

OPTIMAL INSTÄLLNING AV REGULATORER

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Abstract

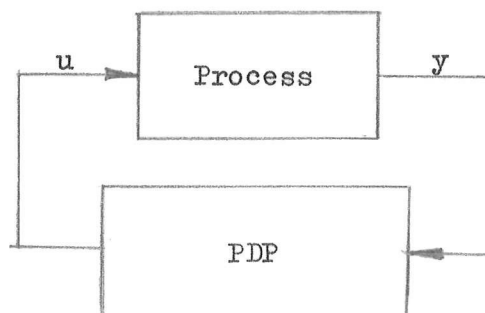
At manual tuning of a regulator you make an experiment, for example a stepresponse and then you vary the regulator parameters in order to improve the performance of the system. The aim has been to make this adjusting procedure automatic. Program developments and experiments have been made at the processor PDP-15 with the realtime system RSX-15 Plus. Programming language: FORTRAN.

Sammanfattning

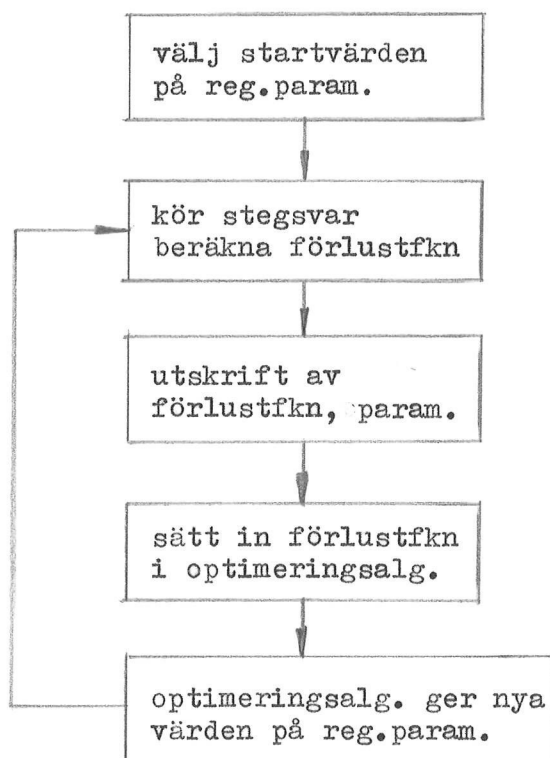
Vid manuell tuning av en regulator gör man ett experiment t.ex. stegsvar och gör sedan förändringar av regulatorparametrarna för att förbättra systemets uppförande. Uppgiften har varit att automatisera denna inställningsprocedur med hjälp av dator. Programutveckling och experiment har skett på processdatorn PDP-15 med reelltidssystemet RSX-15 Plus. Programmeringsspråk: FORTRAN.

1. Systembeskrivning

Optimeringen av regulatorparametrarna sköts helt automatiskt av processdatorn.



Funktionen framgår av följande flödesplan:



Optimeringsprogrammet består av tre huvudprogram med subrutiner: START, REGUL (PIDREG, BBREG), REGO (NUFLE2, PRINT (HESS)). Optimering kan ske för PI-, PD-, PID- och "Bang-Bang"-regulatorer samt för alla kombinationer av dessa.

1.1 Program START

Tilldelning av startvärden sker genom vektorn USER i blocket IOCOM.

Ex. MCR>SET USER 7 0.5

USER 1	N	antal variabler
USER 2	G1	
USER 3	TI1 A1	
USER 4	TD1 K1	
USER 5	GD1 DZ1	regulatorparametrar
USER 6	G2 K2	se listning av PIDREG resp. BBREG
USER 7	TI2 DZ2	
USER 8	TD2 A2	
USER 9	GD2	
USER 10	IREG(1)	regulator typer PI: 1 PD: 2 PID: 3 BB: 4
USER 11	IREG(2)	en regulator - IREG(2): 0
USER 12	TSAMP	samlingsperiod
USER 13	NSAMP	antal samlingar
USER 14	UR	steg
USER 15	ALFA	P-I-D: 0 BB: 1 konstanter för modifiering
USER 16	FIV	servo: 0 bom: 0,1 av förlustfkn
USER 19	DFN	se listning
USER 20	HH	av NUFLE2
USER 21	DEIMAX(1)	
USER 22	DEIMAX(2)	
USER 23	DEIMAX(3)	
USER 24	DEIMAX(4)	max. deländring av parametervärdet
USER 25	DEIMAX(5)	se NUFLE2
USER 26	DEIMAX(6)	
USER 27	DEIMAX(7)	
USER 28	DEIMAX(8)	

Request name sätts till REGUL vilket innebär att programmet ALIO (utför logisk och analog in/utmatning mellan datorn och datorns interface) vid varje exekvering (sampling) aktiverar programmet REGUL (logisk insignal 7 eller 16 sann).

TDV>TKB

OPT

>SZ,EXM

TU

>START

) DP

>200

PAR

>TDV

SCP

>I0COM(72000,400),RTCOM(75000,1000)

RC

>START,.ER#

L&S

>

START 20020-21022 01003

STOP 21023-21036 00014

.FPP 21037-21445 00407

CTSER 21446-21716 00251

CPNSG 21717-22042 00124

COTO 22043-22070 00026

.DA 22071-22171 00101

.BP 22172-22201 00010

) EXIF.1 22202-22204 00003

.FP 22205-22206 00002

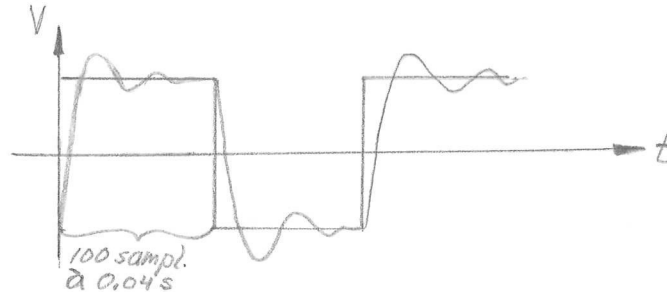
CORE REQ'D

20000-22206 02207

1.2 Program REGUL

Programmet REGUL sköter regleringen av processen. REGUL aktiveras varje samplingsintervall av ALIO och kallar på regulatorerna (subrutinerna PIDREG och BBREG).

En stegstörning sker och förlustfunktionen beräknas under ett



antal samplingsintervall, varefter programmet REGO aktiveras. Inkopplingen av REGO styrs från datorns interface genom logisk insignal 14. REGUL måste (liksom ALIO) fixeras i kärnminnet då programmet exekverar var 40:e ms. Regul har högre prioritet än REGO för att inte beräkningar och tidskrävande utskrifter skall påverka regleringen.

```

TDV>TKB
.
OPT
>SZ,EXN
)TN
>REGUL
DP
>150
PAR
>PIPUS
SCB
>IIOCOM(72000,400),RTCOM(75000,1000)
RC
>REGUL,PIDREG,BBREG
L&S
>
REGUL 13000-13526 00527
PIDREG 13527-13777 00251
BBREG 14000-14157 00160
.BC 14160-14305 00126
STOP 14306-14321 00014
)FPP 14322-14730 00407
)SPMSG 14731-15054 00124
GETBIT 15055-15102 00026
REQF.2 15103-15131 00027
.DA 15132-15232 00101
FTS.3 15233-15322 00070
.BP 15323-15332 00010
EXIF.1 15333-15335 00003
.FP 15336-15337 00002

CORE REQ'D
13000-15337 02340

```

1.3 Program REGO

Programmet sköter optimeringen av regulatorparametrarna och resultatutskrifterna genom subrutinerna NUFLE2 resp. PRINT. Dessutom sker en kontroll av att de nya parametervärdena inte passerar noll, dvs byter tecken, vilket skulle få ödesdigra konsekvenser för regleringen av processen.

1.3.1 Subrutinen NUFLE2

Algoritmen beräknar nya värden på regulatorparametrarna utifrån de förändringar i förlustfunktionen som parametervariationerna ger. Algoritmen beräknar således minimum av en funktion med tillgång endast till funktionsvärdena.

Givet $f(x)$ där $x=(x_1, x_2, x_3)$

Gradienten beräknas

$$\tilde{x}=(x_1+h, x_2, x_3) \quad f(\tilde{x})$$

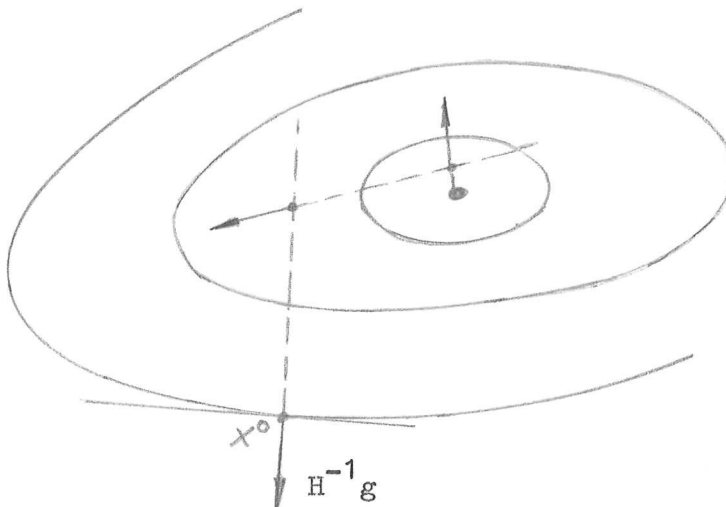
$$g_1=(f(\tilde{x})-f(x))/h \approx \frac{\partial f}{\partial x_1} \quad g_2 \approx \frac{\partial f}{\partial x_2} \quad g_3 \approx \frac{\partial f}{\partial x_3}$$

Algoritmen bildar sökriktningen

$s = -\bar{H}^{-1}g$ där H är Hessianen (approx. av 2:a deriv.)

Därefter sker beräkning av min. $f(x)$ i sökriktningen.

I min.punkten bildas en ny sökriktning osv.



