OBSERVABILITY OF POWER SYSTEMS

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Sammanfattning - Summary

This work consists firstly of a study of the general observability problem and secondly of a presentation of an algorithm for determination of observability in power systems.

The literature survey showed that the general observability problem still is a field where much research remains to be done. There are no criteria of observability that can be conveniently applied to non-linear deterministic or stochastic systems in practice.

The presented algorithm determines observable subsystems in a power system. The algorithm performs an investigation of network topology and measurement configuration.

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KY KY-bibl

Observerbarhet i kraftsystem.

Mitt examensarbete har bestått dels i en litteraturstudie av det allmäna observerbarhetsproblemet och dels av framtagningen av en algoritm för bestämning av observerbarhet i kraftsystem.

Litteraturundersökningen visade att mycket forskning återstår inom det allmäna observerbarhetsproblemet.Det finns inga observerbarhetskriterier som enkelt och praktiskt kan tillämpas på icke-lineära deterministiska eller stokastiska problem.

Den framtagna algoritmen bestämmer observerbara delsystem i ett kraftsystem. Algoritmen grundar sig på en undersökning av kraftnätets topologi och mätinstrumentens placering.

CONTENTS

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1 INTRODUCTION

In the last years much interest has been shown to the problem of state estimation. A basic part of this problem is to determine if the system considered is observable, that is whether it is possible to uniquely determine the state of the system using the available input data.

The purpose of the work reported here has been to find a method for the determination of the observable parts of a power network (observability for a part of a system is defined in Section 2.1.4). The purpose of this determination is to find groups of nodes to which a state estimation algorithm can be applied. The measurement system is described as a non-linear stochastic one. Since the power and measurement system is variable in time the observability is not a constant property. It also depends upon the used algorithm for estimation.

The literature on the general observability problem is discussed in Section 2.

Section 3 treats the special problem of observability in power systems and contains an algorithm for finding observable parts of a power network.

Section 4 contains an example of a power system. A program built on the algorithm in Section 3 has been applied to the example and the results are presented in Section 4.

2 THE GENERAL OBSERVABILITY PROBLEM

2.1 Statement of the General Observability Problem

A system is said to be observable if the state of the system can be determined in a well defined sense from observations of the system during a finite time.

We describe the general system S in the following way:

$$S: \begin{cases} \frac{dx}{dt} = f(t,x,p,u) & x = (x_1, \dots\, x_n)^T \\ y = h(t,x,p)+w & y = (y_1, \dots\, y_m)^T \end{cases}$$

where

x = the state vector of the system

t = time

p = the vector of unknown parameters

u = the vector of control signals

y = the observation vector

w = the vector of measurement errors

The latter is often assumed to consist of white noise with zero mean.

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No general solution to the observability problem for general non-linear systems seems to have been presented. Observability for special classes of non-linear systems is treated in the literature. Some papers on the subject will be discussed in this report. The observability problem for deterministic systems will be treated in Section 2.2 and for stochastic systems in Section 2.3. In Section 2.4 the least squares estimate and observability is discussed.

2.2 Deterministic Systems

Necessary and sufficient conditions for observability of the linear deterministic system:

$$\frac{dx}{dt} = A(t)x + B(t)u \qquad x = (x_1 \ldots , x_n)^T$$

$$y = C(t)x$$

are well known.

The condition is that the observability matrix W_o will have rank n. W_o is defined by

$$W_o = \int_{t_o}^{t_1} \Phi^T(t,t_o)C^T(t)\, C(t)\, \Phi(t,t_o)dt$$

where Φ is the state transition matrix associated with $A(t)$.

$$\frac{\partial\Phi}{\partial t}(t,t') = A(t) \quad \Phi(t,t')$$

$$\Phi(t,t) = I$$

For the special case of time invariant matrices A, B and C we get

$$W_o = (C^T \; A^T C^T \; \ldots \; (A^T)^{p-1}\, C^T)$$

where p is the degree of the minimal polynom of $A(p \leq n)$.

Observability of non-linear systems is a much more difficult problem because the observability is no longer a property of the system only but depends on the applied control variables. General observability conditions for non-linear systems was first reported by Kostyukovskii [1, 2].

In Ref. [3] L Johnsson gives eighteen references (among these [4-8]) treating the non-linear observability problem from different points of view, with comments regarding their applicability to the observability problem in power systems.

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2.3 Stochastic Systems

There is much less literature available treating observability in stochastic systems than in deterministic systems. But it is a problem of great interest because many control systems can be regarded as stochastic systems, for example a system with noisy measurements.

A linear stochastic system can be treated in the same way as a linear deterministic system [3].

The observability problem for non-linear stochastic systems is more complicated. Again the reference list in [3] is of interest. Specially the articles by Fitts [8] and by Figuerido et. al. [9] treat the stochastic observability problem.

