x(kh + h) = \Phi x(kh) + \Gamma u(kh) + \Gamma_1 \frac{(u(kh + h) - u(kh))}{h}$$

where

$$\begin{aligned} \Phi &= e^{Ah} \\ \Gamma &= \int_0^h e^{As} ds B \\ \Gamma_1 &= \int_0^h e^{As}(h - s) ds B \end{aligned}$$

Φ, Γ and Γ_1 satisfy the differential equations

$$\begin{aligned} \frac{d\Phi(t)}{dt} &= \Phi(t)A \\ \frac{d\Gamma(t)}{dt} &= \Phi(t)B \\ \frac{d\Gamma_1(t)}{dt} &= \Gamma(t) \end{aligned}$$

which can also be expressed as

$$\frac{d}{dt} \begin{bmatrix} \Phi(t) & \Gamma(t) & \Gamma_1(t) \\ 0 & I & It \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} \Phi(t) & \Gamma(t) & \Gamma_1(t) \\ 0 & I & It \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A & B & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}$$

Φ, Γ and Γ_1 can finally be calculated from

$$\begin{bmatrix} \Phi & \Gamma & \Gamma_1 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \end{bmatrix} exp\left\{ \begin{bmatrix} A & B & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} h \right\}$$

The Φ and Γ matrices will be identical to those obtained by ZOH sampling. Using the value of h calculated in the previous section the discrete time system matrices will be

$$\Phi = \begin{bmatrix} 0.9769 & 0.2120 & 0.0231 & 0.0017 \\ -0.2120 & 0.9469 & 0.2120 & 0.0239 \\ 0.0252 & 0.0018 & 0.9747 & 0.2208 \\ 0.2191 & 0.0239 & -0.2191 & 0.9357 \end{bmatrix}$$

$$\Gamma = 10^{-2} \begin{bmatrix} 0.1755 \\ 1.6029 \\ 0.0008 \\ 0.0131 \end{bmatrix}$$

$$\Gamma_1 = 10^{-4} \begin{bmatrix} 0.1609 \\ 2.2159 \\ 0.0004 \\ 0.0090 \end{bmatrix}$$

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Chapter 3

PID control

It is possible to control the system with a PID or even a PI controller. This will, however, not give a system with a very fast response. In this chapter design and simulation of continuous time PI and PID controllers is considered. The PI controller design is limited to placing a single controller zero and this is done by setting the desired damping of the closed loop system. For the PID control the design will instead be done from looking at the root locus for the system. Theory for PI and PID control and the certain issues discussed in this chapter are described in [5], [6] and [7].

3.1   PI control

Specifications

The plant has four poles; one at the origin, one on the negative real axis and two oscillatory poles just barely in the negative half plane. The PI controller adds one pole at the origin and a zero that can be placed anywhere on the real axis. The location of the zero will determine the locations of the closed loop poles. The first aim is to move the two poles at the origin into the complex half plane but any attempt to do so also moves the pole on the negative real axis towards and eventually past the origin. A reasonable choice is then to place all three poles at the same distance α from the origin. The two oscillatory poles of the process will then be almost unaffected. They will also not particularly influence the behaviour of the closed loop system. There is more than one way to place the three slow poles at the same distance from the origin. Two of the three poles will be complex and can be chosen with different damping ζ. Neglecting the oscillatory poles the desired closed loop characteristic polynomial will then be

$$(s + \alpha)(s^2 + 2\zeta\alpha s + \alpha^2) = s^3 + (2\zeta + 1)\alpha s^2 + (2\zeta + 1)\alpha^2 s + \alpha^3$$

Controller design

The PI controller can be described by

$$U(s) = K_c(1 + \frac{1}{sT_i})(U_c(s) - Y(s))$$

and the considered part of the process, ie without the oscillatory poles, by

$$Y(s) = \frac{b}{s(s+a)}U(s)$$

b is K_p compensated for the magnitude of the two neglected poles:

$$b = \frac{K_p}{\omega_p^2}$$

where ω_p is the magnitude of the poles. The characteristic polynomial will then be

$$s^2(s+a) + bK_c(s + \frac{1}{T_i}) = s^3 + as^2 + bK_cs + \frac{bK_c}{T_i}$$

which is to be made equal to the desired polynomial. Equating the two polynomials and solving for K_c and T_i gives

$$K_c = \frac{a^2}{b(2\zeta+1)} = \frac{a^2\omega_p^2}{K_p(2\zeta+1)}$$

$$T_i = \frac{(2\zeta+1)^2}{a}$$

Note that α will be equal to

$$\alpha = \frac{a}{2\zeta+1}$$

thus the speed of the system decreases when the damping increases. The actual closed loop system still depends on the poles omitted in the design.

$$G_{cl} = \frac{G_cG_p}{1 + G_cG_p} = \frac{K_cK_p(s + 1/T_i)}{K_cK_p(s + 1/T_i) + s^2(s+a)(s^2 + 2\zeta_p\omega_ps + \omega_p^2)}$$

The influence of the oscillatory poles causes the locations of the three poles placed by the design to differ a little bit from the desired locations. This is crucial if ζ is chosen close to zero, since the two complex poles will be pushed into the right half plane, resulting in an unstable system.

Nominal design

The two complex poles of the desired closed loop system could be placed arbitrarily by choosing α and ζ but this would also affect the real pole. Placing all three poles at the same distance α leaves the single design parameter ζ. For the nominal design ζ has been chosen to $\zeta = 0.5$. That gives an $\alpha = 0.6423$ and controller parameters equal to $K_c = 0.3326$ and $T_i = 3.1140$. An analysis of the entire closed loop system, including the oscillatory poles of the process gives the closed loop poles

$$
\begin{aligned}
-0.6511 & \quad \pm 11.4119i \\
-0.3211 & \quad \pm 0.5598i \\
-0.6402 &
\end{aligned}
$$

This corresponds to $\alpha = 0.6453$ and $\zeta = 0.4976$ which is very close to the desired values. The oscillatory process poles have been moved very little as expected. The controller zero is located in $z_c = -0.3211$. A simulation of the system with the nominal K_c and T_i is shown in Figure 3.1. The simulation shows the response to a unit step input at time 0. A load disturbance of magnitude -0.1 is introduced at time 20 and finally a measurement noise with the frequency 20 rad/s and the amplitude 0.01 is applied to the system at time 40.
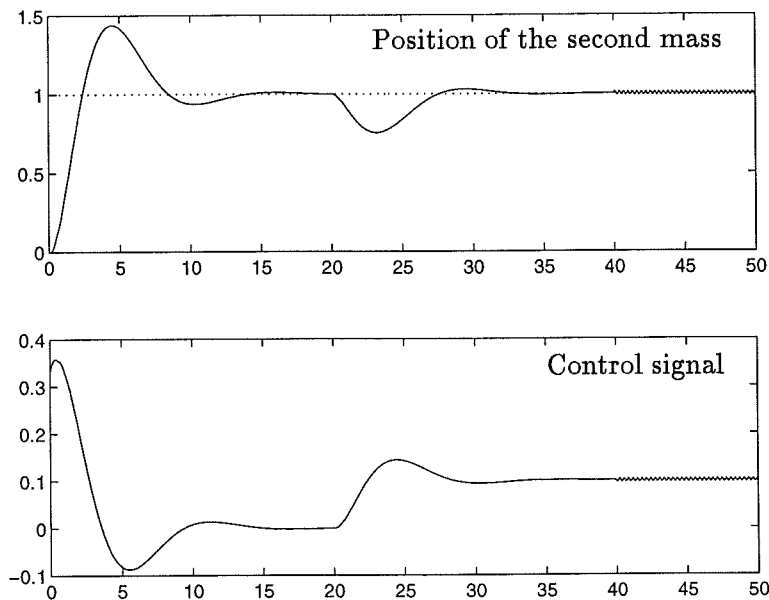

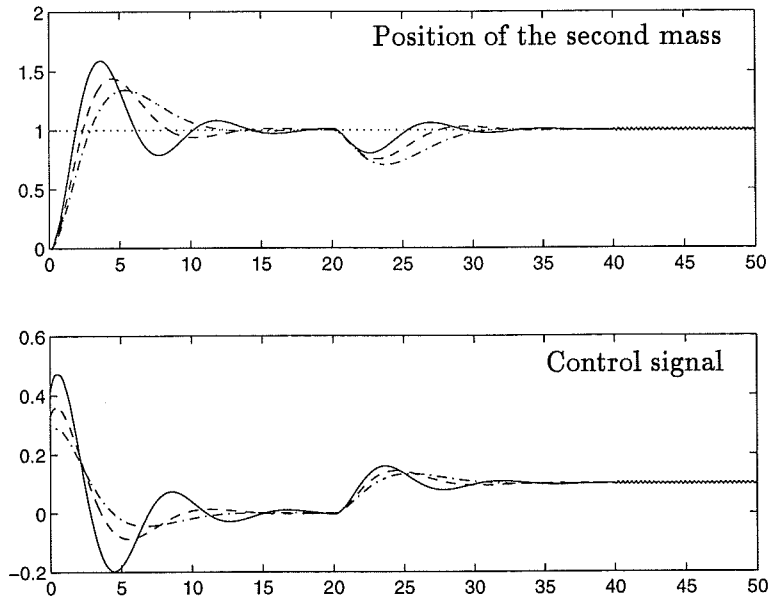Figure 3.1: Simulation of the system with $\zeta = 0.5$

Figure 3.2: Simulation of the system with $\zeta = 0.3$ (full line), 0.5 (dashed line) and 0.7 (dashed/dotted line).

Changing ζ

Figure 3.2 shows the effects of changing ζ; notice, however, that the true damping of the system will be slightly smaller than ζ due to the oscillatory poles of the process which have been neglected in the design. The figure also shows that increasing ζ decreases α which corresponds to the speed of the response. Table 3.1 shows the corresponding α, location of the controller zero and controller gain to each ζ of the figure.