(se Jacoby, Kowalik, Pizzo: Iterative methods for nonlinear optimization problems sid 143 - 145)

1.3.2 Subrutin PRINT

Rutinen ger utskrift av process (DC-servo, Bom), regulatortyp, parametrar, gradienter, och Hessianen.

Hessianen överförs från produktform till enkel matris med subrutinen HESS (se listning av HESS).

```

,TDV>TKB

OPT
>SZ,EXM
IN
>REGO
DP
>200
PAR
>TDV
SCF
>RTCON(75000,1000)
RC
>REGO,NUFLE2,PRINT,HESS
L&S
>
REGO      20020-20354 00335
NUFLE2    20355-25201 04625
PRINT     25202-27624 02423
HESS      30020-30421 00402
BCDIO     30422-34220 03577
ABS       27625-27637 00013
.BC       27640-27765 00126
STOP      34221-34234 00014
FIOPS     34235-35132 00676
INTEAE    35133-35263 00131
.FPP      35264-35672 00407
OTSER     35673-36143 00251
SPMSG     36144-36267 00124
DOTSS     36270-36406 00117
COTO      36407-36434 00026
.DA       36435-36535 00101
.BP       27766-27775 00010
EXIF.1    36536-36540 00003
.FP       27776-27777 00002

CORE REQ'D
      20000-36540 16541

```

2. Exempel

Optimeringsprogrammet har körts på DC-servo och Bom.

Lämpliga värden på USER-parametrarna:

TSAMP=0.04

NSAMP=200

UR=1

DFN=-0.5

HH=0.05

2.1 DC-servo

Bang-Bang regulator (A=0.9)

Startvärden: K=1 DZ=0.1 Delmax(1)=0.2 Delmax(2)=0.2

$K_{opt}=0.9595$ $DZ_{opt}=0.3801$

Diagram 1 visar förlustfunktionen som fkn av antalet exekveringar av optimeringsalgoritmen.

Diagram 2 — stegsvar för startvärden

Diagram 3 — stegsvar för optimala värden

PI-regulator

Diagram 4 — stegsvar för G=1 TI=1

Diagram 5 — stegsvar för $G_{opt}=1.1450$ $TI_{opt}=2.2998$

(Då servot är integrerande kan ett stort värde på TI användas, $RI \approx 0$)

PID-regulator

Diagram 6 — stegsvar för G=1 TI=1 TD=1 GD=1

Diagram 7 — stegsvar för $G_{opt}=1.7365$ $TI_{opt}=1.3644$

$TD_{opt}=0.0852$ $GD_{opt}=0.8875$

Om startvärdena skiljer sig väsentligt från de optimala kan algoritmen finna ett lokalt minimum som ej är optimalt. Om DELMAX då är litet kan det hända att algoritmen ej kan bryta sig ur denna omgivning. Variera startvärdena eller öka DELMAX.

För PI- och PID-reg. har genomgående använts DELMAX(I)=0.4

2.2 BomPD — PD (läge — vinkel)

Lämpliga startvärden:

	reg.1(läge)	reg.2(vinkel)
G	-0.5	1.2
TI	10^6	10^6
TD	0.9	0.25
GD	30	30

DELMAX bör ej vara större än 0.1 för G1 och TD2 (DELMAX(1) resp. DELMAX(5)) då större förändringar av dessa parametrar påverkar regleringen så pass kraftigt att kulan ramlar av bommen.

	reg.1(läge)	reg.2(vinkel)
G _{opt}	-0.8390	1.4388
TI _{opt}	10^6	10^6
TD _{opt}	1.0465	0.3159
GD _{opt}	30.0068	30.0311

Kommentar

Vid rimligt val av startvärden och få parametrar finner algoritmen snabbt en minimivå på förlustfunktionen (se diagram 1, diagram 8). En svag variation av de optimala parametervärdena erhålls dock pga. friktion, glapp m.m. i systemet.

Som regel bör alternativa startvärden prövas då algoritmen kan finna lokala minimipunkter som ej är optimala.

OPTIMERING AV REGULATORMPARAMETRAR

(se diagram 1)

PROCESS: DC-SERVO REGULATOR: B-B (LAGE) A= 0.90

ALOSS	K1	DZ1
136.780	1.0000	0.1000
156.707	1.0000	0.1000
128.504	1.0500	0.1000
127.958	0.9500	0.1000
137.342	1.0000	0.1500
150.714	1.0000	0.0500
GRADIENT:	44.5201	-131.6881
	HESSIAN MATRIX	
	1.0000	0.0000
	0.0000	1.0000
90.630	0.9662	0.2000
61.626	0.9324	0.3000
62.275	0.9824	0.3000
57.303	0.8824	0.3000
72.191	0.9324	0.3500
96.047	0.9324	0.2500
GRADIENT:	59.6565	-246.0415
	HESSIAN MATRIX	
	1.0000	0.0000
	0.0000	1.0000
54.434	0.9081	0.4000
58.151	0.8839	0.5000
58.842	0.9581	0.4000
61.918	0.8581	0.4000
66.364	0.9081	0.4500
71.288	0.9081	0.3500
GRADIENT:	-11.6545	-49.2344
	HESSIAN MATRIX	
	238.4770	-655.2882
	-655.2882	1809.1862
113.314	1.1081	0.4726
111.745	1.0081	0.4363
GRADIENT:	-11.6545	-49.2344
	HESSIAN MATRIX	
	238.4770	-655.2882
	-655.2882	1809.1862
56.300	0.9181	0.4036
43.697	0.9131	0.4018
64.807	0.9631	0.4018
58.161	0.8631	0.4018
54.359	0.9131	0.4518
61.319	0.9131	0.3518
GRADIENT:	59.7051	-41.2825
	HESSIAN MATRIX	
	13955.3711	871.4693
	871.4693	1979.1197
38.916	0.9074	0.4252
59.288	0.9017	0.4486
55.252	0.9574	0.4252
51.575	0.8574	0.4252
59.280	0.9074	0.4752
34.076	0.9074	0.3752
GRADIENT:	61.7149	356.9735
	HESSIAN MATRIX	

13981.6663 3516.8975
 3516.8975 17892.7314
 34 → 32.735 0.9080 0.4051
 31.753 0.9087 0.3851
 39.215 0.9587 0.3851
 46.959 0.8587 0.3851
 60.377 0.9087 0.4351
 59.695 0.9087 0.3351
 GRADIENT: -101.1626 5,5641

HESSIAN MATRIX
 15355.5044 4542.8793
 4542.8793 8895.9690

39.590 0.9167 0.3804
 59.509 0.9127 0.3827
 GRADIENT: -101.1626 5,5641

HESSIAN MATRIX
 15355.5044 4542.8793
 4542.8793 8895.9690

39.299 0.9091 0.3848
 RESTART

59.092 0.9091 0.3848
 32.694 0.9087 0.3851
 44 → 31.721 0.9587 0.3851
 39.212 0.8587 0.3851
 68.737 0.9087 0.4351
 39.170 0.9087 0.3351
 GRADIENT: -50.0434 291.1633

HESSIAN MATRIX
 1.0000 0.0000
 0.0000 1.0000

60.359 0.9259 0.2851
 37.712 0.9173 0.3351
 GRADIENT: -50.0434 291.1633

HESSIAN MATRIX
 1.0000 0.0000
 0.0000 1.0000

32.356 0.9095 0.3801
 51 → 31.665 0.9595 0.3801
 39.210 0.8595 0.3801
 68.470 0.9095 0.4301
 40.251 0.9095 0.3301
 GRADIENT: -50.4348 278.9326

HESSIAN MATRIX
 3.5309 78.8845
 78.8845 2459.6967

55.687 0.9208 0.3797
 60.512 0.9152 0.3799
 GRADIENT: -50.4348 278.9326

HESSIAN MATRIX
 3.5309 78.8845
 78.8845 2459.6967

58.778 0.9101 0.3800
 RESTART

33.185 0.9101 0.3800
 33.241 0.9095 0.3801
 32.279 0.9595 0.3801
 39.324 0.8595 0.3801
 67.784 0.9095 0.4301
 49.237 0.9095 0.3301
 GRADIENT: -45.4261 189.9918