In [10] Yu and Seinfeldt study linear distributed parameter systems whose solutions can be represented by eigenfunction expansions. They prove a theorem, saying that there always exist spatial measurement points such that a system of the regarded class is completely observable. They also give sufficient conditions for observability with respect to measurement location. However, the result is not applicable in power system state estimation.

2.4 The Least Squares Estimate

Let us regard the system model where the measurements, y, are related to the state vector, x, by the non-linear relation: $y = h(x)$.

The dimension of y is usually greater than the dimension of x in order to get a good estimate. According to the reference [12] we can in the case of power systems perform the non-linear transformation: $y'(y,x) = H \cdot x$. If we also include the measurement noise we get $y'(y,x) = H \cdot x + w$. The purpose of this transformation is to make the right side of the equation linear in x, i.e. H is independent of x. If we disregard terms of higher order we get the weighted least squares solution:

$$x = (H^T W^{-1} H)^{-1} H^T W^{-1} \cdot y'(y,x) = F(x)$$

where W is the weighting matrix.

The Picard algorithm now gives us the iteration scheme: $x_{n+1} = F(x_n)$.

If we can compute the state with this algorithm (i.e. the iterations converge), then we know that the system is observable. Thus one way to determine the observability is to carry out the computations. However, we can also analyse the properties of $F(\cdot)$. The conditions for convergence are stated in the following theorem.

Theorem [11] Assume that

a) for every $x \in \Omega \subset R^n$, $F(x) \in \Omega \subset R^n$

b) $F(x)$ is continuous for all $x \in \Omega$

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c) $\|F(x_1) - F(x_2)\| \leq L \| x_1 - x_2 \|, L < 1$

(i.e. the Lipschitz condition) holds for all vectors x_1 and x_2 in Ω.

Then

a) A solution vector x^* to the equation $x = F(x)$ exists in Ω.

b) The solution vector x^* is unique.

c) The Picard algorithm: $x_{k+1} = F(x_k)$
k=0, 1, ... converges to x^*.

However, this theorem is difficult to use in practice because it is difficult to verify that the Lipschitz condition holds for all pairs of vectors x_1 and x_2 in the set Ω.

Let us now assume that the function F(x) in addition to being continuous also has continuous partial derivatives with respect to the components of the vector x. Then we can apply the mean-value theorem [11] to F(x): Let x_1 and x_2 be arbitrary vectors in R^n. Then there exists a vector $\hat{x}$ in the line segment joining x_1 and x_2 such that

$$F(x_1) = F(x_2) + \left(\frac{\partial F}{\partial x}\bigg|_{x=\hat{x}}\right)(x_1 - x_2)$$

The vector $\hat{x}$ can be written $\hat{x} = \alpha x_1 + (1-\alpha)x_2$ for some α,

$0 \leq \alpha \leq 1$. We get immediately ($\|\cdot\|$ is a vector norm).

$$\|F(x_1) - F(x_2)\| = \left\| \left(\frac{\partial F}{\partial x}\bigg|_{x=\hat{x}}\right)(x_1 - x_2)\right\| \leq \left\| \frac{\partial F}{\partial x}\bigg|_{x=\hat{x}} \right\| \cdot \|x_1 - x_2\|$$

We can now reformulate the theorem above:

Theorem [11] Assume that

a) F(x) is continuous for all $x \in R^n$.

b) $F(x) \in \Omega$ for all $x \in \Omega$.

c) F(x) has continuous derivatives for all $x \in \Omega$.

d) $\|\partial F/\partial x\| < 1$ for all $x \in \Omega$ where $\|\cdot\|$ denotes the spectral radius.

Then

a) There exists a unique solution $x^* \in \Omega$, that is $x^* = F(x^*)$

b) The Picard algorithm converges to x^*.

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However, these convergence theorems have four disadvantages if they shall be used for determination of observability:

1) One has to compute $(H^T W^{-1} H)^{-1}$, at the observability determination. This means that a big part of the computations for the state estimation has to be performed before it is certified that state estimation is possible on the current set of data. This should be avoided.

2) The Lipschitz condition is in both its forms above very difficult to check.

3) In order to find observable parts of the network, one has to search for these parts by a trial and error method.

4) This check only tells us if the used algorithm gives a correct state estimate. Thus only a sufficient condition for observability has emerged. This is not very constructive.

3 OBSERVABILITY OF POWER SYSTEMS

3.1 Power System Model and Definitions

Let us assume that the power system can be described as a stationary system where the measurements are corrupted by noise.

3.1.1 Network model

The power network is represented by a number of nodes connected to each other by branches. These two entities, node and branch, will be used in the following to describe the network.

3.1.2 Definition of the state of the system

We can choose the complex nodal voltages as components of the state vector. Since we can choose the phase arbitrarily at any one node in the network (the reference node) the dimension of the state vector for a system with n nodes is 2n-1.

3.1.3 Measurement system

Let us assume that we have three kinds of measurements available from the power system:

1) complex branch power flows,
2) complex power injections,
3) voltage magnitudes.