ζ	α	z_c	K_c
0.3	0.8028	-0.5018	0.4157
0.5	0.6423	-0.3211	0.3326
0.7	0.5352	-0.2230	0.2771

Table 3.1: ζ, α, z_c and K_c for different designs

Robustness

A look at the systems response to a disturbance applied at the output of the process gives the sensitivity function

$$S = 1 - G_{cl} = \frac{s^2(s+a)(s^2 + 2\zeta_p\omega_p s + \omega_p^2)}{K_c K_p(s + 1/T_i) + s^2(s+a)(s^2 + 2\zeta_p\omega_p s + \omega_p^2)}$$
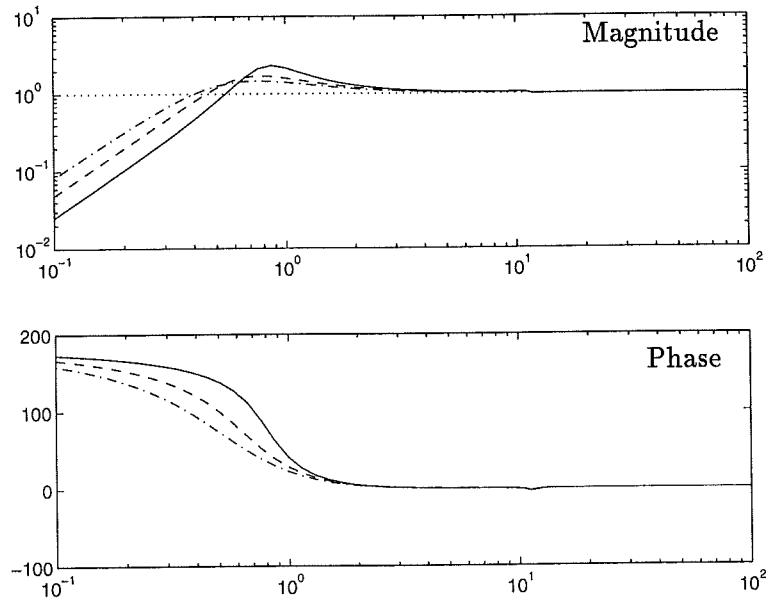
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Figure 3.3: Sensitivity of the system with $\zeta = 0.3$ (full line), 0.5 (dashed line) and 0.7 (dashed/dotted line).

The Bode plot of the sensitivity function is shown in Figure 3.3 for $\zeta = 0.3$, 0.5 and 0.7. All three controllers amplify the disturbances above a certain frequency and this has to be, according to Bode's integral (See eg [8]). The highest peak is for the fastest controller, ie the one with $\zeta = 0.3$. This controller is also most sensitive to high frequent disturbances and least sensitive to very low frequent disturbances. The peak values are shown in table 3.1.

ζ	S_{max}
0.3	2.3
0.5	1.7
0.7	1.5

Table 3.2: Sensitivity function maximum for different ζ.

Weighting of the reference value

The overshoot of the step response can be reduced by letting only a fraction β of the reference signal act on the proportional part of the controller.

$$U(s) = K(\beta U_c(s) - Y(s)) - \frac{K}{sT_i}(U_c(s) - Y(s))$$

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This means that the controller does not immediately respond fully to a reference signal change but gradually as the integral part discovers the change. This is similar to low pass filtering of the reference signal. The effects of varying β is shown in Figure 3.4.
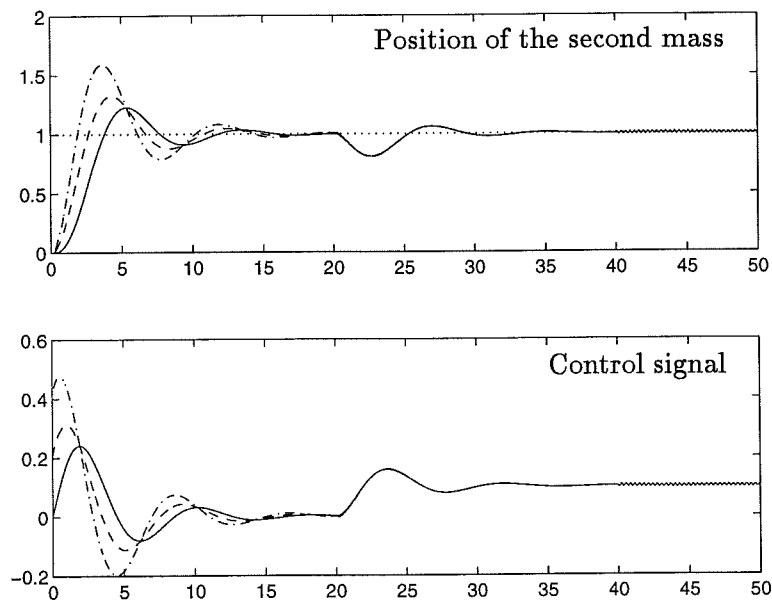

Figure 3.4: Simulation of the system with $\beta = 0$ (full line), 0.5 (dashed line) and 1 (dashed/dotted line); $\zeta = 0.3$.

3.2 PID control

With a PID controller there is a possibility to cancel the real pole of the plant in s=-a by placing a controller zero in the same location. Doing so gives a greater freedom in placing the closed loop poles and the system can be made faster.

Specifications

When using a PI controller the speed of the closed loop system was limited by the real pole of the process at s=-a; when the closed loop poles at the origin were moved into the negative half plane the pole at s=-a moved towards the origin. With a PID controller there is one extra zero which can be used to cancel this pole. The two poles at the origin can now be moved further into the negative half, limited instead by the oscillatory poles which will move towards and eventually past the imaginary axis. Instead of giving a desired

closed loop transfer function it will be easier to look at the root locus of the system for different sets of controller parameters.

Controller design

The first step in the controller design is to cancel the process pole in -a. The PID controller can simplest be described by

$$U(s) = K_c(1 + \frac{1}{sT_i} + sT_d)(U_c(s) - Y(s))$$

For the same reason as in the PI controller the influence of the reference signal on the proportional part of the controller is weighted by the parameter β. Neither the derivative of the reference signal should fully affect the control signal and it is therefore weighted with the parameter γ. γ should always be set to zero if the reference signal changes stepwise. In that case the derivative is infinite in the transitions. To decrease the influence of high frequent noise the derivative gain can be limited to N at high frequencies by rewriting the derivative part of the controller as

$$sT_d \approx \frac{sT_d}{1 + sT_d/N}$$

and the total expression for controller becomes

$$U(s) = K_c(\beta + \frac{1}{sT_i} + \gamma\frac{sT_d}{1 + sT_d/N})U_c(s) - K_c(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_d/N})Y(s)$$

With a zero in -a the part acting on the feedback can also be expressed as

$$U_{fb}(s) = -K_c(N + 1)\frac{(s + a)(s + z_2)}{s(s + N/T_d)}Y(s)$$

where $s = -z_2$ is the location of the second controller zero. Relating T_i to T_d and N by equating the above expressions gives

$$T_i = \frac{aT_d - N}{a(aT_d(N + 1) - N)}$$

This condition ensures cancelling of the pole in s=-a regardless of how the parameters K_c, T_d and N are chosen.

Nominal design

β and γ are set to zero to make the proportional and derivative parts of the controller independent of the reference signal. N is initially set to infinity but this will later be changed in order to deal with high frequency measurement noise. If T_d is set to zero a PI controller with cancelling of the pole in s=-a
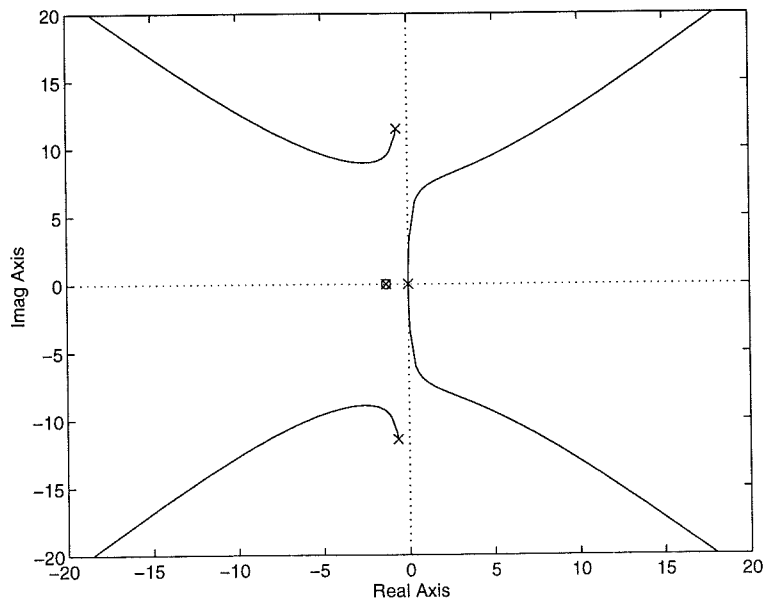
Figure 3.5: Root locus for $T_d = 0$; $T_i = 1/a$.

is obtained. As can be seen in Figure 3.5 it will not be possible to get a stable system with this controller. Figure 3.6 shows what will happen with the root locus when T_d is increased. It is interesting to see that increasing the controller gain will, at least initially, increase both the speed and the damping of the two complex poles closest to the origin. It will, however, also push the two oscillatory process poles towards instability. The maximum stable gains of the systems in Figure 3.6 are shown in Table 3.2 With an

T_d	K_{c-max}
0.1/a	5.96
0.2/a	3.19
0.3/a	2.18
0.5/a	1.33

Table 3.3: Maximum gain for different T_d

infinite N the value of T_d must also not exceed 1/a in order to keep T_i positive. The nominal design parameters have been chosen as $T_d = 0.2$ and $K_c = 2.4$. A simulation of the system with this controller can be seen in Figure 3.7. As with the PI controller simulations the input is a unit step at time 0. A load disturbance of magnitude -0.1 is introduced at time 20 and a measurement noise with the frequency 20 rad/s and the amplitude 0.01 is applied to the system at time 40.
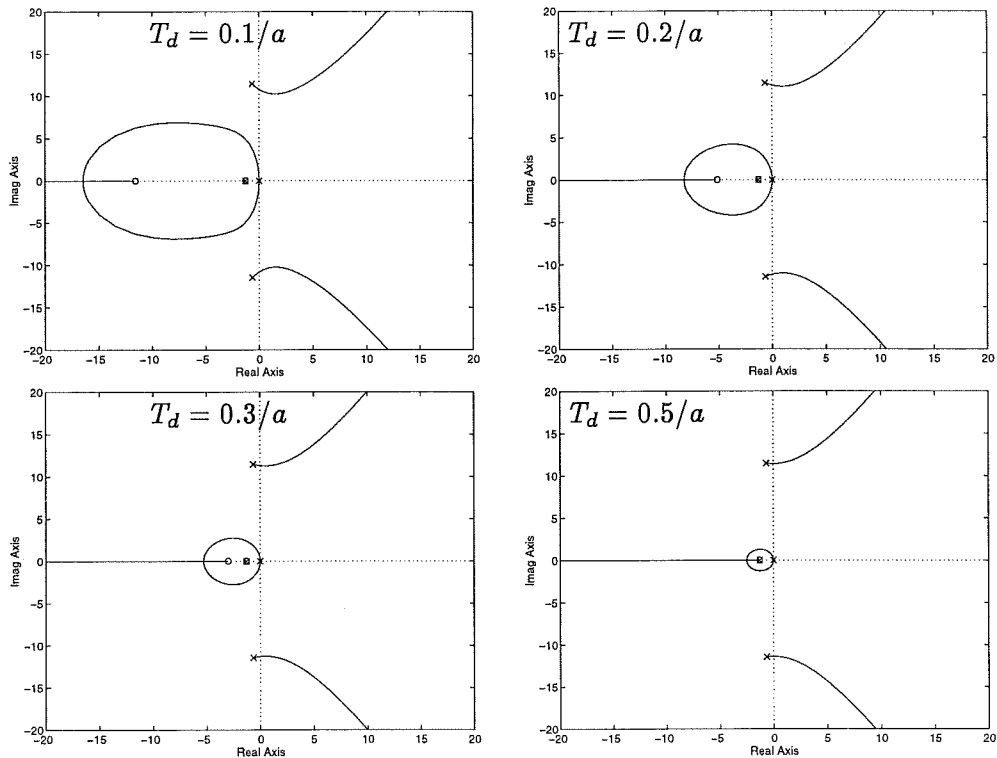
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Figure 3.6: Root loci for different T_d.