HESSIAN MATRIX
 1.0000 0.0000
 0.0000 1.0000

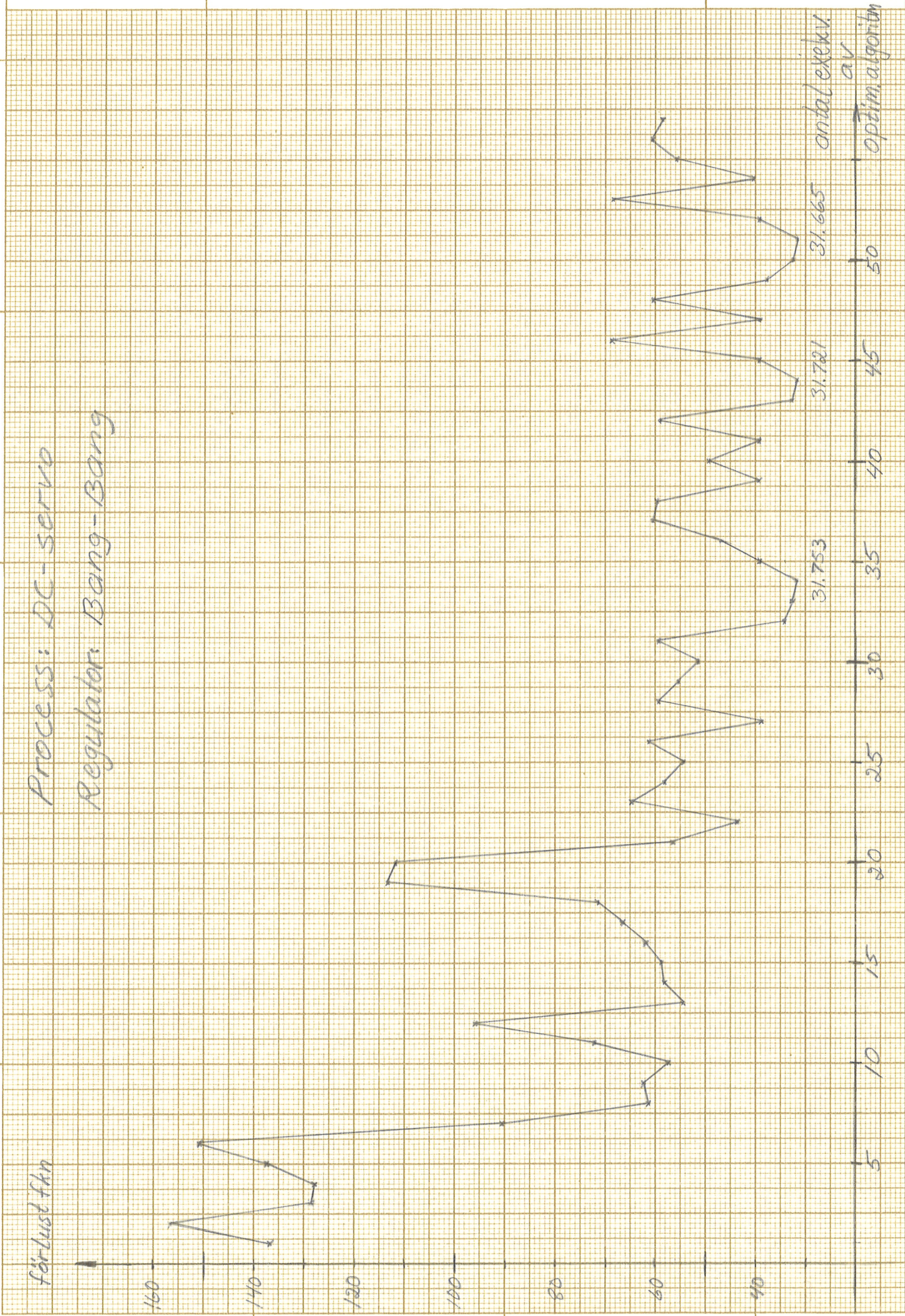
86.947 0.9334 0.2801

Förlustfunktion som fkn av antalet
exekveringar av optimeringsalgoritm

Diagram 1

Process: DC-serve
Regulator: Bang-Bang

förlust fkn



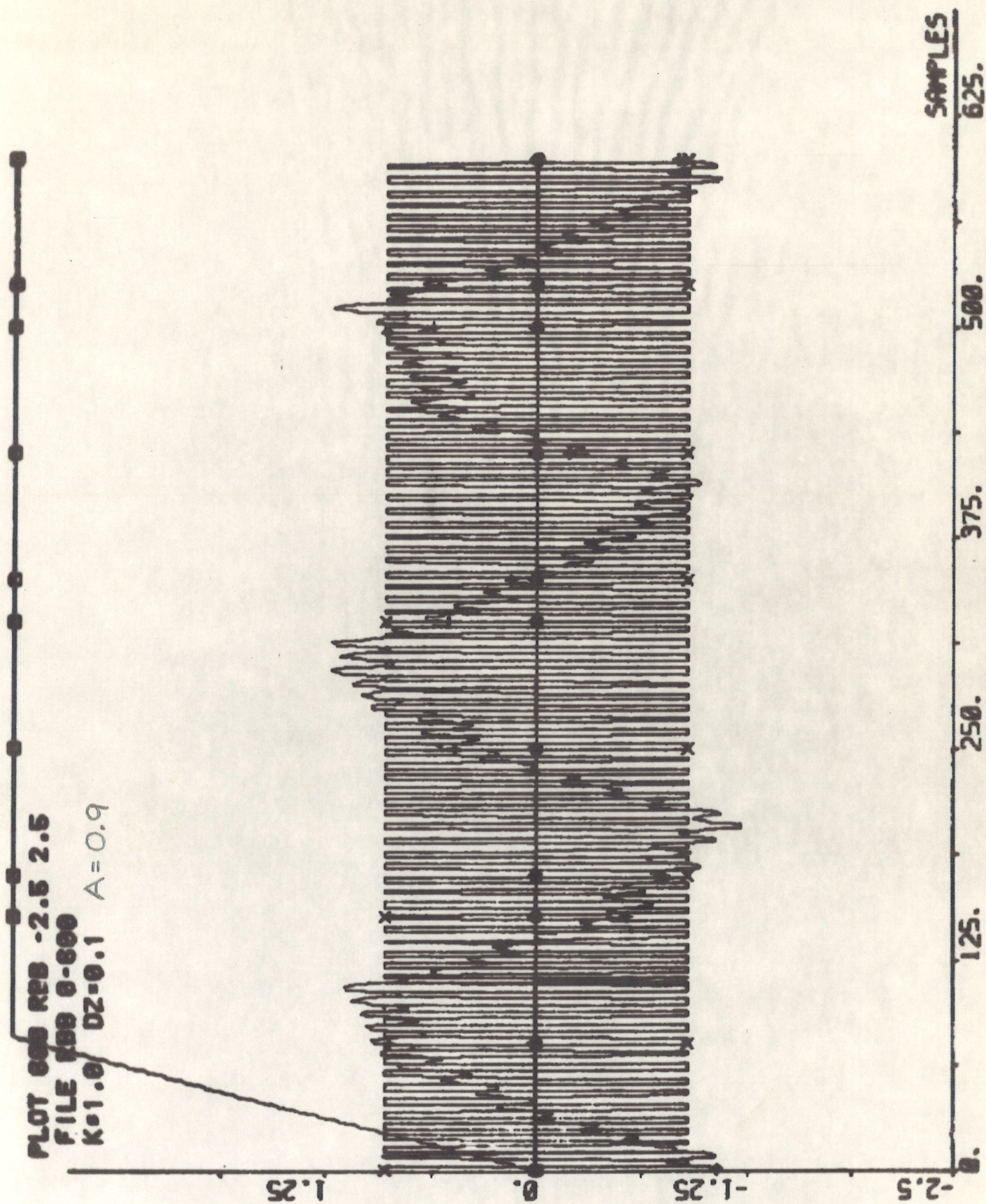


Diagram 2. Stegsvär för servo med Bang-Bang reg. (ref.)

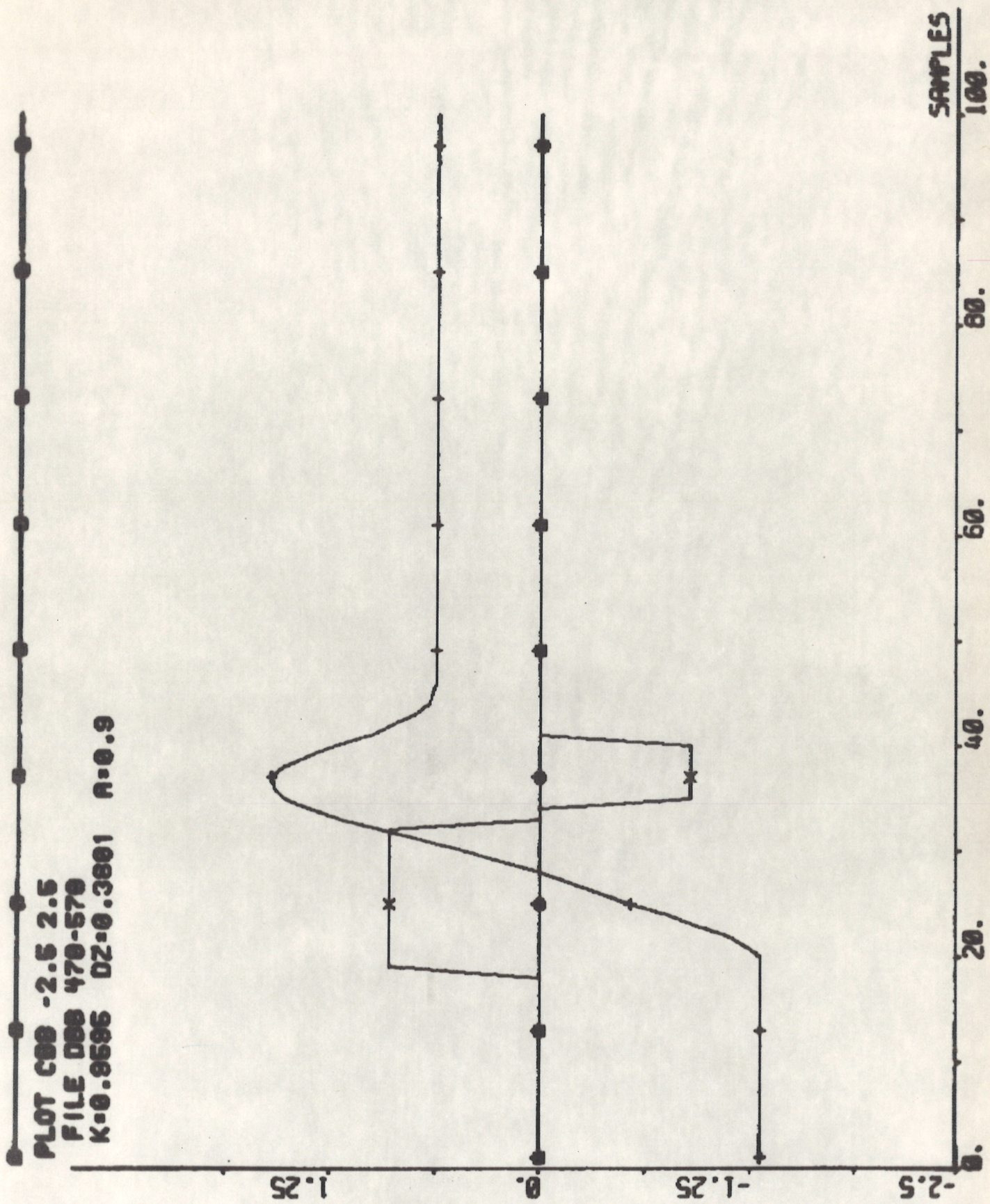


Diagram 3. Stegsvär för servo med Bang-Bang reg. (opt.)