It may happen that only active (real) or reactive (imaginary) power is measured in types 1 and 2.

The power measurements above can be written as linear combinations of squares and products of the components in the state vector. The coefficients in these linear combinations are the admittances of the network.

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The voltage magnitude is the square root of such a linear combination. Thus we get a non-linear system of equations relating the components of the state vector to the measurements.

3.1.4 Definition of observability

The system is observable if the entire state vector can be uniquely determined from the measurements. One usually regards observability as a property of the entire system. Here observability of the parts of a system will also be considered.

Let us define a subsystem as a set of connected nodes in the network. An observable subsystem is then defined as the greatest possible subsystem of nodes, whose corresponding components of the state vector can be uniquely determined from the measurements. Two nodes belong to the same observable subsystem, if it is possible to relate the phases of the complex voltages of the two nodes to each other.

Then one can compute the power flows in connecting branches. During the search procedure for observable subsystems two temporary constructs, "observable islands" and "unobservable islands", will be used.

An "observable island" is a part of the network in which we can compute the power flows in all branches. Thus the observable subsystems are the greatest possible "observable islands". The areas outside the "observable islands" are called "unobservable islands".

A node, connected both to a branch with a power flow that can be computed and to a branch where it can not, belongs to both an "observable" and an "unobservable island". Such nodes will be denoted "boundary nodes". All the other nodes will be denoted "inner nodes" (within "observable" or unobservable islands").

3.1.5 Statement of the problem

The task has been to create an algorithm for finding groups of nodes in the network that form observable subsystems. This algorithm is going to be used on line in a computer system. The regarded network may change both topology and measurement configuration. Thus the algorithm shall pick sets of nodes to which a state estimation algorithm can be applied. The algorithm shall be able to treat a network where the three earlier mentioned types of measurements (3.1.3) are mixed. It shall also be able to treat power measurements, where only active or reactive power is measured.

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3.2
An Algorithm for Observability Determination in Power Systems

3.2.1
Earlier presented algorithms

An algorithm for observability determination is earlier presented by Månson [13]. This algorithm is built on the necessary condition, that a node belongs to an observable subsystem, if there is a power flow measurement on a connected branch or an injection measurement at the node or at a node connected to the regarded node by a single branch.

This condition is not sufficient, for example the algorithm collects all neighbouring nodes to an injection measured node into one subsystem. But this subsystem is not observable in general.

Clements and Wollenberg [14] present an algorithm, which first treats only power flow measurements and then only injection measurements. The "unobservable islands" are regarded as networks without branch flow measurements. In these islands one can check the injection measurements. It is a good idea to treat one kind of measurement at the time but there are two disadvantages in the algorithm:

1) The unobservable islands are not built in such a way, that the check of the injection measurements always leads to the correct maximal "observable islands".

2) The algorithm cannot treat power measurements, where only active or reactive power is measured.

The algorithm presented in this work is built upon [14] but the "unobservable islands" are built in such a way that correct results for all network configurations are obtained.

The algorithm presented here is also able to treat the case, where only active or reactive power is measured. The algorithm in [14] repeats the observability determination for all islands if any island changes. The algorithm presented here makes these repetitions selectively, in order to save time.

The arguments leading to the new algorithm are as follows. Let us try to find some basic rules by which we can pick nodes belonging to the same observable subsystem. We assume that we know the configuration of the network and the measurements.

3.2.2
Networks containing only active and reactive branch power flows and voltage magnitude measurements

Let us select one node with voltage magnitude measurement as a reference node. Now it is possible to compute the complex nodal voltages in all nodes connected to the reference node by a sequence of branches with power flow measurements.

It is obvious that a sufficient condition for observability of the entire considered network is that the set of measured

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branches is able to form a tree including all nodes and that there is at least one node with voltage magnitude measurement.

3.2.3
Networks containing only active and reactive node injection and voltage magnitude measurements

A necessary but not sufficient condition for a network with n nodes to be observable is that there are at least n-1 complex injection power measurements present. Measurements of n-1 injections and one voltage magnitude are sufficient for observability.

3.2.4
An assumption concerning the voltage magnitude measurements

Let us assume that there is at least one voltage magnitude measurement among the nodes forming each subsystem that below will be denoted "observable island". It is necessary that this assumption holds for the final "observable islands" (the observable subsystems), we have after having finished the search procedure.

3.2.5
Building of "observable" and "unobservable islands".

Let us now apply the rules for the two special cases above to a general network. In order to do this we build "observable" and "unobservable islands" in the network. We first build "observable islands" by grouping nodes connected by branches with power flow measurements.

When we are building "unobservable islands" we are searching for subsystems outside the "observable islands". The "unobservable islands" shall be built in such a way that they can be regarded as networks with only injection measurements upon which we can apply the rules in Section 3.2.3. In order to find all parts of the network, which can be found observable by these rules, we have to build the "unobservable islands" in a special way, that makes them somewhat more difficult to build than the "observable islands".