Changing K_c and T_d

Increasing K will move the two complex poles closest to the origin away from the origin making them faster but also better damped (Figure 3.6). At the same time though, the two far poles will move toward the imaginary axis, eventually making the system unstable. A simulation of the system with a K_c closer to the stability limit is shown in Figure 3.8. The same figure also contains the plot of the simulation of the nominal design for comparison. It can be seen that the step response is better damped but that the oscillatory behaviour of the process also becomes significant. The sensitivity to measurement noise seems to be greater with a larger K_c as well, although part of the increased amplitude of the noise in the control signal is due to the ringing that remains from the setpoint change and the load disturbance introduction. The effects of changing T_d can be seen in Figure 3.9. With a smaller T_d the system can be made even faster before the oscillatory poles start to get significant. This can also be understood by looking at the root loci in Figure 3.6.
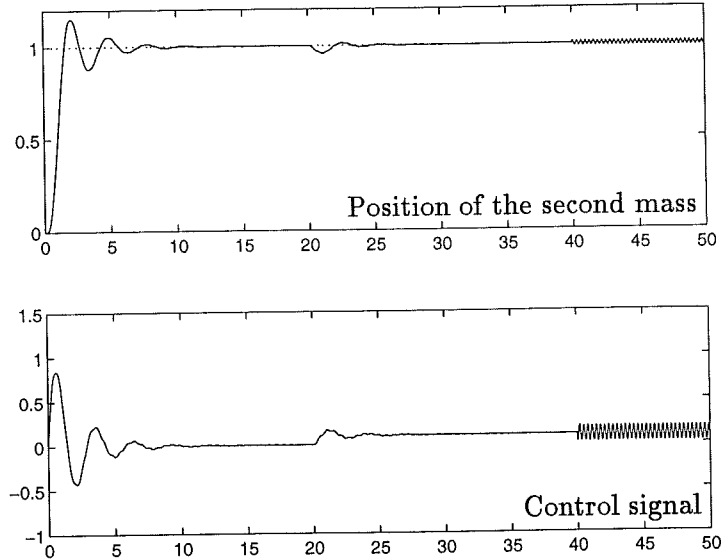
Figure 3.7: Simulation with PID control. $K_c = 2.4$, $T_d = 0.2/a$.

Changing T_i

The value of a cannot be measured exactly. It may also vary a little bit in time due to nonlinear behavior of the springs and of friction and other damping in the system. T_i can thus not be expected to put a zero in the exact location of s=-a. An important issue is then how much the behavior of the closed loop system is influenced by small changes in T_i. Figure 3.10 shows simulations of the system with rather larger deviations ($\pm 10\%$) of T_i. A change in a doesn't seem to be crucial to the behavior in this case.

Limited high frequency gain

As the frequency of the measurement noise is increased the control signal will be very noisy. For high frequencies there will be considerable problems. This can be dealt with, as mentioned, by limiting the gain of the derivative part of the controller to a value N. N is normally set to about 10 and in this case such a value of N will just barely influence the behaviour of the system at low frequencies, ie the speed and the damping of the step and load disturbance responses, but it will effectively keep the amplification low at high frequencies. An example of this is shown in Figure 3.11. In this simulation the noise frequency is 200 rad/s. The control signal dependency on measurement disturbances can also be analysed mathematically. With a measurement noise v(t) added to the measured output signal y(t) the
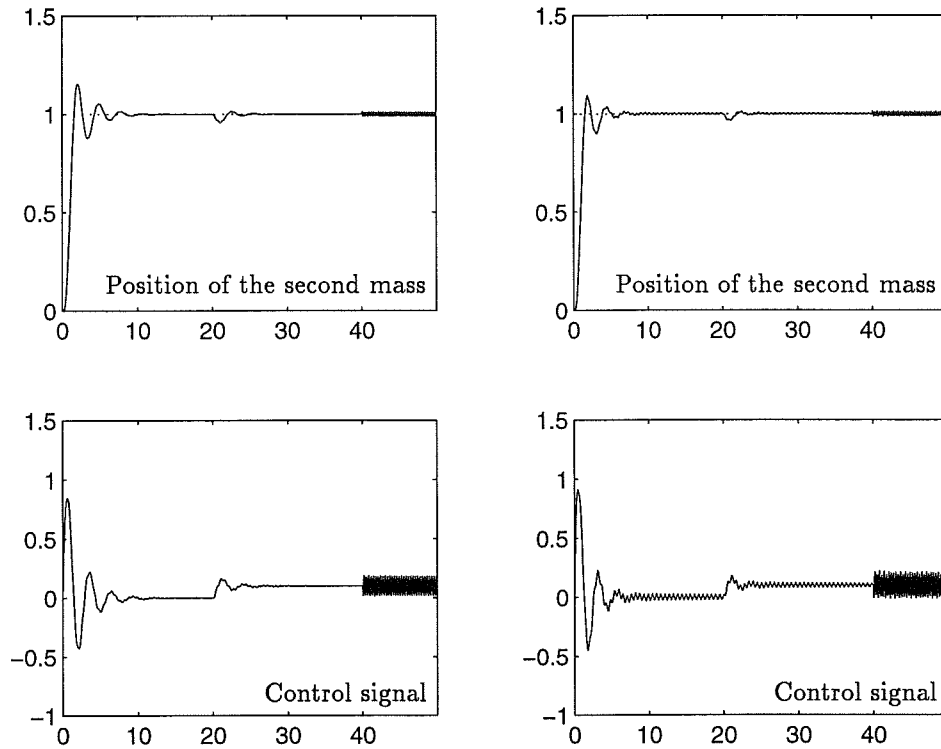
Figure 3.8: The effects of increasing K_c. $K_c = 2.4$ (left) and $K_c = 3$ (right); $T_d = 0.2/a$.

transfer function from v to u will be given by

$$
\begin{aligned}
U(s) &= \frac{G_c}{1 + G_c G_p} V(s) \\
&= \frac{\overline{K_c} s^2 (s + a)(s^2 + p_1 s + p_2)(s^2 + 2\zeta_p \omega_p s + \omega_p^2)}{\overline{K_c} K_p (s^2 + p_1 s + p_2) + s^2 (s + a)(s + N/T_d)(s^2 + 2\zeta_p \omega_p s + \omega_p^2)}
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{K_c} &= K_c(1 + N) \\
p_1 &= \frac{N T_i + T_d}{T_i T_d (1 + N)} \\
p_2 &= \frac{N}{T_i T_d (1 + N)}
\end{aligned}
$$

The effect of introducing a high frequency gain limitation is very clear in the bode diagram of Figure 3.12 which shows the same system with different values of N.
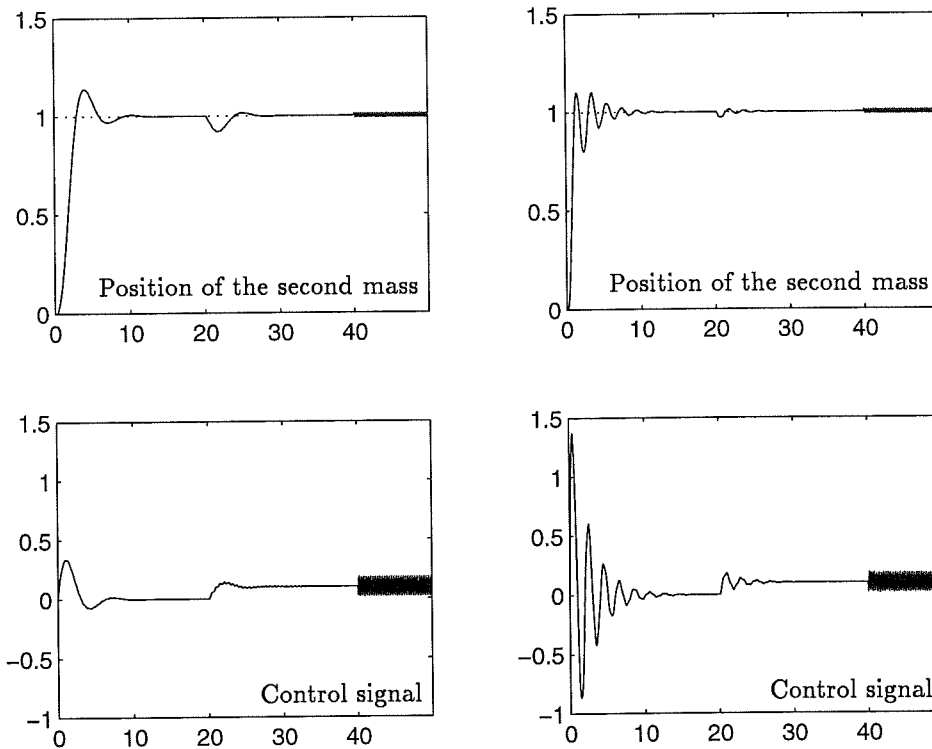
Figure 3.9: PID control with $K_c = 1$, $T_d = 0.5/a$ (left) and $K_c = 4$, $T_d = 0.1/a$ (right).

Robustness

The sensitivity function for the system with a PID controller is

$$S = \frac{1}{1 + G_c G_p}$$

$$= \frac{s(s + a)(s + N/T_d)(s^2 + 2\zeta_p \omega_p s + \omega_p^2)}{\overline{K_c} K_p (s^2 + p_1 s + p_2) + s^2(s + a)(s + N/T_d)(s^2 + 2\zeta_p \omega_p s + \omega_p^2)}$$