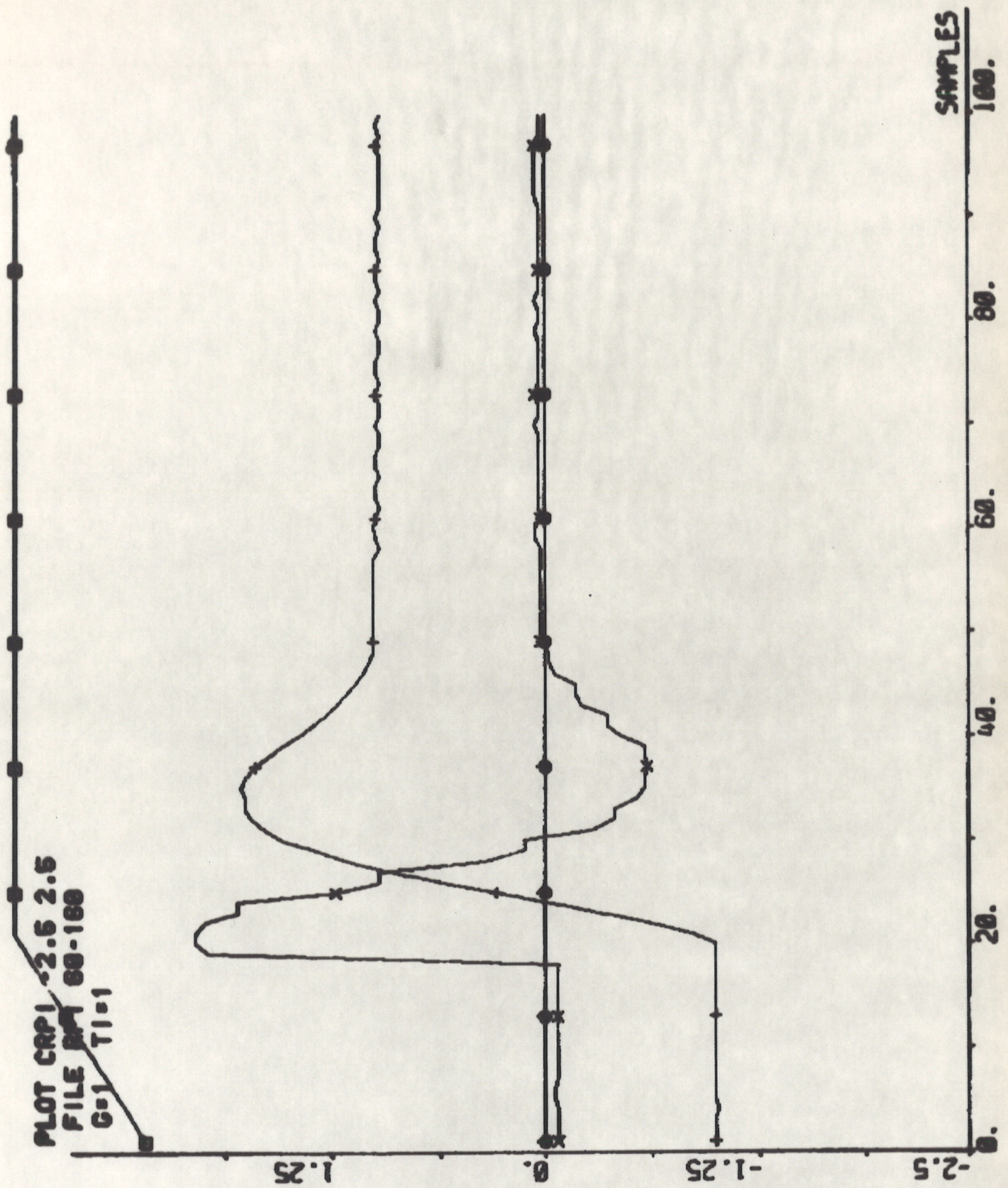


Diagram 4. Stegsvär för servo med PI-reg. (ref.)

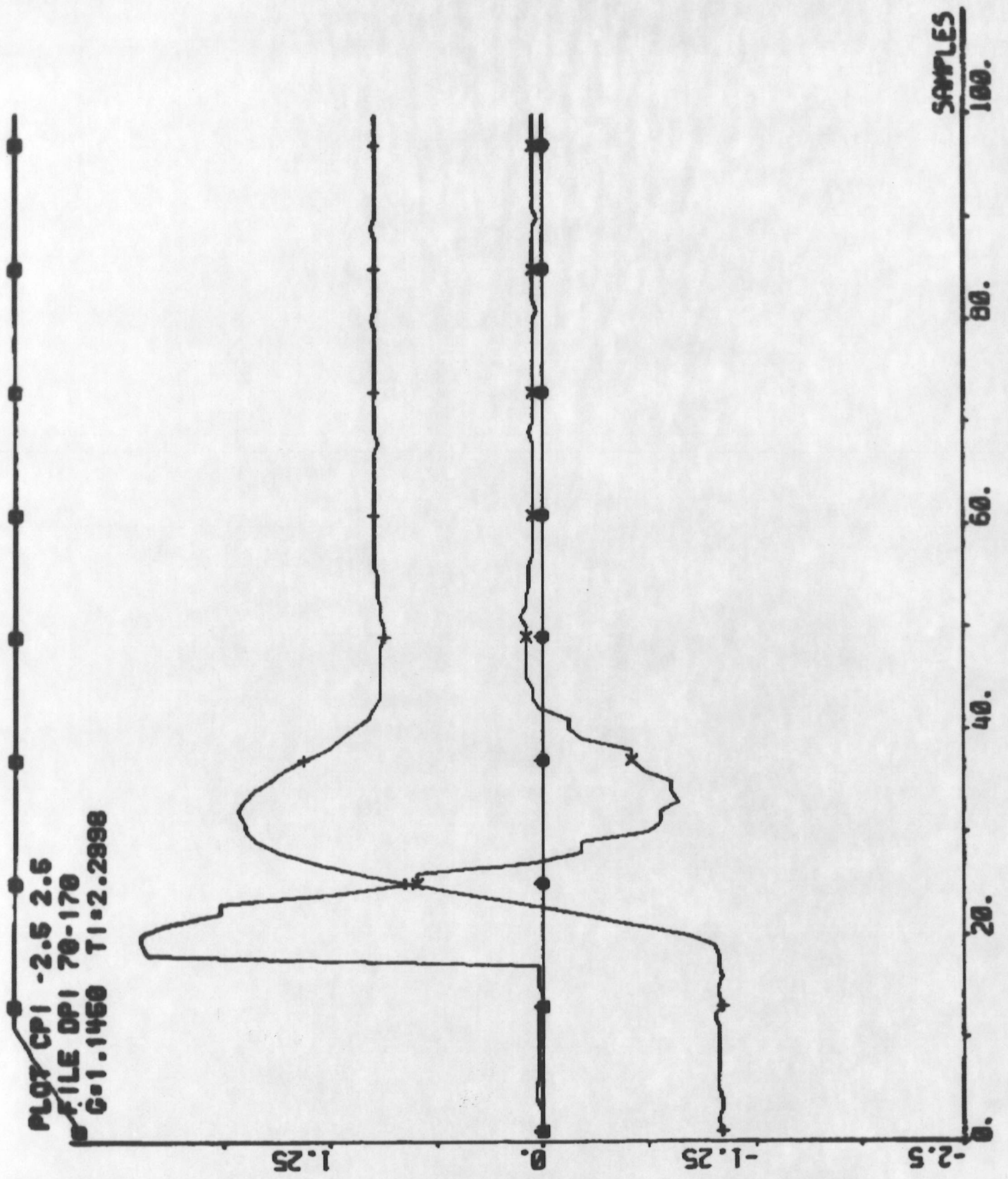


Diagram 5. Stegsvär för servo med PI-reg. (opt.)