We use unmeasured branches in the same way as we used measured branches when we constructed the "observable islands", but when we reach a node which already belongs to an observable island (thus a boundary node), we have to make the following investigation.

If we shall continue the building of the "unobservable island" from the node, it has to fulfil the following two conditions:

1) It has to be an injection measurement at the node. According to the rules that will be given in Section 3.2.7, the injection measurements at boundary nodes are important. In order to find all "unobservable islands" that can be classified as "observable islands" due to injection measurements we stop the building of the regarded "unobservable island" at a node without power injection measurement.

2) There must not be a branch from the regarded boundary node leading to another "observable island". This is because

even if there is an injection measurement at the boundary node this injection measurement cannot be used if there is a branch with unknown power flow to another observable area.

If one bilds two "unobservable islands" instead of one in such cases, one can be sure of finding all parts of the network that can be rendered observable.

If the node does not fulfil these two conditions we call it a "stop node". Thus a node can temporarily only belong to one "observable island", but it can (if it is a stop node) belong to many "unobservable islands". It has to be emphasized that the "observable" and the "unobservable islands" are only temporary constructions to be used during the search procedure.

3.2.6 Methods for increasing "observable islands"

There are two rules that can be used in order to increase the "observable islands". These two rules will be denoted COLL1 and COLL2 in the following.

COLL1 — An unmeasured branch between two nodes both belonging to the same "observable island" can be marked as measured (we know the voltages in both ends and the admittance of the branch). This rule will not directly add any new nodes to an "observable island" but it may indirectly do it by altering the measurment configuration in COLL2.

COLL2 — An unmeasured branch from an injection measured node with all the other connected branches measured can be marked as measured and the connected node can be included in the "observable island" (the injection flow into a node equals the sum of all the power flows from the node).

When we use this rule, it may happen that two "observable islands" will unite into one or that one "unobservable island" splits into two. This means that a repetition of the investigations is necessary.

3.2.7 The observability conditions for an "unobservable island"

Let us now try to apply the rules in Section 3.2.3 to the "unobservable islands".

There are three conditions the "unobservable island" has to fulfil in order to be classified as "observable".

1) All inner nodes have to be injection measured.

2) There must be at least one boundary node with injection measurement belonging to each neighbouring "observable island".

3) One can accept that either one inner node lacks injection measurement or that one neighbouring "observable island" lacks an injection measured boundary node.

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The check of these three conditions will in the following be denoted INJ.

When we find an "unobservable island" that fulfil the conditions above, two or many "observable islands" may unite into one. These are perhaps neighbours to another "unobservable island". This means that we have to repeat both the two COLL-checks and INJ until no island changes.

The algorithm for observability determination will be presented in two steps. In Section 3.2.8 the algorithm is presented in a simplified form with almost the same structure as the algorithm presented in [14]. This version gives a good survey of the steps in the algorithm and will make it easier to understand the complete algorithm in Section 3.2.9.

3.2.8 Simplified version of the algorithm

Let us sum up the rules above into the following algorithm.

1) Build "observable islands" using branch flow measurements only.

2) Do COLL1 and COLL2 for all nodes in the network.

3) If COLL2 in 2) included any new node in an "observable island" go back to 2).

4) Build "unobservable islands".

5) Do INJ for every "unobservable island" and update them if they are found observable.

6) If any "unobservable island" was found "observable" in 5) go to 2).

7) Check if there is a voltage magnitude measurement in every "observable island".

3.2.9 The complete algorithm

It is possible to do the algorithm above faster by doing the repetitions (point 3) and point 6)) more selective. This makes the algorithm somewhat more complex.

1) Build "observable islands".

2) Do COLL1 and COLL2 for all the nodes. When COLL2 includes a new node in an "observable island" repeat COLL1 and COLL2 for the found node if it has been treated earlier. Denote all these checks COLL.

3) Build "unobservable islands".

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4) Regard the first "unobservable island".

5) Do INJ for this "unobservable island".

6) If the island is not found "observable" go to 10.

7) Unite the "unobservable island" and its "observable neighbours" into one "observable island".

8) If the "unobservable island" did not have more than one "observable" neighbour go to 10.

9) Do CONT for all the other "unobservable islands". CONT denotes a routine that selectively applies COLL1, COLL2 and INJ to the regarded "unobservable island". For details see the program description [15].

10) Pick the next "unobservable island" and go to 5). If all islands are treated continue to 11.

11) If no further repetition is necessary go to 14). (Further repetition can be demanded from CONT in 9) and 12)).

12) Do CONT for the "unobservable islands" for which repetition is necessary.

13) Go to 11).

14) Check if there is a voltage magnitude measurement in every "observable island".

The flow chart for this algorithm is given in Figure 1.

It sometimes occurs that one measures only active or reactive power at some points in the network. One can handle this problem by treating first only the active power measurements and then the reactive power measurements. Finally one constructs the "observable islands" as the intersection of the "observable islands" one has found in the two investigations. This procedure is included in the program [15] that has been applied to the example in Section 4.