A plot of the sensitivity function for $K_c = 1.0$, $T_d = 0.5/a$ and N=10 is shown in Figure 3.13. The output signal sensitivity is of course closely related to the control signal sensitivity discussed in the last section. Notice the peak in magnitude when the frequency is slightly larger than 10 rad/s. The peak value is about 10.8dB, ie disturbances with this frequency are amplified by a factor 3.5.
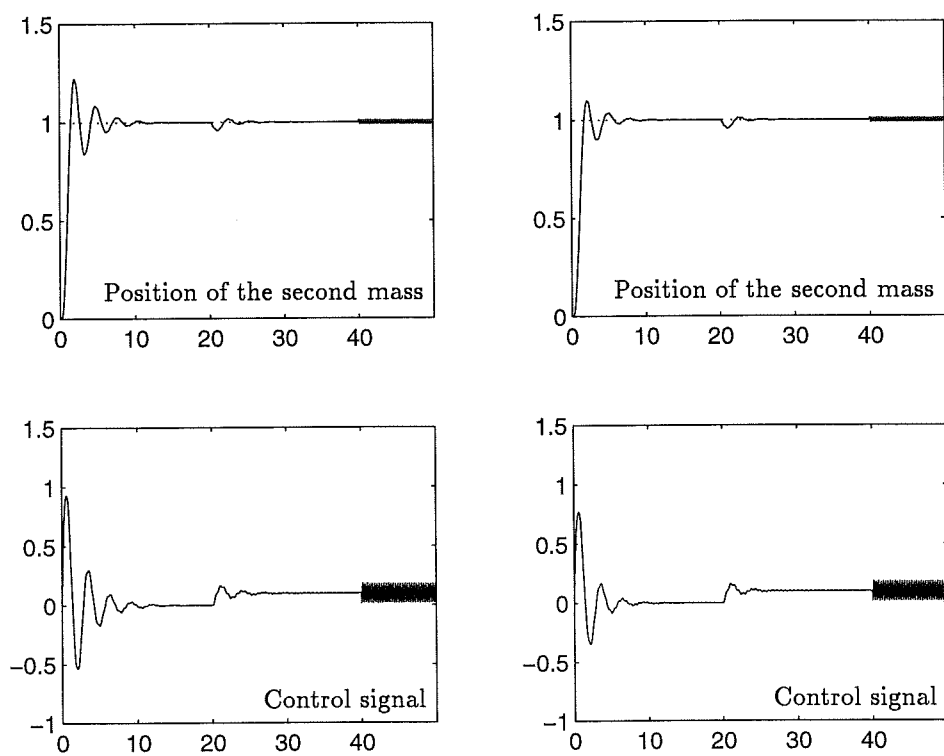
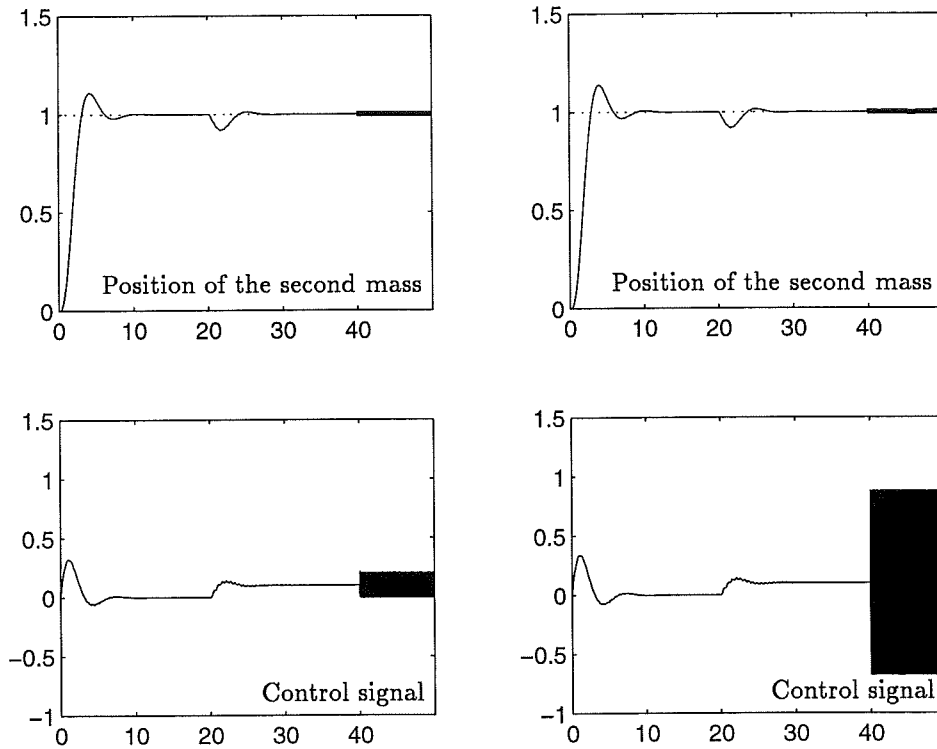Figure 3.10: Simulation with 10% deviation of T_i, down (left) and up (right); $K_c = 2.4$, $T_d = 0.2/a$.

Figure 3.11: The effect of limiting the derivative gain for high frequencies. N=10 (left) and infinity (right); $K_c = 1.0$, $T_d = 0.5/a$

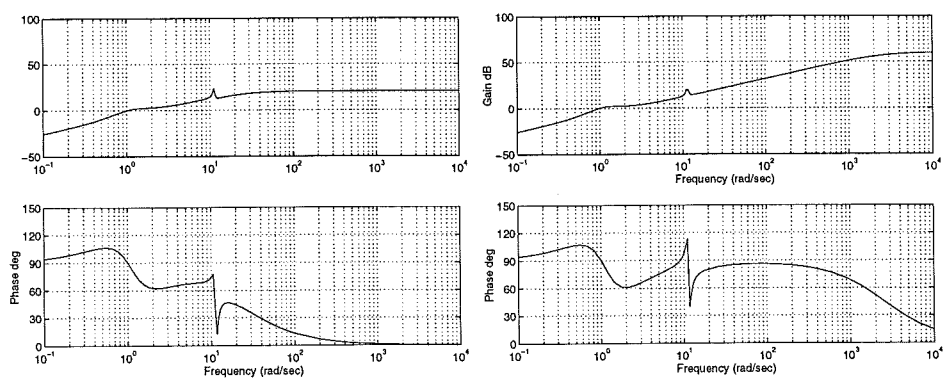
Figure 3.12: Control signal sensitivity to measurement noise. $K_c = 1.0$, $T_d = 0.5/a$, N=10 (left) and 1000 (right).
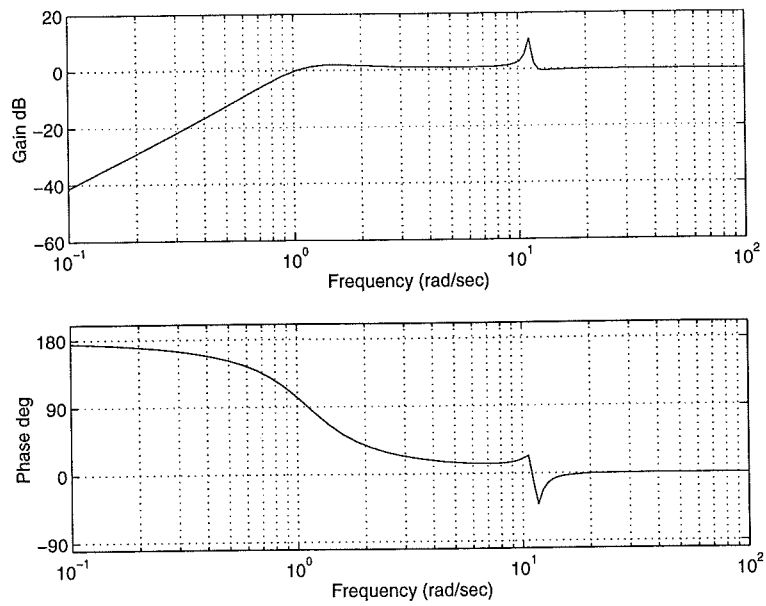
Figure 3.13: Sensitivity function with PID control. $K_c = 1.0$, $T_d = 0.5/a$, N=10.

Chapter 4

State feedback control

State feedback control uses more information about the process than PID control. There is a possibility to measure the positions of both masses and with proper models of the process dynamics the velocities of the masses and even some disturbances can be estimated. In this chapter pole placement design by state feedback is considered. First the process model is modified in section 4.1 to include disturbances. The basic structure of the controller is presented in section 4.2. Section 4.3 is on design with zero-order-hold sampling; a controller with state estimation and feedback and feedforward control is designed and examined. Section 4.4 covers design with predictive first-order-hold sampling. The focus is on design and comparisons with zero-order-hold.

4.1 Process model

The process model used in this chapter is

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\xi \xi(t)$$
$$y(t) = Cx(t) + e(t)$$

where $\xi(t)$ and e(t) are stochastic disturbances. B_ξ is equal to B, ie the state noise is considered to be entering the system in the same way as the control signal.

4.2 Controller structure

Both the ZOH and the PFOH designs will use the same basic controller structure with separate feedback and feedforward parts and an observer that estimates the four states of the process and any constant state disturbances. The general structure is presented in Figure 4.1. There will be some minor changes for the PFOH which are explained in section 4.4.
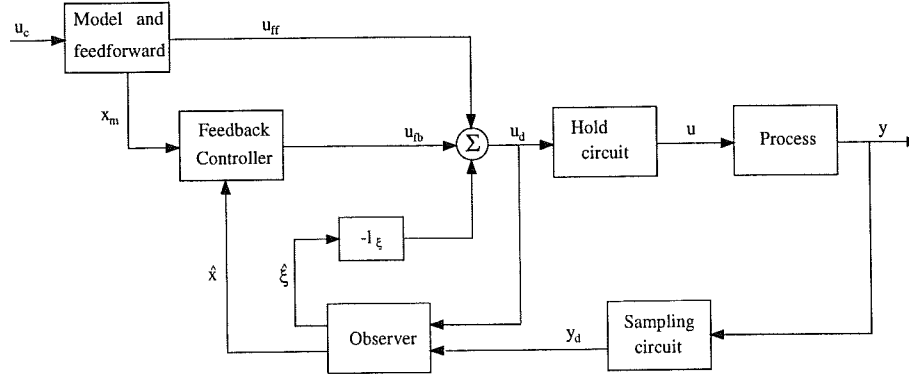
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Figure 4.1: Structure of the state feedback controller

4.3 Zero-order-hold

Specifications

The controller will be designed with a linear quadratic approach in continuous time. The goal is then to choose the matrices Q_{1c}, Q_{12c} and Q_{2c} to minimize the loss function

$$J = E\left\{ \int_0^{Nh} \left[x^T(t)Q_{1c}x(t) + 2x^T(t)Q_{12c}u(t) + u^T(t)Q_{2c}u(t) \right] dt \right.$$
$$\left. + x^T(Nh)Q_{0c}x(Nh) \right\}$$

This will be translated into its discrete time counterpart

$$J = E\left\{ \sum_{k=0}^{N-1} \left[x^T(kh)Q_1x(kh) + 2x^T(kh)Q_{12}u(kh) + u^T(kh)Q_2u(kh) \right] \right.$$
$$\left. + x^T(Nh)Q_0x(Nh) \right\}$$

which is minimized by choosing the feedback matrix L. L is chosen as the steady state solution of L(k) to the Riccati equation. An observer will be used for reconstruction of the states. It is developed as a Kalman filter but the feedback matrix K is only chosen as the steady state value of the filter matrix K(k). The design parameters will be the continuous time weighting matrices Q_{1c}, Q_{12c} and Q_{2c} and the approximated disturbance covariances σ_v^2 and σ_e^2. Q_{0c} will not influence the steady state solution of the equations and doesn't have to be set in the design.