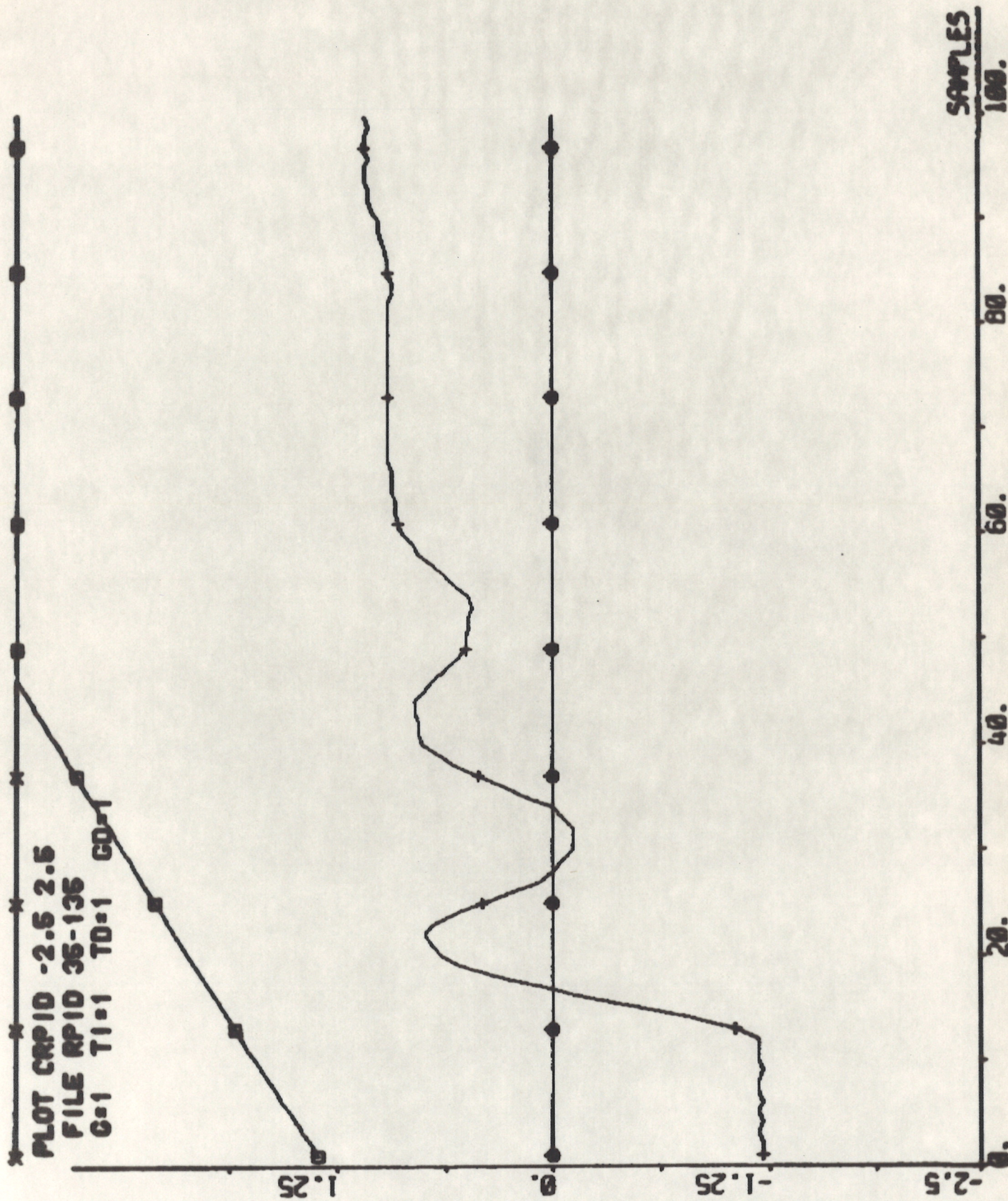


Diagram 6. Stegsvär för servo med PID-reg. (ref.)

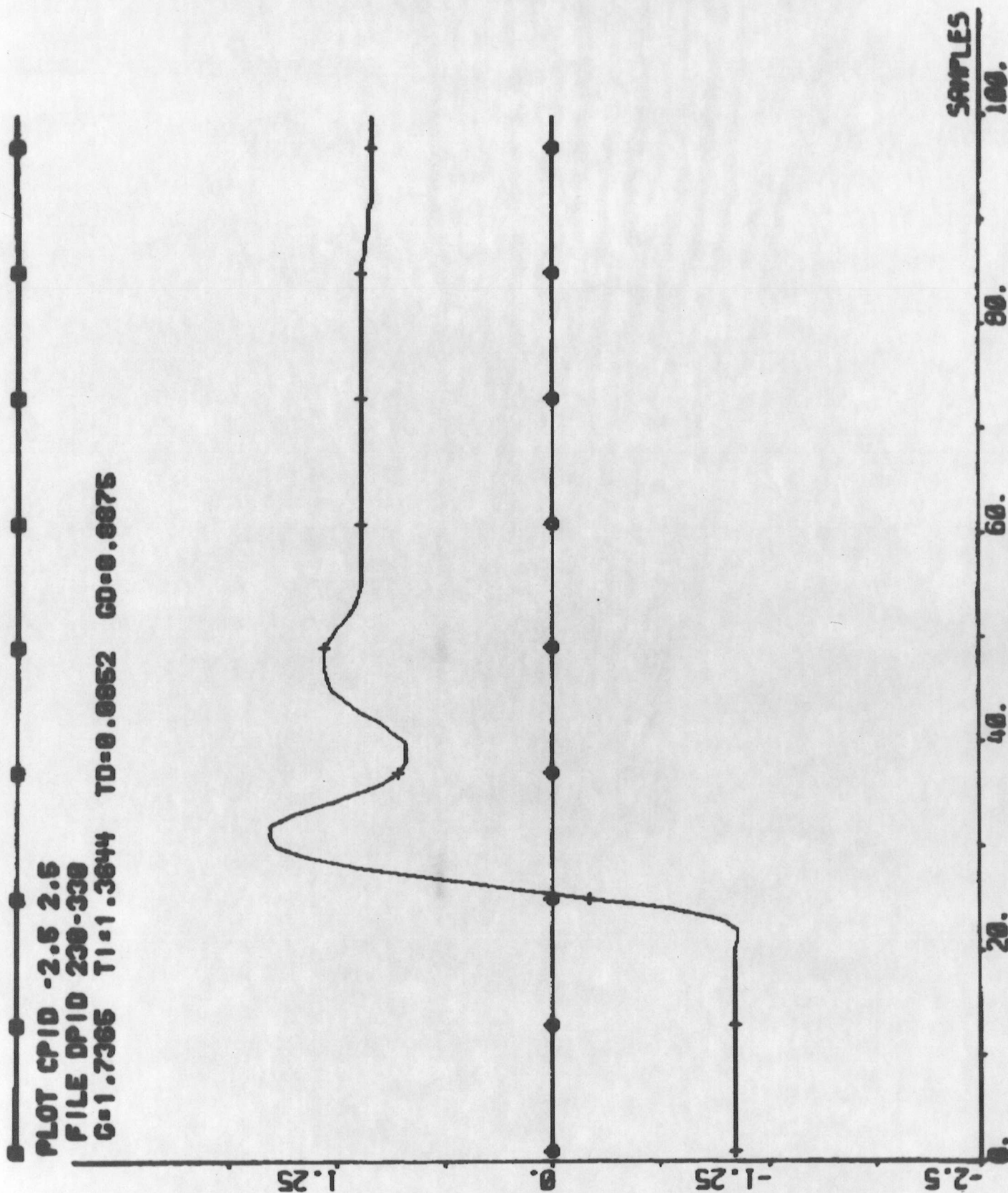


Diagram 7. Stegsvär för servo med PID-reg. (opt.)

Förlustfunktion som fkn av antalet
exekveringar av optimeringsalgoritm

Diagram 8



Appendix

Programlistningar

START	A1
REGUL	B1
PIDREG	B3
BBREG	B4
REGO	C1
NUFLE2	C2
PRINT	C8
HESS	C10

```

C      TASK START
C
C      ALLOTS CONSTANTS AND VARIABLES THEIR INITIAL VALUES.
C      SETS REQUEST NAME TO 'REGUL'.
C
      DIMENSION USER(36),X(8),XM(8),DELMAX(8),G(8),RP(8),IREG(2)
      COMMON /I0COM/ICOM(256)
      COMMON /RTCOM/IBUFF(512)
      EQUIVALENCE
      1(ICOM(122),REQNM),(ICOM(185),USER(1)),
      1(IBUFF(1),IND),(IBUFF(2),N),(IBUFF(3),MODE),(IBUFF(4),IDIFF),
      2(IBUFF(46),X(1)),(IBUFF(62),G(1)),(IBUFF(78),XM(1)),
      3(IBUFF(114),ALOSS),(IBUFF(118),DFN),(IBUFF(120),HH),
      4(IBUFF(122),EPS),(IBUFF(204),DELMAX(1)),
      5(IBUFF(361),NSAMP),(IBUFF(362),MSAMP),(IBUFF(363),UR),
      6(IBUFF(365),Y01),(IBUFF(367),RI1),(IBUFF(369),D1),
      7(IBUFF(371),E01),(IBUFF(373),Y02),(IBUFF(375),RI2),
      8(IBUFF(377),D2),(IBUFF(379),E02),(IBUFF(381),IREG(1)),
      9(IBUFF(384),RP(1)),
      1(IBUFF(400),A1),(IBUFF(402),A2),(IBUFF(404),TSAMP),
      2(IBUFF(406),ALFA),(IBUFF(408),FIV),(IBUFF(410),IST)
C
      REQNM=5HREGUL
      ALOSS=0.
      Y01=0.
      RI1=0.
      D1=0.
      Y02=0.
      RI2=0.
      D2=0.
      E01=0.
      E02=0.
      IND=1
      MODE=1
      IDIFF=2
      EPS=0.001
C
      N=USER(1)
      A1=USER(3)
      A2=USER(8)
      IREG(1)=USER(10)
      IREG(2)=USER(11)
      TSAMP=USER(12)
      NSAMP=USER(13)
      MSAMP=USER(13)/2.
      UR=USER(14)
      ALFA=USER(15)
      FIV=USER(16)
      DFN=USER(19)
      HH=USER(20)
      DO 1 I=1,N
1      DELMAX(I)=USER(20+I)
C
      J=0
      M=1
      IF(IREG(1).EQ.4) M=3
      DO 2 I=M,8
      J=J+1
      G(J)=0.
      RP(J)=USER(1+I)
2      XM(J)=1.
C
      MREG=1