This program is also able to disregard small observable subsystems. The least number of nodes in an observable subsystem of interest is given in a parameter declaration (MINA) to the program. All the electrical connected areas less than this parameter will be disregarded during the search procedure in order to do the searching faster and more simple. In the output from the program all the other observable subsystems less than this parameter are disregarded.

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4 EXAMPLE

A FORTRAN-program for the ASEA H6000 computer built on the algorithm in Section 3.2 has been written [15]. This program has been tested on the following example.

The network used for the example is shown in Figure 2. The nodes are represented by points and the branches by lines. The connected areas are identified by the number in the circles.

There are 46 nodes and 60 branches in the network.

The measurement system is shown in Figure 3. The arrows are representing injection measurements at the nodes and the crosses are representing power flow measurements in the branches. P indicates active and Q indicates reactive power measurement.

Voltage magnitude measurements ar not regarded in the program and the check mentioned in Section 3.2.4 is not performed.

In Figure 4 the observable subsystems are shown. The parameter (MINA) used for disregarding small subsystems (see Section 3.2.9) was given the value 4 in this example. This means that the connected area 3 and node 11 was disregarded from the beginning of the search procedure.

In Figure 5 the output from the program is shown when MINA is not applied to the results. In Figure 6 the output is shown when MINA = 4 is applied to the results.

The input data to the program consists of a description of the network and measurement system configuration. Each node and branch in the network is given a specific number. The network connections are described by the numbers of the nodes in each end of every branch. The input data on the measurement system describes the power measurements.

For each branch the input data tells if the power flow is measured at any of its ends. For every node the input data also tells if the power injection is measured.

For both types of measurements the input data carries information telling whether only active, reactive or both types of power are measured.

The program consists of 976 FORTRAN statements including comments. The core memory requirements for data are

- six parameters for the main routine and each of the ten subroutines. The parameters include the maximal number of nodes, NODMAX, and of branches, LINMAX, in the considered power network.
- for input data: 2*NODMAX+4*LINMAX words,
- for internal arrays: 9*NODMAX+8*LINMAX words,
- for other internal variables: 125 words.

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The example described here had the values

NODMAX = 46
LINMAX = 60

which gives a memory requirement for data of 1417 words ≃ 1.4k words.

The total core memory requirement for an execution of the program on the ASEA H6000 computer was 11k words for the described example. The processor time for execution totalled 0.0006 hours = 2.16 seconds.

5 CONCLUSIONS

Further investigations in the field of observability of non-linear systems, deterministic as well as stochastic, are needed. The conditions presented in the literature are very difficult to apply to a practical problem.

In the special case of observability of a power system it is possible to determine observable parts of the system from the knowledge of network topology and measurement configuration.

The algorithm presented in Section 3.2 finds all observable subsystems in a power system with complete complex power measurements.

When there are measurements of only the active or the reactive power, the algorithm builds the observable areas as the intersection of the islands that have been found "half observable" with respect to the measurements of active and reactive power respectively. This method will give a sufficient condition, but it is difficult to investigate necessity.

The problem with incomplete complex measurements is even more complicated. For example, one has the question, whether a voltage magnitude measurement in combination with an active power measurement can replace a reactive power measurement. This and similar problems call for further investigation.