Understanding the design parameters

Setting the design parameters requires at least a basic understanding of the influence of the weighting and covariance matrices on the LQ and Kalman

filter equations respectively. From the continuous time loss function equation it is clear that a large value of Q_{2c}, which is a scalar, will require small values of the control signal u to keep the loss function small. The same way of thinking is applicable to Q_{1c}, although it needs some further interpretation since it is a matrix. The diagonal elements correspond to one each of the four states of the model in exactly the same way as Q_{2c} corresponds to u. The other elements will influence the allowed differences in value between the states. For example a large value of the third element of the first row will give a design that doesn't allow one of the masses to move unless the other one follows. Notice that the Q_{1c} matrix has to be symmetric. The elements of Q_{12c} will couple the states of the system to the control signal. The values of the covariances for the filter design, σ_v^2 and σ_e^2, are in this case not based on knowledge of the true noise but will be treated in the same way as the weighting matrices for the control design. Thus a large value of σ_e^2 will give an estimator that anticipates insecure measurements and therefore is rather slow. A large σ_v^2 would instead give a faster but more sensitive estimator. Of course the absolute values of the weighting and covariance matrices have no meaning, but only the values relative to each other.

Controller design

A simple way of looking at the disturbances $\xi(t)$ and e(t) in the process model is as gaussian white noise with zero mean value and covariances σ_ξ^2 and σ_e^2 respectively. This structure will, however, not be able to eliminate errors due to constant disturbances. If instead $\xi(t)$ is considered to be the integral of a white noise v(t) with zero mean value and the covariance σ_v^2 the process model will be

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) + B_\xi \xi(t) \\
\dot{\xi}(t) &= v(t) \\
y(t) &= Cx + e(t)
\end{aligned}
$$

where e(t) is still white noise with zero mean and covariance σ_e^2. Constant disturbances can then be eliminated by simply subtracting a term $B^{-1}B_\xi \xi(t)$ from the control signal. In this case v(t) is introduced in the same point as the control signal and $B_\xi = B$, thus the control signal will be compensated with $-\xi(t)$ or rather, as the disturbance can't be directly measured, by the estimate $-\hat{\xi}(t)$.

Linear quadratic control design

The position and velocity feedback matrix L will be computed as the matrix L(k) of an LQG controller in steady state. If the weighting matrices Q_1,

Q_{12} and Q_2 are known the Riccati equation

$$S(k) = [\Phi - \Gamma L(k)] S(k+1) [\Phi - \Gamma L(k)] + Q_1 - L^T(k)Q_{12}^T - Q_{12}L(k)$$
$$+ L^T(k)Q_2 L(k)$$

where

$$L(k) = (Q_2 + \Gamma^T S(k+1)\Gamma)^{-1}(\Gamma^T S(k+1)\Phi + Q_{12}^T)$$

can be solved or iterated to give the steady state values S and L.

State estimation

The states can be estimated with a Kalman filter. If only the position and velocity states are reconstructed the filter can be described by

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + \Gamma u(k) + K(k)(y(k) - C\hat{x}(k|k-1))$$

But a value of the disturbance ξ is also needed as stated above. The state vector is extended with the ξ and the extended state vector is denoted $z(t)$.

$$z(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}$$

With this transformation the state-space model becomes

$$\dot{z}(t) = A_z z(y) + B_z u(t) + B_v v(t)$$
$$y(t) = C_z z(t) + e(t)$$

with the matrices

$$A_z = \begin{bmatrix} A & B_\xi \\ 0 & 0 \end{bmatrix}$$

$$B_z = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$B_v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_z = \begin{bmatrix} C & 0 \end{bmatrix}$$

Sampling the transformed system without regarding the disturbances gives a discrete time system

$$z_{k+1} = \Phi_z z_k + \Gamma_z u_k$$
$$y_k = C_z z_k$$

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and the Kalman filter becomes

$$
\begin{aligned}
\hat{z}_{k+1} &= \Phi_z \hat{z}_k + \Gamma_z u_k + K_k(y_k - C\hat{z}_k) \\
K_k &= \Phi_z P_k C_z^T (R_2 + C_z P_k C_z^T)^{-1} \\
P_{k+1} &= \Phi_z P_k \Phi_z^T + R_1 - \Phi_z P_k C_z^t (R_2 + C_z P_k C_z^T)^{-1} C_z P_k \Phi_z^T
\end{aligned}
$$

where R_1 and R_2 are the discrete time representations of R_{1c} and R_{2c} respectively and $R_{1c} = \sigma_v^2 B_\xi B_\xi^T$ and $R_{2c} = \sigma_e^2 I$.

Feedforward

Introduction of the reference signal will be done in terms of a desired state behaviour and a feedforward control signal. The states of the model are denoted $x_m(k)$ and the feedforward control signal $u_{ff}(k)$. The first mass is modelled as a second order system independent of the second mass and with the reference signal u_c as its input. The transfer function from the reference signal to the position, denoted x_1^m, is

$$
G_{m1} = \frac{\alpha^2}{(s + \alpha)^2}
$$

where α is a design parameter, setting the speed of the model. Similarly the second mass will be viewed as a second order system with the position of the first mass as its input. The second mass position is denoted x_3^m and the transfer function from the first to the second mass position is

$$
G_{m3} = \frac{\beta^2}{(s + \beta)^2}
$$

where β is another design parameter. x_2^m and x_4^m are related to the positions in the same way as in the process model used in the feedback controller. The feedforward model can then be expressed as a state space system

$$
\dot{x}_m = A_m x_m + B_m u_c
$$

where

$$
A_m = \begin{bmatrix}
0 & 1/c_1 & 0 & 0 \\
-\alpha^2 c_1 & -2\alpha & 0 & 0 \\
0 & 0 & 0 & 1/c_2 \\
\beta^2 c_2 & 0 & -\beta^2 c_2 & -2\beta
\end{bmatrix}
$$

$$
B_m = \begin{bmatrix}
0 \\
\alpha^2 c_1 \\
0 \\
0
\end{bmatrix}
$$

and

$$c_1 = \sqrt{\frac{m_1}{k}}$$
$$c_2 = \sqrt{\frac{m_2}{k}}$$

The feedforward control signal u_{ff} is created from

$$u_{ff} = \frac{G_{m1}}{\bar{G}_p} u_c$$

$\bar{G}_p$ is chosen to be the simplified process model transfer function used in Chapter 3 for PI control design

$$\bar{G}_p = \frac{b}{s(s+a)}$$

u_{ff} can then be expressed in terms of x_m and u_c as

$$u_{ff} = -\frac{\alpha^2}{b} x_1^m + \frac{a - 2\alpha}{c_1 \alpha^2} x_2^m + \frac{\alpha^2}{b} u_c$$

and the complete state space system for the feedforward model becomes

$$\dot{x}_m = A_m x_m + B_m u_c$$
$$u_{ff} = C_m x_m + D_m u_c$$

with A_m and B_m as above and

$$C_m = \begin{bmatrix} -\frac{\alpha^2}{b} & \frac{a-2\alpha}{c_1\alpha^2} & 0 & 0 \end{bmatrix}$$
$$D_m = \frac{\alpha^2}{b}$$

Nominal design

The sampling interval h is set to 0.0274 s. The weighting matrices Q_{1c} and Q_{2c} are both set to unity. The state noise covariance σ_v is assumed to be 1 and the measurement noise covariance σ_e is assumed to be 0.00001 to get an observer that is considerably faster than the controller. The parameters of the feedforward model are set to $\alpha = 2$ and $\beta = 4$. A simulation of the system with this controller can be seen in Figure 4.2. The inputs are the same as in the previous chapters, ie a unit step at time 0, followed by a load disturbance of magnitude -0.1 at time 20 and a measurement noise with the frequency 20 rad/s and the amplitude 0.01 at time 40. How well the positions follow the feedforward model is shown in Figure 4.3. The poles of the closed loop system, assuming the states to be directly measured, are

$$-0.6974 \quad \pm 11.4475i$$
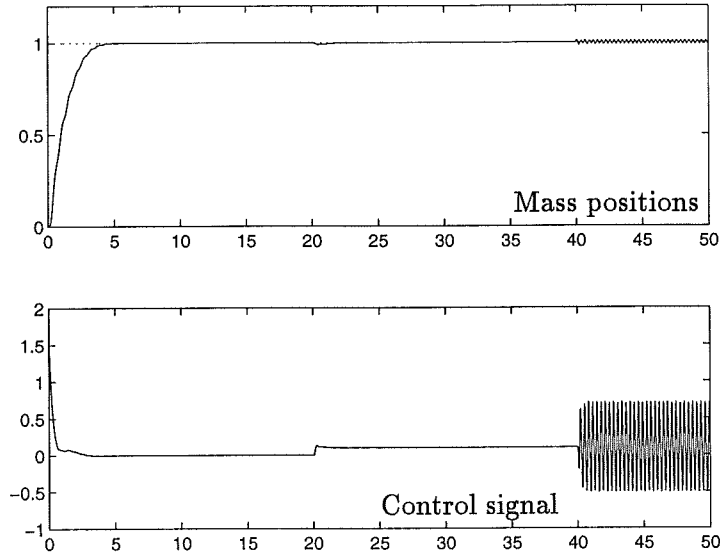$$-1.4882 \quad \pm 1.1367i$$

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Figure 4.2: Simulation of the system with state feedback and zero-order-hold sampling.

and the poles of the observer are

$$
\begin{array}{ll}
-9.6679 & \pm 18.9098i \\
-3.8795 & \pm 9.0700i \\
-20.0756 &
\end{array}
$$

Compared with the simulations of the PID controllers in the previous chapter the result is much better from reference signal and load disturbance points of view. The control signal seems to be more sensitive to measurement noise though. Other designs could probably be superior to PID control in all the three contexts.

4.4 Predictive first-order-hold

The sampled system with a predictive first-order-hold circuit and with disturbances taken into account looks like

$$
\begin{aligned}
x_{k+1} &= \Phi x_k + \Gamma u_k + \Gamma_1 v_k + \Gamma_\xi \xi_k \\
y_k &= C x_k + e_k
\end{aligned}
$$

where

$$
v_k = \frac{u_{k+1} - u_k}{h}
$$

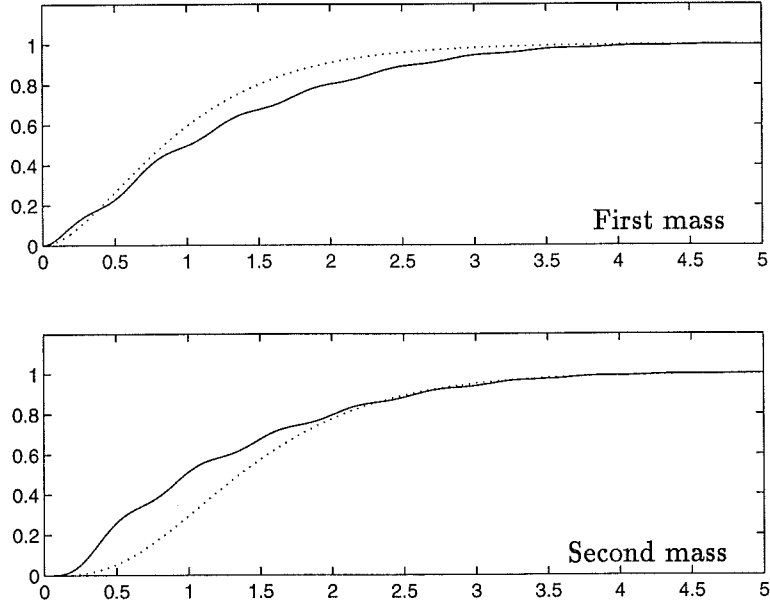and ξ_k and e_k are the state and measurement noises respectively.