```

```
IX=1
IRP=1
3 X(IX)=RP(IRP)
  IX=IX+1
  IRP=IRP+1
  IF(IX.GT.N) GO TO 7
  GO TO (4,4,5,6,3,4,3,3),IRP
4 IF(IREG(MREG).EQ.2) IRP=IRP+1
  GO TO 3
5 IF(IREG(1).EQ.1) IRP=IRP+2
  GO TO 3
6 IF(IREG(1).EQ.4.AND.IREG(2).EQ.2) IRP=IRP+1
  MREG=MREG+1
  GO TO 3
C
7  IST=0
C
  CALL EXIT
  END
```

```

C      TASK REGUL
C
C      PERFORMS PID- AND "BANG-BANG" CONTROL,
C      ESTIMATES LOSS-FUNCTION,
C      ACTIVATES TASK REGO.
C
C      RUNS IN PARTITION PIPUS
C
C      TASK FIXED IN CORE
C
C      DEFAULT PRIORITY 150
C
C      SUBROUTINE REQUIRED
C          PIDREG
C          BBREG
C
C      LOGICAL OPT
C      DIMENSION USER(36),FLAI(16),FLAO(8),RP(8),RPO(8),IREG(2)
C      COMMON /I0COM/ICOM(256)
C      COMMON /RTCOM/IBUFF(512)
C      EQUIVALENCE
C      1(ICOM(26),LIWRD),
C      2(ICOM(137),FLAI(1),Y1),(ICOM(139),Y2),
C      3(ICOM(169),FLAO(1),U1),(ICOM(171),U2),
C      4(ICOM(185),USER(1)),
C      1(IBUFF(114),ALOSS),
C      2(IBUFF(361),NSAMP),(IBUFF(362),MSAMP),(IBUFF(363),UR),
C      3(IBUFF(365),Y01),(IBUFF(367),RI1),(IBUFF(369),D1),
C      4(IBUFF(371),E01),(IBUFF(373),Y02),(IBUFF(375),RI2),
C      5(IBUFF(377),D2),(IBUFF(379),E02),(IBUFF(381),IREG(1)),
C      6(IBUFF(384),RP(1)),(IBUFF(400),A1),
C      7(IBUFF(402),A2),(IBUFF(404),TSAMP),(IBUFF(406),ALFA),
C      8(IBUFF(408),FIV),(IBUFF(411),ALOSO),(IBUFF(413),RPO(1))
C
C      ALOSS=ALOSS+(Y1-UR)**2+ALFA*U1**2+FIV*(Y2**2+ALFA*U2**2)
C      IF(NSAMP.NE.MSAMP) GO TO 2
C
C      CHECK IF THE OPTIMIZATION ROUTINE SHOULD BE CALLED
C      CALL GETBIT(LIWRD,14,OPT)
C      IF(.NOT.OPT) GO TO 1
C      SAVE DATA FOR SUBROUTINE PRINT
C      ALOSO=ALOSS
C      DO 10 I=1,8
10     RPO(I)=RP(I)
C      CALL REQST(4HREGO,200)
C
C      UR=-UR
C
C      IFI=1
C      IF(IREG(1).EQ.4) GO TO 4
C      CALL PIDREG(RP,IFI,TSAMP,UR,Y1,U1,Y01,RI1,D1)
C      IF(IREG(2).EQ.0) GO TO 6
C      IFI=5
C      IF(IREG(2).EQ.4) GO TO 5
3     CALL PIDREG(RP,IFI,TSAMP,U1,Y2,U2,Y02,RI2,D2)
C      GO TO 6
4     CALL BBREG(RP,IFI,TSAMP,UR,Y1,U1,E01,A1)
C      IF(IREG(2).EQ.0) GO TO 6
C      IFI=3
C      IF(IREG(2).NE.4) GO TO 3
5     CALL BBREG(RP,IFI,TSAMP,U1,Y2,U2,E02,A2)
6     NSAMP=NSAMP-1
C      IF(NSAMP.LE.0) GO TO 7

```

C

CALL EXIT

C

7

ALOSS=0.

NSAMP=USER(13)

UR=USER(14)

C

CALL EXIT

END

```

C      PID-REGULATOR
C
C      SUBROUTINE PIDREG(RP,IFI,TSAMP,UR,Y,U,YO,RI,D)
C
C      INTEGRATING TERM RI,DERIVATIVE TERM D AND OLD PROCESS OUTPUT
C      ARE UPDATED.
C
C      RP      A REAL ARRAY IN WHICH THE REGULATOR-PARAMETERS ARE
C              STORED.
C      IFI     INDICATES THE POSITION OF THE FIRST PARAMETER (G) IN RP.
C      G       GAIN
C      TI      INTEGRATING TIME CONSTANT
C      TD      DERIVATING TIME CONSTANT
C      GD      DERIVATING FILTER CONSTANT
C      TSAMP   SAMPLING PERIOD
C      UR      INPUT REFERENCE VALUE
C      Y       OUTPUT SIGNAL FROM PROCESS
C      U       INPUT SIGNAL TO PROCESS
C      YO     OLD VALUE OF PROCESS OUTPUT
C      RI     INTEGRATING TERM (INTERNAL VARIABLE)
C      D      DERIVATIVE TERM (INTERNAL VARIABLE)
C
C      DIMENSION RP(1)
C
C      G=RP(IFI)
C      TI=RP(IFI+1)
C      TD=RP(IFI+2)
C      GD=RP(IFI+3)
C
C      E=UR-Y
C      ALFA=G*TD/TSAMP
C      BETA=TD/(TD+GD*TSAMP)
C
C      P=G*E
C      RI=RI+G*TSAMP/TI*E
C      D=BETA*D+ALFA*(1.-BETA)*(YO-Y)
C      U=P+RI+D
C
C      YO=Y
C
C      RETURN
C      END

```

```
C      "BANG-BANG"-REGULATOR
C
C      SUBROUTINE BBREG(RP,IFI,TSAMP,UR,Y,U,E0,A)
C
C      OLD ERROR IS UPDATED.
C
C      RP      A REAL ARRAY IN WHICH THE REGULATOR-PARAMETERS ARE
C              STORED.
C      IFI     INDICATES THE POSITION OF THE FIRST PARAMETER (K) IN RP.
C      K       SWITCHING LINE CONSTANT
C      DZ      DEATH ZONE CONSTANT
C      TSAMP   SAMPLING PERIOD
C      UR      INPUT REFERENCE VALUE
C      Y       OUTPUT SIGNAL FROM PROCESS
C      U       INPUT SIGNAL TO PROCESS
C      E0      OLD VALUE OF ERROR (INTERNAL VARIABLE)
C      A       "BANG-BANG" CONSTANT (+A OR -A)
C
C      DIMENSION RP(1)
C
C      K=RP(IFI)
C      DZ=RP(IFI+1)
C
C      E=Y-UR
C      EP=(E-E0)/TSAMP
C      ALFA=E+K*EP
C
C      U=0.
C      IF(ALFA.GT.DZ) U=-A
C      IF(ALFA.LT.-DZ) U=A
C
C      E0=E
C
C      RETURN
C      END
```

```
C      TASK REGO
C
C      PERFORMS OPTIMIZATION OF REGULATOR-PARAMETERS
C
C      RUNS IN PARTITION TDV
C
C      DEFAULT PRIORITY 200
C
C      SUBROUTINE REQUIRED
C          NUFLE2
C          PRINT
C
C      DIMENSION P(8),RP(8),IREG(2)
C      COMMON /RTCOM/IBUFF(512)
C      EQUIVALENCE
C      1(IBUFF(2),N),(IBUFF(30),P(1)),
C      2(IBUFF(381),IREG(1)),(IBUFF(384),RP(1))
C
C      CALL THE OPTIMIZATION ROUTINE
C      CALL NUFLE2
C
C      PRINT DATA ON LINE-PRINTER
C      CALL PRINT
C
C      PREVENT PARAMETERS FROM PASSING ZERO
C      MREG=1
C      IP=1
C      IRP=1
1      IF(P(IP)/RP(IRP).GT.0.) GO TO 2
C      P(IP)=0.1
C      IF(RP(IRP).LT.0.) P(IP)=-0.1
2      RP(IRP)=P(IP)
C      IP=IP+1
C      IRP=IRP+1
C      IF(IP.GT.N) GO TO 6
C      GO TO (3,3,4,5,1,3,1,1),IRP
3      IF(IREG(MREG).EQ.2) IRP=IRP+1
C      GO TO 1
4      IF(IREG(1).EQ.1) IRP=IRP+2
C      GO TO 1
5      IF(IREG(1).EQ.4.AND.IREG(2).EQ.2) IRP=IRP+1
C      MREG=MREG+1
C      GO TO 1
6      CONTINUE
C
C      CALL EXIT
C      END
```


SUBROUTINE NUFLE2

ADAPTION OF NUFLET FOR RSX

ROUTINE FOR FINDING THE MINIMUM OF A FUNCTION F(X), WHEN ONLY FUNCTION VALUES ARE AVAILABLE

AUTHOR, T. GLAD 1973-07-01.

REVISED, C. KALLSTROM 1973-08-10.

REVISED (ADAPTED FOR SIMNON), T. GLAD 1974-07-06

REVISED (ADAPTED FOR RSX), T. GLAD 1974-11-16

REFERENCE: R. FLETCHER, FORTRAN SUBROUTINES FOR MINIMIZATION BY QUASI-NEWTON METHODS, REPORT AERE-R7125, HARWELL

THE FIELD IBUFF IS DIVIDED ACCORDING TO THE EQUIVALENCE STATEMENT BELOW

ALOSS VALUE OF LOSS FUNCTION

P NEW PARAMETER VALUE

IND INDICATES TO WHICH POINT IN NUFLE1 A RETURN SHOULD BE MADE

X A REAL ARRAY OF N ELEMENTS IN WHICH THE CURRENT ESTIMATE OF THE SOLUTION IS STORED. AN INITIAL APPROXIMATION MUST BE SET IN X ON ENTRY TO NUFLET AND THE BEST ESTIMATE OBTAINED WILL BE RETURNED ON EXIT

N THE NUMBER OF VARIABLES (MIN 2, NO MAX)

F A REAL NUMBER IN WHICH THE BEST VALUE OF F(X) CORRESPONDING TO X ABOVE WILL BE RETURNED

G A REAL ARRAY OF N ELEMENTS USED TO STORE AN ESTIMATE OF THE GRADIENT VECTOR. NOT TO BE SET ON ENTRY

H A REAL ARRAY OF $N*(N+1)/2$ ELEMENTS IN WHICH AN ESTIMATE OF HESSIAN MATRIX IS STORED. THE MATRIX IS REPRESENTED IN THE PRODUCT FORM $LD(L)T$ WHERE L IS A LOWER TRIANGULAR MATRIX WITH UNIT DIAGONALS AND D IS A DIAGONAL MATRIX. THE LOWER TRIANGLE OF L IS STORED BY COLUMNS IN H EXCEPTING THAT THE UNIT DIAGONAL ELEMENTS ARE REPLACED BY THE CORRESPONDING ELEMENTS OF D. THE SETTING OF H ON ENTRY IS CONTROLLED BY THE PARAMETER MODE.