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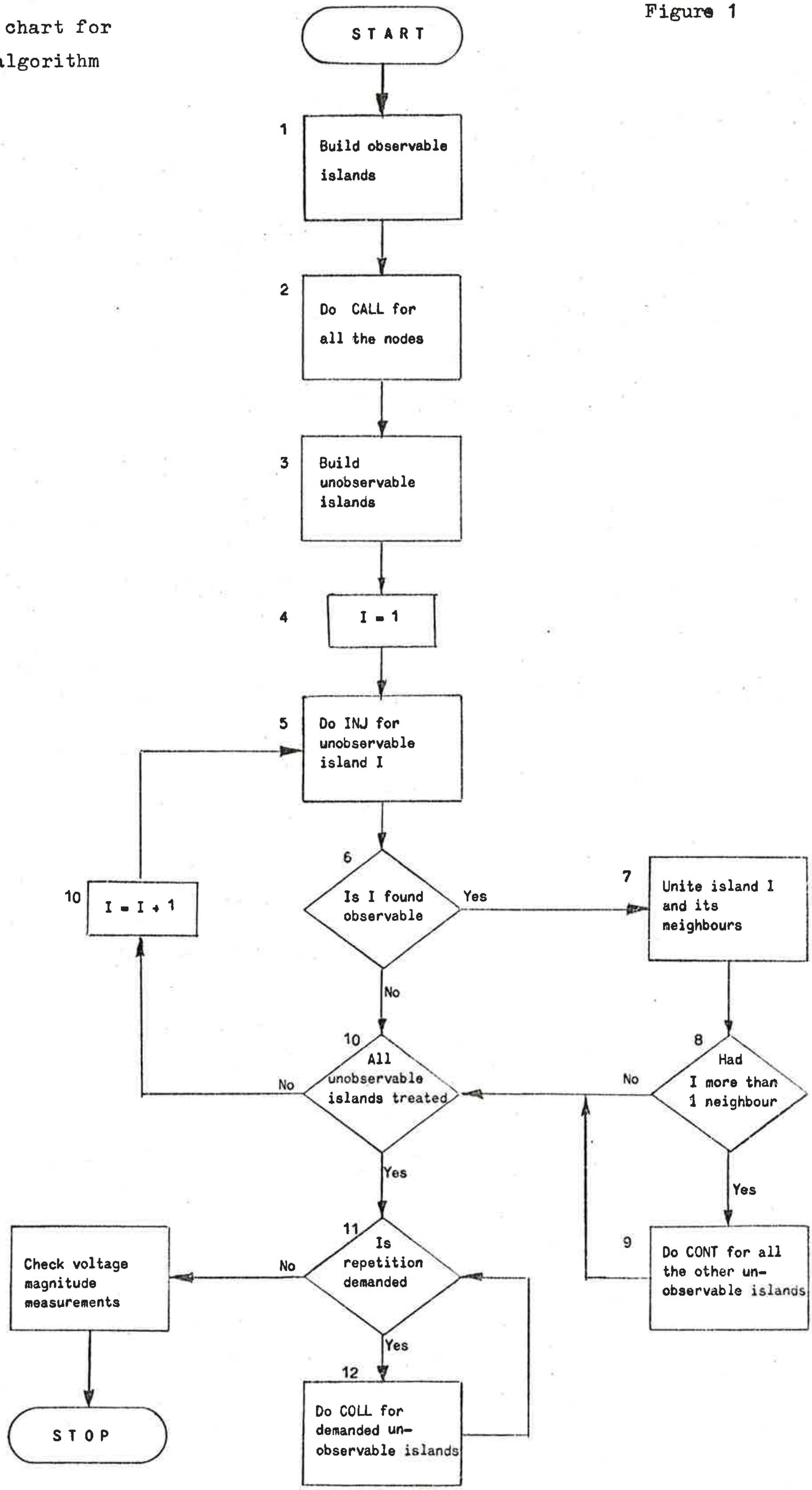