Figure 4.3: Positions with corresponding model states. Full lines are the true positions; dotted lines are the reference model positions.

Specifications

As with the zero-order-hold a linear quadratic controller will be designed in continuous time and then translated to discrete time. The loss function is the same, ie

$$
J = E\left\{ \int_0^{Nh} \left[x^T(t)Q_{1c}x(t) + 2x^T(t)Q_{12c}u(t) + u^T(t)Q_{2c}u(t) \right] dt \right.
$$
$$
\left. + x^T(Nh)Q_{0c}x(Nh) \right\}
$$

In discrete time though, the loss function will be different.

$$
J = E\left\{ \sum_{k=0}^{N-1} \left[x^T(kh)Q_1x(kh) + 2x^T(kh)Q_{12}u(kh) + u^T(kh)Q_2u(kh) \right. \right.
$$
$$
\left. + 2x^T(kh)Q_{13}v(kh) + 2u^T(kh)Q_{23}v(kh) + v^T(kh)Q_3v(kh) \right]
$$
$$
\left. + x^T(Nh)Q_0x(Nh) \right\}
$$

Again, only the steady state solution will be used for both the feedback controller and the state estimator.

Controller design

In analogy with the ZOH the state noise $\xi(t)$ is assumed to be the integral of a white noise w(t) with zero mean and the covariance σ_w^2. Also, e(t) is assumed

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to be white noise with zero mean and the covariance σ_e^2. The coupling between u_k and v_k has been discussed earlier. If v(t) is the derivative of the control signal u(t) in continuous time, as is the case with the first-order-hold between the sampling instants, the system can be described by

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) + B_\xi \xi(t) \\
\dot{u}(t) &= v(t) \\
\dot{\xi}(t) &= w(t) \\
y(t) &= Cx(t) + e(t)
\end{aligned}
$$

The discrete time counterpart of this is

$$
\begin{aligned}
x_{k+1} &= \Phi x_k + \Gamma u_k + \Gamma_1 v_k + \Gamma_\xi \xi_k \\
u_{k+1} &= u_k + h v_k \\
\xi_{k+1} &= \xi_k + w_k \\
y_k &= Cx_k + e_k
\end{aligned}
$$

Structure

The structure of the controller is the same as for the ZOH with three additional inputs to the observer: the feedback and feedforward parts of the control signal, u_{fb} and u_{ff} respectively, and the desired states, x_m. Since the the same calculations are made in the estimator as in the feedback controller they could very well be integrated into one block.

Linear quadratic control design

For the feedback control design the disturbances can be omitted from the model. The multiple inputs, u and v, would be a problem though, but this is solved by the transformation

$$
z(t) = \left[\begin{array}{c} x(t) \\ u(t) \end{array} \right]
$$

The transformed system becomes

$$
\begin{aligned}
\dot{z}(t) &= A_z z(t) + B_z v(t) \\
y(t) &= C_z z(t)
\end{aligned}
$$

where the matrix coefficients are equal to

$$
A_z = \left[\begin{array}{cc} A & B \\ 0 & 0 \end{array} \right]
$$

$$
B_z = \left[\begin{array}{c} 0 \\ I \end{array} \right]
$$

$$
C_z = \left[\begin{array}{cc} C & 0 \end{array} \right]
$$

The loss function of the transformed system looks like

$$
J = E\left\{\int_0^{Nh} z^T(t)\begin{bmatrix} Q_{1c} & 0 \\ 0 & Q_{2c} \end{bmatrix} z(t)dt \right.
$$
$$
\left. + z^T(Nh)\begin{bmatrix} Q_{0c} & 0 \\ 0 & 0 \end{bmatrix} z(Nh)\right\}
$$

Since $v(t)$ is constant over the sampling intervals, sampling the transformed system with a zero-order-hold is equivalent to sampling the original system with a predictive first-order-hold. The discrete time system is then

$$
\begin{aligned}
z_{k+1} &= \Phi_z z_k + \Gamma_z v_k \\
y_k &= C_z z_k
\end{aligned}
$$

with the matrices

$$
\Phi_z = \begin{bmatrix} \Phi & \Gamma \\ 0 & I \end{bmatrix}
$$

$$
\Gamma_z = \begin{bmatrix} \Gamma_1 \\ hI \end{bmatrix}
$$

and the loss function

$$
J = E\left\{\sum_{k=0}^{N-1}\left[z^T(kh)Q_{1z}z(kh) + 2z^T(kh)Q_{12z}v(kh) + v^T(kh)Q_{2z}v(kh)\right]\right.
$$
$$
\left. + z^T(Nh)Q_{0z}z(Nh)\right\}
$$

where

$$
Q_{1z} = \begin{bmatrix} Q_1 & Q_{12} \\ Q_{12}^T & Q_2 \end{bmatrix}
$$

$$
Q_{12z} = \begin{bmatrix} Q_{13} \\ Q_{23} \end{bmatrix}
$$

$$
Q_{2z} = Q_3
$$

If the Riccati equation yields an L_z that consists of $L_z = \begin{bmatrix} L & l_u \end{bmatrix}$ the control signal will be computed from

$$
\begin{aligned}
u_{k+1} &= u_k + hv_k \\
v_k &= L(x_k^m - \hat{x}_{k+1}) - l_u u_{k+1}
\end{aligned}
$$

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which can be rewritten as

$$u_{k+1} = su_k + shL(x_k^m - \hat{x}_{k+1})$$

where

$$s = \frac{1}{1 + hl_u}$$

State estimation

State estimation in the PFOH case is not quite as straightforward as with the ZOH. The state prediction with constant disturbances taken into account is described by

$$\hat{x}_{k+1} = \Phi\hat{x}_k + \Gamma u_k + \Gamma_1 \frac{u_{k+1} - u_k}{h} + \Gamma_\xi \hat{\xi}_k + K_x(y_k - C\hat{x}_k)$$

$$\hat{\xi}_{k+1} = \hat{\xi}_k + K_\xi(y_k - C\hat{x}_k)$$

To get rid of the u_{k+1} term the calculation of the control signal must be taken into account in the estimator. The signal consists of three parts; the feedforward signal u_{ff}, the feedback signal u_{fb} and the compensation for the disturbance ξ. The relationship is

$$u_{k+1} = u_k^{ff} + u_{k+1}^{fb} - \hat{\xi}_{k+1}$$

Of these three signals u_{ff} is not a problem because it is outside the control loop; $\hat{\xi}$ is calculated within the estimator and is also no trouble. u_{fb}, though, has to be calculated in the estimator in the same way as in the feedback controller.

$$u_{k+1}^{fb} = su_k^{fb} + shL(x_k^m - \hat{x}_{k+1})$$

Substituting this into the observer equation and rearranging yields

$$\hat{x}_{k+1} = W(\Phi - K_x C + \frac{1}{h}\Gamma_1 K_\xi C)\hat{x}_k + W(\Gamma_\xi - \frac{1}{h}\Gamma_1)\hat{\xi}_k + W(\Gamma - \frac{1}{h}\Gamma_1)u_k$$

$$+ \frac{s}{h}W\Gamma_1 u_k^{fb} + \frac{1}{h}W\Gamma_1 u_k^{ff} + W(K_x - \frac{1}{h}\Gamma_1 K_\xi)y_k + sW\Gamma_1 L x_k^m$$

$$\hat{\xi}_{k+1} = -K_\xi C\hat{x}_k + \hat{\xi}_k + K_\xi y_k$$

where $W = (I + s\Gamma_1 L)^{-1}$. The estimator can be made as a state-space system

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{\xi}_{k+1} \end{bmatrix} = \Phi_E \begin{bmatrix} \hat{x}_k \\ \hat{\xi}_k \end{bmatrix} + \Gamma_E \begin{bmatrix} y_k \\ u_k \\ u_k^{fb} \\ u_k^{ff} \\ x_k^m \end{bmatrix}$$

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with $\hat{x}_{k+1}$ and $\hat{\xi}_{k+1}$ as outputs and the matrices

$$\Phi_E = \begin{bmatrix} W(\Phi - K_x C + \frac{1}{h}\Gamma_1 K_\xi C) & W(\Gamma_\xi - \frac{1}{h}\Gamma_1) \\ -K_\xi & I \end{bmatrix}$$

$$\Gamma_E = \begin{bmatrix} W(K_x - \frac{1}{h}\Gamma_1 K_\xi) & W(\Gamma - \frac{1}{h}\Gamma_1) & W(\frac{s}{h}\Gamma_1) & \frac{1}{h}W\Gamma_1 & sW\Gamma_1 L \\ K_\xi & 0 & 0 & 0 & 0 \end{bmatrix}$$

The K matrix consisting of

$$K = \begin{bmatrix} K_x \\ K_\xi \end{bmatrix}$$

is calculated as the matrix for an ordinary stationary Kalman filter for the loop and disturbance parts of the system:

$$\begin{bmatrix} x_{k+1} \\ \xi_{k+1} \end{bmatrix} = \begin{bmatrix} W\Phi & W(\Gamma_\xi + l_u s\Gamma_1) \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ \xi_k \end{bmatrix} + \begin{bmatrix} \Gamma - l_u s\Gamma_1 \\ 0 \end{bmatrix} u_k$$

$$+ \begin{bmatrix} -\frac{1}{h}\Gamma_1 \\ I \end{bmatrix} w_k$$

$$y_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \xi_k \end{bmatrix} + e_k$$

where w_k is the state noise and e_k is the measurement noise.

Feedforward

The feedforward is identical to the one described in Section 4.3. The feedforward control signal and the desired states signal will not be one sample ahead as is the case with the feedback control signal and the estimated states.