W A REAL ARRAY OF $4*N$ ELEMENTS USED AS WORKING SPACE

DFN A REAL NUMBER WHICH MUST BE SET SO AS TO GIVE NUFLET AN ESTIMATE OF THE LIKELY REDUCTION TO BE OBTAINED IN F(X). DFN IS USED ONLY ON THE FIRST ITERATION SO AN ORDER OF MAGNITUDE ESTIMATE WILL SUFFICE.
 DFN>0 THE SETTING OF DFN ITSELF WILL BE TAKEN AS THE LIKELY REDUCTION TO BE OBTAINED IN F(X)
 DFN=0 IT WILL BE ASSUMED THAT AN ESTIMATE OF THE MINIMUM VALUE OF F(X) HAS BEEN SET IN ARGUMENT F, AND THE LIKELY REDUCTION IN F(X) WILL BE COMPUTED ACCORDING TO THE INITIAL FUNCTION VALUE
 DFN<0 A MULTIPLE ABS(DFN) OF THE MODULUS OF THE INITIAL FUNCTION VALUE WILL BE TAKEN AS AN ESTIMATE OF THE LIKELY REDUCTION.

XM A REAL ARRAY OF N ELEMENTS USED IN SCALING, SEE BELOW

HH A REAL NUMBER. THE STEP LENGTH USED WHEN CALCULATING G(I) BY DIFFERENCES IS $HH*XM(I)$. HINT GIVEN BY FLETCHER: SET HH EQUAL TO $2**(-T/2)$ WHERE T IS THE NUMBER OF SIGNIFICANT BINARY DIGITS IN THE CALCULATION OF F.

EPS A REAL NUMBER. THE ACCURACY REQUIRED IN X(I) IS $EPS*XM(I)$.
 MODE AN INTEGER WHICH CONTROLS THE SETTING OF THE INITIAL ESTIMATE OF THE HESSIAN MATRIX IN THE PARAMETER H.

MODE=1 AN ESTIMATE CORRESPONDING TO THE UNIT MATRIX IS SET IN H BY NUFLET

MODE=3 NUFLET ASSUMES THAT THE HESSIAN MATRIX HAS BEEN SET IN H IN PRODUCT FORM.

C IDIFF INDICATES IF FORWARD DIFFERENCES (IDIFF=1) OR CENTRAL
 C DIFFERENCES (IDIFF=2) SHALL BE USED IN NUMERICAL
 C DIFFERENTIATION
 C IEXIT AN INTEGER GIVING THE REASON FOR EXIT FROM NUFLET, THIS WILL
 C BE SET BY NUFLET AS FOLLOWS
 C IEXIT=0 THE NORMAL EXIT IN WHICH $ABS(DX(I)) < EPS * XM(I)$ FOR
 C $I=1,2,..,N$ WHERE $DX(I)$ IS THE CHANGE IN X ON AN ITERATION
 C IEXIT=1 (MODE=2 ONLY) HESSIAN MATRIX IS NOT POSITIVE DEFIN
 C IEXIT=2 $GT * DX \geq 0$ EITHER DUE TO ROUNDING ERRORS
 C BECAUSE EPS IS SET TOO SMALL FOR THE COMPUTER WORD LENGTH
 C OR THE TRUNCATION ERROR IN THE FINITE DIFFERENCE FORMULA
 C FOR G BEING DOMINANT.
 C IEXIT=3 FUNCT CALLED MAXFN TIMES.
 C IEXIT=4 MINIMIZING TERMINATED FROM FUNCT.

C SUBROUTINE REQUIRED
 C (FUNCT)

C DIMENSION X(8),G(8),H(36),W(32),XM(8),P(8),XACT(8),
 C 1 AMAX(8),AMIN(8),DELMAX(8)
 C COMMON/RTCOM/IBUFF(512)

C EQUIVALENCE (IBUFF(1),IND),(IBUFF(2),N),(IBUFF(3),MODE),
 C 1(IBUFF(4),IDIFF),(IBUFF(5),IEXIT),(IBUFF(6),NP),
 C 2(IBUFF(7),N1),(IBUFF(8),NN),(IBUFF(9),IS),(IBUFF(10),IU),
 C 3(IBUFF(11),IV),(IBUFF(12),IB),(IBUFF(13),ITN),
 C 4(IBUFF(14),INT),(IBUFF(15),I),(IBUFF(16),LINK),
 C 5(IBUFF(30),P(1)),(IBUFF(46),X(1)),(IBUFF(62),G(1)),
 C 6(IBUFF(78),XM(1)),
 C 7(IBUFF(114),ALOSS),(IBUFF(116),F),(IBUFF(118),DFN),
 C 8(IBUFF(120),HH),(IBUFF(122),EPS),(IBUFF(124),DMIN),
 C 9(IBUFF(126),Z),(IBUFF(128),DF),(IBUFF(130),GS0),
 C 1(IBUFF(132),AEPS),(IBUFF(134),ALPHA),(IBUFF(136),FF),
 C 2(IBUFF(138),TOT),(IBUFF(140),F1),(IBUFF(142),F2),
 C 3(IBUFF(144),GYS),(IBUFF(146),DGS),(IBUFF(148),SIG),
 C 4(IBUFF(150),ZZ),(IBUFF(152),XACT(1)),(IBUFF(168),ZACT),
 C 5(IBUFF(170),ZZACT),(IBUFF(172),AMAX(1)),(IBUFF(188),AMIN(1)),
 C 6(IBUFF(204),DELMAX(1)),(IBUFF(220),H(1)),(IBUFF(292),W(1))

C GO TO (1,205,212,222,232,242,252,262),IND
 1 CONTINUE
 NP=N+1
 N1=N-1
 NN=N*NP/2
 IS=N
 IU=N
 IV=N+N
 IB=IV+N
 IEXIT=1
 IF(MODE.EQ.3) GO TO 15
 IJ=NN+1
 DO 5 I=1,N
 DO 6 J=1,I
 IJ=IJ-1
 6 H(IJ)=0.
 5 H(IJ)=1.
 15 CONTINUE
 IJ=NP
 DMIN=H(1)
 DO 16 I=2,N
 IF(H(IJ).GE.DMIN) GO TO 16
 DMIN=H(IJ)
 16 IJ=IJ+NP-1
 IF(DMIN.LE.0.) RETURN

```

Z=F
ITN=0
IND=2
DO 202 I=1,N
202 P(I)=X(I)
RETURN
205 F=ALOSS
DO 207 I=1,N
207 XACT(I)=P(I)
DF=DFN
IF(DFN.EQ.0.) DF=F-Z
IF(DFN.LT.0.) DF=ABS(DF*F)
IF(DF.LE.0.) DF=1.
17 CONTINUE
DO 19 I=1,N
19 W(I)=X(I)
LINK=1
IF(IDIFF-1) 100,100,110
18 CONTINUE
20 CONTINUE
ITN=ITN+1
W(1)=-G(1)
DO 22 I=2,N
IJ=I
I1=I-1
Z=-G(I)
DO 23 J=1,I1
Z=Z-H(IJ)*W(J)
23 IJ=IJ+N-J
22 W(I)=Z
W(IS+N)=W(N)/H(NN)
IJ=NN
DO 25 I=1,N1
IJ=IJ-1
Z=0.
DO 26 J=1,I
Z=Z+H(IJ)*W(IS+NP-J)
26 IJ=IJ-1
25 W(IS+N-I)=W(N-I)/H(IJ)-Z
Z=0.
GS0=0.
DO 29 I=1,N
IF(Z*XM(I).GE.ABS(W(IS+I))) GO TO 29
Z=ABS(W(IS+I))/XM(I)
29 GS0=GS0+G(I)*W(IS+I)
AEPS=EPS/Z
IEXIT=2
IF(GS0.GE.0.) GO TO 92
ALPHA=-2.*DF/GS0
IF(ALPHA.GT.1.) ALPHA=1.
FF=F
TOT=0.
INT=0
IEXIT=0
30 CONTINUE
DO 329 I=1,N
IF(ALPHA*ABS(W(IS+I)).GT.DELMAX(I)) ALPHA=
1 DELMAX(I)/ABS(W(IS+I))
329 CONTINUE
DO 31 I=1,N
31 W(I)=X(I)+ALPHA*W(IS+I)
IND=3
DO 210 I=1,N
210 P(I)=W(I)

```

```

RETURN
212 F1=ALOSS
    IF(F1.GE.F) GO TO 40
    F2=F
    TOT=TOT+ALPHA
32 CONTINUE
    DO 33 I=1,N
    XACT(I)=P(I)
33 X(I)=W(I)
    F=F1
    IF(INT-1) 35,49,50
35 CONTINUE
    DO 334 I=1,N
    IF(ALPHA*ABS(W(IS+1)).GT.DELMAX(I)) ALPHA=
1 DELMAX(I)/ABS(W(IS+1))
334 CONTINUE
    DO 34 I=1,N
34 W(I)=X(I)+ALPHA*W(IS+1)
    IND=4
    DO 220 I=1,N
220 P(I)=W(I)
    RETURN
222 F1=ALOSS
    IF(F1.GE.F) GO TO 50
    IF(F1+F2.GE.F+F.AND.7.*F1+5.*F2.GT.12.*F) INT=2
    TOT=TOT+ALPHA
    ALPHA=2.*ALPHA
    GO TO 32
40 CONTINUE
    IF(ALPHA.LT.AEPS) GO TO 92
    ALPHA=.5*ALPHA
    DO 41 I=1,N
41 W(I)=X(I)+ALPHA*W(IS+1)
    IND=5
    DO 230 I=1,N
230 P(I)=W(I)
    RETURN
232 F2=ALOSS
    IF(F2.GE.F) GO TO 45
    TOT=TOT+ALPHA
    F=F2
    DO 42 I=1,N
    XACT(I)=P(I)
42 X(I)=W(I)
    GO TO 49
45 CONTINUE
    Z=.1
    IF(F1+F.GT.F2+F2) Z=1.+5*(F-F1)/(F+F1-F2-F2)
    IF(Z.LT..1) Z=.1
    ALPHA=Z*ALPHA
    INT=1
    GO TO 30
49 CONTINUE
    IF(TOT.LT.AEPS) GO TO 92
50 CONTINUE
    ALPHA=TOT
    DO 56 I=1,N
    W(I)=X(I)
56 W(IB+1)=G(I)
    LINK=2
    IF(IDIFF-1) 100,100,110
54 CONTINUE
    GYS=0.
    DO 55 I=1,N