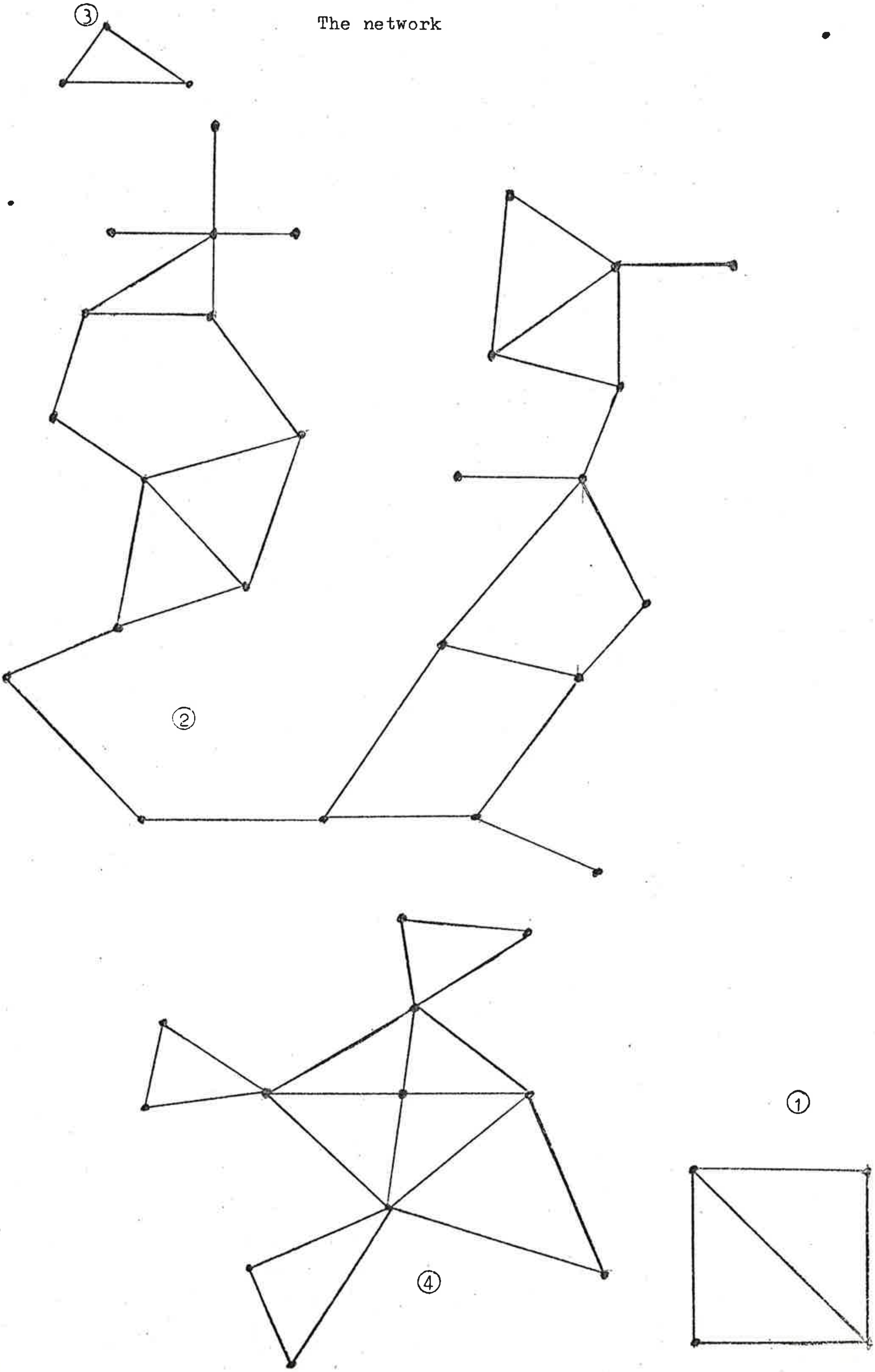
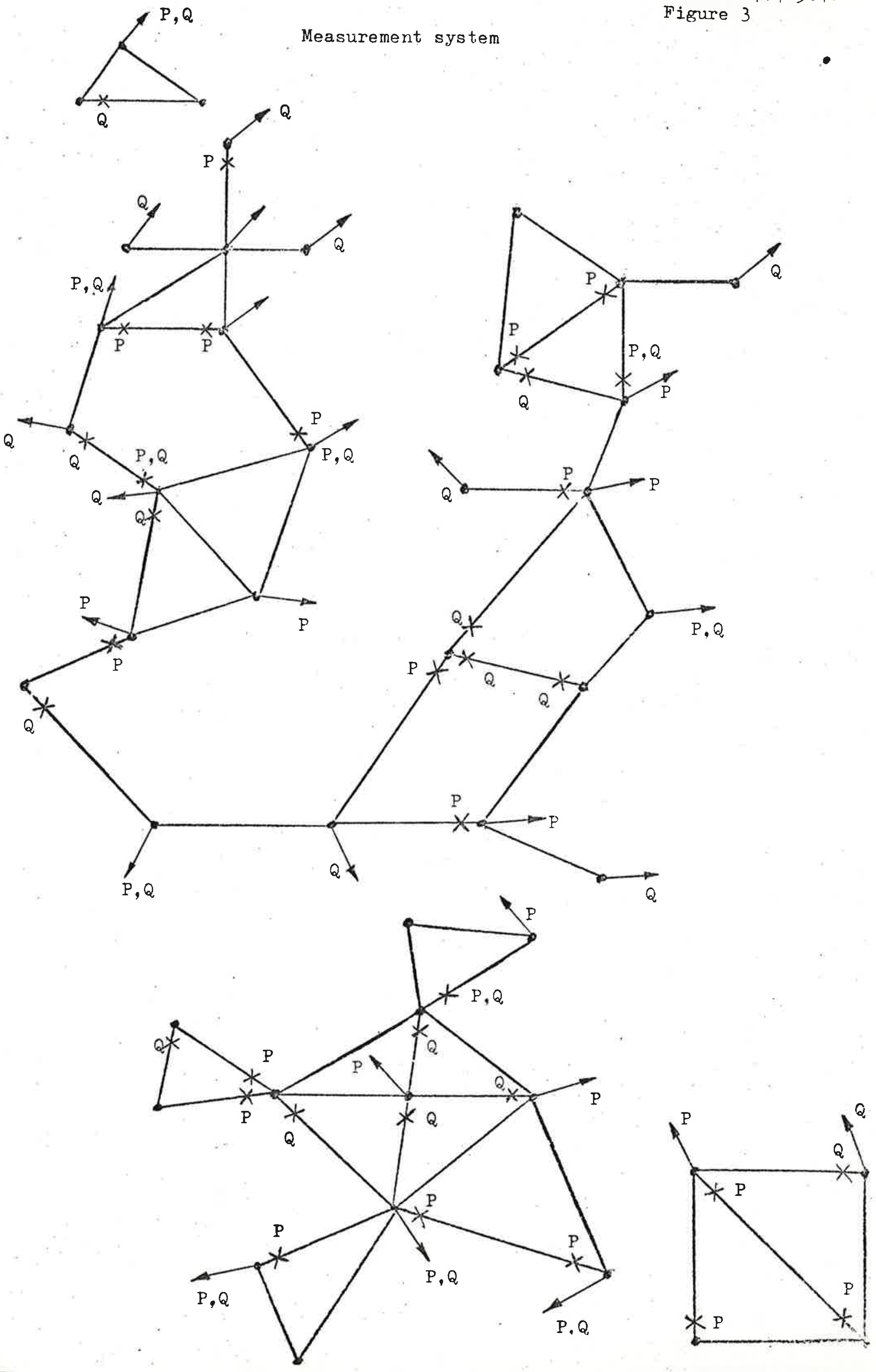
Flow chart for
the algorithm