Nominal design

The nominal design parameters are chosen to be exactly the same as for the ZOH, ie $h = 0.0274s$, $Q_{1c} = I$, $Q_{2c} = I$, $\sigma_v = 1$, $\sigma_e = 0.00001$, $\alpha = 2$ and $\beta = 4$. The closed loop poles assuming directly mesured states are then

$$\begin{array}{ll} -0.6942 & \pm 11.4476i \\ -1.4878 & \pm 1.1387i \\ -28.8820 & \end{array}$$

and the poles of the observer are

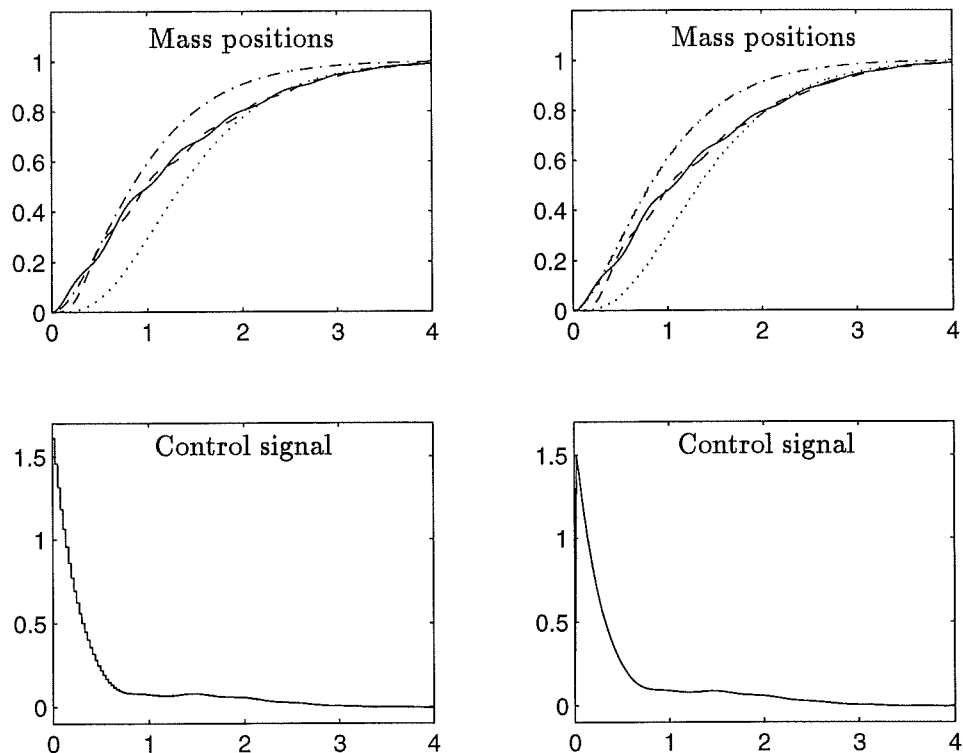$$\begin{array}{ll} -23.9309 & \pm 36.7502i \\ -3.8455 & \pm 9.0825i \\ -36.3741 & \end{array}$$

Figure 4.4: Reference signal response with ZOH (left) and PFOH (right). The lines in the position figures are first mass actual position (full line) and model position (dashed/dotted line) and second mass actual position (dashed line) and model position (dotted line).

A simulation of this gives roughly the same result as with a ZOH but a closer examination of the three more revealing parts shows some differences. In Figure 4.4 the reference signal responses are compared. The main difference is here in the control signal which is somewhat larger in the beginning for the ZOH. Another observation from this figure is that the feedforward model is unrealistic. A sudden load disturbance, as in Figure 4.5, is clearly dealt with faster with the PFOH and with a smoother control signal during the first half second. Figure 4.6, finally, shows that at least the first mass is a little bit better damped with the PFOH when a measurement noise is introduced and also, except for in the beginning, the control signal oscillation is slightly smaller in amplitude.
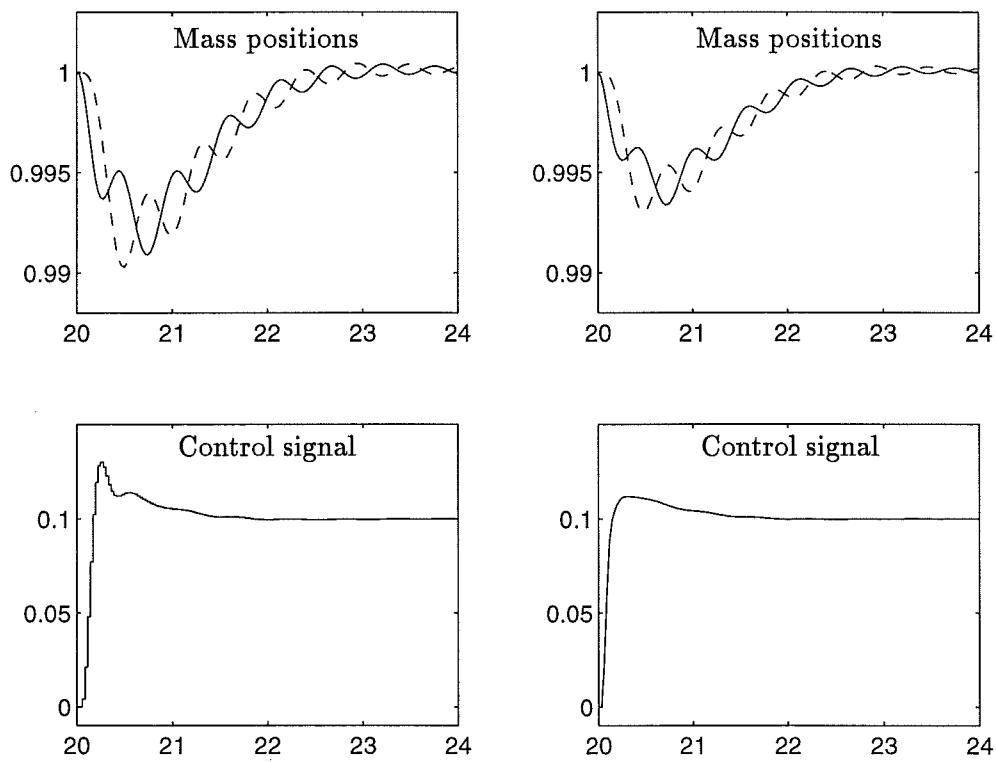
Figure 4.5: Load disturbance response with ZOH (left) and PFOH (right). Upper figure full lines are first mass positions; dashed lines are second mass positions.
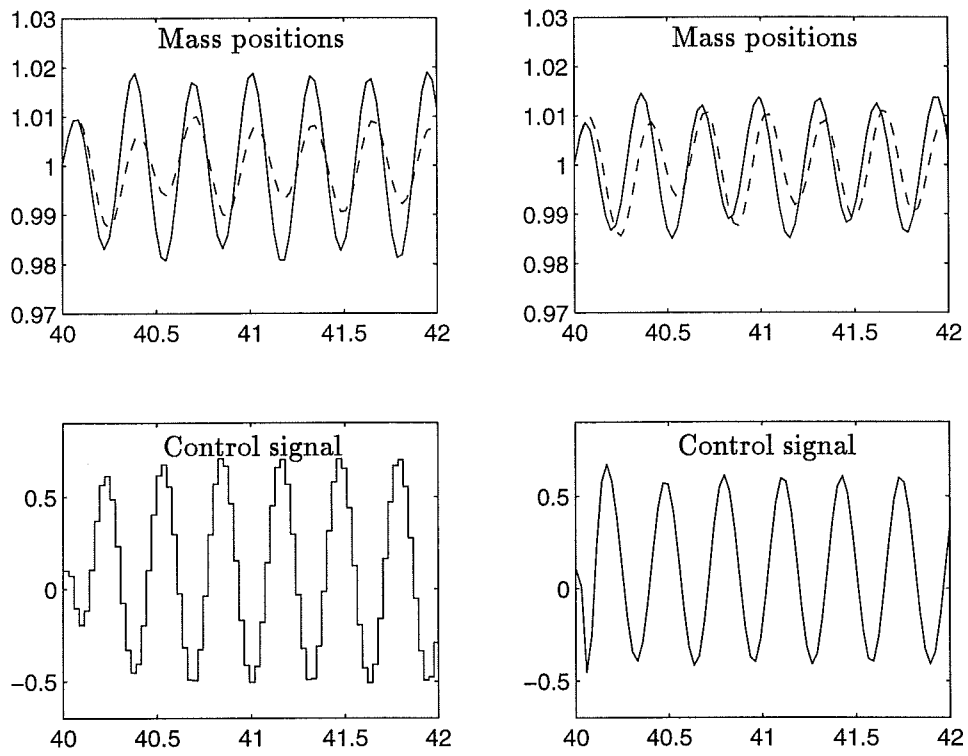
38

Figure 4.6: Measurement error sensitivity with ZOH (left) and PFOH (right). Upper figure full lines are first mass positions; dashed lines are second mass positions.

Chapter 5

Conclusions and future work

This thesis has examined control of a typical oscillatory system. Controllers of different complexity have been designed and tried out on a simulation model of the system. A simple PI controller gave a slow closed loop system with a large overshoot at setpoint changes, but also very insensitive to noise. With setpoint weighting the overshoot was effectively reduced but still quite large. The PI controller couldn't be made fast enough to reveal the oscillatory behaviour of the system. With PID control a faster system could be obtained. The oscillatory behaviour of the system became significant as the controller gain was increased. The overshoot at setpoint changes was further reduced but still rather large. Faster responses were obtained at the price of a larger noise sensitivity. Digital state feedback control gave remarkably improved results for setpoint changes and response to constant disturbances. Use of a reference model and feedforward control to deal with setpoint changes eliminated the overshoot. The price for these improvements was an increased sensitivity to noise and much more complex controllers. A comparison of the load disturbance responses of three different controllers can be seen in Figure 5.1. Although not examined in this thesis it is clear that state feedback control will give a much greater freedom in choosing the performance of the system. The noise sensitivity could for example be reduced considerably with a different design. An important point about the state feedback control is that it is very dependent on the models that it uses for the process dynamics. In this thesis the models used for the controller design and for the simulations have been identical (with one exception in the PID chapter). In reality there will be differences which will influence the controller performance, especially for the state feedback controllers. Finally a comparison was made between zero-order-hold and predictive first-order-hold sampling when using state feedback control. With the same control design, the predictive first-order-hold controller gave both a faster response and less sensitivity to noise than the zero-order-hold. Another great benefit was a smoother control signal. The predictive first-order-hold is, though,
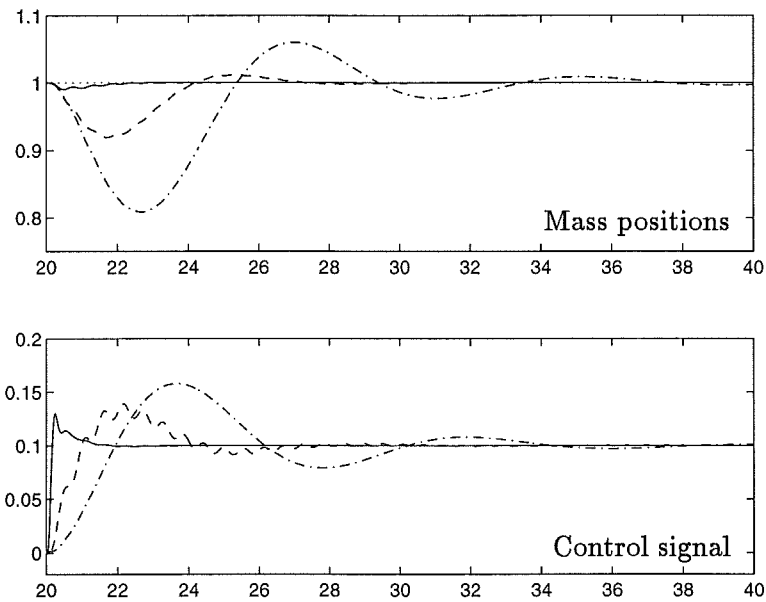
Figure 5.1: Load disturbance response for three different controllers; PI (dashed/dotted line), PID (dashed line) and ZOH state feedback (full line).

even more complex and it also gives less freedom to separate different tasks in the controller. The estimator has to know the feedback control law and might thus just as well be integrated with the feedback controller. Predictive first-order-hold state feedback controllers would also be even more dependent on a correct model of the plant.