```

```

W(I)=W(IB+I)
55 GYS=GYS+G(I)*W(IS+I)
    DF=FF-F
    DGS=GYS-GS0
    IF(DGS.LE.0.) GO TO 20
    LINK=1
    IF(DGS+ALPHA*GS0.GT.0.) GO TO 52
    DO 51 I=1,N
51 W(IU+I)=G(I)-W(I)
    SIG=1./(ALPHA*DGS)
    GO TO 70
52 CONTINUE
    ZZ=ALPHA/(DGS-ALPHA*GS0)
    Z=DGS*ZZ-1.
    DO 53 I=1,N
53 W(IU+I)=Z*W(I)+G(I)
    SIG=1./(ZZ*DGS**2)
    GO TO 70
60 CONTINUE
    LINK=2
    DO 61 I=1,N
61 W(IU+I)=W(I)
    IF(DGS+ALPHA*GS0.GT.0.) GO TO 62
    SIG=1./GS0
    GO TO 70
62 CONTINUE
    SIG=-ZZ
70 CONTINUE
    W(IV+1)=W(IU+1)
    DO 71 I=2,N
        IJ=I
        I1=I-1
        Z=W(IU+I)
        DO 72 J=1,I1
            Z=Z-H(IJ)*W(IV+J)
72 IJ=IJ+N-J
71 W(IV+I)=Z
        IJ=1
        DO 75 I=1,N
            Z=H(IJ)+SIG*W(IV+I)**2
            IF(Z.LE.0.) Z=DMIN
            IF(Z.LT.DMIN) DMIN=Z
            H(IJ)=Z
            W(IB+I)=W(IV+I)*SIG/Z
            SIG=SIG-W(IB+I)**2*Z
75 IJ=IJ+NP-1
        IJ=1
        DO 80 I=1,N1
            IJ=IJ+1
            I1=I+1
            DO 80 J=I1,N
                W(IU+J)=W(IU+J)-H(IJ)*W(IV+I)
                H(IJ)=H(IJ)+W(IB+I)*W(IU+J)
80 IJ=IJ+1
        GO TO (60,20),LINK
90 CONTINUE
    IEXIT=3
    GO TO 94
92 CONTINUE
94 CONTINUE
    IND=1
    RETURN
100 CONTINUE
    I=0

```

```
102 I=I+1
    Z=HH*XM(I)
    W(I)=W(I)+Z
    IND=6
    DO 240 K=1,N
240 P(K)=W(K)
    RETURN
242 F1=ALOSS
    ZACT=P(I)-XACT(I)
    G(I)=(F1-F)/ZACT
101 W(I)=W(I)-Z
    IF(I.LT.N) GO TO 102
    GO TO (18,54),LINK
110 CONTINUE
    I=0
112 I=I+1
    Z=HH*XM(I)
    W(I)=W(I)+Z
    IND=7
    DO 250 K=1,N
250 P(K)=W(K)
    RETURN
252 F1=ALOSS
    ZACT=P(I)-XACT(I)
    W(I)=W(I)-Z-Z
    IND=8
    DO 260 K=1,N
260 P(K)=W(K)
    RETURN
262 F2=ALOSS
    ZZACT=XACT(I)-P(I)
    G(I)=(F1-F2)/(ZACT+ZZACT)
111 W(I)=W(I)+Z
    IF(I.LT.N) GO TO 112
    GO TO (18,54),LINK
END
```

```

SUBROUTINE PRINT
C
C PRINTS LOSS-FUNCTION,REGULATOR-PARAMETERS,GRADIENTS AND
C HESSIAN MATRIX.
C
C SUBROUTINE REQUIRED
C HESS
C
C DIMENSION FT(14),VFT(8),A(8,8),H(36),G(8),RPO(8),TEXT(7),TAB(13),
1 IREG(2)
COMMON /RTCOM/IBUFF(512)
EQUIVALENCE
1( IBUFF(1),IND), (IBUFF(2),N), (IBUFF(62),G(1)),
2( IBUFF(220),H(1)), (IBUFF(381),IREG(1)),
3( IBUFF(400),A1), (IBUFF(402),A2), (IBUFF(410),IST),
4( IBUFF(411),ALOSO), (IBUFF(413),RPO(1))
C
C DATA TEXT/5HDC-SE,3HRVO,3HBOM,3HPI ,3HPD ,3HPID,3HB-B/
DATA TAB/5HALOSS,3HG1 ,3HTI1,3HTD1,3HGD1,3HG2 ,3HTI2,3HTD2,3HGD2,
1 3HK1 ,3HDZ1,3HK2 ,3HDZ2/
DATA FT(1),FT(3),FT(14)/4H(1X,, 5HF13.4, 1H)/
DATA VFT/5HF9.3,, 3H9X,, 1H,, 5HE13.1, 1H3, 3H13X, 1H2, 4H26X,/
C
C IF(IST.NE.0) GO TO 10
C
C WRITE(16,100)
100 FORMAT(1H1,33HOPTIMERING AV REGULATORPARAMETRAR/1X,33(1H*)////
1 1X,8HPROCESS:,15X,10HREGULATOR:)
IF(IREG(2).NE.0) GO TO 1
WRITE(16,101) TEXT(1),TEXT(2),TEXT(3+IREG(1))
101 FORMAT(1H+,9X,A5,A3,17X,A3,7H (LAGE))
IF(IREG(1).EQ.4) WRITE(16,102) A1
102 FORMAT(1H+,48X,2HA=,F5.2)
GO TO 2
1 WRITE(16,103) TEXT(3),TEXT(3+IREG(1))
103 FORMAT(1H+,9X,A3,22X,A3,7H (LAGE))
IF(IREG(1).EQ.4) WRITE(16,102) A1
WRITE(16,104) TEXT(3+IREG(2))
104 FORMAT(35X,A3,9H (VINKEL))
IF(IREG(2).EQ.4) WRITE(16,102) A2
C
2 IF(IREG(1).EQ.4) GO TO 3
WRITE(16,105) TAB(1),(TAB(1+I),I=1,4)
105 FORMAT(//4X,A5,8X,8(A3,10X))
GO TO 4
3 WRITE(16,105) TAB(1),TAB(10),TAB(11)
4 IF(IREG(2).EQ.0) GO TO 6
IF(IREG(2).EQ.4) GO TO 5
WRITE(16,106) (TAB(5+I),I=1,4)
106 FORMAT(1H+,68X,4(A3,10X))
GO TO 6
5 WRITE(16,106) TAB(12),TAB(13)
6 WRITE(16,107)
107 FORMAT(1H )
IST=1
C
10 FT(2)=VFT(1)
FT(4)=VFT(3)
FT(5)=FT(3)
IF(IREG(1).EQ.2) FT(5)=VFT(4)
FT(6)=VFT(3)
FT(7)=VFT(5)

```

```

IF(IREG(1).EQ.4) FT(7)=VFT(8)
FT(8)=FT(3)
FT(9)=VFT(3)
FT(10)=FT(3)
IF(IREG(2).EQ.2) FT(10)=VFT(4)
FT(11)=VFT(3)
FT(12)=VFT(7)
FT(13)=FT(3)
NR=N
IF(IREG(1).GE.3) GO TO 11
NR=NR+1
IF(IREG(1).EQ.1) NR=NR+1
11 IF(IREG(2).EQ.0.OR.IREG(2).GE.3) GO TO 12
NR=NR+1
IF(IREG(2).EQ.1) NR=NR+1
12 WRITE(16,FT) ALOSO,(RPO(I),I=1,NR)
C
IF(IND.EQ.1) WRITE(16,110)
110 FORMAT(1X,7HRESTART)
IF(IND.EQ.3) GO TO 13
C
RETURN
C
13 FT(2)=VFT(2)
IF(IREG(1).EQ.1) FT(7)=VFT(8)
IF(IREG(1).EQ.2) FT(5)=VFT(6)
IF(IREG(2).EQ.2) FT(10)=VFT(6)
WRITE(16,FT) (G(I),I=1,N)
WRITE(16,111)
111 FORMAT(1H+,10H GRADIENT:/16X,14HHHESSIAN MATRIX)
CALL HESS(N,H,A)
DO 14 I=1,N
14 WRITE(16,FT) (A(I,K),K=1,N)
C
RETURN
END

```


SUBROUTINE HESS(N,H,A)

ROUTINE FOR TRANSFORMING THE HESSIAN MATRIX FROM THE PRODUCT
FORM LD(L)T TO A SINGEL MATRIX.

N THE NUMBER OF VARIABLES (MIN 2,NO MAX)
H A REAL ARRAY OF $N*(N+1)/2$ ELEMENTS IN WHICH AN ESTIMATE OF
THE HESSIAN MATRIX IS STORED,THE MATRIX IS REPRESENTED IN
THE PRODUCT FORM LD(L)T WHERE L IS A LOWER TRIANGULAR
MATRIX WITH UNIT DIAGONALS AND D IS A DIAGONAL MATRIX,
THE LOWER TRIANGEL OF L IS STORED BY COLUMNS IN H EXCEPTING
THAT THE UNIT DIAGONAL ELEMENTS ARE REPLACED BY THE
CORRESPONDING ELEMENTS OF D.
A A REAL FIELD IN WHICH THE TRANSFORMED HESSIAN MATRIX IS
STORED.

DIMENSION H(1),A(8,8)

COMPUTE THE LOWER TRIANGULAR MATRIX L

M=0

DO 1 I=1,N

DO 1 J=1,N

A(I,J)=0.

DO 2 J=1,N

DO 2 I=J,N

M=M+1

A(I,J)=H(M)

DO 3 I=1,N

A(I,I)=1.

COMPUTE LD

K=1

DO 4 J=1,N

M=K

DO 4 I=J,N

K=K+1

A(I,J)=A(I,J)*H(M)

COMPUTE LD(L)T

NJ=N-1

DO 5 J=1,N

JN=N+1-J

NK=JN-1

DO 5 I=1,N

M=0

DO 5 K=1,NK

M=M+N+1-K

MM=M+1-J

A(I,JN)=A(I,JN)+A(I,K)*H(MM)

RETURN

END