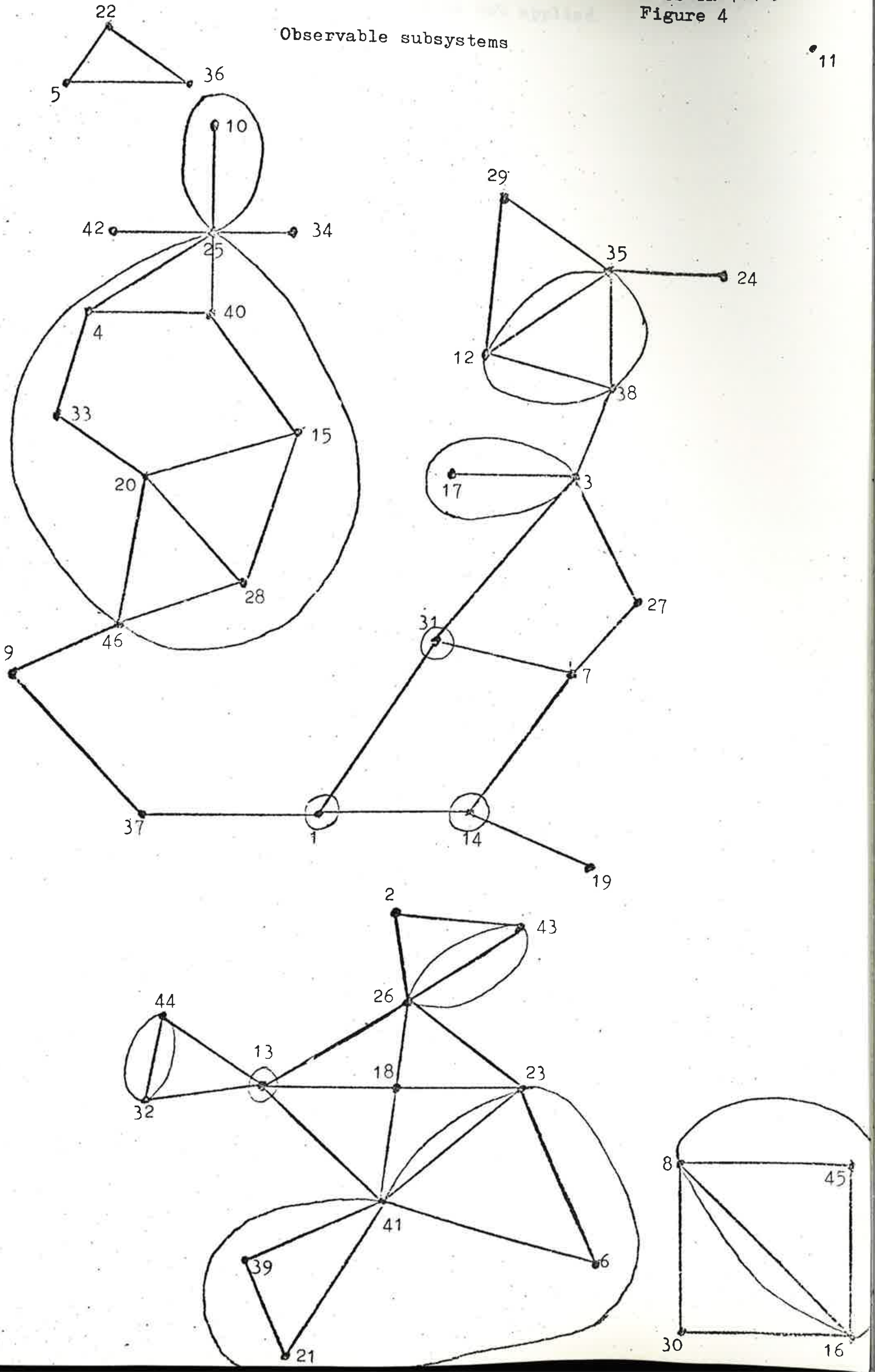
The network



Measurement system



Observable subsystems



Output when MINA is not applied
to the results

```
OBSAREA  1
 1

OBSAREA  2
 3
17

OBSAREA  3
 4
10
15
20
25
28
33
40
46

OBSAREA  4
 6
21
23
39
41

OBSAREA  5
 8
16
45

OBSAREA  6
 9

OBSAREA  7
12
35
38

OBSAREA  8
13

OBSAREA  9
14

OBSAREA 10
26
43

OBSAREA 11
31

OBSAREA 12
32
44
```

Output when MINA is applied to the results

```
OBSAREA  3
 4
10
15
20
25
28
33
40
46

OBSAREA  4
 6
21
23
39
41
```