Some important issues are not covered in this thesis. Interesting future work on the topic could be:

- Refinement of the simulation model including physical limitations like the bumpers at the ends of the rail and control signal saturation.

- Further investigation of state feedback control, examining the possibilities and the limitations.

- Analytical exploration of sensitivity and robustness for state feedback control.

- Development of a good feedforward model to improve reference signal response.

- Experiments on the real plant.

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Control of an Oscillatory System

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Karl Johan Åström

Introduction

Oscillatory systems:

- Robot arms with harmonic drives
- Arms of disk drives and cd players
- Drive trains for cars

A progression of controllers:
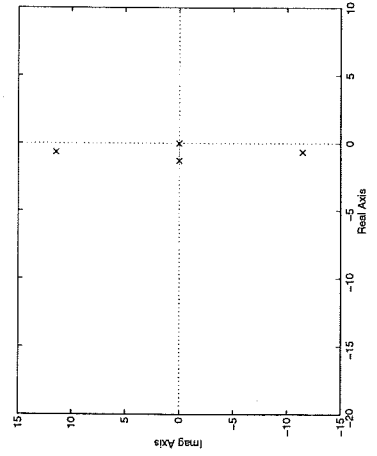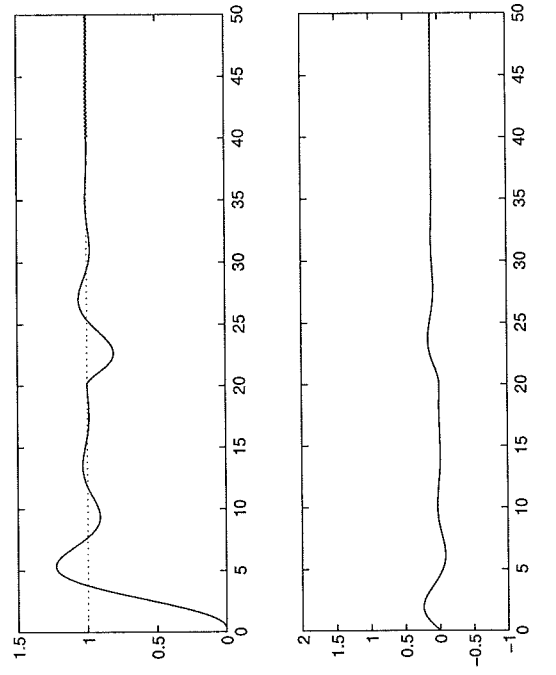
- PI
- PID
- State feedback
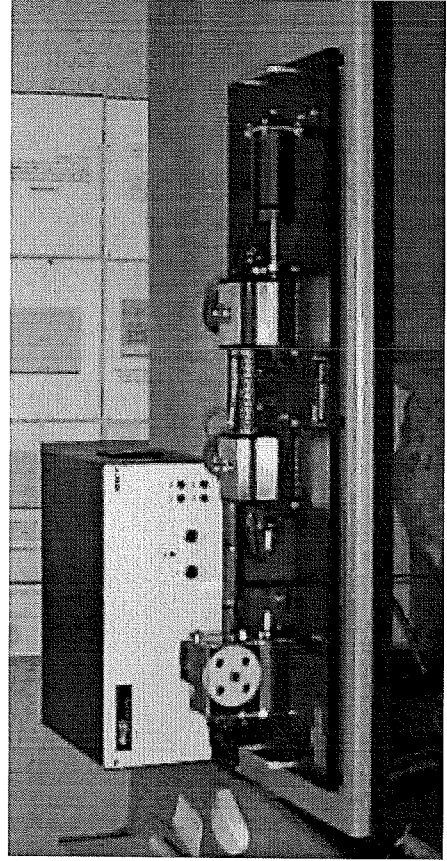
System equations

$$\begin{cases} m_1 \ddot{y}_1 + c_1 \dot{y}_1 + k(y_1 - y_2) &= F(t) \\ m_2 \ddot{y}_2 + c_2 \dot{y}_2 + k(y_2 - y_1) &= 0 \end{cases}$$
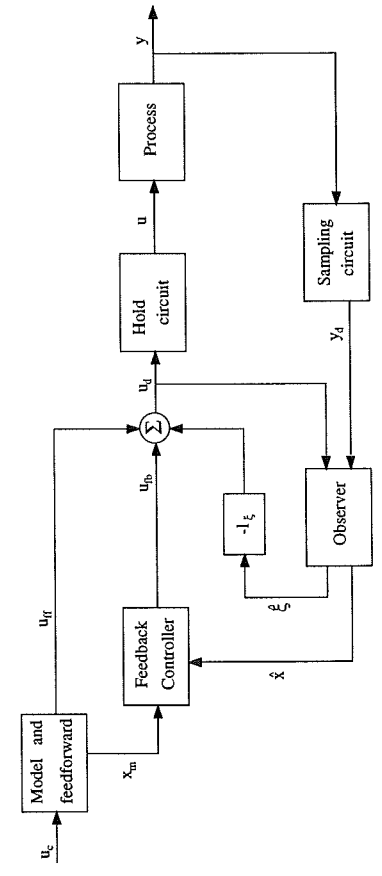
System poles

The plant

PID root loci
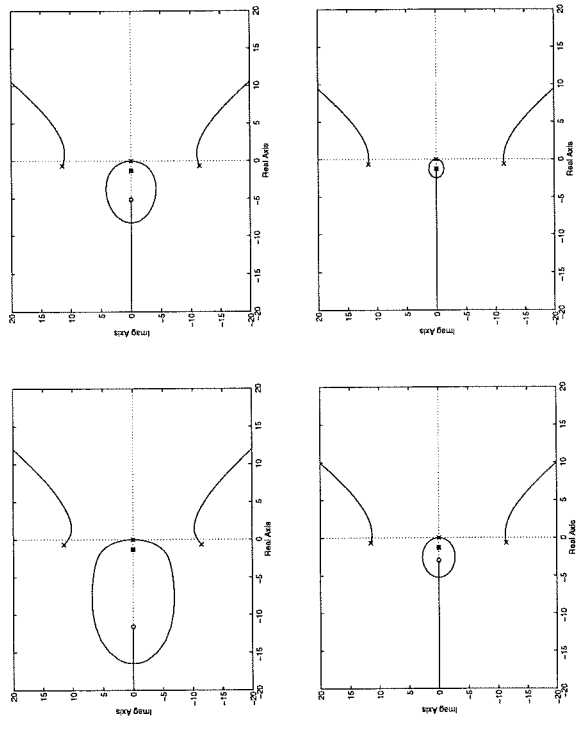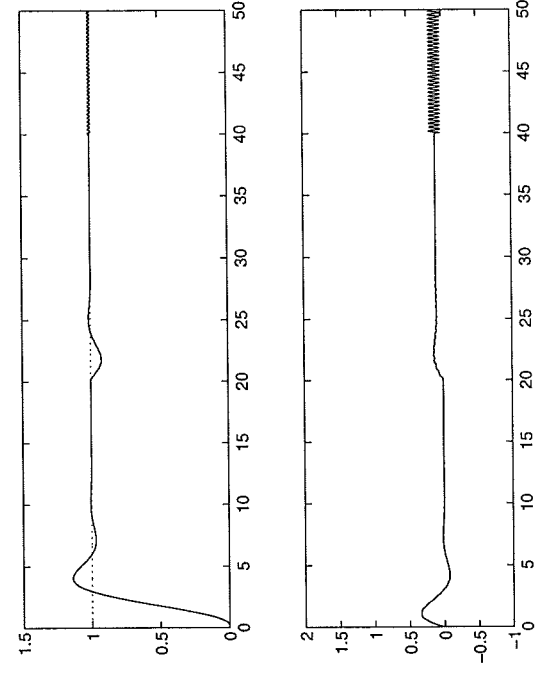
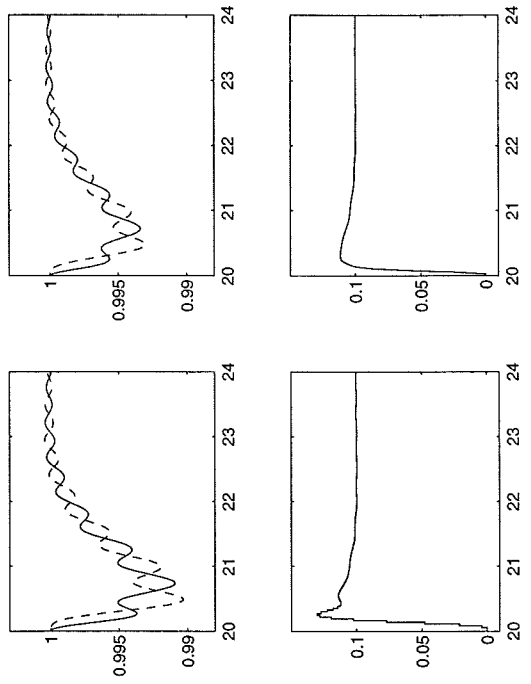# PI simulation

State feedback controller structure

PID simulation

ZOH vs PFOH

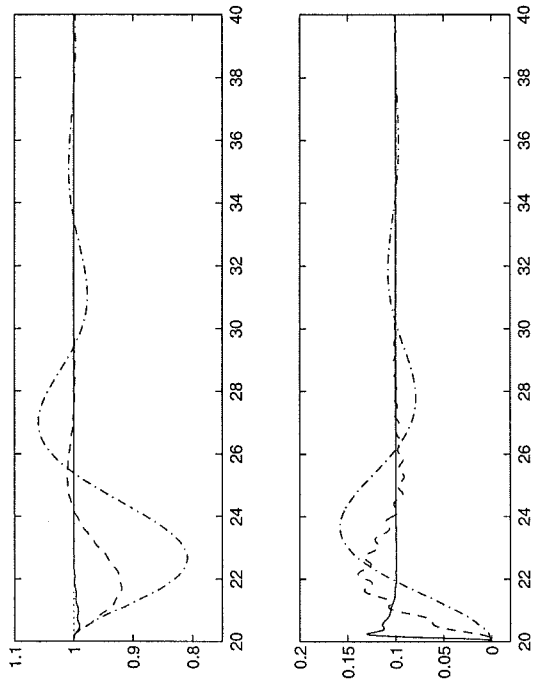**PFOH:**

- Smoother control signal
- Better damped responses
- More complex
- More model-dependent

Load disturbance responses

State feedback simulation (ZOH)

ZOH and PFOH load disturbance responses

Future work

- Refinement of the simulation model
- Further investigation of state feedback control
- Improved feedforward
- Experiments on the real plant

Conclusions

- Oscillatory systems
- Large performance differences between different controllers
- It pays to use state feedback
- Useful for education